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## Navigating the sea of natural real interest rate estimates

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Bank of Finland

# BoF Economics Review

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## Navigating the sea of natural real interest rate estimates

Mikael Juselius\*

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Recent inflationary trends have reignited debates about the natural real interest rate ( $r^*$ ). This paper reviews the literature on estimating  $r^*$  and its drivers, highlighting significant uncertainty and variability in estimates. The challenges in achieving consensus on  $r^*$  is mostly due to differing methodologies and model assumptions. Understanding these sources of uncertainty is essential for shaping future monetary policy, especially in a low interest rate environment where conventional policy tools may be limited.

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# Navigating the sea of natural real interest rate estimates

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## Abstract

Recent inflationary trends have reignited debates about the natural real interest rate ( $r^*$ ). This paper reviews the literature on estimating  $r^*$  and its drivers, highlighting significant uncertainty and variability in estimates. The challenges in achieving consensus on  $r^*$  is mostly due to differing methodologies and model assumptions. Understanding these sources of uncertainty is essential for shaping future monetary policy, especially in a low interest rate environment where conventional policy tools may be limited.

## 1 Introduction

*“But I know, somehow, that only when it is dark enough can you see the stars.”* – Martin Luther King Jr in his last speech, 1968.

Central banks help guide the economy by setting the short-term nominal risk-free interest rate. Changing this rate affects the expected real (inflation adjusted) return people get from saving or investing because prices don't adjust instantly. It therefore influences how much people spend or save, and how much businesses invest. But to set the right rate, central banks need a reference point for the real interest rate at which policy is neither expansionary nor contractionary. This is called the natural interest rate (*natural rate* for short), often written as  $r^*$ . Unfortunately, the natural rate is not directly observed and has to be estimated from data.

For more than a decade, natural rate estimates have been at historically low levels. Combined with low inflation rates, this makes it more difficult for central banks to steer the economy. However, the recent surge in inflation

following the pandemic has sparked discussions about whether the natural rate has risen again (e.g., Brand, Lisack, and Mazelis (2024), Benigno et al. (2024), and Davis et al. (2023)). From a longer-term perspective, factors such as massive fiscal spending during the pandemic, technological advancements in artificial intelligence, the green transition and the retirement of the large baby-boomer generation may all create upward pressure on the rate. Yet, the subsequent and equally swift decline in inflation from its post-pandemic peak has led others to conclude that  $r^*$  remains at a low level, pointing out that many of the factors that have suppressed rates over the past decades are still in effect (Holston, Laubach, and Williams (2023), (IMF) and Obstfeld (2023)). Determining which of these perspectives holds more merit is important for shaping future monetary policy.

A rising natural rate would facilitate the normalization of monetary policy, reducing reliance on unconventional tools such as quantitative easing and forward guidance. Conversely with a low  $r^*$  and a given inflation target, there is a greater risk that monetary policy runs out of room to expand the economy in response to adverse shocks (Williams (2016)). This is because the nominal rate is constrained from below by the effective lower bound (ELB). For example, suppose the natural rate is -3%,<sup>1</sup> current inflation expectations are 2%, and the ELB is 0%. In this case, monetary policy can at most achieve a current ex-ante real interest rate of -2%, which is above the assumed value of  $r^*$ . Hence, policy would inadvertently be contractionary.

Unfortunately, there is little consensus in the academic literature on the current value and future direction of the natural rate, complicating policy decisions. Numerous different models and approaches deliver a wide range of estimates that span most observed values of the real interest rate itself. In other words,  $r^*$  can be anywhere. Similarly, studies into the drivers of  $r^*$  display substantial disagreement and often deliver conflicting assessments. For example, the findings in Rachel and Smith (2017) suggest that demographic trends are a key driver of the natural rate while Marx, Mojon, and Velde (2021) downplays the role of demographics and highlight the productivity slowdown and increasing risk aversion instead. To make matters worse, purely empirical studies typically fail to find stable long-run correlations between proposed drivers of the natural rate and measures of real rates (see

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<sup>1</sup>While a negative natural rate is possible theoretically, this seems unlikely in practice. Nevertheless, common narratives about the low-inflation environment following the 2008 global financial crisis implicitly rely on the assumption of a negative natural rate.

e.g., Borio et al. (2022)). This lack of consensus makes it hard to gauge the monetary policy stance and safeguard sufficient policy space going forward.

In this note, I will review the extensive literature on the natural rate, highlighting the reasons behind varied results and conflicting conclusions. The lessons that emerge can be summarized as follows: (i) There is a general consensus that the real risk-free natural rate has declined by several percentage points over the past four decades. (ii) Point estimates are highly uncertain and can differ by several percentage points. (iii) The bulk of this uncertainty reflects various modeling choices but there is also a high degree of statistical uncertainty within specific models. (iv) While a comprehensive list of potential drivers of  $r^*$  has been proposed in the literature, there is little consensus about which are quantitatively important and which are not. These lessons largely reflect the empirical weaknesses of the real interest channel and the Phillips Curve, rendering estimates of  $r^*$  less useful in practice.

Before commencing with the review, I provide a brief overview of the natural rate and related concepts. Readers that are familiar with this subject can jump to Section 3.

## 2 The natural interest rate, expectations, risk and horizon

The natural rate is the real (inflation-adjusted) interest rate consistent with full employment and stable inflation, assuming no short-term shocks. It's not directly observable, but it serves as a conceptual anchor. If the central bank sets the real policy rate below  $r^*$ , it is likely stimulating the economy; if it sets it above  $r^*$ , it is likely restraining it. This makes  $r^*$  a critical reference point for assessing the stance of monetary policy.

Because economic decisions depend on expected real returns, central banks focus on the ex-ante real interest rate, i.e., the nominal rate minus expected inflation. In contrast, the ex-post real rate, i.e., the nominal rate adjusted for actual inflation, is only known after the fact and shouldn't influence current behavior. This distinction is important at least in theory: central banks aim to influence expectations and forward-looking decisions, not just outcomes. In practice, however, the ex-ante ex-post distinction is less sharp as expectations of future inflation tend to move with the current

inflation rate rather than ahead of it (Figure 1). For instance, the correlation between current inflation and the SPF and Tealbook 1-year ahead inflation expectations in the figure is 0.89 and 0.84, respectively, which is higher than the correlation between them and one year ahead inflation, which are 0.80 and 0.79, respectively. This suggest that agents are either to a large extent backward-looking or lack information that would help them forecast inflation more accurately.<sup>2</sup>

By design, central banks directly control the short-term risk-free nominal rate – the return on money – which then influences broader financial conditions. However, most real-world investment decisions involve risky rates that include various premia for uncertainty. When risk premia are relatively stable, the distinction between risk-free and risky natural rates is less important. But when risk premia fluctuate, for example due to financial stress or changes in investor sentiment, the gap between risk-free and risky rates can widen. For instance, during periods of heightened demand for safe assets, such as U.S. Treasuries during so called risk-off episodes, these assets may carry a convenience premium; a lower yield due to their perceived safety and liquidity. In such cases, the risk-free rate may understate the true cost of capital facing firms, complicating assessments of the natural rate.

Although central banks set short-term rates, their goal is also to influence long-term economic outcomes. This means they must understand how short-term rates affect long-term interest rates, which are more relevant for investment, housing, and other key economic decisions. The natural rate is often thought of as a long-run concept as it is the rate that prevails once the effects of various frictions have run their course. But even so, it helps central banks determine the appropriate path for short-term rates over time.

### 3 Estimates

The literature offers various approaches for deriving the natural rate, resulting in widely differing estimates. The bulk of the variation stems from methodological aspects rather than from statistical uncertainty within a given model.

One important dimension is how closely they relate to underlying theoretical concepts of the neutral rate. The natural rate is commonly defined as

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<sup>2</sup>This is broadly in line with the conclusions in Verbrugge and Zaman (2021) who study the forecasting performance of US inflation expectations.

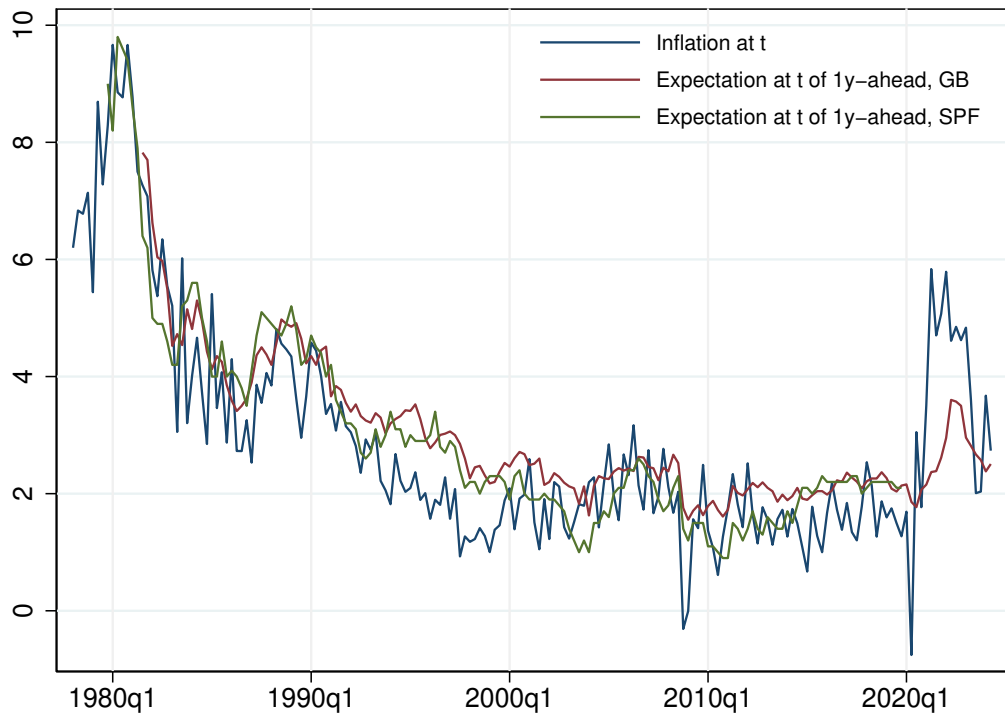


Figure 1: Inflation at time  $t$  compared with 1-year ahead expectations formed at  $t$ . SPF: Survey of Professional Forecasters, Philadelphia Fed. GB: Philadelphia Fed's Tealbook (formerly Greenbook) forecasts of headline inflation four quarters into the future formed at the end of each quarter (x-axis).

*the real interest rate that is consistent with output being at its potential level and inflation being stable.*<sup>3</sup> Hence, at one end of the spectrum are approaches that impose such equilibrium conditions, delivering estimates that are fully in line with some conceptual definition. At the other end are approaches that rely more heavily on the statistical properties of real interest rate and their measurement to infer the value of  $r^*$ .

The second key dimension is whether a given approach delivers short-run or long-run notions of the natural rate (see discussion in Brand, Lisack, and Mazelis (2024)). Long-run versions of the natural rate can be estimated from observed long-term real interest rates,<sup>4</sup> such as the real ex-ante yield of 10-year government bonds, by extracting their low-frequency components using long-run restrictions in a statistical filter. The latter essentially limit how quickly  $r^*$  is allowed to change in response to new data in the filter. Short-run versions, on the other hand, give  $r^*$  more freedom (but still imposes some limits on its movement) and often use short-term real rates as inputs.

A final dimension of relevance is how closely a given approach ties the natural rate to observed (ex-ante) real interest rate series. Some filters strictly impose the restriction that actual real rates should fluctuate around the natural rate over the long run while others do not. This can make a big difference in terms of outcome.

Given these different dimensions, it is useful to divide approaches into the following classes:

1. Estimates from statistical models
2. Estimates derived from fitted theoretical models
3. Estimates from semi-structural filters

I will discuss each of these in turn.

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<sup>3</sup>Reis (2022) extend the concept further by distinguishing between the natural rates on risky and risk-free assets, while Akinci et al. (2023) add a natural rate that covers financial stability aspects.

<sup>4</sup>It should be noted that ex-ante real ex-ante rates are not fully observed as expectations are not perfectly recorded. Instead, researchers often rely on survey data or empirical forecasts to proxy real ex-ante rates.

### 3.1 Estimates from statistical models

I begin by considering estimates from “pure” statistical filters. These filters use statistical restrictions to impose mean reversion between observed real rates and the natural rate estimate. The justification is that actual real rates cannot deviate from the natural rate over the long-run when the central bank acts to stabilize inflation. The resulting  $r^*$  estimate from a statistical filter is typically a smooth representation of the observed real rate, around which the latter tends to fluctuate. Other approaches differ from pure statistical models by including additional information and assumptions that can increase the variability of the  $r^*$  estimate and weaken its connection to observed rates.

The easiest way of obtaining a natural rate estimate is to apply a (univariate) statistical filter, such as the standard Hodrick–Prescott (HP) filter, to a real interest rate series. Applying this filter to euro area real interest rates data (Figure 2, bottom left panel, green line) produces estimates that on average follow the real rates closely. Aside from this reversion property, there is little that relates such estimates to the underlying natural rate concept. For example, there is no guarantee that the deviations of the real rates from their smoothed trend are associated with activity or inflation. Hence, it is impossible to say at the outset if the resulting measure provides a rate that is consistent with output at its long-run equilibrium and stable inflation.

Statistical filtering does not necessarily produce similar natural rate estimates. Substantial variation can already come from using different real interest rate measures (Figure 2, upper panels). For instance, the dynamics of long-rates are often much more smooth than corresponding short-rates and, hence, vary less over the cycle (upper panels, green vs blue and red lines). Risky rates have mostly been stable since the 2000s (upper right panel, orange line) whereas risk-free rates have come down. In theory, ex-post (or backward looking) real rates should also differ substantially for ex-ante (or forward looking) real rates, but in reality this difference is smaller at least for short rates.

Another source of variation arises from assumptions regarding how much wiggle room the natural rate estimate should have relative to the observed rate.<sup>5</sup> The statistical uncertainty associated with the  $r^*$  estimate increases

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<sup>5</sup>Such assumptions are also imposed on estimated theory models as one would obtain perfect “fit” for as many equations as there are “star” variables without them. The difference is that, theory models tend to use less restrictive assumptions on the movement of  $r^*$ , leading to more short-term notions of the rate, while statistical filters typically impose

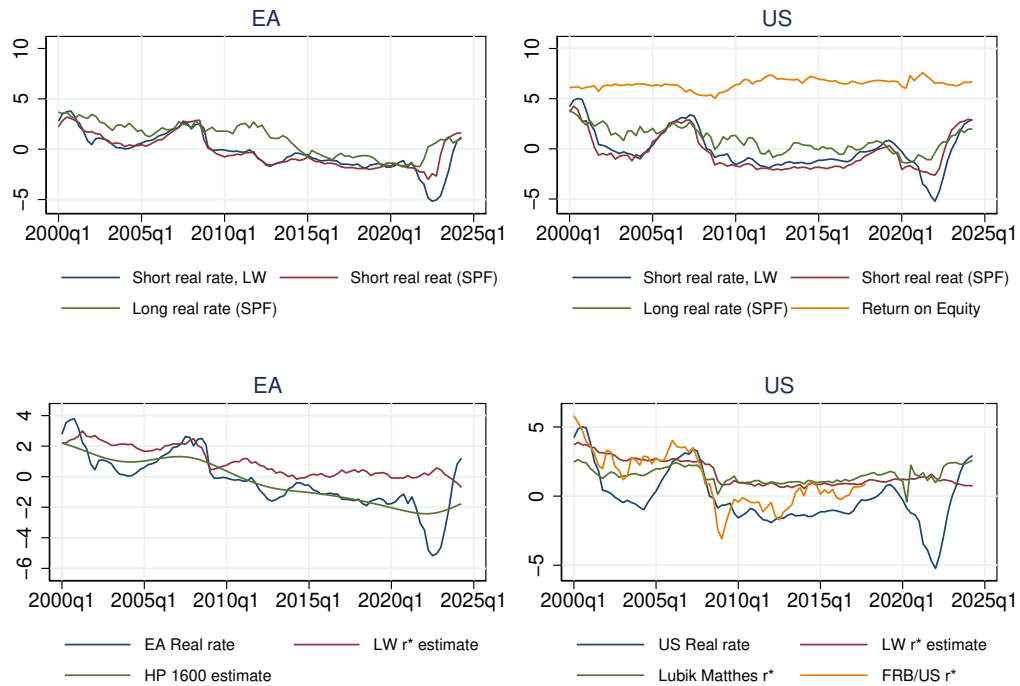


Figure 2: Real interest rates (top panels) and natural rate estimates (bottom panels) for the EA (left panels) and the US (right panels). Sources and definitions: The Laubach-Williams data set contains both real rates and  $r^*$  estimates (<https://www.newyorkfed.org/research/policy/rstar>). The SPF based real rates use either 1-year or 10-years ahead inflation expectations to short-term or long-term nominal interest rates, respectively, obtained from Macrobond. The real return on equity is from Gomme, Ravikumar, and Rupert (2011). The Lubik-Matthes Natural Rate of Interest can be found at [https://www.richmondfed.org/research/national\\_economy/natural\\_rate\\_interest](https://www.richmondfed.org/research/national_economy/natural_rate_interest); The FRB/US  $r^*$  estimate is available until 2017q4 from Roberts (2018).

the more wiggle room is given to it. But given that all safe real rates have been trending down over the last 40 years or so, applying pure statistical filtering to them generally delivers very similar  $r^*$  estimates. The same will generally be true for estimates from both theory models and semi-structural filters as long as a close connection between  $r^*$  and the observed real rate is maintained.

Filtering comes with two statistical problems. The first is the well known end point problem. In essence, filters (both empirical and theoretical) tend to place too much weight on the last few observations when estimating  $r^*$ . Hence, the uncertainty surrounding natural rate estimates from filters is the largest at the most critical point; the current period. The second issue is the so called pile-up problem; estimates of the degree of smoothness in filters for  $r^*$  are biased and give too high values.<sup>6</sup>

Other, more elaborate, statistical models can produce vastly different estimates. For example the Time-Varying Vector Autoregression (TV-VAR) model by Lubik and Matthes (2015) and Lubik and Matthes (2023) uses an empirical forecasting model to make longer-term projections of interest rates and inflation. Together these projections provide an estimate of  $r^*$  (Figure 2, lower right panel, green line).<sup>7</sup> While it does not make explicit assumptions on how  $r^*$  is allowed to behave, it implicitly does so by setting the forecast horizon and priors on the parameters. Also, the model has a large number of parameters that need to be estimated, which introduces additional noise in the  $r^*$  estimates.

## 3.2 Estimates from theory models

Theory models can be readily used for estimating the natural rate. The approach is as follows: take a specific theory model, for example a mainstream Dynamic Stochastic General Equilibrium (DSGE) model or term-structure

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more restrictive assumptions leading to longer run estimates.

<sup>6</sup>See e.g., Stock and Watson (1998). The pile-up problem arises when estimating the variability of an unobserved trend component within a statistical filter, and the likelihood function peaks at zero. In such cases, the filter 'prefers' a model where the trend remains constant over time, effectively piling up at zero variability. This can occur due to small sample sizes, low signal-to-noise ratios, or a flat or ill-behaved likelihood surface near zero, which can cause the optimizer to get stuck.

<sup>7</sup>This approach is in the spirit of the famous Hamilton (2018) filter.

models,<sup>8</sup> estimate its parameters, and use it filter out  $r^*$  from the data.

The main advantage of this approach is that it offers highly interpretable estimates that by construction correspond to a sensible  $r^*$  concept. For example, standard New Keynesian monetary models will produce estimates of the equilibrium real interest rate that would prevail in the absence of pricing frictions. This interest rate level is also consistent with stable inflation and output at its potential, as monetary policy and price rigidities only have transitory effects in such models.

However, there are also two major sources of uncertainty with this approach: parameter uncertainty and model validity.

Theory models are notoriously hard to estimate.<sup>9</sup> Technically, the likelihood surface is often flat or wobbly, with many local peaks and troughs. This means that starting values and priors – that is, restrictions or weights on the values parameter estimates can take based on the investigators beliefs – matter a lot for the final estimates. As a consequence, the outcomes can differ substantially depending on the preferences of the investigator.

More importantly, model validity is often questionable and rarely tested in a serious manner. The restrictions that standard models impose on the data tend to deliver very different dynamics from the unrestricted dynamics in the data. For example, the relationships between inflation and activity, as well as between activity and the real rate, as given by the the forward-looking IS and Phillips curves in the New Keynesian model, do not fit the data very well. But despite this, it is possible to force such relationship to hold in the estimated models by giving enough flexibility to estimates of unobserved “star” variables such as  $y_t^*$  and  $r_t^*$ .<sup>10</sup>

The problem is that theory models with too many unobserved “star” variables cannot be statistically tested and therefore belong more to the realm of

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<sup>8</sup>Examples of the former are Edge, Kiley, and Laforte (2008), Neri and Gerali (2019) and Del Negro et al. (2017), whereas examples of the latter are Joslin, Singleton, and Zhu (2011) and Ajevskis (2020).

<sup>9</sup>see Canova and Sala (2009) and a recent review by Fernandez-Villaverde and Guerron-Quintana (2021)

<sup>10</sup>For instance, consider a “empirical” Phillips curve relationship of the form:  $\pi_t = \alpha \cdot (y_t - y_t^*) + \varepsilon_t$ . If we give full flexibility to the *estimate* of  $y_t^*$  we can make this relationship fit perfectly for any  $\alpha$ ; the filter will simply impose  $y_t^* = y_t - \frac{1}{\alpha}\pi_t$  which leads to  $\varepsilon_t = 0$ . In general, we can arbitrarily place looser or tighter restrictions on any “star” variable to obtain as close a fit as we desire of the relationship where it enters. While investigators often report e.g., a t-statistic for the parameter alpha from such filtering, this does not have the usual meaning as a statistical test.

philosophy than science. How then should we assess natural rate estimates from such models? A good common-sense “smell-test” is that the more variability you have to give to the  $r^*$  estimate in order for the the IS-curve to “emerge”, the less should you trust it.

Estimates of  $r^*$  from mainstream DSGE models typically come with a lot of variability (see e.g., Barsky, Justiniano, and Melosi (2014) and the references above). For example, Brand, Lisack, and Mazelis (2024) report that estimates of  $r^*$  estimates from a euro area DSGE model move from above 2% in 2008, to less than -3% in 2016 and the back to 1% in recent periods. This can also be seen from the FRB/US model  $r^*$  estimate that displays similar (or even higher) variability than the observed real interest rates themselves (Figure 2, bottom right panel, orange line).

### 3.3 Semi-structural estimates

Semi-structural estimates lie between estimates from theoretical and empirical models. They typically incorporate some smoothing and assumptions found in statistical filters but also include information from economic relationships. Like the use of Structural Vector Autoregressions in empirical macroeconomics, they reflect a reluctance to rely too heavily on any given theory model.

The main advantage of incorporating economic relationships in a statistical filters is that it increases the connection between the resulting natural rate estimate and the concept it is intended to capture. But compared to fully specified economic models, the investigator is free to adjust the relationships to better fit the data. For example, the famous Laubach and Williams (2003) filter – LW filter for short – incorporates both an IS-curve and a Phillips curve, but uses empirical proxies for expectations and include more lags than corresponding theory versions.<sup>11</sup> Figure 2 plots the most recent LW filter estimates for the EA and US (bottom panels, red lines).

The drawback of semi-structural filters is that they suffer from all of the sources of variation listed above. These include measurement variation from the input series, poorly fitting or misspecified economic relationships, the

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<sup>11</sup>Recent estimates for several countries appear in Holston, Laubach, and Williams (2017) and Holston, Laubach, and Williams (2023). Similar filters appear in Wynne and Zhang (2018), Brand, Bielecki, and Penalver (2018), Carvalho (2023) and Davis et al. (2023). Hördahl and Tristani (2014) provide a semi-structural framework that incorporates both common macro relationships and take term-structure dynamics into account.

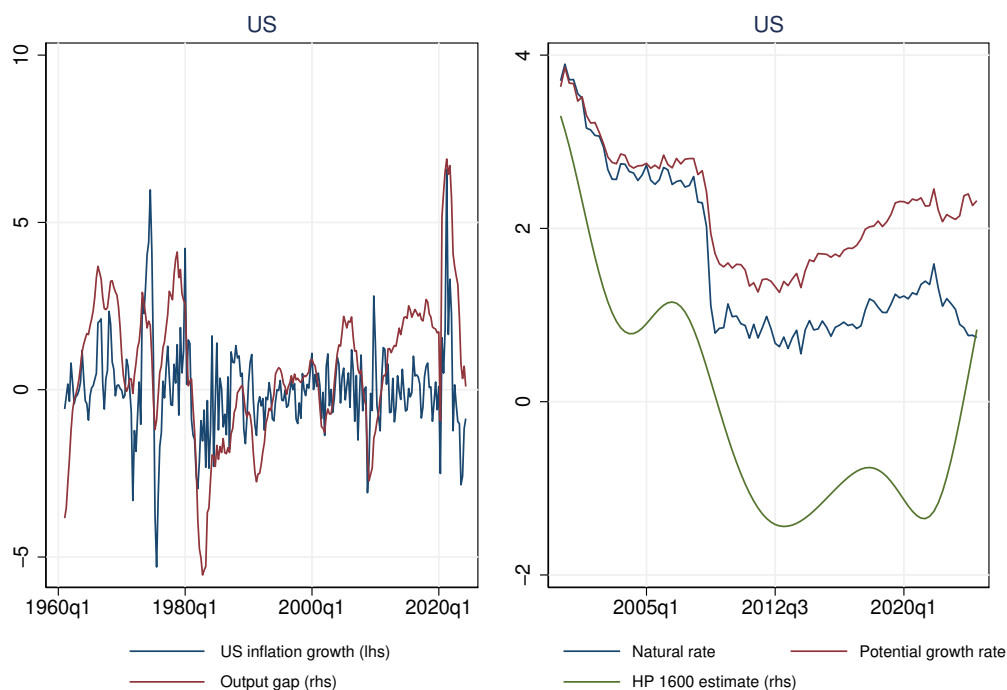


Figure 3: Understanding the LW filter. Left panel: US inflation changes and the LW output gap. Right panel: natural rate, potential output growth, and HP trend component ( $\lambda = 1600$ ) from real interest rate.

weights accorded to them, the room for maneuver that is given to the “star” variables, end-point problems and so on.

Many of these issues can be illustrated with the LW model on US data.<sup>12</sup> The filter places a large weight on the Phillips curve to ensure a close correlation between the output gap and (approximately) the change in inflation (Figure 3, left panel), ultimately ensuring that the obtained  $r^*$  measure is conceptually sound. But it turns out that even the change in US inflation shows substantial low-frequency variation which is therefore transmitted to the output gap. For instance, inflation accelerated in the 1960’s and 1970’s, ie its change was positive, and decelerated in the 1980’s and early 1990’s when inflation came back down (left-hand panel, Figure 1). The last decade

<sup>12</sup>The latest estimates are available at <https://www.newyorkfed.org/research/policy/rstar>.

has seen more positive changes in inflation although these changes have been rather small. To match these dynamics, the filter produces a very persistent output gap measure, for instance with a more than a decade long boom in the early period (some high-frequency spikes notwithstanding) followed by a similarly long recession afterward. These patterns also translate to more stable potential than actual output growth over time.

Stable potential output growth, in turn, has implications for the estimated natural rate. In the filter, the natural rate is proportional to potential output growth,  $g^*$ , plus some other unobserved factor,  $z$ , that captures other forces such as the role of shifting time preferences due to demographic concerns (technically,  $r_t^* = cg_t^* + z_t$ ). But with relatively stable  $g_t^*$  over time (Figure 3, right panel, red line) it is harder to match the long-run (smooth) dynamics of the actual real money market rate (note the gap between the red and green lines), which has declined over time. The solution is to allow a lot of flexibility to the  $z_t$  factor, technically unit-root dynamics, which then drops toward the end of the sample and therefore lowers the LW estimates of  $r_t^*$  currently (blue line).<sup>13</sup> However, this comes with a price; the unit-root dynamics blows up the statistical variance around the  $r^*$  estimate.

As in standard theory models, the problem is that the core relationships fit poorly. The only way to reconcile the model with the data is to give a lot of freedom to at least one “star” variable. In the LW filter, that variable is the hard to interpret  $z$  factor.

## 4 Drivers

Understanding the future trajectory of the natural rate is crucial for central banks to effectively adjust their monetary policy strategies. For instance, if  $r_t^*$  is expected to remain low for an extended period, central banks may need to adjust their strategies to secure sufficient policy space. Options include raising their inflation targets or, even more radically, targeting some other variable like the price level or nominal GDP (Williams (2016)). In contrast, a higher future  $r_t^*$  would allow central banks to maintain their current strategies more or less intact, unless the increase is primarily driven by irresponsible government spending.

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<sup>13</sup>Kiley (2020), Buncic (2021) and Morley, Tran, and Wong (2023) shows that misspecification in the LW model can lead to both over- and under-estimation of the  $z$  factor.

The set of potential long-term drivers of  $r_t^*$  is well understood. While theoretical details and modeling choices may differ, the fundamental principle is that anything that increases savings or reduces the appetite for investment will drive down  $r_t^*$ , and vice versa.

## 4.1 Theoretical channels

The literature has suggested many potential drivers for the decline in the natural rate over the past decades. These include:

- (i) Technological growth slowdown which lowers productivity and, consequently, also the demand for investment (e.g., Cesa-Bianchi, Harrison, and Sajedi (2023), Marx, Mojon, and Velde (2021), and Eggertsson, Mehrotra, and Robbins (2019)).
- (ii) Demographic factors (e.g., Carvalho, Ferrero, and Nechio (2016), Gagnon, Johannsen, and Lopez-Salido (2021), and Lisack, Sajedi, and Thwaites (2021)): Declining labor growth which increases the capital-to-labor ratio and, hence, reduces the marginal product of capital. Similarly longevity increases savings as people prepare for retirement.
- (iv) Wage or wealth inequality, as wealthier individuals tend to save more (e.g., Mian, Straub, and Sufi (2021) and Platzer and Peruffo (2022)).
- (v) Declining relative price of capital, as the cost of investment goods has decreased in relative terms, which means that the same output can be achieved with less investment (e.g., Sajedi and Thwaites (2016) and Rachel and Smith (2017)).
- (vi) A global savings glut that increase the demand for (primarily advanced country) safe assets stemming from many causes, such as global demographic forces, heightened uncertainty related to wars or fears of financial crises etc., risk of sudden stops and capital flight and so on (e.g., Bernanke (2005), Caballero, Farhi, and Gourinchas (2017a), Rachel and Smith (2017), DelNegro et al. (2019), and Ferreira and Shousha (2023)).

Some countervailing forces which are expected to drive up  $r^*$  have also been suggested. For instance:

- (i) Increasing dependency ratios, as the dependent population consume more than they save (e.g., Carvalho, Ferrero, and Nechio (2016)). This is likely to become increasingly important as e.g., the large baby-boomer generations retire.
- (ii) Fiscal deficits, as government borrowing competes for funds with the private sector and drive up  $r_t^*$  (e.g., Rachel and Summers (2019)).

In addition to these channels, a few papers argue that monetary policy could influence the natural rate (see e.g., discussion in Benigno et al. (2024)). For example, one way monetary policy could affect  $r_t^*$  is by raising inflation risk premia, although this may not be particularly desirable. Monetary policy could also effect  $r_t^*$  via hysteresis effects, that is endogenous interaction between nominal variables and technological progress.<sup>14</sup> However, a one-off permanent loss of output generated, say, by bad policies would only lead to a temporary drop in  $r_t^*$  (e.g., Jordà, Singh, and Taylor (2020)).<sup>15</sup> Additionally, gross irrationality, such as agents being unable to form unbiased inflation expectations, could lead to a self-fulfilling long-lasting effect of monetary policy on real rates which could bias  $r^*$  estimates. In sum, monetary policy is highly unlikely to have anything but temporary effects on  $r^*$ .

Looking ahead, the rise of artificial intelligence (AI), the green transition, and increasing defense spending which could potentially also reverse the technological slowdown and therefore drive up the natural rate. However, there is a substantial risk that, for example, AI will exacerbate both wealth and income inequality as workers are increasingly displaced.

## 4.2 Which drivers are important?

How potent are the different channels for explaining movements in the natural rate? There is substantial disagreement in the literature.

Virtually all of the channels mentioned above have at least one paper arguing it is the key channel. For example, Carvalho, Ferrero, and Nechio

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<sup>14</sup>See e.g., Elfsbacka Schmöller and Spitzer (2021) and references therein.

<sup>15</sup>For a more lasting effect, monetary policy would need to have a lasting effect on the potential growth rate of the economy, i.e. so called *super-hysteresis*. While the available empirical evidence on hysteresis provide at least some support for occasional endogenous and long-lasting output losses in a many economies, there is much less evidence for similar effects on the upside (so called reverse hysteresis) let alone for super-hysteresis (see Aikman et al. (2022)).

(2016), Rachel and Smith (2017), Eggertsson, Mehrotra, and Robbins (2019), and Gagnon, Johannsen, and Lopez-Salido (2021) find that the demographic factors have pushed down  $r^*$  by anything from one to four percentage points over the past decades. Rachel and Summers (2019) find huge positive effects of fiscal spending on  $r^*$ , up to 4 percentage points over the past decades, implying that drivers that lower the rate have had much larger negative effects than commonly thought. DelNegro et al. (2019) highlights the role of a declining convenience yield on safe assets in line with the global savings glut hypothesis. Marx, Mojon, and Velde (2021) find that productivity slowdown and increasing risk aversion account for the drop in risk free rates, but do not find a big role demographics and inequality. Kiley (2020a) studies natural rates across 13 economies and finds that global factors dominate. Cesa-Bianchi, Harrison, and Sajedi (2023) show that productivity slowdown and longevity drove down  $r^*$  by 3%. Platzer and Peruffo (2022) claim that inequality, demographics and the productivity slowdown account for a 2.2% decline in the US natural rates, while government spending has a small but positive effect. Using micro-data, Mian, Sufi, and Straub (2021) challenges the notion that demographics is behind the decline in the natural rate and suggest instead that rising inequality is the main culprit.

Why do these studies differ so widely as to the relevant drivers?

Most studies assess the relative importance of just a few channels within a theoretical model, leading to seemingly incompatible results across models that contain alternative channels. Such assessments do not provide a test of whether some specific channel *is* a driver of the natural rate; some channel will always be important in each model as it attempts to explain the declining real rates. This will be the case even if some other channel outside the model were to be the “true” driver.

A different and smaller strand of literature attempts to link the real interest rate to its theoretical drivers but often find limited and inconsistent correlations. This approach involves collecting a real interest rate measure (either a long-run or short-run one; risky or risk-free) and proxies for the different drivers suggested above. For example, life expectancy at retirement would be a good measure for longevity. Researchers can then check the significance and stability of the long-run correlation between the real rate and each of these variables, and whether the sign of the correlation aligns with theoretical predictions. Most of these papers find few significant correlations and even fewer that are in line with theoretical predictions (e.g., Eichengreen (2015), Hamilton et al. (2016), Borio et al. (2022), and Rogoff, Rossi, and

Schmelzing (2024)). In contrast, Lunsford and West (2019) find that real rates are significantly correlated with demographic factors over the long-run, but not with other measures.<sup>16</sup>

The lack of empirical long-run correlation between drivers and real interest rates may be due to measurement issues, persistent deviations, or misaligned expectations. The simplest explanation is that the real rates and/or their drivers are poorly measured. While this may be, this would also present a problem for the various theoretical studies that are largely predicated on similar input data. Another explanation, is that deviations between actual rates and the natural rate are very persistent. This may also be true but given the long sample used in several of the studies, it would imply that convergence between the real and natural rate is so slow that the latter lacks practical relevance for policymakers. For instance, declining rates for the last decades could just as likely be due to transitory movements in the real rate as to changes in  $r^*$ . A third possibility could be that agents lack the ability to form unbiased (inflation) expectations over the relevant horizons and that equilibrium between savings and investment internalizes such misaligned expectations.

Even if there were wide agreement on the relevant drivers of the natural rate, projecting it forward in time would still be fraught with difficulties. The reason is that such assessments rest on projections of the underlying drivers which come with varying degrees of uncertainty. For instance, demographic trends are relatively easy to project forward. But even so, birth rate projections often miss their mark with actual numbers declining much faster than expected. Increasing life expectancy is usually interpolated from past (almost linear) trends. But while humanity has managed to reduce the risk of early death, it has not yet managed to extend the maximum lifespan of humans (see Olshansky et al. (2024)). Hence, unless new innovations come along, this channel for driving down real rates has likely run its course. Most other explanations are much harder to project forward. The drivers of the global savings glut are poorly understood and hard to predict. The same goes for income inequality and technological trends.

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<sup>16</sup>Glick (2019) presents suggestive correlations based on shorter time series arguing, that several factors are important for explaining declining real rates. However, Borio et al. (2022) show that these supportive correlations are likely spurious.

## 5 Implications

The overwhelming message from the literature is that assessments of the value of the natural rate and its future development come with a high degree of uncertainty that partly reflect methodological choices and model assumptions. And even if some approaches or narratives may be more plausible or sound than others, the statistical uncertainty is also large. It does not seem likely that accuracy will increase in the future. Hence, policymakers will need to adapt to this uncertainty when gauging the appropriate stance for monetary policy.

There are at least a few, not mutually exclusive, ways in which monetary policy can respond to the high uncertainty surrounding the natural rate.

The first option is simply to ignore all estimates and instead respond to inflationary outcomes and, importantly, expectations thereof in the spirit of the Martin Luther King quote above. This is a fairly robust approach. For example, when inflation is expected to be above target and/or accelerating, the real interest rate is likely to be below  $r_t^*$ , and vice versa. This type of “the proof of the pudding” approach is arguably how monetary policy is often conducted in practice. However, there are at least two possible drawbacks to this approach. First, it implicitly places a lot of emphasis on the real interest rate channel for monetary policy. That is, it assumes closer connections between the real interest rate, activity, and inflation than are warranted from the data. A second drawback is that monetary policy runs the risk of being reactionary rather than proactive as inflation expectations largely move with current inflation (see Figure 1).

The second option is to pick a preferred family of natural rate estimates and take averages among these estimates. It makes less sense to average across different families as this is akin to comparing apples and oranges. For example, if one has a high degree of faith in mainstream NK models, estimates from them make the most sense. If one has a higher degree of faith in the real interest rate channel but not in specific theory models, semi-structural models may be the best alternative. If one has less faith in models and believes that any measure of  $r_t^*$  should not persistently stray far from actual real interest rates (however measured), then pure statistical filters would deliver the most accurate estimates. Of course, choosing the wrong family could lead to big policy mistakes, so it is essential to monitor performance.

The third option is to make the overall monetary policy framework robust to miss-measurement of the natural rate (Williams (2016)). This can be

done, for example, by raising the inflation target to minimize the chance of hitting the effective lower bound, or by changing the target from inflation to the price level or nominal output. One can also increase the degree of gradualism if more faith is placed on some long-term (sluggish) concept of  $r_t^*$  or decrease it if more faith is placed on fully structural estimates that tend to be more volatile and, hence, demand more rapid monetary policy responses. Nevertheless, changes to current frameworks can come with substantial costs in terms of increased inflation volatility and reduced credibility.

More broadly, the difficulties in obtaining reliable natural rate estimates may be a reflection of more fundamental shortcomings in conventional frameworks used for monetary policy analysis. For example, could it be that the IS-curve, which is key for the transmission of monetary policy, is a poor reflection of how people make consumption-savings decisions?<sup>17</sup> Or could other core elements of the conventional models be misspecified? Such questions are rarely asked by practitioners who tend to view the uncertainty in  $r_t^*$  estimates as intrinsic. Nevertheless, posing such questions might eventually lead to a better understanding of the transmission of monetary policy, and with it, more accurate estimates of the natural rate.

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<sup>17</sup>The literature that attempts to estimate the IS-curve in a more data-drive way, find almost no real interest rate effect. See e.g., Ascari, Magnusson, and Mavroeidis (2021), and references therein. Reis (2022) argues that it is the return on equity that matters for consumption and that monetary policy cannot do much to affect it. The author therefore downplays the role of  $r_t^*$  for monetary policy.

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