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Valuating consumer credit portfolios

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ABSTRACT

This paper proposes a model that associates borrower credit risk with the cash flow method to assess the economic value of a consumer credit portfolio. A Monte Carlo simulation applying the method to an illustrative loan reveals that the lending standards of the institution, captured in the model by the expected and unexpected losses of the contract according to Basel II Internal Rating-Based Approach, is a key driver of the portfolio's intrinsic value, lending support to the evidence that a bank's credit policy and a bank's valuation are associated.

1. Introduction

How should the fair value of a credit portfolio be assigned? While consensus seems to exist on the valuation of non-financial firms through the discounted cash flow method, when it comes to assessing the economic value of a bank the debate is still open. One plausible explanation for this relies on the fact that, in the specific case of financial firms, the cash flows related to their core business are more uncertain due to the delinquency probability of the borrowers, together with the lending standards of the bank (Dell'ariccia and Marquez, 2006). Not surprisingly, the empirical evidence on banks' valuation and loan growth presents contradictory findings. Although some studies have found that banks' expansion of credit portfolios is associated with future decrease of their stocks price (Foos et al., 2010; Colomiris and Nissim, 2014; Fahlenbrach et al., 2018), others document that such an increase in loans is accompanied by growth in banks' valuation (Niu, 2016; Zemel, 2018). Because loans are the majority of the assets held by banks (Niu, 2016), the debate on how to assign the fair value of a loan while considering the idiosyncrasies of the lending activity can shed light into this discussion.

In this paper, I propose a model to valuate a consumer credit portfolio that embodies the principles of the discounted cash flow method together with Basel II regulatory guidelines on credit risk. A Monte Carlo simulation of a hypothetical loan employing the model reveals how the default probability of the borrower is a key driver to assess the fair value of the contract, indicating that changes to the credit policies of banks (Rajan, 1994; Dell'ariccia and Marquez, 2006) could offer a plausible explanation for the conflicting findings in the literature. The model suggested here can also be of interest for practitioners and regulators.

The paper is structured as follows: Section 2 presents the economic foundation of the Basel II guideline on capital measurement of credit portfolios. Section 3 presents the model proposed to valuate credit portfolios using the DCF method. Section 4 presents illustrative applications of the method, first for a single contract and then for an entire portfolio. The work is brought to a conclusion in Section 5.

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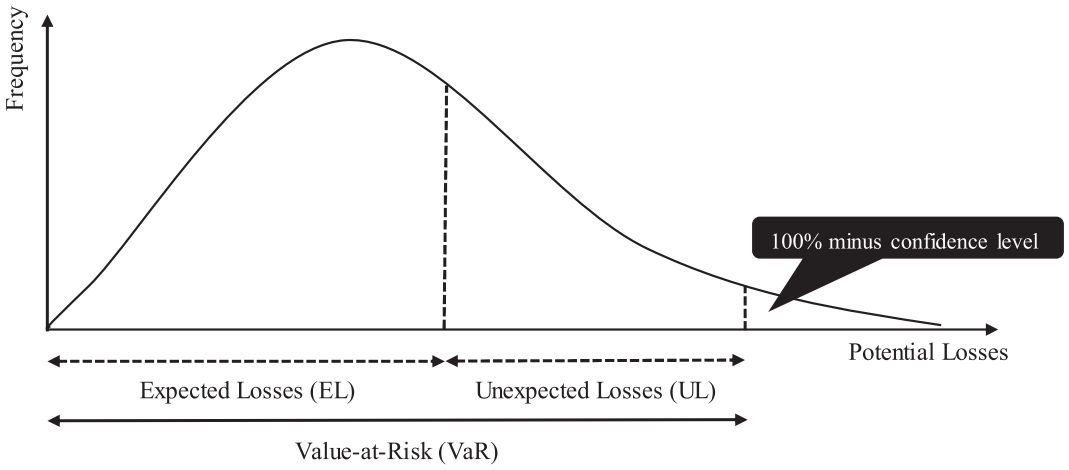

Fig. 1. Expected and unexpected Losses in the probability density curve.
The figure displays the probability density curve of potential losses in a hypothetical loan portfolio, identifying the expected and unexpected Losses as well as the value-at-risk areas of the curve according to the IRB framework.

2. Basel II and loan default probabilities

In 2004, the Basel II committee published a Revised Framework on International Convergence of Capital Measurement and Capital Standards. The main focus of the document was to serve as a guide for financial institutions to determine the capital requirements of their lending activity based on internal measures of the likelihood of a borrower defaulting under specific conditions, in addition to supervisory approval, in a framework named the Internal Rating-Based Approach (hereafter, IRB).

Losses are an intrinsic part of the lending activity. Although predicting when a loss will occur is impossible, a financial institution is able to estimate potential average losses based on the particular behavior of its credit portfolio. Based on this predictable characteristic, the IRB defines estimates of this type of default risk as Expected Losses (EL), which can be measured by statistical models, such as logistic regression or artificial neural networks. According to the IRB, these losses are frequent, given that they are the cost of the lending activity, are small in magnitude, and can be covered by the contract's risk premium embedded in the interest on the loan.

A second source of losses in the credit business is related to events that can affect borrowers as a whole, such as a financial crisis. Given this systemic feature, losses related to this type of event cannot be captured by internal statistical models (since they only capture idiosyncratic risks), making them very difficult to predict. In the IRB, such losses are known as Unexpected Losses (UL). Because extreme events are a recurrent fact despite the relative rarity of such events, the capital held by financial institutions must be substantial enough to protect them against these adverse circumstances that cause infrequent but significant losses. Nevertheless, holding capital to cover every possible UL would be economically inefficient, because it would escalate the cost of borrowing money to protect the financial institution from all types of rare events. To address this point, the IRB assumes that the capital requirement for financial institutions should be sufficient to face a shock that has a 0.1% chance of happening (i.e., 99.9% Value-at-Risk event; VaR). In other words, the reserves held by a bank to face future losses resulting from its lending activity must cover all the EL together with the UL due to systemic events with a magnitude lower than 99.9% VaR.

To illustrate this framework better, Fig. 1 shows the probability density distribution of potential credit losses for a hypothetical institution. As previously described, ELs are frequent, albeit of low magnitude, filling the right tail of the distribution, whereas ULs are more unlikely but more sizable, appearing to fill the right tail of the curve. For capital requirement purposes, the UL distribution is truncated at the 0.1% confidence level (99.9% VaR) to avoid the aforementioned prohibitive costs of the lending activity. A detailed explanation of the method used to determine the UL of a portfolio is provided in Appendix 1. The model that incorporates the parameters to estimate the present value of the contract is developed in the next section.

3. The model

Consider a single consumer credit loan L with no collateral that generates a series of payments depending on the interest rate i, the number of payments n and the amount of the Principal P. Assume no taxes or transaction costs. The balance of the loan on date t (where $t < n$) corresponds to the present value of the payments discounted at the rate i. However, this amount does not correspond to the discounted cash flow of the contract from the bank's standpoint for two main reasons. First, there is a chance that part of the loan will not be repaid (credit losses), and second, the cost of capital k of the loan is distinct from the interest rate i implicit in the contract, (bank's spread). As addressed in the previous section, Basel II understands that the default probability of a loan is given by two components: an EL that can be predicted by the bank with the use of statistical models, and an UL that refers to unpredicted

systematic exposures. In the former case, such an EL can be captured by the default probability of the statistical model ρ_L for the given contract, while in the latter, the UL is defined, according to Basel II requirements briefly described in the previous section.[1]

Consequently, the payments CF_t predicted for the loan must be diminished by the default probability ρ_L as well as by the UL of the contract to correspond to the cash flow of period t generated by the loan, which will be discounted at the cost of capital k, leading to the first component of the model, as expressed by Eq. (1),

$$DCF'_L = \sum_{t=1}^{n} \frac{CF_t \times \left(1 - \rho_L - UL\right)}{(1+k)^t} \tag{1}$$

where, DCF'_L refers to the first component of the discounted cash flow of the loan L. Eq. (1) assumes that all cash flows (CF_t) generated by the loan will be penalized by the default probability, which could be seen as too conservative since it is reasonable to believe that any default will occur only after a given period n' (where $1 \leq n' < n$), and not throughout the payment series. However, it is important to realize that when a consumer defaults, all remaining payments will be lost and not only the portion $CF_t \times (\rho_L + UL)$ of each payment as assumed by the model, which can be seen as too optimistic. Therefore, the model seems to provide a moderate proposal that counterbalances the conservatism (optimism) of the reality observed if the consumer pays (defaults on) the remaining principal of the loan. In addition, in Eq. (1), since the idiosyncratic and systemic risks of the loan are captured by $\rho_L$and UL, respectively, the cost of capital k corresponds to the rate at which the bank raises money (e.g., certificate of deposit rate). Finally, the default probability of the loan ρ_L can be determined by a binary response model (e.g., probit, logit, artificial neural networks, etc.[2]) applied in the portfolio itself in which the dependent variable assumes 1 if the given loan defaulted and 0 otherwise, whereas the explanatory variables are composed by registration and behavior information of the respective customers.

If a borrower defaults, the bank still has a chance to recover, at least, part of the pending balance. Hence, a second component of the loan's valuation model will be composed by the present value of the amount recovered by the bank after the default.[3] Since the bank is not able to recover the entire pending balance of every defaulted contract, this second component must incorporate the bank's recovery rate μ_B of consumer credit loans that will affect only the defaulted portion implicit in each payment, which is captured by $CF_t \times (\rho_L + UL)$. Consequently, the second component of the valuation of loan L is expressed as follows:

$$DCF''_L = \sum_{t=1}^{n} \frac{CF_t \times (\rho_L + UL) \times \mu_B}{(1+k)^t} \tag{2}$$

The addition of Eqs. (1) and (2) provides the valuation of the loan L by the discounted cash flow method, as exhibited below:

$$DCF_L = DCF'_L + DCF''_L = \sum_{t=1}^{n} \frac{CF_t \times \left(1 - \rho_L - UL\right) + CF_t \times (\rho_L + UL) \times \mu_B}{(1+k)^t} \tag{3}$$

The sum of the result of Eq. (3) applied to every loan L of the portfolio p provides the fair value of the entire book loan DCF_p, as follows:

$$DCF_p = \sum_{L=1}^{N} DCF_L \tag{4}$$

It is easy to see that the default probability and the ULs are important drivers of the portfolio's fair value. Hence, if the expansion of the bank's book loan is made at the cost of the quality of the borrower the overall expected losses ($\rho_L + UL$) will increase, which could lead to a decrease in the fair value of the portfolio.[4] If such a practice is extended to the bank's other credit products, it could result in a decrease in the bank's profitability and market value, which is in line with findings in the literature (e.g., Foos et al., 2010; Fahlenbrach et al., 2018). However, if the increase of the consumer credit portfolio is accompanied by a high-quality credit risk, both ρ_L and UL will stabilize, and the increase in N would lead to an increase in the fair value of the portfolio, raising the market value of the bank, giving support to a number of studies (e.g., Niu, 2016; Zemel, 2018) that have documented the association between a bank's loan growth and its higher valuation. Hence, the proposed model can accommodate both contrary findings reported by the literature. The next section provides an illustrative application of the model in two steps. First, for a single contract, to supply more granular view of the valuation process, and then, for an entire portfolio.

[1] See Basel Committee on Banking Supervision (2005) An Explanatory Note on the Basel II IRB Risk Weight Functions for a detailed explanation.

[2] See Crook et al. (2007) for a review of methods used to assess consumer credit risk.

[3] The amount recovered in the model does not refer to the cash flow generated by the collateral, because the model assumes a contract with no collateral. This amount actually refers to part of the future payments that are recovered by the institution's collection task force after the default. This assumption is in line with the common approach in credit scoring, which defines a default as payment that remains overdue after a given number of days, rather than after ceasing all efforts to recover the amount under delinquency. Consequently, it is reasonable to assume that in the days following the "official" default, the institution will take action to recover part of the amount classified as a loss.

[4] A more attentive reader could conjecture that under specific circumstances, UL and EL could move in opposite directions. According to the IRB framework, in this scenario the overall losses (i.e., UL + EL) would still increase, diminishing the DCF of the loan as assumed by the model. Appendix 2 provides a detailed demonstration of this argument.

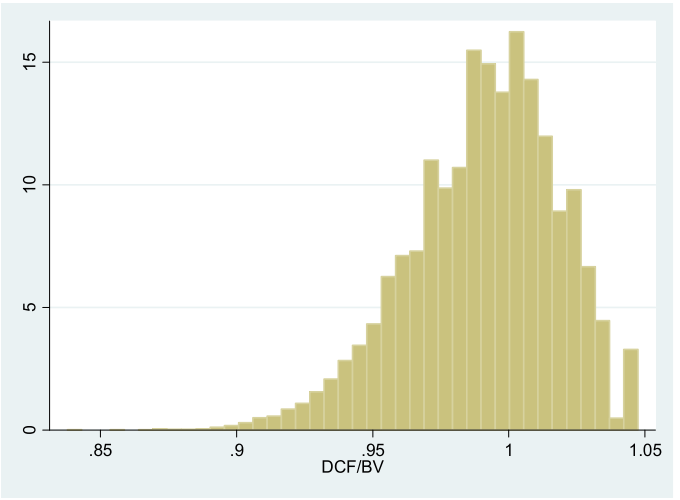

Fig. 2. Histogram of DCF/BV ratio of a Monte Carlo simulation in an illustrative consumer credit loan.
The figure displays the histogram of 10,000 Monte Carlo simulation of the ratio between discounted cash flow and book-value (DCF/BV) of one revolver contract under de assumptions described on Section 4.1.

4. Illustrative application

4.1. Valuating a single contract

Suppose a revolving consumer loan contract with a principal of $20,000, interest rate of 1.0% per month, and 12 monthly payments. Under these assumptions, the cash flow of the contract would be the following:

-20,000.00	1,776.98	1,776.98	1,776.98	1,776.98
0	**1**	**2**	**3**	**12**

Assume a default probability of 5% for the loan, a recovery rate of 20% and a cost of capital of 0.25% per month for the bank. For a ρ_L of 5%, the UL deriving from the Basel II IRB Approach would be 9.73%.[5] Hence, using Eq. (3), the discounted cash flow of the contract would be the following:

$$DCF_L = \frac{1,776.98 \cdot (1 - 0.05 - 0.0973) + 1,776.98 \cdot (0,05 + 0.0973) \cdot 0,20}{(1 + 0.25)^1} + \dots$$

$$+ \frac{1,776.98 \cdot (1 - 0.05 - 0.0973) + 1,776.98 \cdot (0,05 + 0.0973) \cdot 0,20}{(1 + 0.25)^{12}} = 18,508.38$$

In this case, the discounted cash flow of the contract is lower than the book value of the loan at time 0, due to the default probability and unexpected loss. Notwithstanding, this is not a mandatory condition, because this amount could be higher than the book value depending on the credit quality of the borrower. For example, in an extreme scenario, if there was no credit loss probability (e.g.: $\rho_L = UL = 0$) the $DCF_L = 20,981.20$.[6]

To illustrate this point, Fig. 2 shows the distribution of the ratio between the discounted cash flow (DCF) and the book value (BV) of the above-mentioned loan, obtained from a Monte Carlo simulation on 10,000 contracts changing ρ_L and UL, while keeping the remaining characteristics of the loan. To be consistent with the IRB approach, the simulation was achieved assuming a normal distribution for ρ_L with a mean of 2% and a standard deviation of 1.5%.[7] It is observable that the changes in ρ_L and UL can lead to a variety of DCF/BV ratios, lending support to previous studies documenting that banks' valuation are sensitive to the quality of the loans in their portfolio (Dell'ariccia and Marquez, 2006; Foos et al., 2010; Collomiris and Nissim, 2014; Niu 2016; Fahlenbach et al., 2018). Section 4.2 presents an application of the model to a hypothetical portfolio.[8]

[5] Appendix 1 provides a detailed example on the calculation of UL.

[6] $DCF_L = \frac{1,776.98 \cdot}{(1+0.10)^1} + \dots + \frac{1,776.98}{(1+0.10)^{12}} = 20,981.20$

[7] According to IRB framework, UL is a function of the loans' correlation (R), maturity (MA) and expected losses (ρ_L), as reported in Appendix 1. Because R and MA are constant in the simulation, the UL distribution will be a function of the values stemming from the ρ_L distribution.

[8] The application of the model into a real consumer credit portfolio is not straightforward because it involves a significant effort in calibrating the default probability model on the registration and behavior information of the respective customers, which is not the main purpose of the paper. This effort is a suggestion for future papers.

Table 1
Default probability and unexpected loss of an illustrative retail portfolio.

ID	Portfolio information							Losses parameters							
	Age	Ln income	Dsr	Married	Sex	Relation	Credit Limit	Z	ρ_L	Factor M	Factor R	Nρd	N99.9%	N	UL%
1	49	10.82	13	1	1	69	11,348	-5.6564	0.0035	1.0000	0.0400	-2.7541	-0.6308	0.0169	0.0142
2	47	11.01	26	1	1	20	8352	-0.2281	0.4432	1.0147	0.0400	-0.1458	-0.6308	0.6862	0.2422
3	48	11.20	9	0	1	95	13,464	-9.1446	0.0001	0.9093	0.0400	-3.7788	-0.6308	0.0008	0.0007
4	29	11.25	17	1	0	66	6418	-1.9296	0.1268	1.0147	0.0400	-1.1652	-0.6308	0.2965	0.1768
5	63	10.90	39	0	0	58	10,612	-1.9901	0.1202	1.0224	0.0400	-1.1980	-0.6308	0.2853	0.1638
6	60	10.90	39	1	0	55	10,750	-3.8388	0.0211	0.9842	0.0400	-2.0741	-0.6308	0.0745	0.0519
7	41	11.00	34	0	0	72	12,366	-2.9990	0.0475	1.0161	0.0400	-1.7043	-0.6308	0.1415	0.0974
8	61	10.98	17	0	0	39	12,146	-3.9994	0.0180	1.0287	0.0400	-2.1403	-0.6308	0.0656	0.0476
9	44	10.78	37	1	1	16	8826	2.2267	0.9026	1.0249	0.0400	1.3234	-0.6308	0.9747	0.0724
10	43	11.11	22	1	1	28	8457	-1.2925	0.2154	1.0187	0.0400	-0.8039	-0.6308	0.4313	0.2154
11	68	10.72	17	0	0	77	13,423	-6.4828	0.0015	1.0489	0.0400	-3.0233	-0.6308	0.0084	0.0068
12	28	10.74	10	1	0	58	10,196	-1.7584	0.1470	1.0000	0.0400	-1.0711	-0.6308	0.3299	0.1807
13	49	11.23	23	1	1	15	7429	-0.9683	0.2752	0.9962	0.0400	-0.6094	-0.6308	0.5085	0.2360
14	53	10.82	34	1	1	44	9715	-1.6088	0.1668	1.0045	0.0400	-0.9870	-0.6308	0.3608	0.1985
15	41	11.08	44	0	1	60	14,884	-3.9107	0.0196	1.0121	0.0400	-2.1039	-0.6308	0.0704	0.0534
16	27	10.97	21	1	1	18	6884	2.9011	0.9479	1.0073	0.0400	1.6583	-0.6308	0.9890	0.0409
17	26	10.99	31	1	1	39	13,030	-2.4054	0.0828	1.0885	0.0400	-1.4153	-0.6308	0.2164	0.1321
18	59	11.23	43	1	0	64	7490	-3.4654	0.0303	0.9856	0.0400	-1.9149	-0.6308	0.0995	0.0682
19	53	10.90	16	0	1	16	10,159	0.1700	0.5424	1.0000	0.0400	0.1087	-0.6308	0.7702	0.2278
20	37	10.84	31	0	0	92	9718	-1.0475	0.2597	0.9883	0.0400	-0.6575	-0.6308	0.4893	0.2274
...	...							...							
100	51	11.02	32	1	0	71	8216	-3.2595	0.0370	1.0103	0.0400	-1.8236	-0.6308	0.1165	0.0795

The table exhibits the portfolio information and loss parameters for a hypothetical retail credit portfolio. The default probability ρ_L was determined using the following logit model, described in Section 4.2. $UL\%$ derives from the value of ρ_L associated with the risk metrics set by the Basel II Internal Rating-Based Approach for a consumer credit portfolio (see Appendix 1 for details). The complete portfolio is available in the online appendix (http://bit.ly/2PiePBx).

4.2. Valuating an illustrative portfolio

Consider a hypothetical revolving consumer credit portfolio with 100 customers.[9] To assess the default probability of each contract, the financial institution runs a logit model using internal data where the dependent variable assumes 1 if the given customer has defaulted and 0 otherwise. After employing this method, suppose that the final model has the following expression:

$$Ln\left(\frac{\rho_L}{1-\rho_L}\right) = 78.596 - 0.100Age - 6.01LnIncome + 0.079dsr - 2.258Married - 0.062Relation - 7.796 \times 10^{-4}Credit\ Limit$$

where, Age refers to the customer's Age, $LnIncome$ is the natural log of the client's annual salary, dsr is the ratio between contract payment and monthly salary (debt-to-salary ratio), $Married$ is a dummy variable that equals 1 if the customer is married and 0 otherwise, $Relation$ is the number of months that the customer operates with the institution, and $Credit\ Limit$ is the limit on the credit that the institution provided to the customer. The replacement of individual information in the expression above would provide the ρ_L associated with each contract.

The signs of the logit model indicate that older and married clients are less likely to default, as well as borrowers with higher wages, higher credit limits and a longer relationship with the financial institution. Finally, the positive sign of dsr indicates that clients with a larger proportion of their salary compromised to make payments on a loan have a higher default probability. The multiplication of the logit model's coefficient by the corresponding variable of a given client will provide an estimate of his default probability ρ_L. In this regard, Table 1 presents the relevant individual information (i.e., significant variables of the logit model) that will produce the two parameters that compose the potential losses, namely EL and UL, of an illustrative part of the contracts. Once more, the EL estimates are obtained by replacing the individual information in the logit model, whereas the UL estimates are based on the IRB framework. The online appendix shows the original spreadsheet with the detailed calculation of each parameter of Table 1. The EL and UL estimates for each contract are highlighted in the table and represent the relevant inputs that are used to find the expected cash flow for each loan.

Knowing the amount of the contract, the number of the remaining payments and the interest rate for each contract, it is easy to find the cash flow corresponding to each contract in each period (CF_t). Assuming a recovery rate of 20% and a cost of capital of 0.25% per month, the discounted cash flow for each contract would be given by employing Eq. (3). The results for the DCF_L are illustrated in Table 2, which aggregates the loan information together with the valuation of each contract based on the model proposed in the paper (column DCF).

[9] The relevant characteristics of each individual is available in the online appendix (http://bit.ly/2PiePBx).

Table 2
Discounted cash flow of the contracts of a consumer credit portfolio.

id	Loan information				Valuation				
	Amount	Total N	Remaining N	Int. Rate	CFt	DCF'	DCF"	DCF	Book value
1	25,000	36	22	1.10%	844.76	17,748.89	63.79	17,812.68	16,427.17
2	30,300	28	10	1.34%	1305.53	4052.36	1765.52	5817.88	12,140.48
3	36,400	24	7	0.70%	1652.92	11,448.20	1.79	11,449.99	11,253.17
4	38,600	48	29	1.40%	1109.81	21,605.40	1883.67	23,489.07	26,303.48
5	27,100	19	9	0.80%	1543.15	9821.56	779.41	10,600.97	13,348.71
6	27,000	14	5	1.10%	2091.45	9622.69	151.52	9774.21	10,120.82
7	29,900	34	23	0.70%	991.27	18,931.40	641.34	19,572.74	20,990.87
8	29,200	26	12	0.80%	1248.39	13,776.28	193.40	13,969.69	14,229.95
9	24,000	57	16	1.60%	645.39	252.64	1972.06	2224.70	9045.16
10	33,300	24	11	1.42%	1646.19	10,155.89	1537.42	11,693.31	16,660.09
11	22,500	22	11	0.90%	1131.90	12,166.51	20.41	12,186.92	11,803.94
12	23,100	26	7	0.90%	1000.44	4661.82	454.53	5116.35	6757.60
13	37,800	32	18	1.78%	1558.54	13,396.35	2802.16	16,198.51	23,834.17
14	25,100	32	22	0.80%	892.17	12,113.12	1393.75	13,506.87	17,932.02
15	32,400	40	25	1.40%	1063.37	23,870.41	376.02	24,246.43	22,300.34
16	29,100	20	8	1.65%	1719.77	152.90	2690.76	2843.65	12,791.48
17	29,600	12	8	0.90%	2613.34	16,233.35	888.64	17,121.99	20,084.76
18	37,800	18	8	1.10%	2326.25	16,591.34	362.76	16,954.10	17,721.57
19	27,000	35	12	1.30%	965.11	2619.29	1755.70	4374.99	10,659.29
20	25,500	14	7	1.30%	2003.99	7124.73	1353.16	8477.89	13,325.99
...	...				...				
100	30,600	39	12	0.70%	899.31	9383.68	247.40	9631.08	10,316.31

The table provides the DCF of the contracts of the hypothetical retail credit portfolio using Eq. (3), together with the default probability ρ_L and UL of each contract shown in Table 1. This example considers a cost of capital (k) of 0.25% per month and a recovery rate μ_B of 20%.

The last column shows the current book value (i.e., debit balance) of each loan. In some cases (e.g., ID 1, 3, 11), the DCF is higher than the book value due to the bank spread not being jeopardized by credit losses, because these clients exhibit low ρ_L and UL. However, in the case of clients with a high probability of delinquency (e.g., id 2, 9, 16), the DCF is much lower than the corresponding book value, due to the high values of ρ_L and UL. The result indicates the model is consistent with the literature (e.g., Foos et al., 2010; Fahlenbrach et al., 2018) reporting that the quality of credit is an important driver for banks' valuation. This difference between the book value and the DCF indicates that the debit balance of a loan is not a consistent proxy for its fair value because it ignores the potential losses that are intrinsic to the lending activity. As previously mentioned, this paper contributes to the field because the proposed model incorporates these potential losses into the estimates of future cash flows.

Furthermore, since the bank spread aims to cover these potential losses, the comparison between the book value and the fair value (DCF) of a portfolio indicates whether the spread of the institution has been sufficient to achieve its goal. If the spread is not enough to cover the risk of lending, the fair value of the portfolio will be under its book value, meaning the financial institution is destroying value for its shareholders. From a regulator's perspective, this situation would mean the institution is negligent about the quality of the loans in its portfolio, possibly suggesting the need to increase the capital requirement of the institution. This is the second contribution that the paper makes for practitioners, as well as regulators.

5. Conclusion

The model incorporates the default probability of the loan together with the corresponding unexpected loss indicated by the Basel II IRB approach into the discounted cash flow of the loan and allows the financial institution's lending standards to play a central role in the bank's valuation. The challenge for future research is to apply this method in a real context, and preferably contrasting distinct credit policies to test the effect of a bank's credit guidelines on its portfolio's fair value. In this respect, portfolios containing loans with non-fixed interest rates (e.g., LIBOR) are especially welcomed, beacause the dynamics of the yield curve will affect some of the drivers of the DCF method proposed here, such as future payments and cost of capital. Consequently, the empirical evidence in this approach would provide an auspicious agenda on the degree of sensitivity that the fair value of the credit portfolio has to this additional source of risk.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix 1

Based on Fig. 1 of Section 2, we can define UL as the difference between the VaR at a given confidence level (e.g., 0.1%) and EL in the probability density distribution. Since the IRB assumes a normal distribution for the losses of the lending activity,[10] this definition can be expressed in the following terms:

$$N^{-1}(UL) = N^{-1}(VaR) - N^{-1}(EL) \tag{5}$$

where, N^{-1} refers to the inverse of the normal distribution. Consequently, the probability of UL in a contract is given by the following equation:

$$UL = N\left[N^{-1}(VaR) - N^{-1}(EL)\right] \tag{6}$$

To illustrate this definition, assume a contract with a default probability of 5% according to the internal model of the institution (EL). For a confidence level of 0.1% (99.9% VaR), the unexpected loss would be[11]

$$UL = N\left[N^{-1}(0.001) - N^{-1}(0.05)\right] = N[(-3.0902) - (-1.6448)] = N(-1.4454) \ = \ 0.07418$$

Consequently, in this illustrative example, the overall default probability of the contract would be 12.418%, because it is modeled on both idiosyncratic (EL = 5%) and systemic (UL = 7.418%) potential losses. Based on the theoretical model and empirical evidence from G10 countries, in the specification of the IRB approach, the regulators included two adjustments to the framework described above; namely, loans correlations and loans maturities, whose economic foundation is as follows.

According to the IRB, a consumer credit portfolio consists of a large number of contracts with low individual exposures, which, by the law of large numbers, would lead to a highly diversified portfolio whose exposure could be captured by a single systemic (non-diversifiable) factor. Under this assumption, this portfolio is consistent with Merton's model of the firm (1974), in which the default probability of a contract (PD_i) due to a single systemic factor (y) can be express as follows:

$$PD_i(y) = N\left[(1-R)^{-0.5} \times K_i - \left(\frac{R}{1-R}\right)^{0.5} \times y\right] \tag{7}$$

where, R is the contract's correlation with the systemic factor and K_i is the default threshold for the contract. In the IRB approach, K_i is operationalized by the inverse of the normal distribution to the contract's EL, and y is operationalized by the inverse of the normal distribution of an extreme event with a 0.1% chance of happening. Consequently, the default probability of a loan due to a systemic factor under the IRB framework can be calculated as follows:

$$PD_i(y) = N\left[(1-R)^{-0.5} \times N^{-1}(EL) - \left(\frac{R}{1-R}\right)^{0.5} \times N^{-1}(0.1)\right] \tag{8}$$

Because $N^{-1}(0.1\%) = -N^{-1}(99.9\%)$, Eq. 8 can be rewritten in the following way:

$$PD_i(y) = N\left[(1-R)^{-0.5} \times N^{-1}(EL) + \left(\frac{R}{1-R}\right)^{0.5} \times N^{-1}(99.9)\right] \tag{9}$$

Given that EL arise from the internal statistical model, the focus relies on how to estimate the contract's correlation with the systemic factor. Based on empirical evidence from G10 countries, the regulator indicates the following correlation, depending on the nature of the loan:

(a) Residential mortgages: $R = 0.15$

(b) Revolving retail exposures: $R = 0.04$

(c) Other retail exposures:

$$R = 0.03\left(\frac{1-e^{-35EL}}{1-e^{-35}}\right) + 0.16\left[1 - \left(\frac{1-e^{-35EL}}{1-e^{-35}}\right)\right] \tag{10}$$

It can be seen that for other retail contracts (option "c"), the correlation is a function of EL, with an upper (lower) limit of 0.16 (0.03) if the expected loss reaches 0% (100%). This inverse relation between R and EL relies on the intuition that the higher the expected losses are, the more important the idiosyncratic factor to measure the risk of a borrower is. Consequently, the overall default would depend more on individual risk drivers and less on the overall state of the economy (UL factor).

[10] Some studies (e.g., Daníelsson, 2002; Yamai and Yoshiba, 2005; Périgon, 2008; Alexander et al., 2012) advocate that this assumption is too optimistic, as there is evidence that credit losses exhibit fat tail distributions. Addressing possible drawbacks of the IRB framework is beyond the scope of this paper. For a more detailed discussion regarding some of the economic fundamentals of the Basel II guidelines, see also Kaplanski and Levy (2007) and Daníelsson et al. (2012).

[11] N^{-1} can be obtained using the function "NORMSINV" in Microsoft Excel, whereas N is given by "NORMSDIST".

The last adjustment proposed by the IRB framework refers to the maturity of the contract. Long-term contracts are more likely to have future downgrades and/or systemic exposures. Consequently, the default probability of a loan should increase with maturity. According to the IRB, the specification of the maturity adjustment (MA) is as follows:

$$MA = \frac{1 + (M - 2.5) \cdot (0.11852 - 0.05478 \cdot \log(EL))^2}{1 - 1.5 \cdot (0.11852 - 0.05478 \cdot \log(EL))^2} \tag{11}$$

where, M is the maturity of the loan in years. It should be noted that for 1 year maturity contracts, the maturity adjustment is equal to 1, to be consistent with Merton's model, which is based on a one-year corporate debt. Accordingly, in a 1 year loan, there will be no maturity adjustments in the estimation of UL. Eq. (12) presents the final specification of UL for a systemic factor under the IRB framework:

$$UL = \left\{ N\left[(1-R)^{-0.5} \times N^{-1}(EL) + \left(\frac{R}{1-R}\right)^{0.5} \times N^{-1}(99.9)\right] - EL \right\} \times MA \tag{12}$$

To illustrate the operationalization of UL, suppose a 5-year educational loan with a default probability of 5%, according to the internal model of the institution (EL). Since this loan is classified as "other retail exposures" for correlation purposes, the correlation adjustment will be given by Eq. (10):

$$R = 0.03\left(\frac{1 - e^{-35*0.05}}{1 - e^{-35}}\right) + 0.16\left[1 - \left(\frac{1 - e^{-35*0.05}}{1 - e^{-35}}\right)\right] = 0.0248 + 0.0278 = 0.0526$$

While the maturity adjustment for this loan would be

$$MA = \frac{1 + (5 - 2.5) \cdot (0.11852 - 0.05478 \cdot \log(0.05))^2}{1 - 1.5 \cdot (0.11852 - 0.05478 \cdot \log(0.05))^2} = \frac{1 + (5 - 2.5) \cdot 0.0360}{1 - 1.5 \cdot 0.9460} = 1.1523$$

Finally, the UL for this contract would be given by

$$UL = \left\{ N\left[(1 - 0.0526)^{-0.5} \times N^{-1}(0.05) + \left(\frac{0.0526}{1 - 0.0526}\right)^{0.5} \times N^{-1}(99.9)\right] - 0.05 \right\} \times \ 1.1523$$

$$UL = \{N[-1.6899 + 0.7281] - 0.05\} \times \ 1.1523 = \{0.1681 - 0.05\} \times \ 1.1523 = 13.61\%$$

Consequently, the overall default probability of the loan would be 18.61% (13.61% from UL plus 5% from EL), which, in other words, would mean that the institution should plausibly expect to receive 81.39% of contracts' future cash flows.

In the illustrative contract of Section 4.1, $\rho_L = 0.05$, the maturity of the loan is of one year ($M = 1$), which leads to $MA = 1$ independent of the ρ_L. Because it is a revolving contract, $R = 0.04$. Replacing these values on Eq. (8), we have,[12]

$$UL = \left\{ N\left[(1 - 0.04)^{0.5} \times N^{-1}(0.05) + \left(\frac{0.04}{1 - 0.04}\right)^{0.5} N^{-1}(0.999)\right] - 0.05 \right\} \cdot 1.0,$$

$$UL = N[-1.6448 \cdot 1.0206 - 3.0902 \cdot 0.2041] - 0.05 = N(-1.0480) - 0.05,$$

$$UL = 0.1473 - 0.05 = 0.0973.$$

Appendix 2

According to IRB framework, above a given default probability (i.e., EL), EL and UL can deviate in opposite directions. To illustrate this, I simulated a series of loans with the same characteristics of the example in the paper (i.e., revolver loan with an interest rate of 1% per month and a cost of capital of 0.25% per month), changing only the EL and calculating the corresponding UL using Eq. (12) presented in Appendix 1. The simulation results are shown in Fig. A.1.

It can be seen that if EL is higher than 40%, the UL decreases with the increase of the EL, indicating that when the quality of borrowers is poor, individual characteristics are more important to credit risk than systemic factor is. However, in terms of overall losses (UL + EL), the aggregated defaults always increase with EL, albeit at a decreasing rate, as shown in Fig. A.2.

Consequently, for the DCF, the inverse relation between UL and EL still diminishes the DCF, as shown in Fig. A.3.

Hence, it is correct to assume that if the expansion of the book loan is made at the cost of the quality of the borrower, the DCF will decrease, as assumed by the model.

[12] The Online Appendix provides the spreadsheet with the calculus for the entire portfolio of Section 4.2 as well as for an alternative retail credit portfolio in which Eq. (10) is used to find R. This appendix is available at: http://bit.ly/2PiePBx.

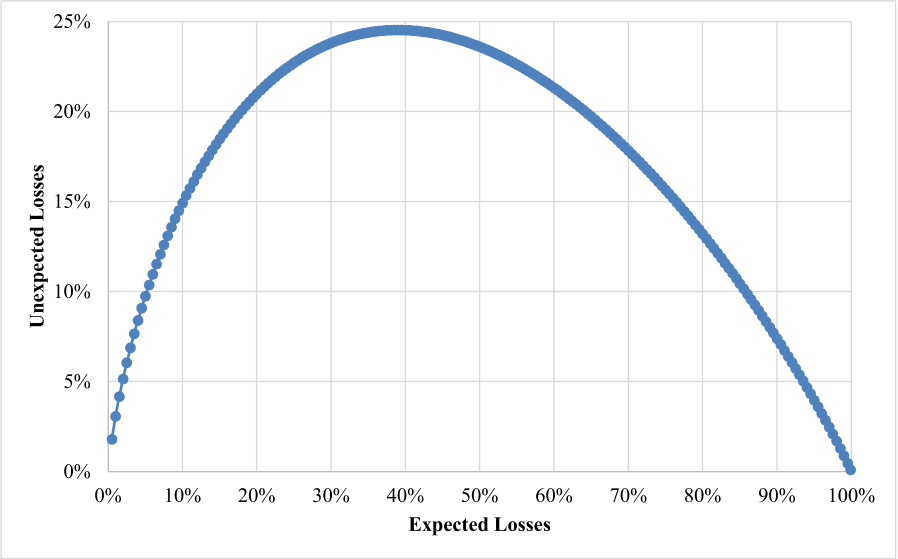

Fig. A.1. The relationship between expected and unexpected losses.
The figure displays the relation between EL and UL from the simulation of a series of loans with the same characteristics as the example in the paper (i.e., revolver loan with an interest rate of 1% per month and a cost of capital of 0.25% per month), changing only the EL, and calculating the corresponding UL using Eq. 12 from Appendix 1.

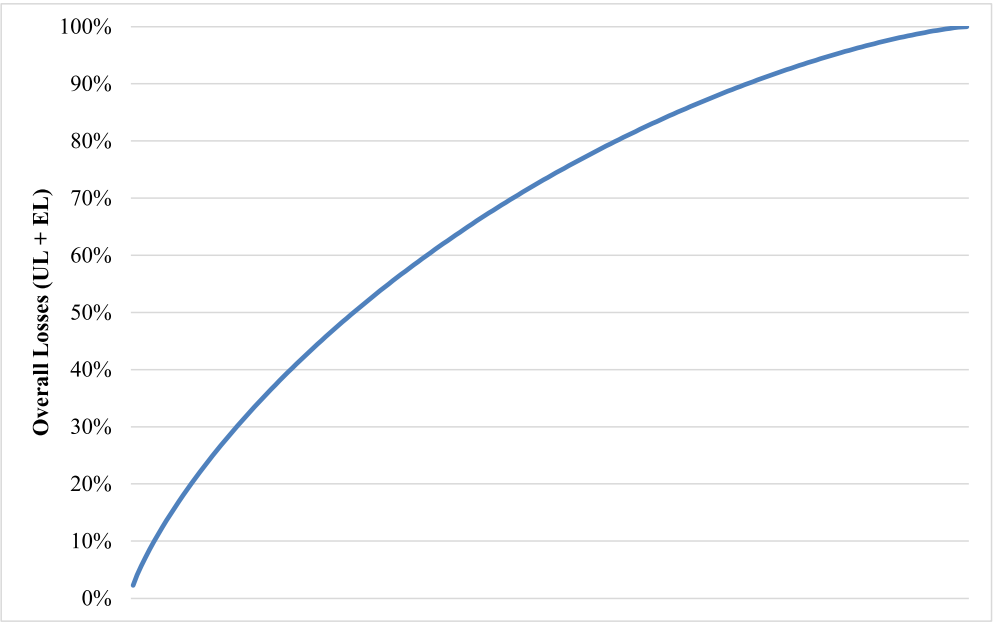

Fig. A.2. Overall losses.
The figure displays the overall losses (i.e., EL + UL) from the simulation of a series of loans with the same characteristics as the example in the paper (i.e., revolver loan with an interest rate of 1% per month and a cost of capital of 0.25% per month).

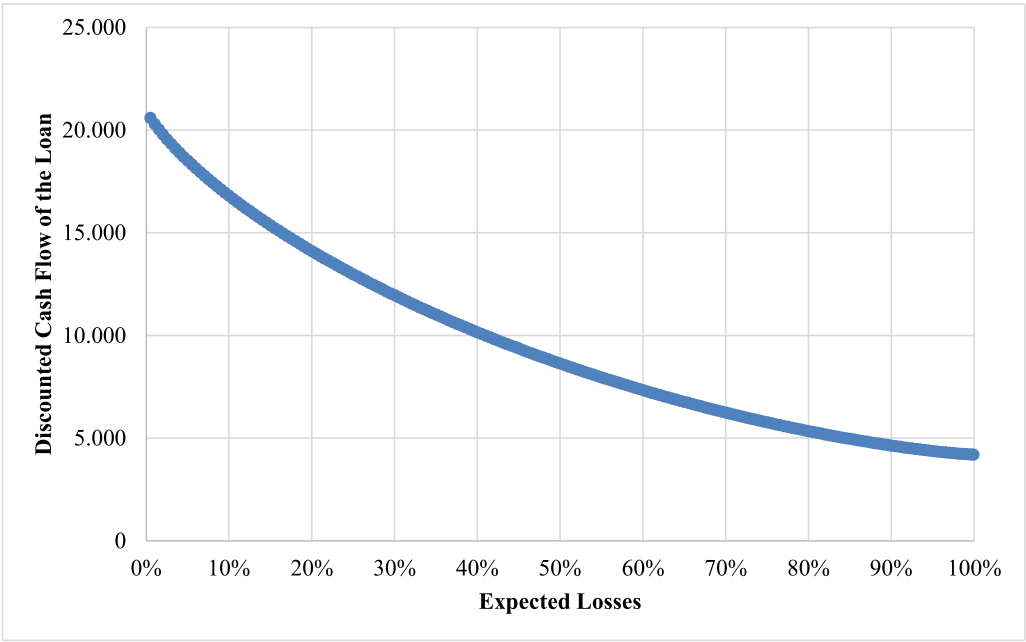

Fig. A.3. Expected losses and discounted cash flow of the loan.
The figure displays the corresponding discounted cash flow of the loan from the simulation of a series of loans with the same characteristics as the example in the paper (i.e., revolver loan with an interest rate of 1% per month and a cost of capital of 0.25% per month), changing only the EL and calculating the corresponding DCF using Eq. (3).

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