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## Start-to-low drawdown as a risk measure and its application to portfolio optimization for levered investors under solvency regimes

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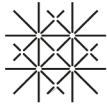
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# Start-to-Low Drawdown as a Risk Measure and its Application to Portfolio Optimization for Levered Investors under Solvency Regimes

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## Abstract

Drawdown is an important risk measure in both theory and practice. Most drawdown measures use the running peak as the reference point from which to calculate the drawdown. Instead, the start-to-low drawdown (SLD), which references the start of the period, is firstly proposed as a relevant measure for levered investors. Secondly, an application to a levered investor who is also subject to regulatory capital requirements, as seen in the banking or insurance industry, is proposed. Such an investor is faced with regulatory sanctions as soon as their own funds no longer cover capital requirements, i.e., even before equity is exhausted. Portfolio optimization objectives are developed that consider return, cost of capital, and cost of drawdown together: the solvency cost-adjusted return (SCAR) including the cost of drawdown (SCARD). This is applied to the European insurance industry, with capital requirement calculations following the Solvency II standard model. For the empirical analysis, models of life and non-life insurance companies are constructed using EIOPA market overview data, and their investments are optimized for SCAR and SCARD as objectives. The investment universe consists of equity, corporate bond, and government bond indices with data ranging from 2005 to 2024. The characteristics and performance of SCARD-optimal portfolios of the modeled companies are compared to those of SCAR-optimal and equally weighted portfolios. Out-of-sample SCAR and SCARD following both objectives are higher than those of the equally weighted reference portfolio. SCARD-optimal portfolios show lower cost of solvency capital and lower drawdown than their SCAR-optimal counterparts, but also lower returns. The differences in return outweigh those of the other components, resulting in the SCAR and SCARD of SCAR-optimal portfolios tending to be higher than those of SCARD-optimal portfolios.

**Keywords:** Portfolio Optimization; Drawdown Risk; Cost of Solvency Capital; Leveraged Investors; Solvency II

**JEL Classification:** C63; G11; G22; G28; G32

## 1. Introduction

Risk and return are central to portfolio selection, with return generally viewed as desirable and risk as undesirable. The trade-off between the two has been a fundamental subject of financial theory since the seminal work of [Markowitz \(1952\)](#), which introduced the mean–variance framework.

In industries that are subject to capital requirement regulation, such as banking and insurance, investment risk is directly linked to capital requirements. Under such regulatory frameworks, risk exposures are quantified using regulatory prescribed methodologies, whereby higher risk generally translates into higher required capital. If the own funds of the undertaking are insufficient vis-à-vis the capital requirement (CR), measures or sanctions from the regulator may follow, alongside possible other negative consequences such as negative press regarding the undertaking’s capital position.

If undertakings must adhere to capital requirements on a continuous basis, draw-downs may pose a distinct risk. A large decline in the value of a firm’s assets, *ceteris paribus*, may cause its own funds to fall below regulatory thresholds. In such cases, the firm may be forced to raise additional equity or sell riskier assets to restore compliance. Both responses come at a cost: the cost of additional capital or the opportunity cost of foregone risk premia. This paper integrates three components into the portfolio selection problem: (i) return, (ii) the cost of regulatory capital, and (iii) the cost of drawdown.

A popular path-dependent risk measure in both research and industry is maximum drawdown (MDD), which represents the largest cumulative loss from a peak to a subsequent trough. While many academics link its importance to the fund industry, for example by suggesting that investors would shy away from funds that experience large drawdowns (e.g., [Magdon-Ismail \*et al.\* \(2003\)](#), [Chekhlov \*et al.\* \(2005\)](#), [Malekpour & Barmish \(2013\)](#)), the line of reasoning this paper engages with is that of [Goldberg & Mahmoud \(2017\)](#), which holds that an MDD perspective can be relevant to any leveraged investor who is at risk of getting caught in a liquidity trap and being forced to sell low after an abrupt market decline because funding cannot be secured. The start-to-low drawdown (SLD) is proposed in this paper as an alternative downside path-dependent risk measure tailored to such scenarios, assuming that the investor’s liabilities are fixed over the examined window. In that case, the relevant amount of drawdown is the amount of equity at the start of the window and not a function of the running maximum. Thus, unlike other popular path-dependent risk measures, the SLD refers to the start of the period rather than the peak.

In the first part of this paper, some properties of the SLD are examined based on a Brownian motion model.

Second, the application of the SLD is developed. The scenario used in this paper is that of an investor subject to capital requirements regulation under Solvency II. If drawdown resistance were the only objective, a simple solution would be to hold

large amounts of equity or choose low-risk investment strategies. However, equity is costly, and low-risk strategies may come at the expense of lower expected returns. Therefore, first an objective accounting for return minus the cost of solvency capital (CoSC), the solvency cost adjusted return (SCAR), is studied. The concept of SCAR is related to that of economic value added (EVA) or risk adjusted return on risk adjusted capital (RARORAC). To trade off expected return and the cost of capital against the consequences of drawdown, the cost of drawdown is added, resulting in the solvency cost adjusted return including [the cost of] drawdown (SCARD). To the best of our knowledge, the SCARD composition is novel. It is worth noting that all three components of SCARD are affected by investment decisions. To bring the investment decision situation into the context of a company, modeled life and non-life insurance organizations are constructed using market overview data published by the European Insurance and Occupational Pensions Authority (EIOPA). In particular, modeled balance sheets and business risk CR are derived from EIOPA data. The CR for business risks interacts with the market risk through diversification, which alters the impact of investment decisions on the company's overall CR.

Third, in the experimental chapters, the investment portfolios of these modeled undertakings are optimized using SCAR and SCARD as objectives, and the properties of the resulting portfolios are studied.

The remainder of this paper is organized as follows: section 2 introduces the SLD, explaining its motivation, definition, and key properties in comparison with those of traditional risk measures. Section 3 discusses solvency capital regimes, focusing on Solvency II, and relates the cost of covering regulatory capital requirements to return and drawdown costs. In addition, SCAR and SCARD are introduced. Section 4 provides information on empirical data, assumptions, and methodology relevant to the experimental chapters. Modeled life and non-life insurance organizations are constructed from EIOPA data. Section 5 presents the experimental results for SCAR- and SCARD-optimal portfolios and section 6 concludes the paper.

## 2. Definition and properties of the start-to-low drawdown

### 2.1. Motivation

The maximum drawdown (MDD), like many other drawdown measures presented in the literature, has the characteristic that the starting point for measuring drawdown is the running peak. Other measures include the Conditional Drawdown (CDD) introduced by Chekhlov *et al.* (2005), which is the mean of the  $(1 - \alpha) \cdot 100\%$  worst drawdowns, the Conditional Expected Drawdown (CED) introduced by Goldberg & Mahmoud (2017), which is the tail mean of maximum drawdown distributions, and the end-of-period drawdown (eopDD) introduced in Möller (2018), which adapts the drawdown definition such that it no longer considers the trough but the end of the period. A recent review on drawdown risk measures is presented in (Geboers *et al.* 2023).

While the loss from peak to trough may be relevant for many applications and considerations, for example investor sentiment towards a fund or because it may mark the end of a successful trading strategy, it may not, under certain conditions, be the most relevant point of reference for levered investors. If, in balance sheet terms, a levered investor does not adapt its liabilities after reaching a peak, equity increases with the rising valuation of assets. In this case, the possible loss before forced liquidation is measured not by the decline from the peak, but by the equity level fixed at the last adjustment of the financing side. The importance of drawdown for levered investors has already been proposed by [Goldberg & Mahmoud \(2017\)](#).

An example is used to illustrate this point. A balance sheet of 100, with 90 units of liabilities (L) and 10 units of equity (E), is assumed. If liabilities remain unchanged during the period over which the drawdown is calculated, then any increase or decrease in assets (A) is directly reflected in equity. Under these assumptions, equity is depleted when the value of assets drops below 90. The lower limit of 90 here does not take into account a possible earlier increase in asset value, i.e., the drawdown measured from the start must not exceed 10. Measuring from the peak in this case would not generally indicate when equity was depleted. An MDD of 10 might or might not lead to equity being depleted if preceded by an increase in asset value, but it would if there were no prior increase. Fig. 1 illustrates the difference between MDD and SLD.

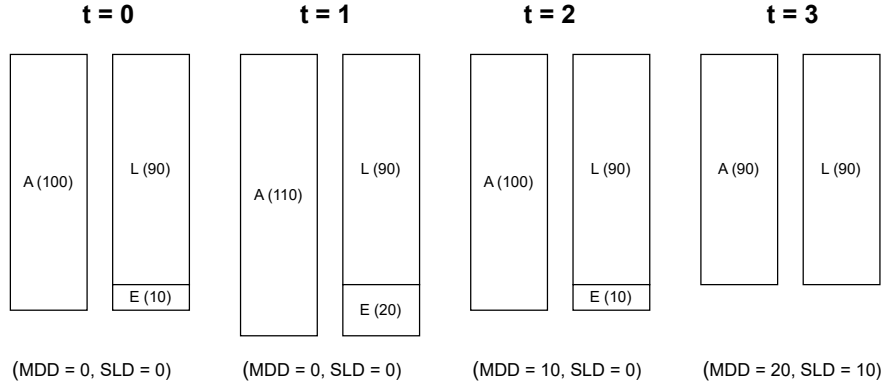


Fig. 1. Example Balance Sheet Movements with Corresponding MDDs and SLDs

Thus, bearing levered investors with period-fixed liabilities in mind, we propose the start-to-low Drawdown (SLD) as an appropriate measure to quantify the said investors' risk of being forced to sell because of a lack of equity.

## 2.2. Definitions

The SLD uses the value at the start of the period as a reference point to calculate drawdown, instead of the peak used by other drawdown measures (MDD, CDD, CED, eopDD).

The relative (percentage) SLD, hereafter used, is thus defined as

$$SLD(S) = \max_{t \in [0, T]} \frac{S_0 - S_t}{S_0} = \frac{S_0 - \min_{t \in [0, T]} S_t}{S_0} = \left| \min_{t \in [0, T]} R_{0,t} \right| \quad (2.1)$$

where  $R_{0,t}$  is the cumulative return from 0 to a point in time  $t$ . The absolute of  $\min R_t$  is taken to display the SLD as a positive number, which is in line with the definitions of other drawdown measures. Note that because  $t = 0$  falls within the interval, the lower bound of the SLD is 0.

For completeness, it is noted that the absolute SLD equals the numerator without the denominator in the price path-based definition in Eq. 2.1, i.e.,  $\max_{t \in [0, T]} S_0 - S_t$ , or the largest negative cumulative price movement. The absolute SLD is not, however, further investigated in this paper.

Depending on the mechanism used during a drawdown, other SLD measures besides the expected SLD might be considered to manage risk. If it is crucial not to exceed some level of SLD, a probability measure of not falling below that level may be more relevant. That risk measure is referred to as Quantile-SLD (QSLD) and is defined as

$$QSLD_\alpha(S) = \inf\{x \mid \mathbb{P}(SLD(S) > x) < \alpha\} \quad (2.2)$$

If falling below a certain level triggers costs that increase with the amount of the shortfall, a measure of the expected amount by which the threshold is breached, a conditional expected SLD (CSLD), may be of interest

$$CSLD_\alpha(S) = \mathbb{E}[SLD(S) \mid SLD(S) \geq QSLD_\alpha(S)] \quad (2.3)$$

This measure is similar in nature to the SLDs, as is the expected shortfall (ES) to returns and the conditional expected drawdown (CED) to MDDs.

## 2.3. Properties under Brownian motion and the impact of time discretization

The expected SLD of Brownian motions is presented in this section, and its dependency on drift, volatility, and time discretization is illustrated. A comparison with the sensitivities of other risk measures to volatility and the level of return is provided in (Stahli 2024).

For the arithmetic case, the expected SLD corresponds to the absolute value of the expected minimum of the arithmetic Brownian motion (ABM), irrespective of the starting value  $S_0$ .

A drifted ABM is a process that follows

$$\widetilde{W}_t = \mu dt + \sigma dW \quad (2.4)$$

where  $\mu$  is the average return per unit of time,  $\sigma$  is the volatility of returns, and  $dW$  is the usual Wiener increment, i.e.,

$$W_t = \mathcal{N}(0, t) \quad (2.5)$$

The expected minimum of an ABM is

$$\begin{aligned} \mathbb{E} \left[ \min_{t \in [0, T]} \widetilde{W}_t \right] &= -\sigma \sqrt{\frac{T}{2\pi}} e^{-\mu^2 T / (2\sigma^2)} + \left( \mu T + \frac{\sigma^2}{2\mu} \right) \Phi \left( \frac{-\mu \sqrt{T}}{\sigma} \right) \\ &\quad - \frac{\sigma^2}{2\mu} \Phi \left( \frac{\mu \sqrt{T}}{\sigma} \right) \end{aligned} \quad (2.6)$$

where  $\Phi$  is the cumulative distribution function of the normal distribution. The derivation of the expected minimum of an ABM from its probability density function is shown in the appendix under [Start-to-low drawdown derivation](#).

Two special cases must be considered: the limits as  $\sigma \rightarrow 0$  and  $\mu \rightarrow 0$ . The expression for the case  $\sigma \rightarrow 0$  is taken from [Privault \(2024, Exercise 10.2, p. 187\)](#):

$$\mathbb{E} \left[ \min_{t \in [0, T]} \widetilde{W}_t \right] = \begin{cases} \mu T \Phi(+\infty) = \mu T & \text{if } \mu \leq 0 \\ \mu T \Phi(-\infty) = 0 & \text{if } \mu \geq 0. \end{cases} \quad (2.7)$$

Based on a transformation of the formula for the corresponding case of the expected maximum of an ABM, as presented in [Privault \(2024, Exercise 10.2, p. 186\)](#), the expected minimum of an ABM in the limit as  $\mu \rightarrow 0$  is given by:

$$\mathbb{E} \left[ \min_{t \in [0, T]} W_t \right] = -\sigma \sqrt{\frac{2T}{\pi}}. \quad (2.8)$$

Moving to the case of a geometric Brownian motion (GBM). A GBM is a continuous-time stochastic process in which the logarithm of a randomly varying quantity follows a Brownian motion with drift. The corresponding stochastic differential equation (SDE) represents a stochastic process that follows

$$dS_t = \mu S_t dt + \sigma S_t dW_t \quad (2.9)$$

Thus, the expected value of an increment, as well as its variance, is not independent of  $S_t$ . SDE (2.9) has an analytic solution, which is (under Itô's interpretation):

$$S_t = S_0 \exp \left( \left( \mu - \frac{\sigma^2}{2} \right) t + \sigma W_t \right) \quad (2.10)$$

Following a simplification of Proposition 10.10. in [Privault 2022](#) on page 318, the expected minimum of a GBM is

$$\begin{aligned} \mathbb{E}^* [m_0^T] &= S_0 \Phi \left( \frac{r - \sigma^2/2}{\sigma} \sqrt{T} \right) - S_0 \frac{\sigma^2}{2r} \Phi \left( \frac{r - \sigma^2/2}{\sigma} \sqrt{T} \right) \\ &\quad + S_0 e^{rT} \left( 1 + \frac{\sigma^2}{2r} \right) \Phi \left( \frac{-\sigma^2/2 + r}{\sigma} \sqrt{T} \right). \end{aligned} \quad (2.11)$$



The limiting cases are also discussed in the same source and are as follows: if  $r \rightarrow 0$

$$\mathbb{E}^*[m_0^T] = 2S_0 \left(1 + \frac{\sigma^2 T}{4}\right) \Phi\left(\frac{-\sigma^2 T/2}{\sigma\sqrt{T}}\right) - \sigma S_0 \sqrt{\frac{T}{2\pi}} e^{-\sigma^2 T/8} \quad (2.12)$$

and if  $\sigma \rightarrow 0$

$$\mathbb{E}\left[\min_{t \in [0, T]} M_0^T\right] = \begin{cases} S_0 e^{\mu T} & \text{if } \mu \leq 0 \\ S_0 & \text{if } \mu \geq 0 \end{cases} \quad (2.13)$$

the equations simplify as shown in Eq. (2.12) and (2.13), respectively.

Note that by convention, Brownian motion (BM) minima are negative. In contrast, the SLD is shown as a positive number.

Having determined the expected SLD of an ABM and a GBM, we can study its dependencies on volatility and drift. In addition, it is possible to investigate the impact of time discretization. Because the SLD is path-dependent, it becomes smaller as the data becomes less granular. Similar to MDD, having more granular data may increase the measure because a lower minimum might be observed. Conversely, a minimum observed with less granular data cannot become less significant as more granular data becomes available. This is an aspect where path-dependent measures may differ from end-of-period measures.

Fig. 2 (ABM) and Fig. 3 (GBM) present the mean SLD for different levels of volatility, drift, and time discretization. For each combination of the three, 10,000 simulated price paths were generated. For comparison, the theoretical SLD following a BM is added.

The drift ranges from  $-20\%$  to  $40\%$ . The time discretizations are 12, 50, 250 and 2000 steps. Thinking of  $T$  as one year, the step sizes correspond to monthly, weekly, daily, and hourly observations of the price path. Volatility levels of  $20\%$  and  $40\%$  are depicted, and the zero-volatility case is added for reference.

The SLD increases with lower drift, higher volatility, and the greater granularity of observations. The impact of time discretization increases, *ceteris paribus*, with higher volatility. This is intuitive because greater variation leads to larger expected movements being omitted by less granular data. For the same reason, it is intuitive that the impact of discretization increases with the absolute size of the time interval for which observations are not available.

Within the range of parameters presented in Fig. 2 (ABM), the impact of time discretization does not notably vary with the level of drift. In contrast, in the GBM case depicted in Fig. 3 the impact of discretization tends to decrease with lower levels of drift. The size of the next increment in a GBM and its volatility depends on the current level of the price path. As drift becomes more negative, prices tend to be lower and thus their upcoming movements. Consequently, the size of what could be omitted by less granular data decreases.

Notably, the upper bound of the SLD under a GBM is  $100\%$ , i.e., the price path of a GBM does not become negative. In contrast, there is no upper bound for the SLD of an ABM.

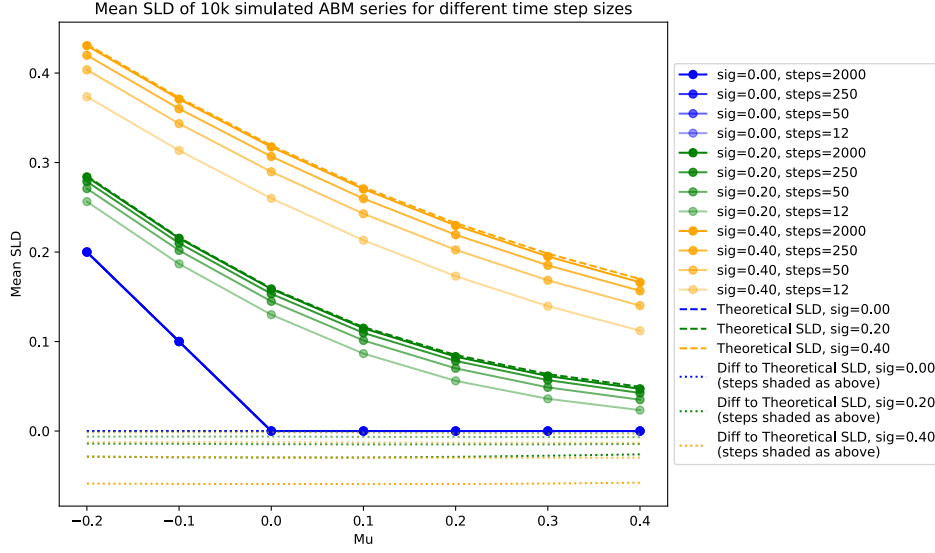


Fig. 2. SLD of an ABM as a Function of Drift, Volatility and Time Interval Size

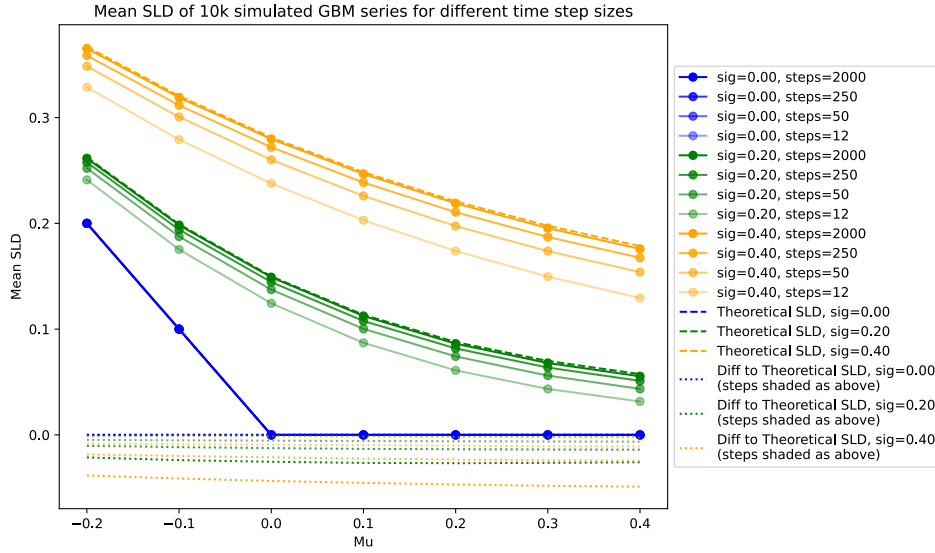


Fig. 3. SLD of a GBM as a Function of Drift, Volatility and Time Interval Size

Downside risk measures typically benefit from higher levels of return and so does the SLD. As is shown in more detail in (Stahli 2024), the SLD is more sensitive to

the level of return than, e.g., maximum drawdown (MDD), expected shortfall (ES) and value-at-risk (VaR). The high sensitivity of the SLD to the level of return will be of relevance in section 4.3. An intuitive reason for the high sensitivity is that the SLD is likely to remain zero if returns are sufficiently positive at the start of the period. Some days of negative returns would not lead to an above-zero SLD if they were smaller in magnitude compared to the strong start. In contrast, the VaR and ES of daily returns and MDD would increase with negative returns following a positive start. Thus, the SLD might be at risk of losing paths in which it remains zero or very low as mean returns decrease, because it decreases the chances of a strong start to the period.

### 3. Solvency capital regimes, leverage and drawdown

#### 3.1. *Solvency capital regimes and levered investors*

Solvency capital regimes, such as Solvency II for the European insurance sector, Basel III for global banking, and the Insurance Capital Standard (ICS) for internationally active insurance groups, impose requirements on the amount and quality of capital to hold against risks. For simplicity's sake, this paper adheres to Solvency II terms: the capital eligible to cover solvency requirements is referred to as eligible own funds (EOF), and the required amount of EOF by regulation is referred to as the solvency capital requirements (SCR).

When financial institutions violate solvency capital requirements, they face regulatory interventions, including the requirement to submit corrective action plans and possibly undergo increased supervision. Regulators may impose restrictions on business activities, financial penalties, or mandate capital raising to restore compliance. Severe breaches can lead to forced restructuring, or in extreme cases, revocation of the institution's license to operate. Such violations may harm the institution's reputation, possibly leading to a loss of market share and customer confidence.

The calculation methodologies applied by Solvency II, Basel III, and the ICS, while tailored to their specific sectors, share a fundamental principle: the use of risk-based assessments.

Important risk types typically include market risk and business-specific risk. Market risk involves the assessment of potential losses following from fluctuations in, for example, interest rates, equity prices, property values, and currency exchange rates. Business-specific risk may, for example, include the risk of loss arising from a borrower's or counterparty's inability to meet contractual obligations, as well as insurance risks (such as underwriting risk, reserving risk, and catastrophe risk) for insurance companies.

Thus, companies subject to solvency capital regimes must hold EOF based on the riskiness of their assets and business. EOF are a function of the difference between assets and liabilities; they therefore decrease in the case of market drawdowns. Following a sufficiently large drawdown, the company may no longer hold enough EOF to meet regulatory requirements. The company may then be forced to de-risk

its investments or business, both leading to foregone profits, or to raise additional capital with some type of equity character, leading to higher financing costs. The third option, becoming non-compliant with regulations, usually incurs a “double cost”: the cost following regulatory intervention (e.g., work on a corrective plan and increased supervision) in addition to that regulatory compliance still must be restored.

As detailed in section 2.1, drawdown is a risk for levered investors if the shortage of own funds after drawdown forces liquidation or other costly action. In the presence of solvency capital requirements, equity may already be lacking even before it is depleted. That is when regulatory requirements are no longer met. Thus, the mechanic of how a drawdown is a risk to an investor subject to regulatory capital requirements is similar to how it is a risk to the a leveraged investor in general, but the acceptable drawdown<sup>a</sup> is, ceteris paribus, smaller if equity must meet regulatory requirements rather than just being non-negative. Notably, similar considerations apply to any threshold of own funds that should not be breached.

Note that the above statement refers to the impact of the drawdown on the types of investor mentioned. Technically, investors subject to solvency capital requirements do not have to have a levered balance sheet, although it is usually the case in practice.

### 3.2. *Solvency capital requirement under Solvency II*

There are two metrics that define the levels of capital requirement under Solvency II: the minimum capital requirement (MCR) and the solvency capital requirement (SCR). The MCR is computed in a simplified fashion and is lower than the SCR, as it is capped at 45% of the SCR. Hence, the SCR is discussed and used in this paper.

While the regular frequency of reporting Solvency II data to supervisors is not higher than quarterly, it is important to note that, following Article 138(1) of the EIOPA Solvency II Directive ([European Parliament and Council 2024](#)), insurance and reinsurance organizations must immediately inform the supervisory authority if they observe non-compliance with the SCR.

The SCR is conceptually set up such that insurance and reinsurance companies hold sufficient capital to withstand adverse scenarios over a one-year period with a 99.5% confidence level (value-at-risk 99.5%). The SCR is calculated using either a standard formula prescribed by the regulatory framework or an internal model approved by the supervisory authorities. For generality, the standard formula situation is referred to in this paper.

The SCR calculation aggregates capital requirements across several risk modules. The so-called basic SCR (BSCR) modules consist of market risk, counterparty

<sup>a</sup>The acceptable drawdown refers to the drawdown that occurs before a threshold of own funds, which is intended not to be undercut, is breached

default risk, underwriting risks for life, non-life and health insurance, and intangible asset risk. The aggregation of these categories considers both diversification across and within BSCR modules. The standard formula for calculating the BSCR employs correlation matrices to account for these diversification benefits. Correlation coefficients are given by the regulation.

$$BSCR = \sqrt{\sum_{i=1}^n \sum_{j=1}^n SCR_i \cdot SCR_j \cdot \rho_{i,j}} \quad (3.1)$$

The contribution of one module to the overall capital requirement thus depends on the level of the other modules.

The total SCR then is the linear summation of BSCR, operational risk ( $OP_{\text{risk}}$ ), and adjustments for the loss-absorbing capacities of deferred taxes ( $Adj_{DT}$ ) and of technical provisions ( $Adj_{TP}$ ).

$$SCR = BSCR + OP_{\text{risk}} - (Adj_{DT} + Adj_{TP}) \quad (3.2)$$

The operational risk is a function of earned premiums or technical provisions and expenses. LAC DT originates from scenarios in which potential losses reduce future tax, while LAC TP originates from the organization's option to reduce future discretionary benefits in the case of negative developments. As explained in more detail in [Appendix B](#),  $OP_{\text{risk}}$ ,  $Adj_{DT}$ , and  $Adj_{TP}$  are not considered in the objective in this paper.

In the empirical part of this paper, optimization is performed on the asset portfolio but not on the insurance business part. More specifically, investment selection is performed over a universe of equity, corporate bond, and government bond indices described in section 4.1. Thus, the capital requirement from any module other than the market risk module is assumed to be constant in this paper, and for the market risk module, only the sub-modules on equity risk, interest rate risk, and spread risk are calculated. Additional details are provided in [Appendix C](#).

### 3.3. Solvency cost adjusted return

This section discusses the solvency cost adjusted return (SCAR) as a measure of the profitability of investments and businesses. SCAR is the return less the cost of solvency capital (CoSC) resulting from those investments or businesses. The concept behind SCAR is not new and is consistent with that of economic value added (EVA) and risk-adjusted return on risk-adjusted capital (RARORAC). Its place in the literature and how it differs from other popular concepts is explained later in this section, along with comments on the CoSC.

In the literature, various perspectives have been proposed to bring together expected return or profit, risk and the cost of capital.

In the classic Markowitz portfolio optimization, the objective is to identify the optimal combination of assets that minimizes risk for a given level of desired return. This results in an efficient frontier, which consists of a set of optimal portfolios. The

canonical measure of risk in this context is portfolio variance; however, efficient frontiers are drawn with a variety of risk measures

In addition, it is common practice to define ratios of (excess) return over risk, such as the Sharpe ratio. Under solvency capital regimes, capital requirement (CR) is a function of risk calculated following the rules of the regulation. Thus, the term of risk can be substituted with CR imposed to cover that risk. Scholars draw return to capital requirement frontiers, for example in the context of the Basel regulatory framework (e.g. [Drenovak et al. \(2017\)](#)) or Solvency II (e.g. [Kouwenberg \(2018\)](#), [Heinrich & Wurstbauer \(2018\)](#) and [Drenovak et al. \(2021\)](#)). In many cases, the translation from portfolio properties to CR is subject to the specific rules of the regulatory framework.

Similarly, ratios of profit to CR are formulated. Ratios to relate profit to economic capital used by banks, insurance companies, and other financial institutions include the return on risk-adjusted capital (RORAC) and the ratio of risk-adjusted return on capital (RAROC).

The formula for RORAC is:

$$\text{RORAC} = \frac{\text{Expected Profit}}{\text{Economic Capital}} \quad (3.3)$$

For the RAROC, there is an additional adjustment for risk in the numerator:

$$\text{RAROC} = \frac{\text{Expected Profit} - \text{Expected Loss}}{\text{Economic Capital}} \quad (3.4)$$

As pointed out by [Brüggemann et al. \(2017\)](#), the economic capital here is not the equity of the balance sheet but the capital necessary to cover unexpected losses from different types of risk. The use of capital required by regulation as the denominator is described as common in [Wilkens et al. \(2006\)](#), and [Baione et al. \(2021\)](#) use the Solvency II SCR as the RORAC-denominator.

None of the above, however, factors in a weighting between return and risk or capital requirements. If, for example, an investor is not sensitive to risk or capital is cheap, moving to the upper right of the efficient frontier would be more optimal than if the investor is risk averse or capital is expensive. This holds true irrespective of whether or not that movement improves return-to-risk (or capital) ratios.

A concept for weighing up return against the cost of capital is the well-known economic value added (EVA). The EVA is defined as the expected profit minus the cost of financing.

$$\text{EVA} = \text{Profit} - (\text{WACC} \cdot \text{CE}) \quad (3.5)$$

Usually, profit refers to the net operating profit after taxes (NOPAT), and the cost of capital is the product of the capital employed (CE) and the weighted average cost of capital (WACC). There is no explicit dependency of EVA on the level of risk. However, it can be implicitly assumed that the cost of capital increases with higher risk. A known measure that relates the risk-dependent RORAC to a required

level of return is the RARORAC, which is the return on economic capital minus the hurdle rate applied to it:

$$\text{RARORAC} = \text{RORAC} - \text{Hurdle-Rate} \quad (3.6)$$

Both EVA and RARORAC provide a direct interpretation of whether or not a given profit exceeds the cost of capital or demanded return. This is the case when both EVA and RARORAC are above zero.

The SCAR aligns with the concepts of EVA and RARORAC, in that it represents the difference between return and the cost of (solvency) capital.

For the cost to consider in SCAR calculation, recall that for solvency purposes, the capital that is eligible to cover solvency requirements is called eligible own funds (EOF). EOF consists of capital with an equity character but does not necessarily have to be composed entirely of tier 1. There are rulings that specify the quality and the percentage of total EOF that must be represented.

The CoSC is thus calculated as the product of EOF and the cost of EOF (Co-EOF).

$$\text{CoSC} = \text{EOF} \cdot \text{CoEOF} \quad (3.7)$$

In Eq. (3.7), only the cost of EOF is considered. Depending on the situation, a further term for the cost of liabilities not attributable to EOF, which is omitted in Eq. (3.7), may or may not be appropriate. If the implementation of an investment or business decision requires taking on additional liabilities, including their cost may be appropriate. However, if the decision only involves allocating a given amount, allocating an additional cost may not be necessary.

Note that Eq. (3.7) deliberately considers EOF and not the SCR directly. This is because companies may choose to hold EOF above the SCR by rule or policy. If a general risk add-on by policy (GRAP) is adhered to, the cost of the EOF held is relevant, i.e., SCR plus risk add-on rather than the SCR alone.

$$\text{EOF} = \text{SCR} \cdot (1 + \text{GRAP}) \quad (3.8)$$

For example, if a company adheres to a minimum target SCR ratio of 140%, i.e., a GRAP of 40%, a decision that results in an SCR increase of 100 requires an EOF increase of 140 to remain policy compliant.

The absolute SCAR is then defined as the expected return (ER) minus the CoSC required to pursue the underlying business or investments.

$$\text{SCAR} = \text{ER} - \text{CoSC} \quad (3.9)$$

The relative (percentage) SCAR used in the rest of this paper is employed as a measure to describe the profitability of investment decisions and is defined as the absolute SCAR divided by the amount invested.

For practical application: Companies may apply multiple criteria when making investment and business decisions. While the SCAR can always be used as a measure to describe the profitability of investment or business decisions in relation to the solvency costs they generate, for the SCAR to be relevant for decisions, the regulatory capital requirement it uses should be a binding constraint on capital. A situation in which this would not be the case would be if another decision criterion always resulted in EOF levels above those required for regulatory reasons. An example of such a decision criterion would be the objective of achieving a target rating (see e.g. [Höring \(2013\)](#)).

It may also be worth noting that within a holding structure, the holding company may be subject to different decision criteria and regulations from its subsidiaries. If, firstly, both the holding company and its subsidiaries are subject to regulatory capital requirements and, secondly, capital cannot flow freely and without cost between the holding company and its subsidiaries, then the application of SCAR at both the level of the holding company and that of the subsidiaries may be considered.

#### **3.4. Solvency cost adjusted return and drawdown**

The concept of SCAR is next combined with the aspect of drawdown. To the best of our knowledge, this is a novel combination.

In the case of a drawdown, EOF may no longer be sufficient to cover CR, leading to the issues described in section 3.1. Thus, the cost portion of SCAR is expanded by the cost of drawdown (CoD), leading to the solvency cost adjusted return including [the cost of] drawdown (SCARD).

$$SCARD = SCAR - CoD \quad (3.10)$$

In cases where EOF no longer cover CR, actions to reduce CR or increase EOF will be necessary. Quantifying the CoD can be a complex task, as it depends on assumptions about the consequences that follow a drawdown. Cost may consist of multiple components, each of which may take different functional forms.

A linear component of cost, which starts after a certain buffer is depleted, can be motivated by considering an optimal economic capital allocation principle, as for example done by [Baione et al. \(2021\)](#). If capital is allocated only to activities that earn at least their cost of capital, being forced to de-risk (reduce CR) may be expected to lead to forgone income that at minimum matches CoEOF. This holds true irrespective of whether the reduction comes from market risk, for example by selling stock, or insurance risk, for example by realizing lower business volumes. If the mismatch between CR and EOF is addressed not by de-risking but by providing additional EOF, the cost would again be CoEOF. In practice, the speed at which actions can be implemented varies. An emergency reaction premium (ERP) may be added to the CoEOF, assuming that a forced reaction would likely not take place under the best possible terms. In addition, the cost of managing the situation may be added to the ERP.



For simplicity, a linear drawdown cost function is assumed in Eq. (3.11). There might be grounds for incorporating terms of different functional forms or for establishing that costs become effective en bloc if the start-to-low drawdown (SLD) exceeds a particular value. The latter is related to the concept of quantile SLD presented in section 2.1.

Assuming a linear cost function and that the company fixes its tactical fund, business, and asset allocation over a particular period, responding only in case of an issue leads to the following quantification of the CoD:

$$CoD = \max(SLD - PFB, 0) \cdot (CoEOF + ERP) \quad (3.11)$$

The buffer indicates the extent of losses before drawdown costs start to accrue. For the calculation of the CoD, the buffer is defined relative to the portfolio used for calculating the SLD. Thus, it is referred to as a portfolio buffer (PFB). The portfolio can lose value by the amount of PFB before CoD starts to accrue.

Furthermore, management may consider SCR-to-EOF ratios that should not be breached, prompting action when such thresholds are reached. The buffer between this threshold and the current ratio is referred to as the capital requirement quota buffer (CRQB) here and corresponds to how much the SCR quota can fall before CoD starts to accrue. To revisit the numeric example in section 3.3: assuming the current ratio is 140% and action is taken if the quota falls below 120%, then the CRQB would be 20%.

The mathematical relationship between PFB and CRQB is a function of the size of the portfolio used for the drawdown calculation ( $V_{pf}$ ) and the SCR in conjunction with the quota buffer.

$$SCR \cdot CRQB = V_{pf} \cdot PFB \quad (3.12)$$

$$PFB = SCR \cdot CRQB \cdot \frac{1}{V_{pf}} \quad (3.13)$$

If action is only taken when the SCR quota reaches 100%, then EOF equals  $SCR \cdot (1 + CRQB)$  and thus the following holds in addition:

$$PFB = \frac{EOF}{1 + CRQB} \cdot CRQB \cdot \frac{1}{V_{pf}} \quad (3.14)$$

$$PFB = \frac{CRQB}{(1 + CRQB)} \cdot \frac{EOF}{V_{pf}} \quad (3.15)$$

It is worth noting that the SCAR(D) concepts could also be used to calculate an optimal level of buffer. As long as the cost per unit of drawdown is the same as that of holding EOF upfront, the solution will always be to hold a buffer of zero.

For practical application: The inclusion of drawdown in the objective is proposed for situations where there is a mechanism in place whereby large drawdowns within a given window incur additional costs. Possible drivers of drawdown costs are not limited to regulatory requirements but may also arise from other external or

even internal requirements. Furthermore, with regard to the frequency of changes used to calculate drawdown, in practice it is not necessary for an organization to perform such calculations daily to apply the model proposed here. It is sufficient to assume that data is readily available and that internal mechanisms would detect and perform calculations when drawdown becomes noticeably large, i.e., when it approaches the point of exhausting the buffers.

## 4. Data, assumptions and methodology

### 4.1. *Investment universe*

The empirical investment universe consists of indices of equities, government bonds, and corporate bonds.

For bonds, important features are their type (corporate or a particular type of government), their credit rating, and their duration. A spread risk stress of 0% is assigned to the bonds of the central governments and central banks of EU Member States, denominated and funded in the domestic currency of the respective central governments and banks, if the exposure is fully, unconditionally, and irrevocably guaranteed by the counterparty.

To represent a euro stock market, the following 14 MSCI “investable universe” indices of European euro countries have been used: Austria, Belgium, Croatia, Estonia, Finland, France, Germany, Ireland, Italy, Lithuania, Netherlands, Portugal, Slovenia, and Spain.

Corporate bonds are represented by 20 iBoxx EUR corporates provided by IHS Markit, which belong to S&P Global. Indices combining different ratings (AAA, AA, A, and BBB) and durations (1–3, 3–5, 5–7, 7–10, and 10+ years) were selected. The maximum average duration among 10+ corporate bond indices is 11.6 years. For government bonds, seven FTSE eurozone government bond indices were used with durations of 1–3, 3–5, 5–7, 7–10, 10–15, 15–25, and 25+ years.

If data for a particular index is not available throughout the entire training and out-of-sample window, that index is excluded from the investment universe for that point in time.

### 4.2. *Assumptions on the decision situation and model undertakings*

The range of decisions regarding business and investments that an organization may take is extensive. To illustrate and study the impact of considering capital requirements and drawdown during optimization, certain assumptions and simplifications must be made.

The first assumption made in the empirical part of this paper is that business decisions are not part of the optimization itself. Rather, the decision situation is that of tactical asset allocation, i.e., optimization is performed on the asset portfolio but not on the insurance business part. This asset allocation is assumed to remain

fixed for the decision horizon, which is one quarter in this paper. Likewise, asset positions, such as strategic participations resulting from processes other than the tactical asset allocation subject to optimization, are considered constant. Thus, SCAR and SCARD measured in section 5 are based on the balance sheet position of investments (excl. HiRU) displayed in Fig. 4.

The impact of investment decisions on the solvency capital requirement (SCR) is not independent of the business of an insurance company.

First, there is diversification among the market risk basic SCR (BSCR) module, the underwriting risk modules and the counterparty default risk module (cf. section 3.2). Second, the duration of other balance sheet positions, in particular that of technical provisions, is relevant to the SCR related to interest rate risk. If the shock-weighted duration of liabilities is larger than that of assets, an increase in asset duration reduces interest rate risk. Conversely, an increase in asset duration increases interest rate risk. Third, the assumption regarding the SCR quota at the beginning of a period is important for quantifying the cost of drawdown (CoD) as defined in this paper.

The model for life and non-life insurance companies used in this paper includes both a model balance sheet and model assumptions regarding their BSCR modules, excluding market risk. Following the considerations described in Appendix B, the total SCR is assumed to be equal to the BSCR.

EIOPA collects a variety of information through quarterly and annual Solvency II reports and provides aggregated information on insurance companies to the public.

#### *Model balance sheets*

The EIOPA Insurance Statistics<sup>b</sup> (EIS) provide a time series of the average life and non-life insurance undertakings' balance sheet. The model companies' simplified balance sheets presented in Fig. 4 are based on rounded values of the mentioned source. The reference date is 31.12.2022. Index- and unit-linked assets and liabilities have been canceled out as their risk and return is that of the policyholder, not of the insurance undertaking. The length of the balance sheet of both model companies is scaled to be 100 Euro.

The part of the balance sheet that is subject to optimization is "Investments excl. HiRU", where HiRU is "Holdings in Related Undertakings including Participations". HiRU and other assets are not part of the decision situation.

The balance sheet of the presented life model company is not too far from that of Høring (2013), to which later papers also refer (e.g., Kouwenberg (2018)): their technical provisions are 75 also, and their own funds are 10 (vs. our 12.5). On the asset side, their market risk portfolio is 75, that number matching our investments excl. HiRU position in size. Scope, however, differs to some extent.

<sup>b</sup>Link: <https://www.eiopa.europa.eu/tools-and-data/insurance-statistics.en>

Non-Life		Life	
Investments excl. HiRU (60)	Technical Provisions (60)	Investments excl. HiRU (75)	Technical Provisions (75)
HiRU (15)	Other Liab (10)	HiRU (10)	Other Liab (12.5)
Other Assets (25)	Own Funds (30)	Other Assets (15)	Own Funds (12.5)

Fig. 4. Balance Sheets of Model Undertakings

An additional assumption made relates to the duration of technical provisions (TP). Typically, TP relating to non-life business are substantially shorter than those of life. In this paper, the duration of non-life TP is assumed to be three years and that of life 15 years. TP duration is relevant for the calculation of interest rate risk.

#### *Model capital requirements and solvency ratios*

In the next step, the euro value of the BSCR modules of the model corporations is derived. These values will form the input to the BSCR calculation for all BSCR modules other than market risk (MR). For this, the equity of the model corporation in conjunction with a target SCR quota is employed.

An important aspect of the model proposed in this paper is the assumption that undertakings associate capital cost with their own funds. Thus, for the purpose of this study, the target SCR quota of both our model corporations is 140%.

The total amount of the BSCR<sup>c</sup> is set to a value such that the SCR-quota is 140%. The relative weights of the BSCR-modules then follow the typical distribution of undiversified BSCR as at the end of 2022 presented in the European Insurance Overview report 2023<sup>d</sup>. The resulting absolute BSCR values and their relative size are shown in Fig. 5.

In the experiments in this paper, MR is calculated using the results of investment optimization and does not usually match the value following the typical distribution shown in Fig. 5. When MR is subject to the result of optimization, it may be larger or smaller than that of the model undertaking. Therefore, a decision needs to be made regarding the assumptions: first, whether to maintain the original balance

<sup>c</sup>Recall that the assumption is made that BSCR equals total SCR.

<sup>d</sup>Link: [https://www.eiopa.europa.eu/publications/european-insurance-overview-report-2023\\_en](https://www.eiopa.europa.eu/publications/european-insurance-overview-report-2023_en)  
Please refer to “Figure 24 BSCR Composition - Standard Formula users”.

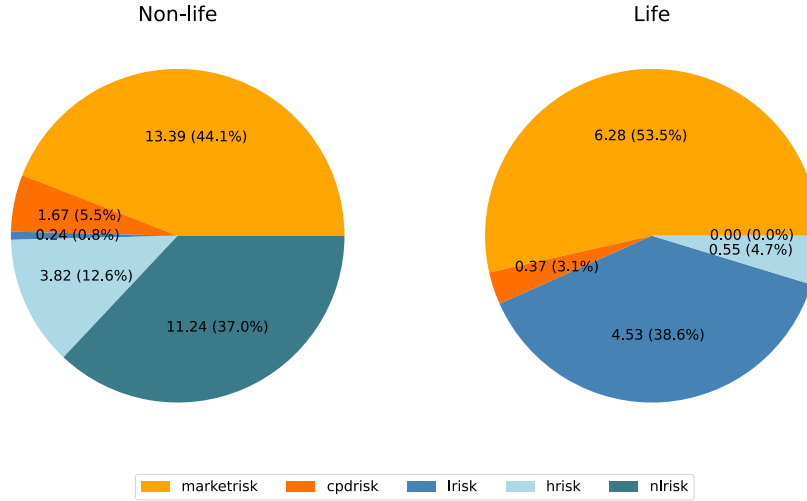


Fig. 5. Model Undertakings BSCR Modules

sheet, resulting in an SCR ratio different from the assumed 140%; second, whether to adjust the equity such that the quota returns to 140%; or third, whether to use a lower or higher capital requirement elsewhere to balance changes in market risks.

In a situation in which the cost of capital is derived from the amount of capital required, there is an implicit assumption that it could be used elsewhere at the CoEOF rate. This means that saving a particular amount of required EOF in the optimized part reduces SCAR and SCARD, *ceteris paribus*, by CoSC (and vice versa for higher required EOF). Thus, in the context of this paper, the third of the above-mentioned options is selected. This implies that the undertaking is *somehow* capable of restoring its balance sheet in conjunction with a SCR ratio of 140% and that the higher or lower CoSC of optimized portfolios adequately describes the financial impact of moving to said situation, for example by increasing or decreasing business and its CR. However, this leads to imprecision in the valuation of diversification effects between BSCR modules, as BSCR is still calculated based on model undertaking underwriting and counterparty default risk values. This imprecision tends to increase the more MR caused by optimization deviates from the model value shown in Fig. 5.

In addition, if MR values deviate substantially from those of the model undertakings', the assumption that the undertaking would be able to adapt in other areas at the CoSC to restore its balance sheet, while maintaining an SCR ratio of 140%, may become more challenging to uphold.<sup>e</sup>

<sup>e</sup>For a truly realistic picture of trade-offs on capital consumption between business and investments, the business part would also require some modeling, as well as the potential inclusion of information or assumptions on the speed at which changes can be made within the business part. This, while

A possible remedy within the setting used in this paper could be to impose bounds on the deviation of MR from its model undertaking value. Such a restriction could also have a feasible interpretation in practice: that the portfolio should not be shifted too much from one period to another. In practice, the reference point from which not to deviate excessively would be the current asset allocation of the actual undertaking. Note that in the context of this paper, optimization at each point in time is performed independently of the previous asset allocation. This approach fosters independence among the optimization results at the different points in time, which is beneficial for later comparative studies of optimization results and portfolio properties following different objectives. However, it may not be a realistic assumption because, first, it neglects transaction costs, and second, there are typically other processes and bounds in place that lead to asset allocation that cannot be completely neglected.

#### **4.3. Solvency cost based optimization and cardinality**

Low cardinality is a known challenge when optimizing for Solvency II capital requirement (CR) under the standard model (e.g., [Drenovak et al. \(2021\)](#)). Generally, there is one asset that offers the best return-to-capital-requirement trade-off for a particular market risk sub-module, leading to a significant allocation of weight to those assets that offer the highest return per respective risk. It is important to keep in mind that while there is diversification between sub-modules and modules, no diversification is realized at the most granular level of capital requirement calculation for a specific type of risk, such equity type 1 (eqt1) or spread risk (cf. section 3.2). For example, investing 100 euros in a single exchange-traded stock results in the same eqt1 CR as investing ten euros in ten different stocks. Thus, optimizing for a difference between return and cost based on eqt1 CR, without further modification or constraint, will lead to all weight being allocated to the stock promising the highest expected return.

Diversification is realized between market risk sub-modules: interest rate, equity, and spread risk (and the others assumed to be zero in this paper). However, if the expected standalone return minus the cost of capital for a particular asset type (e.g., equities) is higher than that of other asset types (e.g., bonds) by a sufficient amount, the impact of the cost-of-capital reduction on the objective owing to diversification may not be sufficient to offset the difference in return. In that case, most to all weight might flow into one asset class, and within that class into one asset owing to the reason described at the beginning of this section. The higher the correlation, the lower, ceteris paribus, the possible diversification and the smaller the interval in which diversification plays a role vis-à-vis the differences in the expected return. In this context, it is worth noting that the correlation between equity- and spread

likely challenging in all cases, seems to be something that may be considered when aiming to study and gain insights into such information in the case of a specific organization.

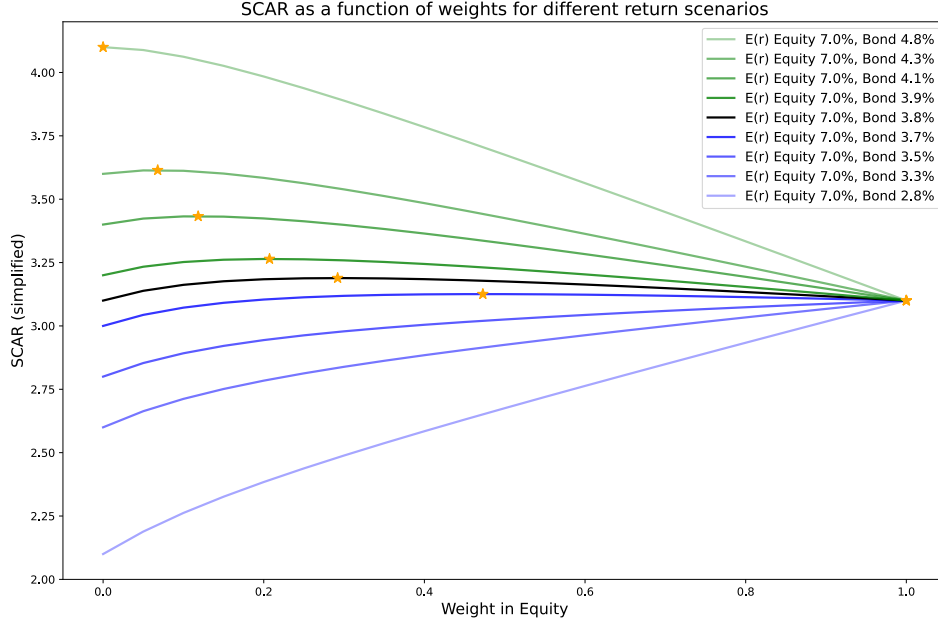


Fig. 6. SCAR (simplified) for a bond and equity portfolio as a function of equity weight ( $w_{eq}$ ) and bond weight ( $w_b = 1 - w_{eq}$ ). The standalone capital requirement for the equity risk is 39% and the spread risk for a bond with a rating of 'A' and a duration of five years is 7% (interest rate risk is disregarded). The cost of capital is assumed to be 10% of the market risk CR. Equity return is assumed to be 7%. Expected bond returns vary from 2.8% to 4.8% to show the sensitivity of SCAR-optimal portfolios (marked with stars) towards the magnitude of return expectation differences. The black line is the situation in which standalone equity and bond SCAR are equal to each other:  $7\% - 39\% \cdot 10\% = 3.1\%$  for equity and  $3.8\% - 7\% \cdot 10\% = 3.1\%$  for the bond. Green marks the scenarios in which the standalone SCAR of the bond is higher than that of the equity and blue refers to the opposite scenarios.

risk is rather high with a coefficient of 0.75. In summary, the differences between the expected return of investments vis-à-vis their impact on CR are crucial for the amount of diversification resulting from ER and CoSC optimization.

To illustrate, a two-asset case is assumed: one equity of type 1 and one corporate bond with a rating of A and a duration of five years. The symmetric adjustment is assumed to be zero; thus, the equity capital charge is 39% and the spread risk of the bond is 7%. Fig. 6 presents the SCAR as a function of equity and bond weights. The SCAR calculation is simplified to the extent that only equity type 1 and spread risk are considered in the CR calculation.

From this example, it is evident that the optimal portfolio consists of a combination of the two assets only if their individual SCARs do not differ considerably. The standalone cost of capital per-value invested for both assets does not vary; thus, a change in expected returns translates directly into a corresponding change in standalone SCAR. If the individual SCARs deviate sufficiently, diversification

ceases to play a role, and it is optimal to invest entirely in the asset with the higher individual SCAR.

The scenario selected is one in which the standalone capital charges of assets are quite different: the capital charge for equity is five times higher than that of the bond. Note that the x-axis in Fig 6 is not 'CR in equity' but 'weight in equity'. This is why the fall into 'all equity' appears to occur more rapidly in this figure compared to 'all bond'. If the individual CRs of both assets were similar, this would be more symmetrical (see [SCAR optimal weights of two equal-CR assets](#) for an example where the individual CR of the bond matches that of the equity).

In practice, such extreme weight allocation may not be seen as desirable unless one is able to predict returns with great accuracy. Several remedies would be possible. For instance, in a practical scenario involving multiple asset classes and assets per class (or indices representing assets with particular properties in a class), a first intuition may be to define bounds – minimum or maximum weights for the asset class or asset with particular properties. These bounds may already be established by management decisions, making their inclusion a natural choice. While weight restrictions would enforce diversification to some extent, they would not solve the problem of a lack of diversification at the most granular level of CR calculation for a particular type of risk. Therefore, diversification would only be added to the extent enforced by these bounds, while the portion of the weight that can be allocated freely would still flow into a few assets.

A second solution is to impose weights-penalization. One approach e.g. used by [DeMiguel \*et al.\* \(2009\)](#), [Still & Kondor \(2010\)](#) and [Zhang \*et al.\* \(2022\)](#) involves adding a penalty for the weights'  $\ell_2$ -norm  $\|w\|_2$  to the objective, thus penalizing large weights.

$$\|w\|_2 = \sqrt{\sum_{i=1}^n w_i^2} \quad (4.1)$$

The advantage of this approach lies in the fact that the penalty is a gradual function of the weight, creating a trade-off between the relative SCAR advantage of an asset and the penalty for its high weight. The greater the comparative advantage, the more the penalty for a high weight can be accepted by the objective. This means that an asset with a significant advantage can have a larger weight compared to one with only a slight comparative advantage, which can be seen as favorable behavior. An important aspect of this approach is the configuration of the penalty weight, typically denoted as  $\lambda$ . It should be large enough to have an impact on the outcome but not make the actual objective insignificant. This would for example be the case if all weights were nearly equal despite clear reasons for them to differ. Such reasons are present in SCAR and SCARD if there are relevant differences between individual SCAR(D)s. The choice of lambda in this paper is described in section 4.4.

A third option is to add a risk measure to the objective that actually benefits from diversification itself. With the addition of the cost of drawdown and, hence, the



SLD to the SCAR, a risk measure that values both higher return and diversification at the same time is added to the objective. Because SLD values both higher return and diversification, it is not clear, without further information, whether adding CoD will lead to higher or lower concentration in high-return asset weights. The relative weight of CoD compared to the other objective components is determined by the SCARD parameters discussed in section 4.4.

#### 4.4. *SCARD parameters, calculation and training*

##### *SCARD parameters*

SCAR and SCARD components are detailed in section 3.3 and section 3.4. The parameters are selected as follows:

- General risk addon by policy: 40% (target SCR-ratio 140%)
- Cost of eligible own funds (CoEOF): 10% p.a. (2.5% per quarter)
- Capital requirement quota [equity] buffer: 20%
- Emergency reaction premium: 2.5%

The general risk addon by policy is used for the calculation of the cost of solvency capital (CoSC). The capital requirement quota [equity] buffer and the emergency reaction premium flow into the calculation of the cost of drawdown (CoD). The CoEOF is used in both the CoSC and CoD calculations.

##### *Comments on SCARD calculation*

SCAR and SCARD are measured based on the investment portfolio. Therefore, the undertakings overall capital requirement (CR) is not used for the calculation of the CoSC, but rather the additional CR from the selected investments. Specifically, the market risk associated with investments subject to optimization, as well as the overall interest rate risk, was assumed to be zero in the reference case. This assumption was made because, in the absence of any assets to balance the duration of technical provisions, the reference case would reveal a significantly high interest rate risk, particularly for life insurance. Economically, this translates into the investment portfolio consisting of risk- and return-free assets that balance duration in a way that the interest rate risk is zero.

For CoD and return calculations, only the price movements of investments are considered. One could argue that the changes in value in technical provisions that follow fluctuations in the yield curve should also be considered, as those fluctuations may be an important driver of fixed income value changes, and there may be hedging between these assets and technical provisions.

### *Training hyperparameters and objective*

The training window ends the day before the out-of-sample window begins. Expected return and capital requirement calculations are based on market data on the last training window day.

As detailed in section 4.3, larger differences in expected returns between asset classes –even more so between assets with the same risk properties– lead to a significant concentration of asset weights. As a first measure, expected returns are set such that they do not vary too much. For all asset indices, the same expected return was used at a particular point in time: the current weekly Euribor rate ( $r_f$ ) plus a market risk premium (MRP) of 6%. Following [Fernandez & del Campo Baonza \(2010\)](#), the average MRP for Europe used by companies in 2010 (the year our out-of-sample period begins) was 5.7%. This was rounded up to the MRP used in this paper. For simplicity, no updates to the MRP were assumed.<sup>f</sup> For bonds, the yield to maturity (YtM) was used as the expected return. Assuming the YtM decomposes into  $r_f$  and a spread, the return expectations of equities as bonds move together with  $r_f$ .

Second, regularization was applied by adding the  $\ell_2$ -norm of weights to the SCAR and SCARD objectives to promote diversification. To find a good training window for expected SLD and a feasible value of the weight of the  $\ell_2$ -penalty ( $\lambda$ ), a validation sample was used prior to the actual test sample. The validation sample consisted of eight quarters in 2008 and 2009. For the expected SLD, a training window of three years was found to be a good choice. With regard to  $\lambda$ , a value of 0.2 was found optimal in the validation sample. These results hold true for both the life and non-life insurance cases.

The CoD calculation was adjusted during training. As CoD only starts after a certain threshold is reached, selecting high-return assets in training can result in a training CoD of zero. However, this selection may not be stable out-of-sample. Therefore, during training the buffer was reduced to zero and, to compensate, the drawdown cost factor was halved.

No constraints were applied other than that weights must be non-negative and must add to one.

## **5. Empirical SCARD-optimal portfolio properties**

The properties of SCARD-optimal portfolios are compared to those of SCAR-optimal portfolios, and a portfolio where each index within the investment universe described in section 4.1 is assigned an equal weight. The equal-weights portfolio is selected as a reference because of the simplicity and clarity it offers: it is clear

<sup>f</sup>For readers interested in investigating MRP movements in more detail, similar surveys to that of [Fernandez & del Campo Baonza \(2010\)](#) were published in the following years, providing information on country-by-country market risk premiums. The latest so far in 2024: [Fernández et al. \(2024\)](#). Links to previous surveys are embedded in that most recent version.

what it is and what it is not. It is neither optimized for return nor for regulatory or other risks. Some additional considerations on the choice of the 1/N portfolio as a benchmark and alternatives to it are presented in [Appendix E](#). Portfolios are optimized separately for the life and the non-life model undertaking. Table 1 presents the out-of-sample SCARDs and their components for life and non-life model undertaking, respectively. In both cases, higher returns are observed for SCAR and

Table 1. Out-of-Sample SCARD, its Components and SLD

	R	CoSC	CoD	SLD	SCAR	SCARD
Part 1: Life Model Undertaking						
E <sub>qref</sub>	1.007	0.569	0.023	1.684	0.439	0.416
SCAR <sub>opt</sub>	2.267	0.409	0.050	2.598	1.858	1.808
$\Delta$ E <sub>qref</sub>	1.260**	-0.159***	0.027**	0.914***	1.419***	1.392***
SCARD <sub>opt</sub>	2.077	0.383	0.045	2.343	1.694	1.649
$\Delta$ E <sub>qref</sub>	1.070**	-0.186***	0.022**	0.659**	1.255***	1.233***
SR-SD	0.190	0.026	0.005	0.255**	0.164	0.159
Part 2: Non-Life Model Undertaking						
E <sub>qref</sub>	1.007	0.334	0.003	1.684	0.673	0.670
SCAR <sub>opt</sub>	2.204	0.515	0.014	2.622	1.689	1.675
$\Delta$ E <sub>qref</sub>	1.196***	0.180***	0.011	0.939***	1.016***	1.005***
SCARD <sub>opt</sub>	2.033	0.435	0.011	2.294	1.598	1.587
$\Delta$ E <sub>qref</sub>	1.025***	0.101***	0.008	0.611***	0.925***	0.917***
SR-SD	0.171	0.080***	0.003	0.328***	0.091	0.088

\*\*\*, \*\*, \* statistically significant at 0.01, 0.05 and 0.1, respectively (paired t-test, df=59)

Rows: E<sub>qref</sub>: equally weighted reference portfolio, SCAR<sub>opt</sub> and SCARD<sub>opt</sub>: SCAR and SCARD optimal portfolios. SR-SD: Difference SCAR to SCARD optimal.

Columns: Cumulative return (R), Cost of Solvency (CoSC) and Cost of Drawdown (CoD) and its aggregation to SCAR and SCARD. SLD: start-to-low Drawdown. All measures over the horizon of a quarter. Values are in %.

SCARD-optimal portfolios compared to the equally weighted reference portfolio. The higher returns are also the largest determinant by magnitude for optimized portfolios with higher out-of-sample SCAR and SCARD.

Between SCAR- and SCARD-optimal portfolios, the out-of-sample returns of SCAR-optimal portfolios tend to be higher than those of SCARD-optimal portfolios, while the Cost of Solvency Capital (CoSC) and the Cost of Drawdown (CoD) of SCARD-optimal portfolios are lower than those of their SCAR-optimal counterparts. The magnitude of the difference in returns outweighs the differences in other

components, which is why the out-of-sample SCAR and SCARD of SCAR-optimal portfolios tend to be higher than those of SCARD-optimal portfolios.

In the life model undertaking, it is worth noting that the CoSC of investments is lower for optimized portfolios than it is for the equally weighted reference portfolio. The reason for this is that the optimized portfolios contribute more to closing the duration gap than do the equally weighted reference portfolios, and thus the interest rate risk is lower for the optimized portfolios.

A notable difference between the life and non-life cases is that even though the difference between the start-to-low drawdowns (SLDs) of SCAR- and SCARD-optimal portfolios is significant in both cases, the CoD difference is only significant in the life case. Recall that under the model assumptions made here, SLD only begins to count towards CoD as it exceeds 20% of equity. The equity-ratio of the non-life model undertaking is more than double that of the life-undertaking (cf. section 4.2 for model undertaking balance sheets). The non-life case indicates that even if drawdown is reduced, the extent to which this translates into lower CoD depends, among other things, on the capital structure (cf. section 3.4).

Table 1 already showed that SLDs of SCARD-optimal portfolios are lower than those of their SCAR-optimal counterparts. In the next step, Table 2 compares other out-of-sample risk metrics. The overall picture is coherent in that all of the additionally investigated risk metrics (volatility, value-at-risk, expected shortfall, and maximum drawdown) are lower for SCARD-optimal than for SCAR-optimal portfolios. It is interesting to note that the differences regarding the risk metrics between the life and non-life cases tend to be of small magnitude compared to the magnitude of the difference between the equally weighted reference portfolio and the two optimized portfolios.

While the difference in out-of-sample risk metrics between life and nonlife cases might be deemed comparatively small, the differences in asset classes selected between life and non-life cases are somewhat larger. Table 3 displays the weight of asset classes. SCAR- and SCARD-optimal portfolios of the non-life model undertaking have a 48.6% and 40.6% share of equities on average over time, respectively, while the share of equities in the life organization is approximately half: 25.0% and 22.1%, respectively. On the other hand, the fraction of government bonds is substantially larger in life undertaking portfolios. Recall that no bounds of asset class weights have been imposed, other than the requirement that the sum of weights must equal one for the overall portfolio.

Between SCAR- and SCARD-optimal portfolios, the former tend to have higher allocations of equity and lower allocations of corporate bonds than their SCARD counterparts in both the life and non-life cases. Are equities assumed to be typically associated with higher risk metrics than bonds, this observation aligns with the results in Table 2, which show that out-of-sample risk metrics of SCARD-optimal portfolios are lower than those of SCAR portfolios. In addition to the information provided on asset class weights in Table 3, Table 4 presents the distributional properties of overall asset weights. As a measure to assess the concentration of weight

Table 2. Out-of-Sample Risk Metrics

	R	Vola	VaR	ES	MDD	SLD
Part 1: Life Model Undertaking						
Eqref	1.007	0.310	0.407	0.441	2.848	1.684
SCAR <sub>opt</sub>	2.267	0.559	0.689	0.777	4.535	2.598
$\Delta$ Eqref	1.260**	0.249***	0.282***	0.335***	1.686***	0.914***
SCARD <sub>opt</sub>	2.077	0.494	0.612	0.688	4.060	2.343
$\Delta$ Eqref	1.070**	0.184***	0.205***	0.247***	1.212***	0.659**
SR-SD	0.190	0.065***	0.077***	0.089***	0.475***	0.255**
Part 2: Non-Life Model Undertaking						
Eqref	1.007	0.310	0.407	0.441	2.848	1.684
SCAR <sub>opt</sub>	2.204	0.537	0.679	0.757	4.431	2.622
$\Delta$ Eqref	1.196***	0.227***	0.272***	0.316***	1.583***	0.939***
SCARD <sub>opt</sub>	2.033	0.470	0.579	0.658	3.921	2.294
$\Delta$ Eqref	1.025***	0.160***	0.173***	0.217***	1.073***	0.611***
SR-SD	0.171	0.067***	0.100***	0.099***	0.510***	0.328***

\*\*\*, \*\*, \* statistically significant at 0.01, 0.05 and 0.1, respectively (paired t-test, df=59)

Rows: Eqref: equally weighted reference portfolio, SCAR<sub>opt</sub> and SCARD<sub>opt</sub>: SCAR and SCARD optimal portfolios. SR-SD: difference SCAR to SCARD optimal.

Columns: quarterly cumulative return (R), volatility (Vola), mean value-at-risk 90% (VaR) and expected shortfall 80% (ES) of daily returns, maximum drawdown (MDD) and start-to-low drawdown (SLD) over the horizon of a quarter and based on daily price observations. Values are in %.

in a few assets, the non-normalized Herfindahl index (Herf) is used.

$$H = \sum_{i=1}^n w_i^2 \quad (5.1)$$

This index increases as the concentration of weights in a few assets increases. Its value ranges from  $1/N$  to 1, where  $N$  is the number of assets. The lower bound is reached when all weights are the same, while the upper bound is reached when all weights lie in one asset. In addition, maximum weights and the fraction of assets with weights greater or equal to 2% are displayed in Table 4. Higher maximum weights can also be seen as a measure of stronger concentration and a higher fraction of assets with weights equal to or above 2% as a sign of lower concentration.

Weight concentration differs between life and non-life cases. The average maximum asset weight of SCAR-optimal portfolios is 32.1% (SCARD-optimal 28.9%) for the life case and 17.2% (17.9%) for the non-life case. Two aspects are worth noting here: first, the maximum weight is lower for SCARD- than for SCAR-optimal portfolios, and second, the maximum weight in the non-life case is lower than in the life

Table 3. Asset Class Weights

	Life			Non-Life		
	Eq	CB	GB	Eq	CB	GB
Eqref	0.270	0.541	0.189	0.270	0.541	0.189
SCAR <sub>opt</sub>	0.250	0.115	0.635	0.486	0.169	0.346
$\Delta$ Eqref	-0.020	-0.426***	0.446***	0.215***	-0.372***	0.157***
SCARD <sub>opt</sub>	0.221	0.151	0.628	0.406	0.216	0.378
$\Delta$ Eqref	-0.049	-0.389***	0.438***	0.136***	-0.324***	0.189***
SR-SD	0.029**	-0.037***	0.008	0.080***	-0.048***	-0.032***

\*\*\*, \*\*, \* statistically significant at 0.01, 0.05 and 0.1, respectively (paired t-test, df=59)  
 Rows: Eqref: equally weighted reference portfolio, SCAR<sub>opt</sub> and SCARD<sub>opt</sub>: SCAR and SCARD optimal portfolios. SR-SD: Difference SCAR to SCARD optimal.  
 Columns: Weight of asset class equity (Eq), corporate bond (CB) and government bond (GB)

Table 4. Distributional Properties of Asset Weights

	Life			Non-Life		
	Max	Ge2%	Herf	Max	Ge2%	Herf
Eqref	0.027	1.000	0.027	0.027	1.000	0.027
SCAR <sub>opt</sub>	0.321	0.211	0.231	0.172	0.356	0.104
$\Delta$ Eqref	0.294***	-0.789***	0.204***	0.145***	-0.644***	0.077***
SCARD <sub>opt</sub>	0.289	0.245	0.198	0.179	0.355	0.105
$\Delta$ Eqref	0.262***	-0.755***	0.171***	0.152***	-0.645***	0.078***
SR-SD	0.032***	-0.034***	0.033***	-0.007	0.001	-0.002

\*\*\*, \*\*, \* statistically significant at 0.01, 0.05 and 0.1, respectively (paired t-test, df=59)  
 Rows: Eqref: equally weighted reference portfolio, SCAR<sub>opt</sub> and SCARD<sub>opt</sub>: SCAR and SCARD optimal portfolios. SR-SD: Difference SCAR to SCARD optimal.  
 Columns: Mean of maximum weights (Max), fraction of weights greater than or equal to 2% (Ge2%) and Herfindahl index (Herf).

case. As introduced in section 4.3, it is not clear a priori whether the addition of CoD to the SCAR objective will lead to higher or lower asset concentration. On the one hand, drawdown tends to be reduced by diversification, which supports lower concentration; on the other hand, higher returns reduce drawdown, which increases the objective's preference for assets with high expected returns, which supports higher concentration. In the life case, where weight concentration was higher, adding the CoD resulted in reduced concentration, while this effect was not observed in the non-life case.

A possible explanation for the higher concentration observed in the life case may be the construction of the investment universe described in section 4.1. The assumed duration of the life model undertakings liabilities is 15 years compared to three years for the non-life case. Thus, for life, selecting assets of long duration benefits the objective by reducing the interest rate risk and thus lowering the CoSC. However, there are only four government bond indices in the investment universe with a duration exceeding ten years, which may lead to a relatively high concentration of weight in those assets.

Table 5 provides a regulatory view on market risk (MR) and its components. It

Table 5. Duration, Credit Quality and Market Risk Capital Requirement

	Dur	CQS	EqR	IRR	SpR	MR
Part 1: Life Model Undertaking						
Eqref	4.763	1.500	9.506	6.189	2.952	16.043
SCAR <sub>opt</sub>	10.868	1.817	8.853	3.884	0.844	12.162
$\Delta$ Eqref	6.105***	0.317***	-0.653	-2.305***	-2.108***	-3.881***
SCARD <sub>opt</sub>	10.492	1.854	8.111	4.002	0.994	11.649
$\Delta$ Eqref	5.729***	0.354***	-1.395*	-2.187***	-1.958***	-4.394***
SR-SD	0.376	-0.037	0.742**	-0.118**	-0.150***	0.513
Part 2: Non-Life Model Undertaking						
Eqref	4.763	1.500	8.942	1.574	2.362	11.680
SCAR <sub>opt</sub>	6.079	2.018	13.790	2.079	1.080	15.230
$\Delta$ Eqref	1.316***	0.518***	4.848***	0.505***	-1.282***	3.550***
SCARD <sub>opt</sub>	6.688	2.047	12.058	2.080	1.290	13.636
$\Delta$ Eqref	1.925***	0.547***	3.116***	0.506***	-1.072***	1.956***
SR-SD	-0.609***	-0.029	1.732***	-0.001	-0.210***	1.594***

\*\*\*, \*\*, \* statistically significant at 0.01, 0.05 and 0.1, respectively (paired t-test, df=59)

Rows: Eqref: equally weighted reference portfolio, SCAR<sub>opt</sub> and SCARD<sub>opt</sub>: SCAR and SCARD optimal portfolios. SR-SD: Difference SCAR to SCARD optimal.

Columns: average duration of selected investments (Dur), CQS: average credit quality step of corporate bonds ('AAA': 0, 'AA': 1, 'A': 2, 'BBB': 3, ...), capital requirement for Equity Risk (EqR), Interest Rate Risk (IRR), Spread Risk (SpR) and its aggregation to Market Risk (MR).

displays capital requirements for the MR modules of equity (EqR), spread (SpR) and interest rate risk (IRR). As important determinants of SpR and IRR, information on bond duration and corporate bond credit quality is also provided; the latter by means of the so-called credit quality step (CQS), where ratings are mapped to numeric values: AAA corresponds to a CQS of 0, AA to 1, A to 2, BBB to 3, etc.

In both the life and non-life cases, equity risk is, firstly, the biggest market risk and, secondly, is higher in SCAR- than in SCARD-optimal portfolios. The latter is in line with Table 3, where a similar pattern in asset class weights is displayed: the weight of equities is lower in SCARD- than in SCAR-optimal portfolios. Unsurprisingly, the duration of assets is larger in the life than in the nonlife case. The average CQS of corporate bonds is 1.8 in the life and 2.0 in the non-life case, and little difference is observed between SCAR- and SCARD-optimal portfolios.

Fig. 7 presents the weights of asset classes for both life and non-life cases over time. Time series on interest rates and symmetric adjustment are added.

Several questions can be addressed using this information. First, it is observed that the weight of each asset class is close to zero at some point in time and high at others. As explained in section 4.3 and section 4.4, a regularization term in the form of the  $\ell_2$ -norm of weights is added to the SCAR and SCARD objectives to promote diversification. A sign that the penalty might be excessive would be if asset (class) weights did not change much over time and remained close to their fraction of the investment universe. According to the equal-weighted portfolio displayed in Table 4, the investment universe comprises 18.9% government bonds, 27% equities and 54.1% corporate bonds. However, government bonds accounted for almost 100% of the weight in life portfolios around 2014, while corporate bonds comprised nearly 0% from 2017 to 2020 in both life and non-life cases. This aligns with the mean maximum weights in a single asset being 32.1% in life undertakings SCAR-optimal portfolios (cf. Table 4). Thus, the selected penalization parameter of  $\lambda = 0.2$  does not show signs of being overly prohibitive on assets and asset class concentration.

Next, it can be observed that the changes in asset class weights can occur rapidly from one quarter to another. Penalties for portfolios deviating largely from one point in time to another, such as transaction costs, have purposely not been put in place in order to illustrate optimization results that are solely based on the situation found at a given point in time and are otherwise independent of each other. However, the speed of changes in asset class weights may suggest that the inclusion of such a penalty may be interesting for future research. In addition, considering, for example, that the fraction of equities exceeds 80% at some point in time for both life and non-life cases may suggest that adding bounds to asset class weights would be worth considering in future research. This might be in the form of a hard bound or penalties that increase with the degree of deviation from a specified bound. Similar considerations for restricting or penalizing asset class weights may apply, as were discussed in section 4.3 with regard to individual asset weights.

The symmetric adjustment (SA) and interest rates influence the individual SCARs of asset classes, affecting their relative desirability in SCAR- and SCARD-optimization. The SA is of an anti-cyclical nature and is used in the calculation for the equity capital charge, which is set at  $39\% + SA$  for exchange-traded equities that are not classified as strategic participation (cf section 3.2).

As the SA can range from  $-10\%$  to  $+10\%$ , resulting in an equity capital charge between 29% to 49%, with the upper bound being around two-thirds higher than



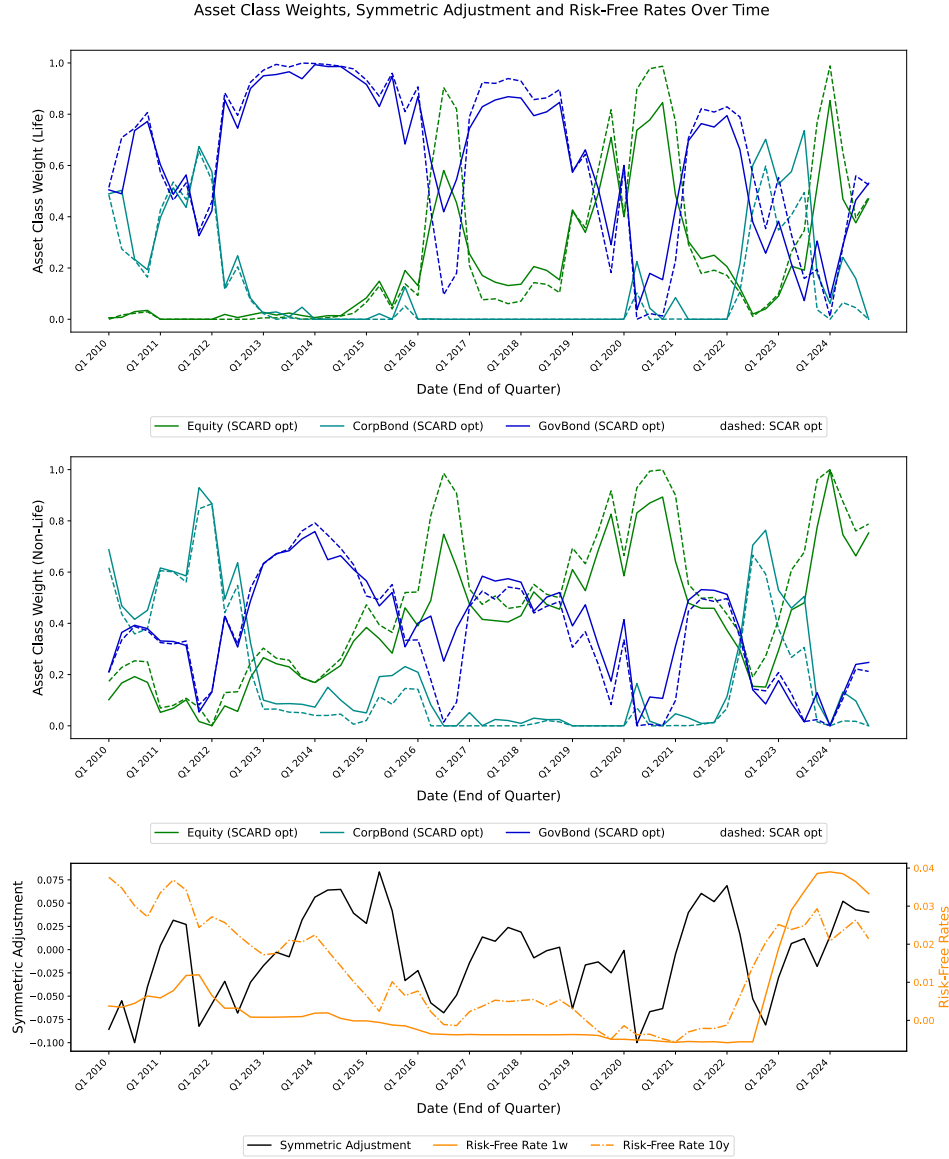


Fig. 7. Weights of asset classes: equities, corporate bonds (CorpBond) and government bonds (GovBond) of SCARD- and SCAR-optimal (dashed) portfolios over time. Upper panel: life model undertaking, mid panel: non-life model undertaking. Bottom panel: symmetric adjustment and EUR risk-free interest rates over time. The one-week Euribor (1w) and the European Central Bank's ten years governmental AAA-rated spot rate (10y) are displayed.

the lower bound. A high SA thus makes equities less favorable from a CoSC point of view, and the magnitude of impact of varying SA may be substantial. Expected returns for equities, as constructed in this paper, do not depend on SA; thus, *ceteris paribus*, the individual SCAR of equities decreases as SA increases. For bonds, higher risk-free interest rates –considering the duration of the bond– lead to higher expected returns, *ceteris paribus*. The extent to which the capital charge for interest rate increases depends on the (shock-weighted) duration gap between assets and liabilities. Generally, a low SA in conjunction with low interest rates on bonds tends to increase the relative attractiveness of equities. This situation occurs mid-2020 for example, and the observed shifts in asset classes are in line with expectation. High SA generally reduces the attractiveness of equities and, *ceteris paribus*, bonds are selected instead. As already mentioned, the extent to which this holds depends on the duration gap. With its longer liabilities duration, the life organization may generally be assumed to have a greater capacity for increasing duration in assets without any negative impact on interest rate risk. A notable instance of comparatively high bond interest rates and varying SA occurred between 2010 to 2012. Note that only the risk-free interest rate for ten years is shown in Fig. 7 and that interest rates for other bond durations may not shift in parallel. In addition, the development of credit spreads is not depicted. Remember that the short-term risk-free interest rate is used to form the expected returns for equities ( $r_f + 6\%$ ); therefore, the strong increase in the short-term risk-free rate from mid-2022 onwards may add to the high share of equities within this time range.

## 6. Conclusion

The contribution of this paper is twofold. First, the start-to-low drawdown (SLD) was proposed as a downside path-dependent risk measure tailored to situations where an investor, with fixed liabilities over the window under consideration, is at risk of being forced to act under unfavorable conditions after an abrupt market decline because their own funds become insufficient.

Second, an application of the SLD was developed in an environment where the investor is subject to regulations relating to capital requirements, such as those in the banking and insurance industries. In this context, if capital requirements must be met on an ongoing basis, drawdowns must not only remain below equity levels but should also avoid a situation where regulatory requirements are breached. To trade off the objective of lowering drawdowns with a desire for higher expected returns and lower costs of capital, the solvency cost adjusted return (SCAR) including [the cost of] drawdown (SCARD) is proposed as a measure and an objective for portfolio optimization. The specific regulatory framework employed in this paper is that of the European insurance industry, Solvency II. Model life and non-life insurance undertakings are constructed from EIOPA market overview data.

A challenge when considering the Solvency II capital requirement in the objective is that certain CR calculation aspects drive the solution towards low cardinal-

ity. No diversification is realized on the most granular level of capital requirement calculation, such as equity type 1 risk or spread risk. Therefore, without further modification or constraint, SCAR-optimal portfolios place high weights on the few assets that offer the best return to the submodule-CR trade-off. Although there is diversification among the sub-modules, in order for it to have an impact on the optimization result, the differences in expected asset returns must not be so large that the reduction in CR through diversification becomes irrelevant vis-à-vis the difference in expected returns. The addition of actual risk may be used to promote diversification, such as when adding the cost of drawdown (CoD) to the objective. However, while diversification reduces drawdown, so too do higher returns. Thus, the impact of adding CoD into return dependency is not known a priori.

In the empirical part of this paper, data on European stock, corporate bond and government bond indices from 2005 to 2024 was employed. Empirical results indicate that SCARD encourages diversification by incorporating the benefits of reduced drawdown, which arise with better diversification. In addition, the imposition of a weight penalty was investigated as a further measure to address the all-in tendency of SCAR, and was found to be a sound measure for improving SCAR optimization.

Out-of-sample properties of SCAR and SCARD optimal portfolios were analyzed empirically and compared with those of reference portfolios. Out-of-sample SCAR and SCARD following both SCAR and SCARD as objectives are higher than those of the equally weighted reference portfolio. SCARD-optimal portfolios show lower cost of solvency capital (CoSC) and lower drawdown than SCARD-optimal portfolios, but also lower returns. Return differences outweigh those of other components, thus the SCAR and SCARD of SCAR-optimal portfolios tend to be higher than those of their SCARD-optimal counterparts. Like SLD, other risk measures such as volatility, value at risk (VaR), expected shortfall (ES), and maximum drawdown (MDD) are also lower in SCARD- compared to SCAR-optimal portfolios. SCARD-optimal portfolios tend to allocate less weight to equities as an asset class than their SCAR-optimal counterparts.

Several areas for future research remain. The modeling of the CoD was kept simple on purpose here, with one linear cost function only. In practice, the slope of the CoD function may become steeper as critical levels are undercut more severely. Also, it may be justified to add on-hit penalties, for example if the personnel cost of having to start dealing with the situation is deemed relevant.

In the empirical section of this paper, optimization with regard to CoD was conducted using three years of historical data for training, irrespective of the market regime. If the focus is on reducing drawdown within negative market environments, training data that replicates or simulates adverse scenarios may be more suitable.

In practice, optimal asset allocation is often influenced by additional constraints, and incorporating these may lead to interesting results for academia as well as application in practice. First, asset allocations may be subject to restrictions or limits on individual weights or class weights, for example that equities in sum must not exceed a certain fraction. Such limits may, for example, flow from organizational

risk policies. Second, if other processes are leading the search for optimal portfolios, that portfolio could serve as a starting point. An attempt could then be made to improve it from a SCAR(D) point of view by reducing the CoD or the CoSC while limiting the deviation from the starting point. The corresponding objective function could be constructed in a way that the SCAR(D) is expanded by a penalty term which increases with deviation from said starting point. Third, the current SCAR and SCARD targets do not yet take transaction costs into account. Given the magnitude of the changes in asset weights observed over time in the empirical section, the inclusion of transaction costs could be a justified extension.

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## Appendix A. Start-to-low drawdown derivation

This appendix provides an exemplary derivation of the start-to-low drawdown (SLD). The example employed is that of an arithmetic Brownian motion (ABM). In textbooks on stochastic finance, the probability density function (pdf) of the maximum of an ABM is probably the common starting point for deriving an extremum of an ABM. In this case, finding the expectation of the maximum of a BM with drift  $\mu$  is the same as finding the negative of the minimum of a BM with  $-\mu$ ; thus, conversion from expected maximum to minimum will be relatively easy in the arithmetic case.

Following [Shreve \*et al.\* \(2004\)](#) Eq. (7.2.5) on page 297 and [Privault \(2022\)](#) Eq. (10.14) on page 308, the pdf of the maximum of an ABM is

$$\varphi_{X_0^+}(a) = \sqrt{\frac{2}{\pi T}} e^{-\frac{(a-\mu T)^2}{2T}} - 2\mu e^{2\mu a} \Phi\left(-\frac{a-\mu T}{\sqrt{T}}\right), \quad a \geq 0. \quad (\text{A.1})$$

where the notion used here is aligned with that in [Privault \(2022\)](#).

Following the approach used in [Privault \(2024\)](#), Exercise 10.2 on page 185,<sup>g</sup> the problem is reformulated as follows:

$$\varphi(y) = \sigma \left[ \sqrt{\frac{2}{\pi T}} \cdot e^{-\frac{\left(\frac{y-\mu T}{\sigma}\right)^2}{2T}} - 2\frac{\mu}{\sigma} e^{\left(\frac{2\mu y}{\sigma}\right)} \cdot \Phi\left(\frac{-y-\mu T/\sigma}{\sqrt{T}}\right) \right] \quad (\text{A.2})$$

The expected maximum

$$\mathbb{E}[\varphi(y)]_{y \geq 0} = \int_0^\infty \varphi(y) \cdot y \, dy \quad (\text{A.3})$$

then is

$$= \underbrace{\sigma \int_0^\infty \sqrt{\frac{2}{\pi T}} \cdot y \cdot e^{s_1} \, dy}_\text{I} - \underbrace{\sigma \int_0^\infty \frac{2\mu y}{\sigma} \cdot e^{\left(\frac{2\mu y}{\sigma}\right)} \cdot \Phi(s_2) \, dy}_\text{II} \quad (\text{A.4})$$

with

$$s_1 = -\left(\frac{\left(y - \frac{\mu T}{\sigma}\right)^2}{2T}\right) \quad \text{and} \quad s_2 = \left(\frac{-y - \frac{\mu T}{\sigma}}{\sqrt{T}}\right)$$

which is split into two parts. For better readability, part II is investigated first. Part II is addressed using integration by parts

$$\int_0^\infty u(y) v'(y) \, dy = [u(y) v(y)]_0^\infty - \int_0^\infty v(y) u'(y) \, dy \quad (\text{A.5})$$

<sup>g</sup>The step-by-step derivation presented in this section was conducted because an imprecision was identified in the original source. The final result in [Privault \(2024\)](#) did not match the outcomes of our simulation study. To locate the source of the discrepancy, a full re-derivation was carried out. The version presented here corrects that imprecision.

where  $u(y)$  and  $v'(y)$  are defined as follows:

$$\Pi = \sigma \int_0^\infty \frac{2\mu y}{\sigma} \cdot e^{\left(\frac{2\mu y}{\sigma}\right)} \cdot \Phi(s_2) dy = 2\mu \int_0^\infty \underbrace{ye^{\left(\frac{2\mu y}{\sigma}\right)}}_{v'(y)} \underbrace{\Phi(s_2)}_{u(y)} dy \quad (\text{A.6})$$

Comment: the definition of  $v'(y)$  here differs from that in [Privault \(2024\)](#) page 185: constants are placed in front of the integral in this document.

To solve Part II,  $v(y)$  and  $u'(y)$  have to be found. First,  $v(y)$  is addressed, again using integration by parts. For clarity of terms,  $f$  and  $g$  represent the respective parts instead of  $u$  and  $v$ .

$$v(y) = \int v'(y) = \int \underbrace{y}_f \cdot \underbrace{e^{(s_3 \cdot y)}}_{g'} dy \quad \text{with} \quad s_3 = \frac{2\mu}{\sigma} \quad (\text{A.7})$$

and accordingly:

$$\begin{aligned} f' &= 1 \\ g &= \frac{1}{s_3} e^{(s_3 \cdot y)} \end{aligned}$$

Integration by parts:

$$\begin{aligned} v(y) &= fg - \int f'g dy = \frac{y}{s_3} e^{(s_3 \cdot y)} - \int \frac{1}{s_3} \cdot e^{(s_3 \cdot y)} dy \\ &= \frac{y}{s_3} e^{(s_3 \cdot y)} - \frac{1}{s_3^2} e^{(s_3 \cdot y)} \\ &= \frac{1}{s_3} e^{(s_3 \cdot y)} \cdot \left(y - \frac{1}{s_3}\right) \end{aligned}$$

replacing  $s_3$  with its full expression leads to

$$v(y) = \frac{\sigma}{2\mu} e^{\left(\frac{2\mu}{\sigma} y\right)} \left(y - \frac{\sigma}{2\mu}\right). \quad (\text{A.8})$$

The second additional term for solving Part II is  $u'(y)$ . To do so, the known derivative of the normal cdf is applied, which is the normal pdf.

Probably the best known relationship is that for

$$\Phi\left(\frac{x - \mu}{\sigma}\right)$$

the pdf is

$$\phi(x) = \frac{1}{\sigma} \cdot \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}.$$

Applying the same relationship to  $u(y)$  obtains the following results:

$$\begin{aligned} u(y) &= \Phi\left(\frac{(-y - \frac{\mu T}{\sigma})}{\sqrt{T}}\right) \\ u'(y) &= \frac{-1}{\sqrt{T}} \cdot \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{(-y - \frac{\mu T}{\sigma})^2}{2T}\right)} \end{aligned} \quad (\text{A.9})$$

Comment: It is  $-1/\sqrt{T}$  and not  $1/\sqrt{T}$  here following the application of the chain rule, because it is  $-y$ , not  $+y$  in  $u(y)$ .

With the results of Eq. (A.8) and Eq. (A.9) as well as the definitions in Eq. (A.6), the components needed to solve Part II (Eq. (A.6)) using integration by parts (Eq. (A.5)) are known:

$$\begin{aligned} \text{II} &= 2\mu \int_0^\infty \underbrace{ye^{\left(\frac{2\mu y}{\sigma}\right)}}_{v'(y)} \underbrace{\Phi(s_2)}_{u(y)} dy \\ &= 2\mu \left[ \underbrace{\Phi(s_2)}_{u(y)} \underbrace{\frac{\sigma}{2\mu} e^{\left(\frac{2\mu}{\sigma}y\right)} \left(y - \frac{\sigma}{2\mu}\right)}_{v(y)} \right]_0^\infty \\ &\quad - 2\mu \int_0^\infty \underbrace{\frac{\sigma}{2\mu} e^{\left(\frac{2\mu}{\sigma}y\right)} \left(y - \frac{\sigma}{2\mu}\right)}_{v(y)} \underbrace{\frac{-1}{\sqrt{2\pi T}} e^{-\frac{(-y - \frac{\mu T}{\sigma})^2}{2T}}}_{u'(y)} dy \end{aligned} \quad (\text{A.10})$$

Recall that

$$s_2 = \left( \frac{-y - \frac{\mu T}{\sigma}}{\sqrt{T}} \right)$$

For  $y \rightarrow \infty$  the value of  $s_2$  goes to  $-\infty$ . The value of the normal cdf is zero at minus infinity, i.e.  $u(y \rightarrow \infty) = 0$  and consequently  $u(\infty)v(\infty) = 0$ . Thus

$$u(\infty)v(\infty) - u(0)v(0) = 0 - \Phi\left(-\frac{\mu\sqrt{T}}{\sigma}\right) \cdot \left(-\frac{\sigma^2}{4\mu^2}\right)$$

simplifies the first half of Part II. For the second half, the constant term is taken out before the integral, and the remainder multiplied out

$$\begin{aligned}
\Pi &= 2\mu \cdot (-1)\Phi\left(-\frac{\mu\sqrt{T}}{\sigma}\right)\left(-\frac{\sigma^2}{4\mu^2}\right) \\
&\quad + 2\mu \cdot \frac{\sigma}{2\mu\sqrt{2\pi T}} \int_0^\infty \left(y - \frac{\sigma}{2\mu}\right) e^{\left(\frac{2T \cdot 2\mu y}{\sigma} - \frac{y^2}{\sigma} - \frac{2T\mu y}{\sigma} - \frac{\mu^2 T^2}{\sigma^2}\right)} dy \\
&= \frac{\sigma^2}{2\mu} \Phi\left(-\frac{\mu\sqrt{T}}{\sigma}\right) + \frac{\sigma}{\sqrt{2\pi T}} \int_0^\infty \left(y - \frac{\sigma}{2\mu}\right) e^{\left(\frac{2T\mu y}{\sigma} - \frac{y^2}{2T} - \frac{\mu^2 T^2}{\sigma^2}\right)} dy \\
&= \frac{\sigma^2}{2\mu} \Phi\left(-\frac{\mu\sqrt{T}}{\sigma}\right) + \frac{\sigma}{\sqrt{2\pi T}} \int_0^\infty \left(y - \frac{\sigma}{2\mu}\right) e^{\left(-\frac{\left(y - \frac{\mu T}{\sigma}\right)^2}{2T}\right)} dy
\end{aligned}$$

Recall that

$$s_1 = -\left(\frac{\left(y - \frac{\mu T}{\sigma}\right)^2}{2T}\right)$$

Applying the definition of  $s_1$  and splitting the integral leads to

$$\Pi = \frac{\sigma^2}{2\mu} \Phi\left(-\frac{\mu\sqrt{T}}{\sigma}\right) + \frac{\sigma}{\sqrt{2\pi T}} \int_0^\infty y \cdot e^{s_1} dy - \frac{\sigma^2}{2\mu\sqrt{2\pi T}} \int_0^\infty e^{s_1} dy \quad (\text{A.11})$$

With  $s_1$  now being present in both, Part I and II, it is useful to merge both parts at this stage. Note that the sign will change on Part II because it is part I minus part II (cf. Eq. (A.4)). The resulting complete equation for the expected maximum

$$\begin{aligned}
\mathbb{E}[\varphi(y)]_{y \geq 0} &= \sigma \sqrt{\frac{2}{\pi T}} \int_0^\infty y e^{s_1} dy - \frac{\sigma^2}{2\mu} \Phi\left(-\frac{\mu\sqrt{T}}{\sigma}\right) \\
&\quad - \frac{\sigma}{\sqrt{2\pi T}} \int_0^\infty y \cdot e^{s_1} dy + \frac{\sigma^2}{2\mu\sqrt{2\pi T}} \int_0^\infty e^{s_1} dy
\end{aligned}$$

can then be simplified to

$$\mathbb{E}[\varphi(y)]_{y \geq 0} = \frac{\sigma}{\sqrt{2\pi T}} \int_0^\infty y \cdot e^{s_1} dy - \frac{\sigma^2}{2\mu} \Phi\left(-\frac{\mu\sqrt{T}}{\sigma}\right) + \frac{\sigma^2}{2\mu\sqrt{2\pi T}} \int_0^\infty e^{s_1} dy \quad (\text{A.12})$$

Recall the definition of the normal cdf

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{1}{2}a^2} da \quad (\text{A.13})$$

In order to solve  $e^{s_1}$ , a change of variables is employed. First,  $e^{s_1}$  is slightly reformulated

$$e^{s_1} = e^{-\frac{\left(y - \frac{\mu T}{\sigma}\right)^2}{2T}} = e^{-\frac{1}{2}\left(\frac{\left(y - \frac{\mu T}{\sigma}\right)}{\sqrt{T}}\right)^2}$$



then, the following substitution is applied:

$$u = \frac{y - \frac{\mu T}{\sigma}}{\sqrt{T}} \rightarrow y = u\sqrt{T} + \frac{\mu T}{\sigma} \quad (\text{A.14})$$

$$du = \frac{dy}{\sqrt{T}} \rightarrow dy = du\sqrt{T} \quad (\text{A.15})$$

$$\text{bounds : } [0, \infty] \rightarrow \left[ \frac{0 - \frac{\mu T}{\sigma}}{\sqrt{T}}, \frac{\infty - \frac{\mu T}{\sigma}}{\sqrt{T}} \right] = \left[ -\frac{\mu T}{\sigma\sqrt{T}}, \infty \right] \quad (\text{A.16})$$

Using the substitution just described in conjunction with the definition of the normal cdf and considering the symmetry of the normal pdf, it is possible to apply the following transformations:

$$\begin{aligned} \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-\frac{(y - \frac{\mu T}{\sigma})^2}{2T}} dy &= \frac{1}{\sqrt{2\pi}} \int_{-\frac{\mu\sqrt{T}}{\sigma}}^\infty e^{-\frac{1}{2}u^2} du \cdot \sqrt{T} \\ &= \sqrt{T} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\mu\sqrt{T}}{\sigma}} e^{-\frac{1}{2}u^2} du \end{aligned}$$

and consequently

$$\frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-\frac{(y - \frac{\mu T}{\sigma})^2}{2T}} dy = \sqrt{T} \Phi\left(\frac{\mu\sqrt{T}}{\sigma}\right) \quad (\text{A.17})$$

Applying Eq. (A.17) to the overall equation (A.12) transforms it to

$$\mathbb{E}[\varphi(y)]_{y \geq 0} = \frac{\sigma}{\sqrt{2\pi T}} \int_0^\infty y \cdot e^{s_1 y} dy - \frac{\sigma^2}{2\mu} \Phi\left(-\frac{\mu\sqrt{T}}{\sigma}\right) + \frac{\sigma^2}{2\mu} \Phi\left(\frac{\mu\sqrt{T}}{\sigma}\right) \quad (\text{A.18})$$

A similar substitution is applied to the remaining integral.

$$\frac{\sigma}{\sqrt{2\pi T}} \int_0^\infty y e^{-\frac{1}{2}\left(\frac{y - \frac{\mu T}{\sigma}}{\sqrt{T}}\right)^2} dy = \frac{\sigma}{\sqrt{2\pi T}} \int_{-\frac{\mu\sqrt{T}}{\sigma}}^\infty \left(u\sqrt{T} + \frac{\mu T}{\sigma}\right) e^{-\frac{1}{2}u^2} du \sqrt{T}$$

The resulting term then can be split into two part and each processed separately:

$$\begin{aligned}
& \frac{\sigma}{\sqrt{2\pi T}} \int_{-\frac{\mu\sqrt{T}}{\sigma}}^{\infty} \left( u\sqrt{T} + \frac{\mu T}{\sigma} \right) e^{-\frac{1}{2}u^2} du \sqrt{T} \\
&= \frac{\sigma}{\sqrt{2\pi}} \int_{-\frac{\mu\sqrt{T}}{\sigma}}^{\infty} \sqrt{T} \cdot u e^{-\frac{1}{2}u^2} du + \frac{\sigma}{\sqrt{2\pi}} \cdot \frac{\mu T}{\sigma} \int_{-\infty}^{\frac{\mu\sqrt{T}}{\sigma}} e^{-\frac{1}{2}u^2} du \\
&= \frac{\sigma\sqrt{T}}{\sqrt{2\pi}} \int_{-\frac{\mu\sqrt{T}}{\sigma}}^{\infty} u e^{-\frac{1}{2}u^2} du + \mu T \Phi\left(\frac{\mu\sqrt{T}}{\sigma}\right) \\
&= \frac{\sigma\sqrt{T}}{\sqrt{2\pi}} \left[ -e^{-\frac{1}{2}u^2} \right]_{-\frac{\mu\sqrt{T}}{\sigma}}^{\infty} + \mu T \Phi\left(\frac{\mu\sqrt{T}}{\sigma}\right) \\
&= \frac{\sigma\sqrt{T}}{\sqrt{2\pi}} \left[ e^{-\frac{1}{2}\infty} - \left( -e^{-\frac{1}{2}\left(\frac{-\mu\sqrt{T}}{\sigma}\right)^2} \right) \right] + \mu T \Phi\left(\frac{\mu\sqrt{T}}{\sigma}\right) \\
&= \frac{\sigma\sqrt{T}}{\sqrt{2\pi}} \left( 0 + e^{-\frac{1}{2}\frac{\mu^2 T}{\sigma^2}} \right) + \mu T \Phi\left(\frac{\mu\sqrt{T}}{\sigma}\right)
\end{aligned}$$

thus in summary:

$$\frac{\sigma}{\sqrt{2\pi T}} \int_0^{\infty} y e^{-\frac{1}{2}\left(\frac{y-\mu T}{\sqrt{T}}\right)^2} = \frac{\sigma\sqrt{T}}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{\mu^2 T}{\sigma^2}} + \mu T \Phi\left(\frac{\mu\sqrt{T}}{\sigma}\right) \quad (\text{A.19})$$

Comment on  $\int u e^{0.5u^2}$ : no explicit way of solving is provided. Considering that  $f(x) = 2au e^{au^2}$  is the derivative of  $F(x) = e^{au^2}$  and applying  $a = -0.5$  leads to the relationship used.

Merging Eq. (A.19) back into the latest complete equation (A.18) leads to

$$\begin{aligned}
\mathbb{E}[\varphi(y)]_{y \geq 0} &= \frac{\sigma\sqrt{T}}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{\mu^2 T}{\sigma^2}} + \mu T \Phi\left(\frac{\mu\sqrt{T}}{\sigma}\right) \\
&\quad - \frac{\sigma^2}{2\mu} \Phi\left(\frac{-\mu\sqrt{T}}{\sigma}\right) + \frac{\sigma^2}{2\mu} \Phi\left(\frac{\mu\sqrt{T}}{\sigma}\right)
\end{aligned}$$

which can be summarized as

$$\mathbb{E}[\varphi(y)]_{y \geq 0} = \frac{\sigma\sqrt{T}}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{\mu^2 T}{\sigma^2}} + \left( \mu T + \frac{\sigma^2}{2\mu} \right) \Phi\left(\frac{\mu\sqrt{T}}{\sigma}\right) - \frac{\sigma^2}{2\mu} \Phi\left(\frac{-\mu\sqrt{T}}{\sigma}\right) \quad (\text{A.20})$$

Because of the symmetry of the ABM, the expected low (minimum)  $\mathbb{E}[L]$  of an ABM with drift  $\mu$  is the same as the expected high (maximum)  $\mathbb{E}[H]$  with drift  $-\mu$  (e.g. Magdon-Ismail *et al.* (2004)), i.e.

$$\mathbb{E}[L|\mu, \sigma] = -\mathbb{E}[H|-\mu, \sigma] \quad (\text{A.21})$$

if the minimum was interpreted as a negative number. In the definition of the SLD used in this paper, drawdown is displayed as a positive number, thus the minus sign is not applied.

The expected SLD of an ABM is therefore

$$\mathbb{E}[SLD]_{ABM} = \frac{\sigma\sqrt{T}}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{\mu^2 T}{\sigma^2}} - \left(\mu T + \frac{\sigma^2}{2\mu}\right) \Phi\left(\frac{-\mu\sqrt{T}}{\sigma}\right) + \frac{\sigma^2}{2\mu} \Phi\left(\frac{\mu\sqrt{T}}{\sigma}\right) \quad (\text{A.22})$$

Note that the start value is not relevant for calculating the SLD of a given ABM process but only the running difference to the start value.

## Appendix B. Assumptions and remarks on operational risk and loss absorbing capacities

For  $OP_{\text{risk}}$ , this is an easy assumption to make within the context of this paper, as there is no direct dependency between that risk and the optimized investments, although there might be some direct dependency for  $Adj_{DT}$ . However, both  $OP_{\text{risk}}$  and  $Adj_{DT}$  are often not of very large magnitude.

A component that may be of high absolute magnitude in practice is  $Adj_{TP}$ . Recall that  $Adj_{TP}$  originates from the undertaking's option to reduce future discretionary benefits (FDB) in the case of negative developments.  $Adj_{TP}$  may have a substantial impact on MR, life and health similar to life techniques SLT underwriting, and counterparty default risk. However, it is highly dependent on the assumptions of how FDB are reduced in case of losses, on management action, and possibly also on lapse risk as a function of reduced FDB. In summary, this is specific to the situation in each organization and thus difficult to add to a general model undertaking. A reader intending to use the SCARD with  $Adj_{TP}$  may also want to consider the conceptual question of the meaning of the return in SCAR: if additional returns are to be passed on to policyholders in the form of a high fraction of FDB, should the return of the entire portfolio be used, or rather just the fraction that the organization would retain?

## Appendix C. Solvency II market risk calculation

Market risk captures the risk of losses due to fluctuations in the value of assets and liabilities resulting from changes in financial markets. It is further divided into several sub-modules:

- (1) Interest rate risk: the risk of loss from changes in interest rates affecting the value of assets and liabilities.
- (2) Equity risk: the risk associated with fluctuations in stock markets, divided into type 1 equities (listed) and type 2 equities (unlisted).
- (3) Property risk: reflects the risk of loss from changes in the value of real estate investments.
- (4) Spread risk: associated with the impact of changes in the credit spread of marketable securities over the risk-free interest rate.
- (5) Currency risk: the risk of loss from changes in foreign exchange rates.

- (6) Concentration risk: reflects the risk from high exposure to a particular counterparty (corporate group).

In the empirical part of this paper, only equity, spread, and interest rate risk are calculated, as the other risks are assumed to be zero. Therefore, emphasis is hereafter placed on these three.

### C.1. Equity risk

The total equity risk capital requirement ( $SCR_{\text{equity}}$ ) under Solvency II, which includes both, equities of type 1 (eqt1) and type 2 (eqt2), is calculated using the formula:

$$SCR_{\text{equity}} = \sqrt{SCR_{\text{eqt1}}^2 + 2 \cdot 0.75 \cdot SCR_{\text{eqt1}} \cdot SCR_{\text{eqt2}} + SCR_{\text{eqt2}}^2} \quad (\text{C.1})$$

The value invested in a particular type of equity is denoted as  $V$ , while  $SA$  denotes the symmetric adjustment. For equity risk, differentiated by type 1 (listed) and type 2 (unlisted) equities, the SCR is as follows:

$$SCR_{\text{eqt1}} = (39\% + SA) \cdot V \quad (\text{C.2})$$

$$SCR_{\text{eqt2}} = (49\% + SA) \cdot V \quad (\text{C.3})$$

This is unless the equity is a strategic investment in a related undertaking, in which case the capital requirement is  $22\% \cdot V$ .

The  $SA$  is based on an equity index that measures the market price of a diversified portfolio of equities that is representative of the nature of equities typically held by insurance and reinsurance undertakings.

$$SA = \frac{1}{2} \cdot \left( \frac{CI - AI}{AI} - 8\% \right) \quad (\text{C.4})$$

where  $CI$  denotes the current level of the equity index and  $AI$  denotes the weighted average of the daily levels of the index over the last 36 months. Thus, the capital charge per  $V$  is reduced if index levels are below average, providing some relief on the SCR to EOF ratio during drawdown. The value of  $SA$  is capped at plus 10% and minus 10%. The value and time series of the equity index as well as the value of  $SA$  are provided by the authority behind Solvency II.<sup>h</sup>

### C.2. Spread risk

Spread risk calculation assesses the impact of a specified widening of credit spreads. Rulings are in place for three different types of instruments: (i) bonds and loans ( $SCR_{\text{bonds}}$ ), (ii) securitization positions ( $SCR_{\text{securitization}}$ ) and (iii) credit derivatives ( $SCR_{\text{cd}}$ ). The SCRs for all three types are summed up as follows

$$SCR_{\text{spread}} = SCR_{\text{bonds}} + SCR_{\text{securitisation}} + SCR_{\text{cd}} \quad (\text{C.5})$$

<sup>h</sup><https://www.eiopa.europa.eu/tools-and-data/symmetric-adjustment-equity-capital-charge>

Under certain conditions, the spread shock is zero. This applies, for example, to bonds or loans of the European Central Bank and those from EU Member States' central government and central banks, if they are denominated and funded in the domestic currency of the respective central government and central bank. Apart from special rulings<sup>i</sup> the spread shock is a function of the adjusted duration of the bonds or loans and their credit rating. Capital requirements increase with longer modified duration and with worse credit ratings.

As only bonds form part of assets for this publication, the spread risk will be equal to  $SCR_{\text{bonds}}$ , which is

$$SCR_{\text{bonds}} = \sum_i \text{Spread Shock Bond}_i \times V_i \quad (\text{C.6})$$

Indices are used as asset data without any look-through being performed. In particular, the average rating of an index is used to calculate the credit quality step for the entire value invested in that index.

### C.3. Interest rate risk

The capital requirement for interest rate risk considers the scenarios of (i) an increase and (ii) a decrease in the term structure of interest rates. In both scenarios, the shocks to interest rates for each maturity are provided by regulation, including rules for intra- and extrapolation. In addition, bounds are set: the increase in basic risk-free interest rates at any maturity is at least one percentage point, and in a decrease scenario, for negative basic risk-free interest rates, the decrease is nil. The interest rate risk to be considered then is the one that results in a higher capital charge.

For simplicity, it is assumed in this paper that all technical provisions originate from one line of business, with the calculation being based on an assumed average duration. The up and down scenarios of the interest rate are calculated as follows:

$$IR_{up} = \sum_i V_i \cdot dur_i \cdot r_i \cdot str_{(i,up)} - BE \cdot dur_{BE} \cdot r_{BE} \cdot str_{(BE,up)} \quad (\text{C.7})$$

$$IR_{down} = \sum_i V_i \cdot dur_i \cdot r_i \cdot str_{(i,down)} - BE \cdot dur_{BE} \cdot r_{BE} \cdot str_{(BE,down)} \quad (\text{C.8})$$

where  $V_i$  denotes the value of position  $i$ ,  $dur_i$  the duration of position  $i$  and  $r_i$  the relevant risk-free rate for the duration of position  $i$ .  $str_{(i,up)}$  and  $str_{(i,down)}$  the relative upward and downward stresses of interest rate for the duration of  $i$  and  $BE$  the best estimate liability.

No other element of liabilities is modeled in Interest Rate Risk. Thus,  $BE$  is all that is considered on the financing side of the balance sheet.

<sup>i</sup>EIOPA Delegated Regulation Article 180

#### C.4. Not used market risk sub-modules

All market risk modules other than equity, spread, and interest rate risk are assumed to be zero in this paper for simplicity.

#### Appendix D. SCAR optimal weights of two equal-CR assets

The setup for Fig. 8 matches that for Fig. 6 in section 4.3, with one exception: the standalone capital requirement (CR) for the bond matches now that of the equity, i.e., is 39%. This corresponds to the CR of a BBB bond with a duration of 38 years. While these properties may seem exceptional, the scenario is chosen to illustrate the claim made in section 4.3 that the convergence to 'all-equity' and 'all-bond' becomes symmetrical (in terms of weights-distance to the most diversified portfolio, which lies at 0.5:0.5) if standalone CRs are also symmetrical.

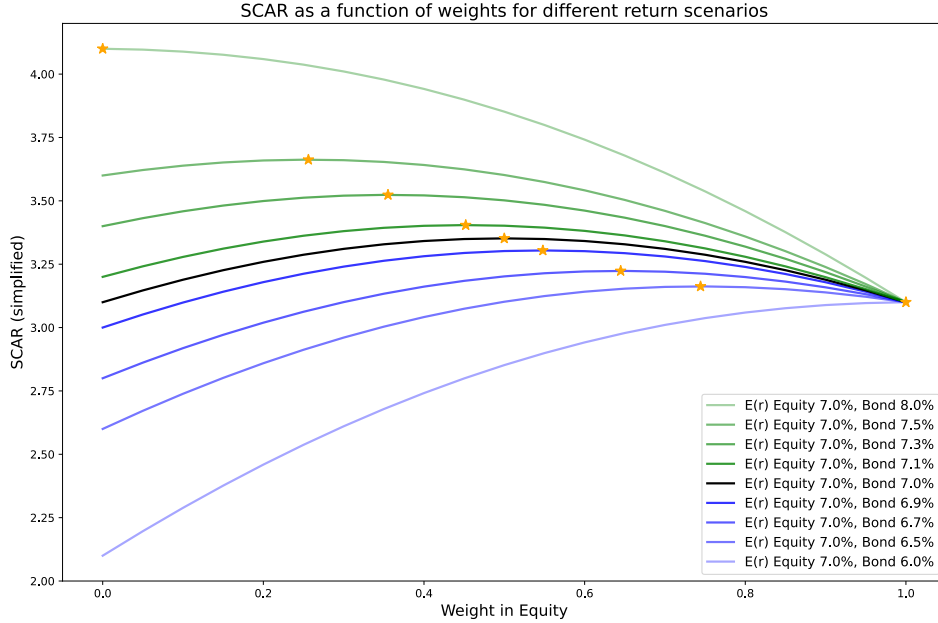


Fig. 8. SCAR (simplified) for a bond and equity portfolio as a function of equity weight ( $w_{eq}$ ) and bond weight ( $w_b = 1 - w_{eq}$ ). The standalone CR for the equity is 39% and the spread risk for a bond with a rating of BBB and a duration of 38 years is also 39% (interest rate risk is ignored). The cost of capital is assumed to be 10% of the market risk CR and equity return is assumed to be 7%. Expected bond returns vary from 6% to 8% to show the sensitivity of SCAR-optimal portfolios (marked with stars) to the magnitude of differences in expected return. The black line is the situation in which standalone equity and bond SCAR are equal to each other. Green indicates scenarios in which the standalone SCAR of the bond is higher than that of the equity and blue indicates the opposite scenarios.

## Appendix E. Reference portfolio choice

The 1/N portfolio is selected as a benchmark in this paper. Some considerations on this choice and alternatives are presented here: The equal-weights portfolio is selected as a reference because of the simplicity and clarity it offers: it is clear what it is and what it is not. It is neither optimized for return nor for regulatory or other risks. Another option may be to choose minimum risk portfolios (e.g., minimum-volatility or minimum-VaR) for reference. SCAR and SCARD reflect return and regulatory capital requirements. Any minimum-risk portfolio would neglect the return component, which is the largest contributor to SCAR and SCARD (cf. Table 1). Obviously, a reference portfolio could be constructed that minimizes risk, subject to the condition that it matches the level of return. Still, optimizing only within the investment portfolio would, for example, neglect the aspect of the duration gap, which has a major impact on market risk and thus CoSC. Therefore, to establish a reference that accounts for duration gap considerations, a penalty for a higher duration gap was needed, and a penalty weight had to be defined. While this approach may be possible and feasible, it requires several assumptions and adds complexity. In practice, investment selection processes may reasonably be assumed to consider expected return and the duration gap. Consequently, the reference for comparing SCAR and SCARD optimization is likely to be the portfolio resulting from the investment decision-making process otherwise in place at the company.

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