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Article

Commercial and banking credit network in Uruguay

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ABSTRACT

We build a commercial credit network, identify the most central economic sectors in terms of commercial debt, and provide a more complete idea of total indebtedness and financial interlinks between firms and banks in Uruguay. “Commerce,” “manufacturing,” and “transportation, storage, and communication” are the most central sectors in the commercial credit network. In a stress testing exercise, “transportation, storage, and communication” and “hotels and restaurants” are deeply affected in all cases. These sectors are the most exposed in terms of contagion. “Commerce” and “manufacturing” are central and have the highest level of indebtedness, but they have a large amount of liquid assets, which allows them to overcome shocks from other sectors. The results highlight the importance of having a good estimation of commercial credit interlinks for financial stability analysis.

1. Introduction

Contagion through increasingly complex and interrelated credit networks was at the core of the global financial crisis of 2007-08. In its aftermath, a growing strand of the literature study the structure of these networks and its effects on the propagation of shocks. The contagion links among financial institutions, the interlinks between the financial sector and firms in the real sector, and the input-output network among firms have been deeply studied. However, research on the contagion through commercial indebtedness among firms in the real sector has received less attention (see, for instance, [Acemoglu et al., 2016](#)). This paper aims to contribute in filling that gap.

We construct a network of commercial credit indebtedness (i.e., credit among firms in the real sector) using unique data from a unique survey of firms in Uruguay. Interruptions in the chain of payments among firms have shown to have a deep, negative impact on financial stability during past crises episodes. This source of financial instability may be particularly important in countries with relatively underdeveloped financial systems, where a large proportion of firms rely on commercial credit to finance their activities. However, lack of reliable and timely data generally prevents a complete assessment of this risk in a proactive manner. Hence, our first step is to construct a data set with the information required to build the commercial credit network in Uruguay. More precisely, we conduct a survey to a sample of firms that is representative of the universe of firms with more than 50 employees. Working at a sector level, we provide a series of measures of interconnectedness and identify the most central sectors in terms of commercial credit. We also perform a stress test exercise to analyze the vulnerability of the network to a default shock.

To the best of our knowledge, there is no previous work of this kind studying financial interconnections via the commercial credit network among sectors in the economy. There is, however, a large strand of literature studying the input-output network. We show that considering the input-output matrix as a proxy for commercial credit interlinks among economic sectors may be misleading:

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The production network is generally much denser than the commercial credit network, and the former might overestimate the indebtedness linkages because the fact that one sector uses an input from another does not necessarily mean that it is financed by commercial credit.

With the data gathered through the survey, we build a commercial credit network among economic sectors.¹ We analyze the structure of the network and its main characteristics by using traditional measures such as centrality, size, density, reciprocity, and transitivity. As a result, we identify which sectors are the most connected in terms of commercial credit and which sectors impose a larger threat to the stability of the network. According to the results, “commerce,” “manufacturing industry,” and “transportation, storage, and communication” are the most central sectors in the commercial credit network, whereas “real estate” is the least central sector.² We also identify a high level of indebtedness of the “commerce” sector to the “manufacturing” sector.

Next, we perform a stress test exercise to complete our analysis of how credit risk is transferred through the commercial credit network. Stress tests are an important tool to assess the vulnerability of a given financial network. To this end, detailed data on financial interactions are required. However, it is difficult to collect such data in full and to make them readily available to researchers, such that granular interaction-specific data generally remain unavailable (Anand et al., 2018). We contribute by filling that gap for the case of commercial credit among firms in Uruguay. To reconstruct the commercial credit network, we follow the minimum density method proposed by Anand et al. (2015) because, according to the authors, when used in a stress testing context, this approach performs better than alternative reconstruction methods such as the maximum entropy proposed by Upper and Worms (2004), and also permits more robust analysis.

In the stress test, we assume that a given sector defaults in its obligations and then consider its effects on the other sectors in the network. To simplify the analysis, we assume that firms cannot borrow if their short-term assets are less than their short-term liabilities. Once the trigger sector defaults, current assets of the other sectors are reduced by an amount equivalent to their credit exposure to the sector that has defaulted times the loss given default. As a benchmark, we assume an extreme stress scenario where the trigger sector defaults in full and then the loss given default is 100%. In turn, sectors suffering contagion from the sector originally going into default will be able to honor their debt if their current assets are larger than their short-term liabilities. Otherwise, these sectors will also default, and the propagation through the commercial network will continue.

The results from the benchmark stress test exercise show that the sectors “transport, communication, and storage” and “hotels and restaurants” are affected by the default of any of the other sectors. These sectors are the most exposed in terms of contagion. Although the “manufacturing” and “commerce” sectors are the most central according to centrality measures and have the highest level of indebtedness, they have a large amount of liquid assets, which allows them to survive all the shocks from other sectors in the short term. We also try different thresholds for the loss given default and find that for values of less than 90%, there is no shock propagation through the network. Overall, this implies that the dangerous threshold for the Uruguayan economy in its commercial credit network is relatively high.

Finally, we link the survey data with data from the Bank Credit Registry, a database containing the indebtedness of firms to banks. With this information, we are able to have a more complete picture of the credit network by considering both commercial and banking credit. The complete network enables a more complete estimation of the exposition of banking institutions to each economic sector by considering not only the direct indebtedness, but also the exposure of that sector to others through the commercial indebtedness. This second channel is important because commercial credit is larger than banking loans for most of the sectors; in cases like “construction” and “commerce,” the former is more than twice the latter. We also identify the degree of centrality of banks in the network. The results show that the top banks in this ranking are the ones facing higher capital requirements due to their contribution to systemic risk. Overall, the effort of constructing a commercial and bank credit network provides a useful tool for the analysis of financial stability.

The rest of the paper is organized as follows. In Section 2, we present related literature. In Section 3, we analyze the production network of Uruguay using information of the input-output matrix. In Section 4, we describe the data. In Section 5, we introduce the different measures used in the literature to describe networks and their topology. We discuss some of the implications of these measures in terms of identifying relevant sectors and expected propagation results. In Section 6, we present the commercial debt and total debt network structure in Uruguay. In Section 7, we present our stress testing general framework. In Section 8, we present the main results of the exercise over the commercial debt network. Finally, in Section 9, we discuss the main conclusion of this work and further developments that could be carried out with the networks introduced in this paper.

2. Related literature

Since the global financial crisis of 2007-08, network contagion has gained particular attention in the literature. However, the topic is not new. At least since 1932, networks have been applied in different fields of social science (Borgatti et al., 2009).

¹ In a companion paper using the same data set, Landaberry et al. (2021) complete the intra-firm commercial credit network by using alternative reconstruction methods.

² “Real estate” is the least central sector in terms of commercial credit. This does not imply, however, that the sector is not important in terms of financial stability. For instance, it may hold excessive banking credit (although that is not the case in Uruguay), and a sudden drop in real estate prices can trigger other financial risks, as has been the case in several developed economies during the last decade. In this paper, we focus on the commercial credit channel and find that the sector is not central in that network. The study of the other channels is, however, outside the scope of the paper.

We use several approaches to network analysis in this paper. The first one is about topology and network structure. We use different measures to characterize the commercial and banking credit networks in Uruguay. These measures, which are explained in detail in Section 5, were developed by Freeman (1978-1979), Katz (1953), Taylor (1969), Barabási et al. (2002), and Stephenson and Zelen (1989), among others. Borgatti (2004) presents a topology of network flows and discusses the measure of centrality of a network that best matches the type of flow represented in the network. Valenti et al. (2008) analyze the correlation between the different centrality measures. They conclude that in general, these measures are positively correlated, but the magnitude of the correlation is not high enough to assume they are redundant.

There is a theoretical strand of literature about financial contagion through networks. In particular, Acemoglu et al. (2014) develop a unified framework for the study of how network interactions can function as a mechanism for propagation and amplification of microeconomic shocks. The framework nests various classes of games over networks, models of macroeconomic risk originating from microeconomic shocks, and models of financial interactions. Elliot et al. (2014) develop a general model to study financial contagion and cascades of failures among organizations linked through a network of financial interdependence. They find that integration (greater dependence per organization) and diversification (more counterparties per organization) have different, non-monotonic effects on the extent of cascades. Diversification connects the network initially, permitting cascades to travel. However, as it increases further, organizations are better insured against another's failures. Integration also faces trade-offs: increased dependence on other organizations versus less sensitivity to one's own investments. Acemoglu et al. (2015) relate shock magnitude and network structures. According to their results, if negative shocks affecting a financial institution are sufficiently small, then a more densely connected financial network enhances financial stability. However, when shocks are large enough, a denser network serves as a mechanism for the propagation of shocks, leading to a more fragile financial system. In this paper, we compare the density and connection measures in terms of the commercial credit network structure based on these ideas.

Another strand of the literature has focused on contagion among banking institutions. Calomiris et al. (2019) investigate the role of contagion on bank distress during the Great Depression by mapping in detail the inter-bank network and examining how network connections affect the risk of bank failure. They find that banks with more correspondents (hence with greater liquidity risk) and banks whose correspondents closed down are more likely to fail. Silva et al. (2016) analyze how the Brazilian inter-bank network evolves with respect to different types of market participants using well-known complex network measures. Souza et al. (2013) analyze the Brazilian inter-bank market to study (through simulation) the potential contagion among institutions, the contagion losses, and the contagion route associated with financial institution bankruptcies. They also compute the possibility of contagion to other markets triggered by financial institution defaults in the inter-bank markets. Battiston et al. (2012) introduce DebtRank as a measure for estimating the systemic importance of financial institutions. It considers the impact of the default of one or more institutions to their counterparts across the whole network. In this paper, we perform a stress test exercise to study the contagion through commercial credit among economic sectors in the real economy.

The literature considering the bipartite links between banks and firms is extensive. For instance, Aoyama et al. (2013), Marotta et al. (2015), and De Masi et al. (2011) build a bipartite bank-firm network for Japan. De Masi and Gallegati (2007) represent the Italian bank-firm system as a network, using an approach based on graph theory. Lux (2016) proposes a stochastic model of a bipartite credit network between banks and the non-bank corporate sector. The author focuses on the effect of the failure of one non-bank corporate unit on the banking sector and concludes that contagion due to joint exposures to counterparty risk via loans to firms is more relevant than contagion due to inter-bank credit. Poledna et al. (2018) reconstruct and analyze the financial liability network, combining the firm-bank network and the inter-bank network in the Austrian banking system. They find that all firms together create more systemic risk than the entire financial sector. Delli-Gatti et al. (2006) model a network economy with firms and banks, where firms are connected through productive relationships, while banks and firms are connected through bank credit. According to this literature, the risk imposed by firms to the banking sector is relevant. Differently from our study, it only considers the bipartite financial network between banks and firms through banking credit. We focus on the financial interlinks of commercial debt among economic sectors.

Research on interlinks through commercial indebtedness among different industries or economic sectors has been less developed (Acemoglu et al., 2016), possibly due to the lack of reliable and timely data (Anand et al., 2018). Most of the literature on interlinks among economic sectors consider their production relationships, capturing the input and output relationships in the economy. For instance, Acemoglu et al. (2012) build the inter-sector network corresponding to the U.S. by using data from the input-output matrix. Atalay (2017) uses input-output tables to estimate the relevant elasticities of substitution among products in the U.S. and study how industry-specific shocks have the potential to contribute to aggregate output volatility. Hausmann and Hidalgo (2011) look at the built-in output structure in the network that connects countries with the products they export. DePaolis et al. (2020) apply network analysis techniques to large input-output systems. Using these techniques, they identify key sectors in the economy. Although the network measures used in that paper are similar to ours, its focus is on economic relationships. Instead, we focus on identifying key sectors in the commercial credit network. We also study the input-output relationships among sectors and show that the linkage in terms of economic activity may be related to the commercial debt between sectors. However, to use the former as a proxy for the latter may be misleading because the production network might overestimate the indebtedness linkages among economic sectors. Hence, our paper helps to fill the data gap regarding commercial credit networks and to understand the limitations of proxy indicators.

In general, there is only partial information to estimate the structure of the networks. A strand of the literature proposes different methods of reconstruction to complete the interconnection matrix starting from partial (e.g., aggregate) information. Anand et al. (2018), compare seven network reconstruction methods. They find that the best method depends on the final purpose of the reconstructed network. Anand et al. (2015) propose the minimum density (MD) methodology based on the rationale that "economic linkages are costly to add and maintain." This reconstruction method minimizes the number of links necessary to distribute something



Fig. 1. Production network in Uruguay. *Note:* Edge width represents the amount of input trade, and the size of the nodes represent gross value added from the sector. 2012 current Uruguayan pesos.

(e.g., a loan). When performing stress test exercises over networks resulting from using this method, contagion is overestimated. On the other side, the maximum entropy (ME) reconstruction method proposed by [Upper and Worms \(2004\)](#) will result in a denser interconnection matrix, and it tends to create complete networks. In this case, contagion is underestimated. In this paper, we use information that is partial in two aspects. First, the survey does not include firms belonging to some sectors. In this case, interlinks are inferred from the answers of other firms that declare having debt or credit with firms in the non-surveyed sectors. Second, we have information about the three main debtors and creditors of each firm in the survey, as well as about the aggregate positions. To reconstruct the complete credit network, we follow the spirit of the MD reconstruction method, where the number of links is minimized between sectors according to the information available from the survey. In this paper, we reconstruct the commercial credit network at a sector level. Using the same data set, in a companion paper, [Landaberry et al. \(2021\)](#) use network reconstruction methods to obtain a bipartite network between firms and banks and the intra-firm networks. DebtRank is used to evaluate systemic risk. As we do in this paper at a sector level, they find that ignoring intra-firm exposures might result in an important underestimation of systemic risk.

Finally, the data collected to build the commercial credit network enables the realization of a simple stress test exercise to analyze the propagation of credit risk through the commercial debt network. In doing so, we follow [Ong and Jobst's \(2020\)](#) guidance for stress testing.

3. Input-output network

As mentioned in the literature review, previous work generally uses an input-output matrix to analyze and describe the interdependence among economic sectors. This matrix represents the static equilibrium of the technological conditions of the total production of an economy during a specific period. In that sense, it can be used as a proxy of the production network, where the nodes represent the economic sectors and the links represent the production traded among them.

In this section, we estimate this production network to describe the interdependence among economic sectors. It also serves as a benchmark for comparison with the commercial credit network that we compute in [Section 6](#). Indeed, at least part of the trade of goods and services among economic sectors is supported by credit from suppliers to users. [Figure 1](#) shows the graph of the Uruguayan production network in 2012, the latest year with available information, based on [Brun and Lalanne \(2017\)](#).³ Detailed information on the input-output matrix is presented in the Appendix.

As can be seen, almost all sectors are connected through the input-output trade. The only two sectors that are not connected are financial intermediation and primary activities because the former does not use inputs from the latter. In other words, the production network is extremely dense and, as we will see in [Section 6](#), the commercial credit network is much less dense than the production network; that is, not all trade relationships are backed by commercial credit.

The interdependence that exists between economic activity sectors implies that when the demand to a specific sector changes, a chain of reactions will affect the other sectors. The coefficients of the Leontief inverse matrix, constructed from the data in the input-output matrix, enables quantification of not only the direct effects but also the indirect ones in the production of all the sectors in the

³ The productive structure of the Uruguayan economy has suffered changes in recent years (structural changes, modifications of relative prices and technological change); therefore, this input-output matrix may not reflect the current production network in Uruguay.

Table 1
Total commercial credit and debt (Uruguayan pesos).

Sector	Firms	Sales credit	Commercial debt
Manufacturing industry	101	65.494.037.217	30.165.504.698
Electricity, gas and water	1	29.244.106	18.172.739
Construction	2	1.269.045.595	494.193.982
Commerce	43	24.454.984.231	21.451.037.893
Hotels and restaurants	8	441.909.218	365.789.382
Transportation, storage and communications	38	10.579.487.571	9.353.670.484
Teaching	9	2.330.114.133	246.611.246
Others	38	16.076.815.742	7.047.796.714
Total	240	120.675.637.813	69.142.777.138

Source: Survey of Economic Expectation, October 2018.

Table 2
Three major creditors and debtors in total (percentage).

Sector	Creditors	Debtors
Manufacturing industry	44.4	49.6
Electricity, gas and water	68.9	21.2
Construction	17.9	52.2
Commerce	60.3	30
Hotels and	38.1	86.6
Transportation, storage and communications	48.7	57.8
Teaching	39.1	4.8
Others	45.5	60.7
Average	45.4	45.4

Source: Survey of Economic Expectation, October 2018.

economy when the final demand of products of a determined sector changes. The Leontief inverse matrix for the case of Uruguay is presented in Table 9 of the Appendix. The sum of each column in this matrix shows the total demand generated by a unit demand of that sector. Thus, the sectors with the largest totals are the ones that generate a higher increase in the total production of the economy given a rise in their demand. The data for Uruguay indicate that the activity sectors that generate a bigger multiplier effect are electricity, gas and water, manufacturing industry and construction.

4. Data to build the commercial and banking credit network

We survey a sample of firms that is representative of the universe of Uruguayan firms with more than 50 employees. More precisely, in October 2018, we add specific questions regarding commercial credit to the Survey of Economic Expectations that is collected monthly by the Instituto Nacional de Estadísticas (the national statistic agency) on behalf of Banco Central del Uruguay (the central bank).⁴ We collect information on the amount of each firm's commercial debts and credits, and are able to identify each firm's three major debtors and creditors. We can also identify the sector of the economy to which each firm belongs to, allowing us to aggregate the results at the sector level. The number of firms in the survey and the total amount of commercial credit and debt outstanding in each sector are shown in Table 1. In Table 2, we show the average debt and credit shares of the three major creditors and debtors. These represent, on average, 45.4% of total credits and debits. There is heterogeneity across sectors.

With this information, we build a commercial credit network considering a total of twelve sectors in the economy. The survey does not include firms belonging to the primary activity sector, financial intermediation, public sector, or real estate activities. Hence, the information about the connections with these sectors is incomplete. Interlinks are reconstructed from the answers of other firms that declare having debt or credit with firms of the aforementioned sectors. In Table 3, we present the total commercial credit that results from the sum of the three major commercial debtors of each sector, as well as the total commercial debt that results from the sum of the three major creditors of each sector.⁵ Figures for the primary activity sector, financial intermediation, public sector, and real estate activities are reconstructed from the answers from firms in the other surveyed sectors.

We obtain additional balance sheet information using data from the Economic Activity Survey. Because the latest survey available is from 2014, we update the balance sheet items until 2018 using the Consumer Price Index (CPI). In particular, we are interested in identifying short-term assets and short-term liabilities other than commercial credit. The total of cash available, temporary investments, and other short-term liabilities, adjusted by CPI, are presented in Table 4.

⁴ The survey is conducted on a sample according to a statistical criteria to ensure that the expansion of the answers is representative of the universe of firms with more than 50 employees in Uruguay.

⁵ For all sectors, except "electricity, gas, and water," the total debt and credit is bigger than the result from adding the three major creditors and debtors. For this particular sector (that is represented by only one firm in the survey), we substitute total debt and credit with the result from the sum of the three major creditors and debtors.

Table 3
Commercial credit and debt considering the three major debtors and creditors (Uruguayan pesos).

Sector	Sales credit	Debt
Primary activities *	1.791.310.985	1.370.693.495
Manufacturing industry	7.563.863.587	4.391.350.620
Electricity, gas, and water	287.842.268	873.712.455
Construction	877.426.410	331.366.354
Commerce	3.825.718.914	7.172.426.364
Hotels and restaurants	241.727.909	91.352.714
Transportation, storage, and communications	2.716.065.669	729.138.071
Financial intermediation *	117.921.741	1.473.859.781
Real estate *	1.580.979	300.279
Public sector *	30.713.827	4.787.851.624
Teaching	7.959.769	51.783.967
Others	7.094.123.862	3.282.420.196
Total	24.556.255.920	24.556.255.919

Source: Survey of Economic Expectation, October 2018 *: Reconstructed from answers provided by firms in surveyed sectors.

Table 4
Short-term assets and liabilities (Uruguayan pesos).

Sector	Cash	Temporary investment	Other short-term liabilities
Manufacturing industry	16.999.512.755	6.762.554.557	11.555.202.093
Electricity, gas, and water	70.088.439	NA	112.138.566
Construction	188.174.834	NA	575.026.439
Commerce	16.444.577.276	3.198.887.316	7.234.864.743
Hotels and restaurants	920.846.913	1.827.794	745.909.160
Transportation, storage, and communications	7.537.318.433	884.691.713	7.692.839.710
Teaching	62.166.078	NA	NA
Others	8.394.321.570	1.420.693.597	8.479.923.827
Total	50.617.006.298	12.268.654.976	36.395.904.538

Source: Survey of Economic Activity 2014 updated to 2018 using CPI.

Table 5
Financial Debt (Uruguayan pesos).

Sector	Financial debt
Manufacturing industry	88.082.844.993
Electricity, gas, and water	0
Construction	215.885.823
Commerce	17.327.411.631
Hotels and restaurants	2.948.169.769
Transportation, storage, and communications	12.860.617.317
Teaching	672.936.419
Others	8.664.398.114
Total	130.772.264.066

Source: Central Risk Database, October 2018.

Finally, we also consider banking credit. On top of the 11 banks, we define a group of other financial institutions. This group contains information about financial houses, credit management companies, and financial intermediation cooperatives. Using these data, we obtain an enlarged network that represents the linkage between economic sectors and each one of those financial institutions. The data about the total financial debt for each economic sector are obtained from the Credit Registry provided by the Superintendency of Financial Services (SSF). For that, we consider the debt with the financial sector of the firms included in the survey and, using the corresponding sample weight, obtain an estimation of the total financial debt for each economic sector. Financial debt by sector is presented in [Table 5](#).

5. Network structure measures

In this section, we briefly describe the network structure measures that will be used in the next section to characterize the commercial credit network. The network structure is defined by the nodes and edges that compose it. In the commercial credit network, the nodes are the economic sectors, and the edges serve as the commercial credit linkage between two sectors. In this case, edges represent whether one sector owes debt to another one.

Mathematically, one can represent a network by an $n \times n$ adjacency matrix A . The matrix A has elements $A_{ij} = 1$ if there is an edge between i and j , and 0 otherwise. Matrix is symmetric if there is no direction between nodes, and if there is an edge between i and j , then there is also an edge between j and i . In some cases, when the direction of the relationship between nodes is represented by the edge matters, the matrix may not be symmetric. For instance, this may be the case when we work with commercial credit among sectors and our analysis is based on a matrix that is not symmetric. In this situation, the edges in our graphs are directed from the sector that owes the debt to the corresponding creditor sector.

We use conventional measures of the topology of the network to identify the nodes that are more central and to characterize the commercial credit network. Regarding centrality measures, we consider degree centrality, in-degree centrality and out-degree centrality, centrality closeness, betweenness centrality, and eigenvector centrality.⁶ As a consequence of the different key concepts that are considered to be central in each measure, as well as the different assumptions made about the manner in which flow through the network occurs, they may provide different and complementary information (Newman, 2008). Following Valenti et al. (2008), in general, these measures have a positive correlation, but the magnitude of the correlation is not high enough for one to assume that they are redundant. Hence, we compute all of them to assess the relative importance of a particular node in the network.

Other measures, such as size, density, reciprocity, and transitivity, are used to describe the structure of the network.⁷ The specific structure of the network has implications for the way that shock propagates. Elliot et al. (2014) show that the way in which defaults trigger further failures depends on the network structure. If integration in a network (defined as the level of exposure of sectors to one other) is intermediate and the network is partly diversified, then the network is more susceptible to widespread financial failures. In terms of network measures, a network with medium values of density, reciprocity, and transitivity would be more susceptible to widespread shocks. Moreover, the structure may have different effects for different shock sizes as pointed out by Acemoglu et al. (2015). A perfect diversified network is optimal for moderate shocks but it is the worst possible one in the context of a large shock. Finally, it is worth noting that not only is the topology of the underlying graph relevant for the propagation of shocks but also, as shown in our empirical application, firms' economic situation is important. In this sense, firms' ability to absorb shocks is also key to stopping or alleviating the propagation of the shocks.

6. Commercial and banking credit networks

6.1. Commercial credit network

For the commercial credit network in Uruguay, we define a total of 12 nodes representing economic sectors: primary activities; manufacturing; electricity, gas and, water; construction; commerce; hotels and restaurants; transportation, storage, and communication; financial intermediation; real estate; public sector; teaching; and others.⁸ The edges represent the debt of one sector to another one. The sectors identified and the linkage between them are obtained from the October 2018 Economic Expectations Survey as described in Section 4.

As mentioned in Section 4, the survey does not include firms belonging to the primary activity sector, financial intermediation, public sector or real estate activities. The information on the connections among these sectors is incomplete, so connections are reconstructed from the answers provided by other firms that claim to have debt or are creditors of some firms from these sectors. In general, we define node n_i as economic sector i , and we define edge e_{ij} as the connection resulting from the fact that sector i owes debt to sector j . To obtain the total owed by sector i to sector j , we add the weighted amount of debt with sector j that is informed by the surveyed firms in sector i .⁹ Considering only the three main creditors and debtors, we obtain on average 45.4% of the aggregate position. For the remaining amount of debt, we assign it proportionally to the sectors for which we already have information that there is a link derived from the three main debtors and creditors.

To reconstruct the complete commercial credit network presented in Fig. 2, we follow the spirit of the MD reconstruction method (Anand et al., 2015), where the number of links is minimized between sectors based on the information that is available from the survey. According to the authors, when used in a stress testing context such as the one we perform in Sections 7 and 8, this approach works better than alternative reconstruction methods do (e.g., the ME proposed by Upper and Worms (2004)), and also permits a more robust analysis.

Figure 2 shows a graph of the resulting commercial credit network. Edges are directed from the sector that owes debt to the corresponding creditor sector. The diameter size of the network is 3, and the mean distance is 1.52 edges. The network has a medium density of 0.51. This means that approximately half of all possible connections that could exist are active in the reconstructed commercial credit network. The reciprocity level is 0.77, and the transitivity is 0.69. According to Acemoglu et al. (2015), this medium connectivity in the network is better for accommodating large shocks, but the widespread contagion of a particular sector default may be bigger than that in other networks (Elliot et al., 2014).

The results of centrality measures imply that "commerce," "manufacturing," and "transportation, storage, and communication" are the most central sectors. "Real estate" is the least central one in terms of commercial credit. There is a high correlation between the centrality measures that have been estimated. The "transportation, storage, and communication" sector is very central in terms of the connections it has with other economic sectors, but its debt is more diversified, and its indebtedness level is smaller than that

⁶ A detailed explanation of these measurements is in the Appendix.

⁷ Definitions are presented in the Appendix. To compute these measures, we use CINNA and igraph packages in R.

⁸ We follow the Instituto Nacional de Estadística's (INE) classification.

⁹ Weights provided by the survey are representative of the universe of Uruguayan firms with more than 50 employees.

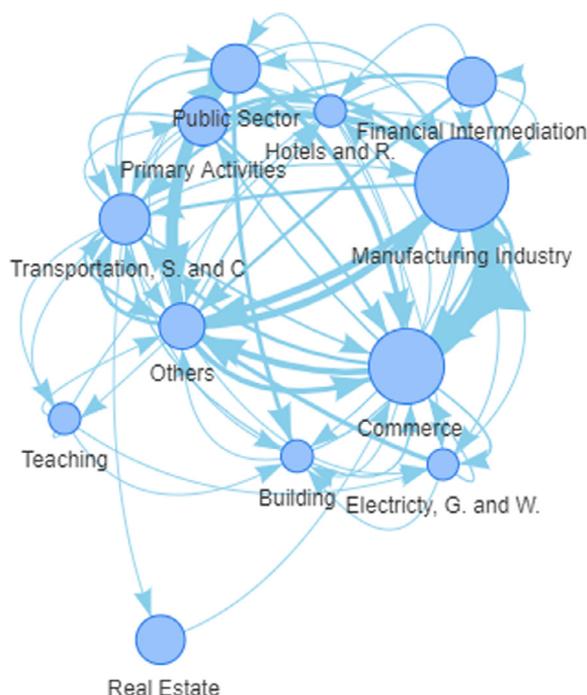


Fig. 2. Commercial credit network in Uruguay. *Note:* Edges' widths represent the amount owed, and the sizes of the nodes represent the total commercial debt from the sector.

Table 6

Economic sector centrality measures ranking.

Sector	Centrality degree	In-degree centrality	Out-degree centrality	Closeness centrality	Betweenness centrality	Eigenvector centrality
Manufacturing industry	2	2	2	2	3	1
Commerce	3	1	3	3	2	3
Transportation, S. and C.*	1	2	1	1	1	2
Others	4	2	3	3	4	4
Hotels and restaurants	5	3	3	3	5	5
Construction	6	4	5	5	6	6
Electricity, G. and W.*	7	5	5	6	7	8
Public sector	7	6	4	4	8	7
Teaching	7	7	4	4	8	11
Primary activities	7	6	6	6	9	9
Financial intermediation	7	7	4	4	9	10
Real estate	8	8	7	7	9	12

* S: Storage, C: Communications, G: Gas, W: Water.

observed for “commerce” and “manufacturing.” On the other hand, there is a high level of indebtedness from the “commerce” sector to the “manufacturing” sector.

In Table 6, we present a descendent rank, where 1 is the most central node according to the centrality measure. In the Appendix (Table 10), we present the results of all of the centrality measures estimated for each economic sector.

The commercial credit network is a lot much less dense than the production network is in Section 3. The latter has a medium density of 0.51, whereas for the production network, this measure reaches 0.99, meaning that almost all sectors are connected through the input-output trade¹⁰. Moreover, the production network has a smaller diameter and mean distance than the commercial network does (2 versus 3, and 1.01 versus 1.52, respectively).

For most economic sectors, there exists a relatively high, positive correlation between the production and the commercial credit networks. On average, however, the correlation among the interlinks in networks is below 50%. This highlights the importance of building the commercial credit network. Although the information contained in the input-output matrix may be used as a proxy of the credit exposure among economic sectors, it might oversize the indebtedness linkages: It is a much denser network, and a trade

¹⁰ The only two sectors that are not connected are financial intermediation and primary activities because the former does not use inputs from the latter.

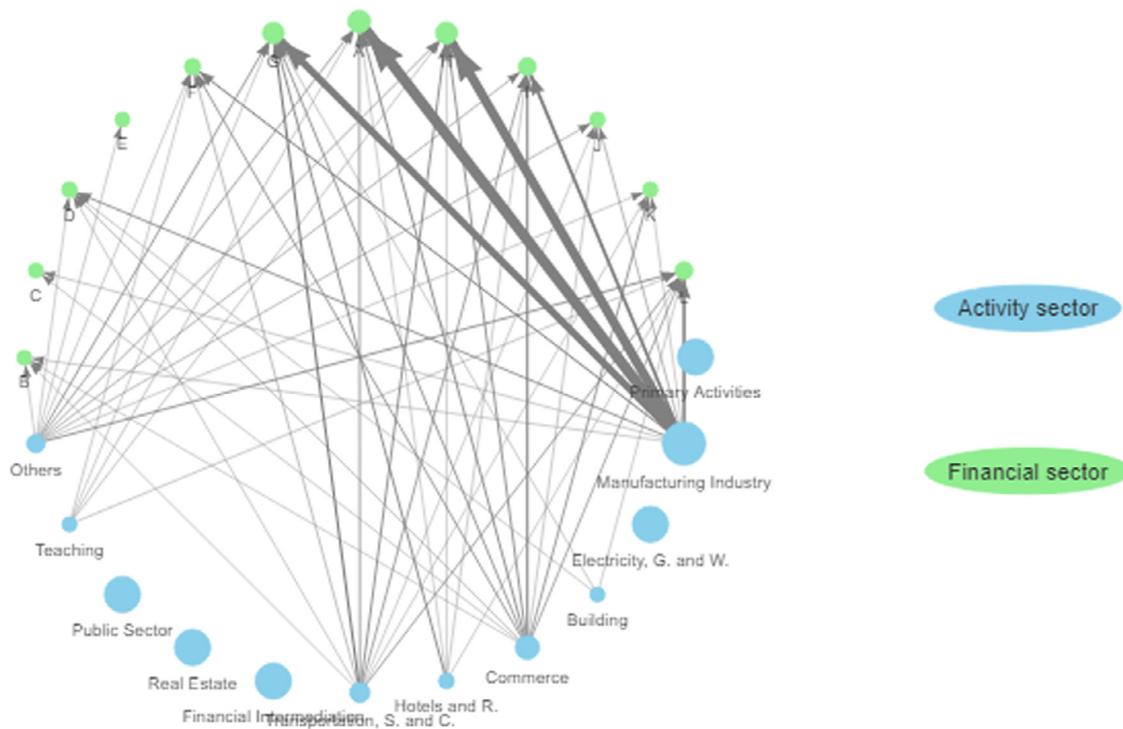


Fig. 3. Bipartite network between activity sectors and financial institutions. *Note:* Edges, widths represent the amount owed. The sizes of the nodes of the activity sectors represent the total debt (commercial and financial) of the sectors, and the sizes of the nodes of the financial sectors represent their credit.

Table 7
Financial to commercial credit ratio.

Sector	Ratio
Manufacturing industry	0.74
Electricity, gas and water	0.00
Construction	0.35
Commerce	0.45
Hotels and restaurants	0.00
Transportation, storage and communication	1.00
Teaching	1.28
Others	0.30

relationship does not necessarily imply commercial credit. For the purposes of financial stability, it is more appropriate to directly use information on commercial credit links when it is available because a sector might use an input from another one but that does not necessarily mean that the sectors are linked through commercial credit.

6.2. Banking credit network

We complete the analysis by estimating the banking credit network. As mentioned in Section 4, we consider the debt with the financial sector held by the firms that are included in the survey, and using the corresponding sample weight, we obtain an estimation of the total financial debt for each economic sector. Figure 3 shows the networks among economic sectors and among banking institutions.¹¹ The sizes of the nodes of the sectors represent the amount of commercial and financial debt, whereas the sizes of the nodes of the financial institutions represent the amount of debt owed to each of them. We do not consider linkages between the banks, as they are relatively not important in Uruguay.

A first observation is that for most sectors, commercial credit is larger than banking credit is. More precisely, the ratio of financial to commercial credit is lower than 1 (see Table 7). The amounts of both types of credit are similar in the manufacturing, transportation, and teaching sectors. Meanwhile, in the remaining sectors, the commercial indebtedness is more relevant than the financial debt is.

¹¹ The economic activity sector of financial intermediation was consolidated with the banking institutions.

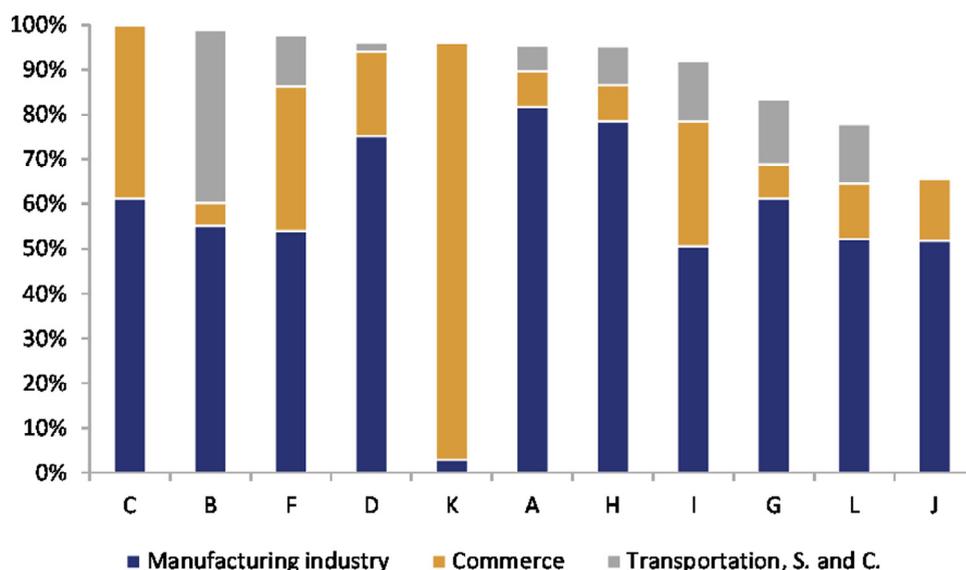


Fig. 4. Exposure of banking institutions to the most central sectors. Note: The bars represent the percentage of the total credit of financial institutions owed by each sector.

Table 8
Banking centrality measures ranking.

Institution	Centrality degree	Eigenvector centrality	High capital requirements for systemic risk
L	1	2	✓
G	2	3	✓
A	3	4	✓
H	4	5	✓
D	5	7	
F	6	8	
I	7	6	✓
K	8	1	
B	9	9	
J	10	10	✓
C	11	11	
E	12	12	

This result alerts us of the importance of seriously considering commercial indebtedness in general, as well as assess the stability of the banking system in particular. In this regard, commercial credit interlinks may amplify shocks and impair the quality of banking credit.

In Figure 4, we focus on the exposure of financial institutions to the most central economic activity sectors. For each institution, the bar represents the percentage of the total credit owed by each of the most central sectors. The banks with high capital requirements due to systemic risk are also the most exposed to the economic activity sectors identified as the most central in terms of commercial indebtedness. This suggests the importance of considering commercial credit networks to assess the contributions of individual banks to systemic risk.

For obtaining the centrality measures of the banks in the network, the interconnections were weighted by the use of credit by the firms of each institution, as well as the total credit granted to firms by the financial system. As shown in Table 8, the most central banks identified by centrality measures coincide with those that have a higher capital requirements for systemic risk according to the regulations in Uruguay. This result is in line with the findings of Umut et al. (2013), which indicated that centrality measures work well for identifying and monitoring systemically important financial institutions. The banks are ordered in a descendent order according to the centrality measure, where 1 represents the most central bank. In Table 11 in the Appendix, we present the values of the centrality measures estimated for banks.

7. General stress test framework

To complete our analysis and facilitate our understanding of how credit risk is transferred through the commercial debt network, we perform a default contagion exercise. We follow Ong and Jobst (2020) framework and principles for stress testing to propose a balance sheet stress test approach. More precisely, we consider the effects of the default of each sector on the network in the

context of financial credit restriction. With this purpose, we consider that each economic sector individually enters into default and affects the creditor sectors, whose current assets are reduced by the amount equivalent to the amount owed by the sector that enters into default, times the loss given default. As a benchmark, we assume a loss given default of 100%. Hence, this is an extreme-case stress test exercise, where it is assumed that all firms in the trigger sector default and the loss given default is full. Although these assumptions are extreme and are unlikely in practice, they allow us to show a type of stress testing analysis that is possible using the credit network. Moreover, following [Ong and Jobst \(2020\)](#), we focus on an extreme value risk considering the fact that the loss given default typically “double[s] in the case of a severe/extreme stress” from an average of around 40-60%. Next, we perform the exercise with other loss given default assumptions to identify a dangerous threshold in the case under study.

We build an $n \times n$ matrix A , where element A_{ij} is the total amount owed from sector i to j . The total commercial debt (CD_i) of sector i is given by [Eq. \(1\)](#), and we assume that commercial debt is a component of short-term liabilities of sector i . The total sales credit (SC_i) for sector i is given by [Eq. \(2\)](#), and we assume that sales credit is a component of short-term assets of sector i :

$$CD_i = \sum_j A_{ij}, \quad (1)$$

$$SC_i = \sum_i A_{ij}. \quad (2)$$

When sector i defaults, it is assumed that it does not pay the amount owed to the other sectors. The Counterpart k will be affected and the total credit sales asset will be reduced by the amount owed by sector i to sector k . The direct effect of sector i default in the total credit of sector k is represented in [Eq. \(3\)](#). We use t_0 to refer to the initial moment before the default, and we use t_1 to refer to the moment after the first default:

$$SC_{kt_1} = SC_{kt_0} - A_{ki}. \quad (3)$$

The sector affected will be able to honor its debts if its current assets are larger than its short-term liabilities are; if not, this sector will also default, and the propagation through the network continues. The financial credit restriction is imposed by the fact that firms cannot borrow money from the financial system if their current assets are less than their commercial debt is.

We define matrix S of dimension $n \times 1$ representing short-term assets. S_{k1} are the short-term assets of sector k . Short-term assets are the sum of sales credit, available cash and temporary investments. These last two items are obtained from the Annual Economic Activity Survey.

We also define matrix L of dimension $n \times 1$ representing short-term liabilities. L_{k1} are the short-term liabilities of sector k . Short-term liabilities are the sum of commercial debt and other short term liabilities. These balance sheet items are also obtained from the Annual Economic Activity survey. The default of sector i affects the short-term assets of all the other sectors that are creditors of i : its short-term assets are reduced by the amount A_{ki} . Sector k will go into default in $t = 1$ if

$$S_{k1} - L_{k1} < 0. \quad (4)$$

If this sector defaults, the propagation through the network continues. We perform this exercise simulating the default in $t = 0$ for each node in the network (a total of n simulations). A financial credit restriction is assumed: firms cannot borrow money from the financial system if their current assets are less than their commercial debt is. As a result of this exercise, we can identify the sectors that default, how the default propagates through the network and the extent of the propagation.

8. Commercial credit stress test results

In this section, we present the results of the hypothetical, extreme-case default shock exercise introduced in [Section 7](#). Given that we need additional balance sheet information to perform the exercise, and given that the Annual Economic Activity Survey does not include data for the sectors of real estate, primary activities, public sector and financial intermediation, we exclude these sectors from the analysis.

The results of the default shock propagation are presented in [Fig. 5](#).¹² For example, in [Fig. 5](#) panel (1), we describe the result of a hypothetical initial default shock of the manufacturing sector. The y axis shows the total number of sectors in default, and the x axis shows the periods of the propagation of shocks. This is represented in $t = 1$ by the default of the “exercise default” sector as well as other sectors that go into default as a result of direct contagion (“shock propagation”). We continue adding bars until the shock propagation ends and no other sectors go into default. The last bar of each graph is the final propagation period and indicates the total number of sectors that defaulted as a result of the initial default of the sector considered. In panel (1), the default of the manufacturing sector produces a total of two defaults as a result of the propagation effect, and the contagion lasts two periods. At the end, there are three sectors affected: the manufacturing industry that is in default by assumption, and two other sectors affected by contagio. One is affected in period $t = 1$ and the other in $t = 2$. These two sectors are “hotels and restaurants” and then “transportation, storage and communication,” respectively.

In all cases, the total time of default propagation is less than or equal to 2. This means that at a maximum, there are second-round effects on default contagion in the network. The maximum contagion, by the propagation of the shock, is produced when

¹² “Electricity, gas and water,” “construction,” and “others” in the initial moment already have short-term assets lower than their short-term liabilities are. This means that in the context of bank credit restrictions, these sectors will enter into default in all the exercises. For this reason, we focus on the impact of an initial default on each of the remaining sectors.

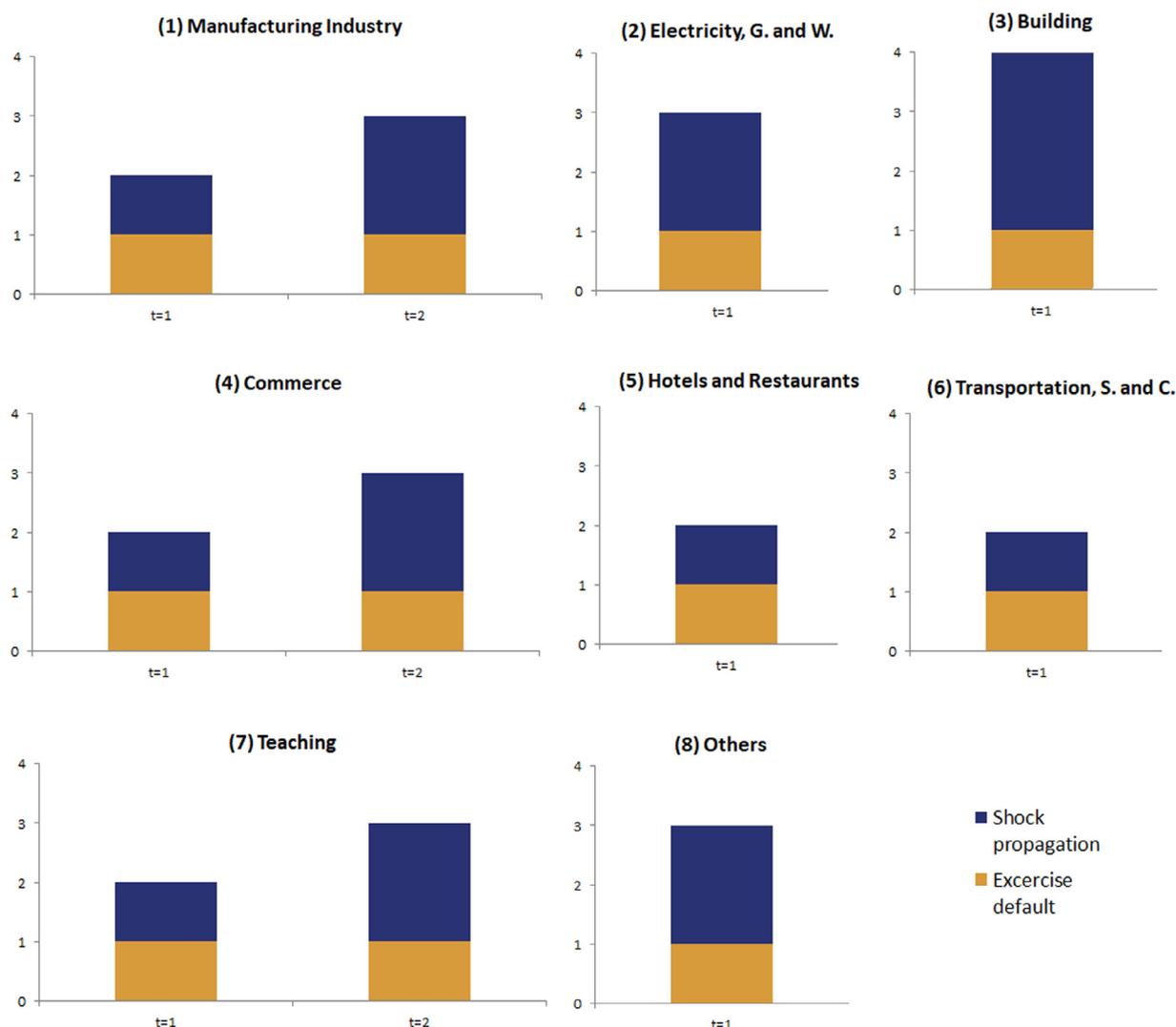


Fig. 5. Defaults by sector.

the “construction” sector defaults. In this situation, the following sectors also default: “teaching,” “transportation,” and “hotels and restaurants”. The most exposed sectors to contagion are “transportation, storage, and communication” and “hotels and restaurants”. These sectors default by the contagion effects in all scenarios. We simulate the default of all of the individual sectors and consider the effects of direct and indirect contagion. The results change according to the sector that is affected first.

Although the “manufacturing” and “commerce” sectors are the most central and have the highest levels of indebtedness, they have large amounts of liquid assets in the short term, which allows them to survive all of the shocks coming from other sectors. As consequence, we do not observe the default of all sectors in this exercise.

We run the same exercise for other values of the loss given default. The results show that for a loss given default of less than 90%, there is no propagation of shocks through the economic sectors. At an aggregate level, this implies that the dangerous threshold for the loss given default in the commercial credit network of Uruguay is relatively high.

9. Final remarks

In this paper, we build a commercial debt network using a sector-level analysis for Uruguay. Using traditional centrality and structure measures for networks, we identify the manufacturing industry; commerce; and transportation, storage, and communications as the most central sectors in the network. We also identify a high level of indebtedness of the commerce sector with the manufacturing sector. In addition, we obtain a more complete and complex network by adding the sectors’ financial debt to banks. This allows us to estimate the centrality measures for banking institutions. Most of the banks are connected to all sectors; four banks are not. According

to the results, the most central banks identified with centrality measures coincide with those that have higher capital requirements for systemic risk.

Considering simultaneously the financial and commercial debt by economic sector, we obtain a more adequate measure of the indebtedness structure of each sector. The financial to commercial credit ratio is similar to the one in the manufacturing industry, transportation, and teaching sectors. In the remaining sectors, the commercial indebtedness is more relevant than the financial debt is.

To complete our analysis and facilitate our understanding of how credit risk is transferred through the network of commercial debt, we perform default contagion exercises among the economic sectors. We consider the effects of the default of each sector on the network in the context of financial credit restriction. For this purpose, we consider that each economic sector individually goes into default and affects the creditors sectors, whose current assets are reduced by the amount owed by the sector that goes into default. The sectors affected will be able to honor their debts if their current assets are bigger than their short-term liabilities. If not, these sectors will also default, and propagation through the network continues. The financial credit restriction is imposed by the fact that firms cannot borrow money from the financial system if their current assets are less than their short-term liabilities are. Although these are extreme and unlikely assumptions in practice, they allow us to demonstrate a type of stress testing exercise that is possible with the credit network.

As a result of this exercise, we find that the sectors of “transport, communication, and storage” and “hotels and restaurants” are affected when any of the other sectors default. These sectors are the most exposed in terms of contagion. Although the manufacturing industry and commerce are the most central, according to centrality measures, and although they have the highest level of indebtedness, they have large amounts of liquid assets in the short term, which allows them to survive all of the shocks coming from other sectors. Because of the relevance of the manufacturing industry and commerce, the shock could be amplified considering the default of this sector simultaneously with some liquidity shock, which may affect one or both sectors.

The results highlight the importance of having a good estimation of commercial credit interlinks for financial stability analysis. Moreover, using an input-output matrix as a proxy of commercial credit may be misleading because the production network might oversize the indebtedness linkages among the economic sectors. Hence, our paper helps to fill the data gap regarding commercial credit networks and facilitates an understanding of the limitations of proxy indicators. Further development of network analysis in Uruguay includes the buildup of a commercial debt network at the firm level, as well as an analysis of the increase in banking credit risk as a result of sector defaults and contagion.

Declaration of Competing Interest

None.

Appendix A

A. Tables

B. Network topology

Centrality measures are one of the most fundamental measures used for characterizing a network and identifying the most important or central node in the network. The simplest centrality measure is *degree centrality* of a node, defined as the number of edges attached to it. The Degree centrality DG_i of node i when matrix A is not symmetric¹³ is:

$$DG_i = \sum_{j=1}^n (A_{ij} + A_{ji}). \quad (5)$$

When there is a direction defined in the linkage between nodes, we can also define in-degree centrality or out-degree centrality (Donglei, 2012). *In-degree centrality*, defined in Eq. (6), considers only the edges that go to node i (all of the sectors that owes debt to node i):

$$In - DG_i = \sum_{j=1}^n (A_{ji}). \quad (6)$$

Out-degree centrality, defined in Eq. (7), considers only the edges that originate in node i (all of the sectors that i owes debt to):

$$Out - DG_i = \sum_{j=1}^n (A_{ij}). \quad (7)$$

For comparing centrality measures between two or more different networks, it is necessary to standardize the values by dividing the result by the total of nodes minus 1.

¹³ When matrix A is symmetric and the direction of the relationship between nodes is not relevant, $DG_i = \sum_{j=1}^n A_{ij}$.

Table 9

Total requirements coefficients matrix (Leontief inverse matrix).

	Prim. Act.	Manu. Ind.	Elect.,G., and W.	Const.	Comm.	Hot. & Rest.	Trans., S. and C.	Finan.Interm.	Pub. Sect.	Real State	Teach.	Oth.
Prim. Act.	1.23	0.48	0.29	0.14	0.07	0.16	0.10	0.01	0.02	0.05	0.03	0.07
Manu. Ind.	0.38	1.52	0.84	0.40	0.20	0.45	0.30	0.04	0.05	0.14	0.08	0.20
Elect., G., and W.	0.03	0.05	1.24	0.01	0.03	0.03	0.02	0.01	0.00	0.04	0.02	0.02
Const.	0.03	0.03	0.02	1.30	0.03	0.02	0.02	0.01	0.13	0.01	0.01	0.01
Comm.	0.08	0.09	0.08	0.06	1.05	0.07	0.08	0.01	0.01	0.02	0.01	0.04
Hot. & Rest.	0.00	0.01	0.01	0.00	0.01	1.01	0.01	0.00	0.00	0.02	0.01	0.00
Trans., S. and C.	0.07	0.08	0.07	0.03	0.08	0.04	1.14	0.02	0.01	0.03	0.01	0.06
Finan. Interm.	0.04	0.06	0.09	0.03	0.05	0.05	0.05	1.24	0.06	0.08	0.06	0.02
Pub. Sect.	0.02	0.02	0.02	0.01	0.03	0.04	0.02	0.01	1.03	0.02	0.01	0.03
Real Estate	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.00	0.00	1.00	0.00	0.00
Teach.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
Oth.	0.04	0.08	0.11	0.04	0.13	0.07	0.12	0.06	0.01	0.13	0.03	1.17
Total	1.93	2.41	2.77	2.02	1.71	1.94	1.88	1.41	1.31	1.54	1.27	1.63

Table 10
Economic sector centrality measures.

Sector	Centrality degree	In-degree centrality	Out-degree centrality	Closeness centrality	Betweenness centrality	Eigenvector centrality
Primary Activity	7	4	3	0,6	0,0	0,5
Manufacturing Industry	18	9	9	0,8	0,1	1,0
Electricity, G. and W.	9	5	4	0,6	0,0	0,6
Construction	10	6	4	0,6	0,0	0,6
Commerce	17	10	7	0,7	0,1	0,9
Hotels and Restaurants	13	6	7	0,7	0,0	0,8
Transportation, S. and C.	19	9	10	0,9	0,2	1,0
Financial Intermediation	7	2	5	0,6	0,0	0,5
Public Sector	2	1	1	0,6	0,0	0,6
Real Estate	9	4	5	0,5	0,0	0,1
Teaching	7	2	5	0,6	0,0	0,4
Others	16	9	7	0,7	0,1	0,9

Table 11
Banking centrality measures.

Institution	Weigthed eigenvector centrality	In-degree centrality
L	0,35	7
G	0,35	7
A	0,32	6
D	0,27	5
F	0,27	5
I	0,29	5
K	0,59	5
B	0,24	4
J	0,24	4
C	0,12	2
E	0,06	1

The node in the network with more degree centrality would be the most central. *Centrality closeness* takes into account not only the number of the linkages or nodes related to define the centrality of the node but also the distance between the different nodes. Following this measure, a node is more central when the distance between this node and all of the other nodes in the network is the lowest. Centrality closeness, defined in Eq. (8), is the inverse of the sum of the distances of the geodesic path (shortest path d) between node i and all of the other nodes in the network (Freeman, 1978-1979):

$$CC_i = \frac{1}{\sum d(i, j)}. \quad (8)$$

Betweenness centrality is defined as the proportion of times that node i is necessary for node k to reach node j following the geodesic path between nodes k and j . This measure of centrality is important when we want to consider the importance of a node in terms of the flow that is transmitted through that node. If we define g_{kj} as the number of geodesic paths between i and j , g_{kij} , as the number of these geodesic paths that pass through node i , then the betweenness centrality of node i is presented in Eq. (9):

$$BC_i = \sum_k \sum_j \frac{g_{kij}}{g_{kj}}. \quad (9)$$

Eigenvector centrality considers the influence of a node. It assigns a score to each node in the network considering that having connections with nodes that also have high levels of connections makes that node more central (Sola et al., 2013). For a given graph $G := (V, E)$ with $|V|$ vertices, let $A = (a_{v,t})$ be the adjacency matrix. The relative centrality, x , and the score of vertex v can be defined as:

$$x_v = \frac{1}{\lambda} \sum_{t \in M(v)} x_t = \frac{1}{\lambda} \sum_{t \in G} a_{v,t} x_t \quad (10)$$

where $M(v)$ is a set of the neighbors of v , and λ is a constant.

Size measures include diameter and mean distance. They are used to compare two networks as a whole or to evaluate the evolution of the same network in two time moments. The diameter is the longest geodesic distance between any ordered pair of nodes in the network, and the mean distance is the average of the distance for all pair of nodes in the underlying network.

The density of the network measures the interconnection levels among all nodes. It is defined as the proportion of connections in the network over the maximum number of connections that could exist in it.

Reciprocity is the proportion of the pair of nodes in the network where edges have a direction defined from one node to the other (are mutually related) over those that are related by an arc.

Finally, **transitivity** is the proportion of arcs uw that exist over all directed trajectories uvw .

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