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**Working Paper**

## A Model of the Model: Unpacking CGE Results on Carbon Leakage

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# A Model of the Model: Unpacking CGE Results on Carbon Leakage

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## **Abstract**

Computational general equilibrium (CGE) models can evaluate detailed tax reforms, trade restrictions, or environmental policy. These models can capture many complexities, but these complexities can make results difficult to interpret. Analytical general equilibrium (AGE) models provide better intuition and interpretation but cannot capture relevant complexities. We propose a method that employs AGE models to understand CGE models – a “model of the model”. We apply this idea to climate policy and carbon leakage – the increase in emissions elsewhere. Our AGE models identify seven key economic determinants of leakage within any one outcome. We then unpack results from three existing CGE models.

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“Highly detailed models are complex, and the mechanisms driving their results can be opaque, especially to outside observers. The use of CGE models for regulatory purposes will need to be accompanied by significant efforts to make the key drivers behind modeling results clear for both policymakers and the public.” – U.S. EPA (2017, p.iii)

Sixty years ago, Arnold Harberger (1962) and Ronald Jones (1965) built simple two-sector, two-factor models and differentiated all equations to linearize them and to solve for analytical general equilibrium (AGE) effects of a small change in tax rates or trade policy. A closed-form expression for each AGE outcome helps show the role of each parameter. They were followed by other AGE efforts to add sectors, other factor inputs, market imperfections, and other policies. For practical policy analysis, however, AGE models have been largely supplanted by computational general equilibrium (CGE) models starting with Shoven (1976). Solved numerically on a computer, they can handle any number of sectors, factors, constraints, market failures, large policies, and other complications. CGE models also study development, energy, and environment. They can take advantage of new data sources, advances in theory, and additional computational capacity.

Yet complexity comes at some cost of transparency. Sometimes, a large computer model can seem like a “black box,” used and understood only by its developers who insert a particular set of parameters and crank out a set of general equilibrium outcomes. These outcomes may or may not be accompanied by intuitive explanations, and readers may have difficulty understanding what features of the model or of the policy are most important for determining the size and even the sign of key predicted outcomes.<sup>1</sup> The earlier analytical models are more intuitive but cannot allow for adequate detail to undertake practical policy analysis. Probably the best research would consider the wide range from simple intuitive models to complex, realistic models.

This paper develops AGE techniques – not to predict effects of policies, but to understand CGE models. We show how to “unpack” any CGE model’s results into relevant and intuitive economic mechanisms that drive the effects of a policy, by rearranging a mathematical expression for the change in a key outcome into multiple component terms that add up to the total effect on that outcome. Each positive or negative term shows exactly how primitive parameters of the model determine the size of that component effect. To be sure, others have used analytical models to help understand their own complex computational model.<sup>2</sup> Here, we use the approach not to explain our own numerical results, but to show how AGE models can be used to interpret and compare

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<sup>1</sup> As Bergman (2005) observes, “In particular CGE models are aimed at *quantifying* [emphasis added] the impact of specific policies on the equilibrium allocation of resources and relative prices of goods and factors” (p.1276).

<sup>2</sup> In Bovenberg and Goulder (1996), “a stylized analytical model uncovers the major mechanisms at play, while a numerical model explores the empirical significance of these mechanisms in a more realistic setting” (p.985). Since then, other papers also employ both AGE and CGE models (*e.g.*, Goulder *et al.*, 2024).

others' CGE results. We illustrate the approach for three existing CGE models of climate policy.

Absent worldwide agreement to restrict all sources of greenhouse gas emissions, any actual climate policy is likely to cover only some sectors in some countries. Detail is essential, and many CGE models have been used to quantify effects of actual or proposed policies. A key question is about leakage – the increase in emissions by countries or sectors not subject to the policy. Why is leakage important? Early discussion focused on how unilateral carbon policy would provide a competitive advantage to other countries, *i.e.*, a terms of trade effect that shifts jobs overseas. Also, some papers grabbed attention by noting that high leakage could substantially offset intended abatement from domestic policy; leakage of 100 percent or more makes unilateral efforts pointless (because total emissions rise).<sup>3</sup> Other important problems include distributional effects from raising the price of electricity, or from border carbon adjustments (BCAs) that tax carbon-intensive imports from low-income countries. Our AGE approach helps assimilate and explain the large ensuing literature that adds many features to elaborate CGE models.<sup>4</sup>

To build an AGE model that identifies the major CGE effects on leakage, we start with the simple model of Baylis *et al* (2014) that identifies three effects. In one interpretation, their model has two countries; each produces one good, and they trade because consumers want both goods. When one country's carbon tax raises the price of its output, demand shifts to the other country's output and increases their emissions (the terms of trade effect, TTE). For international trade, that model is too simple. A better interpretation of the same model is a two-sector closed economy. We use this interpretation and study sectoral leakage. However, the same TTE arises when a carbon tax increases one sector's output price and shifts demand to the uncovered sector's output.

Second, Baylis *et al* (2014) identify a new “abatement resource effect” (ARE): a carbon tax can induce the taxed sector to substitute away from carbon fuel toward primary factors, which can reduce primary factors available to the other sector – reducing their output and emissions.

Their third effect is also included in our models, but for clarity here, we change their name of it. Here, we call it a “factor reallocation effect” (FRE): the carbon tax can reduce demand for primary factors, especially in fuel production, which can reduce the relative price of mobile

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<sup>3</sup> Babiker (2005) finds a leakage rate of 130 percent. At the other extreme, Felder and Rutherford (1993) and Burniaux and Oliveria Martins (2012) find cases with negative leakage (where emissions fall in uncovered sectors).

<sup>4</sup> Typically, these papers find that leakage is 10 percent (Paltsev, 2001) or 20 percent (Elliot *et al*, 2010a). Fischer and Fox (2010) find negative domestic leakage and zero total leakage. A related paper by Fischer and Fox (2012) finds a 7 percent overall leakage rate. A review of BCA and intersectoral leakage is in Cosbey *et al* (2019). For a recent review of all leakage literature, see Grubb *et al* (2022) and their 186 cited references.

primary factors and allow the untaxed sector to expand both output and emissions.

This paper develops more elaborate AGE models to identify additional effects on leakage. These effects are carefully defined below, but their names are meant to be indicative. We find that a policy normalization effect (PNE) occurs when a pre-existing carbon tax in the other sector is assumed to be fixed relative to a numeraire factor price instead of being fixed relative to the overall price level. A pure income effect (PIE) occurs if additional carbon tax revenue is spent rather than rebated, which can reduce consumer incomes, purchases, output, and emissions. An input-output effect (IOE) occurs if the newly taxed sector's output is used as an input to the uncovered sector, raising its costs and reducing both its output and emissions. A fuel price effect (FPE) occurs in a model with a fixed natural resource like oil. Then a tax that reduces demand for carbon fuel reduces resource rents and fossil fuel prices (which can increase fossil fuel use in the untaxed sector). Finally, as explained below, we identify an interaction effect (IE) that occurs in both CGE and AGE models when nonlinear interactions of all effects above cannot be attributed to any one effect.

We show that the CGE models find small TTE but large positive FPE and large negative IOE effects on leakage. The latter two effects offset each other, so models with an input-output feature (for the IOE) and a fixed natural resource (for the FPE) find moderate overall leakage rates. But a model with only one of those two effects could find positive or negative overall leakage.

Our results can help provide guidance to academic researchers about what features to build into their models, and to policymakers about what features to build into their policy proposals. We shed light on key issues of policy design, because results depend on parameters representing the size and scope of the sectors taxed or exempted, behaviors in both the covered and uncovered sectors, strictness and enforcement of the policy, the number and size of provinces or nations in a coalition, plus border carbon adjustments and other details of policy design.

Our AGE approach can also be applied to any set of CGE models used to analyze any policy in any country. A major example is the new attention to tariffs. Computational models are critically important to understand the effects of tariffs that apply only to a subset of traded commodities imported from a subset of other countries.<sup>5</sup> When several sets of researchers each build their own CGE model, others will wonder what features are key to differences in results. Our approach can help answer such questions, not only for leakage (environmental econ), but also for

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<sup>5</sup> Kehoe *et al* (2017) reviews CGE trade models. Like other CGE models, important new quantitative trade models estimate structural equations, simulate large shocks, and calculate welfare effects (*e.g.*, Costinot *et al* 2016, Galle *et al* 2023, and Mérel *et al* 2023). Some show decompositions of component effects (*e.g.*, Larch *et al* 2018, or Gouel and Laborde 2021). Unlike those papers, we use small changes in AGE models to explain and to compare CGE models.

tariffs (international econ), domestic taxes (public econ), partial coverage of minimum wage hikes (labor econ), land use and food policy (agricultural econ), poverty and income distribution (development econ), and other policies in other fields.<sup>6</sup>

Our paper makes two major contributions. First, we build a model of the model. We start with the AGE model of Baylis *et al* (2014) that differentiates all equations to obtain 11 linear equations in 11 unknowns and finds closed-form solutions for outcomes in terms of exogenous parameters and an exogenous policy shock. Here, we extend that model to add key features of CGE models, and we find elaborate closed-form solutions (with derivations in online appendices, all available [here](#)). Our final model with 21 equations allows us to create a decomposition of total leakage that unpacks all components. We discuss what assumptions give rise to each effect and what parameters determine its size.

Our second contribution is to use these analytical expressions to unpack leakage results from three different CGE models: the Energy-Environment-Economy (E3) model of Goulder and Hafstead (2017), the CIM-EARTH model of Elliott *et al* (2010a,b), and the “BCR” model used by Carbone (2013), based on Böhringer, Carbone, and Rutherford (2011, 2018).

We calculate values for parameters in our AGE model that best summarize the many parameters in each CGE model. If a CGE model is used to simulate a carbon tax on several industries, for example, then we define those industries as one sector of our AGE model. We then determine how that sector in the CGE model substitutes from carbon toward other inputs, yielding an elasticity of substitution for our AGE model. Similar steps identify other parameters for our model, used to calculate the sign and size of each of seven effects in our AGE expression. The combination of these component effects yields an overall number for leakage in the AGE model.

Good CGE papers do provide intuition for results. First, authors use their own assumptions and model structure to explain an outcome of interest with a particular sign or size. Second, they calculate primary results and then conduct sensitivity analysis by varying behavioral parameters, model structure, and even policy design.<sup>7</sup> If such changes lead to large deviations in results, researchers can infer what effects are important. Using our mathematical decomposition, however,

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<sup>6</sup> In six separate requests, we asked ChatGPT to “name a dozen academic economics works that use computational general equilibrium (CGE) models to study effects of policy” in each of those six fields. It found 12 such works in each field. We have neither space here nor the expertise to review 72 papers in those six fields, but it does indicate that many topics with CGE results could be interpreted and compared using our AGE approach.

<sup>7</sup> Using the model of Böhringer *et al* (2011, 2018), Carbone (2013) finds a leakage rate that varies from 15 to -8.5 percent depending on parameter values. In Winchester and Rausch (2013), the leakage rate varies from 3 to 10 percent.

we find that each component effect depends on several parameters (and a particular parameter may enter several component effects). Thus, varying one parameter does not show the entire logic of each leakage mechanism. Third, a CGE model can be modified to remove a feature that affects the outcome of interest, and thus to calculate the size of a component effect (*e.g.*, Adams, 2005, Carbone, 2013, Larch *et al*, 2018). For example, removing a CGE model's fixed natural resource can indicate the numerical size of a FPE. In contrast, our analytical methods decompose the full mathematical solution into terms that each show the share and elasticity parameters within each component effect and thus how parameters *determine* the numerical size of each effect.<sup>8</sup>

We motivate the coming analysis with a summary here of our results for sectoral leakage; full comparisons are in Table 7 below. Climate policy proposals often raise the price of emissions only for electricity.<sup>9</sup> Using their CGE model called E3, Goulder and Hafstead (2017) simulate a U.S. carbon tax within the electricity sector. In contrast, Elliott *et al* (2010a,b) build a worldwide model called CIM-EARTH. To focus on sectoral leakage without international leakage, we unpack a CIM-EARTH simulation of a worldwide agreement for a carbon tax on all countries' electricity sectors (described in Elliott and Fullerton, 2014). The BCR model of Carbone (2013) is similar to CIM-EARTH; we unpack his simulation of the world with a carbon tax on the electricity sector.

The Goulder and Hafstead (2017) E3 model finds an overall leakage rate of 0.81 percent, while their parameters in our full AGE model yields a 2.27 percent rate. Their U.S. model assumes a fixed world price of oil, so they logically have no fixed natural resource and no FPE. Their largest driver of leakage in our AGE model is 3.72 percent from the factor reallocation effect (FRE). The TTE contributes 1.89 percentage points, and IOE is the largest negative term (-2.58 points).

The CIM-EARTH model finds overall leakage is 3.58 percent. Their parameter values in our full AGE model results in a 4.97 percent rate. We find that their fixed natural resource causes a large FPE of 13.67 percent and a large negative IOE of -5.39 percent. The largest negative effect is a -7.94 percent interaction effect (IE), explained below. In contrast, the BCR simulation by Carbone (2013) finds a large CGE leakage rate of 9.45 percent, but his parameters in our AGE model yield an overall 5.56 percent rate. That difference is larger than for the other two CGE

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<sup>8</sup> With a CGE model, a computer might help find closed-form expressions, but they can be too complicated to help intuition. Also, Böhringer *et al* (2014) provide a decomposition of *motives* within unilateral carbon policy, but their terms of trade motive relates to beggar-thy-neighbor trade policy that shifts abatement burden to trading partners.

<sup>9</sup> The U.S. EPA's 2014 Clean Power Plan would have restricted carbon emission from electric generating facilities starting in 2015 (<http://www2.epa.gov/carbon-pollution-standards/clean-power-plan-proposed-rule>). Also, increasing attention has turned to regional leakage within countries (see Fell and Maniloff, 2018, and Caron *et al*, 2015).

models. Again, the FPE is the largest positive effect, while the IOE and IE are large and negative.

No AGE calculation of total leakage can replicate a detailed CGE calculation, but that comparison is not important for the break-down of component effects. Four aspects of our results are encouraging for the use of AGE models to represent CGE results. First, an AGE model is used to represent each of the three CGE models, and the rank order of AGE overall leakage rates matches the rank order of the CGE overall leakage rate (from smallest to largest).

Second, one CGE calculation does not show components – economic forces that drive overall leakage. Even when the CGE calculation of leakage is small, our AGE calculations show large but offsetting leakage effects that are still operating within each CGE calculation.

Third, this kind of unpacking has policy implications. For example, control of the TTE via border carbon adjustments (BCAs) may not have large impact – consistent with the Böhringer *et al* (2012) that BCAs reduce only some carbon leakage and yield only small efficiency gains.

Fourth, our AGE calculations are also important for understanding fundamental modeling choices and how they differ across models. Fischer and Fox (2010) find overall negative leakage in a model with a large IOE, consistent with our finding of a large negative IOE. And our finding that the FPE is a large positive leakage effect is consistent with other CGE literature.

Section 1 outlines our initial model from Baylis *et al* (2014) with three effects. Section 2 extends that model by including three additional leakage effects. Section 3 proceeds to unpack the calculation of leakage in the E3 model of Goulder and Hafstead (2017). Section 4 then derives the FPE and our full AGE model. Section 5 uses the full AGE model to unpack all seven effects in the two other CGE models. Section 6 compares all three CGE models, and Section 7 concludes.

## **1 The Initial Model**

We begin with a model in Baylis *et al* (2014) to derive the terms of trade effect (TTE), abatement resource effect (ARE), and factor reallocation effect (FRE).

### **1.1 Initial Setup with Three Effects**

This initial AGE model has three sectors: two final output sectors and a fuel sector.<sup>10</sup> The two final outputs' production functions are  $X = X(K_X, C_X)$  and  $Y = Y(K_Y, C_Y)$ , where a primary factor ( $K_i$ ) and carbon-based fuel ( $C_i$ ) have decreasing marginal products ( $i = X, Y$ ). The primary factor represents a composite of all clean inputs (*e.g.*, capital, labor, land). We define one unit of fuel as

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<sup>10</sup> We collapse all fuels into one for simplicity, even though they have different carbon content per unit of energy.

the amount that emits one unit of carbon, so  $C_i$  represents both fuel input and emissions. Since the production of fuel itself requires primary factors, its production function is  $C = K_C$ , where  $K_C$  is the clean input to the production of  $C$  (but an extension below adds a fixed factor). This model has no storage, so supply of  $C$  equals demand  $C = C_X + C_Y$ . The total clean input is fixed ( $\bar{K}$ ) and perfectly mobile.<sup>11</sup> Its resource constraint is  $\bar{K} = K_X + K_Y + K_C$ .

This closed economy has an initial carbon tax in each sector.<sup>12</sup> The fuel price is  $p_C$ , but sector  $i$  faces a per-unit carbon tax  $\tau_i$  and a cost of  $p_C + \tau_i$  per unit of fuel. All sectors are perfectly competitive, with constant returns to scale (CRTS) production, which means we can scale the entire economy so that all units are defined per household (including  $X, Y, C, \bar{K}$  and allocations  $K_i$ ). With millions of identical households,  $n$ , each household disregards its own contribution to total emissions, so  $nC$  is viewed as fixed. The utility function  $U = U(X, Y; nC)$  is homothetic, where total carbon is separable in utility.<sup>13</sup> Carbon tax revenue per household is  $R \equiv \tau_X C_X + \tau_Y C_Y$  and is rebated in a lump sum to consumers who face a budget ( $p_K \bar{K} + R = I = p_X X + p_Y Y$ ) in their choice of  $X$  and  $Y$ . The factor price is  $p_K$  is numeraire, while final goods prices are  $p_X$  and  $p_Y$ .

To determine leakage, we use the model to solve for the effects of a small increase in  $\tau_Y$ , with no change in  $\tau_X$ . Next, we define parameters and derive a set of linear equations.

## 1.2 Log-Linearization

Totally differentiate the resource constraint and use the “hat” notation to denote a proportional change in any variable (e.g.,  $\hat{K}_X \equiv dK_X/K_X$ ), which yields:

$$0 = \alpha_X \hat{K}_X + \alpha_Y \hat{K}_Y + \alpha_C \hat{K}_C \quad (1)$$

where  $\alpha_i \equiv K_i/\bar{K}$  is the share of primary factor in sector  $i$  ( $i = X, Y, C$ ), and  $1 = \alpha_X + \alpha_Y + \alpha_C$ . Due to the fixed total supply  $\bar{K}$ , using more of it in one sector means less elsewhere.<sup>14</sup>

Next, we differentiate the fuel sector’s production function ( $C = K_C$ ), zero-profits

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<sup>11</sup> CGE models often assume factors are perfectly mobility between sectors within a region but perfectly immobile between regions (see Elliott *et al* 2010a, Böhringer *et al* 2018, and Winchester *et al* 2011). A different extension in Baylis *et al* (2014) has an imperfect degree of mobility, with effects between perfect mobility and immobility.

<sup>12</sup> Modeling a small increase in one sector’s carbon tax requires an initial, non-zero tax rate. Also, initial tax rates help close the model such that initial carbon emissions are bounded. Finally, even if a sector does not have an explicit carbon tax, emissions may be limited by an implicit carbon tax, such as a fuel economy standard.

<sup>13</sup> Separability of  $nC$  means that climate does not affect the  $X/Y$  ratio (e.g., use of air conditioners, which may affect emissions). It is useful for tractability in AGE models but has been studied in CGE models (Carbone and Smith, 2008). Whether we include this  $nC$  in utility has no impact on results below, but it reminds us of the reason for climate policy.

<sup>14</sup> See Baylis *et al* (2014) for an extension with a variable supply of the clean input.

condition ( $p_C C = p_K K_C$ ), and market clearing condition ( $C = C_X + C_Y$ ) to find:

$$\hat{C} = \hat{K}_C \quad (2)$$

$$\hat{p}_C + \hat{C} = \hat{p}_K + \hat{K}_C \quad (3)$$

$$\hat{C} = \rho_X \hat{C}_X + \rho_Y \hat{C}_Y \quad (4)$$

where  $\rho_X \equiv (C_X/C)$  and  $\rho_Y \equiv (C_Y/C)$ , so that  $\rho_X + \rho_Y = 1$ . In words,  $\rho_X$  is the fraction of fuel initially used in sector  $X$ . A firm in the fuel sector earns only a normal return.

Totally differentiate production functions for  $X$  and  $Y$ , and substitute in the first-order condition from each profit maximization problem to show how changes to inputs affect outputs:

$$\hat{X} = \theta_{XK} \hat{K}_X + \theta_{XC} \hat{C}_X \quad (5)$$

$$\hat{Y} = \theta_{YK} \hat{K}_Y + \theta_{YC} \hat{C}_Y \quad (6)$$

where  $\theta_{ij} > 0$  is the factor share for input  $j$  in the production of good  $i$  (where all inputs are necessary for production). Then CRTS implies  $\theta_{XK} + \theta_{XC} = 1$  and  $\theta_{YK} + \theta_{YC} = 1$ .

The carbon tax applies per unit, so each zero-profit condition is  $p_i i = p_K K_i + (p_C + \tau_i) C_i$  (for  $i = X, Y$ ). Total differentiation yields:<sup>15</sup>

$$\hat{p}_X + \hat{X} = \theta_{XK} (\hat{p}_K + \hat{K}_X) + \theta_{XC} \hat{C}_X + \theta_{XC}^p \hat{p}_C + \theta_{XC}^\tau \hat{\tau}_X \quad (7)$$

$$\hat{p}_Y + \hat{Y} = \theta_{YK} (\hat{p}_K + \hat{K}_Y) + \theta_{YC} \hat{C}_Y + \theta_{YC}^p \hat{p}_C + \theta_{YC}^\tau \hat{\tau}_Y \quad (8)$$

where  $\theta_{XC}^p \equiv p_C C_X / p_X X$  is the fuel-price factor share in  $X$ , and  $\theta_{XC}^\tau \equiv \tau_X C_X / p_X X$  is the carbon-tax factor share, so that  $\theta_{XC} = \theta_{XC}^p + \theta_{XC}^\tau$ . Thus,  $\theta_{XC}$  is the total-fuel factor share in sector  $X$  that accounts for both the underlying fuel cost as well as the carbon tax. Definitions are analogous for sector  $Y$ . Summing (7) and (8) yields the differentiated consumer budget constraint.

Differentiate each sector's definition of the elasticity of substitution in production:

$$\hat{C}_X - \hat{K}_X = \sigma_X \left( \hat{p}_K - \frac{\theta_{XC}^p}{\theta_{XC}} \hat{p}_C - \frac{\theta_{XC}^\tau}{\theta_{XC}} \hat{\tau}_X \right) \quad (9)$$

$$\hat{C}_Y - \hat{K}_Y = \sigma_Y \left( \hat{p}_K - \frac{\theta_{YC}^p}{\theta_{YC}} \hat{p}_C - \frac{\theta_{YC}^\tau}{\theta_{YC}} \hat{\tau}_Y \right) \quad (10)$$

where  $\sigma_i$  is the elasticity of substitution in sector  $i$ . In each sector, the only two inputs must be substitutes ( $\sigma_i > 0$ ). For instance, equation (10) shows that an increase in the carbon tax in sector

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<sup>15</sup> These equations help count all unknown price and quantity changes but will be combined with other equations like (5) and (6) to reduce the number of unknowns.

$Y$  ( $\hat{\tau}_Y > 0$ ) leads to a fall in the fuel-to-primary-factor input ratio ( $\hat{C}_Y - \hat{K}_Y < 0$ ), all else equal.

Because pollution is separable in utility, a single parameter  $\sigma_U$  defines the elasticity of substitution in utility between  $X$  and  $Y$ . Differentiating the definition of  $\sigma_U$  yields:

$$\hat{X} - \hat{Y} = \sigma_U(\hat{p}_Y - \hat{p}_X) \quad (11)$$

where  $\sigma_U > 0$ . Then equations (1)-(11) are a linear system to solve for general equilibrium effects of a small change in policy (assuming interior solutions).

### 1.3 Equilibrium Effects

We choose the primary factor as numeraire ( $\hat{p}_K = 0$ ) and consider a small exogenous increase in one carbon tax ( $\hat{\tau}_Y > 0$ ) while the other tax is constant ( $\hat{\tau}_X = 0$ ). Then  $\hat{C} = \hat{K}_C$  in equation (2) and  $\hat{p}_C + \hat{C} = \hat{p}_K + \hat{K}_C$  in (3) imply  $\hat{p}_C = 0$ . Any increase in demand for the primary factor raises its relative price, however, so the fuel sector's marginal cost curve is upward-sloping in general equilibrium. Since  $\hat{\tau}_X = \hat{p}_K = \hat{p}_C = 0$ , sector  $X$  experiences no change in relative input prices, so (9) means  $\hat{C}_X = \hat{K}_X$ . Unchanged input prices imply no change in the breakeven output price, so  $\hat{p}_X = 0$  (see equations (5) and (7)). Production in  $X$  is not Leontief, but relative input choices remain unchanged because relative input prices are unchanged (and  $\sigma_X$  drops out). This result also implies that output changes in the same proportion ( $\hat{X} = \hat{K}_X = \hat{C}_X$ ).

Equations (6) and (8) imply  $\hat{p}_Y = \theta_{YC}^\tau \hat{\tau}_Y > 0$ . Increasing  $\tau_Y$  always raises the price of  $Y$  relative to the price of  $X$ . Online Appendix A shows that the equilibrium quantity of good  $Y$  falls:

$$\hat{Y}^0 = -[(\alpha_X \sigma_U + \alpha_Y \sigma_Y) \theta_{YC} + \alpha_C \rho_X (\sigma_U \theta_{YC} + \sigma_Y \theta_{YK}) - \alpha_C \sigma_Y \theta_{YK}] \frac{\theta_{YC}^\tau}{\theta_{YC}} \hat{\tau}_Y \quad (12)$$

where the superscript “0” indicates that these solutions are from the initial “model-0”. More algebra reveals that the change in emissions in the sector with the increased carbon tax is:

$$\hat{C}_Y^0 = - \left[ \underbrace{(\alpha_X \sigma_U + \alpha_Y \sigma_Y) \theta_{YC} + \alpha_C \rho_X (\sigma_U \theta_{YC} + \sigma_Y \theta_{YK}) - \alpha_C \sigma_Y \theta_{YK}}_{\text{Output Effect on } \hat{Y} \text{ (and thus on } C_Y)} + \underbrace{\sigma_Y \theta_{YK}}_{\substack{\text{Substitution} \\ \text{Effect on } C_Y}} \right] \frac{\theta_{YC}^\tau}{\theta_{YC}} \hat{\tau}_Y \quad (13)$$

where we decompose the change in emissions into the usual output and substitution effects. Indeed, the output effect in (13) exactly matches the relative change in output ( $\hat{Y}^0$ ) in (12). In this model, the output and substitution effects on covered-sector emissions  $C_Y$  are both negative.

Finally, “leakage” is the change in emissions in the other sector:

$$\hat{C}_X^0 = \left[ \underbrace{\sigma_U \alpha_Y \theta_{YC}}_{\text{TTE}} - \underbrace{\sigma_Y \alpha_Y \theta_{YC}}_{\text{ARE}} + \underbrace{\alpha_C \rho_Y (\sigma_U \theta_{YC} + \sigma_Y \theta_{YK})}_{\text{FRE}} \right] \frac{\theta_{YC}^\tau}{\theta_{YC}} \hat{\tau}_Y. \quad (14)$$

The added carbon tax ( $\hat{\tau}_Y$ ) applies to the taxed portion of the fuel price ( $\theta_{YC}^\tau/\theta_{YC}$ ). The first term in large brackets is a terms of trade effect (TTE) on leakage: the tax induces consumers to substitute away from good  $Y$  into  $X$  (via  $\sigma_U$ ). The TTE increases production of  $X$  and raises  $C_X$ , so it is a positive effect on leakage. The second effect in brackets is an abatement resource effect (ARE) that arises as firms in sector  $Y$  substitute toward clean inputs (*e.g.*, abatement capital). Thus, they bid labor and capital – the composite input – away from sector  $X$ . Firms in sector  $X$  choose to reduce both  $K_X$  and  $C_X$  in equal proportion, so the ARE is a negative effect on leakage. The third term is a factor reallocation effect (FRE). It is positive, because sector  $Y$  reduces primary factor use, which allows for sector  $X$  to expand output and associated emissions.

## 2 Three Model Variants

This section adds equations necessary to construct three more extensions. Each adds one effect on leakage. We provide solutions and intuition here, but derivations are in online appendices.

### 2.1 Policy Normalization Effect

Steps above follow prior AGE and CGE literatures by choosing a primary factor as numeraire when changing a tax in one sector (*e.g.*,  $\tau_Y$ ) while holding constant any other tax (*e.g.*,  $\tau_X$ ). Yet, Garnache and Mérel (2022) point out that those studies effectively assume a particular policy normalization: holding  $\tau_X$  constant means constant relative to the numeraire. If so, it is not constant in real terms, relative to the overall price level. To ensure that outcomes depend only on relative prices and not on an arbitrarily chosen policy normalization, they recommend using the overall price level as numeraire. Here, to show how results in prior CGE models may depend on policy normalization, we follow Garnache and Mérel (2022, eq.13) by defining  $p_I$  as an index of final goods prices,  $p_I \equiv p_X^{\beta_X} p_Y^{\beta_Y}$ , where  $\beta_X \equiv \frac{p_X X}{I}$  and  $\beta_Y \equiv \frac{p_Y Y}{I}$  (with  $\beta_X + \beta_Y = 1$ ). The solution for  $\hat{C}_X^0$  above holds  $\tau_X/p_K$  constant, as did prior models, but we now solve for a different policy shock with  $\tau_X/p_I$  fixed, so that  $\tau_X$  is constant in real terms. Thus, we have:

$$\hat{\tau}_X = \hat{p}_I \quad (15)$$

$$\hat{p}_I = \beta_X \hat{p}_X + \beta_Y \hat{p}_Y. \quad (16)$$

Appendix B solves equations (1)-(11) along with (15) and (16). All of our models use the factor  $K$

as numeraire ( $\hat{p}_K = 0$ ) and impose an exogenous policy shock ( $\hat{\tau}_Y > 0$ ), but here the former assumption ( $\hat{\tau}_X = 0$ ) is replaced (by  $\hat{\tau}_X = \hat{p}_I$ ). The added  $\tau_Y$  raises  $p_Y$ , which raises  $p_I$  in (16), which raises  $\tau_X$  in (15), which raises  $p_X$ . Thus, both output prices rise relative to the numeraire. Using superscript “1” for outcomes in this first extension (model-1), leakage is:

$$\hat{C}_X^1 = \left[ \underbrace{\sigma_U \alpha_Y \theta_{YC}}_{\text{TTE}} - \underbrace{\sigma_Y \alpha_Y \theta_{YC}}_{\text{ARE}} + \underbrace{\alpha_C \rho_Y (\sigma_U \theta_{YC} + \sigma_Y \theta_{YK})}_{\text{FRE}} \right] \frac{\theta_{YC}^\tau}{\theta_{YC}} \hat{\tau}_Y$$

$$- \underbrace{\left[ \underbrace{(\alpha_Y \sigma_U + \alpha_X \sigma_X) \theta_{XC} + \alpha_C \rho_Y (\sigma_U \theta_{XC} + \sigma_X \theta_{XK}) - \alpha_C \sigma_X \theta_{XK}}_{\text{Output Effect of } \tau_X \text{ on } X \text{ (and thus on } C_X)} + \underbrace{\sigma_X \theta_{XK}}_{\text{Substitution Effect of } \tau_X \text{ on } C_X} \right]}_{\text{PNE}} \frac{\theta_{XC}^\tau}{\theta_{XC}} \underbrace{\Omega \theta_{YC}^\tau \hat{\tau}_Y}_{=\hat{\tau}_X} \quad (17)$$

where  $\Omega \equiv \beta_Y / (1 - \beta_X \theta_{XC}^\tau)$ , and  $\hat{\tau}_X = \Omega \theta_{YC}^\tau \hat{\tau}_Y$  depends on the tax share ( $\theta_{XC}^\tau \equiv \tau_X C_X / p_X X$ ). The first line of eq. (17) is exactly  $\hat{C}_X^0$  from the initial model (eq. 14), so leakage has the same first three effects plus a long new term we call the policy normalization effect (PNE). With the negative sign out front, the increase in  $\tau_X$  has negative output and substitution effects on  $C_X$ . These effects of  $\tau_X$  on  $C_X$  inside the PNE are perfectly symmetrical to the effects of  $\tau_Y$  on  $C_Y$  in equation (13).

Instead of holding  $\tau_X$  fixed as in the initial model, this new model raises  $\tau_X$  somewhat and thus *diminishes* shifts toward  $C_X$ . The PNE unambiguously reduces  $\hat{C}_X^1$  relative to the initial model. Conversely, this result means that measurement of leakage is enlarged in the initial model above. The size of leakage clearly depends on the policy experiment, but our calculation of the PNE below allows us to show the extent to which leakage in prior CGE models would have been smaller if those simulations had kept the tax rate in the other sector constant in real terms.

## 2.2 Pure Income Effect

For one extension at a time, we revert to the initial model to add each extension. In model-0, all added carbon tax revenue is rebated lump sum. In this extension, our AGE model accounts for any CGE simulation that does *not* rebate all extra revenue. No simulation can say government throws away funds, however, because every agent must be on their budget constraint. Hence, we assume that government spends a fraction  $\gamma$  of revenue per household ( $R$ ) on a public good,  $Z$ . The total public good ( $nZ$ ) is nonrival and separable in utility, so  $U = U(X, Y; nZ, nC)$ . If consumers do not get back all new carbon tax revenue, they must reduce spending on both consumer goods via this pure income effect (PIE). Reduced spending on  $X$  gives rise to another negative effect on leakage. The purpose here is not to investigate effect of public goods, *per se*, but to find the pure effect of

reduced consumer income.<sup>16</sup>

We assume that government spending on this public good is  $p_Z Z = \gamma R$ , where  $0 \leq \gamma \leq 1$  and  $R \equiv \tau_X C_X + \tau_Y C_Y$ . The rest of the carbon tax revenue is a lump-sum transfer to consumers,  $T = (1 - \gamma)R$ . Moreover, the public good must have a resource cost, so its production function is  $Z = K_Z$  (one unit of  $Z$  is produced using one unit of primary factor). This model has an additional use for the primary factor, so the constraint in equation (1) is modified:

$$0 = \alpha_X \widehat{K}_X + \alpha_Y \widehat{K}_Y + \alpha_C \widehat{K}_C + \alpha_Z \widehat{K}_Z \quad (18)$$

where  $\alpha_Z \equiv K_Z/\bar{K}$  is the share of primary factor in production of  $Z$ , and  $\alpha_X + \alpha_Y + \alpha_C + \alpha_Z = 1$ . We differentiate the production function for  $Z$  and the government balanced-budget constraint:<sup>17</sup>

$$\hat{Z} = \widehat{K}_Z \quad (19)$$

$$\hat{p}_Z + \hat{Z} = \delta_X (\hat{\tau}_X + \hat{C}_X) + \delta_Y (\hat{\tau}_Y + \hat{C}_Y) \quad (20)$$

where  $\delta_X \equiv \gamma \tau_X C_X / p_Z Z$  and  $\delta_Y \equiv \gamma \tau_Y C_Y / p_Z Z$  (those parameters sum to 1, because  $\gamma \tau_X C_X$  and  $\gamma \tau_Y C_Y$  sum to  $p_Z Z$ ). Thus,  $\delta_i$  is the share of tax revenue collected from sector  $i$ . Online appendix C solves equations (2)-(11) along with (18)-(20) and retains the composite factor as numeraire. This model has 15 equations and 15 unknowns. Leakage in this model-2 with superscript 2 is:

$$\begin{aligned} \hat{C}_X^2 = & \left[ \underbrace{\sigma_U \alpha_Y \theta_{YC}}_{\text{TTE}} - \underbrace{\sigma_Y \alpha_Y \theta_{YC}}_{\text{ARE}} + \underbrace{\alpha_C \rho_Y (\sigma_U \theta_{YC} + \sigma_Y \theta_{YK})}_{\text{FRE}} \right] \frac{\theta_{YC}^\tau}{\theta_{YC}} \hat{\tau}_Y \\ & + \underbrace{\alpha_Z \delta_Y \left( (\sigma_U \theta_{YC} + \sigma_Y \theta_{YK}) \left( \frac{\theta_{YC}^\tau}{\theta_{YC}} \right) - 1 \right)}_{\text{PIE}} \hat{\tau}_Y. \end{aligned} \quad (21)$$

Equation (21) matches the initial model's equation (14) but for the last added term, the PIE. A reduction in consumer income reduces purchases of  $X$ , with a negative effect on leakage, but the sign of the PIE term in (21) is ambiguous because the higher carbon tax rate might reduce total revenue. If a higher rate does raise revenue, on the normal side of the Laffer curve, then Appendix C shows that the PIE term is negative – a negative effect on leakage in (21). Interestingly, in this model where production of  $Z$  does not emit carbon, the use of only a primary factor for production of  $Z$  diverts resources away from sector  $X$ , which contracts and emits less carbon.

<sup>16</sup> A detailed treatment of public spending could identify additional effects. For example, emissions depend on whether government spends marginal revenue on reforestation or on military transport! Also, separability in utility means  $Z$  does not affect  $X/Y$ , but public spending could support low-carbon private activities.

<sup>17</sup> The added equations for government include a budget condition ( $p_Z Z = p_K K_Z$ ) that is totally differentiated to yield  $\hat{p}_Z + \hat{Z} = \hat{p}_K + \widehat{K}_Z$ . Using (20) to cancel terms leaves  $\hat{p}_Z = \hat{p}_K$ . If  $K$  is numeraire, then  $\hat{p}_K = 0$  implies  $\hat{p}_Z = 0$ .

### 2.3 Input-Output Effect

In many CGE models, the taxed sector's output is used as an input by other sectors, so the extra carbon tax raises other sectors' costs (reducing their output and emissions). For this reason, our next extension uses  $Y_X$  to represent the use of good  $Y$  in sector  $X$ . Production is now modeled as  $X = \min(V(K_X, C_X), Y_X/b)$ , where  $V$  is a sub-production function, and  $b$  is the fixed ratio of  $Y_X$  per unit of  $X$ . Many CGE models also use fixed coefficients for intermediate inputs, and this simple function for  $X$  allows us to avoid modeling the cross-price elasticities of a more general function with more inputs (*e.g.*, Fullerton and Heutel, 2007). Although this function fixes the ratio of intermediate input to output, it is still more general than other models that fix the ratio of emissions to capital (*e.g.*, Ogawa and Wildasin, 2009). In our model,  $\sigma_X$  is the elasticity of substitution between input  $K_X$  and carbon  $C_X$  (within the sub-production function  $V$ ).

We show here just enough equations to define needed parameter. The production function for sector  $X$  above implies  $\hat{V} = \hat{Y}_X$  and  $\hat{V} = \theta_{XK}\hat{K}_X + \theta_{XC}\hat{C}_X$ , where its share of sales revenue used to buy  $Y_X$  is  $\theta_{XY} \equiv p_Y Y_X / p_X X$ . Then CRTS implies  $1 = \theta_{XK} + \theta_{XC} + \theta_{XY}$ . Production can still be written generally as  $X = X(K_X, C_X, Y_X)$ , which we linearize and substitute to find  $\hat{X} = \hat{V} + \theta_{XY}\hat{Y}_X$ . Finally, the change in output  $Y$  matches the change in demand,  $\hat{Y} = \lambda_U \hat{Y}_U + \lambda_X \hat{Y}_X$ , where  $\lambda_U \equiv Y_U/Y$  is the share of  $Y$  used in utility,  $\lambda_X \equiv Y_X/Y$  (and  $\lambda_U + \lambda_X = 1$ ).<sup>18</sup> This model modifies initial equations, so online Appendix D shows the entire new set of equations.

In this model-3 (with superscript “3”), good  $X$  is no longer an equivalent numeraire to the composite input, because  $\hat{p}_X^3 = \theta_{XY}\hat{p}_Y$  (where  $\hat{p}_Y = \theta_{YC}^T \hat{t}_Y > 0$ ). That is, the price of  $X$  increases by the share of  $Y$  in its production. Leakage is:

$$\hat{C}_X^3 = \frac{\left[ \underbrace{\sigma_U \alpha_Y \theta_{YC}}_{\text{TTE}} - \underbrace{\sigma_Y \alpha_Y \theta_{YC}}_{\text{ARE}} + \underbrace{\alpha_C \rho_Y (\sigma_U \theta_{YC} + \sigma_Y \theta_{YK})}_{\text{FRE}} - \underbrace{\Phi \sigma_U \theta_{YC}}_{\text{IOE Term}} \right] \left( \frac{\theta_{YC}^T}{\theta_{YC}} \right) \hat{t}_Y}{1 - \underbrace{\Phi \theta_{XY}}_{\text{IOE Scalar}}} \quad (22)$$

where  $\Phi \equiv (\alpha_Y + \alpha_C \rho_Y)(\lambda_X + \lambda_U \theta_{XY})$  and  $0 < \Phi < 1$ . This input-output model collapses to the initial model when  $\theta_{XY} = 0$  and  $\lambda_X = 0$  (such that  $\Phi = 0$ ).

The input-output effect (IOE) enters (22) in two places, unlike the previous extensions where each model has one new additive leakage term. The IOE is a non-linear effect because both the numerator and denominator are modified from the initial leakage solution. The “IOE Term” is

<sup>18</sup> We then must redefine household expenditure shares  $\beta_Y \equiv p_Y Y_U / I$  and  $\beta_X \equiv p_X X / I$  (with  $\beta_Y + \beta_X = 1$ ), where  $I$  is total final expenditures (*i.e.*, national income).

negative, reducing leakage. Intuitively, the raised input price  $p_Y$  raises the cost of producing  $X$ , so this IOE reduces substitution into  $X$ . But the denominator in (22) also has an “IOE Scalar” where  $0 < (1 - \Phi\theta_{XY}) < 1$ . The denominator is less than one, so it enlarges each term in the numerator (whether positive or negative). Thus, the overall impact of the IOE is ambiguous. With reasonable parameters, however, it reduces total leakage. In summary, having  $Y$  in production of  $X$  transmits part of the tax increase from sector  $Y$  into sector  $X$ , meaning that the implicit tax difference between the two sectors is smaller than in model-0, and thus overall leakage is smaller.

### 3 Unpacking Goulder and Hafstead (2017)

We first unpack CGE results from the Energy-Environment-Economy (E3) model described in the book by Goulder and Hafstead (2017). It is a dynamic, multi-sector model of the U.S. economy, and it has many features that enable detailed distributional analysis of US climate policies.<sup>19</sup> We worked with those authors to determine how best to represent the many parameters from their E3 model, using the fewer parameters of our AGE models. We unpack their carbon leakage result from a particular E3 simulation that imposes a \$30 per metric ton CO<sub>2</sub> tax on the U.S. electricity sector – with no changes in other policies – which finds a steady-state 0.81 percent leakage rate in other U.S. sectors.<sup>20</sup> This E3 simulation most closely matches the setup of our AGE models.<sup>21</sup>

To adapt their detailed data to our simpler model, we first map their 35 production sectors into three: electricity  $Y$ , other goods  $X$ , and carbon-fuel  $C$  (used both in  $X$  and in  $Y$ ). Goulder and Hafstead (2017) calibrate their model to 2013 U.S. data (in 2013 dollars). **Table 1** provides all their level values used to calculate our parameters. The first column shows model notation, while the second column provides a description. For example, the first row in panel A (for the electricity sector) shows  $p_K K_Y$ , described as “value of all clean inputs”. The third column shows the numerical value (183.2), and the fourth column shows units (\$billions). The last column provides the source for the numerical value. Remaining data are from Goulder and Hafstead (2017) or are derived from other cells in Table 1 (except for the initial carbon tax rate).

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<sup>19</sup> E3 is not a worldwide model with trade in all goods. Instead, it ensures that international trade flows in the model match the observable U.S. data by assuming that a representative foreign firm produces an aggregate good.

<sup>20</sup> In Goulder and Hafstead (2017), p.135 begins a study of the proposed U.S. Clean Power Plan (which was never actually implemented). Page 140 concludes: “At the low level of stringency, we find leakage to be about 1.2 percent.” The CPP has a different carbon price for each U.S. state, but a newer simulation not in their book uses a common tax rate of \$30/tonne in all states, where they find a 0.81 leakage rate. We thank Marc Hafstead for this extra calculation and for sending us this leakage rate as well as initial values of  $C_X$  and  $C_Y$  (used below).

<sup>21</sup> The E3 model has adjustment costs and fixed capital within each period; short-run leakage rates are higher. Our AGE model has mobile factors, comparable to their long-run equilibrium where the leakage rate is 0.81 percent.

The value of all primary inputs is allocated across sectors: the electricity sector in Panel A ( $p_K K_Y = \$183.2$  billion), the fuel sector in Panel B ( $p_K K_C = \$476.4$  billion), and other goods in Panel C ( $p_K K_X = \$12,016.3$  billion).<sup>22</sup> The total \$476.4 billion of fuel output is purchased by electricity (sector  $Y$ , \$41.3 billion in row 2) and by the rest of the economy (sector  $X$ , \$435.1 billion in row 9). The rest of the economy includes direct-to-household consumption of fuels.

The E3 model's CO<sub>2</sub> emissions in billions of metric tons include  $C_Y = 1.03$  from electricity and  $C_X = 2.09$  from other industries. All three CGE models start with no carbon taxes, but multiple sectors have energy taxes and fuel taxes. The only tax rates in our AGE model are the two carbon tax rates, so the effects of these other energy and fuel taxes are approximated by setting initial CO<sub>2</sub> taxes at  $\tau_X = \tau_Y = \$15$  per metric ton (as recorded in Table 1, rows 4 and 11).<sup>23</sup> Thus, the implied initial values of carbon emissions are  $\tau_Y C_Y = \$15.5$  billion and  $\tau_X C_X = \$31.4$  billion.

The total value of electricity is  $p_Y Y = \$240.0$  billion, of which \$177.4 billion is purchased by the  $X$  sector.<sup>24</sup> Thus, our unpacking of this E3 model will include an IOE. Competition means that the sum of inputs equals the value of output for each sector, and total household expenditure is  $p_Y(Y - Y_X) + p_X X$ , which \$12,722.8 billion (national income).<sup>25</sup>

The E3 simulation adds a CO<sub>2</sub> tax of \$30 per metric ton on electricity only, where all added tax revenue is returned to consumers by lump-sum transfer (so  $\gamma = 0$ ,  $Z = 0$ , and the pure income effect, PIE, will be zero). From the E3 data, existing government spending is grouped into sector  $X$  for our simpler model. The policy normalization effect (PNE) is small, but it is not zero because E3 uses labor as numeraire and holds the small unit tax rates in sector  $X$  constant.

Input share parameters can be calculated directly from information in Table 1. Consumer expenditure shares are calculated as  $\beta_Y = 0.005$  and  $\beta_X = 0.995$  (from rows 6, 13, and 14). Next, rows 6 and 13 imply  $\lambda_U \equiv Y_U/Y = 0.261$ , so 26 percent of electricity  $Y$  is purchased for consumer utility (and the rest is used by sector  $X$ ). To match the E3 model's elasticity of substitution in utility, we set  $\sigma_U = 1.0$  in our AGE model. To match their other parameters, we set our elasticities of substitution in production to  $\sigma_X = 0.7$  and  $\sigma_Y = 0.4$ , which means the electricity sector  $Y$  is less

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<sup>22</sup> Other CGE models have a fixed factor in production of fuel, so they have three rows in Panel B: value of all clean inputs, resource rents, and their sum (sales revenue). The E3 model has only one row in Panel B because the 2017 book has no fixed factor nor resource rents in any of its three fuel extraction sectors (oil, coal, and natural gas).

<sup>23</sup> This initial tax rate of \$15/ton is arbitrary and only approximate, but it has little effect on results below.

<sup>24</sup> E3 separately reports generation (placed into our sector  $Y$ ) from transmission and distribution (with no emissions, placed into  $X$ ). Thus, our spending on electricity in Table 1 is less than the typical total of household electricity bills.

<sup>25</sup> In our E3 calculations without the IOE, the value of  $Y_X$  is added instead to the primary factor input in sector  $X$ .

flexible than  $X$  in substituting between the primary factor and carbon fuel.

Using parameters that correspond to these E3 values, **Table 2** provides AGE results for initial model-0 and its three variants above. Column headings provide each AGE model's name and our equation number for its solution. Our AGE solutions are more valid for small changes than for large changes, so we set  $\hat{\tau}_Y = 0.10$ , which increases the electricity sector's carbon tax by 10 percent (from \$15/tonne to \$16.5/tonne). Panel A shows the proportional changes for key AGE model outcomes, including the fuel price ( $\hat{p}_C$ ), covered emissions ( $\hat{C}_Y$ ), and leakage ( $\hat{C}_X$ ). The fuel price  $p_C$  is constant across these AGE models, because they are built to represent the E3 model. The E3 fuel sector has no fixed factor and uses only the primary factor (our numeraire).

Starting with initial model-0 in column [0], the increase in  $\tau_Y$  leads to a 1.45 percent decline in carbon emissions from the covered sector and a small 0.024 percent increase in emissions elsewhere (*i.e.*, leakage). The values for these effects are found simply by inserting the parameter values derived above into each term of equation (14). Then these three additive values from the initial model are carried forward to each extended model.

We could discuss our AGE model's decomposition of leakage as reported in proportional changes ( $\hat{C}_X \equiv -dC_X/C_X$ ), but results from E3 and other CGE models are normally reported as a leakage *rate*, defined as the change in the other sector's emissions ( $dC_X$ ) as a fraction of covered sector abatement ( $-dC_Y$ ). From our AGE model results, we calculate:

$$\text{"Leakage Rate"} \text{ (in percent)} \equiv (dC_X/-dC_Y) \times 100 = -(\hat{C}_X/\hat{C}_Y) \times (C_X/C_Y) \times 100$$

where  $C_X/C_Y = 2.09/1.03 = 2.03$ . For comparability with results reported from all CGE models, we therefore convert all AGE leakage amounts in panel A to leakage rates in panels B and C. For the initial model, panel A says that the leakage amount is a 0.024 percent increase in  $C_X$ , but panel B with the leakage rate says that the increase in  $C_X$  is 3.29 percent of the  $C_Y$  reduction.

Use of this leakage rate makes AGE results comparable to CGE results because it takes the ratio of  $\hat{C}_X$  to  $\hat{C}_Y$ . Each depends directly on  $\hat{\tau}_Y$ , so the  $\hat{\tau}_Y$  cancel, and leakage rates do not depend on the size of the added tax. In general, CGE models can capture large tax shocks better than the linear approximations of AGE models. But, \$30/tonne is not a huge carbon tax.

Table 2 panel C shows components of that leakage rate. For these calculations, the total  $\hat{C}_X$  is first broken down into component parts using the key equation from each model, and then each component of  $\hat{C}_X$  replaces total  $\hat{C}_X$  in the leakage rate formula above (with no change in the formula's  $C_X$ ,  $C_Y$ , or  $\hat{C}_Y$ ). For additive components of  $\hat{C}_X$ , the component leakage rates sum to the

total leakage rate. Column [0] for the initial model-0 shows that the 3.29 percent leakage *rate* includes a TTE of +1.28, ARE of  $-0.51$ , and FRE of +2.52 (where  $1.28 - 0.51 + 2.52 = 3.29$ ). Thus, the terms of trade effect is only a third of the overall leakage rate, while the factor reallocation effect is most of it. We later discuss more comparisons across all effects in all models.

Column [1] shows results for the model-1 extension with the policy normalization effect, which shows the effect on the leakage rate when defining the policy experiment as a change in  $\tau_Y$  with no change in the real  $\tau_X$ , defined as  $\tau_X/p_I$  (Garnache and Mérel, 2022).<sup>26</sup> This PNE component is  $-0.02$ , so it reduces the total leakage rate from 3.29 to 3.27 percent.<sup>27</sup>

Column [2] reports results for our AGE model-2, modified from the initial model only by the pure income effect. The E3 simulation by Goulder and Hafstead (2017) returns all marginal revenue to consumers, so the PIE here is zero. Thus, column [2] results are identical to column [0].

Column [3] reports calculations from AGE model-3, where the only change from the initial model is the IOE. The E3 model's input-output table is collapsed into one parameter,  $\theta_{XY}$  (the sector  $X$  input share for electricity, from Table 1 row 13 divided by row 14). Panel A shows that the same 10% carbon tax hike in this model generates less covered-sector abatement ( $-\hat{C}_Y = 0.986$  instead of 1.452 percent) and less leakage ( $\hat{C}_X = 0.011$  instead of 0.024). When panel C reports our decomposition in column [3], components of  $\hat{C}_X$  are the same as in earlier columns, but each is divided by a smaller denominator ( $-\hat{C}_Y$ ), which enlarges all effects. The TTE rises from 1.28 to 1.89, the ARE rises from  $-0.51$  to  $-0.76$ , and the FRE rises from 2.52 to 3.72 (all in percent).

Because the input-output effect is non-additive, we calculate it by residual: total  $\hat{C}_X$  in column [3] minus the sum of the first three additive components of  $\hat{C}_X$  from the initial model. Thus, the IOE component of the leakage rate is  $-2.58$  (given independent rounding). The IOE is a large negative effect on the leakage rate, because this model specifies how the carbon tax raises the cost of the covered sector's output  $Y$  – which is used as an input to the  $X$  sector.

We next discuss the fuel price effect, which does not appear in the E3 model of Goulder and Hafstead (2017), but which is necessary to unpack the other two CGE models.

## 4 Fuel Price Effect and Full AGE Model

<sup>26</sup> The formula for  $\hat{C}_Y$  in the PNE model differs from our initial model, but the value reported in column [1] is the same to three decimals, so the other leakage rate components (TTE, ARE, and FRE) are unchanged to two decimal places.

<sup>27</sup> All three CGE models have no initial carbon taxes, but they have energy taxes or fuel taxes in multiple sectors (which we represent using initial  $\tau_X = \tau_Y = \$15/\text{tonne}$ ). This  $\tau_X$  is small, so the PNE is small, but this approximation means that their actual PNE is not measured precisely.

This section adds the FPE and then discusses our full AGE model to compare all CGE models.

#### 4.1 Fuel Price Effect

Fuel production in our initial model uses only primary factors, so its cost is constant relative to our numeraire (*i.e.*, the primary input). Some CGE models specify a fixed nonrenewable resource, so any increase in demand bids up the rents to those fixed resources. It makes the marginal cost of producing fuel rise with output. If so, a carbon tax that reduces demand for fossil fuels has a negative effect on fuel price (encouraging leakage). Here, the initial AGE model's fuel production function is altered so that  $C = C(\bar{Q}, K_C)$ , where  $\bar{Q}$  is a fixed factor such as the quantity of fossil resources available for extraction. The CRTS assumption means that increasing *both* inputs would raise output in the same proportion. But  $\bar{Q}$  is fixed, so  $K_C$  has a decreasing marginal product. Rents to the fixed factor must also appear in consumer income ( $p_K \bar{K} + R + p_Q \bar{Q}$ ).

Using linearization as above, we differentiate the new fuel production function. Since  $\bar{Q}$  is fixed, changes in fuel output depend only on changes in primary factor input:

$$\hat{C} = \theta_{CK} \hat{K}_C \quad (23)$$

where  $\theta_{CK} \equiv p_K K_C / p_C C$  is the primary factor share in production of carbon fuel. Similarly, define  $\theta_{CQ} \equiv p_Q \bar{Q} / p_C C$  as the resource factor share (with  $1 = \theta_{CK} + \theta_{CQ}$ ). Next, the new zero-profit condition in the fuel sector is  $p_C C = p_Q \bar{Q} + p_K K_C$ , and differentiation yields:

$$\hat{p}_C + \hat{C} = \theta_{CQ} \hat{p}_Q + \theta_{CK} (\hat{p}_K + \hat{K}_C). \quad (24)$$

Next, differentiate the definition of  $\sigma_C$ , the elasticity of substitution in the carbon-fuel sector:<sup>28</sup>

$$-\hat{K}_C = \sigma_C (\hat{p}_K - \hat{p}_Q). \quad (25)$$

This  $\sigma_C$  is not a fuel supply elasticity; it is the elasticity of substitution within the fuel sector. When we solve the FPE model to find leakage in this model, the price of  $X$  is no longer constant relative to the numeraire. Instead,  $\hat{p}_X = \theta_{XC}^p \hat{p}_C$ , where  $\theta_{XC}^p \equiv p_C C_X / p_X X$  is the share of sales revenue that sector  $X$  uses to pay for fuel. Similarly,  $\theta_{XC}^r \equiv \tau_X C_X / p_X X$  with  $\theta_{XC} = \theta_{XC}^p + \theta_{XC}^r$ .

The FPE in model-4 replaces equations (2) and (3) with equations (23) and (24), and it adds (25) to represent trade-offs within the fuel sector. Online Appendix E reduces all equations to show an intermediate equation for  $\hat{C}_X^4$  in terms of the endogenous  $\hat{p}_C^4$  and exogenous  $\hat{\tau}_Y$ :

<sup>28</sup> Equation (25) has the same form as linearized equations (9) and (10) for  $\sigma_X$  and  $\sigma_Y$ , but it is much simpler because one input is fixed ( $\hat{Q} = 0$ ), and sale of fuel is not subject to a tax.

$$\Gamma \hat{C}_X^4 = \left[ \underbrace{\frac{\sigma_U \alpha_Y \theta_{YC}}{\text{TTE}}}_{\text{TTE}} \underbrace{-\sigma_Y \alpha_Y \theta_{YC}}_{\text{ARE}} + \underbrace{\frac{\alpha_C \rho_Y}{\theta_{CK}} (\sigma_U \theta_{YC} + \sigma_Y \theta_{YK})}_{\text{FRE}} \right] \left( \frac{\theta_{YC}^\tau}{\theta_{YC}} \hat{t}_Y + \frac{\theta_{YC}^p}{\theta_{YC}} \hat{p}_C \right) \quad (26)$$

$$- \underbrace{\left[ \underbrace{(\alpha_Y \sigma_U + \alpha_X \sigma_X) \theta_{XC} + \frac{\alpha_C \rho_Y}{\theta_{CK}} (\sigma_U \theta_{XC} + \sigma_X \theta_{XK}) - \alpha_C \sigma_X \theta_{XK}}_{\text{Output Effect on } X \text{ (and thus } C_X)} + \underbrace{\frac{\sigma_X \theta_{XK}}{\text{Substitution Effect on } C_X}}_{\text{Substitution Effect on } C_X} \right]}_{\text{FPE}} \frac{\theta_{XC}^p}{\theta_{XC}} \hat{p}_C^4.$$

On the left side,  $\Gamma \equiv (\alpha_X + \alpha_Y + (\alpha_C/\theta_{CK})) > 1$  could be shown as a denominator on the right side.<sup>29</sup> Also,  $\hat{p}_C$  on the right is a function of the exogenous  $\hat{t}_Y$  shock. Comparing (14) and (26) shows that the initial model is a special case of this FPE model: when  $\theta_{CK} = 1$ , the fuel price effect goes to zero, so  $\Gamma = 1$  and  $\hat{p}_C = 0$ . Then equation (26) collapses to (14).

The first line in equation (26) mimics the solution of model-0 in (14) with the original TTE, ARE, and FRE (but the FRE is divided by  $\theta_{CK}$  to account for the fixed-factor). Using definitions that appear after equation (8) above, these component effects are then all multiplied by  $((\theta_{YC}^\tau/\theta_{YC})\hat{t}_Y + (\theta_{YC}^p/\theta_{YC})\hat{p}_C)$ . The first part includes  $\hat{t}_Y > 0$  as before, and the second part includes  $\hat{p}_C$ , the fuel price change. The solution for  $\hat{p}_C$  is shown below, with its ambiguous sign, but we then discuss why that sign is likely negative. If so, then the two parts have opposite signs; for reasonable parameters, the term with  $\hat{t}_Y > 0$  dominates the term with  $\hat{p}_C < 0$ .

The minus sign in front of the second line of (26) means that  $\hat{p}_C < 0$  has a positive output effect on both  $X$  and  $C_X$ , plus a positive substitution effect on  $C_X$ . We call this term the FPE even though the first line also has a  $\hat{p}_C$ . It is scaled by  $\theta_{XC}^p/\theta_{XC}$ , the relative carbon share attributable to the fuel price. If a fall in sector  $Y$ 's fuel use reduces the fuel price, then sector  $X$  can increase its use of fuel (positive leakage).<sup>30</sup> Next, we show the closed-form solution for the fuel price change:

$$\hat{p}_C^4 = \frac{-1}{\Gamma\eta + \Delta_1 - \Delta_2} \left[ \underbrace{(\alpha_X \rho_Y - \alpha_Y \rho_X) (\sigma_U \theta_{YC} + \sigma_Y \theta_{YK})}_{\text{Output Effect on } C \text{ (and thus } p_C)} + \underbrace{\sigma_Y \alpha_Y}_{\text{Substitution Effect on } C} \right] \frac{\theta_{YC}^\tau}{\theta_{YC}} \hat{t}_Y \quad (27)$$

where  $\eta \equiv (\theta_{CK}/\theta_{CQ})\sigma_C \geq 0$  (and so  $\Gamma\eta \geq 0$ ). The online Appendix E shows that  $\eta =$

<sup>29</sup> The collection of parameters  $\Gamma$  arises because the fixed factor in the fuel sector reduces movement of primary factors (since the marginal product of the primary factor goes to infinity as its level in the fuel sector goes to zero).

<sup>30</sup> Interestingly, the long term for the FPE in (26) matches closely the long term for the PNE in (17), but for one scalar ( $\theta_{CK}$ ). The PNE adjusts leakage for the effect of prices on the tax rate by the appearance of  $\theta_{XC}^\tau$  outside the bracket, while the FPE adjusts leakage for the effect of fuel prices by the appearance of  $\theta_{XC}^p$  outside the bracket.

$(dC/dp_C)/(C/p_C)$ , the fuel supply elasticity in partial equilibrium (if no other price changes). Thus, as either  $\sigma_C$  or  $\eta$  goes to infinity, then the entire denominator goes to infinity, and  $\hat{p}_C^4$  goes to zero (fuel supply becomes perfectly elastic). In general, our parameters in  $(\theta_{CK}/\theta_{CQ})\sigma_C$  can be used to replicate any value of  $\eta$  from a CGE model. The other terms in the denominator have ambiguous signs.<sup>31</sup> For a fuel supply elasticity large enough, the denominator is positive.

The numerator of (27) also has ambiguous sign, but we interpret components. In the second term, the substitution effect of  $\hat{t}_Y$  reduces  $C_Y$ . This shift reduces total fuel demand and leads to a fall in the worldwide fuel price (given a positive denominator). In the first term, the shock  $\hat{t}_Y$  also has output effects on both final outputs, but in opposite directions. These terms together are not the full general equilibrium change in  $C$ , only the direct effects.<sup>32</sup> The net direct effect on all carbon in the numerator of  $\hat{p}_C^4$  in (27) can only be positive if the tax hike  $\hat{t}_Y$  has positive direct effect on leakage ( $dC_X$ ) that exceeds 100% of direct abatement ( $-dC_Y$ ). For these reasons,  $\hat{p}_C^4 < 0$  and  $FPE > 0$  are highly likely. Those signs are not guaranteed, however, so we search across parameters to find at least one extreme example that yields the opposite sign.<sup>33</sup>

In summary, for the parameters derived in this paper to represent the CGE models we unpack, we get the likely result that increasing the carbon tax on electricity reduces demand for carbon-based fuel, so it reduces the equilibrium price of carbon fuel ( $\hat{p}_C^4 < 0$ ) and induces the other sector to use more carbon fuel (positive leakage,  $\hat{C}_X^4 > 0$ ).

## 4.2 Full Model

Finally, we include all features discussed above in a single “model-5” with 21 linearized equations. Online Appendix F discusses a closed-form expression for leakage. It is complex, but we have explained the intuition of all seven effects within it. For this reason, we omit further interpretation here, but we use the full AGE model to solve for leakage and to compare the three CGE models.

The input-output effect and fuel price effect were each identified by adding two different

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<sup>31</sup>  $\Delta_1 \equiv [(\alpha_X\rho_Y - \alpha_Y\rho_X)(\sigma_U\theta_{YC} + \sigma_Y\theta_{YK}) + \alpha_Y\sigma_Y]\frac{\theta_{YC}^p}{\theta_{YC}}$  and  $\Delta_2 \equiv [(\alpha_X\rho_Y - \alpha_Y\rho_X)(\sigma_U\theta_{XC} + \sigma_X\theta_{XK}) - \alpha_X\sigma_X]\frac{\theta_{XC}^p}{\theta_{XC}}$ . We also substitute (27) into (26) to derive a closed-form solution for  $\hat{C}_X^4$  shown in online Appendix E, but it is unnecessarily complex. The best interpretation of  $\hat{C}_X^4$  uses (26) and appears above.

<sup>32</sup> Substitution and output effects are identified in online Appendix E in a solution for  $\hat{C}$  as a function of  $\hat{p}_C^4$  and  $\hat{t}_Y$ .

<sup>33</sup> Starting with CIM-EARTH parameters in the FPE model, we reduce  $\rho_Y \equiv C_Y/C$  from 0.341 to 0.005 (raising  $\rho_X \equiv C_X/C$  to 0.995). In this case,  $\hat{t}_Y > 0$  does not change the fuel price. Any  $\rho_Y < 0.005$  leads to a fuel price increase. With  $\rho_Y = 0.001$ , for example,  $\hat{t}_Y > 0$  leads to  $\hat{p}_C^4 = 0.003$ . In fact, not just the direct effects on total carbon are positive, but the full general equilibrium  $C$  rises, which means leakage is more than 100% of abatement.

features into the same initial model. But each one changes both the numerator and denominator of the initial closed-form solution for leakage. These non-additive effects mean that the combination of all extensions yields a solution for leakage that is not equal to the sum of the seven parts measured separately. Thus, in addition to the effects identified above, model-5 includes an Interaction Effect (IE). It is defined numerically as the full AGE model's leakage result minus the sum of the component effects of leakage from the separate formulas above. This difference cannot be attributed to any particular component effect.

Any CGE model with the same nonlinear equations must have the same interaction effects that we identify here, but those papers do not unpack and identify all effects separately. The size of any non-additive effect (and especially the interaction effect) must depend on what nonlinear features are already in the AGE or CGE model. The size of each non-additive effect depends on the order in which they are added, because each is measured as the difference between the leakage rates in two different versions of the model. We show below that the IE can be large but does not affect the relative sizes of the other components of interest.

## **5 Unpacking Other CGE Models**

Here, we use analytical models to decompose leakage in the remaining two CGE models, and we compare all three CGE model results to our AGE results. We show how the overall leakage rate from each CGE model is the net impact of positive and negative effects, and how the size of each effect is driven by a distinct economic mechanism. Section 3 above gets AGE parameters from the E3 model's data of Goulder and Hafstead (2017), while the next two sections use data from the CIM-EARTH model of Elliott *et al.* (2010a,b) and the BCR model used by Carbone (2013).

### **5.1 Unpacking Results from CIM-EARTH**

As for the E3 model, we talked with CIM-EARTH authors Elliott *et al.* (2010a,b) to determine values of the few parameters in our AGE models that best summarize their many parameters. Their worldwide model has 16 regions and complicated patterns of substitution. Each region has primary inputs that are mobile across industries but not across regions. Here, we analyze a simulation (described in Elliott and Fullerton, 2014) that closely matches the setup of our analytical models: a worldwide agreement to impose a carbon tax of \$30/tonne on electricity sectors in all regions.

The CIM-EARTH model is calibrated using 2004 GTAP7 data (in 2004 dollars). We aggregate their 16 regions into a worldwide coalition, and we map their many production sectors into electricity  $Y$ , other goods  $X$ , and carbon-fuel  $C$  (used in both  $X$  and  $Y$ ). **Table 3** provides all

their level values used to calculate our parameters (with a format similar to Table 1).<sup>34</sup> The table uses CIM-EARTH data with three exceptions. First, we assume initial carbon prices are  $\tau_Y = \tau_X = \$15$  per metric ton CO<sub>2</sub>. Second, we add their primary factor use for public spending ( $Z$ ) to the rest-of-economy sector  $X$ . Their *marginal* revenue from the increase in carbon tax is returned to consumers by lump-sum transfer (so the PIE will be zero). The PNE is not zero, because CIM-EARTH uses labor as numeraire. Third, we calculate their resource rents for Panel B.

Electricity in Panel A shows fuel inputs  $p_C C_Y = \$260.2$  billion.<sup>35</sup> The fuel sector in Panel B shows clean inputs  $p_K K_C = \$583.4$  billion. We need to aggregate CIM-EARTH's three fuel sectors (oil, coal, and gas). According to Elliot *et al* (2010b), all three are assigned 0.6 as the elasticity of substitution between two inputs ( $K_C$  and  $Q$ ). Thus, we use  $\sigma_C = 0.6$  for our one fuel sector. They assign each fuel sector a fixed natural resource factor share  $\theta_{CQ}$  (0.239 for oil, 0.229 for coal, and 0.115 for gas). These imply different fuel supply elasticities  $\eta = (\theta_{CK}/\theta_{CQ})\sigma_C$ . We calculate  $\eta = 2.87$  as a weighted average of their three fuel supply elasticities.<sup>36</sup> Then, from the  $\eta$  equation and  $\sigma_C = 0.6$ , we calculate our one resource share as  $\theta_{CQ} = 0.173$  (so  $\theta_{CK} = 0.826$ ).<sup>37</sup> Given that clean inputs are \$583.4 billion, the remaining  $\theta_{CQ} = 0.173$  of resource rents must be  $p_Q Q = \$122.2$  billion. Panel B also shows the fuel sector's implied total inputs of \$705.6 billion. Finally, panel C provides values for the rest-of-economy, sector  $X$ .

Our share parameters are calculated from levels in Table 3. Given values of the two final outputs in rows 6 and 16 (adjusting for intermediate sales in row 15), we calculate the consumer expenditure shares as  $\beta_Y = 0.002$  and  $\beta_X = 0.998$ . Then, rows 6 and 15 imply  $\lambda_U \equiv \frac{Y_U}{Y} = 0.081$ , so 8 percent of electricity is purchased for consumer utility (and the rest is an input to  $X$ ). Third, we set  $\sigma_U = 1.0$  and  $\sigma_X = \sigma_Y = 0.5$  to reflect the CIM-EARTH simulation we unpack.

**Table 4** reports results for each AGE model, using parameters that correspond to CIM-EARTH (assuming  $\hat{\tau}_Y = 0.10$ ). As in Table 2, Panel A shows key AGE model outcomes: fuel price

<sup>34</sup> See Narayanan and Walmsley (2008) for GTAP 7 Data Base documentation.

<sup>35</sup> The Petroleum Refining sector in GTAP is separate from the Oil Extraction sector and does not employ resources directly, so we include Petroleum Refining in the rest-the-economy.

<sup>36</sup> For oil, the fixed resource share of 0.239 and  $\sigma_C = 0.6$  together imply  $\eta = 1.91$ . Coal has  $\eta = 2.02$ , and gas has  $\eta = 4.63$ . The weights are total sales from each of the three fuel sectors to the electricity sector in the CIM-EARTH data (as fractions of their sum). The values for oil, coal, and natural gas are 84.0, 86.8, and 87.5, respectively (in \$billions).

<sup>37</sup> This method is not the only way to combine the three CIM-EARTH fuel sectors into one sector for our AGE model. We could have calculated a weighted average of the natural resource factor-input shares (equal to 0.194), implying a fuel supply elasticity of 2.50. Or, we could use a different choice of weights, such as gross receipts of each fuel sector.

$(\hat{p}_C)$ , covered emissions ( $\hat{C}_Y$ ), and leakage ( $\hat{C}_X$ ). The fuel price changes only in models with the FPE. With model-0 in column [0], the tax increase leads to a 2.463 percent decline in covered-sector emissions and a small 0.027 percent increase in emissions elsewhere ( $\hat{C}_X$ ). As before, these effects are found simply by inserting our parameter values into each term of equation (14). These additive components from model-0 are carried forward to each extended model.

The leakage rate is  $-(\hat{C}_X/\hat{C}_Y)(C_X/C_Y)$ , where  $C_X/C_Y=1.93$  (Table 3). Table 4 Panel B reports only this total AGE leakage rate from each model. In the last column, the full AGE model leakage rate is 4.97 percent (the abatement in  $Y$  offset by added emissions from  $X$ ). In comparison, the single leakage rate reported from the CIM-EARTH simulation is 3.58 percent.<sup>38</sup>

Perhaps this 4.97 percent rate in our full AGE model is “close enough” to the 3.58 percent overall rate in CIM-EARTH, but a section below describes three reasons this comparison is not important. We have no doubt that a good CGE model can use detail to calculate overall leakage better than a tractable AGE model with analytical solutions using fewer sectors. Rather, the point here is to understand the component forces within a complicated CGE model.

Panel C of Table 4 shows components of leakage using CIM-EARTH parameters. The first five effects are additive, so they are carried forward as components in the full model. The last two effects are not additive, so each is calculated as a residual. We carry forward those IOE and FPE components for use in the full AGE model in column [5].<sup>39</sup>

Column [1] shows results for model-1 with the PNE, the effect of how the policy shock is defined. It reduces the total leakage rate from 2.16 to 2.11 percent. CIM-EARTH holds the carbon tax rate in the other sector fixed relative to the price of labor as numeraire, but leakage is lower if the tax in the other sector is held constant relative to the (rising) overall price level.

Column [2] reports results for the AGE model-2 modified only by the pure income effect, but the CIM-EARTH simulation returns all marginal revenue (so the PIE here is zero).

Column [3] reports results for model-3 with the IOE, where the CIM-EARTH input-output table is collapsed into one parameter,  $\theta_{XY}$  (use of electricity in  $X$ ). The IOE is a large negative effect on leakage, so Panel B shows the overall leakage rate turns negative (−0.63 percent). In

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<sup>38</sup> Elliot and Fullerton (2014) conduct simulations for different coalitions and parameter values. We unpack the result in their Figure 7 for  $\sigma_U = 1$  and  $\sigma_Y = 0.5$ . In correspondence, they report that the depicted leakage rate is 3.58 percent.

<sup>39</sup> Each of those eight component parts of  $\hat{C}_X$  is inserted into the leakage rate formula (all using the same model-5 result for  $\hat{C}_Y$  and the CIM-EARTH data for  $C_X/C_Y$ ). By construction, the total model-5 leakage rate of 4.97 equals the sum of its eight components (including the IE).

Panel C with the IOE, each component of  $\hat{C}_X$  is divided by a smaller  $-\hat{C}_Y$ , enlarging all component effects. The TTE rises from 1.97 to 4.21, the ARE from  $-0.98$  to  $-2.11$ , and the FRE from 1.18 to 2.51. The IOE is measured as a residual equal to  $-5.25$  percentage points. It is a large negative effect because the carbon tax raises the cost of electricity ( $Y$ ) used in sector  $X$ .

Column [4] reports results for model-4 with the FPE, a different non-additive leakage effect calculated as a residual. Panel A shows a fuel price change of  $-0.255$  percent, which quadruples the change in leakage amount  $\hat{C}_X$  (from 0.027 to 0.107). The carbon tax reduces demand for fuel, and the falling fuel price induces the uncovered sector to use more fuel. In Panel B, the overall leakage rate is 8.81 percent. Subtracting the additive effects (TTE, ARE, and FRE) yields the residual FPE of 6.54 percent, which in this column is the dominant effect on leakage.

Column [5] shows results for the full AGE model-5. For interpretation, note that full model components each have the same numerator (part of  $\hat{C}_X$ ) as in column [4], but abatement ( $-\hat{C}_Y$ ) in Panel A falls from 2.347 to 1.123 percent. Because each denominator is cut by half, each additive leakage effect is doubled. Moving down column [5], the IOE shifts the tax to the other sector and makes the full model's leakage rate fall by 5.39 percentage points. Counteracting that effect, the FPE reduces the fuel price, encourages use of fuel, and enlarges the full model's leakage rate by 13.67 percentage points (the highest number in the entire table). The IE is discussed below.

## **5.2 Unpacking Results from Carbone (2013)**

Next, we demonstrate the generality of our analysis by unpacking another CGE model. Derivations of parameters from Carbone (2013) are analogous to those for CIM-EARTH. Carbone's BCR model aggregates the globe into 14 output sectors and three fuel sectors that each use capital, labor, energy, and materials. Here again, fuel production is disaggregated into oil, coal, and natural gas.<sup>40</sup> His simulation aggregates all world regions into a single closed economy with a covered sector (electricity) and uncovered sector (all other industries). Clean inputs are mobile between sectors.

Carbone (2013) simulates a 20 percent reduction in emissions of the electricity sector (for which the necessary carbon tax rate is \$15.40 per metric ton). Leakage occurs in the other 13 sectors. Carbone's BCR model structure is broadly similar to CIM-EARTH, and it uses GTAP 7.1 data that are nearly identical to the GTAP 7 data in CIM-EARTH. We show these BCR level values in **Table 5**. These data are so similar to Table 3 that we do not repeat explanations.

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<sup>40</sup> Furthermore, the CGE leakage analysis in Carbone (2013) was intentionally structured like the analysis here, in order to calculate the ARE (which was introduced in a working paper later published as Baylis *et al.* 2014).

Instead, the difference is elasticity assumptions. For substitution between the fixed resource and mobile primary factor, Carbone (2013) assumes substitution elasticities of 0.50 for oil, 1.0 for coal, and 0.25 for natural gas. Using these elasticities and his resource shares, we calculate a supply elasticity for each fuel type and a weighted-average fuel supply elasticity of 2.30 (smaller than 2.87, used for CIM-EARTH).<sup>41</sup> The single elasticity of substitution is 0.583. Finally, using  $\eta \equiv (\theta_{CK}/\theta_{CQ})\sigma_C$ , we calculate  $\theta_{CQ} = 0.202$  (somewhat larger than the 0.173 value for CIM-EARTH).

Other key elasticities for Carbone are different from those in CIM-EARTH. First,  $\sigma_U = 0.5$  for Carbone is smaller than 1.0 in CIM-EARTH. Second,  $\sigma_Y = 0.256$  in Carbone is smaller than 0.5 in CIM-EARTH. Meanwhile,  $\sigma_X = 0.5$  is the same in both CGE models.<sup>42</sup>

Results in Panel B of **Table 6** (column [5]) shows that Carbone’s parameters in our full AGE model yield an overall AGE leakage rate of 5.56 percent. His single CGE leakage rate is 9.45 percent. For most components in column [5], Carbone’s model has results similar to those for CIM-EARTH in Table 4. Carbone (2013) uses factor income as numeraire (PNE –0.21 percent) and returns all added revenue to consumers (PIE zero). Carbone and CIM-EARTH have very similar values for other additive components as well (TTE, ARE, and FRE).

This similarity may be surprising, because Carbone’s key elasticities ( $\sigma_U = 0.5$ ,  $\sigma_Y = 0.256$ ) are half the size of those in CIM-EARTH ( $\sigma_U = 1.0$ ,  $\sigma_Y = 0.5$ ). These elasticities enter directly into leakage  $\hat{C}_X$  in equation (14):  $TTE = \sigma_U \alpha_Y \theta_{YC}$ ,  $ARE = \sigma_Y \alpha_Y \theta_{YC}$ , and  $FRE = \alpha_C \rho_Y (\sigma_U \theta_{YC} + \sigma_Y \theta_{YK})$ . These terms for Carbone are indeed half the size of those for CIM-EARTH. But these terms are components of leakage amounts, whereas leakage rates in the tables are those amounts divided by abatement ( $-\hat{C}_Y$ ). Comparing the row for  $\hat{C}_Y$  in Tables 4 and 6 indicates that Carbone’s smaller elasticities also result in abatement that is about half of CIM-EARTH’s abatement. Both numerator and denominator are cut in half, so components of the leakage rate are similar to CIM-EARTH. In other words, those component effects are the same percentages of the smaller abatement.

With the IOE, the carbon tax in  $Y$  raises the cost of electricity to firms in  $X$  (reducing their production). Thus, a large negative component of the leakage rate is the IOE – just over 5 percent in both columns [3] and [5] that contain the IOE (for both CIM-EARTH in Table 4 and Carbone

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<sup>41</sup> We use electricity sales as weights. The fixed factor input shares are 0.239 for oil, 0.229 for coal, and 0.115 for gas.

<sup>42</sup> CGE models have more flexibility, which affects leakage. Our  $\sigma_Y$  reflects substitution between primary factors and carbon fuel, but all three CGE models also allow substitution between labor and capital. Our  $\sigma_X$  reflects substitution within our uncovered sector, but CGE models have multiple uncovered sectors (some with low-carbon technology). All three CGE models also allow substitution between coal and natural gas or other low-carbon fuels.

in Table 6). For Carbone in Table 6, Panel A shows it reduces  $C_X$  in both these columns (relative to other columns). In addition, for those two columns with the IOE, direct abatement in the covered sector ( $-\hat{C}_Y$ ) is less than *half* the abatement in other columns. In effect, sharing the cost of the tax across sectors means sharing the effort to reduce emissions. But this shared cost in the other sector is not effective, because it is on an intermediate input and not on emissions. Direct abatement in  $Y$  is halved, and sector  $X$  abates a little, but total abatement  $\hat{C}$  is also halved.

The biggest effects are the positive fuel price effect and negative interaction effect. The FPE for Carbone is 17.15 percent, larger than 13.67 in CIM-EARTH, because Carbone has a larger fixed resource share in fuel production ( $\theta_{CQ}$ ) and smaller fuel supply elasticity ( $\eta$ , in the denominator of the FPE of equation 27). Also, Carbone's IE (-10.72 percent) is larger than in CIM-EARTH (-7.94 percent). The positive FPE and negative IE offset each other, so the overall AGE leakage from Carbone's model is 5.56 percent and in CIM-EARTH is 4.97 percent.

The large negative IE seems important. It arises from the way we unpack the components in the full AGE model-5. Here, we use a footnote to explain how details of this breakdown imply that the interaction effect is not important for the relative sizes of the components of leakage.<sup>43</sup>

## 6 Overall Comparisons

To compare all AGE and CGE results, **Table 7** summarizes the size and logic of many positive and negative effects on overall leakage.<sup>44</sup> For the E3 model of Goulder and Hafstead (2017), the bottom of the first column shows that the overall AGE leakage rate is 2.25 percent, where the CGE model finds 0.81 percent. For CIM-EARTH, the second column shows that overall AGE leakage is 4.97, where the CGE model finds 3.58 percent. In both columns, the AGE rate is fairly close to

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<sup>43</sup> For the breakdown in model-5, we carry forward each component portion of the overall leakage amount ( $\hat{C}_X$ ) from each of the prior models (that is, the TTE, ARE, and FRE are taken from model-0, the PNE from model-1, the IOE from model-3, and the FPE from model-4). The sum of those amounts is subtracted from overall leakage in the full model-5 to get the size of the IE. Thus, in effect, the IE is not important for the relative sizes of the component *amounts* in the breakdown of model-5. But these component amounts are all divided by direct abatement ( $-\hat{C}_Y$ ) to get components of the leakage rates in the tables. As just described, these  $\hat{C}_Y$  in Table 6 with the IOE (columns [3] and [5]) are half the size of abatement in other columns. Thus, the IOE component of the leakage *rate* in model-5 matches the one in model-3. All of the other component effects in model-5 have the same numerator as in their earlier models 1-to-4, but the denominator is half the size, so the components of the leakage rate are approximately doubled. Thus, the FPE is approximately doubled (from 8.23 percent in model-4 to 17.15 percent in model-5). Finally, the IE is a large negative number because it is calculated as the overall leakage *rate* from model-5 minus the sum of components of the leakage rate that are almost all doubled in size from their earlier calculation.

<sup>44</sup> We have modeled eight effects, including the IE, but the pure income effect (PIE) does not arise in any of the three CGE simulations studied here (because all of them return all marginal revenue from the increase in carbon tax).

the CGE rate. The third column shows results from Carbone's use of the BCR model, where overall AGE leakage is 5.56 and the CGE leakage is 9.45 percent.

Are these overall comparisons "close enough"? Consider three points. First, our earlier tables show that the full AGE model-5 provides a much better approximation of each CGE leakage rate than either model-3 or model-4 (each with the initial three effects plus one non-linear effect). Using CIM-EARTH, model-3 adds only a large negative IOE and finds an overall leakage rate of -0.63 percent (net negative leakage); model-4 has the same three initial effects but adds only the large positive FPE to find an overall leakage rate of 8.81 percent. Our full Model-5 result is 4.97 percent, much closer to the reported 3.58 percent from their CGE model. Thus, adding both non-linear effects to the full AGE model does much better than adding only one.

Second, in our efforts to approximate a variety of CGE models' results, our full AGE model has incorporated considerable complexity. We use a set of parameters and a tax policy experiment that are close to those used in each CGE simulation. But even our full AGE model has only two outputs, one primary factor, and one type of fossil fuel. It is still a crude approximation of a CGE model with 16 sectors, separate labor and capital inputs, and three fossil fuel sectors. Overall, we are gratified that our full AGE model yields a leakage rate close to the CGE result (in two cases showing a difference of less than two percentage points). Furthermore, across the three models, our AGE rate increases as the reported CGE rate increases.

But third, the comparison of overall AGE and CGE leakage rates is not relevant to our goal. A good CGE model can always better predict overall leakage. We ask only if the AGE approach is useful for breaking down the components of a CGE model's leakage rate, to obtain intuition that cannot be provided by a complex CGE model. This decomposition is discussed next.

Starting with the E3 model of Goulder and Hafstead (2017) in Table 7, our decomposition finds two positive leakage effects totaling 5.61 percent, and four negative leakage effect total 3.36 percent. This total of negative leakage effects is larger in absolute value than the total leakage rate in either the AGE model (2.25 percent) or CGE model (0.81 percent). In this column for the E3 model, the -2.58 percent input-output effect (IOE) is the only non-linear component. In the other two CGE models, the fixed resource input to fuel production leads to a large positive fuel price effect (FPE, in the first row) but large offsetting interaction effects (IE, in the eighth row).

The second column of Table 7 unpacks CIM-EARTH, where the subtotal of positive effects would be a substantial amount of leakage, at 20.57 percent, but that subtotal is reduced by four substantial negative effects on leakage that sum to -15.60 percent. The sum of these negative

effects is larger than the largest positive effect (the FPE, 13.67 percent). Because negative leakage effects are operating within all three CGE models, our analysis can help explain why the single reported rate in CGE models is so low (*e.g.*, 0.81 in E3 and 3.58 in CIM-EARTH). In other words, results here are useful for interpretations of all prior CGE results.

The net effect in CIM-EARTH is an overall AGE leakage rate of 4.97 percent, about a third of the size of the FPE. The terms of trade effect (TTE) was an initial focus of the leakage literature but is shown here to be relatively small (4.32 percent). The factor reallocation effect (2.58 percent), and the abatement resource effect (−2.16 percent) are similar across CGE models. But no numerical decomposition of a CGE model calculates all these component effects simultaneously.

The third column of Table 7 unpacks Carbone’s use of the BCR model, where our full AGE model finds an overall leakage rate of 5.56 percent (about half of the CGE model’s 9.45 percent). But the individual component effects in this column are quite similar to those in other columns.

Because Carbone’s own CGE calculation of leakage (9.45 percent) is higher than the other CGE calculations of leakage, a reader might incorrectly infer that the models are substantially different from each other. In this case, the primary difference is Carbone’s larger FPE (17.15 percent) compared to CIM-EARTH (13.67). And a CGE model can be used to gauge the numerical size of the FPE: calculate leakage in the full model, then remove resource rents from the model, calculate leakage again, and take the difference between the two numbers.

Yet these CGE models are quite similar in terms of structure and many parameters. Thus, our first major contribution listed in the Introduction is that we build AGE models to unpack all components of leakage simultaneously and show them in analytical expressions. Our second major contribution is that we can show not just the size of each effect but what parameters together *determine* the size of each effect. For example, equation (27) shows the fuel supply elasticity  $\eta \equiv (\theta_{CK}/\theta_{CQ})\sigma_C$  in the denominator of  $\hat{p}_C$ . Our calculated weighted-average elasticity of substitution  $\sigma_C$  is 0.583 for Carbone and 0.6 for CIM-EARTH, a small difference that slightly enlarges Carbone’s fall in fuel price (and enlarges his positive FPE) relative to CIM-EARTH. But our calculated weighted average of the resource rent share  $\theta_{CQ}$  is 0.202 for Carbone and 0.173 for CIM-EARTH, a larger difference that has a larger impact enlarging Carbone’s fall in fuel price (and his positive FPE) relative to CIM-EARTH. Thus, our analytical model shows that the larger FPE is mostly from a larger rent share, and not from the smaller substitution elasticity.

A first key overall point is that this analysis is not just a breakdown of components in our analytical GE model. The same major forces are operating in these CGE models as in our full AGE

models, so each reported CGE leakage rate must be based on a combination of the same conceptual component parts. Thus, the relative sizes of the true CGE components must be similar to those of the AGE components we derive here.

A second point is that the focus on leakage rates might be somewhat misplaced. We analyze leakage rates because they are the focus in CGE studies of leakage. But our breakdown shows the value of looking at the leakage amounts in the numerator first, before division by direct abatement in the covered sector – which vary and can greatly affect rates.

In a third point, a major value of this kind of “model of the model” is to alert practitioners to consider closely the underlying theory in designing their detailed CGE models, helping to capture the key forces that have the most important effects on relevant outcomes.

And finally, detailed CGE simulations of policy changes certainly have important value in discussing effectiveness of policy and its unintended consequences. But a model of the model has further policy implications; it can strengthen those insights and add others. One example is that we showed how the power of a sector-specific policy is diluted when that sector’s output is used as an input to other sectors. The reduction in total emissions is much lower in this case, because the policy raises general costs in the other industry rather than raising the cost of emissions.

## **7 Conclusion**

We build AGE models with increasing complexity that are a close approximation to a CGE model used to calculate leakage. Our AGE model is effectively a model of a more complicated model, useful to unpack component effects on leakage from a climate policy that does not cover all sectors. We identify and measure seven components, of which three are positive and four are negative. We apply parameters from three existing CGE models to measure and compare component effects.

We find that the fuel price effect (FPE) is often the largest positive effect on leakage, with a size that depends on parameter values. A carbon tax that reduces rents paid to a fixed natural resource can reduce the market price of carbon fuel and increase emissions elsewhere. We also find that the input-output effect (IOE) is an important negative leakage effect. Thus, excluding input-output analysis from policy models could greatly overstate leakage – just as excluding fuel price effects could understate leakage. Our results help inform policymakers about the relative size of leakage effects and thus help clarify discussion of policies. And they help researchers identify salient mechanisms to include when building a CGE model intended to measure leakage.

The usefulness of our approach extends well beyond our example of intersectoral leakage.

Our closed-economy AGE model with two sectors can be interpreted as two countries that each produces one good, so the terms of trade effect (TTE) in our intersectoral model is fundamentally similar to the TTE in a trade model. They have the same structure, but an AGE trade model would have its own parameters and could certainly involve more goods in each country.

Furthermore, our approach is applicable to analysis of any outcome from any policy, especially where multiple CGE models obtain answers that differ for unclear reasons. To explain these differences, our approach suggests building AGE models to unpack results in complicated CGE models of other tax or environmental policies, trade policies, development policies, market regulations, and other measures intended to enhance economic development and social welfare.

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**Table 1: Level Values from Goulder and Hafstead (2017) for E3 Model (in 2013 dollars)**

Row #	Notation	Description	Value	Unit	Source
<b>Panel A: Electricity Sector (Y)</b>					
1	$p_K \times K_Y$	Value of all clean inputs	183.2	\$billions	Goulder and Hafstead (2017)
2	$p_C \times C_Y$	Value of all fuel inputs	41.3	\$billions	Goulder and Hafstead (2017)
3	$C_Y$	Carbon/fuel level	1.03	Billion metric tons CO <sub>2</sub>	Goulder and Hafstead (2017)
4	$\tau_Y$	Initial carbon tax	15	\$ per metric ton CO <sub>2</sub>	Authors' assumption
5	$\tau_Y \times C_Y$	Value of carbon tax in Y	15.5	\$billions	Calculation: Row 3 $\times$ Row 4
6	$p_Y \times Y$	Value of all inputs; Total sales revenue	240.0	\$billions	Calculation: Sum of Rows 1+2+5
<b>Panel B: Fuel Sector (C)</b>					
7	$p_K \times K_C = p_C \times C$	Value of all clean inputs; Total sales revenue	476.4	\$billions	Goulder and Hafstead (2017)
<b>Panel C: Rest-of-Economy (X)</b>					
8	$p_K \times K_X$	Value of all clean inputs	12,016.3	\$billions	Goulder and Hafstead (2017)
9	$p_C \times C_X$	Value of all fuel inputs	435.1	\$billions	Calculation: Row 7- Row 2
10	$C_X$	Carbon/fuel level	2.09	Billion metric tons CO <sub>2</sub>	Goulder and Hafstead (2017)
11	$\tau_X$	Initial carbon tax	15	\$ per metric tons CO <sub>2</sub>	Authors' assumption
12	$\tau_X \times C_X$	Value of carbon tax X	31.4	\$billions	Calculation: Row 10 $\times$ Row 11
13	$p_Y \times Y_X$	Value of all electricity inputs	177.4	\$billions	Goulder and Hafstead (2017)
14	$p_X \times X$	Value of all inputs; Total sales revenue	12,660.2	\$billions	Calculation: Sum of Rows 8+9+12+13

Notes: Each sector is assumed to have an initial carbon price of \$15 per metric ton of CO<sub>2</sub>. National income can be calculated two ways. First, add values of primary inputs ( $p_K \bar{K}$ ) and carbon taxes ( $\tau_Y C_Y + \tau_X C_X$ ) in rows (1), (5), (7), (8), and (12). Alternatively, add consumer expenditures on final goods,  $p_X X$  in row (14) plus final sales of Y to consumers (row (6) minus (13), equal to  $p_Y Y_U = \$62.6$  billion). Both ways show national income of \$12,722.8 billion. In our calculations without the IOE, the value of intermediate goods sold from sector Y to sector X are added instead to the primary factor input in sector X. All cells in the table are subject to independent rounding.

**Table 2: Relative Changes and Leakage Decomposition using Goulder & Hafstead (2017)**  
 ( $\hat{\tau}_Y=0.1$ , but all results reported in percent)

	[0]	[1]	[2]	[3]
Analytical GE (AGE) Model:	Initial Model <sup>a</sup>	Policy Normalization Effect (PNE) <sup>b</sup>	Pure Income Effect (PIE)	Input-Output Effect (IOE)
Equation:	14	17	21	22
<b>Panel A: Key Endogenous Outcome Changes</b>				
$\hat{p}_c$	0.000	0.000	0.000	0.000
$\hat{C}_Y$	-1.452	-1.452	-1.452	-0.986
$\hat{C}_X$	0.024	0.023	0.024	0.011
<b>Panel B: Total Leakage Rate</b>				
Our AGE Model:	3.29	3.27	3.29	2.27
<b>Panel C: Leakage Rate Decomposition</b>				
TTE	1.28	1.28	1.28	1.89
ARE	-0.51	-0.51	-0.51	-0.76
FRE	2.52	2.52	2.52	3.72
PNE	-	-0.02	-	-
PIE	-	-	0.00	-
IOE	-	-	-	-2.58

Notes: Parameters are based on Goulder and Hafstead (2017). All cells in the table are subject to independent rounding. A cell is grey when that row's component effect is not included in that column's AGE model.

<sup>a</sup> Our initial model includes three additive terms: the Terms of Trade Effect (TTE), Abatement Resource Effect (ARE), and Factor Reallocation Effect (FRE). All extended models include these effects plus another effect that may not be additive.

<sup>b</sup> The PNE shows how results are affected by fixing  $\tau_X/p_I$  in real terms instead of  $\tau_X/p_K$  relative to the numeraire. All three CGE models have no initial carbon taxes, but they have energy taxes or fuel taxes in multiple sectors (which we represent using initial  $\tau_X = \tau_Y = \$15/\text{tonne}$ ). This  $\tau_X$  is small, so the PNE is small, but this approximation means that the actual PNE for each CGE model is not precise.

**Table 3: Level Values from CIM-EARTH Data (in 2004 dollars)**

Row #	Notation	Description	Value	Unit	Source
<b>Panel A: Electricity Sector (Y)</b>					
1	$p_K \times K_Y$	Value of all clean inputs	569.3	\$billions	CIM-EARTH database
2	$p_C \times C_Y$	Value of all fuel inputs	260.2	\$billions	CIM-EARTH database
3	$C_Y$	Carbon/fuel level	9.44	Billion metric tons CO <sub>2</sub>	CIM-EARTH database
4	$\tau_Y$	Initial carbon tax	15	\$ per metric ton CO <sub>2</sub>	Authors' assumption
5	$\tau_Y \times C_Y$	Value of carbon tax in Y	141.5	\$billions	Calculation: Row 3 $\times$ Row 4
6	$p_Y \times Y$	Value of all inputs; Total sales revenue	971.1	\$billions	Calculation: Sum of Rows 1+2+5
<b>Panel B: Fuel Sector (C)</b>					
7	$p_K \times K_C$	Value of all clean inputs	583.4	\$billions	CIM-EARTH database
8	$p_Q \times Q$	Resource rents	122.2	\$billions	Author's calculation
9	$p_C \times C$	Value of all inputs; Total sales revenue	705.6	\$billions	Calculation: Row 7 + Row 8
<b>Panel C: Rest-of-Economy (X)</b>					
10	$p_K \times K_X$	Value of all clean inputs	31,046.3	\$billions	CIM-EARTH database
11	$p_C \times C_X$	Value of all fuel inputs	445.4	\$billions	Calculation: Row 9 - Row 2
12	$C_X$	Carbon/fuel level	18.26	Billion metric tons CO <sub>2</sub>	CIM-EARTH database
13	$\tau_X$	Initial carbon tax	15	\$ per metric tons CO <sub>2</sub>	Authors' assumption
14	$\tau_X \times C_X$	Value of carbon tax in X	273.8	\$billions	Calculation: Row 12 $\times$ Row 13
15	$p_Y \times Y_X$	Value of all electricity inputs	892.2	\$billions	CIM-EARTH database
16	$p_X \times X$	Value of all inputs; Total sales revenue	32,657.8	\$billions	Calculation: Sum of Rows 10+11+14+15

Notes: Each sector is assumed to have an initial carbon price of \$15 per metric ton of CO<sub>2</sub>. The CIM-EARTH model employs GTAP7 expenditure and revenue data for the 2004 baseline year. The Petroleum Refining sector in GTAP is separate from the Oil Extraction sector and does not employ resources directly, so we include Petroleum Refining in the rest-the-economy. Worldwide income can be calculated two ways. First, add values of primary inputs ( $p_K \bar{K}$ ) and carbon taxes ( $\tau_Y C_Y + \tau_X C_X$ ) plus rents ( $p_Q Q$ ). Alternatively, add consumer expenditures on final goods,  $p_X X$  plus final sales of Y to consumers (row (6) minus (15), equal to  $p_Y Y_U = \$78.9$  billion). Both ways show worldwide income of \$32,736.6 billion. In cases without the IOE, the value of intermediate goods sold from sector Y to sector X is added instead to the primary factor input in sector X. All cells in the table are subject to independent rounding.

**Table 4: Relative Changes and Leakage Decomposition using CIM-EARTH**  
 ( $\hat{\tau}_Y=0.1$ , but all results reported in percent)

	[0]	[1]	[2]	[3]	[4]	[5]
Analytical GE (AGE) Model:	Initial Model <sup>a</sup>	Policy Normalization Effect (PNE) <sup>b</sup>	Pure Income Effect (PIE)	Input-Output Effect (IOE)	Fuel Price Effect (FPE)	Full AGE Model
Equation(s):	14	17	21	22	26/27	Appendix
<b>Panel A: Key Endogenous Outcome Changes</b>						
$\hat{p}_c$	0.000	0.000	0.000	0.000	-0.255	-0.127
$\hat{C}_Y$	-2.463	-2.463	-2.463	-1.151	-2.347	-1.123
$\hat{C}_X$	0.027	0.027	0.027	-0.004	0.107	0.029
<b>Panel B: Total Leakage Rate</b>						
Our AGE Model:	2.16	2.11	2.16	-0.63	8.81	4.97
<b>Panel C: Leakage Rate Decomposition</b>						
TTE	1.97	1.97	1.97	4.21	2.07	4.32
ARE	-0.98	-0.98	-0.98	-2.11	-1.03	-2.16
FRE	1.18	1.18	1.18	2.51	1.23	2.58
PNE	-	-0.05	-	-	-	-0.11
PIE	-	-	0.00	-	-	0.00
IOE	-	-	-	-5.25	-	-5.39
FPE	-	-	-	-	6.54	13.67
IE <sup>c</sup>	-	-	-	-	-	-7.94

Notes: Parameters are based on the CIM-EARTH model. All cells in the table are subject to independent rounding. A cell is grey when that row's component effect is not included in that column's AGE model.

<sup>a</sup> Our initial model includes three additive terms: the Terms-of-Trade Effect (TTE), Abatement Resource Effect (ARE), and Factor Reallocation Effect (FRE). All extended models include these three effects plus another effect that may not be additive.

<sup>b</sup> The PNE shows how results are affected by fixing  $\tau_X/p_I$  in real terms instead of  $\tau_X/p_K$  relative to the numeraire. All three CGE models have no initial carbon taxes, but they have energy taxes or fuel taxes in multiple sectors (which we represent using initial  $\tau_X = \tau_Y = \$15/\text{tonne}$ ). This  $\tau_X$  is small, so the PNE is small, but this approximation means that the actual PNE for each CGE model is not precise.

<sup>c</sup> The interaction effect (IE) is non-zero in the full AGE model, because the two non-linear terms (IOE and FPE) are not additive and thus interact with all other terms.

**Table 5: Level Values from Carbone (2013) (in 2004 dollars)**

Row #	Notation	Description	Value	Unit	Source
<b>Panel A: Electricity Sector (Y)</b>					
1	$p_K \times K_Y$	Value of all clean inputs	569.3	\$billions	CIM-EARTH database
2	$p_C \times C_Y$	Value of all fuel inputs	260.2	\$billions	CIM-EARTH database
3	$C_Y$	Carbon/fuel level	9.44	Billion metric tons CO <sub>2</sub>	CIM-EARTH database
4	$\tau_Y$	Initial carbon tax	15	\$ per metric ton CO <sub>2</sub>	Authors' assumption
5	$\tau_Y \times C_Y$	Value of carbon tax in Y	141.5	\$billions	Calculation: Row 3 $\times$ Row 4
6	$p_Y \times Y$	Value of all inputs; Total sales revenue	971.1	\$billions	Calculation: Sum of Rows 1+2+5
<b>Panel B: Fuel Sector (C)</b>					
7	$p_K \times K_C$	Value of all clean inputs	583.4	\$billions	CIM-EARTH database
8	$p_Q \times Q$	Resource rents	148.0	\$billions	Author's calculation
9	$p_C \times C$	Value of all inputs; Total sales revenue	731.4	\$billions	Calculation: Row 7 + Row 8
<b>Panel C: Rest-of-Economy (X)</b>					
10	$p_K \times K_X$	Value of all clean inputs	31,046.3	\$billions	CIM-EARTH database
11	$p_C \times C_X$	Value of all fuel inputs	471.2	\$billions	Calculation: Row 9 - Row 2
12	$C_X$	Carbon/fuel level	18.26	Billion metric tons CO <sub>2</sub>	CIM-EARTH database
13	$\tau_X$	Initial carbon tax	15	\$ per metric tons CO <sub>2</sub>	Authors' assumption
14	$\tau_X \times C_X$	Value of carbon tax in X	273.8	\$billions	Calculation: Row 12 $\times$ Row 13
15	$p_Y \times Y_X$	Value of all electricity inputs	892.2	\$billions	CIM-EARTH database
16	$p_X \times X$	Value of all inputs; Total sales revenue	32,683.6	\$billions	Calculation: Sum of Rows 10+11+14+15

Notes: Each sector is assumed to have an initial carbon price of \$15 per metric ton of CO<sub>2</sub>. The BCR model underlying Carbone (2013) employs GTAP 7.1 expenditure and revenue data for the 2004 baseline year. The Petroleum Refining sector in GTAP is separate from the Oil Extraction sector and does not employ resources directly, so we include it in the rest-the-economy. Worldwide income can be calculated two ways. First, add values of primary inputs ( $p_K \bar{K}$ ) and carbon taxes ( $\tau_Y C_Y + \tau_X C_X$ ) plus rents ( $p_Q Q$ ). Alternatively, add consumer expenditures on final goods,  $p_X X$  plus final sales of Y to consumers (row (6) minus (15), equal to  $p_Y Y_U = \$78.9$  billion). Both ways show worldwide income \$32,762.5 billion. In our calculations without the IOE, the value of intermediate goods sold from sector Y to sector X are added instead to the primary factor input in sector X. All cells in the table are subject to independent rounding.

**Table 6: Relative Changes and Leakage Decomposition using Carbone (2013)**  
 ( $\hat{\tau}_Y=0.1$ , but all results reported in percent)

	[0]	[1]	[2]	[3]	[4]	[5]
Analytical GE (AGE) Model:	Initial Model <sup>a</sup>	Policy Normalization Effect (PNE) <sup>b</sup>	Pure Income Effect (PIE)	Input-Output Effect (IOE)	Fuel Price Effect (FPE)	Full AGE Model
Equation(s):	14	17	21	22	26/27	Appendix
<b>Panel A: Key Endogenous Outcome Proportion Changes</b>						
$\hat{p}_c$	0.000	0.000	0.000	0.000	-0.160	-0.081
$\hat{C}_Y$	-1.244	-1.244	-1.244	-0.588	-1.206	-0.578
$\hat{C}_X$	0.014	0.013	0.014	-0.002	0.065	0.017
<b>Panel B: Total Leakage Rate</b>						
Our AGE Model:	2.13	2.03	2.13	-0.64	10.42	5.56
<b>Panel C: Leakage Rate Decomposition</b>						
TTE	1.95	1.95	1.95	4.12	2.01	4.19
ARE	-1.00	-1.00	-1.00	-2.11	-1.03	-2.15
FRE	1.18	1.18	1.18	2.48	1.21	2.53
PNE	-	-0.10	-	-	-	-0.21
PIE	-	-	0.00	-	-	0.00
IOE	-	-	-	-5.14	-	-5.23
FPE	-	-	-	-	8.23	17.15
Interaction <sup>c</sup>	-	-	-	-	-	-10.72

Notes: Total CGE leakage is taken from Carbone's (2013) reference case with carbon tax sufficient to reduce carbon emissions by 20 percent in the covered sector. All cells in the table are subject to independent rounding. A cell is grey when that row's component effect is not included in the column's model.

<sup>a</sup> Our initial model includes three additive terms: the Terms-of-Trade Effect (TTE), Abatement Resource Effect (ARE), and Factor Reallocation Effect (FRE). All extended models include these three effects plus another effect that may not be additive.

<sup>b</sup> The PNE shows how results are affected by fixing  $\tau_X/p_I$  in real terms instead of  $\tau_X/p_K$  relative to the numeraire. All three CGE models have no initial carbon taxes, but they have energy taxes or fuel taxes in multiple sectors (which we represent using initial  $\tau_X = \tau_Y = \$15/\text{tonne}$ ). This  $\tau_X$  is small, so the PNE is small, but this approximation means that the actual PNE for each CGE model is not precise.

<sup>c</sup> The interaction effect (IE) is non-zero in the full AGE model, because the two non-linear terms (IOE and FPE) are not additive and thus interact with all other terms.

**Table 7: Comparisons and the Logic of Offsetting Effects on the Leakage Rate**  
(all results reported in percent)

AGE “Effect”	E3 Goulder & Hafstead	CIM- EARTH Elliot <i>et al.</i>	BCR Carbone	What drives this effect?
Fuel Price (FPE)	N.A.	13.67	17.15	Reduced demand for carbon pushes fuel sector down its rising cost curve
Terms of Trade (TTE)	1.89	4.32	4.19	Raised cost of $Y$ shifts consumers towards purchase of $X$
Factor Reallocation (FRE)	3.72	2.58	2.53	Reduced production of carbon fuel frees resources for production of $X$
<i>Positive Subtotal</i>	5.61	20.57	23.87	Positive effects on the leakage rate emphasized in the literature
Policy Normalization (PNE)	-0.02	-0.11	-0.21	Carbon tax in $X$ is fixed relative to factor price instead of overall prices
Abatement Resource (ARE)	-0.76	-2.16	-2.15	Substitution in $Y$ from $C_Y$ to $K_Y$ draws primary factors away from $X$
Input Output (IOE)	-2.58	-5.39	-5.23	Raised cost for input of $Y$ raises cost of producing $X$ , reducing demand
Interaction (IE)	0.0	-7.94	-10.72	Nonlinear interactions of all effects above cannot be attributed to any one
<i>Negative Subtotal</i>	-3.36	-15.60	-18.31	Negative effects on the leakage rate, not as emphasized in the literature
Overall AGE	2.25	4.97	5.56	Total increase in emissions in sector $X$ as percent of abatement in $Y$
Overall CGE	0.81	3.58	9.45	This row shows the one leakage rate available from the CGE model

Note: The order of rows is from largest to smallest by the CIM-EARTH effects. The Pure Income Effect (PIE) does not appear in this table because it is zero for all three CGE models. Cells are subject to independent rounding.