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Risk diversification and vote decisions in mixed-member electoral systems

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Abstract

This paper builds on the literature about mixed-member electoral systems, exploring how ballot design interacts with voter behavior. We present a theoretical model for vote decision-making in mixed-member systems that takes into account the interaction between both tiers. The model is grounded in a spatial model for vote decision-making under risk and inspired by the logic of portfolio diversification under risk. Accordingly, voters are modeled as risk-averse decision-makers who may prefer diversified vote packages (i.e. split-ticket) when party and candidate uncertainties are highly correlated. The risk diversification strategy abates when voters cast their votes sequentially. This finding provides a potential explanation for the impact of vote sequence in mixed-member systems, an under-investigated topic in the literature. It thus links the established literature on mixed-member systems with scholarship on ballot design and its effects. Additionally, the paper's analysis explores the implications of combining the proposed model with the well-established wasted vote model.

Keywords Mixed-member systems · Vote decision under uncertainty · Risk diversification · Vote sequence.

1 Introduction

Mixed-member electoral systems typically allow voters to cast two ballots, one for plurality and the other for proportional representation. While many voting models exist for each tier, theoretical models that incorporate both tiers are scarce. For example, models based on voters' expected utilities predict strategic voting in the form of split-ticket voting, but they model voting behavior in each tier independently and disregard any possible interactions between them.

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This paper introduces a theoretical model that examines vote decisions in mixed-member systems, taking into account different interactions based on the system type. At the individual voter level, the model expands on conventional approaches by incorporating joint preferences for both ballots. By considering risk aversion, certain voters choose to reduce risk by splitting their ticket, even if their preference would be to cast a straight ticket if their decisions were independent. The model also explores the impact of ballot casting methods, distinguishing between simultaneous and sequential voting, and predicts varying levels of split-ticket voting. This finding offers a potential explanation for the influence of vote sequence in mixed-member systems, which remains an under-investigated topic. Additionally, while the proposed model is based on sincere voting, the analysis also explores the implications of incorporating strategic voting models, such as wasted voting.

2 Related literature

There is a substantial body of literature exploring the impact of mixed-member systems on voting behavior and election outcomes (for an overview, see Herron et al. 2018). These studies can generally be categorized into two approaches: controlled comparison and contamination. The controlled comparison approach treats mixed-member systems as controlled experimental settings, where two distinct electoral systems operate within the same social, economic, and cultural contexts (e.g. Moser and Scheiner 2012). This approach assumes that the micro-level voting decision processes in each tier are independent of each other. One straightforward implication of this approach is that voting behavior in the proportional representation (PR) tier tends to be sincere, while strategic considerations, such as avoiding wasted votes, may influence voting behavior in the first-past-the-post (FPTP) tier (see e.g. Bawn 1999). Consequently, this could lead to a specific type of split-ticket voting pattern. However, it is important to note that this is just one possible implication. Alternatively, it can also be postulated that voting behavior in the PR tier is strategic with the aim of achieving certain coalition governments (Pappi and Thurner 2002; Shikano et al. 2009). Regardless of how voting behavior is modeled in each tier, models based on this approach commonly treat the voting decisions in each tier as independent of each other.

In contrast to the controlled comparison approach, the contamination approach emphasizes the interdependencies between the two tiers of mixed-member systems (Ferrara et al. 2005). Early studies in this approach have demonstrated that running a candidate in a FPTP district can boost the party's list votes in the same district, indicating a contamination effect (Herron and Nishikawa 2001). This finding suggests that voters' decision-making processes in both tiers are not independent but rather interconnected. Subsequent studies have provided evidence for various types of contamination effects (e.g. Krauss et al. 2012), while others have expressed skepticism about the existence of such effects (e.g. Maeda 2008; Kurella 2016). It is important to note that the empirical evidence on contamination effects is often derived from aggregate-level election outcomes across different countries. This may contribute to the mixed findings. In this regard, Rheault et al. (2020) advanced our understanding of contamination effects at the individual voter level. Their study shows that the decision order between candidate and party votes, as well as the voter's level of political information, plays a crucial role, which possibly explains the mixed empirical findings in the literature. However, while their contribution is primarily empirical, the theoretical foundations of contamination effects at the individual level remain less developed in the literature, particularly in comparison to the more established controlled comparison

approach. Recently, Bräuninger and Pappi (2023) advanced a formal theoretical model addressing contamination effects by explicitly introducing non-separable preferences in mixed-member voting. Yet, their work leaves open the substantive question of why and under what conditions these non-separable preferences arise. We still need a clear micro-foundation for this non-separability.

While studies of mixed-member systems have proliferated at least since the 1990 s, there are still several under-investigated topics within this field. One such topic is the effect of the ballot structure (Barnes et al. 2017). Mixed-member systems can adopt different types of ballot structures. For instance, in Germany, voters have a single ballot for both tiers, whereas in Japan, voters cast separate ballots for each tier. This difference in ballot structure leads to distinct vote sequences, which can potentially impact voting behavior. When voters have a single ballot, they can simultaneously choose a candidate and a party. In contrast, separate ballots require voters to make sequential decisions. These different vote sequences can influence voting behavior in various ways. In multiple survey experiments, Shikano et al. (2023); Herron and Shikano (2025) found that the German-style simultaneous ballots facilitate more split-ticket voting compared to separate ballots for each tier. Their research also provides further insights into the mechanisms underlying vote decisions:

- Simultaneous ballots require less overall decision time compared to separate ballots;
- when faced with separate ballots, voters tend to spend significantly less time on the second ballot compared to the first one;
- with simultaneous ballots, voters allocate their attention to both tiers before making their initial decision.

While these results suggest the existence of an interaction between both tiers, they cannot be explained by either of the aforementioned approaches. Therefore, in the following sections, a new model will be proposed that takes these findings into account and provides a framework for understanding vote decision-making in mixed-member systems.

3 Theoretical model

The proposed model incorporates the basic ideas outlined below. In this model, voters are faced with two decisions: selecting a district candidate in the FPTP tier and choosing a party list in the PR tier. Their choices are driven by the goal of maximizing their utility based on expected outcomes. There are two types of outcomes: district-specific pork and national-level politics. District candidates are responsible for the former, while parties are responsible for the latter.

Candidates and parties make announcements that align with certain outcomes in the event of winning the district race or gaining governmental power. Voters are assumed to have knowledge of their own ideals regarding each type of outcome, enabling them to evaluate the proximity or distance between their positions and those announced by candidates and parties. The closer the alignment between voters and a candidate or party position, the higher the utility for the voter.

Voters in this model are primarily concerned with the actual outcomes that may differ from the announced positions. This is because the realization of announced positions relies on the legislative process following the election. Consequently, there is inherent

uncertainty associated with the announced positions and their distance from the voters' own positions. In this regard, voters are assumed to be risk-averse, meaning they discount utilities of outcomes with higher levels of uncertainty. Additionally, the uncertainty of candidates' and parties' positions may not be independent from each other. For instance, it is reasonable to assume that a party and its candidates have a positive correlation in their uncertainty, while a party and candidates from another party are less correlated or independent. Voters also take into account this covariance when casting both candidate and party list votes. Specifically, a vote combination with a higher covariance will be discounted to reduce the overall risk in the package of outcomes. As a result, some voters may choose to split their ticket, while they would have preferred to cast a straight ticket if the votes were cast independently.

The basic idea above is inspired by the logic of portfolio diversification under risk, as originally formulated by Markowitz (1952). In his seminal work, Markowitz demonstrated that risk-averse investors do not evaluate assets in isolation, but instead consider the expected return and risk of an entire portfolio. Here, the total risk is shaped not only by the variance of individual assets but also by the covariance between them. Analogously, the model presented here treats vote combinations - candidate and party list votes - as a joint decision under uncertainty, where the total utility is influenced by both the expected alignment with a voter's ideal points and the risk associated with deviations in realized policy outcomes. In line with the investment logic, voters are modeled as risk-averse decision-makers who may prefer diversified vote packages when party and candidate uncertainties are highly correlated.

Here, we have to make some points about our model clear: First, regarding institutional differences between corrective mixed-member systems (e.g., Germany) and parallel systems (e.g., Japan) (Massicotte and Blais 1999), our basic setup is more consistent with the latter, where district-level pork and national-level government are determined separately by the FPTP and PR votes, respectively. Nevertheless, we argue that our model can also be applied to corrective systems, provided we assume that voters focus on the perceived consequences of their votes, rather than on the technical details of how votes are aggregated across tiers. Crucially, our model departs from the common assumption that the two votes are decided independently. Instead, we treat vote choice as a joint decision under uncertainty, driven by the voter's concern for coherence and risk in the overall policy outcome. Here, it is important to distinguish between two types of independence: institutional independence in how votes are tallied, and perceptual independence in how voters evaluate the risk that elected representatives may diverge from their announced positions after the election. Our model is agnostic about the former, but fundamentally grounded in the latter. Second, our model is not a game-theoretic, but a decision-theoretic model. This means that individual voters are treated independently, and they make their own decisions based on their personal circumstances which are assumed to be given. Last, but not least, our model deals with one of many different possible types of motivation. Other types of motivation, such as expressive motivation (Schuessler 2000), may also play a role in split-ticket voting. However, the goal of this paper is not to model all potential motives, but rather to isolate and explore one theoretical mechanism: risk-averse utility maximization under uncertainty. By focusing on instrumental motives, we aim to provide a clear and formal account of how voters might evaluate vote combinations as a package when party and candidate behaviors are uncertain. This will never exclude the possibility of another model in which expressive and instrumental motives may interact or compete in such decisions.

In the following discussion, we will first introduce a simple model based on sincere voting without any strategic considerations. Then, we will incorporate the wasted vote model in the FPTP tier.

3.1 Basic set up

Consider three-party competition under a mixed-member system with $P = \{p_1, p_2, p_3\}$. Each party can run one candidate in each district and denote the set of candidates in district d by $C_d \subseteq \{c_{d,1}, c_{d,2}, c_{d,3}\}$ for $d \in \{1, \dots, D\}$. A strategy for voter i residing in d is a pair $v_i = (c, p) \in P \times C_d$.

Assume that voter i has an ideal point on the general policy dimension and the pork dimension. Since each voter can be eligible to vote only in one district, we drop the index d in the following so that: $c_{d,j} = c_j$. Voter i 's benefit from p_j and c_j is the function of the distance between their announced position and i 's ideal point, which we denote by $d_i(c_j)$ and $d_i(p_j)$. Their announced positions have some uncertainty ϵ_{cj} and ϵ_{pj} , which concerns e.g. whether the announced positions will be implemented in legislation after the election. By using the squared loss function, we define i 's utility from candidate j 's winning the district race (w_{cj}) as follows:

$$U_i(w_{cj}) = - (d_i(c_j) + \epsilon_{cj})^2. \quad (1)$$

Analogously, we also define i 's utility from party j 's winning the government power (g_{pj}) as follows:

$$U_i(g_{pj}) = - (d_i(p_j) + \epsilon_{pj})^2. \quad (2)$$

The expected value of ϵ_{cj} and ϵ_{pj} is assumed to be zero and their variances are denoted by σ_{cj} and σ_{pj} , respectively. Further, we assume their independence from the other components in the utility function:

$$\epsilon_{cj} \perp\!\!\!\perp d_i(c_j), \quad \epsilon_{pj} \perp\!\!\!\perp d_i(p_j), \quad \epsilon_{cj} \perp\!\!\!\perp d_i(p_j), \quad \epsilon_{pj} \perp\!\!\!\perp d_i(c_j),$$

The equivalent set-up of spatial voting model with uncertainty can be found in the literature (e.g. Enelow and Hinich 1984). Distinct from the previous models, however, we treat both decisions in FPTP and PR tiers simultaneously. And more crucially, we allow that random terms of both tiers can be correlated with covariance denoted by $\sigma_{cj,pj}$.

3.2 Independent choices in FPTP and PR

Before we discuss simultaneous choice under mixed member systems, we discuss independent choices in FPTP and PR, respectively. In the following, we deal with each voter's choice independently from each other in a decision-theoretic model. Therefore, we drop the index i in the following so that $U_i(\cdot) = U(\cdot)$, $v_i = v$, $d_i(c_j) = d_{cj}$ and $d_i(p_j) = d_{pj}$. Furthermore, without loss of w.l.o.g we assume for FPTP and PR, respectively:

$$d_{c1} < d_{c2} < d_{c3} \quad (3)$$

$$d_{p1} < d_{p2} < d_{p3} \quad (4)$$

For both FPTP and PR, our starting point is the decision-theoretic model of Black (1978) about strategic voting under multipartism. We extend it by using the above set-up with vote decision under uncertainty. This model's most important element is the pivot probability $p_{jj'}$ that a single vote can determine who of candidate c_j and $c_{j'}$ for $j \neq j'$ wins the most votes in the election.

According to his model,¹ a voter would cast her vote in favor of the second closest c_2 if:

- p_{23} is large relative to p_{12} and p_{13} .
- $d_{c_3} - d_{c_2}$ is large relative to $d_{c_2} - d_{c_1}$.

For the latter condition, Black did not explicitly consider uncertainty of the distance to candidate j : $\sigma_{c_j}^2$. By taking it into account, we can add the following conditions:

- $\sigma_{c_3}^2 - \sigma_{c_2}^2$ is large relative to $\sigma_{c_2}^2 - \sigma_{c_1}^2$.

The Black model was originally developed for FPTP elections but can also be extended to vote decision-making under PR. The key distinction lies in the assumption that the pivot probability between any pairs of parties is constant (p_{pr}). However, even in PR systems, there may arise situations where a voter deviates from their closest party in favor of the candidate from their second closest party. This can occur if:

$$d_{p_2} - d_{p_1} < \sigma_{p_1}^2 - \sigma_{p_2}^2 \quad (5)$$

In other words, we expect a PR vote in favor of the second closest party if the difference between the distance to the announced positions is compensated by the second closest party's more certain position in comparison to the closest one's more uncertain position. For more details see Appendix B.

3.3 Simultaneous choice

If we take the controlled comparison approach, we can employ the above models for a single tier. In this subsection, in contrast, we will model simultaneous choice in both tiers with interaction.

We assume that the voter seeks to have the minimum square of the sum of distances to the candidate and parties. This corresponds to the following:

$$U(w_{c_j}, g_{p_k}) = -(d_{c_j} + \epsilon_{c_j} + d_{p_k} + \epsilon_{p_k})^2 \quad (6)$$

This function represents a joint evaluation of the overall policy package at both the local and national levels. The intuition behind summing the distances to the candidate and party before squaring is that voters form a holistic perception of risk across both tiers. They then penalize large overall deviations more, consistent with risk aversion. This structure also captures voters' concerns about coherence or "alignment" between the candidate and the party. For example, the greater the total misalignment across both dimensions, the larger the potential for disappointment or regret in the policy outcomes – especially when voters

¹ See for more details Appendix A.

are uncertain about how closely elected representatives will adhere to their announced positions.

We start with a special situation where $C = \{c_1\}$. That is, there is only one district candidate. Therefore, voter i has only three possible options (c_1, p_1) , (c_1, p_2) and (c_1, p_3) . Apparently, the voter has no need to strategically cast her vote in the FPTP tier.

Now consider whether there exists a situation in which a voter would split the ticket in favor of p_2 . First, we obtain the expected utility of casting the straight ticket (v_{c_1, p_1}) :

$$\begin{aligned} EU(v_{c_1, p_1}) &= -E[(d_{c_1} + \epsilon_{c_1} + d_{p_1} + \epsilon_{p_1})^2] \\ &= -d_{c_1}^2 - d_{p_1}^2 - 2d_{c_1}d_{p_1} - E[(\epsilon_{c_1} + \epsilon_{p_1})^2] \\ &= -d_{c_1}^2 - d_{p_1}^2 - 2d_{c_1}d_{p_1} - \sigma_{c_1}^2 - \sigma_{p_1}^2 - 2\sigma_{c_1, p_1} \end{aligned} \quad (7)$$

Analogously, for a split ticket (v_{c_1, p_2})

$$\begin{aligned} EU(v_{c_1, p_2}) &= -E[(d_{c_1} + \epsilon_{c_1} + d_{p_2} + \epsilon_{p_2})^2] \\ &= -d_{c_1}^2 - d_{p_2}^2 - 2d_{c_1}d_{p_2} - \sigma_{c_1}^2 - \sigma_{p_2}^2 - 2\sigma_{c_1, p_2} \end{aligned} \quad (8)$$

The voter would split if:

$$\begin{aligned} EU(v_{c_1, p_1}) &< EU(v_{c_1, p_2}) \\ -d_{c_1}^2 - d_{p_1}^2 - 2d_{c_1}d_{p_1} - \sigma_{c_1}^2 - \sigma_{p_1}^2 - 2\sigma_{c_1, p_1} &< -d_{c_1}^2 - d_{p_2}^2 - 2d_{c_1}d_{p_2} - \sigma_{c_1}^2 - \sigma_{p_2}^2 - 2\sigma_{c_1, p_2} \\ (d_{p_2} - d_{p_1})(d_{c_1} + d_{p_1} + d_{p_2}) &< \sigma_{p_1}^2 + 2\sigma_{c_1, p_1} - \sigma_{p_2}^2 - 2\sigma_{c_1, p_2} \end{aligned} \quad (9)$$

From here, we learn that the likelihood of a split ticket is higher if:

- the distances to the announced positions of Party 1 and 2 are similar.
- the announced positions of both parties and Candidate 1 are closer to the voter.

Since $d_{p_2} > d_{p_1}$ and all d 's are non-negative, the left side of the last inequality is larger than zero. Therefore we obtain a further **necessary** condition for v_{c_1, p_2} :

$$\begin{aligned} \sigma_{p_1}^2 + 2\sigma_{c_1, p_1} - \sigma_{p_2}^2 - 2\sigma_{c_1, p_2} &> 0 \\ \sigma_{p_2}^2 - \sigma_{p_1}^2 &< 2(\sigma_{c_1, p_1} - \sigma_{c_1, p_2}) \end{aligned} \quad (10)$$

From this inequality, we can obtain some further insights concerning uncertainty of the announced position:

- The larger the uncertainty of Party 1's position and/or the smaller the uncertainty of Party 2's position, the more likely is a split ticket.
- The more consistent the party and its candidate in the stochastic term, the more likely there will be a split ticket. In other words, if the candidates have higher potentials to behave differently from their parties, voters are more likely to cast a straight ticket.²

² Note that this is one of the consequences of using the quadratic loss over the sum of distance from ideal points. If we instead use the Euclidean distance, that is $U(c_1, p_1) = -(x_{c_1} + \epsilon_{c_1} - \theta_{c_1})^2 + (x_{p_1} + \epsilon_{p_1} - \theta_{p_1})^2$, neither σ_{c_1, p_1} nor σ_{c_1, p_2} plays any role.

This result can be extended to the case where we have not only one candidate, but two or three candidates. For illustrative purposes, we position parties and candidates along a one-dimensional spatial axis (e.g., $\{1, 2, 4\}$), respectively. This setup is not meant to represent any specific party system, but rather to capture a stylized case where two options are ideologically closer and one is more distant. This is a simplified, but realistic configuration where parties are positioned in an asymmetrical way. The substantive results of our model, in particular those concerning risk diversification and vote sequence, do not depend on this specific configuration even if we change the relative distance between parties and/or the number of parties. While more complex placements such as two-dimensional structures could yield further insights, our goal here is to isolate and demonstrate the effect of correlated uncertainty in the simplest tractable setting.

Both panels in Fig. 1 present predicted votes for different voter positions in a certain candidate/party constellation on the pork and national policy dimensions. The grey dotted grid lines divide the electorate to different voting patterns based on dimension-by-dimension distance-based decisions. That is, both tiers are treated as independent. By contrast, the solid lines divide the electorate based on the model above which takes into account covariance between candidates' and party's uncertainty. The left and right panels differ in covariance, the left one is based on a higher and the right one is based on a lower covariance. If we focus on the left panel, some voters who are predicted to cast straight tickets in the dimension-by-dimension model would split their tickets in our model. These voters are very close to the grey grid lines, which means the closest and second closest candidates/parties are almost equidistant to them. Furthermore, this area becomes smaller if the distance on the other dimension increases. For example, if we look at the border area at v_{c_1, p_1} and v_{c_1, p_2} , its area is the largest if voters are very close to C_1 's announced position. This area becomes smaller if voters are more distant from C_1 . This corresponds to the second prediction requiring voter closeness of parties' and candidates' positions. If we turn to the right panel, the size of the border areas is much smaller than in the left panel. That is, if voters expect the district candidate would behave differently in the legislature, they are more likely to stick to the straight ticket.

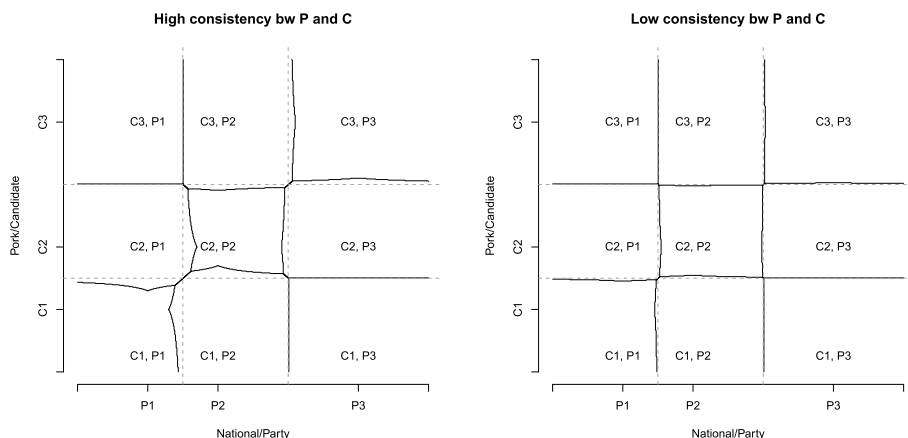


Fig. 1 Expected voting behavior depending on announced positions uncertainty. The left panel is based on a higher correlation between party and candidate announced position ($\rho = 0.8$). The right panel is based on a lower correlation ($\rho = 0.2$). The grey dotted lines divide voters depending on the closest vote combinations

The above analysis does not consider the possibility of wasted voting in the FPTP tier. This can be incorporated by introducing pivot probability denoted by $p_{jj'}$, which means that a single vote can determine the rank of candidate j and j' . Note that we remain in the decision theoretic framework in which $p_{jj'}$ is exogenous. Below, we follow Black's decision theoretic model (Black 1978, for more details see Appendices of this paper). Assume a three candidate/party-race with a voter whose party distance is $d_{p_1} < d_{p_2} < d_{p_3}$. Further assume that her distance to candidates also corresponds to that of parties. Now consider a choice between three options: v_{c_1, p_1} , v_{c_2, p_1} and v_{c_3, p_1} .

From the above exercise, we know:

$$\begin{aligned} EU(w_{c_1}, g_{p_1}) &= -d_{c_1}^2 - d_{p_1}^2 - \sigma_{c_1}^2 - \sigma_{p_1}^2 - 2\sigma_{c_1, p_1} \\ EU(w_{c_2}, g_{p_1}) &= -d_{c_2}^2 - d_{p_1}^2 - \sigma_{c_2}^2 - \sigma_{p_1}^2 - 2\sigma_{c_2, p_1} \\ EU(w_{c_3}, g_{p_1}) &= -d_{c_3}^2 - d_{p_1}^2 - \sigma_{c_3}^2 - \sigma_{p_1}^2 - 2\sigma_{c_3, p_1} \end{aligned} \quad (11)$$

Casting a ballot for c_1 and p_1 has a utility difference to non-voting with κ representing the voting cost:

$$\begin{aligned} EU(v_{c_1, p_1}) - EU(v_0) &= p_{12}(EU(w_{c_1}, g_{p_1}) - EU(w_{c_2}, g_{p_1})) \\ &\quad + p_{13}(EU(w_{c_1}, g_{p_1}) - EU(w_{c_3}, g_{p_1})) - \kappa \\ &= p_{12}(-d_{c_1}^2 + d_{c_2}^2 - \sigma_{c_1}^2 + \sigma_{c_2}^2 - 2\sigma_{c_1, p_1} + 2\sigma_{c_2, p_1}) \\ &\quad + p_{13}(-d_{c_1}^2 + d_{c_3}^2 - \sigma_{c_1}^2 + \sigma_{c_3}^2 - 2\sigma_{c_1, p_1} + 2\sigma_{c_3, p_1}) - \kappa \end{aligned} \quad (12)$$

$$\begin{aligned} EU(v_{c_2, p_1}) - EU(v_0) &= p_{12}(EU(w_{c_2}, g_{p_1}) - EU(w_{c_1}, g_{p_1})) \\ &\quad + p_{23}(EU(w_{c_2}, g_{p_1}) - EU(w_{c_3}, g_{p_1})) - \kappa \\ &= p_{12}(-d_{c_2}^2 + d_{c_1}^2 - \sigma_{c_2}^2 + \sigma_{c_1}^2 - 2\sigma_{c_2, p_1} + 2\sigma_{c_1, p_1}) \\ &\quad + p_{23}(-d_{c_2}^2 + d_{c_3}^2 - \sigma_{c_2}^2 + \sigma_{c_3}^2 - 2\sigma_{c_2, p_1} + 2\sigma_{c_3, p_1}) - \kappa \end{aligned} \quad (13)$$

The voter will split iff:

$$\begin{aligned} EU(v_{c_1, p_1}) - EU(v_0) - EU(v_{c_2, p_1}) + EU(v_0) &< 0 \\ 2p_{12}(EU(w_{c_1}, g_{p_1}) - EU(w_{c_2}, g_{p_1})) + p_{13}(EU(w_{c_1}, g_{p_1}) - EU(w_{c_3}, g_{p_1})) \\ - p_{23}(EU(w_{c_2}, g_{p_1}) - EU(w_{c_3}, g_{p_1})) &< 0 \end{aligned} \quad (14)$$

Here, the equation looks similar to Black's original model. Accordingly, the voter is more likely to cast a ticket in favor of candidate 2 (i.e. split ticket), if the pivot probability between 2 and 3 becomes larger and/or the other pivot probabilities become smaller.

Here, we have to note that there are significant differences between Black's original model and the model here. As we have seen above, for voters whose distance to candidates is $d_{c_1} < d_{c_2} < d_{c_3}$, the following is not always the case:

$$\begin{aligned} EU(w_{c_1}, g_{p_1}) - EU(w_{c_2}, g_{p_1}) &> 0 \\ EU(w_{c_1}, g_{p_1}) - EU(w_{c_3}, g_{p_1}) &> 0 \\ EU(w_{c_2}, g_{p_1}) - EU(w_{c_3}, g_{p_1}) &> 0 \end{aligned} \quad (15)$$

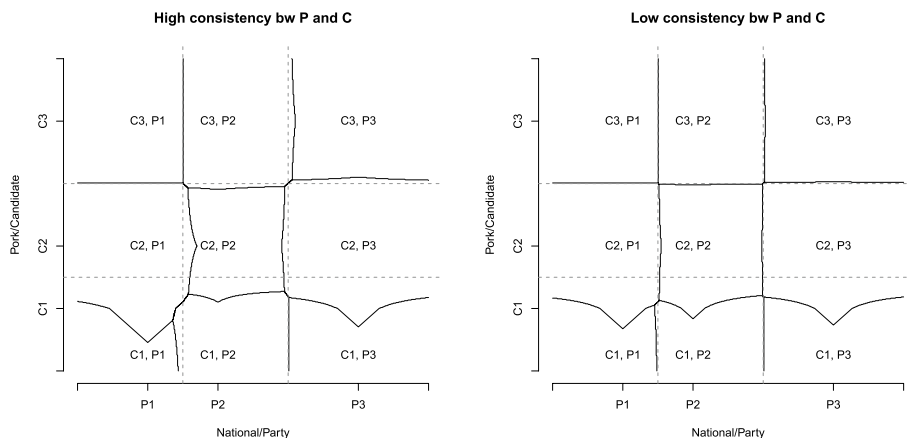


Fig. 2 Expected strategic voting behavior depending on voter positions. The left panel is based on a higher correlation between party and candidate announced position ($\rho = 0.8$). The right panel is based on a lower correlation ($\rho = 0.2$). Pivot probabilities are $p_{23} = 1.5p_{12} = 1.5p_{13}$

The above exercise was based on the very specific situation where voters have only one choice in the PR tier. Its straightforward extension would be to consider all possible pairs of vote combinations with their pivot probabilities. This, however, would be not plausible since the above wasted voting model concerns a decision in the FPTP tier and not for the comparison between two vote combinations which differ only in the PR tier (e.g. v_{c_1, p_1} and v_{c_1, p_2}). A more reasonable way to extend the above model to multipartism in the PR tier should focus on the two most promising vote combinations which differ in FPTP decisions. Suppose two vote combinations have the highest and second highest expected utility $EU(w_{c_1}, g_{p_2}) > EU(w_{c_2}, g_{p_3}) > \dots$ but Candidate 1 is quite unlikely to win the race and Candidates 2 and 3 are the more realistic winners of this district. In this case, we only consider the above two vote combination and the best vote combination with w_{c_3} , e.g. v_{c_3, p_2} .

Figure 2 gives predicted vote combinations depending on voter positions with $p_{23} = 1.5p_{12} = 1.5p_{13}$. A comparison with the previous result without strategic considerations (Fig. 1) leads to an important insight: In general, the share of Candidate 2 increases due to wasted voting, however its volume is different depending on party choice. In particular, voters for Party 2 are less affected by wasted voting. Some voters prefer to split their ticket due to risk-diversification.

3.4 Sequential choice

Sequential choice differs from simultaneous choice at least in two points: First, voters have to explicitly make one decision before coming to the next decision. Therefore, at the time point of the second decision, the first decision is externally given. Second, voters have already paid a certain decision cost before the second decision. For this reason, voters are less willing to pay the cost at the second vote.

Both above elements are related to each other: If the decision in one tier has been made, the choice set will be reduced in the second decision. In the above example with 3 parties with their own candidates, there are 9 vote combinations if a voter decides

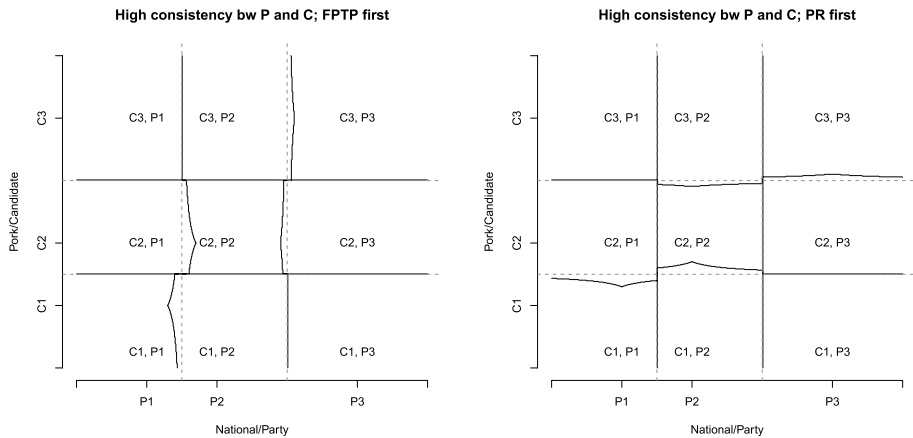


Fig. 3 Predicted voting combinations in sequential choice. The left panel is based on the sequence FPTP first, the right panel is based on the sequence PR first. Both panels are based on a higher correlation between party and candidate announced position ($\rho = 0.8$)

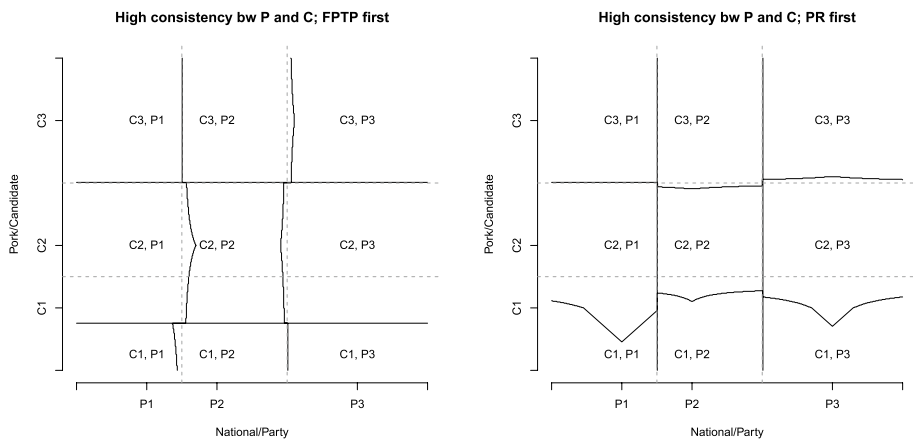


Fig. 4 Expected vote combinations in sequential choice with wasted vote. The left panel is based on the sequence FPTP first, the right panel is based on the sequence PR first. Both panels are based on a higher correlation between party and candidate announced position ($\rho = 0.8$)

simultaneously in both tiers. In sequential choice, by contrast, a voter has a choice set of 3 alternatives in each decision.

Figure 3 gives predicted voting combinations in sequential choice, without considering strategic voting. The left panel is based on the vote sequence with FPTP first. Since voters care only about the candidates in the first choice, their decision is for the closest candidates. Therefore, the border of the candidate choice perfectly corresponds to the grey dotted line. Given this decision, voters can decide for a party. Here, they also consider covariance between the selected candidate's position and party positions. Straight-ticket voting is avoided for certain voters who in particular are very close to the selected candidate position. This is an analogous result as above in the left panel of Fig. 1. The same can be observed in the right panel which only exchanges the candidates with the parties. If we compare simultaneous and sequential voting, the former

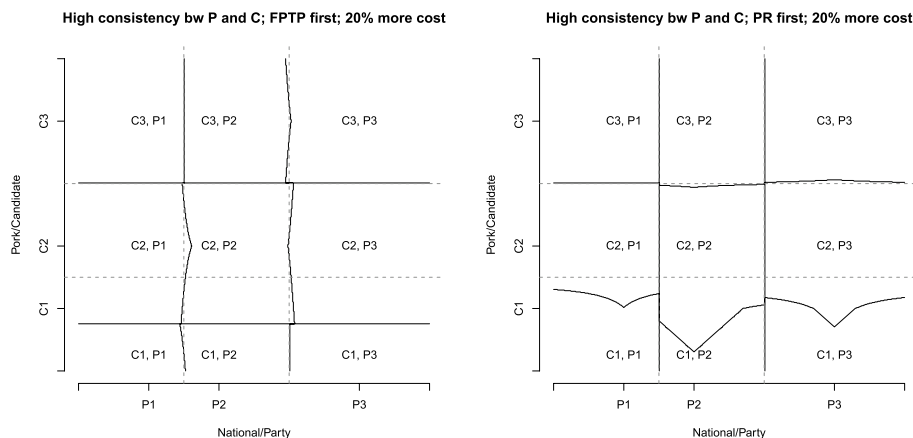


Fig. 5 Expected voting behavior in sequential choice. The left panel is based on the sequence FPTP first, the right panel is based on the sequence PR first. Both panels are based on a higher correlation between party and candidate announced position ($\rho = 0.8$)

predicts more split-ticket voting than the latter, which can provide a possible explanation for the empirical result above.

We can also introduce wasted voting in sequential choice settings (Fig. 4). A comparison with the left panel in Fig. 2 makes clear that sequential voting with FPTP first eliminates the effect of risk diversification in favor of a split ticket v_{c_1, c_2} and Candidate 2 benefits from wasted voting independently from the party decision. The sequence with PR first, by contrast, shows only a marginal difference in favor of a straight ticket. The counter effect of risk diversification against wasted voting remains.

Thus far, the model does not include any explicit cost for the second decision. This can be introduced by differentiating voting costs in the second choice. Suppose that a voter with preference $c_1 > c_2 > c_3$ has already cast a vote for Party 1 in the PR tier. Now she has to decide between the candidates. This is a similar situation above in Equations 12 and 13. In the above equations, we have the same voting cost κ for both vote decisions. In sequential choice, however, a voter can have different costs depending whether she has already decided for the candidate's party or not. Correspondingly, we introduce a voter's decision cost for the candidates whose party has been voted by her: $\kappa' = \kappa - \delta_\kappa$ for $\delta_\kappa > 0$. In this situation, the voter would split the ticket if and only if:

$$\begin{aligned}
 & EU(v_{c_1, p_1}) - EU(v_0) - EU(v_{c_2, p_1}) + EU(v_0) < -\delta_\kappa \\
 & 2p_{12}(EU(w_{c_1}, g_{p_1}) - EU(w_{c_2}, g_{p_1})) + p_{13}(EU(w_{c_1}, g_{p_1}) - EU(w_{c_3}, g_{p_1})) \\
 & \quad - p_{23}(EU(w_{c_2}, g_{p_1}) - EU(w_{c_3}, g_{p_1})) < -\delta_\kappa
 \end{aligned} \quad (16)$$

This is a more severe condition for the same voter to split the ticket than in simultaneous choice if all the differences between expected utilities in Equation 16 obtain. As we have observed above, however, this does not have to be the case. In this case, the impact of δ_κ will be more limited.

We can also consider sequential choice in reverse order, that is FPTP first. Again suppose that a voter with preference $p_1 > p_2 > p_3$ has already decided for Candidate 1.

$$\begin{aligned}
 EU(v_{c_1, p_1}) - EU(v_0) - EU(v_{c_1, p_2}) + EU(v_0) &< -\delta_k \\
 p_{pr}(EU(w_{c_1, g_{p_1}}) - EU(w_{c_1, g_{p_2}})) &< -\delta_k
 \end{aligned}
 \tag{17}$$

Given p_{pr} is larger than the pivot probabilities in the FPTP tier, the impact of the reduced cost is smaller in the sequential choice with FPTP first. This can be also visible if we compare Fig. 4 with Fig. 5 which considers the different voting costs in the second choice. For this figure, the reduced cost corresponds to 20% of the expected utility from straight ticket vote decision ($\delta_k = 0.2EU(v_{c_1, p_1})$). The effect of this differentiated cost in the second vote decision is only marginal if voters make a choice in the FPTP tier first. By contrast, the impact of this additional cost is much higher if they cast their PR votes first.

4 Discussion

In this paper, a theoretical model was proposed to explain vote decisions under mixed-member systems, where voters cast two votes that can be interconnected. The model takes into account the interaction between parties and candidates by incorporating the covariance between their uncertain announced positions. By doing so, voters are able to choose a vote combination that maximizes their utilities, taking into consideration the associated risk. The proposed model yields significant implications regarding the impact of ballot structures and resulting vote sequences. It suggests that simultaneous ballots, where voters can choose a candidate and a party simultaneously, are likely to result in more split-ticket voting compared to sequential ballots, where voters make decisions in a specific order. Additionally, the effect of wasted votes in the FPTP tier varies depending on the party choice made in the other tier when simultaneous ballots are used.

One of the key assumptions in the proposed model is that the joint utility for a vote combination is based on a squared loss function over the combined distances to candidate and party positions. This assumption plays a crucial role in generating the results above. From a substantive standpoint, this assumption can be justified as it allows for the consideration of the covariance between the uncertain positions of parties and candidates. If an alternative metric, such as Euclidean distance, were used, it would not capture the interdependence between these positions. Furthermore, if the squared distance were replaced with the straight linear distance, it would imply that voters are not risk-averse and do not prioritize risk diversification in their decision-making process.

We are fully aware that we can currently justify our model in the Friedman's 'as-if' framework: Independently whether the assumed utility function is empirically valid, the model can explain existing findings (Rheault et al. 2020; Shikano, et al. 2023a; Herron and Shikano 2025). At the same time, empirical results do support one of the crucial points: the utility components of both votes are not independent. For example, the series of studies about contamination effects suggests that both utility components are not independent from each other. But more importantly, the use of a squared loss function over the combined distances to candidate and party positions allows the model to incorporate not only variance but also covariance between tiers, paralleling how total portfolio risk is calculated in modern portfolio theory. By assuming this, our model can offer a conceptual lens through which to understand how risk-averse voters might behave in mixed-member systems when facing correlated uncertainties in party and candidate positions.

The proposed model is based on a simplified setting. While the current model assumes equal weighting of utility in each tier, it is important to note that this setting can be extended

to incorporate additional weights for each tier without fundamentally altering the basic results. A more intricate extension could involve incorporating strategic coalition voting in the PR tier. This would require modeling how voters strategically evaluate the potential coalitions that could be formed based on the party list vote outcomes (e.g. Herrmann 2014). This extension definitively belongs to the future agenda.

The proposed model offers contributions beyond its ability to predict vote outcomes based on different vote sequences. It has the capacity to differentiate between split-ticket voting driven by strategic considerations and one motivated by risk diversification. Moreover, the model can be applied for theory-driven empirical analyses. For instance, survey data can be utilized to obtain individual respondents' vote decisions or intentions, as well as their perceived distances from parties and candidates, which can be approximated using feeling thermometer scores. By leveraging this dataset, it becomes possible to estimate the remaining parameters of the model, including the uncertainty of announced positions and their covariance.

Appendix

A Extending Black's model with uncertainty

Below we summarize Black's model in the paper's set-up of a three-candidate-competition with $\mathbb{C} = \{c_1, c_2, c_3\}$.

The model is based on the Riker and Ordeshook's calculus of voting: $R = E^1 - E^0 = PB - C + D$. Or more specifically, its extension by McKelvey and Ordeshook to the multi-party competition. Accordingly, $R^j = E^j - E^0 = \sum_{j' \neq j} p_{jj'} B_{jj'} - C + D$. In this model, pivot probability $p_{jj'}$ plays a crucial role. It is the probability that a single vote can determine who of candidate c_j and $c_{j'}$ for $j \neq j'$ wins the most votes in the election. It is important to note that Black assumes $p_{jj'}$ to be exogenous.

Assume voter i 's distance to policy outcome implemented by each candidate is $d_i(c_1) < d_i(c_2) < d_i(c_3)$. That is, voter i has a preference $c_1 > c_2 > c_3$.

Given the uncertainty of d and the squared loss function, the utility which voter i can expect in case of candidate j 's winning (w_{c_j}) is:

$$EU(w_{c_j}) = -d_i(c_j)^2 - \sigma_{c_j}^2 \quad (18)$$

Casting ballot for Candidate 1 (v_{c_1}) has utility difference to non-voting (v_0):

$$\begin{aligned} EU(v_{c_1}) - EU(v_0) &= p_{12}(EU(w_{c_1}) - EU(w_{c_2})) + p_{13}(EU(w_{c_1}) - EU(w_{c_3})) - \kappa \\ EU(v_{c_2}) - EU(v_0) &= p_{12}(EU(w_{c_2}) - EU(w_{c_1})) + p_{23}(EU(w_{c_2}) - EU(w_{c_3})) - \kappa \end{aligned}$$

with $\kappa = C - D$.

Voter i votes for c_2 , iff:

$$\begin{aligned}
 & EU(c_1) - EU(v_0) < EU(c_2) - EU(v_0) \\
 & 2p_{12}(EU(c_1) - EU(c_2)) + p_{13}(EU(c_1) - EU(c_3)) \\
 & \quad - p_{23}(EU(c_2) - EU(c_3)) < 0
 \end{aligned}$$

Based on this inequality, Black identifies two conditions for voter i 's defection from her closest candidate in favor of the second-closest one:

- p_{23} is large relative to p_{12} and p_{13} .
- $EU(c_2) - EU(c_3)$ is large relative to $EU(c_1) - EU(c_2)$.

For the latter condition, Black did not explicitly consider uncertainty σ^2 . Therefore, the second condition in his version is as follows:

- $d_{c_3} - d_{c_2}$ is large relative to $d_{c_2} - d_{c_1}$.

By taking uncertainty into account, we can add the following conditions:

- $\sigma_{c_3}^2 - \sigma_{c_2}^2$ is large relative to $\sigma_{c_2}^2 - \sigma_{c_1}^2$.

B Extending Black's model to proportional representation

The set-up is analogous to the above plurality case, but the pivot-probability will be substantially different.

Assume a proportional representation with a d'Hondt system. There are N voters who cast valid votes and M seats to distribute.

A vote is decisive on whether one additional seat goes to Party j or j' if the expected result of j and j' without the vote at stake is $\frac{m \times N}{M} - 1$ and $\frac{m' \times N}{M} - 1$ with $m, m' \in \{1, \dots, M\}$. In other words, both parties need one additional vote to reach the next quota for another seat.

Here, the pivot-probability under plurality and proportional representation is apparently different. Under plurality, both candidates should be expected to have a similar amount of votes so that the voter's pivot probability among them becomes larger. This does not have to be the case under proportional representation since a voter can have a high pivot probability among a party being expected to obtain a large amount of votes and another party being expected to obtain a small amount of votes.

Given a large number for M , we can reasonably assume $p_{p_1 p_2} = p_{p_1 p_3} = p_{p_2 p_3} = p_{pr}$. If we apply this to the above result under plurality, we obtain:

$$\begin{aligned}
 & EU(v_{p_1}) - EU(v_0) < EU(v_{p_2}) - EU(v_0) \\
 & 2p_{pr}(EU(g_{p_1}) - EU(g_{p_2})) + p_{pr}(EU(g_{p_1}) - EU(g_{p_3})) \\
 & \quad - p_{pr}(EU(g_{p_2}) - EU(g_{p_3})) < 0 \\
 & \quad EU(g_{p_1}) - EU(g_{p_2}) < 0 \\
 & \quad -d_{p_1}^2 - \sigma_{p_1}^2 + d_{p_2}^2 + \sigma_{p_2}^2 < 0 \\
 & \quad \quad d_{p_2}^2 - d_{p_1}^2 < \sigma_{p_1}^2 - \sigma_{p_2}^2
 \end{aligned}$$

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Declarations

Conflict of interest: The authors have no Conflict of interest to declare that are relevant to the content of this article.

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