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RESEARCH



On the Correlation and Predictability of Topological Measures in Transportation Networks

Rudy Milani¹ · Marian Sorin Nistor¹ · Maximilian Moll¹ · Stefan Pickl¹

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Abstract

The computation of topological measures in large-scale complex networks, such as those found in transportation systems, is often a resource-intensive process. These measures, however, are critical for a comprehensive understanding of network structures and for optimizing their design. A key challenge lies in selecting the appropriate metrics that encapsulate the essential information of the network, thereby reducing the computational burden. Traditional methods involve identifying correlations between various topological measures to infer missing data. In this paper, we introduce an enhanced analytical framework comprising three stages aimed at selecting a subset of metrics to efficiently summarize network characteristics and predict measures that are costly to compute. The methodology involves: a correlation analysis of topological metrics; a principal component analysis to reduce dimensionality and highlight the essential features; and the application of SHAP and recursive feature elimination to assess the predictive significance of each metric. We demonstrate the utility of this approach using metro and road networks from 46 cities in the EU/EEA region, yielding promising results in identifying relationships between metrics and predicting missing data.

Keywords Network analysis · Metro networks · Road networks · Correlation analysis · Principal component analysis · SHAP · Recursive feature elimination

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1 Introduction

A wide set of real-world systems, ranging from neurobiology to statistical physics [26], has been successfully modeled using complex network theory [6]. This approach allows for an in-depth analysis of the structural properties underlying these systems, which is crucial for understanding their dynamics and optimizing their functionality. To this end, various topological measures and indices have been employed to capture critical network characteristics [12]. However, the computational complexity of these measures becomes a significant challenge, especially when dealing with large-scale networks. Consequently, selecting a subset of informative metrics that can adequately represent the network while minimizing computational costs is essential.

Several studies have explored the relationships between topological measures to address this challenge, often focusing on specific domains such as internet AS- and router-level networks [14, 18], brain networks [16, 17], and other real-world systems [8]. For example, [19] proposed a framework for selecting a subset of topological measures that can sufficiently describe network properties. Subsequently, [12] advanced this methodology by calculating correlation coefficients and principal components across various network types. More recently, further authors took this pipeline and carried out experiments on medium/large-size datasets consisting of both modelgenerated graphs and real networks [2, 13]. These works have laid the foundation for identifying meaningful subsets of metrics that capture the essence of a network's topology.

However, the focus of the methodologies in the literature is restricted to the determination of the statistical relationship between the topological metrics. Thus, no insights on the possibility of predicting specific high-cost measures from a subset of selected metrics are produced. Nevertheless, in order to reduce the complexity burden of computing most of the topological metrics it is necessary to determine a standardized pipeline for the forecasting procedure.

Hence, this paper presents an extended analytical pipeline designed to identify key relationships between topological measures and evaluate those that summarize the most information of the network. We leverage correlation analysis, principal component analysis (PCA), and machine learning (ML) prediction techniques combined with SHAP and recursive feature elimination (RFE) to enhance the efficiency of network characterization. Our analysis spans transportation networks from 46 EU/EEA cities, with the aim of uncovering new insights into the relationships between topological measures and optimizing the prediction of costly metrics.

The primary contributions of this study are as follows:

- Enhancement of an existing analytical pipeline: We refined a pre-existing methodology [12] for analyzing and selecting relevant features that effectively summarize the fundamental characteristics of complex networks. This improvement involved extending the PCA framework by incorporating factor loadings and introducing an additional phase dedicated to selecting the optimal subset of features to enhance the prediction accuracy of the un-computed metrics.
- Comprehensive analysis of topological metrics: We conducted an in-depth examination of the topological metrics and their correlations within metro and road



networks across various European cities, offering insights into the structural properties of these transportation networks.

• Identification of high-accuracy predictive subsets: We uncovered a subset of correlated features that are instrumental in achieving high-accuracy predictions in the context of transportation networks.

The remainder of this paper is organized as follows. In Section 2, we introduce the topological measures and networks used in the analysis. In Section 3, we present the main contribution of the paper: the extended methodology proposed. In Section 4, we first apply the new approach to metro networks, and then to road networks. In this part, we gather all the results of the corresponding steps together with a brief discussion. After reporting all the outcomes for both types of transportation networks, we focus on comparing the results obtained in Section 5. Finally, we summarize the paper and draw possible directions for future development.

2 Background

In this section, we first define the topological measures that are studied throughout this paper in Section 2.1. Then, we introduce the networks investigated, and the procedure that led us to obtain this data in Section 2.2.

2.1 Topological Measures

We mathematically define a real-world system as a graph G = (V, E) representing a finite and connected network with the set of vertices V = V(G), and the set of edges E = E(G). Furthermore, we assume the network to be connected and without selfloops. In the sequel, we present the set of topological measures considered in this paper, along with explanations about their relevance in characterizing the network topology.

The first parameter introduced is the most basic one: the network density (ND) $ND(G) = \frac{2|E|}{|V|(|V|-1)}$, which is the result of the ratio between the actual number of edges and the maximum possible ones. A high ratio denotes a dense network, whereas a low ratio denotes a sparse network [28]. The degree $D_G(v)$, which counts the number of adjacent edges for the vertex v in G, is considered an index of the connectivity strength of a vertex in the network [28]. However, we need topological measures that can describe the whole network. For this purpose, we consider the average degree (AD) $AD(G) = \frac{\sum_{v \in V} D_G(v)}{|V|}$

Another valuable feature for the transportation networks is the path length $d_G(v, w)$ between a pair of vertices $v, w \in G$, which is the length of any shortest path as a number of edges [27]. From this, we can derive the average path length (AVP) of a network G as $AVP(G) = \frac{\sum_{v \in V} \sum_{w \in V \setminus \{v\}} d_G(v, w)}{|V|(|V|-1)}$ [27] which is the average of the shortest paths of all possible pairs of vertices in a network. Closely related to the path length is the eccentricity of a vertex $\epsilon(v, G) = \max_{w \in V \setminus \{v\}} d_G(v, w)$, which represents the distance of the longest shortest path between that vertex and any other vertex in the network [10]. For our scopes, we use the average eccentricity (VE) of a network Gdefined as $VE(G) = \frac{\sum_{v \in V} \epsilon(v, G)}{|V|}$.



We also consider the assortativity parameters which provide more insight into the correlation between vertices. Commonly studied are the average nearest neighbor degree [1, 24] and the degree assortativity [20, 21]. Average nearest neighbor degree nn(k, G) of vertices of degree k in graph G is defined as $nn(k, G) = \sum_{l=1}^{|V|-1} l P_G(l|k)$ where $P_G(l|k)$ is the conditional probability that a vertex of degree k is linked with a vertex of degree l in graph l. This aspect captures the dependencies between degrees of neighboring vertices in a network l in graph l formulation, we can then compute the average nearest neighbor degree (ANN) of a graph l as l and l and l and l are degree assortativity l and l are degree assortativity l and l are degree l are the degrees of the vertices connected with the edge l and l are degree l are the degrees of the vertices connected with the edge l are l and l are l are the degree l are the degree tend to connect more often (assortativity mixing, when l and l are degree l are the degree l are the

The clustering coefficient (CC) c(v,G) of a vertex captures how close its neighbors are to a complete network [29]. Given three vertices $v_i, v_j, v_k \in V$ having v_i connected with v_j and with v_k , it represents the likeliness that v_j is also connected with v_k . If v_j is also connected with v_k then the three vertices form a triangle — complete subgraph or closed triple, and the number of triangles in G that includes vertex v is denoted by $\delta(v,G)$. Otherwise, they form a triple — a path of length two from v_j to v_k with v_i as center vertex, and the number of triples in G that includes vertex v is denoted by $\tau(v,G)$. Hence the CC of a vertex v is defined as $c(v,G) = \frac{\delta(v,G)}{\tau(v,G)}$, where $\delta(v,G) = |\{(v_i,v_j,v_k) \in V^3|\{(v_i,v_j),(v_i,v_k),(v_j,v_k)\} \subseteq E\}|$ and $\tau(v,G) = |\{(v_i,v_j,v_k)\} \subseteq E\}|$. Therefore, we generalize this measure for a network G by considering the average over all the vertices: $CC(G) = \frac{\sum_{v \in V} c(v,G)}{|V|}$.

Remaining on the topic of clustering, we now introduce the rich-club coefficient $RC_k(G)$ which is a structural parameter of a network that captures the tendency of hubs of vertices to connect [5]. The vertices with a high degree also called rich vertices, have a higher likelihood to interconnect into subgraphs (clubs), when compared with low-degree vertices [5]. This measure is calculated using the following formula: $RC_k(G) = \frac{2|E|_{>k}}{|V|_{>k}(|V|_{>k}-1)}$, where $|E|_{>k}$ represents the number of edges between the vertices of degree greater than or equal to k, and $|V|_{>k}$ is the number of vertices with a degree greater than or equal to k.

Moving to centrality measures we focus on the vertex betweenness centrality BV(v, G) [11], or simply vertex betweenness (BV). This measure is a centrality parameter that determines the occurrence of a vertex on the shortest paths between any pair of vertices. In our study, we also consider the edge betweenness centrality BE(e, G) [11], or simply edge betweenness (BE), which is the equivalent of BV(v, G) for edges. Mathematically, these parameters can be expressed as

 $BV(v,G) = \frac{\sum_{s \neq v \neq t} \frac{\sigma_{st,G}(v)}{\sigma_{st,G}}}{(|V|-1)(|V|-2)} \text{ and } BE(e,G) = \frac{\sum_{s \neq t} \frac{\sigma_{st,G}(e)}{\sigma_{st,G}}}{(|V|-1)(|V|-2)} \text{ where } \sigma_{st,G}(\cdot) \text{ is the number of the shortest path (in } G) \text{ between the vertices } s \text{ and } t \text{ that includes the input vertex or edge (depending on the measure used) and } \sigma_{st,G} \text{ is the number of all the shortest}$



path (in G) between the vertices s and t. As mentioned before, we can then define the corresponding general metrics for the network G just by considering the average.

Related to the connectivity of a network, we have the algebraic connectivity (AC) AC(G) which represents the second smallest eigenvalue of the Laplacian matrix of G [7]. This parameter is useful for measuring how difficult it is to cut a graph into independent components (not connected anymore to the remaining part of the graph). The last measure that we used in this work is the average vertex coreness AVC(v, G), which gives an insight into the position of the vertex v and indicates its influence on spreading dynamics [3]. In fact, for larger coreness values we have vertices that are more central in the network. The average vertex coreness is defined as the maximum k such that this vertex is still in the k-core but removed in the (k+1)-core. Finally, in order to have a measure related to the full network, we compute the average and define AVC(G) as the average coreness (AVC) of G.

2.2 Transportation Networks Data

This paper analyzes transportation networks from 46 European cities within the EU/EEA region, specifically focusing on metro and road networks. A comprehensive list of the cities included in the analysis is provided in Appendix. An example of both network types for a single city, Munich, is shown in Fig. 1, where the metro and road networks are depicted graphically. In these graphs, vertices represent either metro stations or street junctions, while edges denote the direct connections between these points. The following paragraphs provide further details on the data collection and processing methodologies essential for the analysis.

The metro network data were collected individually from the official websites of metro operators. In cases where the available information was insufficient, external sources were used. Each of the 46 metro networks is represented by an individual table that lists metro stations per line in each city. A custom algorithm was then developed to generate adjacency matrices based on these tables. In the case of cities with more than one metro system, such as London (which operates both the London Underground



Fig. 1 Graph visualization of Munich metro on the left and of road system on the right



and the Docklands Light Railway), the networks were combined, with shared stations merged into one dataset. This method was applied to other cases, such as the Amsterdam Metro, where light rail line 51 was treated as part of the regular metro network.

Although all networks are required to be connected, exceptions were made for cities like Berlin and Naples, where some metro lines operate as isolated components. In these cases, only the largest connected component was considered for the analysis. A similar approach was taken for networks with walking connections, such as Valencia Metro's line 10, where transfers occur by walking between nearby stations. These connections were treated as regular edges, adding new links between previously unconnected components.

The process of obtaining road network data was more streamlined than for metro networks. The OpenStreetMap package for Python [4] was used to extract road data for the selected cities. However, preprocessing was required before the data could be analyzed. Specifically, we transformed the directed road networks into undirected graphs, eliminating self-loops caused by circular roads. The structure of these graphs was simplified by retaining only the vertices that represent junctions of multiple streets.

3 Methodology

In this section, we propose an extended (from [12]) analytical pipeline designed to analyze the relationships between topological measures in complex networks. This methodology is particularly well-suited for large, complex networks where the computational cost of calculating all topological measures is prohibitive. By understanding the relationships among these measures, it becomes possible to limit the number of metrics that need to be computed, thereby reducing computational demands while maintaining a comprehensive understanding of the network structure.

The examination of feature interdependencies encompasses various methodologies, ranging from elementary statistical correlation to sophisticated ML assessments. Therefore, our comprehensive approach incorporates three distinct analytical phases: correlation analysis, PCA, and training of the predictive models and feature selection.

The preliminary step in the proposed methodology consists of the computation of the topological measures discussed in Section 2.1. These parameters serve as the cornerstone upon which all the subsequent analysis are built.

Moving the focus on the actual pipeline, the first stage, represented in the yellow boxes of Fig. 2, involves a statistical analysis to examine the correlations between the topological measures. By calculating pairwise correlations between metrics, we identify dependencies and potential redundancies. This analysis enables the identification of clusters of correlated measures, facilitating dimensionality reduction while preserving the network's essential structural characteristics [22]. As observed in previous studies [12], the relationships between topological measures may vary across different contexts, making it crucial to reassess these relationships for each application.

In the second step, shown in the orange boxes of Fig. 2, PCA is employed to reduce the dimensionality of the dataset by transforming the correlated measures into a smaller set of uncorrelated variables, known as principal components [23]. This process captures the most significant patterns of variation within the data, allowing the complex



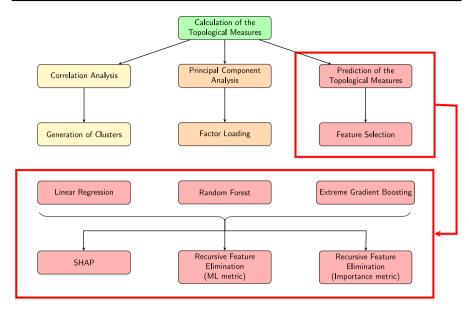


Fig. 2 Proposed pipeline for the analysis of the topological metrics in networks

network's structure to be summarized using a reduced set of orthogonal variables. To ensure accurate results, the data are first standardized to avoid skewed eigenvectors due to varying magnitudes. The factor loadings of the topological measures are also computed, providing insight into the importance of each variable in relation to the principal components. Thereby, this procedure enhances the interpretability of the PCA results and provides further discoveries on the clusters obtained in the correlation analysis.

The final stage, portrayed in the red boxes of Fig. 2, considers the application of ML algorithms to predict the topological measures based on a subset of features. Specifically, linear regression, random forests, and extreme gradient boosting (XGB) are the predictive models used in our study. The selection of these ML algorithms is justified by the limited amount of data points. Linear regression, random forests, and XGB perform efficiently when trained considering small datasets while more complex algorithms, e.g., neural networks, achieve remarkable results only when properly trained with large amount of data.

Additionally, two feature selection techniques are employed: SHAP and RFE. SHAP is a game-theoretic approach to explain the contribution of each feature to the model's predictions [15], while RFE iteratively removes the least important features based on their contribution to the prediction accuracy (evaluated using the coefficient of determination R^2) or the importance weights of the features [9]. These methods allow for the identification of the most informative subset of features, optimizing the trade-off between prediction accuracy and computational efficiency.

By integrating these three components (correlation analysis, PCA, and predictive modeling) the proposed methodology offers a comprehensive approach to efficiently analyze complex networks. It not only reduces the computational burden but also provides a systematic way to predict uncalculated topological measures, ultimately



enhancing our understanding of the network's structure. Moreover, the most appropriate subset of measures summarizing the underlying structure of the complex networks is determined from the comparison of the statistical results and ML outputs.

4 Results

In this section, we present the results of our analysis for both metro and road networks, following the methodology described previously. The computations were performed using parallel processing on a 2x AMD EPYC 7763 CPU, with 256 cores ($2 \times 64 + \text{hyperthreading}$) and 1007GB of RAM. The analysis utilized Python libraries such as *NetworkX 3.1, SHAP 0.41.0*, and *Scikit-learn 1.2.2*.

4.1 Metro Networks

The first step in this analysis was to calculate the topological measures. The values obtained for each metric are listed in Appendix in Table 15, while the computational time for each metric is shown in Table 16. These computations reveal that, despite their complexity, the calculations were completed in under a second due to the relatively small size of the metro networks. The number of vertices ranged between 8 and 309, and the number of edges between 7 and 356, classifying these networks as small to medium in size.

A key observation in this analysis is that the CC for most metro networks is zero. This outcome is likely due to the linear structure of most metro systems, where stations are typically connected in a sequence, forming lines rather than triangles, which are necessary for high clustering values.

Moving to the correlation analysis, Pearson's correlation coefficients for each pair of topological measures are shown in Fig. 3. Strong correlations (absolute values greater than 0.60, as generally considered in the literature [25]) were observed in 25 out of 66 possible comparisons, with 11 positive and 14 negative correlations. To better visualize these relationships, we generated a correlation graph shown in Fig. 4, which revealed two principal clusters. The first cluster includes BE, BV, ND, and AC, while the second contains AVC, ANN, CC, DA, AVP, VE, and AD. The CC and DA are only weakly connected to this second cluster, and the RC remains largely uncorrelated.

The next step in the analysis involved calculating the principal components using PCA. The explained variance for the first 12 principal components is illustrated in Fig. 5. After standardization, we observe that the first principal component accounts for 61.48% of the total variance. However, to adequately represent the dataset (i.e., capturing at least 95% of the explained variance), it is necessary to consider five principal components.

Examining the factor loadings for the topological measures in the first five components, as shown in Table 1, we find that the first principal component is predominantly influenced by a cluster of measures, including ND, BE, BV, and AC. As previously discussed in the correlation analysis, this cluster exhibits a strong negative correlation with another major cluster centered around AD. The second principal component



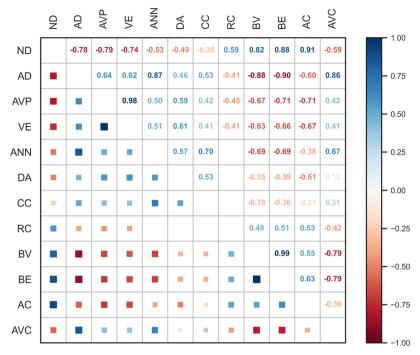


Fig. 3 Pearson's correlation coefficients of the topological measures for the metro networks (colored in blue for the positive correlation and red for negative correlation)

highlights isolated features, showing a strong positive correlation with the CC and the RC, along with a moderate positive correlation with the DA. This suggests that the first component already encapsulates significant information from the second cluster. The remaining three components consist of more diverse combinations of features, but they do not reveal any substantial or influential correlations.

After conducting various statistical analyses of the relationships between features in metro networks, the focus has been turned to predicting individual features using the remaining ones as inputs. Specifically, we compared the performance of linear regression, random forest, and XGB for forecasting, while employing SHAP and RFE for feature selection. The RFE process was conducted using both feature importance weights (RFEI) and changes in the R^2 metric (RFEM). To validate the analysis, we divided the full dataset of 11 topological metrics (with one as the output) from the 46 cities into training and test sets, following a 3: 1 ratio. This resulted in a training set of 34 data points and a test set of 12.

We report the results in terms of the selected features and R^2 values, starting with the linear regression model. For brevity, we focus on the most significant findings, while the full results for the selected feature sets are provided in Table 2, and the corresponding R^2 values are shown in Table 3. Notably, the models struggled to predict features that were disconnected or isolated from the main clusters, such as the DA, CC, and RC. In these cases, the R^2 values were negative, indicating the difficulty of accurately predicting these features. Indeed, these topological measures



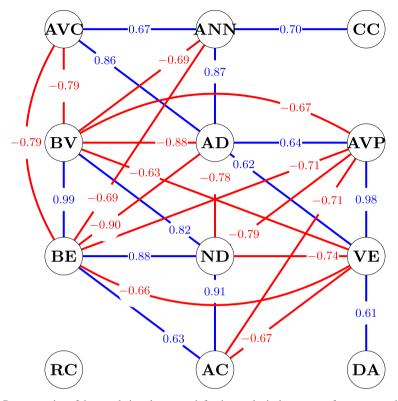


Fig. 4 Representation of the correlation cluster graph for the topological measures of metro networks

| Components | Explained variance | |
|------------|--------------------|-----------------------------------|
| 1 | 0.614798 | 1.00 |
| 2 | 0.137898 | 80 up 5 |
| 3 | 0.115102 | 0.95 |
| 4 | 0.046124 | 0.90 |
| 5 | 0.037489 | 0.85 |
| 6 | 0.019499 | 0.80 |
| 7 | 0.016221 | 0.70 |
| 8 | 0.009424 | 0.65 |
| 9 | 0.002221 | 0.60 |
| 10 | 0.000851 | 2 4 6 8 10 1. |
| 11 | 0.000356 | Number of components |
| 12 | 0.000016 | (b) Cumulative Explained Variance |

Fig. 5 Explained variance for the first 12 principal components for the metro networks

(a) Explained Variance for each Principal Com-



Table 1 Factor loading of the topological measures in the metro network

| | ND | AD | AVP | VE | ANN DA | | CC | RC | BV | BE | AC | AVC |
|----------------------|----------------------|--|-----------|-----------|-------------------|-----------|---|-----------|---------------------|-----------|-------------|-----------|
| Component 1 | 0.916753 | Component 1 0.916753 -0.927415 -0.851200 -0.824763 | -0.851200 | -0.824763 | -0.769048 | -0.581155 | -0.769048 -0.581155 -0.522792 0.508777 | | 0.901286 | 0.933128 | 0.762366 | -0.739380 |
| Component 2 | Component 2 0.248772 | 0.069502 | -0.011106 | 0.035675 | 0.485519 0.561569 | 0.561569 | 9 0.651170 0. | 886502 | 0.135966 | 0.145240 | 0.234948 | -0.135463 |
| Component 3 | -0.175218 | Component 3 -0.175218 -0.309114 0.401758 0.412831 | 0.401758 | 0.412831 | -0.318305 0 | .437010 | -0.071592 | -0.054325 | 0.277288 | 0.210422 | -0.413772 - | -0.574792 |
| Component 4 0.169109 | 0.169109 | -0.046640 0.161675 | 0.161675 | 0.166005 | -0.044749 | -0.226866 | $-0.226866 \ 0.460966 \ -0.380784 \ 0.098675$ | -0.380784 | 0.098675 | 0.118839 | 0.184870 | -0.005339 |
| Component 5 0.138479 | 0.138479 | -0.068125 0.277132 | 0.277132 | 0.318558 | -0.059133 | -0.069858 | -0.228573 0.197411 | | -0.143254 -0.090488 | -0.090488 | 0.340573 | 0.058970 |



 Table 2
 Selected features for linear regression

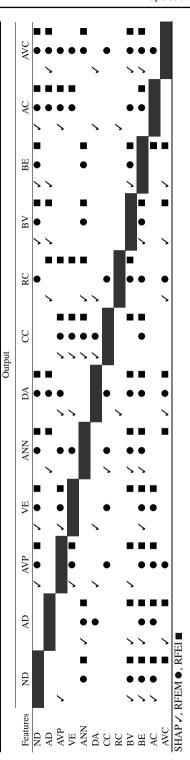




Table 3 Linear regression R^2

| | ND | AD | AVP | VE | ANN | DA | CC | RC | BV | BE | AC | AVC |
|--------------|----------|----------|----------|----------|----------|-----------|------------|-----------|----------|----------|----------|----------|
| R_{SHAP}^2 | 0.950745 | 0.948226 | 0.976000 | 0.972994 | 0.791868 | -0.467867 | -24.254429 | -0.926574 | 0.999126 | 0.999338 | 0.288190 | 0.630156 |
| R_{RFEM}^2 | 0.950686 | 0.831016 | 0.965388 | 0.964198 | 0.516257 | -2.170150 | -22.421979 | -2.628122 | 0.998289 | 0.998659 | 0.812253 | 0.743163 |
| R_{RFEI}^2 | 0.953685 | 0.937340 | 0.982685 | 0.980552 | 0.526916 | -2.699641 | -21.206942 | -1.713796 | 0.998530 | 0.998692 | 0.814438 | 0.762572 |
| R^2 | 0.947773 | 0.906828 | 0.969001 | 0.960048 | 0.683127 | -1.401943 | -18.993547 | -1.036491 | 0.997830 | 0.998360 | 0.787994 | 0.580374 |



are inherently unpredictable through this set of information as attested by the poor correlations shown with the other variables. For this reason, it is necessary to extend the data included in the analysis to obtain acceptable results in their prediction.

A further observation arises from the feature selection process: only for the CC prediction did we identify a clear rule for selecting the subset of features. For the other metrics, the choice of input features varied depending on the method used, largely due to the absence of a definitive optimal subset. This issue was also encountered in the experiments with random forest and XGB.

For the remaining topological measures, prediction errors were generally low. It is worth noting that in some cases, the full model did not provide optimal forecasts, particularly for the ANN and AVC. However, when we compared these metrics with those obtained by selecting specific features, we found that the coefficient of determination could be improved to 0.79 using SHAP for ANN, and to 0.76 using RFEI for AVC. Overall, prediction errors did not vary significantly between using the full dataset or a reduced set of features.

Similar conclusions can be drawn from the random forest results. The selected features and corresponding coefficients of determination for the test set are listed in Tables 4 and 5. In this case, the recursive feature elimination methods tended to include a larger number of features. However, this did not result in improved prediction accuracy. Interestingly, SHAP achieved comparable performance using only four features at a time, indicating that the output can be well approximated with a minimal set of relevant inputs.

Another noteworthy observation is the comparison between the \mathbb{R}^2 values for linear regression and random forest. In most cases, linear regression outperformed random forest. Exceptions occurred with the DA, CC, and AC, where random forest provided better approximations. However, as observed in the linear regression analysis, DA and CC belong to the class of isolated features, making it challenging to select appropriate input features for their prediction. For AC, despite the random forest's better performance, the coefficient of determination for linear regression was still scientifically acceptable at 0.79.

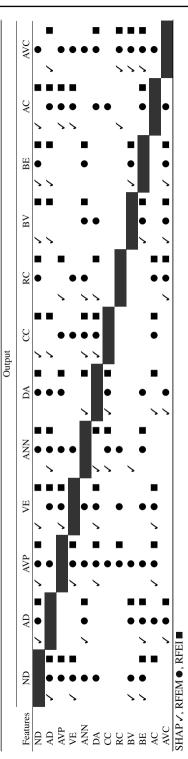
Finally, the selected features varied depending on the regression model used. Only two instances shared the exact same subset of features for both linear regression and random forest: BV with SHAP, and BE with SHAP and RFEI. In other cases, the models selected different combinations of topological measures, although some overlap was observed when predicting the same output.

The considerations mentioned earlier were also validated through experiments using the XGB method. The variables selected for this context are provided in Table 6, with the corresponding R^2 values shown in Table 7. Notably, there was an exceptional case involving the isolated feature DA. For this feature, the RFEI model produced a positive coefficient of determination, a result that had not been achieved with any previous method. Additionally, for the AVC, the model was able to approximate the output reasonably well using only the features selected by the RFEM method.

Moreover, when comparing the results with those obtained using SHAP, it becomes clear that the ANN is a key feature. ANN is included in the RFEI model but not in the SHAP model, highlighting its importance. A similar observation can be made



 Table 4
 Selected features for random forest



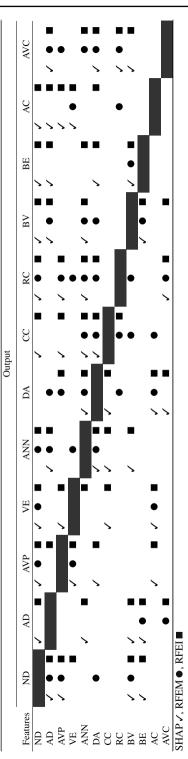


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| | ND | AD | AVP | VE | ANN | DA | CC | RC | BV | BE | AC | AVC |
|--------------|----------|----------|----------|----------|-----------|-----------|------------|-----------|----------|----------|----------|----------|
| R_{SHAP}^2 | 0.938164 | 0.889520 | 0.950552 | 0.921131 | 0.190528 | -0.059183 | -8.206427 | -3.199260 | 0.894742 | 0.936816 | 0.964030 | 0.270231 |
| R_{RFEM}^2 | 0.926842 | 0.891924 | 0.940929 | 0.927943 | 0.206184 | -0.287366 | -13.365895 | -1.879339 | 0.918395 | 0.952274 | 0.957341 | 0.064595 |
| R_{RFEI}^2 | 0.949491 | 0.890548 | 0.951762 | 0.920138 | 0.243719 | -0.096437 | -7.798809 | -3.212647 | 0.900252 | 0.941660 | 0.960443 | 0.239398 |
| R^2 | 0.927336 | 0.894174 | 0.942309 | 0.923821 | -0.060730 | 0.108959 | -9.004830 | -3.366023 | 0.906618 | 0.933599 | 0.942903 | 0.155532 |



 Table 6
 Selected features for XGB





| Table 7 XGB R | 7 |
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| | ND | AD | AVP | VE | ANN | DA | CC | RC | BV | BE | AC | AVC |
|--------------|----------|----------|----------|----------|-----------|-----------|-----------|-----------|----------|----------|----------|----------|
| R_{SHAP}^2 | 0.956054 | 0.905015 | 0.922705 | 0.929900 | 0.125844 | -0.073378 | -5.858772 | -5.879550 | 0.946106 | 0.925570 | 0.946009 | 0.121962 |
| R_{RFEM}^2 | 0.898683 | 0.742185 | 0.898391 | 0.917288 | -0.067203 | -0.147108 | -6.941562 | -4.420400 | 0.902773 | 0.975978 | 0.918739 | 0.570213 |
| R_{RFEI}^2 | 0.902764 | 0.854853 | 0.918221 | 0.891585 | 0.342405 | 0.294456 | -5.429628 | -4.563038 | 0.877861 | 0.915890 | 0.928731 | 0.427139 |
| R^2 | 0.915969 | 0.793333 | 0.921423 | 0.891544 | 0.291111 | -0.360931 | -1.615500 | -4.533205 | 0.885512 | 0.910154 | 0.929637 | 0.367852 |



when comparing RFEI selections, where AVP was chosen instead of BV, resulting in a reduction of approximately 0.14 in the R^2 value.

In conclusion, selecting a specific subset of topological measures can improve accuracy while reducing the time required to compute excluded metrics. However, it is important to note that this selection is influenced by the predictive model used. For metro networks, we found that linear regression provided the best performance across both the restricted and full datasets.

4.2 Road Networks

In this subsection, we present the results from the analysis of road networks for the same 46 European cities previously analyzed in the metro study. However, the road networks differ significantly in scale compared to the metro networks. The size of these graphs ranges from Lausanne, the smallest network with 1, 358 vertices and 1, 859 edges, to London, the largest, with 127, 540 vertices and 160, 010 edges. As a result, this section focuses on the analysis of large-scale graphs. In Appendix, detailed information on the fundamental measures calculated for each network is provided in Table 17, while Table 18 outlines the computation times for each topological measure across cities.

One notable observation is the dramatic increase in computational time compared to metro networks. For instance, calculating BE and BV requires several hours, while VE and AVP can be computed in a few minutes. Consequently, identifying and selecting the most representative features of a graph for computation becomes a critical task to optimize efficiency.

The first step of the proposed analytical pipeline involves calculating the correlation coefficients, illustrated in Fig. 6. In this case, we observed that the interactions between features were relatively weak. Out of 66 possible pairs, only 10 coefficients had an absolute value greater than 0.60. This aspect is justified by the structural difference between the two transportation networks: metro networks generally have simpler, linear structures, where stations are sequentially connected along defined lines, while, road networks exhibit a more complex topology, characterized by a dense interconnection of junctions and streets. Additionally, the latter networks span much larger areas, containing significantly more vertices and edges (up to hundreds of thousands). Such a large scale introduces greater variability and structural heterogeneity. This complexity results in weaker correlations among the topological metrics, since there are many possible routes and structural variations leading to less uniformity across the network. Moreover, all of the strong correlations were positive, while the negative correlations were of moderate magnitude [25], falling within the range of 0.40 to 0.60. Despite this, we were still able to clearly define multiple clusters of topological measures, shown in Fig. 7.

The first prominent cluster comprises the AD, DA, AVC, and ANN. Within this cluster, the relationships are strong, with all four variables being closely interconnected. A second visible group includes ND, BE, and BV. This set of features can be expanded by incorporating AC and the RC through positive medium correlations.

Additionally, this extended cluster has a negative influence on the final pair of metrics: AVP and VE. All of the medium negative correlations occur between these



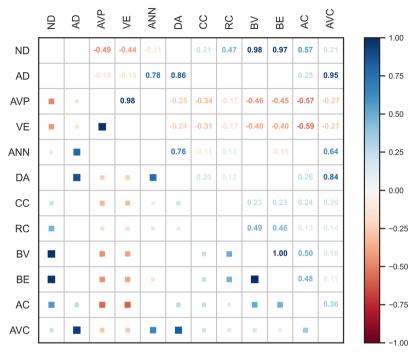


Fig. 6 Pearson's correlation coefficients of the topological measures for the road networks (colored in blue for the positive correlation and red for negative correlation)

two groups. Furthermore, AVP and VE exhibit a very high correlation with each other, forming the last identifiable group of features in road networks. The CC, in contrast, remains completely isolated from all other clusters in this analysis.

To assess the relative importance of each cluster, we examined the results of the PCA. First, it is noteworthy to observe the explained variance of each principal component, as shown in Fig. 8. Unlike the metro network analysis, the first two principal components account for substantial variance, with values of 0.39 and 0.28, respectively. Despite the high explained variance of these components, at least five orthogonal directions are needed to reach an acceptable total variance of approximately 0.93.

Subsequently, we computed the factor loadings of the topological measures, as presented in Table 8. From this analysis, we observe that the first principal component is positively correlated with the cluster comprising VE and AVP. It also shows a negative correlation with the extended group of features. These correlations, particularly in absolute terms, are the strongest, confirming that these two sets of topological measures are closely interconnected.

The second principal component provides insights into the remaining highly correlated cluster. The subsequent components primarily involve combinations of different measures, resulting in medium-level interactions with the other features. Notably, the fourth principal component exhibits a significant factor loading for the CC, an



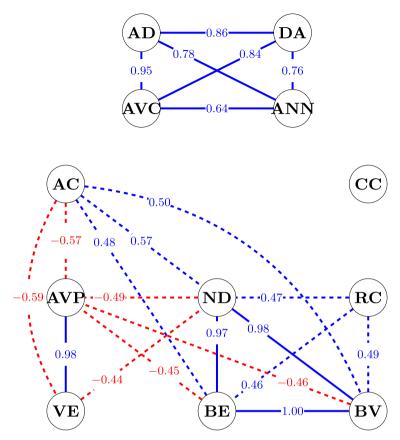


Fig. 7 Representation of the correlation cluster graph for the topological measures of road networks. The dotted lines represent a medium correlation (absolute value between 0.40 and 0.60) while normal lines are the strong correlations (absolute value > 0.60)

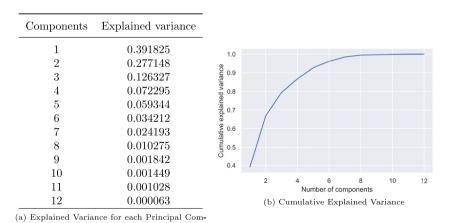


Fig. 8 Explained variance for the first 12 principal components for the road networks

Table 8 Factor loading of the topological measures in the road network

| | ND | AD | AVP | VE | ANN DA | DA | CC | RC | BV | BE | AC | AVC |
|-------------|-----------------------------------|-----------|-----------|-----------|---------------------------------------|--|--------------------|-----------|-----------|-----------|-----------------------------------|-----------|
| Component 1 | Component 1 -0.830784 -0.433530 (| -0.433530 | 0.754811 | 0.725210 | -0.219527 | -0.453105 -0.374711 -0.452145 -0.808894 | -0.374711 | -0.452145 | -0.808894 | -0.777577 | -0.733414 | -0.576986 |
| Component 2 | Component 2 0.422919 -0.858633 | -0.858633 | -0.119435 | -0.094536 | -0.832054 | -0.829109 | 0.055400 | 0.136075 | 0.434472 | 0.498413 | 0.035254 | -0.739405 |
| Component 3 | Component 3 0.289035 | 0.119248 | 0.534376 | 0.560549 | 0.260029 | 0.260029 -0.017133 -0.433386 0.547254 | -0.433386 | 0.547254 | 0.335170 | 0.311539 | -0.223995 | 0.059965 |
| Component 4 | Component 4 0.074395 | 0.014545 | 0.275185 | 0.330661 | -0.099629 0.073856 | | 0.757944 -0.188115 | -0.188115 | 0.110463 | 0.110640 | -0.107074 | 0.124782 |
| Component 5 | Component 5 -0.156346 -0.108907 | -0.108907 | -0.155908 | -0.157396 | -0.155908 -0.157396 0.021430 0.103035 | | 0.275993 | 0.608491 | -0.100855 | -0.107787 | -0.107787 -0.376684 -0.074519 | -0.074519 |



otherwise isolated feature. Additionally, this component has a strong positive correlation with the DA, suggesting a potential relationship between these two metrics.

Following the same training approach used for the metro networks, we validated the earlier results through the prediction phase of the analytical pipeline. In this case, we focus on the most significant findings, while the remaining outcomes are presented in the accompanying tables.

The first model considered was linear regression and the corresponding results are reported in Tables 9 and 10. As shown in Table 10, the predictions did not consistently yield optimal performance. A notable example is the prediction of the ANN. Despite being a highly correlated feature, it remains challenging to predict accurately, regardless of the input features selected. Table 9 demonstrates that all measures correlated with ANN were chosen as key inputs, yet the R^2 value remained negative. Similar results were observed for the CC, though in this case, the poor prediction accuracy can be attributed to the isolation of CC from other feature clusters.

Overall, the trend indicates that prediction accuracy improves with careful feature selection. This was confirmed across multiple tests, where the coefficient of determination increased by approximately 0.25 when a specific set of features was selected.

However, this strong result is not reflected in the other two models (see Tables 11, 12, 13 and 14). In particular, with random forest, there were no significant changes (less than 0.10) when using different subsets of features, as shown in Table 12. The only exceptions, where performance improved beyond this threshold, were for AVP and CC, both using SHAP.

An interesting observation arises when examining the selected features listed in Table 11. The choice of topological measures is fairly consistent across the different methodologies applied. This consistency is also mirrored in the R^2 results: using the same subset of inputs tends to produce similar forecasting outcomes. Furthermore, since the prediction errors are comparable to those obtained with the full model, the most informative features have been successfully selected.

Before transitioning to the next model analysis, it is worth noting the results related to the ANN. While the coefficient of determination was always positive (around 0.64), this value is still not satisfactory.

Better results were achieved with the XGB model. The topological measures selected for each prediction using XGB are presented in Table 13, while the corresponding coefficients of determination from the testing phase are listed in Table 14. As mentioned earlier, the RFEM method yielded good performance in predicting the ANN. Specifically for the XGB case, we observed a notable difference between RFEM and SHAP. However, when comparing SHAP with RFEI (which performs similarly to the full model), the only variation in feature selection was the inclusion of AVP. Despite the lack of a significant correlation between ANN and AVP, it is clear that AVP has a considerable influence on the model's ability to predict ANN.

Overall, the accuracies achieved with XGB and random forests were comparable. However, as in previous cases, poor results were obtained for isolated features, reinforcing the importance of directly calculating these metrics.



Table 9 Selected features for linear regression

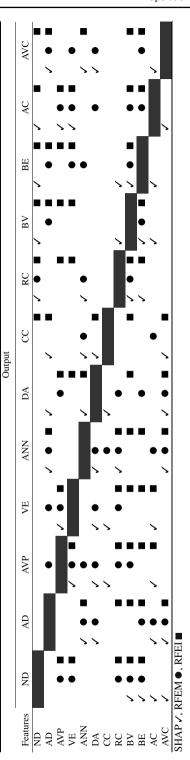




Table 10 Linear regression R^2

| | ND | AD | AVP | VE | ANN | DA | CC | RC | BV | BE | AC | AVC |
|--------------|----------|----------|----------|----------|-----------|----------|-----------|-----------|----------|----------|----------|----------|
| R_{SHAP}^2 | 0.982941 | 0.943263 | 0.925412 | 0.921262 | -1.170507 | 0.643634 | -0.134785 | -0.053726 | 0.996018 | 0.981424 | 0.422221 | 0.894990 |
| R_{RFEM}^2 | 0.763239 | 0.947141 | 0.829198 | 0.902390 | -1.614260 | 0.346339 | -0.169483 | 0.044717 | 0.996673 | 0.996369 | 0.325781 | 0.890164 |
| R_{RFEI}^2 | 0.763239 | 0.880538 | 0.876592 | 0.882708 | -1.763658 | 0.621421 | -0.196800 | 0.050838 | 0.996083 | 0.995319 | 0.463975 | 0.920215 |
| R^2 | 0.748714 | 0.730139 | 0.843971 | 0.852106 | -1.738818 | 0.418725 | -1.356248 | 0.131662 | 0.998584 | 0.997837 | 0.102107 | 0.621594 |



Table 11 Selected features for random forest

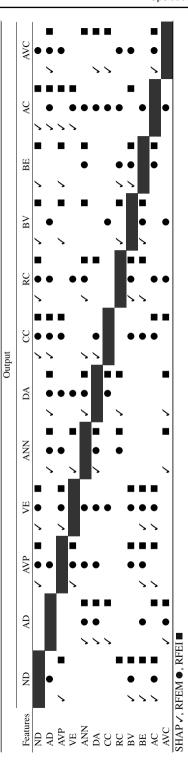




Table 12 Random forest R^2

| ומחוב ול | I able 12 Namuolii 1010st A | V 1 | | | | | | | | | | |
|--------------|-----------------------------|-------------------|----------|----------|----------|----------|-----------|----------|----------|----------|----------|----------|
| | ND | AD | AVP | VE | ANN | DA | CC | RC | BV | BE | AC | AVC |
| R_{SHAP}^2 | 0.826513 0.786998 | 0.786998 | 0.835218 | 0.908193 | 0.647517 | 0.693857 | -0.114572 | 0.095183 | 0.933388 | 0.904024 | 0.490160 | 0.859182 |
| R_{RFEM}^2 | 0.724716 | 0.773535 | 0.901575 | 0.903498 | 0.663890 | 0.495067 | -0.340280 | 0.080388 | 0.913590 | 0.905001 | 0.409934 | 0.807671 |
| R_{RFEI}^2 | 0.816161 | 0.811773 | 0.849848 | 0.909967 | 0.615401 | 0.593065 | -0.242279 | 0.090905 | 0.950692 | 0.908551 | 0.492554 | 0.825163 |
| R^2 | 0.808640 | 0.808640 0.772625 | 0.786533 | 0.887647 | 0.646603 | 0.593095 | -0.268207 | 0.044489 | 0.916335 | 0.915817 | 0.457599 | 0.782199 |



Output ANN AVP

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Table 13 Selected features for XGB

Table 14 XGB R^2

| | ND | AD | AVP | VE | ANN | DA | CC | RC | BV | BE | AC | AVC |
|--------------|----------|----------|----------|----------|----------|----------|-----------|-----------|----------|----------|----------|----------|
| R_{SHAP}^2 | 0.909292 | 0.724856 | 0.903402 | 0.882579 | 0.291743 | 0.630523 | -0.530387 | -0.024946 | 0.961840 | 0.947753 | 0.463920 | 0.816234 |
| R_{RFEM}^2 | 0.805525 | 0.588517 | 0.902173 | 0.888480 | 0.846979 | 0.573294 | -1.113806 | 0.020257 | 0.943276 | 0.824694 | 0.447121 | 0.772353 |
| R_{RFEI}^2 | 0.845489 | 0.743405 | 0.853026 | 0.819157 | 0.794181 | 0.693664 | -1.275593 | -0.015827 | 0.933938 | 0.947408 | 0.408047 | 0.795994 |
| R^2 | 0.830658 | 0.719017 | 0.853922 | 0.800807 | 0.786731 | 0.526808 | -1.251441 | -0.000107 | 0.938814 | 0.949034 | 0.457871 | 0.794754 |



5 Discussion

This section provides a comparative analysis between the results obtained from the transportation networks analyzed here and those found in the existing literature. Beginning with the correlation analysis, certain recurring patterns are evident in both metro and road networks. Specifically, two primary clusters of features exhibit strong correlations in both network types. The first cluster, referred to as the Distance-Connectivity group, includes BE, BV, ND, and AC. In road networks, the correlations involving AC are of moderate strength. Nevertheless, this metric is included due to its multiple negative correlations with VE and AVP, similar to other measures within this group.

The second identifiable cluster consists of Degree-associated features, including AD, AVC, and ANN. For road networks, this cluster also includes a strong correlation with DA, a relationship that is not observed in metro networks. This discrepancy may stem from the structural differences between the two types of networks: metro systems typically consist of long, linear vertex sequences, while road networks exhibit more complex interconnections. As a result, road networks demonstrate more intricate dependency patterns between vertex degrees, offering deeper insights into their connectivity.

A third notable set of correlated features includes VE and AVP, which together form a Distance cluster. An important observation is the lack of significant correlations involving the CC and the RC. These two metrics do not exhibit notable correlations with other features, making them difficult to predict or approximate, as demonstrated in the forecasting analyses. Nonetheless, these measures have been shown to correlate with others in different types of networks [12], suggesting that their lack of significance in transportation networks may be context-specific.

The clusters of highly correlated features identified in this study can be compared with those reported in the literature. For instance, [12] identified four clusters based on distance, degree, intra-connectivity, and inter-connectivity. When comparing these findings with our results, the Distance-Connectivity group observed in our study appears to be a combination of the Distance and Intra-connectivity clusters from their work. Furthermore, the Degree cluster in our analysis includes the ANN rather than the CC. It is important to note that [12] study covered a broad range of networks, including infrastructure and social systems, which may explain the absence of some correlation links observed in our transportation network analysis. This highlights the fact that the relationships between topological measures are highly context-dependent, a conclusion supported by multiple authors [2, 16].

From the PCA, we gathered insights regarding the variance explained by a smaller number of principal components. Factor loadings enabled us to evaluate the relative importance of each feature. In metro networks, the first principal component was strongly correlated with the Distance-Connectivity group, while in road networks, it was more strongly linked to the Distance cluster. Notably, the first principal component explained 61% of the variance in metro networks, compared to only 39% in road networks. This difference is likely due to the varying scale and complexity of the networks analyzed; metro networks tend to be smaller or medium-sized, whereas road networks often encompass a significantly larger number of vertices and edges.



Consequently, it is necessary to consider multiple features in larger graphs, such as those representing road networks.

Finally, when comparing the features selected in the prediction phase with the identified correlation groups, some overlap is evident. However, the most correlated measures are not always selected for improving prediction accuracy. This can be attributed to the fact that having features outside of a specific cluster is often necessary to avoid redundancy. Over-reliance on highly correlated features may result in redundant information, which can limit the effectiveness of the predictive models. Conversely, metrics that exhibit multiple correlations are easier to predict, while those with fewer or no correlations remain challenging. The selection of features and prediction accuracy are thus highly dependent on the specific model and experiment. A generalized approach applicable to all scenarios is difficult to achieve. For instance, linear regression outperformed other models in metro networks, while XGB proved more effective for road networks. Overall, selecting an appropriate subset of features for training machine learning models enhances the predictive performance and reduces computational complexity.

6 Conclusion and Future Work

In this paper, we present an enhanced analytical pipeline for network analysis. Our approach is structured into three distinct phases. First, we compute the correlation coefficients of the topological measures and visualize the resulting clusters to identify patterns and relationships. Next, we apply PCA to assess how much of the data can be explained by a reduced set of components, allowing us to extract relevant information from the previously identified clusters. Finally, a predictive modeling phase is carried out to guide the selection of the most informative features. In particular, we employ a game-theoretic approach known as SHAP and the RFE algorithm (using feature importance and the coefficient of determination as metrics) to select inputs for the training phase. This methodology was tested on 46 EU/EEA metro and road networks, yielding promising results in terms of discovering correlations between features and predicting missing information. Furthermore, our findings are compared with existing literature, offering new perspectives and insights for future research.

In comparison with the work of [12], our study reveals that the relationships between features can vary significantly depending on the network context. This suggests the need for further investigation across different types of networks to better understand the correlation patterns that emerge in various domains. A natural extension of this study would involve exploring railway and tram networks to assess whether similar patterns hold. Additionally, comparing the selected features for prediction with the most highly correlated measures could help validate the reliability of the feature selection algorithms used. Lastly, optimizing the selection of features for ML models by taking into account the computational time required for each metric would be valuable. This would enable a more balanced approach to gaining insights into the network while minimizing the computational burden.



Appendix. Metro and road network data

In this appendix, are reported additional information concerning the metro and road network considered in our study. Specifically, we first list in alphabetical order all the cities included in our analysis:

| 1. | Amsterdam, | 13. | Frankfurt, | 26. | Milan, Italy | 38. | Sevilla, Spain |
|-----|----------------|-----|---------------|-----|---------------|-----|----------------|
| | Netherlands | | Germany | 27. | Munich, | 39. | Sofia, |
| 2. | Athens, | 14. | Genoa, Italy | | Germany | | Bulgaria |
| | Greece | 15. | Glasgow, UK | 28. | Naples, Italy | 40. | Stockholm, |
| 3. | Barcelona, | 16. | Hamburg, | 29. | Nuremberg, | | Sweden |
| | Spain | | Germany | | Germany | 41. | Toulouse, |
| 4. | Berlin, | 17. | Helsinki, | 30. | Oslo, Norway | | France |
| | Germany | | Finland | 31. | Palma de | 42. | Turin, Italy |
| 5. | Bilbao, Spain | 18. | Lausanne, | | Mallorca, | 43. | Tyne and |
| 6. | Brescia, Italy | | Switzerland | | Spain | | Wear, UK |
| 7. | Brussels, | 19. | Lille, France | 32. | Paris, France | 44. | Valencia, |
| | Belgium | 20. | Lisbon, | 33. | Porto, | | Spain |
| 8. | Bucharest, | | Portugal | | Portugal | 45. | Vienna, |
| | Romania | 21. | London, UK | 34. | Prague, | | Austria |
| 9. | Budapest, | 22. | Lyon, France | | Czech | 46. | Warsaw, |
| | Hungary | 23. | Madrid, | | Republic | | Poland |
| 10. | Catania, Italy | | Spain | 35. | Rennes, | | |
| 11. | Charleroi, | 24. | Malaga, | | France | | |
| | Belgium | | Spain | 36. | Rome, Italy | | |
| 12. | Copenhagen, | 25. | Marseille, | 37. | Rotterdam, | | |
| | Denmark | | France | | Netherlands | | |

For the sake of completeness, we include in the following pages the tables reporting the different topological measures computed (in Table 15 for the metro networks and Table 17 for the road networks) and the corresponding time required for the calculation of the specific values (in Table 16 for the metro networks and Table 18 for the road networks).



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|---|--------|----------|----------------|----------------------------|---------------|---|----------|---|----------|-------------------------------------|----------|--|-------------------|----------|
| City, Country | V | V $ E $ | ND | AD | AVP | VE | ANN | DA | CC | RC | BV | BE | AC | AVC |
| Amsterdam Metro, Netherlands | 53 | 53 | 0.038462 | 2.000000 | 11.299710 | 0.038462 2.000000 11.299710 23.264151 2.309524 0.129108 | 2.309524 | | 0.020000 | 0.127313 | 0.201955 | 0.020000 0.127313 0.201955 0.213202 0.007858 1.056604 | 0.007858 | 1.056604 |
| Athens Metro, Greece | 61 | 62 | 0.033880 | 2.032787 | 10.917486 | 0.033880 2.032787 10.917486 22.590164 2.225490 0.163418 | 2.225490 | | 0.000000 | 0.184564 | 0.168093 | 0.000000 0.184564 0.168093 0.176088 0.009957 1.131148 | 0.009957 | 1.131148 |
| Barcelona Metro, Spain | 149 | 149 168 | 0.015237 | 2.255034 | 11.153183 | 2.255034 11.153183 22.751678 2.547139 0.062184 | 2.547139 | 0.062184 | 0.004000 | 0.061021 | 0.069069 | 0.004000 0.061021 0.069069 0.066388 0.010276 1.711409 | 0.010276 | 1.711409 |
| Berlin U-Bahn, Germany | 170 | 170 182 | 0.012670 | 2.141176 | 14.478455 | 0.012670 2.141176 14.478455 31.817647 2.215540 0.079077 | 2.215540 | | 0.004000 | 0.004000 0.037601 0.080229 0.079552 | 0.080229 | 0.079552 | 0.005488 1.400000 | 1.400000 |
| Bilbao Metro, Spain | 48 | 47 | 0.041667 | 1.958333 | 11.775709 | 23.229167 | 2.018293 | 0.041667 1.958333 11.775709 23.229167 2.018293 -0.016216 0.000000 0.029393 0.234255 0.250547 0.008369 1.0000000 0.029393 0.234255 0.250547 0.008369 1.00000000 0.029393 0.234255 0.250547 0.008369 0.0000000 0.0000000000000000000000 | 0.000000 | 0.029393 | 0.234255 | 0.250547 | 0.008369 | 1.000000 |
| Brescia Metro, Italy | 17 | 16 | 0.117647 | 0.117647 1.882353 | 00000009 | 12.235294 | 1.966667 | 6.000000 12.235294 1.966667 -0.066667 0.000000 0.125490 0.333333 0.375000 0.034054 1.0000000 0.00000000000000000000000 | 0.000000 | 0.125490 | 0.333333 | 0.375000 | 0.034054 | 1.000000 |
| Brussels Metro, Belgium | 99 | 09 | 0.033898 | 2.000000 | 10.951412 | 2.000000 10.951412 21.783333 2.080065 0.027778 | 2.080065 | | 0.000000 | 0.079201 | 0.171576 | 0.000000 0.079201 0.171576 0.182524 0.010227 1.283333 | 0.010227 | 1.283333 |
| Bucharest Metro, Romania | 47 | 84 | 0.044403 | 0.044403 2.042553 7.278446 | 7.278446 | 13.468085 | 2.113095 | 13.468085 2.113095 -0.067655 0.000000 0.040572 0.139521 0.151634 0.032201 1.382979 | 0.000000 | 0.040572 | 0.139521 | 0.151634 | 0.032201 | 1.382979 |
| Budapest Metro, Hungary | 84 | 8 | 0.042553 | 2.000000 | 7.662234 | 14.062500 | 2.146742 | 0.042553 2.000000 7.662234 14.062500 2.146742 -0.142020 0.000000 0.023138 0.144831 0.159630 0.023686 1.166667 | 0.000000 | 0.023138 | 0.144831 | 0.159630 | 0.023686 | 1.166667 |
| Catania Metro, Italy | = | 10 | 0.181818 | 0.181818 1.818182 3.745455 | 3.745455 | 7.090909 | 2.000000 | 7.090909 2.000000 -0.333333 0.000000 0.215909 0.305051 0.374545 0.088744 1.000000 | 0.000000 | 0.215909 | 0.305051 | 0.374545 | 0.088744 | 1.000000 |
| Charleroi, Belgium | 46 | 49 | 0.047343 | 2.130435 | 10.282126 | $0.047343 \ \ 2.130435 \ \ 10.282126 \ \ 21.391304 \ \ 2.825351 \ \ 0.380088$ | 2.825351 | | 0.023000 | 0.282904 | 0.210957 | 0.023000 0.282904 0.210957 0.209839 0.010261 1.217391 | 0.010261 | 1.217391 |
| Copenhagen Metro, Denmark | 22 | 21 | 0.090909 | 0.090909 1.909091 6.212121 | 6.212121 | 11.500000 2.000000 0.000000 | 2.000000 | 0.000000 | 0.000000 | 0.098086 | 0.260606 | 0.000000 0.098086 0.260606 0.295815 0.037742 1.000000 | 0.037742 | 1.000000 |
| Frankfurt, Germany | 86 | 108 | 0.022722 | 2.204082 | 12.813802 | 27.602041 | 2.511784 | 0.022722 2.204082 12.813802 27.602041 2.511784 -0.029219 0.000000 0.108352 0.123060 0.118646 0.006217 1.530612 0.006217 0.0062 | 0.000000 | 0.108352 | 0.123060 | 0.118646 | 0.006217 | 1.530612 |
| Genoa Metro, Italy | 8 | 7 | 0.250000 | 0.250000 1.750000 3.000000 | 3.000000 | 5.500000 | 1.916667 | 5.500000 1.916667 -0.166667 0.000000 0.291667 0.333333 0.428571 0.152241 1.000000 | 0.000000 | 0.291667 | 0.333333 | 0.428571 | 0.152241 | 1.000000 |
| Glasgow Subway, | 15 | 14 | 0.133333 | 0.133333 1.866667 5.333333 | 5.333333 | 10.733333 | 1.961538 | 10.733333 1.961538 -0.076923 0.000000 0.143590 0.333333 0.380952 0.043705 1.000000 | 0.000000 | 0.143590 | 0.333333 | 0.380952 | 0.043705 | 1.000000 |



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| City, Country | <u> </u> | E | ND | AD | AVP | VE | ANN | DA | CC | RC | BV | BE | AC | AVC |
|-----------------------------------|----------|-----|-----------|----------------------------|--------------------|--|----------|---|----------|----------|----------|----------|---|----------|
| Hamburg U-Bahn, Germany | 91 | 96 | 0.023443 | 2.109890 | 11.654212 | 23.725275 | 2.145789 | 0.023443 2.109890 11.654212 23.725275 2.145789 -0.012417 0.000000 0.059386 0.119710 0.121398 0.009112 1.450549 | 0.000000 | 0.059386 | 0.119710 | 0.121398 | 0.009112 | 1.450549 |
| Helsinki Metro, Finland | 25 | 24 | 0.080000 | 0.080000 1.920000 8.126667 | | 16.480000 2.000000 0.000000 | 2.000000 | | 0.000000 | 0.085455 | 0.309855 | 0.338611 | 0.000000 0.085455 0.309855 0.338611 0.016836 1.000000 | 1.000000 |
| Lausanne, Switzerland | 29 | 28 | 0.068966 | 0.068966 1.931034 8.344828 | | 16.896552 | 2.000000 | 16.896552 2.000000 —0.000000 0.000000 0.072944 0.272031 0.298030 0.016909 1.000000 | 0.000000 | 0.072944 | 0.272031 | 0.298030 | 0.016909 | 1.000000 |
| Lille Metro, France | 99 | 09 | 0.033898 | 2.000000 | 2.000000 14.337853 | 31.500000 2.037037 | 2.037037 | $-0.034483 \ 0.000000 \ 0.017565 \ 0.229963 \ 0.238964 \ 0.004913 \ 1.183333$ | 0.000000 | 0.017565 | 0.229963 | 0.238964 | 0.004913 | 1.183333 |
| Lisbon Metro, Portugal | 49 | 51 | 0.043367 | 0.043367 2.081633 7.802721 | 7.802721 | 15.346939 | 2.496230 | 15.346939 2.496230 -0.164384 0.000000 0.081951 0.144739 0.152995 0.020441 1.408163 | 0.000000 | 0.081951 | 0.144739 | 0.152995 | 0.020441 | 1.408163 |
| London Undergro- und + DLR, UK | 309 356 | 356 | 0.007481 | 2.304207 | 14.369794 | 0.007481 2.304207 14.369794 28.142395 2.830357 0.126384 | 2.830357 | | 0.047000 | 0.035133 | 0.043550 | 0.040365 | 0.047000 0.035133 0.043550 0.040365 0.005547 1.543689 | 1.543689 |
| Lyon Metro, France | 40 | 40 | 0.051282 | 0.051282 2.000000 6.917949 | 6.917949 | 13.350000 | 2.066667 | 13.350000 2.066667 -0.078431 0.000000 0.027972 0.155735 0.172949 0.029569 1.275000 0.029569 0.02956 | 0.000000 | 0.027972 | 0.155735 | 0.172949 | 0.029569 | 1.275000 |
| Madrid Metro, Spain | 276 | 308 | 0.008116 | 2.231884 | 16.902899 | 2.231884 16.902899 43.221014 2.746507 | 2.746507 | 0.240821 | 0.007000 | 0.033384 | 0.058040 | 0.054880 | 0.007000 0.033384 0.058040 0.054880 0.002174 1.452899 | 1.452899 |
| Malaga, Spain | 22 | 21 | 0.090909 | 0.090909 1.909091 6.147186 | 6.147186 | 12.272727 2.138889 | 2.138889 | -0.009615 0.000000 0.400673 0.257359 0.292723 0.031151 1.000000 | 0.000000 | 0.400673 | 0.257359 | 0.292723 | 0.031151 | 1.000000 |
| Marseille Metro, France | 28 | 28 | 0.074074 | 0.074074 2.000000 6.026455 | 6.026455 | 11.857143 | 2.090909 | 11.857143 2.090909 —0.076923 0.000000 0.040258 0.193325 0.215231 0.037839 1.250000 | 0.000000 | 0.040258 | 0.193325 | 0.215231 | 0.037839 | 1.250000 |
| Milan Metro, Italy | 106 | 109 | 0.019587 | 2.056604 | 12.175741 | $0.019587 \ \ 2.056604 \ \ 12.175741 \ \ 25.349057 \ \ 2.074361$ | 2.074361 | -0.129621 0.000000 0.010384 0.107459 0.111704 0.006967 1.301887 | 0.000000 | 0.010384 | 0.107459 | 0.111704 | 0.006967 | 1.301887 |
| Munich U-Bahn, Germany | 96 | 66 | 0.021711 | 2.062500 | 10.307675 | 0.021711 2.062500 10.307675 19.093750 2.179342 | 2.179342 | -0.013765 0.000000 0.042071 0.099018 0.104118 0.012684 1.354167 | 0.000000 | 0.042071 | 0.099018 | 0.104118 | 0.012684 | 1.354167 |
| Naples Metro, Italy | 18 | 17 | 0.1111111 | 0.111111 1.888889 6.333333 | 6.333333 | 13.000000 | 1.968750 | 13.000000 1.968750 -0.062500 0.000000 0.118056 0.333333 0.372549 0.030384 1.0000000 0.033333 0.372549 0.030384 0.0000000000000000000000000000000000 | 0.000000 | 0.118056 | 0.333333 | 0.372549 | 0.030384 | 1.000000 |
| Nuremberg U-Bahn, Germany | 48 | 84 | 0.042553 | 2.000000 9.118794 | 9.118794 | 18.229167 2.157346 0.032258 | 2.157346 | 0.032258 | 0.000000 | 0.064500 | 0.176496 | 0.189975 | 0.000000 0.064500 0.176496 0.189975 0.014905 1.104167 | 1.104167 |
| Oslo Metro, Norway | 101 102 | 102 | 0.020198 | 2.019802 | 14.494059 | 0.020198 2.019802 14.494059 28.069307 2.089286 0.070312 | 2.089286 | | 0.000000 | 0.041835 | 0.136304 | 0.142099 | 0.000000 0.041835 0.136304 0.142099 0.006010 1.158416 | 1.158416 |



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| City, Country | V $ E $ | E | ND | AD | AVP | VE | ANN | DA | CC | RC | BV | BE | AC | AVC |
|---------------------------------|----------|-----|----------------------------|----------------------------|----------------------------|--|-------------------|--|-------------------|----------------------------|----------|--|-------------------|----------|
| Palma de Mallorca, Spain | 16 | 15 | 0.125000 | 0.125000 1.875000 4.966667 | 4.966667 | 9.812500 | 2.000000 0.000000 | 0.000000 | 0.000000 | 0.000000 0.139423 0.283333 | 0.283333 | 0.331111 0.049940 1.000000 | 0.049940 | 1.000000 |
| Paris Metro, France | 303 358 | 358 | 0.007825 | 2.363036 | 12.020480 | 12.020480 24.184818 | 2.688760 | -0.011342 | 0.016000 | 0.016000 0.010350 0.036613 | 0.036613 | 0.033577 0.011286 1.617162 | 0.011286 | 1.617162 |
| Porto, Portugal | 80 | 80 | 0.025316 | 2.000000 | 15.050316 | 15.050316 31.425000 2.148259 | 2.148259 | 0.175824 | 0.000000 | 0.106587 | 0.180132 | 0.188129 | 0.004280 1.050000 | 1.050000 |
| Prague Metro, Czech Republic | 28 | 58 | 0.035088 | 2.000000 | 9.505747 | 17.396552 | 2.158730 0.101684 | 0.101684 | 0.000000 | 0.000000 0.185243 | 0.151888 | 0.163892 | 0.016093 | 1.086207 |
| Rennes Metro, France 15 | 15 | 14 | 0.133333 | 1.866667 | 5.333333 | 10.733333 1.961538 | 1.961538 | -0.076923 0.000000 0.143590 0.333333 0.380952 | 0.000000 | 0.143590 | 0.333333 | 0.380952 | 0.043705 1.000000 | 1.000000 |
| Rome Metro, Italy | 73 | 72 | 0.027397 1.972603 | 1.972603 | 13.755708 | 13.755708 27.767123 2.015625 | | $-0.016129 \ 0.000000 \ 0.019083 \ 0.179658 \ 0.191051 \ 0.005582$ | 0.000000 | 0.019083 | 0.179658 | 0.191051 | | 1.000000 |
| Rotterdam Metro, Netherlands | 62 | 62 | 0.032787 | 2.000000 | 12.422528 | 2.000000 12.422528 23.758065 2.028302 | 2.028302 | -0.040268 0.000000 0.022834 0.190375 | 0.000000 | 0.022834 | 0.190375 | 0.200363 | 0.007809 | 1.322581 |
| Sevilla, Spain | 22 | 21 | 0.090909 1.909091 7.666667 | 1.909091 | 7.666667 | 16.000000 1.975000 | 1.975000 | -0.050000 0.000000 0.095455 0.333333 0.365079 0.020357 1.000000 | 0.000000 | 0.095455 | 0.333333 | 0.365079 | 0.020357 | 1.000000 |
| Sofia Metro, Bulgaria | 34 | 34 | 0.060606 | 2.000000 | 7.490196 | 14.852941 2.034483 | 2.034483 | -0.044693 0.000000 0.042424 0.202819 | 0.000000 | 0.042424 | 0.202819 | 0.220300 0.018117 1.441176 | 0.018117 | 1.441176 |
| Stockholm Metro, Sweden | 100 | 100 | 0.020202 | 2.000000 | 13.320808 | 2.000000 13.320808 26.600000 | 2.137262 | $-0.029240 \ 0.000000 \ 0.028589 \ 0.125723$ | 0.000000 | 0.028589 | 0.125723 | 0.133208 | 0.005934 1.070000 | 1.070000 |
| Toulouse Metro, France | 37 | 36 | 0.054054 1.945946 8.708709 | 1.945946 | | 16.945946 | 2.020833 | 16.945946 2.020833 -0.011236 0.000000 0.057330 0.220249 0.241909 0.016985 1.000000 | 0.000000 | 0.057330 | 0.220249 | 0.241909 | 0.016985 | 1.000000 |
| Turin Metro, Italy | 21 | 20 | 0.095238 | 1.904762 | 0.095238 1.904762 7.333333 | 15.238095 | 1.973684 | 15.238095 1.973684 -0.052632 0.000000 0.100251 0.333333 0.366667 0.022338 1.000000 | 0.000000 | 0.100251 | 0.333333 | 0.366667 | 0.022338 | 1.000000 |
| Tyne and Wear Metro, UK | 09 | 09 | 0.033898 | 2.000000 | | 12.196045 24.100000 2.083726 | 2.083726 | -0.192053 | 0.000000 0.023421 | | 0.193035 | 0.203267 0.007169 | | 1.450000 |
| Valencia, Spain | 145 150 | 150 | 0.014368 | 2.068966 | 21.081609 | $2.068966 \ \ 21.081609 \ \ 47.103448 \ \ 2.387719 \ \ 0.250911$ | 2.387719 | 0.250911 | 0.026000 | 0.074128 | 0.140431 | 0.026000 0.074128 0.140431 0.140544 0.001858 1.193103 | 0.001858 | 1.193103 |
| Vienna U-Bahn, Austria | 86 | 104 | 0.021881 | 2.122449 | 10.670524 | 20.551020 | 2.347714 | 0.016296 | 0.000000 | 0.055980 | 0.100735 | 0.102601 | 0.012449 | 1.336735 |
| Warsaw Metro, Poland | 27 | 26 | 0.074074 1.925926 7.236467 | 1.925926 | | 15.000000 | 2.030303 | 15.000000 2.030303 -0.015625 0.000000 0.080515 0.249459 0.278326 0.021198 1.000000 | 0.000000 | 0.080515 | 0.249459 | 0.278326 | 0.021198 | 1.000000 |



Table 16 Time (seconds) for the evaluation of the topological measures of metro networks for European cities

| City, Country | ND | AD | AVP | VE | ANN | DA | CC | RC | BV | BE | AC | AVC |
|---------------------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| Amsterdam Metro, Netherlands | 0.000000 | 0.000994 | 0.006017 | 0.005998 | 0.001003 | 0.002000 | 0.013522 | 0.001003 | 0.017000 | 0.016991 | 0.031001 | 0.001000 |
| Athens Metro, Greece | 0.000000 | 0.001001 | 0.005169 | 0.004783 | 0.003015 | 0.001065 | 0.010000 | 0.000000 | 0.016046 | 0.017516 | 0.070009 | 0.000991 |
| Barcelona Metro, Spain | 0.000000 | 0.000000 | 0.021999 | 0.023073 | 0.003028 | 0.004057 | 0.008543 | 0.001425 | 0.084610 | 0.092601 | 0.081525 | 0.002004 |
| Berlin U-Bahn, Germany | 0.000000 | 0.000000 | 0.034006 | 0.040528 | 0.006002 | 0.005998 | 0.012998 | 0.002008 | 0.139882 | 0.175579 | 0.167473 | 0.002881 |
| Bilbao Metro, Spain | 0.000000 | 0.000000 | 0.003000 | 0.002000 | 0.000998 | 0.001004 | 0.008961 | 0.000000 | 0.009036 | 0.009000 | 0.028000 | 0.001002 |
| Brescia Metro, Italy | 0.000000 | 0.000000 | 0.000000 | 0.001000 | 0.000000 | 0.000000 | 0.007996 | 0.000000 | 0.001002 | 0.001999 | 0.009002 | 0.000000 |
| Brussels Metro, Belgium | 0.000000 | 0.000000 | 0.003000 | 0.002001 | 0.001002 | 0.000998 | 0.005997 | 0.000000 | 0.011990 | 0.011508 | 0.021037 | 0.000000 |
| Bucharest Metro, Romania | 0.000000 | 0.000000 | 0.004003 | 0.001997 | 0.001000 | 0.002004 | 0.012995 | 0.001001 | 0.008998 | 0.011004 | 0.123509 | 0.000000 |
| Budapest Metro, Hungary | 0.000000 | 0.000000 | 0.002999 | 0.002010 | 0.001035 | 0.001967 | 0.007031 | 0.000989 | 0.009011 | 0.011000 | 0.110496 | 0.001001 |
| Catania Metro, Italy | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.001000 | 0.000000 | 0.005969 | 0.000998 | 0.000000 | 0.001032 | 0.005967 | 0.001033 |
| Charleroi, Belgium | 0.000000 | 0.000000 | 0.003013 | 0.001997 | 0.001000 | 0.001002 | 0.010500 | 0.000000 | 0.009050 | 0.011003 | 0.034958 | 0.001002 |
| Copenhagen Metro, Denmark | 0.000000 | 0.000000 | 0.000998 | 0.001000 | 0.000000 | 0.001001 | 0.012004 | 0.000000 | 0.002999 | 0.002001 | 0.028296 | 0.000996 |
| Frankfurt, Germany | 0.000000 | 0.000000 | 0.015997 | 0.012003 | 0.002002 | 0.001995 | 0.009002 | 0.000997 | 0.071554 | 0.077785 | 0.048991 | 0.001004 |
| Genoa Metro, Italy | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.001009 | 0.006995 | 0.000000 | 0.000000 | 0.000000 | 0.006030 | 0.000000 |
| Glasgow Subway, UK | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000999 | 0.005999 | 0.000000 | 0.001006 | 0.000999 | 0.008001 | 0.000000 |
| Hamburg U-Bahn, Germany | 0.000000 | 0.000000 | 0.010002 | 0.009002 | 0.002004 | 0.001998 | 0.009001 | 0.000994 | 0.039006 | 0.053559 | 0.142538 | 0.000998 |
| Helsinki Metro, Finland | 0.000000 | 0.000000 | 0.001003 | 0.000000 | 0.001019 | 0.000000 | 0.008012 | 0.000000 | 0.001961 | 0.003007 | 0.008999 | 0.000000 |
| Lausanne, Switzerland | 0.000000 | 0.000000 | 0.003209 | 0.001992 | 0.000000 | 0.001000 | 0.013992 | 0.001003 | 0.005006 | 0.005990 | 0.019003 | 0.001001 |
| Lille Metro, France | 0.000000 | 0.000000 | 0.004004 | 0.003999 | 0.000970 | 0.000998 | 0.008040 | 0.000000 | 0.014000 | 0.014000 | 0.018624 | 0.001000 |
| Lisbon Metro, Portugal | 0.000000 | 0.001000 | 0.003999 | 0.001997 | 0.001000 | 0.002002 | 0.014012 | 0.000993 | 0.010003 | 0.011558 | 0.052962 | 0.001000 |



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| City, Country | ND | AD | AVP | VE | ANN | DA | CC | RC | BV | BE | AC | AVC |
|---------------------------------|----------|----------|----------|-------------------|----------|----------|---|----------|--|----------|----------|----------|
| London Underground + DLR, UK | 0.000000 | 0.000000 | 0.103987 | 0.101185 | 0.006008 | 0.005000 | 0.009036 | 0.001999 | 0.399284 | 0.704214 | 0.281174 | 0.003999 |
| Lyon Metro, France | 0.000961 | 0.000000 | 0.001000 | 0.002008 | 0.001038 | 0.000992 | 0.007000 0.000000 | 0.000000 | 0.005998 | 0.007004 | 0.032980 | 0.001000 |
| Madrid Metro, Spain | 0.000000 | 0.000000 | 0.068048 | 0.064554 | 0.004174 | 0.003998 | 0.008008 | 0.001000 | 0.333636 | 0.308774 | 0.072404 | 0.002962 |
| Malaga, Spain | 0.000000 | 0.000000 | 0.000999 | 0.000000 | 0.000999 | 0.000000 | 0.007003 | 0.000000 | 0.001997 | 0.001964 | 0.013087 | 0.000000 |
| Marseille Metro, France | 0.000000 | 0.000000 | 0.001001 | 0.000000 | 0.001002 | 0.000965 | 0.005996 | 0.001002 | 0.001999 | 0.003001 | 0.022995 | 0.000000 |
| Milan Metro, Italy | 0.000000 | 0.000000 | 0.011006 | 0.010007 | 0.002564 | 0.001964 | 0.008999 | 0.001001 | 0.054532 | 0.066550 | 0.076280 | 0.001034 |
| Munich U-Bahn, Germany | 0.000000 | 0.000000 | 0.011999 | 0.012007 | 0.001995 | 0.001999 | 0.010005 | 0.001001 | 0.055312 | 0.054998 | 0.144193 | 0.002001 |
| Naples Metro, Italy | 0.000000 | 0.000000 | 0.001533 | 0.000461 | 0.000000 | 0.001017 | 0.001017 0.011086 | 0.000000 | 0.002210 | 0.001015 | 0.014378 | 0.001021 |
| Nuremberg U-Bahn, Germany | 0.000000 | 0.000000 | 0.003411 | 0.003534 | 0.001516 | 0.001545 | 0.015476 | 0.000540 | 0.010677 | 0.012285 | 0.044012 | 0.001051 |
| Oslo Metro, Norway | 0.000000 | 0.000000 | 0.013012 | 0.020025 | 0.002997 | 0.002995 | 0.010535 | 0.000998 | 0.082476 0.066527 | 0.066527 | 0.157169 | 0.001001 |
| Palma de Mallorca, Spain | 0.000566 | 0.000000 | 0.000000 | 0.000000 | 0.001009 | 0.001011 | 0.009362 | 0.000000 | 0.001089 | 0.001067 | 0.009296 | 0.000000 |
| Paris Metro, France | 0.000000 | 0.000000 | 0.084479 | 0.080675 | 0.004004 | 0.004591 | 0.006999 | 0.002005 | 0.671602 | 0.707328 | 0.484080 | 0.004000 |
| Porto, Portugal | 0.000000 | 0.001003 | 0.006960 | 0.007039 | 0.000955 | 0.001045 | 966800.0 | 0.000000 | 0.025002 | 0.028208 | 0.028603 | 0.001040 |
| Prague Metro, Czech Republic | 0.000000 | 0.000000 | 0.003002 | 0.003967 | 0.001034 | 0.000997 | 0.006001 | 0.000000 | 0.013049 | 0.015550 | 0.066404 | 0.000999 |
| Rennes Metro, France | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.008998 | 0.000000 | 0.002002 | 0.000995 | 0.013005 | 0.000000 |
| Rome Metro, Italy | 0.000000 | 0.000000 | 0.008973 | 0.007998 | 0.003000 | 0.002024 | 0.008983 | 0.001000 | 0.027668 | 0.033414 | 0.055536 | 0.002006 |
| Rotterdam Metro, Netherlands | 0.000000 | 0.000000 | 0.005001 | 0.005997 | 0.001001 | 0.001000 | 0.010001 | 0.000000 | 0.023999 | 0.019534 | 0.082877 | 0.000000 |
| Sevilla, Spain | 0.000000 | 0.000000 | | 0.000000 0.001003 | 0.000000 | 0.001003 | 0.000000 0.001003 0.007144 | 0.000000 | 0.000000 0.002001 0.002001 0.007960 0.000000 | 0.002001 | 0.007960 | 0.000000 |
| Sofia Metro, Bulgaria | 0.000000 | 0.000000 | 0.000998 | 0.000000 | 0.000999 | 0.000000 | 0.000000 0.000999 0.000000 0.006000 0.000000 0.003000 0.004000 0.008002 0.001000 0.000000 0.001000 0.001000 0.001000 0.001000 0.001000 0.001000 0.001000 0.001000 0.001000 0.001000 0.001000 0.001000 0.001000 0.00100000 0.0010000 0.001000000 0.00100000 0.001000000 0.001000000 0.001000000 0.001000000 0.0010000000 0.001000000 0.0010000000 0.00100000000 0.0010000000000 | 0.000000 | 0.003000 | 0.004000 | 0.008002 | 0.001000 |
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| City, Country | QN | AD | AVP | VE | ANN | DA | AVP VE ANN DA CC RC BV BE AC | RC | BV | BE | AC | AVC |
|---|----------|---|-------------------------------------|-------------------|-------------------|----------|--|----------|-------------------|-------------------|--|----------|
| Stockholm Metro, Sweden 0.000000 0.000000 | 0.000000 | 0.000000 | 0.008965 | 0.008029 | 0.000969 | 0.002038 | 0.008965 0.008029 0.000969 0.002038 0.007197 0.000000 0.034051 0.038559 0.079265 | 0.000000 | 0.034051 | 0.038559 | 0.079265 | 0.001002 |
| Toulouse Metro, France | 0.000000 | 0.000000 | 0.002008 | 0.001992 | 0.001992 0.000000 | 0.001000 | 0.008998 | 0.000000 | 0.007004 0.006003 | 0.006003 | 0.039002 | 0.001001 |
| Turin Metro, Italy | 0.000000 | 0.000000 | 0.000999 | 0.001001 | 0.001016 | 0.000993 | 0.010008 | 0.000000 | 0.002997 | 0.003003 | 0.018119 | 0.001001 |
| Tyne and Wear Metro, UK | 0.000000 | 0.000000 | 0.007998 | 0.007998 0.007005 | 0.001009 | 0.001000 | 0.008999 | 0.001000 | 0.017143 | 0.022534 | 0.021515 | 0.001005 |
| Valencia, Spain | 0.000000 | 0.000000 | 0.019082 | 0.019082 0.022483 | 0.002996 | 0.002000 | 0.008000 | 0.002001 | 0.091573 | 0.091573 0.108531 | 0.104833 | 0.003006 |
| Vienna U-Bahn, Austria | 0.000000 | 0.000997 | 0.000997 0.009000 0.014997 0.001000 | 0.014997 | 0.001000 | 0.002006 | 0.006992 | 0.000000 | 0.042529 | 0.042529 0.037029 | 0.071569 | 0.001003 |
| Warsaw Metro, Poland | 0.000000 | 0.000000 0.000000 0.000998 0.000999 0.001002 0.001000 | 0.000998 | 0.000999 | 0.001002 | 0.001000 | 0.009099 | 0.000000 | 0.003001 | 0.003959 | 0.009099 0.000000 0.003001 0.003959 0.014195 | 0.001037 |



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| City, Country | V | E | ND | AD | AVP | VE | ANN | DA | CC | RC | BV | BE | AC , | AVC |
|--|---------------|---------------|-----------|----------|---|--|-------------------|--|----------|----------------------------|---|-------------------|----------------------------|----------|
| Amsterdam, Netherlands | 12,577 | 12,577 18,497 | 0.000234 | 2.941401 | 2.941401 39.378503 67.615489 | 67.615489 | 3.351498 0.198036 | 0.198036 | 0.065000 | 0.001276 | 0.065000 0.001276 0.003052 0.002129 0.000304 1.857677 | 0.002129 | 0.000304 | .857677 |
| Barcelona, Spain | 8833 13,818 | 13,818 | 0.000354 | 3.128722 | 38.536394 | 0.000354 3.128722 38.536394 73.968867 | 3.314761 0.228239 | 0.228239 | 0.049000 | 0.000782 | $0.049000 \ \ 0.000782 \ \ 0.004251 \ \ 0.002789 \ \ 0.000673 \ \ 1.913846$ | 0.002789 | 0.000673 | .913846 |
| Berlin, Germany | 27,991 | 27,991 41,820 | 0.000107 | 2.988103 | 67.946565 | $0.000107 \ \ 2.988103 \ \ 67.946565 \ \ 134.206173 \ \ 3.240147 \ \ 0.133620$ | 3.240147 | 0.133620 | 0.055000 | 0.000415 | $0.055000 \ \ 0.000415 \ \ 0.002392 \ \ 0.001625 \ \ 0.000122 \ \ 1.863170$ | 0.001625 | 0.000122 | .863170 |
| Big Athens Ring, Greece | 27,504 | 27,504 45,098 | 0.000119 | 3.279378 | 84.909130 | 3.279378 84.909130 167.246800 3.443555 0.160967 | 3.443555 | | 0.034000 | 0.001795 | 0.034000 0.001795 0.003051 0.001883 0.000088 1.945608 | 0.001883 | 0.000088 | .945608 |
| Bilbao, Spain | 2550 | 3611 | 0.0011111 | 2.832157 | 2.832157 27.530846 54.598824 | 54.598824 | 3.175358 0.144757 | 0.144757 | 0.078000 | 0.001399 | $0.078000 \ \ 0.001399 \ \ 0.010412 \ \ 0.007624 \ \ 0.001258 \ \ 1.852549$ | 0.007624 | 0.001258 | .852549 |
| Brescia, Italy | 9029 | 8963 | 0.000399 | 2.673129 | 2.673129 41.448647 78.724277 | 78.724277 | 2.986898 0.045753 | 0.045753 | 0.058000 | 0.058000 0.002328 0.006034 | | 0.004624 | 0.004624 0.000329 1.745750 | .745750 |
| Brussels, Belgium | 2286 | 3407 | 0.001304 | 2.980752 | 31.750002 64.994751 | 64.994751 | 3.189089 0.165696 | 0.165696 | 0.065000 | 0.065000 0.030885 0.013463 | | 0.009319 | 0.009319 0.000414 1.926509 | .926509 |
| Bucharest, Romania | 16,648 24,194 | 24,194 | 0.000175 | 2.906535 | 57.244066 | 2.906535 57.244066 113.576225 3.190907 0.143179 | 3.190907 | | 0.057000 | 0.000624 | 0.057000 0.000624 0.003379 0.002366 0.000190 1.869294 | 0.002366 | 0.000190 | .869294 |
| Budapest, Hungary | 23,999 | 23,999 36,155 | 0.000126 | 3.013042 | 69.755331 | 0.000126 3.013042 69.755331 140.691237 3.213403 0.168701 | 3.213403 | | 0.053000 | 0.000287 | 0.053000 0.000287 0.002865 0.001929 0.000110 1.889995 | 0.001929 | 0.000110 | 366688 |
| Catania, Italy | 7673 | 10,768 | | 2.806725 | 48.972235 | $0.000366 \ \ 2.806725 \ \ 48.972235 \ \ 107.796299 \ \ 3.109059 \ \ 0.091306$ | 3.109059 | 0.091306 | 0.080000 | 0.000435 | $0.080000 \ \ 0.000435 \ \ 0.006254 \ \ 0.004548 \ \ 0.000213 \ \ 1.814675$ | 0.004548 | 0.000213 | .814675 |
| Charleroi, Belgium | 5257 | 7322 | 0.000530 | 2.785619 | 32.492524 | 62.764885 | 3.110573 0.074149 | 0.074149 | 0.059000 | 0.059000 0.002627 0.005993 | | 0.004438 0.000621 | 0.000621 | 1.838501 |
| Copenhagen Municipality, Denmark | 7005 | 10,183 | | 2.907352 | 0.000415 2.907352 51.593797 97.681799 | | 3.378410 0.138623 | | 0.061000 | 0.010568 | 0.061000 0.010568 0.007225 0.005067 0.000215 1.846966 | 0.005067 | 0.000215 | .846966 |
| Frankfurt, Germany | 9469 | 13,669 | 0.000305 | 2.887105 | 2.887105 40.207894 71.291794 | | 3.129995 0.115645 | | 0.073000 | 0.000629 | 0.073000 0.000629 0.004142 0.002942 0.000361 1.845285 | 0.002942 | 0.000361 | .845285 |
| Genoa, Italy | 8963 | 8963 10,813 | 0.000269 | 2.412808 | $0.000269 \ \ 2.412808 \ \ 45.980310 \ \ 83.581502$ | 83.581502 | 3.002511 | $3.002511 \ -0.031012 \ 0.034000 \ 0.002590 \ 0.005020 \ 0.004252 \ 0.000165 \ 1.572911$ | 0.034000 | 0.002590 | 0.005020 | 0.004252 | 0.000165 | .572911 |
| Glasgow City, UK | 16,246 20,861 | 20,861 | 0.000158 | 2.568140 | 2.568140 47.371723 94.032254 | 94.032254 | 3.113136 0.062334 | | 0.048000 | 0.000241 | 0.048000 0.000241 0.002855 0.002271 0.000142 1.665333 | 0.002271 | 0.000142 | .665333 |



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| Table 17 continued | pe | | | | | | | | | | | | |
|--------------------------|---------------|---------|----------|----------|-----------------------------|------------------------------|-------------------|-----------|---|-------------|----------|----------|----------|
| City, Country | V | E | ND | AD | AVP | VE | ANN | DA | CC RC | BV | BE | AC | AVC |
| Hamburg, Germany | 21,867 | 30,372 | 0.000127 | 2.777884 | 2.777884 71.006658 | 144.648557 3.102478 0.119489 | 3.102478 | 0.119489 | 0.083000 0.000289 0.003202 0.002338 0.000116 1.803128 | 39 0.003202 | 0.002338 | 0.000116 | 1.803128 |
| Helsinki, Finland | 9617 | 13,114 | 0.000284 | 2.727254 | 0.000284 2.727254 49.888059 | 99.658001 | 3.115504 0.128433 | 0.128433 | 0.102000 0.000939 0.005085 0.003804 0.000191 1.733389 | 9 0.005085 | 0.003804 | 0.000191 | 1.733389 |
| Lausanne, Switzerland | 1358 | 1859 | 0.002018 | 2.737850 | 0.002018 2.737850 28.240144 | 64.044919 | 3.013090 0.024674 | 0.024674 | 0.078000 0.002492 0.020089 0.015191 0.000533 1.825479 | 0.020089 | 0.015191 | 0.000533 | 1.825479 |
| Lille, France | 3419 | 5090 | 0.000871 | 2.977479 | 0.000871 2.977479 38.698261 | 88.446037 | 3.146971 0.148565 | 0.148565 | $0.071000 \ 0.001581 \ 0.011033 \ 0.007603 \ 0.000188 \ 1.903481$ | 31 0.011033 | 0.007603 | 0.000188 | 1.903481 |
| Lisbon, Portugal | 10,090 14,5 | 14,555 | 0.000286 | 2.885035 | 0.000286 2.885035 38.564306 | 73.061249 | 3.083847 0.151133 | 0.151133 | 0.084000 0.000407 0.003724 0.002650 0.000369 1.865510 | 0.003724 | 0.002650 | 0.000369 | 1.865510 |
| London, UK | 127,540 | 160,010 | 0.000020 | 2.509174 | 134.143548 | 267.821640 | 3.036988 | -0.014970 | $127,540\ 160,010\ 0.000020\ 2.509174\ 134.143548\ 267.821640\ 3.036988\ -0.014970\ 0.038000\ 0.000590\ 0.001044\ 0.000838\ 0.000018\ 1.659448$ | 00 0.001044 | 0.000838 | 0.000018 | 1.659448 |
| Lyon, France | 4244 | 9259 | 0.000730 | 3.098963 | 0.000730 3.098963 35.511524 | 75.198398 | 3.208356 0.171200 | 0.171200 | $0.063000 \ 0.001162 \ 0.008136 \ 0.005400 \ 0.000658 \ 1.910226$ | 52 0.008136 | 0.005400 | 0.000658 | 1.910226 |
| Madrid, Spain | 30,768 | 47,080 | 0.000099 | 3.060322 | 0.000099 3.060322 40.778483 | 75.050020 | 3.228753 0.191071 | 0.191071 | $0.062000 \ 0.001849 \ 0.001293 \ 0.000866 \ 0.000213 \ 1.926872$ | 19 0.001293 | 0.000866 | 0.000213 | 1.926872 |
| Malaga, Spain | 10,952 | 15,850 | 0.000264 | 2.894449 | 0.000264 2.894449 35.367326 | 61.423850 | 3.091381 0.118723 | 0.118723 | $0.055000 \ 0.000929 \ 0.003139 \ 0.002231 \ 0.000301 \ 1.865778$ | 9 0.003139 | 0.002231 | 0.000301 | 1.865778 |
| Marseille, France | 13,000 | 17,587 | 0.000208 | 2.705692 | 0.000208 2.705692 43.356149 | 104.849615 3.106465 0.059054 | 3.106465 | 0.059054 | 0.098000 0.001002 0.003259 0.002465 0.000231 1.765615 | 0.003259 | 0.002465 | 0.000231 | 1.765615 |
| Milan, Italy | 12,884 19,810 | 19,810 | 0.000239 | 3.075132 | 0.000239 3.075132 54.547232 | 105.689149 3.223614 0.177049 | 3.223614 | 0.177049 | $0.110000\ 0.000499\ 0.004157\ 0.002754\ 0.000332\ 1.933716$ | 9 0.004157 | 0.002754 | 0.000332 | 1.933716 |
| Munich, Germany | 14,080 | 20,801 | 0.000210 | 2.954687 | 0.000210 2.954687 45.769784 | 85.598224 | 3.162331 0.120671 | 0.120671 | 0.055000 0.000293 0.003180 0.002200 0.000220 1.876065 | 3 0.003180 | 0.002200 | 0.000220 | 1.876065 |
| Naples, Italy | 7826 | 10,897 | 0.000356 | 2.784820 | 0.000356 2.784820 35.668602 | 66.429083 | 3.082851 0.072177 | 0.072177 | $0.068000 \ 0.000722 \ 0.004431 \ 0.003273 \ 0.000361 \ 1.813953$ | 22 0.004431 | 0.003273 | 0.000361 | 1.813953 |
| Nuremberg, Germany | 8067 | 10,825 | 0.000333 | 2.683773 | 0.000333 2.683773 43.600060 | 90.276187 | 3.104386 0.083331 | 0.083331 | 0.070000 0.000680 0.005282 0.004028 0.000233 1.737201 | 30 0.005282 | 0.004028 | 0.000233 | 1.737201 |
| Oslo, Norway | 8247 | 11,215 | 0.000330 | 2.719777 | 0.000330 2.719777 38.112188 | 76.433612 | 3.135082 0.091428 | 0.091428 | $0.090000 \ 0.000534 \ 0.004501 \ 0.003398 \ 0.000355 \ 1.773493$ | 34 0.004501 | 0.003398 | 0.000355 | 1.773493 |
| Palma, Spain | 7105 | 10,487 | 0.000416 | 2.952006 | 0.000416 2.952006 36.142755 | 74.922449 | 3.308668 0.177013 | 0.177013 | $0.065000\ 0.001386\ 0.004948\ 0.003446\ 0.000285\ 1.854469$ | 36 0.004948 | 0.003446 | 0.000285 | 1.854469 |
| Paris, France | 9605 | 14,966 | 0.000324 | 3.116294 | 0.000324 3.116294 39.397240 | 69.692972 | 3.257076 0.120109 | 0.120109 | $0.062000 \ 0.000502 \ 0.003998 \ 0.002632 \ 0.000696 \ 1.935867$ | 0.003998 | 0.002632 | 0.000696 | 1.935867 |
| Porto, Portugal | 5204 | 7307 | 0.000540 | 2.808224 | 0.000540 2.808224 31.299655 | 57.268447 | 3.052890 0.067935 | 0.067935 | 0.080000 0.001689 0.005825 0.004284 0.000542 1.843966 | 39 0.005825 | 0.004284 | 0.000542 | 1.843966 |



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| City, Country | V | E | ND | AD | AVP | VE | ANN | DA | CC | RC | BV | BE | AC | AVC |
|--|-----------|--------|----------|----------|--|--|-------------------|-----------|----------|----------|----------|----------|---|----------|
| Prague, Czech Republic | 21,503 | 29,804 | 0.000129 | 2.772078 | 50.227571 | 21,503 29,804 0.000129 2.772078 50.227571 102.124494 3.152889 0.096263 0.061000 0.000429 0.002290 0.001685 0.000148 1.778915 | 3.152889 | 0.096263 | 0.061000 | 0.000429 | 0.002290 | 0.001685 | 0.000148 | 1.778915 |
| Rennes, France | 4225 5799 | 5799 | 0.000650 | 2.745089 | 38.685475 | $0.000650 \ \ 2.745089 \ \ 38.685475 \ \ 74.668166 \ \ \ 3.198164 \ \ 0.138099$ | 3.198164 | 0.138099 | 0.072000 | 0.009593 | 0.008924 | 0.006671 | $0.072000 \ 0.009593 \ 0.008924 \ 0.006671 \ 0.000565 \ 1.781775$ | 1.781775 |
| Rome, Italy | 43,164 | 60,026 | 0.000064 | 2.781299 | 62.317524 | 43,164 60,026 0.000064 2.781299 62.317524 132.395098 3.119074 0.094109 | 3.119074 | 0.094109 | 0.068000 | 0.000175 | 0.001421 | 0.001038 | $0.068000 \ \ 0.000175 \ \ 0.001421 \ \ 0.001038 \ \ 0.000066 \ \ 1.801409$ | 1.801409 |
| Rotterdam, Netherlands | 11,302 | 16,228 | 0.000254 | 2.871704 | 42.217502 | 11,302 16,228 0.000254 2.871704 42.217502 101.803398 3.200585 0.162599 | 3.200585 | 0.162599 | 0.060000 | 0.000861 | 0.003648 | 0.002602 | 0.060000 0.000861 0.003648 0.002602 0.000115 1.830649 | 1.830649 |
| Seville, Spain | 10,219 | 15,296 | 0.000293 | 2.993639 | 38.755375 | 10,219 15,296 0.000293 2.993639 38.755375 68.069283 3.109249 0.169285 | 3.109249 | 0.169285 | 0.063000 | 0.000927 | 0.003695 | 0.002534 | 0.063000 0.000927 0.003695 0.002534 0.000421 1.907134 | 1.907134 |
| Sofia, Bulgaria | 11,544 | 16,980 | 0.000255 | 2.941788 | 11,544 16,980 0.000255 2.941788 45.898110 90.123008 | 90.123008 | 3.171503 0.094564 | 0.094564 | 0.036000 | 0.000460 | 0.003890 | 0.002703 | $0.036000 \ \ 0.000460 \ \ 0.003890 \ \ 0.002703 \ \ 0.000314 \ \ 1.856462$ | 1.856462 |
| Stockholm, Sweden | 13,649 | 18,007 | 0.000193 | 2.638582 | 13,649 18,007 0.000193 2.638582 47.924038 91.940215 | 91.940215 | 3.062187 0.066786 | 0.066786 | 0.066000 | 0.000778 | 0.003438 | 0.002661 | 0.066000 0.000778 0.003438 0.002661 0.000057 1.710675 | 1.710675 |
| Toulouse, France 8867 12,442 0.000317 2.806361 37.352932 66.439495 3.073783 0.120726 0.094000 0.001212 0.004101 0.003002 0.000446 1.855193 | 8867 | 12,442 | 0.000317 | 2.806361 | 37.352932 | 66.439495 | 3.073783 | 0.120726 | 0.094000 | 0.001212 | 0.004101 | 0.003002 | 0.000446 | 1.855193 |
| Turin, Italy | 11,307 | 17,784 | 0.000278 | 3.145662 | 47.372613 | 11,307 17,784 0.000278 3.145662 47.372613 95.295215 3.357630 0.241318 0.058000 0.001575 0.004102 0.002664 0.000395 1.904838 0.004102 0.002664 0.000395 0.004102 | 3.357630 | 0.241318 | 0.058000 | 0.001575 | 0.004102 | 0.002664 | 0.000395 | 1.904838 |
| Tyne and Wear, UK | 46,098 | 54,690 | 0.000051 | 2.372771 | 62.502480 | 46,098 54,690 0.000051 2.372771 62.502480 123.414226 2.953304 -0.077966 0.029000 0.001065 0.001334 0.001143 0.000041 1.520196 | 2.953304 | -0.077966 | 0.029000 | 0.001065 | 0.001334 | 0.001143 | 0.000041 | 1.520196 |
| Valencia, Spain | 8005 | 12,186 | 0.000380 | 3.044597 | 41.045532 | 8005 12,186 0.000380 3.044597 41.045532 96.708432 3.191113 0.179617 | 3.191113 | 0.179617 | 0.052000 | 0.001019 | 0.005004 | 0.003368 | $0.052000 \ \ 0.001019 \ \ 0.005004 \ \ 0.003368 \ \ 0.000255 \ \ 1.906683$ | 1.906683 |
| Vienna, Austria | 16,083 | 23,891 | 0.000185 | 2.970963 | 50.355838 | 16,083 23,891 0.000185 2.970963 50.355838 95.532488 | 3.315368 0.123420 | 0.123420 | 0.067000 | 0.000549 | 0.003069 | 0.002108 | $0.067000 \ 0.000549 \ 0.003069 \ 0.002108 \ 0.000200 \ 1.861655$ | 1.861655 |
| Warsaw, Poland | 19,202 | 26,552 | 0.000144 | 2.765545 | 49.564319 | $19,202\ \ 26,552\ \ 0.000144\ \ 2.765545\ \ 49.564319\ \ 93.362306\ \ 3.116886\ \ 0.099272$ | 3.116886 | 0.099272 | 0.076000 | 0.000422 | 0.002529 | 0.001867 | $0.076000 \ 0.000422 \ 0.002529 \ 0.001867 \ 0.000164 \ 1.782210$ | 1.782210 |



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| Table 1 |
| Table ' |

| City, Country | ND | AD | AVP | VE | ANN | DA | CC | RC | BV | BE | AC | AVC |
|--|----------|-------------------|-------------------------------|------------|----------|----------|----------|----------|---|-------------|--------------------|----------|
| Amsterdam, Netherlands | 0.008708 | 0.008708 0.016168 | 208.997041 | 190.898394 | 0.193531 | 0.189099 | 0.007209 | 0.110595 | 899.224999 | 654.944430 | 8.321645 | 0.126936 |
| Barcelona, Spain | 0.005650 | 0.005650 0.010539 | 96.894135 | 74.981631 | 0.134353 | 0.128374 | 0.007443 | 0.077048 | 0.134353 0.128374 0.007443 0.077048 506.827851 | 474.537499 | 5.449561 | 0.050524 |
| Berlin, Germany | 0.017715 | 0.017715 0.030867 | 1096.320881 | 608.371375 | 0.204242 | 0.185225 | 0.004771 | 0.737006 | 3219.699749 | 3709.580312 | 45.075825 | 0.494536 |
| Big Athens Ring, Greece | 0.017349 | 0.031803 | 0.017349 0.031803 1054.899072 | 494.492131 | 0.246797 | 0.235258 | 0.004622 | 0.344825 | 0.246797 0.235258 0.004622 0.344825 2970.116192 | 3498.475964 | 28.184772 | 0.214928 |
| Bilbao, Spain | 0.001520 | 0.001520 0.002972 | 7.274398 | 6.907903 | 0.045215 | 0.044393 | 0.007948 | 0.021677 | 31.101992 | 35.721923 | 2.228243 | 0.022635 |
| Brescia, Italy | 0.003879 | 0.003879 0.007611 | 53.071468 | 43.988352 | 0.116731 | 0.114155 | 0.009479 | 0.062187 | 237.123640 | 275.940767 | 1.908161 | 0.210609 |
| Brussels, Belgium | 0.001266 | 0.001266 0.002478 | 5.482430 | 4.765464 | 0.041526 | 0.039535 | 0.009186 | 0.022058 | 27.265302 | 30.341219 | 0.285780 | 0.027225 |
| Bucharest, Romania | 0.009729 | 0.009729 0.018252 | 328.025847 | 345.830140 | 0.303786 | 0.288712 | 0.009050 | 0.605261 | 0.009050 0.605261 1243.700011 | 1092.888802 | 17.310471 | 0.140947 |
| Budapest, Hungary | 0.015785 | 0.015785 0.029051 | 860.402676 | 454.353126 | 0.175331 | 0.156758 | 0.003479 | 0.093229 | 0.175331 0.156758 0.003479 0.093229 2272.008394 | 2493.980476 | 51.443081 0.175052 | 0.175052 |
| Catania, Italy | 0.004563 | 0.004563 0.008612 | 70.960357 | 67.603036 | 0.138810 | 0.137325 | 0.008250 | 0.073884 | 0.008250 0.073884 361.302687 | 369.906102 | 5.707481 | 0.092450 |
| Charleroi, Belgium | 0.002825 | 0.002825 0.005302 | 30.991562 | 31.067167 | 0.046805 | 0.046012 | 0.005201 | 0.030689 | 0.030689 143.257823 | 172.367118 | 0.836883 | 0.027593 |
| Copenhagen Municipality, Denmark | 0.003636 | 0.003636 0.007411 | 55.263829 | 53.752108 | 0.122891 | 0.117856 | 0.006903 | 0.061547 | 0.061547 265.570585 | 337.819414 | 1.722214 | 0.082325 |
| Frankfurt, Germany | 0.004880 | 0.009904 | 0.004880 0.009904 111.068365 | 77.924856 | 0.141696 | 0.130319 | 0.007261 | 0.073405 | 0.141696 0.130319 0.007261 0.073405 559.224195 | 480.117207 | 4.175030 | 0.055574 |
| Genoa, Italy | 0.004736 | 0.004736 0.009516 | 87.444033 | 67.041272 | 0.162609 | 0.154603 | 0.007341 | 0.077603 | 0.077603 493.426712 | 453.886482 | 2.104708 | 0.132991 |
| Glasgow City, UK | 0.008712 | 0.008712 0.016193 | 321.862709 | 261.460036 | 0.115836 | 0.106714 | 0.003693 | 0.176726 | 0.176726 1217.543967 | 973.596741 | 18.938349 | 0.101694 |



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| City, Country | ND | AD | AVP | VE | ANN | DA | 22 | RC | BV | BE | AC | AVC |
|--------------------------|---------------|----------|--|--------------|----------|----------|---|----------|--|--------------|-------------|----------|
| Hamburg, Germany | 0.013363 0.02 | | 5769 725.580783 | 444.859971 | 0.172730 | 0.159510 | 0.003934 | 0.087607 | 0.172730 0.159510 0.003934 0.087607 1882.973543 | 1949.351104 | 26.902015 | 0.216871 |
| Helsinki, Finland | 0.005319 | 0.010345 | 0.005319 0.010345 100.191947 | 87.017784 | 0.172706 | 0.165426 | 0.007910 | 0.087693 | 0.172706 0.165426 0.007910 0.087693 588.295027 | 461.689851 | 9.210443 | 0.159403 |
| Lausanne, Switzerland | 0.000827 0.00 | 0.001640 | 1640 1.928913 | 1.283558 | 0.011511 | 0.010954 | 0.011511 0.010954 0.003842 0.005701 8.073826 | 0.005701 | 8.073826 | 9.988138 | 0.120324 | 0.008422 |
| Lille, France | 0.001370 | 0.003393 | 0.001370 0.003393 12.627719 | 11.068537 | 0.054377 | 0.052160 | 0.054377 0.052160 0.007748 0.030086 65.666574 | 0.030086 | 65.666574 | 65.335806 | 0.156618 | 0.019134 |
| Lisbon, Portugal | 0.005763 0.01 | | 1266 132.845638 | 99.033283 | 0.182869 | 0.175744 | 0.008399 | 0.090106 | 0.182869 0.175744 0.008399 0.090106 665.705965 | 432.860622 | 5.774817 | 0.101403 |
| London, UK | 0.081546 | 0.159827 | 0.081546 0.159827 14426.115892 16045.692152 2.225395 2.197306 0.015759 2.351946 89389.737012 99169.296649 2127.071927 2.2576767 2.2576767 2.2576767 2.257677 | 16045.692152 | 2.225395 | 2.197306 | 0.015759 | 2.351946 | 89389.737012 | 99169.296649 | 2127.071927 | 1.113487 |
| Lyon, France | 0.002034 | 0.004406 | 0.002034 0.004406 19.524251 | 17.363027 | 0.075222 | 0.085095 | $0.075222 \ 0.085095 \ 0.008744 \ 0.039281 \ 97.298160$ | 0.039281 | 97.298160 | 98.570587 | 0.813995 | 0.049833 |
| Madrid, Spain | 0.012499 0.02 | 0.020972 | 0972 1230.997426 | 629.426697 | 0.216481 | 0.215247 | 0.004756 | 0.968889 | 0.216481 0.215247 0.004756 0.968889 3925.684133 | 4429.026140 | 38.963468 | 0.663908 |
| Malaga, Spain | 0.006003 0.01 | | 2249 141.492853 | 130.506136 | 0.098660 | 0.087803 | 0.004313 | 0.050275 | 0.098660 0.087803 0.004313 0.050275 747.080316 | 495.056826 | 42.324206 | 0.108422 |
| Marseille, France | 0.008642 0.01 | 0.016155 | 6155 197.571037 | 176.909276 | 0.228706 | 0.217401 | 0.008120 | 0.112155 | 0.228706 0.217401 0.008120 0.112155 922.147568 | 674.932282 | 12.705823 | 0.118751 |
| Milan, Italy | 0.007582 0.01 | 0.014704 | 4704 209.467841 | 189.445585 | 0.248847 | 0.239370 | 0.009419 | 0.308553 | 0.248847 0.239370 0.009419 0.308553 919.593745 | 764.411776 | 9.637183 | 0.293980 |
| Munich, Germany | 0.007735 0.01 | | 5168 229.079582 | 232.401819 | 0.263145 | 0.258919 | 0.008992 | 0.140416 | 0.263145 0.258919 0.008992 0.140416 1044.598972 | 771.404656 | 9.846286 | 0.126827 |
| Naples, Italy | 0.005169 | 0.009813 | 0.005169 0.009813 69.397460 | 64.105090 | 0.136178 | 0.140355 | 0.010303 | 0.076749 | 0.136178 0.140355 0.010303 0.076749 386.399691 | 431.106463 | 3.950655 | 0.076470 |
| Nuremberg, Germany | 0.004626 0.00 | 0.009131 | 9131 68.303878 | 70.242816 | 0.136035 | 0.128789 | 0.136035 0.128789 0.007595 0.069122 | 0.069122 | 381.560755 | 475.594117 | 2.984282 | 0.113547 |
| Oslo, Norway | 0.005630 | 0.010577 | 0.005630 0.010577 77.186905 | 57.111298 | 0.069818 | 0.058566 | 0.003382 | 0.030088 | 0.069818 0.058566 0.003382 0.030088 428.585345 | 455.853397 | 1.658191 | 0.047389 |
| Palma, Spain | 0.004053 | 0.008090 | 0.004053 0.008090 51.359697 | 53.444024 | 0.135368 | 0.132307 | 0.008980 | 0.070615 | $0.135368 \ \ 0.132307 \ \ 0.008980 \ \ 0.070615 \ \ 300.788654$ | 358.222173 | 1.286385 | 0.087270 |



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|------------------------------|---------------|-------------------|------------------------------|---|----------|-------------------|--|----------|---|---|-------------|----------|
| Paris, France | 0.005464 0.01 | 0.010834 | 10834 100.602912 | 74.981781 | 0.176433 | 0.169974 | 0.008737 | 0.336858 | 635.516402 | 472.253089 | 1.833786 | 0.053708 |
| Porto, Portugal | 0.002905 | 0.002905 0.005504 | 30.198572 | 29.665719 | 0.092149 | 0.092149 0.088463 | 0.007974 | 0.043253 | 0.007974 0.043253 143.279550 | 173.624362 | 0.333803 | 0.028542 |
| Prague, Czech Republic | 0.013257 | 0.013257 0.025728 | 630.092800 | 512.002287 | 0.181224 | 0.159014 | 0.003815 | 0.096364 | 0.181224 0.159014 0.003815 0.096364 1849.581723 1927.207649 | | 27.475664 | 0.414062 |
| Rennes, France | 0.001943 | 0.003255 | 0.001943 0.003255 18.398222 | 18.360547 | 0.074728 | 0.069203 | 0.074728 0.069203 0.007860 0.038912 | 0.038912 | 99.698286 | 98.033360 | 1.123189 | 0.146416 |
| Rome, Italy | 0.030118 0.05 | 0.055262 | 55262 1837.553791 | $1143.290766 \ 0.298524 \ 0.281754 \ 0.003947 \ 0.152936$ | 0.298524 | 0.281754 | 0.003947 | 0.152936 | 8115.247589 | 8115.247589 10064.383087 1388.824745 0.399801 | 1388.824745 | 0.399801 |
| Rotterdam, Netherlands | 0.007380 0.01 | 0.013766 | 139.228126 | 126.406795 | 0.199255 | 0.197546 | 0.199255 0.197546 0.008420 0.104872 | 0.104872 | 807.154365 | 502.890889 | 6.160848 | 0.119808 |
| Seville, Spain | 0.005354 0.00 | 0.009730 | 09730 125.556233 | 105.806228 | 0.180711 | 0.169387 | 0.008293 | 0.087249 | 0.180711 0.169387 0.008293 0.087249 638.344608 | 482.315488 | 12.070543 | 0.219362 |
| Sofia, Bulgaria | 0.016997 0.01 | 0.012414 | 149.960999 | 136.126625 | 0.200455 | 0.191835 | 0.008368 0.100143 | 0.100143 | 821.974195 | 544.606920 | 9.977724 | 0.118126 |
| Stockholm, Sweden | 0.008485 0.01 | | 6507 241.833753 | 250.934736 | 0.246878 | 0.197184 | 0.008822 | 0.455692 | 0.246878 0.197184 0.008822 0.455692 929.589701 | 728.274954 | 3.828852 | 0.119949 |
| Toulouse, France | 0.004864 | 0.004864 0.009904 | 99.419466 | 70.555064 | 0.159793 | 0.153936 | 0.159793 0.153936 0.008477 0.084128 | | 475.553876 | 465.424704 | 2.071956 | 0.051537 |
| Turin, Italy | 0.007217 0.01 | 0.013630 | 141.821731 | 151.648244 | 0.205652 | 0.202150 | 0.205652 0.202150 0.009111 0.116478 | 0.116478 | 808.815130 | 476.703016 | 12.039782 | 0.116632 |
| Tyne and Wear, UK | 0.029904 | 0.029904 0.058193 | 2139.861492 | 1237.870544 | 0.317945 | 0.285757 | 0.004616 1.314275 | 1.314275 | 9240.494702 | 11716.502484 | 79.690288 | 0.725430 |
| Valencia, Spain | 0.003953 | 0.003953 0.008310 | 66.079262 | 65.093321 | 0.142270 | 0.134132 | 0.142270 0.134132 0.008582 0.070899 | 0.070899 | 373.529783 | 435.697695 | 1.726928 | 0.111545 |
| Vienna, Austria | 0.006057 0.01 | 0.011969 | 333.969349 | 309.329290 | 0.251905 | 0.237454 | 0.006827 | 0.532720 | 0.251905 0.237454 0.006827 0.532720 1249.865738 1030.323555 | 1030.323555 | 6.166231 | 0.139557 |
| Warsaw, Poland | 0.011515 | 0.022370 | 0.011515 0.022370 431.051147 | 481.924582 | 0.351275 | 0.334576 | 0.008214 | 0.177704 | 0.351275 0.334576 0.008214 0.177704 1493.024513 1489.674855 | | 13.039317 | 0.160354 |



Author Contributions R.M.: conceptualization, formal analysis, methodology, software, writing—original draft preparation, writing—review and editing. M.S.N.: conceptualization, supervision, writing—original draft preparation, writing—review and editing. M.M.: supervision, writing—review and editing. S.P.: supervision, writing-review and editing.

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Data Availability Data available on request from the authors.

Declarations

Competing Interests The authors declare no competing interests.

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