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Observable interpersonal utility comparisons

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Abstract

Harsanyi's seminal aggregation theorem axiomatized weighted utilitarianism based on expected utility theory. However, the weights assigned to each individual cannot be separated from the individual's utility. We show that once we depart from the expected utility framework, it is possible to uniquely identify the utilities and the weights. Specifically, we show that in the min-of-means social welfare function if each individual has a cardinal utility, unique up to a positive affine transformation, and any redistribution of utilities changes the social welfare for some initial allocation, then we can uniquely identify the utilities of the individuals and the weights of the social welfare function.

1 Introduction

Harsanyi's seminal aggregation theorem (Harsanyi 1955) axiomatized weighted utilitarianism based on expected utility theory. However, the result has been criticised since the weights attributed to individuals are not meaningful because they cannot be separated from the individuals' utilities (Sen 1976; Broome 1987; Weymark 1991). To overcome this identification issue, Harsanyi (1977) used direct interpersonal utility comparisons. However, interpersonal utility comparisons are difficult to make and have remained controversial in the literature (Elster and Roemer 1991; Greaves and Lederman 2018). Additionally, the assumption of expected utility theory in Harsanyi's aggregation theorem has been criticised (e.g. Diamond (1967); Sen (1970); Broome (1987)).

We show that once we depart from Harsanyi's expected utility framework, the weights and utilities are can be meaningful even without direct interpersonal utility comparisons. Specifically, we show that in the min-of-means social welfare function if each individual has a cardinal utility, unique up to a positive affine transforma-

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tion, and for every utility redistribution, there is some initial allocation such that the redistribution changes the social welfare, then we can uniquely identify the utilities of the individuals and the weights of the social welfare function. This shows that the social observer is behaving as if making interpersonal utility comparisons that we can observe indirectly together with the fairness of the society. Our result formalizes Kaneko's (1984) suggestion for observing interpersonal utility comparisons from the social welfare function.

Here, we identify the interpersonal utility comparisons from the non-linearities of the social welfare function. For example, in the case of the Rawlsian social welfare function (Rawls 1971), the non-linearities capture the change of the worst-off individual that allows us to identify utilities across individuals. We generalize this identification strategy beyond the Rawlsian social welfare function.

We study the identification of individuals' utilities and weights in the min-of-means social welfare function. This has been considered as capturing the ignorant observer in Gajdos and Kandil (2008). Additionally, it has been considered in the context of income inequality in Ben-Porath et al. (1997), Gajdos and Maurin (2004), Crès et al. (2011), and recently in Mongin and Pivato (2021). This representation includes utilitarianism and the Rawlsian social welfare function (Rawls 1971) as special cases.

The min-of-means representation consists of a (von Neumann–Morgenstern) utility function u_i for each member $i \in \mathcal{I} = \{1, \dots, n\}$ and a set of weights for each member $\Lambda \subseteq \Delta(\mathcal{I})$ such that the societal value of an alternative x is

$$\min_{\lambda \in \Lambda} \sum_{i \in \mathcal{I}} \lambda_i u_i(x).$$

We show that the set of weights and the utility functions are identified if and only if any utility redistribution from one member to another changes the welfare in some situation. That is if $(v_i)_{i \in \mathcal{I}} \in \mathbb{R}^n$ is a utility redistribution such that there exist members i and j with $v_i > 0 > v_j$, then there exists an initial utility allocation $(w_i)_{i \in \mathcal{I}} \in \mathbb{R}^n$ such that redistributing the utility by $(v_i)_{i \in \mathcal{I}}$ changes the social welfare,

$$\min_{\lambda \in \Lambda} \sum_{i \in \mathcal{I}} \lambda_i w_i \neq \min_{\lambda \in \Lambda} \sum_{i \in \mathcal{I}} \lambda_i (w_i + v_i). \quad (1)$$

This condition captures in terms of the social welfare function that the societal value of every utility redistribution depends on the context: Formally, for each utility redistribution $(v_i)_{i \in \mathcal{I}} \in \mathbb{R}^n$ such that there exist members i and j with $v_i > 0 > v_j$, there exist weights $\lambda, \lambda' \in \Lambda$ such that

$$\sum_{i \in \mathcal{I}} \lambda_i v_i \neq \sum_{i \in \mathcal{I}} \lambda'_i v_i.$$

Technically, this condition is equivalent to the set of weights Λ having a non-empty interior.

This result shows that once we move away from the expected utility framework and the societal value of every utility redistribution depends on the current utility distribution, interpersonal utility comparisons and weights assigned to individuals can be observable. Especially we show that in the min-of-means representation with affine utilities, our identifying condition Eq. (1) is equivalent to the identification of the set of Pareto weights and equivalent to the identification of interpersonal utility

Mathematically, the separation of weights and utility functions is symmetrical to the separation of probabilities and state dependent utilities in choice under uncertainty. Here, the min-of-means social welfare function corresponds to the state dependent maxmin expected utility. Our identification result follows as a corollary from the identification of probabilities and state dependent utilities in a state dependent maxmin expected utility in Mononen (2024).

Our second contribution is that we characterize the existence of the min-of-means social welfare function by relaxing Harsanyi's assumption that the societal preferences satisfy the expected utility theory. Instead, we allow for violations of expected utility theory when the alternatives involve trade-offs across the members and only assume that the societal preferences satisfy expected utility theory when there are no trade-offs across the members.¹

Our results are closely related to Gajdos and Kandil (2008). They study when an impartial observer's extended preferences have a min-of-means representation. In this extended setting, the observer is especially able to make direct interpersonal utility comparisons and Harsanyi's utilitarianism is fully identified. We instead study observable social preferences that do not include direct interpersonal utility comparisons. This allows us to substantially simplify the axiomatization of Gajdos and Kandil (2008) and clarify further the difference between the min-of-means social welfare function and utilitarianism.

We follow the single-profile formalism pioneered by Harsanyi (1955) that studied preference aggregation over fixed preferences. This approach has been used for example in Mongin (1995), Gilboa et al. (2004), Chambers and Hayashi (2006) and Gajdos et al. (2008). This approach is in contrast to the multi-profile formalism studying preference aggregation over varying preferences as pioneered by Arrow (1951) and Sen (1970) and has been summarized in d'Aspremont and Louis (2002).

In choice under uncertainty, Mongin (1995) and Mongin (1998) study the aggregation of individuals under subjective expected utility theory. Mongin shows that if there is sufficient preference diversity, then under state independent utility and Pareto monotonicity, this leads to a dictatorial choice rule. However, under state dependent utility, non-dictatorial aggregation is possible. Amarante and Ghossoub (2021) shows the possibility of non-dictatorial aggregation when the aggregated preferences do not follow subjective expected utility theory.

The solution in the literature for the lack of identification in Harsanyi (1955) has been to consider non-observable extended lotteries that allow for direct interpersonal utility comparisons. This approach was pioneered in Harsanyi (1977) and used in Karni and Weymark (1998), Gajdos and Kandil (2008), Grant et al. (2010), and discussed in Adler (2014) and Greaves and Lederman (2018).

Another solution to the lack of identification has been to consider relative utilitarianism that was introduced by Dhillon and Mertens (1999) and Segal (2000) and preference satisfaction. Börgers and Choo (2017) and Karni and Weymark (2024) study the elicitation of Pareto weights in relative utilitarianism. These approaches are discussed later in Sect. 2.3.

¹ However, since we do not observe interpersonal utility comparisons, we make a more general assumption and assume there are two different lotteries that satisfy the expected utility theory.

Technically, our results are related to the literature on income inequality measurement Weymark (1981), Yaari (1988), Ben-Porath et al. (1997). However, here we focus on the more general welfare inequality measurement with subjective utility for each member.

The remainder of the paper proceeds as follows: Sect. 1.1 offers a simple example highlighting the intuition for our identification result. Section 2 studies the identifications of the min-of-means social welfare function, Sect. 2.3 compares the identification to relative utilitarianism and utilitarianism. Section 3 axiomatically characterizes the existence of the representations and Sect. 4 concludes. The Appendix proves all the results.

1.1 An example of identification

We begin with a simple example illustrating that with the weighted utilitarian social welfare function individuals' utilities and weights cannot be separated. However, this is only an unidentified special case. In the second part of the example, we show that with the min-of-means social welfare function these can be separated and identified from the violations of the independence axiom.

First, we illustrate the lack of identification in weighted utilitarianism. Consider a society consisting of two individuals 1 and 2 that have preferences over some set of lotteries $\Delta(X)$ over social alternatives X . Each of the individuals has an affine von Neumann–Morgenstern utilities $u_1, u_2 : \Delta(X) \rightarrow \mathbb{R}$. These are aggregated into weighted utilitarian social welfare with equal weight for both of the individuals: The social value of alternative $p \in \Delta(X)$ is

$$0.5u_1(p) + 0.5u_2(p).$$

Now these preferences have an alternative weighted utilitarian representation with any weight $\lambda \in (0, 1)$ for individual 1 since

$$0.5u_1(p) + 0.5u_2(p) = \lambda \left(\frac{0.5}{\lambda} u_1(p) \right) + (1 - \lambda) \left(\frac{0.5}{1 - \lambda} u_2(p) \right) = \lambda \tilde{u}_1(p) + (1 - \lambda) \tilde{u}_2(p),$$

where the terms inside the parentheses define new utility functions \tilde{u}_1, \tilde{u}_2 .

In this alternative representation, we have replaced the weight of the individual for the intensity of preferences. This highlights the impossibility of identification in weighted utilitarianism since the intensities of preferences are inseparable from the weights. This violates our identification condition (1) since for utility redistribution $(v_1, v_2) = (\frac{1}{\lambda}, -\frac{1}{1-\lambda})$, we have for all $(w_1, w_2) \in \mathbb{R}^2$,

$$\lambda w_1 + (1 - \lambda) w_2 = \lambda(w_1 + v_1) + (1 - \lambda)(w_2 + v_2).$$

Next, we move on to min-of-means social welfare over lotteries p defined by two weights $\lambda_* < \lambda^*$ for individual 1 and affine von Neumann–Morgenstern utilities u_1, u_2

$$\min_{\lambda \in [\lambda_*, \lambda^*]} \lambda u_1(p) + (1 - \lambda) u_2(p).$$

We show that utilities are identifiable across individuals from changes in the Pareto weights, that is violations of the independence axiom.² For this, let p and q be two

² The independence axiom from von Neumann and Morgenstern (1947) and Harsanyi (1955) characterizes

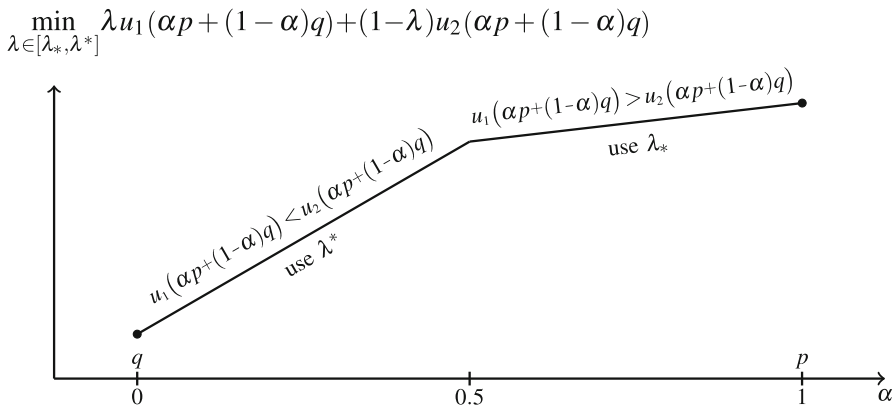


Fig. 1 An example of identifying when the utilities across individuals are equal from the non-linearities. Lotteries p and q are such that $u_1(p) + u_1(q) = u_2(p) + u_2(q)$ and $u_1(q) < u_2(q)$. The x-axis changes p to q with convex combinations. The y-axis is the min-of-means social welfare of $\alpha p + (1-\alpha)q$. At $\alpha = 0.5$ utilities across the individuals are equal that is observable as a violation of the independence axiom

lotteries such that $u_1(p) + u_1(q) = u_2(p) + u_2(q)$ and $u_1(q) < u_2(q)$. Next, we show that there is a violation of the independence axiom at $0.5p + 0.5q$.

We focus on the min-of-means social welfare of $\alpha p + (1-\alpha)q$ when α changes from 0 to 1 as in Fig. 1. First, between 0 and 0.5, by the affine utilities, $u_1(\alpha p + (1-\alpha)q) < u_2(\alpha p + (1-\alpha)q)$. So the min-of-means welfare uses the weight λ^* and it changes linearly at the rate $\lambda^*(u_1(p) - u_1(q)) + (1-\lambda^*)(u_2(p) - u_2(q))$. Second, between 0.5 and 1, $u_1(\alpha p + (1-\alpha)q) > u_2(\alpha p + (1-\alpha)q)$. So the min-of-means welfare uses the weight λ_* and this time it changes linearly at the rate $\lambda_*(u_1(p) - u_1(q)) + (1-\lambda_*)(u_2(p) - u_2(q))$. Since $u_1(p) - u_1(q) > u_2(p) - u_2(q)$, the rate of change switches at $\alpha = 0.5$ and there is a non-linearity at that point. This represents a violation of the independence axiom at the lottery $0.5p + 0.5q$ where both of the individuals have the same utility.

Finally, there can be violations of the independence axiom only if the utilities for both individuals are the same. The only situations where there can be non-linearities as in Fig. 1 are when the used Pareto weight changes. However, the min-of-means social welfare function with two individuals always maximizes the weight for the individual with a lower utility. Thus, the change in the used Pareto weight means that the utility order of the individuals changed. Especially, in here, the utilities for both of the individuals are exactly the same. In summary, the lotteries where the utilities for both individuals are equal are characterized by the violations of the independence axiom and especially they are observable.

The min-of-means social welfare function rules out the previous violations of our identification condition (1). First, for many redistributions, the axiom holds trivially. If (v_1, v_2) is a utility redistribution such that $v_1 < 0 < v_2$ and $\min_{\lambda \in [\lambda_*, \lambda^*]} \lambda v_1 + (1-\lambda)v_2 \neq$

the linearity of weighted utilitarianism. It states that for all lotteries p, q, r and $\alpha \in (0, 1)$,

$$p \succsim q \iff \alpha p + (1-\alpha)r \succsim \alpha q + (1-\alpha)r.$$

0, then the condition holds for $(\tilde{w}_1, \tilde{w}_2) = (0, 0)$. Second, we focus on redistributions that preserve the welfare when redistributing from $(0, 0)$ and show that in another context when redistributing from $(w_1, w_2) = (-0.5v_1, -0.5v_2)$, the redistribution affects the welfare. Now, we have since $v_1 < 0 < v_2$ and the redistribution does not affect welfare when redistributing from $(0, 0)$,

$$\min_{\lambda \in [\lambda_*, \lambda^*]} \lambda(w_1 + v_1) + (1 - \lambda)(w_2 + v_2) = \lambda^*(0.5v_1) + (1 - \lambda^*)(0.5v_2) = 0.$$

Next, since $\lambda_* < \lambda^*$ and $v_1 < 0 < v_2$, we have

$$0 < \lambda_*(0.5v_1) + (1 - \lambda_*)(0.5v_2).$$

Hence,

$$0 > \lambda_*(-0.5v_1) + (1 - \lambda_*)(-0.5v_2) = \min_{\lambda \in [\lambda_*, \lambda^*]} \lambda w_1 + (1 - \lambda)w_2.$$

This shows that the identification condition (1) holds also in this case. This illustrates that for most of the utility redistributions, the identifying condition holds trivially. However, the identifying condition assumes that utility redistributions that are welfare preserving in one context affect welfare in some other context.

This identification example is generalized in our main result, Theorem 1, to finitely many individuals. There we show that if the social value of every redistribution depends on the context, then the individuals' utilities are observable.

2 Identification

2.1 Preliminaries and notation

We follow the setting from Harsanyi (1955, 1977). Society consists of members $\mathcal{I} = \{1, \dots, n\}$. X is a set of social-alternatives. Each member $i \in \mathcal{I}$ has preferences \succsim_i over (simple) social-alternative lotteries $\Delta(X)$ and additionally, we observe societal preferences \succsim_0 over (simple) social-alternative lotteries $\Delta(X)$. (Normalized) weights for the members are probability distributions on the members $\Delta(\mathcal{I})$. $\Delta(\mathcal{I})$ is equipped with the Euclidean topology.

We consider the min-of-means social welfare function following (Ben-Porath et al. 1997; Gajdos and Kandil 2008) over expected utility members as in Harsanyi (1955, 1977).

Definition Affine utilities $u_i : \Delta(X) \rightarrow \mathbb{R}$ for each $i \in \mathcal{I}$ and a convex and closed set of Pareto weights $\Lambda \subseteq \Delta(\mathcal{I})$ is a *min-of-means representation* for $((\succsim_i)_{i \in \mathcal{I}}, \succsim_0)$ if the following two conditions hold:

1. for each $i \in \mathcal{I}$ and $p, q \in \Delta(X)$, we have

$$p \succsim_i q \iff u_i(p) \geq u_i(q).$$

2. for all $p, q \in \Delta(X)$, we have

$$p \succsim_0 q \iff \min_{\lambda \in \Lambda} \sum_{i \in \mathcal{I}} \lambda_i u_i(p) \geq \min_{\lambda \in \Lambda} \sum_{i \in \mathcal{I}} \lambda_i u_i(q).$$

We offer an axiomatic characterization for the min-of-means social welfare function later on in Sect. 3. We focus especially on min-of-means representations with the smallest possible set of Pareto weights as defined next.

Definition Affine utilities $u_i : \Delta(X) \rightarrow \mathbb{R}$ for each $i \in \mathcal{I}$ and a convex and closed set of Pareto weights $\Lambda \subseteq \Delta(\mathcal{I})$ is a *minimal min-of-means representation* for $((\succsim_i)_{i \in \mathcal{I}}, \succsim_0)$ if for any other min-of-means representation with the same utilities $(u_i)_{i \in \mathcal{I}}$ and a set of Pareto weights $\tilde{\Lambda}$, we have $\Lambda \subseteq \tilde{\Lambda}$.

The next example shows the significance of minimal representations since there can be weights that the social welfare function never uses. We connect general and minimal representations in the next section.

Remark (Non-minimal example) If $n = 2$ and for all $p \in \Delta(X)$, $u_1(p) < u_2(p)$, then the set of Pareto weights $\Lambda = \{(\lambda, 1 - \lambda) | \lambda \in [0, \frac{1}{2}]\}$ is not minimal since for all $p \in \Delta(X)$

$$\min_{\lambda \in [0, \frac{1}{2}]} \lambda u_1(p) + (1 - \lambda) u_2(p) = \frac{1}{2} u_1(p) + \frac{1}{2} u_2(p).$$

Next, we define when the minimal set of weights and the utility functions are identified.

Definition The set of weights in the minimal min-of-means representation is identified if for all minimal min-of-means representations $((u_i)_{i \in \mathcal{I}}, \Lambda)$ and $((\tilde{u}_i)_{i \in \mathcal{I}}, \tilde{\Lambda})$, we have

$$\Lambda = \tilde{\Lambda}.$$

In contrast to the set of weights that are only identified for minimal representations, we identify the utility functions for all the min-of-means representations.

Definition The utilities in the min-of-means representation are identified up to a common positive affine transformation if for all min-of-means representations $((u_i)_{i \in \mathcal{I}}, \Lambda)$ and $((\tilde{u}_i)_{i \in \mathcal{I}}, \tilde{\Lambda})$, there exist $\alpha > 0$ and $\beta \in \mathbb{R}$ such that for each $i \in \mathcal{I}$ and $p \in \Delta(X)$

$$u_i(p) = \alpha \tilde{u}_i(p) + \beta.$$

2.2 Uniqueness

Our main result characterizes when the minimal min-of-means representation is fully identified. This identification is characterized by the following condition.

Axiom 1 For any lotteries p and q such that there exist $i, j \in \mathcal{I}$ with $p \succ_i q$ and $q \succ_j p$, there exists a lottery r and $\alpha \in [0, 1]$ such that

$$\alpha p + (1 - \alpha)r \approx_0 \alpha q + (1 - \alpha)r$$

Here, we consider redistributing the utilities by $(u_k(q) - u_k(p))_{k \in \mathcal{I}}$ that benefits the member j and makes the member i worse off. However, we do not make any restrictions on how the redistribution affects other members. Then the axiom assumes that there exists an allocation $\alpha p + (1 - \alpha)r$ with utilities

$$(u_k(\alpha p + (1 - \alpha)r))_{k \in \mathcal{I}}$$

such that performing the utility redistribution to change the utilities to

$$(u_k(\alpha p + (1 - \alpha)r) + \alpha(u_k(q) - u_k(p)))_{k \in \mathcal{I}} = (u_k(\alpha q + (1 - \alpha)r))_{k \in \mathcal{I}}$$

changes the welfare. That is, for any utility redistribution there is some situation such that the redistribution changes the welfare. For this interpretation, it is crucial that the members have affine utilities.

The next result shows that Axiom 1 characterizes the identification of the minimal min-of-means representation.

Theorem 1 Assume that $((\succsim_i)_{i \in \mathcal{I}}, \succsim_0)$ has a minimal min-of-means representation $((u_i)_{i \in \mathcal{I}}, \Lambda)$ such that

$$\text{int} \left\{ (u_i(p))_{i \in \mathcal{I}} \mid p \in \Delta(X) \right\} \neq \emptyset.$$

Then the following five conditions are equivalent.

- (1) $((\succsim_i)_{i \in \mathcal{I}}, \succsim_0)$ satisfy Axiom 1.
- (2) $\text{int } \Lambda \neq \emptyset$.
- (3) For all $v \in \mathbb{R}^{\mathcal{I}}$ such that there exist $i, j \in \mathcal{I}$ with $v_i > 0 > v_j$, there exist $\lambda, \lambda' \in \Lambda$ such that

$$\sum_{i \in \mathcal{I}} \lambda_i v_i \neq \sum_{i \in \mathcal{I}} \lambda'_i v_i.$$

- (4) The set of weights in the minimal min-of-means representation is identified.
- (5) The utilities in the min-of-means representation are identified up to a common positive affine transformation.

First, the equivalency between (1), (4), and (5) shows the identification of weights assigned to members and interpersonal utility comparisons: When any redistribution changes welfare in some situation, then the minimal set of weights assigned to members and interpersonal utility comparisons can be identified in the min-of-means representation from the societal preferences. Here, the social observer is behaving as if making interpersonal utility comparisons that we can observe indirectly. This identification was illustrated in Sect. 1.1. As in the example, Axiom 1 guarantees that any redistribution violates the independence axiom and there is a change in the Pareto weight for any redistribution that is used for the identification.

Second, the equivalency between conditions (1), (2), and (3) characterizes when the minimal min-of-means representation satisfies Axiom 1. This shows that our identifying condition is equivalent to the societal value of every utility redistribution depending on the context or to the set of weights that the social welfare function uses having a non-empty interior. For example, this result shows that if all the Pareto weights agree on the weight of member i or if all the Pareto weights agree that the weight of member i is twice as large as member j , then the min-of-means representation does not satisfy Axiom 1.

The assumption that

$$\text{int} \left\{ (u_i(p))_{i \in \mathcal{I}} \mid p \in \Delta(X) \right\} \neq \emptyset$$

is a standard identification condition in the literature. It has been used e.g. in Harsanyi (1955), Weymark (1991), and Fleurbaey and Mongin (2016). It is characterized by the independent prospects axiom assuming that for each $i \in \mathcal{I}$, there exists lotteries p and q such that $p \succ_i q$ and for each $j \neq i$, $p \sim_j q$.

The next example shows the identification for a convex combination of utilitarianism and the Rawlsian social welfare function as proposed by Gajdos and Kandil

(2008). Here, utilitarianism is the only case where the weights are not meaningful and interpersonal utility comparisons are not observed.

Example The social welfare function defined for all $p \in \Delta(X)$ by

$$(1 - \theta) \min_{i \in \mathcal{I}} u_i(p) + \frac{\theta}{|\mathcal{I}|} \sum_{i \in \mathcal{I}} u_i(p)$$

where $\theta \in [0, 1]$ is fully identified and satisfies Axiom 1 iff $\theta \neq 1$.

The next result connects minimal and non-minimal representations and provides the general identification with Theorem 1.

Proposition 2 Assume that $((u_i)_{i \in \mathcal{I}}, \Lambda)$ is a minimal min-of-means representation for $((\tilde{z}_i)_{i \in \mathcal{I}}, \tilde{z}_0)$ and

$$\text{int} \left\{ (u_i(p))_{i \in \mathcal{I}} \mid p \in \Delta(X) \right\} \neq \emptyset.$$

Denote $\mathcal{U} = \{(u_i(p))_{i \in \mathcal{I}} \mid p \in \Delta(X)\}$ and

$$\Lambda^* = \bigcap_{x \in \mathcal{U}} \{ \lambda \in \Delta(\mathcal{I}) \mid x \cdot \lambda \geq \min_{\delta \in \Lambda} x \cdot \delta \}.$$

Then $((u_i)_{i \in \mathcal{I}}, \tilde{\Lambda})$ is a min-of-means representation for $((\tilde{z}_i)_{i \in \mathcal{I}}, \tilde{z}_0)$ iff $\Lambda \subseteq \tilde{\Lambda} \subseteq \Lambda^*$.³

Especially, this shows that if the set of utilities contains a constant utility in the interior, then all representations are minimal and so the weights for all the representations can be identified.

Corollary 3 If $((u_i)_{i \in \mathcal{I}}, \Lambda)$ is a min-of-means representation for $((\tilde{z}_i)_{i \in \mathcal{I}}, \tilde{z}_0)$ and there exists $a \in \mathbb{R}$, such that

$$(a)_{i \in \mathcal{I}} \in \text{int} \left\{ (u_i(p))_{i \in \mathcal{I}} \mid p \in \Delta(X) \right\},$$

then $((u_i)_{i \in \mathcal{I}}, \Lambda)$ is a minimal min-of-means representation for $((\tilde{z}_i)_{i \in \mathcal{I}}, \tilde{z}_0)$.

2.3 A comparison to weighted relative utilitarianism and utilitarianism

Dhillon and Mertens (1999) and Segal (2000) consider the weighted relative utilitarian representation for $((\tilde{z}_i)_{i \in \mathcal{I}}, \tilde{z}_0)$. This consists of affine and bounded utility functions $u_i : \Delta(X) \rightarrow \mathbb{R}$ and weights $\lambda_i \in \mathbb{R}_+$ for each individual $i \in \mathcal{I}$ such that $\sum_{i \in \mathcal{I}} \lambda_i = 1$ and the following two conditions hold:

(1) for each $i \in \mathcal{I}$ and $p, q \in \Delta(X)$, we have

$$p \tilde{z}_i q \iff u_i(p) \geq u_i(q).$$

(2) for all $p, q \in \Delta(X)$, we have

$$p \tilde{z}_0 q \iff \sum_{i \in \mathcal{I}} \lambda_i \frac{u_i(p) - \inf_{s \in \Delta(X)} u_i(s)}{\sup_{r \in \Delta(X)} u_i(r) - \inf_{s \in \Delta(X)} u_i(s)} \geq \sum_{i \in \mathcal{I}} \lambda_i \frac{u_i(q) - \inf_{s \in \Delta(X)} u_i(s)}{\sup_{r \in \Delta(X)} u_i(r) - \inf_{s \in \Delta(X)} u_i(s)}.$$

³ Geometrically, Λ^* is the intersection of all the half-spaces defined by \mathcal{U} that contain Λ . In other words, Λ^* is the best approximation of Λ using the half-spaces defined by \mathcal{U} .

This corresponds to weighted utilitarianism when each individual's utility function is normalized to have a infimum of 0 and a supremum of 1. In this representation, the weights λ_i are unique.

Börgers and Choo (2017) offer a method for eliciting weights in weighted relative utilitarianism based on the marginal rate of substitution in the probability of the best and worst prize for each individual. Building on this, Karni and Weymark (2024) axiomatize relative utilitarianism when each individual has the same weight in a single-profile setting.

Similarly, Broome (1991) suggests using an unweighted utilitarian representation. Broome argues for the use of unweighted utilitarianism based on the ethical judgement that there is no conceptual difference between the size of a benefit and how much the benefit counts. Here, the utility functions are unique up to a common positive affine transformation.

However, axiomatically, weighted relative utilitarianism and utilitarianism are exactly the same as weighted utilitarianism. Here, it is only the restriction on using utility functions that are normalized to be between 0 and 1 that gives the identification for the weights in relative utilitarianism and the restriction on using equal weights for each member that gives the identification for the utilities in utilitarianism. However, these restrictions are additional ethical judgements beyond the observable preferences. The assumption of preference satisfaction has been discussed in Hausman (1995, 2011) and Barrett (2019). In contrast, our identification result shows the identification among any utility functions and any sets of weights. Our identification reflects an observable property of the societal preferences that for any redistribution from one member to another, there is some situation where the redistribution changes the welfare as captured by Axiom 1.

3 Existence of the min-of-means social welfare function

Finally, in this section, we axiomatize the existence of a minimal min-of-means representation. This result shows how relaxing the independence axiom for the societal preferences to only hold for two lotteries and assuming convexity while maintaining the other axioms from Harsanyi (1955) characterizes the min-of-means representation. Our Axioms 2–6 maintain Harsanyi's assumptions. This axiomatization simplifies substantially the characterization in Gajdos and Kandil (2008) and clarifies the difference between the min-of-means social welfare function and utilitarianism.

First, we assume that each member $i \in \mathcal{I}$ is an expected utility maximizer. This is assumed with the following three standard assumptions.

Axiom 2.1 For each $i \in \mathcal{I}$, \succsim_i is complete and transitive.

Axiom 2.2 For each $i \in \mathcal{I}$ and for all $p, q, r \in \Delta(X)$, the sets $\{\alpha \in [0, 1] \mid \alpha p + (1 - \alpha)q \succsim_i r\}$ and $\{\alpha \in [0, 1] \mid r \succsim_i \alpha p + (1 - \alpha)q\}$ are closed.

Axiom 2.3 For each $i \in \mathcal{I}$ and for all $p, q, r \in \Delta(X)$ and $\alpha \in (0, 1)$,

$$p \succsim_i q \iff \alpha p + (1 - \alpha)r \succsim_i \alpha q + (1 - \alpha)r.$$

Our first two assumptions on societal preferences are that they are a continuous weak order.

Axiom 3 \succsim_0 is complete and transitive.

Axiom 4 For all $p, q, r \in \Delta(X)$, the sets $\{\alpha \in [0, 1] | \alpha p + (1 - \alpha)q \succsim_0 r\}$ and $\{\alpha \in [0, 1] | r \succsim_0 \alpha p + (1 - \alpha)q\}$ are closed.

Next, we maintain the standard weak Pareto monotonicity axiom.

Axiom 5 If for each $i \in \mathcal{I}$, $p \succ_i q$, then $p \succ_0 q$.

Next, we assume the independent prospects axiom that is a commonly used richness assumption on the set of lotteries (Harsanyi 1955; Weymark 1991; Fleurbaey and Mongin 2016).

Axiom 6 For each $i \in \mathcal{I}$, there exist lotteries p and q such that $p \succ_i q$ and for each $j \in \mathcal{I}$, $j \neq i$, $p \sim_j q$.

The last two assumptions relax Harsanyi's assumption that the societal preferences satisfy expected utility theory by taking into account that the social-alternatives might involve trade-offs across members. Diamond (1967) criticized Harsanyi's assumption that the societal preferences satisfy the expected utility theory with the following example. Consider two members with utilities u_1 and u_2 respectively and two lotteries p and q defined as follows:

	u_1	u_2	u'_1	u'_2
p	1	0	1	1
q	0	1	0	2
$1/2p + 1/2q$	$1/2$	$1/2$	$1/2$	$3/2$

Now, under symmetry, $p \sim_0 q$ and so under the independence axiom $p \sim_0 \frac{1}{2}p + \frac{1}{2}q$. However, $\frac{1}{2}p + \frac{1}{2}q$ is more fair than p or q and hence a preference for $\frac{1}{2}p + \frac{1}{2}q$ over p seems reasonable contradicting the independence axiom.

As has been noted by Sen (1970), this example requires full interpersonal utility comparability. If we would consider alternative utility functions u'_1 and u'_2 , where the utility of 2 is increased by 1 for all lotteries as in the above figure, then by the same fairness argument as before, p seems better than $\frac{1}{2}p + \frac{1}{2}q$. This example illustrates the difficulty in determining when there might be a preference for fairness and when the independence axiom might be satisfied. In our relaxation of the independence axiom, we allow for this uncertainty in determining when the independence axiom is satisfied by assuming that there are two lotteries that satisfy the independence axiom without making any assumption on what these lotteries are.

First, we define a lottery as neutral if it satisfies the independence axiom. This means that mixing the neutral lottery with any lottery increases the fairness of the mixture by the same fraction regardless of the lottery. Under Axiom 1, these neutral lotteries give the same utility to each member.

Definition A lottery r is *neutral* if for all lotteries p, q and $\alpha \in (0, 1)$

$$p \succsim_0 q \iff \alpha p + (1 - \alpha)r \succsim_0 \alpha q + (1 - \alpha)r.$$

The independence axiom assumes that every lottery is neutral. We relax this by assuming that there exist two different neutral lotteries such that one is better for each member than the other. Akin to Diamond's example, this axiom allows for freedom on what lottery is neutral with respect to fairness as long as the set of lotteries is rich enough that there are two alternatives that are neutral. Under Axiom 1, this axiom assumes the expected utility theory when there are no trade-offs across members.

Axiom 7 There exist neutral lotteries p and q such that for each $i \in \mathcal{I}$, $p \succ_i q$.

Secondly, we relax the independence axiom by assuming convexity of the societal preferences. For the interpretation of this axiom, it is crucial that each member is an expected utility maximizer. This axiom assumes that smoothing out the utility distribution, increases welfare.

Axiom 8 For all lotteries p, q and $\alpha \in (0, 1)$, if $p \succsim_0 q$, then $\alpha p + (1 - \alpha)q \succsim_0 q$.

These axioms characterize the existence of the min-of-means representation.

Theorem 4 (Existence) *The following two conditions are equivalent:*

- (1) $((\succsim_i)_{i \in \mathcal{I}}, \succsim_0)$ satisfies Axioms 2–8.
- (2) There exist $((u_i)_{i \in \mathcal{I}}, \Delta)$ that is a minimal min-of-means representation for $((\succsim_i)_{i \in \mathcal{I}}, \succsim_0)$ such that $\text{int}\{(u_i(p))_{i \in \mathcal{I}} | p \in \Delta(X)\} \neq \emptyset$ and $(0)_{i \in \mathcal{I}}, (1)_{i \in \mathcal{I}} \in \{(u_i(p))_{i \in \mathcal{I}} | p \in \Delta(X)\}$.

This result shows how relaxing the independence axiom for the societal preferences by convexity and independence axiom only when there are no trade-offs across members while maintaining the other axioms from Harsanyi (1955) characterizes the min-of-means representation.

Our axiomatization provides an alternative to the axiomatization of the min-of-means social welfare function in Gajdos and Kandil (2008). Gajdos and Kandil (2008) use extended preferences that allow for direct interpersonal utility comparisons and for varying the set of weights that the society considers possible. This allows the authors, for example, to directly identify which lotteries have a constant valuation for each member and reducing the weights of the society into a lottery over social-alternatives. Instead, we follow the setting of Harsanyi (1955, 1977) without extended preferences. This allows us to substantially simplify the axiomatization of Gajdos and Kandil (2008) and clarify further the difference between the min-of-means social welfare function and utilitarianism. Especially, our axiomatization shows that neutral lotteries are equivalent to lotteries that give constant value to each member when we do not observe direct interpersonal utility comparisons.

4 Conclusion

Our identification result shows that when we depart from the expected utility framework and move to the min-of-means social welfare function, each individual has a cardinal utility, and the societal value of every utility redistribution depends on the current utility distribution, then we can observe interpersonal utility comparisons and

the set of weights assigned to individuals uniquely. This allows us to dispense with extended preferences for the identification that have been recently criticized in Greaves and Lederman (2018).

Our result follows as a direct application of the identification of subjective probabilities and state dependent utility from decision theory under uncertainty (Mononen 2024). This highlights the close connection between state dependent utilities and social choice. Our result formalizes Kaneko's (1984) suggestion for observing the interpersonal utility comparisons from the social welfare function. It is an open question if this identification strategy generalizes to other social welfare functions.

Here, we have maintained Harsanyi's (1955) assumption that each member of the society is an expected utility maximizer. It is an open question if the state dependent utility theory can be generalized without vNM-utility following the approach of Alon and Schmeidler (2014). Secondly, it is an open question if the identification result generalizes beyond affine transformations using the approach from Fleurbaey and Mongin (2016). Alon and Schmeidler (2014) offer suggestive evidence that Fleurbaey and Mongin's (2016) identification result for utilitarianism generalizes to the maxmin approach.

Appendix

A Proofs

Proof of Theorem 1 Theorem 1, (1) \iff (2) \iff (4) \iff (5) follows directly from Mononen (2024). Mononen uses $\mathcal{U} = \times_{i \in \mathcal{I}} [a_i, b_i]$ where $a_i, b_i \in \mathbb{R} \cup \{\infty, -\infty\}$ and $a_i < b_i$. However, the proof directly generalizes to Theorem 1 where \mathcal{U} is a convex set with $\text{int } \mathcal{U} \neq \emptyset$.

The equivalency (2) \iff (3) follows as a simple corollary of the observation from Boyd and Vandenberghe's (2004) Section 2.5.2 that $\text{int } \Lambda \neq \emptyset \iff \Lambda^\perp = \{\vec{0}\}$. \square

Proof of Proposition 2 We show first the "if" direction. Assume that $\Lambda \subseteq \tilde{\Lambda} \subseteq \Lambda^*$. Now for all $p \in \Delta(X)$,

$$\min_{\lambda \in \tilde{\Lambda}} \sum_{i \in \mathcal{I}} \lambda_i u_i(p) = \min_{\lambda \in \Lambda} \sum_{i \in \mathcal{I}} \lambda_i u_i(p)$$

that shows the claim.

Next, we show the "only if" direction. Since (u, Λ) is a minimal representation, we have $\tilde{\Lambda} \supseteq \Lambda$. By Mononen (2024), since $(u, \tilde{\Lambda})$ and (u, Λ) represent \succsim_0 , we have for all $p \in \Delta(X)$,

$$\min_{\lambda \in \tilde{\Lambda}} \sum_{i \in \mathcal{I}} \lambda_i u_i(p) = \min_{\lambda \in \Lambda} \sum_{i \in \mathcal{I}} \lambda_i u_i(p).$$

Thus for each $p \in \Delta(X)$ and for all $\theta \in \tilde{\Lambda}$,

$$\sum_{i \in \mathcal{I}} \theta_i u_i(p) \geq \min_{\lambda \in \Lambda} \sum_{i \in \mathcal{I}} \lambda_i u_i(p).$$

So for each $p \in \Delta(X)$,

$$\left\{ \lambda \in \Delta(\mathcal{I}) \mid \sum_{i \in \mathcal{I}} \lambda_i u_i(p) \geq \min_{\delta \in \Lambda} \sum_{i \in \mathcal{I}} \delta_i u_i(p) \right\} \supseteq \tilde{\Lambda}.$$

Thus $\Lambda^* \supseteq \tilde{\Lambda}$. □

The standard proofs for the following lemmas are omitted.

Lemma 1 Assume that $U \subset \mathbb{R}^n$ is a convex set such that for each $1 \leq i \leq n$, there exist $p, q \in U$ such that $p_i > q_i$ and for each $j \neq i$, $p_j = q_j$. Then $\text{int } U \neq \emptyset$.

Lemma 2 Assume that \succsim_0 is a weak order and p and q are neutral such that $p \succ_0 q$. Then for all $\alpha, \beta \in [0, 1]$,

$$\alpha \geq \beta \iff \alpha p + (1 - \alpha)q \succsim_0 \beta p + (1 - \beta)q.$$

The last standard lemma is that weak Pareto monotonicity implies semi-strong Pareto monotonicity under continuity and vNM-individuals.

Lemma 3 Assume that $((\succsim_i)_{i \in \mathcal{I}}, \succsim_0)$ satisfies Axioms 2–5 and 8. If for each $i \in \mathcal{I}$, $p \succsim_i q$, then $p \succsim_0 q$.

Proof of Theorem 4 The necessity of the axioms is standard and omitted. By Axiom 7, there exists neutral lotteries $p^*, p_* \in \Delta(X)$ such that for each $i \in \mathcal{I}$, $p^* \succ_i p_*$. By the mixture space theorem (Herstein and Milnor 1953), for each $i \in \mathcal{I}$, there exists an affine $u_i : \Delta(X) \rightarrow \mathbb{R}$ that represents \succsim_i such that $u_i(p^*) = 1$ and $u_i(p_*) = -1$. Denote $\mathcal{U} = \{(u_i(p))_{i \in \mathcal{I}} \mid p \in \Delta(X)\}$. Since each u_i is affine, \mathcal{U} is convex. By Lemma 1 and Axiom 6, $\text{int } \mathcal{U} \neq \emptyset$.

Define the function $I : \mathcal{U} \rightarrow \mathbb{R}$ by the following: Let $u(p) \in \mathcal{U}$. There exists $\alpha \in (0, 1)$ such that $\alpha u(p) \in (-1, 1)^n$ and so by Lemma 3, $p^* \succsim_0 \alpha p + (1 - \alpha)(1/2 p^* + 1/2 p_*) \succsim_0 p_*$. Thus there exists a unique $\beta \in [0, 1]$ such that $\beta p^* + (1 - \beta)p_* \sim_0 \alpha p + (1 - \alpha)(1/2 p^* + 1/2 p_*)$. Define $I(u(p)) = \frac{2}{\alpha}(\beta - \frac{1}{2})$.

By Axioms 7 and 8 and Lemma 3, it is standard to show that for all $p, q \in \Delta(X)$,

$$p \succsim_0 q \iff I(u_i(p)_{i \in \mathcal{I}}) \geq I(u_i(q)_{i \in \mathcal{I}})$$

and I is well-defined, monotonic, C-additive, positively homogeneous, and concave in \mathcal{U} . By Chandrasekher et al.'s (2022) proof of their Theorem 1, there exists $P \subset \Delta(\mathcal{I})$ such that for all $\varphi \in \mathcal{U}$, $I(\varphi) = \min_{p \in P} p \cdot \varphi$. □

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