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Pandemic meltdown and economic recovery – A multi-phase dynamic model, empirics, and policy

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ABSTRACT

In this paper a two-phase pandemic-economic model is proposed, with phase-specific modeling and policy variables – as suggested by the chronicle of pandemic and economic policy developments over the period 2020–2021. In a first phase, the spread of a pandemic disease is the primary concern of authorities, that still also pay attention to economic activity. A dynamic model is introduced, embedding a two-way interaction between an extended epidemic Susceptible-Infected-Recovered (SIR) model and output gap dynamics. In the second phase, posterior to lockdowns when waves fade away, monetary policy becomes the control variable, pursuing again a joint objective, of supporting a non-inflationary recovery without causing significant fatalities. We then use a standard stylized model for the macroeconomy with simplified infection dynamics, that also enter the policy objective. The two phases are thus studied in a regime change model where the control and state variables as well as the objective function are allowed to change across phases. We solve the model over a finite horizon and derive the optimal lockdown or monetary policy path that jointly minimizes pandemic and economic losses. The two-phase finite horizon decision model is empirically calibrated and numerically solved by discretization and optimization methods. In the first phase, albeit with lasting adverse effects on output, lockdown-based control can be effective in reducing infection rates, but less so when starting from a negative output gap. In the second phase, accommodative monetary policy appears to be effective on both fronts, with even an eventual need for a return to tightening as output gap closes and inflation resumes.

1. Introduction

Macroeconomic studies involving the spread of an infectious disease hardly existed before the outbreak of the Corona virus pandemic meltdown in 2020. In particular, there was not much exploration until then of what such a massive meltdown would mean not only for the population's health or the health care system, as already considered by epidemiologists, but also in terms of economic losses. The latter concern

came to the forefront once strict lockdown measures had to be put in place to contain the spread of the virus. At that stage, a burgeoning literature promptly emerged in the macro-modeling field, considering economic output jointly with health outcomes in the health policy decision problem.² At the same time, the literature has remained sparse at best on how to possibly contain and recover from such a generalized pandemic meltdown, for instance by resorting to monetary or fiscal policy measures.

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¹ Views expressed by the author are strictly his own, and do not necessarily represent those of the ECB.

² Just to name a few significant contributions, see [Caulkins et al. \(2020\)](#), [Acemoglu et al. \(2020\)](#), and [Eichenbaum et al. \(2020\)](#), all building on variations of the otherwise standard SIR epidemiology model (see [Karmack and Mc Kendrick \(1927\)](#)), supplemented with an output or labor indicator.

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In line with the FSB (2021) typology (Table 1 on p.4 in a report to the G20 April meeting), four stages can explain how economic policies have been implemented in the course of this crisis: first a market shock resulting from a turn in expectations; second a lockdown period; third lockdown relaxation with uncertainties still; and fourth a recovery starting with support measures then lifted.³ Looking back, this periodization needs revisiting, since the third period was also characterized by subsequent waves and re-instatement of lockdown measures that postponed the recovery, as the main policy focus remained on limiting contagion as long as vaccination was not widely available.

To the extent that pandemic developments especially, but also policy steps varied across countries or areas, it is worth having a closer look at a specific country, in the case at hand we focus on Germany. The health policy can be best illustrated with the corresponding Oxford stringency indicator computed along the lines of Ritchie et al. (2020),⁴ see Fig. 1. Two indicators are reported, one solely referring to mobility restrictions (such as lockdowns), the other one also incorporating broader measures (such as test or mask requirements). The former should be the relevant one from an economic perspective, as the one with a direct impact on activity. Both evolve between 0 and 1, ie between no and total restrictions. In broad terms, the lockdown indicator remains above 0.5 until end-summer 2021, taking especially strong values during two periods for Spring 2020 and Autumn 20 over to Spring 2021. Mobility restrictions then are stable above 0.5 until Spring 2022, with afterwards a sharp decline to about 0.

Some rough periodization for health measures can be derived, split first into before and after widespread vaccinations (Summer 2021), with beforehand two major lockdown episodes, Spring 2020 and Autumn 2020-Spring 2021, L1 and L2 on Fig. 1, responding respectively to the first and second waves of infection. Before Summer 2021, there were only a limited number of vaccine doses available for priority groups, so that a second strong wave could develop. It was brought down by a new lockdown sequence, also increasingly resorting to generalized tracking and testing as well as vaccinations. Once vaccination became widely available (Post-Vax period on Fig. 1), and especially following the sanitary EU-wide certificate – actively used in Germany between Summer 2021 and Spring 2022 – there were no further lockdowns despite high incidence rates, as variants triggering further waves were less severe and pressure on hospitals contained. From Spring 2022 onwards, and possibly until a new variant emerges, the pandemic has not appeared any longer a relevant factor, with a continuous alleviation of containment measures taking place – see the last period shown on the chart.

Turning to the economy, the health policy pattern largely drove activity fluctuations over the pre-vaccination period. There was a sharp drop in GDP during the first lockdown, an equally strong recovery afterwards, however broken by the second lockdown, with then a delayed catch-up back to pre-crisis levels – still to be completed now. In parallel, the combination of supply chain disruptions worldwide with the steep recovery boosted supply side pressures, resulting after a while in a marked inflation surge - which pre-dated the more recent war-triggered energy price shock, beyond the time span covered by this study.

The very sizable real side impact of the pandemic, well beyond that experienced at the beginning of the Great Financial Crisis (GFC), called for strong and prompt policy reactions on the economic side, from both the fiscal and monetary sides. Some periodization can also be defined for support policies, taking inspiration from the above-mentioned four-stage FSB approach, and also aligning it with the introduced pandemic timeline. Focusing on monetary policy, after the market crash in March 2020, the ECB first put in place special schemes, for long-term funding

and asset purchases (the PEPP).⁵ Thereafter, in March and June 2020, resp. at the beginning of and subsequent to the first lockdown, the PEPP was expanded twice, then matching FSB stages two and three. Then, with the occurrence of the second wave (and lockdown) in Autumn 2020, the support lift-off was postponed, conditional on the pandemic end.⁶ It was not until Autumn 2021, after generalised vaccinations, that the end of the PEPP could be announced, and effectively implemented in March 2022, along with a reduction in the pre-pandemic asset purchase programme, thereby materialising an FSB stage four configuration.⁷

Monetary policy was a particular active area already when the pandemic crisis started. Detailed lists of steps taken by major central banks such as the US Fed, the ECB, the Bank of England or the Bank of Japan can be found in eg Cantú et al. (2021) or Penalver and Szczerbowski (2021), the latter also linking measures to specific transmission channels. Initially, especially in March 2020, the focus had chiefly been on stabilizing financial markets via targeted asset purchases and liquidity support, also avoiding exchange rate disturbances via swaps. Thereafter, facilitating banks' funding and supporting credit supply via lending operations became critical. The objective was to avoid an amplification of the lockdown-implied real side shocks, via major increases in financing costs or a subsequent credit crunch. Once out of the lockdown, such support policies also contribute to strengthen and anchor the ensuing recovery, which needs to be backed by sufficient credit and adequate financing conditions.

These monetary policy actions complemented fiscal and supervisory measures, mostly presented as transient steps.⁸ Governments funded furlough schemes to mitigate the unemployment impact and granted targeted subsidies, eg to SMEs in affected sectors. Supervisory authorities put restrictions on dividends, relaxed regulatory constraints on capital and liquidity ratios, released macroprudential buffers, and postponed the implementation of Basel III or the full impact of IFRS9. The two policy domains also met where state guarantees allowed borrowers to benefit from a payment moratorium, and banks to mitigate the impact of the crisis on their loan provisions. In the EU moreover, a specific fiscal envelope has also been agreed upon, for member states to use however at a later stage.

As a matter of fact, monetary policy changes in the euro area broadly coincided with entry and exit of lockdowns, in between major waves. Monetary policy is specifically relevant for Euro area economies that have faced country-specific multiple pandemic waves, with considerable fatalities and output losses. Monetary policy was to some extent restricted by being identical across countries, but could still complement country-level fiscal measures. These were better suited to tackle idiosyncrasies in the virus spread and health policy responses, as well as structural features - such as specialization in strongly exposed sectors e. g. tourism.⁹

⁵ For an overview of ECB measures, see Lane (2022)

⁶ See eg de Guindos (2021) on the links between pandemic and economic policies, in particular to support the recovery.

⁷ This monetary policy timeline is valid by definition for all euro area countries. On the pandemic policy timeline, cross-country differences existed, with eg Italy being particularly early and strongly affected, France instead was very much synchronized with Germany.

⁸ Our exercise focuses on monetary policy as the key support to aggregate demand, while fiscal policy would primarily help affected households or firms and reduce negative spillovers - as discussed in Blanchard (2020); see also Woodford (2020) for sector-specific policies. Fiscal support indeed played an important role cushioning income shocks in Germany, estimated by Hinterlang et al. (2021) at 4 percentage points of GDP. At the same time, fiscal moves have lower flexibility and frequency than monetary steps. In any case, their analysis would require very granular country-specific data and corresponding models.

⁹ Differentiated fiscal steps taken however led to public debt increases that eventually reinforced an already contrasted picture across countries, with differing financing conditions also for the private sector in spite of a common monetary policy. It may also have contributed to the resumption of inflation.

³ Borgioli et al. (2020) similarly consider four phases in their event study, but only covering March to September 2020 in which the 4th step is the announcement of the EU recovery package.

⁴ Data retrieved from: 'https://ourworldindata.org/coronavirus'

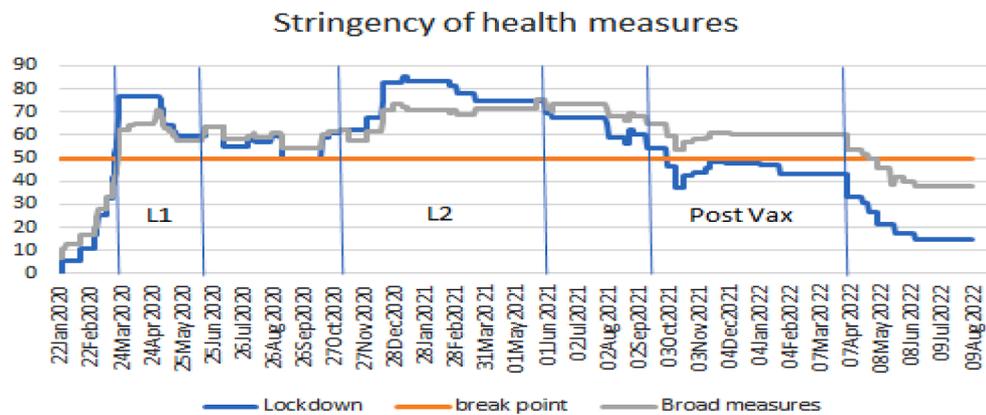


Fig. 1. Oxford two stringency indicators - for mobility restrictions and broader measures.

Against this background, stylized facts and periodization, we will jointly model pandemic and economic developments as well as respective control policies in a multi-phase set-up, with a view to developing an integrated and comprehensive framework. The objective is to characterize two differing phases, in terms of the pandemic meltdown and counteracting policies, each with its phase-specific model. We consider both output and infection outcomes, with a two-way interaction since higher activity pushes up infections while the latter bears on activity. Policy-makers resort primarily either to infection control (via lockdowns) or to economic policy measures (via monetary policy). In both cases, there is a need for multiple objectives, minimizing jointly pandemic and economic losses, by combining infection metrics with macroeconomic measures, accounting for the trade-off between health and activity objectives that results from the two-way interaction. When the focus is mostly on infection outcomes, the model is granular on the pandemic side and sketchy on the economic side. The reverse applies for periods where the attention is mostly on economic outcomes. The first phase design matches lockdown periods, for which we will consider initial conditions with positive or negative output gaps (like before and after the first wave, respectively). The second phase approach suits the period between the two lockdowns, but can also be applied to the period post-summer 2021, ie when, largely supply-driven, inflation pressures mounted, but the pandemic still remained a concern.

More specifically, the first phase can be analyzed in a version of the [Koehler et al. \(2020\)](#) granular epidemiological policy model, that we augment with a single macroeconomic variable (the output gap). At this point, even though fiscal and monetary policies can be in place, they are crisis responses aimed at avoiding a collapse. Subsequently, after pandemic waves moderate and lockdown measures are relaxed, non-health policies can become more active, and then help the recovery, instead of merely cushioning the adverse shocks. We thus introduce for the second phase a model including monetary policy in the spirit of [Faulwasser et al. \(2020\)](#), with a view to studying to what extent and with which speed monetary policy can come and support activity, albeit without re-triggering significant fatalities. As the spread of the disease is no longer the dominant factor and lockdowns not a control policy, we can simplify the epidemiological part, which is then captured only via an aggregate infection rate variable determined inter alia by the macroeconomy. Accounting jointly for all macroeconomic and epidemic features and variables would result in both regimes in an overly complex model, hardly tractable and overly detailed for the end purpose, not least given the documented contrast between lockdown episodes and other periods in terms of policy steps taken. In our simulations, the two

phases will also be connected to reflect the observed sequencing, as some final conditions of the first phase would pin down some initial conditions for the second phase.¹⁰

Formally, the two phases can therefore be studied in some sort of a regime-change model, where the state variables, the policy and objective functions can be allowed to change from the first regime to the second one. Such a more formally oriented two-phase finite horizon decision model can also be empirically calibrated and numerically solved by discretizing the underlying optimal control problems. The resulting large-scale optimization problems are formulated within the Mathematical Programming Language AMPL and solved by the optimization code Ipopt: see [Appendix A](#). How such formal model works is elaborated in [Semmler, Maurer, and Bonen \(2021\)](#), and briefly explained in [Appendix B](#).¹¹

The main findings of such a first phase optimal control problem are, first, that a continuously varying control (i.e. the lockdown intensity) fares better than a pattern with sporadically taken discrete-time decisions with lockdown intensity staying constant over some time intervals, conditional on fixed pandemic-related thresholds.¹² In addition, reviewing the impact of differing initial conditions, i.e. where countries originally stood in the cycle when the pandemic broke out, we show that regardless of whether the economy initially exhibits a positive output gap (boom period), or a negative one (recession period), output does not go back to potential, but persistently stays below it (i.e. the output gap remains negative) at least until about a year after lockdown implementation.¹³ By contrast, the initial state of the economy matters in case of a sequence of epidemic waves. Starting from a negative output gap after a first wave, with the already reduced social distancing in a recession, and therefore less interconnectedness, a further use of policy tools does not turn out to be as effective as in the first wave. Economic and health outcomes are therefore further adversely affected in this second pandemic episode. In both cases, the derived lockdown profile broadly matches what occurred in Germany.

Results for the second phase show that control can then be effective

¹⁰ To recall, a return to pre-crisis GDP levels, similar to a closure of the output gap, was not envisaged until well into 2022 even before the Ukraine conflict (see September projections, [ECB \(2021\)](#)).

¹¹ Global solutions are needed, that we find by using the AMPL method, wherein several variations, for example changing initial conditions, can also be explored. [Appendix B](#) shows moreover how the two phases of the model can theoretically be combined and jointly solved for a piecewise optimal policy path. In this study, we present separate illustrative simulations solving two phases that are only loosely connected.

¹² We thereby confirm [Koehler et al. \(2020\)](#) so-called benchmark results.

¹³ Simulations are conducted over a 60 week horizon, which appears sufficient as German lockdowns 1 and 2 lasted about 15 and 30 weeks respectively, and there was about 20 weeks between the two lockdowns.

on both pandemic and economic outcomes – but also broadly replicate recent stylized facts. Given the initial recessionary conditions after a lockdown, there is an immediate monetary policy reaction, with a one-off drop in interest rates. Following which, as those have reached their lower band, accommodative monetary policy to continue has to resort to QE. With first contained and then declining risk premia and funding costs, growth can restart. The output gap closes gradually, turning positive with then a return of inflation, that triggers a hike in interest rates around the end of the simulation period. Credit reflects these developments with a lag, initially declining until late before eventually rebounding to moderate at horizon end. The overall infection rate continuously decreases over the horizon, as evidence of an effective joint control, even in the absence of vaccination – which to recall represented the decisive step against the pandemic spread.

This two-phase model is defined from a heuristic perspective that allow us to study new economic features and policy challenges brought in by containment policies such as lockdowns and their lagged effects on the economy. Altogether, thanks to its phase and control specific features, this novel modeling set-up can address many issues raised by academics' and policy-makers' equally. It builds into an integrated dynamic process the spread of infectious disease along with macroeconomic developments. Further, the definition of policy objectives combining epidemic and economic ones, with possibly changing weights, provides a framework flexible enough to reflect at different points in time or phases in the pandemic episode what the main focus and policy tools are. The state equations also exhibit non-linearity, which can be specifically relevant at times of crisis. This set-up allows us in particular to discuss complementary policies for informing policy responses to the epidemic risk in its different phases, at the onset and once it is fading out.

The remainder of the paper is structured as follows. Section 2 summarizes the relations between our modeling set-up and the pandemic-economic literature, Section 3 presents the first phase detailed pandemic model and how the infection and mitigation measures interact with economic activity. Section 4 reports major results on the first phase, analyzing pandemic waves contingent on the stages of the business cycle, i.e. an expansion and a recession. Section 5 studies the second phase, with a less dominant spread of the pandemic disease, focusing on a dynamic macroeconomic model including monetary policy decisions, with the spread of disease captured only in one aggregate variable. Section 6 concludes the paper. Appendices provide information on the solution method as implemented in AMPL, describe the linkages between the two phases in more technical terms, and recall our data sources.

2. Related Macro Pandemic - Economic Literature

Our modeling strategy for the two phases borrows from the existing literature, both from epidemiologists and economists, introducing additional required elements in order to match the configuration of the two identified phases. In this section, we elaborate on the extent to which our models relate to previous work, what are the new elements we added, but also flag some limitations of our framework.

For the first phase, on the pandemic side, we employ an extended version of the standard Susceptibility-Infection-Recovery (SIR) epidemiologists' model (originally from Karmack and Mc Kendrick (1927)), namely the Koehler et al. (2020) extension of the eight-dimensional SIDARTHE model for Germany, as introduced by Giordano et al. (2020) for Italy. In this set-up, the spread of the infectious disease across specific sub-groups depends on the degree of lockdown and implied social distancing. The breakdown in sub-groups, in that case with

respect to symptomatic or testing status, has become crucial to model contagion and health policy decisions. The same applies with ICU capacities, as added by Koehler et al. (2020). Like theirs, our model is calibrated on German epidemiological data observed in 2020.¹⁴

We add to this model an economic loss measure, via the output gap, as an additional state variable also impacted by policy (lockdown) decisions.¹⁵ Cross-feedback effects arise between health and economic costs, first from the output gap to the spread of the disease, through rising interconnectedness among agents, in turn the disease affects economic output. While Koehler et al. (2020) also consider economic losses, they proxy them by a purely infection-based measure (the inverse of the contagion risk, itself a decreasing function of the lockdown intensity). Instead, we include an actual economic indicator with its own dynamics, moreover introducing non-linearities in the relation between the output gap and lockdown intensity.

The two types of dynamics, epidemics and economics, can then be jointly controlled by the lockdown intensity, as examined by e.g. Caulkins et al. (2020) or Koehler et al. (2020), also with a focus on when to impose or relax lockdowns, ie by steering the reproduction rate. Policy-makers have thereby two goals to consider when setting the path for policy steps, minimizing fatalities as well as minimizing economic losses¹⁶, resulting in a multi-objective function in our nine state-variable dynamic set-up. In a SIR set-up, lockdown-type of control have also been studied, with also double objectives, but purely health-related. Examples are Kantner and Koprucki (2020), who minimize infection peaks with a final target for susceptibles (in the herd immunity spirit) or Sereno et al. (2022) who jointly minimize eventual susceptibles and fatalities.

Extensions of the model are readily available, albeit not employed in the paper. While we eg take fixed weights of the dual goals of the decision model, one could attach varying weights to the two goals, which could then be taken piece-wise as given. We could then consider what would be best points on the Pareto frontier, see Kaya and Maurer (2013).¹⁷ The model can also easily allow for further variations, in initial conditions or in parameter calibrations, eg based on alternative health data, as the pandemic evolves or recedes or testing requirements are put in place over given periods.

At the same time, we do not take into account demographic structures and age-specific population groups (see e.g. Acemoglu et al. (2020) for such an approach), and how they are differently affected by the lockdown intensity, nor do we consider in what sectors or markets lockdowns may cause disproportionate effects on output or employment (see e.g. Blanchard (2020) and Woodford (2020)). We also do not model human risk perception and behavior to impact the meltdown as well as the recovery (See e.g. Eichenbaum et al. (2020) for a reaction of agents to a consumption tax, resulting in less interactions, or Toxvaerd (2021) who analyzed the effects of individuals' own decisions to go for self-imposed distancing).¹⁸

In the second phase, further feedback from and to monetary and

¹⁴ The model specification and parametrization is not applicable to the period after Spring 2022 anyhow, due to the widespread vaccination that cannot be accounted for in this model.

¹⁵ For the relevance of the output gap in dynamic macro models, see Faulwasser et al. (2020).

¹⁶ Pandemic-related control policies also focus on long-run effect on output, as illustrated by President Lagarde's Twitter message during the G20 Ministers of Finance and Governors' meeting: "The measures that have been taken in response to the pandemic are working but need to be continued to avoid long-term scarring to the economy", October 14, 2020.

¹⁷ An evaluation of the variation of the weights in such an objective function has been undertaken in Faulwasser et al. (2020), finding that a greater weight for an objective will help to increase the adjustment speed toward its target.

¹⁸ While our model is not an Agent-Based Model in the sense that we do not model interactions at the individual level, it can still be interpreted as an aggregated ABM, similar to the SIR in fact.

financial markets need to be considered. All may be involved in second round effects after the pandemic-caused slowdown has faded but left scars, with then a specific role for non-health policy steps, in particular monetary policy. Considering optimal monetary policy against the pandemic background is a new feature with respect to previously conducted work on the interaction between epidemics and economics.

Recent research has also considered the spread of infectious disease jointly with some macroeconomic indicators, using standard macroeconomic models or enhanced versions of pandemic ones, but to derive optimal health policies – as in Eichenbaum et al. (2020), within a DGSE framework, focusing on consumption when assessing the impact of infection on activity. In turn, the ECB BASIR, Angelini et al. (2020), uses a semi-structural model enriched with lockdown impacts, on top of investment consumption and labor feedbacks – also accounting for quarantined population. Both considered a SIR model, but Eichenbaum et al. (2020) used a combined loss function, hence pointing to the trade-off we are investigating. This issue has also been addressed in Alvarez et al. (2020), again with a SIR model and a planner controlling fatalities while minimizing output losses. The latter depend on fatalities and infection, since output obtains as an aggregation of individuals' own production. In this work, an early strong lockdown effectively controls infection within a quarter. They also compare a generalized lockdown to one only affecting those tested positive, with the clear conclusion that selective lockdowns fare better. Piguillem and Shi (2020) also seek the best policy, again using a simple production function where output is simply proportional to healthy individuals, but like Koehler et al. (2020) account for binding hospital capacities as a cause of fatalities. Using a loss function combining output with a non-zero cost of deaths rather than the number of fatalities itself, they find that in the absence of testing lockdown is optimal.

While some of these papers took a loss function approach to derive optimal policies, they then did not address the issue of non-health policies in a pandemic context, which is what would be needed for periods outside lockdowns as previously identified. Our original model for the second phase therefore focuses more on macroeconomic and less on pandemic variables, with a view to analyzing the Central Banks' (CB) decision-making process, including rules to set the interest rate and undertake QE-type policies. It is based on a dynamic decision problem that should be guided by the CBs' policy targets and constrained by dynamic variables, adding the spread of infectious disease to more standard factors. Such a switch in the model type will help us evaluate the impact of CB monetary, financial, credit and inflation control policies, and how the exit from the pandemic-driven recession can play out, once adequate conditions for a recovery are in place. We will explore these dynamic effects by presuming a modified CB's mandate, as has been discussed in academic work accounting for unconventional policies.¹⁹ This CB behavior will be embedded in a basic Non-Linear Quadratic (NLQ) macroeconomic model, where (simplified, aggregate) infection dynamics directly enter the CB objective and also affect its other arguments.²⁰

Our work thus like others highlights the important policy trade-off between limiting the infection and preserving growth, but documents implications for economic policies, moreover in a refined dynamic control framework. In addition, to our knowledge, a regime switching behavior where objective functions as well as state variables can change was not considered beforehand either. Based on this novel set-up, we can study the effects of such an infection-augmented set of CB's goals,

thereby helping address specific serious risks and challenges arising from the spread of infectious diseases. This approach appears particularly suitable over specific periods though.

Despite a rather rich framework, the proposed model and simulation exercise has further limitations. Given the focus put on monetary policy, there is no modelling of fiscal measures. The latter could still be interpreted as a maintained exogenous upward shock to GDP, which may simply shift the output gap up with respect to our experiments. On monetary policy, while QE is accounted for, via its impact on risk premia during recessions and policy interest rates are at their lower band, there is no explicit modelling of the decision to implement QE. Making the response of risk premia conditional on an additional CB decision would however further complicate the model without clear benefits. We also do not explore how time-varying or state-dependent weights in the objective function would affect decisions. More generally, to the extent that a number of key parameters we employ are calibrated, results obtained are not meant to provide direct policy advice, but to illustrate how varied economic and pandemic situations can in broad terms respond to specific policies.

3. Pandemic and Lockdown Policy – The first Phase Model

A core part of our subsequent first phase model is stylizing the spread of an infectious disease, known as the Susceptible-Infected-Recovered (SIR) model in epidemiological studies (see Karmack and Mc Kendrick (1927)). We apply here, however, an extended model first employed for Italy by Giordano et al. (2020), called SIDARTHE, with the introduction of the key distinction between symptomatic and asymptomatic individuals. This model was further enriched by Koehler et al. (2020) to include health policy variables with a corresponding objective function, also facing Intensive Care Unit (ICU) constraints. They also used numerical control methods to obtain solutions of an eight-dimension model over a finite horizon. We include in our model additional pandemic-macroeconomic cross-feedback effects, resulting in a model with nine state variables, the ninth one being the output gap. We further expand the framework by introducing a dual objective function that help answer essential questions on relevant health and economic policy targets.

3.1. Description of the extended Pandemic model

In the original SIR model, the Susceptibles, measured as fraction of the total population, decline proportionally to the product of Susceptibles and Infected, a process representing the probability of contagion via one group encountering the other. Symmetrically, the Infected population increases proportionally to the Susceptibles minus a fraction of Infected (due to Recovery or fatality). The extended version, SIDARTHE, as defined in Giordano et al. (2020) and Koehler et al. (2020), breaks down the infected population into four categories $I, D, A,$ and $R,$ according to their testing and symptom status (see Fig. 2). It also introduces a Threatened category T when symptoms call for ICU treatment. Those amendments are critical to the calibration of health policy decisions - namely lockdowns that mitigate contagions. H stands for Healed, the equivalent of the Recovered population in the SIR model. E for Extinct represents fatal outcomes. These sub-populations evolve over time, with specific parameters governing the transitions between the various states representing the epidemic status. We further add a macroeconomic dimension to this detailed epidemiological model by introducing a dynamic equation for the output gap as our activity indicator $y.$

Our own complete model can then be summarized in a system of nine differential equations, as follows:

$$\dot{S} = -S(\alpha I + \beta D + \gamma A + \beta R), \quad (1)$$

$$\dot{I} = S(\alpha I + \beta D + \gamma A + \beta R) - (\epsilon + \zeta + \lambda)I + \omega(y + q), \quad (2)$$

¹⁹ See for example, in a generic Linear Quadratic model, de Groot et al. (2021) with QE terms in the US Fed policy rule, and De Grauwe (2019) concerning climate risk.

²⁰ The infection is still present albeit weaker in this phase, and could derail an emerging recovery. The ECB for instance envisaged the timing of future monetary policy steps also in relation to the state of the pandemics, e.g. De Guindos (2021).

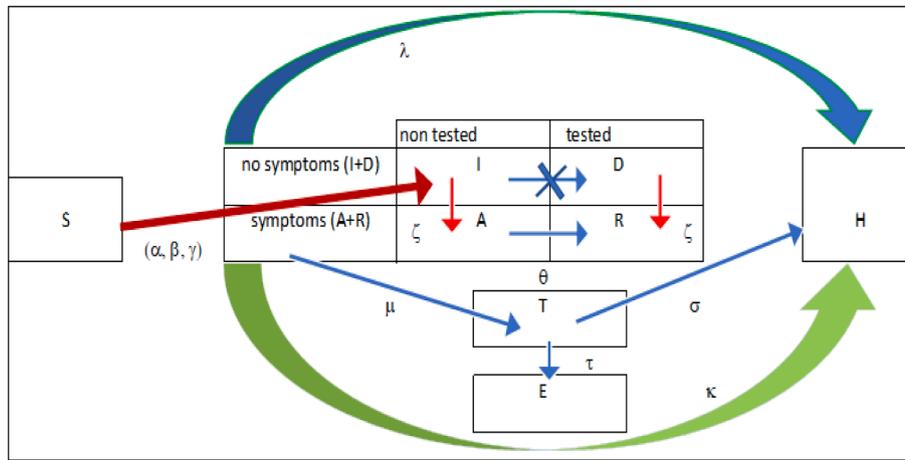


Fig. 2. Flow chart of the Pandemic Model.

$$\dot{D} = \epsilon I - (\zeta + \lambda)D, \tag{3}$$

$$\dot{A} = \zeta I - (\theta + \mu + \kappa)A, \tag{4}$$

$$\dot{R} = \zeta D + \theta A - (\mu + \kappa)R, \tag{5}$$

$$\dot{T} = \mu A + \mu R - (\sigma(T) + \tau(T))T, \tag{6}$$

$$\dot{H} = \lambda I + \lambda D + \kappa A + \kappa R + \sigma(T)T, \tag{7}$$

$$\dot{E} = \tau(T)T, \tag{8}$$

$$\dot{y} = -ay - b(\exp(-c(u(t) - d)^2) - 1) - eI + f. \tag{9}$$

In differential eqs. (1) - (8), the population is partitioned in eight possible states: $S = Susceptible$, $I = Infected$ (asymptomatic, undetected), $D = Diagnosed$ (asymptomatic, detected), $A = Ailing$ (symptomatic, undetected), $R = Recognized$ (symptomatic, detected), $T = Threatened$, (symptomatic with life-threatening symptoms, detected), $H = Healed$ (immune after prior infection, detected or undetected), $E = Extinct$ (dead, detected). Capital letters all stand for the fraction of the whole population in the respective state. At the equilibrium of the epidemiological system, the eight shares for the corresponding states add up to 1²¹:

$$S + I + D + A + R + T + H + E = 1$$

These first eight differential equations reflect contagion by interconnection, and then transitions between various states e.g I into D, A and H ; D into R and H ; A into R, T and H ; R into T and H ; T into H and E , the latter two as final absorbing states. Fig. 1 provides a schematic visualization, with arrows materializing the transitions between states as they occur over time. The starting point is the susceptible fraction of population (S) from which the infected part is originating. From state I , infected either heal directly (H) or develop symptoms (A), in which case they can be recognized after testing (R).²² Symptoms can aggravate and affected individuals then join the threatened population (T). They can heal (H) or eventually result in fatalities (E).

Koehler et al. (2020) enrich Giordano et al. (2020) in a number of ways. First they introduce a measure of lockdown intensity to assess the impact of differing policies. The measure denoted u acts as a control

²¹ Given this constraint, we need only to compute the ratios corresponding to the first seven variables.

²² As in Koehler et al. (2020), testing is assumed to be performed only on symptomatic individuals, as is the case in the absence of compulsory testing.

variable to be modified over time by the policy-maker. It moves between 0 and 1, 1 meaning a complete lockdown and 0 no lockdown at all, hence the resulting constraint: $u(t) \in [0, 1]$. They moreover assume a direct influence of $u(t)$ on infection rates $\alpha(t)$ and $\gamma(t)$, the contagion parameters in eqs. (1) and (2), according to the following eqs. (10) and (11):

$$\alpha(t) = \alpha_{max} + (\alpha_{min} - \alpha_{max})u(t), \tag{10}$$

$$\gamma(t) = \gamma_{max} + (\gamma_{min} - \gamma_{max})u(t). \tag{11}$$

The infection rates (after contacts with undetected infected populations I and A) are now state-dependent, impacted by the decision variable $u(t)$, even though they stay within pre-defined limits that the lockdown intensity cannot influence. In the SIR-based control literature, intensity is also used but affects directly only the overall reproduction rate.

Koehler et al. (2020) also account for a potential overwhelming of hospitals, distinguishing within T ($T = Threatened$, i.e. symptomatic with life-threatening symptoms, detected) those patients requiring treatment in Intensive Care Units (ICU). Both u and ICU then affect transitions directly, so that policy decisions will eventually also reflect structural constraints that may aggravate fatalities. T_2 represents the number of people in need of intensive care and T_1 are the remaining threatened COVID-19 cases, both with specific recovery and fatality rates. Then $T = T_1 + T_2$, with the two sub-populations driven by the respective differential eqs. (12) and (13), that are similar to eq. (6):

$$\dot{T}_1 = \mu_1 A + \mu_1 R - (\sigma_1(T) + \tau_1(T))T, \tag{12}$$

$$\dot{T}_2 = \mu_2 A + \mu_2 R - (\sigma_2(T) + \tau_2(T))T, \tag{13}$$

in which $\mu_1 + \mu_2 = \mu$.²³

Given that T at any point in time is then split into T_1 and T_2 , $\tau(T)$ and $\sigma(T)$, as fraction of the *Threatened*, can accordingly be adjusted, also accounting for two ICU configurations, i.e. depending on whether the latter are fully used or not. Koehler et al. (2020) therefore additionally propose the following two equations:

$$\tau(T)T = (\mu_1/\mu)\tau_1 T + \max\{(\mu_2/\mu)\tau_2 T, \tau_2 T_{ICU} + \tau_{crit}((\mu_2/\mu)T - T_{ICU})\}, \tag{14}$$

$$\sigma(T)T = (\mu_1/\mu)\sigma_1 T + \sigma_2 \min\{(\mu_2/\mu)T, T_{ICU}\}. \tag{15}$$

Eqs. (14) and (15) both comprise two terms reflecting the T_1 and T_2 sub-groups. The term for the latter sub-group has two components, with worst outcomes if T_2 exceeds ICU capacities. In such a case, for those

²³ Assuming as a proxy that $T_2 = (\mu_2/\mu) \cdot T$ yields $T_1 = (\mu_1/\mu_2) \dot{T}_2$.

Table 1
List of pandemic parameters and variables for the SIDARTHE model.

	Description	Values for the German Outbreak
α	Range of rates at which susceptible individuals are infected by state I - lockdown-dependent	$\alpha_{min} = 0.0422$ $\alpha_{max} = 0.3614$
β	Rate at which susceptible individuals are infected by states D or R - identical for all detected	$\beta = 0.0084$
γ	Range of rates at which susceptible individuals are infected by state A - identical to I, as both states undetected	$\gamma_{min} = 0.0422$ $\gamma_{max} = 0.3614$
ϵ	Testing rate, i.e., rate at which asymptomatic infected individuals (state I) go from undetected to detected - set at 0 if no systematic testing policy	$\epsilon = 0.00$
θ	Testing rate, i.e., rate at which (symptomatic) infected individuals go from undetected to detected	$\theta = 0.079$
ζ	Rate of asymptomatic (detected or undetected) infected individuals developing symptoms	$\zeta = 0.1981$
μ	Rate at which (symptomatic, A or R) develop life-threatening symptoms - specific if ICU needed and not	$\mu_1 = 0.0080$ $\mu_2 = 0.0050$
λ	Recovery rates for (asymptomatic infected) I and D individuals	$\lambda = 0.0596$
κ	Recovery rates for (symptomatic infected) A and R individuals	$\kappa = 0.0563$
$\rho(T)$	Recovery rates for T individuals - specific if ICU needed and not	$\rho_1 = 0.0080$ $\rho_2 = 0.0050$
$\tau(T)$	Mortality rate for T individuals - specific if ICU needed and not	$\tau_1 = 0.0080$ $\tau_2 = 0.0050$
u	Different lockdown policies, monotonically increasing from no counter-measures to full lockdown	$0 < u < 1$
S	Rate of Susceptible people	-
I	Rate of Infected, asymptomatic and undetected	$I_0 = 20581/N_{total}$
D	Rate of Diagnosed, infected, asymptomatic and detected	-
A	Rate of Ailing, infected, symptomatic and undetected	$A_0 = 8041/N_{total}$
R	Rate of Recognized, infected, symptomatic and detected	-
T	Rate of Threatened, infected and detected with life-threatening symptoms	-
H	Rate of Healed	-
E	Rate of Extinct, threatened cases that evolve to death	-

that cannot be treated in ICU, τ_{crit} is higher than the standard τ and σ is set at 0, i.e. no recovery.

While our model takes on board Koehler et al. (2020) granular pandemic modeling,²⁴ we also significantly enrich it to model macroeconomic developments in relation to epidemiological ones. Eqs. (2) and (9) are key in this respect, defining respectively the impact of activity on infection and the reverse - the previously referred to two-way interaction. Pandemic parameters and variables are shown in Table 1 while macroeconomic parameters are listed in Table 2.

The additional dynamic equ. (9) defines the path for the output gap, and also captures the macroeconomic effects of the spread of the infectious disease and lockdown. It is inspired by the Exponential Smooth Transition AutoRegressive model of Granger and Terasvirta (1993), ESTAR.²⁵ This specification is particularly suitable for establishing a link between bounded variables, plus allows us to account for some non-linearity in the relation between lockdown intensity and output. Beyond a given critical threshold, we thereby can generate decreasing negative effects on output when closer to a full lockdown. This is line with IMF

²⁴ Koehler et al. (2020) introduce other elements, which we do not use here, such as the cost and impact of testing, or the computation of herd immunity ratios and the long-run stable outcome for susceptibles.

²⁵ see also Terasvirta (1994) for a documented generic presentation of smooth transition auto regressive models, as well as the survey on their exponential variant, ESTAR, by van Kempen (2014).

Table 2
List of economic parameters and variables for the SIDARTHE model.

	Description	Values for the German Outbreak
	Infection rate response to output gap fluctuations	$w = 0.02$
Intercept		$r = 0.3$
a	Output gap autoregressive coefficient	$a = 0.1$
b	Output gap response to lockdown and infection rate - linear component	$b = 0.03$
c	Output gap response to lockdown - exponential function slope	$c = 20$
d	Output gap response to lockdown - exponential function maximum	$d = 0.8$
e	Output gap response to infection rate - linear function	$e = 0.2$
f	ESTAR shift parameter	$f = -0.03$

(2020) which finds that lockdowns have decreasing marginal economic costs. Deb et al. (2022) also find that imposing and relaxing lockdowns have asymmetric effects. The ESTAR specification implies that for low values of lockdown intensity, output costs increase gradually but then accelerate when closer to the critical threshold. The latter we assumed can be set at 0.8 (peak value reached by the Oxford index during both German lockdowns). Finally, the slope parameter – measuring the average output reaction around the threshold – has been calibrated equal to 20 based only on German data observed during lockdown episodes. The intercept and autoregressive parameters ensure a gradual closure of the gap in the absence of pandemic or other shocks.²⁶

Turning to equ. (2), we modified the original Koehler et al. (2020) version, so that the output measure now enters the infection dynamics explicitly, with a coefficient ω , instead of only implicitly via the change induced by the lockdown intensity in contagion parameters. Unless the output gap reaches extreme negative values, as captured by the ρ coefficient, economic activity thereby always contributes to aggravating the infection spread.²⁷

Koehler et al. (2020) moreover suggest that the (health) policy-maker focuses on the expected fatalities, including those to materialize beyond the simulation horizon. Assuming ICU capacities are not exceeded beyond that horizon (thereby simplifying the expressions in eqs. (12) and (13)), the predicted number of eventual fatalities comprises two terms, the Extinct population plus that part of the current and future Threatened unavoidably resulting in additional fatalities, that the planner also needs to account for, even though they occur beyond the planning horizon:

$$F = E + \frac{\tau}{\tau + \sigma} \left(\frac{\mu}{\mu + \kappa} \left(\frac{\zeta}{\zeta + \lambda} (I + D) + A + R \right) + T \right). \tag{16}$$

In Koehler et al. (2020), equ. (16) is the objective to be minimized, which is a function of τ and σ , thereby requiring an explicit modeling of ICU, T_1 and T_2 populations. The cost of lockdowns is only indirectly defined through $1/\alpha(t)$ from equ. (10). Fatalities are the only argument in the objective function, and the (proxied) economic loss is used only as a reference to ensure that the derived optimal control implies a lesser cost than, discrete, ad hoc policies. In their case, a total lockdown with $u = 1$ leads by construction to the highest economic cost, which is a

²⁶ Both parameter values play a crucial role in shaping reported simulation results.

²⁷ Limit cases correspond to $\omega = 0$ where the model largely behaves like a SIR one, and to $S = 0$ which by definition is not a relevant case for this model, as a null value corresponds to a state where the pandemic cannot restart given the definition of susceptibility.

simple function of the minimum contagion rate. In our extension, we also use equ. (16) on fatalities to measure losses from a health policy perspective, but supplement it with a direct measure for economic losses, the output gap y , itself affected in a non-linear fashion by health policies from equ. (9). This will yield a dual policy objective for the policy-maker.

3.2. A dual objective function

We can now specify further the policy challenge with a dual objective in mind. Taking into account the policymakers' attempt to come as close as possible, given the multi-objective decision problem, to the health goal (fatalities minimized) as well as to the output goal (a limited output gap y) – the complete optimal control model for policy decision-making can be described as follows:

$$\min_{u(t)} F(N \cdot T_s) + \nu y(N \cdot T_s)^2 \tag{17}$$

$$u(k \cdot T_s) \in [0, 1], \quad k = 0, 1, \dots, N - 1 \tag{18}$$

$$\sum_{k=0}^{N-1} c_{policy}(u(k \cdot T_s)) \leq \sum_{k=0}^{N-1} c_{policy}(u^b(k \cdot T_s)) \tag{19}$$

T_s is a one-week reference period, over which u remains constant, while the index k refers to the number of weeks. The objective function $F(N \cdot T_s) + \nu y^2$ in equ. (17) considers a horizon of N weeks, with a dual goal. The first part of the objective $F(N \cdot T_s)$ denotes the eventual value of the state of the system simulated through dynamic eqs. (1) - (9), implying a control of the death rate. The second part, νy^2 , is a quadratic penalty on the output gap, with a weight ν , that is also assumed to be set by the policy-maker and equal to 1 in our simulations.²⁸ Minimizing functional (17) implies the human fatalities and the economic losses should jointly be minimized when the intensity of the lockdown, represented by the control $u(t)$, varies.²⁹

The decision variable u varies between specific limits, whereby $u(t) = 1$ represents the full lockdown policy (partly approached in Germany both in mid-April 2020 and early 2021) and $u(t) = 0$ no social distancing or other lockdown measures. These limit values correspond to a policy

fatalities, but this would be at the cost of an output gap y moving further down into negative territory.

Equ. (19) also relates to the optimal path for $u(t)$. It introduces a constraint, whereby the optimal variation in lockdown intensity has to imply an economic cost (c_{policy}) lower than that for a policy with arbitrary discrete steps in lockdowns. The latter represents a baseline case, $u^b(k \cdot T_s)$, against which the optimal lockdown intensity can be compared over the k steps. This issue is investigated by Koehler et al. (2020), in which the baseline policy involves discrete decisions to gradually loosen the lockdown, only if ICUs are partly occupied and the infection has slowed down at each point in time³⁰. The discrete baseline policy can result in successive waves due to its stop-and-go nature (see Figure 6 of Koehler et al. (2020) in case a lockdown is lifted too quickly), whereas the optimal policy path allows the policy-maker to limit the occurrence of such, more costly, outcomes.³¹

4. Pandemic with Multiple Lockdowns – Optimization results

Although the pandemic made itself felt already in January 2020 in China, spilling over Asian countries and then to Europe (Italy, Germany, Spain and France), it did not immediately trigger widespread fears. As a result, the pandemic did not initially lead to either behaviorally or legally imposed social distancing or interpersonal reduced connections. At that point in time, early in the first quarter of 2020, European economies were therefore still on a reasonably dynamic growth path – not yet affected by any containment measures, even though external trade had already been seriously hit. With the subsequent spread of the pandemic disease, the dynamics of the variables of the SIDARTHE system (1) - (8) were set in motion, accompanied by the lockdown measures gradually put in place around March-April 2020, all creating further downward pressure on output, including from the domestic side.

The best policy will be sought in order to achieve the two goals, pandemics and economics together. Following Koehler et al. (2020), the time horizon is finite, of roughly 2 years, while a vaccine is assumed to be still under development and other policies need to be considered. The state vector for the optimal control problem (17) - (19) is (X, y) where $X = (S, I, D, A, R, T, H, E)$ and y is the output gap. Initial conditions for that vector are recalled in (20), reflecting the pandemic situation on April

$$X_0 = (82636256, 20581, 0, 8041, 41931, 11469, 276911, 4810)/N_{total}, \quad y_0 = 0.20 \text{ or } y_0 = -0.2 \tag{20}$$

focused on maximizing health and output objectives in isolation, respectively. In both cases, focusing solely on one objective is heavily detrimental to the other one. Instead, a two-goal optimal policy can be achieved by using $0 < u(t) < 1$, given the trade-off between the two competing objectives. For instance with $u(t)$ rising, the interconnectedness among agents decreases, and so will infection rates and

21st, 2020 – as in Koehler et al. (2020):

The last two numbers in the vector correspond to the alternative calibrations of the initial output gap. There is as often during crises an elevated degree of uncertainty surrounding the value of the output gap. Depending on the extent to which potential output then already reflected the output contraction experienced in March, the output gap could then take a wide range of positive or negative values (see e.g. Bodnár et al., 2020 for illustration). To cover extreme and symmetric

²⁸ The quadratic specification is consistent with a generic objective of output stabilization – in a pandemic episode, lockdown intensity could thereby be stronger if the economy would be in a booming state than if operating already well below potential. To check robustness we also ran slightly alternative specifications (eg considering average fatalities instead of final ones) with little differences in the outcome, the results of which can be obtained from the authors upon request.

²⁹ $u(t)$ impacts S and I , and then A and via A also E , ie the fatalities to be minimized. In comparison with the conventional predator–prey model, which in the basic SIR means that the Susceptibles are reduced (the prey) by increasing the infection (the predator), we have in our model several types of direct and indirect prey effects: first, directly on the susceptible population, S , through I , then indirectly on D, H, A, R, T and E and finally on the economic variable, y .

³⁰ For further details, see Koehler et al. (2020).

³¹ In our set-up the actual economic cost is measured by νy^2 . The inequality in equ. (19) is checked in our AMPL algorithm when we compute the lockdown intensity on a continuous basis. The running program shows us whether equ. (19) holds, which is indeed the case.

cases in the simulations, we set the initial output gap alternatively at $y = 0.2$ (a boom situation) or the opposite (a recession situation)³².

Those initial conditions can be modified to update and explore scenarios, as new information comes in and the epidemic variables as well as the output gap evolve over time, so that the lockdown intensity has to be re-assessed on a quasi-real time basis. The following results show indeed that the interactions between the spread of COVID-19 and the output gap are strongly influenced by initial conditions.

4.1. Dynamics with initial boom and full feedback effects

Our simulations embed a full set of feedbacks, both within epidemic variables and across the output gap and infection rates. We first show results for initial conditions $y(0) = 0.2$. The policy-maker sets the output gap weight in the objective to $v = 1$. State variables are indexed with 1 on reported Figures.

Fig. 3, on the right side, shows that given the initially high levels of infected population and infection rate, see $-S(\dots)$ in equ. (1) and also equ. (2), the lockdown intensity u needs to increase promptly. Shortly after period 12, it then reaches an upper bound at about 0.7 – a high level still below a full lockdown ie $u = 1$. Lockdown relaxation happens only after about 15 more weeks, which compares with what was experienced with the first German lockdown. Restrictions can vanish completely just before period 50. On the left side, the evolution of the output gap is shown, starting from $y(0) = 0.2$ and then responding to the path of u . It first declines steadily, reaching a deep trough around 25 weeks, picking up only once the lockdown is lifted, but then does not close over the horizon.

In Fig. 4, left panel, the infection rate, I , left panel, peaks after around 15 weeks, declines over the lockdown period, and stays on a plateau after the lockdown is removed. The infection from period 35 onwards then progressively recedes but does not vanish, I being down to 6 percent around period 60. The Ailing share A (symptomatic infected undetected), right panel, follows a similar albeit smoother pattern. Fig. 5 shows the evolution of Threatened, T , on the left side. There is a visible delay, with T reaching its maximum well after I did, very close to the end of the horizon. The Extinct, E , on the right side, trend upwards continuously with a further delay and does not stabilize after 60 weeks, even though the infection rate has been declining for a while.

Fig. 6 shows the evolution of Recognized, R (symptomatic infected detected), left panel. Despite the control operating, R goes on increasing until around period 35, before starting to decline. At the same time, the susceptible fraction of population, S , right panel, continuously decreases, stabilizing at a level eventually just below 20 percent – thereby leaving open the risk of a further wave, despite a rather long lockdown and an elevated cost in output terms well beyond historical standards, as seen on Fig. 3.

This sequence would correspond to an optimal lockdown policy decision, in a given continuous-time model as we specified it. Delays in such a set-up may sizably increase the complexity of the problem to be solved (see Silva et al. (2009) for the technical aspects and Casella (2020) for an application to a SIR variant). Moreover, in practice, decisions are impacted by policy considerations other than those accounted for in our model as well as population fatigue.

4.2. Dynamics with initial recession and full feedback effects

The two-way interaction between the output gap and the infection rates, neglected in Koehler et al. (2020) that we have introduced, allows

³² The calibration is informed by the variation in the monthly Industrial Production Index, since GDP is not available at sufficiently high frequency. Observed values over the period 2007–2021 can exhibit deviations from average close to 20 percentage points, which admittedly can be consistent with even stronger shocks at a higher, weekly, frequency

us to study the spread of infectious disease conditional on a variety of initial output gap conditions. We thus explore next the cross-feedback effects between infection and activity with, at the start of the pandemic, a negative output gap. We show results for initial condition $y(0) = -0.2$ with otherwise the same parameters as in the previous expansionary case. Such a simulation can also be interpreted as an illustration of the dynamics of the variables in eqs. (1) - (9) when a second wave – or further waves – occurs, i.e. when the output gap has already turned negative due to first lockdown measures, and the susceptible population group has also remained quite large after the lockdown, as was the case in Germany. State variables are indexed with 2 on reported Figures.

Fig. 7 on the left panel shows the output gap, which is now by assumption from the start significantly negative. In this case activity exhibits a cyclical pattern, with the gap closing before the lockdown, right panel, is imposed, which triggers a decline in output. The restrictions are implemented a bit later than beforehand and more gradually. They reach a similar intensity, of about 0.7, and remain at that strong level for about 10 weeks before again a gradual reduction, with a complete lift-off after period 50. While the lockdown stays at its maximum for less time than earlier, intensity remains higher than 0.5 over about the same period as in the previous case. The gradual relaxation of the lockdown, from period 30 onwards, supports a recovery in output from again a very deep trough, that translates moreover in an eventually still negative output gap at again some -0.2 . Restrictions are lifted now only after period 50. The overall lagged pattern results from the lower initial activity, but also from an implied slower diffusion of the infection.

In Fig. 8, left panel, the infection rate, I increases less markedly than before, reaching a similar peak, but two months later. When the lockdown is reduced, there is first some stabilization and then a slow decline, to an eventual non-zero level slightly higher than in the boom situation. The Ailing share A (symptomatic infected undetected), right panel, follows again a similar pattern. On Fig. 9, left side, the Threatened, T reacts like before with a delay, but without this time a clear peak within the simulation horizon. The Extinct population ratio, E , right side, as beforehand grows continuously with an additional lag. While the final extinct ratio appears smaller than in the boom case, the eventually stronger slope may point to additional future increases. On Fig. 10, developments for Recognized, R (symptomatic infected detected), left panel, and Susceptibles, S , right panel, are comparable to those obtained for the positive gap case, with again a sizable final S .

Summarizing, results for the first phase simulations show that:

- In both cases, starting in a boom or recession, the infection does not disappear but seems controllable through a hump shape control $u(t)$ resulting in a declining output gap, reduced interconnectedness, first rising then falling pandemic variables, with overall limited fatalities. The resulting drop in output however is substantial and activity does not recover fully in sample.
- In the recession case, the lockdown is implemented later, as output is initially lower when the infection starts. Restrictions also last longer as their impact is somewhat limited because of reduced social distancing and interconnectedness – this also gives rise to a delayed infection peak and further similar effects on other variables.³³
- The recession simulation can also be relevant in case a second wave kicks in after a substantial output reduction due to a first lockdown. In that case, we could observe a break in the recovery and also longer lasting fatality rates than for the boom case or first wave. Such higher rates could pose a controllability problem³⁴ to the policy-maker

³³ Delays have been recently studied in the epidemiological literature, see Silva et al. (2009) and Maurer and Do Pinho (2009).

³⁴ This is a typical issue in control theory, for example studied in models of state-dependent debt controllability, see Gruene and Semmler (2005).

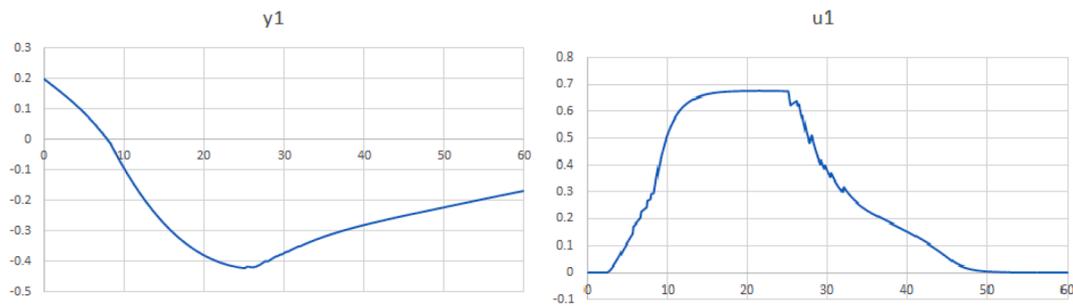


Fig. 3. First phase, first wave: Evolution of output gap y , LHS, lockdown intensity u , RHS.

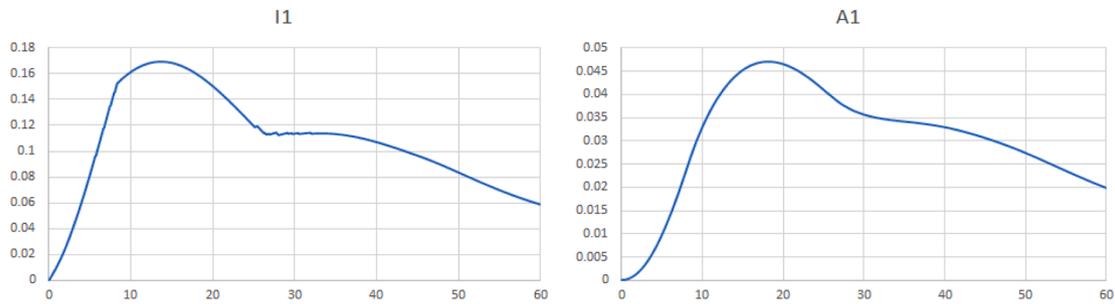


Fig. 4. First phase, first wave: Evolution of Infections, LHS, Ailing, RHS.

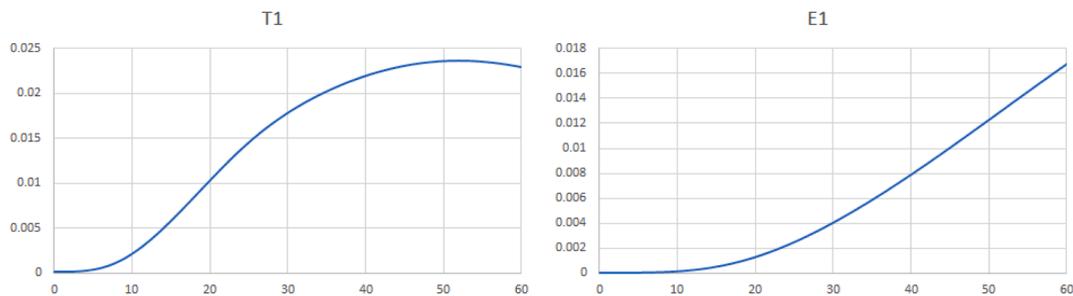


Fig. 5. First phase, first wave: Evolution of Threatened T , LHS, Extinct E , RHS.

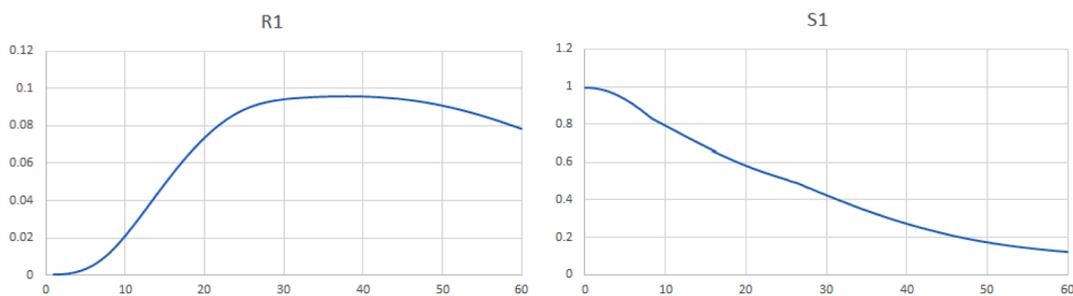


Fig. 6. First phase, first wave: Evolution of Recognized R , LHS, and Susceptible S , RHS.

resorting to social distancing measures, who would then need to design additional health policies (eg targeted to groups or regions).

- Overall focusing on the results for the control variable, the simulations broadly replicated the lockdown timing and strength as was experienced in Germany between Spring 2020 and Spring 2021, albeit to a lesser extent for the second wave.

5. Monetary Policy and Recovery – The Second Phase Model

Our first simulation exercise with its two alternative output gap configurations reflected the initially dominant focus on the pandemic

spread. Once lockdowns are lifted or the scope of vaccination has been sufficiently broad, discussion on extending or exiting economic support measures can resume. The policy debate then returns to discussing more standard monetary policy-oriented targets and relatively less health-oriented objectives.

In modeling terms, we can therefore heuristically at this point identify the beginning of a second phase, that then requires a new model, as recovery policies have become more important and attract primary attention. We can thus consider some regime change, in which the state and control variables as well as the objective functions can be allowed to change from the first (pandemic) to the second (recovery)

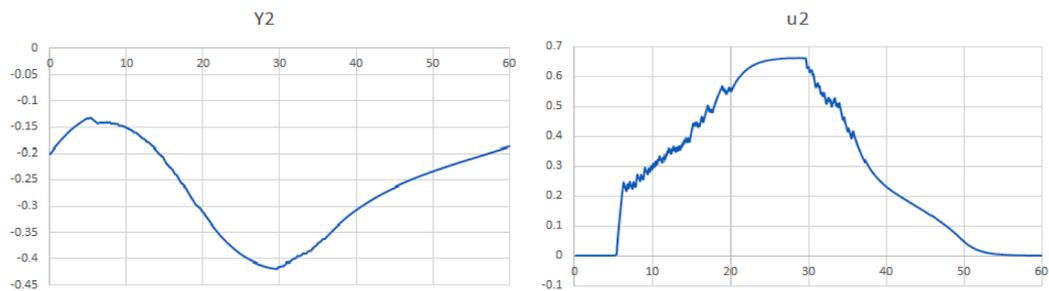


Fig. 7. First phase, second wave: Evolution of output gap y , LHS, lockdown intensity u , RHS.

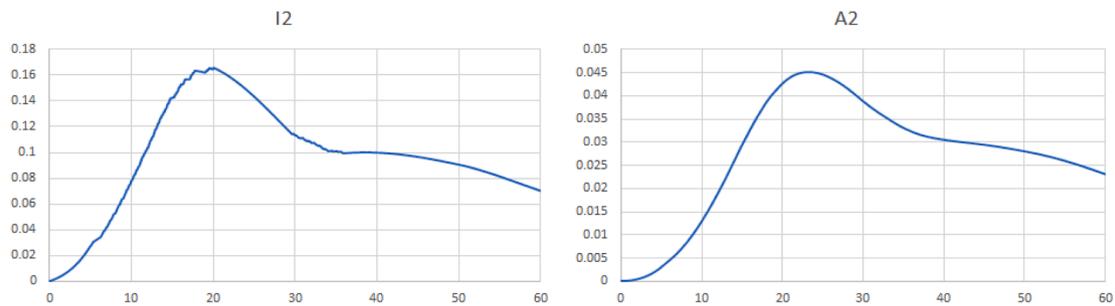


Fig. 8. First phase, second wave: Evolution of Infections, LHS, Ailing, RHS.

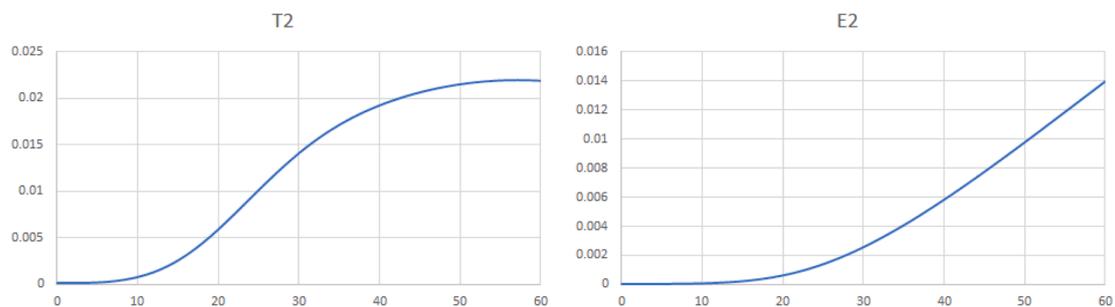


Fig. 9. First phase, second wave: Evolution of Threatened T , LHS, Extinct E , RHS.

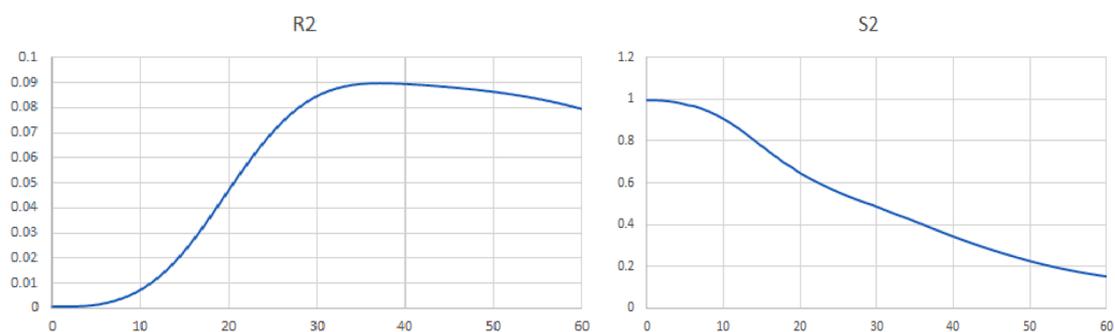


Fig. 10. First phase, second wave: Evolution of Recognized R , LHS, and Susceptible S , RHS.

phase. Accordingly, the policy-maker of interest also changes, from health authorities acting on lockdown intensity to monetary policy ones adjusting interest rates and related similar policy tools.

Indeed, it does not seem adequate to keep using the same dynamic decision model for a long time period, especially at times of different types of crisis, without paying attention to the possibly evolving relevant macro issues to be addressed and solved using varying policy tools. In the case at hand, on top of the change in control, objectives and state variables, the new post-pandemic times may still bear resemblance to the pre-crisis period. In modeling terms, this would call for more

standard and refined behavioral specifications on the macro side in comparison with that used in the first phase.

While considering in the second phase more detailed features of the macro economy, in particular for monetary policy, we can instead neglect some parts of the complex granular pandemic dynamics studied in the previous phase configuration. We include an aggregate indicator for the spread of infections, but we consider further drivers of the output gap, such as interest rates, risk premia, inflation rates and credit flows. As a result, we then assign only a rudimentary role to the epidemics in shaping (monetary) policy decisions – while keeping the cross-feedbacks

and therefore the trade-off between economic and health objectives.

5.1. Stylized macro-financial facts for the Corona crisis

The Covid-19 recession that especially affected GDP in 2020H2 worldwide, was triggered by the fast spread of an infectious disease, as featured in our first phase model, possibly developing into multiple waves, as indeed observed in a wide range of countries. This started as a non-financial crisis, that still eventually had sizable global economic impacts. The latter called for specific policy responses beyond the health remit, thereby coming closer nonetheless to what is implemented after a more standard financial crisis.

As a result of health policy measures, the Corona crisis was characterized by the shutdown of the supply side of particular sectors, that affected all countries globally on both their domestic and external markets. Many service areas and small and medium size businesses were affected by closures, namely travel and transport, entertainment, accommodation, leisure recreational areas and the educational sector. Demand, especially consumption, was also severely hit as social interaction declined due to lockdowns and the fear of infections. Albeit disproportionately affecting different sectors, this big shock caused large lay-offs, hence very sizable hikes in unemployment, and generated income, liquidity and credit for both households and firms as well as insolvencies that all compressed demand further. Another very important characteristic of this pandemic-driven recession are the propagation lags, whereby causes and effects are considerably separated in time, partly due to uncertainties on prospects as well as to mitigating policies that were promptly implemented in a quicker and stronger manner than in the GFC, also with much more coordination among various types of policy-makers, including internationally.

For illustration, and also to be consistent with the source employed to calibrate our first phase model, we now show some data for the German economy, but major other Euro area countries would exhibit similar features. The upper left panel of Fig. 11 represents industrial production. We can observe a large scale meltdown about 2007–09 and then another one starting beginning of 2020 where the drop in output is even quicker and larger, but also followed by a strong and prompt rebound. The HICP inflation rate (upper right panel), the headline inflation, moved below 0 in 2020, also seen during the GFC, indicating deflationary pressures. There was this time though a sharp turnaround

in inflation later on, with a surge until 2022. The data sources for those illustrations are recalled in Appendix C.

Both the non-financial origin of the crisis and the much better solvency and liquidity position of banks at the outset of the crisis helped to mitigate the financial impacts with respect to the GFC. So did the various measures taken by central banks around the globe, making use of the non-conventional monetary policy toolkit actually developed to tackle the previous crisis. The very low short-term euro interest rate (left upper panel in Fig. 12) as set by the ECB was accompanied by a reduction in financial stress and helped limit impacts on the cost of borrowing, which could stay low during the pandemic meltdown (Fig. 11). During the pandemic, unconventional monetary policy tools that were developed and employed during the GFC were also used, such as QE (the PEPP for the pandemic), ie purchases of assets, as well targeted bank funding (TLTRO), ie subsidized loans to banks backing credit to the economy. These measures contributed to stabilize financial markets and keep a lid on financing costs for the economy. Asset purchases translated in a rapid further increase in the balance sheet of the ECB (Fig. 12, upper right panel) from Spring 2020 onward.

This configuration has supported credit flows (lower panels of Fig. 12), as businesses and households faced with cash-flow effective or anticipated shortages, expressed strong demand for short-term credit. Over March-April 2020, spikes in credit flows appear but as the crisis develops, credit falls again (lower right panel of Fig. 12) – as a result of risk premia going up and real demand falling. The risk of a credit crunch in the pandemic crisis has nonetheless been successfully counteracted, inter alia through supervisory measures related to capital requirements, forbearance of loan losses and fiscal guarantees, that all helped prevent a wave of outright defaults.

5.2. A dynamic macro model for the recovery

For the second phase of our macro dynamic model the major empirical macroeconomic features can be observed in Figs. 11 and 12. Yet we want to highlight in our modeling procedure also some dynamics of the infectious disease as studied in Sections 3,4. The major issue when designing a dynamic macro model – as well as the policy – for the recovery is to specify how the output gap $y < 0$, interacts with credit flows, interest rate, financial stress, risk premia and the inflation rate. Those variables are all strongly connected to the output gap – and the output

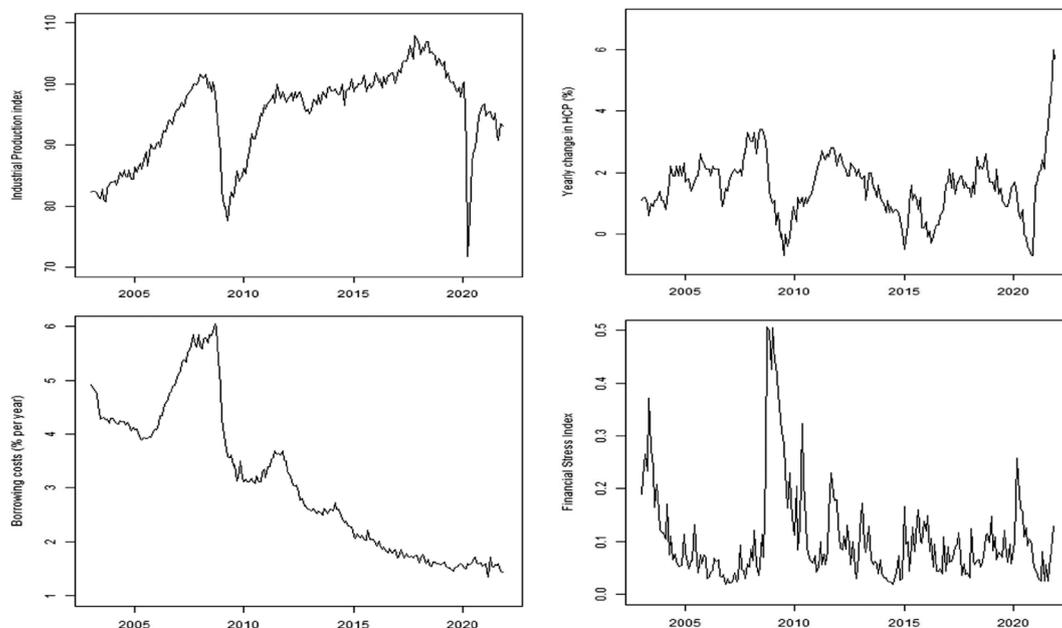


Fig. 11. Large scale meltdowns, for data sources see Appendix C.

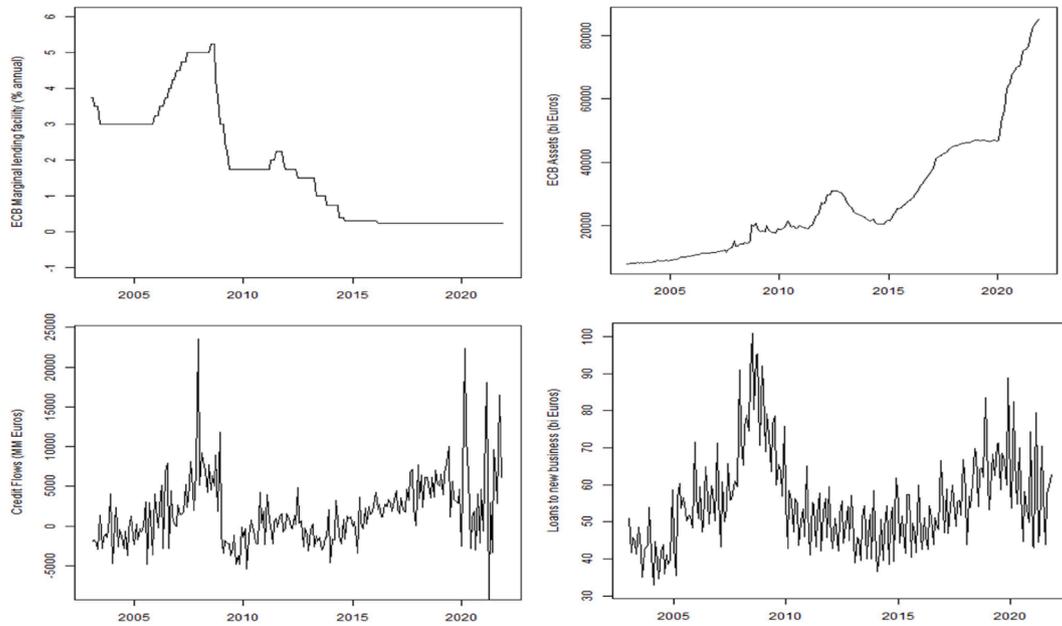


Fig. 12. Large scale meltdowns, for data sources see Appendix C.

gap is itself connected to the diffusion of the pandemic disease. These relations should enter the model. Similarly, the ultimate goal for the policy-maker following a recession is to bring the output gap back to a positive value, $y > 0$ – and with this to reduce the unemployment rate – being aware that in the case at hand, i.e. a pandemic context, such increases in activity speed up the spread of the disease.

To analyse further this transition, from a negative to a positive output gap in a pandemic environment, we will concentrate on monetary policy, which we model in a stylized and illustrative fashion. Along with the inflation and the output gap, the policy-maker considers a range of monetary and financial conditions, such as interest rates, financial stress, credit flows, credit spread and cost of borrowing. We also need to embed monetary policy reactions and their impacts on the economy in a macro model. For that purpose, we will use an extended version of the modeling framework proposed by Gross et al. (2019) and Faulwasser et al. (2020). By extending the latter, non-linear quadratic (NLQ), model, we can discuss policies in the recovery phase after the pandemic-driven recession, but also track aggregate effects of the pandemic. The basics for such a non-linear macro policy model are the dynamics of the inflation rate, π , the output gap, y , and the volume of credit flows, l . Further important drivers of the output gap – and also of credit flows – are financial conditions such as interest rates, risk premia and credit spreads (for details, see Faulwasser et al. (2020)).

The CB’s decision problem is to set the interest rate (i) – as control variable therefore – conditional on an objective function with multiple goals:

$$J(\pi, y, l, i) = \int_0^T e^{-\rho t} (w_\pi (\pi(t) - \pi_s)^2 + w_y (y(t) - y_s)^2 + w_l (l(t) - l_s)^2 + w_i (i(t) - i_s)^2) dt \quad (21)$$

where π is the inflation rate, y is the output gap and l is the volume of credit flows; a specific weight is assigned to each objective (w_j).

Equ. (21) is a weighted functional, with a quadratic penalty on each deviation from stationary values of the objectives considered. There is

inter alia a penalty on high interest rates. It therefore departs from the otherwise standard Taylor rule, by including interest rates and credit terms.³⁵ The latter additional objectives can be interpreted as intermediate ones, helping reach those stated in the CB’s mandate. Such a multi-term quadratic penalty function is also considered in de Groot et al. (2021), presenting a generic set-up to analyze monetary policies, especially QE at the Zero Lower Bound (ZLB) for interest rates. Another example of such a set-up is Ajello et al. (2019), whose loss function also accounts for credit conditions, as driving the risk of a future shift to a crisis regime.

This objective function is to be optimized facing constraints that depend on the macro behavior of each variable, as represented by the model state dynamic equations:

$$\dot{\pi}(t) = -\alpha_1 \pi(t) + \alpha_2 y(t), \quad \pi(0) = \pi_0 \quad (22)$$

$$\dot{y}(t) = -\beta_1 y(t) - \beta_2 (i(t-d) + \delta(y(t)) - \pi(t) - r), \quad y(0) = y_0 \quad (23)$$

$$\dot{l}(t) = \gamma_1 l + \gamma_2 y - \gamma_3 (i(t-d) + \delta(y(t))) - \gamma_4 \pi(t), \quad l(0) = l_0 \quad (24)$$

Equ. (22) represents a Phillips curve for inflation π , whereby α_2 can also vary with the output gap, (see Faulwasser et al., 2020) introducing thereby non-linearities; equ. (23) is an IS curve, with the real interest rate incorporating a risk premium declining with y ; finally equ. (24) is a loan equation, again with the y -dependent risk premium. All coefficients are positive in the above notation, except γ_4 with an uncertain sign.³⁶

Although the CB’s first line of defense is usually the interest rate i , here also the first monetary tool, it has then other instruments to address multiple objectives with additional tools, such as QE to limit and steer risk premia, also in some cases supervisory powers (the latter are not considered in our already rich CB modeling). While not addressing explicitly the ZLB case, the proposed framework can reflect QE by assuming the latter helps dampen the risk premium δ . As long as the policy interest rate is kept on a floor, the corresponding term in equ. (23)

³⁵ The US Fed has a mandate with inflation and employment objectives, whereas the ECB has instead a primary inflation objective. In academic work, both reaction functions have nonetheless been commonly modeled as a standard Taylor rule. We are using here an optimal Taylor rule.

³⁶ The term $(t-d)$ can capture lagged policy effects, but d is set to zero in the present use of the model. For further details on delays, see Chen et al. (2021).

would then be capped, even for strongly negative output gaps. It will also quickly go to 0 when the situation improves.³⁷

To account for the pandemic, we need to extend the system (23) - (24) with the evolution of a new variable, m , namely the fraction of population infected.³⁸ To be in a position to carry out a variety of monetary policy experiments, we also need to complement the monetary policy objective function with an infection-related term, overall leading to eqs. (25) and (26):

$$J(\pi, y, l, i, m) = \int_0^T e^{-\rho t} (w_\pi (\pi(t) - \pi_s)^2 + w_y (y(t) - y_s)^2 + w_l (l(t) - l_s)^2 + w_m (m(t) - m_s)^2 + w_i (i(t) - i_s)^2) dt \tag{25}$$

$$\dot{m}(t) = -\delta_1 m + \delta_2 y \tag{26}$$

The new state equ. (26) defines the dynamics of infection, and can be interpreted as a proxied SIR model whereby contagion risks increase with output. Various versions of the dynamics of equ. (26) could also be explored, by allowing δ_1 to be positive or negative, or being affected in a non-linear way by infection dynamics, through state or time-dependent coefficients, due to lockdown policies, vaccination successes or failures – all affecting δ_1 . The impact of a change in the output gap, y , (through imposing or relaxing a lockdown), can also be modeled via equ. (26), possibly also by a time and state-dependent adjustment coefficient δ_2 .³⁹ This parameter we set to 0.005.

We take here the sign and size of δ_1 fixed, since in line with our periodization, mobility-related health policies do not need to change and we can therefore concentrate on monetary policy – unless a strong infection wave arises again. We assume δ_1 to be positive, equal to 0.001, which can be achieved via either extensive testing or vaccinations (which would match the periods after the first and second lockdowns, respectively). The corresponding counterpart in the SIDARTHE model would be that the Susceptible fraction has substantially declined, for example through extensive vaccination. There could however be further increases in the population vulnerable to infections, due to the appearance of variants of the virus, in which case, a return to the first phase modeling could be warranted with stable economic policies and again varying health ones.⁴⁰

5.3. Numerical simulations of the recovery phase

Assuming that the model of Sections 3 and 4 describes the pandemic meltdown appropriately, due to lockdown measures, the output gap would be negative before the second infection wave kicks in. Based on the parameters employed, the output gap even if initially positive would go down to a marked negative output gap of around $y = -0.2$, at the end of the horizon, after having been twice as big mid-sample. In our second phase or regime, we thus will nonetheless take as initial

conditions an output gap set at $y = -0.2$. This assumption would be consistent with GDP developments observed in Europe before the second wave and the corresponding lockdowns. Even though multiple waves were still experienced that hindered the already started recovery, activity rebounded quite strongly following the lockdowns, part of which corresponded to catching up on postponed spending – ie this occurred without a need for policy support.

For our major macro variables, we wish to roughly mimic the stylized empirical facts from Figs. 11 and 12. To that aim, we can base our second phase initial conditions on the final outcomes of the first phase dynamics – even though they may not always be exactly aligned, albeit realistic, with actual outcomes at a given point in time. In particular, the initial condition for the fraction of infected population can be set at $m(0) = 0.15$, close to what was obtained in Section 4, ie $I = 0.12$.⁴¹ Parameters for economic behaviour - ie except for equ. (26) - are taken from Faulwasser et al. (2020). Namely $\alpha_1 = 0.67$, $\alpha_2 = 0.04$, $\beta_1 = 0.06$, $\beta_2 = 0.157$, $\gamma_1 = 0$, $\gamma_2 = 0.81$, $\gamma_3 = 0.48$, $\gamma_4 = 0.6$, $\mu_1 = 0.15$, $\mu_2 = 0$, $r = 0.03$, and $\rho = 0.03$.

We use again our discretization and optimization methods implemented in AMPL for solving the new phase of our finite horizon macro decision problem, with now differing state variables and objective function, as given by eqs. (22) to (26). This system also captures the policy now supporting the recovery phase, ie that of the CB, on which we now concentrate when summarizing our simulation results. Arguments in the CB objective function (25) have weights w 's equal to unity; all stationary values with subscript s are 0s.

Interest rate-based standard monetary policy affects the output gap with a sizable lag (see Chen et al., 2021). As a result, both in the data and in our simulation, when the economy runs into a recession, it first experiences worsened financial conditions – in the sense that risk premia and credit spreads jump up (see Figs. 11 and 13 for data and simulation respectively) – driving down households and firms' spending, which absent policy reactions would generate a further decline in output, hence a widening of the output gap.

When facing a recession and a large negative output gap, the first line of defense of the CB, namely its conventional monetary policy tool, is the interest rate. Fig. 12, left upper panel, exhibits indeed downward moves in the ECB interest rates. In our simulation, given our initial conditions and parameters, the interest rate even moves into negative territory (Fig. 14, left panel). When interest rates have hit the ZLB, QE can be activated, with a view to keeping a lid on risk premia, which is reflected in increases in the CB balance sheet (see Fig. 12, upper right panel). In our simulation, we assumed accordingly that when the CB interest rate reaches a minimum, QE is employed to reduce state-dependent risk premia and prevent credit spreads to increase beyond a given value, which we set at 5 percent.⁴²

Fig. 13 shows that the output gap gradually recovers, but this reflects factors other than solely low interest rates. With QE operating, the increase in the risk premium can indeed be capped despite the recession, see Fig. 13, left panel, which plots the risk premium against the output gap as per our simulation. Looking at past data, with QE, financial assets became less risky and the cost of borrowing for households and firms declined (see Fig. 11, lower panel). Reflecting a similar improvement, credit spreads also go to zero in our simulation (Fig. 13, left side) as soon as the economy recovers. Lower risk premia thereby slowly stimulates

³⁷ The risk premium is assumed to smoothly switch between 5% and 0 when the output gap moves between -0.005 and 0 (as in Fig. 13 left). See Chen et al. (2021) for a similar approach.

³⁸ The term m is used here to differentiate it from the previous notation I , that in turn referred to only one sub-group among all infected. Now we have only an aggregate variable for all infected m , as in the original SIR model.

³⁹ As envisaged beforehand for monetary policy but now more generally for the whole enlarged model (22) - (26), there could be delay effects in decision and state variables.

⁴⁰ While for illustration purposes, we chose here a simple dynamic formulation with fixed coefficients in Eq. (26), further (epidemic) state variables such as susceptible, infected and recovered segments of the population could be considered, based on the SIR model, without then a need for the SIDARTHE granular model of Sections 3 and 4. Additional differential equations could be introduced, as typically done in infectious disease models, see Maurer and Do Pinho (2009).

⁴¹ The officially declared fraction of infected population, for example in Germany, is lower, but this rate is usually strongly underestimated as testing coverage remains limited, and a number of contagion cases are not declared nor identified.

⁴² With QE, the CB does not target credit flows directly, but instead aims at keeping a cap on risk-driven interest rates, possibly looking along the yield curve, so that financing costs remain affordable, which in turn affects the actual credit cost – and as a result supports credit flows and eventually economic activity via eased funding.

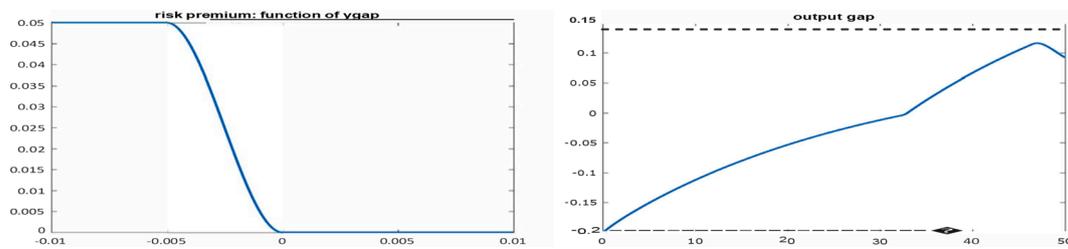


Fig. 13. Second phase: Risk premium, LHS, output gap, RHS.

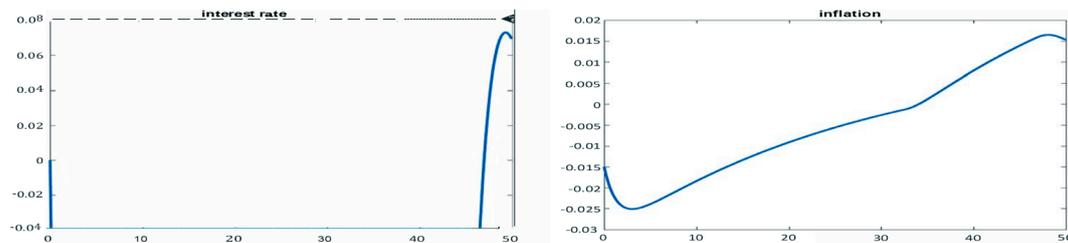


Fig. 14. Second phase: Interest rate, LHS; inflation rate, RHS.

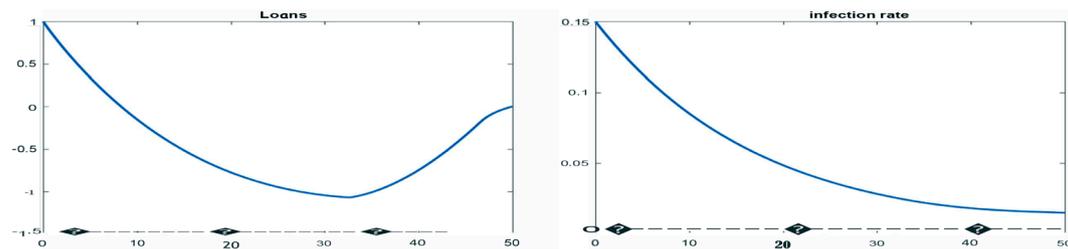


Fig. 15. Second phase: credit flows, LHS; infection rates, RHS (m.dat).

demand and over time, the output gap becomes less negative (Fig. 13, right side). Also similar to actual data (Fig. 12, lower panel), simulated credit flows eventually recover from the negative region (see Fig. 15, left panel).

On the nominal side, inflation initially declines further due to the large negative output gap.⁴³ Given an inflation well below target, a zero risk premia and the still negative output gap, the interest rate (Fig. 14, left panel) has to stay negative for a while.

Along with the recovery in credit flows, output increases further so that the output gap closes around period 35. The continued opening of the output gap triggers a gradual but steady rise in the inflation rate (Fig. 14, right panel), in line with the Phillips curve. At period 45, policy interest rates need to increase, reacting to mounting inflationary pressures and a sizable positive output gap at roughly 0.15. Following the policy move, the output gap then closes somewhat, whereas credit flows stabilize (Fig. 15, left panel) - and inflation also eventually moderates.

In this simulation, the lagged inflationary effect of an effective post-pandemic support to the recovery then warrants an exit for monetary policy when inflation approaches 2 percent – a conjunction which was not yet observed in the data reported on Figs. 11 and 12, where inflation shoots up, but without leading to a monetary policy tightening. Following the pandemic crisis and lockdowns, actual inflation could accelerate further due to pandemic-driven supply constraints or

imported inflation, that are not accounted for in this experiment.

On the sanitary front, over the whole recovery period, total infections decline, as the term $-\delta_1 m$ exerts a negative impact on the level of m (as argued above), Fig. 15 (right panel), also because the output gap remains negative for a while, thereby with still reduced social interaction among economic agents.⁴⁴ In comparison with the first phase documented in Sections 3 and 4, where the focus of policies was on the spread of infectious disease, and output rose whenever the intensity of lockdown was reduced, the opposite effect is now observed – namely a declining infection spread, even if the negative output gap shrinks and the output gap moves into a positive territory.

The decrease of the infectious disease comes from, in principle, the declining reproduction number, an increasing vaccination of the population or lockdowns. Although the parameter δ_1 could be positive or negative, we assumed it here to be negative, due to extensive vaccinations and testing policies. This could also be obtained by keeping some degree of mobility restrictions or with more careful social behavior still prevailing, since both weaken social interconnectedness. At the same time, if the susceptible population fraction does not go to zero, a new infection dynamic could still emerge, as also studies on the lack of the final dissipation of flu infections and as research of virologists on the current pandemic have suggested — in particular given the high eco-

⁴³ Although in the current model, we do not have an open economy Phillips curve possibly triggering imported inflation effects (through commodity prices), it can be included, as in Gross and Semmler (2019); Semmler, Maurer and Bonen (2020) where a nonlinear Phillips curve is estimated. Imported inflation in the aftermath of the pandemic became relevant, even before the Ukraine war.

⁴⁴ Extensive testing and vaccination efforts are indirectly captured by the negative sign and the size of the term $-\delta_1 m$.

conomic activity that leads to intensive social interconnectedness.⁴⁵ As experienced in late 2021, the emergence of particularly contagious or partly vaccine-resistant variants, such as Omicron, can also relaunch the epidemic spread, and new waves cannot be excluded albeit not necessarily calling for a new lockdown implementation.

Summarizing the results of our simulations for Phase 2, it appears that:

- The combination of an immediate interest rate drop and QE capping risk premia gradually brings activity back to potential, while ensuring still a decline in the infection rate thanks to a control policy that considers jointly economic and pandemic objectives - while health measures other than lockdowns may remain in place.
- With the ongoing recovery though, after a while, also inflation resurfaces, up until the point where monetary policy has to tighten again, but this happens far down the horizon, after a full year, completing a cyclical return to more standard conditions. This surge of inflation in the model is however not due to external supply shocks, the effects of which would come on top.
- Credit however reacts with a sizable delay and does not return to its pre-crisis level in sample, so that there appears to be some cost left on the financing side once the infection is controlled, although output and inflation exhibit patterns consistent with the end of the pandemic-induced meltdown.
- These simulation results seem therefore to rather well qualitatively capture the actual macroeconomic developments post-lockdowns, as observed in particular after the second lockdown period, in any case before the additional shock to prices and growth resulting from the Ukraine conflict.

6. Conclusions

We have introduced, calibrated and simulated a two-phase macro model combining epidemic and economic variables, with respective policy objective and associated control variables differing across the two phases. In both phases, there is a bi-directional interaction between pandemic and economic variables. In the first phase, epidemic dynamics and response policies or controls such as lockdown dominate the system, whereas in the second phase, economic activity and associated support policies or controls take precedence. Technically the model is similar to a two-regime model with changes in the objective functions as well as state variables across regimes.⁴⁶ In both regimes, policy-makers consider jointly economic and infection metrics in their loss function, ie they have multiple objectives. We have associated the suggested phases to stylized facts on both the pandemic and economic sides, with a specific focus on Germany.

In the first phase model, the infection rate plays a crucial role. Going beyond the standard epidemiological SIR model, we have modeled the behavior of several types of affected subgroups - using the SIDARTHE model. The susceptible population, S , is where the infected one I starts developing, turning then into a number of sub-populations D, H, A, R, T and E . We have moreover included in our model an effect of the infection and of lockdowns, ie the policy variable in that phase, on the economic variable, y . The latter during this process also feeds back to the infection rates, then propagating through the pandemic variables again.

In the second phase model, policies focus dominantly on the macro variables such as output gap, inflation and credit dynamics, with a control of risk premia and interest rates. Although rescue policies, in particular through monetary policy provided support in the lockdown

⁴⁵ International organizations point the fact that a large part of the world (in particular Asia, Africa and Latin America have been vaccinated only to very small extent and worldwide there is still a large fraction of the population that is susceptible).

⁴⁶ See [Appendix B](#) for a more generic treatment of a regime switching model.

phases, Central Banks' monetary and financial policies (and fiscal policy) can play an even more active role in periods outside lockdowns, as the full impact on the economy can then materialize. At the same time, the increase in activity generated by support policies also results in increased infection rates, which in turn limits economic output.

Our first phase simulations, largely emphasizing the pandemic dynamics, document a variety of control optimal paths and intensities. We illustrate how the cyclical position of countries or multiple waves can affect control and outcomes. We consider alternative initial conditions, distinguishing between positive and negative output gaps (respectively for expansions and recessions). We show that in both cases pandemic variables all decline, but at a cost since output stays eventually below potential.

Further, if the initial output gap is negative, pandemic and economic outcomes worsen, indicating that both can be less easily controlled. In Germany, for some time after the first wave, specific policies were put in place linking lockdown implementation to new cases, targeting local and regional clusters with excess infection rates - which should be captured in the Oxford indicator or equivalently our u control variable. In turn, also mass tests for school, services, travel and retails were introduced which we did not include in our simulations. Even in the expansion case, there is a remaining susceptible fraction of the population vulnerable to infections, hence room for subsequent waves. In such cases, with the already reduced social distancing and lower interconnectedness, as after the first wave - and the general fatigue of the public, a further use of lockdowns do not appear as effective as for a first wave.

What does seem to have been increasingly effective are the widespread vaccination strategies, starting at the end of 2020 in the US and at the beginning of 2021 in Europe. Further infections waves occurred, even when the vaccinations covered a very sizable fraction of the susceptible population, but these waves appeared less damaging on both health and economic fronts. These additional waves nonetheless also triggered new health policy responses, with further lockdowns or other types of restrictions bearing on the un-vaccinated population. We have taken the first two waves as prototypical ones for the first phase of our model, given the then stronger impact of the pandemic on the economic situation and the focus on lockdown measures as a first response to the pandemic developments.

In the model for the second phase, stressing this time the role of macroeconomic drivers and policy variables and focusing on the recovery phase of the macroeconomy, we included only an aggregate variable to capture the dynamics of infections. Some stylized facts helped us calibrate the modeling of this phase. The second phase simulations start with parameters and initial conditions similar to those we were left with at the end of our first phase dynamics. Simulations roughly replicate what we see in the data - on the dynamics of the output gap, the interest rate, the credit flows to businesses and households, the risk premia and credit spreads and inflation rate - all capturing broadly the characteristic features of the observable recovery phases subsequent to lockdowns.

In these second phase simulations, with non-health policies focusing on the recovery, the stock of infected population while eventually very low, does not go fully to zero over the horizon, so a new infection dynamic could - as it did with the second wave - potentially still emerge, as also studies on lack of the final dissipation of flue infections and research studies of virologist on the spread of such a pandemic disease have suggested - in particular given the high economic activity with intensive social interconnectedness in advanced countries and large parts of world not yet sufficiently vaccinated.

Further use of this modeling set-up could be to carry out a full sample optimal control exercise, based on the presented periodization but with specific control associated to given sub-periods. Simulations could also be carried out employing estimates closer to updated data on the pandemic front but also for the economic parts - eg the ESTAR parameters. Incorporating data on testing could also enrich the simulations,

which could then be employed in case a new and stronger variant, eg resistant to currently available vaccines, would emerge, thereby bringing back a phase 1 configuration with again health policies taking precedence over other steps. One can also study a Pareto efficient frontier for multiple goals of policy makers by varying weights for the multiple targets in the objective function. An evaluation of varying weights in such an objective function has been undertaken in [Faulwasser et al. \(2020\)](#), finding that a greater weight for an objective will help to increase the adjustment speed toward its target.

The pandemic may not yet be over, as evidenced with repeated waves worldwide, and there could be some feedback effects from the still remaining large susceptible fraction of the world population to advanced countries. A further important, economic, issue is what long-run scarring effects and structural changes will remain in particular for growth potential and the labor market. Even if the pandemic fades away, over the medium-term a stagflation configuration could emerge from the

pandemic episode. The potential threat of another series of pandemic waves as well as tapering by central banks could push the potential output down, while restricted supply and bottlenecks would keep inflation high long enough to force central banks to raise interest rates and taper QE, which could further exert and entertain negative pressure on the real side of the economy. Such low growth and lasting inflation configuration may become more likely following repeated commodity price shocks, supply chain tensions and mounting geopolitical risks - further pandemic waves would render the situation even more complex to manage for policy-makers.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. The Numerical Procedure AMPL

The optimal control problems in both phases are solved by the numerical approach "First Discretize then Optimize". The discretization of the control problem on a time grid leads to a large-scale nonlinear optimization problem (NLP) that can be conveniently formulated with the help of "A Mathematical Programming Language" (AMPL) developed by [Fourer et al. \(1993\)](#). AMPL can be linked to several powerful optimization solvers. We use the Interior-Point Optimization Solver Ipopt (see [Wächter and Biegler \(2006\)](#)) and the trapezoidal rule as integration method with $N = 600$ grid points for terminal time $T_f = 60$. Choosing the error tolerance $tol = 10^{-8}$ in Ipopt, we can expect that the state variables are correct up to 3–4 decimal digits. The Lagrange multipliers and adjoint variables are obtained as a byproduct of the optimization process in Ipopt which enables us to verify the necessary optimality conditions. More details of (multiprocess) optimal control problems and discretization methods can be found in [Augustin and Maurer \(2000\)](#), [Göllmann, Kern, and Maurer \(2009\)](#), and [Göllmann and Maurer \(2014\)](#).

Appendix B. Multi-Phase Algorithm

Multi-process (multi-phase) optimal control problems have been studied for a macroeconomic application by [Semmler et al. \(2021\)](#) and [Mittnik et al. \(2019\)](#). There are many contributions with respect to many application in economics and game theory and also in the context of DSGE models (see also the toolkits for models without regime changes in [de Groot et al. \(2021\)](#) embedding an extended policy rule for the US Fed in the otherwise standard [Smets and Wouters \(2007\)](#) framework). In the current paper we have linked together two codes with different objective functions and state variables, where the linkage is provided by the fact that the final conditions of the first phase algorithm are employed as initial conditions of the second phase algorithm.

Yet the two algorithm can be integrated to one overall algorithm through time depending switching mechanism as follows, see [Mittnik et al. \(2019\)](#). In general we can state, suppose that a dynamic economic process on a given time interval $[0, T]$ consists of $(s + 1)$ phases (regimes) that switch at the transition times $t_k \in (0, T), k = 1, \dots, s$. The switching times are ordered according to:

$$0 = t_0 < t_1 < t_2 < \dots < t_s < t_{s+1} = T. \tag{27}$$

In each interval $[t_k, t_{k+1}), k = 0, 1, \dots, s$, the dynamics and objectives may be different.

Let $x \in \mathbb{R}^n$ be the state variable and $u \in \mathbb{R}^m$ the control variable. The dimensions of the state vector and control vector may be different in different phases. For simplicity, we refrain here from discussing this general case and assume the same dimensions in each subinterval. Hence, the dynamics of the economic process in the interval $[t_k, t_{k+1}]$ is given by the ordinary differential equation,

$$\dot{x}(t) = f_k(x(t), u(t)), \quad t_k \leq t \leq t_{k+1} \quad (k = 0, 1, \dots, s), \tag{28}$$

where the right hand side of the ODE is a C^1 function $f_k : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$.

Time t_k in (28) is understood from the right, while the time t_{k+1} is taken from the left. The initial condition and terminal constraints are given as

$$x(0) = x_0, \quad \psi(x(T)) = 0. \tag{29}$$

We further impose control constraints in each interval,

$$u_{k,min} \leq u(t) \leq u_{k,max}, \quad t_k \leq t \leq t_{k+1} \quad (k = 0, 1, \dots, s), \tag{30}$$

with $-\infty \leq u_{k,min} < u_{k,max} \leq +\infty$.

A continuous state trajectory $x(t)$ on the whole interval $[0, T]$ is obtained by imposing the continuity condition

$$x(t_k) = x(t_k -), \quad k = 1, \dots, s. \tag{31}$$

Note that the continuity of the state variables in (31) does not automatically ensure the continuity of the control variables; in fact, these can jump, as we demonstrate below, when the system transitions between policy phases are studied. Moreover, we can prescribe interior (transition) conditions for

the state variables by

$$\varphi_k(x(t_k -)) = 0, \quad k = 1, \dots, s, \tag{32}$$

with C^1 functions $\varphi_k : \mathbb{R}^n \rightarrow \mathbb{R}$.

In each interval one may also have different objectives which are defined by functions $L_k : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$, $k = 0, 1, \dots, s$.

Then the optimal multi-phase control problem is defined by the following objective:

$$J(x, u) = \max_u \left\{ \sum_{k=0}^{k=s} \int_{t_k}^{t_{k+1}} e^{-r_k t} L_k(x(t), u(t)) dt \right\}. \tag{33}$$

subject to the constraints (28)–(31), and $r_k > 0$ for $k = 0, 1, \dots, s$.

To solve an optimal multi-phase control problem, with different objective functions, state variables and/or parameters in each phase, we can implement the arc-parameterization in Clarke and Vinter (1989) in conjunction with discretization and nonlinear programming methods through AMPL. Although they apply the arc-parameterization method (APM) only to bang-bang control problems, the APM can be extended to continuous, multi-phase control problems as follows. Let

$$\xi_k = t_{k+1} - t_k, \quad k = 0, 1, \dots, s, \tag{34}$$

denote the *arc-lengths* (or, arc durations) of the multi-process.

The time interval $[t_k, t_{k+1}]$ is mapped onto the fixed interval $[k/(s+1), (k+1)/(s+1)]$ by the linear transformation

$$t = a_k + b_k \tau, \quad \tau \in \left[\frac{k}{s+1}, \frac{k+1}{s+1} \right], \tag{35}$$

where $a_k = t_k - k\xi_k$ and $b_k = (s+1)\xi_k$. Taken together, the complete time interval $[0, T]$ is thereby mapped onto the unit interval $[0, 1]$. Identifying $x(\tau) = x(a_k + b_k \tau) = x(t)$ in the relevant intervals, we obtain the scaled ODE system

$$\frac{dx}{d\tau} = \xi_k(s+1) \cdot f_k(t_k + \tau \cdot \xi_k, x(\tau), u(\tau)) \tag{36}$$

for $\tau \in \left[\frac{k}{s+1}, \frac{k+1}{s+1} \right]$. Note that ξ_k are treated as optimization variables if the transition times t_k are free.

The time transformation leads us to rescale the objective function (33) as follows:

$$J(x, u) = \max_u \left\{ \sum_{k=0}^{k=s} \int_{\frac{k}{s+1}}^{\frac{k+1}{s+1}} \xi_k(s+1) e^{-r_k(a_k + b_k \tau)} L_k(x(\tau), u(\tau)) d\tau \right\}, \tag{37}$$

and subject to (28)–(31) and rescaled according to (35). For the purposes of exposition, we can fix the transition points t_k to reflect the exogenous (and often sub-optimally protracted) nature of introducing and implementing new policies.

Appendix C. Data sources of Section 5.1

Most of the indices are obtained from the ECB website, but others are from Eurostat.

(1) Industrial Production Index - Eurostat <https://ec.europa.eu/eurostat/databrowser/view/teis080/default/table?lang=en>

(2) Yearly change in HCP - Eurostat https://ec.europa.eu/eurostat/data/database?node_code=prc_hicp_manr

(3) Borrowing Cost - ECB <https://sdw.ecb.europa.eu/browse.do?node=9691556>

(4) Financial Stress Index - ECB

<https://sdw.ecb.europa.eu/browse.do?node=9693347>

(5) Credit flow - ECB (Financial transactions (flows)), MFIs excluding ESCB reporting sector - Adjusted loans to non-financial corporations (changing composition)

<https://sdw.ecb.europa.eu/browseExplanation.do?org.apache.struts.taglib.html>

TOKEN=06f55a1018733dc7663fa349bf2ee690&df=true&ec=1&dc=&oc=0&pb=1&rc=0&DATASET=0&removeItem=&removedItemList=&mergeFilter=&activeTab=BSI&showHide=&MAX_DOWNLOAD_SERIES=500&SERIES_MAX_NUM=50&node=SEARCHRESULTS&q=BSI.M.%3F.Y.A.A20T.%3F&type=series&SERIES_KEY=117.BSI.M.DE.Y.A.A20T.A.4.U2.2240.Z01.E

(6) ECB Marginal lending facility (% Annual) - ECB

<https://sdw.ecb.europa.eu/browse.do?node=9691107>

(7) Loan new firms - ECB

<https://sdw.ecb.europa.eu/browseSelection.do?node=9691395>

(8) CB Assets - ECB

<https://www.ecb.europa.eu/pub/annual/balance/html/index.en.html>

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