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## A dynamic model of authority in organizations

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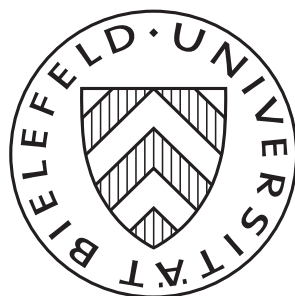


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# A dynamic model of authority in organizations

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Bingbing Li and Manuel Foerster



# A dynamic model of authority in organizations

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## Abstract

In our principal-agent model, the principal can repeatedly delegate authority to an agent with uncertain preferences or take the decisions himself. The principal learns the state at the end of each period and then updates his belief about the agent's bias based on the decision implemented if he delegated authority. We demonstrate that equilibria are characterized by an “imitation” interval of agent types (biases) who mimic less biased types in order to be retained. Interestingly, the principal generally benefits from the agent's imitation compared to a benchmark. Furthermore, comparative statics reveal that, surprisingly, the principal may be worse off with better information. Finally, an extension to finitely many periods shows that the imitation interval gradually shifts, such that agent types within the interval imitate less biased types.

**JEL classification:** D23; D82; D83; D73; C72.

**Keywords:** Delegation, preference uncertainty, private information, dynamic game, organizational design.

## 1 Introduction

In various organizational contexts, decision-makers engage better-informed experts lower in the hierarchy for assistance or advice. For instance, policy-makers seek informational support from subordinate bureaucrats when formulating draft policies. However, bureaucrats often have a partisan orientation, which may not

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align with that of the current policy-maker and affect their behavior.<sup>1</sup> Crucially, the policy-maker—at least initially—faces uncertainty regarding these partisan orientations.

When delegating decision-making authority to subordinates, the policy-maker therefore has to take into account both their expertise and potential conflicts of interest. Over time, the chosen subordinate may then reveal her partisan preferences through her decisions. Another important aspect therefore is the dynamic nature of the delegation process (under preference uncertainty): The policy-maker will not only get to know the environment in her policy domain but also learn the preferences of subordinates, allowing her to reassess her choices.

The trade-off between keeping authority and delegating it to a specialist with different objectives has been extensively discussed in the organizational economics literature. Yet, the preference conflict between principal and agent has been considered common knowledge (e.g., [Aghion and Tirole, 1997](#); [Dessein, 2002](#); [Alonso and Matouschek, 2008](#)). In this paper, we build a theoretical model in which a principal repeatedly decides between keeping authority and granting decision rights to an agent with uncertain preferences. We demonstrate how the prospect of repeated influence on the decision-making process can discipline experts with partisan interests to behave like “good” ones whose objectives are roughly in line with the principal’s.

In our model, a principal (he) has to take a decision whose payoff depends on the evolving state of the world in each of two periods. He can either keep authority and decide himself or delegate authority to a better-informed agent (she) with uncertain preferences. At the end of the first period, the principal observes the true state of the world, thereby acquiring partial insights into the state of the next period (i.e., the state is correlated across periods). If the principal has delegated authority in the first period, he forms expectations regarding the agent’s type based on this information and the decision implemented. At the beginning of the second period, the principal then decides again whether to keep authority or delegate it to a better-informed agent—if he delegated authority in the first period, the principal may either retain or replace the current agent.

Observe that a principal who prefers to keep decision-making authority in the

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<sup>1</sup>In the U.K., bureaucrats have tried to “frustrate this process of Brexit because it goes against the grain so fundamentally”, see [https://www.bbc.com/news/uk-42782637?utm\\_source](https://www.bbc.com/news/uk-42782637?utm_source), accessed August 8, 2025. For similar cases in the U.S., see [https://www.washingtonpost.com/politics/2025/02/07/trump-resistance-federal-workers/?utm\\_source](https://www.washingtonpost.com/politics/2025/02/07/trump-resistance-federal-workers/?utm_source), accessed August 8, 2025. See also [Brehm and Gates \(1999\)](#), who conclude that “the overwhelming evidence [...] indicates that the bureaucrat’s own preferences have the greatest effect on behavior.”

first period will do so also in the second period, as he then is better informed. We thus focus on the case when the principal delegates decision-making authority in the first period. We first establish that, when the discount rate is low, the unique equilibrium is characterized by an imitation interval. While agents who have a low preference conflict with the principal choose their bliss point and are subsequently retained, agents within this interval imitate an agent of the former group in order to be retained as well. Having an intermediate conflict of interest with the principal, they trade off an immediate loss due to a deviation from their bliss point for a future gain due to being retained and thus able to choose their bliss point in the next period. In turn, the principal retains the agent despite the imitation because the expected preference conflict is small enough.

Second, when the discount rate is high, the value of retention for the agents is high, as it allows them to implement their bliss point in the second period. In equilibrium, the principal will then retain the agent upon observing potential mimicking only with a certain probability, and in turn, fewer agent types mimic.

We then show that the principal generally benefits from the agent's imitation. Disciplining the agent, it yields a gain from a better decision in the first period compared with a benchmark in which all agents choose their bliss points. In the second period, it then yields a potential loss from retaining rather biased agents that the principal would not retain if informed about their type. Interestingly, we establish that the loss is, generally, second order.

Next, we examine whether the principal benefits from a more slowly changing environment, such that he has better information about the state in the second period. Our findings reveal that, surprisingly, the principal may be worse off with better information. Some relatively biased agents then cease mimicking a less biased one and instead implement their bliss point in the first period, which reduces the principal's instantaneous, and potentially his total, utility. The reason is that imitation becomes less attractive compared to the alternative of the principal taking the decision himself, as the latter becomes better informed.

Finally, we extend the model to finitely many periods. We establish that the principal is less and less willing to retain the agent upon observing potential mimicking as opportunities to mimic in the future vanish. In equilibrium, the imitation interval thus gradually shifts, such that the agent types within the interval imitate less biased types.

**Related literature.** This paper is situated within the literature on the delegation of formal authority going back to the seminal work by [Aghion and Tirole](#)

(1997). They show that the principal may delegate authority in order to give the agent better incentives to acquire information. [Dessein \(2002\)](#) takes the information structure as given and investigates the principal’s trade-off between keeping authority and delegating it to the agent depending on their preference conflict. While [Dessein](#) shows that the principal prefers delegation over communication if objectives are sufficiently aligned, [Deimen and Szalay \(2019\)](#) show the reverse may hold if the agent has to decide on the amount of information she observes about each of two states. Other contributions have extended the framework to optimal delegation mechanisms ([Alonso and Matouschek, 2008](#)), transfers ([Krähmer, 2006](#); [Lim, 2012](#)), or both ([Foerster and Habermacher, 2025a](#)).<sup>2</sup> In contrast to these papers, we consider uncertainty regarding the preference conflict and show how the prospect of repeated influence disciplines experts with partisan interests.

The disciplining effect of repeated interaction relates our paper to the political agency literature, which has shown that elections can serve the purpose of disciplining a “bad” incumbent politician (agent) to behave like a “good” one to get re-elected by voters (the principal) ([Berganza, 2000](#); [Besley and Smart, 2007](#); [Foerster and Voss, 2022](#)). We show that the same mechanism applies to organizations when a principal faces uncertainty regarding the agent’s preferences and can re-allocate authority depending on her decisions. From a modeling point of view, our model differs from these papers in various aspects. First and foremost, we consider a continuum of types but abstract from differences in ability as in [Foerster and Voss \(2022\)](#). This has an interesting consequence: Since biased agents in [Foerster and Voss \(2022\)](#) imitate less able ones, the disciplining effect may, unlike in our model, be negative from the principal’s point of view. Second, we allow for a correlation of the state across periods, such that the principal may have better information in the second period. It turns out that, surprisingly, more information can sometimes harm the principal. Third, we extend the model to finitely many periods and investigate correlations of the state across periods. Moreover, the extension of our model to finitely many periods shares some features with [Banks and Duggan \(2008\)](#). Instead of a principal-agent model, they consider a general model of repeated elections in which the challenger in each period is randomly chosen from the set of voters, who are assumed to have private policy preferences. [Banks and Duggan](#) show that incumbents choose policy compromises in order to get re-elected—reminiscent of experts imitating less biased ones in order to be retained in our model.

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<sup>2</sup>[Foerster and Habermacher \(2025b\)](#) extend the framework to Bertrand competition between (policy) experts.

In another related paper, [Prendergast \(2007\)](#) studies a principal who may hire a bureaucrat motivated by own objectives to exert effort. In an extension to preference uncertainty, he shows that the wage offered by the principal may serve as a selection device. In contrast, we focus on the allocation of decision rights within an organization and consider selection through repeated interaction.

To the best of our knowledge, we are the first to study the repeated allocation of authority to an agent with uncertain preferences in an organizational context. Our results show how the prospect of repeated influence on the decision-making process can discipline agents with partisan interests to behave like—and thereby imitate—agents whose objectives are roughly in line with the principal’s.

The rest of the paper proceeds as follows. In [Section 2](#), we set up the model. [Section 3](#) derives the pure and mixed equilibria and analyzes the benefits of imitation. [Section 4](#) derives comparative statics on the correlation of the state. [Section 5](#) extends the model to finitely many periods. [Section 6](#) concludes.

## 2 Model and notation

We consider an economy populated by many *ex ante* identical agents and a principal. In period  $t = 1$  the unknown *state (of the world)*  $\theta_1 \in \Theta = \mathbb{R}$  is distributed according to a commonly known distribution  $F$  on  $\Theta$  with expected value  $\mu$  and variance  $\sigma^2$ . In  $t = 2$ , the state  $\theta_2 \in \Theta$  is partially correlated with  $\theta_1$  at a rate  $\lambda \in [0, 1)$ , i.e.,  $\theta_2 = \lambda\theta_1 + (1 - \lambda)\tilde{\theta}_2$  with  $\tilde{\theta}_2$  and  $\theta_1$  i.i.d.

In each period  $t = 1, 2$ , the *principal*  $P$  (he) has to take a decision  $y_t \in \mathbb{R}$ . In the first period,  $P$  (who does not observe the state) decides whether to delegate authority over the decision  $y_1$  to an *agent*  $A_1$  (she), whose type  $b_1 \in B = \mathbb{R}$  is her private information. We assume that  $b_1$  is randomly drawn from a commonly known distribution  $G$  with continuous, strictly positive, and symmetric density  $g$  on  $\mathbb{R}$  with expected value  $\mu_G = 0$  and variance  $\sigma_G^2$ .<sup>3</sup> If  $A_1$  is selected, she observes the true state  $\theta_1$  and then takes the decision  $y_1$ . If  $P$  retains the authority, he takes the decision himself. At the end of the first period,  $P$  observes the state  $\theta_1$  and the outcome  $y_1$ .

At the beginning of the second period,  $P$  decides whether to retain  $A_1$  (if he has delegated the decision to  $A_1$  in the first period), to choose another agent,  $A_2$ , from the pool, or to take the decision himself.  $A_2$ ’s type  $b_2 \in B$  is her private information and randomly drawn from  $G$  (independently of  $b_1$ ).

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<sup>3</sup>We assume symmetry to simplify the exposition. Our results are qualitatively robust to non-symmetric distributions.

The utility of  $P$  is given by

$$v(y_1, y_2 | \theta_1, \theta_2) = - \sum_{t=1}^2 \delta^{t-1} (\theta_t - y_t)^2,$$

where  $0 < \delta < 1$  denotes the discount factor. That is,  $P$  has bliss point  $\theta_t$  in period  $t = 1, 2$ . Similarly, the utility of an agent of type  $b$ , who may or may not have authority over the decision in one or both periods, is

$$u(y_1, y_2 | b, \theta_1, \theta_2) = - \sum_{t=1}^2 \delta^{t-1} (\theta_t + b - y_t)^2,$$

i.e., the agent of type  $b$  has bliss point  $\theta_t + b$  in period  $t = 1, 2$ . To summarize, the timing of events is as follows:

1. Nature draws the state  $\theta_1$  and the type  $b_1$  of  $A_1$ .
2.  $P$  decides whether to delegate authority to  $A_1$ .
- 3a.  $A_1$  takes the decision  $y_1$  if  $P$  has delegated authority to her.
- 3b.  $P$  takes the decision  $y_1$  himself if he keeps authority.
4.  $P$  learns  $\theta_1$  and  $y_1$  at the end of period 1.
5.  $P$  decides whether to retain  $A_1$ , to choose another agent  $A_2$ , or to take the decision himself at the beginning of period 2.
6. Nature draws the state  $\theta_2$ . If  $P$  has delegated authority to another agent  $A_2$ , nature draws her type  $b_2$ .
7.  $P$  ( $A_1$  or  $A_2$ ) takes the decision  $y_2$ .
8. Payoffs realize.

The solution concept we employ is perfect Bayesian equilibrium.

### 3 Equilibrium analysis

We first determine the second-period decision depending on authority and then the allocation of authority. Second, we determine  $A_1$ 's decision in the first period. Note that we will restrict attention to the interesting case where  $P$  delegates authority to  $A_1$  at the beginning of the first period. To conserve on notation, we



do not explicitly define the strategy of  $P$ , who decides whether to retain  $A_1$  based on the observed decision  $\hat{y}_1$  and the state  $\theta_1$  at the end of period 1.

### 3.1 Second-period decision and allocation of authority

If  $P$  keeps the authority in the second period, he maximizes his one-period expected utility given the first-period state  $\theta_1$ , and thus chooses

$$y_{2,P}^* = \arg \max_{y_2} E[-(\theta_2 - y_2)^2 \mid \theta_1] = E[\theta_2 \mid \theta_1] = \lambda\theta_1 + (1 - \lambda)\mu. \quad (1)$$

The residual variance is  $(1 - \lambda)^2\sigma^2$ . If  $P$  has delegated authority to agent  $A_2$  of type  $b$  in the second period, the latter maximizes her one-period expected utility, as revealing bias then is irrelevant, and thus chooses her bliss-point decision

$$y_{2,A}^*(b, \theta_2) = \arg \max_{y_2} E[-(\theta_2 + b - y_2)^2 \mid \theta_2] = \theta_2 + b. \quad (2)$$

The residual variance is zero.

We now turn to the decision of  $P$  whether to delegate authority at the beginning of period 2. Suppose for the moment that  $P$  has kept authority and thus taken the decision himself in the first period. He will then continue to do so in the second period because the state is correlated, such that she has better information than in the first period. Therefore, we henceforth assume that the prior variance is large enough such that  $P$  delegates authority to  $A_1$  in the first period:

**Assumption 1.**  $\sigma^2 \geq \sigma_G^2$ .

If  $P$  has delegated authority to  $A_1$  in the first period and observed the decision  $\hat{y}_1$  and the state  $\theta_1$ , he updates his prior belief about  $A_1$ 's type to the posterior  $\tilde{G} = \tilde{G}(\cdot; \hat{y}_1, \theta_1)$ .

**Proposition 1.** *Suppose that  $P$  has delegated authority to  $A_1$  in the first period. It is (weakly) optimal for  $P$  to retain  $A_1$  at the beginning of period 2 if his posterior belief  $\tilde{G}$  is such that*

$$E_{\tilde{G}}[b^2] \leq \min\{(1 - \lambda)^2\sigma^2, \sigma_G^2\}. \quad (3)$$

*Otherwise, it is weakly optimal to choose another agent from the agent pool if*

$$\sigma_G^2 \leq (1 - \lambda)^2\sigma^2. \quad (4)$$

*Otherwise, it is optimal for  $P$  to take the decision himself.*

$P$  retains  $A_1$  if doing so yields a weakly smaller expected loss (from  $A_1$ 's bias) than both taking the decision himself and choosing another agent from the pool (3). Note also that  $E_{\tilde{G}}[b^2] = E_{\tilde{G}}[b]^2 + \text{Var}_{\tilde{G}}[b]$ . Otherwise, he chooses another agent from the pool if he expects a better decision than the one he would take himself (4). In the remainder of the analysis, we assume that  $P$  retains the selected agent (chooses another agent from the pool) if (3) ((4)) holds with equality, except if a pure-strategy equilibrium does not exist. The next result follows immediately from Proposition 1 and determines the agent types which  $P$  would retain if identified.

**Corollary 1.** *Suppose that  $P$  has delegated authority to  $A_1$  in the first period. If his posterior belief  $\tilde{G}$  is such that the agent is identified as  $b_1$ , then it is (weakly) optimal for  $P$  to retain her at the beginning of period 2 if and only if*

$$b_1 \in B^* \equiv \{b \in B \mid b^2 \leq \min\{(1 - \lambda)^2 \sigma^2, \sigma_G^2\}\}.$$

### 3.2 First-period decision

The first period constitutes a signaling game in which the agent  $A_1$  has decision-making authority and takes the impact of her decision on  $P$ 's subsequent decision whether to retain her in the second period into account. In Proposition 1, we have determined the beliefs that lead  $P$  to retain  $A_1$ . We now investigate the decision of  $A_1$  in equilibrium.

A (*mixed*) *strategy* for agent  $A_1$  of type  $b_1$  (or simply agent  $b_1$ ) is a mapping  $y_1(b_1)$  that assigns a probability distribution over decisions to each type  $b_1$ ; for convenience, we will denote  $A_1$ 's strategy as a pure strategy that assigns a decision (instead of a degenerate probability distribution) to each type  $b_1$  whenever possible. Recall from Section 3.1 that  $P$  behaves according to Proposition 1 and that the second-period decision is determined according to (1) and (2) depending on whether  $P$  has delegated authority or not.

**Definition 1** (Perfect Bayesian equilibrium). *A strategy profile  $y_1^*$  is a (perfect Bayesian) equilibrium of the first-period signaling game if, for some posterior belief  $\tilde{G}$  that is consistent with  $y_1^*$  and for each agent type  $b_1 \in B$  and each state  $\theta_1 \in \Theta$ ,*

$$y_1^*(b_1, \theta_1) \in \Delta \left( \underset{\hat{y}_1}{\operatorname{argmax}} E_{\tilde{G}}[u(\hat{y}_1, y_2^* | b_1, \theta_1, \theta_2) \mid \theta_1] \right).$$

We restrict attention to equilibria in which “good” types whom  $P$  would like to

retain if identified,  $B^*$  (see Corollary 1), separate in the least costly way possible.<sup>4</sup> Define the *costs* of decision  $y_1$  for type  $b_1$  in state  $\theta_1$  as the loss in instantaneous payoff compared to her bliss-point decision  $\theta_1 + b_1$ ,

$$c(y_1 \mid b_1, \theta_1) \equiv E[(\theta_1 + b_1 - y_1)^2 \mid \theta_1, b_1] = (\theta_1 + b_1 - y_1)^2.$$

Let further

$$B^*(y_1^*) \equiv \left\{ b_1 \in B^* \mid \Pr\left(\tilde{G}(b; \hat{y}_1, \theta_1) = 1_{\{b \geq b_1\}} \text{ for all } \hat{y}_1 \in \text{supp}(y_1^*(b_1, \theta_1))\right) = 1 \right\}$$

denote the subset of good types that separate in equilibrium  $y_1^*$  with certainty, and hence are retained by  $P$ . We then select the equilibria in which good types separate whenever possible, minimizing the maximum costs (across all types that separate) to do so.

**Definition 2** (Least-cost separation equilibrium). *The equilibrium  $y_1^*$  is a least-cost separation equilibrium (LCSE) if*

$$y_1^* \in \underset{y_1 \in Y_1^*}{\operatorname{argmin}} \max_{b_1 \in B^*(y_1)} c(y_1(b_1, \theta_1) \mid b_1, \theta_1),$$

where  $Y_1^* \equiv \operatorname{argmax}_{y_1^*} \int_{B^*(y_1^*)} dG$  denotes the set of equilibria in which the largest mass of types separates.

Notice that Definition 2 has bite even if not all good types can separate in equilibrium. In a first step, we determine the optimal behavior of  $A_1$  when  $P$  retains her conditional on the observed decision being close enough to the true state.

**Lemma 1.** *Suppose  $P$  retains  $A_1$  if and only if he observes decision  $y_1 \in [\theta_1 + \underline{b}, \theta_1 + \bar{b}]$ , where  $\underline{b} < 0 < \bar{b}$ . Then, in the first period, agent*

- (i)  $b_1 \in [\underline{b}, \bar{b}]$  chooses her bliss-point decision;
- (ii)  $b_1 > \bar{b}$  ( $b_1 < \underline{b}$ ) takes the decision  $y_1 = \theta_1 + \bar{b}$  ( $y_1 = \theta_1 + \underline{b}$ ) if she wants to be retained in the second period.

Agents  $b_1 \in [\underline{b}, \bar{b}]$  are retained even if they choose their bliss point, and thus do so in equilibrium. Agents  $b_1 \notin [\underline{b}, \bar{b}]$  have to imitate an agent  $b'_1 \in [\underline{b}, \bar{b}]$  in order to

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<sup>4</sup>Cf. Foerster and Voss (2022). The idea that such equilibria are particularly reasonable goes back to Cho and Kreps (1987), see also Tirole (1990) for a discussion.

be retained. They will choose the agent whose preferences are the most aligned with theirs, that is the agent  $b'_1 = \bar{b}$  ( $\underline{b}$ ) if  $b_1 > \bar{b}$  ( $b_1 < \underline{b}$ ).

We next derive the *value of retention* for  $A_1$ , that is, the difference between her benefit from retention and that from decision-making by  $P$  or a randomly drawn agent  $A_2$ .

**Lemma 2.** *The value of retention for agent  $A_1$  is given by*

$$R(b_1) \equiv \delta(b_1^2 + \min\{\sigma_G^2, (1 - \lambda)^2 \sigma^2\}).$$

Note that the value of retention for agent  $A_1$  is strictly increasing in her bias  $b_1$ . Note also that it is weakly decreasing in the correlation of the state across periods ( $\lambda$ ). High correlation means the environment changes slowly, such that  $P$  can take a well-informed decision himself in the second period.

By symmetry of the type distribution  $G$ , we henceforth restrict attention to agent types  $b \geq 0$ . Given Lemma 1 and 2, we can then calculate, for any given  $\bar{b} > 0$ , the net value of retention for the selected agent (by imitating agent  $b'_1 = \bar{b}$  if  $b_1 > \bar{b}$ ), which is the difference between the future benefit and the current cost,

$$NR(b_1, \bar{b}) = R(b_1) - 1_{\{b_1 > \bar{b}\}} c(\theta_1 + \bar{b} \mid b_1, \theta_1) = R(b_1) - 1_{\{b_1 > \bar{b}\}} (b_1 - \bar{b})^2.$$

Note that only the agents  $b_1 > \bar{b}$  whose net value is positive want to be retained in the second period, and thus have incentives to imitate  $\bar{b}$ . In equilibrium,  $P$  will choose after which decisions to retain the agent, i.e.,  $\bar{b}$ , as to maximize his expected utility.

Let  $\bar{\beta}(\delta, \bar{b})$  denote the agent  $b_1$  who is indifferent between choosing her bliss point and imitating agent  $b'_1 = \bar{b}$  conditional on that imitation implies retention, i.e.,  $NR(b_1, \bar{b}) = 0$ . Let  $0 < \delta^* \leq 1$  denote the largest discount factor under which  $P$  is willing to retain the agent if he observes the bliss point of the neutral agent conditional on that all agents below  $\bar{\beta}(\delta, 0)$  imitate  $\bar{b} = 0$ , which is given by the unique solution to

$$E_G[b^2 \mid b \in [0, \bar{\beta}(\delta^*, 0)]] = \min\{\sigma_G^2, (1 - \lambda)^2 \sigma^2\}.$$

We establish that there is a unique pure equilibrium in which agents between  $\bar{b}$  and  $\bar{\beta}(\delta, \bar{b})$  imitate  $\bar{b}$  whenever the discount factor does not exceed  $\delta^*$ .

**Proposition 2.** *If  $\delta \leq \delta^*$ , then there exist unique  $\bar{b} = \bar{b}(\delta) > 0$  and  $\bar{\beta} = \bar{\beta}(\delta, \bar{b}) > \bar{b}$  such that in the unique pure LCSE,*

(i) agent  $b_1$  imitates  $\bar{b}$  if  $b_1 \in (\bar{b}, \bar{\beta}]$ ;

(ii) agent  $b_1$  chooses her bliss point  $\theta_1 + b_1$  else.

$P$  retains the agent if and only if he observed  $y_1 \in [\theta_1, \theta_1 + \bar{b}]$ . There is no pure equilibrium if  $\delta > \delta^*$ .

To understand the intuition behind Proposition 2, observe first that  $P$  prefers to retain agents with a small bias. By the LCSE refinement, those agents, say with bias below  $\bar{b}$ , will choose their bliss point, and thus disclose their type. By Lemma 1, agents with larger bias then need to deviate from their bliss point and imitate agent  $b'_1 = \bar{b}$  in order for  $P$  to retain them. By definition of  $\bar{\beta} = \bar{\beta}(\delta, \bar{b})$ , the net value of doing so is positive,  $NR(b_1, \bar{b}) > 0$ , for all  $b_1 \in (\bar{b}, \bar{\beta})$ , such that we can refer to  $(\bar{b}, \bar{\beta}]$  as the “imitation interval”. In turn, it must be optimal for  $P$  to retain the agent upon observing  $y_1 \in [0, \theta_1 + \bar{b}]$ . In particular, all agents in the imitation interval choosing  $y_1 = \theta_1 + \bar{b}$ , the threshold  $\bar{b}$  is such that  $P$  is indifferent between retaining and not retaining the agent upon observing  $y_1 = \theta_1 + \bar{b}$ . Note that not retaining the agent means either replacing her with  $A_2$  from the pool or taking the decision himself in the second period, which yields expected second-period utility  $\sigma_G^2$  or  $(1 - \lambda)^2 \sigma^2$ , respectively. Since it becomes more attractive to do so the higher the discount factor (as then more agent types imitate), a pure equilibrium exists as long as the discount factor is not too large. In case  $P$  prefers to retain the agent even if he did not learn anything about her type, then there always exists a pure equilibrium:

**Remark 1.** If  $\sigma_G^2 \leq (1 - \lambda)^2 \sigma^2$ , then  $\delta^* = 1$ , i.e., there always exists a pure LCSE, see Figure 1. In particular, even if each agent imitates the neutral one,  $P$  retains the agent since neither choosing another one from the agent pool nor taking the decision himself yields higher expected utility.

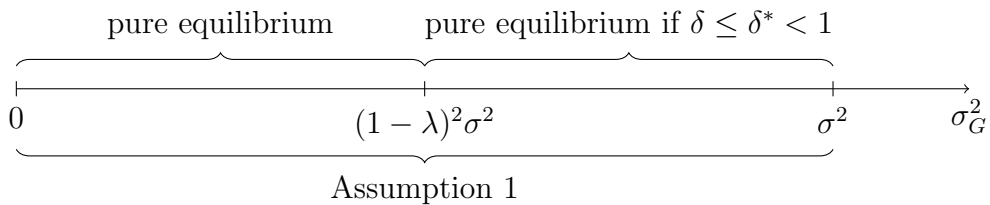


Figure 1: Existence of a pure LCSE depending on the prior variance of the agent’s type distribution,  $\sigma_G^2$ .

Now we consider the case where  $\delta$  is large such that there is no pure equilibrium. Then  $P$  is *not* willing to retain the agent if he observes the bliss point of the neutral

agent conditional on that all agents below  $\bar{\beta}(\delta, 0)$  imitate the neutral agent with  $\bar{b} = 0$ . The reason is that the value of retention is high, incentivizing some agents with relatively high bias to imitate the neutral agent (high  $\bar{\beta}(\delta, 0)$ ).  $P$  then is better off with either choosing another agent or taking the decision himself. We establish that, then, there is a mixed equilibrium in which  $P$  retains the agent upon observing the bliss point of the neutral type with less than probability 1 and in turn fewer agent types imitate.

**Proposition 3.** *If  $\delta > \delta^*$ , then there exists a unique  $0 < \bar{\beta}' = \bar{\beta}'(\delta) < \bar{\beta}(\delta, 0)$  such that there is a mixed equilibrium in which*

- (i) *agent  $b_1$  imitates the neutral one if  $b_1 \in [0, \bar{\beta}']$ ;*
- (ii) *agent  $b_1$  chooses her bliss point  $\theta_1 + b_1$  else.*

$P$  retains the agent with probability  $P_R = \frac{\bar{\beta}'}{R(\bar{\beta}')}$  if and only if he observed  $y_1 = \theta_1$ .

In the mixed equilibrium,  $P$  retains the agent with a probability  $P_R < 1$  upon observing  $y_1 = \theta_1$ . Agents thus have less incentives to imitate the neutral agent, such that we obtain a lower threshold  $\bar{\beta}'(\delta) < \bar{\beta}(\delta, 0)$ . The agents  $b_1 \in (0, \bar{\beta}'(\delta)]$  imitate the neutral one and are retained with probability  $P_R$ , while the agents  $b_1 > \bar{\beta}'(\delta)$  choose their bliss point.

### 3.3 The benefits of imitation

We next investigate whether  $P$  benefits from  $A_1$ 's imitation in equilibrium. We consider  $\delta \leq \delta^*$  and compare  $P$ 's ex ante expected utility in the pure LCSE with a *benchmark without imitation*  $y_1^{\text{bench}}$ , in which all agent types implement their bliss points,  $y_1^{\text{bench}}(b_1, \theta_1) = \theta_1 + b_1$ . Note first that, then,  $P$  will retain the agent upon observing  $\hat{y}_1 = \theta_1 + b_1$  if and only if  $b_1 \leq \bar{b}^{\text{bench}} \equiv \min\{\sigma_G, (1 - \lambda)\sigma\}$ . Therefore, the benchmark does not constitute an equilibrium, because agents  $b_1 \in (\bar{b}^{\text{bench}}, \bar{\beta}(\delta, \bar{b}^{\text{bench}}))$  have incentives to imitate  $\bar{b}^{\text{bench}}$ .

**Proposition 4.** (i) *Suppose  $\delta \leq \delta^*$ . Then the pure LCSE yields a higher ex ante expected utility to  $P$  than the benchmark without imitation.*

(ii) *Suppose  $\delta > \delta^*$ . Then the mixed equilibrium described in Proposition 3 yields a higher ex ante expected utility to  $P$  than the benchmark without imitation.*

The proposition demonstrates that  $P$  generally benefits from the agent's imitation in equilibrium. It yields a gain from a better decision in the first period and

a loss from retaining rather biased agents  $b_1 > \bar{b}^{\text{bench}}$  in the second period, which  $P$  would not retain in the benchmark, i.e., if informed about their type. This loss, however, is second-order: First, the agent's imitation in equilibrium delays her bliss-point decision to the second period. Second, imitation yields a better decision compared with the second period of the benchmark, where  $P$  does not retain  $A$  (as  $\bar{b} < \min\{\sigma_G, (1 - \lambda)\sigma\}$ ).

## 4 Comparative statics on the correlation of the state

We now explore whether  $P$  can improve his payoff when the environment changes more slowly, such that he has better information about the state in the second period. We first investigate the effect of changes in the correlation of the state across periods, measured by the parameter  $\lambda$ , on the imitation interval. We restrict attention to  $\delta \leq \delta^*$ , such that we have a unique pure LCSE. We fix  $\delta$  and henceforth write the imitation interval as  $(\bar{b}(\lambda), \bar{\beta}(\lambda)]$ .

**Lemma 3.** *Suppose that  $\delta \leq \delta^*$ . When*

- (i)  $\lambda \leq 1 - \sqrt{\frac{\sigma_G^2}{\sigma^2}}$ , *then  $\bar{b}(\lambda)$  and  $\bar{\beta}(\lambda)$  are constant in  $\lambda$ , i.e., the imitation interval  $(\bar{b}(\lambda), \bar{\beta}(\lambda)]$  remains unchanged as  $\lambda$  increases.*
- (ii)  $\lambda > 1 - \sqrt{\frac{\sigma_G^2}{\sigma^2}}$ , *then  $\bar{\beta}(\lambda)$  and  $\bar{\beta}(\lambda) - \bar{b}(\lambda)$  are continuously decreasing in  $\lambda$ , i.e., the length of the imitation interval  $(\bar{b}(\lambda), \bar{\beta}(\lambda)]$  decreases as  $\lambda$  increases.*

When the correlation of the state across periods is low ( $\lambda$  small),  $P$  faces high uncertainty in the second period, such that the best alternative to retaining the agent is choosing another agent from the agent pool ( $\sigma_G^2 \leq (1 - \lambda)^2 \sigma^2$ ). Since the imitation interval is such that  $P$  is indifferent between retaining the agent and his best alternative upon observing  $y_1 \in [0, \theta_1 + \bar{b}]$ , it remains unchanged as  $\lambda$  increases.

When the correlation of the state across periods is high ( $\lambda$  high), then the best alternative to retaining the agent is choosing by himself ( $\sigma_G^2 > (1 - \lambda)^2 \sigma^2$ ). In this case, the value of  $P$ 's outside option of taking the decision himself increases as  $\lambda$  increases. In turn, the value of retention for the selected agent is decreasing, which leads to the upper bound of the imitation interval  $\bar{\beta}(\lambda)$  being decreasing in  $\lambda$ . The lower bound of the imitation interval  $\bar{b}(\lambda)$ , however, might either increase or decrease. We show that if it decreases, it does so more slowly than the upper bound, such that the length of the interval is decreasing. Intuitively, agents are not willing to deviate as much as before from their bliss points in order to be

retained. The following example illustrates the change in the imitation interval as  $\lambda$  increases.

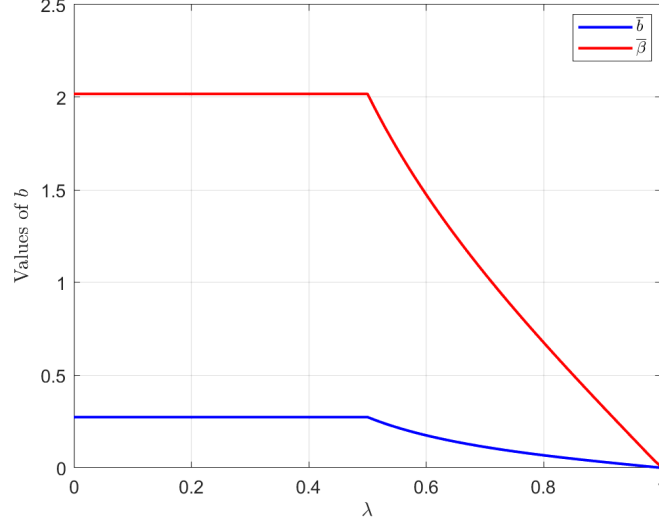


Figure 2: Imitation interval  $(\bar{b}(\lambda), \bar{\beta}(\lambda)]$  depending on  $\lambda$  in Example 1.

**Example 1.** Suppose that  $b \sim \mathcal{N}(0, 1)$ , i.e.,  $\sigma_G^2 = 1$ ,  $\sigma^2 = 4$ , and  $\delta = 0.6$ . First, notice that Assumption 1 holds since  $\sigma_G^2 = 1 < 4 = \sigma^2$ , such that  $P$  always chooses one agent in the first period. Second, we have  $\delta \leq \delta^*$  for all  $\lambda \in [0, 1]$ , such that a pure LCSE always exists. When  $\lambda \leq 0.5$ , the condition  $\sigma_G^2 \leq (1 - \lambda)^2 \sigma^2$  is satisfied, such that  $\bar{b}(\lambda) \approx 0.3$  and  $\bar{\beta}(\lambda) \approx 2$  are constant in  $\lambda$ . However when  $\lambda > 0.5$ , the condition  $\sigma_G^2 > (1 - \lambda)^2 \sigma^2$  holds. In this range, both  $\bar{b}(\lambda)$  and  $\bar{\beta}(\lambda)$  decrease as  $\lambda$  increases, ultimately reaching 0 when  $\lambda = 1$ . In particular, the imitation interval vanishes as  $\lambda \rightarrow 1$ , see Figure 2 for an illustration. Note that although  $\bar{b}(\lambda)$  may in principle increase in  $\lambda$ , this requires particular prior distributions  $G$ .

After having determined the effect of changes in the correlation of the state across periods on the imitation interval, we next analyze the effect on  $P$ 's equilibrium payoff. Observe that by Proposition 2, the unique pure LCSE is characterized by the imitation interval. It thus follows from Lemma 3 that  $P$ 's expected utility remains constant as  $\lambda$  increases when the correlation of the state across periods is low. When it is high, however, his ex ante expected utility in equilibrium can be calculated as

$$-2 \left( \int_0^{\bar{b}} (b^2 + \delta b^2) dG + \int_{\bar{b}}^{\bar{\beta}} (\bar{b}^2 + \delta b^2) dG + \int_{\bar{\beta}}^{\infty} (b^2 + \delta(1 - \lambda)^2 \sigma^2) dG \right).$$



First, the agents  $b_1 \in [0, \bar{b}]$  choose their bliss points and are retained by  $P$ . Second, the agents  $b_1 \in [\bar{b}, \bar{\beta}]$  imitate agent  $\bar{b}$  in the first period and choose their bliss points in the second period. All other agents  $b_1 > \bar{\beta}$  choose their bliss points in the first period, upon which  $P$  does not retain them and instead takes the decision himself in the second period. On the one hand,  $P$ 's expected payoff may decrease as  $\lambda$  increases because some relatively biased agents who did imitate  $b'_1 = \bar{b}$  before will not do so anymore and instead choose their bliss points  $b_1 > \bar{b}$  ( $\bar{\beta}(\lambda)$  decreases by Lemma 3). On the other hand,  $P$ 's expected payoff may increase when the gain from a better decision in the second period, in case he decides by himself, dominates the former loss.

**Proposition 5.** (i) When  $\lambda \leq 1 - \sqrt{\frac{\sigma_G^2}{\sigma^2}}$ ,  $P$ 's ex-ante expected utility is constant in  $\lambda$ .

(ii) When  $\lambda > 1 - \sqrt{\frac{\sigma_G^2}{\sigma^2}}$ ,  $P$ 's ex-ante expected utility may increase or decrease in  $\lambda$ .

The following example illustrates the change in  $P$ 's payoff due to changes in the correlation of the state.

**Example 2.** Suppose that  $b \sim \mathcal{N}(0, 1)$ , i.e.,  $\sigma_G^2 = 1$ ,  $\sigma^2 = 4$ , and  $\delta = 0.6$ . Figure 3 illustrates  $P$ 's payoff as a function of  $\lambda$ . The payoff remains constant when  $\lambda \leq 0.5$ . For  $\lambda > 0.5$ , we have that  $\bar{b}(\lambda)$  and  $\bar{\beta}(\lambda)$  are decreasing in  $\lambda$  (cf. Example 1).  $P$ 's expected payoff is decreasing for  $\lambda \in (0.5, 0.83)$ , and increasing for  $\lambda \in [0.83, 1]$ .

Example 2 reveals a possible counterintuitive outcome:  $P$  may be worse off in equilibrium when the environment changes less across periods, although he then is better informed. Note that we can also construct such examples in case  $\delta > \delta^*$  for the mixed equilibrium considered in Proposition 3.

## 5 Extension to finite horizon

So far, we have analyzed how the prospect of influence in the second period affects  $A_1$ 's behavior in the first period. We now extend the model to finitely many periods  $T \geq 3$ .

In period  $t = 1$  the unknown state  $\theta_1 \in \Theta = \mathbb{R}$  is distributed according to a commonly known distribution  $F$  on  $\Theta$  with expected value  $\mu$  and variance  $\sigma^2$ . In  $t = 2, 3, \dots, T$ , the state  $\theta_t \in \Theta$  is partially correlated with the previous period at a rate  $\lambda \in [0, 1)$ , i.e.,  $\theta_t = \lambda\theta_{t-1} + (1 - \lambda)\tilde{\theta}_t$  with  $\theta_1$  and  $(\tilde{\theta}_t)_{t=2}^T$  i.i.d.

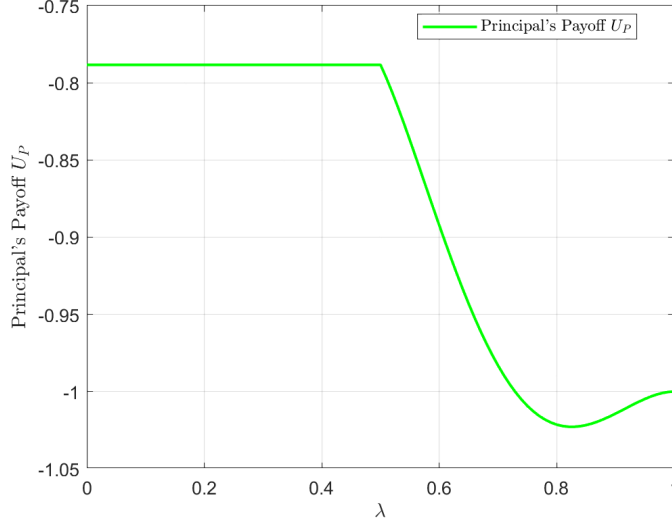


Figure 3: Change of  $P$ 's expected payoff in Example 2.

The game otherwise proceeds as described in Section 2; in particular,  $P$  observes the state of the current period and the decision implemented at the end of each period. At the beginning of the next period,  $P$  then decides whether to retain the agent, choose a new agent from the pool, or take the decision himself.

We restrict attention to Markov perfect Bayesian equilibrium, where at the beginning of each period  $t$ ,  $P$  makes the authority delegation decision based solely on the state and the decision implemented in the last period  $t-1$  (ignoring previous periods  $t-2, t-3, \dots$ ). Let

$$u^t(\hat{y}_t, y_{t+1}, \dots, y_T | b_t, \theta_t, \theta_{t+1}, \dots, \theta_T) = - \sum_{t'=t}^T \delta^{t'-1} (\theta_{t'} + b_t - y_{t'})^2$$

denote the restriction of agent  $b_t$ 's utility function to periods  $t' \geq t$ .

**Definition 3** (Markov perfect Bayesian equilibrium). *A strategy profile  $y_t^*$  is a Markov (perfect Bayesian) equilibrium of the signaling game in period  $t$  if, for some posterior belief  $\tilde{G} = G(\cdot | \theta_t, \hat{y}_t)$  in period  $t+1$  that is consistent with  $y_t^*$  and for each agent type  $b_t \in B$  and each state  $\theta_t \in \Theta$ ,*

$$y_t^*(b_t, \theta_t) \in \Delta \left( \underset{\hat{y}_t}{\operatorname{argmax}} E_{\tilde{G}}[u^t(\hat{y}_t, y_{t+1}^*, \dots, y_T^* | b_t, \theta_t, \theta_{t+1}, \dots, \theta_T) | \theta_t] \right), \forall t \leq T-1.$$

It is straightforward to extend the definition of LCSE (Definition 2) to Markov perfect Bayesian equilibria. Similarly to the baseline model, we proceed by backward induction. We restrict attention to small enough discount factors  $\delta$  such that

we can apply the results from Proposition 2 and let  $[\bar{b}, \bar{\beta}]$  denote the imitation interval in the pure LCSE of the two-period game. We establish that the boundaries of the imitation interval are decreasing over time and eventually coincide with the imitation interval of the two-period game.

**Proposition 6.** *(Finite-horizon pure Markov LCSE) If  $\delta \leq \delta^*$ , then there exists a unique pure Markov LCSE such that in each period  $t = 1, 2, \dots, T - 1$ ,*

- (i) *agent  $b_t$  who got the decision delegated imitates  $\tilde{b}_t$  if  $b_t \in (\tilde{b}_t, \tilde{\beta}_t]$  and chooses her bliss point  $\theta_t + b_t$  else;*
- (ii)  *$P$  retains the agent if and only if  $y_t \in [\theta_t, \theta_t + \tilde{b}_t]$ ;*

*with  $[\tilde{b}_{T-1}, \tilde{\beta}_{T-1}] = [\bar{b}, \bar{\beta}]$  and  $\tilde{b}_t < \tilde{b}_{t-1} < \tilde{\beta}_t < \tilde{\beta}_{t-1}$  for  $2 \leq t \leq T - 1$ .*

In the second but last period, strategic incentives are identical to the two-period game under Markov strategies, so that we obtain the imitation interval  $[\bar{b}, \bar{\beta}]$  as in the baseline model. In earlier periods,  $P$  benefits more from retaining an agent because the latter may continue to imitate in the future. This shifts the lower bound and as a result also the upper bound of the imitation interval to the right as we move back to the beginning of the game. Hence,  $P$  tightens the requirements for retaining the current agent over time by requiring decisions that are less biased.

**Example 3.** *Suppose that  $b \sim \mathcal{N}(0, 1)$ , i.e.,  $\sigma_G^2 = 1$ ,  $\sigma^2 = 4$ ,  $\delta = 0.6$ ,  $\lambda = 0.8$ , and  $T = 15$ . Figure 4 illustrates the specific changes of the imitation interval's boundaries over time. Note that  $\lambda$  is large enough that  $P$  takes the decision forever if he doesn't retain the selected agent. The boundaries  $\tilde{b}_t$  and  $\tilde{\beta}_t$  are decreasing over time until being equal to  $\bar{b}$  and  $\bar{\beta}$  (cf. Example 1) at period 14 (the last period where imitation takes place). Observe also that the changes are becoming increasingly rapid over time.*

## 6 Conclusion

In our model, the principal faces a trade-off between keeping authority and delegating it to an informed agent with uncertain preferences, while the selected agent compares the future benefits of repeated influence on the decision-making process with the instantaneous loss of deviating from her bliss point. We focus on the case where the principal is willing to delegate authority in the first period. When the

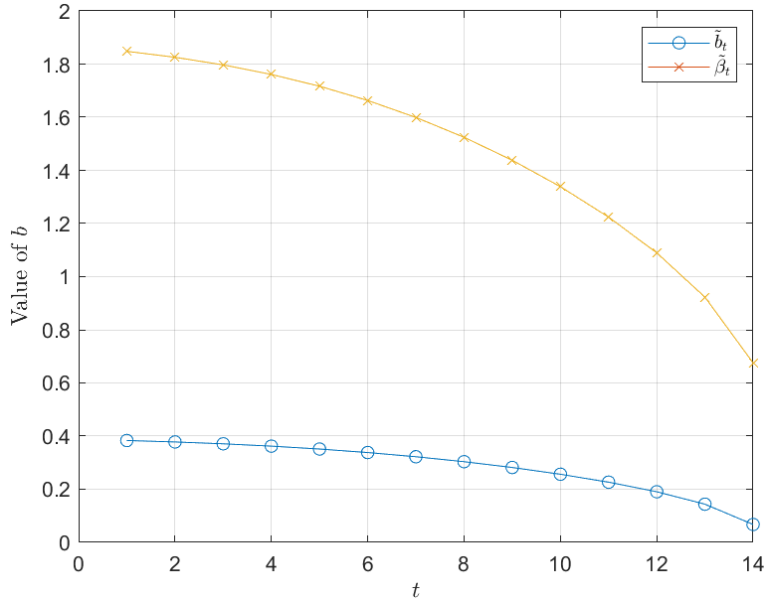


Figure 4: Imitation interval  $(\tilde{b}_t, \tilde{\beta}_t]$  over time in Example 3.

discount rate is low, the unique LCSE is characterized by an imitation interval such that the agents within this interval imitate the “good” ones whose preferences are roughly aligned with the principal. When the discount rate is high, there is a mixed equilibrium in which the principal retains the agent upon observing potential mimicking only with a certain probability. Compared with the benchmark in which all agents choose their bliss points and reveal their bias in the first period, imitation allows the principal to obtain higher ex ante expected utility.

Furthermore, we demonstrate that the principal may be worse off with better information, as he incurs a loss from some relatively biased agents ceasing to mimic a less biased one in the first period. Interestingly, a principal may thus have an incentive to strategically limit their access to information, thereby avoiding the adverse incentive effects that full observability can generate. In other words, choosing to remain partially uninformed can serve as a commitment device that induces more favorable behavior from the agent. Finally, we extend the two-period model to finitely many periods and show that the imitation interval gradually shifts over time.

In the future, this paper can be extended in the following ways. First, we can examine how the equilibrium changes when we incorporate cheap talk and monetary transfers, which, of course, complicates the analysis. Second, we can also design a mechanism that allows the principal to improve his payoff as he knows more, such as committing to the behavior of agents during the authorization decision stage in the decision-making process.

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## References

- Aghion, P. and J. Tirole (1997). Formal and real authority in organizations. *Journal of Political Economy* 105(1), 1–29.
- Alonso, R. and N. Matouschek (2008). Optimal delegation. *The Review of Economic Studies* 75(1), 259–293.
- Banks, J. S. and J. Duggan (2008). A dynamic model of democratic elections in multidimensional policy spaces. *Quarterly Journal of Political Science* 3(3), 269–299.
- Berganza, J. C. (2000, Oct). Two roles for elections: Disciplining the incumbent and selecting a competent candidate. *Public Choice* 105(1), 165–194.
- Besley, T. and M. Smart (2007). Fiscal restraints and voter welfare. *Journal of Public Economics* 91(3-4), 755–773.
- Brehm, J. O. and S. Gates (1999). *Working, shirking, and sabotage: Bureaucratic response to a democratic public*. University of Michigan Press.
- Cho, I.-K. and D. M. Kreps (1987, May). Signaling games and stable equilibria. *The Quarterly Journal of Economics* 102(2), 179–221.
- Deimen, I. and D. Szalay (2019). Delegated expertise, authority, and communication. *American Economic Review* 109(4), 1349–1374.
- Dessein, W. (2002). Authority and communication in organizations. *The Review of Economic Studies* 69(4), 811–838.
- Foerster, M. and D. Habermacher (2025a). Authority, communication, and internal markets. SSRN Working Paper.

- Foerster, M. and D. Habermacher (2025b). Policy-advising competition and endogenous lobbies. *Journal of Public Economics* 245.
- Foerster, M. and A. Voss (2022). Believe me, I am ignorant, but not biased. *European Economic Review* 149.
- Krähmer, D. (2006). Message-contingent delegation. *Journal of Economic Behavior & Organization* 60(4), 490–506.
- Lim, W. (2012). Selling authority. *Journal of Economic Behavior & Organization* 84(1), 393–415.
- Prendergast, C. (2007). The motivation and bias of bureaucrats. *American Economic Review* 97(1), 180–196.
- Tirole, J. (1990). In honor of david kreps, winner of the john bates clark medal. *Journal of Economic Perspectives* 4(3), 149–170.

## A Appendix: Proofs

**Proof of Proposition 1.** Suppose  $P$  has selected an agent in the first period. Given posterior  $\tilde{G}$ , his expected utility from retaining the selected agent is

$$E_{\tilde{G}}[-(\theta_2 - y_{2,A}^*(b, \theta_2))^2] = -E_{\tilde{G}}[b^2].$$

The expected utility of  $P$  from choose a new agent from the agent pool is

$$E_G[-(\theta_2 - y_{2,A}^*(b, \theta_2))^2] = -E_G[b^2] = -\sigma_G^2.$$

Finally, the expected utility of  $P$  from taking the decision himself is  $-(1 - \lambda)^2 \sigma^2$ , which proves the claim.  $\square$

**Proof of Lemma 1.** The selected agent  $b_2$  would choose her bliss point  $y_{2,A}^*(b_2, \theta_2) = \theta_2 + b_2$  in the second period, and is thus ceteris paribus weakly better off when she is being retained than when she is not being retained. Since  $P$  retains the agent if and only if  $y_1 \in [\theta_1 + \underline{b}, \theta_1 + \bar{b}]$ , choosing her bliss point  $y_{1,A}^*(b_1, \theta_1) = \theta_1 + b_1$  in the first period is optimal if  $b_1 \in [\underline{b}, \bar{b}]$ .

If  $b_1 > \bar{b}$  ( $b_1 < \underline{b}$  is analogous), then the agent has to take a decision  $y_1 \in [\theta_1 + \underline{b}, \theta_1 + \bar{b}]$  in order to be retained. To establish the claim, observe that

$$\arg \min_{y_1 \in [\theta_1 + \underline{b}, \theta_1 + \bar{b}]} (\theta_1 + b_1 - y_1)^2 = \theta_1 + \bar{b}.$$

□

**Proof of Lemma 2.** Fix any decision  $y_1$  of the selected agent in the first period. Suppose first that  $(1 - \lambda)^2 \sigma^2 \geq \sigma_G^2$ , such that by Proposition 1  $P$  chooses another agent from the agent pool if he does not retain the selected agent. Using (2), the value of retention for agent  $A_1$  then is:

$$\begin{aligned} R(b_1) &= E[u(y_1, y_{2,A}^*(b_1, \theta_2) | b_1, \theta_1, \theta_2) | b_1, \text{retained}] - E[u(y_1, y_{2,A}^*(b_2, \theta_2) | b_1, \theta_1, \theta_2) | b_1, \text{not retained}] \\ &= 0 - \delta E_G[-(\theta_2 + b_1 - y_{2,A}^*(b_2, \theta_2))^2 | b_1] \\ &= \delta(b_1^2 + \sigma_G^2). \end{aligned}$$

Second, suppose that  $(1 - \lambda)^2 \sigma^2 < \sigma_G^2$ , such that by Proposition 1  $P$  takes the decision himself if he does not retain the selected agent. Using (1), the value of retention for agent  $A_1$  then is:

$$\begin{aligned} R(b_1) &= E[u(y_1, y_{2,A}^*(b_1, \theta_2) | b_1, \theta_1, \theta_2) | b_1, \text{retained}] - E[u(y_1, y_{2,P}^*(\lambda, \theta_1) | b_1, \theta_1, \theta_2) | b_1, \text{not retained}] \\ &= 0 - \delta E_F[-(\theta_2 + b_1 - y_{2,P}^*(\lambda, \theta_1))^2 | b_1] \\ &= \delta(b_1^2 + (1 - \lambda)^2 \sigma^2). \end{aligned}$$

□

**Proof of Proposition 2.** Let  $(1 - \lambda)^2 \sigma^2 < \sigma_G^2$ , so that  $P$  prefers to take the decision himself in the second period instead of choosing another agent from the pool ( $(1 - \lambda)^2 \sigma^2 \geq \sigma_G^2$  is analogous). Suppose that  $P$  retains the agent if and only if  $y_1 \in [\theta_1, \theta_1 + \bar{b}]$ , where  $\bar{b} > 0$ . By Lemma 1, agents  $b_1 \in [0, \bar{b}]$  then choose their bliss point, while agents  $b_1 > \bar{b}$  take the decision  $y_1 = \theta_1 + \bar{b}$  if they want to be retained. We first prove the following lemma.

**Lemma 4.** *If  $\delta \leq \delta^*$ , then there exist unique  $\bar{b} = \bar{b}(\delta) > 0$  and  $\bar{\beta} = \bar{\beta}(\delta, \bar{b}) > \bar{b}$  such that*

$$E_G[b^2 | b \in [\bar{b}, \bar{\beta}]] = \min\{\sigma_G^2, (1 - \lambda)^2 \sigma^2\}.$$

*Proof.* Fix any  $\bar{b} > 0$ . We first prove that there exists a unique  $\bar{\beta} > \bar{b}$  such that  $NR(\bar{\beta}, \bar{b}) = 0$ , i.e., agents  $b_1 \in (\bar{b}, \bar{\beta}]$  prefer to choose  $y_1 = \theta_1 + \bar{b}$  and are retained. Let  $b_1 > \bar{b}$ , then Lemma 1 and Lemma 2 yield

$$NR(b_1, \bar{b}) = R(b_1) - c(\theta_1 + \bar{b} \mid b_1, \theta_1) = (\delta - 1)b_1^2 + 2\bar{b}b_1 + \delta(1 - \lambda)^2\sigma^2 - \bar{b}^2.$$

which is a concave quadratic function. And because

$$NR(b_1, \bar{b}) = R(b_1) = \delta((1 - \lambda)^2\sigma^2 + b_1^2) > 0$$

for  $b_1 \in [0, \bar{b}]$ , there exists a unique  $\bar{\beta} = \bar{\beta}(\delta, \bar{b}) > \bar{b}$  such that  $NR(\bar{\beta}, \bar{b}) = 0$  by continuity.

So the agents  $b_1 \leq \bar{\beta}$  prefer to imitate  $b'_1 = \bar{b}$  in order to be retained in the second period, while the agents  $b_1 > \bar{\beta}$  instead prefer to choose their bliss point (according to Lemma 1).

Second, we prove that there exists  $\bar{b} > 0$  such that  $E_G[b^2 \mid b \in [\bar{b}, \bar{\beta}]] = (1 - \lambda)^2\sigma^2$  if  $\delta \leq \delta^*$ . By definition of  $\bar{\beta}$  and given  $\delta = \delta^*$ , we have

$$NR(b_1, 0) = (\delta^* - 1)(\bar{\beta}(\delta^*, 0))^2 + \delta^*(1 - \lambda)^2\sigma^2 = 0$$

for  $b_1 = \bar{\beta}(\delta^*, 0)$ . Then we get  $\bar{\beta}(\delta^*, 0) = \sqrt{\frac{\delta^*}{1 - \delta^*}}(1 - \lambda)\sigma$  if  $\delta^* < 1$  and  $\bar{\beta}(1, 0) = +\infty$ , where  $\partial\bar{\beta}(\delta^*, 0)/\partial\delta > 0$ . By definition of  $\delta^*$  and the latter,

$$E_G[b^2 \mid b \in [0, \bar{\beta}(\delta, 0)]] \leq (1 - \lambda)^2\sigma^2 \text{ if } \delta \leq \delta^*.$$

Next, note that, given any  $\bar{b}$ ,  $\bar{\beta} = \bar{\beta}(\delta, \bar{b})$  satisfies

$$(\delta - 1)\bar{\beta}^2 + 2\bar{b}\bar{\beta} + \delta(1 - \lambda)^2\sigma^2 - \bar{b}^2 = 0,$$

which yields

$$\bar{\beta}(\delta, \bar{b}) = \frac{\bar{b} + \sqrt{\delta\bar{b}^2 + \delta(1 - \delta)(1 - \lambda)^2\sigma^2}}{1 - \delta}$$

and thus  $\partial\bar{\beta}(\delta, \bar{b})/\partial\bar{b} > 0$ . Therefore,  $E_G[b^2 \mid b \in [\bar{b}, \bar{\beta}(\delta, \bar{b})]]$  is increasing in  $\bar{b}$ , which establishes the claim. Note that there cannot be two solutions  $\bar{b} < \bar{b}'$ , as then  $\bar{\beta}(\delta, \bar{b}) < \bar{\beta}(\delta, \bar{b}')$  and thus

$$(1 - \lambda)^2\sigma^2 = E_G[b^2 \mid b \in [\bar{b}, \bar{\beta}(\delta^*, \bar{b})]] < E_G[b^2 \mid b \in [\bar{b}', \bar{\beta}(\delta^*, \bar{b}')] = (1 - \lambda)^2\sigma^2,$$

which is a contradiction.  $\square$



We next establish the equilibrium. If  $\delta \leq \delta^*$ , then the considered strategies constitute an equilibrium by Lemma 1 and definition of  $\bar{b} > 0$  and  $\bar{\beta}(\delta, \bar{b}) > \bar{b}$ . In particular, all agents  $b_1 < \bar{b}$  separate at least cost and there cannot be another equilibrium in which a larger mass of agents separates by definition of  $\bar{b}$ , which implies that it is the unique LCSE.

Next, consider  $\delta > \delta^*$  and suppose that there is a closed interval  $[\underline{b}', \bar{b}']$ ,  $\underline{b}' \leq \bar{b}'$ , such that  $P$  retains the agent upon  $\hat{y}_1 \in [\theta_1 + \underline{b}', \theta_1 + \bar{b}']$  and that there is no decision  $\hat{y}_1 > \theta_1 + \bar{b}'$  for which this is the case. Then each agent  $b_1 \in [\underline{b}', \bar{b}']$  chooses her bliss point and is retained by Lemma 1. Recall that by definition of  $\delta^*$ ,  $E_G[b^2 \mid b \in [0, \bar{\beta}(\delta, 0)]] > (1 - \lambda)^2 \sigma^2$ . Since further  $\partial \bar{\beta}(\delta, \bar{b}) / \partial \bar{b} > 0$ , we have

$$E_G[b^2 \mid b \in [\bar{b}', \bar{\beta}(\delta, \bar{b}')] ] > (1 - \lambda)^2 \sigma^2 \text{ for any } \bar{b}' > 0,$$

a contradiction. Finally, suppose that  $P$  does not retain any agent. Then all agents choose their bliss points and disclose their type in the first period. But then  $P$  retains the selected agent when observing  $y_1 \in [\theta_1, \theta_1 + (1 - \lambda)\sigma]$ , as then  $b_1 \in [0, (1 - \lambda)\sigma]$  and thus  $b_1^2 \leq (1 - \lambda)^2 \sigma^2$ , a contradiction. This establishes that there is no pure equilibrium if  $\delta > \delta^*$ , which finishes the proof.  $\square$

**Proof of Proposition 3.** If  $\delta > \delta^*$  (in this case,  $\sigma_G^2 > (1 - \lambda)^2 \sigma^2$ , see Remark 1), we consider the mixed equilibrium that  $P$  retain  $A_1$  only when observing  $y_1 = \theta_1$  with a positive probability  $P_R$ , otherwise he take the decision himself.

Similar to pure equilibrium, the net value function of agent  $b_1$  to imitate the neutral one is

$$NR(b_1, \delta) = -b_1^2 + P_R \cdot \delta((1 - \lambda)^2 \sigma^2 + b_1^2).$$

Then we can find the indifferent agents  $\bar{\beta}'(\delta) = \sqrt{\frac{P_R \cdot \delta}{1 - P_R \cdot \delta}}(1 - \lambda)\sigma < \sqrt{\frac{\delta}{1 - \delta}}(1 - \lambda)\sigma = \bar{\beta}(\delta, 0)$  from  $NR(\bar{\beta}'(\delta), \delta) = 0$ . The agent  $b_1 \in (0, \bar{\beta}']$  imitates the neutral one because  $NR(b_1, \delta) \geq 0$ , while the agent  $b_1 \in (\bar{\beta}', \infty)$  chooses her bliss points (the proof is analogous to Proposition 2) given  $P_R$ .

When determining the strategies of  $A_1$ , the expected utility of  $P$  to retain  $A_1$  equals that to take the decision himself when observing  $y_1 = \theta_1$ , which is

$$\int_0^{\bar{\beta}'(\delta)} -b^2 dG(b) + \int_{\bar{\beta}'(\delta)}^{+\infty} -\delta(1 - \lambda)^2 \sigma^2 dG(b) = -(1 - \lambda)^2 \sigma^2.$$

Then we can get  $\bar{\beta}'(\delta)$  from  $E_G[b^2 \mid b \in [0, \bar{\beta}']] = (1 - \lambda)^2 \sigma^2$  and  $P_R = \frac{\bar{\beta}'}{\delta(\bar{\beta}' + (1 - \lambda)^2 \sigma^2)} = \frac{\bar{\beta}'}{R(\bar{\beta}')}$  from  $NR(\bar{\beta}', \delta) = 0$ , which finishes the proof.

□

**Proof of Proposition 4.** If all agents implement their bliss points  $y_1^{\text{bench}}(b_1, \theta_1) = \theta_1 + b_1$  in the benchmark case,  $P$  retains the agent upon observing  $\hat{y}_1 = \theta_1 + b_1$  if  $b_1 \leq \bar{b}^{\text{bench}} \equiv \min\{\sigma_G, (1 - \lambda)\sigma\}$ . Otherwise, he selects a new agent from the pool or takes the decision himself, and obtains  $-\delta(\bar{b}^{\text{bench}})^2$  in the second period.

When  $\delta \leq \delta^*$ , Proposition 2 applies, i.e., there exists an imitation interval  $(\bar{b}, \bar{\beta}]$  such that the agent  $b_1 \in (\bar{b}, \bar{\beta}]$  imitates agent  $\bar{b}$ , and  $E_G[b_1^2 \mid b_1 \in (\bar{b}, \bar{\beta}]] = \min\{\sigma_G^2, (1 - \lambda)^2\sigma^2\}$ . Thus,  $\bar{b} < \bar{b}^{\text{bench}} \equiv \min\{\sigma_G, (1 - \lambda)\sigma\} < \bar{\beta}$ .

We proceed by case distinction with respect to  $b_1$ :

- (i)  $b_1 \in [0, \bar{b}]$ . Since  $\bar{b}^{\text{bench}} > \bar{b}$ ,  $A$  implements her bliss point in the first period and is subsequently retained in both the pure LCSE and the benchmark without imitation, such that  $P$ 's utility is the same in both cases.
- (ii)  $b_1 > \bar{\beta}$ . When  $\bar{b}^{\text{bench}} < \bar{\beta}$ ,  $A$  implements her bliss point in the first period and is subsequently not retained in both the pure LCSE and the benchmark without imitation, such that  $P$ 's utility is the same in both cases.
- (iii)  $b_1 \in (\bar{b}, \bar{\beta}]$ .  $P$ 's ex-ante expected utility conditional on  $b_1 \in (\bar{b}, \bar{\beta}]$  in the pure LCSE is

$$-(\bar{b}^2 + \delta E_G[b_1^2 \mid b_1 \in (\bar{b}, \bar{\beta}]] = -(\bar{b}^2 + \delta(\bar{b}^{\text{bench}})^2), \quad (5)$$

while in the benchmark without imitation it is

$$\begin{aligned} & -(E_G[b_1^2 \mid b_1 \in (\bar{b}, \bar{\beta}]] + \delta(Pr(b \leq \bar{b}^{\text{bench}} \mid b_1 \in (\bar{b}, \bar{\beta}]) E_G[b_1^2 \mid b_1 \in (\bar{b}, \bar{b}^{\text{bench}})] \\ & + (Pr(b > \bar{b}^{\text{bench}} \mid b_1 \in (\bar{b}, \bar{\beta}]) \min\{\sigma_G^2, (1 - \lambda)^2\sigma^2\})) \\ & = -((\bar{b}^{\text{bench}})^2 + \delta((Pr(b \leq \bar{b}^{\text{bench}} \mid b_1 \in (\bar{b}, \bar{\beta}]) E_G[b_1^2 \mid b_1 \in (\bar{b}, \bar{b}^{\text{bench}})] \\ & + Pr(b > \bar{b}^{\text{bench}} \mid b_1 \in (\bar{b}, \bar{\beta}]) (\bar{b}^{\text{bench}})^2)). \end{aligned} \quad (6)$$

The difference between (5) and (6) is

$$\begin{aligned} & (\bar{b}^{\text{bench}})^2 - \bar{b}^2 - \delta Pr(b \leq \bar{b}^{\text{bench}} \mid b_1 \in (\bar{b}, \bar{\beta}]) ((\bar{b}^{\text{bench}})^2 - E_G[b_1^2 \mid b_1 \in (\bar{b}, \bar{b}^{\text{bench}})]) \\ & > (\bar{b}^{\text{bench}})^2 - \bar{b}^2 - \delta Pr(b \leq \bar{b}^{\text{bench}} \mid b_1 \in (\bar{b}, \bar{\beta}]) ((\bar{b}^{\text{bench}})^2 - \bar{b}^2) \\ & > (1 - \delta)((\bar{b}^{\text{bench}})^2 - \bar{b}^2) \\ & > 0, \end{aligned}$$

where the inequalities hold because  $E_G[b_1^2 \mid b_1 \in (\bar{b}, \bar{b}^{\text{bench}})] > \bar{b}^2$ ,  $Pr(b \leq \bar{b}^{\text{bench}} \mid b_1 \in (\bar{b}, \bar{\beta}]) < 1$ , and  $(\bar{b}^{\text{bench}})^2 > \bar{b}^2$ , respectively.

When  $\delta > \delta^*$ , Remark 1 implies  $\sigma_G^2 > (1 - \lambda)^2 \sigma^2$ . Proposition 3 provides the mixed equilibrium in which there exists an imitation interval  $(0, \bar{\beta}']$  such that the agent  $b_1 \in (0, \bar{\beta}']$  imitates the neutral agent, and  $E_G[b_1^2 \mid b_1 \in [0, \bar{\beta}']] = (1 - \lambda)^2 \sigma^2$ . Thus,  $0 < \bar{b}^{\text{bench}} \equiv (1 - \lambda)\sigma < \bar{\beta}'$ .

Similar to the proof of  $\delta \leq \delta^*$ ,  $P$ 's utility is the same in the mixed equilibrium and the benchmark without imitation when  $b_1 > \bar{\beta}'$  and  $b_1 = 0$ , while the mixed equilibrium yields a higher ex-ante utility to  $P$  than the benchmark without imitation when  $b_1 \in (0, \bar{\beta}']$ .

This proves the claim.  $\square$

**Proof of Lemma 3.** If  $\delta \leq \delta^*$ , there exists a pure LCSE (according to Proposition 2). We first prove the following lemma.

**Lemma 5.** Suppose that  $\delta \leq \delta^*$ . Then  $\bar{b}(\lambda)$  and  $\bar{\beta}(\lambda)$  are continuous in  $\lambda$ .

*Proof.* According to the Proposition 2, if  $(1 - \lambda)^2 \sigma^2 = \sigma_G^2$ , the value of  $\bar{b}(\lambda = 1 - \sqrt{\frac{\sigma_G^2}{\sigma^2}})$  and  $\bar{\beta}(\lambda = 1 - \sqrt{\frac{\sigma_G^2}{\sigma^2}})$  can be got from

$$\begin{cases} (\delta - 1)\bar{\beta}^2 + 2\bar{b}\bar{\beta} + \delta\sigma_G^2 - \bar{b}^2 = 0 \\ E_G[b^2 \mid b \in [\bar{b}(\lambda = 1 - \sqrt{\frac{\sigma_G^2}{\sigma^2}}), \bar{\beta}(\lambda = 1 - \sqrt{\frac{\sigma_G^2}{\sigma^2}})]] = \sigma_G^2. \end{cases}$$

If  $(1 - \lambda)^2 \sigma^2 < \sigma_G^2$ , the value of  $\bar{b}(\lambda \searrow (1 - \sqrt{\frac{\sigma_G^2}{\sigma^2}}))$  and  $\bar{\beta}(\lambda \searrow (1 - \sqrt{\frac{\sigma_G^2}{\sigma^2}}))$  can be got from

$$\begin{cases} (\delta - 1)\bar{\beta}^2 + 2\bar{b}\bar{\beta} + \delta(1 - \lambda)^2 \sigma^2 - \bar{b}^2 = 0 \\ E_G[b^2 \mid b \in [\bar{b}(\lambda \searrow (1 - \sqrt{\frac{\sigma_G^2}{\sigma^2}})), \bar{\beta}(\lambda \searrow (1 - \sqrt{\frac{\sigma_G^2}{\sigma^2}})]] = (1 - \lambda)^2 \sigma^2. \end{cases}$$

The solutions of these two systems of equations are the same at  $\lambda = 1 - \sqrt{\frac{\sigma_G^2}{\sigma^2}}$ , so  $\bar{\beta}(\lambda)$  are continuous at  $\lambda = 1 - \sqrt{\frac{\sigma_G^2}{\sigma^2}}$ . We consequently prove that  $\bar{b}(\lambda)$  and  $\bar{\beta}(\lambda)$  are continuous in  $\lambda$  since the density function  $g$  of distribution  $G$  is continuous.  $\square$

According to Lemma 5,  $\bar{b}(\lambda)$  and  $\bar{\beta}(\lambda)$  are continuous in  $\lambda$ . When  $\lambda \leq 1 - \sqrt{\frac{\sigma_G^2}{\sigma^2}}$ , which means  $\sigma_G^2 \leq (1 - \lambda)^2 \sigma^2$ , the values of  $\bar{b}$  and  $\bar{\beta}$  can be got from

$$\begin{cases} (\delta - 1)\bar{\beta}^2 + 2\bar{b}\bar{\beta} + \delta\sigma_G^2 - \bar{b}^2 = 0 \\ E_G[b^2 \mid b \in [\bar{b}(\lambda), \bar{\beta}(\lambda)]] = \sigma_G^2. \end{cases}$$

It is not related to  $\lambda$ , so the solution is not changed with the increase of  $\lambda$ . When  $\lambda > 1 - \sqrt{\frac{\sigma_G^2}{\sigma^2}}$ , we can get  $\bar{b}(\lambda)$  and  $\bar{\beta}(\lambda)$  from

$$\begin{cases} (\delta - 1)\bar{\beta}^2 + 2\bar{b}\bar{\beta} + \delta(1 - \lambda)^2\sigma^2 - \bar{b}^2 = 0 \\ E_G[b^2 \mid b \in [\bar{b}(\lambda), \bar{\beta}(\lambda)]] = (1 - \lambda)^2\sigma^2. \end{cases}$$

Let  $f(\bar{b}(\lambda), \bar{\beta}(\lambda)) = (\delta - 1)\bar{\beta}^2 + 2\bar{b}\bar{\beta} + \delta(1 - \lambda)^2\sigma^2 - \bar{b}^2 = 0$ , then we get

$$f_{\bar{b}}(\bar{b}, \bar{\beta}) = 2(\bar{\beta} - \bar{b}) > 0, \quad f_{\bar{\beta}}(\bar{b}, \bar{\beta}) = 2(\bar{b} - (1 - \delta)\bar{\beta}),$$

and

$$\frac{df(\bar{b}, \bar{\beta})}{d\lambda} = 2(\bar{\beta} - \bar{b})\bar{b}'(\lambda) + 2(\bar{b} - (1 - \delta)\bar{\beta})\bar{\beta}'(\lambda) - 2\delta(1 - \lambda)\sigma^2 = 0$$

Let  $g(\bar{b}(\lambda), \bar{\beta}(\lambda)) = E_G[b^2 \mid b \in [\bar{b}(\lambda), \bar{\beta}(\lambda)]] - (1 - \lambda)^2\sigma^2 = 0$ , then we get

$$\frac{dg(\bar{b}, \bar{\beta})}{d\lambda} = \frac{\partial E_G[b^2 \mid b \in [\bar{b}(\lambda), \bar{\beta}(\lambda)]]}{\partial \bar{b}} \bar{b}'(\lambda) + \frac{\partial E_G[b^2 \mid b \in [\bar{b}(\lambda), \bar{\beta}(\lambda)]]}{\partial \bar{\beta}} \bar{\beta}'(\lambda) + 2(1 - \lambda)\sigma^2 = 0,$$

where we have known  $\frac{\partial E_G[b^2 \mid b \in [\bar{b}(\lambda), \bar{\beta}(\lambda)]]}{\partial \bar{b}} > 0$  and  $\frac{\partial E_G[b^2 \mid b \in [\bar{b}(\lambda), \bar{\beta}(\lambda)]]}{\partial \bar{\beta}} > 0$ .

First, suppose that  $\bar{\beta}'(\lambda) \geq 0$ . We get the following result from  $f(\bar{b}, \bar{\beta}) = 0$ ,

$$\bar{\beta}(\lambda) = \frac{\bar{b}(\lambda) + \sqrt{\delta\bar{b}(\lambda)^2 + \delta(1 - \delta)(1 - \lambda)^2\sigma^2}}{1 - \delta}.$$

The first-order derivative is

$$\bar{\beta}'(\lambda) = \frac{\bar{b}'(\lambda)(\sqrt{\delta\bar{b}(\lambda)^2 + \delta(1 - \delta)(1 - \lambda)^2\sigma^2} + \delta\bar{b}(\lambda)) - \delta(1 - \delta)(1 - \lambda)\sigma^2}{(1 - \delta)\sqrt{\delta\bar{b}(\lambda)^2 + \delta(1 - \delta)(1 - \lambda)^2\sigma^2}}.$$

Obviously,  $\bar{b}'(\lambda) \leq 0$  implies  $\bar{\beta}'(\lambda) < 0$ , so a necessary condition for  $\bar{\beta}'(\lambda) \geq 0$  is  $\bar{b}'(\lambda) > 0$ . But if  $\bar{b}'(\lambda) > 0$  and  $\bar{\beta}'(\lambda) \geq 0$ , then  $\frac{dg(\bar{b}, \bar{\beta})}{d\lambda} > 0$ , a contradiction to  $\frac{dg(\bar{b}, \bar{\beta})}{d\lambda} = 0$ . Therefore,  $\bar{\beta}'(\lambda) < 0$ .

Second, if  $\bar{b}'(\lambda) \geq 0$ , then obviously  $\bar{\beta} - \bar{b}$  is decreasing with the increase of  $\lambda$ .

Third, if  $\bar{b}'(\lambda) < 0$ , then from  $\frac{df(\bar{b}, \bar{\beta})}{d\lambda} = 0$  we get

$$(\bar{b} - (1 - \delta)\bar{\beta})\bar{\beta}'(\lambda) = -(\bar{\beta} - \bar{b})\bar{b}'(\lambda) + \delta(1 - \lambda)\sigma^2 > 0.$$

So we have  $\bar{b} < (1 - \delta)\bar{\beta}$ , which yields  $\bar{\beta} - \bar{b} > (1 - \delta)\bar{\beta} - \bar{b} > 0$ . Thus, we have  $|\bar{\beta}'(\lambda)| > |\bar{b}'(\lambda)|$ . So  $\bar{\beta} - \bar{b}$  is decreasing in  $\lambda$ .  $\square$

**Proof of Proposition 5.** If  $\delta \leq \delta^*$ , there exists a pure equilibrium (according to Proposition 2). When  $\lambda \leq 1 - \sqrt{\frac{\sigma_G^2}{\sigma^2}}$ , then  $\bar{b}(\lambda)$  and  $\bar{\beta}(\lambda)$  are continuous in  $\lambda$  and the imitation interval remains unchanged as  $\lambda$  increases by Lemma 3, so that the expected payoff of  $P$  is constant in  $\lambda$ .

When  $\lambda > 1 - \sqrt{\frac{\sigma_G^2}{\sigma^2}}$ , the expected payoff of  $P$  is

$$U_P(\lambda) = -2 \left( \int_0^{\bar{b}} (b^2 + \delta b^2) dG + \int_{\bar{b}}^{\bar{\beta}} (\bar{b}^2 + \delta \bar{b}^2) dG + \int_{\bar{\beta}}^{+\infty} (b^2 + \delta(1-\lambda)^2 \sigma^2) dG \right).$$

The corresponding first-order derivative with respect to  $\lambda$  is

$$\begin{aligned} U'_P(\lambda) = & -2 \left( ((\bar{b}^2 - \bar{\beta}^2) + \delta(\bar{\beta}^2 - (1-\lambda)^2 \sigma^2)) g(\bar{\beta}) \bar{\beta}'(\lambda) \right. \\ & \left. + 2\bar{b} Pr\{\bar{b} \leq b \leq \bar{\beta}\} \bar{b}'(\lambda) - 2\delta(1-\lambda)\sigma^2 Pr\{b > \bar{\beta}\} \right). \end{aligned}$$

Because  $E_G[b^2 \mid b \in [\bar{b}(\lambda), \bar{\beta}(\lambda)]] = (1-\lambda)^2 \sigma^2$ , we get  $\bar{b}^2 < (1-\lambda)^2 \sigma^2 < \bar{\beta}^2$ . So we have  $(\bar{b}^2 - \bar{\beta}^2) + \delta(\bar{\beta}^2 - (1-\lambda)^2 \sigma^2) < \bar{b}^2 - (1-\lambda)^2 \sigma^2 < 0$ . According to Lemma 3, we have  $\bar{\beta}'(\lambda) < 0$ , so the first part of the first-order derivative is positive. The second part is sign-consistent with  $\bar{b}'(\lambda)$ , while the third part is negative. Therefore, the final sign of  $U'_P(\lambda)$  is unclear and depends on the specific distribution of the bias and the value of the parameters.  $\square$

**Proof of Proposition 6.** Similarly to the two-period case (see the proof of Proposition 2), we can prove that there exists a unique imitation interval in each period (except the last period) of the  $T$ -period ( $T \geq 3$ ) case if  $\delta$  is small enough, since we consider Markov LCSE.

We use the method of mathematical induction to prove  $\tilde{b}_t < \tilde{b}_{t-1} < \tilde{\beta}_t < \tilde{\beta}_{t-1}$  ( $2 \leq t \leq T-1$ ) for any  $T \geq 3$ . Without loss of generality, suppose that  $\lambda$  is large enough such that  $P$  prefers to take a decision himself in the subsequent period if he doesn't retain the selected agent.

**Base case.** We prove that the conclusion holds for  $T = 3$  ( $\tilde{b}_2 < \tilde{b}_1 < \tilde{\beta}_2 < \tilde{\beta}_1$ ). In the second period, we have  $[\tilde{b}_2, \tilde{\beta}_2] = [\bar{b}, \bar{\beta}]$  according to the two-period equilibrium (Proposition 2). We also have  $E_G[b_2^2 \mid b_2 \in [\tilde{b}_2, \tilde{\beta}_2]] = (1-\lambda)^2 \sigma^2$  and  $(\tilde{\beta}_2 - \tilde{b}_2)^2 = \delta((1-\lambda)^2 \sigma^2 + \tilde{\beta}_2^2)$ .

In the first period, if the indifferent agent  $\tilde{\beta}_1$  is selected, in equilibrium, we

have

$$\delta(1+\delta)((1-\lambda)^2\sigma^2 + \tilde{\beta}_1^2) = \begin{cases} (\tilde{\beta}_1 - \tilde{b}_1)^2, & \text{if } \tilde{\beta}_1 \leq \tilde{b}_2 < \tilde{\beta}_2 \\ (\tilde{\beta}_1 - \tilde{b}_1)^2 + \delta(\tilde{\beta}_1 - \tilde{b}_2)^2, & \text{if } \tilde{b}_2 < \tilde{\beta}_1 \leq \tilde{\beta}_2 \\ (\tilde{\beta}_1 - \tilde{b}_1)^2 + \delta^2((1-\lambda)^2\sigma^2 + \tilde{\beta}_1^2), & \text{if } \tilde{\beta}_1 > \tilde{\beta}_2 \end{cases}$$

because agent  $\tilde{\beta}_1$  is indifferent between choosing her bliss point in the first period and imitating the agent  $\tilde{b}_1$ .

If  $\tilde{b}_1 < \tilde{\beta}_1 \leq \tilde{b}_2 < \tilde{\beta}_2$ , we have

$$(1-\lambda)^2\sigma^2 = E_G[b_1^2 \mid b_1 \in [\tilde{b}_1, \tilde{\beta}_1]] < E_G[b_2^2 \mid b_2 \in [\tilde{b}_2, \tilde{\beta}_2]] = (1-\lambda)^2\sigma^2, \quad (7)$$

i.e., a contradiction. The left equality in (7) holds since  $P$  is indifferent between making the decisions himself and retaining  $A_1$  if he observes  $y_1 = \theta_1 + \tilde{b}_1$  in the first period, i.e.,  $(1+\delta)(1-\lambda)^2\sigma^2 = (1+\delta)E_G[b_1^2 \mid b_1 \in [\tilde{b}_1, \tilde{\beta}_1]]$ . If  $P$  retains  $A_1$  in the first period,  $A_1$  chooses her bliss point in the second period and is retained again in the third period because  $\tilde{\beta}_1 \leq \tilde{b}_2$ . So the expected utility of  $P$  to retain the agent is  $(1+\delta)E_G[b_1^2 \mid b_1 \in [\tilde{b}_1, \tilde{\beta}_1]]$ . The inequality in (7) holds because  $\tilde{b}_1 < \tilde{\beta}_1 \leq \tilde{b}_2 < \tilde{\beta}_2$ . The right equality in (7) follows from the two-period case since  $[\tilde{b}_2, \tilde{\beta}_2] = [\bar{b}, \bar{\beta}]$ .

If  $\tilde{b}_2 < \tilde{\beta}_1 \leq \tilde{\beta}_2$  and  $\tilde{b}_1 \leq \tilde{b}_2$ , we have

$$\begin{aligned} & (1+\delta)(1-\lambda)^2\sigma^2 \\ &= Pr\{\tilde{b}_1 \leq x \leq \tilde{b}_2 \mid \tilde{b}_1 \leq x \leq \tilde{\beta}_1\} \cdot E_G[b_1^2 \mid b_1 \in [\tilde{b}_1, \tilde{b}_2]] \\ & \quad + Pr\{\tilde{b}_2 < x \leq \tilde{\beta}_1 \mid \tilde{b}_1 \leq x \leq \tilde{\beta}_1\} \cdot \tilde{b}_2^2 + \delta E_G[b_1^2 \mid b_1 \in [\tilde{b}_1, \tilde{\beta}_1]] \\ &< Pr\{\tilde{b}_1 \leq x \leq \tilde{b}_2 \mid \tilde{b}_1 \leq x \leq \tilde{\beta}_1\} \cdot E_G[b_1^2 \mid b_1 \in [\tilde{b}_1, \tilde{b}_2]] \\ & \quad + Pr\{\tilde{b}_2 < x \leq \tilde{\beta}_1 \mid \tilde{b}_1 \leq x \leq \tilde{\beta}_1\} \cdot E_G[b_1^2 \mid b_1 \in [\tilde{b}_2, \tilde{\beta}_1]] + \delta E_G[b_1^2 \mid b_1 \in [\tilde{b}_1, \tilde{\beta}_1]] \\ &= (1+\delta)E_G[b_1^2 \mid b_1 \in [\tilde{b}_1, \tilde{\beta}_1]] \\ &\leq (1+\delta)E_G[b_2^2 \mid b_2 \in [\tilde{b}_2, \tilde{\beta}_2]] \\ &= (1+\delta)(1-\lambda)^2\sigma^2, \end{aligned} \quad (8)$$

a contradiction. The first equality in (8) holds because  $P$  is indifferent between making the decisions himself in the future and retaining  $A_1$  when observing  $y_1 \in [\theta_1, \theta_1 + \tilde{b}_1]$ . If he retains  $A_1$ , the agent  $b_1 \in [\tilde{b}_1, \tilde{b}_2]$  chooses her bliss point, while  $b_2 \in (\tilde{b}_2, \tilde{\beta}_2]$  imitates agent  $\tilde{b}_2$  in the second period, and  $A_1$  is retained in the third period because  $\tilde{\beta}_1 \leq \tilde{\beta}_2$ . Therefore, we can determine the expected utility

of  $P$  in retaining  $A_1$  (the second row). The first inequality in (8) holds since  $\tilde{b}^2 < E_G[b_1^2 \mid b_1 \in [\tilde{b}_2, \tilde{\beta}_1]]$ . The second equality in (8) holds because of the mathematical calculation. The second inequality in (8) holds since  $\tilde{b}_1 \leq \tilde{b}_2 < \tilde{\beta}_1 \leq \tilde{\beta}_2$ . The last equality in (8) holds from the two-period case.

If  $\tilde{b}_2 < \tilde{\beta}_1 \leq \tilde{\beta}_2$  and  $\tilde{b}_2 < \tilde{b}_1$ , we know from the two-period case that  $(\tilde{\beta}_2 - \tilde{b}_2) = \delta((1 - \lambda)^2\sigma^2 + \tilde{\beta}_2^2)$ . So we get  $(\tilde{\beta}_1 - \tilde{b}_2) \leq \delta((1 - \lambda)^2\sigma^2 + \tilde{\beta}_1^2)$  since  $\tilde{\beta}_1 \leq \tilde{\beta}_2$ . We get  $(\tilde{\beta}_1 - \tilde{b}_1) < \delta((1 - \lambda)^2\sigma^2 + \tilde{\beta}_1^2)$  since  $\tilde{b}_2 < \tilde{b}_1$ . Therefore,  $(\delta + \delta^2)((1 - \lambda)^2\sigma^2 + \tilde{\beta}_1^2) > (\tilde{\beta}_1 - \tilde{b}_1)^2 + \delta(\tilde{\beta}_1 - \tilde{b}_2)$ . By definition of  $\tilde{\beta}_1$ ,  $(\delta + \delta^2)((1 - \lambda)^2\sigma^2 + \tilde{\beta}_1^2) = (\tilde{\beta}_1 - \tilde{b}_1)^2 + \delta(\tilde{\beta}_1 - \tilde{b}_2)$ , as agent  $b_1 = \tilde{\beta}_1$  is indifferent between choosing her bliss point in the first period and imitating agent  $\tilde{b}_1$ , a contradiction.

If  $\tilde{\beta}_1 > \tilde{\beta}_2$  and  $\tilde{\beta}_2 \leq \tilde{b}_1$ , we have

$$(1 - \lambda)^2\sigma^2 = E_G[b_1^2 \mid b_1 \in [\tilde{b}_1, \tilde{\beta}_1]] > E_G[b_2^2 \mid b_2 \in [\tilde{b}_2, \tilde{\beta}_2]] = (1 - \lambda)^2\sigma^2, \quad (9)$$

a contradiction. The left equality in (9) holds since  $P$  is indifferent between taking the decisions himself and retaining  $A_1$  if he observes  $y_1 = \theta_1 + \tilde{b}_1$  in the first period, which is  $(1 + \delta)(1 - \lambda)^2\sigma^2 = E_G[b_1^2 \mid b_1 \in [\tilde{b}_1, \tilde{\beta}_1]] + \delta(1 - \lambda)^2\sigma^2$ . Suppose  $P$  retains  $A_1$  in the first period. In that case,  $A_1$  chooses her bliss point in the second period and is not retained in the third period because  $\tilde{\beta}_1 > \tilde{\beta}_2$ . So the expected utility of  $P$  to retain  $A_1$  in the first period is  $E_G[b_1^2 \mid b_1 \in [\tilde{b}_1, \tilde{\beta}_1]] + \delta(1 - \lambda)^2\sigma^2$ . The inequality in (9) holds because  $\tilde{b}_2 < \tilde{\beta}_2 \leq \tilde{b}_1 < \tilde{\beta}_1$ . The right equality in (9) holds in the two-period case.

If  $\tilde{\beta}_1 > \tilde{\beta}_2$  and  $\tilde{b}_1 \leq \tilde{b}_2 < \tilde{\beta}_2$ , we know from the two-period case that  $(\tilde{\beta}_2 - \tilde{b}_2) = \delta((1 - \lambda)^2\sigma^2 + \tilde{\beta}_2^2)$ . So we get  $(\tilde{\beta}_1 - \tilde{b}_2) > \delta((1 - \lambda)^2\sigma^2 + \tilde{\beta}_1^2)$  since  $\tilde{\beta}_1 > \tilde{\beta}_2$ . We get  $(\tilde{\beta}_1 - \tilde{b}_1) > \delta((1 - \lambda)^2\sigma^2 + \tilde{\beta}_1^2)$  since  $\tilde{b}_1 \leq \tilde{b}_2$ . By definition of  $\tilde{\beta}_1$ ,  $(\tilde{\beta}_1 - \tilde{b}_1) = \delta((1 - \lambda)^2\sigma^2 + \tilde{\beta}_1^2)$ , as agent  $b_1 = \tilde{\beta}_1$  is indifferent between choosing her bliss point in the first period and imitating agent  $\tilde{b}_1$ , a contradiction. Therefore, we have proven  $\tilde{b}_2 < \tilde{b}_1 < \tilde{\beta}_2 < \tilde{\beta}_1$ , which establishes the claim for  $T = 3$ .

**Induction step.** Suppose that  $\tilde{b}_t < \tilde{b}_{t-1} < \tilde{\beta}_t < \tilde{\beta}_{t-1}$  holds for any  $2 \leq t \leq m-1$  when  $T = m$  ( $m \geq 4$  is any natural number). We prove that  $\tilde{b}_t < \tilde{b}_{t-1} < \tilde{\beta}_t < \tilde{\beta}_{t-1}$  holds for any  $2 \leq t \leq m$  when  $T = m + 1$ .

Consider the second period and suppose that  $P$  has delegated the decision to agent  $A_2$  drawn from  $G$ . Note that by the Markov assumption,  $A_2$  will be judged at the end of period 2 solely based on the state  $\theta_2$  and the implemented decision  $\hat{y}_2$ . By the induction basis, we have  $\tilde{b}_t < \tilde{b}_{t-1} < \tilde{\beta}_t < \tilde{\beta}_{t-1}$  holds for all  $3 \leq t \leq m$ . So it is left to prove  $\tilde{b}_2 < \tilde{b}_1 < \tilde{\beta}_2 < \tilde{\beta}_1$ .

Let  $v_{retain}^t(b_{t-1})$  denote the continuation payoff of  $P$  at period  $t \geq 2$  if he retains  $A_{t-1}$  conditional on her bias  $b_{t-1}$ . Since  $P$  in equilibrium will be indifferent between retaining  $A_2$  and deciding himself in subsequent periods upon observing  $\hat{y}_2 = \theta_2 + \tilde{b}_2$ , we obtain

$$E[v_{retain}^3(b_2) \mid b_2 \in (\tilde{b}_2, \tilde{\beta}_2]] = -\frac{1 - \delta^{m-1}}{1 - \delta}(1 - \lambda)^2 \sigma^2.$$

Note that  $v_{retain}^t(b'_{t-1}) < v_{retain}^t(b''_{t-1})$  if  $b'_{t-1} > b''_{t-1}$ . Let  $u_{retained}^t(b_{t-1})$  denote the continuation payoff of  $A_{t-1}$  with bias  $b_{t-1}$  if she is retained at period  $t \geq 2$ .

Next, consider the first period and suppose that the indifferent agent  $b_1 = \tilde{\beta}_1$  is selected. Then, in equilibrium, we have

$$\frac{\delta - \delta^{m+1}}{1 - \delta}((1 - \lambda)^2 \sigma^2 + \tilde{\beta}_1^2) = \begin{cases} (\tilde{\beta}_1 - \tilde{b}_1)^2 - \delta^2 u_{retained}^3(\tilde{\beta}_1), & \text{if } \tilde{\beta}_1 \leq \tilde{b}_2 < \tilde{\beta}_2 \\ (\tilde{\beta}_1 - \tilde{b}_1)^2 + \delta(\tilde{\beta}_1 - \tilde{b}_2)^2 - \delta^2 u_{retained}^3(\tilde{\beta}_1), & \text{if } \tilde{b}_2 < \tilde{\beta}_1 \leq \tilde{\beta}_2 \\ (\tilde{\beta}_1 - \tilde{b}_1)^2 + \frac{\delta^2 - \delta^{m+1}}{1 - \delta}((1 - \lambda)^2 \sigma^2 + \tilde{\beta}_1^2), & \text{if } \tilde{\beta}_1 > \tilde{\beta}_2 \end{cases}$$

because agent  $\tilde{\beta}_1$  is indifferent between choosing her bliss point in the first period (left-hand side) and imitating the agent  $\tilde{b}_1$  (right-hand side).

If  $\tilde{\beta}_1 \leq \tilde{b}_2 < \tilde{\beta}_2$ , we have

$$\begin{aligned} & \frac{1 - \delta^m}{1 - \delta}(1 - \lambda)^2 \sigma^2 \\ &= E_G[b_1^2 \mid b_1 \in [\tilde{b}_1, \tilde{\beta}_1]] - \delta E[v_{retain}^3(b_2) \mid b_2 \in [\tilde{b}_1, \tilde{\beta}_1]] \\ &< (1 - \lambda)^2 \sigma^2 - \delta E[v_{retain}^3(b_2) \mid b_2 \in [\tilde{b}_2, \tilde{\beta}_2]] \\ &= (1 - \lambda)^2 \sigma^2 + \delta \frac{1 - \delta^{m-1}}{1 - \delta}(1 - \lambda)^2 \sigma^2 \\ &= \frac{1 - \delta^m}{1 - \delta}(1 - \lambda)^2 \sigma^2, \end{aligned} \tag{10}$$

a contradiction. The first equality in (10) holds since  $P$  is indifferent between making the decisions himself and retaining  $A_1$  if he observes  $y_1 = \theta_1 + \tilde{b}_1$  in the first period. Suppose  $P$  retains  $A_1$  in the first period. In that case,  $A_1$  chooses her bliss point in the second period and is retained again in the third period because  $\tilde{\beta}_1 \leq \tilde{b}_2$ . So the expected utility of  $P$  to retain the agent  $A_1$  at the beginning of period 2 is  $-E_G[b_1^2 \mid b_1 \in [\tilde{b}_1, \tilde{\beta}_1]] + \delta E[v_{retain}^3(b_2) \mid b_2 \in [\tilde{b}_1, \tilde{\beta}_1]]$ . The second equality in (10) holds because  $E[v_{retain}^3(b_2) \mid b_2 \in [\tilde{b}_2, \tilde{\beta}_2]] = -\frac{1 - \delta^{m-1}}{1 - \delta}(1 - \lambda)^2 \sigma^2$ . The final equality in (10) holds because of the calculation. Next, we prove the inequality in (10) holds. Obviously,  $E[v_{retain}^3(b_2) \mid b_2 \in [\tilde{b}_1, \tilde{\beta}_1]] > E[v_{retain}^3(b_2) \mid b_2 \in [\tilde{b}_2, \tilde{\beta}_2]]$ , since



$\tilde{b}_1 < \tilde{\beta}_1 \leq \tilde{b}_2 < \tilde{\beta}_2$ . Suppose  $E_G[b_1^2 \mid b_1 \in [\tilde{b}_1, \tilde{\beta}_1]] > (1 - \lambda)^2 \sigma^2$ , then we have

$$\begin{aligned} -\frac{1 - \delta^{m-1}}{1 - \delta} (1 - \lambda)^2 \sigma^2 &> E[v_{retain}^3(b_2) \mid b_2 \in [\tilde{b}_1, \tilde{\beta}_1]] \\ &> E[v_{retain}^3(b_2) \mid b_2 \in [\tilde{b}_2, \tilde{\beta}_2]] = -\frac{1 - \delta^{m-1}}{1 - \delta} (1 - \lambda)^2 \sigma^2. \end{aligned}$$

The first inequality holds since  $E_G[b_1^2 \mid b_1 \in [\tilde{b}_1, \tilde{\beta}_1]] > (1 - \lambda)^2 \sigma^2$  and  $\frac{1 - \delta^m}{1 - \delta} (1 - \lambda)^2 \sigma^2 = E_G[b_1^2 \mid b_1 \in [\tilde{b}_1, \tilde{\beta}_1]] - \delta E[v_{retain}^3(b_2) \mid b_2 \in [\tilde{b}_1, \tilde{\beta}_1]]$ . There is a contradiction, so  $E_G[b_1^2 \mid b_1 \in [\tilde{b}_1, \tilde{\beta}_1]] \leq (1 - \lambda)^2 \sigma^2$ , such that the inequality in (10) holds.

Similarly to the three-period case, we can exclude other cases and prove that  $\tilde{b}_2 < \tilde{b}_1 < \tilde{\beta}_2 < \tilde{\beta}_1$ , which establishes the claim.

Now, we have proven that when  $\delta$  is small enough, we get  $\tilde{b}_t < \tilde{b}_{t-1} < \tilde{\beta}_t < \tilde{\beta}_{t-1}$  ( $2 \leq t \leq T - 1$ ) for any  $T \geq 3$ . Finally, note that when  $\delta = \delta^*$ , we have  $0 = \tilde{b}_{T-1} < \tilde{\beta}_{T-1}$  and  $0 < \tilde{b}_t < \tilde{\beta}_t$  for any  $1 \leq t \leq T - 2$ . This implies that there exists a unique Markov LCSE when  $\delta \leq \delta^*$ .

Finally, we prove that if  $P$  doesn't retain the agent, there exists  $1 \leq t^* \leq T$  such that he chooses another agent from the pool if  $t < t^*$ , or he takes the decision himself if  $t \geq t^*$ . Let  $v_{pool}^t(b_{t-1})$  denote the continuation payoff of  $P$  at period  $t \geq 2$  if he doesn't retain  $A_{t-1}$ , then the equivalent proof is that:  $\square$