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Article

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The effects of climate change on a small and open economy: Economic and monetary perspectives[☆]

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ABSTRACT

This policy note analyses the impacts of climate change on a small and open economy, emphasizing its effect on monetary policy. We use a New-Keynesian model to evaluate short- and long-term effects of transition climate change shocks – a permanent and anticipated shock to total factor productivity and an increase in carbon taxes – on main macroeconomic variables, such as potential output, natural interest rate, inflation, and real exchange rate. The results suggest a gradual decline in potential output attributable to climate change, leading to increased production costs. Regarding the short term, the natural interest rate and inflation decrease. The timing of climate change effects on productivity will dictate the extent of their impacts and the response required from monetary policy. In a small and open economy such as Colombia, carbon taxes have a quantitatively low impact on macroeconomic variables in both the short and long run.

Introduction

Climate change poses one of the most pressing contemporary threats to humanity, reshaping our planet through global warming and environmental degradation. The ramifications of this crisis extend far beyond the environment, affecting global economic and financial stability. For instance, increased frequency and intensity of extreme weather events could lead to financial losses. Likewise, the immediate and system-wide transition required to fight climate change could affect several sectors of the economy. These climate-related shocks could threaten central banks' price and financial stability mandates. Consequently, several efforts have recently been made to understand the intricate relationship between climate change, monetary policy, and financial risks (NGFS, 2020).

There is a broad consensus regarding the environmental implications of wide-ranging physical and transition risks. However, there is a lack of agreement on the economic impacts of climate-related risks, given the uncertainty over baseline estimates due to the complexity of the climate change phenomenon (Bernal-Ramírez et al., 2022). While an accurate assessment of the effects of climate change on the economy poses an enormous challenge given its non-linear and scenario-dependent nature, efforts have been made to disentangle the direction of these effects. Overall, the literature suggests that climate change will negatively impact economic output, and the magnitude of this effect depends on the implementation of mitigation policies. These impacts will also be non-linear

[☆] The views expressed in this paper are those of the authors and should not be interpreted as representing the positions of the Banco de la República or its board members.

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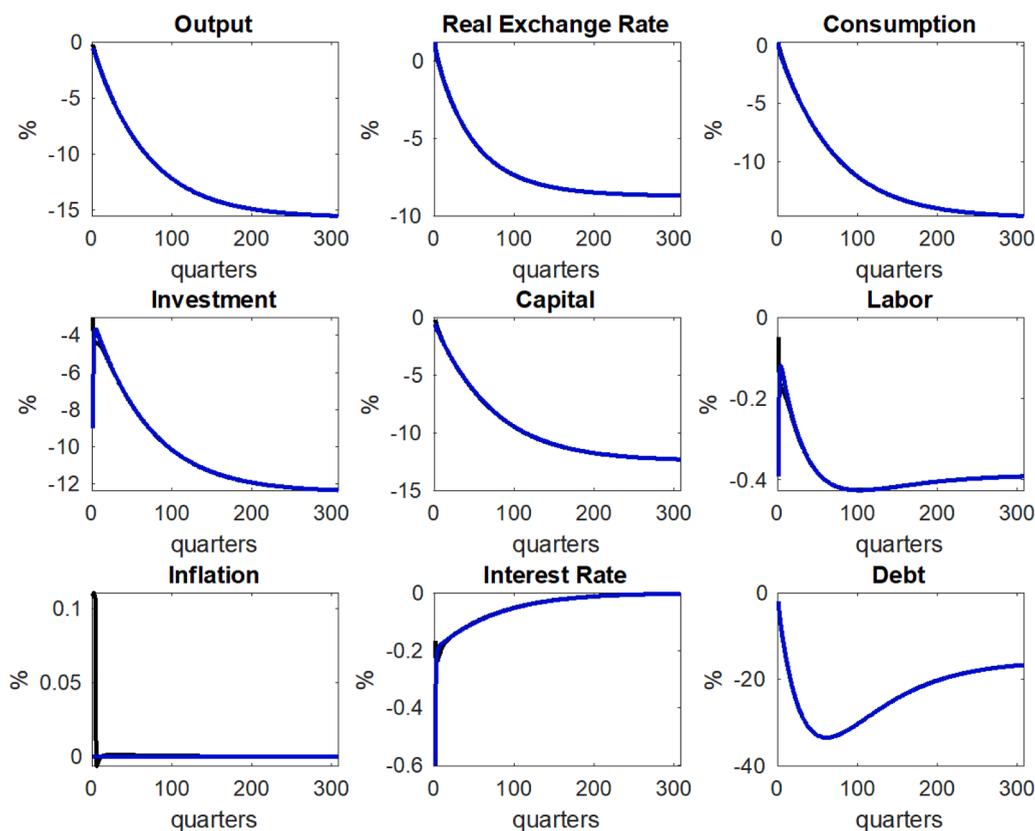


Fig. 1. Long term effects of a 15 % decline in output (shock to TFP due to climate change) on main macroeconomic variables. Blue lines illustrate variable trajectories under flexible prices, while black lines depict the results under sticky prices.

(Bernal-Ramírez et al., 2022), suggesting that more substantial temperature changes lead to disproportionate effects on economic output.

According to several studies, climate change can disrupt economic equilibrium, resulting in supply-side shocks that affect the main economic variables such as GDP, interest rates, prices, and exchange rates (Nordhaus and Moffat, 2017; Hsiang et al., 2017; Kompas et al., 2018; Tol, 2018; Kahn et al., 2019; Szweczyk et al., 2020; Alestra et al., 2020; NFGS, 2021; Mukherjee and Ouattara, 2021; Bernal-Ramírez et al., 2022; Mongelli et al., 2022). For instance, materializing of climate-related risks could lead to output losses ranging from 7.2 % to 23 % of global output (Burke et al., 2015; Kahn, 2019).

Climate change can affect not only economic output but also natural interest rates, given their relationship with several factors such as demographic trends, productivity, household risk aversion, and the need for substantial fiscal policies in climate change (Mongelli et al., 2022). The literature suggests that climate change may exert downward pressure on the natural interest rate. According to ECB (2021), climate-induced migration outflows could reduce labor supply, thereby decreasing the marginal product of capital and lowering the natural interest rate. Furthermore, increased precautionary savings driven by uncertain income prospects due to climate disasters may further contribute to this downward push on the natural interest rate (Cantelmo, 2020).

To analyze the effects of climate change on several macroeconomic variables, some studies have focused on assessing the impact and consequences of environmental policies on the cyclical components of macroeconomic variables such as output, capital, consumption, and employment (Fischer and Springborn, 2011; Heutel, 2012; Angelopoulos et al., 2010, 2013). Another branch has employed DSGE models to assess the impact of climate change on macroeconomic variables and public policy responses. These studies often focus on the interaction between environmental policies to mitigate climate change and monetary policy, with notable work from Annicchiarico et al. (2015) and Economides et al. (2019).

In this policy note we intend to explore the macroeconomic impacts of climate change and mitigation policies, such as carbon taxes, using a Dynamic Stochastic General Equilibrium (DSGE) model tailored for a small open economy. Our model is similar to that presented in Annicchiarico et al. (2015) and Economides (2019); however, it differs in two significant aspects. Firstly, our model comprises two blocks: one simulating an economy under sticky prices, where monetary policy plays a significant role, and the other representing the same economy under flexible prices, where monetary policy does not exert influence. These two blocks are interconnected through the Taylor Rule, which operates exclusively in the sticky prices' world.

As outlined in Woodford (2003), the Taylor Rule dictates that the nominal interest rate responds to the natural interest rate (determined in the flexible prices block), the output gap (measured as deviations between output under sticky prices and flexible

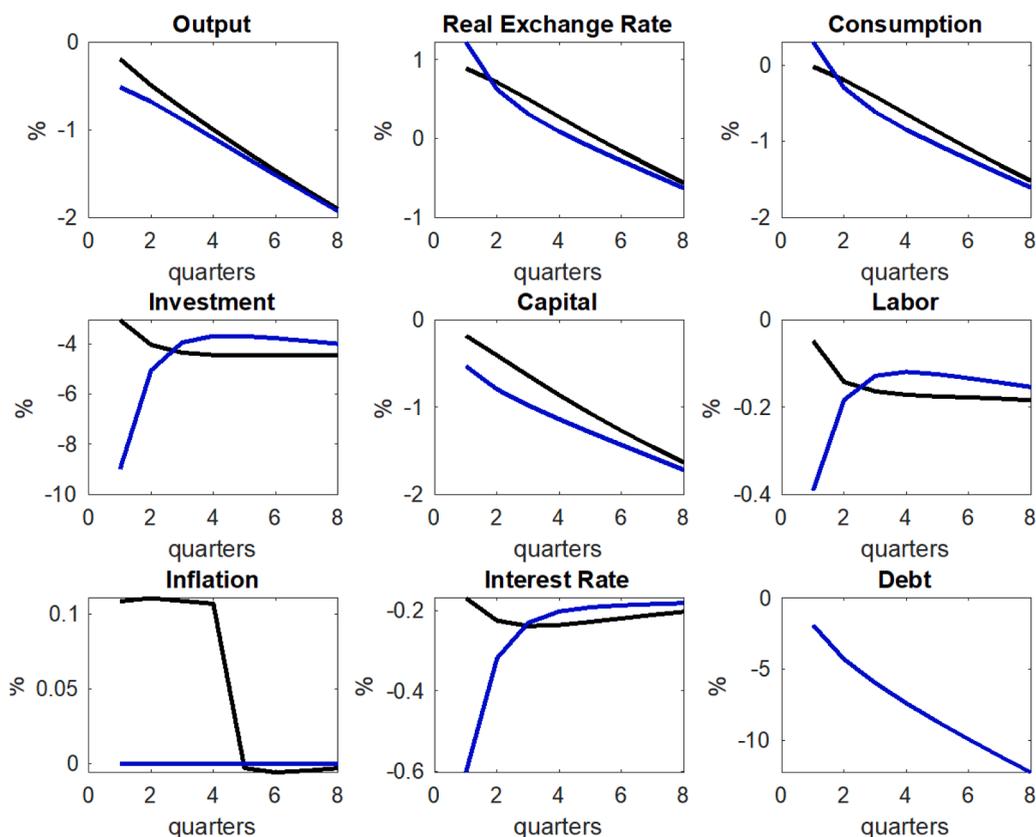


Fig. 2. Short term effects (monetary policy horizon) of a 15 % decline in output (shock to TFP due to climate change) on main macroeconomic variables. Blue lines illustrate variable trajectories under flexible prices, while black lines depict the results under sticky prices.

prices) and the expected deviations of inflation from its target. Secondly, we conduct a quantitative assessment of the impact of climate change and mitigation policies, aiming to compare the optimal responses of economic agents under flexible and sticky prices. The distinction between flexible and sticky prices is pivotal to discern the stance of monetary policy. It is relevant to analyze whether it is contractive or expansive under different dynamics of climate change or in response to several mitigation policies, such as the implementation of carbon taxes.

Our goal is to assess the impact of climate change on monetary policy in a small and open economy. Therefore, we use data from Colombia to calibrate our model. Colombia is a relevant case of study given that it is characterized by being a small and open economy with a net debtor status. Additionally, the nation relies heavily on the export of raw materials, exhibits imperfect capital mobility, and embraces inflation targeting as a core element of its monetary policy. Regarding climate change, Colombia is recognized as one of the most vulnerable countries in the world to the impacts of climate change despite its relatively low carbon dioxide emissions (it accounts for only 0.6 % of global carbon dioxide emissions).

Specifically, we evaluate the effects on main macroeconomic variables under sticky and flexible price frameworks of permanent and anticipated shocks on total factor productivity (TFP) and increases in carbon taxes. Importantly, Our analysis omits modeling a complete fiscal block, aligning with our aim to unravel and quantify the influence of climate change and mitigation policies on monetary policy. The technical appendix presents the equations and assumptions of the entire model and its calibration.

Regarding the studies that use data from Colombia, ours differs from Bernal and Ojeda (2022) given that we follow a structural approach rooted in a DSGE model, while they employ semi-structural models to estimate the trajectories of the natural interest rate and potential GDP in response to climate change. We intend to enhance the understanding of how climate change impacts a small and open economy, its monetary policy responses, and the broader economic context, illuminating the complex relationship between climate change and the economy. Likewise, we intend to provide valuable insights for policymakers and researchers alike.

Methodology

We employ a New Keynesian model tailored for a small and open economy, which we calibrate using data from Colombia to examine the impacts of climate change on monetary policy. Even though most of the effects of climate change will materialize in the long run, we assess short- and long-term effects. In the model, shocks to the total factor productivity will be anticipated and permanent, so variables such as potential output and natural interest rate will be indirectly affected in the short run. Consequently, short-term

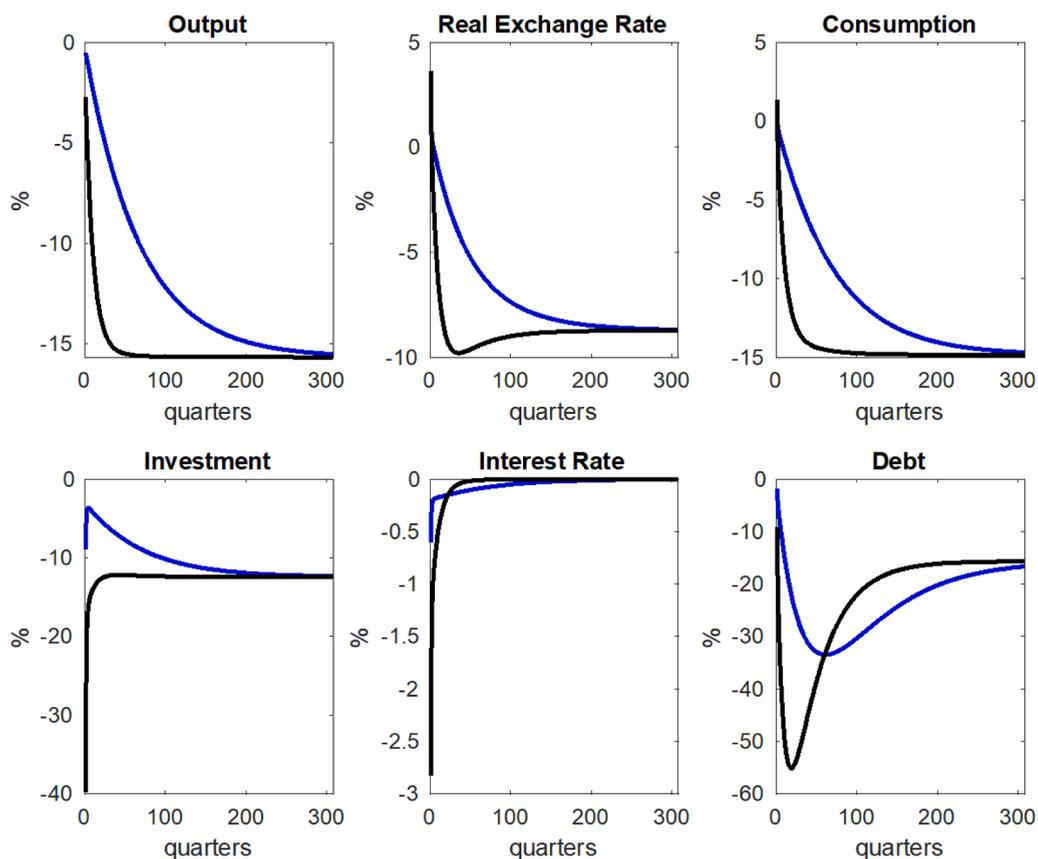


Fig. 3. Effects of a 15 % decline in output (shock to TFP due to climate change) on main macroeconomic variables under flexible prices. Blue lines depict variable trajectories when climate change effects materialize in 2100, while black lines represent variable paths when these effects occur in 2033.

effects on the output gap and monetary policy stance, defined as the difference between the policy and natural interest rates, will be relevant. Likewise, the increase in carbon taxes will affect macroeconomic variables in the short and long run, although it represents a non-anticipated shock.

The Technical Appendix provides a detailed explanation of the model. However, in broad terms, model consists of two sectors: domestic and imported goods, characterized by Calvo rigid prices. It includes investment adjustment costs, consumption habits, separable preferences between consumption and leisure, incomplete markets for external financial assets and a monetary policy that systematically responds to deviations in inflation expectations from its target and movements in the output gap. As [Woodford \(2003\)](#) outlined, the definitions of natural interest rate and potential output align with those occurring in a world of flexible prices and without markup shocks. Unlike the small and open economy models that prevail in this literature, our model features two blocks: flexible and sticky prices. Both blocks interact within the framework of the Taylor rule. This interaction is evident since the natural interest rate and potential output come from the flexible price block.

We conduct two exercises to quantify the impacts of climate change on monetary policy. The first assesses transition dynamics, examining the macroeconomic variables under a permanent and anticipated shock on total factor productivity in the domestic production function. The second exercise evaluates the consequences of a sixfold increase in the carbon tax rate. The first shock is calibrated so that GDP by the year 2100 falls 15 % below its initial steady-state level. This value is derived from the midpoint of the range of climate damage on GDP, as reported in the introduction of this article. The second shock is calibrated to reflect the carbon tax rate implemented in Denmark.¹

Results

The results shed light on the impact of climate-related risks on the monetary policy and the economy. The effects should be

¹ Colombia's current carbon tax is approximately \$5 per ton of carbon, whereas it stands at roughly \$26 per ton of carbon in Denmark. Therefore, for Colombia to achieve this level, it is necessary to increase its carbon tax rate by at least six.

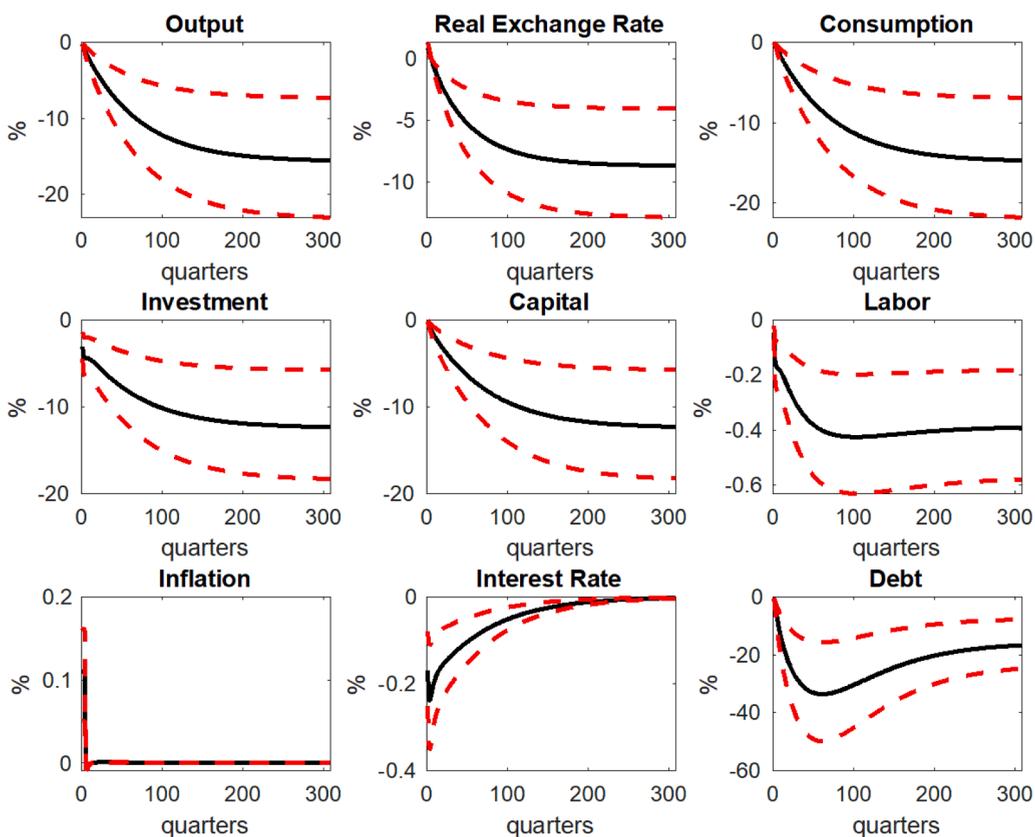


Fig. 4. Sensitivity analysis of the impact of output decline (7.2 % to 23 %) due to shocks on the TFP. The blue lines represent variable trajectories when climate change results in a 15 % output decline, while the dotted red lines depict trajectories for output declines of 7.2 % and 23 %.

carefully analyzed given that the model focused on a small and open country such as Colombia. The impacts of mitigation policies (carbon taxes) cannot be compared to those in a large country, given that their scopes in global emissions are entirely different. Mitigation policies in larger economies can effectively address externalities like the greenhouse effect. In comparison, such policies in smaller economies do not significantly mitigate the greenhouse effect.

Fig. 1 depicts the effects of a permanent and anticipated shock on total factor productivity in the domestic production function. In the long term, the materialization of this climate-related shock leads to a gradual decline in potential GDP, reaching a level 15 % below the initial steady state by 2100. This reduction in potential GDP implies a permanent deterioration in the economy’s income, prompting economic agents to adjust their consumption and reduce debt to lower interest payments. The long-term decrease in total factor productivity increases production costs, leading to increased domestic prices. Consequently, by 2100, the real exchange rate is 8.7 % below its initial steady-state level. This negative impact of climate change on the real exchange rate aligns with the trends in terms of trade reported by Economides (2019).

According to Fig. 2, climate change reduces household debt in the short term as individuals anticipate a permanent decrease in their income, prompting them to reduce their debt and interest expenses. Consequently, the natural interest rate decreases by 0.6 %, given a lower risk premium and an expected real exchange rate appreciation.² Although magnitudes differ, this effect is observed in both frameworks: sticky and flexible prices.

The climate-related shock on the TFP elevates the trajectory of real marginal costs in domestic goods production, which limits the projected output path. As the reduction in potential output (or output under flexible prices) surpasses that under sticky prices, the output gap path becomes positive. Furthermore, under sticky price conditions, the real exchange rate appreciation is less pronounced than flexible prices, resulting in a positive trajectory in the expected real exchange rate gap. Consequently, there is an abrupt surge in inflation of 0.11 % in the short term. This aligns with the New Keynesian model, where inflation is mainly influenced by the anticipated paths of the output and real exchange rate gaps. Following the Taylor rule, interest rates under price stickiness experience an increase

² In this model, the natural interest rate, defined as the rate prevailing under flexible prices and wages in imperfectly competitive markets, is determined by the uncovered interest rate parity (UIP) condition. This condition depends on the expected depreciation of the real exchange rate and the foreign interest rate, which is influenced by the risk-free real interest rate set abroad and a premium component reflecting deviations of foreign debt from its steady-state value.

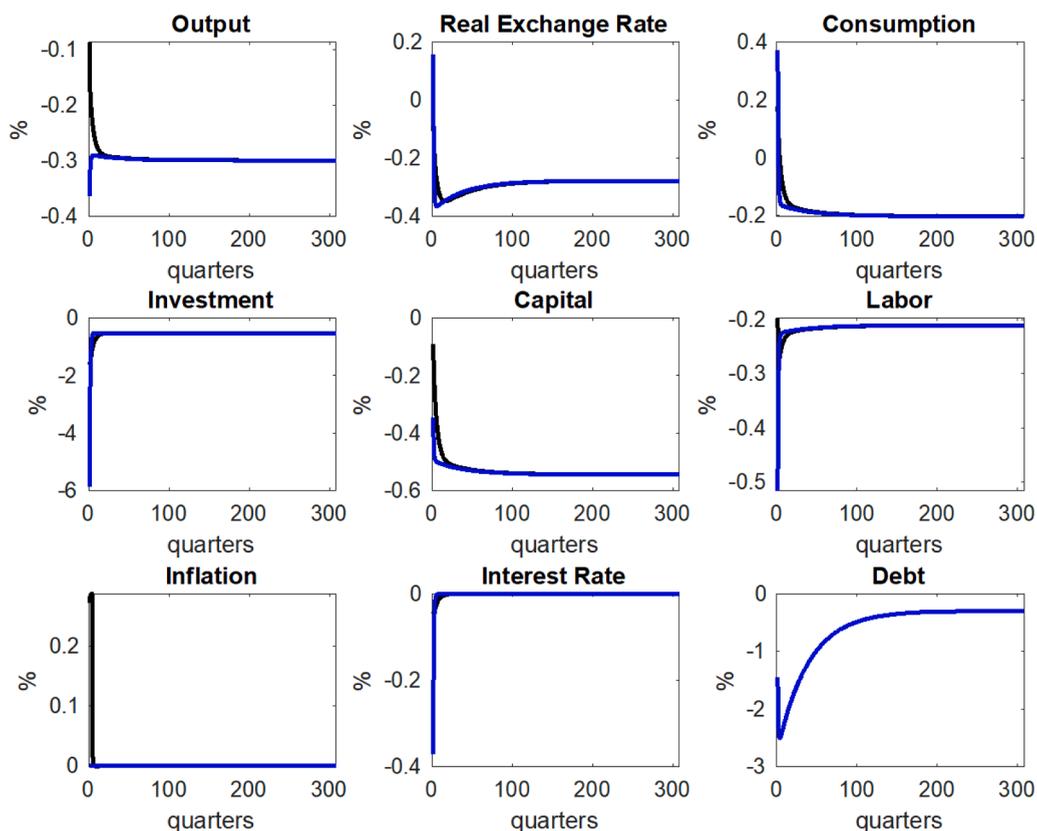


Fig. 5. Long term effects of a sixfold increase in carbon tax on main macroeconomic variables. The blue lines represent variable paths under flexible prices, while the black lines depict the results under sticky prices.

of 0.44 % (44 basis points) from their natural trajectory. This adjustment aims to close output, capital, and labor gaps while guiding inflation back to its target.

The effects of climate change on macroeconomic variables and monetary policy response are initially limited in the short term since, according to our model, the full impact on total factor productivity will not materialize until 2100. However, if the complete effect of climate change on total factor productivity were to manifest by 2033, the short-term effects would be four times more pronounced (Fig. 3). For instance, a 2.5 % decrease in the natural interest rate would be expected in the short term.

In Fig. 4, we present a sensitivity analysis to illustrate the macroeconomic impact of climate change under several climate damage scenarios. The results indicate that climate change has detrimental effects on the economy and its impacts are consistently proportional across all variables in the model. In other words, the damages of climate change exhibit a proportional relationship with its impact on macroeconomic variables, with only minor nonlinearities.

Fig. 5 depicts the results for the long-term horizon regarding the sixfold increase in the carbon tax rate. Potential GDP declines, reaching 0.3 % below its initial steady-state value by 2100. This reduction in potential GDP means a lasting decline in the economy's income, prompting economic agents to adjust their consumption patterns and reduce debt to lower interest payments. The increase in carbon tax leads to higher production costs resulting in a subsequent increase in domestic prices. Consequently, the real exchange rate decreases 0.28 % below its steady-state value by 2100.

A sixfold increase in the carbon tax corresponds to a change of \$21 per ton of carbon, which implies a semi-elasticity of output loss of about 0.014 % (calculated as 0.3 % divided by \$21) per every additional dollar of carbon tax. However, when we look at the literature on Colombia, the estimated semi-elasticity is approximately 0.05 % (calculated as 2.5 % divided by \$50; Calderon et al., 2016).³ The difference between the estimate of the semi-elasticities could be related to the fact that our carbon tax revenue is calibrated to align with the average share of GDP collected by the Central Government of Colombia, based on data available only through 2019. In contrast, Calderon (2016) may have employed a different calibrated parameter based on observed historical carbon tax revenue data from other countries.

As for the short-term effects, there is a 0.45 % increase in consumption, leading to a 5.9 % decrease in investment as agents anticipate a permanent decrease in demand for capital and labor, thus expecting lower remuneration for both inputs due to the carbon

³ Take the average of the output loss for 2050 from Fig. 5 in Calderon et al.'s (2016) study.

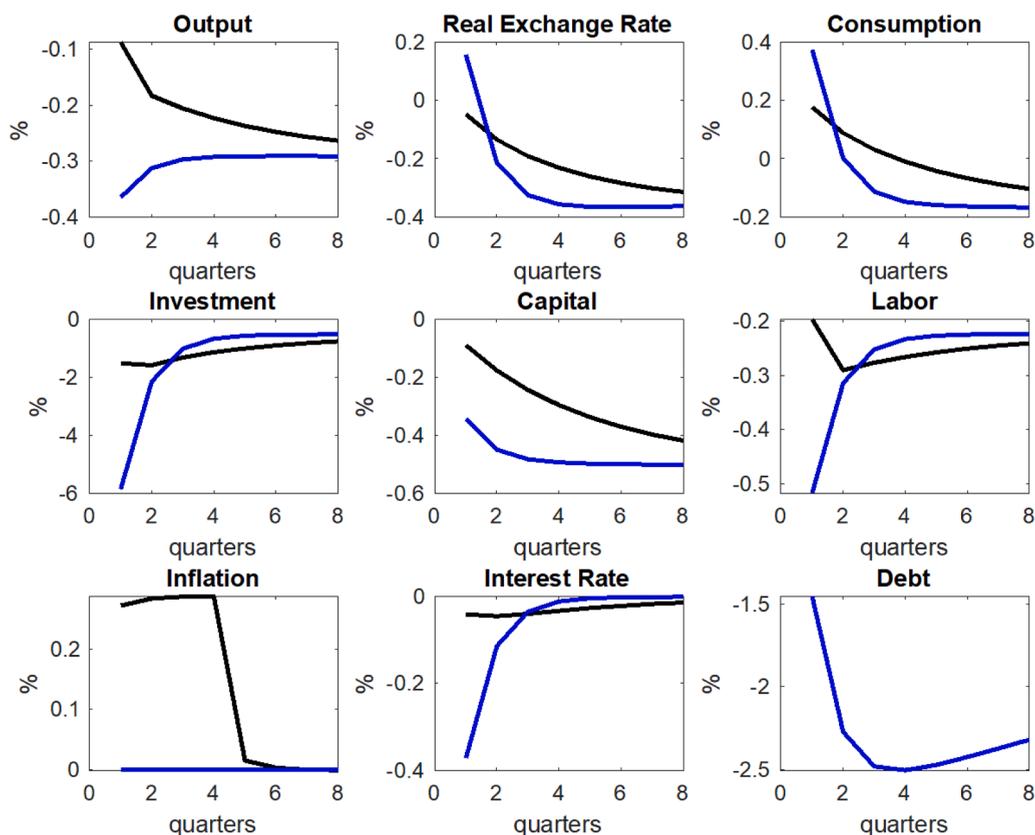


Fig. 6. Short term effects (monetary policy horizon) of a sixfold increase in carbon tax on main macroeconomic variables. The blue lines represent variable paths under flexible prices, while the black lines depict the results under sticky prices.

tax (Fig. 6). As investment falls more significantly than consumption rises, output experiences a 0.35 % decline. Additionally, due to a permanent increase in real marginal costs and a decrease in firms' profits, a decrease in output and an increase in consumption lead to a short-term increase of 0.24 % in the real exchange rate. Since the increase in real marginal costs and firm profits is permanent, households are reducing their debt. These conditions result in a 0.37 % reduction in the natural interest rate in the short term, driven by a lower risk premium and an expected appreciation of the real exchange rate. Although magnitudes differ, this effect is observed in both frameworks: sticky and flexible prices.

Furthermore, inflation registers a significant increase of 0.3 % immediately after the implementation of the carbon tax. By the Taylor rule and considering price stickiness, interest rates observe a 0.33 % (33 basis points) upward adjustment from their natural path. This modification intends to close the output, capital, and labor gaps and guide inflation back to its target.

Overall, the results suggest that the timing of climate change's effects on total factor productivity materializing will determine the magnitude of their impacts on macroeconomic variables in the short term, thereby influencing the response of monetary policy. A carbon tax comparable to that of some European countries, such as Denmark, will have long-term effects that may not be significantly quantitative.

Conclusions and future research

This study contributes to the existing literature on climate change and monetary policy in several meaningful ways. It provides a comprehensive analysis of both short- and long-term effects, explores monetary policy implications, and employs a structured DSGE model.

The results suggest that climate-related shocks (decrease in TFP) lead to a gradual decline in potential GDP in the long term due to increased production costs and a lower real exchange rate. In the short term, inflation rises temporarily while the natural interest rate decreases, impacting output and investment. Overall, short-term effects are less pronounced than their long-term counterparts. Furthermore, monetary policy adjustments are necessary to address the challenges posed by climate change, especially in the short term, when a more contractionary monetary policy will be needed to bring inflation back in line with its target.

Regarding the influence of carbon taxes on macroeconomic variables, their long-term effects are relatively modest. However, in the short term, their impact on inflation and interest rates is more significant, albeit temporary. The magnitude of climate change's economic impact depends on various factors, including the severity of climate change, the pace of technological advances, and the

adopted policy responses.

The study has certain limitations given that it does not consider potential technological advancements or adaptation strategies to mitigate the adverse effects of climate change, nor does it explore climate change’s impact on financial stability. Additional research is needed to address these limitations and explore the transmission channel on the financial sector.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Description of the Model

In this appendix, we provide a detailed description of the model we have developed to analyze the impacts of climate change and mitigation policies on macroeconomic variables and monetary policy. As mentioned before, it is a standard dynamic stochastic general equilibrium model of a small open economy, consisting of two production sectors: one domestic and one external. The model is divided into two blocks: one with sticky prices (à la Calvo) and another with flexible prices. In the case of rigid prices, a Taylor rule is implemented as a monetary policy instrument to keep inflation close to its target value. Additionally, the model considers a government that collects firm taxes on carbon dioxide (CO2) emissions and distributes them to households through lump-sum transfers.

Household Problem

- The households seek:

$$\max_{C_t, h_t, K_{t+1}, B_t, D_t} E_t \sum_{t=0}^{\infty} \beta^t [u(C_t) - v(h_t)]$$

Subject to:

$$C_t + B_t + q_t D_{t-1} (1 + r_{t-1}^*) + p_t^i I_t = \tilde{w}_t h_t + p_t^j R_t K_t + \Pi_t + q_t D_t + (1 + r_{t-1}) B_{t-1} + q_t T_t + S_t \tag{1}$$

$$K_{t+1} = (1 - \delta) K_t + \left[1 + \Gamma \left(\frac{I_t}{I_{t-1}} \right) \right] I_t$$

$$C_t = c(C_t^h, C_t^f) \tag{2}$$

$$I_t = i(I_t^h, I_t^f) \tag{3}$$

- C_t^h represents the consumption of the domestic good.
- C_t^f represents the consumption of the foreign good.
- I_t denotes the investment basket.
- I_t^h refers to domestic goods intended for investment.
- I_t^f refers to imported goods intended for investment.
- T_t represents all incomes other than exports of the domestically produced good h , such as remittances.
- S_t denotes fixed-sum subsidies granted by the government.
- Π_t represents firm dividends.
- h_t denotes the hours worked.
- K_t represents the capital stock in period t .

- B_{t-1} represents the stock of domestic bonds in period $t - 1$.
- D_{t-1} represents the stock of external debt in period $t - 1$.
- q_t represents the real exchange rate.
- \tilde{w}_t represents the real wage.
- r_{t-1}^* represents the interest rate at which the household borrows from abroad.
- r_{t-1} represents the interest rate earned by domestic bonds.
- R_t represents the gross return rate on capital.
- p_t^i represents the relative price of goods intended for investment in units of the consumption basket.
- Where: $R_t, \tilde{w}_t, \Pi_t, r_t, r_t^*, p_t^i, B_{t-1}, D_{t-1}, K_t$, are given.
- All variables are denominated in units of the domestic consumption basket C_t .

Inter-temporal optimization problem

- First-order conditions of the representative household are:

$$[C_t] \quad : \quad u'(C_t) = \Lambda_t$$

$$[h_t] \quad : \quad -v'(h_t) = \Lambda_t \tilde{w}_t$$

$$[K_{t+1}] \quad : \quad \Psi_t = \beta E_t [R_{t+1} p_{t+1}^i \Lambda_{t+1} + (1 - \delta) \Psi_{t+1}]$$

$$[I_t] \quad : \quad \left\{ 1 - \Gamma \left(\frac{I_t}{I_{t-1}} \right) - \Gamma' \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right\} \Psi_t - p_t^i \Lambda_t = -\beta E_t \left[\Gamma' \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right] \Psi_{t+1}$$

$$[B_t] \quad : \quad \Lambda_t = \beta(1 + r_t) E_t \Lambda_{t+1}$$

$$[D_t] \quad : \quad q_t \Lambda_t = \beta E_t (1 + r_t^*) q_{t+1} \Lambda_{t+1}$$

$$[\Lambda_t] \quad : \quad C_t + B_t + q_t D_{t-1} (1 + r_{t-1}^*) + p_t^i K_{t+1} = \tilde{w}_t h_t + p_t^i R_t K_t + \Pi_t + (1 - \delta) p_t^i K_t + q_t D_t + (1 + r_{t-1}) B_{t-1} + q_t T_t$$

Intratemporal optimization problem

Once C_t and I_t are found, the representative household seeks for the optimal combination of domestic and imported goods for consumption and investment, which is:

$$\min_{C_t^h, I_t^h, C_t^f, I_t^f} [p_t^h C_t^h + p_t^f C_t^f + p_t^h I_t^h + p_t^f I_t^f]$$

Subject to (2) y (3)

The first-order conditions for this household's intratemporal problem are as follows:

$$[C_t^h] \quad : \quad p_t^h = \psi_t^c c_h(C_t^h, C_t^f)$$

$$[C_t^f] \quad : \quad p_t^f = \psi_t^c c_f(C_t^h, C_t^f)$$

$$[I_t^h] \quad : \quad p_t^h = \psi_t^i i_h(I_t^h, I_t^f)$$

$$[I_t^f] \quad : \quad p_t^f = \psi_t^i i_f(I_t^h, I_t^f)$$

Firm's Optimization Problem

In contrast to the standard real business cycle model, the New-Keynesian model is built upon a set of assumptions that elucidate why certain firms can determine the selling price of the goods they produce. Within this model, three groups of firms are outlined. The initial group consists of a representative firm that manufactures and distributes its goods under conditions of perfect competition. The second group encompasses a continuum of firms that procure the goods produced by the representative firm of the first group, termed

as homogeneous goods. The firms subtly modify the homogenous products, incurring minimal cost. The resultant goods possess slight variations by firm, although the technology they employ is identical. Each firm sells its resultant product to a third group of firms at a price exceeding the one paid for the homogeneous goods. The third group, represented by a single firm, packages each of these differentiated goods using CES technology. The packaged products are then ready for inclusion in consumption or investment baskets, or for export to the global market.

Producers of Homogeneous Goods (First Group)

- The representative firm seeks to maximize profits:

$$\max_{K_t, h_t} \Pi_t$$

Where:

$$\Pi_t = p_t^h Y_t^{Qh} - p_t^i R_t K_t - \tilde{w}_t h_t \tag{4}$$

$$Y_t^{Qh} = z_t f(K_t, A_t N_t h_t)$$

$$\log(z_{t+1}) = \rho_z \log(z_t) + \varepsilon_t^z$$

$$\log(\Delta A_{t+1}) = \rho_A \log(\Delta A_t) + \varepsilon_t^A$$

Where: $\Delta A_t \equiv \frac{A_t}{A_{t-1}}$

- Y_t^{Qh} represents the quantities produced of the domestic good.
- z_t denotes the total factor productivity.
- First-order conditions for representative firm are:

$$[K_t] \quad : \quad \phi_t^h p_t^h z_t f_k(K_t, A_t N_t h_t) = p_t^i R_t \tag{5}$$

$$[h_t] \quad : \quad \phi_t^h p_t^h z_t f_n(K_t, A_t N_t h_t) = \tilde{w}_t \tag{6}$$

Producers of Domestic Heterogeneous Goods (Second Group, Calvo Pricing)

- There exists a set with an infinite number of firms where each firm j seeks to optimally set the price of good j to maximize its expected profit path⁴.

$$\max_{\bar{P}_t^h} \mathbb{E}_t \sum_{i=0}^{\infty} (\varepsilon^h)^i \Delta_{t,t+i} \left[\frac{\bar{P}_t^h(j)}{\bar{P}_{t+i}^h} Y_{t+i}^h(j) - z_t^h (\varphi_{t+i}^h + \chi \tau_t) Y_{t+i}^h(j) \right]$$

s.a

$$Y_t^h(j) = \left(\frac{\bar{P}_t^h(j)}{P_t^h} \right)^{-\theta^h} Y_t^h$$

$$\bar{P}_{t+i}^h(j) = \bar{P}_{t-1}^h (1 + \pi_{t-1}^h)^i (1 + \bar{\pi})^{(1-i)^h} \tag{7}$$

$$\log(\tau_{t+1}) = (1 - \rho_\tau) \log(\tau) + \rho_z \log(\tau_t) + \varepsilon_t^\tau$$

- Where:
- $Y_t^h(j) = \left(\frac{\bar{P}_t^h(j)}{P_t^h} \right)^{-\theta^h} Y_t^h$ represents the optimal demand for variety j of good h by the packaging firm. (More details in the following subsection)
- ε^h is the probability of not being able to re-optimize prices.

⁴ Note that profit is deflated to sector h prices. Unlike firms in the first group, each of the firms j faces a probability ε^h of being unable to re-optimize the price of its produced good j at period t .

- i^h is the probability of indexing prices to the inflation of the previous period.
- $\Delta_{t,t+i}$ Corresponds to the stochastic discount factor.
- φ_t^h is the real marginal cost of producing an additional unit of the domestic good h .
- z_t^h is the exogenous shock to the marketing margin of good j .
- τ_t stands for the tax per megaton of carbon emitted.
- χ CO2 emissions: Kg per USD of Gross Domestic Product (GDP).
- The solution to the problem of each firm j is represented by the following expression, which arises from the first-order condition of the optimization problem of each firm j :

$$\frac{\bar{P}_t^h}{P_{t+i}^h} = z_t^{ph} \left(\frac{\theta^h}{\theta^h - 1} \right) \frac{\mathbb{E}_t \sum_{i=0}^{\infty} (\epsilon^h)^i \Delta_{t,t+i} \left[\frac{\varphi_{t+i}^h + \chi \tau_{t+i}}{\left\{ \left[\prod_{s=1}^{s=i} (1 + \pi_{t+s-1}^h) \right]^h (1 + \bar{\pi})^{(1-h)i} \right\}^\theta} \left(\frac{P_{t+i}^h}{P_t^h} \right)^{\theta^h} \right] Y_{t+i}^h}{\mathbb{E}_t \sum_{i=0}^{\infty} (\epsilon^h)^i \Delta_{t,t+i} \left[\frac{1}{\left\{ \left[\prod_{s=1}^{s=i} (1 + \pi_{t+s-1}^h) \right]^h (1 + \bar{\pi})^{(1-h)i} \right\}^{\theta-1}} \left(\frac{P_{t+i}^h}{P_t^h} \right)^{\theta^h - 1} \right] Y_{t+i}^h} \quad (8)$$

- Given that the standard solution methods used by DYNARE to solve this family of models require the dynamic equations to be represented sequentially, it is necessary to transform expression (8) as follows:

$$\frac{\bar{P}_t^h}{P_{t+i}^h} = z_t^{ph} \left(\frac{\theta^h}{\theta^h - 1} \right) \frac{F_t^h}{J_t^h}$$

- Where F_t^h represents the numerator, which can be interpreted as the present value of the future path of real marginal costs faced by each firm j .

$$F_t^h = (z_t^{ph} \varphi_t^h + \tau_t \chi) Y_t^h + \mathbb{E}_t \frac{(1 + \pi_{t+1}^h)^{\theta^h}}{\left\{ (1 + \pi_t^h)^h (1 + \bar{\pi})^{(1-h)i} \right\}^{\theta^h}} \Delta_{t,t+1} \epsilon^h F_{t+1}^h$$

- And J_t^h represents the denominator, which can be interpreted as the present value of the future path of real marginal incomes faced by each firm j .

$$J_t^h = Y_t^h + \mathbb{E}_t \frac{(1 + \pi_{t+1}^h)^{\theta^h - 1}}{\left\{ (1 + \pi_t^h)^h (1 + \bar{\pi})^{(1-h)i} \right\}^{\theta^h - 1}} \Delta_{t,t+1} \epsilon^h J_{t+1}^h$$

Packaging Firms of Domestic Goods (Third Group)

- The representative packaging firm seeks to minimize:

$$\min_{Y_t^h(j)} \int_0^1 \frac{P_t^h(j)}{P_t^h} Y_t^h(j) dj$$

$$\text{s.t. } Y_t^h = \left[\int_0^1 [Y_t^h(j)]^{\frac{(\theta^h - 1)}{\theta^h}} dj \right]^{\frac{\theta^h}{\theta^h - 1}} \quad (9)$$

- Where (9) is the technology for variety j of good h , which is produced solely by firm j .
- θ^h is the elasticity of substitution between varieties of the domestic good.
- The solution to this problem is represented by the following expression:

$$Y_t^h(j) = \left(\frac{P_t^h(j)}{P_t^h} \right)^{-\theta^h} Y_t^h$$

- When these optimal demands for each variety j are evaluated in the objective function, we obtain

$$P_t^h = \left\{ \int_0^1 [P_t^h(j)]^{1-\theta^h} dj \right\}^{\frac{1}{1-\theta^h}} \tag{10}$$

Dynamics of Aggregate Domestic Goods Inflation

- Using the Law of Large Numbers, it can be inferred that a fraction $(1 - e^h)$ of distributing firms can re-optimize the selling price of their respective goods while a fraction e^h cannot, so they must resort to the indexing rule (7). Consequently, Eq. (10) can be re-expressed as follows:

$$P_t^h = \left\{ \int_0^{e^h} [\bar{P}_t^{h,r}(j)]^{1-\theta^h} dj + \int_{e^h}^1 [\bar{P}_t^h(j)]^{1-\theta^h} dj \right\}^{\frac{1}{1-\theta^h}}$$

$$P_t^h = \left[e^h (\bar{P}_t^{h,r})^{1-\theta^h} + (1 - e^h) (\bar{P}_t^h)^{1-\theta^h} \right]^{\frac{1}{1-\theta^h}} \tag{11}$$

- Substituting (7) into (11) yields the following expression, reflecting the aggregate dynamics of prices for domestically produced goods (i.e., good h)

$$P_t^h = \left\{ e^h \left[P_{t-1}^h (1 + \pi_{t-1}^h)^h (1 + \bar{\pi})^{(1-\theta^h)} \right]^{1-\theta^h} + (1 - e^h) (\bar{P}_t^h)^{1-\theta^h} \right\}^{\frac{1}{1-\theta^h}}$$

- Firms are divided into two groups to model price rigidities in imported good f . The first group consists of a continuum of firms that acquire the imported good, referred to as homogeneous imported good. These firms slightly transform it into another good, with a negligible cost. Although the technology used by each firm in this group is the same, the goods they produce are slightly different from each other. Each of these firms sells its transformed product to a second group of firms at a price higher than that of the homogeneous good. This second group is represented by a firm that packages each of these differentiated goods using CES technology. The packaged product configures a consumption basket or an investment basket.

Producers of the Heterogeneous Imported Goods (First Group, Calvo Pricing)

- There exists a set with an infinite number of firms where each firm j seeks to optimally set the price of good j such that it maximizes its expected profit path⁵.

$$\max_{\bar{P}_t^f} \mathbb{E}_t \sum_{i=0}^{\infty} (e^f)^i \Delta_{t,t+i} \left[\frac{\bar{P}_t^f(j)}{P_{t+i}^f} Y_{t+i}^f(j) - \phi_{t+i}^f Y_{t+i}^f(j) \right]$$

$$Y_t^f(j) = \left(\frac{\bar{P}_t^f(j)}{P_t^f} \right)^{-\theta^f} Y_t^f$$

$$\bar{P}_{t+i}^f(j) = \bar{P}_{t-1}^f (1 + \pi_{t-1}^f)^{i^f} (1 + \bar{\pi})^{(1-\theta^f)} \tag{12}$$

- Where:
- $Y_t^f(j) = \left(\frac{\bar{P}_t^f(j)}{P_t^f} \right)^{-\theta^f} Y_t^f$ represents the optimal demand for variety j of the good f by the packaging firm. (See in the following subsection)
- e^f is the probability of not being able to re-optimize prices.
- i^f is the probability of indexing prices to the inflation of the previous period.
- ϕ_t^f is the real marginal cost of producing an additional unit of the domestic good f .
- $\Delta_{t,t+i}$ corresponds to the stochastic discount factor.

⁵ Note that profit is deflated to sector f prices. Each of these firms j faces a probability e^f of being unable to re-optimize the price of its produced good j at some point in time.

- The solution to the problem of each firm j is represented by the following expression, which arises from the first-order condition of the optimization problem of each firm j :

$$\frac{\bar{P}_t^f}{P_{t+i}^f} = \left(\frac{\theta^f}{\theta^f - 1} \right) \frac{\mathbb{E}_t \sum_{i=0}^{\infty} (e^f)^i \Delta_{t,t+i} \left[\frac{\varphi_{t+i}^f}{\left\{ \left[\prod_{s=1}^{s=i} (1 + \pi_{t+s-1}^f) \right]^{\theta^f} (1 + \bar{\pi})^{(1-\theta^f)i} \right\}^{\theta^f}} \left(\frac{P_{t+i}^f}{P_t^f} \right)^{\theta^f} \right] Y_{t+i}^f}{\mathbb{E}_t \sum_{i=0}^{\infty} (e^f)^i \Delta_{t,t+i} \left[\frac{1}{\left\{ \left[\prod_{s=1}^{s=i} (1 + \pi_{t+s-1}^f) \right]^{\theta^f} (1 + \bar{\pi})^{(1-\theta^f)i} \right\}^{\theta^f - 1}} \left(\frac{P_{t+i}^f}{P_t^f} \right)^{\theta^f - 1} \right] Y_{t+i}^f} \tag{13}$$

- The standard solution methods for solving this family of models require the dynamic equations to be represented sequentially, so it is necessary to transform expression (13) as follows:

$$\frac{\bar{P}_t^f}{P_{t+i}^f} = \left(\frac{\theta^f}{\theta^f - 1} \right) \frac{P_t^f}{J_t^f}$$

- Where P_t^f represents the numerator, which stands for the present value of the future path of real marginal costs faced by each firm j .

$$P_t^f = \varphi_t^f Y_t^f + \mathbb{E}_t \frac{(1 + \pi_{t+1}^f)^{\theta^f}}{\left\{ (1 + \pi_t^f)^{\theta^f} (1 + \bar{\pi})^{(1-\theta^f)} \right\}^{\theta^f}} \Delta_{t,t+1} e^f P_{t+1}^f$$

- And J_t^f represents the denominator.

$$J_t^f = Y_t^f + \mathbb{E}_t \frac{(1 + \pi_{t+1}^f)^{\theta^f - 1}}{\left\{ (1 + \pi_t^f)^{\theta^f} (1 + \bar{\pi})^{(1-\theta^f)} \right\}^{\theta^f - 1}} \Delta_{t,t+1} e^f J_{t+1}^f$$

Packaging Firms of Differentiated Imported Goods

- The representative packaging firm seeks to minimize:

$$\min_{Y_t^f(j)} \int_0^1 \frac{P_t^f(j)}{P_t^f} Y_t^f(j) dj$$

$$\text{s.t. } Y_t^f = \left[\int_0^1 [Y_t^f(j)]^{\frac{(\theta^f - 1)}{\theta^f}} dj \right]^{\frac{\theta^f}{\theta^f - 1}} \tag{14}$$

- Where (14) represents the technology for variety j of the good f , which is produced solely by firm j .
- θ^f is the elasticity of substitution between varieties of the imported product.
- The solution to this problem is represented by the following expression:

$$Y_t^f(j) = \left(\frac{P_t^f(j)}{P_t^f} \right)^{-\theta^f} Y_t^f$$

- When these optimal demands for each variety j are evaluated in the objective function, we obtain

$$P_t^f = \left\{ \int_0^1 [P_t^f(j)]^{1-\theta^f} dj \right\}^{\frac{1}{1-\theta^f}} \tag{15}$$

Aggregate Dynamics of Imported Goods Inflation

- Using the Law of Large Numbers, it can be inferred that a fraction $(1 - e^f)$ of the distributing firms can re-optimize the selling price of their respective goods while a fraction e^f cannot, so they have to resort to the indexing rule (12). Consequently, Eq. (15) can be reformulated as follows:

$$P_t^f = \left\{ \int_0^{e^f} [\bar{P}_t^f(j)]^{1-\theta} dj + \int_{e^f}^1 [\bar{P}_t^f(j)]^{1-\theta} dj \right\}^{\frac{1}{1-\theta}}$$

$$P_t^f = \left[e^f (\bar{P}_t^f)^{1-\theta} + (1 - e^f) (\bar{P}_t^f)^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{16}$$

- Substituting (12) into (16) yields the following expression, reflecting the aggregate dynamics of prices of domestically produced goods (i.e., good f):

$$P_t^f = \left\{ e^f \left[P_{t-1}^f (1 + \pi_{t-1}^f)^{\beta} (1 + \bar{\pi})^{(1-\beta)} \right]^{1-\theta} + (1 - e^f) (\bar{P}_t^f)^{1-\theta} \right\}^{\frac{1}{1-\theta}}$$

Total Inflation

- Given that p_t^h represents the relative price of the domestic good in units of the consumption basket, we can infer the following relationship between total inflation, defined as the percentage change in the price of the consumption basket, and the inflation of domestic good h .

$$\frac{1 + \pi_t}{1 + \pi_t} = \frac{p_t^h}{p_{t-1}^h}$$

Monetary Policy

- Monetary policy is represented through the following modified Taylor rule, which reflects the central bank’s concerns about reducing volatility in the asset market, responding to deviations of inflation expectations from the target, and deviations of output from its long-term trend.

$$\log(1 + i_t^{PM}) = \rho_{PM} \log(1 + i_{t-1}^{PM}) + (1 - \rho_{PM}) [\log(1 + r_t^n) + \log(1 + \pi_{t+i}^A)]$$

$$(1 - \rho_{PM}) [\phi_{\pi} [\log(1 + \pi_{t+i}^A) - \log(1 + \pi)] + \phi_x \{\log(x_t)\}] + \varepsilon_t^{PM}$$

- Where:

$$(1 + \pi_t^A) = [(1 + \pi_t)(1 + \pi_{t-1})(1 + \pi_{t-2})(1 + \pi_{t-3})]^{\frac{1}{4}}$$

$$x_t \equiv \frac{y_t}{y_t^n}$$

Fisher Identity

$$\log(1 + r_t) = \log(1 + i_t) - \log(1 + \pi_{t+1})$$

Where:

- x_t is the output gap.
- y_t^n and r_t^n denote GDP and the real interest rate under flexible prices.

Resource Constraint

$$Y_t^h = C^h + I_t^h + X_t^h$$

Exports

- The supply curve of final good h to the exterior is determined by the following function:

$$X_t^h = x(p_t^h, q_t) \tag{17}$$

External Borrowing Interest Rate

- To avoid explosive external debt dynamics, [Schmitt-Grohe and Uribe \(2004\)](#), external creditors charge households a risk premium for excessive indebtedness. This risk premium is determined by the following expression:

$$r_t^* = r_t^f + p \left(\frac{q_t D_t}{Y_t} \right)$$

- Where $Y_t \equiv p_t^h Y_t^h$
- r_t^f is the real interest rate in the other country.

Price of Imported Goods

$$\phi_t^f = z_t^m \frac{q_t p_t^{f,*}}{p_t^f}$$

- Where:
- $p_t^{f,*}$ is the relative price of imported goods in terms of the consumption basket of the rest of the world.
- ϕ_t^f is the real marginal cost of imported goods.

Functional Forms

- The functional forms assumed in this model are as follows:

$$u(C_t) = \frac{(C_t - b\bar{C}_{t-1})^{1-\sigma}}{1-\sigma}$$

$$v(h_t) = (A_t N_t)^{1-\sigma} \eta \frac{h_t^{1+\mu}}{1+\mu}$$

$$C_t = \left[(1 - \alpha_F^c)^{\frac{1}{\eta_c}} (C_t^h)^{\frac{\eta_c-1}{\eta_c}} + \alpha_F^c \frac{1}{\eta_c} (C_t^f)^{\frac{\eta_c-1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c-1}}$$

$$I_t = z_t^i \left[(1 - \alpha_F^i)^{\frac{1}{\eta_i}} (I_t^h)^{\frac{\eta_i-1}{\eta_i}} + \alpha_F^i \frac{1}{\eta_i} (I_t^f)^{\frac{\eta_i-1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i-1}}$$

$$f(K_t, A_t N_t h_t) = K_t^\alpha (A_t N_t h_t)^{1-\alpha} \tag{18}$$

$$X_t^h = z_t^x (p_t^h)^{-\eta_c} q_t Y_t^* \tag{19}$$

$$p \left(\frac{q_t d_t}{y_t} \right) = \phi \left[\exp \left(\frac{q_t d_t}{y_t} - \kappa \right) - 1 \right]$$

$$\Gamma \left(\frac{I_t}{I_{t-1}} \right) = \frac{\Upsilon}{2} \left(\frac{I_t}{I_{t-1}} - a \right)^2$$

Competitive Equilibrium

- In equilibrium, all final goods produced by firms are demanded by households and the external sector.
- All hours of work offered by households are demanded by firms.
- All capital supplied by households is demanded by firms.
- Aggregate domestic debt is equal to zero, $B_t = 0$.
- There is no arbitrage between capital, external debt, and domestic debt, implying the price of these financial assets in terms of the domestic goods basket will be the same in equilibrium.
- Each household's consumption is equal to the average consumption of households. (i.e., $C_t = \bar{C}_{t-1}$)

The Model's Representation of Competitive Equilibrium

$$K_{t+1} = (1 - \delta)K_t + \left[1 - \frac{\Upsilon}{2} \left(\frac{I_t}{I_{t-1}} - a \right)^2 \right] I_t \tag{20}$$

$$\left\{ 1 - \frac{\Upsilon}{2} \left(\frac{I_t}{I_{t-1}} - a \right)^2 - \Upsilon \left(\frac{I_t}{I_{t-1}} - a \right) \frac{I_t}{I_{t-1}} \right\} \Psi_t - p_t^i \Lambda_t = -\beta E_t \left[\Upsilon \left(\frac{I_{t+1}}{I_t} - a \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right] \Psi_{t+1} \tag{21}$$

$$\Lambda_t = (A_t N_t)^{1-\sigma} \frac{\eta h_t^\mu}{\bar{w}_t} \tag{22}$$

$$\Lambda_t = z_t^i (C_t - b C_{t-1})^{-\sigma} \tag{23}$$

$$\Psi_t = \beta E_t [R_{t+1} p_{t+1}^i \Lambda_{t+1} + (1 - \delta) \Psi_{t+1}] \tag{24}$$

$$\Lambda_t = \beta E_t [(1 + r_t) \Lambda_{t+1}] \tag{25}$$

$$q_t \Lambda_t = \beta E_t [(1 + r_t^*) q_{t+1} \Lambda_{t+1}] \tag{26}$$

$$q_t D_t = q_t (1 + r_{t-1}^*) D_{t-1} + C_t + p_t^i I_t - Y_t - q_t T_t \tag{27}$$

$$\log(1 + r_t^*) = \log(1 + r_t^f) + \log(1 + r_t^{prem}) + \phi \left[\exp\left(\frac{q_t D_t}{Y_t} - \kappa\right) - 1 \right] \tag{28}$$

$$C_t^h = (1 - \alpha_F^c) (p_t^h)^{-\eta_c} C_t \tag{29}$$

$$C_t^f = \alpha_F^c (p_t^f)^{-\eta_c} C_t \tag{30}$$

$$I_t^h = (1 - \alpha_F^i) \left(\frac{p_t^h}{p_t^i} \right)^{-\eta_c} \frac{I_t}{z_t^i} \tag{31}$$

$$I_t^f = \alpha_F^i \left(\frac{p_t^f}{p_t^i} \right)^{-\eta_c} \frac{I_t}{z_t^i} \tag{32}$$

$$1 = \left[(1 - \alpha_F^c) (p_t^h)^{\eta_c - 1} + \alpha_F^c \frac{1}{\alpha_F^{\eta_c}} (p_t^f)^{\eta_c - 1} \right]^{\frac{\eta_c}{\eta_c - 1}} \tag{33}$$

$$1 = \frac{1}{z_t^i} \left[(1 - \alpha_F^i) \left(\frac{p_t^h}{p_t^i} \right)^{\eta_i - 1} + \alpha_F^i \frac{1}{\alpha_F^{\eta_i}} \left(\frac{p_t^f}{p_t^i} \right)^{\eta_i - 1} \right]^{\frac{\eta_i}{\eta_i - 1}} \tag{34}$$

$$Y_t^{Qh} = z_t K_t^\alpha (A_t N_t)^{1-\alpha} h_t^{1-\alpha} \tag{35}$$

$$\frac{p_t^i R_t}{w_t} = \left(\frac{\alpha}{1 - \alpha} \right) \frac{h_t}{K_t} \tag{36}$$

$$\tilde{w}_t = (1 - \alpha)\mu_t^h \left(\frac{K_t}{h_t}\right)^\alpha (A_t N_t)^{1-\alpha} \tag{37}$$

$$Y_t^h = C_t^h + I_t^h + X_t^h \tag{38}$$

$$x_t^h = z_t^x (p_t^h)^{-\eta_c} (q_t)^{\eta_c+1} Y_t^* \tag{39}$$

$$Y_t = p_t^h Y_t^h \tag{40}$$

$$\phi_t^f = z_t^m q_t \frac{P_t^{f,*}}{P_t^f} \tag{41}$$

$$Y_t^f = C_t^f + I_t^f \tag{42}$$

$$\log(1 + i_t^{PM}) = \rho_{PM} \log(1 + i_{t-1}^{PM}) + (1 - \rho_{PM}) [\log(1 + r_t^n) + \log(1 + \pi_{t+i}^A)] \\ (1 - \rho_{PM}) [\phi_x [\log(1 + \pi_{t+i}^A) - \log(1 + \pi)] + \phi_x \{\log(x_t)\}] + \varepsilon_t^{PM}$$

$$F_t^h = (\phi_t^h + \chi \tau_t) Y_t^h + \mathbb{E}_t \frac{(1 + \pi_{t+1}^h)^{\theta^h}}{\{(1 + \pi_t^h)^h (1 + \bar{\pi})^{(1-h)}\}^{\theta^h}} \Delta_{t,t+1} e^h F_{t+1}^h$$

$$J_t^h = Y_t^h + \mathbb{E}_t \frac{(1 + \pi_{t+1}^h)^{\theta^h-1}}{\{(1 + \pi_t^h)^h (1 + \bar{\pi})^{(1-h)}\}^{\theta^h-1}} \Delta_{t,t+1} e^h J_{t+1}^h$$

$$\frac{\bar{P}_t^h}{P_{t+i}^h} = z_t^{p^h} \left(\frac{\theta^h}{\theta^h - 1}\right) \frac{F_t^h}{J_t^h}$$

$$P_t^h = \left\{ e^h \left[P_{t-1}^h (1 + \pi_{t-1}^h)^h (1 + \bar{\pi})^{(1-h)} \right]^{1-\theta^h} + (1 - e^h) (\bar{P}_t^h)^{1-\theta^h} \right\}^{\frac{1}{1-\theta^h}}$$

$$F_t^f = \phi_t^f Y_t^f + \mathbb{E}_t \frac{(1 + \pi_{t+1}^f)^{\theta^f}}{\{(1 + \pi_t^f)^f (1 + \bar{\pi})^{(1-f)}\}^{\theta^f}} \Delta_{t,t+1} e^f F_{t+1}^f$$

$$J_t^f = Y_t^f + \mathbb{E}_t \frac{(1 + \pi_{t+1}^f)^{\theta^f-1}}{\{(1 + \pi_t^f)^f (1 + \bar{\pi})^{(1-f)}\}^{\theta^f-1}} \Delta_{t,t+1} e^f J_{t+1}^f$$

$$\frac{\bar{P}_t^f}{P_{t+i}^f} = \left(\frac{\theta^f}{\theta^f - 1}\right) \frac{F_t^f}{J_t^f}$$

$$P_t^f = \left\{ e^f \left[P_{t-1}^f (1 + \pi_{t-1}^f)^f (1 + \bar{\pi})^{(1-f)} \right]^{1-\theta^f} + (1 - e^f) (\bar{P}_t^f)^{1-\theta^f} \right\}^{\frac{1}{1-\theta^f}}$$

$$\frac{1 + \pi_t^h}{1 + \pi_t} = \frac{P_t^h}{P_{t-1}^h}$$

$$(1 + \pi_t^A) = [(1 + \pi_t)(1 + \pi_{t-1})(1 + \pi_{t-2})(1 + \pi_{t-3})]^{\frac{1}{4}}$$

Exogenous Processes

$$\log(1 + g_{t+1}^A) = (1 - \rho_{A^*}) \log(1 + g^A) + \rho_{A^*} \log(1 + g_t^A) + \varepsilon_t^{A^*} \tag{43}$$

$$\log(z_{t+1}^{A^*A}) = (1 - \rho_{z^{A^*A}})\log(z^{A^*A}) + \rho_{z^{A^*A}}\log(z_t^{A^*A}) + \varepsilon_t^{A^*A} \tag{44}$$

$$\log(z_{t+1}) = (1 - \rho_z)\log(z) + \rho_z\log(z_t) + \varepsilon_t \tag{45}$$

$$\log(z_{t+1}^{ph}) = (1 - \rho_{ph})\log(z^{ph}) + \rho_{ph}\log(z_t^{ph}) + \varepsilon_t^{ph} \tag{46}$$

$$\log(z_{t+1}^d) = (1 - \rho_d)\log(z^d) + \rho_d\log(z_t^d) + \varepsilon_t^d \tag{47}$$

$$\log(z_{t+1}^i) = (1 - \rho_i)\log(z^i) + \rho_i\log(z_t^i) + \varepsilon_t^i \tag{48}$$

$$\log(z_{t+1}^m) = (1 - \rho_m)\log(z^m) + \rho_m\log(z_t^m) + \varepsilon_t^m \tag{49}$$

$$\log(z_{t+1}^x) = (1 - \rho_x)\log(z^x) + \rho_x\log(z_t^x) + \varepsilon_t^x \tag{50}$$

$$\log(y_{t+1}^*) = (1 - \rho_{y^*})\log(y^*) + \rho_{y^*}\log(y_t^*) + \varepsilon_t^{y^*} \tag{51}$$

$$\log(tr_{t+1}) = (1 - \rho_{tr})\log(tr) + \rho_{tr}\log(tr_t) + \varepsilon_t^{tr} \tag{52}$$

$$\log(p_{t+1}^{f,*}) = (1 - \rho_{f,*})\log(p^{f,*}) + \rho_{f,*}\log(p_t^{f,*}) + \varepsilon_t^{p^{f,*}} \tag{53}$$

$$\log(r_{t+1}^f) = (1 - \rho_{r^f})\log(r^f) + \rho_{r^f}\log(r_t^f) + \varepsilon_t^{r^f} \tag{54}$$

$$\log(r_{t+1}^{prem}) = (1 - \rho_{prem})\log(r^{prem}) + \rho_{prem}\log(r_t^{prem}) + \varepsilon_t^{r^{prem}} \tag{55}$$

$$\log(\tau_{t+1}) = (1 - \rho_\tau)\log(\tau) + \rho_\tau\log(\tau_t) + \varepsilon_t^\tau \tag{56}$$

$$\frac{z_t^A}{z_{t-1}^A} = \frac{g_t^A}{g_t^A} \tag{57}$$

Stationary Representation of the Competitive Equilibrium in the Model (Sticky Prices)

- To solve the model, the variables must become stationary (i.e., expressed in effective units per worker).
- As presented previously, the model has a non-stationary productivity process, A_t .
- To make the model stationary, it is necessary to rescale all non-stationary real variables by the level of productivity and the size of the population.
- It should be noted that real quantities from the other country must be deflated by the productivity parameter of the other country, A_t^* , unless adjusted by the real exchange rate.
- For clearer notation, variables determined under sticky prices will be denoted with a superscript s .

$$k_{t+1}^s(1+n)(1+g_{t+1}^A) = (1-\delta)k_t^s + \left[1 - \frac{\Upsilon}{2} \left(\frac{\dot{i}_t^s}{\dot{i}_{t-1}^s} (1+n)(1+g_t^A) - a \right)^2 \right] \dot{i}_t^s \tag{58}$$

$$\left\{ 1 - \frac{\Upsilon}{2} \left[\frac{\dot{i}_t^s}{\dot{i}_{t-1}^s} (1+n)(1+g_t^A) - a \right]^2 - \Upsilon \left[\frac{\dot{i}_t^s}{\dot{i}_{t-1}^s} (1+n)(1+g_t^A) - a \right] \frac{\dot{i}_t^s}{\dot{i}_{t-1}^s} (1+n)(1+g_t^A) \right\} \psi_t^s - p_t^{i,s} \lambda_t^s = -\tilde{\beta} E_t \left\{ \Upsilon \left[\frac{\dot{i}_{t+1}^s}{\dot{i}_t^s} (1+n)(1+g_{t+1}^A) - a \right] \left[\frac{\dot{i}_{t+1}^s}{\dot{i}_t^s} (1+n)(1+g_{t+1}^A) \right]^2 \right\} (1+g_{t+1}^A)^{-\sigma} \psi_{t+1}^s \tag{59}$$

$$\lambda_t^s = \frac{\eta(h_t^s)^\mu}{w_t^s} \tag{60}$$

$$\lambda_t^s = z_t^d \left[c_t^s - b \frac{c_{t-1}^s}{(1+n)(1+g_t^A)} \right]^{-\sigma} \tag{61}$$

$$\psi_t^s = \tilde{\beta} E_t [R_{t+1}^s p_{t+1}^{i,s} \lambda_{t+1}^s + (1-\delta)\psi_{t+1}^s] (1+g_{t+1}^A)^{-\sigma} \tag{62}$$

$$\lambda_t^s = \tilde{\beta} E_t [(1 + r_t^s) \lambda_{t+1}^s (1 + g_{t+1}^A)^{-\sigma}] \tag{63}$$

$$q_t^s \lambda_t^s = \tilde{\beta} E_t [(1 + r_t^{*,s}) (1 + g_{t+1}^A)^{-\sigma} q_{t+1}^s \lambda_{t+1}^s] \tag{64}$$

$$d_t^s = \frac{q_t^s}{q_{t-1}^s} \frac{(1 + r_{t-1}^{*,s})}{(1 + n)(1 + g_t^A)} d_{t-1}^s + c_t^s + p_t^{i,s} i_t^s - y_t^s - tr_t \tag{65}$$

$$\log(1 + r_t^{*,s}) = \log(1 + r_t^f) + \log(1 + r_t^{prem}) + \phi \left[\exp\left(\frac{d_t^s}{y_t^s} - \kappa\right) - 1 \right] \tag{66}$$

• Recalling that $d_t = \frac{q_t D_t}{A_t N_t}$, $tr_t = \frac{q_t T_t}{A_t N_t}$, $y_t^* = \frac{Y_t^*}{A_t N_t}$, $\tilde{\beta} = \beta(1 + n)^{-\sigma}$

$$c_t^{h,s} = (1 - \alpha_F^c) (p_t^{h,s})^{-\eta_c} c_t^s \tag{67}$$

$$c_t^{f,s} = \alpha_F^c (p_t^{f,s})^{-\eta_c} c_t^s \tag{68}$$

$$i_t^{h,s} = (1 - \alpha_F^i) \left(\frac{p_t^{h,s}}{p_t^{i,s}} \right)^{-\eta_i} \frac{i_t^s}{z_t^i} \tag{69}$$

$$i_t^{f,s} = \alpha_F^i \left(\frac{p_t^{f,s}}{p_t^{i,s}} \right)^{-\eta_i} \frac{i_t^s}{z_t^i} \tag{70}$$

$$1 = \left[(1 - \alpha_F^c) (p_t^{h,s})^{\eta_c - 1} + \alpha_F^c \frac{1}{\eta_c} (p_t^{f,s})^{\eta_c - 1} \right]^{\frac{\eta_c}{\eta_c - 1}} \tag{71}$$

$$1 = \frac{1}{z_t^i} \left[(1 - \alpha_F^i) \left(\frac{p_t^{h,s}}{p_t^{i,s}} \right)^{\eta_i - 1} + \alpha_F^i \frac{1}{\eta_i} \left(\frac{p_t^{f,s}}{p_t^{i,s}} \right)^{\eta_i - 1} \right]^{\frac{\eta_i}{\eta_i - 1}} \tag{72}$$

$$y_t^{Qh,s} = z_t (k_t^s)^\alpha (h_t^s)^{1-\alpha} \tag{73}$$

$$\frac{p_t^{i,s} R_t^s}{w_t^s} = \left(\frac{\alpha}{1 - \alpha} \right) \frac{h_t^s}{k_t^s} \tag{74}$$

$$w_t^s = \mu_t^h (1 - \alpha) z_t \left(\frac{k_t^s}{h_t^s} \right)^\alpha \tag{75}$$

$$y_t^{h,s} = c_t^{h,s} + i_t^{h,s} + x_t^{h,s} \tag{76}$$

$$x_t^{h,s} = z_t^x (p_t^{h,s})^{-\eta_x} (q_t^s)^{\eta_x + 1} y_t^* z_t^{A \cdot A} \tag{77}$$

$$y_t^s = p_t^{h,s} y_t^{h,s} \tag{78}$$

$$\varphi_t^{f,s} = z_t^m \frac{q_t^s p_t^{f,s}}{p_t^s} \tag{79}$$

$$y_t^{f,s} = c_t^{f,s} + i_t^{f,s}$$

$$\log(1 + i_t^{PM}) = \rho_{PM} \log(1 + i_{t-1}^{PM}) + (1 - \rho_{PM}) [\log(1 + r_t^n) + \log(1 + \pi_{t+i}^{A,s})] \\ (1 - \rho_{PM}) [\phi_\pi [\log(1 + \pi_{t+i}^{A,s}) - \log(1 + \pi)] + \phi_x \{\log(x_t)\}] + \varepsilon_t^{PM} \tag{80}$$

$$x_t \equiv \frac{y_t^s}{y_t^n} \tag{81}$$

$$f_t^{h,s} = (z_t^{ph} \varphi_t^{h,s} + \gamma \tau_t) y_t^{h,s} + E_t \frac{(1 + \pi_{t+1}^h)^{\theta^h}}{\{(1 + \pi_t^h)^{\theta^h} (1 + \bar{\pi})^{(1-\theta^h)}\}} \Delta_{t,t+1} (1 + g_{t+1}^A) (1 + n) e^{\eta^h} f_{t+1}^{h,s} \tag{82}$$

$$j_t^{h,s} = y_t^{h,s} + \mathbb{E}_t \frac{(1 + \pi_{t+1}^h)^{\theta^h - 1}}{\left\{ (1 + \pi_t^h)^{\theta^h} (1 + \bar{\pi})^{(1-\theta^h)} \right\}^{\theta^h - 1}} \Delta_{t,t+1} (1 + g_{t+1}^A) (1 + n) e^{\epsilon_h} j_{t+1}^{h,s} \tag{83}$$

$$\frac{\bar{P}_t^h}{P_t^h} = \left(\frac{\theta^h}{\theta^h - 1} \right) \frac{f_t^{h,s}}{j_t^{h,s}} \tag{84}$$

$$1 = \epsilon^h \left[\frac{(1 + \pi_{t-1}^h)^{\theta^h} (1 + \bar{\pi})^{(1-\theta^h)}}{1 + \pi_t^h} \right]^{1-\theta^h} + (1 - \epsilon^h) \left(\frac{\bar{P}_t^h}{P_t^h} \right)^{1-\theta^h} \tag{85}$$

$$f_t^{f,s} = \varphi_t^{f,s} y_t^{f,s} + \mathbb{E}_t \frac{(1 + \pi_{t+1}^f)^{\theta^f}}{\left\{ (1 + \pi_t^f)^{\theta^f} (1 + \bar{\pi})^{(1-\theta^f)} \right\}^{\theta^f}} \Delta_{t,t+1} (1 + g_{t+1}^A) e^{\epsilon_f} f_{t+1}^{f,s} \tag{86}$$

$$j_t^{f,s} = y_t^{f,s} + \mathbb{E}_t \frac{(1 + \pi_{t+1}^f)^{\theta^f - 1}}{\left\{ (1 + \pi_t^f)^{\theta^f} (1 + \bar{\pi})^{(1-\theta^f)} \right\}^{\theta^f - 1}} \Delta_{t,t+1} (1 + g_{t+1}^A) e^{\epsilon_f} j_{t+1}^{f,s} \tag{87}$$

$$\frac{\bar{P}_t^{f,s}}{P_{t+i}^{f,s}} = \left(\frac{\theta^f}{\theta^f - 1} \right) \frac{f_t^{f,s}}{j_t^{f,s}} \tag{88}$$

$$1 = \theta^f \left[\frac{(1 + \pi_{t-1}^f)^{\theta^f} (1 + \bar{\pi})^{(1-\theta^f)}}{1 + \pi_t^f} \right]^{1-\theta^f} + (1 - \theta^f) \left(\frac{\bar{P}_t^f}{P_t^f} \right)^{1-\theta^f} \tag{89}$$

$$\frac{1 + \pi_t^h}{1 + \pi_t} = \frac{p_t^h}{p_{t-1}^h} \tag{90}$$

$$(1 + \pi_t^4) = [(1 + \pi_t)(1 + \pi_{t-1})(1 + \pi_{t-2})(1 + \pi_{t-3})]^{\frac{1}{4}} \tag{91}$$

$$\widetilde{\Delta}_{t,t+1} = \frac{\beta^{-\lambda_{t+1}^s} (1 + g_{t+1}^A)^{-\sigma} (1 + n)^{-\sigma}}{\lambda_t^s} \tag{92}$$

Identities:

$$\frac{tb_t^s}{y_t^s} = \frac{y_t^s - c_t^s - i_t^s}{y_t^s} \tag{93}$$

Stationary Representation of the Competitive Equilibrium in the Model (Flexible Prices)

- The representation of the model under flexible prices consists of the system of Eqs. (58)-(93) when $\epsilon_h = 0$, and the endogenous variables will be denoted with the superscript n . That is:

$$k_{t+1}^n (1 + n) (1 + g_{t+1}^A) = (1 - \delta) k_t^n + \left[1 - \frac{\Upsilon}{2} \left(\frac{i_t^n}{i_{t-1}^n} (1 + n) (1 + g_t^A) - a \right)^2 \right] i_t^n \tag{94}$$

$$\left\{ 1 - \frac{\Upsilon}{2} \left[\frac{i_t^n}{i_{t-1}^n} (1 + n) (1 + g_t^A) - a \right]^2 - \Upsilon \left[\frac{i_t^n}{i_{t-1}^n} (1 + n) (1 + g_t^A) - a \right] \frac{i_t^n}{i_{t-1}^n} (1 + n) (1 + g_t^A) \right\} \psi_t^n - p_t^{i,n} \lambda_t^n = -\widetilde{\beta} E_t \left\{ \Upsilon \left[\frac{i_{t+1}^n}{i_t^n} (1 + n) (1 + g_{t+1}^A) - a \right] \left[\frac{i_{t+1}^n}{i_t^n} (1 + n) (1 + g_{t+1}^A) \right]^2 \right\} (1 + g_{t+1}^A)^{-\sigma} \psi_{t+1}^n \tag{95}$$

$$\lambda_t^n = \frac{\eta h_t^{\mu,n}}{w_t^n} \tag{96}$$

$$\lambda_t^n = z_t^d \left[c_t^n - b \frac{c_{t-1}^n}{(1+n)(1+g_t^A)} \right]^{-\sigma}$$

$$\psi_t^n = \tilde{\beta} E_t [R_{t+1}^n p_{t+1}^{i,n} \lambda_{t+1}^n + (1-\delta)\psi_{t+1}^n] (1+g_{t+1}^A)^{-\sigma} \tag{97}$$

$$\lambda_t = \tilde{\beta} E_t [(1+r_t)\lambda_{t+1}(1+g_{t+1}^A)^{-\sigma}] \tag{98}$$

$$q_t \lambda_t = \tilde{\beta} E_t [(1+r_t^*)(1+g_{t+1}^A)^{-\sigma} Q_{t+1} \lambda_{t+1}] \tag{99}$$

$$q_t^n = \frac{q_t^n}{q_{t-1}^n} \frac{(1+r_{t-1}^{*,n})}{(1+n)(1+g_t^A)} q_{t-1}^n + c_t^n + p_t^{i,n} i_t^n - y_t^n - tr_t^n \tag{100}$$

$$\log(1+r_t^{*,n}) = \log(1+r_t^f) + \log(1+r_t^{prem}) + \phi \left[\exp\left(\frac{d_t^n}{y_t^n} - \kappa\right) - 1 \right] \tag{101}$$

$$c_t^{h,n} = (1-\alpha_F^c) (p_t^{h,n})^{-\eta_c} c_t^n \tag{102}$$

$$c_t^{f,n} = \alpha_F^c (p_t^{f,n})^{-\eta_c} c_t^n \tag{103}$$

$$i_t^{h,n} = (1-\alpha_F^i) \left(\frac{p_t^{h,n}}{p_t^{i,n}} \right)^{-\eta_i} \frac{i_t^n}{z_t^i} \tag{104}$$

$$i_t^{f,n} = \alpha_F^i \left(\frac{p_t^{f,n}}{p_t^{i,n}} \right)^{-\eta_i} \frac{i_t^n}{z_t^i} \tag{105}$$

$$1 = \left[(1-\alpha_F^c) (p_t^{h,n})^{\eta_c-1} + \alpha_F^c \frac{1}{\eta_c} (p_t^{f,n})^{\eta_c-1} \right]^{\frac{\eta_c}{\eta_c-1}} \tag{106}$$

$$1 = \frac{1}{z_t^i} \left[(1-\alpha_F^i) \left(\frac{p_t^{h,n}}{p_t^{i,n}} \right)^{\eta_i-1} + \alpha_F^i \frac{1}{\eta_i} \left(\frac{p_t^{f,n}}{p_t^{i,n}} \right)^{\eta_i-1} \right]^{\frac{\eta_i}{\eta_i-1}} \tag{107}$$

$$y_t^{Qh,n} = z_t (k_t^n)^\alpha (h_t^n)^{1-\alpha} \tag{108}$$

$$\frac{p_t^{i,n} R_t^n}{w_{n,t}} = \left(\frac{\alpha}{1-\alpha} \right) \frac{h_t^n}{k_t^n} \tag{109}$$

$$w_t^n = \mu_t^{h,n} (1-\alpha) z_t \left(\frac{k_t^n}{h_t^n} \right)^\alpha \tag{110}$$

$$y_t^{h,n} = c_t^{h,n} + i_t^{h,n} + x_t^{h,n} \tag{111}$$

$$x_t^{h,n} = z_t^x (p_t^{h,n})^{-\eta_c} (q_t^n)^{\eta_c+1} y_t^* z_t^{A^*A} \tag{112}$$

$$\frac{z_t^A}{z_{t-1}^A} = \frac{g_t^A}{g_t^A}$$

$$y_t^n = p_t^{h,n} y_t^{h,n} \tag{113}$$

$$\phi_t^{f,n} = z_t^m \frac{q_t^n p_t^{f,*}}{p_t^{f,n}} \tag{114}$$

$$y_t^{f,n} = c_t^{f,n} + i_t^{f,n}$$

$$f_t^{h,n} = (z_t^{ph} \phi_t^{h,n} + \chi \tau_t) y_t^{h,n} \tag{115}$$

$$j_t^{h,n} = y_t^{h,n} \tag{116}$$

$$1 = \left(\frac{\theta^h}{\theta^h - 1} \right) \frac{f_t^{h,n}}{j_t^{h,n}} \tag{117}$$

$$f_t^{f,n} = \varphi_t^{f,n} y_t^{f,n} \tag{118}$$

$$j_t^{f,n} = y_t^{f,n} \tag{119}$$

$$1 = \left(\frac{\theta^f}{\theta^f - 1} \right) \frac{f_t^{f,n}}{j_t^{f,n}} \tag{120}$$

$$\tilde{\Delta}_{t,t+1}^n = \frac{\tilde{\beta}^n \lambda_{t+1}^n (1 + g_{t+1}^A)^{-\sigma} (1 + n)^{-\sigma}}{\lambda_t^n} \tag{121}$$

Identities:

$$\frac{tb_t^n}{y_t^n} = \frac{y_t^n - c_t^n - i_t^n}{y_t^n} \tag{122}$$

Calibration

The set of parameters $\{\alpha, \alpha_F^c, \alpha_F^i, \delta, z^{ph}, z^x, t, \tau\}$ of the model is calibrated such that the model can replicate, in its steady state, the historical average values of the economic variables, described below.

Table 1
Target values vs Model.

Variable	Model	Data	Source
$\frac{c}{y}$	0.84	0.86	World Penn Table 1950 - 2019
$\frac{x^h}{y}$	0,08	0.08	World Penn Table 1980 - 2019
$\frac{m}{y}$	0.12	0.11	World Penn Table 1992 - 2019
$\frac{k}{y}$	3.31	3.31	World Penn Table 1980 - 2019
$\frac{i}{y}$	0.20	0.20	World Penn Table 1980 - 2019
$\frac{c^f}{y}$	0.34	0.21	World Penn Table 1992 - 2019
$\frac{wl}{y}$	0.48	0.48	World Penn Table 1992 - 2019
$\frac{\chi^h}{y}$	0.0004	0.0004	Carbon Tax's fiscal revenues, MHCP, 2019.

The resulting values of the calibrated parameters are as follows:

Table 2
Calibrated Parameters.

Parámetro	Valor
α	0.31
α_F^c	0,05
α_F^i	0.4
δ	0.03
z^{ph}	1.07
z^x	1.11
tr	0.93
τ	0.0026

The values of the remaining parameters were determined based on values obtained from the literature, as outlined below:

Table 3
Parámetros fijados.

Parameter	Parameter's Name	Value	Source
μ	Inverse of the Frisch elasticity of labor supply	1.1	Kneip et al. (2013)
χ	CO2 emissions per unit of GDP	0.15	CEIC Data (2023)
η^c	Elasticity of substitution between domestic and foreign consumption goods	0.9	González et al. (2014)
η^i	Elasticity of substitution between domestic and foreign goods for investment	0.9	González et al. (2014)
η^{c^*}	Elasticity of exports to real exchange rate	1.2	Fernández et al. (2015)
ϕ	Elasticity of Interest rate to foreign debt	0.0024	Gertler et al. (2003)
ϵ^h	Probability of keeping prices fixed for firm h	0.75	Gali and Gertler (1999)
ϵ^f	Probability of keeping prices fixed for firm f	0.75	Gali and Gertler (1999)
$\bar{\pi}$	Long run inflation	0.03	Gonzalez et al. (2011)
ϕ_π	Interest rate response to inflation deviations	1.5	Taylor (1993)
ϕ_x	Interest rate response to output gap	0.25	Taylor (1993)
ρ_{PM}	Smoothing parameter in the Taylor rule	0.7	Gonzalez et al. (2011)
r^f	Steady state foreign interest rate	0.01	FRED
r^{pre}	Steady state risk premium (5 years CDS)	0.025	Bloomberg
g^{A^*}	Steady state foreign productivity growth rate	0.02	Jones (2016)
z^{A^*}	Steady state foreign to domestic productivity ratio	1.54	World Penn Table 1954 - 2019
z	Steady state TFP	1	Set by author
z^d	Steady state preferences' factor	1	Set by author
z^i	Steady state investment efficiency's factor	1	Set by author
z^m	Steady state cost push factor: Imported goods	$\frac{1}{\mu_f}$	Set by author in order to hold Law of one price
y^*	Steady state foreign GDP (in effective units per worker)	1	Set by author
$p^{f,*}$	Steady state imports realative price in temrs of foreign goods	1	Set by author

The remaining parameters reported in the model were set to zero as they are not relevant for the conclusions of the exercise developed in the policy note. This is because only two variables, TFP and the carbon tax, were shocked, and these shocks are permanent.

Solution Method

- The solution of this model consists of finding the paths of $c_t^l, i_t^l, k_{t+1}^l, d_t^l, h_t^l, y_t^{Qf,l}, y_t^{f,l}, x_t^{h,l}, y_t^l, y_t^{Qh,l}, y_t^{h,l}, \lambda_t^l, c_t^{h,l}, c_t^{f,l}, R_t^l, \pi_t^A, w_t^l, q_t^l, \pi_t, i_t^{PM}, r_t^*, r_t^{*,l}, \tilde{\Delta}_{t,t+1}^l, p_t^{i,l}, p_t^{h,l}, p_t^{f,l}, \bar{P}_t^h, \bar{P}_t^f, \varphi_t^{h,l}, \varphi_t^{f,l}, \pi_t^h, \pi_t^f, f_t^{h,l}, j_t^{h,l}, j_t^{f,l}, j_t^{f,l}$
- For all $l = \{s, n\}$ that solve the equilibrium of this economy, which is composed of the system of Eqs. (58)–(93) and (94)–(122) for a time horizon T .
- This system of equations is solved using Dynare’s perfect foresight solver. This method is employed to solve economic models in which agents have a perfect understanding of how economic variables will evolve in the future. It means that agents can fully anticipate any changes in future conditions and adjust their behavior accordingly⁶. The perfect foresight solution procedure in Dynare can be summarized by the following steps:
 1. Perfect Anticipation: Agents in the model anticipate all future changes in the economy.
 2. Solving the Current Equilibrium: The current equilibrium of the economy is solved. This involves finding the values of endogenous variables that satisfy equilibrium conditions, given the given values of exogenous variables and any shocks programmed to occur in the future.
 3. Transition Simulation: Once the current equilibrium is determined, the simulation of how the economy will evolve over time is conducted. This entails tracking the path of economic variables from the initial state to the final equilibrium state, taking into account agents’ perfect expectations.
 4. Solution Stability: The stability of the found solution is evaluated to ensure that the economy converges to a stable equilibrium state.

⁶ Note that we compute the new steady state consistent with the nonlinear equilibrium representation of this model. Therefore, our steady state changes keep all the non-linearities of this model. The transition path is solved by the above perfect foresight solution procedure. However, non-linearities issues arise on dynamic path when the model has kinks or there is presence of uncertainty in the agent’s information. In this model we do not have them, therefore our simulations are no going to be significantly affected by the solution method.

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