Sáez-Martí, María; Zenou, Yves

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Cultural Transmission and Discrimination

Maria Saez-Marti
Yves Zenou

December 2005
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Maria Saez-Marti
IUI

Yves Zenou
IUI, GAINS and IZA Bonn

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IZA
P.O. Box 7240
53072 Bonn
Germany
Phone: +49-228-3894-0
Fax: +49-228-3894-180
Email: iza@iza.org

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ABSTRACT

Cultural Transmission and Discrimination*

Each worker belongs to either the majority or the minority group and, irrespective of the group she belongs to, can have good or bad work habits. These traits are transmitted from one generation to the next through a learning and imitation process which depends on parents’ purposeful investment on the trait and the social environment where children live. In a segregated society, we show that, if a high enough proportion of employers have taste-based prejudices against minority workers, their prejudices are always self-fulfilled in steady state. Affirmative Action improves the welfare of minorities without affecting majority workers whereas integration is beneficial to minority workers but detrimental to workers from the majority group. If Affirmative Action quotas are high enough or integration is strong enough, employers’ negative stereotypes cannot be sustained in steady-state.

JEL Classification: J15, J71

Keywords: ghetto culture, overlapping generations, rational expectations, multiple equilibria, peer effects

Corresponding author:

Yves Zenou
IUI, The Research Institute of Industrial Economics
P.O. Box 55665
102 15 Stockholm
Sweden
Email: yvesz@iui.se

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1 Introduction

According to a survey conducted in Chicago in 1988, one of the main reasons employers are not willing to hire inner-city black workers is the lack of basic skills and work ethics. As a suburban employer in Chicago put it, "The experiences that I've run into with it is that they develop bad habits, I guess is the best way to put it. Not showing up to work on time. Not showing up to work. Somewhere down the road they didn't develop good work habits."\(^1\)

This is consistent with more general evidence from sociology and anthropology\(^2\) suggesting the existence of a persistent "ghetto culture", which is transmitted from one generation to the next. The existence of a low work ethic has been pointed out by several scholars as an important element in the set of values defining the prevalent culture in inner-city neighborhoods. These values are in sharp contrast with mainstream American society's working values rooted in the Protestant tradition. As argued by Wilson, it is the social, rather than the physical distance that often separates poor blacks from good jobs. This is particularly true for the African American community, which has experienced high levels of segregation for at least a century (Massey and Denton, 1993, Cutler \textit{et al}., 1999).

"Inner-city social isolation also generates behavior not conducive to good work histories. The patterns of behavior that are associated with a life of casual work (tardiness and absenteeism) are quite different from those that accompany a life of regular or steady work (e.g. the habit of waking up early in the morning to a ringing alarm clock). ... in neighborhoods in which most families do not have a steadily employed breadwinner, the norms and behavior patterns associated with steady work compete with those associated with casual or infrequent work." (Wilson, 1996)

In the words of a counsellor to a training program aiming at exposing black workers to more conventional working values:

"To adopt a regular pattern you have to break with this environment. Your friends laugh at you for going to work, that’s hell, they think you are trying to be better than them! You have to have strong character to resist this pressure."

\(^1\)See Wilson (1996) pages 119-120. Italics are ours.
If all your friends and families went to work they would help you adopt a regular schedule.” (cited in Bonney, 1975)³

Why do minority workers perform worse in the labor market than workers belonging to the majority group? Several explanations have been put forward in the economics literature. In taste-based models (Becker, 1957), discrimination originates from employers’ willingness to reduce profits to avoid hiring workers they are prejudiced against. Minority workers are only hired at lower salaries. The statistical discrimination theory, on the other hand, stresses the role of employers’ beliefs concerning the average quality of workers from different groups. A member of the minority group will be discriminated against if the employer believes he is less qualified or reliable than a worker from the majority group (see Arrow, 1973, Phelps, 1972, Coate and Loury, 1993, and Moro and Norman, 2003). In these models, negative stereotypes are self-fulfilling since workers from the minority become less productive as a result of the negative expectations held by the employers. More recently, it has been argued that the existence of community (or peer) effects can explain the minority’s poor performance. In absence of interaction between communities, minority workers, due to interaction with poorly performing peers, end up with lower levels of education and worse labor market outcomes (see Arnott and Rowse, 1987, De Bartoleme, 1990, and Benabou, 1993).⁴

The importance of family and social environment in the transmission of personality traits has been widely documented (see e.g. Boyd and Richerson, 1985, and Cavalli-Sforza and Feldman, 1981). To the extent that some of those traits are important in determining individual performance in the workplace,⁵ it is important to understand the mechanism of transmission. The evidence from the sociological literature discussed above suggests that children’s families and the communities where they live are important elements in shaping their attitudes towards work. If employers are reluctant, as they declare to be, to hire members from the minority group because of the prevalent values in their communities, then the incentives of parents to transmit the right habits may be affected and policies promoting

³This is related to the idea of “acting white” where economic success of blacks induces peers’ rejection (Austen-Smith and Fryer, 2005)
⁴For a general overview of the issue of race in the labor market, see Altonji and Blank (1999) and Lundberg and Startz (2000).
⁵Kohn (1969) concludes that parents generalize their experiences on the job and pass them to their children. More recently, Osborne Groves (2005) suggests that intergenerational transmission of personality may be a channel to explain intergenerational persistence of income.
integration may have a positive effect on minorities.

The objective of this paper is to put forward these ideas in a dynamic model of cultural transmission. The model explains differences in performance between minority and majority workers and allows the study of the impact of integration and affirmative action policies on economic performance.

We assume that parents are forward looking and invest resources in order to prepare their children for their future working experiences. Parents’ efforts and children’s preferences are also affected by the environment where children interact. As a result, the transmission of the ‘work ethic’ trait is the result of socialization inside and outside the family.

In our model, workers belong either to a majority or to a minority group. All individuals are born equal but, depending on the parents’ investments and the social environment where they live, they acquire either a good or a bad work habit. When deciding how much effort to exert on shaping their children’s attitudes towards work, parents must form expectations about the working opportunities their children are going to face in the future.

We assume that, each worker is randomly matched to an employer who has to assign the worker to a job. Employers know the group (minority or majority) a worker belongs to but cannot perfectly observe his trait. A proportion of employers are taste-based prejudiced against minority workers and systematically allocate them to the worst job. All other employers use an (imperfect) signal concerning the worker’s trait to allocate them to one of two available jobs. This second group of employers are profit maximizers. The different treatment workers from the majority and the minority groups are subject to creates a discrepancy between minority and majority workers’ expected value of the good trait.

We first focus on a segregated society where workers from the minority and the majority groups do not interact and show that, if the fraction of prejudiced employers is high enough, their beliefs are always self-fulfilled. In steady-state, the work habit of minority workers is (on average) bad and the profit maximizing strategy is to allocate them to the worst job. Due to the worse opportunities their children are going to face, minority parents do not find worthwhile exerting effort to transmit “good” values. As a result, more minority workers have bad work habits. This, in turn, influences members of the next generations in the same community and the initial negative beliefs are confirmed in steady state.

We then study the effect of different policies aiming at weakening this “ghetto” culture, which perpetuates bad work habits. We study, in particular, the effect of affirmative action
and integration policies.

We first analyze Affirmative Action programs consisting in (i) imposing a quota of minority workers in *good* jobs, (ii) a quota of minority workers who are treated as the majority workers. We show that the first policy has a negative long run effect while the second does improve the welfare of the disadvantaged workers leaving that of the majority workers unchanged. If the affirmative action imposes high enough quotas, negative beliefs cannot be sustained in steady state and minority workers will develop better work habits.

We then analyze the effect of integration policies. In this case, minority children are to some degree influenced by peers belonging to the majority group and vice versa. We show that integration is beneficial for the minorities but detrimental for the advantaged workers. This result helps us to understand why the latter may have an incentive to resist integration and may be reluctant to accept social mixing with minorities.

The paper is organized as follows. In the next section, we present the model. In section 3, we characterize the steady-state equilibria in the complete segregation case. The policy issues are addressed in sections 4 (Affirmative Action) and 5 (Integration). Section 6 concludes.

2 The model

There is a continuum of workers who belong either to the majority (identified by $M$) or minority group (identified by $m$).\textsuperscript{6} Individuals differ also by an unobservable trait that determines their behavior on the job.

We consider a principal-agent model in which the principal (employer) can observe the group the agent (worker) belongs to but not his type. At each time $t$ ($-\infty < t < \infty$), every active worker is randomly matched with an employer. The employer decides which task to give to the worker while the worker decides whether to work or to shirk. There are two possible tasks: Task 1 is better paid and gives the employer a payoff $W$ if the worker works hard and a payoff $S$ if the worker shirks ($W > S$). Task 2 is a low paid job that gives the employer a payoff $w$ if the worker works hard and $s$ when the worker shirks. The payoffs to the employer in both tasks are related as follows

\[ W > w \geq s > S. \]  

\textsuperscript{6}To avoid monotony, the terms “minority group” and “disadvantaged group” as well as “majority group” and “advantaged group” are used interchangeably throughout this paper.
Agents are paid $\Omega$ in task 1 and $\varpi$ in task 2 ($0 < \varpi < \Omega$) but, depending on their type, some workers prefer working over shirking and some others shirking over working. From the employers’ point of view, good workers (i.e. workers with good work habits) are those who prefer working while bad workers (i.e. workers with bad work habits) are those who prefer shirking. Workers’ payoffs are as follows:

<table>
<thead>
<tr>
<th>Good worker</th>
<th>Task 1</th>
<th>Task 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work</td>
<td>$\Omega$</td>
<td>$\varpi$</td>
</tr>
<tr>
<td>Shirking</td>
<td>$\Omega - c$</td>
<td>$\varpi - c$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bad worker</th>
<th>Task 1</th>
<th>Task 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work</td>
<td>$\Omega - e$</td>
<td>$\varpi - e$</td>
</tr>
<tr>
<td>Shirking</td>
<td>$\Omega$</td>
<td>$\varpi$</td>
</tr>
</tbody>
</table>

where $c, e > 0$. These payoffs capture, in a stylized way, the fact that workers have different utilities from leisure, different disutilities from effort or different ethics and that these differences affect their behavior in the work-place. In particular, $c$ can be interpreted as a moral cost to shirk for a good worker and $e$ as an effort cost to work for a bad worker.

We assume that there is a proportion $q_M$ of workers from the majority group who are good and a proportion $1 - q_M$ who are bad. Similarly, for the minority group, $q_m$ and $1 - q_m$ are the proportion of good and bad workers. The proportion of the workforce from the advantaged group is $\gamma$. Finally, workers from the minority and the majority group can interact with each other because, for example, they live in the same area. We denote the degree of interaction between these two groups by $0 \leq \sigma \leq 1$. The way interaction between the two groups takes place in our model will be clear in what follows.\footnote{Even if Bisin and Verdier (2000) focus on a very different issue (homogamy versus heterogamy in marriage), the choice of $\sigma$ in their model is in some sense endogenous since individuals of one group (or religion) can affect the probability of being matched in their restricted pool (i.e. individuals of the same group only). In the present model, we keep $\sigma$ exogenous because we would like to compare different situations with different levels of integration.}

When $\sigma = 0$, we have a completely segregated society where these two groups never meet. If $\sigma = 1$, we have a completely integrated society in which these two groups perfectly interact.

## 2.1 Cultural transmission of preferences

We study the intergenerational transmission of ‘work habit ’ traits using an overlapping generation model. The way this trait is transmitted from one generation to the next is through an education and imitation process that depends on parents’ investment on the
trait and the social environment where children live. The transmission of the trait is here modeled as a mechanism that interacts socialization inside the family (vertical socialization) with socialization outside the family (oblique socialization) via imitation and learning from peers and role models as in Bisin and Verdier (2001) and Hauk and Saez-Martí (2002).

We assume a Poisson birth and death process that keeps the population size of active workers constant. With probability $\lambda$ an active worker will be active the next period. With probability $1 - \lambda$ an active worker in $t$ has a child who becomes active in $t + 1$.

Children preferences are shaped, via education, by their parents. A fundamental assumption is that the parents care about their children’s future welfare. Parents evaluate their children’ future utility as if it were their own. For instance a good worker, when computing the utility of his (possibly) shirking child computes a utility loss his child may actually not suffer (if he is a bad worker) while a bad worker whose child is a good one will think his child would be better off shirking.

Education works as follows: The parent chooses an education effort $\tau \in [0, 1]$. With a probability equal to the education effort, education will be successful and the child will be like the parent (good or bad worker). Otherwise, the child remains without the trait and gets randomly matched with somebody else whose trait he will adopt. It is at this second stage, after the parents’ unsuccessful education, that children are influenced by friends, peers, teachers, ... (role models). In a segregated society, majority (minority) children only meet majority (minority) parents whereas in a mixed society, they can also meet parents from the other group.

We denote by $p_{ij}^k$ the probability that a child of type-$i$ parent ($i \in \{g, b\}$), belonging to group $k \in \{m, M\}$ is socialized to trait $j \in \{g, b\}$. Since there is continuum of agents of each group $k \in \{m, M\}$, by the Law of Large Numbers, $p_{ij}^k$ also denotes the fraction of children of group $k$ with a parent $i$ who has preferences of type $j$. Consider a good worker belonging to group $m$ who has a child and chooses education effort $\tau_m^g$. Then, we have the following transition probabilities:

$$p_{mg}^{gg} = \tau_m^g + (1 - \tau_m^g) [\sigma \gamma q_M + (1 - \sigma \gamma) q_m]$$

(2)

$$p_{mg}^{gb} = (1 - \tau_m^g) [\sigma \gamma (1 - q_M) + (1 - \sigma \gamma)(1 - q_m)]$$

(3)

where $q_m$ and $q_M$ are the proportion of good workers of types $m$ (minority) and $M$ (majority) respectively.\(^8\) The interpretation of these equations is straightforward. Take for instance

\(^8\)We avoid when possible the use of time indices. It should be clear that both $q_m$ and $q_M$ are time
equation (2). The child of a good m-worker will also be good if his parent’s education is successful (with probability $\tau^g_m$) or if the parent is unsuccessful and he learns from good peers. When the parent fails to transmit his trait (and this happens with probability $1 - \tau^g_m$), the child will pick up the working trait from the society. When the degree of integration is $\sigma$, the child will become good if he meets a good minority worker (with probability $(1 - \sigma \gamma) q_m$) or a good majority worker ($\sigma \gamma q_M$), where $\gamma$ is the proportion of workers from the majority group in the society. Note that when $\sigma = 0$, a minority child will only meet minority workers while when $\sigma = 1$, the probability of meeting a minority worker is equal to their share in the population $(1 - \gamma)$.

The probabilities for bad minority workers are:

\begin{align*}
    p_{bb}^m &= \tau^b_m + (1 - \tau^b_m) [\sigma \gamma (1 - q_M) + (1 - \sigma \gamma) (1 - q_m)] \\
    p_{bg}^m &= (1 - \tau^b_m) [\sigma \gamma q_M + (1 - \sigma \gamma) q_m]
\end{align*}

where $\tau^b_m$ is education effort of bad minority workers. Similarly we calculate the corresponding probabilities for parents from the majority group:

\begin{align*}
    p_{gg}^M &= \tau^g_M + (1 - \tau^g_M) [(1 - \sigma (1 - \gamma)) q_M + (1 - \gamma) \sigma q_m] \\
    p_{gb}^M &= (1 - \tau^g_M) [(1 - \sigma (1 - \gamma)) (1 - q_M) + (1 - \gamma) \sigma (1 - q_m)] \\
    p_{bb}^M &= \tau^b_M + (1 - \tau^b_M) [(1 - \sigma (1 - \gamma)) (1 - q_M) + (1 - \gamma) \sigma (1 - q_m)] \\
    p_{bg}^M &= (1 - \tau^b_M) [(1 - \sigma (1 - \gamma)) q_M + (1 - \gamma) \sigma q_m]
\end{align*}

### 2.2 The education choice

As stated above, we assume that parents care about their children’s trait and put effort in an attempt to transmit their own trait. Socialization is costly. Let $C(\tau)$ be the cost of the education effort $\tau$ and assume that\footnote{For simplicity, we assume that the cost function is the same for both types of parents. It can be argued that transmitting some values may be more costly than transmitting others. Assuming different cost functions will leave all the qualitative results of the paper unchanged.}

\begin{equation}
    C'(\tau) > 0 \text{ for all } \tau \in (0, 1], \quad C''(\tau) \geq 0, \quad C(0) = 0, \quad C'(0) = 0 \quad \text{and} \quad C''(1) \geq \Omega/(1 - \lambda)
\end{equation}

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\end{equation}
In order to endogeneize the education effort, we need to analyze the employers’ task (job) allocations. We assume that each time an employer meets a bad worker he knows his type with probability $\alpha$. With probability $1-\alpha$ he (wrongly) believes that the worker is good. A good worker is never mistaken for a bad one.

When the worker and the employer are matched, the employer chooses one of the two following assignment strategies:

**Separating strategy ($\rho^S$):** Offer task 1 to seemingly good workers, i.e. all good workers and some bad ones who have been (mistakenly) taken for good ones, and offer task 2 to workers who were found to be bad.

**Pooling strategy ($\rho^P$):** Offer task 2 to everyone.

For group $k = m, M$, principals prefer strategy $\rho^S$ to $\rho^P$ if and only if:

$$q_k W + (1 - q_k) \left[ \alpha \cdot s + (1 - \alpha)S \right] \geq q_k w + (1 - q_k) s$$

Under the separating strategy, an employer meets a good worker with probability $q_k$ and gets $W$. With probability $1 - q_k$, he meets a bad worker. In this case, with probability $\alpha$, the principal knows for certain that the worker is bad and thus automatically assigns this worker to task 2, obtaining a payoff of $s$, whereas with probability $1 - \alpha$, he does believe the worker is good and gives him task 1, which yields a payoff $S$. We can rewrite the inequality above as follows:

$$q_k \geq \frac{(1 - \alpha)(s - S)}{(W - w) + (s - S)(1 - \alpha)} \equiv \bar{q}$$

(11)

If the proportion of good workers in group $k$ is high enough ($q_k > \bar{q}$), then the separating strategy is optimal. We denote the optimal strategy by $\mu$

$$\mu(q_k) = \begin{cases} \rho^S & \text{if } q_k \geq \bar{q} \\ \rho^P & \text{if } q_k < \bar{q} \end{cases} \quad k = m, M$$

(12)

Let $\rho_k(t)$ be the employers’ assignment strategy at time $t$ when meeting a worker of group $k = m, M$. We shall assume hereafter that a proportion $\theta \geq 0$ of the employers are taste-based prejudiced against minority workers and systematically assign them to task 2,\(^\text{10}\)

\(^\text{10}\)In a Beckerian perspective, this means that their ‘distaste cost’ to allocate workers from the minority group to task 1 is so high that it is always optimal for them not to do so.
namely $\rho_m(t) = \rho^P$ for all $t$, while when meeting a majority worker they follow the optimal strategy, i.e. $\rho_M(t) = \mu(q_M(t))$. All other employers follow the optimal strategy $\mu(q_k(t))$ at any time period $t$.

A parent, who has a child at $t$, in order to compute his child’s well-being, needs to form expectations concerning the future tasks (employers’ future assignment strategies) his child (who will be a worker at $t + 1$) will be assigned to. A “job profile”, from time $t + 1$ onwards, is an (infinite) sequence $\{\rho_k(z)\}_{z=t+1}^{\infty}$, with $\rho_k(z) \in \{\rho^S, \rho^P\}$, for all $z$. We denote by $\pi_k^e(t)$ the expectations parents belonging to group-$k$ form at time $t$.

Let $V^{ij}(\pi_k^e)$ be the utility a parent of type $i \in \{g, b\}$ and group $k \in \{m, M\}$ attributes to his child having preferences $j \in \{g, b\}$ when the expected job profile is $\pi_k^e$. Note that $V^{ij}(\pi_m^e) = V^{ij}(\pi_M^e)$ for all $i, j$ whenever $\pi_m^e = \pi_M^e$. What is crucial here is that $V^{ij}(\pi_k^e)$ depends on the type of jobs (tasks) the parent expects his child to be allocated to during his lifetime. We assume that both parents and employers know the actual proportion of good agents in each group $k = M, m$.

Given a policy expectation $\pi_k^e$, a parent of type $i \in \{g, b\}$ and of group $k \in \{m, M\}$ chooses the education effort $\tau_k^i \in (0, 1]$ that maximizes

$$p^{ii}_k V^{ii}(\pi_k^e) + p^{ij}_k V^{ij}(\pi_k^e) - C(\tau_k^i)$$

where the probabilities $p^{ii}_k$ and $p^{ij}_k$ are defined above.

Given the cost $C(\tau_k^i)$, both types of parents optimally choose $\tau_k^i$ to maximize (13). For good workers, it seems quite natural that they are ready to bear a cost in order to transmit their values to their kids. For bad workers, one may argue that they may not be ready to pay a cost $C(\tau_k^b)$ to transmit their values to their offsprings. In our model, bad workers just invest in their own values\footnote{This is related to the “acting white” literature (Austen-Smith and Fryer, 2005). Individuals exposed to strong peer effects do not invest in certain “good” traits to avoid being rejected by their friends.} as good workers do. In our framework, this means, that parents who do not value work ethic exert effort to transmit this value to their offsprings (for example, by keeping on repeating that employers are exploiting us) in the sense that if they have the possibility to avoid working hard and thus to shirk, they should do it. On the contrary, good workers will condemn any shirking behavior.

Maximizing (13) with respect to $\tau_k^i$ leads to the following first order condition:

$$C'(\tau_k^i) = \frac{dp^{ii}_k}{d\tau_k^i} V^{ii}(\pi_k^e) + \frac{dp^{ij}_k}{d\tau_k^i} V^{ij}(\pi_k^e)$$

(14)
Let $\tau^i_k = \tau^i_k(\pi^e_k, q_m, q_M, \sigma, \gamma)$ be the optimal education effort i.e., the solution to (14). Substituting (2)-(5) in (14) for minority agents (with expectations $\pi^e_m$) and substituting (6)-(9) in (14) for majority agents (with expectations $\pi^e_M$), we easily obtain the optimal education efforts for both types of workers. They are given by:

$$C^0(\tau^g_g) = \left[ V^{gg}(\pi^e_M) - V^{bg}(\pi^e_m) \right] [\sigma \gamma (1 - q_M) + (1 - \sigma \gamma)(1 - q_m)]$$  (15)

$$C^0(\tau^b_g) = \left[ V^{bb}(\pi^e_M) - V^{bg}(\pi^e_m) \right] [\sigma \gamma q_M + (1 - \sigma \gamma) q_m]$$  (16)

$$C^0(\tau^g_M) = \left[ V^{gg}(\pi^e_M) - V^{bg}(\pi^e_M) \right] [(1 - \sigma(1 - \gamma))(1 - q_M) + (1 - \sigma(1 - \gamma)) q_M]$$  (17)

$$C^0(\tau^b_M) = \left[ V^{bb}(\pi^e_M) - V^{bg}(\pi^e_M) \right] [(1 - \sigma(1 - \gamma)) q_M + (1 - \gamma) \sigma q_m]$$  (18)

Note that, since $C(\tau)$ is convex, there is cultural substitution, i.e. socialization inside the family (or direct vertical socialization) and socialization outside the family (or oblique socialization) are cultural substitutes: Parents have less incentive to socialize their children the more widely dominant are their values in the population. At the limit, if $q_m = 1$ and $q_M = 1$, then there is no incentive for good workers to educate their children ($\tau^g_m = \tau^g_M = 0$) since with probability 1 their children will be of type $g$. Note also that when $\sigma = 0$, the optimal education effort only depends on the expectations and on the proportion of good agents in the own population: $\tau^i_k = \tau^i_k(\pi^e_k, q_k)$.

### 2.3 Population dynamics

We are now able to write the dynamic equation for $q_m$ and $q_M$ given $\pi^e_m, \pi^e_M, \lambda, \sigma$ and $\gamma$. We have:

$$q_m(t+1) = \lambda q_m(t) + (1 - \lambda) [q_m p^{gg}_m(t) + (1 - q_m(t)) p^{bg}_m(t)]$$

and

$$q_M(t+1) = \lambda q_M(t) + (1 - \lambda) \left[ q_M(t)p^{gg}_M(t) + (1 - q_M(t)) p^{bg}_M(t) \right]$$

The interpretation of these equations is straightforward. Take the first equation. The proportion of minority good agents at $t+1$ is equal to the proportion of minority good agents who survived from period $t$ (i.e. $\lambda q_m(t)$) plus all new-born minority children whose parents were good workers ($(1 - \lambda)q_m(t)p^{gg}_m(t)$) and all new-born minority children whose parents were bad workers($(1 - \lambda)(1 - q_m(t)) p^{bg}_m(t)$).
Using (2) and (5) for type-\(m\) agents and (6) and (9) for type-\(M\) agents and eliminating time arguments, we can rewrite these equations as:

\[
\Delta q_m = (1 - \lambda)q_m(1 - q_m)(\tau^*_m - \tau^{bs}_m) + (1 - \lambda)\sigma \gamma (q_m - q_M) \left[q_m(\tau^*_m - \tau^{bs}_m) + \tau^{bs}_m - 1\right]
\]

and

\[
\Delta q_M = (1 - \lambda)q_M(1 - q_M)(\tau^*_M - \tau^{bs}_M) + (1 - \lambda)\sigma (1 - \gamma)(q_M - q_m) \left[q_M(\tau^*_M - \tau^{bs}_M) + \tau^{bs}_M - 1\right]
\]

where \(\Delta q_k = q_k(t + 1) - q_k(t)\) and \(\tau^*_k = \tau^*_k(\pi^e_k, q_m, \sigma, \gamma)\).

We now introduce some definitions that will be useful for the remaining part of the paper.

Let \((q_M(t), q_m(t))\) denote the state of the economy at time \(t\).

Definition 1 Assume that \((q_M(t_0), q_m(t_0)) = (q^*_M, q^*_m)\) for some \(t_0\). We say that \((q^*_M, q^*_m)\) is a steady state under rational expectations iff

1. (Profit maximization)

   The proportion \((1 - \theta)\) of non-prejudiced employers choose

   \((\rho_M(t), \rho_m(t)) = (\mu(q^*_M), \mu(q^*_m))\) for all \(t > t_0\)

   and the prejudiced employers choose

   \((\rho_M(t), \rho_m(t)) = (\mu(q^*_M), \rho^P)\) for all \(t > t_0\)

2. (Rational expectations)

   \[\pi^e_M(t_0) = \{\rho_M(t)\}_{t=t_0+1}^\infty\] (21)

   \[\pi^e_m(t_0) = \{\theta \rho^P + (1 - \theta)\rho_m(t)\}_{t=t_0+1}^\infty\] (22)

3. (Rest point)

   \[\Delta q^*_m = \Delta q^*_M = 0\]
Definition 2 The steady state \((q_M^*, q_m^*)\) is reachable from \((q_M(t_0), q_m(t_0))\) under rational expectations if there is a sequence \(\{q_M(t), q_m(t)\}\) with \(\lim_{t \to \infty}((q_M(t), q_m(t))) = (q_M^*, q_m^*)\) such that:

1. All non prejudiced employers maximize their profits at each \(t\), namely choose \(\mu(q_k(t))\).
2. All prejudiced employers follow the pooling strategy with minority workers and \(\mu(q_M(t))\) with the majority ones.
3. Parents optimally choose their education efforts given expectations (21) and (22)
4. The dynamics governed by (19) and (20) generate \(\{q_M(t), q_m(t)\}\)\(\infty_{t=t_0}\).

We say that prejudices are self-fulfilled in steady-state \((q_M^*, q_m^*)\), if all firms employing minority workers maximize their profits under the pooling strategy, namely \(q_m^* < \tilde{q}\).

3 Equilibria with complete segregation

In this section, we characterize the dynamic behavior of the economy when \(\sigma = 0\). The dynamic equations (19) and (20) are now given by:

\[
\Delta q_m = (1 - \lambda)q_m(1 - q_m)(\tau_m^{g*} - \tau_m^{b*}) \tag{23}
\]

and

\[
\Delta q_M = (1 - \lambda)q_M(1 - q_M)(\tau_M^{g*} - \tau_M^{b*}) \tag{24}
\]

Since the dynamics of the minority workers are independent of those of the majority workers, we will first analyze (23) and then (24).

We have the following proposition that characterizes all possible stable steady-states of minority workers.

Proposition 1 Assume \(\underline{q} < \tilde{q} < \overline{q} \leq 1\), \(\sigma = 0\) and (10) holds. Then, the only stable steady-states for the minority workers are characterized as follows:

(i) If \(\theta \leq \overline{q}\),

\(q_m^* \in \{\underline{q}, \theta \underline{q} + (1 - \theta)\overline{q}\}\)
If $\theta > \theta_0$, $q^*_m = \frac{q}{\bar{q}}$

where

$$q = \frac{c}{c + e}, \quad \bar{q} = \frac{c + \alpha (\Omega - \varpi)}{c + e}$$

and $\bar{\theta} = \frac{q - \tilde{q}}{\bar{q} - \tilde{q}}$ \hspace{1cm} (25)

**Proof:** See the Appendix.

Note that this model has multiple equilibria when $\theta \leq \theta_0$. In this case there is a “good” equilibrium with a high level of good workers. In this equilibrium, only the discriminating employers use the pooling strategy. In the “bad” equilibrium all employers, discriminating or not, use the pooling strategy. When the proportion of discriminating employers is high enough, $\theta > \theta_0$, there cannot be an equilibrium where the non discriminating employers follow the separating strategy for the minority workers. The intuition for this result is simple: if parents expect the non discriminatory employers to offer their children the separating contract, the economy would converge to a state, $\theta q + (1 - \theta) \bar{q} < \tilde{q}$, for which the separating strategy wouldn’t be optimal. This “bad” steady-state equilibrium illustrates the concept of “ghetto culture” put forward in the introduction. Because the two communities (minority and majority) are totally separated and parents anticipate discrimination in the labor market, the main value that is transmitted in the “ghetto” is a “bad” work ethic. Interestingly, in his book, Wilson (1987) argued that the isolation of the black community prevents many residents from developing a work ethic because the lifestyle of the employed—daily routine, regular attendance, self-discipline—remains alien.

We have the following corollary:

**Corollary 1** Assume $q < \tilde{q} < \bar{q} \leq 1$, $\sigma = 0$ and (10). Then, under rational expectations:

1. If $\theta \leq \bar{\theta}$, some reachable steady-state equilibria are such that prejudices are not self-fulfilled.

2. If $\theta > \bar{\theta}$, prejudices are always self-fulfilled.

**Proof:** See the Appendix.

If a high enough proportion of employers have prejudices ($\theta \geq \bar{\theta}$), their beliefs are correct in steady state. When $\theta < \bar{\theta}$ prejudices may or may not be fulfilled depending on the behavior of the non prejudiced employers, the expectations and the initial conditions.
Let us now analyze the steady-state equilibria with rational expectations for the majority workers.

**Proposition 2** Assume \( \underline{q} < \tilde{q} < \overline{q} \leq 1 \), \( \sigma = 0 \) and (10) holds. The population of majority workers has multiple stable steady states:

\[
\begin{align*}
q^*_M \in \{ \underline{q}, \overline{q} \}
\end{align*}
\]

where \( \underline{q} \) and \( \overline{q} \) are defined by (25).

**Proof.** As in Proposition 1, just let \( \theta = 0 \).

There are always two stable steady-state equilibria. Note that since employers are not prejudiced against this group of workers, in the good equilibrium, the proportion of good workers from the majority group is higher than from the minority group (\( \overline{q} > \theta \underline{q} + (1 - \theta)\overline{q} \); see (i) in Proposition 1).

Some of the transition dynamics analyzed above involve future changes in the employers’ behavior that have to be, under rational expectations, perfectly forecasted by the parents in previous periods. We here analyze dynamics under less forward-looking parents. As we will see in the following proposition, if expectations are adaptive, the set of reachable equilibria (given \( \theta \)) from a given initial state, \((q_M(t_0), q_m(t_0))\), is a singleton.

Let us denote by \( \{\rho\}^{+\infty} \) the (infinite) sequence with \( \rho(t) = \rho \) for all \( t \). Assume that parents form expectation adaptively, namely at each period \( t \), expect that from \( t + 1 \) onwards all (non prejudiced) employers will choose the strategy that is optimal at \( t \),

\[
\begin{align*}
\pi^e_M(t) &= \{\rho_M(t)\}^{+\infty} \tag{26} \\
\pi^e_m(t) &= \{\theta \rho^P + (1 - \theta)\rho_m(t)\}^{+\infty} \tag{27}
\end{align*}
\]

**Proposition 3** Assume \( \underline{q} < \tilde{q} < \overline{q} < 1 \), \( \sigma = 0 \) and (10). Under adaptive expectations:

1. If \( \theta \leq \overline{q} \), \( \lim_{t \to \infty} q_m(t) = \overline{q} \) for all \( q_m(t_0) \geq \overline{q} \) (prejudices are not self-fulfilled) and \( \lim_{t \to \infty} q_m(t) = \underline{q} \) for all \( q_m(t_0) < \overline{q} \) (prejudices are self-fulfilled).

2. If \( \theta > \overline{q} \), \( \lim_{t \to \infty} q_m(t) = \underline{q} \) for all initial conditions \( q_m(t_0) \), i.e. prejudices are always self-fulfilled.

3. \( \lim_{t \to \infty} q_M(t) = \overline{q} \) for all \( q_M(t_0) \geq \overline{q} \) and \( \lim_{t \to \infty} q_m(t) = \underline{q} \) for all \( q_M(t_0) < \overline{q} \).
Proof: See the Appendix.

When expectations are adaptative, the initial condition pins down the steady state.

Let us now focus on policy implications of the model. We can shed some light on policies aiming at fighting discrimination, such as Affirmative Action (positive discrimination) and integration policies.

4 Affirmative Action

Let us start by considering an affirmative-action policy that consists in giving a preferential treatment to minority groups, for example by imposing minimum hiring quotas to firms. We analyze two possible positive discrimination policies:

In the first one, the policy consists of imposing to all firms a quota $\phi$ of minority workers who are allocated to task 1. We shall call this policy Affirmative Action Policy 1 (AAP1 hereafter).

In the second one, firms are obliged to treat a proportion $\phi$ of minority workers the same way they treat majority workers, namely use with them the separating policy. This is referred to as Affirmative Action Policy 2 (AAP2 hereafter).

As we will see in the following proposition only the second policy is effective.

**Proposition 4** Assume $q < \tilde{q} < \overline{q} \leq 1$, $\sigma = 0$ and (10). Then

**Under Affirmative Action Policy 1:**

(i) If $\theta + \phi(1 - \theta) \leq \overline{q}$ then,

$$q_m^* \in \{q, (\theta + \phi - \phi \theta)q + (1 - \theta)(1 - \phi)\overline{q}\}$$

(ii) If $\theta + \phi(1 - \theta) > \overline{q}$ then,

$$q_m^* = \overline{q}$$

**Under Affirmative Action Policy 2,**

(i) If $(1 - \phi) \leq \overline{q}$, then

$$q_m^* = \theta (1 - \phi)q + (1 - \theta(1 - \phi))\overline{q}$$

(ii) If $(1 - \phi) > \overline{q} \geq (1 - \phi)\theta$, then

$$q_m^* \in \{\theta (1 - \phi)q + (1 - \theta(1 - \phi))\overline{q}, (1 - \phi)q + \phi\overline{q}\}$$

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(iii) If \((1 − φ)θ > \overline{θ}\) then,

\[ q_m^* = (1 − φ)\overline{q} + φ\overline{q} \]

**Proof.** See the Appendix.

Note first that since there is no integration between the majority and the minority groups, an Affirmative Action policy does not affect the former. Second, comparing these results with those of Proposition 1, it is easy to see that AAP1 yields worse equilibrium outcomes than if the policy were not implemented. The opposite happens with AAP2.

To understand the opposite effects of the two policies assume that the economy is in the good steady state, \(θq + (1 − θ)\overline{q}\). In this steady state, all non prejudiced employers offer the separating contract to the workers they employ. These contracts give an advantage to the good workers (since bad workers are detected with positive probability). The probability of obtaining these separating contracts decreases when AAP1 is introduced. As a result of this, good workers put relatively less effort than bad ones. If \(θq + (1 − θ)\overline{q}\) is close enough to \(\overline{q}\) or/and \(φ\) is high enough, the economy will converge to the bad (and in this case unique) steady state \(q\). Instead, when AAP2 is implemented, a larger share of workers are automatically screened and the return is higher for good workers. Thus, good workers put relatively more effort than bad ones and the equilibrium proportion of good workers is higher in equilibrium. If \(φ\) is high enough, prejudices are never self-fulfilled under AAP2 (see case (i) in Proposition 4).

Our results are related to that of Coate and Loury (1993). In their paper, affirmative action is modeled as a government-mandated constraint on employers requiring them to assign workers from each group (minority and majority) to more rewarding jobs at the same rate. Coate and Loury find that there do exist circumstances under which affirmative action will necessary eliminate negative stereotypes but there are also equally plausible circumstances under which minority workers continue to be (correctly) perceived as less capable, despite the affirmative-action constraint. The reason that affirmative action may sometimes fail is as follows. If employers continue to hold negative views about minority workers then, to comply with the affirmative-action policy, they must lower the standard used for assigning these workers to the better jobs. Lowering the standard may reduce investment incentives because the favored (minority) workers see themselves as likely to succeed without acquiring the relevant skills. Thus employers’ negative stereotypes can continued to be confirmed in the equilibrium under affirmative action. Coate and Loury show that this equilibrium is more likely to exist if the proportion of minority workers is relatively rare in the population.
Even if the mechanism is different, this result is close to ours when affirmative action policy 1 is implemented. In our case, compared to the equilibrium without AAP1, good workers put relatively less effort in transmitting their traits than bad workers because a fraction of workers, good or bad, are always guaranteed to obtain the best jobs. Our conclusion is that only AAP2 should be implemented because it gives the right incentives to good workers to transmit their traits.

5 Integration

We focus now on the effect of an integration policy, like for example the Moving to Opportunity” (MTO) programs.\textsuperscript{12} Let us analyze the behavior of the economy when we integrate the minority and the majority groups. For analytical simplicity, we will assume that all employers are prejudiced. It should be clear that if integration has some positive effect in this worse-case scenario ($\theta = 1$) its positive effect on the minority group will be enhanced by the existence of some non prejudiced employers ($\theta << 1$). Thus, assume that $\theta = 1$.

It follows from (19) and (20) that when $\sigma \neq 0$, $q_m = 0$ (resp. $q_m = 1$) is a rest point only if $q_M = 0$ (resp. $q_M = 1$). Similarly, $q_M = 0$ (resp. $q_M = 1$) is a rest point only if $q_m = 0$ (resp. $q_m = 1$). It is however easily checked that these rest points ($q_m = q_M = 0$ and $q_m = q_M = 1$) are unstable. Our main result is as follows:

\textbf{Lemma 1} Assume $\pi^e_M = \{\rho^S\}^\infty$ and $q_m < q_M$, then

\[ \frac{d\Delta q_m}{d\sigma} > 0 \text{ if } \max\{\tau^g_m, \tau^b_m\} \leq 1/2. \]

If $\pi^e_m = \{\rho^P\}^\infty$ and $q_M > q_m$, then

\[ \frac{d\Delta q_M}{d\sigma} < 0 \text{ if } \max\{\tau^g_M, \tau^b_M\} \leq 1/2. \]

\textsuperscript{12}By giving housing assistance (i.e. vouchers and certificates) to low-income families, the MTO programs help them to relocate to better and richer neighborhoods. The results of most MTO programs (in particular for Baltimore, Boston, Chicago, Los Angeles and New York) show a clear improvement of the well-being of participants and better labor market outcomes (see, in particular, Ladd and Ludwig, 2001, Katz et al. 2001, Rosenbaum and Harris, 2001 and Kling et al. 2005). Observe that the MTO programs are not targeted on minority families (such as blacks) by rather on poor families. But since the two are correlated, this is a good example of an integration policy. Another policy is busing since this policy aims at carrying by bus black pupils to a ‘white’ school of a different area.
**Proof.** See the Appendix.

The following figures illustrate this lemma. Figure 1 illustrates how $\Delta q_m$ changes when we increase integration given $q_M = \bar{q}$. The solid line represents $\Delta q_m$ for $\sigma = 0$ and the dashed line for $\sigma = 0.2$. Note that when $\sigma > 0$, $q_m$ is increasing at $\bar{q}$. Figure 2 illustrates $\Delta q_M$ for given $q_m = \bar{q}$ for $\sigma = 0$ (solid line) and $\sigma = 0.2$ (dashed line). When $\sigma > 0$, $q$ and $\bar{q}$ are no longer steady-state values.

![Figure 1: Dynamics of population for the minority group for $q_M = \bar{q}$, $\sigma = 0$ (solid line) and $\sigma = 0.2$ (dashed line)](image1)

![Figure 2: Dynamics of population for the majority group for $q_m = \bar{q}$, $\sigma = 0$ (solid line) and $\sigma = 0.2$ (dashed line)](image2)

When integration occurs ($\sigma \neq 0$), the dynamics of minority and majority workers are given by (19) and (20), respectively, and are more complex than in the case of complete segregation ($\sigma = 0$). Indeed, in the latter, the dynamics of $q_m$ (resp. $q_M$) were totally
determined by the difference in optimal education efforts, \( \tau^g_m - \tau^b_m \) (resp. \( \tau^g_M - \tau^b_M \)).\textsuperscript{13} To solve analytically the dynamic system with integration is complicated since, contrary to the case when \( \sigma = 0 \), the steady states depend on the specific form of the cost function. Lemma 1 gives a characterization of the population dynamics when \( \sigma > 0 \). There are in fact two effects. When \( \sigma > 0 \) and \( q_m < q_M \), the probability that a naive minority child meets a good worker (regardless of group) increases since \( q_m + \sigma \gamma (q_M - q_m) > q_m \) (positive effect). As a result, good minority parents put less effort in education than when \( \sigma \) is lower (negative effect). Lemma 1 gives a sufficient condition for the positive effect to dominate. The same intuition applies for the population dynamics of the majority group.

The previous lemma has two straightforward implications that are given by the following proposition.

**Proposition 5** Consider an integrated society, i.e. \( \sigma \neq 0 \).

(i) Assume \( \pi^e_m = \{p^P\}^\infty \). Then \( q_m = q \) is a rest point only when \( q_M = q \). If (34) holds at \( q_m = q \) and \( q_M > q_m \), then integration increases the proportion of good workers from the minority group.

(ii) Assume \( \pi^e_M = \{p^S\}^\infty \). Then \( q_M = q \) is a rest point only when \( q_m = q \). If (34) holds at \( q_M = q \) and \( q_m < q_M \), integration decreases the proportion of good workers from the majority group.

If integration is introduced when the economy is in a steady state with \( q_M > q_m^\ast \), its effect is to increase the proportion of good disadvantaged workers and to decrease the proportion of good advantaged workers. This is true even in the extreme case of \( \theta = 1 \). With some degree of integration, \( q_m \) converges to a value (which will depend on the value of \( \sigma \)) that is greater than \( q \) and \( q_M \) converges to a value that is lower than \( q \).

This result is in accordance with most empirical studies. For example, a recent paper by Guryan (2004) shows that the desegregation plans that have been implemented in American schools for the last forty years, have mainly benefited the black students by reducing their high school drop out rates while they had no effect on the dropout rates of whites. Peer effects are shown to be one of the main explanations of this result. Studying the Metco program, a long-running desegregation program that sends mostly Black students out of the Boston

\textsuperscript{13}See equation (23) for the minority group and (24) for the majority group.
public school district to attend schools in more affluent suburban districts, Angrist and Lang (2005) find similar results. There is also a growing literature in the fields of public finance, development and urban economics that shows that investments in public goods, tastes for redistribution, and other forms of civic behavior are less common in racially or ethnically diverse communities (see, in particular, Alesina et. al., 1999, Alesina and La Ferrara, 2000, Luttmer, 2001, Vigdor, 2004).

Racial integration is indeed a very sensitive and highly debated policy and our model can help us to understand why the advantaged group (for example whites) may show resistance to integration and be reluctant to accept social mixing with the disadvantaged group (for example nonwhites).14

Interestingly, Chaudhuri and Sethi (2003), who incorporate neighborhood effects into an otherwise standard model of statistical discrimination, find a similar result, even though the mechanism is totally different. In their paper, increasing integration tends to lower the costs of human acquisition in the disadvantaged group while raising those in the advantaged group. Thus, if integration proceeds far enough, the authors show that negative stereotypes cannot be sustained.

6 Concluding remarks

We have introduced a dynamic model of cultural transmission to explain discrimination in the labor market. We have shown that if the proportion of taste-based prejudiced employers is high enough, prejudices are always confirmed in equilibrium. Otherwise, multiple equilibria exist, with and without discrimination. We have also studied different policies aiming at reducing discrimination. Both affirmative action15 and integration policies may work. The mechanisms through which these two policies affect the quality of the workers are different, though. Affirmative action policies directly affect the expected payoff of the different types of minority workers and the parents’ incentives to invest on those traits. By “improving” the

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14For instance, in 1974, federal judge W. Arthur Garrity ordered to integrate Boston’s schools through forced busing (black kids were driven by bus to white schools). Twenty five years after, in juned 1999, facing pressure from a lawsuit by white parents and advocates of neighborhood schools, the city’s school board voted 5-2 to stop the busing policy and to adopt a race-blind admissions policy starting in September 2000 (Education Week, 08/04/99 edition, by Caroline Hendrie).

15In this discussion, we only focus on Affirmative Action Policy 2.
quality of the peers minority children interact with, integration policy has a positive effect on minority workers. The opposite happens for majority children since, after integration, they interact with a “worse” quality peer group. From a political economy perspective, it is likely that all workers will support the affirmative action policy while only the minority may favor the integration policy. As far as employers are concerned, it seems plausible that they may object to affirmative policies that impose too small quotas. The reason for this opposition is that they are forced to offer contracts that are suboptimal given the average composition of the minority workers. When the affirmative action quotas are high enough, both employers and workers benefit from the policy.

References


7 Appendix

7.1 Proof of Proposition 1

We first prove the following lemma.

**Lemma 2** There are three rest points: (i) \( q^*_m = 0 \), (ii) \( q^*_m = 1 \) and (iii) \( q^*_m \in ]0,1[. Only the last one is globally stable.

**Proof.** By solving \( \Delta q_m = 0 \) for (23), we easily obtain the three rest points, 0, 1 and the interior \( q_m \) which solves \( \tau^g_m(\pi^e_m, q_m) = \tau^b_m(\pi^e_m, q_m) \). By differentiating (23), we can see that 0 and 1 are unstable:

\[
\frac{\partial \Delta q_m}{\partial q_m} \bigg|_{q_m=0} = (1 - \lambda) \left[ \tau^g_m(\pi^e_m, 0) - \tau^b_m(\pi^e_m, 0) \right] = (1 - \lambda) \tau^g_m(\pi^e_m, 0) > 0 \quad (28)
\]

\[
\frac{\partial \Delta q_m}{\partial q_m} \bigg|_{q_m=1} = (1 - \lambda) \left[ \tau^g_m(\pi^e_m, 1) - \tau^b_m(\pi^e_m, 1) \right] = -(1 - \lambda) \tau^b_m(\pi^e_m, 1) < 0. \quad (29)
\]

The stability of the interior rest point follows directly from the continuity of (23) and (28) and (29).

When \( \sigma = 0 \), it follows from (15) and (16) that, \( \tau^g_m(\pi^e_m, q_m) \geq \tau^b_m(\pi^e_m, q_m) \) when

\[
q_m \leq \frac{V^{gg}(\pi^e_m) - V^{gb}(\pi^e_m)}{V^{bb}(\pi^e_m) - V^{bg}(\pi^e_m) + V^{gg}(\pi^e_m) - V^{gb}(\pi^e_m)} = q(\pi^e_m).
\]

We now compute \( V^{gg}(\pi^e) - V^{gb}(\pi^e) \) and \( V^{bb}(\pi^e) - V^{bg}(\pi^e) \) for different expectations \( \pi^e \):

\[
V^{gg} \left( \{\rho^S\}^\infty \right) - V^{gb} \left( \{\rho^S\}^\infty \right) = \frac{c + \alpha (\Omega - \pi)}{1 - \lambda} \quad (30)
\]

\[
V^{bb} \left( \{\rho^S\}^\infty \right) - V^{bg} \left( \{\rho^S\}^\infty \right) = \frac{e - \alpha (\Omega - \pi)}{1 - \lambda} \quad (31)
\]

\[
V^{gg} \left( \{\rho^P\}^\infty \right) - V^{gb} \left( \{\rho^P\}^\infty \right) = \frac{e}{1 - \lambda} \quad (32)
\]

\[
V^{bb} \left( \{\rho^P\}^\infty \right) - V^{bg} \left( \{\rho^P\}^\infty \right) = \frac{e}{1 - \lambda} \quad (33)
\]

If \( \pi^e_m = \{\rho^P\}^\infty \), then \( q(\pi^e_m) = q \) since \( \mu(q) = \rho^P \), and \( q_m = q \) is a steady state under rational expectations.

If \( \pi^e_m = \{ \theta \rho^P, (1 - \theta)\rho^S \}^\infty \), \( q(\pi^e_m) = \theta q + (1 - \theta)\pi \).

When \( \theta \leq \bar{\theta} \), \( \theta q + (1 - \theta)\pi \geq q \) and since \( \mu(\theta q + (1 - \theta)\pi) = \rho^S \), \( q_m = \theta q + (1 - \theta)\pi \) is a steady state under rational expectations.

When \( \theta > \bar{\theta} \), \( \mu(\theta q + (1 - \theta)\pi) = \rho^P \) and \( \theta q + (1 - \theta)\pi \) cannot be an equilibrium.
7.2 Proof of Corollary 1

1. Assume \( \theta > \bar{\theta} \). Then \( \theta q + (1 - \theta)\bar{q} < \tilde{q} \). Assume the minority group anticipate that non-prejudiced employers will always implement the separating strategy to the minority workers. From (15) and (16), \( \tau^g \left( \{ \theta \rho^P, (1 - \theta)\rho^S \}^{\infty}, q_m \right) \geq \tau^b \left( \{ \theta \rho^P, (1 - \theta)\rho^S \}^{\infty}, q_m \right) \) when

\[
q_m \lesssim \theta q + (1 - \theta)\bar{q}.
\]

\( \lim_{t \to \infty} q_m(t) = \theta q + (1 - \theta)\bar{q} < \tilde{q} \), and \( q_m(t) > \tilde{q} \) for all \( t \geq t_j \) and \( j \geq 0 \). This cannot be an equilibrium path since from \( t_j \) onwards \( \mu(q_m(t)) = \rho^P \). In the only steady-state prejudices have to be self-fulfilled.

2. Assume now that \( \theta < \bar{\theta} \). Then \( \theta q + (1 - \theta)\bar{q} > \tilde{q} \). Start at \( q_m(t_0) > \tilde{q} \) with minority families anticipating that non-prejudiced employers always implement the separating strategy to minority workers. From (15) and (16), \( \tau^g \left( \{ \theta \rho^P, (1 - \theta)\rho^S \}^{\infty}, q_m \right) \geq \tau^b \left( \{ \theta \rho^P, (1 - \theta)\rho^S \}^{\infty}, q_m \right) \) when

\[
q_m \lesssim \theta q + (1 - \theta)\bar{q}.
\]

\( \lim_{t \to \infty} q_m(t) = \theta q + (1 - \theta)\bar{q} > \tilde{q} \), and \( q_m(t) > \tilde{q} \) for all \( t \geq t_0 \). This is now an equilibrium with rational expectations.

7.3 Proof of proposition 3

1. \( \theta > \bar{\theta} \).
   
i) Assume first that \( q_m(t_0) \geq \tilde{q} \).

   By (12) \( \mu(q_m) = \rho^S \) whenever \( q_m \geq \tilde{q} \). By (27)

   \[
   \pi^e_m(t_0) = \{ \theta \rho^P + (1 - \theta)\rho^S \}^{+\infty}
   \]

   Under these expectations \( \Delta q_m \lesssim 0 \) whenever \( q_m \gtrless \theta q + (1 - \theta)\bar{q} \) and \( q_m \) converges towards \( \theta q + (1 - \theta)\bar{q} \) (whenever \( q_m(t) > \tilde{q} \)).

   Since \( \theta q + (1 - \theta)\bar{q} < \tilde{q} \), there is a \( t_j > t_0 \) such that \( q_m(t_j) < \tilde{q} \leq q_m(t_{j-1}) \). By (12), \( \mu(q_m(t_j)) = \rho^P \) and by (27)

   \[
   \pi^e_m(t_k) = \{ \theta \rho^P + (1 - \theta)\rho^P \}^{+\infty}
   \]

   Under these expectations \( \Delta q_m \lesssim 0 \) whenever \( q_m \gtrless q \). Since \( q_m(t) < \tilde{q} \) for all \( t \geq t_k \), \( \lim_{t \to \infty} q_m(t) = \tilde{q} \).

   ii) Assume now that \( q_m(t_0) < \tilde{q} \). By (12), \( \mu(q_m(t_0)) = \rho^P \) and by (27)

   \[
   \pi^e_m(t_0) = \{ \theta \rho^P + (1 - \theta)\rho^P \}^{+\infty}
   \]

   Under these expectations \( \Delta q_m \lesssim 0 \) whenever \( q_m \gtrless q \). Since \( q(t) < \tilde{q} \) for all \( t \geq t_0 \), \( \lim_{t \to \infty} q_m(t) = \tilde{q} \).
2. \( \theta \leq \overline{q} \).

i) Assume first that \( q_m(t_0) \geq \overline{q} \). Since \( \theta q + (1 - \theta) \overline{q} > \overline{q} \), \( \pi_m^e(t) = \{ \theta \rho^p + (1 - \theta) \rho^s \}^{+\infty} \) for all \( t \geq t_0 \) and \( \lim_{t \to \infty} q_m(t) = \theta q + (1 - \theta) \overline{q} > \overline{q} \).

ii) Assume now that \( q_m(t_0) < \overline{q} \). Part ii) above shows that \( \lim_{t \to \infty} q_m(t) = \overline{q} \).

3. Let \( \theta = 0 \) and apply part 2.

7.4 Proof of Proposition 4

We have:
\[
\tau^g(\pi_k^e, q_k) \geq \tau^b(\pi_k^e, q_k)
\]

when
\[
q_k \geq \frac{V^{gb}(\pi_k^e) - V^{gb}(\pi_k^e)}{V^{gb}(\pi_k^e) - V^{gb}(\pi_k^e) + V^{gb}(\pi_k^e) - V^{gb}(\pi_k^e)}
\]

Calculating the RHS of this equation for the different policies, we obtain the following results:

Under Affirmative Action policy 1:
1. \( \tau^a_k(\pi^e, q_k) \geq \frac{V^{gb}(\pi_k^e)}{V^{gb}(\pi_k^e) - V^{gb}(\pi_k^e) + V^{gb}(\pi_k^e) - V^{gb}(\pi_k^e)}\) when \( q_k \leq \theta(1 - \phi)(1 - \theta)q + (1 - \phi)q \) and \( \overline{q}(1 - \theta)(1 - \phi)q \leq \theta(1 - \phi)q \) when \( (\theta + \phi - \theta \phi)q \leq \theta(1 - \phi)q \).

Under Affirmative Action policy 2:
1. \( \tau^a_k(\pi^e, q_k) \geq \frac{V^{gb}(\pi_k^e)}{V^{gb}(\pi_k^e) - V^{gb}(\pi_k^e) + V^{gb}(\pi_k^e) - V^{gb}(\pi_k^e)}\) when \( q_k \leq (1 - \phi)(\overline{q} - q) \) and \( \overline{q} - \theta(1 - \phi)(\overline{q} - q) \geq \overline{q} \) when \( \theta(1 - \phi) \leq \theta \).

2. \( \tau^a_k(\pi^e, q_k) \geq \frac{V^{gb}(\pi_k^e)}{V^{gb}(\pi_k^e) - V^{gb}(\pi_k^e) + V^{gb}(\pi_k^e) - V^{gb}(\pi_k^e)}\) when \( q_k \leq (1 - \phi)(\overline{q} - q) \) and \( \overline{q} - \theta(1 - \phi)(\overline{q} - q) < \overline{q} \) when \( (1 - \phi) > \theta \).

7.5 Proof of Lemma 1

From (19), we have:
\[
\Delta q_m = (1 - \lambda)q_m(1 - q_m)(\tau^a_m - \tau^b_m) + (1 - \lambda)\sigma \gamma(q_m - q_M)\left[q_m(\tau^a_m - \tau^b_m) + \tau^b_m - 1\right] = (1 - \lambda)[q_m(1 - \tilde{q}_m)\tau^a_m - \tilde{q}_m(1 - q_m)\tau^b_m + \tilde{q}_m - q_m]
\]

where \( \tilde{q}_m = q_m + \sigma \gamma(q_M - q_m) \). Differentiating this equation with respect to \( \sigma \) leads to:
\[
\frac{d\Delta q_m}{d\sigma} = (1 - \lambda)\gamma(q_M - q_m)(1 - q_m)\tau^a_m - (1 - q_m)\tau^b_m - q_m \frac{d\tau^a_m}{dV} V^g_m(1 - \tilde{q}_m) - (1 - q_m) \frac{d\tau^b_m}{dV} V^b_m \tilde{q}_m
\]

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If $q_M > q_m$ then

$$\frac{d\Delta q_m}{d\sigma} > (1 - \lambda)\gamma (q_M - q_m)(1 - 2 \max\{\tau^g_m, \tau^b_m\}).$$

since

$$\frac{d\tau^k_m(x)}{dx} x \leq \tau^k_m(x)$$

by the concavity of $\tau^k_m$. Hence a sufficient condition for $\frac{d\Delta q_m}{d\sigma}$ to be positive at $\widehat{q}_m$ is that

$$\max\{\tau^g_m((V^b_m - V^b_g)(1 - \widehat{q}_m)), \tau^b_m((V^b_m - V^b_g)\widehat{q}_M)\} \leq 1/2$$ (34)

Similarly, from (20), we have:

$$\Delta q_M = (1 - \lambda)q_M(1 - q_M)(\tau^g_M - \tau^b_M) + (1 - \lambda)\sigma(1 - \gamma)(q_M - q_m) [q_M(\tau^g_M - \tau^b_M) + \tau^b_M - 1] = (1 - \lambda)[q_M(1 - \widehat{q}_M)\tau^g_M - \widehat{q}_M(1 - q_M)\tau^b_M + \widehat{q}_M - q_M]$$

where $\widehat{q}_M = q_M + \sigma(1 - \gamma)(q_m - q_M)$. Differentiating with respect to $\sigma$, we obtain:

$$\frac{d\Delta q_M}{d\sigma} = (1 - \lambda)(1 - \gamma)(q_m - q_M)[1 - q_M\tau^g_M - (1 - q_M)\tau^b_M - q_M \frac{d\tau^g_M}{dV} V^g_M(1 - \widehat{q}_M) - (1 - q_M) \frac{d\tau^b_M}{dV} V^b_M \widehat{q}_M]$$

If $q_M > q_m$, then

$$\frac{d\Delta q_M}{d\sigma} < (1 - \lambda)(1 - \gamma)(q_m - q_M)(1 - 2 \max\{\tau^g_M, \tau^b_M\})$$

A sufficient condition for $\frac{d\Delta q_M}{d\sigma}$ to be negative at $\widehat{q}_M$ when $q_M > q_m$ is that

$$\max\{\tau^g_M((V^b_M - V^b_g)(1 - \widehat{q}_M)), \tau^b_M((V^b_M - V^b_g)\widehat{q}_M)\} \leq 1/2$$ (35)