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**UNEMPLOYMENT, UNDEREMPLOYMENT,
AND SECULAR STAGNATION
IN A SMALL OPEN ECONOMY**

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May 2025

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Unemployment, underemployment, and secular stagnation in a small open economy^{*}

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Abstract

In this paper, we develop a small open economy model of secular stagnation that allows for both unemployment and underemployment. While unemployment results from the standard search and matching friction, underemployment in the form of an involuntary shortfall of working hours occurs under stagnation when households have a strong desire to save in the form of insatiable liquidity preferences. This secular stagnation equilibrium is characterized by persistent deflation and aggregate demand shortage. In this steady state, an improvement in the terms of trade reduces global demand for domestic products and lowers working hours, thereby exacerbating deflation and decreasing consumption and aggregate demand. Consequently, firm profits fall and unemployment worsens. These results are in sharp contrast to the case of a non-stagnant economy, where an improvement in the terms of trade increases consumption without affecting the employment rate.

Key Words: unemployment, labor market frictions, underemployment, secular stagnation, open economy

JEL Classification: E24, F31, F41

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1 Introduction

Globalization has had a profound impact on domestic labor markets as changes in international competitiveness of goods and services affect employment and production through fluctuations in the terms of trade (Johnson and Stafford, 1999; Schmitt-Grohé and Uribe, 2018). In addition, with the secular decline in equilibrium interest rates, developed countries have increasingly become at risk of facing prolonged periods of economic stagnation and underemployment. Japan, in particular, has experienced a rapid rise in part-time and irregular employment, despite maintaining a low unemployment rate during its lost decades. Against this background, we develop a small open economy model incorporating search and matching frictions in the labor market and an aggregate demand shortage in the commodity market. Using this model we examine how changes in the international economic environment and policy interventions, such as terms-of-trade shocks and international transfers, affect working hours (the intensive margin), the employment rate (the extensive margin), consumption, and output under persistent stagnation.

Specifically, we expand upon the small open economy model of secular stagnation by Hashimoto and Ono (2020) by incorporating search and matching frictions in the labor market. Unemployment arises from this friction, while underemployment arises from insufficient demand in the form of an involuntary shortfall of working hours. The latter quickly dominates the total employment gap under stagnation. We then characterize and contrast the two equilibrium possibilities, one with a demand shortage and one without, and assess and quantify the effects of global and domestic micro- and macroeconomic shocks on working hours, the unemployment rate, consumption, output and foreign asset holdings.

In our model, households are employed through a search and matching process and, upon employment, supply working hours inelastically. The unemployment rate is determined by this process, in which households evaluate the relative value of employment and unemployment, while firms weigh search costs against expected profits, with the real wage being determined by Nash bargaining over the joint surplus. Secular stagnation occurs in equilibrium when households have a sufficiently strong desire to save, which we model with a liquidity (wealth) preference. When this preference is weak, households have sufficient demand for commodities and working hours are fully utilized. However, when the wealth preference is strong, there is insufficient demand for commodities and working hours cannot be fully utilized. Working hours are then

determined by aggregate demand, and underemployment occurs in addition to unemployment.

The effects of economic shocks and policy interventions differ decisively between these two steady state equilibria. When there is no shortage of demand and working hours are fully utilized, an improvement in the terms of trade increases real income and consumption. The employment rate is determined solely by labor market frictions and firm productivity, and the effect of a change in the terms of trade on unemployment is negligible. In contrast, under secular stagnation, working hours are not fully utilized and firm profits do not scale with productivity. These results are consistent with those in a closed economy setting by Hashimoto, Ono, and Schlegl (2023). Furthermore, we analyze the effects of international aspects such as foreign asset holdings and terms of trade. An improvement in the terms of trade raises the relative prices of domestic commodities and reduces demand for them. Greater foreign asset holdings also lead to smaller demand for domestic commodities. These decreases in demand further suppress hours worked, reduce firm profits, lead to less firm entry, worsen employment, and ultimately reduce domestic consumption.

Finally, we calibrate our model to match key features of Japan’s economy and numerically obtain and quantify the comparative statics of variations in the international environment, macroeconomic shocks, and microeconomic shocks on both underemployment and unemployment. It is underemployment rather than unemployment that is more sensitive to international environment (such as changes in terms of trade and foreign asset holdings) and macroeconomic shocks (such as changes in productivity or government spending). It dominates the total employment gap under stagnation. In contrast, variations in parameters that affect the search and matching process directly (such as the changes in the job destruction rate or workers’ bargaining power) have substantial, but opposing, effects on unemployment and underemployment.

Related Literature: This paper explores interactions between structural unemployment due to search frictions and underemployment due to a persistent lack of aggregate demand in an open economy framework.

A vast body of literature attributes unemployment to labor market frictions. Pissarides (2000) developed search-and-matching models of unemployment, which have been widely applied across various fields. These models typically assume that all commodities produced through successful worker-firm matches are fully demanded after price adjustments. Recently, many

studies have incorporated labor market frictions into DSGE models in open economy settings (e.g., Bertinelli, Cardi, and Restout, 2020; Guerra-Salasa, Kirchnera, Tranamil-Vidal, 2021). However, these studies do not address demand shortages or the intensive margin of labor.¹

Our approach builds on the concept of wealth preference introduced by Ono (1994, Chapter 11). There is an extensive literature on the implications of wealth preferences for secular stagnation. Notable examples in closed economy settings include Michau (2018), Illing, Ono, and Schlegl (2018), Michaillat and Saez (2022), and Hashimoto, Ono, and Schlegl (2023).

Several open economy models are related to our study. Ono (2006, 2014, 2018) utilizes a two-country framework to examine the international effects of changes in liquidity preferences, government expenditures, and other economic policies. In small open economy settings, Hashimoto and Ono (2020) consider a model with perfect international financial markets, while Michau (2023) examines one with imperfect financial markets. However, neither study endogenizes labor market frictions (the extensive margin of labor).

The remainder of this paper is structured as follows. Section 2 outlines the key features of the model. Section 3 examines the equilibrium conditions when there are no aggregate demand shortages, and analyzes the effects of economic policies and international environment on unemployment and consumption. Section 4 considers the secular stagnation steady state and demonstrates that their effects greatly differ from those in the case without demand shortages. Section 5 calibrates the model using Japanese labor market data and numerically simulates the impact of various parameter changes on the employment gap under stagnation. The final section summarizes our findings and concludes.

2 The model

We consider a small open economy with one factor of production, which is labor, and two commodities, 1 and 2, in a continuous infinite-time setting. The home country specializes in commodity 1 while the rest of the world produces both commodities, which are tradable.² The

¹In a closed economy, Kudoh, Miyamoto, and Sasaki (2019) introduce labor market frictions into a business cycle model, allowing working hours to be determined through household utility from leisure. However, their model assumes voluntary changes in working hours and does not address secular stagnation.

²Under free capital movement across countries, which we later assume, a small country must specialize in one of the two commodities. Using a dynamic 2x2x2 model with capital accumulation, Ono and Shibata (2006) show that under free asset trade instead of free capital movement a country with a much smaller population specializes in one of the two commodities while the other country produces both commodities, which we assume here.

nominal domestic prices of the two commodities are P_1 and $P_2 (= \varepsilon P_2^F)$ respectively, where ε is the nominal exchange rate and P_2^F is the international price of commodity 2 in terms of the foreign currency. Due to the small-country assumption, $\bar{\omega}$, denoting the international relative price of commodity 2 in terms of commodity 1, is exogenously given to the home country and thus the nominal exchange rate ε adjusts so that it always satisfies

$$\bar{\omega} \equiv \frac{\varepsilon P_2^F}{P_1} = \frac{P_2}{P_1}.$$

The international financial market features perfect mobility for financial capital, and thus the real interest rate equals the exogenously given world interest rate \bar{r} :

$$r (= R - \pi) = \bar{r}, \quad (1)$$

where $\pi \equiv \dot{P}/P$ represents the inflation (or deflation if negative) rate of the home consumer price index P and R is the nominal interest rate in terms of the home currency.

2.1 Household behavior

Each household consists of a large number of members normalized to unity, with $(1 - u)$ employed and u unemployed members. Each member is endowed with one unit of time. To eliminate any uncertainty regarding each household member's employment, we follow Merz (1995) and Andolfatto (1996) and assume that the household provides perfect consumption insurance for its members, which guarantees equal consumption regardless of the employment status of a specific member. Then, each household receives wage income $wx(1 - u)$ from employed members and unemployment benefits zu from unemployed members, where w is the real wage and $x \in [0, 1]$ represents realized hours per worker. Each household allocates its total assets (a) between international asset holdings (f) and real money balances (m). This gives the following flow and stock budget constraints expressed in real terms:³

$$\dot{a} = \bar{r}a + wx(1 - u) + zu + d - \tau - p_1c_1 - p_2c_2 - Rm, \quad a = m + f, \quad (2)$$

³The nominal flow budget equation is $\dot{A} = RPf + Wx(1 - u) + Zu + Pd - Pc - P\tau$, where $A(\equiv Pa)$ denotes total asset holdings. Using $a = m + f$ and $\bar{r} \equiv R - \pi$, where $\pi \equiv \dot{P}/P$ is the inflation rate, we obtain (2).

where d is dividend income from total profits of the domestic firm sector, τ is a real lump-sum tax, $c_i (i = 1, 2)$ is the consumption of commodity i , $p_i (i = 1, 2)$ is the real price of commodity i , and $x(1 - u) \in [0, 1]$ is the effective employment (total working time), implying that the realized labor supply is determined by either the potential labor supply ($x = 1$) in the absence of demand shortages, or the firms' demand for labor in the presence of demand shortages ($x < 1$).

The lifetime utility of the household is

$$\int_0^\infty \left(\hat{\phi}(c_1, c_2) + \mu(m) \right) e^{-\rho t} dt,$$

where ρ is the subjective discount rate, $\hat{\phi}(c_1, c_2)$ is the utility of consumption, which is homothetic, and $\mu(m)$ is the utility of real money holdings. The household maximizes its lifetime utility subject to the flow budget equation and the asset constraint given by (2). Because $\hat{\phi}(c_1, c_2)$ is homothetic and the same as that of foreign households, real prices $p_i (i = 1, 2)$ depend only on the relative price $\bar{\omega}$ and satisfy

$$p_1 = \frac{P_1}{P} = \frac{P_1^F}{P^F} = p_1(\bar{\omega}), \quad p_2 = \frac{\varepsilon P_2^F}{P} = \frac{P_2^F}{P^F} = p_2(\bar{\omega}),$$

of which the latter property gives

$$P = \varepsilon P^F.$$

Given the current value Hamiltonian function H of the household optimizing behavior:

$$H = \hat{\phi}(c_1, c_2) + \mu(m) + \lambda(\bar{r}a + wx(1 - u) + zu + d - \tau - p_1c_1 - p_2c_2 - Rm),$$

where λ is the costate variable for a , the first-order optimal conditions are

$$\hat{\phi}_1(c_1, c_2) = \lambda p_1, \quad \hat{\phi}_2(c_1, c_2) = \lambda p_2, \quad \mu'(m) = \lambda R, \quad \dot{\lambda} = (\rho - \bar{r})\lambda. \quad (3)$$

Because $\hat{\phi}(c_1, c_2)$ is homothetic, from (3) one has

$$\frac{p_2}{p_1} (= \bar{\omega}) = \frac{\hat{\phi}_2(c_1, c_2)}{\hat{\phi}_1(c_1, c_2)} = \frac{\hat{\phi}_2(1, c_2/c_1)}{\hat{\phi}_1(1, c_2/c_1)},$$

implying that c_2/c_1 is a function of only $\bar{\omega}$. Therefore, the ratio of consumption expenditures

allocated to each commodity depends only on the relative price

$$p_1(\bar{\omega})c_1 = \zeta(\bar{\omega})c, \quad p_2(\bar{\omega})c_2 = (1 - \zeta(\bar{\omega}))c, \quad c = p_1c_1 + p_2c_2, \quad (4)$$

where c is real total consumption and $\zeta(\bar{\omega})$ is the ratio of consumption expenditure allocated to commodity 1.

Because of the homothetic utility, $\hat{\phi}(c_1, c_2)$ in which c_1 and c_2 take the optimal levels given by (4) is independent of the relative price $\bar{\omega}$ so that one can exhibit $\hat{\phi}(c_1, c_2)$ as $\phi(c)$:

$$\phi(c) \equiv \hat{\phi}\left(\frac{\zeta}{p_1}c, \frac{1-\zeta}{p_2}c\right). \quad (5)$$

Noting that $\partial\hat{\phi}/\partial\bar{\omega} = 0$, from (3), (4) and (5), we obtain

$$\phi'(c) = \zeta \frac{\hat{\phi}_1}{p_1} + (1 - \zeta) \frac{\hat{\phi}_2}{p_2} = \lambda,$$

$$\frac{\bar{\omega}p'_1(\bar{\omega})}{p_1(\bar{\omega})} = -(1 - \zeta(\bar{\omega})) < 0,$$

$$\frac{\bar{\omega}p'_2(\bar{\omega})}{p_2(\bar{\omega})} = \zeta(\bar{\omega}) > 0.$$

From (1), (3) and the above first equation, we obtain the following Euler equation and the money demand function:

$$\eta_c \frac{\dot{c}}{c} = R - \pi - \rho = \bar{r} - \rho, \quad \text{where } \eta_c \equiv \frac{-\phi''(c)c}{\phi'(c)}, \quad (6)$$

$$R = \frac{\mu'(m)}{\phi'(c)}. \quad (7)$$

As is standard in small open economy models, we assume

$$\rho = \bar{r},$$

since otherwise this country will eventually be a large country (if $\rho < \bar{r}$) or disappear (if $\rho > \bar{r}$).

By applying this property to (6) we find

$$\frac{\dot{c}}{c} = 0 \Rightarrow c = c^* : \text{constant}, \quad (8)$$

implying that access to the international asset trade makes consumption completely smoothed. Finally, the transversality condition is

$$\lim_{t \rightarrow \infty} \lambda a e^{-\bar{r}t} = 0.$$

2.2 Firm behavior and the labor market

The labor market is described by the standard search and matching model with free market entry and exit and Nash bargaining between workers and firms as a mechanism for wage determination. The government pays benefits \bar{z} to unemployed workers following the policy rule:

$$z = \bar{z}wx, \tag{9}$$

where $\bar{z} \in [0, 1)$ denotes the replacement rate that is proportional to labor income.

2.2.1 Matching mechanism

Output is produced when firms and workers successfully match in the labor market. The matching function $\Gamma(u, v)$ describes the number of matches between vacant firms v and unemployed workers u . $\Gamma(u, v)$ is continuously differentiable, concave, homogeneous of degree one, increasing with both u and v , and satisfies $\Gamma(0, v) = 0, \Gamma(u, 0) = 0$. The probability that a firm with a vacancy matches with an unemployed worker, denoted by $q(\theta)$, is continuously differentiable in $(0, \infty)$ and satisfies

$$q(\theta) \equiv \frac{\Gamma(u, v)}{v} = \Gamma\left(\frac{1}{\theta}, 1\right); \quad q'(\theta) < 0, \quad q(\infty) = 0, \tag{10}$$

where θ is the jobs-to-applicants ratio: $\theta \equiv v/u \in [0, \infty)$, which measures the tightness of the labor market.

In each period a fraction δ of workers becomes unemployed, where δ constitutes an exogenous separation rate. Hence, the flow into unemployment is $\delta(1 - u)$ while the flow out of unemployment is $\Gamma(u, v)$, where from (10) $\Gamma(u, v)$ satisfies

$$\Gamma(u, v) = q(\theta)v = q(\theta)\theta u.$$

Therefore, the dynamics of the unemployment rate u satisfy the following differential equation:

$$\dot{u} = \delta(1 - u) - \theta q(\theta)u. \quad (11)$$

2.2.2 Firm behavior under free entry and exit

There is a large number of identical firms that produce commodity 1 using only labor. Each firm hires exactly one worker and produces output y with the linear production function:

$$y = \bar{y}x,$$

where \bar{y} is labor productivity and x is working hours of the employed worker. An operating firm earns profits $(p_1\bar{y} - w)x$ whereas a vacant firm incurs search costs p_1k when offering a position in work.⁴

We denote the value of employed workers by E , the value of unemployed workers by U , the value of operating firms matched with a worker by J , and the value of vacant firms searching in the labor market by V . They satisfy the following standard equations:

$$\bar{r}E = wx - \delta(E - U) + \dot{E}, \quad (12)$$

$$\bar{r}U = z + \theta q(\theta)(E - U) + \dot{U}, \quad (13)$$

$$\bar{r}J = (p_1\bar{y} - w)x - \delta(J - V) + \dot{J}, \quad (14)$$

$$\bar{r}V = -p_1k + q(\theta)(J - V) + \dot{V}, \quad (15)$$

where $q(\theta)$ is given by (10).

Under free entry and exit, the value of a vacant firm needs to satisfy $V = 0$ in equilibrium. Hence, from (15), the value of an operating firm J is

$$J = \frac{p_1k}{q(\theta)}. \quad (16)$$

Once a vacant firm successfully matches with an unemployed worker, wages are set through a

⁴Even in cases where search costs require commodity 2, p_2k , it will not qualitatively affect the subsequent discussion.

Nash bargaining process between the firm and the worker so as to maximize their joint surplus $(E - U)^\varepsilon(J - V)^{1-\varepsilon}$, where ε is the bargaining power for the worker. Using (12) and (14) with $V = 0$, taking the derivative of the joint surplus with respect to w , and making the result equal to zero leads to the following sharing rule between the firm and the worker:

$$(1 - \varepsilon)(E - U) = \varepsilon J, \quad (17)$$

which implies that the total matching surplus is shared between the parties according to their respective bargaining power.

Using (9), (12), (13), (14), (16), (17), and $V = 0$, we find that the wage rate satisfies

$$wx = \psi \times (p_1 \bar{y}x + p_1 k\theta), \quad \psi = \psi(\underbrace{\varepsilon}_+, \underbrace{\bar{z}}_+) \equiv \frac{\varepsilon}{1 - (1 - \varepsilon)\bar{z}}, \quad (18)$$

where ψ represents the effective bargaining power of the worker.⁵ From (14), (16), (18) and $V = 0$, we have the following dynamic equation for θ :

$$\bar{r} + \delta = \frac{(1 - \psi)\bar{y}x}{k}q(\theta) - \psi q(\theta)\theta + \eta_\theta \frac{\dot{\theta}}{\theta}, \quad (19)$$

where η_θ is the elasticity of a vacant firm's matching probability $q(\theta)$ and satisfies

$$\eta_\theta \equiv \frac{-q'(\theta)\theta}{q(\theta)} \in (0, 1) \quad \text{for } \theta \in (0, \infty).$$

Because the number of operating firms is equal to that of employed workers $(1 - u)$, the total benefit from production is $(p_1 \bar{y} - w)x(1 - u)$. Since the number of vacant firms is v , the total search costs are $p_1 kv$. Therefore, dividend income d earned from the home firm sector is

$$d = (p_1 \bar{y} - w)x(1 - u) - p_1 kv. \quad (20)$$

⁵See Appendix A for the explicit derivation.

2.3 Government

The government imposes a lump-sum tax τ to balance its budget in each period. Its budget constraint therefore satisfies

$$\tau = zu + g, \quad g \equiv p_1(\bar{\omega})g_1 + p_2(\bar{\omega})g_2, \quad (21)$$

where zu and g represent unemployment benefits and government purchases of the two commodities, respectively.

2.4 Dynamics and steady-state conditions

The nominal money supply \bar{M} is assumed to be constant, for simplicity; hence, in the money market we have

$$m = \frac{\bar{M}}{P}, \quad \frac{\dot{m}}{m} = -\pi. \quad (22)$$

Substituting d given by (20), τ given by (21), the dynamics of money given by (22), and $v = \theta u$ into the flow budget equation (2) yields the dynamics of foreign asset holdings f :

$$\dot{f} = \bar{r}f + p_1(\bar{y}x(1-u) - k\theta u) - c - g. \quad (23)$$

Equations (6), (11), (19), (22), and (23) form an autonomous dynamic system with respect to c, m, θ, f and u . We relegate the stability analysis of these dynamics into Appendix B.

Next, we consider the steady-state equilibrium in which the steady-state value of each variable is indicated by an asterisk (*). From (19) with $\dot{\theta} = 0$, we obtain

$$\bar{r} + \delta = \frac{(1-\psi)\bar{y}x^*}{k}q(\theta^*) - \psi q(\theta^*)\theta^*, \quad (24)$$

which gives the property of the steady-state tightness θ^* (see Appendix C for the derivation of the partial derivative of θ with respect to each variable):

$$\theta^* = \theta(\underbrace{x}_{+}; \underbrace{\bar{y}}_{+}, \underbrace{\psi}_{-}, \underbrace{k}_{-}, \underbrace{\delta}_{-}). \quad (25)$$

The labor inflow to, and the outflow from, the unemployment pool are equal, i.e., $\dot{u} = 0$. Hence

from (11) the steady-state level of u is given by

$$u^* = u(\underbrace{\theta}_{-}; \underbrace{\delta}_{+}) \equiv \frac{\delta}{\delta + \theta q(\theta)}. \quad (26)$$

From (25) and (26), we find

$$u^* = u(\theta(x; \bar{y}, \psi, k, \delta); \delta) \equiv \hat{u}(\underbrace{x}_{-}; \underbrace{\bar{y}}_{-}, \underbrace{\psi}_{+}, \underbrace{k}_{+}, \underbrace{\delta}_{+}). \quad (27)$$

Using (23) with $\dot{f} = 0$ and f^* given by (47) in Appendix B, we find that c^* satisfies

$$c^* = c(\theta, x; f_0, g, \bar{\omega}, \bar{y}, k, \delta) \equiv \bar{r}f^* - g + p_1(\bar{y}(1 - u^*)x^* - k\theta^*u^*), \quad (28)$$

$$f^* = f_0 + \frac{p_1(\bar{y}x^* + k\theta^*)}{\bar{r} + \delta + \theta^*q(\theta^*)}(u^* - u_0).$$

Note that foreign asset holdings gradually move from the initial level f_0 even in a small country because the unemployment rate u and output $\bar{y}(1 - u)x$ vary over time. Whether $x = 1$ or $x < 1$, the steady-state foreign asset holdings f^* and the unemployment rate u^* are positively correlated, as indicated by the second equation of (28). Around the steady state ($u^* \approx u_0$), this function of c satisfies the following properties (see Appendix D for the derivation of the partial derivatives of c):⁶

$$c = c(\underbrace{\theta}_{+(-)}, \underbrace{x}_{+}; \underbrace{f_0}_{+}, \underbrace{g}_{-}, \underbrace{\bar{\omega}}_{-}, \underbrace{\bar{y}}_{+}, \underbrace{k}_{-}, \underbrace{\delta}_{-}). \quad (29)$$

In the case of $c_\theta \equiv \partial c / \partial \theta > 0$ (< 0), an equilibrium is described for slack (tight). From (25) and (28), we can rewrite c as a function of x as follows (see Appendix E for the derivation of the partial derivatives of \hat{c}):

$$c^* = c(\theta(x; \bar{y}, \psi, k, \delta), x; f_0, g, \bar{\omega}, \bar{y}, k, \delta) \equiv \hat{c}(\underbrace{x}_{+(?)}, \underbrace{f_0}_{+}, \underbrace{g}_{-}, \underbrace{\bar{\omega}}_{-}, \underbrace{\bar{y}}_{+(?)}, \underbrace{\psi}_{-(+)}, \underbrace{k}_{-(?)}, \underbrace{\delta}_{-(?)}), \quad (30)$$

where the sign conditions in parentheses imply the case of tight.

⁶Whether the equilibrium is slack or tight depends on the shape and curvature of the matching function and the steady-state level of θ . Pissarides (2000, Chapter 8) shows that an increase in labor market tightness raises total final output ($\bar{y}(1 - u(\theta))$) and total search costs ($k\theta u(\theta)$) and provides a discussion of the interactions of labor market tightness and consumption in a matching model in closed economy. We capture whether consumption capacity expands or decreases (slack or tight) by a change in θ in a small open economy with international foreign asset.

3 The steady state without aggregate demand shortage

In the steady state without demand shortages (i.e., $x = 1$), from (25), (27), and (30), the steady-state values of $\theta^* = \theta^{ns}$, $u^* = u^{ns}$ and $c^* = c^{ns}$ satisfy

$$\begin{aligned}\theta^{ns} &= \theta(1; \underbrace{\bar{y}}_+, \underbrace{\psi}_-, \underbrace{k}_-, \underbrace{\delta}_-), & u^{ns} &= \hat{u}(1; \underbrace{\bar{y}}_-, \underbrace{\psi}_+, \underbrace{k}_+, \underbrace{\delta}_+), \\ c^{ns} &= \hat{c}(1; \underbrace{f_0}_+, \underbrace{g}_-, \underbrace{\bar{\omega}}_-, \underbrace{\bar{y}}_{+ (?)}, \underbrace{\psi}_{-(+)}, \underbrace{k}_{-(?)}, \underbrace{\delta}_{-(?)}).\end{aligned}\tag{31}$$

The nominal price P perfectly adjusts so that $x = 1$ anytime. Thus, from (6), (7), (8), and (22), real money m moves as follows:

$$\frac{\dot{m}}{m} = -\pi = \bar{r} - \frac{\mu'(m)}{\phi'(c^{ns})},\tag{32}$$

which equals zero in the steady state. Then, the nominal price P^{ns} satisfies

$$\frac{\mu'(m^{ns})}{\phi'(c^{ns})} = \bar{r}, \quad m^{ns} = \frac{\bar{M}}{P^{ns}}.\tag{33}$$

The following proposition summarizes the above result:

Proposition 1 : In the steady state without aggregate demand shortage, higher international asset holdings (f_0), an improvement in the terms of trade ($\bar{\omega} \downarrow$), and a decrease in government purchases (g) increase consumption without affecting unemployment. An increase in productivity (\bar{y}) and decreases in cost parameters related to the labor market (ψ, k, δ) increase labor market tightness θ and decrease unemployment u , while their effects on consumption are undetermined due to the hump-shaped patterns in (29), allowing for both a “slack” ($c_\theta > 0$) and a “tight” ($c_\theta < 0$) steady state.

This result is consistent with that of the standard search friction model (Pissarides, 2000) in a small open economy.

4 The secular stagnation steady state

Ono (1994, 2001) has shown that persistent stagnation can arise due to a liquidity trap when households have insatiable liquidity preferences. Applying this assumption to our setting, we compare the effects on consumption and the unemployment rate under aggregate demand shortage ($x < 1$) with those with no aggregate demand shortage ($x = 1$) mentioned in Proposition 1. Surprisingly, these effects are reversed under secular stagnation.

With the money demand function $R = \mu'(m)/\phi'(c)$ in (7), a liquidity trap arises if the desire for money holdings is insatiable:

$$\lim_{m \rightarrow \infty} \mu'(m) = \beta > 0,$$

where β is a positive constant,⁷ and R remains at $\beta/\phi'(c) > 0$. In this case, there is no solution for the steady-state m with $x = 1$, given by (33), if

$$\bar{r} < \frac{\beta}{\phi'(c^{ns})} \left(< \frac{\mu'(m)}{\phi'(c^{ns})} \text{ for any } m \right). \quad (34)$$

Instead, the steady state is characterized by aggregate demand shortage ($x < 1$).

In the presence of aggregate demand shortages ($x < 1$), following Ono (2001) and Schmitt-Grohé and Uribe (2016, 2017), we assume downward price rigidities. More precisely, there is a lower bound for the time change rate of the commodity price P_1 , which is $\alpha(x - 1)$, in order to avoid deflationary wage-price-spirals and to allow for the possibility of a stagnation steady state, where α represents the adjustment speed of P_1 . Since relative prices are constant, the price index $P = P(P_1, \varepsilon P_2^*) = P_1 \cdot P(1, \bar{\omega})$ move in parallel with the commodity price P_1 . This property is summarized as follows:⁸

$$\frac{\dot{P}}{P} (\equiv \pi) = \frac{\dot{P}_1}{P_1} = \alpha(x - 1) < 0 \quad \text{if } x < 1. \quad (35)$$

⁷See Ono (1994: 4–8) for a discussion on the insatiable utility of money in the history of economic thought (e.g., Veblen, Marx, Simmel, Keynes) and its economic implications. The validity of this property is empirically supported by Ono (1994: 34–8) and, more extensively, by Ono, Ogawa, and Yoshida (2004) using parametric and nonparametric methods, and by Akasaka, Mikami, and Ono (2024) using a nationally representative survey of Japan.

⁸This assumption is imposed merely to allow disequilibrium to occur in the commodity market; otherwise, the possibility of unemployment due to aggregate demand shortages is intrinsically avoided. Note that under this assumption, the possibility of no aggregate demand shortage is not eliminated. Ono and Ishida (2014) give a micro-foundation to such an adjustment process, and show that under the micro-founded adjustment process an economy also reaches a steady state of which properties are similar to those of the unemployment steady state of the present paper.

Then, from (22),

$$\frac{\dot{m}}{m} = -\pi = -\alpha(x - 1). \quad (36)$$

From (6), (7), (8), and (35),

$$\frac{\mu'(m)}{\phi'(c^*)} = \alpha(x - 1) + \bar{r}, \quad (37)$$

which gives x as a function of m and c^* . In the steady state with demand shortage ($x < 1$), the nominal price P continues to decline, as shown by (35), and real balances continue to increase, making $\mu'(m)$ converge to β and (37) turn to

$$\frac{\beta}{\phi'(\hat{c}(x; f_0, g, \bar{\omega}, \bar{y}, \psi, k, \delta))} = \bar{r} + \alpha(x - 1), \quad (38)$$

where $\hat{c}(x; \dots)$ is given by (30).⁹ (38) solves x^* in the stagnation steady state.

Figure 1 illustrates the two sides of equation (38) as a function of realized working hours x . As is clear from the graphical illustration, for the equilibrium of x to exist under (34), the following condition is required:

$$\bar{r} + \alpha(\underline{x} - 1) > \frac{\beta}{\phi'(\hat{c}(\underline{x}; f_0, g, \bar{\omega}, \bar{y}, \psi, k, \delta))}, \quad (39)$$

where the lowest possible x , denoted \underline{x} , at which there is no supply of commodities from firms to consumption, is obtained from (24), (26) and (28) as follows:¹⁰

$$\underline{x} \equiv \frac{k}{(1 - \psi)\bar{y}} \frac{\bar{r} + \delta}{q(0)}, \quad \theta(\underline{x}; \bar{y}, \psi, k, \delta) = 0, \quad \underline{c}^s = \hat{c}(\underline{x}; f_0, g, \bar{\omega}, \bar{y}, \psi, k, \delta) = \bar{r}f_0 - g.$$

As is also evident from Figure 1, under (39), the left hand side of (38) is more inclined than the right hand side at the intersection point; hence,

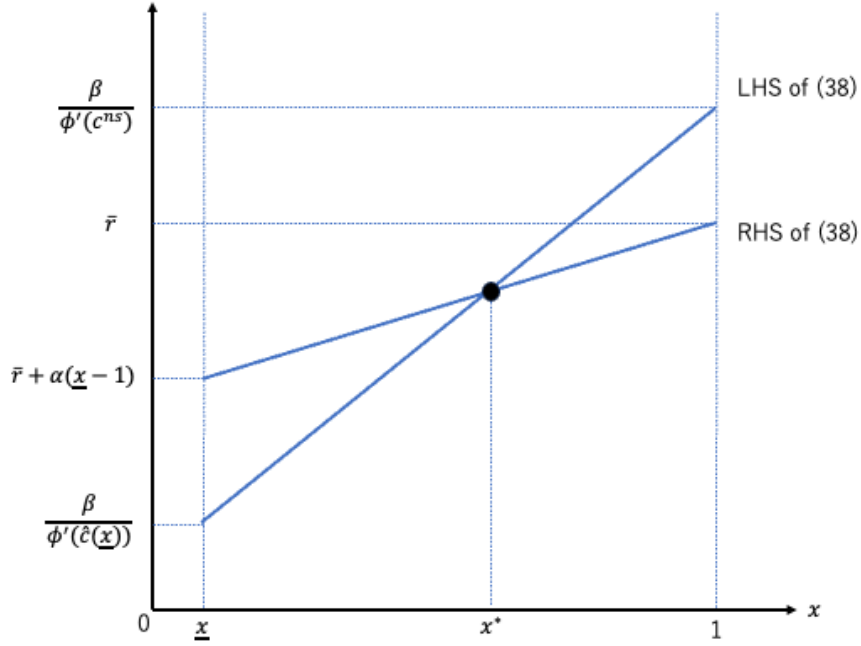
$$\Psi \equiv \frac{-\beta\phi''(c)}{(\phi'(c))^2} \hat{c}_x - \alpha > 0, \quad (40)$$

which implies $\hat{c}_x > 0$ in the stagnation steady state. Therefore, from the first equation in

⁹This deflationary path is compatible with the transversality condition, since the nominal interest rate $R = \beta/\phi'(c)$ is positive, so that $\dot{m}/m = -\pi = -R + \rho < \rho$ holds.

¹⁰In the case of $\Gamma(u, v) = \kappa u^\gamma v^{1-\gamma}$ (Cobb-Douglas form), $\underline{x} = 0$ holds because of $q(\theta) = \kappa\theta^{-\gamma}$ and $q(0) = \infty$.

Figure 1: Secular stagnation equilibrium



Appendix E, we find

$$\hat{c}_{\bar{y}} = \left(\frac{x^*}{\bar{y}} \right) \hat{c}_x > 0. \quad (41)$$

From (38) and (40), we obtain

$$\frac{\Psi}{\frac{-\beta\phi''(c)}{(\phi'(c))^2}} dx = -\hat{c}_h dh \text{ for } h = f_0, g, \bar{y}, \bar{\omega}, \psi, k \text{ and } \delta, \quad (42)$$

where the sign conditions of \hat{c}_h for $h = f_0, g, \bar{y}, \bar{\omega}, \psi, k$ and δ are given by (30) and (41). Also, from (38),

$$\frac{-\beta\phi''(c)}{(\phi'(c))^2} dc = \alpha dx,$$

implying that c moves in the same direction as x . Thus, the comparative statics of x and $c^* = c^s$ are given as follows:

$$\begin{aligned} x^* &= x(\underbrace{f_0}_{-}, \underbrace{g}_{+}, \underbrace{\bar{\omega}}_{+}, \underbrace{\bar{y}}_{-}, \underbrace{\psi}_{+(-)}, \underbrace{k}_{+(?)}, \underbrace{\delta}_{+(?)}) \\ c^s &= c(\underbrace{f_0}_{-}, \underbrace{g}_{+}, \underbrace{\bar{\omega}}_{+}, \underbrace{\bar{y}}_{-}, \underbrace{\psi}_{+(-)}, \underbrace{k}_{+(?)}, \underbrace{\delta}_{+(?)}) \end{aligned} \quad (43)$$

The effects on consumption are opposite to those under no aggregate demand shortage $c^* = c^{ns}$,

shown in (31), which are summarized as follows:

Proposition 2 : The effect of a change in each parameter (i.e., $f_0, g, \bar{\omega}, \psi, k, \delta$) except productivity \bar{y} on consumption under aggregate demand shortages is the opposite of its effect under no shortage. The productivity parameter (\bar{y}) has a negative effect on consumption under aggregate demand shortages (“paradox of toil”), while its effect on consumption depends on “slack” or “tight” under no shortage.

Intuitively, when there is no shortage of aggregate demand, parameter changes that increase the amount of commodities allocatable to consumption naturally expand consumption. However, they reduce consumption by widening the deflationary gap and making asset holdings yield higher real returns under shortages of aggregate demand.

Let us next obtain the comparative statics of the employment rate. From (27) and (43), we immediately obtain

$$\frac{du^s}{df_0} = \underbrace{\hat{u}_x}_{-} \underbrace{\frac{dx^*}{df_0}}_{-} > 0, \quad \frac{du^s}{dg} = \underbrace{\hat{u}_x}_{-} \underbrace{\frac{dx^*}{dg}}_{+} < 0, \quad \frac{du^s}{d\bar{\omega}} = \underbrace{\hat{u}_x}_{-} \underbrace{\frac{dx^*}{d\bar{\omega}}}_{+} < 0.$$

From (26), the second equation in Appendix C, and $dx^*/d\bar{y}$ derived from (40), (41) and (42), we find

$$\frac{du^s}{d\bar{y}} = \underbrace{u_\theta}_{-} \underbrace{\theta_{\bar{y}}}_{+} \left(\frac{\bar{y}}{x^*} \underbrace{\frac{dx^*}{d\bar{y}}}_{-} + 1 \right) = \frac{-\alpha u_\theta \theta_{\bar{y}}}{\Psi} > 0.$$

These results give the following proposition:

Proposition 3 : In the secular stagnation steady state, an increase in productivity \bar{y} , an improvement in the terms of trade ($\bar{\omega} \downarrow$), and greater initial international asset holdings f_0 deteriorate the unemployment rate. Larger government purchases g lower the unemployment rate, irrespective of whether the government spends on the import or export commodity.

The effects of microeconomic parameters related to the labor market, ψ , k , and δ , on the unemployment rate are ambiguous in general, because their effects on x in \hat{u} defined by (27) are ambiguous (see Appendix F for the effects on u^s). The effect of microeconomic shocks on the unemployment rate is generally ambiguous. In the next section, we will present quantitative

results on their overall impacts on hours worked and the employment rate.

5 Quantitative analysis

In this section, we calibrate and simulate the model to quantify the relative magnitude of frictional unemployment and underemployment using the decomposition of the total employment gap:

$$\underbrace{1 - x(1 - u)}_{\text{total employment gap}} = \underbrace{u}_{\text{frictional unemployment}} + \underbrace{(1 - u)(1 - x)}_{\text{underemployment}}.$$

We will also add more insights to the theoretical analysis by quantifying the effects of various parameter variations on the secular stagnation steady state using comparative statics analysis.

5.1 Calibration

We set a time period to be one year. We rely on a Cobb-Douglas matching function, which implies a constant elasticity $\eta_\theta = \gamma$ of the job filling rate $q(\theta)$. Specifically, we assume the following functional form for $\Gamma(u, v)$:

$$\Gamma(u, v) = \varphi u^\gamma v^{1-\gamma},$$

where the scale parameter φ reflects the matching efficiency.

The utility from consumption $\hat{\phi}(c_1, c_2)$ is given by the CES function:

$$\hat{\phi}(c_1, c_2) = \frac{\left(\left(\xi^{\frac{1}{\sigma}} c_1^{\frac{\sigma-1}{\sigma}} + (1-\xi)^{\frac{1}{\sigma}} c_2^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \right)^{1-\varsigma} - 1}{1-\varsigma},$$

where $\varsigma > 0$ is the CRRA parameter, $\xi \in (0, 1)$ the weight of the domestic commodity in the consumption basket, and $\sigma > 1$ the elasticity of substitution between domestic and foreign commodities.¹¹ Substituting c_1 and c_2 from equation (4) into the utility function, together with

¹¹The corresponding consumer price index is given by $P = \left(\xi P_1^{1-\sigma} + (1-\xi) (\varepsilon P_2^F)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$, from which the expressions for the real price p_1 and the expenditure share $\zeta(\bar{\omega})$ of commodity 1 can be derived.

the expressions for $p_1(\bar{\omega})$ and $\zeta(\bar{\omega})$, simplifies the consumption utility function to

$$\hat{\phi}(c) = \frac{c^{1-\varsigma} - 1}{1 - \varsigma}.$$

Given these functional form assumptions, 17 parameters need to be calibrated. Our calibration is summarized in Table 1. In line with the existing literature, we assume an annual time preference rate of $\rho = 0.04$, which determines the (exogenous) global real rate \bar{r} , and a CRRA parameter for consumption of $\varsigma = 2$. Following Backus, Kehoe, and Kydland (1994), we set the elasticity of substitution between domestic and foreign commodities equal to $\sigma = 1.5$, which is a common benchmark in the literature.¹² The weight on home consumption ξ is calibrated to match the share of imported consumption commodities in Japan, which is reported as 0.063 by Katagiri and Takahashi (2023). This gives a value of $\xi = 0.858$, indicating a substantial home bias in consumption. The marginal utility of money β is calibrated to match a target for the shortfall of working hours $1 - x$ in the stagnation steady state. When it comes to the speed of price adjustment, feasibility requires $\alpha < \rho$. We set $\alpha = 0.02$. This implies that a 10 percentage point shortfall of working hours ($1 - x$) is associated with a deflation rate of 0.2%, which equals the average CPI inflation rate in Japan in the period of the binding lower bound on the nominal interest rate (1999-2018). Without loss of generality, labor productivity is normalized to $\bar{y} = 1$. Our baseline calibration also assumes $g = 0$. Variations in \bar{y} and g are used to model supply and demand shocks respectively.

The calibration of the wage negotiation and matching process follows the procedure in Hashimoto, Ono, and Schlegl (2023). We assume equal bargaining power of workers and firms, i.e. $\varepsilon = 0.5$, and a replacement rate of $\bar{z} = 0.6$, which implies an effective bargaining power of $\psi = 0.714$. The job separation is set to $\delta = 0.067$, which implies an average job duration rate of 15 years consistent with the data for Japan.¹³ As is standard in the literature, we calibrate the parameter γ in the matching function equal to the bargaining power parameter of workers, i.e. $\gamma = \varepsilon = 0.5$, thus satisfying the Hosios (1990) efficiency criterion. The scale parameter in the matching function is then calibrated such that the steady state Beveridge curve in (26) is consistent with Japan's average values for the unemployment rate ($\bar{u} = 3.5\%$) and labor market

¹²Bodenstein, Guerrieri, and Gust (2013) estimate a somewhat smaller elasticity of $\sigma = 1.076$ for households in the United States. To our knowledge, no estimates for Japanese households are available.

¹³Miyamoto (2011) reports an average job duration rate of 18 years for Japan (1980-2010), while more recent estimates are closer to 12 years.

Table 1: Parameter calibration

Parameter	Calibration	Reference or moment
Time preference rate	$\rho = 0.04$	global real interest rate \bar{r} of 4%
CRRA parameter in $\hat{\phi}(c)$	$\varsigma = 2$	standard value
Elasticity: Home and foreign commodities	$\sigma = 1.5$	Backus, Kehoe, and Kydland (1995)
Weight on home consumption	$\xi = 0.858$	match share of imported commodities of 6.3%
Marginal utility of money	β	target working hours shortfall $1 - x$
Speed of price adjustment	$\alpha = 0.02$	for feasibility $\rho > \alpha$
Labor productivity	$\bar{y} = 1$	normalized, parameter for supply shocks
Public spending	$g = 0$	parameter for demand shocks
Replacement rate	$\bar{z} = 0.6$	Hashimoto, Ono, and Schlegl (2023)
Bargaining power	$\varepsilon = 0.5$	Hashimoto, Ono, and Schlegl (2023)
Bargaining power (eff.)	$\psi = 0.714$	$\psi = \epsilon(1 - (1 - \varepsilon)z)^{-1}$
Job separation rate	$\delta = 0.067$	δ^{-1} = average job duration of 15 years
Elasticity of $q(\theta)$	$\gamma = 0.5$	Hosios (1990) efficiency condition, $\gamma = \varepsilon$
Scale parameter in $\Gamma(u, v)$	$\varphi = 2.01$	match $\bar{u} = 0.035$ and $\bar{\theta} = 0.85$
Cost of vacancy	$k = 0.435$	match \bar{r} , $\bar{u} = 0.035$ and $\bar{\theta} = 0.85$ for $x = 1$
Relative price of commodities	$\bar{\omega} = 1$	benchmark, robustness analysis
Initial foreign asset position	$f_0 = 0.65$	Japan's net foreign assets (to GDP) in 2019
Initial unemployment rate	$u_0 = 0.024$	Japan's unemployment rate in 2019

Note: We calibrate the strength of the liquidity preference β to target the shortfall of working hours $(1 - x)$ in the secular stagnation steady state. For completeness, the specific values of β are 0.04974 (for $1 - x = 1\%$), 0.05290 (for $1 - x = 5\%$) and 0.05739 (for $1 - x = 10\%$).

Table 2: Steady state

Shortfall of working hours $1 - x^s$ (in %)	1.0	5.0	10.0
Total employment gap $1 - x^s(1 - u^s)$ (in %)	4.46	8.38	13.3
Frictional unemployment u^s (in %)	3.49	3.56	3.66
Underemployment $(1 - u^s)(1 - x^s)$ (in %)	0.97	is given by 4.82	9.64
Total household consumption c^s	0.895	0.859	0.814
Foreign asset position f^s	0.657	0.657	0.658

tightness ($\bar{\theta} = 0.85$) over the period 1980-2018. This gives a value of $\varphi = 2.01$ and an implied job finding rate of $\bar{\theta}q(\bar{\theta}) = 1.85$, which is in line with the quarterly value of 0.426 reported in Miyamoto (2011). The vacancy cost parameter k is chosen to generate an annual steady-state return on equity of $\bar{r} = 4\%$ in the absence of demand shortage based on (24) with $x = 1$, $\bar{\theta} = 0.85$ and $\bar{u} = 3.5\%$, as given above. This yields a vacancy cost of $k = 0.435$. Finally, the relative price $\bar{\omega}$ of the two commodities needs to be determined. Our baseline considers the case of $\bar{\omega} = 1$.

The steady state of our model critically depends on the initial conditions for unemployment u_0 and net foreign asset holdings f_0 . Here, we take Japan's net foreign position (relative to GDP) and unemployment rate in 2019 as the starting point of our analysis. The respective values are $f_{2019} = f_0 = 0.65$ and $u_{2019} = u_0 = 0.024$.

Table 2 summarizes the corresponding secular stagnation steady state for three different targeted values of the working hours shortfall. The total employment gap is decomposed into the contributions of frictional unemployment and underemployment. As the extent of stagnation

Table 3: Comparative statics

Change in	(1) δ	(2) ψ	(3) k	(4) g	(5) \bar{y}	(6) $\bar{\omega}$	(7) f_0
Employment gap (in pp)	-1.84	-0.03	0.38	-1.76	5.02	-0.74	1.06
Frictional unemployment (in pp)	0.47	0.30	-0.10	-0.01	0.01	-0.00	0.00
Underemployment (in pp)	-2.31	-0.33	0.48	-1.75	5.01	-0.74	1.05
Household consumption (in %)	0.41	0.12	-0.08	0.21	-0.61	0.09	-0.13
Foreign asset position (in pp)	0.55	0.34	-0.08	-0.01	0.01	-0.00	15.01

Notes: (1) Reduction in job duration to 12 years (i.e. increase in δ to $\delta' = 1/12$). (2) Increase in workers' effective bargaining power to $\psi' = 0.77$. (3) Reduction in vacancy cost k by 10%. (4) Demand shock. Increase in government spending to $g' = 0.01$. (5) Supply shock. Increase in labor productivity to $\bar{y}' = 1.01$. (6) Increase in $\bar{\omega}$ by 10%. (7) Increase in initial asset position to $f'_0 = 0.8$.

tion worsens, the employment gap widens considerably with both unemployment and underemployment rising. However, the increase in underemployment quantitatively exceeds the rise in unemployment considerably, confirming our previous finding in Hashimoto, Ono, and Schlegl (2023). In addition, private consumption declines while the country's net foreign asset position increases slightly.

5.2 Comparative statics

Let us now investigate how the secular stagnation steady state, and specifically the labor market, is affected by variations in the model parameters. We will use the steady state with a 5% shortfall of working hours in Table 2 as the starting point for each simulation. The results are summarized in Table 3.

Recall from the previous section that the effects of variations in the microeconomic parameters δ , ψ and k , which govern the search and matching process, are ambiguous in general. However, for our calibration to the Japanese economy, we find that changes in these parameters have opposite effects on underemployment and frictional unemployment. Both a higher job destruction rate δ (column 1) and a stronger effective bargaining power of workers ψ (column 2) result in a higher unemployment rate, as is the case in the standard search and matching model, while underemployment declines in both cases due to a rise in effective working hours. Quantitatively, the decline in underemployment exceeds the rise in the unemployment rate so that the total employment gap shrinks, albeit this effect is quantitatively negligible when workers have a stronger effective bargaining power ψ . Household consumption increases as the decline in underemployment stimulates demand, while the foreign asset position improves slightly. Furthermore, a reduction in the vacancy cost k (column 3) causes the employment gap to widen as the increase in underemployment overcompensates for the fall in the unemployment rate.

Household consumption is further depressed and the foreign asset position shrinks.

When it comes to variations in the macroeconomic parameters g , \bar{y} , $\bar{\omega}$ and f_0 , their effects are qualitatively unambiguous as shown in the theoretical analysis of the previous section. Yet, our simulations are indicative of the quantitative impacts on both forms of labour market slack. While all of these shocks have substantial effects on the employment gap, it is the change in underemployment, rather than frictional unemployment, that quantitatively accounts for the overwhelming part of these effects. In fact, the response of frictional unemployment is negligible in all cases. This is in line with our previous findings in Hashimoto, Ono, and Schlegl (2023). As frictional unemployment barely responds to these parameter variations, the steady state foreign asset position is also hardly affected by changes in aggregate demand, supply or the terms of trade $\bar{\omega}$, the notable exception being a change in the initial foreign asset position f_0 (column 7), as is clear from equation (47) in Appendix B.

From a policy perspective, these simulations suggest that the unemployment rate is an insufficient, and potentially misleading, indicator for the effectiveness of labor market policies under secular stagnation. Specifically, traditional labor market measures that aim at improving the efficiency of the matching mechanism, such as a reduction in vacancy costs k (as shown in column 3), can increase the overall employment gap via their effect on underemployment despite lowering the unemployment rate and should consequently be used with caution. In contrast, our analysis highlights the need for and the effectiveness of policies that stimulate aggregate demand, even though their benefits are not primarily reflected in a lower unemployment rate.

6 Conclusion

This study contributes to the literature by integrating the analysis of labor market frictions and demand shortage within a small open economy framework, shedding light on the interplay between international economic shocks and labor market outcomes. It elucidates how variations in international conditions, such as terms of trade and international transfers, influence employment rates and working hours.

Our findings indicate that under demand shortage conditions, a strong wealth preference exacerbates underemployment and reduces the employment rate by diminishing domestic firm profitability and market entry. In this state, an improvement in terms of trade reduces the

demand for domestically produced commodities, which subsequently decreases working hours and firms' profit margins. This, in turn, leads to lower employment rates and reduced consumption. Conversely, in the absence of aggregate demand shortage, an improvement in terms of trade increases real income, leading to higher consumption. Since working hours are already fully utilized in such conditions, firms' profits remain unaffected, and consequently, there is no impact on the employment rate.

Throughout our study, we have demonstrated that the effect of the international economy on the labor market depends on whether a country is experiencing secular stagnation or not.

Appendices

Appendix A. The derivation of (18)

From (12) and (13), we have

$$(r + \delta + \theta q(\theta))(E - U) = wx - z + (\dot{E} - \dot{U}).$$

Using (17), the above equation is rewritten as:

$$(r + \delta + \theta q(\theta)) \frac{\varepsilon}{1 - \varepsilon} J = wx - z + \frac{\varepsilon}{1 - \varepsilon} \dot{J}. \quad (44)$$

Also, using (14) with $V = 0$, the value of an operating firm is

$$\dot{J} = (r + \delta)J - (p_1 \bar{y} - w)x.$$

Substituting this equation into (44) gives

$$\theta q(\theta) \frac{\varepsilon}{1 - \varepsilon} J = wx - z - \frac{\varepsilon}{1 - \varepsilon} (p_1 \bar{y} - w)x.$$

Applying (9) and (16) to the above equation leads to (18).

Appendix B. Stability

The dynamics of consumption (c)

From (8), consumption c is constant over time, whether or not there is an aggregate demand shortage.

The dynamics of the real money (m)

In the case of $x = 1$, (32) forms an autonomous dynamic system with respect to m :

$$\dot{m} = -\pi m = \left(\bar{r} - \frac{\mu'(m)}{\phi'(c^*)} \right) m,$$

where c^* is the constant consumption with or without aggregate demand shortages. Because $\frac{\partial \dot{m}}{\partial m} = -\frac{\mu''(m^*)}{\phi'(c^*)} m^* > 0$ holds around the steady state and the price level can adjust freely when there is no aggregate demand shortage (for $x = 1$), real money balance initially jumps to the steady state ($m = m^{ns}$), which makes $\dot{m} = 0$.

In the case of $x < 1$ with a liquidity trap $\mu'(m) = \beta$ around the steady state, x remains constant at x^* over time, where x^* satisfies $\frac{\beta}{\phi'(c^*)} = \bar{r} + \alpha(x^* - 1)$ from (37). We define $\tilde{m} \equiv 1/m$ with $\tilde{m} = 0$ in steady state. (36) forms an autonomous dynamic system with respect to \tilde{m} :

$$\dot{\tilde{m}} = \pi \tilde{m} = \alpha(x^* - 1) \tilde{m}.$$

Because $\frac{\partial \dot{\tilde{m}}}{\partial \tilde{m}} = \pi = \alpha(x^* - 1) < 0$ holds around the steady state and the price level cannot jump in the presence of aggregate demand shortages ($x < 1$), \tilde{m} converges to zero (or m diverges to infinity).

The dynamics of the labor market tightness (θ)

(19) forms an autonomous dynamic system with respect to θ :

$$\dot{\theta} = \frac{\theta}{\eta_\theta} \left(\bar{r} + \delta - \left(\frac{(1-\psi)\bar{y}x^*}{k} q(\theta) - \psi q(\theta)\theta \right) \right).$$

Because $\frac{\partial \dot{\theta}}{\partial \theta} = \frac{\theta^*}{\eta_\theta} \left(-q'(\theta^*) \frac{(1-\psi)\bar{y}x^*}{k} + \psi q(\theta^*)(1 - \eta_\theta) \right) > 0$ holds around the steady state and θ can jump, θ initially jumps to the steady state: $\theta^* = \theta^{ns}(\bar{y}, \psi, k, \delta)$ with no aggregate demand

shortage and $\theta^* = \theta^s(x^*; \bar{y}, \psi, k, \delta)$ with an aggregate demand shortage.

The dynamics of international asset holdings (f) and unemployment (u)

From (23) and (11), we obtain autonomous dynamics with two (linear) differential equations in f and u :

$$\begin{aligned} \dot{f} &= \bar{r}f + p_1 (\bar{y}(1-u)x^* - k\theta^*u) - c^* - g, \\ \dot{u} &= \delta(1-u) - \theta^*q(\theta^*)u, \\ \Rightarrow \begin{pmatrix} \dot{f} \\ \dot{u} \end{pmatrix} &= \begin{bmatrix} \bar{r} & -p_1(\bar{y}x^* + k\theta^*) \\ 0 & -(\delta + \theta^*q(\theta^*)) \end{bmatrix} \begin{pmatrix} f - f^* \\ u - u^* \end{pmatrix}, \end{aligned} \quad (45)$$

where both the international asset f and the unemployment rate u are non-jumpable. The eigenvalues of this system solve the following characteristic equation:

$$(\lambda - \bar{r})(\lambda + \delta + \theta^*q(\theta^*)) = 0,$$

which gives one negative and one positive eigenvalue; $\lambda_1 = \bar{r} > 0$ and $\lambda_2 = -(\delta + \theta^*q(\theta^*)) < 0$. Thus, the degree of freedom of this dynamics is one. Since c can jump, the equilibrium path is uniquely determined.

Using the negative eigenvalue λ_2 , the dynamics of f and u are explicitly solved as follows:

$$\begin{aligned} f - f^* &= (f_0 - f^*)e^{-(\delta + \theta^*q(\theta^*))t}, \\ u - u^* &= (u_0 - u^*)e^{-(\delta + \theta^*q(\theta^*))t}. \end{aligned} \quad (46)$$

Substituting the two equations in (46) into the first equation of (45) with the steady-state condition:

$$\dot{f} = \bar{r}f^* + p_1 (\bar{y}(1-u^*)x^* - k\theta^*u^*) - c^* - g = 0,$$

and rearranging the result yields

$$u_0 - u^* = \frac{\bar{r} + \delta + \theta^*q(\theta^*)}{p_1(\bar{y}x^* + k\theta^*)}(f_0 - f^*).$$

This equation gives the steady-state level of f as follows:

$$f^* = f_0 + \frac{p_1 (\bar{y}x^* + k\theta^*)}{\bar{r} + \delta + \theta^*q(\theta^*)}(u^* - u_0). \quad (47)$$

Appendix C. The derivation of the partial derivatives of $\theta(x; \bar{y}, \psi, k, \delta)$

From (24), we derive the following properties:

$$\begin{aligned} \theta_x &= \frac{1}{\Phi} \frac{(1-\psi)\bar{y}}{k} q(\theta) > 0, \\ \theta_{\bar{y}} &= \left(\frac{x^*}{\bar{y}} \right) \theta_x = \frac{1}{\Phi} \frac{(1-\psi)x^*}{k} q(\theta) > 0, \\ \theta_\psi &= -\frac{1}{\Phi} \left(\frac{\bar{y}x^*}{k} q(\theta) + q(\theta)\theta \right) < 0, \\ \theta_k &= -\frac{1}{\Phi} \frac{(1-\psi)\bar{y}x^*}{k^2} q(\theta) < 0, \\ \theta_\delta &= -\frac{1}{\Phi} < 0, \\ \Phi &\equiv -q'(\theta) \frac{(1-\psi)\bar{y}x^*}{k} + \psi q(\theta) (1 - \eta_\theta) > 0, \end{aligned}$$

where a subscript of θ represents the partial derivative of θ with respect to each variable.

Appendix D. The derivation of the partial derivatives of $c(\theta, x; f_0, g, \bar{\omega}, \bar{y}, k, \delta)$

From (28), we derive the following properties:

$$\begin{aligned} c_\theta &= p_1 \left(-u_\theta (\bar{y}x^* + k\theta^*) \frac{\delta + \theta^*q(\theta^*)}{\bar{r} + \delta + \theta^*q(\theta^*)} - ku^* \right) > 0 \quad \text{for slack} \quad (c_\theta < 0 \text{ for tight}), \\ c_x &= p_1 \bar{y} (1 - u^*) > 0, \\ c_{\bar{y}} &= \left(\frac{x}{\bar{y}} \right) c_x = p_1 x^* (1 - u^*) > 0, \\ c_{f_0} &= \bar{r} > 0, \\ c_g &= -1 < 0, \\ c_{\bar{\omega}} &= p_1'(\bar{\omega}) (\bar{y}x^* (1 - u^*) - k\theta^*u^*) < 0, \end{aligned}$$

$$c_k = -p_1 \theta^* u^* < 0,$$

$$c_\delta = -p_1(\bar{y}x^* + k\theta^*) \frac{\delta + \theta^* q(\theta^*)}{\bar{r} + \delta + \theta^* q(\theta^*)} u_\delta < 0.$$

Appendix E. The derivation of the partial derivatives of $\hat{c}(x; f_0, g, \bar{\omega}, \bar{y}, \psi, k, \delta)$

Given that $\hat{c}_h = c_\theta \theta_h + c_h$, the partial derivatives of $\theta(x; \bar{y}, \psi, k, \delta)$ in Appendix C and the partial derivatives of $c(\theta, x; f_0, g, \bar{\omega}, \bar{y}, k, \delta)$ in Appendix D yield

$$\hat{c}_x = \left(\frac{\bar{y}}{x}\right) \hat{c}_{\bar{y}}, \quad \hat{c}_{f_0} = \underbrace{c_{f_0}}_{+} > 0, \quad \hat{c}_g = \underbrace{c_g}_{-} < 0, \quad \hat{c}_{\bar{\omega}} = \underbrace{c_{\bar{\omega}}}_{-} < 0,$$

$$\hat{c}_{\bar{y}} = \underbrace{c_\theta}_{+(-)} \underbrace{\theta_{\bar{y}}}_{+} + \underbrace{c_{\bar{y}}}_{+} > 0 \text{ for } c_\theta > 0 : \text{slack} \quad (? \text{ for } c_\theta < 0 : \text{tight}),$$

$$\hat{c}_\psi = \underbrace{c_\theta}_{+(-)} \underbrace{\theta_\psi}_{-} < 0 \text{ for } c_\theta > 0 : \text{slack} \quad (> 0 \text{ for } c_\theta < 0 : \text{tight}),$$

$$\hat{c}_k = \underbrace{c_\theta}_{+(-)} \underbrace{\theta_k}_{-} + \underbrace{c_k}_{-} < 0 \text{ for } c_\theta > 0 : \text{slack} \quad (? \text{ for } c_\theta < 0 : \text{tight}),$$

$$\hat{c}_\delta = \underbrace{c_\theta}_{+(-)} \underbrace{\theta_\delta}_{-} + \underbrace{c_\delta}_{-} < 0 \text{ for } c_\theta > 0 : \text{slack} \quad (? \text{ for } c_\theta < 0 : \text{tight}).$$

Appendix F. The derivation of the effects of microeconomic shocks on u^s

From (25), (27) and (43), we find

$$\frac{du^s}{d\psi} = \underbrace{u_\theta}_{-} \left(\underbrace{\theta_x}_{+} \underbrace{\frac{dx^*}{dk}}_{+(-)} + \underbrace{\theta_\psi}_{-} \right) \gtrless 0 \quad \text{for } c_\theta > 0 : \text{slack} \quad \left(\frac{du^s}{d\psi} > 0 \text{ for } c_\theta < 0 : \text{tight} \right),$$

$$\frac{du^s}{dk} = \underbrace{u_\theta}_{-} \left(\underbrace{\theta_x}_{+} \underbrace{\frac{dx^*}{dk}}_{+ (?) } + \underbrace{\theta_k}_{-} \right) \gtrless 0 \quad \text{for } c_\theta > 0 : \text{slack} \quad \left(\frac{du^s}{d\psi} \gtrless 0 \text{ for } c_\theta < 0 : \text{tight} \right),$$

$$\frac{du^s}{d\delta} = \underbrace{u_\theta}_{-} \left(\underbrace{\theta_x}_{+} \underbrace{\frac{dx^*}{d\delta}}_{+ (?) } + \underbrace{\theta_\delta}_{-} \right) + \underbrace{u_\delta}_{+} \gtrless 0 \quad \text{for } c_\theta > 0 : \text{slack} \quad \left(\frac{du^s}{d\psi} \gtrless 0 \text{ for } c_\theta < 0 : \text{tight} \right).$$

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