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Uncertainty and change: Survey evidence of firms' subjective beliefs

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Uncertainty and Change: Survey Evidence of Firms' Subjective Beliefs

Rüdiger Bachmann Kai Carstensen Stefan Lautenbacher
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October 11, 2025

Abstract

This paper studies how managers plan under uncertainty. In a new panel survey of German manufacturing firms, we observe both forecasts of sales growth and a quantitative measure of subjective uncertainty. We show that subjective uncertainty reflects change experienced by firms: it is high when growth is either unusually low or unusually high. Subjective uncertainty is more than conditional volatility: while more volatile firms are typically more uncertain, experienced change alone increases uncertainty, consistent with a model of learning. Uncertainty over our sample is mostly idiosyncratic but varies substantially over time and leads managers to plan lower employment and prices.

Keywords: expectation formation, firms, measurement, subjective uncertainty, survey data

JEL codes: C83, D22, E20, E23

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1 Introduction

A large literature shows that higher uncertainty perceived by firms implies more cautious intertemporal decisions such as investment, hiring, and price setting.¹ While the mechanisms that translate uncertainty into actions are well understood, it is less clear what makes firms more uncertain in the first place. This is in part because there are few panel data sets to explore the dynamics of subjective uncertainty and how it responds to firm experience. Applications in macroeconomics typically assume rational expectations and exogenous shocks to the conditional volatility of firm fundamentals, such as productivity. As a particular example, uncertainty shocks trigger recessions by increasing the conditional volatility for many firms, consistent with an increase in cross-sectional dispersion when GDP growth is low.

This paper characterizes firm planning under uncertainty using a new panel data set of the German manufacturing sector. We observe quarterly firm-level sales growth as well as one-quarter-ahead forecasts and a quantitative measure of subjective uncertainty about sales growth for 2013-2019, a relatively calm time for the German economy. There are three main findings. First, subjective uncertainty reflects change experienced by firms: it is high when growth is unusually low, but also when it is unusually high. Second, subjective uncertainty is more than just conditional volatility: while more volatile firms are typically also more uncertain, experienced change alone increases uncertainty, consistent with models of learning. Finally, even though uncertainty movements in our sample are essentially all idiosyncratic, they are large and matter for hiring and pricing plans.

Uncertainty reflects change both in the cross section (averaging over time) and the time series (for a given firm). *Across firms*, the relationship between average subjective uncertainty and average growth over our seven-year sample is V-shaped, with a minimum at zero, a steep negative branch and a flatter but significant positive branch. This *cross-sectional V* implies that firms on unusually steep trends perceive more uncertainty, especially when they are shrinking, but also when they are growing. *Over time* for an individual firm, uncertainty is higher after quarters of unusual growth relative to the firm's trend. We document a *time-series V*, again with a minimum at zero: both good and bad unusual realizations increase uncertainty. Both Vs illustrate how experience shapes uncertainty, and not only forecasts or attitudes, as the literature has documented.

Our second finding, the distinction between uncertainty and conditional volatility, similarly emerges both in the cross section and the time series: neither uncertainty V reflects a V-shaped pattern between volatility and growth that should hold under rational expectations. We propose a simple learning framework where uncertainty decouples from volatility because managers are uncertain about information quality in addition to growth. When managers see unfamiliar signals, such as unusual past growth, they trust those signals less, and hence remain more uncertain about future growth. We then observe higher subjective uncertainty in our survey, but we need not observe unusual subsequent growth that elevates measures of volatility: uncertainty rises only because of unusual *past* growth. The learning model also

¹See Bloom (2014), Fernández-Villaverde and Guerrón-Quintana (2020), and Cascaldi-Garcia et al. (2023) for surveys on uncertainty and firm behavior.

delivers predictions for forecast bias that we verify in the data. More broadly, the result says that understanding subjective uncertainty requires going beyond rational expectations models where uncertainty coincides with conditional volatility, with learning as a promising alternative. Since the age of most of the firms in our survey measures in decades, our results suggest that learning is also important for mature firms.

Third, when subjective uncertainty is higher, firms plan for both lower prices and lower employment. What is important here is that our analysis focuses on a period of relatively stable aggregate growth for German manufacturing. As a result, firm-level shocks, as opposed to aggregate or sectoral effects, drive time variation in realized growth rates, forecasts, and subjective uncertainty about growth. The comovement of prices and quantities thus suggests that idiosyncratic uncertainty about demand conditions is prominent. Variation in plans due to uncertainty is comparable in magnitude to that in conditional means. We conclude that mechanisms for how uncertainty matters are relevant much more widely than just in recessions: instead, they should feature even in steady state accounts of firm dynamics and complement variation in conditional means.

Our data come from a new module of the *ifo Business Survey*, a long-established survey administered by the Munich-based ifo institute used to develop business sentiment indicators in Germany.² Our survey module asks top managers not only for a forecast of one-quarter-ahead sales growth, but also for best and worst case sales growth scenarios. The idea behind this survey design is that managers can directly report scenarios developed as part of their firms' regular planning process. Survey responses confirm that most firms engage in scenario analysis and use results from routine quantitative planning in order to fill out the questionnaire. To summarize how a firm's perceived uncertainty is reflected in its planning, we focus on the *span* between its scenarios, that is, the difference between best and worst case sales growth rates. Our focus in this paper is how this robust quantitative measure of subjective uncertainty varies over time and across firms.

To interpret span and relate it to subjective uncertainty in economic models, we present a simple model of managers who perform scenario analysis. Suppose managers face continuous shock distributions, but only have the capacity to think through a finite number of scenarios. They know that their response to shocks will work better when the shock is close to a scenario they have previously thought through. Optimal choice of scenarios then "covers" the space of shock realizations, reflecting the shape of the distribution. We show that when the loss function is quadratic and the distribution of shocks belongs to the location-scale family, span serves as an index of (subjective) conditional volatility. In particular, an increase in volatility entails an increase in span in the same proportion. Since quadratic costs can be viewed as a second-order approximation of a more general loss function, this establishes a close link between span and subjective volatility in a wide range of economic models.

²The survey is well regarded in the German business community: questions are typically answered by senior management and there is a high response rate even from large firms. Recent studies that draw on the ifo Business Survey include [Bachmann, Elstner and Sims \(2013\)](#); [Bachmann and Elstner \(2015\)](#); [Buchheim and Link \(2017\)](#); [Massenot and Pettinicchi \(2018\)](#); [Bachmann, Born, Elstner and Grimme \(2019\)](#); [Enders, Hünnekes and Müller \(2019, 2022\)](#); [Born, Enders, Menkhoff, Müller and Niemann \(2024\)](#). [Bachmann, Carstensen, Menkhoff and Schneider \(2023\)](#) provide an introduction to the data set and further discussion of data quality.

To provide an idea of magnitudes, the mean span between best and worst case quarterly sales growth scenarios is 12.4 percentage points (pp), larger than the mean absolute forecast error: 9.0 pp. Firms differ in their average subjective uncertainty: the cross-firm standard deviation of time-averaged span is 7.4 pp. Our *cross-sectional uncertainty V* says that a one percentage point lower negative (higher positive) average growth rate over our sample goes along with about 0.6 pp higher (0.2 pp higher) subjective uncertainty. At the same time, we document large time variation in subjective uncertainty for individual firms: The time-series standard deviation of span for the average firm is 6.3 pp. Our *time-series uncertainty V* says that, after removing firm fixed effects in both uncertainty and growth, a one percentage point lower negative (higher positive) growth rate experienced over a quarter is followed by 0.2 pp (0.1 pp) higher subjective uncertainty over the next quarter.

Our comparison of subjective uncertainty and volatility is guided by an organizing framework that contrasts rational expectations and learning. A rational expectations model identifies subjective uncertainty with conditional volatility. While it can in principle generate the uncertainty Vs we find in the data, it implies also that forecasts are unbiased and that we should find *volatility Vs* that closely mimic the uncertainty Vs in both the time series and the cross section. Our learning model, by contrast, is designed to generate changes in subjective uncertainty even when conditional volatility is constant. Intuitively, a manager who is not sure about the distribution of growth perceives more uncertainty than the realized volatility an econometrician observes with hindsight. Both subjective uncertainty and forecast bias then depend on the quality of signals as well as managers' prior confidence in their environment.

We study cross-sectional implications of the framework by regressing average uncertainty on firm characteristics. We first show that uncertainty moves strongly with the volatility of sales growth: managers of firms that typically face larger sales growth fluctuations, that is, they face higher turbulence, also report higher spans on average. Under rational expectations, variation in subjective uncertainty should be *entirely* explained by volatility. However, this is not the case along at least two key dimensions, trend growth and size. In particular, shrinking firms are more uncertain, holding fixed volatility. At the same time, their forecasts remain systematically too optimistic. This is consistent with managers learning gradually about the deterioration of business from signals they do not trust. In contrast, large firms are systematically less uncertain, holding fixed volatility. We conclude that more successful—large or fast-growing—firms are more confident about their environment, independently of the size of the shocks they face.

According to our time-series V, a transitory deviation of growth from a firm's average growth predicts higher uncertainty. Under rational expectations, when uncertainty is conditional volatility, past growth deviations should further predict a higher absolute magnitude of the subsequent forecast error. We show that for negative deviations, this is qualitatively true, but relatively weak: firms make somewhat larger forecast errors after low growth. For positive deviations, moreover, there is no systematic relationship at all. The uncertainty V thus reflects two effects. On the one hand, volatility increases in bad times for an individual firm, an idiosyncratic version of the familiar elevated higher volatility in recessions. On the other

hand, learning induces a symmetric V that is unrelated to volatility: managers become more uncertain when they enter unfamiliar territory, whether positive or negative.³

Our paper contributes to a growing literature that uses survey measures to understand firms' "micro" uncertainty about their business.⁴ Following the pioneering work of [Guiso and Parigi \(1999\)](#), the focus has been on the effect of uncertainty on economic activity; see, for example, [Bontempi, Golinelli and Parigi \(2010\)](#) and [Fiori and Scoccianti \(2023\)](#) for Italy, and [Bachmann, Elstner and Sims \(2013\)](#) and [Bachmann, Elstner and Hristov \(2017\)](#) for Germany. An additional attractive feature of the ifo survey is that we observe firms' plans for both price setting and employment, a quantity variable. We thus infer the importance of demand uncertainty from the comovement of two outcomes. This finding is consistent with, and complementary to, the evidence in [Guiso and Parigi \(1999\)](#) and [Bontempi, Golinelli and Parigi \(2010\)](#) who use a survey question that specifically elicits demand uncertainty and show its relevance for investment.

Several recent papers have documented V-shape relationships between measures of firms' subjective uncertainty and lagged sales growth. [Altig, Barrero, Bloom, Davis, Meyer and Parker \(2022\)](#) relate subjective conditional volatility of one-year-ahead sales growth elicited in the Atlanta Fed's monthly Survey of Business Uncertainty, to growth in the previous year. [Bloom, Davis, Foster, Lucking, Ohlmacher and Saporta-Eksten \(2020\)](#) relate subjective conditional volatility to deviations of past sales growth from an historical average in the US Census Bureau's 2015 Management and Organizational Practices Survey (MOPS). [Kumar, Gorodnichenko and Coibion \(2023\)](#), in a New Zealand firm survey, document a V-shape relationship between span for six-months-ahead sales growth and expected as well as past growth.

The presence of V-shapes is thus a robust fact that does not depend on how uncertainty is measured, the quarterly horizon, or the German context we study. Since existing studies use either a single cross section or a pooled sample over a few months, they do not distinguish between the cross-sectional and time-series Vs that we document. Our contribution is to perform that decomposition and further use our panel data to construct volatility Vs that we contrast with the two uncertainty Vs. Beyond sales growth, [Boutros, Ben-David, Graham, Harvey and Payne \(2020\)](#) find that executives who provide confidence intervals for stock returns widen those intervals when the realized return falls out of their last interval—also a response of subjective uncertainty to change.

Our findings also contribute to the large behavioral literature on firm decision making (see [Malmendier, 2018](#), for an overview). The typical application documents systematic biases in managers' forecasts and their relationship to firm actions. Deviations from rational

³Our finding of time-varying firm-level risk that matters for plans is consistent with [Hassan, Hollander, van Lent and Tahoun, 2019](#) who build a measure of firm-level political risk from transcripts of earnings calls and show that such risk affects investment and hiring.

⁴There is also an active literature that studies (managers') expectations and uncertainty about aggregate variables, such as inflation and economic growth (see, for New Zealand, [Kumar, Afrouzi, Coibion and Gorodnichenko, 2015](#); [Coibion, Gorodnichenko and Kumar, 2018](#); [Kumar, Gorodnichenko and Coibion, 2023](#); and, for Italy, [Coibion, Gorodnichenko and Ropele, 2020](#)), economic policy (see [Baker, Bloom and Davis, 2016](#)), and the stock market (see [Alfaro, Bloom and Lin, 2023](#)).

expectations thus typically appear in (conditional) first moments.⁵ An exception is [Ben-David, Graham and Harvey \(2013\)](#), who consider managers' forecast densities for stock returns and show that managers are miscalibrated: their subjective forecast densities are too narrow relative to realizations. [Barrero \(2022\)](#) also finds that managers perceive "too little" uncertainty, consistent with a model where managers overreact to signals because they overestimate their precision. Our results complement these studies by connecting forecast bias to the difference between subjective uncertainty and conditional volatility. In our learning model, managers' perceived precision is endogenous and depends on the realization of the signal. As a result, it can generate overprecision—as we find in response to growth in the short run—but also firms that perceive more uncertainty than conditional volatility.

The paper is structured as follows. Section 2 explains our new survey questions and provides background on data quality as well as basic summary statistics. Section 3 introduces the raw relationship between uncertainty and change and presents our organizing framework. Section 4 studies uncertainty and change in the cross section and Section 5 in the time series. Section 6 studies the effect on employment and prices. Section 7 concludes.

2 Data

The ifo Business Survey, run by the Munich-based ifo Institute at a monthly frequency, is a well-established survey of German businesses. [Hiersemenzel et al. \(2022\)](#) conclude about its representativeness: "*The investigation shows that the business panel is representative for the German economy.*" ifo maintains a representative sample of German businesses by replacing exiting firms with new respondents, which are not necessarily newly founded firms (see [Sauer and Wohlrabe, 2020](#)). In fact, the median firm age in the survey is 45 years ([Best et al., 2025](#)). Responses from the—mostly qualitative—survey are used as inputs for a leading indicator of the German business cycle: the ifo Business Climate Index is widely publicized; it is also part of the EU-harmonized business surveys commissioned by the Directorate General for Economic and Financial Affairs of the European Commission.⁶

In 2012, we designed and added an online module of *quantitative* questions to elicit subjective firm uncertainty. This online module is added every first month of a quarter. A large majority of the firms participates online in the main ifo Business Survey. All manufacturing firms in the online main survey were invited to participate.⁷ An initial pilot wave in December 2012 was met by strong interest. Analysis of text comments submitted by firms further showed

⁵Biases matter for firm actions and do not wash out on aggregate. For example, [Gennaioli, Ma and Shleifer \(2016\)](#) show that managers' expectations are connected to actual firms' investment plans. [Ma, Ropele, Sraer and Thesmar \(2020\)](#) evaluate the aggregate losses from biased forecasts in the structural model of firm dynamics and misallocation developed by [David and Venkateswaran \(2019\)](#).

⁶Aggregate survey results for Germany are presented at <https://www.ifo.de/en/survey/ifo-business-climate-index>, the harmonized European results, including the European Economic Sentiment Indicator, can be found here: https://ec.europa.eu/info/business-economy-euro/indicators-statistics/economic-databases/business-and-consumer-surveys_en.

⁷More recently, firms in the services and trade sectors can also participate in the quantitative uncertainty module. To maintain a longer time series, we focus exclusively on the manufacturing sector.

that firms had no trouble understanding the questions. The module has now been in the field since 2013, with participation of up to 600 firms per wave.⁸

A firm in the survey is either a stand-alone firm or a division of a larger conglomerate. For simplicity, we refer to “firms” throughout this paper. Survey questions are about growth in sales. The German term used in the questionnaire, “Umsatz”, is a well-defined technical term in profit and loss accounting, translated into English as “sales” or “total revenue.” It is commonly used as an accounting statistic at the levels of both a division and an entire firm.

The survey is administered at the beginning of every quarter. Our current sample uses 27 survey waves from 2013:Q2 to 2019:Q4. In fall 2018, we fielded an additional one-time special survey with questions on how firms collect information and arrive at the views expressed in our uncertainty module. We also draw on an additional special survey fielded by ifo in the fall of 2019 that was sent out to all manufacturing firms participating in the main ifo survey.

The composition of respondents to our quantitative uncertainty module closely tracks that of the manufacturing firms in the main ifo survey. Indeed, Appendix A shows that it is essentially impossible to predict participation in the uncertainty module. Our data contains a substantial number of large firms: when we measure firm size by the number of employees, the 75th percentile is at 285 employees. The median firm employs 115 workers while the 25th percentile is at 47.

2.1 Quality of responses

In partnering with ifo, our goal was to develop a high-quality data set that (i) reflects the perception of uncertainty by key decision makers in firms and (ii) allows respondents to draw on quantitative analysis they have already done as part of their in-house planning. Both our own special-survey questions and other papers that use the ifo survey provide direct evidence on data quality along both dimensions. We briefly summarize five key points for the questions about sales growth we use in this paper.⁹

First, in the overwhelming majority of participating manufacturing firms, the survey respondent is a member of top management. [Sauer and Wohlrabe \(2019\)](#) document that 73% of firms mention CEO, CFO or COO, and an additional 13% of survey units refer to a “division head,” the natural label for the top executive if the unit surveyed is not a stand-alone firm. For large firms with more than 500 employees, the shares are only slightly lower: 65% CEOs, CFOs or COOs, and 15% division heads. The findings are consistent with an earlier meta-study conducted by ifo about the trade sector (see [Abberger, Sauer and Seiler, 2011](#)).

Second, the identity of the responder *within* the firm changes rarely. The special survey fielded by ifo in fall 2019 asks who filled out the questionnaire in the past. 83% of firms indicate the responder is “always the same person,” 15% say “mostly the same person,” and less than 2% mention a team of people or that the responder “changes frequently.” This is important for

⁸The raw data can be found under: “IBS-IND (2020a): ifo Business Survey Industry 1/1980 - 6/2020, LMU-ifo Economics & Business Data Center, Munich, doi: 10.7805/ebdc-ibs-ind-2020a.”

⁹[Link et al. \(2023\)](#), [Lehmann \(2023\)](#), and [Kerssenfischer and Schmeling \(2024\)](#) also show the high data quality of the ifo survey. The first paper shows that managers’ macroeconomic forecasts are closely aligned to those of professional forecasters. The other two papers show the strong forecasting power of the ifo survey.

Table 1: Special-survey 2018 answers on quantitative planning

	All obs.	Tiny & Small	Medium	Large
Firms with quantitative sales planning	0.80	0.77	0.79	0.82
Scenario analysis very important or important	0.67	0.59	0.70	0.68
Statistical analysis very important or important	0.52	0.53	0.52	0.50

Notes: The numbers are from the fall 2018 special survey on a sample of 476 firms. Rows two and three are based on the 370 firms that answer both questions on scenario and statistical analysis. The top row presents the share of firms that report that their answers to our uncertainty questions are guided by numbers that the firm has already developed in-house as part of a regular quantitative planning process. Column (1) reports the overall share, while columns (2) to (4) show the share by three size groups. In line with the definition by the German Statistical Office, firms are “tiny” if they have less than 10 employees, “small” if the number of employees is between 10 and 50, “medium” if the number of employees is between 50 and 250, and “large” if the number of employees exceeds 250. The middle and the lower rows contain the results of two follow-up questions for firms that report engaging in regular quantitative planning. We present the shares of firms that consider scenario and statistical analysis, respectively, as “very important” or “important” for their quantitative sales planning. The other answer options were “less important” and “not important.” Columns (2) to (4) shows the sum of the shares answering with “very important” or “important” by size group.

interpreting our time-series results. In particular, changes in subjective uncertainty will thus almost always reflect changing views of one top decision maker, as opposed to differences in opinions between several executives who take turns filling out the survey.

Third, the main Ifo Business Survey requests that firms ignore seasonal fluctuations in their answers. Since the median firms in our sample participated eight times in our quantitative online module, it is reasonable to assume that the firms are well aware of this instruction and indeed deseasonalize their answers. In Appendix B, we corroborate this at both the macro- and micro level.

Fourth, a large majority of survey respondents in firms of all size classes rely on results from routine quantitative planning when filling out the questionnaire. Our own 2018 special survey contains a series of questions on what information respondents use; its original German version can be found in Appendix C. We first ask whether answers to our uncertainty questions are guided by numbers that the firm has already developed in-house as part of a regular quantitative planning process. The results are summarized in the top row of Table 1, both for all firms and broken down by size class.¹⁰ On average, 80% of firms respond that they use results from quantitative planning. The share is remarkably stable across firm size classes.

Fifth, a majority of firms rely on *scenario analysis* in their quantitative sales planning, that is, thinking about the future in terms of a few concrete—often fairly detailed—scenarios without necessarily attaching probabilities.¹¹ We know this from a follow-up question to firms that do quantitative planning: those firms are asked to indicate the importance of either approach on

¹⁰In line with the definition by the German Statistical Office, we define firms as “tiny” if they have less than 10 employees, “small” if the number of employees is between 10 and 50, “medium” if the number of employees is between 50 and 250, and “large” if the number of employees exceeds 250.

¹¹Also, perhaps interestingly, decision makers at the ECB use scenario analysis to deal with an uncertain future, see [Ciccarelli et al. \(2025\)](#).

a four point scale, or possibly fill in an alternative approach. The middle and lower rows of Table 1 show that more than half of the firms viewed scenario analysis and statistical analysis as “important” or “very important.” Interestingly, the firms rely more heavily on scenario analysis than on statistical analysis across all size classes; of the firms that conduct quantitative sales planning, the group of firms that conduct scenario analysis but not statistical analysis is quantitatively the largest: 38%. For firms that routinely compute adverse and favorable scenarios as part of their planning process, filling out the survey thus does not impose an additional forecasting task and is likely to generate more thought-out answers.

We also emphasize that many firms, a little under 30%, employ *both* scenario analysis and statistical analysis. Thus, it makes sense to think of the two sets of planning tools as complementary rather than reflecting, say, radically different ways to look at the world. A plausible reason for this complementarity is the difference between in-sample pattern recognition and out-of-sample forecasting. A firm may study historical patterns, say, how sales respond to certain indicators, with statistical tools. At the same time, it may find it very difficult with the short sample it has available to accurately develop a predictive density for future sales growth, which requires additional confidence in parameter stability. Scenario analysis allows for a less formal way to use statistical results to think through what might happen in the future.¹²

2.2 Eliciting subjective uncertainty

The uncertainty module of the ifo Business Survey asks firms, at the beginning of a quarter, a two-part question. Figure 1 displays the sample questionnaire for April 2014 in the original German. In English, the questionnaire reads:

The following questions refer to changes against the previous quarter.

1. *By how much in percentage terms have your sales changed in the first quarter of 2014?*
2. *By how much in percentage terms will your sales change in the second quarter of 2014?*
 - a. *In the best possible case:*
In the worst possible case:
 - b. *Taking into account all contingencies and risks, I expect for the second quarter of 2014 all in all a change of:*

The questionnaire form contains four boxes for respondents to provide their four numerical answers. Next to every box, there is a reminder to provide positive or negative integers. In addition, respondents are given a “don’t know-”option (“weiß nicht” in German) behind the box, as shown in the figure. Finally, underneath both questions 1 and 2, firms are invited to provide free text comments (“Anmerkungen”).

¹²The well-publicized example of bank stress testing illustrates this: in order to develop scenario forecasts for a stress test, banks have to think through what can go wrong in their own portfolios, given a detailed set of contingencies. Statistical analysis, for example of the sensitivity of loan losses to macroeconomic conditions, is the natural way to do so. It is useful even if the goal is not to forecast stress out-of-sample.

Figure 1: Original survey questionnaire in German

April 2014

Hinweis zu diesen Zusatzfragen:

Das Wirtschaft zu 50% aus Psychologie besteht, wusste schon Ludwig Erhard. Ein wichtiges Element sind dabei Erwartungen über eine unsichere Zukunft, mit der Sie als Unternehmer tagtäglich umgehen müssen. Das haben die Wirtschaftswissenschaften zu lange vernachlässigt. Diese Erwartungen und diese Unsicherheit zu messen und zu evaluieren, ist das Ziel der folgenden Fragen. Mit Ihren Antworten helfen Sie uns sehr.

Für Rückfragen steht Ihnen Frau Wieland zur Verfügung: Tel. 089-9224-1247 - E-Mail: wieland@ifo.de

Die folgenden Fragen beziehen sich auf Änderungen gegenüber dem Vorquartal.

1. Um wieviel Prozent hat sich der Umsatz in Ihrem Bereich im ersten Quartal 2014 verändert?

Veränderung um: % (bitte ganze, positive oder negative Zahlen eingeben) weiß nicht

Anmerkungen:

2. Um wieviel Prozent wird sich der Umsatz in Ihrem Bereich im zweiten Quartal 2014 verändern?

a) Im bestmöglichen Fall: % (bitte ganze, positive oder negative Zahlen eingeben) weiß nicht

Im schlechtestmöglichen Fall: % (bitte ganze, positive oder negative Zahlen eingeben) weiß nicht

b) Unter Berücksichtigung aller Chancen und Risiken erwarte ich im zweiten Quartal 2014 alles in allem eine Veränderung um: % (bitte ganze, positive oder negative Zahlen eingeben) weiß nicht

Anmerkungen:

Notes: Original questionnaire from ifo's online module on subjective uncertainty in German; screenshot from April 2014.

To clarify the timing, consider a firm responding in April 2014, that is, in the first two and a half weeks of 2014:Q2. Question 1 asks for the change in sales between 2013:Q4 and 2014:Q1. This is the most recent sales growth realization that the firm has experienced. Question 2 then asks for the firm's outlook over the current quarter 2014:Q2, as compared to the last quarter 2014:Q1. This is the next growth rate realization that the firm expects.

Our quantitative measure of subjective uncertainty is the *span* between the best and worst case scenarios for sales growth that firms provide in response to question 2.a. A firm's *forecast error* is the difference between its actual sales growth in the current quarter and its sales growth *forecast* at the beginning of that quarter, that is, its answer to part 2.b, which reflects the subjective conditional expectation. At the beginning of every quarter, firms cannot perfectly predict the flow of sales over the entire quarter; forecast errors capture the mistakes they make. We note that in order to observe a forecast error for a firm, we need to observe the firm in two consecutive survey waves.

Sample construction. Our baseline sample consists of 1,005 firms and 8,889 firm-quarter observations from 27 quarters. We describe the sample construction in detail in Appendix D. Briefly, we first focus on firms that have at least five firm-quarter observations for realized previous-quarter sales growth rate (question 1). We then retain firms that also provide sensible answers to the second question on forecasts. In both steps, text comments provided by firms are useful to assess outliers and to drop firms unwilling or unable to provide quarterly forecasts. 533 firms in the baseline sample filled out the fall 2018 special survey.

To study sectoral effects, we form 14 *industries* that are based on two-digit manufacturing codes, but are aggregated further to ensure sufficiently sized populations of surveyed firms.

Details are in Appendix E, where we also show the distribution of firms across industries. Our baseline sample contains at least 247 observations per industry.

2.3 Span as a measure of subjective uncertainty

The premise behind our survey module is that when firms perceive more uncertainty, they contemplate positive and negative scenarios that are further apart, and hence report a larger span. In this section, we provide a simple model that relates scenarios chosen by a firm during scenario analysis to their perceived uncertainty. The purpose of the model is to (i) illustrate a plausible thought process underlying survey answers, and (ii) show how to relate span to belief measures in applications. In particular, we argue that for a large and flexible set of distribution families, span is proportional to standard deviation and can therefore serve as an index of subjective conditional volatility.

There are two dates. At date 1, the firm knows that a stochastic growth rate g will realize at date 2. In order to prepare a contingent plan, managers think through a finite number of n scenarios for sales growth, collected in a vector \hat{g} in ascending order, that is, $\hat{g}_{i-1} < \hat{g}_i$ for $i = 2, \dots, n$. Span is the difference between the best and worst scenarios, or $\hat{g}_n - \hat{g}_1$. Managers know that their response will work relatively well as long as the realized growth rate is fairly close to a scenario they have thought through. We formalize this idea via a quadratic cost function that depends on the distance between the realized growth rate and the closest scenario. We can think of this objective as a second-order approximation of a more general cost function—second order is the lowest order needed to capture risk effects.

The firm chooses the vector of scenarios \hat{g} to minimize expected cost, given its knowledge of the distribution of g :

$$\min_{\hat{g}} E \left[\min_{1 \leq i \leq n} (g - \hat{g}_i)^2 \right] \quad (1)$$

Here, the outer minimum operator reflects that managers' ex ante choice of scenarios minimizes expected cost. The minimum operator inside the expectation captures that, ex post, costs depend on distance to the closest scenario, or equivalently, the minimum distance over all scenarios. The parameter n captures the number of scenarios a firm has the capacity to consider.

The following proposition shows how span responds to the distribution of growth rates, for the location-scale families of distributions that are common in both empirical work and economic modeling.¹³

Proposition 1. Suppose g can be written as $g = \mu + \sigma\varepsilon$ where ε has a continuous density with mean zero and unit variance: Then the optimal span is a linear function of σ , that is, $\hat{g}_n - \hat{g}_1 = \sigma(\hat{s}_n - \hat{s}_1)$, where $(\hat{s}_n - \hat{s}_1)$ is the optimal span for $\sigma = 1$.

¹³Examples of the location-scale family are the normal, Laplace and t-distributions as well as their generalizations such as the exponential power distribution and the asymmetric power distribution.

The proof is in Appendix F. Intuitively, managers would like to optimally “cover” the outcome space of g in order to minimize cost. With quadratic cost, first-order conditions for the optimal vector of scenarios describe conditions on the first two moments, mean and variance. Since the distribution belongs to a location-scale family, the solution can be expressed in terms of “standardized” scenarios $\hat{s}_i = (\hat{g}_i - \mu) / \sigma$ that solve the moment conditions for $\sigma = 1$ and $\mu = 0$. They depend generally on the shape of the density—for example, if the density is skewed, then managers place fewer scenarios in the thin tail.

For general μ and σ , optimal scenarios then follow by an affine transformation: when the location of the random variable is shifted by the mean μ , and the variable is scaled by volatility σ , optimally covering the outcome space applies the same operations to all scenarios. For example, double volatility calls for doubling all distances of scenarios from the mean. This means in particular that span—the difference between the best and worst case scenarios—does not depend on μ and must be proportional to σ .

The proposition provides conditions such that span delivers an index of subjective uncertainty. Its level depends on features of the subjective distribution, captured by $(\hat{s}_n - \hat{s}_1)$, that we cannot infer from our survey question. However, movements in subjective volatility over time will be picked up by movements in span. We thus obtain a simple recipe for using statistics of span to, say, calibrate a model with stochastic subjective uncertainty: relative movements in subjective uncertainty should be consistent with measured movements in span.

We finally note that the proposition also makes precise the sense in which span is a *robust* measure of uncertainty with a relatively low cognitive load. The proof clarifies that the optimal set of scenarios depends on a set of first-order conditions, with the condition for scenario i , say, involving only scenarios that are adjacent to it, but not those further away. We can therefore imagine changing the distribution in the interior of the support, say, and leave span unchanged, or changing little. Managers thus optimally report similar spans for a variety of distributions with quite different histograms that they may not tell apart confidently. By a similar argument, span is robust to changing the shape of the tail of the distribution, a feature that is often hard to form precise views about.

3 Uncertainty and change

In this section, we present summary statistics for sales growth as well as our measures of managers’ subjective beliefs about sales growth, that is, their forecast, their best- and worst-case scenarios, and the span between the two. In particular, we document that subjective uncertainty, as measured by span, varies both in the cross section between firms and in the time series for a given firm. These facts motivate an organizing framework that guides our analysis of *why* subjective uncertainty moves in the following sections.

3.1 Properties of subjective beliefs

Detailed tables of summary statistics are provided in Appendix H, here we present selected numbers as facts highlighted in bold. Specifically, when not otherwise stated, the numbers can be found in Table H2. In this Table, we first compute the time-series mean (upper panel) or

the time-series standard deviation (lower panel) of the respective variables for each firm and then calculate cross-sectional moments.

We begin by describing the nature of variation in growth we observe and how it is reflected in managers' forecasts.

Sales growth is serially uncorrelated and exhibits large cross-sectional heterogeneity. From the perspective of an econometrician, quarterly realized sales growth is serially uncorrelated in the time series dimension: we cannot forecast growth using lagged growth, see Table I1. At the same time, both the time-series mean and the time-series volatility of firm-level growth vary strongly between firms: the time-series mean is 2.2% on average across all firms, its cross-sectional distribution has a standard deviation of 7.1 percentage points (pp) and an interquartile (IQ) range from -0.8% to 5.5% . The time-series standard deviation is 11.6 pp on average across all firms, its cross sectional distribution has an IQ range from 6.1% to 14.7% .

Variation in sales growth is almost entirely idiosyncratic. During our sample 2013-19, Germany did not experience a recession. It is therefore not surprising that when we regress growth on time fixed effects, the R^2 is only about 1%. Moreover, even when we use time-industry fixed effects with our set of 14 industries, the R^2 increases only to about 7%, see Table I2. It follows that the facts below characterize firm-level heterogeneity as well as idiosyncratic variation over time.

Managers have a hard time predicting sales growth, but their forecasts are, on average, accurate and contain valuable information. For the average firm, the manager's forecast error is essentially unbiased and has a standard deviation of 10.3 pp. The average firm thus incurs a mean squared error of $10.3^2 = 106$, about 20% less than an econometrician who simply forecasts the sample mean every period (and thus incurs a mean squared error that equals the time-series standard deviation of growth, $11.6^2 = 135$). This is despite the fact that the manager, forecasting in real time, did not share the econometrician's benefit of hindsight. Part of the variation in forecasts thus reflects valuable information. The distribution of forecasts is compressed relative to that of growth rates, with an IQ range from zero to 5%.¹⁴

We now turn to summary statistics of span, our measure of subjective uncertainty.

On average, best and worst case scenarios bracket the forecast in a slightly asymmetric way. The average worst and best case scenarios are -4.3% and 8.1% , respectively. The midpoint between the scenarios is 1.9% and hence a percentage point below the average forecast of 2.9% . This means that the scenarios are slightly negatively skewed around the forecast.

Skewness of scenarios reflects the skewness in firms' empirical distribution of growth rates. The share of growth realizations that falls outside the range between best and worst case scenarios is one half, with about one quarter above the best scenario and one quarter below the worst (see Table I3). This is what we would expect given our simple model of span in Section 2.3: firms should choose their best (worst) case scenario further away from the forecast than their worst (best) case scenario when their subjective distribution is right (left) skewed.

¹⁴In a cross section of firms, Bloom et al. (2020) document that the standard deviation of annual sales growth rates is much larger than the standard deviation of annual sales growth rate expectations: 40.5 pp vs 9.2 pp. Qualitatively, our cross section has a similar discrepancy but less pronounced: 14.6 pp vs 11.6 pp, see Table H1.

Span is about as large as volatility. The span between the best and worst case scenario—our measure of subjective uncertainty—is similar in magnitude to the unconditional volatility of firm-level growth. Indeed, the average firm has a time-series mean for span of 12.4 pp, while its time-series standard deviation of growth is 11.6 pp. Since growth is hard to predict, the span reported by the average firm is also similar in magnitude to the average absolute forecast error of 9.0 pp it experiences.

Changes in span typically reflect changes in both the best and worst scenarios. When an individual firm increases its span from one quarter to the next, the mean change in the worst case scenario is -4.7 pp whereas the mean change in the best case scenario is $+2.6$ pp. In other words, the average increase in uncertainty thus consists of an outward expansion of span that is slightly asymmetric. The average decrease in span is a symmetric downward compression: conditional on a decrease in span, the worst case increases by 3.9 pp and the best case decreases by 3.4 pp. We conclude that typical changes in span reflect a reassessment of the entire outlook of the firm, and not only, say, a tweak at one end.

In the remainder of this section, we describe key dimensions of variation in subjective uncertainty in our sample.

Subjective uncertainty varies in the cross section. To assess variation of subjective uncertainty between firms in the cross section, we compute the average span for each firm. The cross-sectional standard deviation of firm-specific average span is 7.4 pp. It is similar in magnitude to the cross-sectional standard deviation of the average absolute forecast error of 8.4 pp. Firms thus differ substantially in both the size of the typical shock they experience and in the way their planning deals with perceived uncertainty.

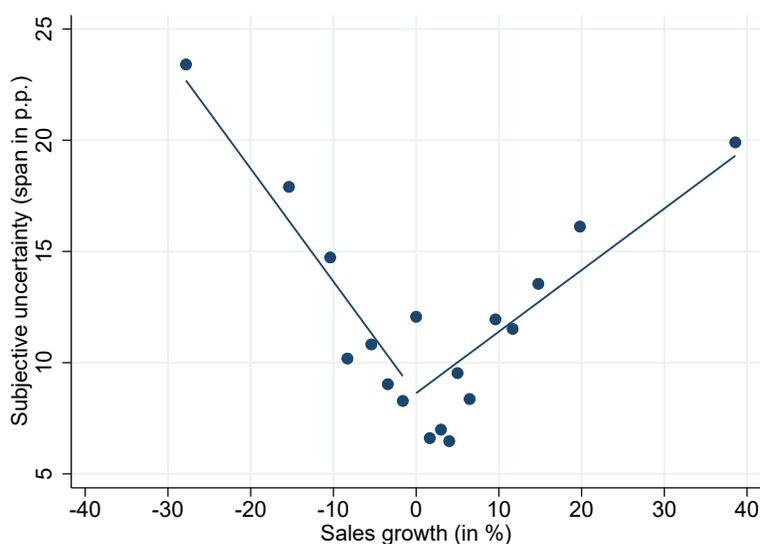
Subjective uncertainty varies in the time series at the firm level. Our data also show substantial within-firm time-series variation in subjective uncertainty. The time-series standard deviation of span for the average firm is 6.3 pp. This time variation in subjective uncertainty is substantial compared to other changes in firms' beliefs. For example, the average firm reports a time-series standard deviation of its forecast of 8.2 pp.

Variation in subjective uncertainty is almost entirely idiosyncratic. We have already seen that growth cannot be explained using time and industry fixed effects. We find the same result for span, as well as for the absolute forecast error. In both cases, a regression on time fixed effects yields an R^2 around 1%, which increases to about 5% when we allow for time-industry fixed effects, see Table I2. The facts below thus speak to firm-level variation of uncertainty in the cross section as well as idiosyncratic heteroskedasticity in the time series.

Subjective uncertainty exhibits an asymmetric V-shape relationship with growth. Figure 2 relates uncertainty and sales growth in a binned scatter plot for our pooled sample. The relationship is an asymmetric V with a minimum near zero, with a steeper negative and a flatter positive branch. Our sample, thus, exhibits similar properties between experienced growth and uncertainty as the literature has documented in different contexts. We now exploit the panel dimension of our data to distinguish cross-sectional and time-series Vs.

Subjective uncertainty exhibits an asymmetric V-shape relationship with growth in the cross section. In the left panel of Figure 3, we now consider variation between firms: we

Figure 2: Subjective uncertainty and sales growth in the pooled sample



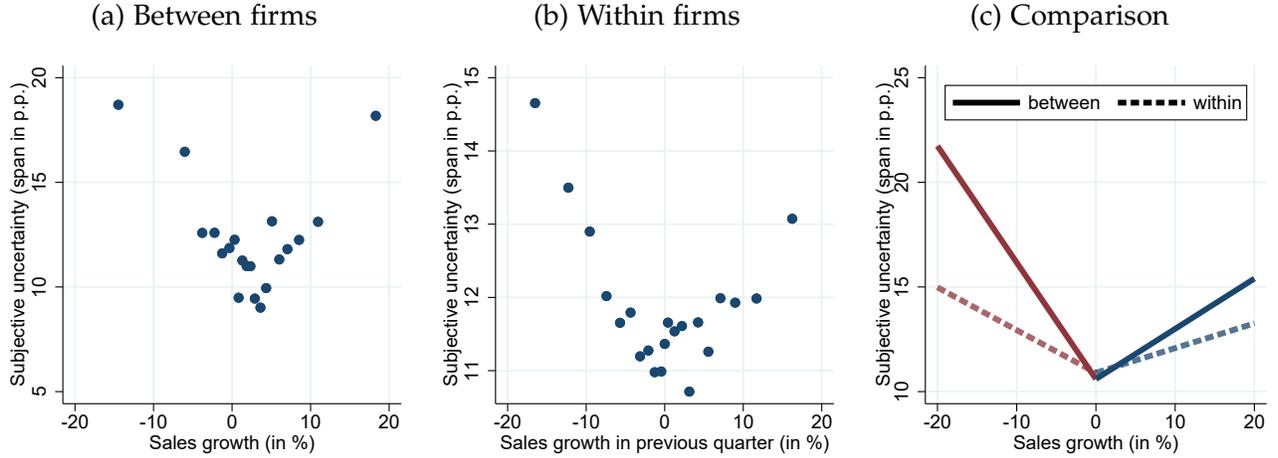
Notes: The figure shows the relationship between subjective uncertainty (span in pp) and sales growth (in %) in the pooled sample in form of a binned scatterplot and the prediction from a piecewise linear regression of subjective uncertainty on sales growth with a break at zero.

show a binned scatterplot of the firm-level average span on the vertical axis against firm-level average growth on the horizontal axis. The relationship is (again) an asymmetric V with a minimum near zero, with a steeper negative and a flatter positive branch, see also the right panel of Figure 3. Subjective uncertainty is thus higher for firms that either grow faster or, even more so, shrink faster on average. Put differently, firms that over the medium run move into unfamiliar territory, in the sense of a different scale of operations, perceive more subjective uncertainty.

Subjective uncertainty exhibits an asymmetric V-shape relationship with growth in the time series. The middle panel of Figure 3 relates uncertainty and sales growth in the time-series (“within”) dimension, taking out level differences between firms. Specifically, we show the deviation of growth from its firm-specific mean along the horizontal axis and the deviation of span from its firm-specific mean along the vertical axis. Again, we obtain an asymmetric V: subjective uncertainty is higher for firms that just experienced an unusually good or bad quarter. Put differently, firms that just dipped into unfamiliar territory in the short run also perceive more subjective uncertainty.

We note that the between-firms and within-firms effects are separate from each other since the within-analysis removes the firm-specific averages that make up the between effects. The right panel compares regression lines for positive and negative growth rates. It shows that the “within V” is flatter than the “between V.” Nevertheless, the slopes of both are sizable compared to the time-series standard deviation of the average firm’s span (6.3 pp) and the cross-sectional standard deviation of firm-specific average span (7.4 pp).

Figure 3: Subjective uncertainty and sales growth: between vs within



Notes: The figure shows the relationship between subjective uncertainty (span in pp) and sales growth (in %): (i) in form of binned scatterplots in panels (a) and (b), and (ii) in form of the prediction from a piecewise linear regression of subjective uncertainty on sales growth with a break at zero in panel (c). Panel (a) shows the cross-sectional (*between*) dimension: observations refer to firm-specific average span in pp and average sales growth in percent. Panel (b) shows the time-series (*within*) dimension: observations refer to deviations of growth and span from their firm-specific means in pp. On the horizontal axis, zero is equal to the firm-specific mean growth. On the vertical axis, the unconditional mean of span is added. Panel (c) compares the relationships between and within firms using piecewise linear regressions fitted to observations in panel (a), solid line, and panel (b), dashed line.

3.2 Subjective beliefs and change: an organizing framework

Our organizing framework relates a firm's subjective uncertainty to the distribution of growth. In particular, we compare different mechanisms for generating cross-sectional and time-series variation in subjective uncertainty and derive additional predictions for those mechanisms that we check in the following sections.¹⁵

We measure time in quarters and define date t as the beginning of a quarter when the survey is in the field. We further write g_{t+1}^i for the growth rate of firm i 's sales over the quarter between dates t and $t + 1$ relative to sales that occurred over the previous quarter between dates $t - 1$ and t . In other words, g_{t+1}^i is the growth rate that firm i forms beliefs about when it answers our survey questions at date t .

Representing firm subjective beliefs. Firm i 's information set at date t consists of a vector of signals s_t^i . It contains in particular past sales growth, but may also include other signals about the future that have arrived up to date t . We represent firm i 's one-quarter-ahead belief about sales growth by the conditional distribution

$$g_{t+1}^i = f^i(s_t^i) + \sigma^i(s_t^i)\varepsilon_{t+1}^i \quad (2)$$

¹⁵Our organizing framework follows a longstanding literature, dating back to [Muth \(1960, 1961\)](#), that studies the relationship between subjective uncertainty and conditional volatility. This relationship is central to our contrast between rational expectations and learning.

where ε_{t+1}^i is an error that has mean zero and variance one under the firms' subjective belief. We add superscripts i to the conditional mean and volatility to highlight that firms' beliefs may depend on firm characteristics that are fixed over our sample, such as parameters of the distribution of growth or signals, as well as the firm's broad size category.

When firm i answers our survey questions at t , it provides in particular a forecast $f^i(s_t^i)$, that is, its subjective conditional expectation. We also observe the subsequent realization g_{t+1}^i and hence the firm's subjective forecast error $f e_t^i = g_{t+1}^i - f^i(s_t^i)$. We further identify span, the difference between firm i 's best and worst case scenarios, with firm i 's subjective conditional standard deviation $\sigma^i(s_t^i)$. The model of scenario analysis in Section 2.3 justifies this step for a large class of distributions.

Firms' subjective beliefs vs the true data generating process. An econometrician also observes data on growth. Under the probability measure that characterizes the true data generating process, growth is distributed as

$$g_{t+1}^i = f^{i*}(s_t^i) + \sigma^{i*}(s_t^i)\varepsilon_{t+1}^{i*} \quad (3)$$

where again the error has mean zero and variance one, but importantly now under the true DGP. We thus allow for two differences between firms' belief and the true DGP. First, firms might make biased forecasts, captured by the difference in conditional means $f^* - f$. Second, subjective uncertainty, captured by σ , may differ from measured volatility σ^* .

Suppose an econometrician has a panel data set of growth rates together with forecasts and subjective uncertainty reported by firms in a survey. Subjective uncertainty—and more generally, beliefs—can now be characterized both in the cross section *between firms* and in the time series *within firms*. For example, comparison of average subjective uncertainty between firms shows how the firm-level unconditional mean of the functions $\sigma^i(s_t^i)$ depends on firm characteristics. Similarly, average forecast errors reflect the mean of the bias $f^i(s_t^i) - f^{i*}(s_t^i)$. To study the variation of beliefs within firms, we can regress subjective uncertainty or forecast errors on observed signals in s_t^i , such as realized growth. We thus infer how σ^i and f^i respond to those signals.

Rational expectations refers to the special case of the framework where the true DGP and the firm's subjective beliefs coincide. Formally, there is no bias ($f^i = f^{i*}$) and subjective uncertainty mirrors actual volatility, that is, $\sigma^i = \sigma^{i*}$. More generally, outside of rational expectations, subjective uncertainty and volatility might differ. When firms learn, for example, subjective uncertainty reflects more than just realized volatility, as it also captures uncertainty about parameters that firms try to learn about.

We now illustrate the distinction between volatility and uncertainty with two concrete simple examples of belief dynamics that capture many of the features we observe in our data below. To ease notation, we omit superscripts i from now on. It is understood, however, that all variables are idiosyncratic to the firm.

Example R: rational expectations and uncertainty as volatility. Suppose growth has a constant mean f , but time-varying volatility that depends on signals s_t . Under rational expecta-

tions, the firm belief (2) coincides with the true DGP (3):

$$g_{t+1} = f + \sigma(s_t)\varepsilon_t. \quad (4)$$

The firm makes the same forecast f every period: past growth cannot be used to predict future growth. Past growth might still be useful, however, to predict the *absolute value* of the forecast error $fe_{t+1} = g_{t+1} - f$ since it is contained in the signal vector s_t . The fixed characteristics of firms here are given by the number f , the function σ , and the distribution of the signals s_t .

Example R captures a number of the facts presented in Section 3.1. To an econometrician, growth looks serially independent. Forecast errors are zero on average. They also look like growth rates themselves, so growth is hard to predict. In fact, the example is too stark in that dimension: it assumes for simplicity that firms have *no* useful information for forecasting growth.¹⁶ Nevertheless, uncertainty varies in the cross section with σ as well as in the time series with s_t .

Uncertainty Vs and volatility Vs. With appropriate assumptions on firm characteristics, Example R generates the Vs in Figure 3. A V-shape relationship of uncertainty and growth between firms obtains if the absolute value of mean growth $|f|$ and mean volatility $E[\sigma(s_t)]$ move together in the cross section of firms. A V-shape relationship between past growth and current uncertainty within firms obtains if we identify the signal with past growth $s_t = g_t$ and the function σ is V-shaped in the signal.

At the same time, Example R is restrictive since rational expectations ties beliefs to the true DGP. For the between-firm analysis, this means that all firm-level mean forecast errors are zero. Moreover, subjective uncertainty reflects only the volatility of forecast errors:

Proposition R1 (Uncertainty looks like volatility in the cross section). Consider two firms A and B with different fixed characteristics, both described by Example R. Firm A has a higher firm-level mean subjective uncertainty than firm B if and only if firm A has higher firm-level mean absolute forecast error than firm B.

We relegate the formal argument of this and the following propositions to Appendix G. Intuitively, under rational expectations, average subjective uncertainty is high if and only if firms experience large surprises on average (that is, large forecast errors in absolute value). The proposition implies in particular that for any two firms that differ in mean growth rates, their subjective uncertainty and mean absolute forecast error must move together. In other words, a cross-sectional uncertainty V must always be accompanied by a cross-sectional volatility V in the data.

The close connection between beliefs and the true DGP also implies restrictions on conditional distributions. In particular, forecasts are unbiased and subjective uncertainty is equal to the conditional volatility of forecast errors, that is $\sigma(s_t) = \sqrt{\text{var}(fe_{t+1}|s_t)}$. We can test these restrictions with regressions. For example, a large literature tests whether signals such as past

¹⁶The assumption is for simplicity; the distinction between uncertainty and volatility that the examples are meant to clarify is also relevant when the forecast depends on signals.

growth help predict forecast errors. Here we check whether past growth predicts the *absolute* forecast error and subjective uncertainty in the same fashion, as would be the case under Example R. We refer to *demeaned* variables as deviations from the time-series mean.

Proposition R2 (Time-series uncertainty V is volatility V). Consider the time series of lagged growth rates $s_t = g_t$, forecasts and subjective uncertainty for a firm described by Example R. A piecewise linear regression of demeaned uncertainty on demeaned growth with a break at zero shows a V-shaped pattern if and only if a regression of the demeaned subjective absolute forecast error on demeaned growth shows the same V-shaped pattern.

Under rational expectations, a signal such as past growth can predict high uncertainty only if it also predicts large surprises. If Example R generated the uncertainty V in the middle panel of Figure 3, we should therefore also see a “conditional-volatility-V,” that is, a V-shape relationship between absolute deviations of forecast errors and deviations of growth from their means.

Example L: learning and uncertainty in unfamiliar territory. Our second example describes firms that learn about future growth from signals of uncertain quality. For tractability and to make the point starkly that heteroskedasticity in subjective uncertainty does not require heteroskedasticity in volatility, we assume that growth is iid normal with mean g^* and standard deviation σ_g^* , and hence in particular homoskedastic. However, firms observe a noisy signal $s_t = g_{t+1} + n_t$ of future growth one period in advance. For example, the signal could be past growth. Under the true DGP, the noise n_t is independent of growth and normally distributed with mean zero and standard deviation $\sigma_g^* \sqrt{(1 - \gamma^*)/\gamma^*}$, where $\gamma^* \in (0, 1]$.

We refer to the parameter γ^* as the *information content* of the signal—it is the R^2 of a regression of growth on the signal. A perfect signal with $\gamma^* = 1$ reveals the growth rate, whereas the signal becomes pure noise as $\gamma^* \rightarrow 0$. The conditional distribution (3) under the true DGP is

$$g_{t+1} = (1 - \gamma^*)g^* + \gamma^*s_t + \sqrt{1 - \gamma^*}\sigma_g^*\varepsilon_{t+1}^*, \quad (5)$$

where the innovation ε_{t+1}^* is standard normal and independent of the signal. If the signal contains more information in the sense of higher γ^* , then the conditional mean, $(1 - \gamma^*)g^* + \gamma^*s_t$, responds more to it, and the conditional volatility is smaller.

Firms are uncertain about both growth g_{t+1} and the information content of the signal. We use a Normal-Gamma prior for tractability. Before observing the signal, firms believe that growth has mean zero and standard deviation σ_g . Moreover, the parameter $\gamma \in (0, 1)$ describes their prior view of the signal’s information content: it determines how the forecast responds to the signal, much like γ^* determines the response under the true DGP. An additional parameter $\nu > 2$ describes prior confidence about information content.¹⁷ In the limit when $\nu \rightarrow \infty$, precision is known and managers learn only about growth.

¹⁷To obtain this parametrization, we assume that growth and the precision of the signal, given by $\gamma^*/(1 - \gamma^*)(\sigma_g^*)^2$, follow a Normal-Gamma distribution $\mathcal{NG}(0, \lambda^{-1}, \Phi^{-2}, \nu)$, with $\lambda = \frac{1-\gamma}{\gamma}$ and $\Phi^2 = \frac{\nu-2}{\nu} \frac{1-\gamma}{\gamma} \sigma_g^2$.

Standard reasoning using Bayes' rule, presented in Appendix G.2, show that the firm's subjective belief (2) is then

$$g_{t+1} = \gamma s_t + \sqrt{\frac{\nu - 2}{\nu + 1} + \frac{\gamma}{\nu + 1} \left(\frac{s_t}{\sigma_g}\right)^2} \sqrt{1 - \gamma} \sigma_g \varepsilon_{t+1}, \quad (6)$$

where the innovation ε_{t+1} follows a t-distribution with $\nu + 1$ degrees of freedom and is independent of the signal. In some respects, the belief looks similar to the true DGP (5). Indeed, its conditional mean, the forecast $f_t \equiv \gamma s_t$, is a convex combination of the signal realization and a constant, here the prior mean of zero. Moreover, the second square root captures the force that higher information content lowers uncertainty.

However, the key difference between the true DGP (5) and the subjective belief comes from learning about precision, captured by the first square root in (6): even though conditional volatility is constant, subjective uncertainty depends on the (squared) realization of the signal s_t , and more so the less confident the manager is in information quality (lower ν). Intuitively, a manager who has observed a signal further away from the prior mean of zero—in either direction—infers lower precision, and hence ends up more uncertain, than a manager who observes a realization close to zero that confirms the prior.¹⁸

Forecasts and bias. To see how Example L accommodates forecast bias, substitute for g_{t+1} from (5). The conditional mean of the forecast error given the signal under the true DGP is

$$E[fe_t | s_t] = E[g_{t+1} - f_t | s_t] = E[g_{t+1} - \gamma s_t | s_t] = (1 - \gamma^*)g^* + (\gamma^* - \gamma)s_t. \quad (7)$$

An econometrician may observe systematic forecast errors for two reasons. First, when the true long run growth rate g^* is different from zero, the manager's forecast is too conservative, that is, biased towards zero. Second, when the manager's prior information content γ is larger than the realized true information content γ^* , then the manager places too much weight on the signal: the econometrician can use the signal to predict the forecast error.

Uncertainty, volatility, and bias between firms. Consider between-firm variation in uncertainty. As for Proposition R1, we ask how differences in fixed characteristics affect uncertainty and volatility. Characteristics are now given by the parameters of growth, the signal as well as the firm's prior, collected in the vector $(g^*, \sigma_g^*, \gamma^*, \sigma_g, \gamma, \nu)$. We describe changes in one dimension holding others fixed, akin to partial derivatives. We begin by varying the volatility of growth:

Proposition L0 (Uncertainty moves with volatility in the cross section). Consider firms described by Example L. Other things equal, if a firm experiences more volatile growth σ_g^* , then both its firm-level mean subjective uncertainty and the firm-level mean absolute deviation (MAD) of its forecast error are higher.

Volatility and uncertainty can thus move together in the cross section, just as they do in Example R. Intuitively, learning managers always take into account differences in volatility to

¹⁸In the limit as $\nu \rightarrow \infty$, this effect vanishes: if the manager is certain about information content, the belief is conditionally normal with constant mean and variance. The belief reduces to the true DGP (5) if $g^* = 0$, $\sigma_g = \sigma_g^*$ and $\gamma = \gamma^*$.

some extent, even if expectations are not rational. The key differences between learning and rational expectations that we can observe in the cross section are summarized by:

Proposition L1 (Uncertainty is more than volatility in the cross section). Consider firms described by Example L. Other things equal,

(a) as the volatility of growth σ_g^ converges to zero, firm-level mean subjective uncertainty converges to a positive constant.*

(b) if a firm's mean growth rate is more positive (negative), then its firm-level mean subjective uncertainty is higher, but the MAD of its forecast error is the same. Moreover, the firm's average forecast error is more positive (negative) and hence further away from zero.

(c) if a manager is more confident about the firm's environment in the sense of lower prior uncertainty about growth σ_g , then its firm-level mean subjective uncertainty is lower, but the MAD of its forecast error is the same.

Part (a) clarifies that learning managers are naturally more uncertain than what it may look like with hindsight. The limiting case where σ_g^* shrinks to zero is one where the manager learns a fixed parameter, the long run trend growth of the business. An econometrician thus observes a constant growth rate. However, the thought experiment holds fixed the manager's prior uncertainty about growth, captured by the parameter σ_g . The manager only finds out about the trend while learning in real time, and reports uncertainty during that time. More generally, whenever the manager does not know the true volatility of growth a priori, even small volatility can go along with substantial uncertainty. The remainder of the proposition then varies two other characteristics, trend growth and confidence, that change uncertainty, but not volatility.

Part (b) shows how learning about trend growth decouples uncertainty from volatility. Subjective uncertainty is higher for firms that consistently experience growth away from the prior mean, or $g^* \neq 0$. Such firms receive many unusual signal realizations and are therefore more suspicious of information content on average. They report high subjective uncertainty, even when the volatility of their forecast errors is not particularly high. In particular, we can observe a cross-sectional uncertainty V without a cross-sectional volatility V . If this mechanism is at work, we further have that this "excess uncertainty" is directly connected to bias: forecasts are always biased towards zero. Importantly, this bias is constant and does not contribute to the volatility of forecast errors.

Part (c) says that managers who are more confident a priori about their environment perceive lower uncertainty, but do not experience more volatility. As for part (a), the key here is the distinction between the manager's subjective prior uncertainty about growth and the true volatility of shocks that is responsible for forecast errors observed by the econometrician. In addition, "other things equal" here means holding fixed the information content of the signal, so a more confident manager uses the same forecasting rule. In other words, a more confident manager who faces the same quality of information starts out less uncertain than a less confident manager, and ends up less uncertain as well. We note that this effect is unrelated to bias: in particular, it is present also when comparing two firms that both make unbiased forecasts.

Subjective uncertainty vs volatility within firms. Consider now within-firm variation under Example L. We would like to compare the dependence of uncertainty and conditional volatility

on past growth. For a learning firm, however, squared forecast errors contain bias, so the simple approach of Proposition R2 would give hard-to-interpret results. To distinguish conditional volatility from bias, we therefore define the *debiased* forecast error as the residual from a regression of forecast error on past growth and a firm fixed effect. Debiasing thus removes both forms of bias discussed above, conservatism during learning and overconfidence. We then have

Proposition L2 (Time-series uncertainty V without volatility V). Consider time series of lagged growth rates s_t , forecasts and subjective uncertainty for a firm described by Example L. Suppose that $|g^| < \sigma_g^* / \sqrt{\gamma^*}$. Up to a second order Taylor expansion, a piecewise linear regression of demeaned subjective uncertainty on demeaned lagged growth with a break at zero generates a V-shaped pattern. In contrast, a regression of the demeaned debiased absolute forecast error on demeaned lagged growth shows no relationship.*

In Example L, a V-shape that relates uncertainty and past growth as in Figure 3 can be purely subjective and unrelated to the volatility of forecast errors. Intuitively, subjective uncertainty of a learning firm is higher after *any* unfamiliar signal, whether positive or negative. This is because an unfamiliar signal leads managers to infer that the unknown precision of the signal is low, so posterior uncertainty about growth remains high. The strength of this effect is governed by the parameter ν , which captures prior uncertainty about precision.¹⁹

The technical caveats are needed to obtain closed form solutions for regression coefficients, as shown in the Appendix, but do not substantially drive the main result that uncertainty decouples from volatility. A second order expansion around the prior mean makes the standard deviation linear in the squared signal. Moreover, in order for the econometrician to find a V with a break at zero, the true growth rate cannot be too far away from the prior mean of zero, a natural assumption in our context where bias is small relative to the volatility of growth. Decoupling of uncertainty from volatility happens more generally: the debiased forecast error is always unrelated to lagged growth, whereas uncertainty moves with lagged growth.²⁰

We finally explore how the potential for overconfidence affects the relationship between uncertainty and volatility. We define a firm's *excess uncertainty* as the difference between its mean firm-level subjective uncertainty and the firm-level mean absolute deviation of its forecast error.

Proposition L3 (Uncertainty and overreaction). Consider time series of lagged growth rates s_t , forecasts and subjective uncertainty for a firm described by Example L.

(a) *If the firm overreacts to signals, $\gamma > \gamma^*$, the covariance between its forecast and forecast error is negative.*

(b) *Other things equal, a decline in the true information content γ^* implies more overreaction to signals and a lower (more negative) covariance between forecast and forecast error. If the manager is not too uncertain about information quality (ν large enough), then it also implies lower excess uncertainty.*

¹⁹As ν converges to infinity, subjective uncertainty becomes independent of the signal and the conditional distribution becomes normal with variance $(1 - \gamma)$ times the prior variance of growth, and hence similar in shape to (5), albeit with possibly different parameter values.

²⁰This is quite different from decoupling in the cross section when managers learn about trend in Proposition L1(b), where the mechanism that leads to decoupling always delivers bias as well.

Part (a) points to a simple statistic that captures bias from overreaction. Intuitively, if a forecast from a manager who systematically overreacts is, say, low, then this is often driven by noise that the manager overreacts to as opposed to actual subsequent low growth. Noise pushes the forecast and the error in opposite directions, and thus induces negative correlation between them.

Part (b) compares firms that overreact to different degrees. Concretely, we lower the true information content. The firm then responds as if its signal was more precise than it actually is, and we obtain more forecasts driven by noise, and lower negative correlation with errors. Provided that the firm is not too uncertain about information quality to begin with, its prior overestimation of information content also makes it *ex post* less uncertain relative to the benchmark of average absolute forecast error.²¹

We close with two remarks about the examples in relation to more general models. To begin, Examples R and L are designed to be stark illustrations of two competing mechanisms for why uncertainty moves around, volatility shocks under rational expectations and learning about information quality. They serve to illustrate the implications of each hypothesis and in particular to suggest tests that only one of them is at work. When we go to the data, we would generally expect that both mechanisms are present to some extent. It is plausible that managers face heterogeneous shock distributions *and* are unsure of information quality. As we will see in Section 5 below, a hybrid model that combines features of both mechanisms delivers our preferred interpretation of the data.

Moreover, the purpose of Example L is to capture in a transparent way how survey beliefs differ from the rational expectations benchmark when information quality is uncertain. We have purposefully abstracted from explicit learning dynamics by assuming that the prior is the same every period and not related to past information. However, the two main qualitative properties of beliefs that we highlight are relevant also in a broader class of models with explicit dynamics.

First, as long as there is *some* uncertainty about the precision of signals, it makes sense for managers to interpret unusual signals more cautiously. This behavior adds a subjective component to uncertainty measured in surveys that can be unrelated to conditional volatility. Second, between firms, for those firms, which are on a longer unusual trajectory, an econometrician, who sees this trajectory with hindsight, has more information than the firm had in real time, and so the firm might look biased.²²

²¹This effect is familiar from models with known precision, that is, the limiting case of our model where $\nu \rightarrow \infty$ so the condition always holds. The qualifier is needed since a manager who is very uncertain a priori might be swayed by unusual signal realizations that information content is in fact much lower than under the prior mean.

²²In a dynamic model of learning, the date- t prior would depend on past information. If unknown parameters are fixed, beliefs typically converge to become concentrated on the parameters of the true DGP. More generally, structural change might cause firms to doubt about their mean growth or their information quality, leading to continual learning. The two key principles, that uncertainty is more than just volatility and that learners look biased with the benefit of hindsight, is shared by these more complex models as well.

4 Uncertainty and change in the cross section

In this section, we ask what type of firms perceive more subjective uncertainty *on average*. In terms of the framework of Section 3.2, we thus characterize the dependence of subjective uncertainty σ on fixed characteristics, assuming that time-averaging removes the effects of information s_t^i .

4.1 Subjective uncertainty, turbulence, trend, and firm size

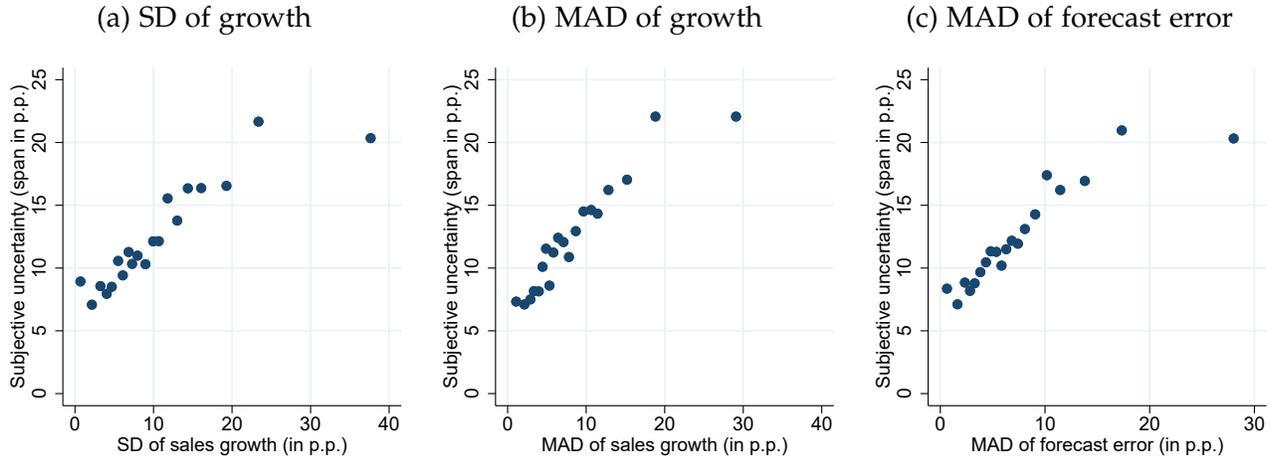
We explore three firm characteristics that plausibly shape subjective uncertainty, as suggested by our framework. We first look at volatility of growth: whether or not firms have rational expectations, it makes sense that firms that face more turbulence—which we measure by the sample time-series standard deviation of sales growth—also report higher uncertainty. Second, we consider long term trend growth. Example L illustrates that when learning firms experience fast positive or negative growth, subjective uncertainty can be high even if volatility is not. Finally, we consider firm size: it is plausible that information processing works differently in large vs small firms, which would affect confidence about information quality and hence uncertainty, holding fixed volatility. We develop a set of facts on each characteristic, again highlighted in bold face.

In the cross section of firms, uncertainty and the volatility of growth move strongly together. Panel (a) of Figure 4 shows a binned scatterplot of average span, our measure of subjective uncertainty, against the standard deviation (SD) of growth, a measure of unconditional volatility. Panel (b) displays the sample time-series mean absolute deviation (MAD) of growth, a more outlier-robust measure of volatility. Here the association is even stronger. In a linear regression, volatility explains about a quarter of the variation in uncertainty. In fact, for mean absolute deviation values below 10 pp, the variables move close to one-for-one. For firms that experience higher volatility, the relationship is less than one-for one, so overall the slope of the regression line is only about 0.6.

To establish more formally that span moves with volatility, we divide firms into “turbulence quartiles,” based on the sample standard deviation of growth. Column (1) of Table 2 shows a cross-sectional regression of span on turbulence dummies for the three highest quartiles, with the lowest quartile as the baseline. The effect of turbulence on uncertainty is highly statistically significant and economically large. Moving from the lowest to highest quartile increases span from 7 pp to 18 pp, by about 1.5 cross-sectional standard deviations. The turbulence dummies alone explain 30% of the cross-sectional variation in uncertainty.

The organizing framework of Section 3.2 says that a strong association of subjective uncertainty and volatility can be consistent with rational expectations, per Proposition R1, or with learning, per Proposition L0. Since span is an index number, we cannot draw strong conclusions about levels. As we have noted above, however, for firms with volatility below 15 pp, about the 75th percentile in the cross section, the regression line has a slope close to one and a nonzero intercept. Proposition L1(a) says that subjective uncertainty in Example L also has an intercept that reflects the firm’s prior uncertainty. Example R instead would call for a line through the origin, requiring a highly nonlinear relationship between uncertainty and span.

Figure 4: Subjective uncertainty and volatility in the cross section



Notes: The figure shows the cross-sectional relationship between subjective uncertainty in percentage points (span) and different volatility measures in binned scatterplots based on firm-level averages. The volatility measure in panel (a) is the standard deviation (SD) of growth in percentage points. The volatility measure in panel (b) is the mean absolute deviation (MAD) of growth in percentage points. The volatility measure in panel (c) is the mean absolute deviation (MAD) of subjective forecast errors in percentage points. There are at least two observations per firm.

In the cross section of firms, uncertainty and the volatility of subjective forecast errors move strongly together. Panel (c) of Figure 4 shows a binned scatterplot of average span against the sample time-series mean absolute deviation of *subjective* forecast errors. The relationship is very similar to that with the volatility of growth. We have already shown that sales growth is hard to predict and average forecast bias is low. As a result, subjective forecast errors behave similarly to those from a forecast that only uses the unconditional mean—the latter is one interpretation of the mean absolute deviation of growth.

Uncertainty is higher for firms that grow or shrink quickly. Panel (a) of Figure 5 displays a binned scatterplot of firm-level average span on the vertical axis against firm-level average growth on the horizontal axis. The relationship is an asymmetric V with a steeper left branch and a flatter right branch. Column (2) of Table 2 presents a cross-sectional regression of span on two “trend dummies” for the top and bottom quartile of the sample average growth rate, with the interquartile range as the baseline. The V-shape is strongly significant, and the statistical significance also pertains to the asymmetry of the V.

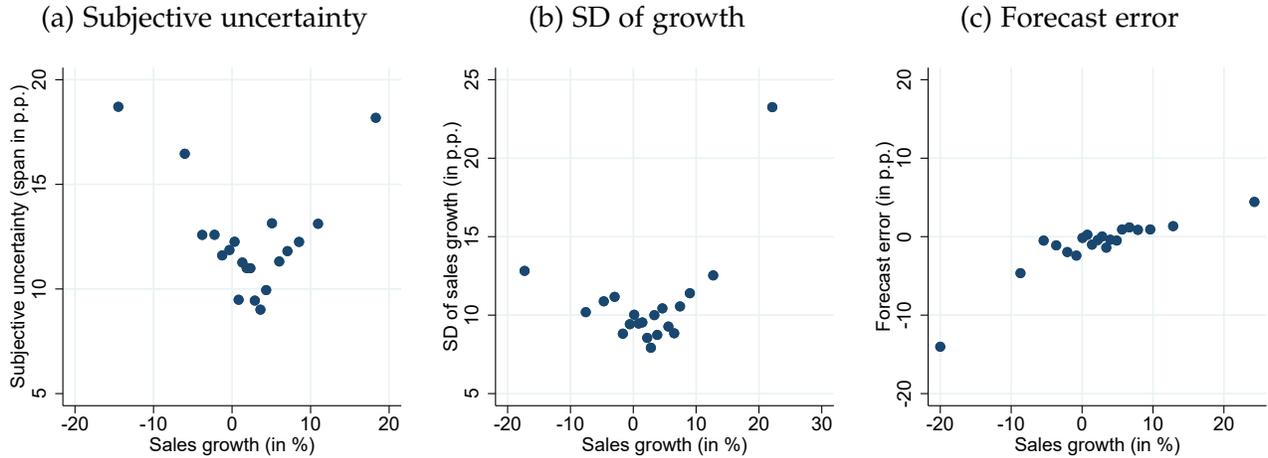
Shrinking firms perceive more uncertainty even controlling for turbulence. Panel (b) of Figure 5 shows that firms with higher absolute trend are also more turbulent. Column (3) of Table 2 thus shows the results from a regression of span on trend dummies together with turbulence dummies. Coefficients change, and in particular the coefficient on the dummy for the top trend quartile now becomes insignificant. However, the dummy for the bottom trend quartile still enters significantly, with a large coefficient of 2 pp. We conclude that, in the cross section, uncertainty is more than just turbulence: it is also higher when firms venture into unfamiliar territory, at least on the negative side.

Table 2: Regressions of firm-level average subjective uncertainty and firm-level average forecast error on firm characteristic dummies

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	avg. span	avg. span	avg. span	avg. FE	avg. span	avg. span	avg. span	avg. FE
Dummy medium low turbulence	2.907*** (0.405)		2.708*** (0.396)	0.323 (0.416)		2.498*** (0.383)	2.509*** (0.395)	0.313 (0.439)
Dummy medium high turbulence	6.161*** (0.441)		5.929*** (0.439)	0.533 (0.530)		5.365*** (0.438)	5.252*** (0.439)	0.579 (0.549)
Dummy high turbulence	10.71*** (0.639)		10.35*** (0.629)	-1.487 (0.984)		9.565*** (0.621)	9.311*** (0.624)	-1.300 (0.976)
Dummy 'bad' sales growth trend		3.727*** (0.616)	2.478*** (0.526)	-4.481*** (0.635)		2.124*** (0.522)	2.179*** (0.523)	-4.613*** (0.664)
Dummy 'good' sales growth trend		2.380*** (0.573)	0.682 (0.488)	4.540*** (0.823)		0.403 (0.468)	0.288 (0.479)	4.610*** (0.834)
Dummy small firms					-2.973* (1.625)		-0.885 (1.389)	1.744 (2.739)
Dummy medium sized firms					-5.971*** (1.558)		-3.166** (1.345)	1.166 (2.610)
Dummy large firms					-8.973*** (1.566)		-5.239*** (1.377)	1.466 (2.598)
Constant	7.421*** (0.242)	10.80*** (0.287)	6.829*** (0.273)	-0.798** (0.314)	18.26*** (1.526)	10.45*** (1.381)	10.54*** (1.668)	0.312 (3.353)
Sector dummies							YES	YES
No. of observations	950	950	950	950	950	950	950	950
No. of firms	950	950	950	950	950	950	950	950
No. of parameters (excl. intercept)	3	2	5	5	3	8	21	21
R-squared	0.30	0.047	0.31	0.11	0.11	0.36	0.38	0.12

Notes: avg. span denotes the time-series average of firm-level span and avg. FE denotes the time-series average of the firm-level forecast error. Results from OLS regressions. Turbulence (trend) dummies are based on quartiles of the time-series standard deviation (average) of firm-level sales growth rates. See Table 1 for explanation of size dummies. Reference groups are, respectively, the lowest quartile of turbulence, the two middle trend quartiles, and tiny firms. Industry dummies are defined in Appendix E. Robust standard errors in parentheses; * p < 0.10, ** p < 0.05, *** p < 0.01.

Figure 5: Uncertainty and trend growth



Notes: The figure shows the cross-sectional relationship between firms’ average sales growth in percent (trend growth) and different firm-level measures in binned scatterplots based on firm-level averages. In panel (a), subjective uncertainty in percentage points (span) is on the vertical axis. In panel (b), the standard deviation of growth in percentage points (turbulence) is on the vertical axis. In panel (c), the forecast error in percentage points (bias) is on the vertical axis.

Higher uncertainty of fast-growing or fast-shrinking firms goes along with biased forecasts.

Panel (c) of Figure 5 plots average forecast errors on the vertical axis against average growth. There are two important patterns. First, for a wide interval of growth rates around zero, average forecast errors are close to zero. This is consistent with our earlier point that unconditional average forecast errors are close to zero as well. Second, we find that for large positive and negative growth rates, average forecast errors are large and have the same sign as the growth rates. Since we define errors as growth minus forecast, this means that *forecasts* are systematically too conservative, that is, too close to zero. Column (4) in Table 2 confirms that bias is significant and symmetric for positive and negative growth, controlling for turbulence.

Taken together, the fact that firms on a steep downward trend make biased forecasts *and* perceive higher uncertainty is evidence in favor of learning, as described by Proposition L1(b). Firms that persistently venture into unfamiliar territory are repeatedly surprised. An econometrician, in contrast, measures trend growth with the benefit of hindsight. The firms thus look “too conservative” to this econometrician. Moreover, firms on steep trends are constantly confronted with unfamiliar signals which make them doubt the precision of those signals and hence perceive more uncertainty. Their higher uncertainty is due to the journey into unfamiliar territory and not to the volatility of their growth rates.

At the same time, firms on steep upward trends also make biased forecasts that are too conservative, but they do not perceive more uncertainty once volatility is controlled for. Fast-growing firms thus look like they also learn about an unknown trend, but are more confident about the signals they receive. Proposition L1(c) offers an explanation: fast-growing firm not only have positive trend growth g^* , but also perceive lower uncertainty about lesa relatively higher confidence parameter ν . One interpretation is that the typical successful firm worries

relatively less about information quality. A shrinking firm, by contrast, while also on an unfamiliar trend, is more suspicious about what the signals mean.

The final firm characteristic we consider is size. The motivation here is that larger firms might have different capabilities to process information. For example, hiring a statistician to aid with planning is a fixed cost component. We divide firms into the four size categories used by the German Statistical Office, with size measured as average employment over our sample.

Larger firms perceive less subjective uncertainty, even controlling for trend and turbulence.

We report the results of regressions on size dummies in columns (5) to (7) of Table 2, with tiny firms as the baseline. Uncertainty declines strongly with size: In column (5), large firms have on average 9 pp lower span than tiny firms. This is partly due to the fact that large firms are less volatile and are less likely to be on strong positive or negative growth trends. Once we control for trend and turbulence in column (6), the coefficients on size dummies decline by about one half.

Nevertheless, holding fixed trend and turbulence, large firms with more than 250 employees report 5 pp lower span, more than half the cross-sectional standard deviation, than the tiny firms with less than 10 employees.²³ In column (7) of Table 2, we regress span on the turbulence, trend growth, and size dummies but in addition include dummies for 14 broad sectors. Coefficients on our three key characteristics remain essentially the same, and sector dummies increase the R^2 only by 2 pp relative to column (6). We conclude that variation due to turbulence, trend, and size is also relevant within industries. This finding is consistent with our earlier point that most variation in growth over our sample is idiosyncratic.

Average forecast errors do not vary by size. Column (8) shows no significant differences across size categories in average forecast errors controlling for turbulence and trend growth. It follows that the relationship between uncertainty and size works differently than that between uncertainty and negative trend growth. In both cases, there is a purely subjective component to uncertainty, not accounted for by volatility. While shrinking firms make biased forecasts, however, small firms do not. In terms of Example L, we can think of firms using a prior mean close to the long-run mean of growth, so no systematic bias arises. Small firms differ from large firms by a smaller parameter ν , which captures lower confidence in the precision of their signals.

4.2 Discussion

We take away two major points from the cross-sectional analysis. First, **firms are acutely aware of the turbulence of their business environment when they plan.** It is well known that idiosyncratic risk varies in the cross section of firms and plants (see for example, [Castro, Clementi and Lee, 2015](#)). What we have shown here is that volatility experienced by firms is also an important determinant of *subjective* uncertainty. This finding supports a key premise of rational expectation models of firm dynamics: firms' perception of uncertainty comoves with volatility measured by an econometrician.

²³[Bloom et al. \(2020\)](#) also document a negative relationship between subjective firm-level uncertainty and firm size and age and a positive relationship with prior realized volatility.

Second, **there are dimensions of cross-sectional firm heterogeneity along which uncertainty is more than just volatility, namely, size and trend growth.** We have shown that small firms and firms that are quickly shrinking are systematically less confident about the future, holding fixed volatility. Example L offers a simple learning mechanism that helps understand where uncertainty might come from beyond volatility, and why in particular it goes along with forecast bias for shrinking firms. The key is that learning firms are unsure about parameters the econometrician observes with the benefit of hindsight. Firms are therefore less confident than if they were planning under rational expectations.

Our second finding thus calls for models of firm dynamics that go beyond rational expectations. [Jovanovic \(1982\)](#) pioneered models where firms learn over time about their “type,” for example the potential of their business idea (see [Baley and Veldkamp, 2022](#) for a survey of Bayesian learning, with many references on firm dynamics). This class of models exhibits the key property we find here because econometricians measure the effect of type on, say, average profitability, and thus see more information than what firms knew in real time.

A novel theme we bring to the table is that learning should matter not only for small and growing firms, but also for shrinking firms. The key underlying principle appears to be that uncertainty is more than volatility when firms enter unfamiliar territory, whether good or bad. Accommodating this pattern calls for models where firms do not learn a fixed type, but types can change, for example as a business idea becomes outdated. Adapting to this new reality then makes firms again more uncertain, in a way that is not captured by changes in volatility alone.

An interesting question for future research is to examine what it is about the technology or organization of the firm that drives behavior towards more or less subjective uncertainty. In other words, what deeper model can microfound the learning parameters of Example L. Our results suggest two plausible directions. The first is increasing returns (e.g., in firms size) in information processing or the management of uncertainty. If some task that helps with planning, such as modeling demand, has a fixed cost, large firms might afford it while small firms do not. This hypothesis is in line with existing evidence that hedging in financial markets—another way to cope with uncertainty—incurs a fixed cost since it requires expertise, and that for this reason it is common only above a size threshold.²⁴

A second hypothesis is more behavioral: it is plausible that success breeds confidence. This idea would help explain not only why larger firms perceive less uncertainty, but also why growing firms do not perceive more uncertainty, even though they appear to learn in unfamiliar territory and make biased forecasts. The behavioral literature has focused a lot of attention on extrapolation in forecasts. It is plausible that confidence is similarly driven by past experience, and that it is therefore also high for large firms that have already grown to be successful.

²⁴For example, [Begenau et al. \(2025\)](#) document that the use of derivatives at US banks is highly concentrated with many small banks not participating.

5 Uncertainty and change over time

In this section, we study variation of subjective uncertainty in the time series, or “within firms.” In terms of the organizing framework of Section 3.2, we are interested in how span responds in the short run to signals the firm receives. We begin by documenting how firm-level heteroskedasticity varies with firm characteristics. We then study the dynamics of uncertainty, focusing in particular on the response of uncertainty to past sales growth. Finally, we analyze how firms react to signals, depending on their prior confidence.

5.1 Heteroskedasticity at the firm level

Time variation in subjective uncertainty is larger for firms in more turbulent environments, firms on positive or negative trends, and smaller firms. Table 3 presents the results of cross-sectional regressions of the firm-level time-series standard deviation of span on dummies for turbulence, trend growth, and size, as introduced in the previous Section 4. Columns (1) to (3) show individual effects. Column (4) shows that those effects largely survive their joint inclusion in the regression, and that the cross-sectional structure of subjective heteroskedasticity is similar to that of average subjective uncertainty.

Turbulence thus goes along not only with higher subjective uncertainty on average, but also with *more variable* conditional subjective uncertainty. In other words, managers who typically report wider spans in a more turbulent environment, that is, best and worst case scenarios that are further apart, also *change* the span between those scenarios relatively more often. Unconditionally, the coefficient of variation for span is about one half for all firms. And average spans increases, as turbulence increases in column (1) of Table 2, by 3 pp, 6 pp, and 11 pp, respectively, relative to the lowest turbulence category; whereas heteroskedasticity in column (1) of Table 3 increases by 1 pp, 3 pp, and 6 pp, respectively.

Changes in uncertainty must be driven by information the firm has one quarter in advance. In principle, it could arise because firms have rational expectations and know a regular pattern that relates a signal to the size of future shocks, as in Example R above. It could also be that changes in uncertainty are a subjective phenomenon, as in Example L: if managers learn from signals not only about future growth, but also about the precision of their signals, we can have heteroskedasticity even if experienced shocks are homoskedastic.

We also see more frequent changes of span among smaller firms. This result provides additional context for our cross-sectional results in Section 4. In particular, we have argued that higher average uncertainty at smaller firms is plausibly due to more costly information processing, captured in Example L by a lower parameter ν . Conditional subjective uncertainty in Example L is described by the subjective volatility in equation (6). When ν is lower, conditional subjective uncertainty becomes more sensitive to the signal s_t .

Holding fixed the distribution of the signal and growth, lower ν thus implies higher subjective conditional heteroskedasticity—exactly what we see in the data. Intuitively, small firms with less information processing capacity are less confident about what the signal means. When they observe a signal realization, they update more about the precision of their signal and, thus, adjust their span relatively more. In the time series, we see span changing more.

Table 3: Regressions of firm-level standard deviation of subjective uncertainty on firm characteristic dummies

	(1)	(2)	(3)	(4)
	std. span	std. span	std. span	std. span
Dummy medium low turbulence	1.258*** (0.283)			1.058*** (0.282)
Dummy medium high turbulence	2.707*** (0.294)			2.322*** (0.296)
Dummy high turbulence	6.107*** (0.545)			5.531*** (0.521)
Dummy 'bad' sales growth trend		2.134*** (0.450)		1.300*** (0.395)
Dummy 'good' sales growth trend		1.526*** (0.436)		0.386 (0.351)
Dummy small firms			-1.891* (1.015)	-0.794 (0.855)
Dummy medium sized firms			-3.705*** (0.955)	-2.156*** (0.801)
Dummy large firms			-4.720*** (0.966)	-2.569*** (0.834)
Constant	3.796*** (0.212)	5.373*** (0.163)	9.730*** (0.927)	5.539*** (0.838)
No. of observations	947	947	947	947
No. of firms	947	947	947	947
No. of parameters (excl. intercept)	3	2	3	8
R-squared	0.19	0.033	0.054	0.23

Notes: std. span denotes the time-series standard deviation of firm-level span. Results from OLS regressions. Turbulence (trend) dummies are based on quartiles of the time-series standard deviation (average) of firm-level sales growth rates. See Table 1 for explanation of size dummies. Reference groups are, respectively, the lowest quartile of turbulence, the two middle trend quartiles, and tiny firms. Robust standard errors in parentheses; * p < 0.10, ** p < 0.05, *** p < 0.01.

5.2 Time variation in subjective uncertainty, volatility, and growth

We now study how conditional subjective uncertainty responds to a particular signal, the last observed growth rate. Throughout this section, we consider within-firm variation. Specifically, we form deviations of span and growth from their firm-level means to remove the cross-sectional effects already studied in Section 4. In terms of the organizing framework, we isolate here effects of time variation in a signal s_t on uncertainty and forecast errors. This allows us to compare the effects of past growth on conditional uncertainty and conditional volatility.

Subjective uncertainty is high after both negative and positive growth. Panel (a) of Figure 6 shows again the asymmetric V-shaped relationship between subjective uncertainty reported at date t and the last realized growth rate g_t , here with regression lines added. Table 4 presents the regression results. We start in column (1) with a linear regression and obtain a small but significantly negative slope. Column (2) presents a piecewise linear specification with a breakpoint at zero. Both branches of the V are strongly statistically significant. The fit also improves by a factor of 5 to achieve an R^2 of about 2%. The negative sign of the linear regression coefficient thus reflects the asymmetry of the V.

The effects of growth on subjective uncertainty are also economically significant. Column (2) says that, for an individual firm, after a one percentage point lower negative sales growth rate, next quarter's span is 20 basis points wider. Similarly, a one percentage point higher positive sales growth rate is followed by a 12 basis points wider span. With a within-firm standard deviation of growth of about 8 pp, this translates into a 1.6 pp (1 pp) increase in span for a one-standard-deviation decrease (increase) in the previous-quarter sales growth rate.

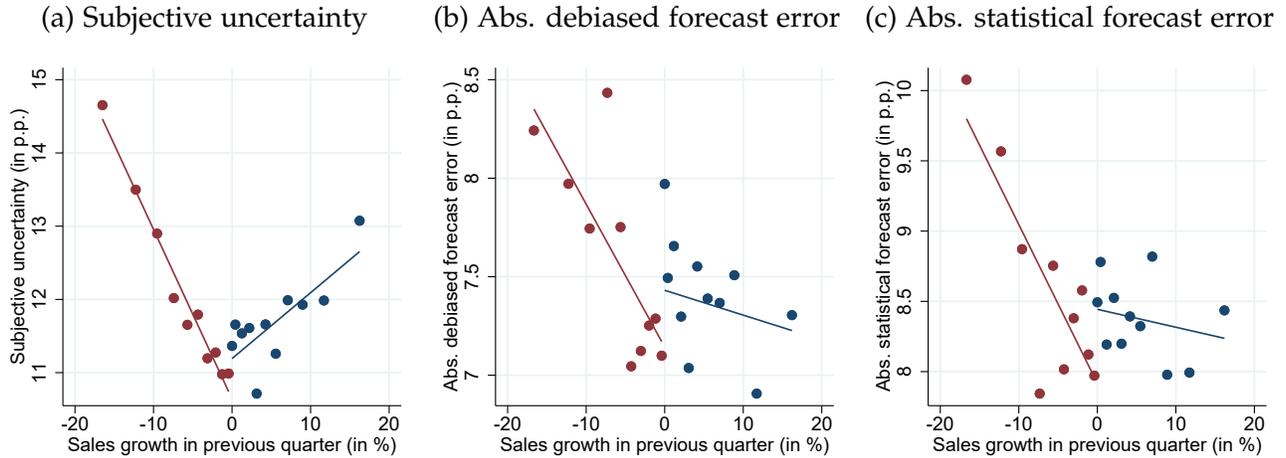
This widening in span after an unusual sales growth rate is driven by changes away from the forecast in the best case and worst case scenarios, as columns (3) and (4) of Table 4 show. For example, after a negative growth rate, both the absolute distance of the worst case from the forecast as well as the absolute distance of the best case from the forecast increase significantly.

In terms of the organizing framework in Section 3.2, the response of uncertainty could be due either to a change in conditional volatility, as in Example R, or to a purely subjective increase in uncertainty, as in Example L. To tell these cases apart, we compare subjective uncertainty to the volatility of forecast errors conditional on growth. We use absolute debiased forecast error as our measure of volatility.

For unbiased forecast errors, a regression of absolute forecast errors on past growth would reveal the effect of growth on volatility. However, forecast errors can be biased, and the bias might itself depend on growth. To eliminate this potential bias, we, thus, demean the firms' forecast error and regress it on the same right-hand side variables we use for the span regression in column (2). We then take the absolute value of the residuals.

Conditional volatility of subjective forecast errors is high after low growth, but not after positive growth. Panel (b) of Figure 6 is a binned scatterplot of the absolute debiased forecast errors against last quarter's growth rate. For negative growth rates, we have a downward sloping line, much like the left branch of a V. For positive growth rates, by contrast, there is no systematic relationship. Column (5) of Table 4 shows that a linear regression has a significantly negative slope. Column (6) shows that this comes from the negative slope at negative growth rates. The slope of -0.06 is about one quarter of the slope for subjective uncertainty.

Figure 6: Uncertainty and past sales growth within firms over time



Notes: The figure shows the relationship between different uncertainty measures and sales growth in the previous quarter in form of binned scatterplots within firms over time. All variables show deviations from their firm-specific means in percentage points. On the horizontal axis, zero is equal to the firm-specific mean growth. On the vertical axis, the unconditional mean of the respective uncertainty measure is added. In panel (a), uncertainty is measured by the subjective uncertainty in percentage points (span). In panel (b), uncertainty is measured by the absolute debiased forecast errors in percentage points. In panel (c), uncertainty is measured by the absolute statistical forecast error in percentage points. See notes to Table 4 for details on variable and sample definitions.

It is interesting to compare heteroskedasticity in errors in subjective forecasts reported by firms to heteroskedasticity measured by an econometrician from growth data alone. The two volatilities could differ, for example, since the former depends on information processing by the firm, whereas the latter depends only on the stochastic properties of the growth process. Since the data suggest that sales growth cannot be predicted using past sales growth within firms, a simple statistical model of firm-level sales growth is its unconditional mean. We, thus, define as the econometrician’s “statistical forecast error” the deviations of firm-level growth rates from this mean and study potential heteroskedasticity of these deviations.²⁵

Conditional volatility of growth is higher after negative growth, but not after positive growth. Panel (c) of Figure 6 shows a binned scatterplot of absolute statistical forecast errors on past growth. The pattern is very similar as for absolute subjective forecast errors in the panel (b): the left branch of a V below zero, and no relationship above. Columns (7) and (8) of Table 4 present linear and piecewise linear regressions, respectively, of the absolute statistical forecast errors on past growth; as the figure suggests, the coefficients are in the same ballpark as for subjective forecast errors. Firms, thus, systematically experience larger shocks in (idiosyncratically) bad times as indicated by low past growth.²⁶

²⁵We have experimented with a number of alternative models of heteroskedasticity using growth data, but it is difficult to improve significantly on this simple model. The basic facts are that (i) growth itself is hard to predict, and (ii) absolute deviations of growth from the firm-specific mean exhibit little persistence.

²⁶The results are robust to a specification, where we require at least 5 span observations for each firm, see Table I4.

Table 4: Within-firm regressions of subjective uncertainty and absolute forecast errors on past sales growth

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Span	Span	DownSpan	UpSpan	Abs. debiased FE	Abs. debiased FE	Abs. stat. FE	Abs. stat. FE
Past sales growth	-0.0472*** (0.0120)				-0.0292** (0.0134)		-0.0384** (0.0155)	
Past sales growth × positive		0.118*** (0.0210)	0.0754*** (0.0143)	0.0422*** (0.0108)		0.00176 (0.0209)		0.0136 (0.0243)
× negative		-0.204*** (0.0200)	-0.116*** (0.0135)	-0.0885*** (0.0119)		-0.0584** (0.0238)		-0.0872*** (0.0270)
Constant	11.91*** (0.0411)	10.92*** (0.101)	6.345*** (0.0693)	4.574*** (0.0542)	7.473*** (0.0389)	7.289*** (0.107)	8.492*** (0.0398)	8.183*** (0.120)
Observations	7999	7999	7999	7999	4855	4855	4746	4746
R ²	0.004	0.020	0.013	0.011	0.002	0.002	0.002	0.004

Notes: The table shows within-firm regressions of different measures of uncertainty and volatility on sales growth rates in the previous quarter. The right-hand side for all regressions is the within-transformed firm-level sales growth rate, in the even columns and column (3) interacted with dummy variables for whether this within-transformed sales growth rate is positive or negative. In columns (1) and (2), the left-hand side variable is span. In column (3), the left-hand side variable is downside span, i.e., the absolute distance from the worst case scenario to the forecast. In column (4), the left-hand side variable is upside span, i.e., the absolute distance from the best case scenario to the forecast. In columns (5) and (6), the left-hand side variable is the absolute debiased subjective forecast error. To be specific, we demean the firms' forecast error and regress it on the same right-hand side variables defined above for column (2). This is to ensure that the results we find are not driven by possible bias in the subjective forecast errors. We then take the absolute value of the residuals of this regression. In columns (7) and (8), the left-hand side variable is the absolute statistical forecast error defined as the absolute difference between realized growth and its firm-specific mean. In all cases, the left-hand side variables are within-transformed (and the overall mean added). In addition, we restrict the sample to p5–p95 of within-transformed past sales growth. Standard errors in parentheses, clustered by firm; * p < 0.10, ** p < 0.05, *** p < 0.01.

We now interpret the results on conditional uncertainty and volatility through the lens of our organizing framework.

The distinction between uncertainty and volatility, already prominent in the cross section in Section 4 reemerges here, when we look at responses to growth. Indeed, the response of subjective uncertainty to growth is more than simply the anticipation of larger shocks. The result is most stark for positive growth rates: firms which have witnessed high growth are more uncertain, but do not see systematically higher absolute forecast errors. This is inconsistent with rational expectations, per Proposition R2. It can be explained, however, by an increase in subjective uncertainty per Proposition L2: firms which see unfamiliar growth rates worry more about the precision of growth as a signal and thus remain more uncertain than firms which have seen more usual growth.

For negative growth, in contrast, lower growth predicts higher uncertainty as well as higher volatility. Since span is an index of subjective uncertainty, we cannot completely rule out that higher uncertainty only reflects higher conditional volatility. We do note, however, that the ratio of span to volatility under this interpretation would have to be much higher than what it was in our between-firm analysis. In the cross section, span moves with volatility about one-for-one for the majority of firms, see Figure 4. In contrast, the ratio of coefficients in Table 4 ranges up to 4.

Our preferred interpretation is therefore that the data reflect a hybrid model that combines elements of both Example R and Example L. Suppose that growth is the sum of two components. The first component has a variance that is a known function of past growth, as in Example R. This function is decreasing in growth for negative values and zero for positive values. This rationalizes the negative branch of a V for the relationship between conditional volatility of forecast errors and growth. The second component is correlated with past growth but the correlation is unknown to managers, as in Example L, so growth also represents a signal of unknown precision that managers learn about, starting from a prior.

The hybrid model, worked out in detail in Appendix G.3, generates an asymmetric V that consists of a symmetric V from learning, plus an extra left branch from changes in conditional volatility. We thus have an additional prediction: if we define *excess span* as the difference between span and the volatility of forecast errors, we should observe a symmetric V-shaped relationship with past growth. Comparing coefficients in Table 4, we see that this is approximately satisfied: taking differences between columns (6) and (2), we have that a one percent increase in the absolute value of growth increases span by 12 bp (14 bp) when growth is positive (negative).

We conclude this section with a discussion of how forecast bias interacts with subjective uncertainty in the time series.

In the cross section, we have seen that excess span, that is, the difference between subjective uncertainty and the volatility of forecast errors measured by an econometrician goes along with forecast bias, plausibly generated by learning. We established this result by comparing average forecast errors to excess span. In the time series, we are interested in whether *conditional* forecast bias is related to excess span. In particular, the previous literature suggests a role for overreaction to signals to generate low uncertainty, as in Example L.

Forecast errors are negatively correlated with forecasts within firms, and more so when excess span is low. In the overall sample, the within-firm correlation between forecasts and forecast errors is -0.27 . On average, managers thus appear to overreact to signals. We further split the sample into managers with above or below median excess span, that is, the difference between span and the absolute mean deviation of the forecast error. We find that the correlation between forecast and forecast error for low excess span firms is 29% larger than for high excess span firms (-0.31 vs -0.24).

We conclude that part of the extra movement in subjective uncertainty beyond movement in conditional volatility is due to overreaction due to overprecision. Indeed, the pattern we find here is consistent with Proposition L3. A positive difference between the prior information content γ and the true information content γ^* gives rise to negative correlation between forecasts and forecast errors, consistent with overreaction to signals. Proposition L3(b) further says that lower γ^* both makes the correlation more negative and generates smaller excess span. Intuitively, managers who are relatively more overconfident in the quality of their information are both more aggressive in their response to signals, thus making more predictable errors, and become less uncertain relative to the benchmark of the conditional volatility of forecast errors.

5.3 Discussion

We draw five main conclusions from this section. First, **even outside of recessions, time variation in subjective uncertainty is large**. This fact is informative for the large literature that studies firm dynamics without aggregate risk, say, to assess effects of policy on growth or the level of output or the size distribution of firms. Such models typically work with a latent source of shocks, for example a persistent TFP process, or shocks that govern success of endogenous innovation efforts.

Our findings imply that such models should have either (a) heteroskedastic innovations, or (b) sufficient nonlinearity in firms' reaction functions so that uncertainty about sales growth can vary over time. In addition, in models with aggregate risk, the steady state distribution of firms' state variables is the key aggregate state variable, which governs the reaction of the economy to aggregate shocks. Our results say that this distribution is shaped by heteroskedastic innovations. A specific example is [Baley and Blanco \(2019\)](#), who stress that idiosyncratic uncertainty cycles may amplify the real effects of aggregate nominal shocks.

Second, **uncertainty moves with past experience**. The fact that growth predicts uncertainty further disciplines the types of shocks and mechanisms that models of firm dynamics should incorporate. We can rule out, for example, a model with purely transitory uncertainty shocks that are uncorrelated with other shocks, say with TFP. In such a model, uncertainty affects intertemporal decisions such as investment or hiring, and thereby future output. Many mechanisms imply that uncertainty lowers future growth. However, the model does not allow for feedback in the other direction: past growth should have nothing to do with future uncertainty.²⁷

In contrast, the effect of experience on beliefs is at the heart of a class of models where agents learn from past data. Several papers have considered feedback effects from growth to uncertainty ([Bachmann and Moscarini, 2012](#); [Fajgelbaum, Schaal and Taschereau-Dumouchel, 2017](#); [Saijo, 2017](#); [Ilut, Kehrig and Schneider, 2018](#); [Baley and Blanco, 2019](#); [Berger and Vavra, 2019](#); [Ilut and Saijo, 2021](#); [Ludvigson, May and Ng, 2020](#)). Our results say that understanding firm dynamics requires either such feedback effects or otherwise uncertainty shocks that are correlated with past growth.²⁸

The third conclusion is that **volatility is high in (idiosyncratically) bad times and firms understand this**. In other words, in bad times, subjective uncertainty mirrors the heteroskedasticity in volatility, as Example R would suggest. In temporarily bad times, indicated by growth below the firm's average, firms experience larger shocks. At the same time, their subjective uncertainty is higher. This finding connects to the large literature on how volatility affects firms in recessions. It is well understood how higher volatility interacts with technology as well as financial or labor market frictions to generate less investment or hiring. Our finding suggests that these mechanisms are not only relevant for thinking about the response to exogenous

²⁷Since we need to generate a link between uncertainty and *past* growth, it is not sufficient to introduce correlated 1st and 2nd moments shocks: such shocks only link expected future growth and uncertainty.

²⁸A persistent uncertainty shock might generate the facts in this section even if it is uncorrelated with TFP. An uncertainty shock that arrives at date $t - 1$, say, reduces investment then, and lowers output growth between $t - 1$ and t . At the same time, uncertainty remains high at t because of persistence.

volatility shocks in recessions, but matter for the propagation of any shock that signals high idiosyncratic volatility in the future.

The fourth conclusion is that, as in the cross section, **time-series variation in subjective uncertainty is more than just variation in conditional volatility**. This finding connects well with the learning mechanisms explored by the literature on feedback from growth to experience. For example, [Ilut and Saijo \(2021\)](#) present a model where learning from experience shapes Knightian uncertainty about firm-specific variables. Subjective uncertainty therefore decouples from conditional volatility, yet responds to past signals of the firm, consistent with the evidence here.

Finally, we find evidence that **firms overreact to signals**. This finding is consistent with the existing literature. In particular, [Barrero \(2022\)](#) emphasizes the effect of overreaction due to overprecision on both forecasts and uncertainty.²⁹ Example L clarifies how this effect arises in a model where managers learn about information quality. Our test based on comparing excess span and the correlation between forecast errors and forecast in the cross section of firms establishes the link between forecast bias and uncertainty decoupled from volatility in our setting.

6 Subjective beliefs and firm plans

In this section, we relate firms' subjective beliefs to their planned actions. The ifo survey elicits firms' plans for two key intertemporal adjustment margins: employment and prices. Every month, firms, in the main ifo Business Survey, are asked whether employment or prices are likely to be increased, decreased or kept the same over the subsequent three months. This is the same time horizon as our subjective uncertainty measure span, which we view as a strength of the ifo Business Survey. We are interested in how subjective beliefs—forecast and uncertainty—alter firms' planned actions. While the ifo survey does not elicit outcome variables in a way that is suitable for our purposes, [Lehmann \(2023\)](#), in a literature survey, reports that *“the ifo indicators [...] accurately forecast labor market variables (for example, the ifo Employment Barometer to forecast employment growth) or inflation (for example, the ifo Price Expectations as leading indicator for producer price development)”*. We include firm and time fixed effects in all regressions, and, therefore, focus on idiosyncratic within-firm variation.

6.1 Employment, price setting, and uncertainty

When uncertainty is higher, firms are more likely to reduce employment, holding fixed expected growth. Columns (1) to (3) of Table 5 present estimates from linear probability models for the event that the firm adjusts employment downward. Columns (1) and (2) show the effects of the expected sales growth and span separately, and column (3) includes both regressors. We find that in both cases, the coefficients are about the same order of magnitude

²⁹[Born et al. \(2024\)](#) also find that firms overreact to micro news in survey data on German firms.

Table 5: Subjective uncertainty and firms' employment and price plans

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Emp down	Emp down	Emp down	P down	P down	P down	P up	P up	P up
Forecast	-0.503*** (0.0526)		-0.530*** (0.0524)	-0.208*** (0.0399)		-0.217*** (0.0408)	0.224*** (0.0519)		0.233*** (0.0514)
Span		0.334*** (0.0566)	0.391*** (0.0575)		0.122*** (0.0452)	0.144*** (0.0453)		-0.101 (0.0641)	-0.126** (0.0632)
Constant	12.61*** (0.179)	7.396*** (0.693)	8.008*** (0.690)	7.030*** (0.130)	5.069*** (0.542)	5.334*** (0.520)	15.84*** (0.175)	17.59*** (0.775)	17.31*** (0.781)
Observations	7943	7943	7943	7946	7946	7946	7946	7946	7946

Notes: The table shows the coefficients of linear probability regressions of an indicator for a planned decrease of employment in the next three months (Columns (1) to (3)), an indicator for a planned decrease of prices in the next three months (Columns (4) to (6)), and an indicator for a planned increase of prices in the next three months (Columns (7) to (9)) on firms' sales growth forecast and subjective uncertainty (span). We control for firm & time fixed effects. We restrict the sample to p5–p95 of within-transformed past sales growth. Standard errors are clustered at the firm level.

and of the opposite sign. Firms plan to reduce employment when they are either pessimistic or uncertain about growth.³⁰

The effects are also economically significant. The within-firm standard deviations of the forecast and span are 8 pp and 6.2 pp, respectively. A one-standard-deviation decrease in expected sales growth and a one-standard-deviation increase in span thus have similar effects, changing the probability of a decline in employment by 4.2 pp and 2.4 pp, respectively. We note that the average probability of decreasing employment is only around 11% to begin with—most firms plan not to reduce employment outside recessions. We conclude that expectations and uncertainty are roughly equally important movers of firms' labor demand decisions and that higher uncertainty has, therefore, similar effects as pessimism.

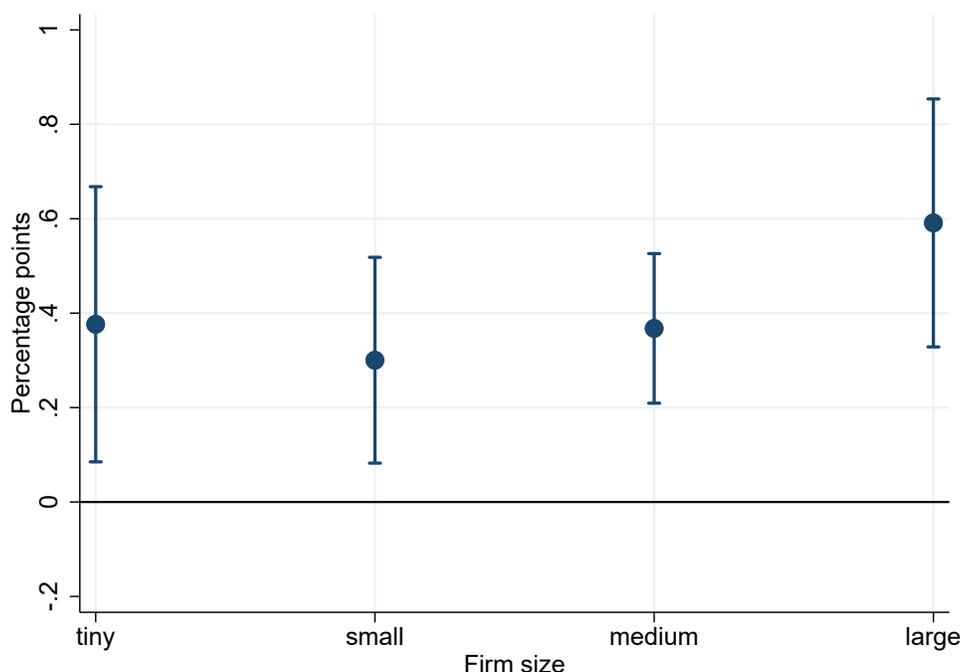
Uncertainty reduces planned employment for all size classes. It is, further, interesting to investigate whether the “uncertainty as pessimism”-effect hits different firms differentially. The large sample size of the Ifo Business Survey allows us to estimate the regressions separately for four size groups. Figure 7 shows the effects of span on employment decrease plans in the specification of column (3) in Table 5. All effects are about the same size, if anything uncertainty has a stronger effect on employment for large firms.

When uncertainty is higher, firms are more likely to plan lower prices, holding fixed expected growth. Table 5 documents this fact by estimating linear probability models for price decreases in columns (4) to (6) and price increases in columns (7) to (9). Again, we begin by including expected sales growth and span separately and then combine them. All coefficients are strongly significant: higher span or lower expected sales growth make it more (less) likely that firms plan to lower (raise) prices.³¹

³⁰For reasons of parsimony, we relegate the results for higher employment to Appendix Table I5. They paint a similar picture: higher sales growth forecasts predict more employment and higher span less.

³¹The results are robust to a specification, where we require at least 5 span observations for each firm, see Table I6.

Figure 7: Subjective uncertainty and probability to reduce employment across size classes



Notes: The figure shows the coefficients of linear probability regressions of an indicator for a planned decrease of employment in the next three months on firms' subjective uncertainty (span). We control for expected sales growth and firm & time fixed effects. Following the definition of the German Statistical Office, we define firms as "tiny" if they have less than 10 employees, "small" if the number of employees is between 10 and 50, "medium" if the number of employees is between 50 and 250, and "large" if the number of employees exceeds 250. We restrict the sample to p5–p95 of within-transformed past sales growth. Standard errors are clustered at the firm level. 95% confidence intervals are shown.

Economically, the changes in probabilities of price changes are smaller than for employment. A one-standard-deviation increase in uncertainty lowers the probability to raise prices by 0.8 pp (average probability of price increases is 16%) and increases the probability of a price decline by 0.9 pp (average probability of price decreases is 6.5%). However, as for employment, the order of magnitude of the coefficients for the forecast and span is the same. Again, the results for price changes suggest that uncertainty acts like pessimism and not as a freezer of firm actions.

6.2 Interpretation

The main takeaway from this section is that for two major intertemporal adjustment margins, **variation in idiosyncratic uncertainty is almost as important for changing firm actions as changes in expectations**. In particular, we find that uncertainty is a force that lowers both employment and prices. There are large literatures that aim to account for labor market churn on the one hand, and price dispersion across firms on the other. Our result suggests that it is fruitful to take into account time variation of idiosyncratic uncertainty at the firm level as a driver of firm actions.

Higher subjective uncertainty works like a negative demand shifter. The fact that employment, a quantity variable, moves in the same direction as prices in response to a change in uncertainty is informative about the type of primitive shocks that managers worry about when they form beliefs about sales growth. For example, in the presence of financial frictions or with risk/ambiguity-averse managers/entrepreneurs, decision-makers place more weight on low payoff states. Specifically, a firm that worries about a shortfall in demand will lower the scale of production by shedding workers and, at the same, time lower its prices. A firm that worries instead about cost, for example the price of a volatile intermediate input in the future, would still reduce employment but instead raise its price. This finding calls for mechanisms that bring out idiosyncratic and subjective demand uncertainty as particularly relevant for labor market churn and price dispersion. In addition, when we take the results of Sections 5 and 6 together, there are complex feedback effects between experienced change, uncertainty, and firm actions, which heterogeneous-firm models should build in.

We are not the first to document partial aspects of the relationship between uncertainty and firms' actions. But we are the first to study jointly the relationships of idiosyncratic and subjective uncertainty with a quantity variable, employment, and prices. [Kumar et al. \(2023\)](#) also document that uncertainty acts like pessimism on employment and prices, but for subjective uncertainty about a *macro* variable, namely, GDP growth. [Bachmann et al. \(2019\)](#) and [Gödl-Hanisch and Menkhoff \(2023\)](#) study the relationship between uncertainty and prices only, and only the latter use a subjective uncertainty measure. [Leahy and Whited \(1996\)](#); [Guiso and Parigi \(1999\)](#); [Bloom, Bond and Van Reenen \(2007\)](#); [Bontempi, Golinelli and Parigi \(2010\)](#); [Alfaro, Bloom and Lin \(2023\)](#); [Fiori and Scoccianti \(2023\)](#); [Diblasi, Mikosch and Sarferaz \(2025\)](#), by contrast, study the relationship between uncertainty and quantities only. None of these papers isolates the effects of idiosyncratic uncertainty in normal times. [Alfaro et al. \(2023\)](#) highlight the role of financial frictions in amplifying the effects of changes in uncertainty, which is one of the key potential mechanisms to rationalize the "uncertainty as pessimism"-effect.

7 Conclusion

To conclude, we review our main findings and discuss their broader implications. We have presented evidence on firms' beliefs that makes three core points: (i) subjective uncertainty reflects change, (ii) subjective uncertainty is more than just conditional volatility, and (iii) variation in (idiosyncratic) uncertainty is quantitatively important for the decisions of (mature) firms even outside recessions. The first point consists of a set of target moments that can serve as a guide for future models of belief formation. We have emphasized that experienced change matters both in the cross section and the time series: it makes managers more uncertain when they are on a persistent path into unfamiliar territory and when they observe an unusually bad quarter in the short run.

Our second point speaks to the broader question of how to quantitatively model subjective uncertainty. The standard approach in many areas of economics is to estimate a process of shocks faced by firms, and then use point estimates to describe agents' beliefs. In our context, this would mean estimating conditional volatility by regressing absolute forecast errors on con-

ditioning information, and using the fitted value as a measure of subjective uncertainty. Two key assumptions underlie this procedure: firms have rational expectations about the shocks *and* perceive no uncertainty beyond the shocks, unlike the econometrician who encounters error when estimating the regression. Our survey results show that this approach measures “too little” uncertainty.

The problem with the standard approach is that managers perceive uncertainty also about features of the environment that an econometrician can observe with hindsight. For example, we can see in our data that a firm was shrinking along a negative trend. The managers whose businesses declined over our sample did not know this for sure at the beginning, but they were worried already then and continued to perceive uncertainty as they were learning. It would be inappropriate to model their actions—or compute their welfare—while ignoring this uncertainty and focusing only on fluctuations around the trend they encounter. Models of learning, such as our simple Example L, make explicit the distinction between volatility measured by an econometrician and uncertainty about the environment. Our results imply that such models should be developed further and applied more widely, in particular also to mature firms.

Since decoupling of uncertainty and volatility matters for welfare, it also matters for policy evaluation. Many models of heterogeneous firms assess misallocation of factors such as labor and capital due to market imperfections such as credit constraints or market power, and propose corrective policies such as subsidies to firms. A recent strand of this literature emphasizes the role of uninsurable firm-level idiosyncratic risk. For example, [Boar et al. \(2025\)](#) measure aggregate welfare effects of undiversified risk, and [Di Tella et al. \(2025\)](#) characterize optimal policy. Our results underscore the importance of this perspective and, in particular, the relevance of uncertainty different from volatility. Our learning example and empirical results further suggest that such risk is particularly relevant when firms experience change, which is relevant for both mature and small firms.³²

Our final set of results, on idiosyncratic uncertainty and its explanatory power for firm plans, imply that both time variation in uncertainty and decision-makers’ aversion to it deserve greater roles in models of firm dynamics. While mechanisms that link uncertainty to firm actions are prominent in the business cycle literature, they typically do not appear in studies of the size distribution of firms or growth with heterogeneous firms. Here the typical approach is to model a balanced growth or transition path without aggregate shocks, and with the firm distribution driven by homoskedastic idiosyncratic shocks. Our results suggest that independent variation in uncertainty, especially about demand for the firm’s products, is another driver of individual firm actions and, hence, the firm distribution, which is also the key aggregate state variable governing the reaction of the economy to aggregate shocks.

In order for idiosyncratic uncertainty to matter, firm decisions must depend on it. When uncertainty is risk, there are no manager-shareholder conflicts and shareholders have expected

³²Moving beyond the idiosyncratic uncertainty featured in our data, the message that uncertainty decoupled from volatility is important to firms extends to uncertainty about policy itself. Consider for example uncertainty about tariffs. Uncertainty can clearly be large even if actual paths of tariffs are not volatile. The basic principle of decoupling which we have emphasized says that welfare calculations need to take such uncertainty into account.

utility preferences, idiosyncratic uncertainty is diversified away and its time variation is irrelevant for firm actions. A model that is consistent with our results should thus adopt one of the plausible departures from this benchmark that have been proposed and tested in the literature. For example, managers may have career concerns or earn compensation through incentive contracts that reward cautious behavior. Firms may also be closely held by owner-managers. Another alternative is that uncertainty is not only risk, but in part ambiguity (or Knightian uncertainty), so standard laws of large numbers do not apply and diversification is not possible (see [Ilut and Saijo, 2021](#); [Ilut and Schneider, 2022](#); [Menkhoff, 2025](#)).

Once we have a model where idiosyncratic uncertainty matters, we know from business cycle applications that uncertainty can interact in interesting and quantitatively relevant ways with standard elements of firm dynamics models. Whenever the choice of inputs is intertemporal, it depends on firms' beliefs about the future. This is a common situation: it takes time to install capital, search for workers, or source intermediates from abroad. Relative to standard benchmarks of static choice or perfect foresight, uncertainty works like a distortion, a wedge between the marginal product and the price of a factor. In a business cycle model, higher aggregate uncertainty increases the wedge and generates recessions. Separately and additionally, higher idiosyncratic uncertainty induces misallocation of factors in the cross section of firms.

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Appendix A Representativeness of the sample

In this appendix, we investigate whether participation in our quantitative uncertainty module is driven by selection, conditional on participation in the main manufacturing survey. We base our analysis on all 69,163 complete firm-quarter responses available in the main survey for the months the uncertainty module was executed. We then ask whether firm size, time dummies, industry dummies, and interacted time-industry dummies are able to predict participation in the uncertainty module. To this end, we run a probit regression of a participation dummy that is 1 for the 14,150 observations of the quantitative uncertainty module and zero otherwise, on these predictors. Since Ifo placed the quantitative uncertainty module more prominently after 2017:Q1, the number of respondents increased considerably, which prompted us to split the sample around that date for the purpose of this analysis. We report the estimated coefficients in column (1) of Tables A1 and A2. We find that there is no significant selection with respect to quarter/survey wave and industry suggesting that the uncertainty sample does not misrepresent specific quarters or industries. While firm size turns out to be significantly negative indicating that large firms are slightly underrepresented in the uncertainty module compared to the main manufacturing survey, the pseudo R-squared of 0.016 shows that this selection is quantitatively irrelevant. This is also reflected by an ROC curve which differs only slightly from the diagonal that indicates no discriminatory power, see the left panels of Figures A1 and A2.³³

We repeat the analysis starting from the subset of 43,211 complete firm-quarter responses available from the online part of the main survey. We thus account for the fact that some firms reply to the main survey by fax and thus, essentially mechanically, do not participate in the uncertainty module, which is solely implemented online. The results of an analogous probit regression are reported in column (2) of Tables A1 and A2. Again, the very low pseudo R-squared suggests that selectivity is not a relevant issue for the uncertainty module. This conclusion is supported by a largely unaltered ROC curve near the non-discriminatory diagonal, see the right panels of Figures A1 and A2.

³³The receiver operating characteristic (ROC) visualizes the discriminatory power of a binary classifier as follows: By varying the classification threshold—here: the probability above which an observation is predicted to participate in the uncertainty module—the classifier can produce any true positive rate (type II error). The ROC curve plots the true positive rate so obtained against its corresponding false positive rate (type I error). In the case of no discriminatory power, true and false positive rates are always the same, the ROC curve equals the diagonal, and the area under the ROC curve (AUC) is 0.5. A good classifier has a ROC curve well above the diagonal and an AUC that is near the maximum of 1.0.

Table A1: Probit regression of a dummy indicating special survey participation on ifo variables until 2017 Q1

	(1) probit vs. ifo all	(2) probit vs. ifo online
Log of number of employees	-0.0754*** (0.0127)	-0.132*** (0.0138)
Dummy survey wave 1	-0.0765 (0.276)	0.0215 (0.295)
Dummy survey wave 2	0.0323 (0.275)	0.185 (0.294)
Dummy survey wave 3	0.215 (0.232)	0.403 (0.245)
Dummy survey wave 4	0.243 (0.236)	0.390 (0.251)
Dummy survey wave 5	0.0933 (0.243)	0.204 (0.255)
Dummy survey wave 6	0.284 (0.214)	0.441** (0.219)
Dummy survey wave 7	0.191 (0.222)	0.338 (0.231)
Dummy survey wave 8	-0.277 (0.269)	-0.198 (0.283)
Dummy survey wave 9	0.0777 (0.213)	0.178 (0.220)
Dummy survey wave 10	0.192 (0.237)	0.312 (0.251)
Dummy survey wave 11	-0.0657 (0.216)	0.0238 (0.226)
Dummy survey wave 12	0.122 (0.205)	0.226 (0.211)
Dummy survey wave 13	0.289* (0.166)	0.285 (0.175)
Dummy survey wave 14	-0.0324 (0.209)	-0.0494 (0.219)
Dummy survey wave 15	0.0954 (0.150)	0.0615 (0.163)
Dummy supersector 1	0.0375 (0.258)	0.394 (0.279)
Dummy supersector 2	-0.432 (0.292)	-0.334 (0.310)
Dummy supersector 3	-0.146 (0.243)	0.0878 (0.260)
Dummy supersector 4	-0.0135 (0.266)	0.0562 (0.284)
Dummy supersector 5	0.0316 (0.253)	0.210 (0.268)
Dummy supersector 6	-0.265 (0.260)	-0.188 (0.275)
Dummy supersector 7	-0.239 (0.270)	-0.00426 (0.290)
Dummy supersector 8	0.00565 (0.274)	0.228 (0.294)
Dummy supersector 9	-0.387 (0.247)	-0.233 (0.263)
Dummy supersector 10	0.0800 (0.269)	0.230 (0.288)
Dummy supersector 11	-0.337 (0.261)	-0.211 (0.277)
Dummy supersector 12	-0.203 (0.238)	-0.110 (0.251)
Dummy supersector 13	0.0698 (0.276)	0.270 (0.295)
Constant	-0.572** (0.228)	-0.216 (0.242)
Additional wave-supersector dummies	YES	YES
No. of observations	38958	26520
No. of firms	3481	2440
No. of parameters (excl. intercept)	224	224
Pseudo R-squared	0.016	0.031

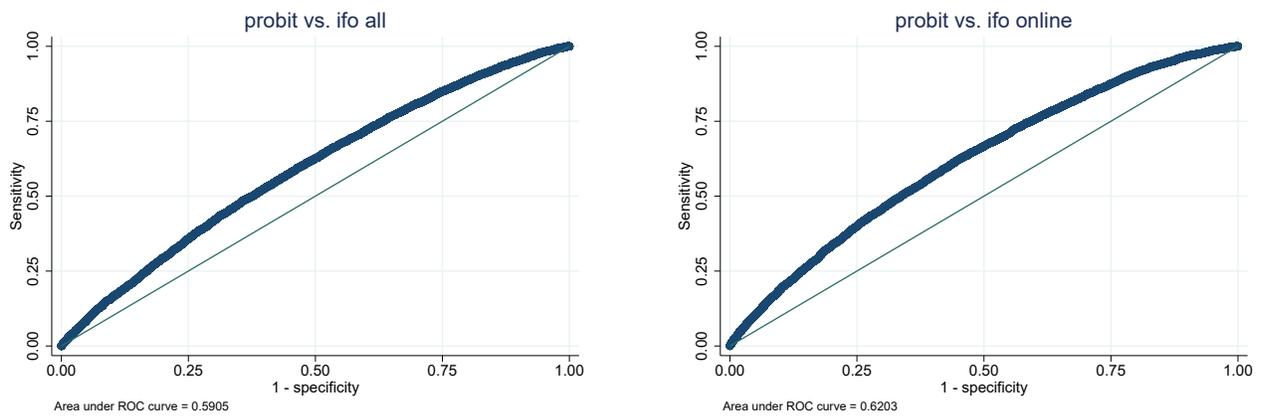
Standard errors in parentheses, clustered by firm; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table A2: Probit regression of a dummy indicating special survey participation on ifo variables since 2017 Q2

	(1) probit vs. ifo all	(2) probit vs. ifo online
Log of number of employees	0.0319*** (0.0118)	-0.0491*** (0.0136)
Dummy survey wave 1	0.166 (0.189)	-0.174 (0.214)
Dummy survey wave 2	0.368* (0.208)	0.00766 (0.233)
Dummy survey wave 3	0.440** (0.175)	0.106 (0.213)
Dummy survey wave 4	0.647*** (0.234)	0.305 (0.266)
Dummy survey wave 5	0.198 (0.191)	0.307 (0.248)
Dummy survey wave 6	0.179 (0.207)	0.223 (0.273)
Dummy survey wave 7	0.130 (0.150)	0.235 (0.217)
Dummy survey wave 8	0.340** (0.173)	0.510** (0.244)
Dummy survey wave 9	0.214 (0.186)	0.186 (0.243)
Dummy survey wave 10	0.315* (0.171)	0.183 (0.226)
Dummy supersector 1	-0.240 (0.201)	0.224 (0.253)
Dummy supersector 2	-0.116 (0.222)	0.0410 (0.273)
Dummy supersector 3	-0.217 (0.194)	-0.114 (0.234)
Dummy supersector 4	-0.358 (0.225)	-0.332 (0.273)
Dummy supersector 5	-0.0230 (0.202)	0.0236 (0.241)
Dummy supersector 6	0.138 (0.196)	0.236 (0.238)
Dummy supersector 7	0.0315 (0.202)	0.243 (0.248)
Dummy supersector 8	-0.150 (0.208)	0.100 (0.255)
Dummy supersector 9	0.0213 (0.185)	0.388* (0.226)
Dummy supersector 10	0.125 (0.199)	0.368 (0.245)
Dummy supersector 11	0.0909 (0.198)	0.278 (0.240)
Dummy supersector 12	-0.178 (0.184)	0.0462 (0.222)
Dummy supersector 13	-0.179 (0.223)	-0.0345 (0.268)
Constant	-0.664*** (0.184)	0.174 (0.219)
Additional wave-supersector dummies	YES	YES
No. of observations	30205	16691
No. of firms	4271	2442
No. of parameters (excl. intercept)	154	154
Pseudo R-squared	0.016	0.016

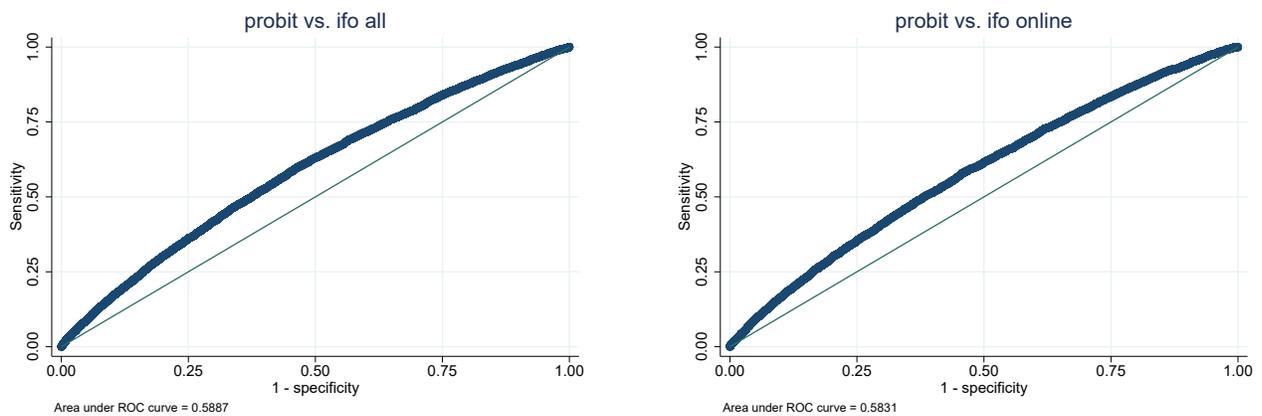
Standard errors in parentheses, clustered by firm; * p < 0.10, ** p < 0.05, *** p < 0.01.

Figure A1: ROC curves for probit estimations until 2017Q1



Notes: The two plots depict ROC curves that correspond to the probit estimations in columns (1) and (2) of Table A1, respectively.

Figure A2: ROC curves for probit estimations since 2017Q1



Notes: The two plots depict ROC curves that correspond to the probit estimations in columns (1) and (2) of Table A2, respectively.

Appendix B Seasonality

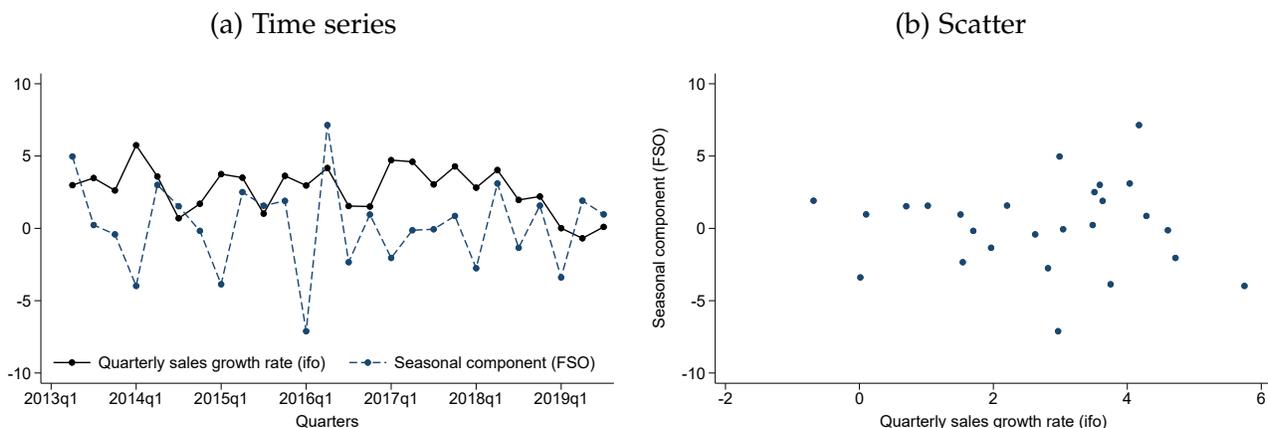
The main Ifo Business Survey requests that firms ignore seasonal fluctuations in their answers. For every survey wave it is emphasized that firms should exclude seasonal fluctuations in their answers. Indeed, the survey instructions point to seasonal patterns due to changes in demand, holidays, and vacations. We also note that firms in our sample have participated for on average of 8.8 waves (more than two years) and the median firm participated 8 times in our quantitative online module. Most of these firms have participated in the general Ifo survey for much longer. It is therefore reasonable to assume that the firms are well aware of this instruction. In fact, we show next that this assumption is borne out in the data.

We begin by comparing the aggregated quarterly sales growth rate from the Ifo survey with the seasonal component of aggregate quarterly sales growth for the German manufacturing sector from the national accounting data (FSO: Federal Statistical Office). Figure B1 in the left panel shows the time series of these two variables and the right panel shows them as a scatterplot. It is clearly visible that they are uncorrelated ($\rho = -0.02$), meaning that the Ifo sales growth rate—at least in the aggregate—has no seasonal pattern.

In a second step, we look at the micro level and we regress firm level quarterly sales growth on seasonal dummies (and firm fixed effects as well as, in some specifications, time fixed effects). Table B1 shows the results: all seasonal dummies are statistically insignificant. Columns (1) and (2) do this for all firms in our sample, while columns (3) and (4) restrict the sample to firms that are observed at least 8 times.

To sum up, both the macro and the micro level analysis corroborate that firms follow the survey's instruction to deseasonalize their answers.

Figure B1: Sales growth—survey vs seasonal component from national accounting data



Notes: The figure shows the aggregated quarterly sales growth rate from the ifo survey (from our final sample) and the seasonal component of aggregate quarterly sales growth for the German manufacturing sector from the national accounting data as published by the Federal Statistical Office (FSO). Source: FSO NRS.

Table B1: Regression of survey sales growth on quarterly dummies

	(1)	(2)	(3)	(4)
Quarter 2	0.732 (0.612)	0.561 (0.619)	1.118 (0.775)	0.933 (0.783)
Quarter 3	0.942 (0.577)	0.697 (0.590)	0.764 (0.738)	0.532 (0.754)
Quarter 4	-0.313 (0.497)	-0.578 (0.507)	-0.185 (0.619)	-0.396 (0.631)
Constant	1.820*** (0.375)	2.000*** (0.384)	1.724*** (0.468)	1.889*** (0.479)
Observations	8882	8882	5660	5660
Firm FE	Yes	Yes	Yes	Yes
Year FE	No	Yes	No	Yes

Notes: The table shows the results of regressing firm level quarterly sales growth on seasonal dummies. All specifications include firm fixed effects. Columns (2) and (4) additionally include time fixed effects. Columns (1) and (2) use all firms in our sample, while columns (3) and (4) restrict the sample to firms that are observed at least 8 times. Standard errors are clustered at the firm level.

Appendix C Questionnaire for the one-time special survey from fall 2018

Meta-Umfrage zur Zusatzumfrage "Unsicherheit"

Kenn-Nr.: kkk-2365-2342
Bereich (XY): 123456 Textilien, Autos und Lebensmittel



Ihre Angaben werden **streng vertraulich** behandelt.
Der gesetzliche **Datenschutz** ist voll gewährleistet.
[Fragebogen als PDF zum Drucken](#)

Zur Erinnerung:
Fragebogen Zusatzumfrage
Unsicherheit

**Erwartungen
Umsatzveränderungen**

Zusatzfrage 2

Zusatzfrage 3

Allgemeine Fragen

Umfrage abschließen

1. In den Zusatzfragen 2 und 3 wurden Sie nach Ihren Erwartungen hinsichtlich der Umsatzveränderung Ihres Bereichs im jeweils begonnenen Quartal gefragt.

a. Wie bedeutend waren die folgenden externen Faktoren typischerweise für Ihre Antwort?

	sehr bedeutend	bedeutend	weniger bedeutend	unbedeutend
Entwicklung der Wettbewerber	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Branchenentwicklung	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Konjunkturelle Entwicklung	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Außenwirtschaftliches Umfeld	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Wirtschaftspolitisches Umfeld	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Sonstiges, und zwar:

b. Wie bedeutend waren die folgenden internen Faktoren typischerweise für Ihre Antwort?

	sehr bedeutend	bedeutend	weniger bedeutend	unbedeutend
Auftragsbestand am Quartalsbeginn	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Für das laufende Quartal erwartete Fertigstellungen/Auslieferungen von Projekten, die vor Quartalsbeginn gestartet wurden	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Erwartete Neuaufträge/-bestellungen im laufenden Quartal	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Sonstiges, und zwar:

c. Wie bedeutend waren die Informationen und Einschätzungen aus den folgenden Funktionsbereichen typischerweise für Ihre Antwort?

	sehr bedeutend	bedeutend	weniger bedeutend	unbedeutend
Vertrieb	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Produktion	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Finanzen / Controlling	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Marketing / Marktforschung	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Sonstiges, und zwar:

Figure C1: Original survey questionnaire in German

Notes: Original questionnaire from ifo's one-time online special survey on its "uncertainty module" in German, from fall 2018.

 Zur Erinnerung: Fragebogen Zusatzumfrage Unsicherheit	Erwartungen Umsatzveränderungen	Zusatzfrage 2	Zusatzfrage 3	Allgemeine Fragen	Umfrage abschließen
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2. Um wieviel Prozent wird sich der Umsatz in Ihrem Bereich im vierten Quartal 2018 verändern?

weiß nicht

a) Im bestmöglichen Fall: % (bitte ganze, positive oder negative Zahlen eingeben)

Im schlechtestmöglichen Fall: % (bitte ganze, positive oder negative Zahlen eingeben)

b) Unter Berücksichtigung aller Chancen und Risiken erwarte ich im **vierten Quartal 2018** alles in allem eine Veränderung um: % (bitte ganze, positive oder negative Zahlen eingeben)

2. In den Zusatzfragen 2.a) und b) wurden Sie gefragt, welche Umsatzveränderung Sie im jeweils begonnenen Quartal im bestmöglichen und schlechtestmöglichen Fall bzw. alles in allem für Ihren Bereich erwarten. Haben Sie bei der Beantwortung der Frage typischerweise Ergebnisse aus einer quantitativen Umsatzplanung verwendet, die ohnehin regelmäßig in Ihrem Bereich stattfindet?

ja
 nein

wenn ja, wie bedeutend waren typischerweise Ergebnisse aus...

	sehr bedeutend	bedeutend	weniger bedeutend	unbedeutend
einer Szenarioanalyse um eine Basisprognose herum	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
einer statistischen Analyse	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Sonstiges, und zwar:

3. In Zusatzfrage 2 a) wurden Sie nach der Umsatzveränderung im bestmöglichen und schlechtestmöglichen Fall gefragt. Welche der folgenden Aussagen beschreiben am ehesten Ihre Antwort?

Diese bestmöglichen und schlechtestmöglichen Fälle sind typischerweise ...

Plausible Szenarien, mit deren Eintreten wir durchaus rechnen müssen.
 Mögliche Szenarien, deren Eintreten wir aber nur in Ausnahmefällen erwarten.

Sonstiges, und zwar:

4. Wenn Sie für Zusatzfrage 2 a) die Umsatzveränderung im bestmöglichen und schlechtestmöglichen Fall ermitteln, wie bedeutend sind dabei typischerweise die nachfolgenden Gesichtspunkte für Sie?

	sehr bedeutend	bedeutend	weniger bedeutend	unbedeutend
Umsatzveränderungen in den letzten ein bis zwei Jahren	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Umsatzveränderungen, die weiter als zwei Jahre zurückliegen	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Überlegungen, die wir aktuell anstellen, unabhängig von der Vergangenheit	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Unsere Risikoeinstellung ("Vorsichtsprinzip")	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Umsatzveränderungen, die wir bei unseren Wettbewerbern beobachten	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Sonstiges, und zwar:

Figure C2: Original survey questionnaire in German

Notes: Original questionnaire from ifo's one-time online special survey on its "uncertainty module" in German, from fall 2018.

Zur Erinnerung: Fragebogen Zusatzumfrage Unsicherheit	Erwartungen Umsatzveränderungen	Zusatzfrage 2	Zusatzfrage 3	Allgemeine Fragen	Umfrage abschließen
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3. Bei den nächsten drei Teilfragen können Sie entweder eine Wahrscheinlichkeit oder ein Wahrscheinlichkeitsintervall angeben.

a) Wie hoch schätzen Sie die Wahrscheinlichkeit ein, dass der Umsatz in Ihrem Bereich im vierten Quartal 2018 steigt?

Wahrscheinlichkeit liegt bei % (bitte ganze Zahlen eingeben)
 Wahrscheinlichkeit liegt zwischen % und % (bitte ganze Zahlen eingeben)
 weiß nicht

5. In der Zusatzfrage 3a wurden Sie gebeten, entweder eine Wahrscheinlichkeit oder ein Wahrscheinlichkeitsintervall dafür anzugeben, dass sich der Umsatz in Ihrem Bereich im jeweils begonnenen Quartal erhöht. Bitte bewerten Sie die Bedeutung der folgenden Gesichtspunkte bei Ihrer Entscheidung, eine Wahrscheinlichkeit oder ein Wahrscheinlichkeitsintervall anzugeben:

Wir entscheiden uns typischerweise, ein Wahrscheinlichkeitsintervall anzugeben, wenn ...

	trifft zu	trifft eher zu	trifft eher nicht zu	trifft nicht zu
sich unser Geschäftsumfeld in den Jahren zuvor stark verändert hat.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
wir für das jeweils begonnene Quartal eine ungewöhnliche Umsatzentwicklung erwarten,	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
uns für das jeweils begonnene Quartal noch eine wichtige Information fehlt.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
wir bei der Planung für das jeweils begonnene Quartal besonders vorsichtig sind.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Sonstiges, und zwar:

Zur Erinnerung: Fragebogen Zusatzumfrage Unsicherheit	Erwartungen Umsatzveränderungen	Zusatzfrage 2	Zusatzfrage 3	Allgemeine Fragen	Umfrage abschließen
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6. Zum Schluss noch drei allgemeine Fragen zu Ihrem Bereich:

a. Wie viele Wettbewerber hat Ihr Bereich (bitte geben Sie eine Zahl ein)?

b. Wie viele dieser Wettbewerber beobachten Sie regelmäßig (bitte geben Sie eine Zahl ein)?

c. Die Kunden unseres Bereichs, gemessen am Umsatz, sind hauptsächlich

- aus der eigenen Unternehmensgruppe
- andere Unternehmen des produzierenden Gewerbes
- Handelsunternehmen (inklusive Onlinehandel)
- andere Dienstleistungsunternehmen
- die öffentliche Hand
- private Endverbraucher
- Sonstige, und zwar:

Figure C3: Original survey questionnaire in German

Notes: Original questionnaire from ifo's one-time online special survey on its "uncertainty module" in German, from fall 2018.

Appendix D Sample creation

In this appendix, we describe the construction of our baseline sample and also explain the number of observations from specific regressions in this paper. The observation numbers that remain after each step are listed in Table D1. Our starting sample from 27 survey waves consists of 14,150 firm-quarter observations. A firm-quarter observation is in the starting sample if the firm at that point in time answered at least one question of our online survey module and pressed the send button to return it.

We start by excluding 1,228 firm-quarter observations that were lacking an answer to question 1, realized sales growth. Then we carefully read the free text comments respondents can give below each of the questions (see Figure 1 in the main text for the questionnaire). We exclude 166 observations for which a comment indicates that the respondent was not able to calculate sales growth rates on a quarterly basis. For example, some firms in some quarters stated that they responded with annual growth rates. Moreover, we drop 60 observations for which the comment raises doubts about the validity and quality of the answer. For example, some firms in some quarters were not able to state realized past growth rates and used estimates instead. Overall, we exclude 1,454 firm-quarter observations based on missing or low-quality answers to question 1, leaving us with 12,696 firm-quarter observations.

Next, we exclude outliers which we define for question 1 (sales growth rate realizations) as lying outside the interval $[-100\%, 100\%]$. 59 firm-quarter observations are thus excluded. We have also experimented with a $[-15\%, 15\%]$ cutoff, that is the bottom and top deciles of the final baseline sample, and found very similar results. We set the upper bounds quite high because large (two-digit) growth rates typically appear to be deliberate responses as many text comments reveal. Firms give explanations such as “Many projects were moved into this quarter” and “Invoice of a major project.” This leaves us with 12,637 firm-quarter observations. After these cleaning steps, we require for the firm-quarter observations of a firm to remain in the sample that it have at least five clean firm-quarter observations on question 1, leaving us with 10,059 firm-quarter of 1,014 firm observations. It is this sample that we base the calculation of the trend and turbulence dummies on.

Subsequently, we exclude, respectively, outliers and inconsistencies related to question 2. Question 2-outliers were excluded according to the following two criteria:

1. The best case and worst case sales growth rates elicited in question 2.a lie outside the intervals $[-100\%; 300\%]$ and $[-100\%; 100\%]$, respectively.
2. The forecast growth rate elicited in question 2.b lies outside the interval $[-100\%; 100\%]$.

Then we check whether respondents order the numbers in question 2 consistently, that is, as worst case $<$ forecast $<$ best case. We exclude firm-quarter observations with the orderings worst case \geq forecast \leq best case or worst case \leq forecast \geq best case because it is unclear what the respondents had in mind with these answers. However, we keep those firm-quarter observations with the inverse ordering worst case \geq forecast \geq best case and simply swap the worst case and best case numbers. Most likely inverse orderings were not intended by

Table D1: Sample creation

	firm-quarter obs. in sample	firm-quarter obs. excluded	firms in sample	firms excluded
Original sample	14,150		2,361	
<i>Require response to question 1</i>				
Response to Q1 exists	12,922	1,228	2,266	95
<i>Text comment</i>				
Wrong reference time excluded	12,756	166	2,257	9
Uncertain data quality excluded	12,696	60	2,252	5
<i>Outliers to question 1</i>				
Outliers in Q1 responses excluded	12,637	59	2,248	4
<i>Number of observations by firm</i>				
At least 5 clean responses to Q1	10,093	2,544	1,014	1,234
<i>Outliers and inconsistencies to question 2</i>				
Inconsistent & outlier responses to question 2 excluded	9,701	392	1,014	0
<i>Require responses to question 2.a and 2.b</i>				
Baseline sample: Response to Q2a and Q2b exists	8,889	247	1,005	6
Forecast error exists alongside span observations	5,379	3,510	961	44
Response to Q2a and Q2b and one forecast error	8,659	230	950	55

the respondent and rather a simple clerical error. Altogether, we eliminate 392 firm-quarter observations in this step.

In a final step, we eliminate those 247 firm-quarter observations which do not have answers to question 2.a or 2.b, the best or worst case scenarios for sales growth and the forecast, leaving us with our baseline sample of 8,889 of firm-quarter observations for 1,005 firms (*baseline sample*).

Starting from the baseline sample, some further exercises require also observations of forecast errors. In one subsample, the forecast errors exist in the same quarterly survey waves as the span observations, which leaves us with 5,379 firm-quarter observations. For other, purely cross-sectional exercises, we require at least one forecast error in the sample. This subsample contains 950 firm observations.

Appendix E Definition of manufacturing industries

Table E1 presents the definition of the 14 industries we use. They are based on the original 24 two-digit manufacturing industries, which are defined by the WZ08 (WZ stands for the German *Wirtschaftszweig*) code of the German Statistical Office. Since not all these industries have a large number of observations in the survey, we aggregate, for the purposes of this paper, some of them, resulting in 14 manufacturing industries. The column with the number of observations by industry refers to our baseline sample of 8,889 observations.

Table E1: Definition of industries

Industry	Industry WZ08	Industry WZ08 name	No. of obs.
1	10, 11, 12	Food products; Beverages; Tobacco products	439
2	13, 14, 15	Textiles; Wearing apparel; Leather and related products	247
3	16, 17, 31	Wood, products of wood and cork except furniture, articles of straw and plaiting materials; Paper and paper products; Furniture	871
4	18	Printing and reproduction of recorded media	497
5	19, 20, 21	Coke and refined petroleum products; Chemicals and chemical support; Basic pharmaceutical products and pharmaceutical preparations	754
6	22	Rubber and plastic products	730
7	23	Other non-metallic mineral products	473
8	24	Basic metals	391
9	25	Fabricated metal products, except machinery and equipment	1,073
10	26	Computer, electronic and optical products	493
11	27	Electrical equipment	822
12	28	Machinery and equipment n.e.c.	1,475
13	29, 30	Motor vehicles, trailers and semi-trailers; Other transport equipment	340
14	32, 33	Other manufacturing; Repair and installation of machinery and equipment	284
All			8,889

Appendix F Proof of proposition in Section 2.3

This appendix provides the proof of the proposition in Section 2.3, restated here:

Proposition. Suppose g can be written as $g = \mu + \sigma\varepsilon$ where ε has a continuous density with mean zero and unit variance: Then the optimal span is a linear function of σ , that is, $\hat{g}_n - \hat{g}_1 = \sigma(\hat{s}_n - \hat{s}_1)$, where $(\hat{s}_n - \hat{s}_1)$ is the optimal span for $\sigma = 1$.

Proof. Since $g = \mu + \sigma\varepsilon$ and ε has density \mathfrak{g} , the density of g is $\mathfrak{g}((g - \mu) / \sigma) \sigma^{-1}$. A scenario \hat{g}_i contributes to cost only when g realizes in an interval where \hat{g}_i is the closest scenario. For an interior scenario other than the best or worst case, that interval is bounded by $\frac{1}{2}(\hat{g}_{i-1} + \hat{g}_i)$ and $\frac{1}{2}(\hat{g}_i + \hat{g}_{i+1})$. We can therefore write the firm's expected cost as:

$$\int_{-\infty}^{(\hat{g}_1 + \hat{g}_2)/2} (g - \hat{g}_1)^2 \mathfrak{g}\left(\frac{g - \mu}{\sigma}\right) \sigma^{-1} dg + \sum_{i=2}^{n-1} \int_{(\hat{g}_{i-1} + \hat{g}_i)/2}^{(\hat{g}_i + \hat{g}_{i+1})/2} (g - \hat{g}_i)^2 \mathfrak{g}\left(\frac{g - \mu}{\sigma}\right) \sigma^{-1} dg + \int_{(\hat{g}_{n-1} + \hat{g}_n)/2}^{\infty} (g - \hat{g}_n)^2 \mathfrak{g}\left(\frac{g - \mu}{\sigma}\right) \sigma^{-1} dg.$$

The first-order conditions with respect to $\hat{g}_1, \dots, \hat{g}_n$, and using Leibniz' rule, are:

$$\int_{(\hat{g}_{i-1} + \hat{g}_i)/2}^{(\hat{g}_i + \hat{g}_{i+1})/2} (g - \hat{g}_i) \mathfrak{g}\left(\frac{g - \mu}{\sigma}\right) \sigma^{-1} dg = 0; \quad i = 2, \dots, n - 1$$

$$\int_{(\hat{g}_{n-1} + \hat{g}_n)/2}^{\infty} (g - \hat{g}_n) \mathfrak{g}\left(\frac{g - \mu}{\sigma}\right) \sigma^{-1} dg = \int_{-\infty}^{(\hat{g}_1 + \hat{g}_2)/2} (g - \hat{g}_1) \mathfrak{g}\left(\frac{g - \mu}{\sigma}\right) \sigma^{-1} dg = 0$$

We note that increasing any given \hat{g}_i increases the upper bound of the interval containing \hat{g}_i as well as the lower bound of the next higher interval. Since the distribution is continuous, these effects cancel, so that the marginal benefit of increasing \hat{g}_i is proportional to expected marginal cost over the interval containing \hat{g}_i .

Now consider the change of variable $s = (g - \mu) / \sigma$:

$$\int_{(\hat{g}_{i-1} + \hat{g}_i - 2\mu)/2\sigma}^{(\hat{g}_i + \hat{g}_{i+1} - 2\mu)/2\sigma} \left(s - \frac{\hat{g}_i - \mu}{\sigma}\right) \mathfrak{g}(s) ds = 0; \quad i = 2, \dots, n - 1$$

$$\int_{(\hat{g}_{n-1} + \hat{g}_n - 2\mu)/2\sigma}^{\infty} \left(s - \frac{\hat{g}_n - \mu}{\sigma}\right) \mathfrak{g}(s) ds = \int_{-\infty}^{(\hat{g}_1 + \hat{g}_2 - 2\mu)/2\sigma} \left(s - \frac{\hat{g}_1 - \mu}{\sigma}\right) \mathfrak{g}(s) ds = 0$$

Define the vector \hat{s} of "standardized scenarios" that solves this system of equations for $\mu = 0$ and $\sigma = 1$. The first order conditions for given μ and σ imply that $\hat{g}_i = \mu + \sigma\hat{s}_i$. It follows in particular that span is $\hat{g}_n - \hat{g}_1 = \sigma(\hat{s}_n - \hat{s}_1)$. ■

Appendix G Modeling learning and uncertainty in unfamiliar territory

G.1 Proofs of the propositions for Example R

Under rational expectations, the firm belief coincides with the true DGP

$$g_{t+1} = f + \sigma(s_t)\varepsilon_t, \quad (8)$$

where ε_t has moments $E(\varepsilon_t) = 0$, $E(|\varepsilon_t|) = c$, and $\text{Var}(\varepsilon_t) = 1$, which are all three unrelated to firm characteristics and independent of information known by the firm when it forms beliefs concerning the next period. In particular, it is independent of the signal s_t the firm receives. The firm forecasts next period's growth as f and perceives uncertainty $\sigma(s_t)$, where both f and the function $\sigma(\cdot)$, that maps the signal to the conditional standard deviation of growth, may depend of firm characteristics.

In the following, we restate and prove propositions R1 and R2.

Proposition R1 (Uncertainty looks like volatility in the cross section). Consider two firms A and B with different fixed characteristics, both described by Example R. Firm A has a higher firm-level mean subjective uncertainty than firm B if and only if firm A has higher firm-level mean absolute forecast error than firm B.

Proof. The forecast error of a firm is $fe_t = g_{t+1} - f = \sigma(s_t)\varepsilon_t$. Its mean absolute forecast error is

$$E(|fe_t|) = E(\sigma(s_t)|\varepsilon_t|) = E(\sigma(s_t))E(|\varepsilon_t|), \quad (9)$$

where the last step follows from independence of ε_t from s_t . The result shows that the mean absolute forecast error, $E(|fe_t|)$, equals the mean uncertainty a firm perceives, $E(\sigma(s_t))$, rescaled by the factor $c > 0$.³⁴ Two firms A and B may differ in their fixed characteristics reflected in different mean growth rates f^A and f^B as well as different conditional standard deviations of growth, $\sigma^A(s_t^A)$ and $\sigma^B(s_t^B)$. Using (9), we obtain the the relation

$$\frac{E(|fe_t^A|)}{E(|fe_t^B|)} = \frac{E(\sigma^A(s_t^A))c}{E(\sigma^B(s_t^B))c} = \frac{E(\sigma^A(s_t^A))}{E(\sigma^B(s_t^B))}.$$

If firm A has a higher mean subjective uncertainty than firm B, $E(\sigma^A(s_t^A))/E(\sigma^B(s_t^B)) > 1$, it thus also has a higher mean absolute forecast error than firm B, $E(|fe_t^A|)/E(|fe_t^B|) > 1$, and vice versa. ■

Proposition R2 (Time-series uncertainty V is volatility V). Consider the time series of lagged growth rates $s_t = g_t$, forecasts and subjective uncertainty for a firm described by Example R. A piecewise linear regression of demeaned uncertainty on demeaned growth with a break at zero shows a V-shaped pattern if and only if a regression of the demeaned subjective absolute forecast error on demeaned growth shows the same V-shaped pattern.

³⁴This factor depends on the shape of the distribution of ε_t . For example, under normality, it is $\sqrt{2/\pi}$.

Proof. Let us consider the general case of some regressor vector x_t with invertible variance matrix $\text{Var}(x_t)$. The regressors we are interested in in this paper are $x_t = [g_t^+, g_t^-]'$, where g_t^+ is g_t if $g_t \geq f$ and zero otherwise, and g_t^- is g_t if $g_t < f$ and zero otherwise. Next, note that a time-series regression of mean-adjusted subjective uncertainty on an intercept and some regressors, x_t , yields the same slope parameters as a time-series regression of undadjusted subjective uncertainty, $\sigma(s_t)$, on an intercept and the same regressors. Hence, let us consider the latter. Under standard conditions, the slope estimates converge in probability towards the slope parameters of a linear projection of subjective uncertainty on the regressors which can be shown to be

$$\beta = \text{Var}(x_t)^{-1} \text{Cov}(x_t, \sigma(s_t)).$$

By the same argument, the slope parameters of a regression of the deviation of the subjective absolute forecast error from its firm-level mean on an intercept and the regressors x_t converge towards

$$\tilde{\beta} = \text{Var}(x_t)^{-1} \text{Cov}(x_t, |fe_t|).$$

Now,

$$\text{Cov}(x_t, |fe_t|) = \text{Cov}(x_t, \sigma(s_t)|\varepsilon_t|) = \text{Cov}(x_t, \sigma(s_t))c,$$

where the latter equality is a general result that follows from the independence of ε_t from information known in period t .³⁵ Substituting into the expression for $\tilde{\beta}$ yields

$$\tilde{\beta} = \text{Var}(x_t)^{-1} \text{Cov}(x_t, \sigma(s_t)) E(|\varepsilon_t|) = \beta c.$$

Since $c > 0$ is a positive constant, the two regressions yield, on average, the same shape of the regression line, adjusted by a scale factor. In particular, if the slopes for $x_t = [g_t^+, g_t^-]'$ have a V-shape in the first regression, they also have a V-shape in the second. ■

G.2 Proofs of the propositions for Example L

G.2.1 Preliminary results

For later use, we state and prove two lemmas that will be useful for proving the propositions for Model L.

Lemma 1. Consider the population regression of the random variable Y on an intercept, Z^+ , and Z^- , where Z is a normally distributed random variable with zero mean and variance σ^2 , Z^+ is Z if $Z \geq 0$ and zero otherwise, and Z^- is Z if $Z < 0$ and zero otherwise. Then the regression coefficients, β^+ and β^- , of Z^+ and Z^- , respectively, are given by

$$\begin{pmatrix} \beta^+ \\ \beta^- \end{pmatrix} = \frac{2/\sigma^2}{(\pi - 2)} \begin{bmatrix} (\pi - 1) \text{Cov}(y, Z^+) - \text{Cov}(y, Z^-) \\ (\pi - 1) \text{Cov}(y, Z^-) - \text{Cov}(y, Z^+) \end{bmatrix}. \quad (10)$$

³⁵Consider random variables X , Y , and Z with means μ_x , μ_y , and μ_z , respectively. Assume Z is independent of X and Y such that, e.g., $E(YZ) = \mu_y\mu_z$. Then $\text{Cov}(X, YZ) = E[(X - \mu_x)(YZ - \mu_y\mu_z)] = E[(X - \mu_x)YZ] - E[(X - \mu_x)]\mu_y\mu_z = E[(X - \mu_x)Y]\mu_z = \text{Cov}(X, Y)\mu_z$, where the last equality follows from the fact that $\text{Cov}(X, Y) = E[(X - \mu_x)(Y - \mu_y)] = E[(X - \mu_x)Y] - E[(X - \mu_x)]\mu_y = E[(X - \mu_x)Y]$ because $E[(X - \mu_x)] = 0$.

If, in addition, $\text{Cov}(y, Z^+) = -\text{Cov}(y, Z^-) > 0$, the regression line has a symmetric V-shape.

Proof. The regression parameters are, in general, given by

$$\begin{pmatrix} \beta^+ \\ \beta^- \end{pmatrix} = \begin{pmatrix} \text{Var}(Z^+) & \text{Cov}(Z^+, Z^-) \\ \text{Cov}(Z^+, Z^-) & \text{Var}(Z^+) \end{pmatrix}^{-1} \begin{pmatrix} \text{Cov}(Z^+, Y) \\ \text{Cov}(Z^-, Y) \end{pmatrix}$$

Using the properties of the half-normal distribution, we obtain

$$\begin{aligned} \text{Var}(Z^+) &= E[(Z^+)^2] - E(Z^+)^2 = E(Z^2|Z \geq 0)P(Z \geq 0) - E(Z|Z \geq 0)^2P(Z \geq 0)^2 \\ &= 0.5\sigma^2 - 0.25\frac{2\sigma^2}{\pi} = \frac{\pi - 1}{2\pi}\sigma^2. \end{aligned}$$

By symmetry, we also obtain

$$\text{Var}(Z^-) = \frac{\pi - 1}{2\pi}\sigma^2.$$

Noting that $E[(Z^+)(Z^-)] = 0$ and using $E(Z^+) = -E(Z^-) = \sqrt{2/\pi}\sigma$, we obtain the covariance as

$$\text{Cov}(Z^+, Z^-) = -E(Z^+)E(Z^-) = E(Z|Z \geq 0)^2P(Z \geq 0)^2 = 0.25 \left(\frac{\sqrt{2}\sigma}{\sqrt{\pi}} \right)^2 = \frac{1}{2\pi}\sigma^2.$$

Substituting these results into the regressor variance matrix yields

$$\begin{pmatrix} \text{Var}(Z^+) & \text{Cov}(Z^+, Z^-) \\ \text{Cov}(Z^+, Z^-) & \text{Var}(Z^+) \end{pmatrix}^{-1} = \frac{\sigma^2}{2\pi} \begin{pmatrix} \pi - 1 & 1 \\ 1 & \pi - 1 \end{pmatrix}^{-1} = \frac{2/\sigma^2}{(\pi - 2)} \begin{pmatrix} \pi - 1 & -1 \\ -1 & \pi - 1 \end{pmatrix}.$$

Substituting yields the regression parameters

$$\begin{pmatrix} \beta^+ \\ \beta^- \end{pmatrix} = \frac{2/\sigma^2}{\pi - 2} \begin{pmatrix} \pi - 1 & -1 \\ -1 & \pi - 1 \end{pmatrix} \begin{pmatrix} \text{Cov}(Z^+, Y) \\ \text{Cov}(Z^-, Y) \end{pmatrix} = \frac{2/\sigma^2}{\pi - 2} \left[\begin{array}{l} (\pi - 1)\text{Cov}(y, Z^+) - \text{Cov}(y, Z^-) \\ (\pi - 1)\text{Cov}(y, Z^-) - \text{Cov}(y, Z^+) \end{array} \right].$$

If, in addition, $\text{Cov}(y, Z^+) = -\text{Cov}(y, Z^-) > 0$, the parameters simplify to

$$\begin{pmatrix} \beta^+ \\ \beta^- \end{pmatrix} = \frac{2\pi/\sigma^2}{(\pi - 2)} \begin{pmatrix} \text{Cov}(y, Z^+) \\ -\text{Cov}(y, Z^+) \end{pmatrix}.$$

From the assumption $\text{Cov}(y, Z^+) > 0$ we conclude that $\beta^+ > 0$, $\beta^- < 0$, and $\beta^+ = -\beta^-$. Hence, the regression line is upward sloping in the positive support and downward sloping with the same slope in the negative support. The branches meet at zero and thus show a symmetric V-shape. ■

Lemma 2. Consider the population regression of the random variable $Q = c_0 + c_1(Z + \mu) + c_2(Z + \mu)^2$ on an intercept, Z^+ , and Z^- , where Z is a normally distributed random variable with zero mean and variance σ^2 , Z^+ is Z if $Z \geq 0$ and zero otherwise, and Z^- is Z if $Z < 0$ and zero otherwise. Then the

regression coefficients, β^+ and β^- , of Z^+ and Z^- , respectively, are given by

$$\begin{pmatrix} \beta^+ \\ \beta^- \end{pmatrix} = \begin{pmatrix} c_1 + 2c_2\mu + \frac{\sqrt{2\pi}}{\pi-2}c_2\sigma \\ c_1 + 2c_2\mu - \frac{\sqrt{2\pi}}{\pi-2}c_2\sigma \end{pmatrix}. \quad (11)$$

Proof. To apply the results of Lemma 1, we need to compute $\text{Cov}(Q, Z^+)$ and $\text{Cov}(Q, Z^-)$. To this end, we re-write $Q = a_0 + a_1Z + c_2Z^2$, where $a_0 = c_0 + c_1\mu + c_2\mu^2$ and $a_1 = c_1 + 2c_2\mu$. Then we compute $E(Q) = a_0 + c_2\sigma^2$ and $E(QZ|Z \geq 0) = a_0E(Z|Z \geq 0) + a_1E(Z^2|Z \geq 0) + c_2E(Z^3|Z \geq 0) = \sqrt{2/\pi}a_0\sigma + a_1\sigma^2 + 2\sqrt{2/\pi}c_2\sigma^3$, where in the last step we used $E(Z|Z \geq 0) = -E(Z|Z < 0) = \sqrt{2/\pi}\sigma$, $E(Z^2|Z \geq 0) = E(Z^2|Z < 0) = \sigma^2$, and $E(Z^3|Z \geq 0) = -E(Z^3|Z < 0) = 2\sqrt{2/\pi}c_2\sigma^3$, which follow from the properties of the half-normal distribution. Similarly, we obtain $E(QZ|Z < 0) = -\sqrt{2/\pi}a_0\sigma + a_1\sigma^2 - 2\sqrt{2/\pi}c_2\sigma^3$. Using these results, it is straightforward to compute

$$\text{Cov}(Q, Z^+) = \frac{1}{2}E(QZ|Z \geq 0) - \frac{1}{2}E(Z|Z \geq 0)E(Q) = \frac{1}{2}a_1\sigma^2 + \frac{1}{2}\sqrt{2/\pi}c_2\sigma^3$$

and

$$\text{Cov}(Q, Z^-) = \frac{1}{2}E(QZ|Z < 0) - \frac{1}{2}E(Z|Z < 0)E(Q) = \frac{1}{2}a_1\sigma^2 - \frac{1}{2}\sqrt{2/\pi}c_2\sigma^3.$$

Substituting these covariances into the framework derived in Lemma 1 yields

$$\begin{pmatrix} \beta^+ \\ \beta^- \end{pmatrix} = \frac{2/\sigma^2}{\pi-2} \begin{bmatrix} (\pi-1)\text{Cov}(y, Z^+) - \text{Cov}(y, Z^-) \\ (\pi-1)\text{Cov}(y, Z^-) - \text{Cov}(y, Z^+) \end{bmatrix} = \begin{pmatrix} c_1 + 2c_2\mu + \frac{\sqrt{2\pi}}{\pi-2}c_2\sigma \\ c_1 + 2c_2\mu - \frac{\sqrt{2\pi}}{\pi-2}c_2\sigma \end{pmatrix}.$$

Note that a sufficient condition for a symmetric V is that the two shift parameters of the quadratic function Q are zero, i.e., $c_1 = 0$ and $\mu = 0$. ■

G.2.2 Example L: the Bayesian learning model

Example L is a basic Bayesian mean-variance learning approach as discussed in standard textbooks such as [Koop \(2003\)](#). Importantly, under Bayesian learning, objective and subjective expectations do not necessarily coincide. In this section, we therefore use $E^*(\cdot)$, $\text{Var}^*(\cdot)$, and $\text{Cov}^*(\cdot)$ to denote moments under the objective probability measure and $E(\cdot)$, $\text{Var}(\cdot)$, and $\text{Cov}(\cdot)$ to denote moments under the firm's subjective probability measure.

Next-quarter growth, g_{t+1} , is iid normal with mean g^* and variance σ_g^{*2} . Firms observe a noisy signal $s_t = g_{t+1} + n_t$ in period t , where noise n_t is independent of growth and normally distributed with mean zero and variance σ_n^{*2} (and thus precision σ_n^{*-2}). Hence, $s_t \sim \mathcal{N}(g^*, \sigma_g^{*2} + \sigma_n^{*2})$. We define

$$\gamma^* = \frac{\text{Var}^*(g_{t+1})}{\text{Var}^*(s_t)} = \frac{\text{Var}^*(g_{t+1})}{\text{Var}^*(g_{t+1}) + \text{Var}^*(n_t)} = \frac{\sigma_g^{*2}}{\sigma_g^{*2} + \sigma_n^{*2}} = \frac{\text{SNR}^*}{1 + \text{SNR}^*} \in (0, 1) \quad (12)$$

as a measure of the information content of the signal with $SNR^* = \sigma_g^{*2}/\sigma_n^{*2}$ being the signal-to-noise ratio.

Under the true DGP, the conditional distribution of g_{t+1} given the signal s_t is, by the rules of the normal distribution, conditional normal with moments

$$E^*(g_{t+1}|s_t) = g^* + \frac{\text{Cov}^*(g_{t+1}, s_t)}{\text{Var}^*(s_t)}(s_t - g^*) = g^* + \gamma^*(s_t - g^*) = (1 - \gamma^*)g^* + \gamma^*s_t \quad (13)$$

and

$$\text{Var}^*(g_{t+1}|s_t) = \text{Var}^*(g_{t+1}) - \frac{\text{Cov}^*(g_{t+1}, s_t)^2}{\text{Var}^*(s_t)} = (1 - \gamma^*)\sigma_g^{*2} = \gamma^*\sigma_n^{*2}. \quad (14)$$

We can write this conditional distribution in regression form as

$$g_{t+1} = (1 - \gamma^*)g^* + \gamma^*s_t + \sqrt{(1 - \gamma^*)\sigma_g^{*2}} \varepsilon_{t+1}^*,$$

where the innovation ε_{t+1}^* is standard normal and independent of s_t .

Firms update beliefs concerning g_{t+1} and precision σ_n^{*-2} in a Bayesian fashion using the normal-gamma prior $\mathcal{NG}(0, \frac{\gamma}{1-\gamma}, \Phi^{-2}, \nu)$ where $\gamma \in (0, 1)$, $\Phi^2 > 0$, and $\nu > 2$.³⁶ Concerning the noise precision, this implies an a priori belief of $E(\sigma_n^{*-2}) = \Phi^{-2}$ and $\text{Var}(\sigma_n^{*-2}) = 2\Phi^{-4}\nu^{-1}$. Hence, Φ^{-2} defines the a priori expectation, while ν governs the tightness of this belief with larger ν signaling higher certainty.³⁷

Concerning future growth, the prior implies an a priori expectation of zero. To interpret γ , it is straightforward to show that γ is the prior belief concerning the information content of the signal, i.e.,

$$\gamma = \frac{\text{Var}(g_{t+1})}{\text{Var}(g_{t+1}) + E(\sigma_n^2)}$$

under the probability measure of the subjective prior belief. Furthermore, the implied prior variance of next-quarter growth is $\text{Var}(g_{t+1}) = \frac{\nu}{\nu-2} \frac{\gamma}{1-\gamma} \Phi^2$. In the following, we denote this prior variance as σ_g^2 and use it instead of Φ^2 to characterize the parameterization of the learning model.

³⁶We use the parameterization of the gamma and normal-gamma distributions as defined in [Koop \(2003\)](#). Specifically, a gamma distributed random variable z with parameters Φ^{-2} and ν , $z \sim \mathcal{G}(\Phi^{-2}, \nu)$, has moments $E(z) = \Phi^{-2}$, $\text{Var}(z) = 2\Phi^{-4}\nu^{-1}$, and $E(z^{-1}) = \frac{\nu}{\nu-2}\Phi^2$. The joint distribution of $y|z \sim \mathcal{N}(\mu, \lambda^{-1}z^{-1})$ and z is normal-gamma, $(y, z) \sim \mathcal{NG}(\mu, \lambda^{-1}, \Phi^{-2}, \nu)$. It implies that the marginal distribution of y is a location-scale t distribution, $y \sim t(\mu, \Phi^2/\lambda, \nu)$, with moments $E(y) = \mu$ and $\text{Var}(y) = \frac{\nu\Phi^2}{(\nu-2)\lambda}$ if $\nu > 2$.

³⁷Taking the expressions for the prior moments $E(\sigma_n^{*-2})$ and $\text{Var}(\sigma_n^{*-2})$ together yields $\nu/2 = \left(\frac{\text{Var}(\sigma_n^{*-2})}{E(\sigma_n^{*-2})^2}\right)^{-1} = (\text{SCV})^{-1}$, where SCV is the prior squared coefficient of variation of σ_n^{*-2} . Hence, a higher coefficient of variation in prior beliefs and thus a larger uncertainty with respect to the inverse noise variance is reflected in a smaller prior parameter ν .

After observing the signal s_t , firms' posterior beliefs are normal-gamma, $(g_{t+1}, \sigma_n^{*-2})|s_t \sim \mathcal{NG}(\gamma s_t, \gamma, \Phi_1^{-2}, \nu + 1)$, where

$$\Phi_1^2 = \frac{\nu}{1+\nu} \Phi^2 + \frac{1-\gamma}{1+\nu} s_t^2 = \frac{(\nu-2)(1-\gamma)}{(1+\nu)\gamma} \sigma_g^2 + \frac{1-\gamma}{1+\nu} s_t^2.$$

By the rules of the normal-gamma distribution, the marginal posterior of g_{t+1} is a location-scale t distribution with $\nu + 1$ degrees of freedom and moments

$$E(g_{t+1}|s_t) = \gamma s_t$$

and

$$\text{Var}(g_{t+1}|s_t) = \frac{\nu+1}{\nu-1} \Phi_1^2 \gamma = \frac{(\nu-2)(1-\gamma)}{\nu-1} \sigma_g^2 + \frac{\gamma(1-\gamma)}{\nu-1} s_t^2.$$

We can write this subjective conditional distribution in regression form as

$$g_{t+1} = \gamma s_t + \sqrt{\frac{(\nu-2)(1-\gamma)}{\nu+1} \sigma_g^2 + \frac{\gamma(1-\gamma)}{\nu+1} s_t^2} \varepsilon_{t+1},$$

where the innovation ε_{t+1} follows, under the probability measure of the subjective posterior belief, Student's t distribution with $\nu + 1$ degrees of freedom and is independent of s_t .

To forecast g_{t+1} , firms use the posterior mean of their subjective belief,

$$f_t \equiv E(g_{t+1}|s_t) = \gamma s_t. \quad (15)$$

Hence, their forecast error is $f e_t = g_{t+1} - \gamma s_t$. To express their uncertainty with respect to next-quarter growth, firms use the square root of their posterior variance, i.e.,

$$\omega_t \equiv \omega(s_t) \equiv \sqrt{a + b s_t^2}. \quad (16)$$

where $a \equiv \frac{(\nu-2)(1-\gamma)}{\nu-1} \sigma_g^2$ and $b \equiv \frac{\gamma(1-\gamma)}{\nu-1}$. Note that $a > 0$ and $b > 0$ except in borderline cases such as $\sigma_g^2 = 0$ or $\gamma = 0$.

An econometrician observes sample moments that relate to population moments under the objective probability measure. In the following, we characterize some of these moments. The forecast error is a linear function of growth and the signal, both of which are normally distributed. Given the signal, it is thus normally distributed with conditional expectation

$$E^*(f e_t | s_t) = E^*(g_{t+1} | s_t) - \gamma s_t = (1 - \gamma^*) g^* + \gamma^* s_t - \gamma s_t = (1 - \gamma^*) g^* + (\gamma^* - \gamma) s_t$$

and conditional variance

$$\text{Var}^*(f e_t | s_t) = \text{Var}^*(g_{t+1} | s_t) = \gamma^* \sigma_n^{*2}.$$

Unconditionally, the forecast error is normally distributed with moments

$$E^*(fe_t) = (1 - \gamma^*)g^* + (\gamma^* - \gamma)g^* = (1 - \gamma)g^* \quad (17)$$

and

$$\begin{aligned} \text{Var}^*(fe_t) &= \text{Var}^*(g_{t+1}) + \gamma^2 \text{Var}^*(s_t) - 2\gamma \text{Cov}^*(g_{t+1}, s_t) \\ &= (1 - 2\gamma + \gamma^2) \sigma_g^{*2} + \gamma^2 \sigma_n^{*2} \end{aligned} \quad (18)$$

$$= [(\gamma - \gamma^*)^2 + \gamma^* (1 - \gamma^*)] \sigma_s^{*2}, \quad (19)$$

where we used $\gamma^* = \sigma_g^{*2} / (\sigma_g^{*2} + \sigma_n^{*2})$ and $\sigma_s^{*2} \equiv \sigma_g^{*2} + \sigma_n^{*2} = \sigma_g^{*2} / \gamma^*$ in the last step.

Squared subjective uncertainty has, under the true DGP, expectation

$$E^* [\omega_t^2] = a + b (\sigma_g^{*2} + \sigma_n^{*2} + g^{*2}), \quad (20)$$

while the expectation of its square root does not have a simple closed-form solution.

G.2.3 Proof of Proposition L0

In the following, we restate and prove Proposition L0.

Proposition L0 (Uncertainty moves with volatility in the cross section). Consider firms described by Model L. Other things equal, if a firm experiences more volatile growth σ_g^ , then both its firm-level mean subjective uncertainty and the firm-level mean absolute deviation (MAD) of its forecast error are higher.*

Proof. While it is obvious from (20) that expected squared uncertainty, $E^*(\omega_t^2)$, is an increasing function of σ_g^* , a few steps are needed to show it for $E^*(\omega_t)$. Let us start by noting that $\omega_t \equiv \omega(s_t)$ is a convex function of s_t as its second derivative w.r.t. s_t is $\omega''(s_t) = ab / (a + bs_t^2)^{3/2}$ which is positive because $a > 0$ and $b > 0$. Hence, Jensen's inequality, $E^*[\omega(s_t)] \geq \omega(E^*[s_t])$, holds. Now consider two signals, $s_{1t} \sim \mathcal{N}(g^*, \sigma_{1s}^{*2})$ and $s_{2t} \sim \mathcal{N}(g^*, \sigma_{2s}^{*2})$, where $\sigma_{2g}^* > \sigma_{1g}^*$ and thus $\sigma_{2s}^{*2} = \sigma_{2g}^{*2} + \sigma_n^{*2} > \sigma_{1s}^{*2} = \sigma_{1g}^{*2} + \sigma_n^{*2}$. Write $s_{2t} = s_{1t} + z_t$, where $z_t \sim \mathcal{N}(0, \sigma_{2g}^{*2} - \sigma_{1g}^{*2})$ is independent of s_{1t} . Hence, $E^*[s_{2t}|s_{1t}] = s_{1t} + E^*[z_t|s_{1t}] = s_{1t}$. Now use Jensen's inequality for the conditional moments, $E^*[\omega(s_{2t})|s_{1t}] \geq \omega(E^*[s_{2t}|s_{1t}]) = \omega(s_{1t})$. By the law of iterated expectations, we finally have $E^*[\omega(s_{2t})] = E^*(E^*[\omega(s_{2t})|s_{1t}]) \geq E^*[\omega(s_{1t})]$. Hence, if the firm experiences the more volatile signal s_{2t} it exhibits higher average uncertainty $E^*[\omega(s_{2t})]$.

The mean absolute deviation of the forecast error is defined as

$$\text{MAD}^*(fe_t) = E^* |fe_t - E^*(fe_t)|.$$

The mean-adjusted forecast error, $fe_t - E^*(fe_t)$, is normally distributed with a mean of zero and the variance $\text{Var}^*(fe_t)$ given in (18). The expectation of its absolute value is, by the rules

of the half-normal distribution,

$$\text{MAD}^*(fe_t) = \sqrt{2/\pi} [\text{Var}^*(fe_t)]^{\frac{1}{2}} = \sqrt{2/\pi} \left[(1 - 2\gamma + \gamma^2) \sigma_g^{*2} + \gamma^2 \sigma_n^{*2} \right]^{\frac{1}{2}}.$$

Since $1 - 2\gamma + \gamma^2 = (1 - \gamma)^2 > 0$, the mean absolute deviation of the forecast error is an increasing function of σ_g^* . ■

G.2.4 Proof of Proposition L1

In the following, we restate and prove Proposition L1.

Proposition L1 (Uncertainty is more than volatility in the cross section). Consider firms described by Model L. Other things equal,

(a) *as the volatility of growth σ_g^* converges to zero, firm-level mean subjective uncertainty converges to a positive constant.*

(b) *if a firm's mean growth rate is more positive (negative), then its firm-level mean subjective uncertainty is higher, but the MAD of its forecast error is the same. Moreover, the firm's average forecast error is more positive (negative) and hence further away from zero.*

(c) *if a firm is more confident about its environment in the sense of lower prior uncertainty about growth σ_g , then its firm-level mean subjective uncertainty is lower, but the firm-level MAD of its forecast error is the same.*

Proof of part (a). By (20), as σ_g^* converges to zero, mean squared subjective uncertainty as observed by an econometrician converges to $a + b(\sigma_n^{*2} + g^{*2})$ which is strictly positive because $a > 0$, $b > 0$, $\sigma_n^{*2} > 0$, and $g^{*2} \geq 0$. While there is no simple closed-form expression for mean subjective uncertainty, $E^*(\omega_t)$, note that ω_t is a strictly positive random variable even if $\sigma_g^* = 0$. Hence, its expectation is strictly positive as well. ■

Proof of part (b). Let us consider the mean absolute deviation of the forecast error, $\text{MAD}^*(fe_t) = E^* |fe_t - E^*(fe_t)|$ first. The mean-adjusted forecast error, $\tilde{fe}_t = fe_t - E^*(fe_t)$, is a normal random variable with a mean of zero and the same variance as fe_t , which does not depend on g^* , see (19). Hence, the distribution of \tilde{fe}_t does not depend on g^* which is why the expectation of its absolute value, and thus the MAD of fe_t , is not affected by a change in g^* .

Second, consider the firm's average forecast error, $E^*(fe_t) = (1 - \gamma)g^*$. As long as $\gamma < 1$, it is further away from zero if a firm's mean growth rate is further away from zero.

Finally, consider the firm's average uncertainty. Again, it is straightforward to see that a mean growth rate, g^* , further away from zero leads to higher average squared uncertainty because its expectation is a linear function of $(g^*)^2$ with slope $b = \gamma(1 - \gamma)/(v - 1) > 0$, see (20). To show this for average uncertainty, let us define the mean-adjusted signal $\tilde{s}_t = s_t - g^*$. Hence,

$$E^*[\omega(s_t)] = E^*[\omega(\tilde{s}_t + g^*)] = \int_{-\infty}^{\infty} \omega(\tilde{s}_t + g^*) \frac{1}{\sigma_s^*} \phi\left(\frac{\tilde{s}_t}{\sigma_s^*}\right) d\tilde{s}_t,$$

where $\phi(\cdot)$ is the pdf of the standard normal distribution which does not depend on g^* . Hence, the first derivative w.r.t. g^* is

$$\frac{\partial E^*[\omega(s_t)]}{\partial g^*} = \int_{-\infty}^{\infty} \frac{\partial \omega(\tilde{s}_t + g^*)}{\partial g^*} \frac{1}{\sigma_s^*} \phi\left(\frac{\tilde{s}_t}{\sigma_s^*}\right) d\tilde{s}_t = E^*[\omega'(s_t)],$$

where $\omega'(s_t) = bs_t/\sqrt{a + bs_t^2}$. Due to $\omega'(-s_t) = -\omega'(s_t)$, we can split the integral into the positive and negative support which yields

$$\begin{aligned} \frac{\partial E^*[\omega(s_t)]}{\partial g^*} &= \int_{-\infty}^0 \omega'(s_t) \frac{1}{\sigma_s^*} \phi\left(\frac{s_t - g^*}{\sigma_s^*}\right) ds_t + \int_0^{\infty} \omega'(s_t) \frac{1}{\sigma_s^*} \phi\left(\frac{s_t - g^*}{\sigma_s^*}\right) ds_t \\ &= - \int_0^{\infty} \omega'(s_t) \frac{1}{\sigma_s^*} \phi\left(\frac{-s_t - g^*}{\sigma_s^*}\right) ds_t + \int_0^{\infty} \omega'(s_t) \frac{1}{\sigma_s^*} \phi\left(\frac{s_t - g^*}{\sigma_s^*}\right) ds_t \\ &= \int_0^{\infty} \omega'(s_t) \frac{1}{\sigma_s^*} \left[\phi\left(\frac{s_t - g^*}{\sigma_s^*}\right) - \phi\left(\frac{-s_t - g^*}{\sigma_s^*}\right) \right] ds_t. \end{aligned}$$

Now, if $g^* > 0$ then $\phi\left(\frac{s_t - g^*}{\sigma_s^*}\right) - \phi\left(\frac{-s_t - g^*}{\sigma_s^*}\right) > 0$, so expected uncertainty increases as g^* increases away from zero. Conversely, if $g^* < 0$ then $\phi\left(\frac{s_t - g^*}{\sigma_s^*}\right) - \phi\left(\frac{-s_t - g^*}{\sigma_s^*}\right) < 0$, so expected uncertainty increases as g^* decreases away from zero. ■

Proof of part (c). Let us first consider the firm's average uncertainty. Again, it is straightforward to see that average squared uncertainty is a linear function of σ_g^2 with slope $(\nu - 2)(1 - \gamma)/(\nu - 1) > 0$, see (20). Hence, lower prior uncertainty about growth reduces average squared uncertainty. For average uncertainty, a simple closed-form solution is not feasible. Nevertheless, uncertainty $\omega_t = \sqrt{a + bs_t^2}$ is, due to $a = \sigma_g^2(\nu - 2)(1 - \gamma)/(\nu - 1)$, shifted downwards at every point s_t as σ_g decreases. Consequently, its expectation $E^*[\omega_t]$ also goes down as σ_g decreases.

The forecast error $fe_t = g_{t+1} - \gamma s_t$ is normally distributed with mean and variance stated in (17) and (19), respectively. Both moments are unrelated to the subjective prior uncertainty σ_g which is why fe_t and all functions of it, such as its mean absolute deviation, are unaffected by a change in σ_g . ■

G.2.5 Proof of Proposition L2

In the following, we restate and prove Proposition L2.

Proposition L2 (Time-series uncertainty V without volatility V). Consider time series of lagged growth rates s_t , forecasts and subjective uncertainty for a firm described by Example L. Suppose that $|g^*| < \sigma_g^*/\sqrt{\gamma^*}$. Up to a second order Taylor expansion, a piecewise linear regression of demeaned subjective uncertainty on demeaned lagged growth with a break at zero generates a V-shaped pattern. In contrast, a regression of the demeaned debiased absolute forecast error on demeaned lagged growth shows no relationship.

Proof of the first part. Let us first consider a regression of the deviation of squared subjective uncertainty from its firm-level mean on the deviation of past growth from its firm-level mean, run separately for positive and negative growth deviations, because this yields a simple closed-form solution. Note that this regression is equivalent to the regression

$$\omega_t^2 = \beta_0 + \beta_1^+ \tilde{s}_t^+ + \beta_1^- \tilde{s}_t^- + u_t,$$

where $\tilde{s}_t^+ = \tilde{s}_t$ if $\tilde{s}_t \geq 0$ and zero otherwise, $\tilde{s}_t^- = \tilde{s}_t$ if $\tilde{s}_t < 0$ and zero otherwise, and $\tilde{s}_t \equiv s_t - E^*(s_t)$ is the deviation of the signal (which we identify with past growth in the empirical part of this paper) from its firm-level mean.

Using the definition $\omega_t^2 = a + bs_t^2$ and applying Lemma 2, we obtain

$$\beta_1^+ = 2bg^* + \frac{\sqrt{2\pi}}{\pi - 2} b\sigma_s^*$$

and

$$\beta_1^- = 2bg^* - \frac{\sqrt{2\pi}}{\pi - 2} b\sigma_s^*.$$

We obtain an asymmetric V if the slope of the right (left) branch is positive (negative). This requires $2g^* + \frac{\sqrt{2\pi}}{\pi-2}\sigma_s^* > 0$ and $2g^* - \frac{\sqrt{2\pi}}{\pi-2}\sigma_s^* < 0$. Noting that $\frac{\sqrt{2\pi}}{\pi-2} \approx 2$, it approximately requires $-\sigma_s^* < g^* < \sigma_s^*$ and thus $-\sigma_g^*/\sqrt{\gamma^*} < g^* < \sigma_g^*/\sqrt{\gamma^*}$. A symmetric V around zero additionally requires g^* .

An equivalent regression of $\omega_t = \sqrt{a + bs_t^2}$ on the same regressors does not allow a simple closed-form solution. Since ω_t looks approximately quadratic in the neighborhood of $s_t = 0$, we apply a second-order Taylor expansion around zero and use it instead of ω_t . Noting that $\omega'(s_t) = bs_t/\sqrt{a + bs_t^2}$ and $\omega''(s_t) = ab/(a + bs_t^2)^{3/2}$, the Taylor polynomial is

$$T_2(s_t) = \omega(0) + \omega'(0)s_t + \frac{1}{2}\omega''(0)s_t^2 = \sqrt{a} + \frac{b}{2\sqrt{a}}s_t^2.$$

The coefficients of the regression

$$T_2(s_t) = \beta_0 + \beta_1^+ \tilde{s}_t^+ + \beta_1^- \tilde{s}_t^- + u_t$$

can again be found by applying Lemma 2. This yields

$$\beta_1^+ = \left[2bg^* + \frac{\sqrt{2\pi}}{\pi - 2} b\sigma_s^* \right] / (2\sqrt{a})$$

and

$$\beta_1^- = \left[2bg^* - \frac{\sqrt{2\pi}}{\pi - 2} b\sigma_s^* \right] / (2\sqrt{a}).$$

Hence, compared to the piecewise regression fitted to ω_t^2 , the slopes of both branches are simply multiplied by the factor $1/(2\sqrt{a}) > 0$.

This implies that we expect an asymmetric V under the same condition $-\sigma_g^*/\sqrt{\gamma^*} < g^* < \sigma_g^*/\sqrt{\gamma^{**}}$. It means that long-run growth g^* must not be too big compared to the standard deviation of the signal which is the square root of $\sigma_g^{*2} + \sigma_n^{*2}$. Hence, we obtain an asymmetric V as long as either the volatility of growth or the standard deviation of the noise are not small. A perfectly symmetric V around zero additionally requires g^* but for $|g^*| \ll \sigma_s^*$, the V will look approximately symmetric. ■

Proof of the second part. Now consider a regression with the same regressors as in part (a) but with the absolute deviation of the debiased forecast error from its firm-level mean on the left-hand side. Debiasing here means that we use the residual of a regression of the forecast error on an intercept, \tilde{s}_t^+ , and \tilde{s}_t^- . Since the forecast error is, by construction, a linear function of \tilde{s}_t , this effectively takes out the conditional mean, $E(fe_t|\tilde{s}_t)$, from the forecast error. Consequently, the debiased forecast error is uncorrelated with the signal. Due to normality, it is also independent from it. Hence, any regression of some function of the debiased forecast error, such as the absolute deviation, on functions of \tilde{s}_t will yield a slope of zero. ■

G.2.6 Proof of proposition L3

In the following, we restate and prove proposition L3.

Proposition L3 (Uncertainty and overreaction). Consider time series of lagged growth rates s_t , forecasts and subjective uncertainty for a firm described by Example L.

(a) If the firm overreacts to signals, $\gamma > \gamma^*$, the covariance between its forecast and forecast error is negative.

(b) Other things equal, a decline in the true information content γ^* implies more overreaction to signals and a lower (more negative) covariance between forecast and forecast error. If the manager is not too uncertain about information quality (v large enough), then it also implies lower excess uncertainty.

Proof of part (a). While a rational firm reacts to signals with weight γ^* , see (13), a learning firm uses the weight γ , see (15). Hence, $\gamma > \gamma^*$ characterizes a firm that overreacts compared to a rational firm. The covariance between forecast and forecast error is, in general,

$$\text{Cov}^*(f_t, fe_t) = \text{Cov}^*(\gamma s_t, g_{t+1} - \gamma s_t) = \gamma \sigma_g^2 - \gamma^2 (\sigma_g^2 + \sigma_n^2). \quad (21)$$

Using the definition (12) of γ^* to substitute $\sigma_g^2 = \gamma^* (\sigma_g^2 + \sigma_n^2)$ yields

$$\text{Cov}^*(f_t, fe_t) = (\gamma \gamma^* - \gamma^2) (\sigma_g^2 + \sigma_n^2) = \gamma (\gamma^* - \gamma) (\sigma_g^2 + \sigma_n^2). \quad (22)$$

Since $0 < \gamma, \gamma^* < 1$ and $\sigma_g^2, \sigma_n^2 > 0$, the covariance is negative if and only if $\gamma > \gamma^*$. ■

Proof of part (b). The true information content of the signal is measured by $\gamma^* = \sigma_g^2 / (\sigma_g^2 + \sigma_n^2)$. Reducing γ^* , keeping the variance of growth, σ_g^2 , unchanged, implies an increase in

the noise variance σ_n^2 . Formally, we are interested in $d\gamma^* < 0$ which is equivalent to $d\sigma_n^2 = -\frac{(\sigma_g^2 + \sigma_n^2)^2}{\sigma_g^2} d\gamma^* > 0$.

For a given prior belief, i.e., invariant γ , the firm overreacts more if γ^* is reduced, so $\gamma - \gamma^*$ is higher.

Concerning the covariance between forecast and forecast error, we use (21) and obtain

$$\frac{\partial \text{Cov}^*(f_t, fe_t)}{\partial \sigma_n^2} = -\gamma^2 < 0.$$

Hence, if a firm overreacts more to the signal, then the covariance between forecast and forecast error becomes more negative.

Concerning the firm-level mean absolute deviation of the forecast error, first note that, by (18), the variance of the forecast error increases:

$$\frac{\partial \text{Var}^*(fe_t)}{\partial \sigma_n^2} = \gamma^2 > 0.$$

Now, by the rules of the half-normal distribution, the mean absolute deviation of the forecast error is an increasing function of the variance,

$$\text{MAD}^*(fe_t) = \text{E}^* |fe_t - \text{E}^*(fe_t)| = \sqrt{\frac{2}{\pi}} [\text{Var}^*(fe_t)]^{1/2}.$$

Hence, it also increases as the firm overreacts more. The derivative is

$$\frac{\partial \text{MAD}^*(fe_t)}{\partial \sigma_n^2} = \frac{\gamma^2}{\sqrt{2\pi}} [\text{Var}^*(fe_t)]^{-1/2} = \frac{\gamma^2/\sigma_s^*}{\sqrt{2\pi}} [(\gamma - \gamma^*)^2 + \gamma^*(1 - \gamma^*)]^{-1/2}.$$

Concerning firm-level uncertainty, let us again consider squared uncertainty first. By (20), its expectation increases as the firm overreacts more:

$$\frac{\partial \text{E}^*[\omega_t^2]}{\partial \sigma_n^2} = b = \frac{\gamma(1 - \gamma)}{\nu - 1} > 0.$$

For uncertainty, ω_t , there is no simple closed-form solution, hence we resort to the second-order Taylor approximation, $T_2(s_t) = \sqrt{a} + \frac{b}{2\sqrt{a}}s_t^2$, introduced for the proof of Proposition L2 above. Its expectation is $\text{E}^*[T_2(s_t)] = \sqrt{a} + \frac{b}{2\sqrt{a}}(\sigma_g^{*2} + \sigma_n^{*2} + g^{*2})$. Thus, its derivative is

$$\frac{\partial \text{E}^*[\omega_t]}{\partial \sigma_n^2} \approx \frac{\partial \text{E}^*[T_2(s_t)]}{\partial \sigma_n^2} = \frac{b}{2\sqrt{a}} = \frac{\gamma/\sigma_g}{2} \frac{\sqrt{1 - \gamma}}{\sqrt{(\nu - 1)(\nu - 2)}}.$$

What increases more, the volatility of forecast errors or subjective uncertainty? For the squared measures, $\text{Var}^*(fe_t)$ and $E^*[\omega_t^2]$, the answer is straightforward. The relative increase,

$$\frac{\partial \text{Var}^*(fe_t)/\partial \sigma_n^2}{\partial E^*[\omega_t^2]/\partial \sigma_n^2} = (\nu - 1) \frac{\gamma}{1 - \gamma},$$

is larger than 1, i.e., the variance of the forecast error increases more than average squared uncertainty, if ν is large. This condition implies that firms are relatively confident regarding the precision of the signal.

For our preferred measures, $\text{MAD}^*(fe_t)$ and $E^*[\omega_t]$, the relative increase is

$$\frac{\partial \text{MAD}^*(fe_t)/\partial \sigma_n^2}{\partial E^*[\omega_t]/\partial \sigma_n^2} = \sqrt{\frac{2}{\pi}} \frac{\gamma \sqrt{(\nu - 1)(\nu - 2)}}{\sqrt{(1 - \gamma) [(\gamma - \gamma^*)^2 + \gamma^* (1 - \gamma^*)]}} \frac{\sigma_g}{\sigma_s^*}.$$

Again, it is larger than 1 if ν is large, i.e., firms are relatively confident regarding the precision of the signal. ■

G.3 Hybrid learning model

The hybrid learning model combines elements of both Example R and Example L without adding new learning mechanisms. We simply assume that next-quarter growth, now denoted by h_{t+1} , consists of two mutually independent shocks, a learnable shock $g_{t+1} \sim \mathcal{N}(g^*, \sigma_g^2)$ and an unlearnable shock $z_{t+1} \sim \mathcal{N}(0, \sigma_z^2)$, i.e.,

$$h_{t+1} = g_{t+1} + z_{t+1}.$$

In the same fashion as in Example L, firms receive a signal concerning the learnable shock, $s_t = g_{t+1} + n_t$. In contrast, as in Example R, they have no a priori knowledge concerning the realization of z_{t+1} but know the mechanism that generates z_{t+1} and come up with a rational forecast of it. For concreteness, reflecting the results of our empirical exercise, let us assume that firms are aware that, under the true DGP, σ_z^2 is a deterministic function of current growth:

$$\text{Var}^*(z_{t+1}|g_t) = \sigma_z^2(g_t).$$

This information structure reflects that firms typically have a good understanding of their own business environment which enables them, unlike the econometrician, to distinguish between learnable and unlearnable components of future sales growth. For example, they might observe signals concerning their stance in their home market but not in new or external markets with which they are not familiar.

Under the true DGP, the conditional distribution of h_{t+1} given the signal s_t is conditionally normal. Due to independence of its components and unpredictability of z_{t+1} , we obtain a

simple extension of the models in Example L and R:

$$E^*(h_{t+1}|s_t) = E^*(g_{t+1}|s_t) + E^*(z_{t+1}) = (1 - \gamma^*)g^* + \gamma^*s_t$$

and

$$\text{Var}^*(h_{t+1}|s_t) = \text{Var}^*(g_{t+1}|s_t) + \text{Var}^*(z_{t+1}) = \gamma^*\sigma_n^2 + \sigma_z^2(g_t).$$

Learning of g_{t+1} works as in the Example L, while z_{t+1} is unlearnable which is why firms form rational expectations as in Example R. Since firms know that the two shocks are mutually independent, their beliefs after observing s_t are characterized by the location-scale t distribution of g_{t+1} , discussed in the Example L, and the true DGP of z_{t+1} . Hence, the subjective posterior of growth, h_{t+1} , is characterized by the sum of a t and a normal distribution. From this distribution, firms derive the forecast

$$f_t \equiv E(h_{t+1}|s_t) = E(g_{t+1}|s_t) + E^*(z_{t+1}) = \gamma s_t$$

and uncertainty

$$\sigma^2(s_t) \equiv \text{Var}(h_{t+1}|s_t) = \text{Var}(g_{t+1}|s_t) + \text{Var}^*(z_{t+1}) = \frac{(\nu - 2)(1 - \gamma)}{\nu - 1} \sigma_g^2 + \frac{\gamma(1 - \gamma)}{\nu - 1} s_t^2 + \sigma_z^2(g_t).$$

Now suppose the unlearnable shock, z_{t+1} , shows the specific form of heteroskedasticity documented in the empirical part of this paper:

$$\sigma_z^2(g_t) = \begin{cases} b_0 + b_1 g_t^2 & \text{if } g_t < 0 \\ b_0 & \text{if } g_t \geq 0 \end{cases}$$

where $b_0, b_1 > 0$. Under the true DGP, growth will inherit this shape of heteroskedasticity because the learnable shock is homoskedastic. Hence, the variance of growth g_{t+1} increases with last period's absolute growth rate, g_t , if that was negative, while it is homoskedastic if g_t was positive. Moreover, the forecast error, $fe_t = g_{t+1} - \gamma s_t$, is a linear combination of growth and the homoskedastic signal which is why its variance exhibits the same basic heteroskedastic shape as growth.

Let us additionally assume that the signal used by firms to learn g_{t+1} is last period's growth rate, i.e., $s_t \equiv g_t$. Then subjective uncertainty becomes the square root of

$$\omega^2(g_t) = \frac{(\nu - 2)(1 - \gamma)}{\nu - 1} \sigma_g^2 + b_0 + \begin{cases} (a_1 + b_1) g_t^2 & \text{if } g_t < 0 \\ a_1 g_t^2 & \text{if } g_t \geq 0 \end{cases}$$

where $a_1 \equiv \frac{\gamma(1-\gamma)}{\nu-1} > 0$. Hence, firms experience uncertainty approximately as an asymmetric V with a steeper left branch.

Appendix H Detailed summary statistics

In this appendix, we report summary statistics for the answers to the questions in our survey module. Table H1 pools all firm-quarter observations and reports mean, standard deviation, and key quantiles for this pooled sample. The numbers here reflect variation both in the time series and in the cross section of firms. For Table H2, we compute, for each individual firm, the time-series mean and standard deviations. The panel reports mean, standard deviation and quantiles of the cross-sectional distributions of firm-level statistics. The number of observations for (functions of) forecast errors naturally drops because, in order to compute firm-level forecast errors, we need to observe the expected sales growth rate and the realized sales growth rate of a firm in two consecutive quarters; for details see Appendix D. In addition, we present in this appendix the same two summary statistics tables split by our three firm characteristics: firm size (Tables H3 to H10), trend (Tables H11 to H16), and turbulence (Tables H17 to H24).

Table H1: Summary statistics of survey answers, span, and forecast errors; pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	8889	2.18	14.62	-14	-5	3	10	16
Forecast sales growth rate for the current quarter	8889	2.65	11.61	-10	0	2	5	12
Worst case sales growth rate for the current quarter	8889	-4.52	12.17	-20	-10	-2	0	5
Best case sales growth rate for the current quarter	8889	7.79	12.99	-1	2	5	10	20
Span between worst and best case forecast	8889	12.31	10.08	4	5	10	15	25
Forecast error	5379	-.85	13.38	-15	-6	0	5	11
Forecast error from random walk model	5379	.56	17.94	-15	-5	0	5	17
Forecast error from iid process	5379	.7	11.19	-9.53	-3.56	.43	4.56	10.8
Absolute forecast error	5379	8.49	10.37	0	2	5	10	20

Notes: The summary statistics for the first five rows in the table above are based on the *baseline sample* with 8,889 firm-quarter observations. The last four rows are based on the *baseline sample* with a forecast error, leading to 5,379 firm-quarter observations (see Table D1 in Appendix D). For comparison, we also compute the forecast error from a random walk model and from an *iid* process, respectively, based on the same sample as for the firms' forecast errors. P10 to P90 denote the corresponding percentiles of the distribution.

Table H2: Summary statistics of survey answers, span, and forecast errors; by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
<i>Time series mean by firm</i>								
Mean by Firm: Sales growth rate in the previous quarter	1005	2.23	7.05	-5	-83	2.12	5.5	9.8
Mean by Firm: Forecast case sales growth rate for the current quarter	1005	2.91	6.66	-3	0	2.5	5	9.08
Mean by Firm: Worst sales growth rate for the current quarter	1005	-4.3	6.94	-12.22	-7.56	-3.33	-2.5	2.36
Mean by Firm: Best case sales growth rate for the current quarter	1005	8.06	8	1.43	3.88	6.7	10.69	15.83
Mean by Firm: Span between worst and best case forecast	1005	12.36	7.38	5.18	7.33	10.73	15.08	21.67
Mean by Firm: Forecast error	950	-9.6	9.54	-9.9	-4	-6.7	2.5	6.25
Mean by Firm: Forecast error from random walk model	950	.99	10.45	-7.11	-3.25	0	3.75	9.96
Mean by Firm: Forecast error from iid process	950	1.1	7.58	-6.01	-2.14	.46	3.33	8.58
Mean by Firm: Absolute forecast error	950	8.98	8.41	2.5	4.14	6.71	11	17.5
<i>Time series standard deviation by firm</i>								
Std. Dev. by Firm: Sales growth rate in the previous quarter	998	11.57	8.34	3.92	6.06	9.13	14.72	21.73
Std. Dev. by Firm: Forecast case sales growth rate for the current quarter	998	8.16	7.36	2.03	3.51	6.21	10.16	16.44
Std. Dev. by Firm: Worst sales growth rate for the current quarter	998	8.82	7.25	2.48	3.95	6.97	11.14	17.67
Std. Dev. by Firm: Best case sales growth rate for the current quarter	998	8.91	8.07	2.45	4.04	6.63	10.8	17.45
Std. Dev. by Firm: Span between worst and best case forecast	998	6.29	5.17	2.24	3.2	5.01	7.6	11.29
Std. Dev. by Firm: Forecast error	835	10.3	8.61	2.75	4.8	8.1	13.06	20.41
Std. Dev. by Firm: Forecast error from random walk model	835	12.83	12.73	2.12	4.51	8.96	16.67	28.21
Std. Dev. by Firm: Forecast error from iid process	835	7.56	7.53	1.52	2.89	5.45	9.79	15.6
Std. Dev. by Firm: Absolute forecast error	835	6.98	6.38	1.71	3.06	5.29	8.62	13.93

Notes: The upper panel shows summary statistics of the cross-sectional distribution of firm-specific time series averages. To this end, we first compute the mean of the respective variables for each firm and then calculate moments of this cross-sectional distribution. The lower panel shows summary statistics of the cross-sectional distribution of firm-specific time series standard deviations. To this end, we first compute the standard deviation of the respective variables for each firm and then calculate moments of this cross-sectional distribution. The summary statistics for the first five rows in the upper panel are based on the *baseline sample* with 1005 firms. The summary statistics for the last four rows in the upper panel are based on the *baseline sample* with a forecast error, leading to 950 firms (see Table D1 in Appendix D). For the summary statistics in the lower panel, we use the same observations. However, not all firms have a sufficient number to compute a standard deviation, which is why the number of firms shrink to 998 and 835, respectively. For comparison, we also compute the forecast error from a random walk model and from an *iid* process, respectively, based on the same sample as for the firms' forecast errors. P10 to P90 denote the corresponding percentiles of the distribution.

Table H3: Summary statistics of survey answers and derived variables from tiny firms, pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	289	-1.65	18.18	-20	-10	0	10	20
Expected case sales growth rate for the current quarter	289	.73	14.34	-15	-5	1	5	15
Worst sales growth rate for the current quarter	289	-	16.15	-30	-20	-10	0	5
		10.34						
Best case sales growth rate for the current quarter	289	9.18	16.43	-10	0	10	15	30
Span between worst and best case forecast	289	19.52	15.32	4	10	15	30	40
Forecast error	187	-4.6	18.51	-30	-12	-2	5	15
Forecast error from random walk model	187	3.24	22.13	-20	-5	0	10	30
Forecast error from iid process	187	3.39	13.52	-	-3	2.09	8.57	18.5
				10.79				
Absolute forecast error	187	12.85	14.06	0	3	10	17	30

Table H4: Summary statistics of survey answers and derived variables from tiny firms, by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
<i>Time series mean by firm</i>								
Mean by Firm: Sales growth rate in the previous quarter	31	.86	8.91	-11.5	-4.21	.5	7.22	11.82
Mean by Firm: Expected sales growth rate for the current quarter	31	3.1	6.08	-6.1	-.62	4.11	7.8	9.88
Mean by Firm: Worst sales growth rate for the current quarter	31	-6.86	7.74	-16.32	-12.5	-6.93	0	1.5
Mean by Firm: Best case sales growth rate for the current quarter	31	11.29	7.49	2	7.11	11.07	15	23.44
Mean by Firm: Span between worst and best case forecast	31	18.15	8.36	9.57	11	16.5	24.17	30.63
Mean by Firm: Forecast error	30	-4.73	13.42	-20.2	-13.57	-2.25	3	6.5
Mean by Firm: Forecast error from random walk model	30	3.48	11.49	-5.37	-3.5	.76	9.18	15.83
Mean by Firm: Forecast error from iid process	30	4.39	9.68	-7.43	-21	3.2	9.23	16.83
Mean by Firm: Absolute forecast error	30	14.21	11.08	4.1	5.5	9.24	20.55	31.8
<i>Time series standard deviation by firm</i>								
Std. Dev. by Firm: Sales growth rate in the previous quarter	30	15.61	8.52	6.28	8.49	13.26	23.18	26.96
Std. Dev. by Firm: Expected sales growth rate for the current quarter	30	11.79	7.2	4.37	6.88	10.24	13.87	25.83
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	30	11.98	7.36	3.37	6.89	11.24	15.05	23.23
Std. Dev. by Firm: Best case sales growth rate for the current quarter	30	12.58	8.52	3.52	6.65	11.4	13.8	24.65
Std. Dev. by Firm: Span between worst and best case forecast	30	9.64	5.01	4.69	6.98	8.51	11.81	16.63
Std. Dev. by Firm: Forecast error	28	15.73	10.85	4.55	7.71	11.95	22.42	31.91
Std. Dev. by Firm: Forecast error from random walk model	28	15.54	14.64	2.36	5.83	10.03	22.47	35.49
Std. Dev. by Firm: Forecast error from iid process	28	9.51	6.24	2.89	4.86	8.59	13.26	17.69
Std. Dev. by Firm: Absolute forecast error	28	11.14	7.9	3.25	4.94	8.69	19.19	21.21

Table H5: Summary statistics of survey answers and derived variables from small firms, pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	2019	2.88	16.48	-15	-5	3	10	20
Expected case sales growth rate for the current quarter	2019	3.7	11.56	-5	0	3	8	15
Worst sales growth rate for the current quarter	2019	-5.04	12.2	-20	-10	-3	0	5
Best case sales growth rate for the current quarter	2019	9.87	13.4	0	3	10	15	20
Span between worst and best case forecast	2019	14.91	11.62	5	7	10	20	30
Forecast error	1222	-.59	16.02	-18	-8	0	5	15
Forecast error from random walk model	1222	.94	18.27	-15	-5	0	7	20
Forecast error from iid process	1222	.76	12.22	-	-4	.52	5.29	12.2
				10.33				
Absolute forecast error	1222	10.41	12.19	0	3	6	15	25

Table H6: Summary statistics of survey answers and derived variables from small firms, by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
<i>Time series mean by firm</i>								
Mean by Firm: Sales growth rate in the previous quarter	243	2.72	7.96	-5.62	-1.67	2.27	6.41	11.6
Mean by Firm: Expected sales growth rate for the current quarter	243	3.8	7.38	-3.2	-.27	2.8	6.42	12
Mean by Firm: Worst sales growth rate for the current quarter	243	-5.2	7.75	-14.4	-9.5	-4.17	-.83	2.5
Mean by Firm: Best case sales growth rate for the current quarter	243	10.15	9.37	2	4.63	8.1	13.29	20
Mean by Firm: Span between worst and best case forecast	243	15.35	8.59	6.78	9.29	13.4	19.38	26.67
Mean by Firm: Forecast error	236	-.51	13.39	-15	-5	-5	3.55	8.43
Mean by Firm: Forecast error from random walk model	236	1.5	12.56	-7.67	-4.84	0	4.93	13.91
Mean by Firm: Forecast error from iid process	236	.97	8.92	-7.5	-2.93	.17	4.12	10
Mean by Firm: Absolute forecast error	236	11.25	11.1	2.73	5	8.5	15	21.5
<i>Time series standard deviation by firm</i>								
Std. Dev. by Firm: Sales growth rate in the previous quarter	241	13.13	8.9	4.71	7.07	10.59	17.09	24.2
Std. Dev. by Firm: Expected sales growth rate for the current quarter	241	8.27	6.46	2.12	4.04	6.99	10.37	15.65
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	241	9.15	6.56	2.74	4.35	8.16	11.14	19.92
Std. Dev. by Firm: Best case sales growth rate for the current quarter	241	9.32	7.29	2.92	4.7	7.49	11.05	16.15
Std. Dev. by Firm: Span between worst and best case forecast	241	7.81	6.28	2.74	4.35	6.38	9.58	14.14
Std. Dev. by Firm: Forecast error	201	12.64	8.58	3.86	7.07	10.07	17.03	22.63
Std. Dev. by Firm: Forecast error from random walk model	201	13.23	11.59	2.89	5	10.02	17.64	27.18
Std. Dev. by Firm: Forecast error from iid process	201	7.83	7.59	1.53	3.45	6	9.7	15.56
Std. Dev. by Firm: Absolute forecast error	201	8.66	7.05	2.12	4.17	7.07	10.67	17.31

Table H7: Summary statistics of survey answers and derived variables from medium sized firms, pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	3961	2.15	14.6	-14	-5	2	10	16
Expected case sales growth rate for the current quarter	3961	2.51	11.86	-10	0	2	6	12
Worst sales growth rate for the current quarter	3961	-4.66	12.41	-20	-10	-2	0	5
Best case sales growth rate for the current quarter	3961	7.62	12.92	-2	2	5	10	20
Span between worst and best case forecast	3961	12.28	9.18	4	6	10	15	21
Forecast error	2481	-.92	13.03	-15	-6	0	5	11
Forecast error from random walk model	2481	.49	17.67	-15	-5	0	5	17
Forecast error from iid process	2481	.62	10.73	-8.87	-3.44	.61	4.31	10.24
Absolute forecast error	2481	8.42	9.99	0	2	5	10	20

Table H8: Summary statistics of survey answers and derived variables from medium sized firms, by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
<i>Time series mean by firm</i>								
Mean by Firm: Sales growth rate in the previous quarter	431	2.1	7.66	-5	-1	2.18	5.5	9.43
Mean by Firm: Expected sales growth rate for the current quarter	431	2.54	6.57	-3.2	-.43	2.36	5.06	8.75
Mean by Firm: Worst sales growth rate for the current quarter	431	-4.63	7.26	-12.5	-8.12	-3.75	-.6	2.25
Mean by Firm: Best case sales growth rate for the current quarter	431	7.65	7.62	1.29	3.57	6.7	10.67	15
Mean by Firm: Span between worst and best case forecast	431	12.28	6.44	5.6	7.8	11.14	15	21.33
Mean by Firm: Forecast error	409	-1.25	8.38	-9.5	-4.22	-1	1.86	6.8
Mean by Firm: Forecast error from random walk model	409	.69	9.98	-7.5	-2.86	.25	3.5	8.5
Mean by Firm: Forecast error from iid process	409	1.12	7.49	-5.78	-1.9	.69	3.25	9
Mean by Firm: Absolute forecast error	409	9.01	7.75	2.58	4.46	6.71	10.86	18
<i>Time series standard deviation by firm</i>								
Std. Dev. by Firm: Sales growth rate in the previous quarter	429	11.63	8.24	3.75	6.06	9.32	14.75	22.5
Std. Dev. by Firm: Expected sales growth rate for the current quarter	429	8.51	7.85	2.12	3.7	6.24	10.38	17.72
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	429	9.14	7.69	2.51	4.01	6.83	12.1	18.07
Std. Dev. by Firm: Best case sales growth rate for the current quarter	429	9.25	8.4	2.44	4.08	6.69	11.58	18.6
Std. Dev. by Firm: Span between worst and best case forecast	429	6.07	4.59	2.42	3.52	5	7.47	10.46
Std. Dev. by Firm: Forecast error	368	10.55	9.38	2.83	4.97	8.04	13.14	20.03
Std. Dev. by Firm: Forecast error from random walk model	368	13.27	12.69	2.28	4.84	9.43	16.43	30.61
Std. Dev. by Firm: Forecast error from iid process	368	7.82	7.67	1.53	3.02	5.5	9.82	16.82
Std. Dev. by Firm: Absolute forecast error	368	7.09	6.51	1.83	3.23	5.62	8.55	14.04

Table H9: Summary statistics of survey answers and derived variables from large firms, pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	2620	2.12	12.46	-10	-4	3	8	15
Expected case sales growth rate for the current quarter	2620	2.25	10.84	-8	0	2	5	10
Worst sales growth rate for the current quarter	2620	-3.27	10.99	-15	-8	-2	1	5
Best case sales growth rate for the current quarter	2620	6.29	12.08	-3	1	5	10	15
Span between worst and best case forecast	2620	9.55	8.32	3	5	7	10	20
Forecast error	1643	-.42	11.08	-11	-5	0	4	10
Forecast error from random walk model	1643	.15	17.74	-15	-5	0	5	14
Forecast error from iid process	1643	.46	10.85	-9.73	-3.47	-1.17	3.88	10
Absolute forecast error	1643	6.86	8.72	0	2	5	9	15

Table H10: Summary statistics of survey answers and derived variables from large firms, by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
<i>Time series mean by firm</i>								
Mean by Firm: Sales growth rate in the previous quarter	300	2.17	4.75	-3.41	0	2	4.72	7.46
Mean by Firm: Expected sales growth rate for the current quarter	300	2.7	6.18	-2.07	.39	2.37	4.32	6.87
Mean by Firm: Worst sales growth rate for the current quarter	300	-2.84	5.26	-9.05	-5.67	-2.29	.25	2.38
Mean by Firm: Best case sales growth rate for the current quarter	300	6.63	6.91	1.44	3.61	5.69	8.15	12.01
Mean by Firm: Span between worst and best case forecast	300	9.47	6.1	4	5.65	8.32	11.4	16.1
Mean by Firm: Forecast error	286	-7.6	5.96	-7	-3.5	-1.7	2.1	5
Mean by Firm: Forecast error from random walk model	286	1.17	10.65	-6.33	-2.67	0	3.2	8
Mean by Firm: Forecast error from iid process	286	.87	6.19	-4.83	-1.9	0	2.69	6.7
Mean by Firm: Absolute forecast error	286	6.87	5.26	2	3.43	5.83	8.67	12.5
<i>Time series standard deviation by firm</i>								
Std. Dev. by Firm: Sales growth rate in the previous quarter	298	9.83	7.62	3.16	5.42	7.89	11.52	18.32
Std. Dev. by Firm: Expected sales growth rate for the current quarter	298	7.2	7.21	1.75	3.01	5.16	8.95	14.27
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	298	7.78	7	2.19	3.71	5.94	9.68	14.33
Std. Dev. by Firm: Best case sales growth rate for the current quarter	298	7.73	7.99	2.16	3.32	5.35	9.16	14.35
Std. Dev. by Firm: Span between worst and best case forecast	298	5.06	4.52	1.7	2.57	4.02	6.21	8.86
Std. Dev. by Firm: Forecast error	257	7.96	6.57	2.12	3.58	6.36	10.41	14.68
Std. Dev. by Firm: Forecast error from random walk model	257	11.73	13.07	1.85	3.79	7.81	15.28	25.2
Std. Dev. by Firm: Forecast error from iid process	257	6.82	7.25	1.41	2.52	4.49	9.27	14.14
Std. Dev. by Firm: Absolute forecast error	257	5.3	4.51	1.5	2.38	4.1	6.56	10.55

Table H11: Summary statistics of survey answers and derived variables from 'bad' trend firms, pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	2129	-4.49	14.67	-20	-10	-5	4	10
Expected case sales growth rate for the current quarter	2129	-.12	13.12	-12	-5	0	5	10
Worst sales growth rate for the current quarter	2129	-8.7	14.12	-25	-15	-6	0	3
Best case sales growth rate for the current quarter	2129	5.96	14.53	-5	0	5	10	20
Span between worst and best case forecast	2129	14.66	11.83	5	7	10	20	30
Forecast error	1329	-4.29	14.58	-20	-10	-3	3	10
Forecast error from random walk model	1329	3.93	18.45	-12	-4	2	10	22
Forecast error from iid process	1329	4.25	11.77	-7.78	-6	3.31	8.32	16.18
Absolute forecast error	1329	10.11	11.34	0	3	7	14	23

Table H12: Summary statistics of survey answers and derived variables from 'bad' trend firms, by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
<i>Time series mean by firm</i>								
Mean by Firm: Sales growth rate in the previous quarter	250	-4.82	6.72	-10.75	-6.6	-3.56	-1.5	-5.4
Mean by Firm: Expected sales growth rate for the current quarter	250	.3	8.51	-6.39	-3	0	2.5	5.9
Mean by Firm: Worst sales growth rate for the current quarter	250	-8.32	8.66	-17.33	-11.8	-7.25	-3.18	-3.1
Mean by Firm: Best case sales growth rate for the current quarter	250	6.34	9.6	-1	2	5	8.75	15
Mean by Firm: Span between worst and best case forecast	250	14.66	8.4	6.06	9.24	12.5	17.63	26.44
Mean by Firm: Forecast error	242	-5.42	8.95	-16	-8.96	-3.68	-5	3
Mean by Firm: Forecast error from random walk model	242	5.73	14.27	-4.89	-2.5	2.55	8.5	16.33
Mean by Firm: Forecast error from iid process	242	5.4	8.63	-2.43	1.1	3.35	8	14.95
Mean by Firm: Absolute forecast error	242	10.72	8.13	3.38	5.25	8.33	14.31	19.75
<i>Time series standard deviation by firm</i>								
Std. Dev. by Firm: Sales growth rate in the previous quarter	247	12.14	7.74	4.65	7.07	10.05	15.56	22.63
Std. Dev. by Firm: Expected sales growth rate for the current quarter	247	9.25	8.39	2.4	4.47	7.16	10.84	17.96
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	247	10.3	8.43	2.95	5	8.55	12.7	17.92
Std. Dev. by Firm: Best case sales growth rate for the current quarter	247	10	9.16	2.64	4.35	7.44	11.93	19.66
Std. Dev. by Firm: Span between worst and best case forecast	247	7.52	6.36	2.79	3.98	5.8	8.65	13.13
Std. Dev. by Firm: Forecast error	211	11.9	9.8	3.54	5.8	9.08	15.19	21.79
Std. Dev. by Firm: Forecast error from random walk model	211	12.89	11.96	3.54	5.5	9.07	16.28	27.46
Std. Dev. by Firm: Forecast error from iid process	211	8.15	7.82	2	3.46	6.12	9.79	15.56
Std. Dev. by Firm: Absolute forecast error	211	8.16	7.14	2.12	3.54	6.5	10.05	16.49

Table H13: Summary statistics of survey answers and derived variables from 'normal' trend firms, pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	4695	2.04	11.43	-10	-4	2	7	14
Expected case sales growth rate for the current quarter	4695	2.31	9.31	-6	0	2	5	10
Worst sales growth rate for the current quarter	4695	-4.01	9.87	-15	-10	-2	0	5
Best case sales growth rate for the current quarter	4695	6.84	10.55	0	2	5	10	20
Span between worst and best case forecast	4695	10.85	8.51	3	5	10	15	20
Forecast error	2973	-.68	10.72	-12	-5	0	4	10
Forecast error from random walk model	2973	.21	15.06	-14	-5	0	5	15
Forecast error from iid process	2973	.46	8.89	-7.78	-2.43	.27	3.5	9
Absolute forecast error	2973	6.95	8.19	0	2	5	10	16

Table H14: Summary statistics of survey answers and derived variables from 'normal' trend firms, by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
<i>Time series mean by firm</i>								
Mean by Firm: Sales growth rate in the previous quarter	503	1.99	2.88	-.33	.67	2	3.55	4.83
Mean by Firm: Expected sales growth rate for the current quarter	503	2.49	4.74	-1.46	.33	2.29	4.18	6.73
Mean by Firm: Worst sales growth rate for the current quarter	503	-3.81	5.1	-10	-6.12	-3.2	-.6	1.2
Mean by Firm: Best case sales growth rate for the current quarter	503	7.03	6.3	2	3.71	6	8.75	13
Mean by Firm: Span between worst and best case forecast	503	10.83	6.33	4.71	6.14	9.17	13.92	19.22
Mean by Firm: Forecast error	482	-1.01	5.78	-7	-3.25	-.59	1.6	5
Mean by Firm: Forecast error from random walk model	482	.52	7.61	-6	-2.6	0	3	7
Mean by Firm: Forecast error from iid process	482	.78	5.04	-4.4	-1.27	.33	2.79	5.7
Mean by Firm: Absolute forecast error	482	7.05	5.49	1.89	3.5	5.94	8.73	13.67
<i>Time series standard deviation by firm</i>								
Std. Dev. by Firm: Sales growth rate in the previous quarter	501	9.99	6.66	3.39	5.62	8.48	12.61	17.85
Std. Dev. by Firm: Expected sales growth rate for the current quarter	501	6.81	5.74	1.75	3.09	5.3	8.87	13.33
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	501	7.48	5.61	2.12	3.71	6.12	9.64	13.82
Std. Dev. by Firm: Best case sales growth rate for the current quarter	501	7.36	6.36	2.12	3.51	5.53	9.35	14.75
Std. Dev. by Firm: Span between worst and best case forecast	501	5.38	3.6	1.89	2.84	4.63	7.21	9.54
Std. Dev. by Firm: Forecast error	433	8.7	6.78	2.12	4.11	7.21	10.8	16.01
Std. Dev. by Firm: Forecast error from random walk model	433	11.28	9.47	1.85	4.32	8.8	15.28	25.01
Std. Dev. by Firm: Forecast error from iid process	433	6.29	5.29	1.41	2.56	4.78	8.87	12.89
Std. Dev. by Firm: Absolute forecast error	433	5.79	4.75	1.6	2.78	4.82	7.36	10.74

Table H15: Summary statistics of survey answers and derived variables from 'good' trend firms, pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	2065	9.39	17.42	-8	2	8	15	25
Expected case sales growth rate for the current quarter	2065	6.26	13.58	-5	2	5	10	17
Worst sales growth rate for the current quarter	2065	-1.36	13.47	-15	-5	0	5	10
Best case sales growth rate for the current quarter	2065	11.82	15.28	0	5	10	15	25
Span between worst and best case forecast	2065	13.19	10.84	4	6	10	15	25
Forecast error	1231	2.58	16.84	-13	-5	1	9	20
Forecast error from random walk model	1231	-2.15	22.84	-20	-8	-2	3	15
Forecast error from iid process	1231	-2.58	14.19	-	-6.9	-2.87	1.64	8.33
				14.11				
Absolute forecast error	1231	10.68	13.27	0	3	6	14	25

Table H16: Summary statistics of survey answers and derived variables from ‘good’ trend firms, by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
<i>Time series mean by firm</i>								
Mean by Firm: Sales growth rate in the previous quarter	252	9.72	5.58	5.5	6.32	8.2	11.41	17.13
Mean by Firm: Expected sales growth rate for the current quarter	252	6.34	6.46	.13	3.3	5.21	8.48	13.14
Mean by Firm: Worst sales growth rate for the current quarter	252	-1.3	6.36	-9	-4.31	-.87	2.12	5.4
Mean by Firm: Best case sales growth rate for the current quarter	252	11.83	8.1	4.94	7.13	10	14.82	22
Mean by Firm: Span between worst and best case forecast	252	13.13	7.57	6	8.21	11.39	15.33	22.44
Mean by Firm: Forecast error	237	3.41	13.33	-6	-1.43	2	6.33	14.5
Mean by Firm: Forecast error from random walk model	237	-2.37	11.08	-13.43	-5.37	-2.58	.5	7.25
Mean by Firm: Forecast error from iid process	237	-2.59	8.64	-12.08	-6.07	-2.29	0	3.7
Mean by Firm: Absolute forecast error	237	11.54	11.95	3	4.92	7.5	13.5	24
<i>Time series standard deviation by firm</i>								
Std. Dev. by Firm: Sales growth rate in the previous quarter	250	14.2	10.88	4.03	6.3	10.2	19.15	30.83
Std. Dev. by Firm: Expected sales growth rate for the current quarter	250	9.81	8.58	2.67	3.97	6.99	12.55	21.78
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	250	10.05	8.38	2.69	4.16	7.59	12.89	21.3
Std. Dev. by Firm: Best case sales growth rate for the current quarter	250	10.94	9.3	3.19	4.73	8.38	13.37	24.78
Std. Dev. by Firm: Span between worst and best case forecast	250	6.92	6.11	2.49	3.43	5.2	8.19	12.93
Std. Dev. by Firm: Forecast error	210	12.55	10.22	3.34	5.84	9.2	16.85	28.13
Std. Dev. by Firm: Forecast error from random walk model	210	16.14	17.49	2.44	4.24	10.19	20.51	36.48
Std. Dev. by Firm: Forecast error from iid process	210	9.66	10.1	1.63	3.3	6.25	12.58	23.01
Std. Dev. by Firm: Absolute forecast error	210	8.53	7.67	1.63	3.77	6.16	10.61	17.58

Table H17: Summary statistics of survey answers and derived variables from low turbulence firms, pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	2243	2.52	5.98	-5	0	3	5	10
Expected case sales growth rate for the current quarter	2243	2.57	4.82	-2	0	2	5	8
Worst sales growth rate for the current quarter	2243	-1.85	5.66	-10	-5	0	1	5
Best case sales growth rate for the current quarter	2243	5.73	5.98	0	3	5	10	10
Span between worst and best case forecast	2243	7.57	6.11	2	4	6	10	15
Forecast error	1405	-.33	5.91	-7	-3	0	3	5
Forecast error from random walk model	1405	.13	6.12	-6	-2	0	2	7
Forecast error from iid process	1405	.31	4.85	-4.33	-1.82	.18	2.25	5.24
Absolute forecast error	1405	3.99	4.37	0	1	3	5	9

Table H18: Summary statistics of survey answers and derived variables from low turbulence firms, by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
<i>Time series mean by firm</i>								
Mean by Firm: Sales growth rate in the previous quarter	253	2.5	4.3	-2	.29	2.33	4.56	7.63
Mean by Firm: Expected sales growth rate for the current quarter	253	2.7	3.02	-.4	.94	2.36	4.33	6.33
Mean by Firm: Worst sales growth rate for the current quarter	253	-1.73	3.55	-6.67	-3.54	-1.37	.33	2.4
Mean by Firm: Best case sales growth rate for the current quarter	253	5.81	3.71	2.22	3.36	5	8	10.73
Mean by Firm: Span between worst and best case forecast	253	7.54	4.06	3.56	4.91	6.56	9.33	12.22
Mean by Firm: Forecast error	244	-.59	4.51	-4.83	-2.21	0	1.5	3
Mean by Firm: Forecast error from random walk model	244	.38	4.93	-3.33	-1.8	-.1	2	4
Mean by Firm: Forecast error from iid process	244	.56	4.35	-3.1	-1.32	.03	1.88	3.7
Mean by Firm: Absolute forecast error	244	4.22	3.48	1.33	2.5	3.63	5	7
<i>Time series standard deviation by firm</i>								
Std. Dev. by Firm: Sales growth rate in the previous quarter	253	4.39	1.55	2.18	3.24	4.47	5.68	6.22
Std. Dev. by Firm: Expected sales growth rate for the current quarter	253	3.48	2.24	1.26	1.89	2.97	4.55	6.23
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	253	4.12	2.42	1.41	2.32	3.74	5.29	7.31
Std. Dev. by Firm: Best case sales growth rate for the current quarter	253	4.17	3.1	1.64	2.31	3.61	5.03	6.88
Std. Dev. by Firm: Span between worst and best case forecast	253	3.88	3.41	1.55	2.14	3.06	4.58	7.14
Std. Dev. by Firm: Forecast error	223	4.5	2.74	1.41	2.53	4.04	6.36	7.83
Std. Dev. by Firm: Forecast error from random walk model	223	4.38	3.03	.84	2.12	3.68	5.79	8.8
Std. Dev. by Firm: Forecast error from iid process	223	3.15	2.34	.82	1.53	2.74	3.86	6.36
Std. Dev. by Firm: Absolute forecast error	223	3.11	2.17	.71	1.62	2.65	4.13	5.57

Table H19: Summary statistics of survey answers and derived variables from medium low turbulence firms, pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	2289	1.53	8.9	-10	-5	2	7	12
Expected case sales growth rate for the current quarter	2289	1.75	7.08	-5	0	2	5	10
Worst sales growth rate for the current quarter	2289	-4.21	8.26	-15	-10	-3	0	5
Best case sales growth rate for the current quarter	2289	5.94	7.7	0	2	5	10	15
Span between worst and best case forecast	2289	10.15	7.05	4	5	10	13	20
Forecast error	1398	-.72	8.54	-11	-5	0	5	10
Forecast error from random walk model	1398	.61	9.97	-10	-5	0	5	12
Forecast error from iid process	1398	.64	6.58	-6.42	-2.67	.58	3.88	7.73
Absolute forecast error	1398	6.27	5.84	0	2	5	9	14

Table H20: Summary statistics of survey answers and derived variables from medium low turbulence firms, by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
<i>Time series mean by firm</i>								
Mean by Firm: Sales growth rate in the previous quarter	252	1.32	5.24	-5.4	-1.13	1.3	4.15	7
Mean by Firm: Expected sales growth rate for the current quarter	252	1.62	4.4	-3.5	-.7	2.29	4.18	5.86
Mean by Firm: Worst sales growth rate for the current quarter	252	-4.45	5.65	-10.53	-6.65	-3.77	-.89	1.43
Mean by Firm: Best case sales growth rate for the current quarter	252	5.86	4.63	.73	2.94	5.88	8.04	11.22
Mean by Firm: Span between worst and best case forecast	252	10.31	5.08	5.67	6.92	9.13	12.62	16
Mean by Firm: Forecast error	238	-.79	4.9	-6.71	-3.12	-.1	1.5	4.57
Mean by Firm: Forecast error from random walk model	238	.89	5.09	-.5	-2.25	.71	3.71	7.33
Mean by Firm: Forecast error from iid process	238	.77	4.08	-4.57	-1.08	.93	2.73	5.83
Mean by Firm: Absolute forecast error	238	6.33	3.15	2.67	4.35	6	7.75	10.75
<i>Time series standard deviation by firm</i>								
Std. Dev. by Firm: Sales growth rate in the previous quarter	251	7.85	1.48	6.4	7.17	8.05	8.82	9.36
Std. Dev. by Firm: Expected sales growth rate for the current quarter	251	5.49	2.99	2.43	3.54	4.97	6.88	9.46
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	251	6.31	3.27	2.89	3.93	5.48	7.99	10.92
Std. Dev. by Firm: Best case sales growth rate for the current quarter	251	6.04	3.2	2.58	3.85	5.48	7.31	10.14
Std. Dev. by Firm: Span between worst and best case forecast	251	5.08	2.97	2.47	3.17	4.46	6.03	8.54
Std. Dev. by Firm: Forecast error	213	7.55	3.39	3.54	5.26	7.64	9.19	11.47
Std. Dev. by Firm: Forecast error from random walk model	213	8.65	5.11	3.06	5.51	7.57	11.68	14.67
Std. Dev. by Firm: Forecast error from iid process	213	5.22	3.71	1.73	2.89	4.47	6.65	9.25
Std. Dev. by Firm: Absolute forecast error	213	5.02	2.63	2.07	3.37	5	6.24	7.64

Table H21: Summary statistics of survey answers and derived variables from medium high turbulence firms, pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	2255	1.89	12.41	-15	-7	3	10	17
Expected case sales growth rate for the current quarter	2255	2.12	9.97	-10	-3	3	7	12
Worst sales growth rate for the current quarter	2255	-6.11	11.17	-20	-10	-5	0	5
Best case sales growth rate for the current quarter	2255	7.8	10.46	-3	2	7	10	20
Span between worst and best case forecast	2255	13.91	8.82	5	8	10	20	25
Forecast error	1424	-.63	12.49	-15	-8	0	5	15
Forecast error from random walk model	1424	.16	15.39	-18	-7	0	8	20
Forecast error from iid process	1424	.37	10.05	-	-4.79	.71	5.55	11.33
				11.25				
Absolute forecast error	1424	9.19	8.48	0	3	7	13	20

Table H22: Summary statistics of survey answers and derived variables from medium high turbulence firms, by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
<i>Time series mean by firm</i>								
Mean by Firm: Sales growth rate in the previous quarter	250	2.14	5.38	-4.11	-1.4	2.1	5	8.42
Mean by Firm: Expected sales growth rate for the current quarter	250	2.38	4.94	-3.6	-5	2.32	5	8.67
Mean by Firm: Worst sales growth rate for the current quarter	250	-5.67	6.08	-14.2	-9.55	-4.95	-1.75	1.5
Mean by Firm: Best case sales growth rate for the current quarter	250	7.81	5.62	1.32	4.5	7.38	11.06	15
Mean by Firm: Span between worst and best case forecast	250	13.48	5.58	7.64	9.75	12.8	15.67	21.62
Mean by Firm: Forecast error	236	-.49	7.21	-9	-5	-.55	4.09	7.38
Mean by Firm: Forecast error from random walk model	236	.08	7.98	-7.67	-5	-.1	4.42	10
Mean by Firm: Forecast error from iid process	236	.27	5.88	-6.55	-3.45	.39	3.38	8.04
Mean by Firm: Absolute forecast error	236	9.35	4.79	4.89	6.07	8.65	11.6	15
<i>Time series standard deviation by firm</i>								
Std. Dev. by Firm: Sales growth rate in the previous quarter	249	11.68	2.66	9.07	10.23	11.52	13.24	14.75
Std. Dev. by Firm: Expected sales growth rate for the current quarter	249	8.33	4.31	3.4	5.42	8.04	10.37	12.99
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	249	9.11	4.87	3.31	6.07	8.77	11.7	14.43
Std. Dev. by Firm: Best case sales growth rate for the current quarter	249	8.66	4.31	3.77	5.77	8.26	10.79	14.14
Std. Dev. by Firm: Span between worst and best case forecast	249	6.49	3.22	2.75	4.47	6.26	8.24	10.04
Std. Dev. by Firm: Forecast error	205	11.18	5.57	4.97	7.64	10.51	13.91	19.09
Std. Dev. by Firm: Forecast error from random walk model	205	13.66	7.64	4.99	8.49	12.75	17.68	22.63
Std. Dev. by Firm: Forecast error from iid process	205	7.94	4.86	2.83	4.79	7.07	10.31	13.6
Std. Dev. by Firm: Absolute forecast error	205	7.26	3.84	2.89	4.55	7.07	9.07	11.93

Table H23: Summary statistics of survey answers and derived variables from high turbulence firms, pooled

Variable	N	Mean	SD	P10	P25	P50	P75	P90
Sales growth rate in the previous quarter	2102	2.84	24.76	-25	-13	3	15	30
Expected case sales growth rate for the current quarter	2102	4.26	19.5	-15	-5	3	10	25
Worst sales growth rate for the current quarter	2102	-5.99	19.27	-30	-15	-5	3	10
Best case sales growth rate for the current quarter	2102	11.99	21.62	-10	0	10	20	30
Span between worst and best case forecast	2102	17.99	13.74	5	10	15	20	33
Forecast error	1306	-1.68	21.95	-25	-12	0	8	21
Forecast error from random walk model	1306	1.48	31.1	-33	-14	0	15	37
Forecast error from iid process	1306	1.52	18.73	-	-7.57	.54	10	21.67
Absolute forecast error	1306	15.18	15.95	1	5	10	20	35

Table H24: Summary statistics of survey answers and derived variables from high turbulence firms, by firm

Variable	N	Mean	SD	P10	P25	P50	P75	P90
<i>Time series mean by firm</i>								
Mean by Firm: Sales growth rate in the previous quarter	250	2.99	11.1	-8	-1.89	2.93	8.83	15
Mean by Firm: Expected sales growth rate for the current quarter	250	4.94	10.93	-4.78	-.75	3.58	9.17	16.11
Mean by Firm: Worst sales growth rate for the current quarter	250	-5.39	10.12	-15.69	-10.67	-5.32	.2	5.19
Mean by Firm: Best case sales growth rate for the current quarter	250	12.8	12.58	1.49	5.69	10.83	17.5	27.08
Mean by Firm: Span between worst and best case forecast	250	18.19	9.08	8.75	11.77	16.19	22.33	30.25
Mean by Firm: Forecast error	243	-2.24	16.17	-20	-10	-2.3	4	13
Mean by Firm: Forecast error from random walk model	243	3.09	18.83	-14.75	-6.25	.4	9.69	25
Mean by Firm: Forecast error from iid process	243	2.82	12.52	-11.01	-5	1.25	9	18.88
Mean by Firm: Absolute forecast error	243	16.42	12.33	5.25	8.2	13.67	20.77	30.5
<i>Time series standard deviation by firm</i>								
Std. Dev. by Firm: Sales growth rate in the previous quarter	245	22.71	8.96	15.35	17.07	20	26.78	35.25
Std. Dev. by Firm: Expected sales growth rate for the current quarter	245	15.57	10.15	5.77	8.57	13.06	20.19	29.24
Std. Dev. by Firm: Worst case sales growth rate for the current quarter	245	15.97	9.65	6.34	9.66	13.72	20.69	27.46
Std. Dev. by Firm: Best case sales growth rate for the current quarter	245	17	11.36	5.73	9.31	14.29	21.73	32.47
Std. Dev. by Firm: Span between worst and best case forecast	245	9.83	7.58	4.16	5.43	7.84	11.73	17.15
Std. Dev. by Firm: Forecast error	213	18.82	11.77	6.29	10.61	16.07	24.11	35.39
Std. Dev. by Firm: Forecast error from random walk model	213	25.21	17.39	7.07	13.4	22.41	33.23	43.91
Std. Dev. by Firm: Forecast error from iid process	213	14.21	10.64	3.54	7.27	11.58	18.26	27.14
Std. Dev. by Firm: Absolute forecast error	213	13.01	8.89	4.24	6.66	10.61	16.49	26.89

Appendix I Additional tables

Table I1: Within-firm regressions of sales growth on past sales growth

	Sales growth in $t + 1$	
	(1)	(2)
Sales growth in t	-0.0253 (0.0286)	
Sales growth in t × positive		0.0297 (0.0482)
× negative		-0.0775 (0.0526)
Observations	4445	4445
R^2	0.000	0.001

Notes: The table shows within-firm regressions of realized sales growth in $t + 1$ on realized sales growth in the previous quarter t . Sales growth is within-transformed and in column (2) interacted with a dummy variable for whether this within-transformed sales growth rate is positive or negative. We restrict the sample to p5–p95 of within-transformed past sales growth. Standard errors in parentheses, clustered by firm; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table I2: Variance decomposition of sales growth and span

	(1)	(2)	(3)
<i>Sales growth</i>			
Constant	2.182*** (0.199)	2.182*** (0.202)	2.182*** (0.200)
R^2	0.006	0.014	0.071
<i>Span</i>			
Constant	12.31*** (0.239)	12.31*** (0.243)	12.31*** (0.239)
R^2	0.016	0.006	0.056
Observations	8889	8889	8889
Industry FE	Yes	No	No
Time FE	No	Yes	No
Industry \times Time FE	No	No	Yes

Notes: The table shows the results of fixed effects regressions of sales growth and span: industry fixed effects in column (1), time fixed effects in column (2), and industry \times time fixed effects in column (3). Standard errors in parentheses, clustered by firm; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table I3: Properties of span

	Shares	N
<i>Skewness of span</i>		
Symmetric	28.63 %	1540
Negative skew	50.94 %	2740
Positive skew	20.43 %	1099
<i>Realized growth outside of span</i>		
Above	22.66 %	1219
Below	21.94 %	1180

Notes: The upper panel of the table shows the shares of observations, where (i) the forecast of sales growth is at the midpoint between the best and worst case scenario (symmetric), (ii) the worst case scenario of sales growth is further away from the forecast than the best case scenario (negative skew), and (iii) the best case scenario of sales growth is further away from the forecast than the worst case scenario (positive skew). The lower panel shows the shares of observations, where realized growth fell outside of span.

Table I4: Within-firm regressions of subjective uncertainty and absolute forecast errors on past sales growth (robustness)

	(1) Span	(2) Span	(3) DownSpan	(4) UpSpan	(5) Abs. debiased FE	(6) Abs. debiased FE	(7) Abs. stat. FE	(8) Abs. stat. FE
Past sales growth	-0.0511*** (0.0123)				-0.0311** (0.0137)		-0.0377** (0.0157)	
Past sales growth × positive		0.124*** (0.0214)	0.0804*** (0.0143)	0.0435*** (0.0111)		-0.00408 (0.0210)		0.0125 (0.0252)
× negative		-0.216*** (0.0211)	-0.121*** (0.0141)	-0.0942*** (0.0127)		-0.0561** (0.0247)		-0.0843*** (0.0275)
Constant	12.00*** (0.0364)	10.97*** (0.105)	6.386*** (0.0709)	4.586*** (0.0563)	7.449*** (0.0374)	7.292*** (0.109)	8.453*** (0.0380)	8.160*** (0.122)
Observations	7355	7355	7355	7355	4573	4573	4518	4518
R ²	0.004	0.023	0.016	0.013	0.002	0.002	0.002	0.004

Notes: The table shows within-firm regressions of different measures of uncertainty and volatility on sales growth rates in the previous quarter. The right-hand side for all regressions is the within-transformed firm-level sales growth rate, in the even columns and column (3) interacted with dummy variables for whether this within-transformed sales growth rate is positive or negative. In columns (1) and (2), the left-hand side variable is span. In column (3), the left-hand side variable is downside span, i.e., the absolute distance from the worst case scenario to the forecast. In column (4), the left-hand side variable is upside span, i.e., the absolute distance from the best case scenario to the forecast. In columns (5) and (6), the left-hand side variable is the absolute debiased subjective forecast error. To be specific, we demean the firms' forecast error and regress it on the same right-hand side variables defined above for column (2). This is to ensure that the results we find are not driven by possible bias in the subjective forecast errors. We then take the absolute value of the residuals of this regression. In columns (7) and (8), the left-hand side variable is the absolute statistical forecast error defined as the absolute difference between realized growth and its firm-specific mean. In all cases, the left-hand side variables are within-transformed (and the overall mean added). In addition, we restrict the sample to p5–p95 of within-transformed past sales growth and require at least 5 span observations for each firm. Standard errors in parentheses, clustered by firm; * p < 0.10, ** p < 0.05, *** p < 0.01.

Table I5: Subjective uncertainty and firms' employment plans

	(1) Emp down	(2) Emp down	(3) Emp down	(4) Emp up	(5) Emp up	(6) Emp up
Forecast	-0.503*** (0.0526)		-0.530*** (0.0524)	0.379*** (0.0455)		0.390*** (0.0459)
Span		0.334*** (0.0566)	0.391*** (0.0575)		-0.116** (0.0540)	-0.158*** (0.0529)
Constant	12.61*** (0.179)	7.396*** (0.693)	8.008*** (0.690)	13.55*** (0.143)	15.86*** (0.647)	15.41*** (0.630)
Observations	7943	7943	7943	7943	7943	7943

Notes: The table shows the coefficients of linear probability regressions of an indicator for a planned decrease of employment in the next three months (Columns (1) to (3)) and an indicator for a planned increase of employment in the next three months (Columns (4) to (6)) on firms' sales growth forecast and subjective uncertainty (span). We control for firm & time fixed effects. In addition, we restrict the sample to p5–p95 of within-transformed past sales growth. Standard errors are clustered at the firm level. * p < 0.10, ** p < 0.05, *** p < 0.01.

Table I6: Subjective uncertainty and firms' employment and price plans (robustness)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Emp down	Emp down	Emp down	P down	P down	P down	P up	P up	P up
Forecast	-0.562*** (0.0616)		-0.582*** (0.0611)	-0.223*** (0.0425)		-0.233*** (0.0429)	0.255*** (0.0599)		0.262*** (0.0594)
Span		0.293*** (0.0596)	0.344*** (0.0613)		0.156*** (0.0452)	0.175*** (0.0453)		-0.0956 (0.0700)	-0.117* (0.0687)
Constant	12.83*** (0.201)	7.902*** (0.736)	8.762*** (0.748)	7.126*** (0.136)	4.693*** (0.541)	5.050*** (0.519)	15.68*** (0.192)	17.48*** (0.849)	17.07*** (0.857)
Observations	7304	7304	7304	7309	7309	7309	7309	7309	7309

Notes: The table shows the coefficients of linear probability regressions of an indicator for a planned decrease of employment in the next three months (Columns (1) to (3)), an indicator for a planned decrease of prices in the next three months (Columns (4) to (6)), and an indicator for a planned increase of prices in the next three months (Columns (7) to (9)) on firms' sales growth forecast and subjective uncertainty (span). We control for firm & time fixed effects. In addition, we restrict the sample to p5–p95 of within-transformed past sales growth and require at least 5 span observations for each firm. Standard errors are clustered at the firm level.