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# Deriving nonlinear pricing schemes using modified least deviations spline regression

Dubravko Radic<sup>1,2</sup>

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## Abstract

Nonlinear pricing is a form of price differentiation in which the average price per unit is a nonlinear decreasing function of demand. It is widely used in many industries such as telecommunications or utilities. Rationales for using nonlinear pricing are manifold, ranging from taking account of decreasing marginal utilities of customers to building up switching costs. As with other forms of price differentiation, the potential for capturing consumer surplus and increasing profits is considerable. Nonlinear pricing schemes come in various forms, such as discounts, buy-one-get-one, power shopping, two-part tariffs, and block-tariffs. All these forms are well known. However, little is said about how exactly to systematically derive an optimal nonlinear pricing scheme. In this paper, I propose a new systematic empirical approach using a modified least deviations Spline regression-estimator.

**Keywords** Price differentiation · Nonlinear pricing · Optimization · Spline regression

## Introduction

Nonlinear pricing is a pricing scheme in which the calculated average price of a unit is a nonlinear function of demand (Wilson 1997; Lambrecht et al. 2012). Typically, it decreases as demand increases. Nonlinear pricing can be applied to any industry, such as telecommunications or utilities, with multivariable demand. Since the consumer decides on the quantity, nonlinear pricing is a form of second-degree price differentiation based on demand.

Nonlinear pricing schemes come in several forms, such as price points for different quantities, standard quantity discounts, or all-units discounts (Iyengar and Gupta 2009). Very common are two-part tariffs and  $k$ -block tariffs. Two-part tariffs take the form:

$$P(q) = f + pq$$

with  $P$  as the total price paid,  $q$  as the quantity demanded,  $f$  as a fixed sum, and  $p$  as the unit price. The calculated average price in this case is equal to  $f/q + p$  and decreases with  $q$ . Linear pricing is a special case of a two-part tariff where  $f=0$  and flat rates are a special case where  $q=0$ .

Combining  $k$  different two-part tariffs from which a consumer can choose leads to a  $k$ -block tariff. Combining two two-part tariffs, leads to a two-block tariff:

$$P(q) = \begin{cases} f_1 + p_1q, & \text{for } q < q_b \\ f_2 + p_2q, & \text{for } q \geq q_b \end{cases}$$

Assuming  $f_1 < f_2$  and  $p_1 > p_2$ , the two tariffs intersect at  $q_b$ , so that the consumer is better off with the first two-part tariff if she consumes less than  $q_b$ . In this case, she pays a fixed sum of  $f_1$  and an additional  $p_1$  per unit. For  $q \geq q_b$  the second two-part tariff would be more favorable with fixed costs of  $f_2$  and unit costs of  $p_2$ .

Common are two-block tariffs which combine a two-part tariff with a flat rate or tariffs which consist of a linear pricing scheme combined with a two-part tariff. In all these cases, the calculated average price not only decreases with demand but also exhibits discontinuity points where consumers have even stronger incentives to expand demand.

There are a number of reasons why nonlinear pricing is so popular among companies (Simon 1989). Nonlinear pricing increases consumers' incentives to increase their demand,

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thereby reducing sellers' uncertainty. It also increases switching costs and leads to lock-in-effects that reduce price elasticity. The most important reason, however, is the diminishing marginal utility of demand. Assuming that consumers' willingness-to-pay corresponds to their marginal utility, prices for additional units should decrease with demand.

Although nonlinear pricing schemes, such as a two-part or  $k$ -block tariffs, are widely used in many industries, little is said about how exactly to derive them. Notable exceptions are Bousquet and Ivaldi (1997) and Iyengar et al. (2008), who both estimate simultaneous models of consumer behavior and optimal pricing schemes, and ultimately arrive at a polynomial model with quantity and squared quantity included as exogenous variables.

In this paper, I will focus on deriving optimal  $k$ -block tariffs using Spline functions, which are flexible enough to approximate any arbitrary distribution of optimal nonlinear price points. These spline functions lead to simpler nonlinear pricing schemes than the derived optimal price points and are far more flexible than polynomial regressions.

Diminishing marginal willingness-to-pay is best captured by nonlinear decreasing prices as we will demonstrate in the next section. Section three introduces Spline regression and how it can be used to derive optimal nonlinear pricing tariffs. We will also argue that the standard estimation approach of minimizing squared residuals should be replaced by minimizing the absolute deviations of willingness-to-pay, prices, and costs. Finally, section four discusses the trade-off between simplicity and profitability of such derived nonlinear pricing schemes and concludes.

## Empirical illustration

We start with a simple hypothetical example to illustrate the profit potential of nonlinear pricing. Suppose market research yields the following estimates of willingness-to-pay for three customers A, B, and C, for quantities  $q = 1, 2, \dots, 5$  (see Table 1). These estimates may come from a Conjoint-analysis where different price and quantity levels were combined into hypothetical profiles, which were then rated by customers (Löffler 2015). Or they were elicited using direct questions, such as: "How much would you pay for  $q$  units of the following product or service?"<sup>1</sup> Given these estimates an average price per unit can be derived for each quantity,  $q$ . These figures are depicted in Table 1.

<sup>1</sup> I think that such a question and interpretation of the estimates is more realistic and easier to grasp than asking for a marginal willingness-to-pay, e.g. "Given that you already bought one unit of a product/service, how much would you be willing to pay for an additional unit of that product/service?"

**Table 1** Hypothetical nonlinear pricing example

$q$	Willingness-to-pay			Optimal price	Profit
	Customer A	Customer B	Customer C		
1	9	10	12	9	27
2	6	7.5	10	6	18
3	3.5	5.5	8	5.5	11
4	2	4	6	4	8
5	1.1	1.5	3.5	3.5	3.5
				Total	67.5

We will interpret these figures as a marginal willingness-to-pay in the following. For example, customer A is willing to pay 9 for one unit and 12 for two units, i.e., his average willingness-to-pay per unit decreases to 6 for the second unit. If the price for the first unit is below 9 and the average price for the second unit below 6, he will buy two units. In the following, we will assume that each customer prefers more units to fewer units as long as his net utility is positive.

Assuming for convenience no costs, an optimal undifferentiated price of  $p^* = 5.5$  can be derived, yielding a profit of  $9 \cdot 5.5 = 49.5$ . Customer A buys two units, customer B buys three units, and customer C buys four units.

With rate fences preventing arbitrage between different quantity segments, optimal prices can be derived separately for each  $q$ . For the first unit, this would be a price of  $p_1 = 9$ , giving a profit of 27 (see the last two columns in Table 1). Similarly, the other optimal price points can be derived, resulting in an optimal nonlinear pricing scheme that yields profits of 67.5, an increase of 36%.

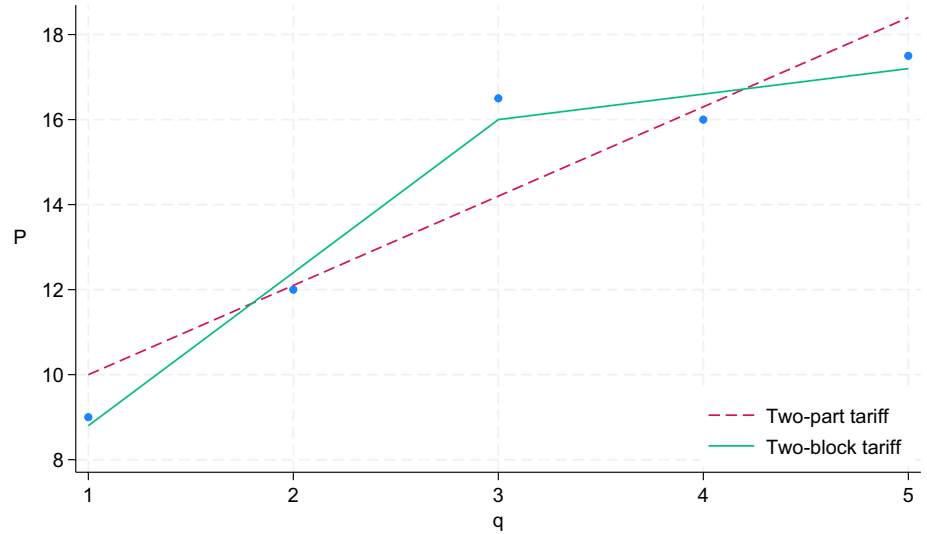
Note, that nonlinear pricing is more profitable, the more heterogeneous the demand. The 36% increase in profits is due to a larger share of consumer surplus captured through nonlinear price differentiation and by serving more consumers with willingness-to-pay above costs. Note also, that even nonlinear pricing does not lead to perfect price differentiation, since, e.g., consumers B and C would be willing to pay more for the first unit than a price of 9. On the other hand, consumers A and B are excluded from buying a fifth unit, even though they would contribute positively to the bottom line.

## Spline regression

Spline regression is a way to interpolate between fixed points of independent variables, called knots (Greene 2017; Poirier 2018). Splines are a series of piecewise linear or polynomial regressions joined at the knots. Compared to polynomial regression, they don't show a tendency to overfit the data and are even more flexible since they can capture different functional forms between the knots.



**Fig. 1** Linear and Spline regression of optimal nonlinear price points



Formally, a linear spline regression with total price paid by the consumer,  $P$ , as the dependent and quantity demanded,  $q$ , as the independent variable for one knot at  $k_1$  is defined as follows (we skip the individual index  $i$  for convenience and assume  $\epsilon \sim i.i.d.N$ ):

$$P = \beta_0 + \beta_1 q + \epsilon \text{ for } q < k_1$$

$$P = (\beta_0 + \gamma_0) + (\beta_1 + \gamma_1)q + \epsilon \text{ for } q \geq k_1$$

Note that when applied to nonlinear pricing, each knot separates one block from another in a  $k$ -block tariff. One knot corresponds to a 2-block tariff, where a consumer is better off with the first two-part tariff if she consumes less than  $k_1$ . In this case, she pays a fixed sum of  $\beta_0$  and an additional  $\beta_1$  per unit consumed. For  $q \geq k_1$  the second two-part tariff is more favorable with fixed costs of  $(\beta_0 + \gamma_0)$  and unit costs of  $(\beta_1 + \gamma_1)$ .

Defining  $d_1 = 1$  for  $q \geq k_1$  and zero otherwise, the two previous equations can be written more compactly as:

$$P = \beta_0 + \beta_1 q + \gamma_0 d_1 + d_1 \delta_1 p q + \epsilon \quad (1)$$

Since the two splines are required to join at knot  $k_1$ , the following linear constraint on the parameters must hold:

$$\beta_0 + \beta_1 k_1 = (\beta_0 + \gamma_0) + (\beta_1 + \gamma_1) k_1$$

Solving (2) for  $\gamma_0 = -\gamma_1 k_1$  and plug in (1) yields:

$$P = \beta_0 + \beta_1 q + \gamma_1 d_1 (q - k_1) + \epsilon \quad (3)$$

which can be estimated using OLS of  $P$  on  $q$  and the newly created variable  $d_1(q - k_1)$ .

As an example, consider the previously derived optimal average price points for different quantities,  $p^*$ . Multiplying

**Table 2** Linear and Spline regression-estimates

	Linear regression		Spline regression	
	Coefficient	Standard error	Coefficient	Standard error
(Intercept)	7.9	1.62	5.2	1.01
$q$	2.1	.49	3.6	.45
$d_1(q-k)$			−3	.80
Adj.-R <sup>2</sup>	.81		.96	

them by  $q$  yields optimal total prices  $P = p^*q$ , which are shown as blue dots in Fig. 1.

Estimating a simple linear regression model ( $k = 0$ ) yields the following estimation results in Table 2. The fitted prediction-line is also depicted in Fig. 1. The estimated coefficients correspond to a two-part tariff with a fixed sum of 7.9 and a unit price of 2.1. More than 80% of the variation of the customer's willingness-to-pay are captured with this simple two-part tariff.

Looking at the scatterplot in Fig. 1 there seems to be a structural break at  $q = 3$ . We therefore assume one knot at  $k_1 = 3$  and estimate the spline regression given in Eq. (3).<sup>2</sup> The estimation results are given in the last two columns of Table 2.

Again, we plot the fitted values in Fig. 1 and notice a considerable increase in prediction accuracy which is also reflected in an adjusted R<sup>2</sup>-value of 0.96. The spline regression with one knot at  $k_1 = 3$  corresponds to a two-block tariff. The first block is a two-part tariff with fixed fee of 5.2 and unit price of 3.6. For the second block, we can calculate

<sup>2</sup> More formally a Chow-test on structural breaks could be conducted or one could run a Spline regression with automatic knot selection proposed by Goepp et al. (2018).



**Table 3** Profits for optimal price points, two-part tariff, and two-block tariff (estimated using standard and the Modified Least Deviation Spline regression)

Willingness to pay				Optimal price points		OLS estimator				MLD-S estimator			
						Two-part tariff		Two-block tariff		Two-part tariff		Two-block tariff	
Q	A	B	C	p*	Profit	p*	Profit	p*	Profit	p*	Profit	p*	Profit
1	9	10	12	9	27	10	20	8.8	26.4	10	20	8.5	25.5
2	6	7.5	10	6	18	6.05	12.1	6.2	12.4	5.75	17.25	6	18
3	3.5	5.5	8	5.5	11	4.73	9.46	5.33	10.66	4.33	12.99	5.17	10.34
4	2	4	6	4	8	4.08	4.08	4.9	4.9	3.62	7.24	4.75	9.5
5	1.1	1.5	3.5	3.5	3.5	3.68	0	4.64	0	3.19	3.19	3.49	0
Total Profit					67.5		45.64		54.36		60.67		63.34

a unit price of  $(\beta_1 + \gamma_1) = 3.6 - 3 = .6$  and a fixed fee equal to  $(\beta_0 + \gamma_0) = (\beta_0 - \gamma_1 k_1) = (5.2 + 3 \cdot 3) = 14.2$ .

More interesting than R2-values are losses in the form of foregone consumer surplus and unrealized profit contributions. Since any regression is only an approximation of the optimal nonlinear price points, we should expect less profits. The average prices and profits calculated for the two-part and two-block tariffs in Table 3 confirm this. For the two-block tariff, e.g., the approximated optimal average price for the first unit is  $P(1) = 5.2 + 3.6 \cdot 1 = 8.8$  which deviates from the optimal price point of 9. Thus, the profit made from the first unit amounts to  $3 \cdot 8.8 = 26.4$  compared to 27.

$$\sum \epsilon^2 = \sum (P - \hat{P})^2 = \sum (P - \beta_0 - \beta_1 q - \gamma_1 d_1(q - k_1))^2$$

As a better alternative, I propose an estimator which minimizes profit deviations instead. Profit is lost for two reasons: either when prices charged are lower than the willingness-to-pay, resulting in a profit loss of  $(WTP - \hat{P})$ , or when prices are higher than the willingness-to-pay, resulting in a lost profit contribution, i.e.,  $(WTP - c)$ , as long as the willingness-to-pay is above costs  $c$ .

Choosing parameters so to minimize these two losses yields the following modified least deviation Spline regression-estimator (MLD-S):

$$\arg \left( \min_{\beta_0, \beta_1, \gamma_1} \sum \left\{ (WTP - \hat{P}) I(WTP \geq \hat{P}) + (\hat{P} - c) I(WTP < \hat{P} \wedge WTP > c) \right\} \right)$$

Similarly, the approximated optimal price for the second unit equals  $P(2)/2 = (5.2 + 3.6 \cdot 2)/2 = 6.2$  instead of 6, which leads to the exclusion of consumer A for all  $q \geq 2$ , since A has a willingness-to-pay of only 6. Thus, the profit for the second unit decreases from 18 to 12.4 and to 54.36 in total.

Note that this reduction in profit compared to the optimal nonlinear pricing scheme is also due to our simplified example with only three consumers. However, this loss could be further minimized by modifying the Spline regression-estimator to an estimator which explicitly minimizes profit losses. This estimator will be referred to as the modified least deviation Spline regression-estimator (MLD-S) and introduced in the next chapter.

with  $WTP$  as the willingness-to-pay,  $\hat{P} = \beta_0 + \beta_1 q + \gamma_1 d_1(q - k_1)$ ,  $I(\cdot)$  as an indicator function, and cost of  $c$  which we set to zero for convenience.<sup>3</sup>

Applying MLD-S to our example for  $k = 0$  knots yields an optimal two-part tariff with fixed fee  $f_1 = 8.51$  and unit price of  $p_1 = 1.49$ . Assuming  $k = 1$  knot at  $q = 3$  yields an optimal 2-block tariff with  $f_1 = 5; p_1 = 3.5$  for  $q \leq 3$  and  $f_2 = 14.9; p_2 = .2$  for  $q > 3$ . Table 3 contains the profits realized under these modified least deviation Spline regression-estimates. It can be seen that profits increase considerably. The MLD-S two-block tariff is close to the realized profit under optimal nonlinear price points.

## Modified least deviation spline regression

Estimates of the parameters of the linear Spline regression are obtained by minimizing the following sum of squared residuals (again, individual indices are skipped):

<sup>3</sup> Estimation was done using STATA. Codes available from the author upon request. Estimation can also be done using MS Excel's Solver.



## Conclusion

A major advantage, and probably the reason why we see more two-part and two-block tariffs than specific optimal price points used by companies, is the simplicity and ease of communicating the pricing scheme to customers. Using Spline regression or my newly proposed MLD-S estimator allows a satisfactory close approximation to the optimal nonlinear price points.

Another reason for the popularity of  $k$ -block tariffs may be that customers show a systematic bias when choosing between them. Most consumers overestimate their usage, which leads to higher costs when choosing a tariff. DellaVigna and Malmendier (2006) empirically demonstrate that consumers subscribing to health club services engage in irrational behaviors when considering their contractual choices and actual usage frequencies. The study found that consumers who chose to pay monthly flat-fee tariffs pay an average of 70% more than what they would have paid had they chosen the pay-as-you-go option for the same number of visits.

In a similar study on tariff options for local telephone services, 65% of consumers who opted for the flat tariff option of unlimited local calls would have saved money with alternative tariffs, e.g., measured billing services (Kridel et al. 1993). Miravete (2003) also found that 59% of households overestimated their future phone service consumption to select the flat rate option, thereby resulting in more than 20% overspending.

Finally, 48% of consumers who subscribed to Internet services with flat tariff options would have been better off had they elected to pay-per-use (Lambrecht and Skiera 2006). Approximately 50–70% of consumers who choose flat tariffs appear to be “cursed” by their ex-ante demand overestimation.

Although these biased consumer choices are good for the company in the short run, unhappy customers who find out they are making bad choices may retaliate by switching to the competition (McGovern and Moon 2007). Companies are on a slippery slope if their most profitable customers are those who make poor choices. Nonlinear pricing, especially  $k$ -block tariffs with small  $k$ , offers companies a simple way to communicate complicated nonlinear pricing schemes and to capture consumer surplus. However, nonlinear  $k$ -block tariffs with large  $k$  are prone to customer backlash if they become too complex to find the optimal one (Iyengar and Gupta 2009). Using my MLD-S approach for different  $k$ 's helps companies to better understand the trade-off between profit loss and simplicity and to derive an optimal two-part or  $k$ -block tariff.

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**Data availability** Dataset used for the example is contained in Table 1.

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