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Belief Distortions and Uncertainty About Inflation

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Belief Distortions and Uncertainty about Inflation*

Stefano Fasani[†]

Giuseppe Pagano Giorgianni[‡]

Valeria Patella[§]

Lorenza Rossi[¶]

October 20, 2025

Abstract

This paper studies the macroeconomic effects of a belief distortion shock—defined as the unexpected component of household inflation expectations after accounting for professional forecasts and observable fundamentals. Using survey data, U.S. macroeconomic variables, and machine-learning methods, we identify this shock and examine its effects both within and outside the zero lower bound (ZLB), conditioning on household inflation uncertainty. The shock raises inflation, uncertainty, and unemployment in normal times. At the ZLB, the shock reduces real interest rates and becomes expansionary; however, the accompanying rise in inflation uncertainty dampens or can even reverse these effects. A New Keynesian model with belief shocks replicates these dynamics and matches the empirical patterns of inflation uncertainty.

Keywords: Inflation; Belief Distortion Shock; Inflation Uncertainty; Households Expectation; Machine Learning; Local Projections; New Keynesian model; Monetary Policy, ZLB.

JEL Codes: E31, C220, D840, C320

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[†]Lancaster University, E-mail address: s.fasani@lancaster.ac.uk

[‡]Sapienza University Rome, E-mail address: giuseppe.paganogiorgianni@uniroma1.it

[§]Sapienza University Rome, E-mail address: valeria.patella@uniroma1.it

[¶]Lancaster University, Corresponding author, Bailrig LA14YX, Lancaster, UK, E-mail address: l.rossi@lancaster.ac.uk

1 Introduction

Understanding how expectations shape macroeconomic outcomes has become increasingly important in a world of persistent inflation surprises and increased uncertainty.

Most of the literature on inflation expectations focuses mainly on explaining survey microdata and understanding the formation, determinants, and heterogeneity of inflation expectations (see [Coibion, Gorodnichenko and Weber, 2022](#), for a survey). While examining the endogenous formation of agents' beliefs is crucial, our paper takes a different, yet complementary approach by assessing the macroeconomic effects of a belief distortion shock. We define this shock as the unexpected component of household inflation expectations after accounting for professional forecasts and observable fundamentals.

Recently, [D'Acunto et al. \(2024\)](#) show that households act on their inflation beliefs and may depart from the predictions of conventional economic models. They document that household expectations, on average, deviate noticeably from Central Bank targets and contrast with expert forecasts, such as those from the Survey of Professional Forecasters, particularly during recessions. [Bhandari, Borovička and Ho \(2024\)](#) show that, consistent with survey evidence, increases in pessimism generate upward biases in unemployment and inflation forecasts, leading to lower economic activity. By developing a theory of subjective beliefs, they demonstrate that biases in household beliefs have quantitatively large negative effects on macroeconomic aggregates.

Our paper builds on this literature by providing new empirical and theoretical insights into how belief distortions and household uncertainty about future inflation affect macroeconomic outcomes—particularly unemployment and key labor market variables—while explicitly accounting for *state-dependent effects*. Unlike previous studies, we analyze how these shocks operate under different macroeconomic regimes, including periods when conventional monetary policy is constrained at the Zero Lower Bound (ZLB), and how inflation uncertainty interacts with belief distortions to shape the response of the economy.

On the empirical side, we identify a belief distortion shock and assess its macroeconomic effects on the US economy both in normal times and during deep recessions, with particular attention to the Global Financial Crisis and the COVID-19 recession, when policy rates were at the ZLB. On the theoretical side, we develop a model that reproduces the state-dependent effects of belief distortion shocks documented in the empirical analysis and generates inflation uncertainty dynamics consistent with those observed in the data.

The identification of the shock is split in two steps. First, we construct a measure of the belief wedge, defined as the difference between household inflation expectations from the Michigan Survey of Consumers (MSC) and those from the Survey of Professional Forecasters (SPF), similar to the approach in [Bhandari, Borovička and Ho \(2024\)](#).

Second, drawing on [Coibion, Gorodnichenko and Wieland \(2012\)](#), who show that although household forecast errors are larger, even professional forecasters update their expectations infrequently and sluggishly—deviating from full rationality—we further isolate the component of household beliefs that departs from a *model-based full-information rational forecast*. To do so, we apply a ridge regression machine learning algorithm to the belief wedge, controlling for a large set of macroeconomic and financial variables from the FRED-MD database, which represent the information available to households at the moment of expectation formation.

This second step goes beyond the identification strategy of [Bhandari, Borovička and Ho \(2024\)](#), allowing us to better isolate the unexpected component of household inflation expectations. The estimated residual from the ridge regression is then interpreted as our identified belief distortion shock, as it captures the unanticipated component of household inflation expectations that is orthogonal to all information available at the time the shock occurs.

We use the identified belief distortion shock to examine its transmission to key macroeconomic variables, including uncertainty about future inflation. We measure households' expected inflation uncertainty following [Binder \(2017\)](#), who constructs an uncertainty measure based on the prevalence of round-number responses in household inflation expectation surveys. We compare this measure with an alternative one constructed using the approach of [Jurado, Ludvigson and Ng \(2015\)](#), which estimates the conditional volatility of the unforecastable component of household inflation expectations once controlling for a large set of macroeconomic variables. We find that these two measures of inflation uncertainty are strongly correlated.

Our analysis of the transmission of the belief distortion shock is divided into two steps. First, we run linear local projections and show that a positive belief distortion shock leads to a significant and persistent increase in inflation, inflation uncertainty, and unemployment. By highlighting the effects of belief shocks on unemployment, we refer to the recent literature on the interaction between aggregate fluctuations driven by inflation expectations and the labor market ([Pappa, Ravn and Sterk, 2023](#)).

Second, we extend our analysis to investigate the effects of the belief distortion shock under state-dependent scenarios. After identifying regimes characterized by high levels of uncertainty about future inflation, we find that these regimes partially overlap with periods when the economy is constrained by the ZLB. We use local projection to conduct a state-dependent analysis to examine the effects of the shock across different states—both within and outside the ZLB—while conditioning on the degree of inflation uncertainty.

Our results show that the shock increases unemployment during normal times. In contrast, when the Central Bank is constrained by the zero lower bound (ZLB) and maintains a prolonged accommodative policy stance, the shock becomes expansionary and reduces unemployment. As inflation expectations increase, the real interest rate

falls, producing expansionary effects that accelerate recovery.

However, once we control for inflation uncertainty under ZLB conditions, the expansionary effect of the shock on unemployment is significantly weakened. We interpret this as evidence that inflation uncertainty dampens the positive transmission of belief distortion shocks to the economy. From a policy perspective, although higher expected inflation can facilitate a quicker escape from a liquidity trap, this benefit may vanish when households remain uncertain about future inflation. This highlights the importance of clear and credible communication by the central bank regarding its short-run inflation targets.

In the second part of the paper, we consider a simple New Keynesian model with search and matching frictions, where inflation expectations slightly depart from rationality due to the presence of an exogenous belief distortion shock. Although not derived from explicit microfoundations, our exogenous shock to short-term inflation expectations falls into the broader category of belief shocks.¹

By taking a third-order approximation of our simple model, we construct a measure of the expected variance of inflation expectations, which we take as the model counterpart of inflation uncertainty. We show that our model is able to rationalize the dynamics of inflation, unemployment, and inflation uncertainty observed in the empirical part, both in normal times and during crises with the monetary policy at the ZLB.

A key validation of our approach lies in the model’s ability to generate simulated data that closely match the empirical results. By feeding the identified belief distortion shock into our theoretical framework, we obtain simulated paths for inflation uncertainty that, particularly during the last two crises, align with those observed in the historical decomposition from the empirical analysis. We also show that, consistent with the empirical counterpart, inflation uncertainty per se increases the unemployment rate in the ZLB, thus contributing to dampen the expansionary effect of the belief distortion shock.

The main contribution of our paper is to demonstrate that any event that unexpectedly raises households’ inflation beliefs —relative from a model-based full-information rational forecast—can potentially facilitate recovery from the ZLB. Importantly, we are the first to show that this effect crucially depends on the level of uncertainty: when the increase in expectations is accompanied by greater uncertainty, the stimulus may be substantially weakened or even reversed, as households respond more cautiously to expected price changes. These results highlight the need for central banks to manage not only the level of inflation expectations, but also the uncertainty surrounding them, through consistent, transparent, and credible short-run communication.

To sum up, our paper combines empirical rigor with a simple and interpretable theo-

¹In the same vein, [Angeletos, Collard and Dellas \(2018\)](#) introduce a shock to short-run expectations of real variables as a reduced-form shock to capture sentiment shifts due to higher-order beliefs. They emphasize that their short-run expectational shock differs from optimism/pessimism in the news shock literature, which typically applies to expected TFP increases in the distant future.

retical framework to shed light on how household beliefs and uncertainty shape macroeconomic outcomes—especially in crisis periods when conventional policy tools are constrained. Beyond advancing the literature, our findings offer clear policy lessons for the design of monetary policy under the ZLB.

Related literature. The paper draws primarily on three strands of the literature. First, it refers to studies that examine distortions in inflation expectations and the role of belief distortion shocks as a source of business cycle fluctuations.

Different papers provide evidence of deviations from rationality for households and firms (Bachmann, Berg and Sims, 2015; D’Acunto et al., 2023, 2021; D’Acunto, Malmendier and Weber, 2021; Bhandari, Borovička and Ho, 2024; Diegel and Nautz, 2021). Capistrán and Timmermann (2009) and D’Acunto, Malmendier and Weber (2023) show that inflation expectations are upwardly biased. Binder and Kamdar (2022) examine the relationship between expected and realized inflation on an historical perspective. Coibion and Gorodnichenko (2015) and Bordalo et al. (2020) test the predictability of forecast errors, and find aggregate underreaction and individual-level overreaction, respectively. Angeletos, Huo and Sastry (2020) reconcile the two views with a theory that combines dispersed information with over-extrapolation. The literature is not decisive about the impact of belief distortion shocks in the data [Fève and Guay (2019), Chahrour and Jurado (2018), Enders, Kleemann and Müller (2021), Lagerborg, Pappa and Ravn (2023), Adams and Barrett (2024)] and in the models [Lorenzoni (2009); Benhabib, Wang and Wen (2015); Angeletos, Collard and Dellas (2018); Ascari et al. (2023)]. We contribute here by using a machine learning regularization algorithm to identify a belief distortion shock entailing a deviation from rational expectations. Its effects are highly nonlinear in normal and crisis times.

Second, our paper connects with the literature measuring uncertainty in agents’ expectations. Bloom (2009) identifies uncertainty as the unconditional volatility of a time series. Using survey data, Jurado, Ludvigson and Ng (2015) focus on the variance of the forecast errors. This measure is generalized with Rossi and Sekhposyan (2015), which compute a distribution-based measure of uncertainty, based on the unconditional likelihood of the realized forecast error. Binder (2017) estimate uncertainty with respect to the rounding behavior of households. Finally, Manski (2018) offers a survey of probabilistic proxies, including the Survey of Consumer Expectations (Armantier et al., 2013).

While no consensus has been reached on the measurement front, the literature agrees on the implications of uncertainty for macroeconomic aggregates. Specifically, uncertainty is countercyclical, increasing in recessions, and dampening spending and investment decisions (Bloom et al., 2018; Ascari et al., 2023; Coibion et al., 2024; Georgarakos et al., 2024), as agents become more cautious when uncertain. We first compute a rounding-based measure of inflation uncertainty, which highly correlates with alternative exist-

ing definitions (both time series volatility measures, probabilistic measures, and cross-sectional disagreement proxies), and then use a general equilibrium model with belief distortion shocks and endogenous uncertainty, as to match both the unconditional and conditional properties of inflation uncertainty in response to the belief distortion shock.

In addition, we refer to the vast literature that enhances the role of nonlinearities in the transmission of macroeconomic shocks. [Auerbach and Gorodnichenko \(2012, 2013\)](#) and [Ramey and Zubairy \(2018\)](#) estimate different fiscal multipliers in recessions and expansions. [Jo and Zubairy \(2025\)](#) establish interaction states between inflation and unemployment to discipline the state dependence of fiscal multipliers with respect to the demand/supply nature of recessions. [Bianchi and Ilut \(2017\)](#) and [Bianchi and Melosi \(2017, 2019\)](#) show that monetary and fiscal policy interaction regimes (including the ZLB) significantly shape inflationary and macroeconomic outcomes. They recommend a temporary fiscal dominance regime, increasing inflation expectations, as an exit strategy from crises. [Falck, Hoffmann and Hürtgen \(2021\)](#) evidence that monetary policy effectiveness changes with the degree of disagreement in inflation expectations.

Finally our paper is related to the recent work of [Bianchi, Faccini and Melosi \(2023\)](#) which has sought to explain the rise in households' inflation expectations by examining unfunded fiscal shocks, which—by raising inflation expectations in the absence of monetary policy adjustments—lower real interest rates and help lift the economy out of recessions, particularly following the Global Financial Crisis and the COVID-19 downturn. This result is also consistent with [Coibion, Gorodnichenko and Ropele \(2020\)](#), who provide micro-level evidence of the potential expansionary effects of rising inflation expectations at the ZLB, using firm-level survey data from the Italian economy. Our work empirically and theoretically shows that inflation expectation shocks help escape ZLB states, where we condition the recovery on inflation uncertainty. It dampens the expansionary impulse of the shock.

The remainder of the paper is organized as follows. [Section 2](#) presents preliminary evidence on inflation expectations and uncertainty. [Section 3](#) identifies the belief distortion shock ([Section 3.1](#)), conducts an impulse response analysis via linear local projections ([Section 3.2](#)), and introduces a state-dependent analysis by examining the role of ZLB periods and the contribution of inflation uncertainty ([Section 3.3](#)). Finally, [Section 4](#) presents the theoretical model and its implied conditional dynamics in response to the belief distortion shock. [Section 5](#) concludes. Robustness and technical details are left in the Appendix at the bottom of the paper.

2 Inflation Expectations and Uncertainty

This section describes the time series used to identify the belief distortion shock and to measure households inflation uncertainty. First, we refer to the mean difference between households' and professional forecasters' 1-year-ahead inflation expectations as the inflation belief wedge. Household expectations are taken from the Michigan Survey of Consumer Attitudes (MSC), whereas the professional forecasters' from the Survey of Professional Forecasters (SPF) by the Philadelphia FED.²

Bhandari, Borovička and Ho (2024) refer to the average expectation of the professional forecaster as a proxy for rational expectations, as professional forecasters have greater access to data, rely on sophisticated forecasting techniques (Bonham and Dacy, 1991), and therefore are considered good benchmarks, also in Faust and Wright (2013). We extend this literature by taking an additional step. Building on Coibion, Gorodnichenko and Wieland (2012)—who show that although household forecast errors are larger, even professional forecasters update their expectations infrequently and sluggishly, deviating from full rationality—we further isolate the component of household beliefs that departs from a model-based rational forecast. To do so, we apply a ridge machine learning algorithm to the belief wedge, controlling for a broad set of macroeconomic and financial variables from the FRED-MD database. This step marks a key departure from Bhandari, Borovička and Ho (2024). The estimated residual from this model is interpreted as our identified belief distortion shock.

Figure 1 reports the data used to compute the wedge. It compares the dynamics of the mean inflation expectations of the MSC and the SPF, taken at the quarterly frequency over the 1981:Q4-2024:Q2 sample. The figure shows that household inflation forecasts tend to systematically exceed the forecasts of professional forecasters, resulting in predictable errors. The literature shows that economic agents tend to associate the prospect of higher future inflation with adverse macroeconomic outcomes. Households anticipating higher inflation also expect higher levels of unemployment (Bhandari, Borovička and Ho, 2024), while reduced inflation expectations are associated with an increase in the consumption of durable goods (Kumar, Gorodnichenko and Coibion, 2023). Coibion, Gorodnichenko and Ropele (2020) reach similar conclusions for firms. On the basis of this evidence, we interpret positive deviations from rational expectations as indicating consensus pessimism, while negative ones as signaling consensus optimism.

The difference (or vertical gap) between the average household forecast and that of the professional forecasters represents the inflation belief wedge. It provides a measure

²The MSC survey is a rotating panel of approximately 500 to 1500 households, with respondents from the general public being either new participants or returning respondents from the previous 6 and 12 months. Data are collected monthly, and the second month of each quarter is selected to create a quarterly series. The Survey of Professional Forecasters collects data on a quarterly basis in the second month of each quarter, with respondents ranging from 9 to 83 professionals, including economists, bankers, and forecasters.

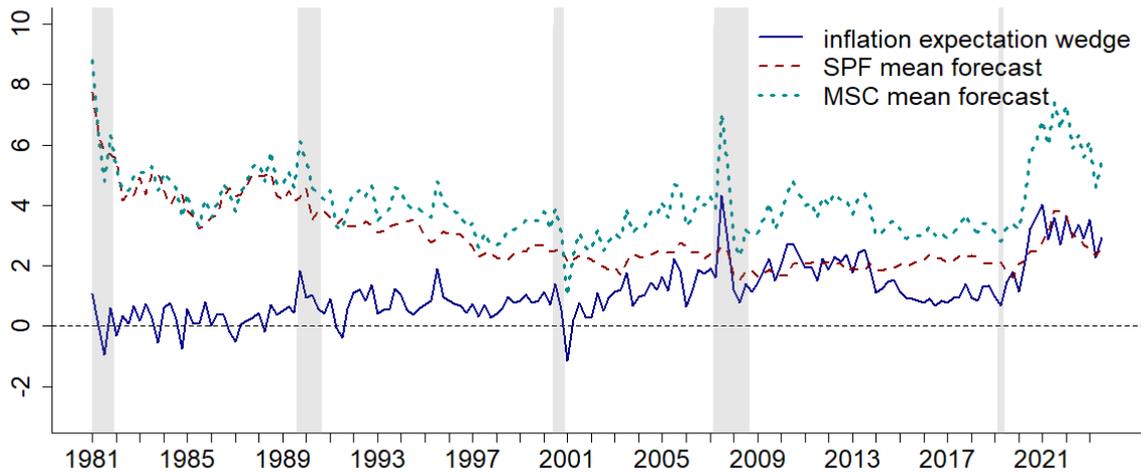


Figure 1: **Inflation expectations and belief wedge.** The figure shows the average 1-year-ahead inflation expectations for households (MSC) and professional forecasters (SPF). The difference defines the inflation belief wedge. *Source:* Michigan Survey of Consumers, Philadelphia Survey of Professional Forecasters. *Sample:* 1981Q4-2024Q2.

of the degree of belief distortion regarding inflation. This measure displays significant temporal variation, with the two main surges coinciding with major economic disruptions: the 2007-2009 Global Financial Crisis and the post-2020 Covid-19 crisis.

Inflation Expectation Uncertainty. To measure inflation uncertainty among households, we exploit both the time series and cross-sectional variation in the Michigan Survey of Consumers. Our approach follows the methodology proposed by [Binder \(2017\)](#), which builds on the observation that consumers frequently round their inflation point forecasts to multiples of five.³ As emphasized by [Krifka \(2009\)](#), “*round numbers suggest round interpretations*”.

Formally, each survey respondent is assumed to hold a subjective probability distribution over one-year ahead expected inflation and can be classified either as a low-uncertainty type (l) or a high-uncertainty type (h). High-uncertainty types exhibit higher dispersion in their subjective distributions; as a result, their forecasts are less precise and more likely to be rounded. Rounding behavior alone, however, does not uniquely identify individual uncertainty. We therefore treat the proportion of high-uncertainty agents in each survey wave as a parameter to be estimated from the data.

Specifically, let the overall probability mass function of the survey responses be given by $\phi_t = \lambda_t \phi_t^h + (1 - \lambda_t) \phi_t^l$, where ϕ_t^x is the probability mass of x -type responses and λ_t is the time-varying fraction of h -type respondents. We estimate λ_t and the parameters of

³[Binder \(2017\)](#) verifies that consumers who provide multiple of 5% forecasts are those whose density forecasts are more dispersed when using the beta estimates of the Survey of Consumer Expectations.

ϕ_t^h and ϕ_t^l via maximum likelihood. Then, our measure of inflation uncertainty is given by:

$$U_t = \frac{\lambda_t N_t + \theta N_t^{DK}}{N_t + \theta N_t^{DK}} \quad (1)$$

where $N_t = N_t^h + N_t^l$ is the total amount of respondents providing a numerical forecast to the survey in each period, N_t^{DK} is the amount of “don’t know” respondents, and θ is a weighting parameter governing the extent to which “don’t know” responses are interpreted as signals of uncertainty. We set $\theta = 1$.

As a further check, we follow the methodology of [Jurado, Ludvigson and Ng \(2015\)](#) and [Ludvigson, Ma and Ng \(2020\)](#) to construct a measure of inflation uncertainty, defined as the conditional volatility of the unforecastable component of one-year ahead MSC household inflation expectations. This approach leverages a large set of macroeconomic variables—the FRED-QD database—to generate direct econometric estimates of time-varying uncertainty. Further details on the construction of this inflation uncertainty measure are provided in [Appendix B](#).

A key strength of this methodology lies on its focus on the overall predictability of the economy, rather than on the variability or dispersion of individual indicators. By minimizing reliance on specific theoretical structures or a narrow set of observables, it aims to capture a broad, model-free notion of uncertainty that is particularly relevant for household decision-making.

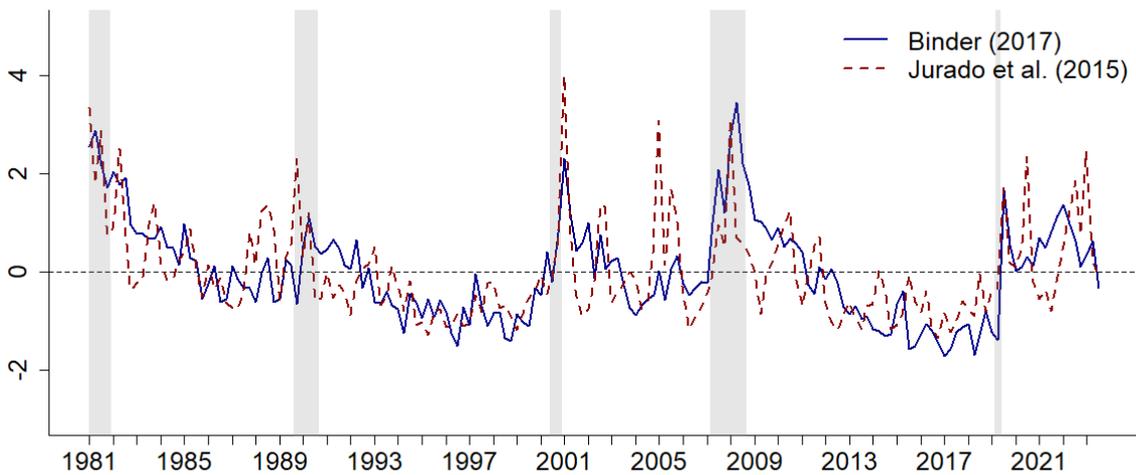


Figure 2: Inflation Uncertainty. The figure reports two alternative measures of inflation uncertainty, both based on the MSC one-year-ahead inflation expectations. The blue solid line is computed following the rounding-based method of [Binder \(2017\)](#). The second measure applies the methodology of [Jurado, Ludvigson and Ng \(2015\)](#). Both measures are normalized to have zero mean and unitary standard deviation and are taken in the second month of each quarter. Sample: 1981:Q4–2024:Q2.

In [Figure 2](#), we compare the two uncertainty measures for MSC expected inflation.

The two indexes share similar dynamics: they are countercyclical, rise sharply during episodes of macroeconomic and financial stress (September 2001 terrorist attack, the Global Financial Crisis), and comove closely over the full sample (correlation coefficient of 0.7).

For additional validation, we compare the Binder-based index with subjective probability densities from the Survey of Consumer Expectations produced by the New York Fed from 2013 onward (Manski, 2018). As shown in Appendix C, the two series exhibit a correlation of 0.86 over the overlapping sample of available time series (2013:Q1–2024:Q2), reinforcing the interpretation of rounding-based responses and forecast errors-based measures as informative indicators of household-level uncertainty.⁴

3 Impulse responses to a belief distortion shock

In this section, we estimate the belief distortion shock and study its impact on the economy both unconditionally—in a linear environment—and conditionally, in the presence of ZLB states, controlling for their interaction with inflation uncertainty. Specifically, Section 3.1 details the methodology used to estimate the belief distortion shock, including an evaluation of its correlation with a set of macroeconomic and policy shocks from the literature. Section 3.2 presents impulse responses generated by our identified belief distortion shock using a linear local projections analysis. Section 3.3 deepens the analysis by providing a nonlinear investigation on the transmission of the belief distortion shock in and out ZLB periods, highlighting also the interaction with inflation uncertainty states.

3.1 Inflation Belief Distortion shock

We derive a novel strategy for identifying belief distortion shocks. We proceed in two steps:

Step 1: Constructing the inflation belief wedge. We first compute the difference between average one-year-ahead inflation forecasts from the Michigan Survey of Consumers (MSC) and those from the Survey of Professional Forecasters (SPF), following Bhandari, Borovička and Ho (2024). Note that the SPF are fielded in the second month of each quarter, so to align the timing of our data, we take the cross-sectional mean of MSC households’ forecasts from the same month. The belief wedge is so defined as the difference in first-moment forecasts,

$$y_{m=2,t}^x = \tilde{E}_{m=2,t}(x_{t+h}) - \bar{E}_{m=2,t}(x_{t+h}) \quad (2)$$

where x_{t+h} denotes the forecasted variable (here, consumer inflation π at horizon $h = 4$),

⁴The New York Fed series is available at <https://www.newyorkfed.org/microeconomics/sce/>.

$\tilde{E}_{m=2,t}(x_{t+h})$ is the cross-sectional mean of MSC households' forecasts, and $\bar{E}_{m=2,t}(x_{t+h})$ is the corresponding SPF forecast, both formed in month $m = 2$ of quarter t . The belief wedge collapses to zero in the absence of distortions in belief formation, i.e., when $\tilde{E}_{m=2,t}(x_{t+h}) = \bar{E}_{m=2,t}(x_{t+h})$.

SPF forecasts are likely to provide informed expectations, as professionals employ more sophisticated methods to form predictions (Bonham and Dacy, 1991; Faust and Wright, 2013) and households' forecast errors are typically larger than those of professionals. However, it has been shown that even SPF forecasts deviate from full rationality due to infrequent and sluggish updating (Coibion, Gorodnichenko and Wieland, 2012). This justifies the introduction of our second step.

Step 2: Isolating the belief distortion shock. The extent to which the belief wedge can be used as a measure to identify a pure belief distortion shock depends on our ability to clean it from variations attributable to predictable or observable changes in macroeconomic variables, including fluctuations associated with the business cycle. Therefore, to isolate the portion of household beliefs that departs from a fully rational/predictable benchmark, we regress the MSC–SPF wedge on the full set of macroeconomic information contained in the FRED-MD database, employing ridge regularization.

We adopt a machine learning regularization method to predict changes in the belief wedge using a large set of regressors from the FRED-MD dataset, also controlling for inflation uncertainty.⁵ See also Aruoba and Drechsel (2025) for ridge regressions used for monetary policy causal analysis. Our approach serves to extract the component of the belief wedge that is orthogonal to observable information and inflation uncertainty. Formally, we estimate:

$$y_{m=2,t}^\pi = c + \sum_{l=0}^8 \tilde{X}_{m=1,t-l}' \beta_l + \theta_{m=2,t} \quad (3)$$

where $y_{m=2,t}^\pi$ denotes the inflation belief wedge of Equation (2), c is a constant, and $\tilde{X}_{m=1,t-l}$ is the vector of FRED-MD regressors observed in month $m = 1$ of quarter $t - l$ for l lags, with $0 \leq l \leq 8$. We include the inflation uncertainty measure we built following Binder (2017) as a control to orthogonalize our shock, defined on first-moment expectations, from its second-moment counterpart.

In this setup, the residual term, $\theta_{m=2,t}$, measured at month $m = 2$ of quarter t , captures the component of the inflation belief wedge that cannot be explained by the large set macroeconomic predictors, $\tilde{X}_{m=1,t-l}$ measured at month $m = 1$ of quarter $t - l$. The timing of the predictors—which is also our shock identifying assumption—reflects the notion that households form or revise inflation expectations using the information available

⁵See Appendix A for data description and sources. Controls include 8 lags per regressor. Non-stationary regressors are transformed into annual percentage changes. All regressors are standardized.

at the previous period. Accordingly, we condition households' inflation expectations in the second month of each quarter on predictors observed in month $m = 1$ of the same quarter. This delivers the orthogonality condition:

$$\mathbb{E}\left[\theta_{m=2,t} \mid \tilde{X}_{m=1,t}, \tilde{X}_{m=1,t-1}, \dots, \tilde{X}_{m=1,t-8}\right] = 0,$$

so that the residual isolates the component of the wedge that is not predictable from the forecast-relevant information set.

Despite this method is novel in its ability to isolate belief distortion shocks, under this information-set timing, our strategy is conceptually equivalent to a recursive identification in a VAR where $y_{m=2,t}^\pi$ is ordered last with respect to the information available: all variables contemporaneously (in information terms) enter households' forecast formation, but $y_{m=2,t}^\pi$ does not contemporaneously feed back into them. By construction, this residual is orthogonal to all publicly available information contained in the FRED-MD dataset at the time households form expectations.

We interpret $\theta_{m=2,t}$ as a belief distortion shock—that is, the component of household inflation forecasts which is not predictable from professional forecasts and publicly available information, i.e., observable fundamentals. Intuitively, this residual represents shifts in household inflation expectations that cannot be rationalized by standard information channels, such as business cycle conditions, financial variables, or past inflation dynamics, and thus reflect a structural distortion shock in the process of belief formation about future inflation.

Differently from a standard recursive identification, our approach conditions our belief shock on a broad information set. We achieve this by estimating [Equation 3](#) using a ridge regression approach.⁶ Formally, ridge regression minimizes the residual sum of squares augmented by a penalty on the squared magnitude of the coefficients:

$$L(\beta) = \|y - \tilde{X}\beta\|_2^2 + \lambda\|\beta\|_2^2 \tag{4}$$

where the hyperparameter λ controls the degree of shrinkage. When $\lambda = 0$, the constraint imposed in the minimization problem is not binding, and the solution is equivalent to OLS. As λ increases, the stronger is the shrinkage. The optimal value of λ is selected using time-series cross-validation, a standard method in the machine learning literature for validating a model's predictive ability when dealing with time-series data.⁷ Hence, the

⁶We adopt ridge regression because, unlike LASSO or elastic net, it shrinks all coefficients without performing variable selection. Conceptually, ridge regression can be viewed as a Bayesian OLS estimator with normal priors centered at zero on the coefficients. This produces a dense specification, which is more appropriate in the presence of highly correlated macroeconomic regressors ([Giannone, Lenza and Primiceri, 2021](#)).

⁷This technique preserves the temporal structure of the data by training the model on past observations and validating it on future ones. At each iteration, the training sample expands to include all

identification assumption in Equation 3 does not rely on traditional exclusion restrictions; instead, we use a data-driven ridge regression to downweight less informative predictors while capturing the relevant information for forecasting the belief wedge.

Results from the ridge regression are reported in Figure 3. Panel (a) shows that the model performs well in predicting the inflation belief wedge, with an adjusted $R^2 = 0.76$. Panel (b) shows the time series of the belief distortion shock. This is centered around zero, and its distribution is closer to normality, as shown in Panel (c). Panel (d) presents the correlograms of the inflation wedge and the belief distortion shock, demonstrating that the ridge regression effectively removes autocorrelation from the latter.

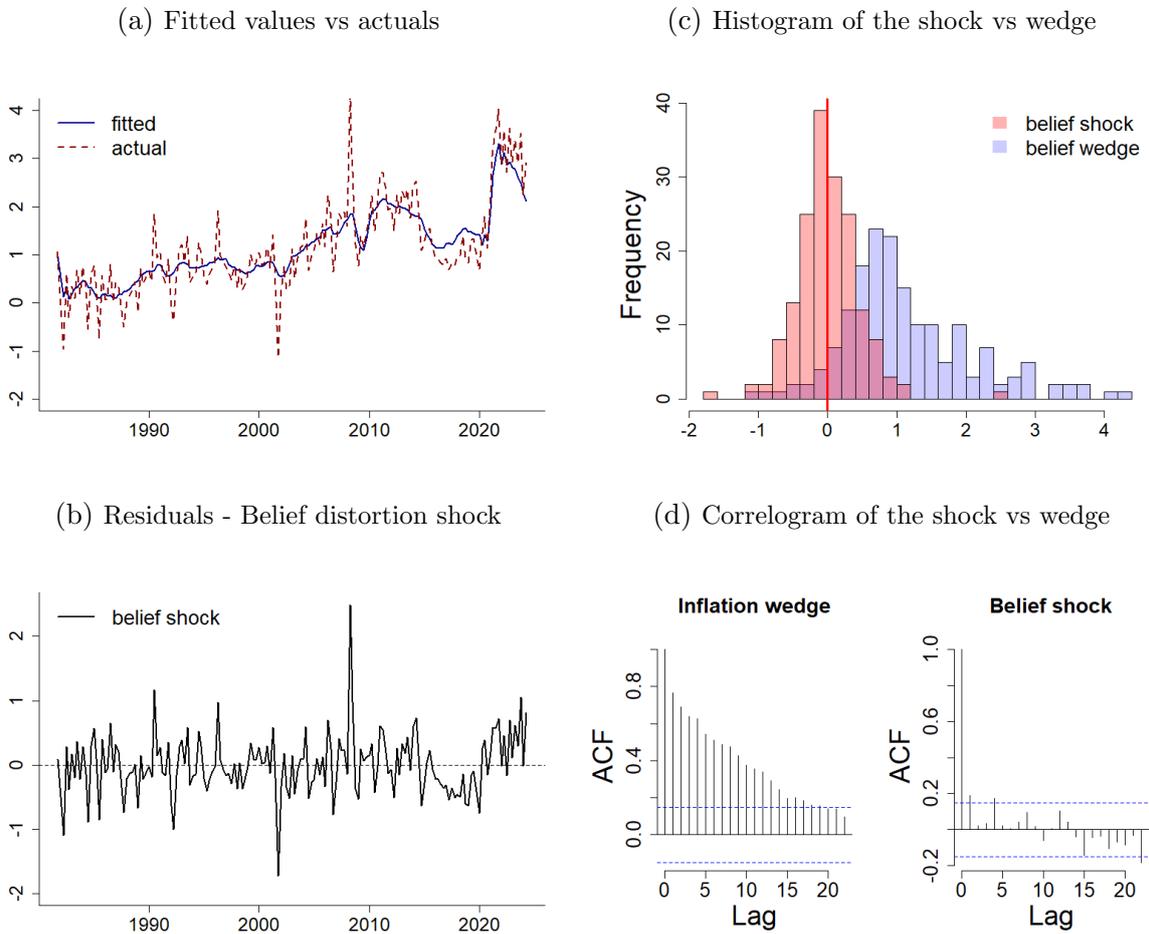


Figure 3: **Identification of the Belief distortion shock.** The figure presents histograms comparing the inflation belief wedge and the inflation belief distortion shock, with the latter being the residual of a ridge regression where the former is the dependent variable. It also displays the fitting regression performance, by comparing actual and fitted values, the estimated residuals, their distribution, and the autocorrelation functions (ACF) associated to both measures. *Source:* Michigan Survey of Consumers, Survey of Professional Forecasters, FRED-MD. *Sample:* 1981Q4–2024Q2.

observations up to the current split, while the test set moves forward. The model is then evaluated over a grid of candidate values for λ , and the one minimizing the out-of-sample RMSE is selected.

Exogeneity tests. We perform a set of exogeneity tests on the identified belief distortion shock, based on a database of identified policy and non-policy shocks from the literature. Specifically, we regress our shock measure on contemporaneous and four-lagged values of each structural innovation in the database, then test for exogeneity using Wald tests of joint significance.

Table 1: **Wald test results.**

| Innovation | Sample | Wald statistic | P-value |
|---|-----------------|----------------|---------|
| Känzig (2021) Oil price news | 1981Q4 - 2024Q2 | 10.8065 | 0.055 |
| Caldara, Cavallo and Iacoviello (2019) Oil demand | 1987Q1 - 2015Q4 | 1.9382 | 0.858 |
| Caldara, Cavallo and Iacoviello (2019) Oil supply | 1987Q1 - 2015Q4 | 7.7228 | 0.172 |
| Baumeister and Hamilton (2019) Oil demand | 1981Q4 - 2024Q2 | 9.5069 | 0.090 |
| Baumeister and Hamilton (2019) Oil inventories | 1981Q4 - 2024Q2 | 3.0312 | 0.695 |
| Baumeister and Hamilton (2019) Oil supply | 1981Q4 - 2024Q2 | 5.4147 | 0.367 |
| Baumeister (2023) Oil price expectation | 1986Q1 - 2023Q1 | 8.2507 | 0.143 |
| Francis et al. (2014) Technology shock | 1981Q4 - 2009Q4 | 7.8373 | 0.165 |
| Jarociński and Karadi (2020) Monetary policy | 1990Q1 - 2024Q2 | 6.3281 | 0.276 |
| Jarociński and Karadi (2020) Central bank information | 1990Q1 - 2024Q2 | 12.6145 | 0.027* |
| Romer and Romer (2004) Monetary policy | 1981Q4 - 2007Q4 | 3.6899 | 0.595 |
| Jarociński (2024) Standard Monetary policy | 1991Q1 - 2024Q2 | 5.9810 | 0.308 |
| Jarociński (2024) Odyssean forward guidance | 1991Q1 - 2024Q2 | 0.8691 | 0.972 |
| Jarociński (2024) Long term asset purchase | 1991Q1 - 2024Q2 | 8.2549 | 0.143 |
| Jarociński (2024) Delphic forward guidance | 1991Q1 - 2024Q2 | 4.2885 | 0.509 |
| Aruoba and Drechsel (2025) Monetary policy | 1982Q4 - 2008Q3 | 1.5629 | 0.906 |
| Miranda-Agrippino and Ricco (2021) Monetary policy | 1991Q1 - 2018Q4 | 7.2192 | 0.205 |
| Miranda-Agrippino and Ricco (2021) Information | 1991Q1 - 2018Q4 | 4.3545 | 0.500 |
| Gertler and Karadi (2015) Monetary policy | 1991Q1 - 2012Q2 | 1.7861 | 0.879 |
| Romer and Romer (2010) Tax shock | 1981Q4 - 2007Q4 | 3.3104 | 0.652 |
| Mertens and Ravn (2012) Unexpected tax liabilities changes | 1981Q4 - 2007Q4 | 2.9637 | 0.706 |
| Mertens and Ravn (2013) Personal income tax | 1981Q4 - 2006Q4 | 1.9508 | 0.856 |
| Mertens and Ravn (2013) Corporate income tax | 1981Q4 - 2006Q4 | 4.9613 | 0.421 |
| Ramey (2011) Defense news | 1981Q4 - 2013Q4 | 7.0389 | 0.218 |
| Ben Zeev and Pappa (2017) Defense news | 1981Q4 - 2007Q4 | 1.0755 | 0.956 |
| Basu and Bundick (2017) Financial uncertainty | 1987Q1 - 2014Q4 | 2.6597 | 0.752 |
| Baker, Bloom and Davis (2016) Economic policy uncertainty | 1981Q4 - 2024Q2 | 7.3743 | 0.194 |
| Jurado, Ludvigson and Ng (2015) - Financial uncertainty | 1981Q4 - 2024Q2 | 6.4347 | 0.266 |
| Jurado, Ludvigson and Ng (2015) - Real uncertainty | 1981Q4 - 2024Q2 | 3.6113 | 0.607 |
| Jurado, Ludvigson and Ng (2015) - Macroeconomic uncertainty | 1981Q4 - 2024Q2 | 5.3939 | 0.370 |

Notes: The table reports Wald test statistics for the joint significance of coefficients from regressions of the belief distortion shock on contemporaneous and four lagged values of each structural innovation considered in the database. The null hypothesis is that all coefficients are zero, indicating no contamination. * indicates significance at the 5% level.

We include the oil price news shock of Känzig (2021), the oil demand and supply shocks of Caldara, Cavallo and Iacoviello (2019), the oil shocks of Baumeister and Hamilton (2019), and the oil price expectation shock of Baumeister (2023). Technology effects are captured through the technology shock of Francis et al. (2014). For monetary policy, we include the monetary policy and Central Bank information shocks of Jarociński and Karadi (2020), the narrative-based monetary policy shock of Romer and Romer (2004), the conventional and unconventional monetary policy shocks of Jarociński (2024), the

monetary policy shock of [Aruoba and Drechsel \(2025\)](#), the monetary policy and information shocks of [Miranda-Agrippino and Ricco \(2021\)](#), and the monetary policy shock of [Gertler and Karadi \(2015\)](#). Fiscal policy is included by considering the tax shocks of [Romer and Romer \(2010\)](#), [Mertens and Ravn \(2012\)](#), and [Mertens and Ravn \(2013\)](#), as well as the defense news shocks of [Ramey \(2011\)](#) and [Ben Zeev and Pappa \(2017\)](#). Finally, we include uncertainty measures through the financial uncertainty shock of [Basu and Bundick \(2017\)](#), as well as the residuals of AR(2) processes fitted to the economic policy uncertainty index of [Baker, Bloom and Davis \(2016\)](#), and to the financial, real, and macroeconomic uncertainty indexes of [Jurado, Ludvigson and Ng \(2015\)](#).

Results are displayed in [Table 1](#). The Wald tests provide strong evidence supporting the exogeneity of our belief distortion shock, as we cannot reject the null hypothesis of joint coefficient insignificance for the vast majority of structural innovations. The only exception occurs with the central bank information shock of [Jarociński and Karadi \(2020\)](#), which shows significance at the 5% level. Nevertheless, purging our belief distortion shock from this particular innovation leaves our main results unchanged.⁸ Additionally, in [Appendix D](#), we provide additional evidence through dynamic correlation analysis, computing lead-lag correlations between our belief distortion measure and each structural innovation across a range of ± 6 quarters.

3.2 Linear Local Projections Analysis

To estimate the macroeconomic effects of an increase in inflation belief distortions, we run lag-augmented local projections ([Jordà, 2005](#); [Montiel Olea and Plagborg-Møller, 2021](#)).⁹ First, to validate the first step of our belief identification strategy - that relies on computing the households' belief wedge in deviation from the SPF forecast - we estimate the impulse responses of the difference between realized inflation and inflation expectations (forecast errors), for both MSC households and SPF professional forecasters. Then, we investigate the effects on inflation, inflation uncertainty, and the unemployment rate.

Hence, we estimate a collection of $H + 1$ forecasting regressions:

$$x_{t+h} = \beta_x(h)\tilde{\theta}_t + controls + \epsilon_{t+h}, \quad h = 0, 1, 2, \dots, H \quad (5)$$

where h is the horizon at which we run each regression, x_{t+h} is the response variable,

⁸The results remain robust also when we purge our shock from the oil news shock from [Känzig \(2021\)](#) for which we cannot reject the null at 10% significance.

⁹This approach offers several advantages over standard vector autoregressions (VARs). First, impulse responses in VARs are estimated from the Wold representation, which requires invertibility of parameter estimates—a condition that holds only if the model is correctly specified. By contrast, local projections abstract from assumptions about the data-generating process, delivering more robust estimates of impulse responses. Second, local projections are highly flexible: they conveniently accommodate nonlinearities and allow for the computation of state-dependent responses.

$\tilde{\theta}_t$ denotes the belief distortion shock, *controls* stands for the vector of control variables, ϵ_{t+h} is the residual. The sample period spans from 1981:Q4 to 2024:Q2.

When we specify $x_t = \{FE_t^{HH}, FE_t^{SPF}\}$, where $FE_t^i = \pi_{m=2,t+4} - E_{m=2,t}^i \pi_{t+4}$ indicates the inflation forecast error of agent i , as *controls*, we include a constant, up to four lags of the belief shock, and four lags of the variables $\{\hat{\Delta}_{m=2,t} \pi_{m=2,t}, u_{m=2,t}\}$, where $\hat{\Delta}_{m=2,t}$ is Binder (2017)'s measure for inflation uncertainty, $\pi_{m=2,t}$ is the annualized consumer inflation, $u_{m=2,t}$ is the unemployment rate. We also include a decaying COVID indicator that takes a value of 1 in 2020:Q2 and decreases exponentially to 0 over the subsequent quarters.

Response of Forecast Errors. First, we provide a validation exercise for our identification strategy. Specifically, we estimate the impulse responses of household (HH) and SPF forecast errors—that is, the difference between realized future inflation and HH or SPF inflation expectations, respectively. Under the assumption that professional forecasters' expectations are closer to rational expectations, the corresponding differential should exhibit little or no response to the shock. By contrast, for households, we expect the null hypothesis of no response to be rejected. Figure 4 rejects the null hypothesis that households' forecast errors do not respond. The response is negative on impact and remains so for about one year, indicating that households tend to overestimate inflation and that the shock indeed reflects an upward distortion in their inflation beliefs. By contrast, the response of SPF forecast errors is small and closer to zero, consistent with the view that professional forecasters form better-informed expectations using more sophisticated methods, though still not fully in line with FIRE models.

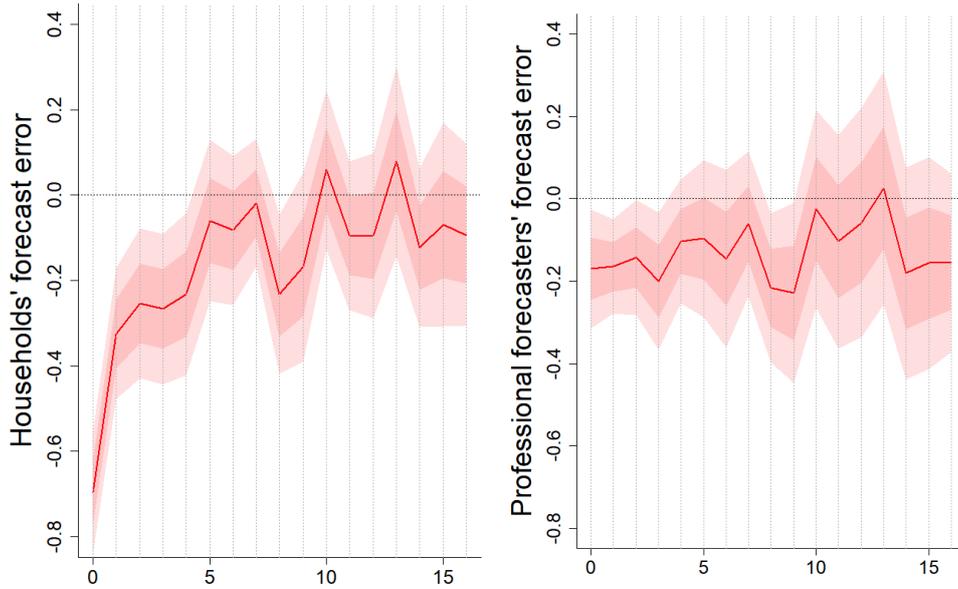


Figure 4: **Impulse responses of MSC and SPF forecast errors to a belief distortion shock in linear local projections.** The figure shows the impulse responses of households and professional forecasters inflation forecast error to a one standard deviation belief distortion shock. They are estimated by lag-augmented local projections. The bands show the 90% and 68% confidence intervals based on Newey-West standard errors. *Sample:* 1981Q4-2024Q2.

Response of Inflation Uncertainty, Inflation and Unemployment. Next, we compute the responses of our main variables of interest, that is: inflation uncertainty, actual inflation and unemployment to a belief distortion shock, that is when we specify $x_t = \{\hat{\Delta}_{\bar{m}_{2,3,t}}, \pi_{\bar{m}_{2,3,t}}, u_{\bar{m}_{2,3,t}}\}$, for the dependent variable we take the quarterly average excluding the first month, ensuring consistency with the identification restrictions we imposed in Section 3.1.¹⁰ As controls, we re-use those of the previous exercise by setting $m = \bar{m}_{2,3}$.

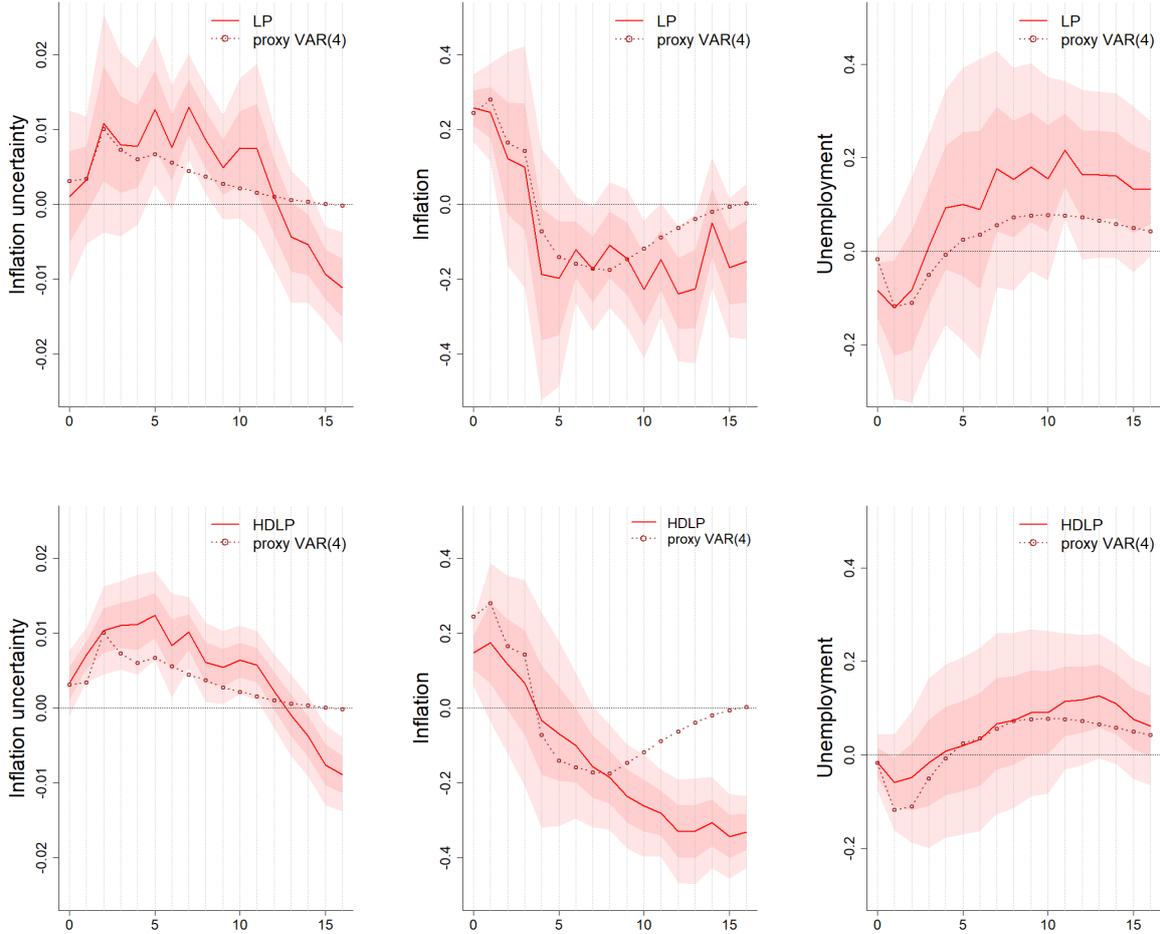


Figure 5: Impulse responses of inflation uncertainty, inflation, and unemployment to a belief distortion shock in linear local projections. The figure shows the responses of inflation uncertainty, inflation, and unemployment to a one standard deviation belief distortion shock. The top row reports results from lag-augmented local projections, while the bottom row shows results from a one-step high-dimensional local projection (HDLP) approach (Adamek, Smeekes and Wilms, 2024). The bands show the 90% and 68% confidence intervals based on Newey-West standard errors. For comparison, we also report responses obtained from a recursive proxy-internalized VAR(4). *Sample:* 1981Q4-2024Q2.

We test the robustness of the lag-augmented local projection analysis with two exercises. First, we specify a Proxy VAR(4) in the vector of variables $\hat{y}'_t = (\tilde{\theta}_t, \hat{\Delta}_t, \pi_t, u_t)'$, imposing a recursive structure (Bruns and Lütkepohl, 2022).¹¹ Hence, local projections and the Proxy-VAR are specified in order to have an equivalent lag structure (Plagborg-Møller

¹⁰Taking the dependent variables in the second month of the quarter conveys similar results.

¹¹According to Plagborg-Møller and Wolf (2021), the advantage of this approach over the equivalent

and Wolf, 2021). Second, we study impulse responses obtained from a one-step high-dimensional local projection (HDLP) approach (Adamek, Smeekes and Wilms, 2024). In this framework, the response variable is directly regressed on the belief wedge, while controlling for the large set of regressors used in the ridge regression, including the eight lags of each regressor. This allows us to estimate impulse responses in a high-dimensional setting without relying on a two-step procedure, thereby providing a robustness check on our baseline results.¹²

Figure 5 shows the results. A belief distortion shock significantly and persistently raises households' inflation uncertainty. Inflation surges on impact but quickly reverts, partly reflecting the contractionary effects triggered by the ensuing recession. Unemployment remains muted on impact but increases persistently after a few periods, displaying the recessionary nature of the belief distortion shock.

These dynamics are fairly robust across identification strategies. As for the Proxy VAR, our evidence is consistent with Plagborg-Møller and Wolf (2021), especially at short horizons. As for the high-dimensional local projections, results also display similar patterns, confirming the medium run recessionary consequences of belief distortion shocks when analyzed within linear specifications.

3.3 State-dependent Local Projections Analysis

Belief distortions—defined as the wedge between households' inflation expectations and those of professional forecasters, as shown in the upper panels of Figure 1—exhibit significant temporal variation, with notable spikes during the Global Financial Crisis and the post-COVID19 period, when both inflation was increasing and the U.S. economy reached the zero lower bound (ZLB) for nominal interest rates in response to the unfolding impact of the crises. We here verify whether the recessionary effects caused by the belief distortion shock—raising uncertainty and unemployment while exerting temporary upward pressure on inflation—, as observed in the linear local projections in Section 3.2, may hide underlying nonlinearities and highlight crisis-specific dynamics.

3.3.1 Inflation Uncertainty and ZLB states

We first extract states based on the prevailing level of inflation uncertainty. We define states of high and low inflation uncertainty using two alternative measures: the rounded-based index of Binder (2017) and the forecast error-based measure of Jurado, Ludvigson

SVAR-IV method of Stock and Watson (2018), besides its simplicity, is that the estimated impulse responses are valid even if the shock of interest is non-invertible.

¹²Lloyd and Manuel (2024) show that the widely used two-step approach for estimating impulse response functions—in which we identify the shock externally and then estimates the response by regressing the variable of interest on the identified shock—may, in some circumstances, generate inflated confidence bands and biased responses. We adopt a one-step procedure to mitigate these concerns.

and Ng (2015), alternatively. Following Falck, Hoffmann and Hürtgen (2021), we smooth the inflation uncertainty measure using a seven-period backward-looking weighted moving average filter and scale the smoothed series by average inflation expectations to ensure comparability across periods of elevated and subdued inflation.¹³

The transition between states of high and low inflation uncertainty is governed by probabilities defined by a logistic function $S(\hat{\Delta}_{t-1}) \in [0, 1]$ of the lagged uncertainty measure, defined as:

$$S(\hat{\Delta}_{t-1}) = \frac{\exp\left(\eta \frac{\hat{\Delta}_{t-1} - \mu}{\sigma_{\hat{\Delta}}}\right)}{1 + \exp\left(\eta \frac{\hat{\Delta}_{t-1} - \mu}{\sigma_{\hat{\Delta}}}\right)} \quad (6)$$

where η determines the steepness of the transition—hence, how strongly the state probabilities react to changes in inflation uncertainty—while μ and $\sigma_{\hat{\Delta}}$ represent the median and standard deviation of the state variable, respectively.¹⁴ As a robustness check, we also consider a simpler regime classification based on a threshold rule: the economy is in the high-uncertainty state $S(\cdot)$ with probability 1 whenever $\hat{\Delta}_{t-1} > 0$, and 0 otherwise.

Figure 6 plots—on the left axis—the probabilities of being in the high inflation uncertainty state when using Binder (2017)’s measure.¹⁵ The smooth transition probabilities are compared with those generated by the threshold rule, together with states when the nominal interest rates hit the ZLB. On the right axis, we plot the smoothed inflation uncertainty state variable.

Probabilities of being in the high inflation uncertainty state exhibit a clear time-varying pattern: they are high in the first part of the sample up to the early 1990s, during the 2008–09 crisis, persist for a few years thereafter, and rise again with the most recent post-COVID inflationary surge. Unsurprisingly, inflation uncertainty—in blue solid line in the upper panel—follows a similar pattern. Notably, the last two high-inflation uncertainty episodes—coinciding with the Global Financial Crisis and the recovery following the post-COVID19 pandemic—partially align with periods when the federal funds rate reached the ZLB, limiting Central Bank operations.

Indeed, if we consider the two ZLB periods realizing in the sample period, our belief distortion shock displays clear evidence of state dependency in its conditional first moments. Notably, as reported in Table 2, the conditional mean of the belief shock is on average close to zero in normal times, when the interest rate is unconstrained; and it is significantly positive during ZLB periods—particularly in the post-COVID period, signaling that households becomes pessimistic about inflation in period of crisis and overreact with respect to non-crisis period.

¹³We thus lose the first seven observations, and the state-dependent local projections are estimated over 1983:Q2–2024:Q2.

¹⁴As in Falck, Hoffmann and Hürtgen (2021), we set $\eta = 5$.

¹⁵Similar results are obtained by using the Jurado, Ludvigson and Ng (2015)’s measure.

In contrast, professional forecasters may anchor their expectations to medium-term dynamics, beyond the immediate impact of expansionary shocks, remaining more conservative in revising their inflation expectations upward during ZLB periods. The different impact of anticipated measures on households versus professional forecasters may help explain the positive mean of the belief distortion shock during ZLB episodes.

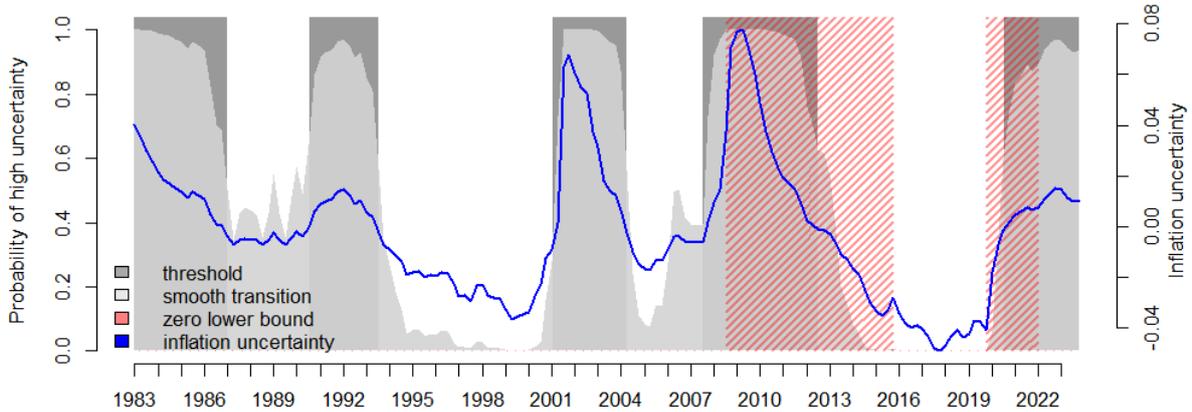


Figure 6: **Inflation Uncertainty and ZLB states.** The figure displays the estimated probability of the high inflation uncertainty state, based on the measure by [Binder \(2017\)](#), and as defined by the smooth transition logistic function (light gray areas) and the threshold rule (dark gray areas). The blue line denotes the smoothed measure of inflation uncertainty, while red shaded regions indicate ZLB periods. Sample: 1983Q2–2024Q2.

Table 2: **Conditional Moments of the Belief distortion shock.**

| Normal Times | ZLB Episodes | ZLB Great Recession | ZLB COVID-19 |
|---------------|---------------|---------------------|---------------|
| 1981Q4–2024Q2 | 1981Q4–2024Q2 | 2009Q2–2016Q1 | 2020Q3–2022Q2 |
| -0.06 | 0.32 | 0.21 | 0.68 |

Notes: Conditional means of the belief distortion shock across different interest rate regimes. Column (1) reports the mean during normal times - out of ZLB. Columns (2)–(4) report averages during episodes when the U.S. economy was at the ZLB: all ZLB periods, the Great Recession episode (2009Q2–2016Q1), and the Post-COVID episode (2020Q3–2022Q2). *Sample:* 1983Q2–2024Q2.

When the economy is constrained at the ZLB, an increase in inflation expectations—such as the one caused by the belief distortion shock—leads to a decrease in the ex-ante real interest rate, stimulating the economy and generating an expansionary effect. In such a case, we would observe an increase in inflation and a decrease in unemployment due to the interest rate channel. This result speaks also with [Coibion, Gorodnichenko and Ropele \(2020\)](#), who provide micro-level evidence of the possibility of expansionary effects from rising inflation expectations at the ZLB. The authors use firm survey data to document that firm profitability increased during the ZLB period. This result is also in line with the work of [Bianchi and Melosi \(2017, 2019\)](#), who propose fiscal strategies aimed at raising inflation expectations to lift the economy out of the ZLB, resulting in negative real rates, higher inflation, and lower debt accumulation.

Given the potential impact of higher inflation expectations at the ZLB, we test whether the transmission of the belief distortion shock exhibits state dependence. However, as [Figure 6](#) shows, ZLB episodes are also characterized by high inflation uncertainty. Hence, we run local projections, which depart from the linear specification in [Equation 5](#), and resort to nonlinear specifications to study the effects of the belief distortion shock across different states—within and outside the ZLB—while conditioning on the level of inflation uncertainty.

3.3.2 State-dependent local projections

We run state-dependent local projections by incorporating both the probability of high inflation uncertainty, $S(\hat{\Delta}_{t-1})$, as specified in [Figure 6](#), and a state variable defined as $Z(FFR_{t-1})$, which takes the value 1 when the federal funds rate, FFR_{t-1} , is constrained at the zero lower bound, and 0 otherwise.¹⁶ We label the two states as *ZLB* and *Normal Times*, respectively.

The baseline specification of the state-dependent local projections, labeled as *Baseline*, are given by the following set of regressions for each horizon $h = 0, 1, 2, \dots, H$:

$$\begin{aligned}
 x_{t+h} = & \left[\beta_x^Z(h) \tilde{\theta}_t + \text{controls}(x, Z) + \sum_{l=0}^j \gamma_{\text{int}, t-l}^Z \tilde{\theta}_{t-l} S(\hat{\Delta}_{t-1-l}) \right] Z(FFR_{t-1}) \\
 & + \left[\beta_x^{\bar{Z}}(h) \tilde{\theta}_t + \text{controls}(x, \bar{Z}) + \sum_{l=0}^j \gamma_{\text{int}, t-l}^{\bar{Z}} \tilde{\theta}_{t-l} S(\hat{\Delta}_{t-1-l}) \right] \bar{Z}(FFR_{t-1}) \quad (7) \\
 & + \epsilon_{t+h}, \quad \text{with } \bar{Z} \equiv 1 - Z(FFR_{t-1})
 \end{aligned}$$

Consistent with the linear specification, h denotes the horizon at which each regression is run, x_{t+h} is the response variable, $\tilde{\theta}_t$ represents the belief distortion shock, and ϵ_t is the forecast error. As *control* variables, we include a state-specific constant, the COVID dummy, and lagged values of both the shock and the response variable. For the sake of tractability, in the baseline specification, we avoid including lagged values of additional variables beyond the one for which we estimate the response to the shock, x .¹⁷ We allow the number of lags for the control variables to be both variable- and state-dependent, as determined by the Akaike Information Criterion.

In the state-dependent local projections of [Equation \(7\)](#), the approach results in a triple interaction term in each state.¹⁸ This term captures the effect of inflation un-

¹⁶We use the smooth transition probability as a benchmark to capture variation in inflation uncertainty within ZLB states. Using a threshold-based state variable for inflation uncertainty delivers similar responses.

¹⁷We verify that the empirical findings are robust to the inclusion of additional controls, specifically the variables used in the lag-augmented linear local projection in [Section 3.2](#).

¹⁸ $\tilde{\theta}_t S(\hat{\Delta}_{t-1}) Z(FFR_{t-1})$ and $\tilde{\theta}_t S(\hat{\Delta}_{t-1}) \bar{Z}(FFR_{t-1})$, respectively.

certainty on the dependent variable when the shock occurs at the ZLB or in Normal Times.

To assess the recessionary impact and labor market disruptions induced by the belief distortion shock, as indicated by the linear local projection in [Section 3.2](#), we estimate the state-dependent local projections of [Equation \(7\)](#) for unemployment only.¹⁹

The first two panels of [Figure 7](#) show unemployment responses to the belief shock (sequence of β_x , panel a) and to the triple interaction term (sequence of γ_{int} , panel b). Results based on [Binder \(2017\)](#)'s uncertainty measure appear in the top row, and those based on [Jurado, Ludvigson and Ng \(2015\)](#) in the bottom row. Responses are state-dependent: blue responses denote Normal Times and red ones denote ZLB periods.

In Normal Times, the unemployment response to the belief distortion shock—captured by $\beta^{\bar{Z}}x$, panel a)—is positive, consistent with the linear local projection in [Section 3.2](#) and indicative of adverse effects on the economy. At the ZLB, however, the response—captured by β^Zx , panel a)—reverses sign, becoming negative and larger in magnitude, implying labor market gains when households form higher inflation expectations. This result holds for the pure effect of the shock, i.e., net of the influence of elevated inflation uncertainty.

In contrast, the unemployment response to the interaction term exhibits opposite dynamics across states. At the ZLB, the contribution of inflation uncertainty—captured by $\gamma^Z\text{int}$, panel b)—is positive and statistically significant, indicating that uncertainty attenuates the expansionary effect of the belief distortion shock. In Normal Times, by contrast, the response—captured by $\gamma^{\bar{Z}}\text{int}$, panel b)—is negative but not significant.

Our empirical evidence supports the interpretation of belief distortion shocks under the ZLB as expansionary policy shocks, as perceived by households ([Bianchi, Faccini and Melosi, 2023](#)). Given the distressing times during which they occur, belief distortion shocks raise inflation expectations, but high inflation uncertainty dampens their expansionary effect, as reflected in the positive unemployment response to the interaction term.

Based on the evidence obtained from the state-dependent local projections of [Equation \(7\)](#), we test our findings using an alternative specification, which we label as *Misspecified*. This specification is identical to the *Baseline* but omits the triple interaction term among the belief distortion shock, the probability of high inflation uncertainty, and the state of the economy. The *Misspecified* specification is given by the following set of regressions for each horizon $h = 0, 1, 2, \dots, H$:

¹⁹The Akaike Information Criterion selects 4 lags for the shock, 2 lags for the dependent variable, and $j = 3$ lags for the interaction term in the ZLB state (Z); in Normal Times (\bar{Z}) it selects 1 lag for the shock, 4 lags for the dependent variable, and $j = 1$ lag for the interaction term.

$$\begin{aligned}
x_{t+h} = & \left[\tilde{\beta}_x^Z(h) \tilde{\theta}_t + \text{controls}(x, Z) \right] Z(FFR_{t-1}) \\
& + \left[\tilde{\beta}_x^{\bar{Z}}(h) \tilde{\theta}_t + \text{controls}(x, \bar{Z}) \right] \bar{Z}(FFR_{t-1}) \\
& + \epsilon_{t+h}, \quad \text{with } \bar{Z} \equiv 1 - Z(FFR_{t-1})
\end{aligned} \tag{8}$$

where the terms in the regression equation are the same as in the *Baseline*; however, the control variables no longer include any lags of the interaction term, as this is not considered here. Panel c) in [Figure 7](#) shows the response of unemployment to the belief shock under the *Misspecified* state-dependent local projections—represented by the sequence of coefficients $\tilde{\beta}_x$. The responses of unemployment in ZLB and Normal Times follow a pattern similar to those in panel a), which depicts the *Baseline* specification’s responses via the sequence of coefficients β_x from [Equation \(7\)](#). However, while the positive response during Normal Times remains comparable to that in panel a), the negative response under ZLB conditions is notably attenuated in the *Misspecified* case—shifting from approximately -1.5 in the *Baseline* to slightly above -0.4 .

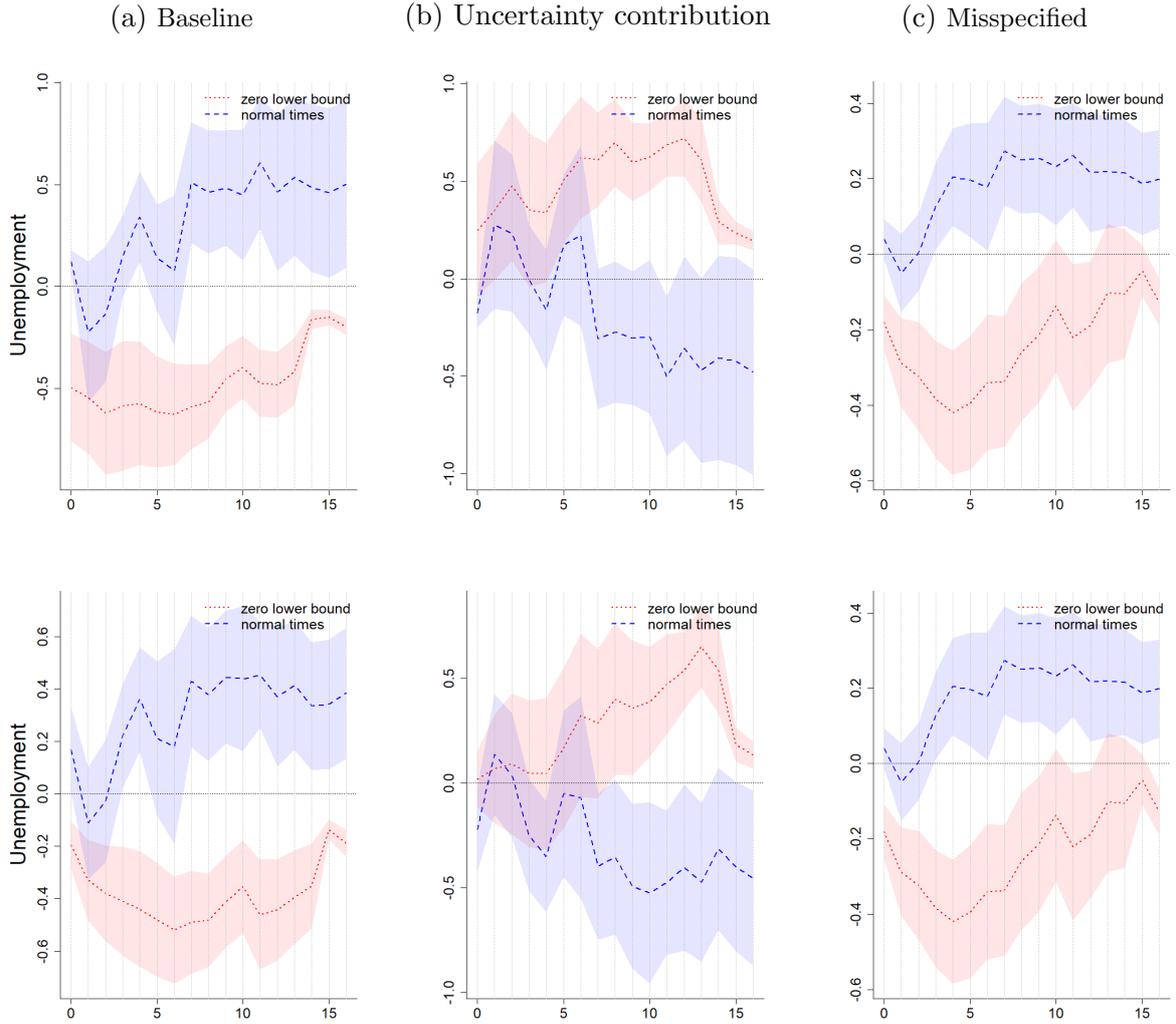


Figure 7: Unemployment state-dependent response to a belief distortion shock. Panels (a) and (b) report results from the *baseline* local projections in Equation (7), while panel (c) is based on the *misspecified* specification in Equation (8). Responses in the top row are based on the inflation uncertainty measure of Binder (2017), while responses in the bottom row are based on the measure of Jurado, Ludvigson and Ng (2015). Estimation follows the lag-augmentation method of Montiel Olea and Plagborg-Møller (2021), with lag length selected via AIC. Shaded bands represent 68% confidence intervals based on Newey-West standard errors. *Sample: 1983Q3–2024Q2.*

These findings underscore the importance of isolating the effect of the belief distortion shock from the effects of higher inflation uncertainty under ZLB conditions. During periods of economic distress, inflation uncertainty exerts a disruptive effect on unemployment, and the expansionary impact of the belief distortion shock cannot be fully appreciated without disentangling the two.

To provide economic reasoning behind the expansionary effect of the belief distortion shock at the ZLB, when controlling for inflation uncertainty, we examine the state-dependent transmission of the shock to additional variables. We focus on the vacancy rate and the Shiller price-to-earnings ratio (CAPE), which serves as a measure of firm profitability.²⁰ The vacancy rate helps assess whether dynamics similar to those observed

²⁰We retrieve data on the vacancy rate provided by Barnichon (2010). As for the Shiller price-to-

for unemployment are common to the broader labour market. In contrast, tracking the response of a profitability measure provides insight into state-contingent firm behavior. [Figure E.6](#) in [Appendix E.2](#) shows the impulse responses obtained from the two state-dependent local projection specifications, as for unemployment in [Figure 7](#).

The results for both the vacancy rate and the CAPE ratio align with those for unemployment under both the *Baseline* and *Misspecified* specifications. These results indicate expansionary effects when the pure contribution of the belief distortion shock is considered. However, the interaction between the shock and inflation uncertainty under ZLB conditions generates recessionary effects, confirming the detrimental implications of high inflation uncertainty during times of economic distress.

Finally, the *Misspecified* state-dependent local projections deliver results for vacancy and CAPE that are consistent with those for unemployment: the effect of the belief distortion shock is state-dependent and positive under ZLB conditions, though smaller than in the *Baseline* specification, as the contribution of inflation uncertainty is not disentangled in the alternative local projections equation.

3.3.3 Validity of State-dependent Local Projections

Recently, [Gonçalves et al. \(2024\)](#) raised concerns about endogeneity and the potential bias of impulse response functions in the context of state-dependent local projections. Moreover, the same contribution highlights the role of large shocks as a leading factor driving the results.

We address these concerns in three steps. First, we note that while the probabilities of high-inflation uncertainty states tend to increase with inflation, there is significant co-movement between these probabilities and the identified belief distortion shock. Thus, in our case, the local projection estimator may still yield marginal impulse responses that accurately approximate average conditional responses. Second, we standardize the empirical shock series entering the state-dependent local projection. This ensures that the magnitude of the shock, relative to the standard deviation of the empirical series, remains small. Third, as a preliminary analysis in a nonlinear framework, we test the contribution of large shocks by running a nonlinear local projections in the spirit of [Caravello and Martinez-Bruera \(2024\)](#), to evaluate whether larger shocks may have disproportionately different effects.²¹

earnings ratio (CAPE), it is computed as the ratio of the S&P500 index to average inflation-adjusted earnings over the past 10 years, reflects the price investors are willing to pay per dollar of average inflation-adjusted earnings, implicitly incorporating market expectations of future profitability. Data on the Shiller CAPE ratio are freely available in the Shiller database. Source: <https://shillerdata.com/>.

²¹[Caravello and Martinez-Bruera \(2024\)](#) demonstrate that, under the assumption of symmetric shock distributions—which we verify holds for the belief distortion shock—introducing nonlinear transformations within the local projection framework effectively captures both size and sign asymmetries. Specifically, even-order nonlinear transformations capture sign asymmetries, while odd-order nonlinear transformations capture size asymmetries.

Figure E.5 in Appendix E compares the impulse response functions obtained from nonlinear but state-independent local projections. The results show that no size asymmetry in the transmission of the shock prevails in our case, further alleviating concerns that large shocks may bias the results. However, we do observe sign asymmetry in the transmission of the shock. Specifically, positive shocks are relatively stronger than negative shocks for real variables such as unemployment. This finding aligns with our interpretation of positive—and potentially relevant—shocks as those occurring during ZLB periods, as indicated by the conditional mean of our shock series.

4 Theoretical Model

The model considered is a basic New Keynesian model featuring search and matching frictions (SAM) in the labour market, labelled as NKSAM and modified to embed belief distortion shocks. The economy is populated by households, firms, and a monetary policy authority. Households consist of a continuum of worker members. They consume a basket of differentiated retail-goods and their consumption is characterized by internal habits formations. They own a continuum of firms, each of which uses one worker to produce an intermediate-good under monopolistic competition and flexible prices. The labour market is characterized by search and matching frictions. In each period, a fraction of workers is unemployed and searches for jobs. Firms post vacancies at a fixed cost. The number of successful matches is produced with a Cobb-Douglas matching technology. Real wages are determined by Nash bargaining between firms and workers. Real wages are however sticky and adjust slowly to their Nash optimal value. The government finances workers' unemployment benefits through lump-sum taxes. Retail sector firms compete under monopolistic competition and set their prices using quadratic Rotemberg (1982) adjustment costs. Finally, the monetary policy is described by a standard Taylor rule, where the nominal interest rate responds to deviations of inflation from their long-run target.

The model equations are reported in Table 3. We now present a very brief description of the model, underlying the way in which we model inflation expectations and belief distortion shocks.

Inflation Expectations and Belief Distortion Shocks The primary shock analyzed in the model economy is a shock to inflation expectations. In the spirit of Adams and Barrett (2024) and Ascari et al. (2023) we slightly deviate from the standard rational expectation assumption by positing that agents' forecast for the next period, Π_{t+1}^e , follows the rational expectation hypothesis but is subject to a shock, as:

$$\Pi_t^e \equiv E_t[\Pi_{t+1}]e^{\varepsilon_{bd,t}}, \quad (9)$$

with $E_t[\Pi_{t+1}]$ being the rational expectation in t of $t+1$ gross inflation and $\varepsilon_{\pi,t}$ being an exogenous process that allows inflation expectations to deviate from their rational expectation solution. By taking the logs of the previous equation and subtracting the agent expectation from the rational expectation we get a measure of the belief distortion, BD, which is entirely exogenous and function of the shock:

$$BD = \pi_t^e - E_t\pi_{t+1} = \varepsilon_{bd,t}, \quad (10)$$

The belief distortion shock has zero mean and a constant standard deviation equal to σ_{bd} :

$$\varepsilon_{bd,t} = \rho_{bd}\varepsilon_{bd,t-1} + \sigma_{bd}u_{\pi^e,t}. \quad (11)$$

Endogenous Inflation Uncertainty A novel part of our analysis is that we focus on how shocks to inflation expectations affect uncertainty around agents' expected inflation in the model. To this scope, as in [Basu and Bundick \(2017\)](#) and [Mumtaz and Theodoridis \(2020\)](#), we alternatively use the term volatility of expected variables or measured uncertainty to refer to the heteroskedastic response of a variable. In the model, we define the inflation uncertainty as:

$$\hat{\sigma}_{\pi,t} = 100 \log \left(\frac{\sigma_{\pi,t}}{\sigma_{\pi}} \right), \quad \text{where} \quad \sigma_{\pi,t} = \text{var}_t(\pi_t) = E_t[\pi_{t+1} - E_t\pi_{t+1}]^2, \quad (12)$$

Despite our focus on the belief distortion shock, we aim to examine the transmission of this shock under both normal times and distress periods, characterized by the zero-lower bound constraint in the interest rate. For this reason, we also introduce a risk premium shock, as specified in [Smets and Wouters \(2007\)](#). This shock allows the economy to enter a recession and enables monetary policy to reach the zero lower bound.

In particular, we denote the risk premium by $\varepsilon_{rp,t}$, which has a zero mean and a constant standard deviation equal to σ_{rp} . The AR(1) process characterizing the risk premium shock is presented in [Table 3](#), along with the other equations describing the model.

Model Calibration The model is calibrated using standard values from the literature. The discount factor ($\beta = 0.99$) is set accordingly, while the intertemporal elasticity of substitution is unitary. The steady-state unemployment rate is $U = 0.064$ close to the mean value in the US over the sample period considered in the empirical analysis. Labor market parameters include a vacancy filling probability ($q^v = 0.7$), a Nash bargaining weight ($\chi = 0.5$), and a real wage rigidity parameter ($\gamma = 0.8$). The separation rate is set at $\rho = 0.1$, while vacancy posting costs are given by $\kappa = 0.02(Y/v) = 0.14$. The elasticity of substitution among varieties is $\eta = 6$, and price adjustment costs are $\Omega_P = 50$, which

corresponds to a probability of resetting price every three quarters. Monetary policy follows a Taylor rule with inflation weight ($\rho_{\Pi} = 1.5$). The steady-state gross inflation rate is $\Pi = 1.005$.

For shock processes, the autoregressive parameter is set to 0.9 for both shocks ($\rho_{\pi^e} = \rho_{\varepsilon} = 0.9$) and the steady-state standard deviations to 0.01 ($\sigma_{\pi^e} = \sigma_{\varepsilon} = 0.01$).

Table 3: **Model Equations**

| Description | Equation |
|------------------------------|--|
| Euler Equation | $\frac{1}{R_t} = \beta e^{\varepsilon_{rp,t}} E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \frac{1}{\Pi_t^e} \right]$ |
| Nash Bargaining Real Wage | $w_t = \chi \left(q_t Z_t + (1 - \rho) \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\kappa v_{t+1}}{u_{t+1}} \right) + (1 - \chi) \left(\frac{\chi}{\Lambda_t} + \phi \right)$ |
| Production Function | $Y_t = Z_t N_t$ |
| Resource Constraint | $\left(1 - \frac{\Omega_P}{2} \left(\frac{\Pi_t}{\Pi} - 1 \right) \right)^2 Y_t = C_t + \kappa v_t$ |
| Phillips Curve | $q_t = \frac{\eta-1}{\eta} + \frac{\Omega_P}{\eta} \left(\frac{\Pi_t}{\Pi} - 1 \right) \frac{\Pi_t}{\Pi} - \frac{\Omega_P}{\eta} \beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \frac{Y_{t+1}}{Y_t} \left(\frac{\Pi_t^e}{\Pi} - 1 \right) \frac{\Pi_t^e}{\Pi} \right]$ |
| Job Creation Condition | $\frac{\kappa}{q_t^v} = q_t Z_t - w_t + (1 - \rho) \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\kappa}{q_{t+1}^v} \right\}$ |
| Matching Function | $m_t = \mu u_t^\alpha v_t^{1-\alpha}$ |
| Job Finding Probability | $q_t^u = \frac{m_t}{u_t}$ |
| Vacancy Filling Probability | $q_t^v = \frac{m_t}{v_t}$ |
| Employment Law of Motion | $N_t = (1 - \rho) N_{t-1} + m_t$ |
| Unemployment Rate | $U_t = 1 - N_t$ |
| Job Searcher Population | $u_t = 1 - (1 - \rho) N_{t-1}$ |
| Risk Premium Shock | $\varepsilon_{rp,t} = \rho_{rp} \varepsilon_{rp,t-1} + \sigma_{rp} u_{b,t}$ |
| Belief Distortion Shock | $\varepsilon_{bd,t} = \rho_{bd} \varepsilon_{bd,t-1} + \sigma_{bd} u_{bd,t}$ |
| Government Budget Constraint | $(1 - N_t) \phi = T_t$ |
| Taylor Rule | $\frac{R_t}{R} = \max \left(1, \left(\frac{\Pi_t}{\Pi} \right)^{\rho_{\Pi}} \right)$ |

4.1 Impulse Response Functions

In this section, we take the third-order approximation of the model economy and show the impulse response function of unemployment, inflation, and inflation uncertainty to a belief distortion shock that hits the economy in normal times and in a recessionary scenario, where the Central Bank faces the ZLB constraint. In the latter case, we hit the economy by a belief distortion shock together with a sequence of risk premium shocks that drag the economy into a liquidity trap for several periods.²²

Figure 8 shows the contribution of a belief distortion shock in both normal times and during the ZLB scenario. As for the empirical findings, the response of the unemployment rate is state-dependent. During normal times, the belief distortion shock behaves like a negative supply shock: it is both inflationary and recessionary, leading to higher inflation uncertainty and increased unemployment. Although the shock remains inflationary when

²²To solve the model in a stochastic environment, we use Dynare and the OBC toolbox developed by Tom Holden. See <https://github.com/tholden/dynareOBC>.

the economy is at the ZLB, the increase in inflation uncertainty is significantly higher than in normal times, while the unemployment response turns negative, resulting in an economic boom. Therefore, unemployment follows a similar state-dependent dynamic as observed in the data, confirming the model’s ability to replicate the empirical evidence described in [Section 3.2](#).

The responses to the belief distortion shock under the ZLB in [Figure 8](#) are obtained by isolating its contribution from that of the recessionary risk-premium shocks that drive the economy to the ZLB. To enhance clarity, we generate impulse response functions (IRFs) for inflation uncertainty, inflation, and unemployment in response to a risk-premium shock that brings the nominal interest rate to the ZLB, as well as IRFs for the same shock combined with a one standard deviation increase in the belief distortion shock. The responses to the belief distortion shock under the ZLB, shown in [Figure 8](#), are then derived as the difference between the IRFs generated by both shocks together and those generated by the risk-premium shock alone.

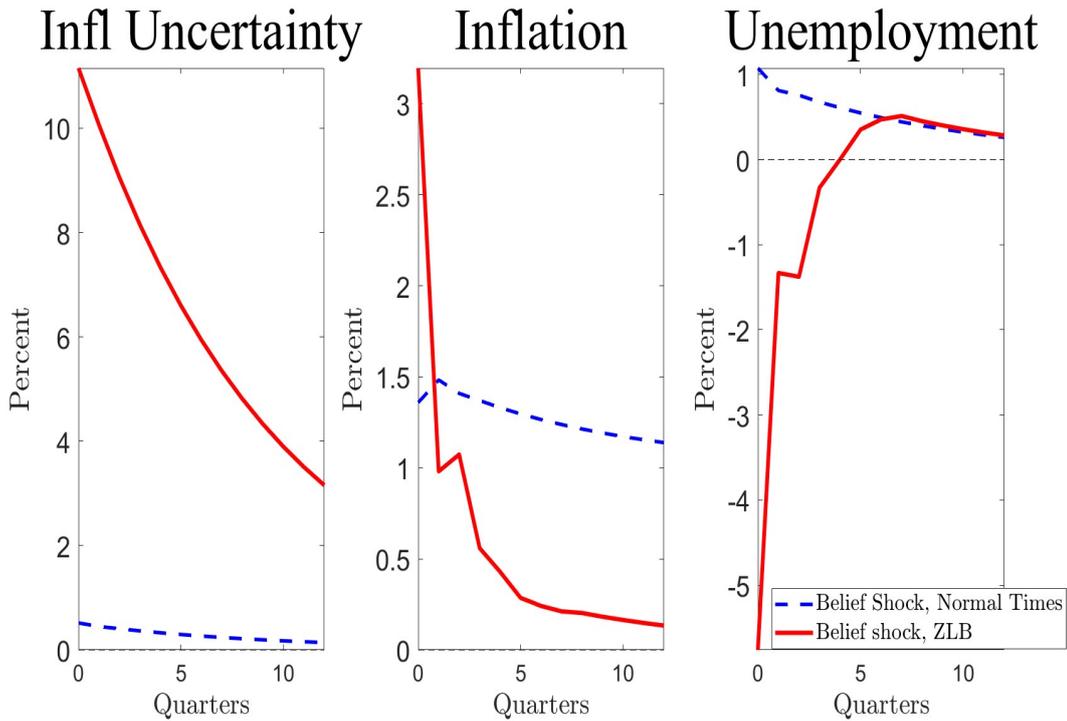


Figure 8: **Model Dynamics.** Simulated responses to a one standard deviation in *belief distortion shock* in Normal Times (blue dashed lines) and in ZLB (red solid lines).

[Figure 9](#) presents the responses of unemployment and the nominal interest rate to the risk-premium shock that drives the economy to the ZLB, with (solid lines) and without (dashed lines) the additional positive belief distortion shock. The belief distortion shock mitigates the severity of the recession, as indicated by the smaller increase in unemployment (left panel). Overall, the belief distortion shock supports an earlier exit from the ZLB, as reflected in the nominal interest rate remaining at the lower bound for fewer

periods when the shock is present (right panel).

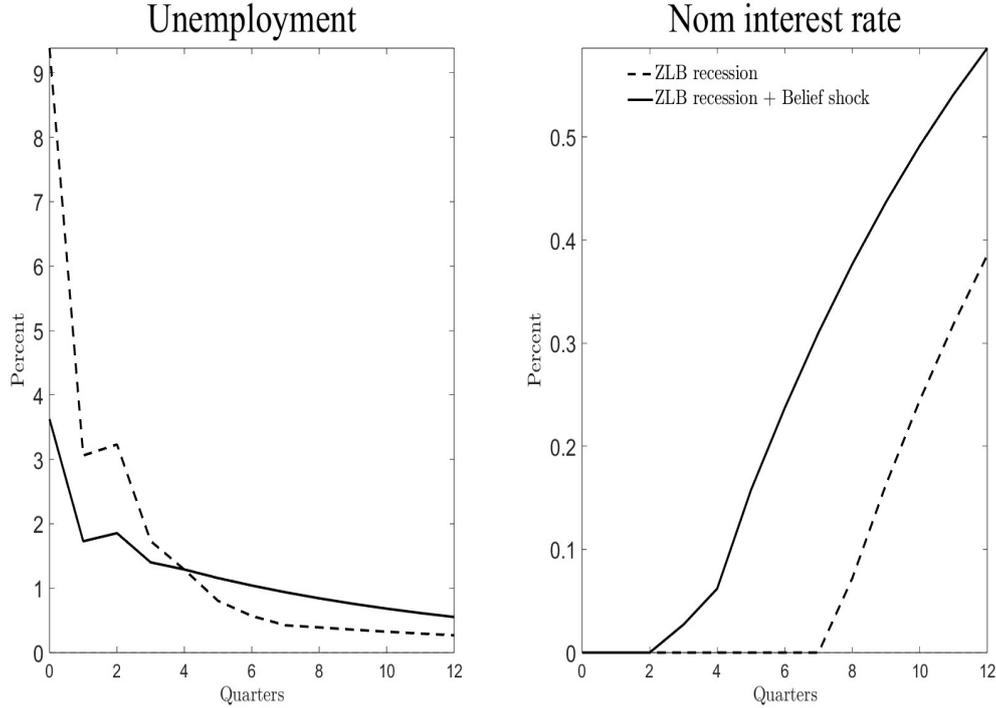


Figure 9: **Model Dynamics.** Simulated responses to a one standard deviation increase in the *risk premium shock* that brings the economy into the ZLB (black dashed lines); simulated responses to the same same risk premium shock combined with a one standard deviation in *belief distortion shock* (black solid lines).

The key driver of this result is inflation expectations. An increase in inflation expectations, induced by our identified belief distortion shock in ZLB, lowers the ex-ante real interest rate, stimulating the economy and generating expansionary effects that accelerate the recovery. As policymakers—specifically the Central Bank in this case—adopt a prolonged accommodative stance that does not offset the effects triggered by elevated inflation expectations, the conditional effect of the belief distortion shock becomes expansionary, resulting in lower unemployment in ZLB.

4.2 Model Validation

In this section, we evaluate the model’s ability to generate endogenous inflation uncertainty consistent with its empirical counterpart and to reproduce a state-dependent response of the unemployment rate to shocks. To achieve this, we feed the model with empirical values of the belief distortion shock and generate model-implied simulated data for the sample 1983Q3-2024Q2. For consistency with the data, we hit the economy by a risk premium shock in order to get the same number of ZLB periods that the US economy has experienced during the Global Financial Crisis and Covid-19 crises.

Simulated vs. Empirical Inflation Uncertainty To validate the model’s ability to generate inflation uncertainty in line with empirical data, we simulate the third-order solution of the model using identified belief distortion shocks. We obtain the simulated path for the shock’s contribution to inflation uncertainty and compare it with the shock’s contribution to the [Binder \(2017\)](#)’s inflation uncertainty from the historical decomposition of the proxy VAR model estimated in [Section 3.2](#). [Figure 10](#) shows that the simulated series closely aligns with the empirical data with a correlation of 0.77.²³

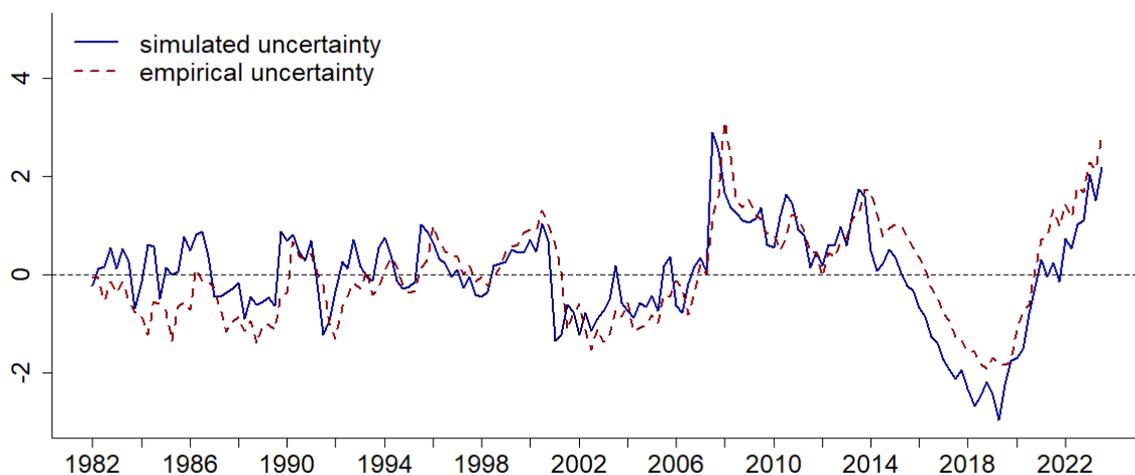


Figure 10: **Simulated versus Empirical Inflation Uncertainty.** The simulated series is obtained by feeding the theoretical model with the empirical belief distortion shocks (red dashed lines) and evaluating its contribution to inflation uncertainty. The empirical series is obtained from the historical decomposition of inflation uncertainty from a proxy-VAR (black solid lines). Both measures are scaled for comparability. *Sample: 1982Q4-2024Q2.*

State-Dependent Local Projection with Simulated Data To further validate the performance of the model, we now use the simulated data from the model to run state-dependent local projections. As in the empirical part, we estimate [Equation 7](#) using the simulated data for unemployment. [Figure 11](#) (panel a) shows the response β_x^Z , which captures the effect of the shock on the unemployment rate at the ZLB, and $\beta_x^{\bar{Z}}$, which represents the effect of the shock on unemployment when the economy is outside the ZLB period.

²³The correlation is 0.72 when using the [Jurado, Ludvigson and Ng \(2015\)](#)’s uncertainty measure.

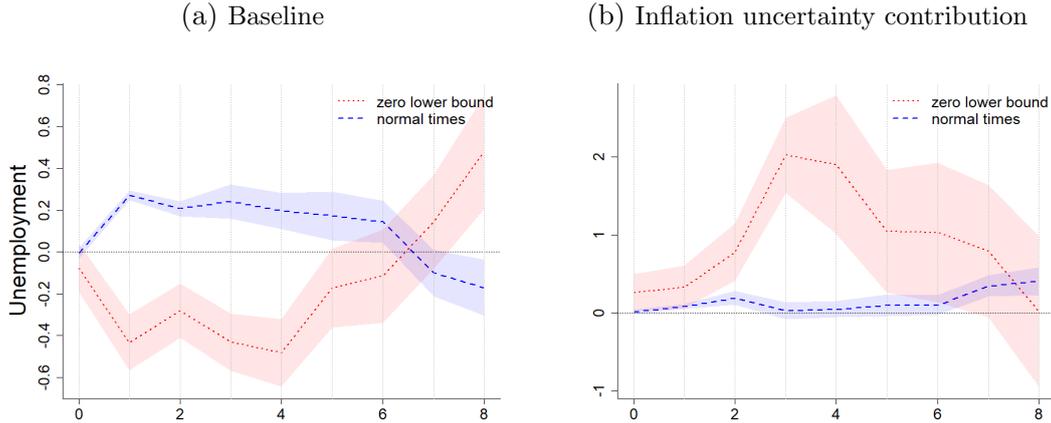


Figure 11: **Unemployment response to a belief distortion shock under Normal Times and ZLB states.** Local projections are estimated on model-simulated data using the lag-augmentation method of [Montiel Olea and Plagborg-Møller \(2021\)](#) and the specification in [Equation \(7\)](#), where lags are selected by minimizing the Akaike Information Criterion. At ZLB states (state Z), it selects the inclusion of 3 lags of the shock, 4 lags of the dependent variable, and $j = 4$ lag for the interaction term. In normal times (state \bar{Z}), the AIC recommends 2 lags of the shock, 2 lags of the dependent variable, and $j = 2$ lags for the interaction term. The bands show the 68% confidence intervals, based on Newey-West standard errors. *Sample: 1983Q3-2024Q2.*

[Figure 11](#) (panel b) instead reports the inflation uncertainty contribution, which is given by the coefficient γ_{int}^Z at the ZLB and by $\gamma_{\text{int}}^{\bar{Z}}$ outside the ZLB period. It shows that, consistently with the empirical counterpart, inflation uncertainty per se increases the unemployment rate in the ZLB, thus contributing to dampen the expansionary effect of the belief distortion shock.

5 Conclusions

This paper examines the macroeconomic effects of a belief distortion shock, defined as the unexpected component of household inflation expectations after accounting for professional forecasts and observable fundamentals. Using survey-based expectations alongside machine learning techniques, we identify this shock and study its impact through linear and state-dependent local projections. We find that it significantly raises inflation uncertainty and has state-dependent effects: it is inflationary and recessionary in normal times, but expansionary during crises when monetary policy at the ZLB. Inflation uncertainty dampens and can even revert this expansionary effect.

To interpret these findings, we develop a New Keynesian model incorporating belief distortion shocks. The model replicates the observed dynamics in inflation, unemployment, and inflation uncertainty, confirming that these shocks depress activity in normal times but support recovery in deep recessions—such as the Global Financial Crisis and the COVID-19 downturn—by lowering real rates. A key validation of our results comes from the model’s ability to generate simulated data that closely match empirical results. By feeding the identified belief distortion shock into our theoretical framework, we obtain

simulated paths for inflation uncertainty that align with those observed in the historical decomposition from the empirical analysis.

Our study shows that belief distortions are a key source of inflation uncertainty and business cycle fluctuations. These findings have important policy implications: Central Banks should account for the expansionary effects of inflation expectations in crisis times, while controlling for inflation uncertainty, when responding to inflation. Future research could explore optimal policy design and communication strategies that target belief uncertainty and its macroeconomic impact.

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Appendices

A Data

A.1 Survey expectations data

To measure households' inflation expectations, we use US data from the Michigan Survey of Consumer Attitudes (MSC), conducted by the Survey Research Center at the University of Michigan, over the period 1981Q4-2024Q2. The MSC survey is a rotating panel of approximately 500 to 1500 households, with respondents from the general public being either new participants or returning respondents from the prior six and twelve months. Data are collected monthly, with the second month of each quarter selected to create a quarterly series.

One-period-ahead inflation forecasts are derived from responses to the question: “By about what percent do you expect prices to go (up/down), on the average, during the next 12 months?”.²⁴ We follow the data preparation approach of [Curtin \(1996\)](#), applying a truncation rule of $[-5, 50]$ to remove outliers. We exclude responses without directional information on expected inflation. Categorical responses indicating only an increase/decrease without a specific rate are imputed, proportionally distributed across positive and negative values in the sample.

For rational expectations, we utilize CPI inflation and unemployment rate annual forecasts from the Survey of Professional Forecasters (SPF) of the Philadelphia Fed.²⁵ These forecasts are collected quarterly during the second month of each quarter, with respondents ranging from 9 to 83 professionals, including economists, bankers, and forecasters. Given their access to advanced forecasting methods, we consider these professionals to approximate rational agents relative to households ([Bonham and Dacy, 1991](#)).

A.2 Fred-MD data

In order to identify the belief distortion shock through a ridge regression approach as shown in [Section 3.1](#), we control for a large set of variables from the FRED-MD (Federal Reserve Economic Data - Monthly Data) dataset.²⁶ It is a comprehensive macroeconomic dataset sourced from the Federal Reserve Bank of St. Louis, and it is widely used in empirical macroeconomics for forecasting and structural analysis. The dataset consists of 128 variables at monthly frequency, covering key macroeconomic and financial indicators. Most of the series are available starting from January 1959. The series are divided into 8 groups: Output and Income; Labor Market; Housing; Consumption, Orders, and Inventories; Money and Credit; Interest and Exchange Rates; Prices; and Stock Market.

²⁴[Michigan Survey of Consumers Questionnaire](#).

²⁵[Philadelphia FED Survey of Professional Forecasters](#).

²⁶The dataset is freely accessible at <https://research.stlouisfed.org/econ/mccracken/fred-databases/>

B Measuring Inflation Expectations Uncertainty

Our measures of inflation expectations uncertainty are derived using the methodology proposed by [Jurado, Ludvigson and Ng \(2015\)](#) and [Ludvigson, Ma and Ng \(2020\)](#). We employ econometric estimates in a data-rich environment to capture the time-varying nature of inflation expectations uncertainty. To formalize the concept of uncertainty, let $x_{jt} \in X_t = (x_{1t}, \dots, x_{N_t})'$ be a variable, and $U_{jt}^x(h)$ represent the h -period ahead uncertainty, defined as the volatility of the purely unforecastable component of the future value of the series, conditional on all the information available:

$$U_{jt}^C(h) = \sqrt{E \left[\left(y_{jt+h}^C - E \left[y_{jt+h}^C \mid I_t \right] \right)^2 \mid I_t \right]} \quad (\text{B.13})$$

where I_t represents the information available, and C indicates a category, for example, real activity, financial, or in our case, inflation expectations. The information set I_t is represented by the whole FRED-QD dataset.

The time-varying forecast error is computed by allowing the prediction error to have time-varying volatility; to account for the predictable component using information from a large dataset, the forecast $E \left[y_{jt+h}^C \mid I_t \right]$ is taken from a factor-augmented forecasting model. Using a stochastic volatility model, uncertainty is calculated as the conditional expectation of the time-varying squared forecast error.

C Survey of Consumer Expectations' measure of uncertainty

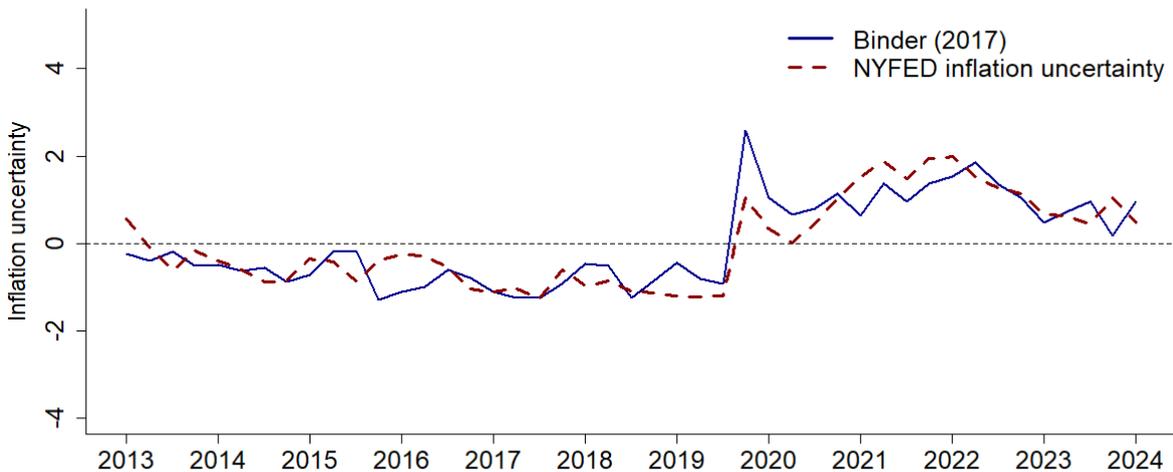


Figure C.1: **Inflation Uncertainty.** The figure reports two alternative measures of inflation uncertainty. The blue solid line is computed following the rounding-based method of [Binder \(2017\)](#) using MSC data. The second measure is obtained from the NY FED survey of consumers expectations. Both measures are normalized to have zero mean and unitary standard deviation and are taken in the second month of each quarter. Sample: 2013:Q1–2024:Q2.

D Shock exogeneity

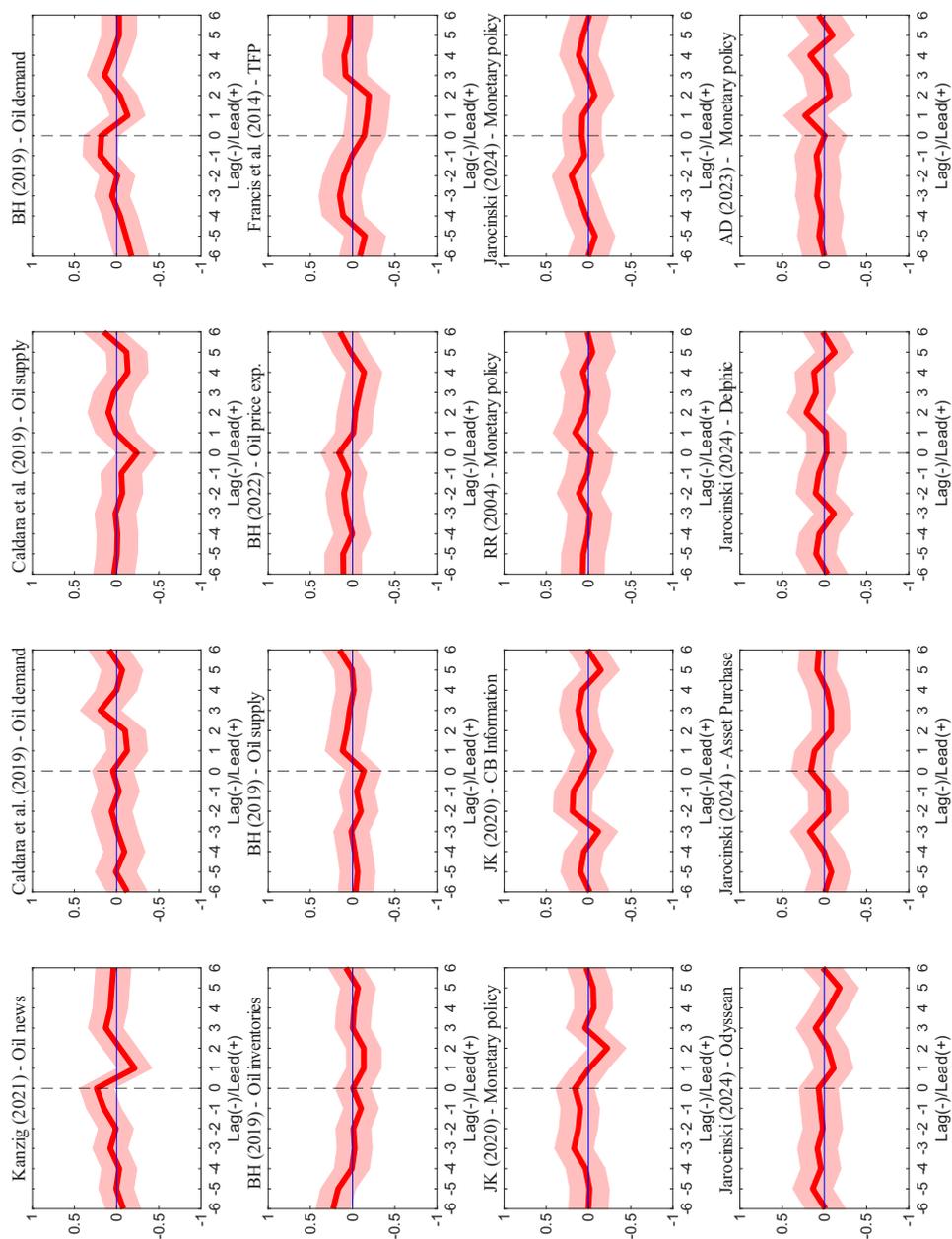


Figure D.2: **Dynamic Correlations Between Belief Shock and Structural Innovations.** The figure displays correlation coefficients between the belief distortion shock and various structural innovations from the literature at different leads and lags (from -6 to +6 quarters), together with the 95% confidence intervals (red shaded area).

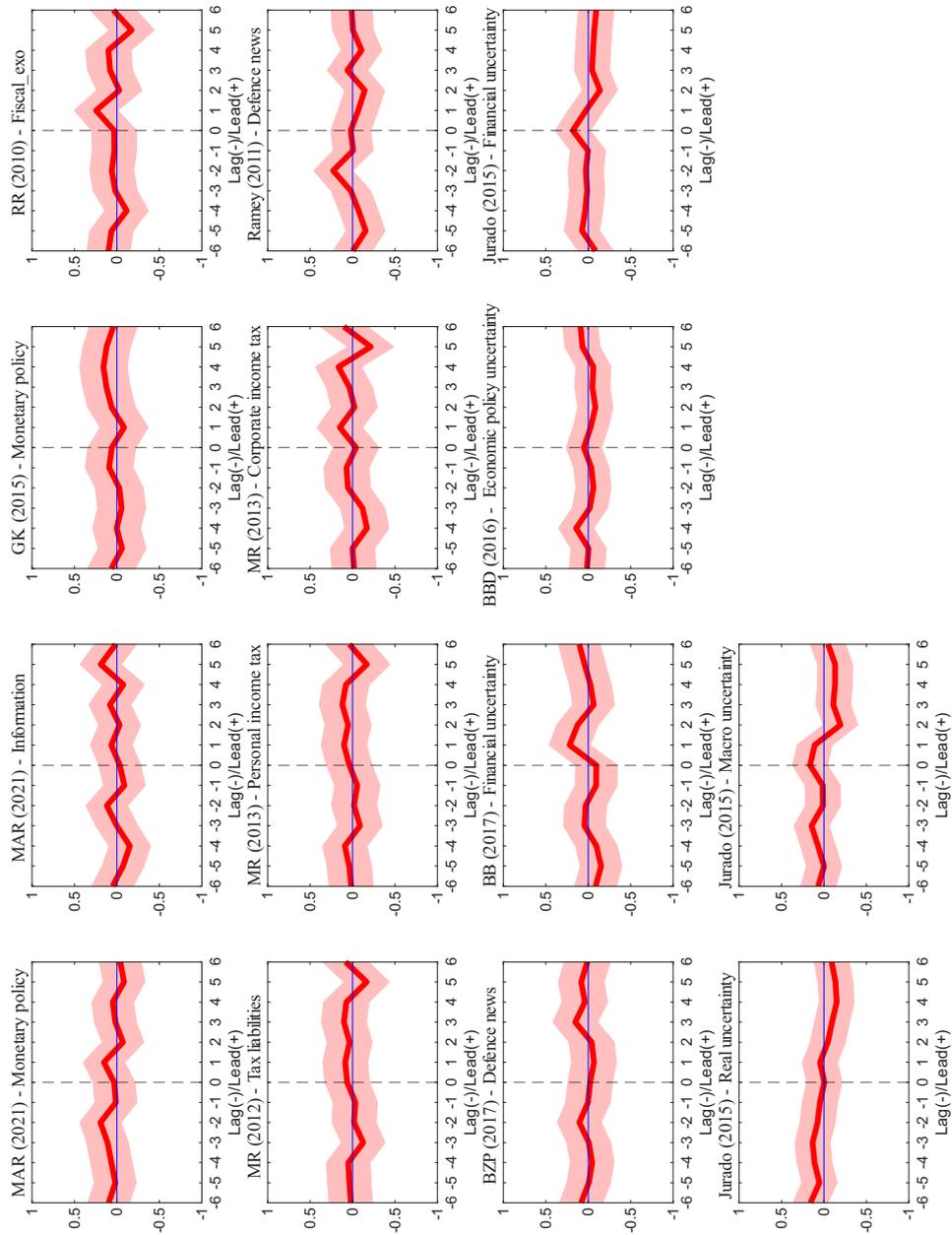


Figure D.3: **Dynamic Correlations Between Belief Shock and Structural Innovations.** The figure displays correlation coefficients between the belief distortion shock and various structural innovations from the literature at different leads and lags (from -6 to +6 quarters), together with the 95% confidence intervals (red shaded area).

Figure D.2 and Figure D.3 displays correlation coefficients between the belief distortion shock and various structural innovations from the literature at different leads and lags (from -6 to +6 quarters). Each subplot shows the correlation coefficient (red line) with 95% confidence bands (shaded area). The vertical dashed line at lag 0 indicates contemporaneous correlation. Lag(-) indicates that the structural innovation leads the BS shock, while Lead(+) indicates that the BS shock leads the structural innovation.

In Figure D.4 we compare the impulse responses of Binder (2017) inflation uncertainty, inflation and unemployment when we use the original shock versus a purged version obtained by regressing our measure on the Jarczyński and Karadi (2020) central bank information shock and its four lags, then using the residual. The results show that any possible contamination from the central bank information shock does not drive our findings.

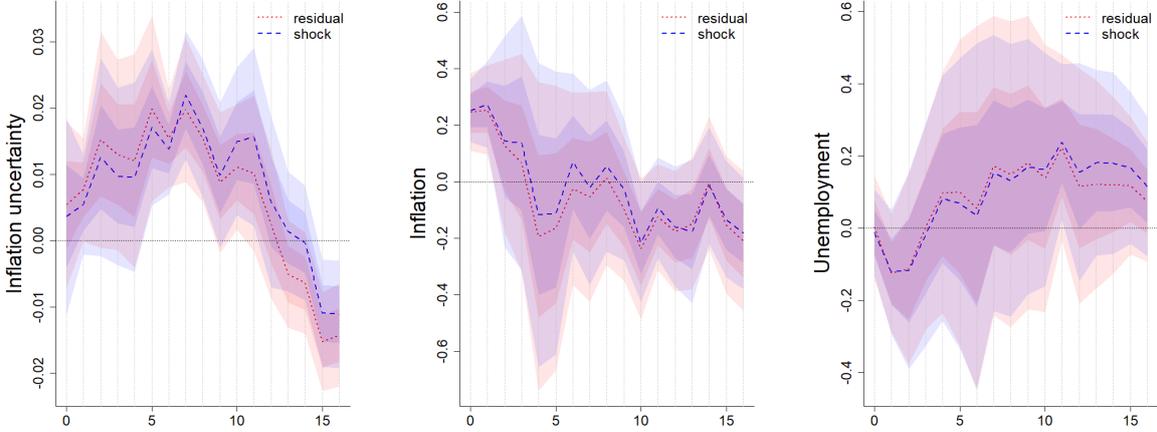


Figure D.4: **Impulse responses to a belief distortion shock in linear local projections.** The figure compares the impulse responses of inflation uncertainty, inflation, and unemployment to a one standard deviation belief distortion shock. The blue response is estimated by using the belief distortion shock, while the red response is obtained by purging the shock from the central bank information shock of Jarczyński and Karadi (2020). The bands show the 90% and 68% confidence intervals based on Newey-West standard errors. *Sample:* 1991Q1-2024Q2.

E Local Projections

E.1 Size and Sign Asymmetry

In this section, we extend our analysis using nonlinear local projections, following Caravello and Martinez-Bruera (2024). They show that, under symmetric shock distributions, nonlinear transformations in local projections can capture both **sign** and **size** asymmetries: *even* functions capture sign asymmetries, while *odd* functions capture size asymmetries.

Following Gonçalves et al. (2024), we first assess whether larger belief distortion shocks have disproportionate effects before turning to state-dependent projections. After testing for shock symmetry using the triples test Randles et al. (1980), we estimate the following for each horizon $h = 0, 1, \dots, H$:

$$x_{t+h} = \beta_{1,x}(h)\tilde{\theta}_t + \beta_{2,x}(h)f_1(\tilde{\theta}_t) + controls(x) + \epsilon_{t+h}, \quad (\text{E.14})$$

where $f_1(\tilde{\theta}_t) = |\tilde{\theta}_t|$ captures **sign asymmetries**, or alternatively $f_1(\tilde{\theta}_t) = \tilde{\theta}_t^3$ to capture **size asymmetries**. We also consider both simultaneously:

$$x_{t+h} = \beta_{1,x}(h)\tilde{\theta}_t + \beta_{2,x}(h)|\tilde{\theta}_t| + \beta_{3,x}(h)\tilde{\theta}_t^3 + \text{controls}(x) + \epsilon_{t+h}, \quad (\text{E.15})$$

In [Equation \(E.14\)](#) and [Equation \(E.15\)](#), x_{t+h} denotes the response variable (inflation uncertainty $\hat{\Delta}_t$, inflation π_t , or unemployment u_t), and $\tilde{\theta}_t$ is the belief distortion shock. The sample spans 1981:Q4–2024:Q2. Controls include a constant, four lags of the shock, and all dependent variables.

[Figure E.5](#) shows IRFs from these models with 95% confidence intervals:

- The first column shows the sequence of $\beta_{1,x}(h)$, capturing the linear effect of the shock. The shaded areas represent the linear specification without nonlinear terms, while the lines within each panel show the median of $\beta_{1,x}(h)$ under alternative nonlinear specifications.
- The second column displays the responses of $\beta_{2,x}(h)$ from [Equation \(E.14\)](#) when $f_1(\tilde{\theta}_t) = |\tilde{\theta}_t|$, highlighting **sign asymmetries**.
- The third column shows $\beta_{2,x}(h)$ from [Equation \(E.15\)](#) when both absolute value and cubic terms are included, isolating the effect of the absolute value term and highlighting **sign asymmetries** when controlling for size asymmetries.
- The fourth column presents the response of $\beta_{3,x}(h)$ from [Equation \(E.14\)](#) when $f_1(\tilde{\theta}_t) = \tilde{\theta}_t^3$, to assess **size asymmetries**.

The response of the linear coefficients aligns with the findings from standard local projections, indicating increasing uncertainty, inflation, and unemployment in response to belief distortion shocks. However, when focusing on nonlinear effects, increases in belief distortion appear to cause more pronounced effects than decreases. This is evident in the second and third columns, where the coefficients on the nonlinear term $|\tilde{\theta}_t|$ are positive at the impact for inflation uncertainty and unemployment, while the estimates are less clear for inflation. This suggests that at least for uncertainty and unemployment, positive shocks generate stronger responses than negative ones.

In contrast, the fourth column provides no evidence of size asymmetry: the coefficients on the cubic term $\tilde{\theta}_t^3$ are not statistically significant for any variable. We interpret this as evidence that the magnitude of the shock (size) does not matter beyond its sign; i.e., the responses are symmetric in size but asymmetric in sign.

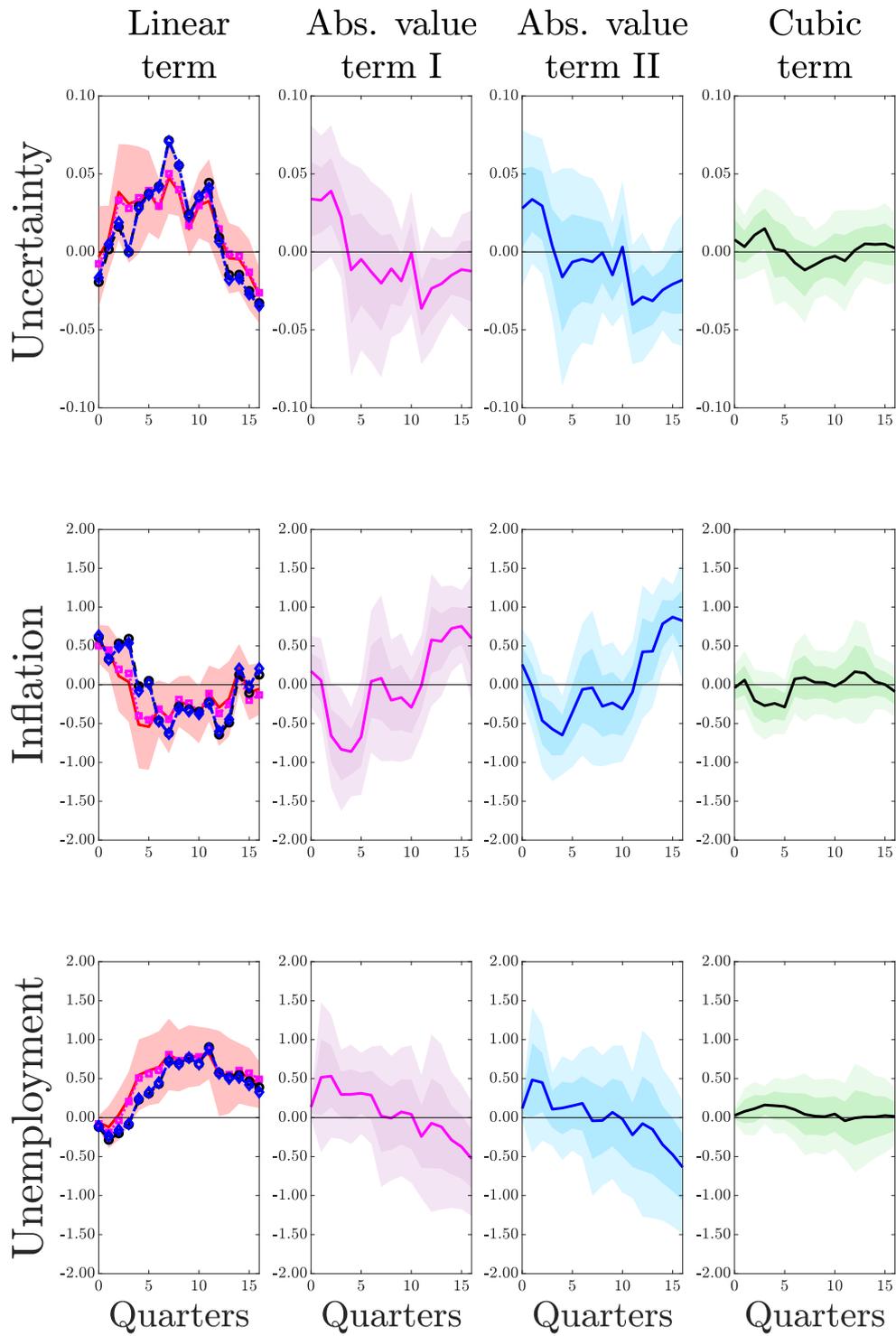


Figure E.5: LPs to belief distortion shocks. Column 1 shows β_i on ε_t , Column 2 shows β_i on $|\varepsilon_t|$, Column 3 shows β_i on $|\varepsilon_t|$ when a cubic term is included as regressor, Column 4 shows β_i on $(\varepsilon_t)^3$; Shaded areas are 68% and 95% confidence intervals.

E.2 State-dependent local projections: vacancy and CAPE

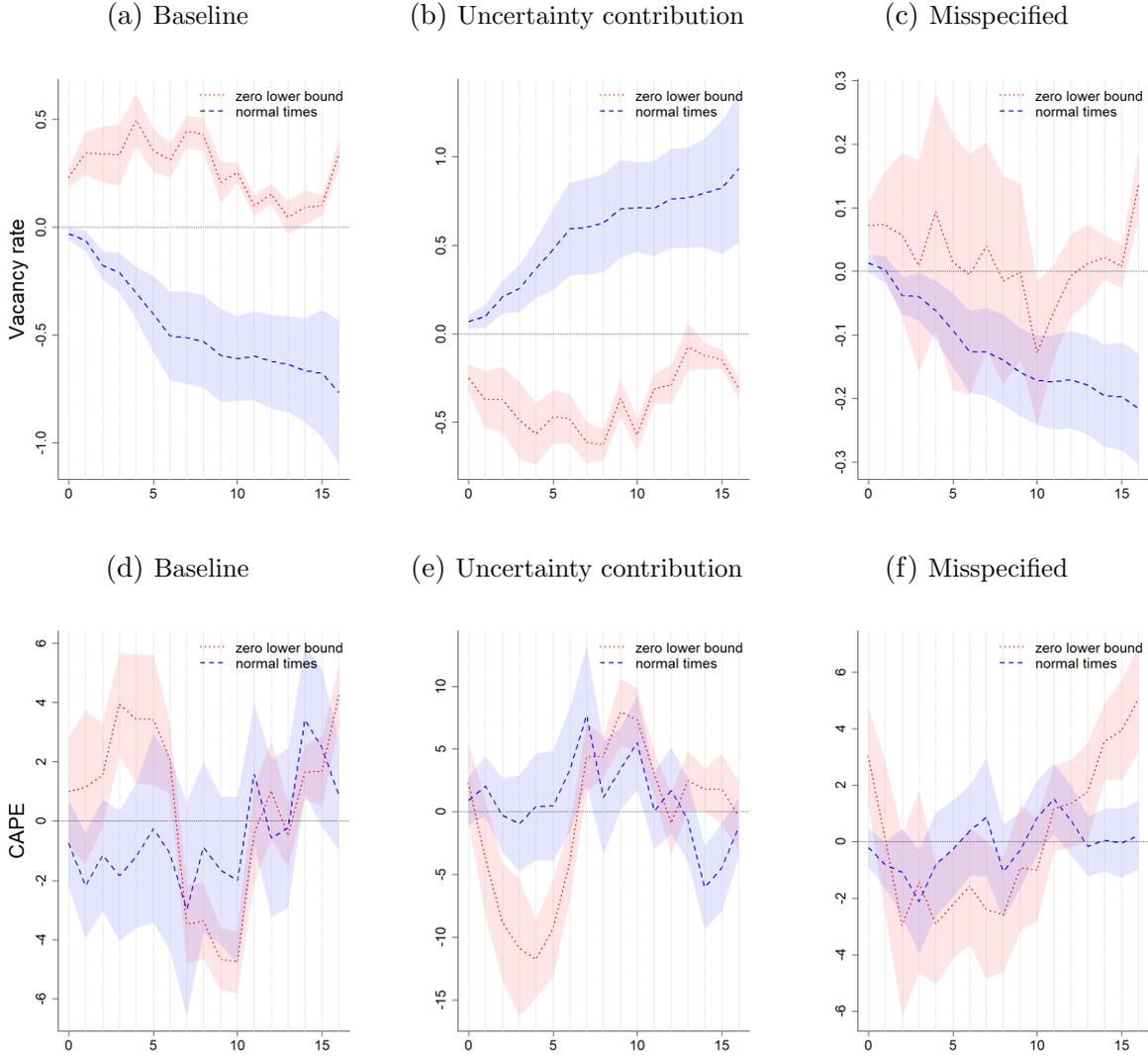


Figure E.6: **Vacancy rate and CAPE state-dependent responses to a belief distortion shock.** Panels (a),(b), (d), and (e) report results from the *baseline* local projections in Equation (7), while panels (c) and (f) are based on the *misspecified* specification in Equation (8). Local projections are estimated using the lag-augmentation method of Montiel Olea and Plagborg-Møller (2021), where lags are selected by minimizing the AIC. In the baseline local projections, for the *vacancy rate*, in the ZLB state (Z), we select 4 lags of the shock, 4 lags of the dependent variable, and $j = 1$ lag for the interaction term. In normal times (state \bar{Z}), 2 lags of the shock, 2 lags of the dependent variable, and $j = 2$ lags for the interaction term. For the *CAPE*, we select 4 lags of the shock, 2 lags of the dependent variable, and $j = 3$ lags for the interaction term in the ZLB state (Z), whereas in normal times (\bar{Z}), 2 lags for both the shock and the dependent variable, and $j = 2$ lags for the interaction term. In the misspecified local projections, for the vacancy rate, we select 4 lags of the shock and 2 lags of the dependent variable in the ZLB state, and 1 lag of the shock and 4 lags of the dependent variable in normal times. For the CAPE, 4 lags of the shock and 2 lags of the dependent variable in the ZLB state, and 1 lag of the shock and 4 lags of the dependent variable in normal times. The bands show the 95% and 68% confidence intervals, based on Newey-West standard errors. *Sample: 1983Q3-2024Q2.*