

Cervellati, Matteo; Meyerheim, Gerrit; Sunde, Uwe

**Working Paper**

## Decomposing Comparative Development 1880-2020: A Quantitative Dynamic Analysis

CESifo Working Paper, No. 12176

**Provided in Cooperation with:**

Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

*Suggested Citation:* Cervellati, Matteo; Meyerheim, Gerrit; Sunde, Uwe (2025) : Decomposing Comparative Development 1880-2020: A Quantitative Dynamic Analysis, CESifo Working Paper, No. 12176, Munich Society for the Promotion of Economic Research - CESifo GmbH, Munich

This Version is available at:

<https://hdl.handle.net/10419/331641>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

# Working Papers

## **Decomposing Comparative Development 1880-2020: A Quantitative Dynamic Analysis**

Matteo Cervellati, Gerrit Meyerheim, Uwe Sunde

Imprint:

**CESifo Working Papers**

ISSN 2364-1428 (digital)

Publisher and distributor: Munich Society for the Promotion  
of Economic Research - CESifo GmbH

Poschingerstr. 5, 81679 Munich, Germany  
Telephone +49 (0)89 2180-2740

Email [office@cesifo.de](mailto:office@cesifo.de)  
<https://www.cesifo.org>

Editor: Clemens Fuest

An electronic version of the paper may be downloaded free of charge

- from the CESifo website: [www.ifo.de/en/cesifo/publications/cesifo-working-papers](http://www.ifo.de/en/cesifo/publications/cesifo-working-papers)
- from the SSRN website: [www.ssrn.com/index.cfm/en/cesifo/](http://www.ssrn.com/index.cfm/en/cesifo/)
- from the RePEc website: <https://ideas.repec.org/s/ces/ceswps.html>

# Decomposing Comparative Development 1880-2020: A Quantitative Dynamic Analysis\*

Matteo Cervellati  
University of Bologna  
IZA, Bonn  
CEPR

Gerrit Meyerheim  
LMU Munich

Uwe Sunde  
LMU Munich  
IZA, Bonn  
CEPR

## Abstract

This paper demonstrates that a tractable heterogeneous agent endogenous growth model can quantitatively match the stylized empirical facts of long-run development trajectories of income, life expectancy, and fertility for 86 countries over the past 140 years. A decomposition of comparative development differences into contributions of country-specific “deep determinants”, accumulation forces during the historical development process, and balanced growth dynamics sheds new light on the mechanisms leading to country-specific differences in development and establishes a link between the largely disparate literatures on endogenous growth, comparative development, and growth accounting. Structural estimation results show that historical accumulation dynamics explain most of today’s comparative development patterns. A quantification of the demographic dividend suggests implications for future growth dynamics.

*JEL-Classification:* O10

*Keywords:* Long-Run Growth Dynamics, Quantitative Growth Dynamics,  
Growth Accounting, Development Accounting,  
Comparative Development, Structural Estimation

---

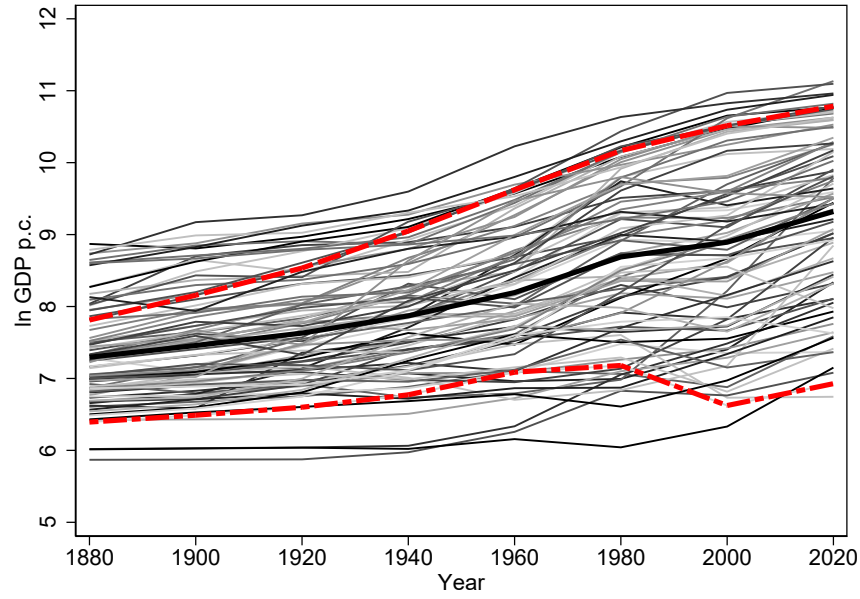
\*The authors thank Mathilde Esposito, Martin Haas, Nathan Vieira, and seminar participants at Brown University, the AEA Winter Meetings, LSE, LMU Munich, the University of Groningen, and the University of Manchester for helpful comments. Gerrit Meyerheim and Uwe Sunde gratefully acknowledge support by the German Science Foundation (Deutsche Forschungsgemeinschaft, Project 491578970). Matteo Cervellati: m.cervellati@unibo.it, Gerrit Meyerheim: gerrit.meyerheim@econ.lmu.de, Uwe Sunde: uwe.sunde@lmu.de.

# 1 Introduction

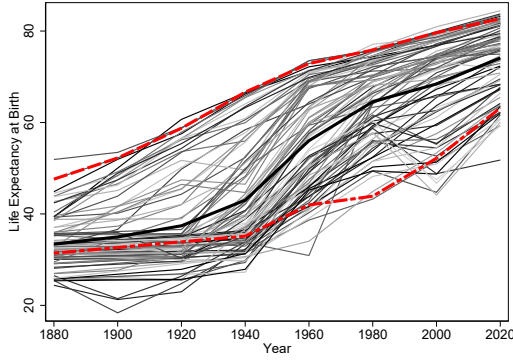
Figure 1 depicts the patterns of economic development across the world over the past 140 years. Figure 1(a) shows raw data for log income per capita across countries and illustrates four stylized facts. First, incomes, and hence economic living conditions, have increased substantially around the world over the past century – incomes and growth rates today are much larger than in the past. Second, this improvement in living conditions was not a steady process: economic development is characterized by an acceleration from very modest to sustained rates of growth that was related to demographic change, structural transformation, and productivity increases. Third, the global pattern of development is characterized by pronounced heterogeneity. The considerable inequality across the world at the beginning of the observation period in 1880 has even widened by 2020. Fourth, the evolution of heterogeneity in development is closely linked to the non-linear growth dynamics. Some countries experienced an earlier acceleration than others, giving rise to changes in the ordering of countries with some countries having improved their relative position in the distribution while others have fallen behind.

Panels (b) and (c) of Figure 1 illustrate that development over the past 140 years was not confined to income dynamics. The stylized facts also extend analogously to population dynamics, reflected by mortality and fertility. On average, life expectancy effectively doubled, whereas fertility almost halved. The increase in life expectancy appears to have preceded the development in incomes, while the decline in fertility occurred at about the same time.

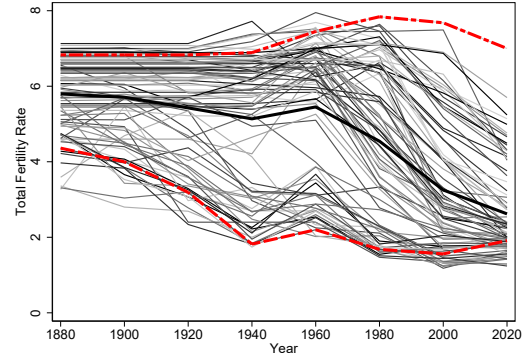
These stylized facts are exemplified by the two highlighted examples of the development trajectories of Sweden and Niger in comparison to the sample mean. Sweden was a middle-income country in the late 19<sup>th</sup> century. Over the course of the 20<sup>th</sup> century, Sweden experienced a pronounced acceleration of development, and by 2020, Sweden has become one of the richest countries in the world. In contrast, Niger remained among the poorest countries, and the gap in income relative to Sweden has widened as Niger did not experience a similar acceleration in economic development. These patterns are also reflected in the demographics. Sweden experienced an early and pronounced increase in life expectancy and a decline in fertility, whereas in Niger life expectancy only began to increase very late and fertility remains at high levels in global comparison.



(a) ln GDP p.c.



(b) Life Expectancy



(c) Fertility

Figure 1: The Dynamics of Long-Run Development

*Note:* Panel (a) plots the natural logarithm of GDP per capita for constant 2011 dollars for 86 countries. The upper thick dashed line corresponds to data for Sweden. The lower dot-dashed line corresponds to data for Niger. Data source: gapminder (based on data from World Bank, Maddison, and IMF, see: [www.gapminder.org/sources/gdp-per-capita/](https://www.gapminder.org/sources/gdp-per-capita/)). Panel (b) plots life expectancy at birth for the same countries. The upper thick dashed line corresponds to data for Sweden. The lower dot-dashed line corresponds to data for Niger. Data source: gapminder (based on data from the Global Burden of Disease Study 2019 of the Institute for Health Metrics and Evaluation, University of Washington, Seattle, see: <https://www.gapminder.org/tag/life-expectancy/>). Panel (c) plots the total fertility rate for the same countries. The lower thick dashed line corresponds to data for Sweden. The upper dot-dashed line corresponds to data for Niger. Data source: gapminder (based on data from the UN World Population Prospects 2022, see <https://www.gapminder.org/data/documentation/gd004/>). All panels: Each line corresponds to the time series for a different country. The thick (black) line corresponds to the sample mean for each time period. See Appendix Table A1 for the list of 86 countries.

Despite substantial progress in understanding the mechanisms of growth over the past decades, these stylized facts of development and their interactions remain poorly understood. Evidence from growth accounting suggests that physical capital plays a minor role for explaining growth, and similarly evidence from development accounting suggests that differences in physical capital do not account for much of the comparative development differences. Instead, human capital and productivity account for a majority of the variation in growth and comparative development differences in the world. Nonetheless, the reasons for the differences in productivity remain a matter of debate. The same applies to the acceleration in growth. Long-run growth models often view human capital as a key driver for this acceleration, and the secular acceleration of growth is closely associated with a sharp, and non-recurrent, expansion of human capital (Jones and Romer, 2010; Jones, 2016). Yet the empirical results of growth and development accounting studies suggest that the direct effects of human capital for explaining comparative development are limited. What explains this discrepancy? If human capital plays a key role in the progressive amplification of productivity differences (see, e.g., Hsieh and Klenow, 2010), which factors account for the substantial and persistent heterogeneity in productivity, human capital, and economic development? And how are initial differences in productivity related to the observed patterns of overtaking and divergence? The dynamics of the demographic variables add an additional complication: Is the heterogeneity in productivity and income growth, for instance between Sweden and Niger, linked to these demographic dynamics, and if so, how? Finally, what is the outlook for today's low-income countries such as Niger that have fallen further behind the high-income countries in relative terms over the past decades?

To the best of our knowledge, no attempt has been made to address these questions within a single and comprehensive quantitative framework. This paper demonstrates a tractable and parsimonious model with three country-specific structural parameters that are estimated can reconcile the quantitative patterns of development illustrated in Figure 1. Our analysis is based on the conjecture that the long-run trajectory of economic development is characterized by non-linear dynamics: economies transition from a long phase of quasi-stagnant development with slow improvements in productivity and economic living conditions to a regime of balanced growth. This transition is the result of interactions between economic and demographic development. These interactions are key to understanding the reasons underlying the great divergence in incomes observed over the past century, and the corresponding heterogeneity in comparative development. This view accounts for the long-run development dynamics experienced by today's high-income countries. At the same time, it also accounts

for the observation that many low-income countries still exhibit living conditions in various dimensions that resemble those characterizing Europe a century ago. Accordingly, the key reason that parts of the world have not experienced similarly dramatic improvements in economic development, and that substantial differences in development persist until today, is the delay in the acceleration of development. This suggests the need for a better understanding of the factors and mechanisms that are responsible for this delay.

Our analysis proceeds in several steps. First, we present a tractable canonical heterogeneous agents model of endogenous long-run growth that nests models of endogenous growth with a long phase of quasi-stagnant growth. This model builds on the insight that technology (reflected in the productivity of resources) is the key driver of output per capita growth. Productivity is crucially influenced by human capital, and growth dynamics are closely linked to the dynamics of human capital accumulation. In turn, human capital and fertility choices of heterogeneous individuals are closely intertwined and interact with aggregate dynamics that relate to population, mortality, and productivity. Comparative development differences are the result of delays in the endogenous onset of a transition in education and fertility behavior that drives the acceleration from quasi-stagnation to balanced growth. The timing, extent, and steepness of this transition is related to three country-specific parameters.

In a second step, we structurally estimate these country-specific parameters together with other parameters that are common to all countries, using a quantitative version of our long-run growth model. To discipline the analysis, we restrict attention to the non-linear dynamics (and the timing) of the transition to balanced growth as the only explanation for contemporaneous cross-country development patterns. The results reveal a remarkably good fit of simulations of the estimated model to actual time series of different countries. In comparison to a reduced-form empirical two-way fixed-effects specification, the model exhibits a similar goodness of fit in terms of explained variance. At the same time, the model unfolds the dynamic mechanisms behind the development patterns of the specific countries, in contrast to reduced-form evidence. The estimation of the structural parameters thereby allows us to assess the quantitative performance of the model. We also document that the model performs remarkably well in matching the long-run dynamics of non-targeted moments in the data and show that the timing of the transition plays a critical role for comparative development differences in various dimensions. The structural estimates of country-specific parameters establish a direct link to the deep determinants of development. Finally, the structural estimation of the model also allows us to analyze the quantitative implications of changes in the structural parameters for comparative development differences.



In a third step, we develop an analytical decomposition of comparative development differences into the contributions of different phases of the non-linear development path. Specifically, we show how comparative development differences can be decomposed into the contributions of time-invariant country-specific differences, of development dynamics related to the transition from stagnation to growth, and of growth along the balanced growth path. The results suggest that variation in initial conditions reflected in the “deep determinants” alone accounts for less than 10% of the variation in comparative development, whereas delays in the accumulation accounts for more than 90%. We illustrate these insights using the example of Sweden versus Niger. The decomposition of development can also rationalize episodes of overtaking and divergence via differences in initial conditions and in the accumulation dynamics. Moreover, the decomposition illustrates that the interaction between country-specific heterogeneity and dynamic accumulation forces is key to understand comparative development. This also suggests that country-specific time-invariant heterogeneity explains more of the development of low-income countries like Niger than of high-income countries, whose development is shaped more by accumulation dynamics. Finally, since the acceleration in human capital accumulation during the demographic transition is inherently temporary, population and education dynamics have implications for future growth. We show how a decomposition of population and aging dynamics can be used to quantify the demographic dividend for each country during various phases of development. This also allows us to provide an outlook for the implied future growth potential in different countries. The findings reveal sizable heterogeneity in the growth potential. Countries with low levels of life expectancy and income still exhibit considerable unrealized growth potential – according to our quantitative results, countries with average incomes below 22,000 USD have only realized about 40% of the demographic dividend, on average. In contrast, high-income countries are expected to face a drag on future development due to the demographic “debt” that has built up during the past phases of development. Our results suggest that, on average, countries with average incomes of more than 22,000 USD have overdrawn their demographic growth potential by 25%, on average.

The remainder of the paper is structured as follows. After discussing the contribution in light of the literature in Section 2, Section 3 describes the model and characterizes the long-run equilibrium dynamics. Section 4 describes the quantitative implementation and the estimation approach. Section 5 presents the results of the structural estimation and of the model fit. Section 6 presents a decomposition analysis of comparative development differences and the results of various quantitative exercises. Section 7 concludes.

## 2 Contribution to the Literature

This paper contributes to several strands of literature. We develop a parsimoniously parameterized prototype heterogeneous agent model of endogenous long-run growth whose core building blocks are human capital and technology. The model comprises the endogenous determination of human capital, mortality, and fertility. Human capital is determined in an occupational choice heterogeneous agents model with choices of acquiring unskilled human capital or skilled human capital. Endogenous population dynamics are related to individual fertility decisions. By determining the available time for education and work, mortality is the crucial state variable for investments in human capital and children at the individual level. Mortality evolves endogenously as consequence of human capital externalities. The aggregate production technology determines the demand for human capital and evolves dynamically. The quantitative model combines different building blocks of technological change and fertility as in the seminal unified growth framework by Galor and Weil (2000), of differential fertility as in de la Croix and Doepke (2003), and occupational choice and finite longevity as in Cervellati and Sunde (2005, 2015), but in contrast to these earlier contributions allows for a systematic quantitative analysis on the basis of a structural estimation approach.

The modern economic growth literature (e.g., Solow, 1956; Lucas, 1988; Romer, 1990; Aghion and Howitt, 1992) has traditionally focused on studying the mechanisms behind sustained income growth along a balanced growth with all relevant state variables growing at constant rates. Empirical work in this field is also conceptually rooted in the notion of a balanced growth path and convergence dynamics to this path (see, e.g., Mankiw, Romer, and Weil, 1992; Dalgaard and Strulik, 2013), with a particular focus on the role of human capital (Krueger and Lindahl, 2001; Cohen and Soto, 2007; Portela, Alessie, and Teulings, 2010; Sunde and Vischer, 2015). The exclusive focus on balanced growth dynamics is inconsistent with the first stylized fact, namely the secular acceleration of economic development and the simultaneous expansion of education attainment (see also Jones and Romer, 2010; Jones, 2016). Work that has considered an acceleration in growth as consequence of accumulation processes includes the seminal contribution by Hansen and Prescott (2002). However, their focus on capital accumulation is inconsistent with evidence suggesting that physical capital plays a minor role for explaining growth and growth accelerations, and their approach does not account for endogenous demographic dynamics. Unified growth theories instead rationalize the growth acceleration by emphasizing the interplay between economic and demographic development (see, e.g., the seminal work by Galor and Weil, 2000; Galor, 2011).

However, these contributions are typically restricted to qualitative analysis. They remain largely silent about the roots of the global income divergence and about the reasons for the income differences and their persistence. Moreover, they do not address the role of mortality and thus remain limited in accounting for fertility dynamics. Our approach explicitly addresses these issues. We present a quantitative and empirical analysis that can reconcile the insights of the endogenous balanced growth literature with the insights of unified growth theories regarding long-run dynamics. In particular, our analysis explicitly accounts for non-linear development dynamics and delays in attaining the balanced growth path while providing a quantitative methodology that allows estimating the structural parameters that are responsible for the differences in the long-run dynamics of economic and demographic development across countries.

In an attempt to explain the heterogeneity in development and growth dynamics today, the literature on growth accounting has isolated productivity growth as the key to explaining growth dynamics (see, e.g., Jones, 2016). Likewise, when focusing on the empirical determinants for the existing income differences across the world, the development accounting literature has pointed towards a critical role of productivity differences (see, e.g., Jones, 2016). Evidence also suggests that productivity differences are amplified through the accumulation processes behind factor endowments (see, e.g., Hsieh and Klenow, 2010; Comin and Mestieri, 2018). In addition, recent work indicates that skill bias in technology is a quantitatively important explanation for higher productivity of high-skilled workers in high-income countries (Rossi, 2022). This paper provides a rationale for these patterns and their emergence, by relating differences in productivity to the accumulation of productivity associated with long-run development dynamics, especially related to human capital. Our decomposition results show how initial conditions and country-specific parameters can have a fundamental impact on development by affecting the timing and the intensity of the growth take-off. Specifically, our approach demonstrates how the dynamics of human capital accumulation are associated with the growth take-off and the compounding of productivity differences, and how they can explain the observed development dynamics.

A distinct strand of literature has explored the “deep determinants” of development differences, which manifest themselves as differences in total factor productivity (Hall and Jones, 1999), but which are related to time-invariant factors that imply persistence in development differences over long periods of time. This literature has documented a large variety of latent country-specific characteristics such as geographical, climatological, historical, cultural, or institutional factors that explain historical and contemporaneous development differences,

typically in reduced form and separately from each other (e.g., Hall and Jones, 1999; Gallup, Sachs, and Mellinger, 1999; Olsson and Hibbs, 2005; Tabellini, 2010; Ashraf and Galor, 2013; Galor and Özak, 2016, belong to a non-exhaustive list of examples), see Spolaore and Wacziarg (2013) for a survey. While the considerable persistence of these factors can influence the development trajectory over long periods of history, this makes it hard to disentangle the different candidates and their influence on comparative development (for a discussion of this point see, e.g., Voth, 2021; Durlauf, 2023), and the link to the transition to modern growth and its timing is typically not addressed. Our framework provides an interpretative lens for the mechanisms through which the deep determinants affect comparative development differences. Moreover, our quantitative analysis documents the close link between the deep determinants that have been isolated in reduced-form empirical work and the structural parameters that influence the timing, extent, and steepness of the transition. This helps closing the gap between the empirical work on comparative development and theories of long-run development, which has been largely neglected in the existing literature. In addition, our approach allows us to unfold the dynamic mechanisms through which the latent characteristics affect comparative development and thereby establish a link between the deep determinants and heterogeneity in historical as well as in contemporaneous development. We also show how comparative development differences can be decomposed into the contributions of time-invariant factors and of the non-linear development dynamics that characterize the transition to modern growth. The quantitative decomposition of development also allows us to contribute to recent discussions about the consequences of demographic change for future growth (see, e.g. Cervellati, Sunde, and Zimmermann, 2017; Jones, 2022a,b).

In spirit, our analysis is close to early work by Rostow (1956), who decomposed the growth process into three distinct phases – a long, quasi-stagnant phase, the rapid transition that entails a growth take-off, and finally a phase of sustained growth – and who emphasized the crucial role of the pre-transitional phase in establishing the conditions required for the take-off. Few recent studies emphasize the empirical relevance of variation in the timing of the growth take-off (Cervellati and Sunde, 2015; Cervellati, Meyerheim, and Sunde, 2019) and explore the implications of cross-country spillovers in technology (Delventhal, Fernandez-Villaverde, and Guner, 2021; Cervellati, Meyerheim, and Sunde, 2023). We provide the first comprehensive quantitative framework that explicitly links comparative development differences to the non-linear development dynamics through a structural estimation of the parameters that govern the timing, steepness, and extent of the transition. In contrast to existing work, our study is concerned with the non-monotonic long-run dynamics of aggre-

gate economic development and the estimation exploits the heterogeneity in the non-linear dynamics of economic and demographic development across a sample of 86 countries. A key contribution is to link long-run growth dynamics and comparative development differences in a coherent and internally consistent way, providing a new perspective for studying development differences.

Our quantitative analyses of the demographic dividend at different stages of development contributes to a recent literature that has investigated the consequences of demographic change for growth, either using reduced form approaches (Kotschy and Sunde, 2018; Maestas, Mullen, and Powell, 2023) or quantitative models (Doepke, 2004; Cooley and Henriksen, 2018; Aksoy, Basso, Smith, and Grasl, 2019). The quantitative decomposition of development differences into the contributions of productivity gains during the different phases of long-run development complements work that has tried to account for development differences by considering differences in human capital accumulation (Erosa, Koreshkova, and Restuccia, 2010; Manuelli and Seshadri, 2014; Hanushek, Ruhose, and Woessmann, 2017) and how this complements initial differences in productivity (Hsieh and Klenow, 2010).

### 3 A Quantitative Model of Long-Run Growth

#### 3.1 Production

We consider a sequence of overlapping generations of individuals  $t \in \mathbb{N}^+$ . Each generation of individuals produces a unique consumption good with an aggregate production function

$$Y_t = A_t [x_t (H_t^s)^\eta + (1 - x_t) (H_t^u)^\eta]^\frac{1}{\eta} \quad (1)$$

with  $A_t$  denoting total factor productivity representing the technology available to generation  $t$ ,  $H_t^s$  and  $H_t^u$  denoting the aggregate stocks of skilled human capital and unskilled human capital of the generation, respectively,  $\eta \in (0, 1)$ , and the relative weight of skilled human capital in production,  $x_t \in (0, 1) \forall t$ , reflecting the degree of skill bias in the production process.

Wages are determined on competitive markets and thus equal marginal products

$$w_t^s = \frac{\partial Y_t}{\partial H_t^s} \quad \text{and} \quad w_t^u = \frac{\partial Y_t}{\partial H_t^u} \quad (2)$$

### 3.2 Demographics and Individual Decisions

The life of each individual of a generation  $t$  consists of two periods, childhood and adulthood. During childhood, which lasts for a fixed time  $k$ , members of generation  $t$  make no choices but face a child survival probability,  $\pi_t \in (0, 1)$ . During adulthood, individuals make all their decisions. Reproduction takes place after the fixed time interval  $m \geq k$ . Individuals are heterogeneous with respect to their innate ability, with individual  $i$  being endowed with ability  $a^i \in [0, 1]$ . Ability is drawn at birth from a truncated normal distribution on the closed interval between zero and one, with mean  $\mu$  and standard deviation  $\sigma$ . All draws are assumed to be independent from parental ability.

Individuals have preferences defined over their lifetime consumption  $c$  and over the quantity  $n$  and quality  $q$  of (surviving) children. We assume that time is continuous,  $z \in \mathbb{R}^+$  to incorporate variation in the length of life of a generation. Concretely, we assume that adulthood lasts for a duration of  $T_t$ , which corresponds to the deterministic life expectancy at age  $k$ . We abstract from life cycle considerations by assuming no discounting and merely ensuring a balanced lifetime budget.<sup>1</sup> Lifetime utility of individual  $i$  is then given by

$$U(c_t^i, \pi_t n_t^i q_t^i) = \int_0^{T_t} \ln c_t^i(\tau) dz + \gamma \ln(\pi_t n_t^i q_t^i) \quad \text{with } \gamma > 0 \quad (3)$$

Individuals maximize their lifetime utility by deciding about the number of children  $n_t^i$ , the time spent raising each child  $r_t^i$ , which maps into child quality  $q_t^i$  as described below, and by making an occupational choice that amounts to acquiring unskilled human capital or skilled human capital. When making these decisions, individuals take the economic and demographic environment, in terms of technology, wages, and mortality as given.

Individuals face two constraints, a time constraint and a budget constraint. The effective productive time available during adulthood is bounded from above by adult longevity  $T_t$ , which gives rise to the time constraint

$$T_t \geq l_t^i + e^i + \pi_t n_t^i r_t^i \quad (4)$$

with  $l_t^i$  denoting lifetime labor supply,  $e^i$  denoting the time spent on acquiring human capital, and the time  $r_t^i$  denoting the time spent raising each child that survives infancy.<sup>2</sup>

<sup>1</sup>For this reason, it suffices to assume a zero interest rate and the existence of perfect capital markets that are not modeled explicitly. Allowing for discounting would affect the results quantitatively but not qualitatively. The same applies to assuming age-dependent mortality. Details are available upon request.

<sup>2</sup>The inclusion of a mandatory retirement age  $R$  implies an effective length of productive life of  $\bar{T}_t = \min\{T_t, R\}$ .

The expenditure constraint requires lifetime consumption to not exceed lifetime labor income

$$w_t^j h_t^{j,i} l_t^i \geq T_t c_t^i \quad (5)$$

where  $w_t^j$  denotes the wage per time unit of labor supply of an individual with human capital  $h_t^{j,i}$ .

Human capital is modeled using a simplified version of a standard Mincerian human capital production function. The acquisition of skilled human capital is facilitated by individual ability  $a$ . An individual with ability  $a$  deciding to acquire skilled human capital acquires

$$h^{s,i} = e^{\alpha a^i} \quad (6)$$

units of human capital, with  $\alpha$  denoting the return to ability. In exchange, becoming skilled implies foregoing a fixed cost in terms of time  $e^s > 0$ . Acquiring unskilled human capital involves no fixed cost in terms of time and delivers

$$h^u = e^{\alpha \mu} \quad (7)$$

units of human capital, where  $\mu$  is the mean of the ability distribution. This implies that individuals with a high ability have a comparative advantage in becoming skilled.

Other than labor supply or acquiring human capital, time can also be invested in the quality of children. The quality of offspring is modeled as

$$q_t(\underline{r}, \tilde{r}_t^i, g_{t+1}) = [\tilde{r}_t^i \delta (1 + g_{t+1}) + \underline{r}]^\beta \quad \text{with } \delta > 0, \beta \in (0, 1) \quad (8)$$

where  $\tilde{r}_t^i$  denotes the parental choice of how much time to spend on educating each child that survives to adulthood,  $g_{t+1}$  denotes productivity growth, and  $\underline{r}$  denotes a fixed cost of raising children. This fixed cost is crucial for the parental decisions about fertility and child investments (see, e.g., Strulik and Weisdorf, 2014). In the analysis below,  $\underline{r}$  constitutes a country-specific structural parameter that will be estimated.

### 3.3 The Intra-Generational Equilibrium

For a generation  $t$  of individuals, the demographic environment in terms of  $T_t$  and  $\pi_t$ , as well as the technological environment, in terms of  $A_t$  and  $x_t$  are given. Individuals maximize their lifetime utility by choosing the type of human capital they acquire (i.e., becoming

skilled or unskilled workers) and their fertility in terms of quantity and quality of their children,  $\{j = \{u, s\}, n, r\}$ , while respecting their lifetime budget and time constraints, taking wages as given. Consequently, the intra-generational general equilibrium reflects the fixed point at which optimal individual decisions are consistent with the aggregate allocation in terms of the skill composition of the population that determines wages. As a result of the comparative advantage of individuals with high ability in becoming skilled, the equilibrium is characterized by the ability threshold  $\tilde{a}_t$  that represents the ability of the marginal individual that is indifferent between becoming skilled or unskilled. This threshold pins down the skill share as

$$\lambda_t = \int_{\tilde{a}_t}^1 f(a) da = \Lambda(T_t, x_t) \quad (9)$$

with  $f(\cdot)$  denoting the pdf of ability. The function  $\Lambda(\cdot, \cdot)$  is increasing in longevity,  $T_t$ , and in the skill bias in technology as reflected by  $x_t$ . Intuitively, the incentive to becoming skilled increases with a longer amortization period for the time investment in skill acquisition, and with a larger skill premium due to higher demand for skills. The corresponding fertility in equilibrium is given by

$$n_t = N(T_t, \lambda_t, \pi_t) \quad (10)$$

and is thus determined by parental education choice, which exhibits a fertility differential between individuals acquiring unskilled and skilled human capital. Under the present setting, existence and uniqueness of this equilibrium are ensured.

### 3.4 Dynamics

The model is closed by defining the laws of motion for demographics and technology. To discipline the analysis, we adopt the usual view of a country as the unit of interest. This constitutes a natural starting point for an analysis of comparative development differences from the perspective of long-run growth dynamics and allows us to explore the capability of the model to replicate the empirical patterns for different countries in the world. It should be expected from the outset that this setting works better empirically for some countries than for others. In particular, today's low-income countries are confronted with a much greater potential to benefit from an advanced frontier in health and production technologies than Western forerunner countries did during their early phases of development.

Mortality dynamics are modeled as follows. In line with empirical evidence from devel-



opment economics, the child survival probability is assumed to depend on living conditions (at birth), which can be represented as

$$\pi_t = 1 - \frac{1 - \underline{\pi}}{1 + \kappa \lambda_{t-1} y_{t-1}} \quad (11)$$

where  $\underline{\pi}$  is the extrinsic (baseline) child survival rate in the absence of parental education, and  $\kappa$  is the elasticity of child survival to economic conditions (with income per capita denoted by  $y_{t-1} = Y_{t-1}/N_{t-1}$ ).

Adult longevity is assumed to depend on the medical achievements of the previous generation, which boils down to a human capital externality of the generation  $t - 1$  on the longevity of generation  $t$

$$T_t = \underline{T} + \rho \lambda_{t-1} \quad (12)$$

with  $\rho$  denoting the strength of this inter-generational externality, and  $\underline{T}$  reflecting the extrinsic (baseline) mortality environment in the absence of any medical knowledge or health technology. Through its crucial role for the individual occupational choice of acquiring skilled human capital, the extrinsic mortality environment is an important determinant for the timing of the education expansion. In the analysis below,  $\underline{T}$  constitutes the second country-specific structural parameter that will be estimated.

The dynamics of the production technology also evolve endogenously as the result of inter-generational externalities. Specifically, the weight of skilled human capital in production is assumed to increase in the skill share of the parent generation, but at a rate that is declining in the existing weight, such that

$$\frac{x_t - x_{t-1}}{x_{t-1}} = \lambda_{t-1}(1 - x_{t-1}) \quad (13)$$

Total factor productivity also increases with skilled human capital

$$g_{t+1} = \frac{A_{t+1} - A_t}{A_t} = \phi \lambda_t \quad (14)$$

with  $\phi > 0$  denoting the strength of the externality. The production environment is strongly influenced by the country-specific environmental conditions. In the estimation, this will be reflected by treating the initial condition  $A_0$  as the third country-specific structural parameter that will be estimated.

### 3.5 The Dynamic Equilibrium

The dynamic equilibrium comprises the sequence of endogenous state variables that emerge from a set of initial conditions as the consequence of intra-generational equilibrium conditions for each generation and the resulting inter-generational spill-overs. Specifically, the dynamic evolution of the economy can be characterized by a non-linear dynamic system among the central state variables – the skill composition of the population  $\lambda_t$ , adult longevity  $T_t$ , the skill bias of technology  $x_t$ , and total factor productivity  $A_t$  – which is described by conditions (9), (12), (13), and (14). The corresponding accumulation processes are

$$\begin{aligned}\lambda_t &= \Lambda(x_t, T_t) \\ T_{t+1} &= \underline{T} + \rho \lambda_t \\ x_{t+1} &= [1 + \lambda_t(1 - x_t)]x_t \\ A_{t+1} &= (1 + \phi \lambda_t)A_t\end{aligned}$$

and iterating forward yields

$$\begin{aligned}x_{t+1} &= x_0 \prod_{\tau=1}^{t+1} (1 + \lambda_{\tau-1}) - x_0^2 \sum_{\tau=1}^{t+1} \lambda_{\tau-1} \prod_{m=0}^{\tau-1} (1 + \lambda_m) \\ A_{t+1} &= A_0 \prod_{\tau=1}^{t+1} (1 + \phi \lambda_{\tau-1})\end{aligned}$$

where initial TFP,  $A_0$ , as well as baseline mortality,  $\underline{T}$ , are proxies for deep determinants. Hence, our heterogeneous agents endogenous growth model characterizes long-run development dynamics as the consequence of the interplay between deep determinants and an accumulation process that crucially relies on human capital (reflected by the share skilled,  $\lambda$ ) as the engine of development. This simple, generic, and parsimonious model incorporates a phase of slow development, an endogenous acceleration to a regime of long-run balanced growth and a role for deep determinants.<sup>3</sup> Given a set of initial conditions and time-invariant parameters, the system endogenously produces the dynamic evolution of income as the result of the intertwined dynamics of the human capital composition of the population and technology (in terms of skill bias and TFP), as well as the other endogenous state variables, including adult longevity, fertility, and child survival.

---

<sup>3</sup>The properties of the dynamic equilibrium path can be illustrated by the evolution of the (conditional) dynamic system in the  $\{\lambda, T, x\}$ -space as detailed in the Appendix in Figure A1.

The model allows for a decomposition of output per capita along the development path in terms of variation in productivity and human capital. Notice that output per capita can be written as

$$y_t = \frac{Y_t}{N_t} = A_t \left\{ x_t \left( \int_{\tilde{a}_t}^1 h^s f(a) da \right)^\eta + (1 - x_t) [(1 - \lambda_t) h^u]^\eta \right\}^{\frac{1}{\eta}} \quad (15)$$

Hence, the income dynamics are governed by the dynamics of TFP,  $A_t$ , the dynamics of sectoral composition  $x_t$  (reflecting industrialization), and the dynamics of human capital composition,  $\lambda_t$ . While the dynamics of sectoral composition and human capital composition are of crucial importance for the transition from stagnant development to a balanced growth regime, the balanced growth path is governed by the dynamics of TFP. The model thus reflects the insights of modern growth theory that sustained growth relies on productivity improvements. At the same time, the model illustrates that the dynamics in the sectoral and educational composition are crucial for the non-recurrent acceleration of growth during the transition. The intuition for this result is easily seen when considering the limit of  $\lambda_t \rightarrow 1$  along the balanced growth path, which implies  $x_{t+1} \rightarrow x_t(2 - x_t)$ , and hence a steady state at  $x_{t+1} = x_t = x \rightarrow 1$ .

It is worth noting that the model does not rely on scale effects, which implies that population size is not important for the long-run dynamics of output per capita, since  $A_{t+1} = (1 + \phi\lambda_t)A_t$ .<sup>4</sup> Unlike in most existing models of long-run growth, there are no non-convexities that imply corner solutions. Instead, the dynamics are given by the smooth evolution of the dynamic system that results from interior solutions in all variables.

In the following, we conduct the thought experiment that all countries follow the same qualitative development path. This approach is conservative in the sense that the development dynamics in all countries rely on the same economic trade-offs and mechanisms, which disciplines the analysis and limits the degrees of freedom. Contemporaneous development differences are then the result of a delay in the timing of the transition to balanced growth. In the next section, we develop a methodology that allows us to estimate the structural parameters of the model. In particular, we estimate the parameter  $\underline{T}$  that is responsible for heterogeneity in the timing of the take-off in general, as well as the structural parameters  $A_0$  and  $\underline{r}$  that govern the steepness of the take-off and the scope for the human capital expansion, as well as the remaining parameters that are common to all countries. The three structural

---

<sup>4</sup>Note that scale effects along the lines of semi-endogenous growth models as in Jones (2022a) could be added. However, we refrain from specifying a particular scale effect as it would not alter the main qualitative results.

parameters capture heterogeneity in the constraints of the optimization problem that underlies the development process and are linked to country-specific determinants of development differences that have been isolated in empirical work, as detailed below.

## 4 Empirical Implementation and Estimation

### 4.1 Data and Calibration

The parameters are estimated to match panel data for a balanced panel data set of 86 countries over the period 1880 – 2020 in 20-year intervals. In particular, the data set comprises information about GDP per capita, life expectancy at birth, and total fertility rates from Gapminder.<sup>5</sup> As a result, there is one unique time series per country to identify each of the three parameters.

We calibrate parameters that are of no immediate relevance for the main mechanism of non-linear long-run growth and that we assume to be identical across countries. Concretely, we set  $\mu$  and  $\sigma$  to values corresponding to the conventional scale of an average IQ of 100 points and a standard deviation in IQ scores of 15 points. We proceed to calibrate parameters that are targeting long-run moments on the balanced growth path using a restricted sample of 18 Western high-income countries (the “Forerunners”), while calibrating parameters that are targeting pre-transitional moments using the full sample. The length of childhood,  $k$ , is set to 5 years and the retirement age  $R$  is calibrated using data from the OECD on the average retirement age of developed countries for the period 2013 – 2018. We next set the value of  $e^s$  to the average years of secondary schooling for developed countries in 2010 using data from Lee and Lee (2016). The parameter  $\alpha$  governs the stationary distribution of income in the long run.<sup>6</sup> We calibrate  $\alpha$  by using data of the standardized world income inequality database by Solt (2020) for the average (market) Gini coefficient of income of developed countries in 2020. The parameter  $\underline{\pi}$  is calibrated using data from Gapminder on the average rate of child survival in the full sample in 1800. The parameter  $\kappa$  is calibrated by using (11) evaluated at the previously calibrated  $\underline{\pi}$ ,  $\lambda = 1$ , and the average GDP per capita of the developed

---

<sup>5</sup>Gapminder aggregates long-run data from multiple sources, for information on the data and the respective primary sources consult the documentation available at <https://github.com/Gapminder-Indicators>.

<sup>6</sup>In particular, the stationary distribution of income in the long run of the model is characterized by a log-normal distribution with a Gini coefficient given by

$$\text{Gini} = \text{erf}\left(\frac{\alpha\sigma}{2}\right)$$

where  $\text{erf}(\cdot)$  is the Gaussian error function.

Table 1: Calibrated Parameters

| Parameter | Value | Explanation                   | Matched Moment               | Sample                   |
|-----------|-------|-------------------------------|------------------------------|--------------------------|
| $\mu$     | 0.5   | Ability distribution          | Mean IQ of 100               |                          |
| $\sigma$  | 0.075 | Ability distribution          | Std. deviation of IQ of 15   |                          |
| $k$       | 5     | Duration of childhood         | End of child mortality       |                          |
| $R$       | 60    | Retirement age                | Mean retirement age          | Forerunners, 2013 – 2018 |
| $e^s$     | 12    | Time cost of becoming skilled | Mean years of sec. schooling | Forerunners, 2010        |
| $\alpha$  | 7.76  | Mincerian return of ability   | Mean Gini coefficient        | Forerunners, 2020        |
| $\pi$     | 0.63  | Child survival (baseline)     | Mean child survival          | Full sample 1800         |
| $\kappa$  | 0.002 | Child survival (improvement)  | Mean child survival          | Forerunners, 2020        |
| $\gamma$  | 8     | Utility weight of children    | Mean total fertility rate    | Forerunners, 2020        |

countries in 2020 while targeting the average child survival rate in developed countries in 2020. Finally,  $\gamma$  is calibrated by using the model’s long-run solution of fertility and targeting the average total fertility rate of the developed countries in 2020. The respective parameters, values, and targets are listed in Table 1.

## 4.2 Estimation Approach

In this section, we develop an estimation methodology that allows us to assess the role of the non-linear growth dynamics implied by the model for matching actual data. The structural estimation of the model delivers two sets of parameters: a set of three structural parameters that are country-specific and a set of global parameters that are the same across all countries.

The crucial structural parameters for the long-run growth dynamics in the core dimensions of the model are estimated separately for each country. In particular, countries are allowed to differ in three time-invariant parameters: the extrinsic level of adult longevity,  $\underline{T}$ , the minimum fertility cost in terms of time that has to be devoted to raising of children,  $\underline{r}$ , and the initial level of total factor productivity,  $A_0$ . In contrast to the calibrated parameters, which are set so as to match moments along the balanced growth path, these “deep” parameters reflect heterogeneity in latent structural dimensions across countries in terms of their initial development.  $A_0$  represents development (income) in the absence of a skilled population (when  $\lambda = 0$ ). Likewise,  $\underline{r}$  effectively determines the population (dynamics) and  $\underline{T}$  governs development in terms of longevity in the absence of a skilled population (when  $\lambda = 0$ ), i.e., in an initial state of the world without (prior) knowledge accumulation. With a comparable target of a balanced growth path thanks to the identical calibration of other parameters, the structural parameters indirectly thereby also determine the timing of the transition, the extent of the human capital expansion, and the steepness of the take-off.

The vector of country-specific parameters to be estimated is denoted by  $\Theta = (A_0, \underline{T}, \underline{r})$ . The goal of the estimation is to determine the levels of the country-specific parameters that provide the closest fit of the model-generated data to the observed cross-country panel data. In practice, we use the parametric version of the model developed in the previous section as data generating process and simulate an artificial world composed of 86 countries. In the artificial world, countries differ only in terms of the three “deep” parameters  $\Theta = (A_0, \underline{T}, \underline{r})$ . Each country is simulated over the period 0 A.D – 2100 using a 20-year frequency of subsequent generations.

To illustrate the empirical approach, denote by  $Z$  the matrix of actual data in the three dimensions of interest: log GDP per capita, life expectancy at birth, and the total fertility rate, with observations for the observation period 1880 – 2020 in 20-year frequencies in each dimension. Analogously, denote by  $\tilde{Z}$  the corresponding matrix of simulated values in the three dimensions over the same observation period. The vector of parameter estimates  $\hat{\Theta}_j = (A_0^j, \underline{T}^j, \underline{r}^j)$  is the solution of a minimization of distance in the residuals that is conducted for each country  $j = 1, \dots, 86$  separately according to

$$\hat{\Theta}_j = \arg \min_{\Theta_j} \psi(\Theta_j)' W \psi(\Theta_j) \quad (16)$$

with  $\psi(\Theta_j) = \frac{\tilde{Z}_j - Z_j}{Z_j}$  denoting the vector of residuals, which depends on the set of structural parameters  $\Theta_j$  and is conditional on a set of parameters that are common for all countries,  $\hat{\Phi}$ , and  $W$  denoting a weighting matrix.<sup>7</sup>

Parameters and initial conditions that are assumed to be common for all countries and for which direct empirical estimates or a straightforward calibration are not readily available, are estimated using the same procedure outlined above. These include the initial weight of skilled human capital in production, initial population size and the initial ability threshold, as well as parameters determining the elasticity of substitution between production factors, the maximum span of longevity improvements, the evolution of TFP, and investments in child quality,  $\Phi = (x_0, N_0, \tilde{a}_0, \eta, \rho, \phi, \beta, \delta)$ . The vector of estimates of the country-common parameters  $\hat{\Phi} = (x_0, N_0, \tilde{a}_0, \eta, \rho, \phi, \beta, \delta)$  is the solution to

$$\hat{\Phi} = \arg \min_{\Phi} \psi(\Phi)' W \psi(\Phi) \quad (17)$$

---

<sup>7</sup>The weighting matrix here is given by the identity matrix. For iterated estimation, an alternative would be the weighted squared error terms of the previous stage estimation. Unreported experiments show that this does not change the results as  $\psi(\Theta_j)$  already measures relative distances. Details are available upon request.

where  $\psi(\Phi) = \frac{\tilde{Z}-Z}{Z}$  denotes the vector of residuals, which depends on  $\Phi$  and is conditional on a set of 86 country-specific parameter vectors  $\hat{\Theta}_j$ , which we denote by  $\hat{\Theta}$ .

The country-specific and country-common parameters are estimated jointly via an iterative process. Since the estimates of country-specific and country-common parameters are interdependent, we apply a sequential estimation approach. For a given set of values for the calibrated parameters, we assign an initial value to each of the country-common and country-specific parameters.<sup>8</sup> Next, we estimate the vector of country-specific parameters,  $\hat{\Theta}$ , for this initial setting of country-common parameters. Subsequently, the country-common parameters  $\hat{\Phi}$  are estimated, holding the vector of country-specific parameters  $\hat{\Theta}$  fixed. These are re-estimated in a next round conditional on the estimates of country-common parameters,  $\hat{\Phi}$ , which are then re-estimated subsequently. This is repeated until convergence in terms of a pre-defined level of parameter stability is achieved. These estimates serve as benchmark and proof of concept for the methodology. Below we discuss the sensitivity of the results with respect to modifications in the assumptions about parameter homogeneity.

## 5 Structural Estimation Results

### 5.1 Estimation Results

The structural estimation of the model delivers a set of global parameters that are the same across all countries and three structural parameters that are estimated separately for each country. While most of the estimates refer to model-specific parameters, some have a straightforward interpretation and can be readily assessed in terms of the adequacy and external validity of the estimates. For instance, an estimate of the CES coefficient  $\eta$  of around 0.3 corresponds to an elasticity of substitution between skilled human capital and unskilled human capital of approximately 1.4, which is in the range that is typically considered reasonable in the literature (see, e.g. Acemoglu and Autor, 2011; Rossi, 2022). Likewise, an estimate of 44 years for the scope of improvement in adult longevity from extrinsic levels of  $\underline{T}$  of 30 to 35 years implies approximately an upper bound for life expectancy at birth of 84 ( $= 5 + 35 + 44$ )

---

<sup>8</sup>For the country-specific parameters, the initial values are identical across countries. Specifically, we estimate the model for the set of 18 developed countries to obtain an estimate of the country-specific parameters and a benchmark estimate for the country-common parameters as well as for the country-common initial conditions. Then, we estimate the country-specific parameters and the country-common parameters (excluding the country-common initial conditions) for the full set of 86 countries while refraining from re-estimating the global initial conditions in order to discipline the analysis. Re-estimating the global initial conditions does not change the results, details are available on request.

years (for  $k = 5$ ) and is in line with values that have been considered in earlier work (see, e.g. Cervellati and Sunde, 2015). Finally, an estimate of a growth externality of the skill share,  $\phi$ , of 0.67 corresponds to an annual growth rate of approximately 2.5% per year along the balanced growth path. Table 2 contains the estimation results for the global parameters.<sup>9</sup>

Table 2: Estimated Global Parameters (World Sample)

| Parameter     | Value   | Explanation   |
|---------------|---------|---|
| $x_0$         | 0.0339  | Initial weight of skilled human capital in production |
| $N_0$         | 16.2554 | Initial population size                               |
| $\tilde{a}_0$ | 0.9858  | Initial ability threshold                             |
| $\eta$        | 0.2915  | CES coefficient                                       |
| $\rho$        | 43.836  | Scope for improvements in adult longevity             |
| $\phi$        | 0.6664  | Growth along the BGP (over one generation)            |
| $\beta$       | 0.8740  | Return of investments in child quality                |
| $\delta$      | 0.7617  | Weight of time investments in child quality           |

Table 3 contains the summary statistics of the estimated country-specific parameters  $A_0$ ,  $\underline{T}$ , and  $\underline{r}$ . Figure A2 in the Appendix plots the corresponding distributions of the estimates for the three country-specific parameters as well as the distribution of the country-specific fit in terms of a Pseudo- $R^2$  that captures the variation in the observed data explained by the simulated data.<sup>10</sup> The estimates reveal considerable heterogeneity. Initial productivity,  $A_0$ , ranges from 0.031 to 0.893. Estimates for the extrinsic adult longevity parameter  $\underline{T}$  vary between 30 and 35 years. There is also considerable variation in child cost,  $\underline{r}$ .

Table 3: Estimated Country-Specific Parameters (World Sample)

| Parameter       | Mean  | Std. Dev. | Min.  | Max.  |
|-----------------|-------|-----------|-------|-------|
| $A_0$           | 0.171 | 0.144     | 0.031 | 0.893 |
| $\underline{T}$ | 31.91 | 1.027     | 30.44 | 34.47 |
| $\underline{r}$ | 2.98  | 0.648     | 2.140 | 4.796 |

<sup>9</sup>Other parameters do not have a direct empirical counterpart but nevertheless serve to discipline the model. For instance, the estimate of a common initial weight of skilled human capital in production,  $x_0$ , is equivalent to specifying a common initial share skilled  $\lambda$ .

<sup>10</sup>In particular, the Pseudo- $R^2$  is computed as  $1 - \frac{\psi(\hat{\Theta}_j)' W \psi(\hat{\Theta}_j)}{\Psi' W \Psi}$ , where  $\Psi = \frac{Z - \bar{Z}}{\bar{Z}}$  is the deviations from the mean in the actual data.



## 5.2 Results: Model vs. Data

Figure 2 plots the long-run dynamics of log income per capita, life expectancy, fertility, and the skill share, in terms of period averages for the entire sample of countries. Consistent with the discussion of the dynamic equilibrium in Section 3.5, the figure illustrates that the model dynamics exhibit a long phase of slow development and eventually a rapid transition in incomes, longevity, and fertility, all of which are crucially linked to a transition in the education composition of the population.

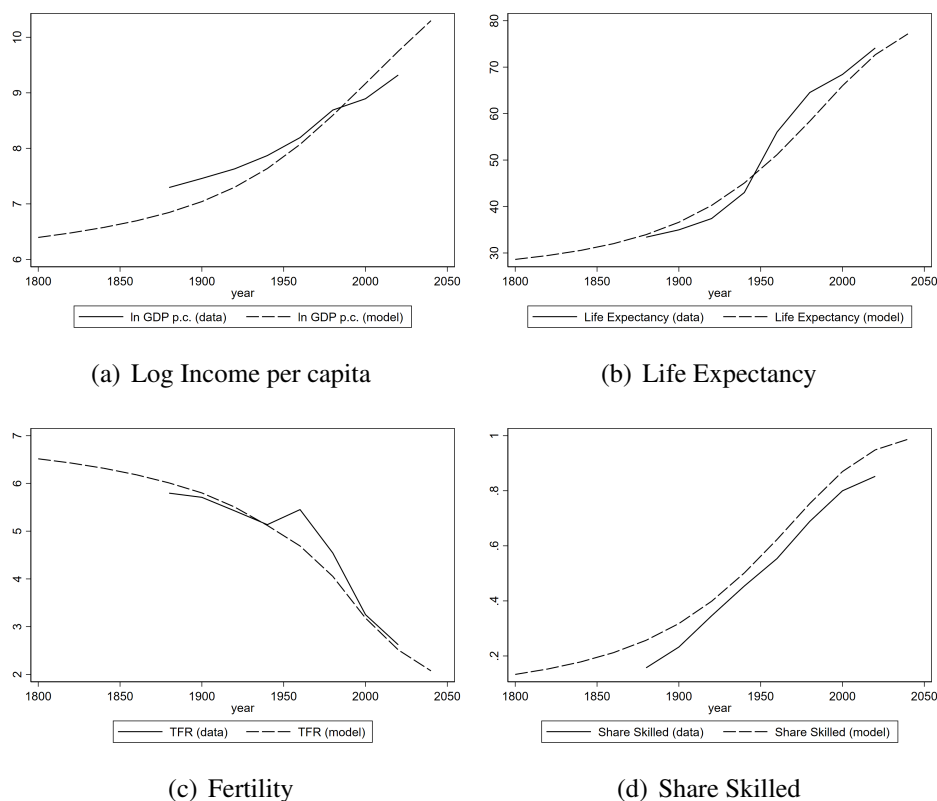


Figure 2: Long-Run Development Dynamics: Model vs. Data

*Note:* Each panel plots averages of the respective variable across countries for a given time period. The model is based on estimates of country-specific parameters that target the dynamics in log income per capita, life expectancy, and fertility. Fertility is measured by TFR. The skill share is measured by the population share with at least secondary education and is not targeted in the estimation.

In general, the model fit to the data is remarkable over the period of 140 years, particularly in light of the parsimonious parametrization and the fact that only three of the estimated parameters are country-specific. It is worth recalling that the education composition has not been used at all in the estimation procedure, but constitutes the central dimension governing

the accumulation dynamics in the model. The share skilled thus represents a dimension that can be used to gauge the model performance. In fact, the estimated model matches the corresponding non-targeted empirical moment closely.

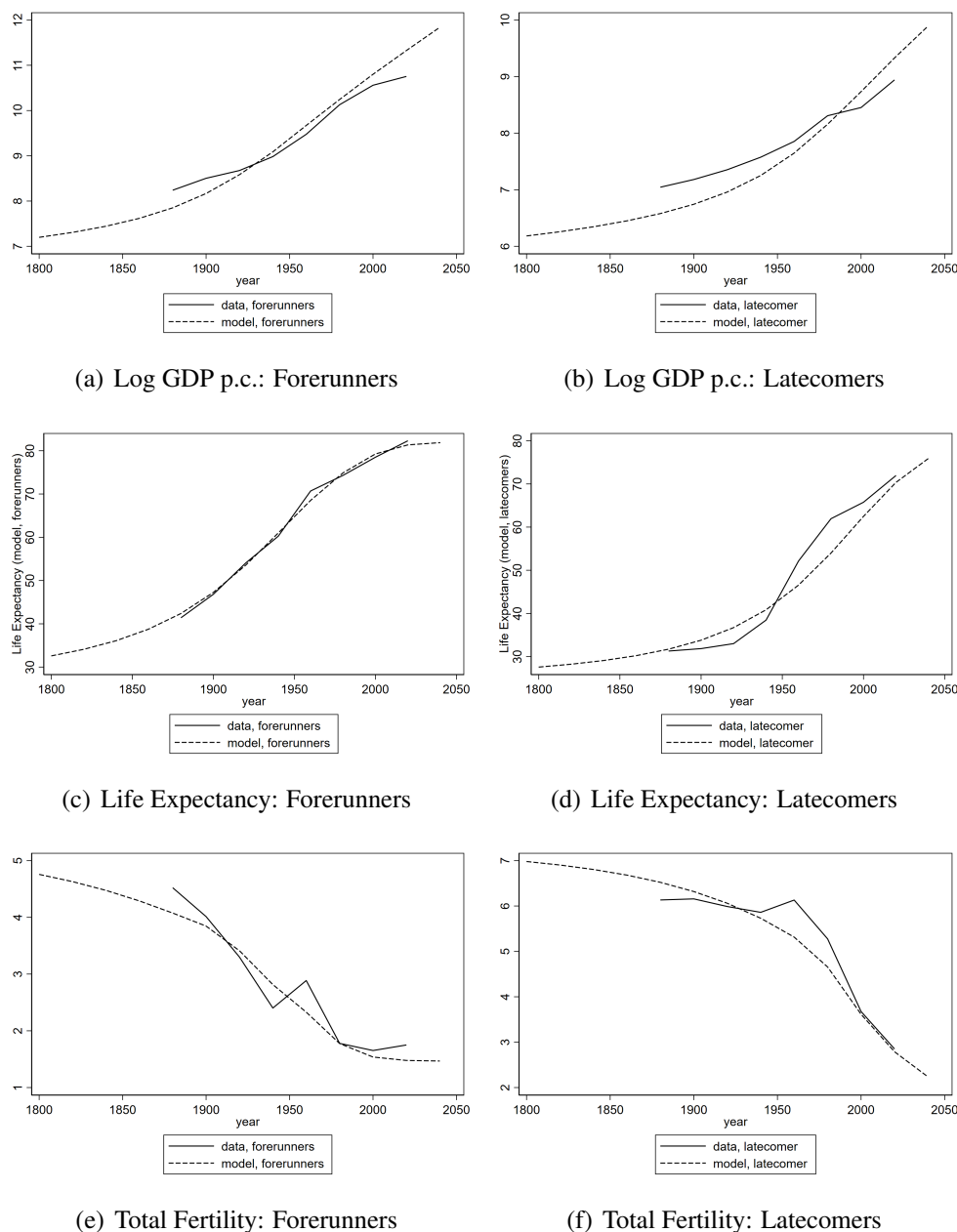
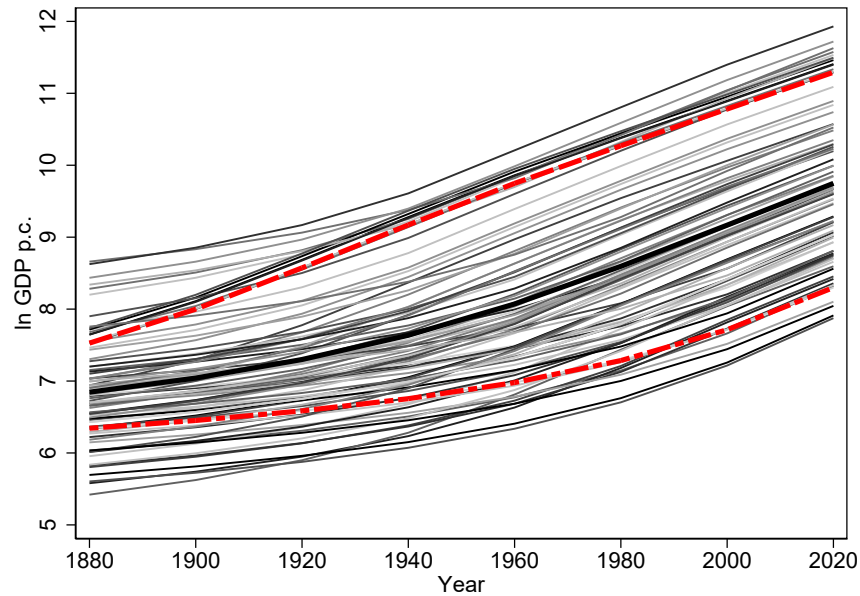


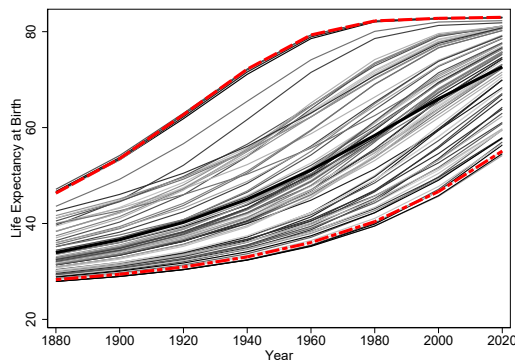
Figure 3: Long-Run Dynamics: Forerunner vs. Latecomers

Figure 3 shows the corresponding period averages when splitting the sample into “fore-runner” countries with an onset of the demographic transition before 1945, and “latecomers”

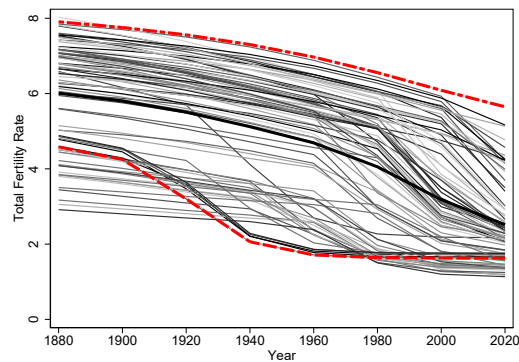
with an onset after 1945 following the classification of transition dates by Reher (2004). The figure illustrates that the quantitative fit of the model is comparable for countries with considerably different historical development trajectories.



(a) ln GDP p.c.



(b) Life Expectancy



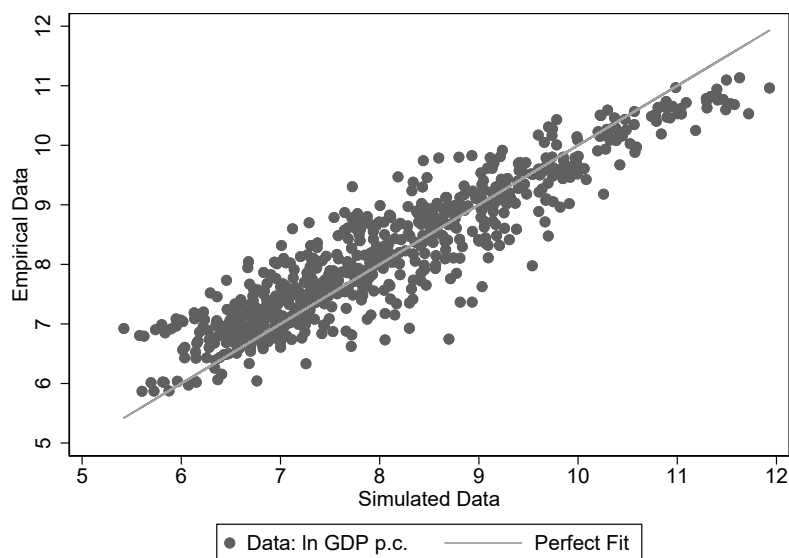
(c) Fertility

#### Figure 4: The Dynamics of Long-Run Development – Model Simulation

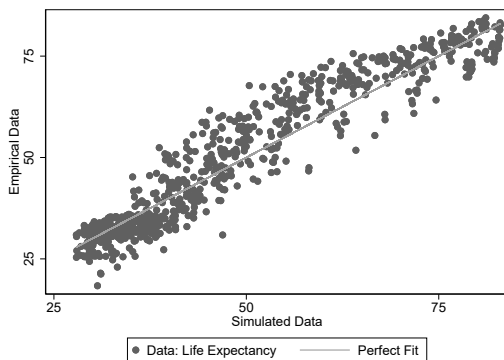
*Note:* Panel (a) plots the natural logarithm of GDP per capita. Panel (b) plots life expectancy at birth. Panel (c) plots the total fertility rate. Data source: Baseline model, parameterized based on structural estimation results. All panels: Each line corresponds to the time series for a different country. The thick (black) line corresponds to the sample mean for each time period. The thick dashed lines correspond to Sweden and the thick dot-dashed corresponds to Niger, respectively. See Appendix Table A1 for the list of 86 countries.

Figure 4 shows the simulated time series of income, life expectancy, and total fertility

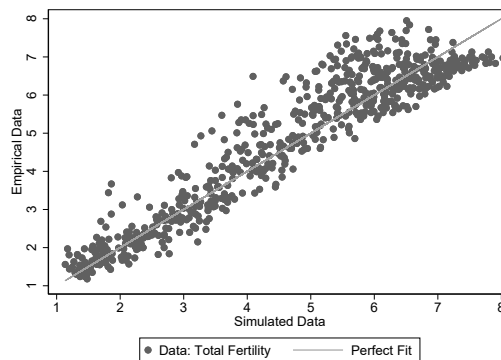
rates, based on the structural estimation results for all countries in the sample. The model generates country-specific non-linear dynamics in income, life expectancy, fertility, and human capital that exhibit considerable heterogeneity in terms of levels and timing. The comparison to Figure 1 illustrates that, despite its parsimony, the model is able to capture the stylized facts of long-run development. It is also remarkable that the model performance is comparable across subsets of countries with considerably different levels of contemporaneous development.<sup>11</sup>



(a) Comparative Development



(b) Life Expectancy



(c) TFR

Figure 5: Model Fit: Comparison of Model Data and Empirical Data

<sup>11</sup>See also Appendix Figure A3, which distinguishes between the dynamics of forerunners and latecomers.

To assess the model fit, we compare the simulated data for a given country and time period to the corresponding empirical data, pooling all countries and time periods. This point-by-point comparison is shown in Figure 5(a) for comparative development, as reflected by GDP per capita. The figure documents that the model closely fits the data. This is also true in the demographic dimensions life expectancy and fertility, as shown in Figure 5(b) and (c). The simulated data exhibit similar variation as the actual data, and the overall fit, including at the extremes of the distribution, is fairly accurate. Moreover, there does not seem to be a systematic overestimation or underestimation.<sup>12</sup>

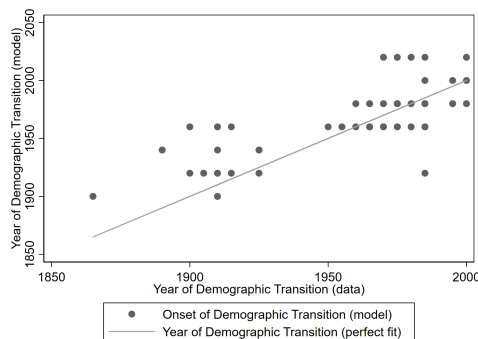


Figure 6: The Timing of the Demographic Transition: Model vs. Data

In addition, the model captures heterogeneity in the demographic transition. This is illustrated in Figure 6, which plots the years of the transition onset in the model, as measured by the first year in which life expectancy at birth exceeds 50 years as in the conventional classification in demography (see, e.g. Chesnais, 1992), against the empirical onsets, using the standard classification of the year of the onset of the demographic transition by Reher (2004).

The analysis suggests that comparative development outcomes are closely related to the timing of the demographic transition, with later transitions associated with lower levels of development. Figure 7 illustrates this in terms of scatter plots for comparative development differences measured in the year 2020 relative to the year of the demographic transition, for data and model estimates. Besides documenting the model performance, these results also show that the model captures the non-linearities of the development process reflected in the data.

<sup>12</sup>Appendix Figure A4 provides an alternative illustration in terms of scatter plots that contrast the relation between life expectancy and income, and between life expectancy and fertility, in the data and in the simulated data.

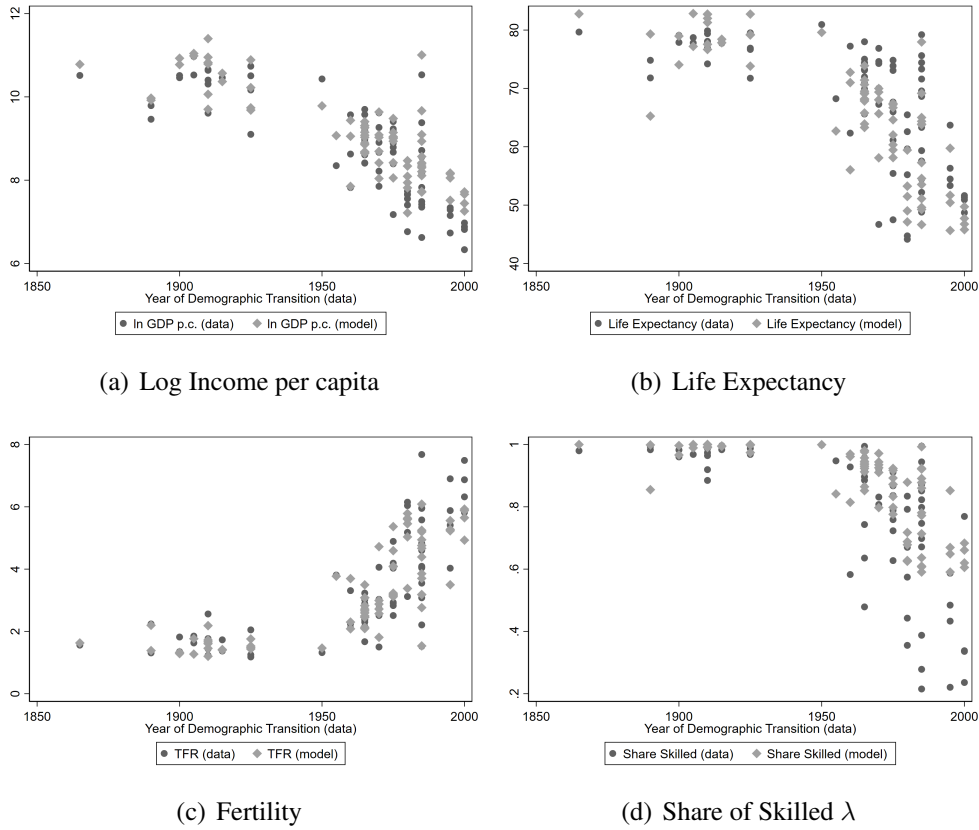


Figure 7: Timing of the Transition and Comparative Development  
Scatter plots of cross-sectional variation in the respective variables in the year 2000, relative to the timing of the demographic transition. Fertility is measured by TFR. The skill share is measured by the population share with at least secondary education.

### 5.3 Panel Data Accounting

To analyze the performance of the structural estimation of the model, we compare the fit to regression models that account for country-fixed effects or country-fixed and period-fixed effects. These models are frequently used in empirical work to account for systematic heterogeneity across countries in “deep”, time invariant factors, and systematic time trends. We compare the performance of these models, as reflected by the  $R^2$ -measure, to a regression of the observed data on the simulated data from the estimated model with no other controls.

The results are contained in Table 4. The three panels correspond to regression results for the three outcomes, log GDP per capita, life expectancy, and total fertility rates, respectively. The results in Columns (1), (4), and (7) show that country-fixed effects are able to explain a considerable share of the variation in incomes and fertility, but not so much in life expectancy.

Table 4: Model Fit (Variance Explained)

|                             | ln GDP p.c. |       |                     | Life Expectancy |       |                     | Total Fertility Rate |       |                     |
|-----------------------------|-------------|-------|---------------------|-----------------|-------|---------------------|----------------------|-------|---------------------|
|                             | (1)         | (2)   | (3)                 | (4)             | (5)   | (6)                 | (7)                  | (8)   | (9)                 |
| ln GDP p.c. (simulated)     |             |       | 0.805***<br>(0.013) |                 |       |                     |                      |       |                     |
| Life Expectancy (simulated) |             |       |                     |                 |       | 1.037***<br>(0.013) |                      |       |                     |
| TFR (simulated)             |             |       |                     |                 |       |                     |                      |       | 0.969***<br>(0.012) |
| Country Fixed Effects       | Yes         | Yes   | No                  | Yes             | Yes   | No                  | Yes                  | Yes   | No                  |
| Period Effects              | No          | Yes   | No                  | No              | Yes   | No                  | No                   | Yes   | No                  |
| Observations                | 688         | 688   | 688                 | 688             | 688   | 688                 | 688                  | 688   | 688                 |
| Adjusted $R^2$              | 0.516       | 0.902 | 0.853               | 0.088           | 0.924 | 0.903               | 0.439                | 0.822 | 0.898               |

*Note:* Estimation results based on the panel data over the period 1880-2020 in 20-year periods for 86 countries that serves as target data set for the structural estimations. See Section 4.1 for details.

Once period-fixed effects are added, the explained variance increases substantially to above 90% for income and life expectancy and to 82% for fertility, as documented in Columns (2), (5), and (8). Columns (3), (6), and (9) show the results from regressing the observed data on the model simulation in the respective country and year, but no other controls. The model closely fits the data, both in terms of Adjusted  $R^2$ , which are comparable to those in a two-way fixed effects specification, and in terms of coefficient estimates. In particular, the coefficient estimates are close to one in all dimensions, indicating that the simulated data closely track the observed data in terms of their variation across and within countries.

## 5.4 Discussion

In sum, the estimated model performs remarkably well in matching the long-run dynamics of targeted and non-targeted moments in the data. Specifically, the model generates dynamics of the skill composition that resemble those in the data, although they have not been used for the estimation. In light of the parsimony of the model that restricts attention to the well-established core building blocks of long-run dynamics – health, human capital, and technology – and abstracts from other potential first order drivers of development differences, such as country-specific heterogeneity in cultural, religious, or institutional factors, the model fit is surprisingly good. These omitted factors will influence the quantitative estimates of the country-specific parameters, but to the extent that the precise point estimates of these parameters are irrelevant for the relative ordering of the timing of the transition across countries,

this will not affect the overall fit of the model regarding the dynamics of long-run development. Likewise, the estimated model deliberately abstracts from heterogeneity in dynamics across countries, such as spill-overs in technology, health, or institutions. This imposes discipline while still delivering a comparable fit to estimates that account for country-specific heterogeneity and common global dynamics (in the sense of country-fixed effects and period effects). This way, our model provides a single and comprehensive quantitative framework that is able to account for 85-90% of the variation in income, life expectancy, and fertility across countries over the past 140 years.

In the following, we provide a brief discussion of the implications of allowing for additional heterogeneity in the structural parameter estimates that sheds further light on these issues.

**Global Parameters.** To assess the sensitivity of the results, we replicated the analysis allowing for different global parameters for different sub-samples of countries. Specifically, instead of estimating all global parameters using data for the entire sample of 86 countries (“World”), we re-estimated both the global parameters and the country-specific parameters for the sample of 18 developed forerunner countries (“Forerunners”). We then re-estimated the country-specific parameters of the sub-sample of latecomer countries while fixing the global parameters to the levels obtained from the forerunner sample (“Latecomers – fixed”). Alternatively, we re-estimated the global parameters and the country-specific parameters for the sub-sample of 68 latecomer countries (“Latecomers – variable”).<sup>13</sup> Contrasting these estimates provides a possibility to assess the quantitative relevance of allowing for heterogeneity in the global parameters, e.g., in view of systematic differences in other dimensions such as cross-country spillovers that lead to heterogeneous dynamics of development.

Comparing the results of these different estimations shows that the world estimates of global parameters are a convex combination of the global parameters obtained from the sub-sample estimations, see Table 5. Interestingly, while the estimates of some parameters, such as the production function parameter  $\eta$ , are essentially unaffected, the estimates for other global parameters differ more substantially across specifications. For example, the estimated scope for longevity improvements due to an increase in the skill share,  $\rho$ , is larger when estimated for the subsample of latecomers. For the forerunner sample, the scope for improvement in adult longevity is 36.8 years, whereas the scope is close to 49 years for the sample of latecomer countries. This could be an indication for health spillovers that im-

---

<sup>13</sup>For comparability and to discipline the analysis, we kept the initial conditions for  $x_0$ ,  $N_0$ , and  $\tilde{a}_0$  fixed at the same levels as for the baseline estimation.



ply a greater human capital elasticity in health, and thus a steeper transition trajectory, for these countries. A similar comment applies to the returns to investments in child quality ( $\beta$ ). Inversely, the parameter linking human capital to income growth,  $\phi$ , is estimated to be smaller for the latecomer sample (0.62, compared to 0.77 in the forerunner sample), suggesting a lower influence of domestic human capital on GDP growth rates, and correspondingly smaller growth rates along the balanced growth path in latecomer economies.

Table 5: Estimated Global Parameters: Different Sample Specifications

| Global Parameter       | Sample |             |            |          |
|------------------------|--------|-------------|------------|----------|
|                        | World  | Forerunners | Latecomers |          |
|                        |        |             | fixed      | variable |
| $\eta$                 | 0.2915 | 0.3085      | 0.3085     | 0.2928   |
| $\rho$                 | 43.846 | 36.833      | 36.835     | 48.935   |
| $\phi$                 | 0.6664 | 0.7673      | 0.7673     | 0.6201   |
| $\beta$                | 0.8740 | 0.6864      | 0.6864     | 0.8705   |
| $\delta$               | 0.7617 | 1.0100      | 1.0100     | 0.8091   |
| Pseudo- $R^2$ (global) | 0.824  | 0.855       | 0.750      | 0.832    |

Correspondingly, the estimates of the country-specific structural parameters are quantitatively slightly different when global parameters are allowed to vary across sub-samples, but without affecting the ranking of estimates across countries.<sup>14</sup> In fact, the different estimates of global parameters, such as the scope for improvement in longevity ( $\rho$ ) or the human capital-elasticity of growth ( $\phi$ ) do not seem to affect the ranking of country-specific estimates ( $\underline{T}$  and  $\underline{r}$ ) or imply a modest re-ranking in favor of latecomer countries ( $A_0$ ). Likewise, the model performance is comparable for the different specifications.<sup>15</sup> This suggests that the scope for improvements in the model fit by introducing heterogeneity in the dynamics that reflects, e.g., spillovers in technology or health, is limited. Taken together, these findings are reassuring regarding the reliability and robustness of the baseline results. In the remaining analysis, we therefore stick with the extreme but parsimonious version of estimating the global parameters with the entire sample of countries (“World”), noting that relaxing

<sup>14</sup>This is illustrated by Appendix Figures A5 and A6. Figure A5 shows a scatter plot of the estimates of  $\underline{T}$ ,  $\underline{r}$ , and  $A_0$  for latecomer countries obtained with the set of global parameters estimated for the sample of forerunners (“Latecomers – fixed”, horizontal axis) and with the set of global parameters estimated for the sample of latecomers (“Latecomers – variable”, vertical axis). Figure A6 shows a scatter plot of the ranks of countries in the distribution of the country-specific parameters for the separate estimates of the global parameters on the respective sub-samples of forerunner and latecomer countries relative to the ranks from the baseline estimation of the global parameters on the pooled sample (world). The figure documents that the ranks are mostly preserved.

<sup>15</sup>Specifically, the model fit for forerunners and latecomers is similar as for the world sample even when allowing for group-specific estimates of the global parameters, see Appendix Table A2 and Figure A7.

this assumption and re-estimating the model for high and low income countries separately would imply only moderate differences in the estimates of the country-specific parameters and slight increases in the model fit, but at the cost of more limited comparability by allowing for additional degrees of freedom in the parametrization for different sub-samples.

**Country-Specific Parameters.** The considerable heterogeneity in the estimates of the country-specific structural parameters summarized in Table 3 implies heterogeneity in development. A visual comparison of the development patterns of the model to that in the data suggests a comparable fit for different groups of countries (forerunners and “latecomer” countries; see Figure 3). Nevertheless, the heterogeneity in the country-specific structural parameters is systematic. Specifically, the estimates of  $A_0$ ,  $\underline{T}$ , and  $\underline{r}$  exhibit higher values for the forerunners, but similar values for the latecomers as compared to the baseline estimates. At the same time, the distribution of the goodness of fit (in terms of the Pseudo- $R^2$ ) does not reveal considerable differences in model performance across forerunner and latecomer countries in comparison to the baseline.<sup>16</sup> Thus, overall the model performs well in replicating the data patterns in the different subsets of countries.

Since all model economies are identical except for the structural parameters governing the transition timing ( $\underline{T}$ ), the scope of the expansion ( $\underline{r}$ ), and the slope of the development trajectory ( $A_0$ ), these parameters are natural candidates to capture deep-rooted factors that have been associated with comparative development differences in previous literature. These factors typically reflect time-invariant, geographic variables, including temperature, climate, disease environment, and land quality related to its suitability for agricultural production, historical contingencies such as the timing of the Neolithic transition, the (predicted) genetic diversity, or cultural factors. The existing literature has typically considered these deep-rooted factors in terms of reduced-form estimates while focusing on one single factor in isolation and treating the importance and significance of a factor as a primarily empirical question. However, the interplay of different deep-rooted factors is not well understood. Likewise, it is not clear how the different factors are related to the long-run process of development and the timing of the transition to balanced growth.

The structural estimates allow for an alternative perspective on the deep-rooted factors. A first indication for this is the observation that the structural parameters exhibit a high correlation with the coefficient estimates for the respective country-fixed effects in the regressions

---

<sup>16</sup>Appendix Figure A8 reports the distributions of the estimated country-specific parameters, which also suggests a closer overlap of the corresponding estimates for the full sample and the forerunner sample.

of ln GDP per capita.<sup>17</sup> However, these correlations only provide a very indirect link between the structural parameters and the deep-rooted factors. The main problem in this respect is the large number of candidates for deep-rooted factors.

To explore the relationship between the estimated country-specific structural parameters and observable country-specific and time-invariant deep determinants of development documented in the recent literature, we perform a descriptive analysis. To establish comparability with the existing literature on this topic, we combine information on a collection of deep determinants of development from Spolaore and Wacziarg (2013) and Nunn and Puga (2012) and add information on the extrinsic disease environment from Cervellati, Sunde, and Valmori (2012, 2017). To tackle the issue of different dimensionality – a limited number of structural parameters and the potentially large number of empirical measures of deep determinants of development that are correlated with each other – we analyze the mapping between structural parameters and empirical measures using canonical correlations as descriptive methodology. Canonical correlations reflect the linear combinations of two sets of variables, in this case between the structural parameters ( $A_0$ ,  $\underline{T}$ , and  $\underline{r}$ ) and the empirical deep determinants, which contain potentially many country-specific characteristics. The canonical correlations are computed so that the correlation between the two sets of variables is maximized, where the dimensionality of the linear combinations corresponds to the dimensionality of the smaller set of variables (here three) and the canonical dimensions are orthogonal to each other by construction. The results reveal that the structural parameters exhibit a close correlation with the empirical deep determinants, with correlations of almost 0.9, 0.63, and more than 0.5 for the best three linear combinations. Moreover, the results based on subsets of the empirical deep determinants show that the gain of information from adding additional empirical measures is limited.<sup>18</sup> This finding raises the question about which of the empirical variables exhibit the highest relation with the different structural parameters. The respective coefficients for the different dimensions of the linear combinations do not reveal a clear pattern in this respect, reflecting the fact that all of the deep determinants (and structural parameters) are highly correlated with each other.<sup>19</sup> Thus, as one might expect, the analysis does not deliver evidence for a single, unique empirical deep determinant of long-run development, while, at the same time, providing an interpretative lens for the role of deep determinants for comparative development differences.

---

<sup>17</sup>The respective correlation coefficients are 0.83 for  $\underline{T}$ , 0.61 for  $\underline{r}$ , and 0.52 for  $A_0$ .

<sup>18</sup>See Appendix Table A3, Figure A9, and the discussion in Appendix B.

<sup>19</sup>See Appendix Table A4 and Figure A10 for details.

**Counterfactual Scenarios.** The methodology allows investigating the consequences of heterogeneity in the deep-rooted factors for the entire development path by simulating the model for counterfactual scenarios. This provides a quantitative assessment of the relative importance of differences the country-specific parameters  $A_0$ ,  $\underline{T}$ , and  $\underline{r}$  for comparative development and explicitly addresses the main critiques that have been expressed regarding the empirical literature on deep-rooted factors and the strong implicit identifying assumptions (see, e.g., Durlauf, 2023, for a discussion of this point). Concretely, we conducted three counterfactual exercises, simulating each country with the estimated  $A_0$  for Sweden while keeping the country-specific variation in  $\underline{T}$  and  $\underline{r}$ , or, alternatively, doing the analogous exercise with the estimated  $\underline{T}$  and  $\underline{r}$  of Sweden, while keeping the country-specific estimates in the respective other parameters unchanged. The respective results show that variation in the country-specific parameters has heterogeneous effects on comparative development in income and population. Specifically, for some countries, the counterfactual levels of development are significantly improved in income and reduced in terms of fertility in the counterfactual simulations.<sup>20</sup> Similarly, the counterfactual simulations imply a concentration in the world income distribution at higher levels of income than observed in the data, and an analogous shift in the distribution of fertility rates.<sup>21</sup>

Together, these results suggest that variation in deep-rooted environmental factors closely relates to comparative development differences. At the same time, the counterfactual simulations suggest that variation in deep-rooted factors that affect productivity, health, or fertility has heterogeneous implications for development. The next section presents a systematic analysis of how comparative development differences relate to heterogeneity in deep-rooted factors and accumulation dynamics.

## 6 Decomposing Development

This section investigates the implications for comparative development differences today, their roots in past development, and their foreseeable consequences for future development.

---

<sup>20</sup>Appendix Figure A11 plots the resulting comparative development patterns in income per capita and total fertility rates, respectively, when comparing actual levels in 2020 to counterfactual levels.

<sup>21</sup>Figures A12 and A13 plot the corresponding distributions of global income and fertility against the respective counterfactuals.

## 6.1 Decomposing Comparative Development Differences

The analysis so far implies that development differences are related to differences in the country-specific parameters. It remains unclear, however, how exactly this heterogeneity maps into comparative development differences. That the timing of the transition plays an important role already becomes apparent when distinguishing between subsamples of fore-runner countries with an early onset of the transition and latecomer countries with a delayed transition.<sup>22</sup>

In this section, we provide a decomposition in light of the different phases of development. Focusing attention on comparative development in terms of income per capita, it is clear from (15) that differences in income per capita across countries are closely related to productivity differences in terms of  $A$ .<sup>23</sup> Formally, from the results in Section 3.5 it follows that

$$A_{t+1} = A_t(1 + \phi\lambda_t) = A_0 \prod_{\tau=1}^{t+1} (1 + \phi\lambda_{\tau-1})$$

As a further direct implication of the discussion in Section 3.5, the level of productivity prevailing in a country at a given point in time is the result of the long-run development process and, in particular, depends on a country's position in that process. Note that the development process conceptually comprises three different phases: first, a phase of slow development, with small shares of skilled individuals and small increments in the shares of skilled from one generation to the next. Second, the rapid transition that goes along with a fast expansion of the skill share over the course of few generations. Third, the balanced growth path, to which the economy eventually converges with a skill share converging to  $\bar{\lambda} \approx 1$ .<sup>24</sup>

For the sake of illustration, suppose that the economy reaches the balanced growth path at time  $\tilde{t}$  when the skill share exceeds a level  $\bar{\lambda}$ , which is in the vicinity of 1. Then, the (log) level of productivity of an economy that has already converged to the balanced growth path at the time of observation  $t$  can be decomposed into influences of country-specific factors related to the initial condition  $A_0$ , the influence of long-run development dynamics before and during the transition to balanced growth, and productivity gains on the balanced growth

---

<sup>22</sup>See again Appendix Figure A3.

<sup>23</sup>To see this, note that as discussed in Section 3.5, differences in income per capita closely track differences in productivity on the balanced growth path (i.e., if  $\lambda_t \rightarrow 1$  and hence  $x_t \rightarrow 1$ ). The fit of the respective series to the data is illustrated in Appendix Figure A14.

<sup>24</sup>For a graphical illustration of the three different phases of development, see Appendix Figure A1.

path

$$\ln A_{t+1} = \underbrace{\ln A_0}_{\text{Initial Cond.}} + \underbrace{\sum_{\tau=1}^{\tilde{t}+1} \ln \left( 1 + \phi \cdot \frac{T_{\tau-1} - \underline{T}}{\rho} \right)}_{\text{Development Dynamics}} + \underbrace{(t+1 - \tilde{t}) \ln(1 + \phi)}_{\text{Balanced Growth}} \quad (18)$$

Notice that  $\underline{T}$  has two effects, one direct through its influence on skill acquisition via affecting life expectancy, and one more indirect through its influence on the onset of balanced growth (reflected by  $\tilde{t}$ ).<sup>25</sup>

Figure 8 illustrates the implications of this decomposition for the simulated data for Sweden and Niger as example cases, thus resembling the patterns of long-run development in income. The figure shows that the development dynamics until the onset of the balanced growth path, which occurs in 1960 for Sweden, constitute a major component of the accumulated productivity, whereas the contribution of the accumulation along the balanced growth path is steadily increasing, but initially moderate. According to the structural estimates, Niger, as an example of a development latecomer, has a comparable level of  $A_0$  (the baseline estimate is 0.1192764) as that for Sweden (baseline estimate 0.1192608), but a substantially lower  $\underline{T}$  (30.44067 compared to 34.47222 for Sweden), which implies a slower accumulation of education and productivity, and a later onset of balanced growth (which only occurs in 2080 in the simulation). This suggests that comparing countries focusing exclusively on the balanced growth path might deliver a misleading representation of the forces underlying comparative development differences. With this decomposition, we can characterize the comparative development differences between two economies,  $i$  and  $j$ , and assess the quantitative importance of variation in  $A_0$  and  $\underline{T}$ , and the corresponding differences in the timing of the onset of the balanced growth path.

Without loss of generality, suppose that  $\tilde{t}^i < \tilde{t}^j$ , then the development difference between

---

<sup>25</sup>Analogously, suppose for a moment that the average skill share during the first phase of development is given by  $\underline{\lambda}$  and let us denote the time of the onset of growth acceleration by  $\underline{t}$ . For concreteness, suppose that this is the time when the change in the skill share from one generation to the next exceeds 1%. Thus, distinguishing the period before and after the onset of the transition as marked by  $\underline{t}$ , one can decompose further as

$$A_{t+1} = \underbrace{A_0 \cdot (1 + \phi \underline{\lambda})^{\underline{t}}}_{\substack{\text{Initial Cond.} \\ \text{Pre-Transitional Dynamics}}} \cdot \underbrace{\prod_{\tau=\underline{t}}^{\tilde{t}+1} \left( 1 + \phi \cdot \frac{T_{\tau-1} - \underline{T}}{\rho} \right)}_{\text{Transition Dynamics}} \cdot \underbrace{(1 + \phi)^{t+1 - \tilde{t}}}_{\text{BGP}}$$

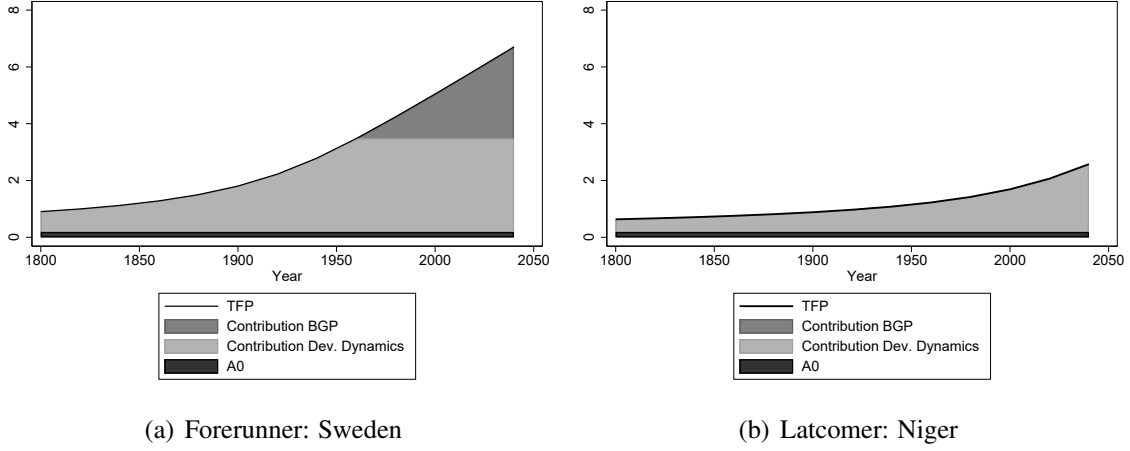


Figure 8: Decomposition of Long-Run Dynamics: Sweden as Benchmark

Note: Simulated data, TFP normalized to Sweden in 1800.

country  $i$  and  $j$  can be decomposed as

$$\begin{aligned}
 \ln A_{t+1}^i - \ln A_{t+1}^j &= \underbrace{(\ln A_0^i - \ln A_0^j)}_{\text{Initial Cond.}} \\
 &+ \underbrace{\sum_{\tau=1}^{\tilde{t}^i+1} \ln \left( \frac{\rho + \phi \cdot (T_{\tau-1}^i - \underline{T}^i)}{\rho + \phi \cdot (T_{\tau-1}^j - \underline{T}^j)} \right)}_{\text{Development Dynamics (pre BGP)}} \\
 &+ \underbrace{(\tilde{t}^i + 1 - \tilde{t}^j) \ln(1 + \phi) - \sum_{\tau=\tilde{t}^i}^{\tilde{t}^j+1} \ln \left( 1 + \phi \cdot \frac{(T_{\tau-1}^j - \underline{T}^j)}{\rho} \right)}_{\text{Development Dynamics (BGP vs. pre-BGP)}}
 \end{aligned} \tag{19}$$

Thus, economy  $i$ 's advantage in comparative development derives from a differential in initial productivity  $A_0$ , a different speed of accumulation dynamics before the entry to balanced growth of the forerunner economy  $i$  as reflected in the second line, and a delay in economy  $j$  entering the balanced growth path that is due to the later onset (the delay in  $\tilde{t}^j$  relative to  $\tilde{t}^i$ ) in the last line. Both, the difference in speed and the delay in the onset, are the result of the differential in  $\underline{T}$ . Notably, the entire comparative development gap between the two economies is the result of a delay in the accumulation – the gap neither widens nor closes once both economies have reached their balanced growth path.<sup>26</sup>

<sup>26</sup>Notice that both countries exhibit the same growth along the balanced growth path, so that the corresponding component drops out from the expression.

Figure 9 illustrates this again for the case of Niger relative to Sweden as benchmark. Following the usual practice of illustration in development accounting, we plot the gap in TFP as in (19) of Niger relative to Sweden, which constitutes an upper bound benchmark for the relative productivity. Both panels show the same data, but Panel (a) shows the entire simulation period whereas Panel (b) focuses on the phase during the transition to the balanced growth path. Since the difference in  $A_0$  is small, it does not contribute much to the differential in productivity, and hence to comparative development – there is no gap as  $A_0$  in Niger is essentially the same as in Sweden. Instead, the main bulk of the productivity gap arises due to the differential accumulation prior to the transition to balanced growth represented by the dashed line, corresponding to the second line in (19). The gap widens further once the forerunner country (Sweden) has reached its balanced growth path, while the latecomer economy (Niger) is still in transition, as shown by the dash-dot line that corresponds to the last line of (19). However, as becomes visible from comparing the total productivity gap (the solid line), the differential that arises during this last phase is quantitatively smaller than the differential that arises during the preceding phase.

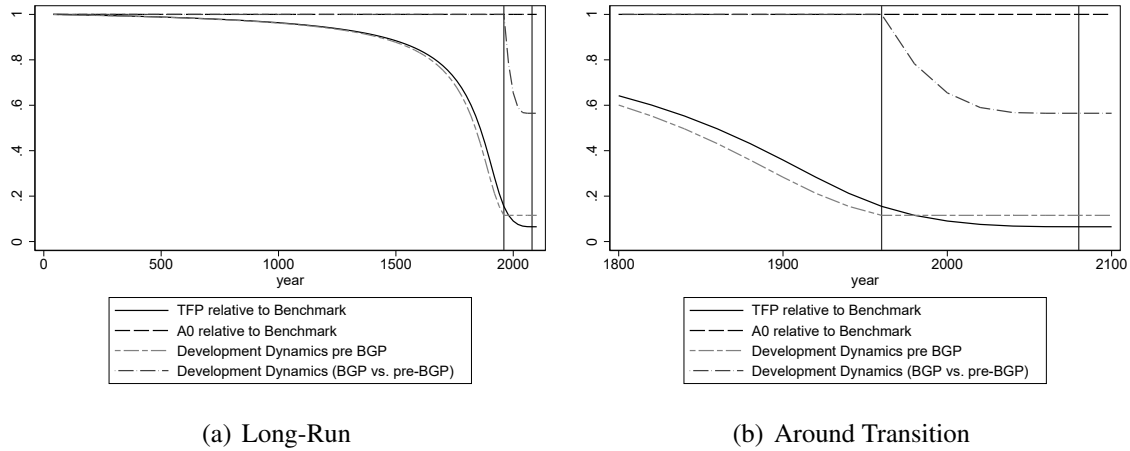


Figure 9: Decomposition of Comparative Development Dynamics Relative to Benchmark  
*Note:* Simulated data, TFP normalized to Sweden (Sweden = 1) as benchmark.

With this methodology, we can relate contemporaneous comparative development differences across the world to the heterogeneous development dynamics and the associated delays in the transition towards the balanced growth path. To do so, we apply the decomposition of development in terms of productivity, as implied by the expressions in (18) and (19). Figure 10(a) plots the components contributing to the development gap relative to the benchmark of Sweden as of 2020. Specifically, we plot the contribution of initial conditions as opposed to



the contribution of the accumulation process reflected by the two bottom lines of (19). The results show that accumulation during the development process accounts for more than 90% of the level of development and hence for considerably more than initial conditions. Moreover, initial conditions account for relatively more of development for countries that exhibit lower levels of development relative to Sweden. Figure 10(b) considers the patterns of comparative development since 1960. In view of the fact that Sweden has reached the balanced growth path in 1960, this reflects the development patterns captured by the last line in (19). During the period 1960-2020, the development dynamics exhibit a pattern of divergence that is the result of the delayed transition to the balanced growth path in most countries as compared to Sweden, and the associated discrepancy in the accumulation process.<sup>27</sup> The figure shows that the delay leads to an amplification of the comparative development differences: countries that are relatively poorer than the benchmark in 1960 fall even further behind as the result of the slower accumulation.

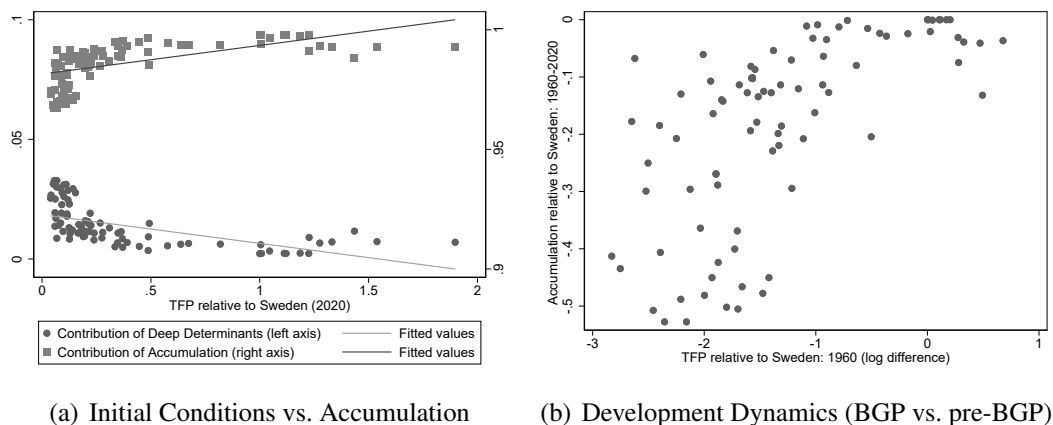


Figure 10: Decomposition of Comparative Development

*Note:* Simulated data, normalized to Sweden. Panel (a): Cross-section 2020. Panel (b): Changes 1960-2020.

## 6.2 Decomposing Population and Aging Dynamics

This section illustrates the consequences of the changes in the age composition of the population that are associated with the transition to balanced growth. In light of the pace and force of demographic development, this analysis also sheds light on the foreseeable implications for development in the future.

<sup>27</sup>This pattern is illustrated in Appendix Figure A15.

From the analysis so far it is obvious that variation in deep-rooted factors that affects income and fertility costs is an important determinant of population dynamics. Figure 11 illustrates this by presenting the model simulations for the differential dynamics of the total fertility rate, again using the example of Niger relative to Sweden. The graph illustrates that over a long phase of development, simulated fertility in Niger is higher than in Sweden, whereas the levels of life expectancy and the skill share were initially comparable. This pattern is the result of a lower estimate of the cost for children,  $\underline{r}$ , in Niger. In fact, the corresponding parameter estimates are 2.318 in Niger as compared to 3.263 in Sweden. When Sweden enters the transition towards balanced growth, Niger lags behind, which implies an increasing gap in life expectancy and skill share  $\lambda$ , and a corresponding hump in relative fertility. This hump, which is crucially related to differential fertility across skill groups, only abates as Niger enters its own transition.

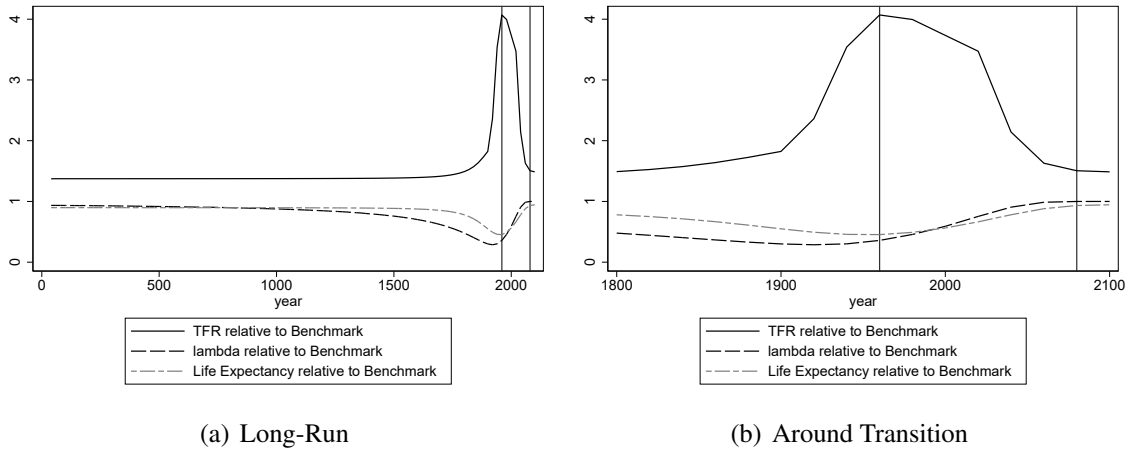


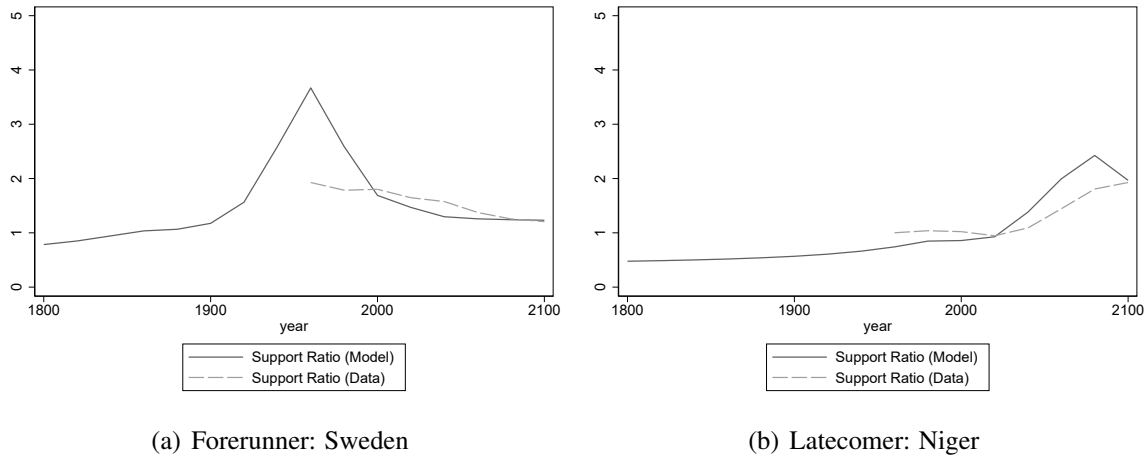
Figure 11: Decomposition of Population Dynamics Relative to Benchmark

*Note:* Simulated data for TFR, skill share ( $\lambda$ ) and life expectancy of Niger relative to Sweden (Sweden = 1) as benchmark. Vertical lines mark the entry to the balanced growth path for Sweden and Niger, respectively.

A less obvious aspect concerns the implications for economic development that arise from these heterogeneous population dynamics. It is well known that the demographic transition and the associated shifts in the age composition of the population can entail a “demographic dividend” that is related to a temporary increase in the population share made up by individuals in working ages (Bloom, Canning, and Sevilla, 2003). The magnitude of this dividend has been suspected to vary across countries and continents (see, e.g., Eastwood and Lipton, 2011), and the relative contribution of variation in the age structure relative to other aspects, in particular the expansion of human capital, has been a matter of intense debate (see, e.g.,

Cuaresma et al., 2014; Lutz et al., 2019; Kotschy et al., 2020). However, existing work has been largely based on reduced-form empirical evidence.

Our quantitative model provides a natural alternative to study the magnitude of the demographic dividend. Formally, the demographic dividend can be rationalized as the ratio between income per capita and income per worker (or per working age person). This ratio is equivalent to the support ratio, i.e., the ratio of individuals of working age over dependent individuals (who are either too young to work, or too old and retired). During the demographic transition, the support ratio first increases as consequence of a combination of population growth and increasing life expectancy. The onset of a fertility decline together with an increasing life expectancy leads to a temporary acceleration of the support ratio. Eventually, however, the increase in life expectancy induces an increase in the old-age dependency ratio, i.e., the population share of individuals beyond retirement age relative to the population share of working age individuals. This leads to a decline in the support ratio and, ultimately, a stabilization at a steady state level that is determined by the balanced growth path.



**Figure 12: Population Dynamics: The Support Ratio**

*Note:* Simulated data of the support ratio, computed as the population-weighted sum of individuals in working age relative to the population-weighted sum of individuals in adolescence and in old-age inactivity.

Figure 12 illustrates these dynamics for the examples of Sweden and Niger. The model fits the data well despite the fact that this has not been targeted in the estimation.

Figure 13 plots the corresponding pattern for the cross-section of countries in 2020, relative to their life expectancy at birth (Panel a). The demographic dividend related to the temporary increase in the support ratio is also visible here, both in the model and in the data. The figure also demonstrates the reversal that has occurred for countries with high levels of

life expectancy and economic development.

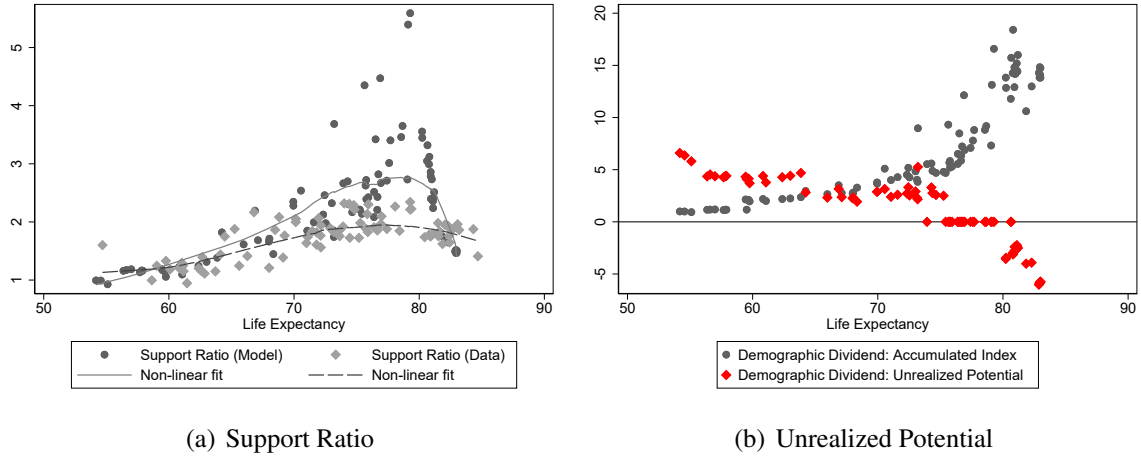


Figure 13: The Demographic Dividend: Cross-Section 2020

Simulated data of the support ratio relative to life expectancy at birth; cross-section for 2020. Both Panels: Support ratio computed as the population-weighted sum of individuals in working age relative to the population-weighted sum of individuals in adolescence and in old-age inactivity, evaluated for the median cohort in a given year. Panel B: Demographic Dividend (Accumulated Index) is computed as the cumulative sum of the support ratio between the onset of the demographic transition and 2020. Demographic Dividend (Unrealized Potential) is computed as the difference between the accumulated index and the cumulative sum of the support ratio from the onset of the demographic transition until the balanced growth path.

In addition to illustrating the demographic dividend, the model enables us to uncover the unrealized potential of the demographic dividend for countries that have not completed the demographic transition and reached the balanced growth path. In particular, we can construct an index of the accumulated support ratio from the onset of the demographic transition until the present for each country. In addition, we can compute the corresponding index until the balanced growth path is reached. This allows computing the unrealized potential of the demographic dividend by calculating the difference between the total dividend and the accumulated dividend as of the present. These numbers are illustrated in Panel b of Figure 13. The figure shows that countries with low life expectancy today exhibit substantial potential for realizing a demographic dividend in the future, whereas the potential is negative for countries with high levels of life expectancy, which suggests a negative growth effect of population aging in the future. Specifically, countries with a life expectancy of less than 75 years in 2020 have, on average, only realized about 56% of their so-measured demographic potential, whereas countries with a life expectancy of 75 years and more have over-exhausted their demographic potential by 17% (on average). Holding productivity and hence output per working-age person fixed, this is equivalent to a drop of 17% in income per capita as the

result of a declining support ratio that reflects the fading of the demographic dividend into a demographic drag.

Figure 14 shows the corresponding figure for different levels of economic development in terms of log GDP per capita. The overall pattern is similar, with low income countries (log GDP per capita less than 10, which is equivalent to an income of approximately 22,000 USD) only having realized about 40% of their demographic potential on average, in contrast to high income countries that have overdrawn their overall demographic potential by about 25% on average.

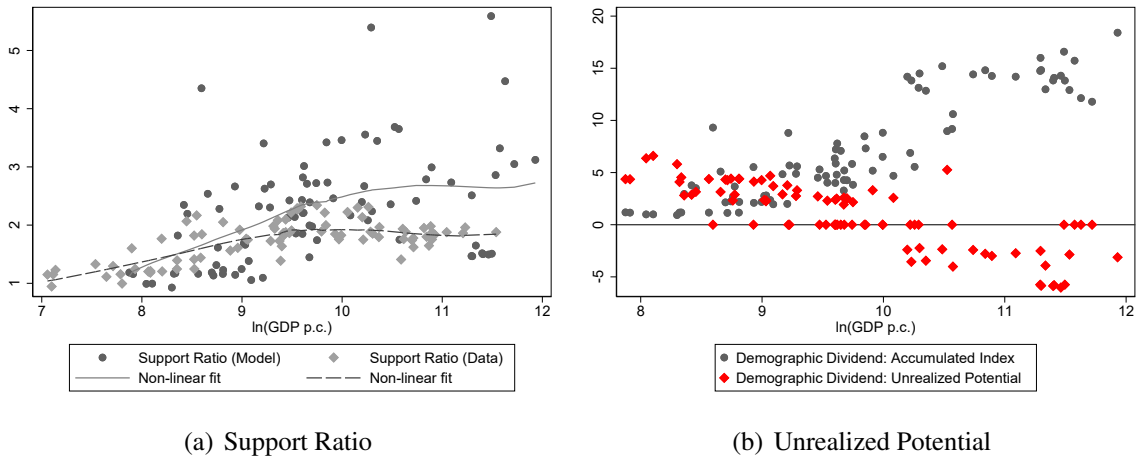


Figure 14: The Demographic Dividend: Cross-Section 2020

*Note:* Simulated data of the support ratio relative to ln GDP per capita; cross-section for 2020. Both Panels: Support ratio computed as the population-weighted sum of individuals in working age relative to the population-weighted sum of individuals in adolescence and in old-age inactivity, evaluated for the median cohort in a given year. Panel B: Demographic Dividend (Accumulated Index) is computed as the cumulative sum of the support ratio between the onset of the demographic transition and 2020. Demographic Dividend (Unrealized Potential) is computed as the difference between the accumulated index and the cumulative sum of the support ratio from the onset of the demographic transition until the balanced growth path.

## 7 Concluding Remarks

This paper provides a new perspective on comparative development. We reconcile the theoretical literature on growth with empirical work on the reasons for growth and development differences across the world, and with work on the deep determinants of development. Our work highlights the importance of non-linear long-run development dynamics, specifically the timing of the transition from quasi-stagnation to balanced growth, for comparative devel-

opment differences today. Based on a quantitative model and exploiting variation along the non-linear development path, we present the first attempt to structurally estimate country-specific parameters that govern the dynamic trajectories of the main dimensions of development – the timing of the transition from quasi-stagnation to growth, the extent of the expansion after the transition, and the steepness of the take-off – for 86 countries over the past 140 years. The estimated model is able to closely match the corresponding observed non-linear long-run development dynamics. At the same time, our analysis of panel data accounting shows that a parsimonious model, characterized by a low-dimensional vector of structural parameters, is able to account for empirical long-run development patterns. We also established a link between the structural parameters and the “deep determinants” of development that have been identified in the empirical literature.

Our methodology also sheds new light on contemporaneous comparative development differences and how they are linked to non-linear development dynamics in the past. Specifically, the results of a model-based decomposition of comparative development differences into their cumulative components illustrate the critical role of the different phases of the long-run development trajectory. While variation in initial conditions only accounts for less than 10% of the differences in comparative development, delays in the accumulation accounts for more than 90%. Moreover, the model enables the quantification of the demographic dividend at different points of the development process, and a quantification of the growth potential related to the past and future trajectory of demographic change of countries at different points in time.

These findings also show that comparative development differences are not equivalent to comparative population dynamics, as sometimes claimed in the literature. Instead of being a prerequisite or determinant of long-run development, the declines in fertility and population growth are a consequence of investment decisions and the corresponding opportunity cost considerations. This implies that the focus on fertility as the key to long-run development in the existing long-run growth literature might need to be modified. Likewise, the focus on a fertility decline driven by changes in the quantity-quality trade-off needs to be broadened to the consideration of differential fertility patterns across skill groups of parents. In sum, our results suggest that investment decisions such as education, and the resulting accumulation consequences, play a crucial role for development patterns. This indicates a need for shifting the focus on the determinants of these investments, as exemplified by variation in health and longevity.

The methodology developed in this paper provides new opportunities for future research

on growth and development by offering a tighter link between theory and empirical applications than the existing literature on long-run growth.

## References

- ACEMOGLU, D., AND D. AUTOR (2011): “Skills, Tasks and Technologies: Implications for Employment and Earnings,” *Handbook of Labor Economics*, 4b, 1043–1171.
- AGHION, P., AND P. HOWITT (1992): “A Model of Growth Through Creative Destruction,” *Econometrica*, 60(2), 323–351.
- AKSOY, Y., H. S. BASSO, R. P. SMITH, AND T. GRASL (2019): “Demographic Structure and Macroeconomic Trends,” *American Economic Journal: Macroeconomics*, 11(1), 193–222.
- ASHRAF, Q., AND O. GALOR (2013): “The Out of Africa Hypothesis, Human Genetic Diversity, and Comparative Economic Development,” *American Economic Review*, 103(1), 1–46.
- BLOOM, D. E., D. CANNING, AND J. SEVILLA (2003): *The Demographic Dividend: A New Perspective on the Economic Consequences of Population Change*. Rand Corporation, Santa Monica, CA.
- CERVELLATI, M., G. MEYERHEIM, AND U. SUNDE (2019): “The timing of the demographic transition and economic growth,” *Economics Letters*, 181, 43–46.
- (2023): “The Empirics of Economic Growth Over Time and Across Nations: A Unified Growth Perspective,” *Journal of Economic Growth*, 28, 173–224.
- CERVELLATI, M., AND U. SUNDE (2005): “Human Capital, Life Expectancy, and the Process of Development,” *American Economic Review*, 95(5), 1653–1672.
- (2015): “The Economic and Demographic Transition, Mortality, and Comparative Development,” *American Economic Journal: Macroeconomics*, 7(3), 1–39.
- CERVELLATI, M., U. SUNDE, AND S. VALMORI (2012): “The Distribution of Infectious Diseases and Extrinsic Mortality Across Countries,” *Mathematical Population Studies*, 19(2), 73–93.
- (2017): “Pathogens, Weather Shocks, and Civil Conflicts,” *Economic Journal*, 127(607), 2581–2616.



- CERVELLATI, M., U. SUNDE, AND K. F. ZIMMERMANN (2017): “Demographic dynamics and long-run development: insights for the secular stagnation debate,” *Journal of Population Economics*, 30, 401–432.
- CHESNAIS, J.-C. (1992): *The Demographic Transition: Stages, Patterns and Economic Implications. A Longitudinal Study of Sixty-Seven Countries Covering the Period 1720–1984*. Clarendon Press, Oxford.
- COHEN, D., AND M. SOTO (2007): “Growth and Human Capital: Good Data, Good Results,” *Journal of Economic Growth*, 12(1), 51–76.
- COMIN, D., AND M. MESTIERI (2018): “If Technology Has Arrived Everywhere, Why Has Income Diverged?,” *American Economic Journal: Macroeconomics*, 10(3), 137–178.
- COOLEY, T., AND E. HENRIKSEN (2018): “The demographic deficit,” *International Economic Review*, 93, 45–62.
- CUARESMA, J. C., W. LUTZ, AND W. SANDERSON (2014): “Is the Demographic Dividend an Education Dividend?,” *Demography*, 51(1), 299–315.
- DALGAARD, C.-J., AND H. STRULIK (2013): “The History Augmented Solow model,” *European Economic Review*, 63(1), 134–149.
- DE LA CROIX, D., AND M. DOEPKE (2003): “Inequality and Growth: Why Differential Fertility Matters,” *American Economic Review*, 93(4), 1091–1113.
- DELVENTHAL, M. J., J. FERNANDEZ-VILLAYERDE, AND N. GUNER (2021): “Demographic Transitions Across Space and Time,” *NBER Working Paper*, 29480.
- DOEPKE, M. (2004): “Accounting for Fertility Decline During the Transition to Growth,” *Journal of Economic Growth*, 9(3), 347–383.
- DURLAUF, S. N. (2023): “The Journey of Humanity by Oded Galor: A Review Essay,” *Population and Development Review*, 49(2), 403–421.
- EASTWOOD, R., AND M. LIPTON (2011): “Demographic transition in Sub-Saharan Africa: How big will the economic dividend be?,” *Population Studies*, 65(1), 9–35.
- EROSA, A., T. KORESHKOVA, AND D. RESTUCCIA (2010): “How Important Is Human Capital? A Quantitative Theory Assessment of World Income Inequality,” *Review of Economic Studies*, 77(4), 1421–1449.

- GALLUP, J. L., J. D. SACHS, AND A. D. MELLINGER (1999): “Geography and Economic Development,” *International Regional Science Review*, 22(2), 179–232.
- GALOR, O. (2011): *Unified Growth Theory*. Princeton University Press, Princeton, N.J.
- GALOR, O., AND O. ÖZAK (2016): “The Agricultural Origins of Time Preference,” *American Economic Review*, 106(10), 3064–3103.
- GALOR, O., AND D. N. WEIL (2000): “Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond,” *American Economic Review*, 90(4), 807–828.
- HALL, R. E., AND C. I. JONES (1999): “Why Do Some Countries Produce So Much More Output Per Worker Than Others?,” *Quarterly Journal of Economics*, 114(1), 83–116.
- HANSEN, G. D., AND E. C. PRESCOTT (2002): “Malthus to Solow,” *American Economic Review*, 92(4), 1205–1217.
- HANUSHEK, E. A., J. RUHOSE, AND L. WOESSMANN (2017): “Knowledge capital and aggregate income differences: development accounting for US states,” *American Economic Journal: Macroeconomics*, 9(1), 184–224.
- HSIEH, C.-T., AND P. J. KLENOW (2010): “Development Accounting,” *American Economic Journal: Macroeconomics*, 2(1), 207–223.
- JONES, C. I. (2016): “The Facts of Economic Growth,” in *Handbook of Macroeconomics*, ed. by J. Taylor, and H. Uhlig, vol. 2A, chap. 1, pp. 3–69. Elsevier Science B.V., Amsterdam.
- (2022a): “The End of Economic Growth? Unintended Consequences of a Declining Population,” *American Economic Review*, 112(11), 3489–3527.
- (2022b): “The Past and Future of Economic Growth: A Semi-Endogenous Perspective,” *Annual Review of Economics*, 14, 125–152.
- JONES, C. I., AND P. ROMER (2010): “The New Kaldor Facts: Ideas, Institutions, Population and Human Capital,” *American Economic Journal: Macroeconomics*, 2(1), 224–245.
- KOTSCHY, R., P. SUAREZ-URTAZA, AND U. SUNDE (2020): “The Demographic Dividend is More Than an Education Dividend,” *Proceedings of the National Academy of Sciences*, 117(4), 25982–25984.

- KOTSCHY, R., AND U. SUNDE (2018): “Can Education Compensate the Effect of Population Aging on Macroeconomic Performance?,” *Economic Policy*, 33(86), 587–634.
- KRUEGER, A. B., AND LINDAHL (2001): “Education for Growth: Why and for Whom?,” *Journal of Economic Literature*, 39(4), 1101–1136.
- LEE, J.-W., AND H. LEE (2016): “Human capital in the long run,” *Journal of Development Economics*, 122, 147–169.
- LUCAS, R. E. (1988): “On the Mechanics of Economic Development,” *Journal of Monetary Economics*, 22(1), 3–42.
- LUTZ, W., J. CRESPO CUARESMA, E. KEBEDE, A. PRSKAWETZ, AND W. C. SANDERSON (2019): “Education Rather Than Age Structure Brings Demographic Dividend,” *Proceedings of the National Academy of Sciences*, 116(26), 12798–12803.
- MAESTAS, N., K. J. MULLEN, AND D. POWELL (2023): “The Effect of Population Aging on Economic Growth, the Labor Force, and Productivity,” *American Economic Journal: Macroeconomics*, 15(2), 306–332.
- MANKIW, N., D. ROMER, AND D. WEIL (1992): “A Contribution to the Empirics of Economic Growth,” *Quarterly Journal of Economics*, 107, 407–437.
- MANUELLI, R. E., AND A. SESHADRI (2014): “Human Capital and the Wealth of Nations,” *American Economic Review*, 104(9), 2736–2762.
- NUNN, N., AND D. PUGA (2012): “Ruggedness: The Blessing of Bad Geography in Africa,” *Review of Economics and Statistics*, 94(1), 20–36.
- OLSSON, O., AND D. A. HIBBS (2005): “Biogeography and Long-Run Economic Development,” *European Economic Review*, 49(4), 909–938.
- PORTELA, M., R. ALESSIE, AND C. TEULINGS (2010): “Measurement Error in Education and Growth Regressions,” *Scandinavian Journal of Economics*, 112(3), 618–639.
- REHER, D. S. (2004): “The Demographic Transition Revisited as a Global Process,” *Population, Space and Place*, 10(1), 19–41.
- ROMER, P. M. (1990): “Endogenous Technological Change,” *Journal of Political Economy*, 98(5), S71–S102.

- ROSSI, F. (2022): “The Relative Efficiency of Skilled Labor across Countries: Measurement and Interpretation,” *American Economic Review*, 112(1), 235–266.
- ROSTOW, W. (1956): “The Take-Off Into Self-Sustained Growth,” *Economic Journal*, 66(261), 25–48.
- SOLOW, R. (1956): “A Contribution to the Theory of Economic Growth,” *Quarterly Journal of Economics*, 70(1), 65–94.
- SOLT, F. (2020): “Measuring Income Inequality Across Countries and Over Time: The Standardized World Income Inequality Database,” *Social Science Quarterly*, 101(3), 1183–1199.
- SPOLAORE, E., AND R. WACZIARG (2013): “How Deep are the Roots of Economic Development?,” *Journal of Economic Literature*, 51(2), 325–369.
- STRULIK, H., AND J. WEISDORF (2014): “How Child Costs and Survival Shaped the Industrial Revolution and the Demographic Transition,” *Macroeconomic Dynamics*, 18(1), 114–144.
- SUNDE, U., AND T. VISCHER (2015): “Human Capital and Growth: Specification Matters,” *Economica*, 82(326), 368–390.
- TABELLINI, G. (2010): “Culture and Institutions: Economic Development in the Regions of Europe,” *Journal of the European Economic Association*, 8(4), 677–716.
- VOTH, H.-J. (2021): “Persistence – Myth and Mystery,” in *The Handbook of Historical Economics*, ed. by A. Bisin, and G. Federico, chap. 9, pp. 243–267. Elsevier.

## Supplementary Online Appendix

### A Illustration of the Dynamics of Development

The properties of the dynamic equilibrium path can be illustrated by the evolution of the (conditional) dynamic system in the  $\{\lambda, T, x\}$  space as in Figure A1. The figure plots the phase diagram for  $\{\lambda, T\}$ , conditional on different levels of technological development in terms of  $x$ . The linear curve depicts  $T$  as in (12), whereas the S-shaped curve constitutes the combinations of  $\lambda$  and  $T$  that are consistent with the intra-generational equilibrium for a given level of  $x$ , as reflected by  $\lambda = \Lambda(T, x)$  in (9). Starting from a low level of technological development (low  $x$ ), the dynamic system is characterized by a stable steady state with low adult longevity and a correspondingly small but positive skill share as depicted in Figure A1 (a). Due to this positive share, the demand for education slowly increases across generations as consequence of a larger skill bias due to (13). This leads to a counter-clockwise rotation of the intra-generational equilibrium locus  $\Lambda(\lambda, x)$ . As consequence of the local stability of the steady state, the emergence of a second steady state does not affect the dynamic equilibrium. Development in terms of growth in education, longevity, and income, is slow, as implied by Figure A1 (b). Once the initial steady state disappears, a rapid transition to the second stable steady state occurs and brings the economy to a development regime that is characterized by high longevity and a large skill share as in Figure A1 (c). This transition in turn crucially depends on the structural parameters that govern the location of (12) and (9). A larger  $\underline{T}$  implies an earlier transition due to a right-shift of the locus characterized by (12), everything else equal.

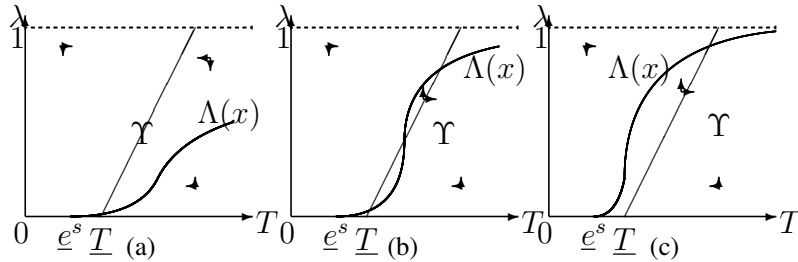


Figure A1: Long-Run Growth Dynamics

## B Additional Figures and Tables

### List of Countries

Table A1: List of Countries

|                    |             |                     |
|--------------------|-------------|---------------------|
| Afghanistan        | Guatemala   | Panama              |
| Albania            | Guyana      | Paraguay            |
| Algeria            | Honduras    | Peru                |
| Argentina          | Hungary     | Philippines         |
| Australia          | India       | Poland              |
| Austria            | Indonesia   | Portugal            |
| Bangladesh         | Iran        | Senegal             |
| Belgium            | Iraq        | Sierra Leone        |
| Benin              | Ireland     | South Africa        |
| Bolivia            | Italy       | Spain               |
| Brazil             | Japan       | Sri Lanka           |
| Bulgaria           | Jordan      | Sudan               |
| Cameroon           | Kenya       | Swaziland           |
| Canada             | Lesotho     | Sweden              |
| Chile              | Liberia     | Switzerland         |
| China              | Malawi      | Syria               |
| Colombia           | Malaysia    | Thailand            |
| Costa Rica         | Mali        | Togo                |
| Cote d'Ivoire      | Mexico      | Trinidad and Tobago |
| Denmark            | Morocco     | Tunisia             |
| Dominican Republic | Mozambique  | Turkey              |
| Ecuador            | Myanmar     | Uganda              |
| Egypt              | Nepal       | United Kingdom      |
| Finland            | Netherlands | United States       |
| France             | New Zealand | Uruguay             |
| Gambia             | Nicaragua   | Venezuela           |
| Germany            | Niger       | Zambia              |
| Ghana              | Norway      | Zimbabwe            |
| Greece             | Pakistan    |                     |

## Estimation Results

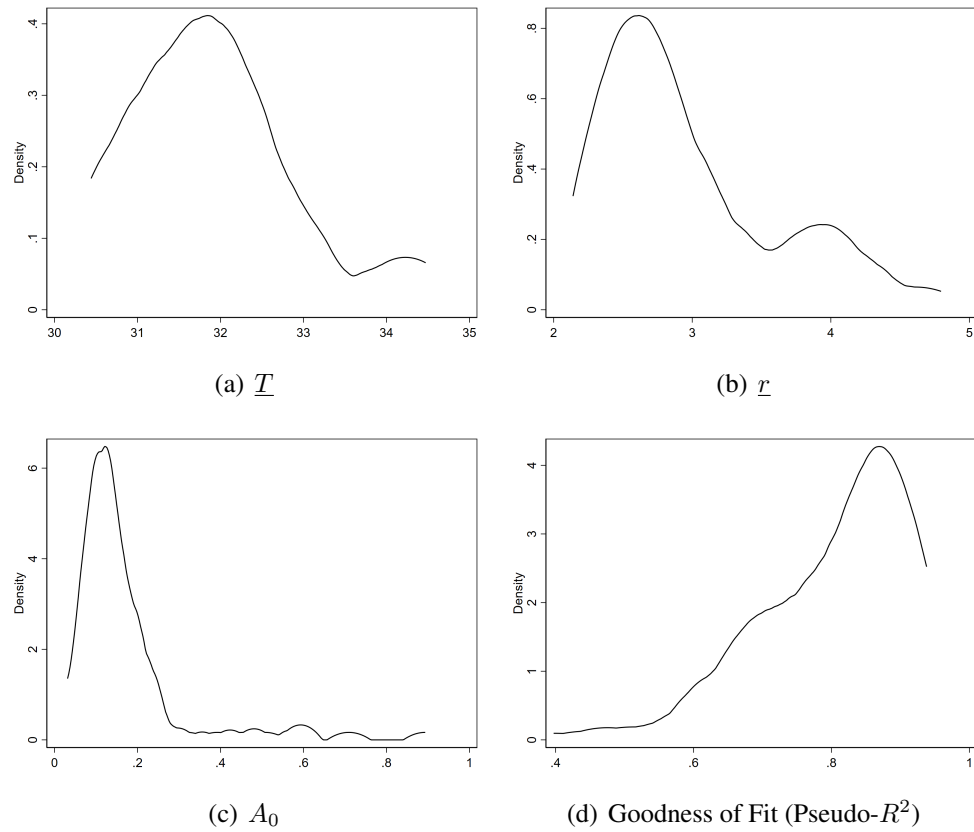


Figure A2: Distributions of Structural Parameter Estimates – World Sample

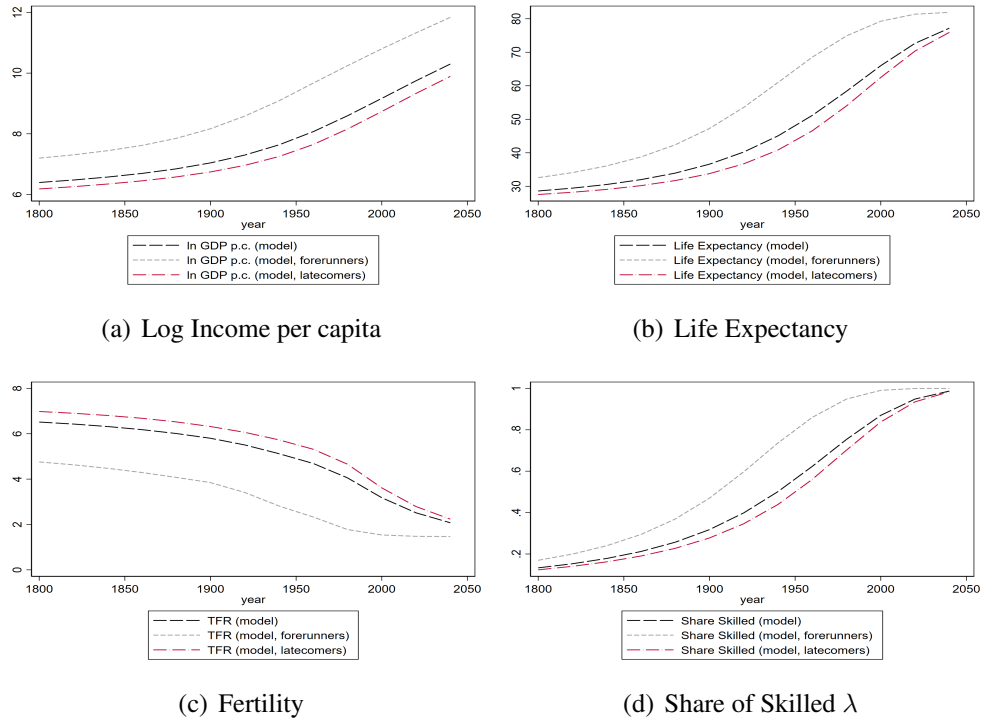
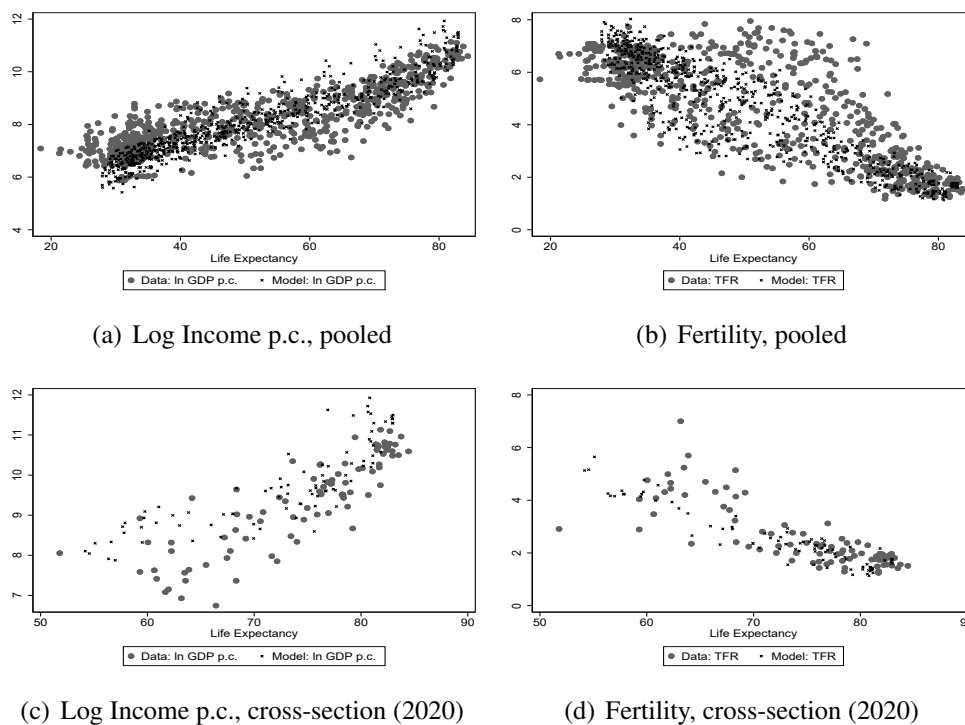


Figure A3: Variation in  $T$  and the Timing of the Transition

*Note:* Each panel plots averages of the respective variable across countries for a given time period. The baseline model estimates (dashed line) are as in Figure 2, the remaining lines show the corresponding estimates for the samples of forerunner countries and latecomer countries separately. All models are based on estimates of country-specific parameters that target the dynamics in log income per capita, life expectancy, and fertility. Global parameters are the same for all models. Fertility is measured by TFR. The skill share is measured by the population share with at least secondary education and is not targeted in the estimation.





**Figure A4: Life Expectancy and Development**

*Note:* Panel (a) plots ln GDP per capita against life expectancy for the pooled panel data (1880-2020) for all countries for data and model simulation. Panel (b) plots TFR against life expectancy for the pooled panel data (1880-2020) for all countries for data and model simulation. Panel (c) plots ln GDP per capita against life expectancy for cross section of countries in 2020 for all countries for data and model simulation. Panel (d) plots TFR against life expectancy for the cross section of countries in 2020 for all countries for data and model simulation.

## Additional Results

### Alternative Estimates of Global Parameters

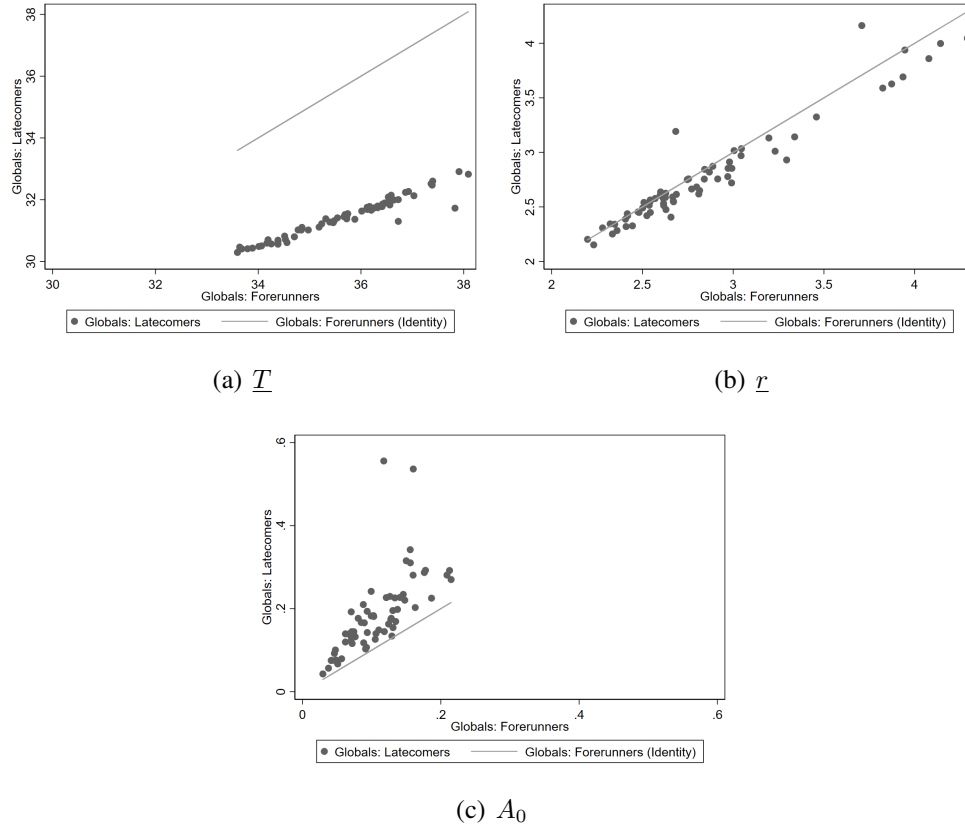
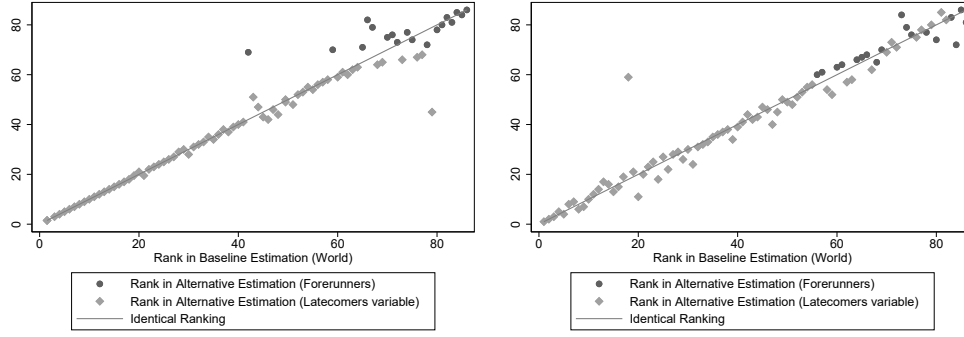


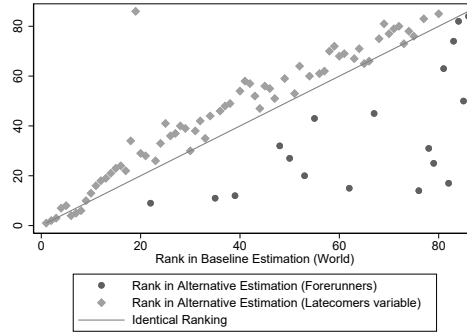
Figure A5: Comparison of Structural Parameter Estimates (for Latecomer Countries) for Different Estimates of Global Parameters

*Note:* Each panel plots the estimate of the respective country-specific parameter for the 68 latecomer countries obtained with global parameters fixed at the estimates obtained for the sub-sample of 18 forerunner countries (Globals: Forerunners) against the estimates obtained with global parameters estimated for the sub-sample of 68 latecomer countries (Globals: Latecomers). The corresponding Spearman rank-order correlations are 0.96 for  $\underline{T}$ , 0.95 for  $\underline{r}$ , and 0.85 for  $A_0$ .



(a)  $\underline{T}$

(b)  $\underline{r}$



(c)  $A_0$

Figure A6: Comparison of Ranking of Structural Parameter Estimates for Different Estimates of Global Parameters

*Note:* Each panel plots the ranking of the respective country-specific parameter for the sub-sample of 18 forerunner countries obtained with global parameters estimated on this sub-sample (Globals: Forerunners), as well as the ranking of the respective country-specific parameter for the 68 latecomer countries obtained with global parameters estimated on their sample (Globals: Latecomer variable), in comparison to the ranking obtained for the baseline estimates of global parameters based on the full sample (Globals: World). The corresponding rank correlations are 0.99 for  $\underline{T}$ , 0.99 for  $\underline{r}$ , and 0.93 for  $A_0$ .

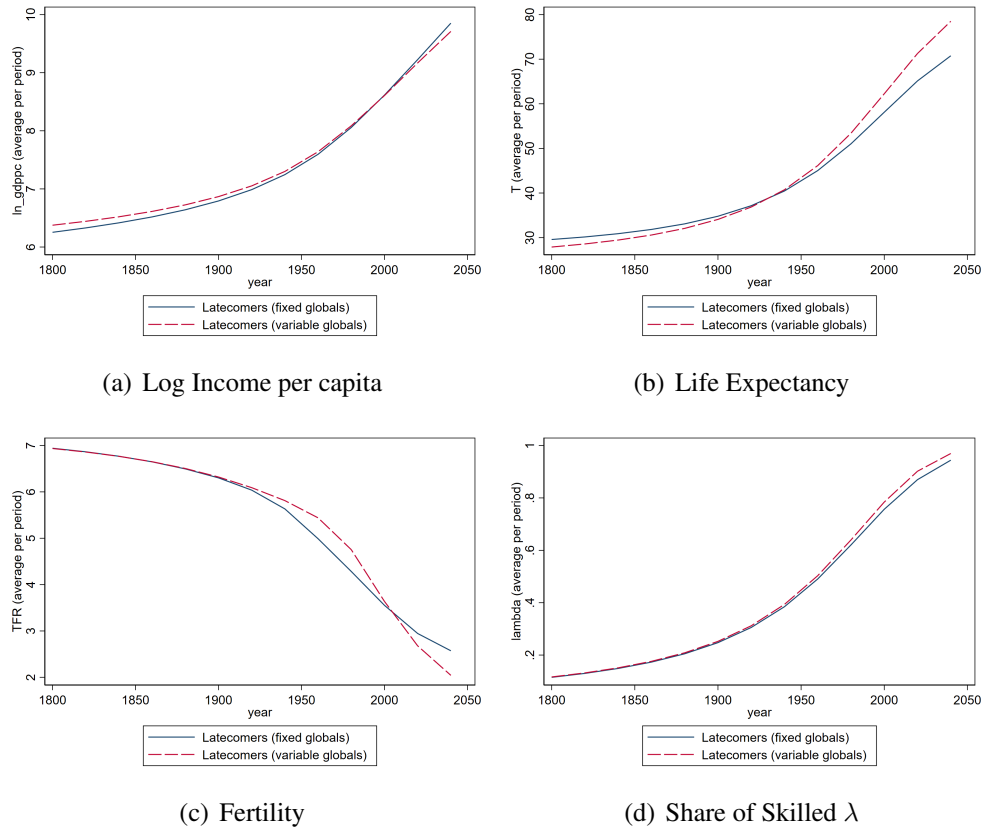
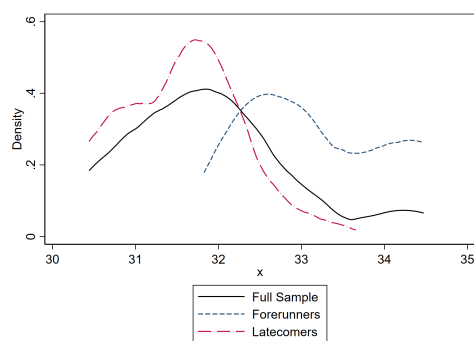
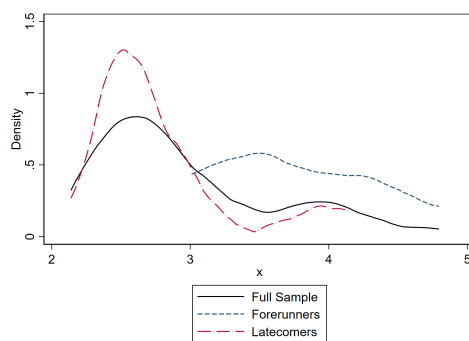


Figure A7: Development Trajectories of Latecomer Countries for Different Estimates of Global Parameters

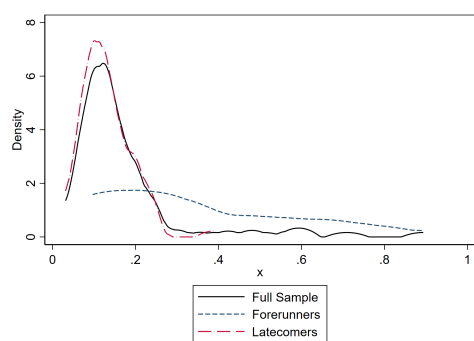
*Note:* Each panel plots averages of the respective variable across latecomer countries for a given time period. The models are based on estimates of country-specific parameters that target the dynamics in log income per capita, life expectancy, and fertility. The graphs show the respective models for two different set of global parameters (see text for details). Fertility is measured by TFR. The skill share is measured by the population share with at least secondary education and is not targeted in the estimation.



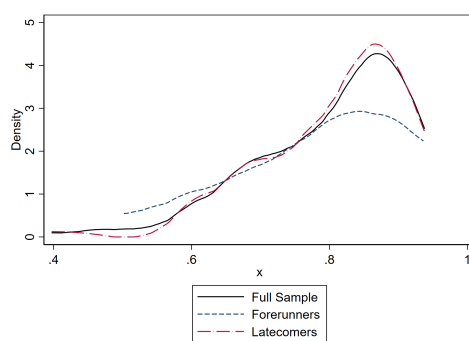
(a)  $\underline{T}$



(b)  $\underline{r}$



(c)  $A_0$



(d) Goodness of Fit (Pseudo- $R^2$ )

Figure A8: Distributions of Structural Parameter Estimates – Across Sub-samples

Table A2: Model Fit and Variance Explained – Different Specifications

| Global Estimates            | World               |                     | Forerunners         |                     | Latecomers          |
|-----------------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| Sample                      | Forerunners         | Latecomers          | Forerunners         | Latecomers          | Latecomers          |
|                             | (1)                 | (2)                 | (3)                 | (4)                 | (5)                 |
| Panel A                     | ln GDP p.c.         |                     |                     |                     |                     |
| ln GDP p.c. (sim.)          | 0.734***<br>(0.021) | 0.787***<br>(0.018) | 0.687***<br>(0.020) | 0.833***<br>(0.020) | 0.859***<br>(0.019) |
| $R^2$                       | 0.900               | 0.771               | 0.895               | 0.758               | 0.790               |
| Panel B                     | Life Expectancy     |                     |                     |                     |                     |
| Life Expectancy (sim.)      | 0.922***<br>(0.027) | 1.097***<br>(0.016) | 1.032***<br>(0.024) | 1.245***<br>(0.021) | 1.087***<br>(0.017) |
| $R^2$                       | 0.890               | 0.898               | 0.931               | 0.867               | 0.887               |
| Panel C                     | TFR                 |                     |                     |                     |                     |
| TFR (sim.)                  | 0.933***<br>(0.043) | 0.959***<br>(0.016) | 1.048***<br>(0.041) | 0.929***<br>(0.020) | 0.954***<br>(0.015) |
| $R^2$                       | 0.769               | 0.871               | 0.825               | 0.801               | 0.885               |
| All Panels:<br>Observations | 144                 | 544                 | 144                 | 544                 | 544                 |

*Note:* Estimation results based on regressions of actual data on simulated data and a constant in different dimensions. Each entry corresponds to the results of a separate regression. Specification in Column (4) refers to Globals: Latecomers fixed and Column (5) refers to Globals: Latecomers variable.

## Canonical Correlations

For the analysis, we select a set of 17 variables that reflect time-invariant characteristics related to cultural or genetic factors, geography, climate, and disease environment. The canonical correlation is constructed from the linear combinations of the structural parameters ( $\underline{T}$ ,  $\underline{r}$ , and  $A_0$ ) in three orthogonal dimensions with the linear combinations of these components in the same three orthogonal dimensions. Table A3 presents the results from this analysis.

The results of OLS estimates of regressions of each of the structural parameters on different dimensions of the deep determinants (in terms of the respective principal components) confirm this and show that the different dimensions of country-specific deep-rooted factors do not exhibit a clear pattern of correlations.<sup>28</sup> Still, together the deep determinants explain about 60% of the variation in  $\underline{T}$ , 30% of the variation in  $\underline{r}$ , and about 50% of the variation in  $A_0$ . However, these estimates are likely to be misleading as they disregard the cross-correlations of the structural parameters.

Table A3: Estimated Parameters and Deep Determinants: Canonical Correlations

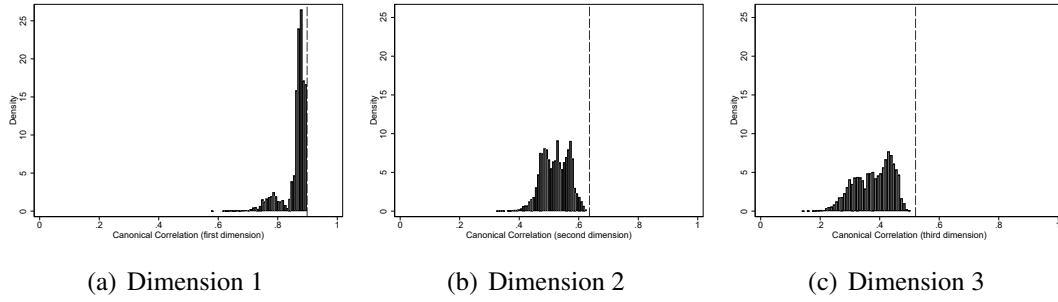
|                        | Dimension 1 | Dimension 2 | Dimension 3 |
|------------------------|-------------|-------------|-------------|
| Canonical Correlations | 0.8986      | 0.6361      | 0.5205      |
| p-Value                | 0.0001      | 0.0012      | 0.0752      |

*Note:* Canonical correlations obtained with baseline (world) estimates for the vector of structural parameters  $\{\underline{T}, \underline{r}, A_0\}$  and a set of 17 deep determinants of development (average precipitation, average temperature, the number of multihost vector-transmitted diseases, % tropical climate, absolute latitude, terrain ruggedness, mean elevation, ethnic fractionalization, linguistic fractionalization, crop yield, land suitability for agriculture, % arable land, % fertile soil; % desert, landlock dummy, predicted genetic diversity, and timing of the neolithic transition). See Appendix Table A4 for the corresponding estimates of loadings for the linear components. P-values refer to tests of significance of canonical correlations in dimensions 1-3 (Column 1), in dimensions 2-3 (Column 2), and in dimension 3 (Column 3).

To test the sensitivity of the results for canonical correlations, we replicated the analysis for subsets of the empirical variables. It is very likely that, due to the high correlations across the different empirical measures, a subset already contains substantial information that is relevant to account for comparative development differences. To test this conjecture, we replicated the analysis of canonical correlations for subsets of the 17 deep determinants. Specifically, we constructed all possible subsets of 10 variables that can be drawn from the

<sup>28</sup>Results available upon request.

set of 17 variables and computed the corresponding canonical correlations for each of these subsets. The resulting distributions of correlations in the three dimensions of linear combinations are shown in Figure A9, together with the canonical correlations obtained with the full set of 17 empirical proxies. The results show that the gain of information from adding additional empirical measures is limited, especially when considering the best linear combination of the structural parameters and their empirical counterparts. The majority of canonical correlations of the respective best linear combinations exceeds 0.85. This is indicative for one dimension that is captured by both, the structural model parameters and the empirical counterparts.



**Figure A9: Canonical Correlations for Different Dimensions of Empirical Determinants**  
*Note:* Each panel plots a histogram of the respective dimension of the canonical correlation for all 19,448 combinations of 10 of the 17 deep determinants. Dashed lines correspond to the respective canonical correlations obtained with the full set of 17 deep determinants.



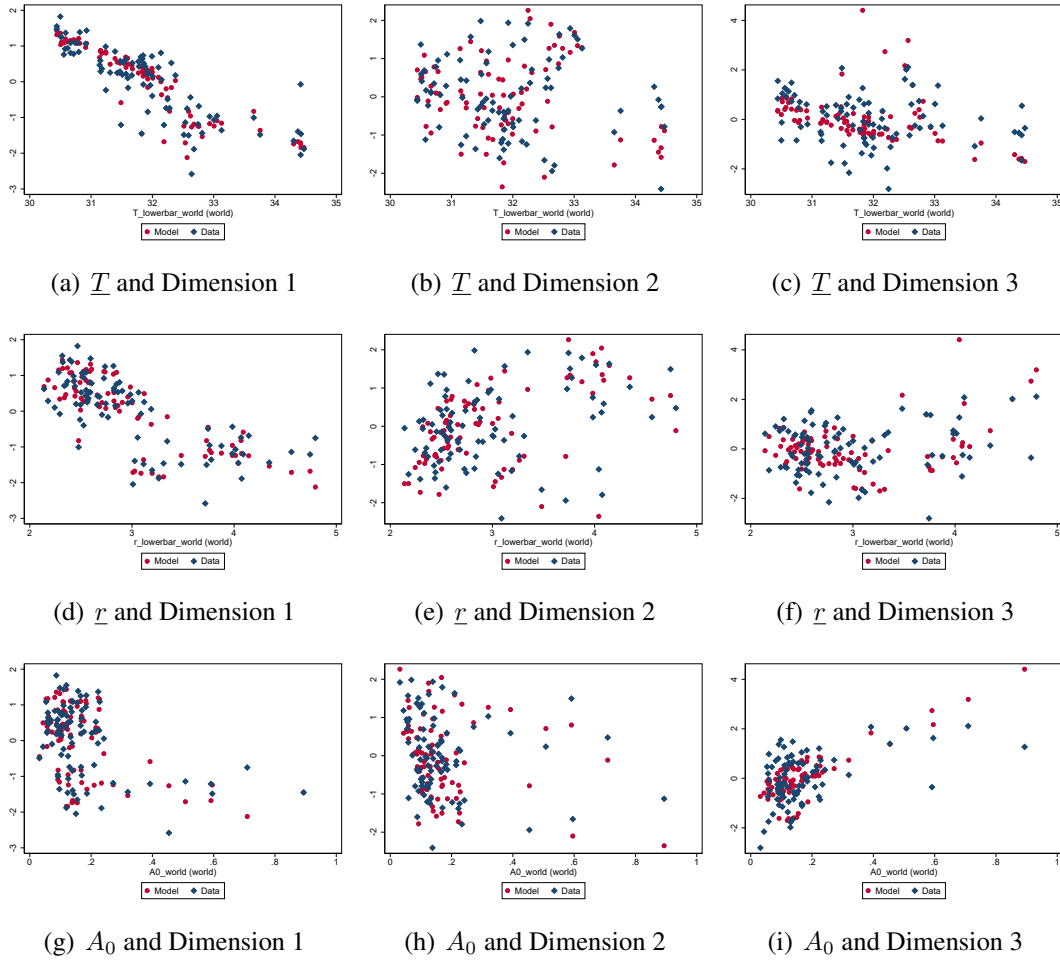


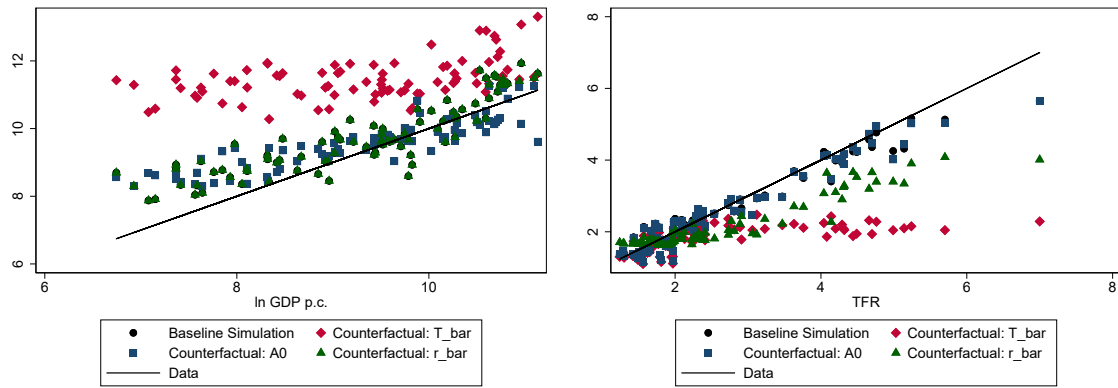
Figure A10: Structural Parameters: Canonical Correlations vs. Model and Data  
*Note:* Each panel plots a scatter plot of the structural parameter and the respective dimension of the canonical correlation (model-side and data-side).

Table A4: Estimated Parameters and Deep Determinants: Canonical Correlations

|                        | (Dim. 1)              | (Dim. 2)             | (Dim. 3)             |
|------------------------|-----------------------|----------------------|----------------------|
| Canonical Correlations | 0.8991                | 0.6530               | 0.5409               |
| $\underline{T}$        | -0.705***<br>(-11.11) | -0.629***<br>(-4.16) | -0.707***<br>(-3.49) |
| $\underline{r}$        | -0.356***<br>(-4.67)  | 1.305***<br>(7.18)   | 0.427<br>(1.75)      |
| $A_0$                  | -0.174*<br>(-2.60)    | -1.048***<br>(-6.59) | 0.644**<br>(3.02)    |
| Precipitation          | -0.1535               | 0.0322               | 0.7636               |
| Temperature            | 0.7331                | -0.6924              | -0.0357              |
| Multihost              | -0.1436               | 0.9920               | -1.0150              |
| Multihost-Vector       | 0.1127                | -0.5397              | 0.6614               |
| Tropics                | 0.1381                | 0.1211               | -0.4229              |
| Absolute Lat.          | -0.1098               | -0.3616              | 0.6106               |
| Ruggedness             | -0.1563               | 0.2837               | 0.1072               |
| Elevation              | 0.4099                | -0.1460              | -0.4770              |
| Ethnic Fr.             | 0.1217                | 0.0746               | 1.0726               |
| Linguistic Fr.         | 0.0040                | -0.3644              | 0.0613               |
| Crop Yield             | -0.0980               | 0.8454               | 0.2488               |
| Suit. f. Agr.          | 0.0776                | 0.1781               | 0.1526               |
| Arable Land            | -0.0494               | 0.1426               | -0.4510              |
| Soil Quality           | -0.0305               | -0.1846              | 0.4153               |
| Desert                 | -0.0121               | 0.4550               | 0.2849               |
| Landlocked             | 0.0868                | -0.1447              | 0.2015               |
| Pred. Gen. Div.        | 0.1980                | 0.8838               | 0.1086               |
| Neolithic Trans.       | -0.0307               | 0.2446               | 0.2403               |
| Obs.                   | 86                    | 86                   | 86                   |

Standardized coefficients for structural parameters and empirical deep determinants. t-values in parentheses. \*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$ .

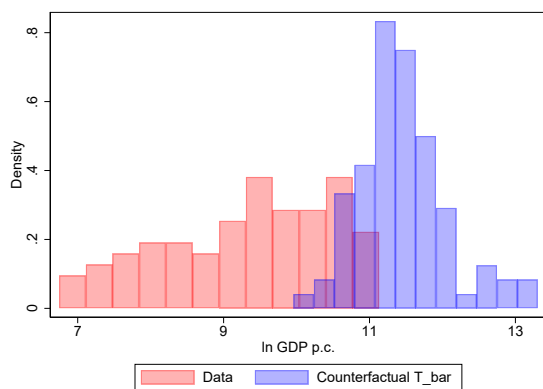
## Counterfactual Simulations



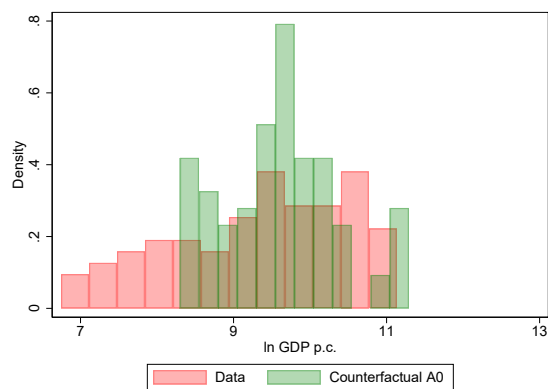
(a) Counterfactual Income: 2020

(b) Counterfactual Population Dynamics (TFR): 2020

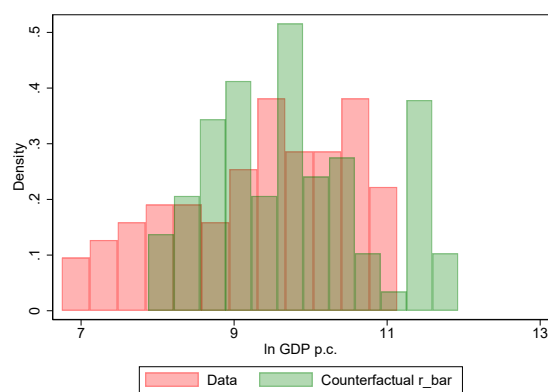
Figure A11: Comparative Development: Counterfactual Simulations



(a) Counterfactual:  $\underline{T}$

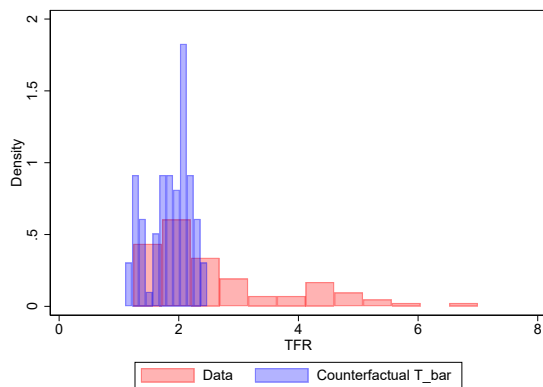


(b) Counterfactual:  $A_0$

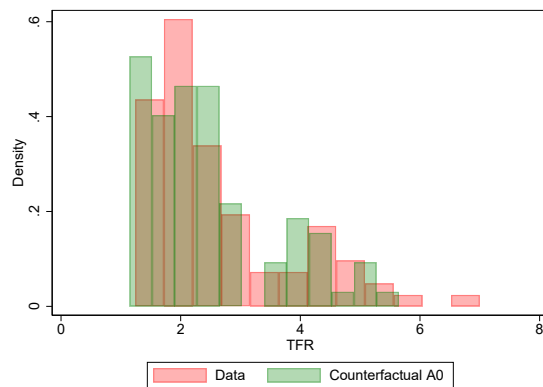


(c) Counterfactual:  $\underline{r}$

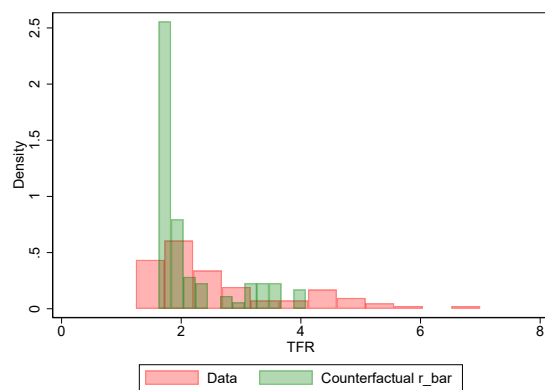
Figure A12: Counterfactual Comparative Development (2020)



(a) Counterfactual:  $\bar{T}$



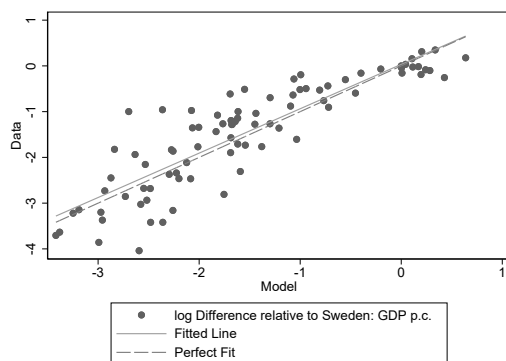
(b) Counterfactual:  $A_0$



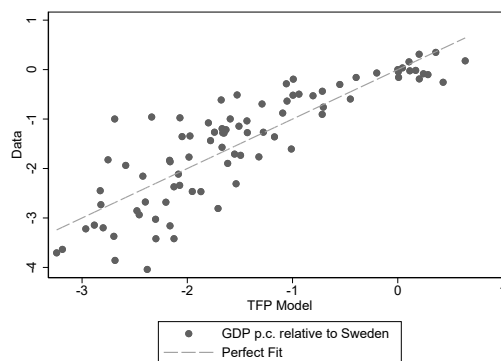
(c) Counterfactual:  $\bar{r}$

Figure A13: Counterfactual Comparative Population Dynamics (2020)

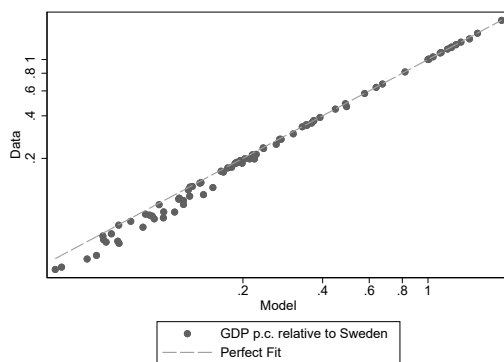
## Comparative Development



(a) Model Fit: GDP



(b) Model Fit: GDP vs. TFP (simulated)



(c) Model Fit: GDP vs. TFP (both simulated)

Figure A14: Comparative Development: Model vs. Data (2020)  
Observed and simulated data, GDP p.c. and productivity normalized to Sweden.

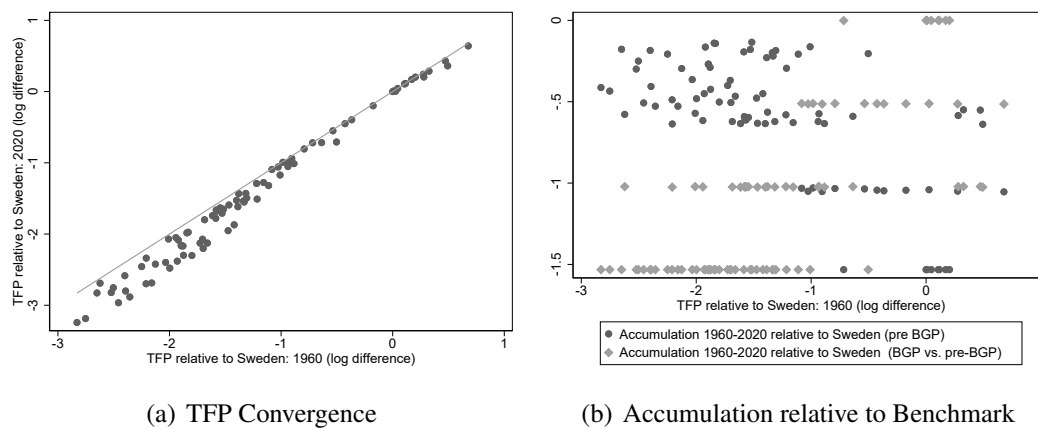


Figure A15: Comparative Development Relative to Benchmark 1960–2020  
Simulated data, GDP p.c. and productivity normalized to Sweden.