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Article

Strategic Complementarities in a Model of Commercial Media Bias

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Abstract: Media content is an important privately supplied public good. While it has been shown that contributions to a public good crowd out other contributions in many cases, the issue has not been thoroughly studied for media markets yet. We show that in a standard model of commercial media bias, qualities of media content are strategic complements, whereby investments into quality can *crowd in* further investments and engage competitors in a race to the top. Therefore, financially strong public service media can mitigate commercial media bias: the content of commercial media can be more in line with the preferences of the audience and less advertiser-friendly in a dual (mixed public and commercial) media system than in a purely commercial media market.

Keywords: commercial media bias; public service media; advertising; two-sided markets; supermodular games; strategic complements; public goods

JEL Classification: C70; H41; L13; L51; L82



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1. Introduction

Media content belongs to the most important cases of privately supplied public goods. Its consumption is non-rival, and in many cases like free TV or freely available Internet content no exclusion is taking place. Media content differs markedly from other public goods, though, because media outlets typically rely on advertising revenues instead of charging their consumers a pecuniary price. The economic analysis of the private supply of media content as a public good must take this multi-sided nature of media markets into account (Anderson & Coate, 2005). Recent literature has made major progress in this research area (see Anderson & Jullien, 2015; Jullien et al., 2021).

One important result from the theory of private public good supply is that, under fairly general conditions, private contributions to a public good are strategic substitutes, i.e., higher private contributions to the public good are crowding out other private contributions (for overviews, see Chapter 6 of (Batina & Ihori, 2005), and Finding F9 in (Buchholz & Sandler, 2021)). Surprisingly, this issue has not been thoroughly examined for media markets, even though it is highly relevant for the welfare analysis of media policy. E.g., in discussions about the proper role and scope of public service media (PSM), one crucial question is whether raising the quality of a regulated PSM will increase or decrease the quality of its commercial—i.e., profit-maximizing—competitors.

There are two conflicting views. On the one hand, PSM could crowd out private investment and innovation in media markets. E.g., the existence of PSM may lead to less entry of

commercial media; see (Berry & Waldfogel, 1999) for empirical evidence. Similarly, (Armstrong & Weeds, 2007a) show that in a duopoly where a PSM and a commercial broadcaster compete, raising the quality of PSM partially crowds out the commercial broadcaster and lowers its quality. This reasoning is echoed by regulation authorities like Ofcom (Ofcom, 2004) in the UK and the Scientific Advisory Board at the Federal Ministry of Finance in Germany (Wissenschaftlicher Beirat beim Bundesministerium der Finanzen, 2014).

However, PSM might also foster a “competition for quality”, whereby public and commercial media compete for audiences. This reasoning goes back to (Coase, 1947), pondering that PSM might induce a “natural rivalry to furnish the most attractive programs” (p. 197). Indeed, recent empirical evidence suggests that in countries where PSM invest into high-quality media content, the quality of commercial media tends to be high, too (Simon, 2013). Similarly, (Sehl et al., 2020) find that, controlling for GDP, per capita revenues of PSM and commercial broadcasters are positively correlated across EU countries. These correlations are in line with a *crowding in* effect of PSM, i.e., the presence of strong PSM coincides with flourishing commercial media.¹

In this paper, we show that in a model of commercial media bias, program qualities in terms of unbiased reporting are strategic complements rather than strategic substitutes.² Unbiased reporting here refers to a program that fully and truthfully reports facts as opposed to withholding information. E.g., advertisers might prefer the media to hide unfavorable facts about their products; prime examples include the tobacco and carbon-emitting industries. Viewers prefer high program quality, while advertisers prefer the opposite. The strategic complementarity stems from the media’s fundamental trade-off in these models: Raising program quality increases the value of the program for the audience but decreases the willingness to pay of the advertisers to reach consumers.³ The latter effect becomes less important when a media company has a smaller audience; hence, its incentives to raise program quality are higher. Thus, in a media market with both PSM and commercial media, raising the PSMs’ program quality reduces the commercial media’s audiences and thereby also their implicit cost of increasing their own program quality. As a result, the PSM *crowd in* program quality and engage the commercial media in a race to the top.

Our main model focuses on PSM and commercial media whose programs are freely available to the consumers. However, our results generalize to a model featuring both freely available and pay media, when program quality involves revealing information that the media already possess. We also discuss conditions under which our findings generalize to multidimensional strategy spaces, spillover effects of program quality on advertising revenue of other media outlets, endogenous entry and exit, and biases of PSM.⁴

Our paper relates to four strands of literature. First, we contribute to the literature on commercial media bias. Several empirical papers document the effect of advertising on media coverage in terms of mutual fund recommendations (Reuter & Zitzewitz, 2006), product mentions (Gambro & Puglisi, 2015), coverage of government scandals (Tella & Franceschelli, 2011) and climate change (Beattie, 2020). We present a fairly standard model of commercial media bias. Our model is in many ways similar to the models studied by (Ellman & Germano, 2009), (Germano & Meier, 2013), and (Kerkhof & Münster, 2015), as it captures bias through a program that caters to the preferences of advertisers rather than consumers. Our paper is especially close to (Ellman & Germano, 2009) and (Germano & Meier, 2013) who show that competition in media markets mitigates commercial bias, and to (Kerkhof & Münster, 2015) who find that competition between media outlets increases the likelihood that a cap on advertising quantities is welfare enhancing. Relatedly, (Blasco et al., 2016) find that if the media can raise their audience share through reducing their bias, then competition in the market may also increase the expected program quality.⁵

These predictions are in line with the empirical results of (Beattie et al., 2021) who find that newspapers provide less coverage of car recalls by their advertisers, but competition for readers mitigates this bias. Similarly, (Focke et al., 2016) show that commercial media bias is likely mitigated by reputational concerns on behalf of the media, e.g., if they face a demanding audience.

In contrast to the existing literature, the present study considers competition between commercial media and PSM, where PSM are not profit-maximizing, potentially regulated, and do not depend on advertising revenue to fund their operations. This allows us to inform policy debates regarding the proper role and scope of PSM in media markets. Moreover, in contrast to previous work, the present paper explicitly models competition between media outlets as a supermodular game, enabling us to draw fairly general conclusions regarding the impact of raising PSMs' budget on the program quality provided by commercial media.

Second, we advance the broad research on the private supply of public goods (Batina & Ihori, 2005; Bergstrom et al., 1986). The provision of public goods via advertising is studied by (Anderson & Coate, 2005; Luski & Wettstein, 1994). These papers do not study media bias, however.

Third, our paper relates to the literature on supermodular games, i.e., games in which the best response of any player is increasing in the actions of its competitors (Frankel et al., 2003; Milgrom & Roberts, 1990; Topkis, 1979; Van Zandt & Vives, 2007; Vives, 1985, 1990, 2005a, 2005b). To the best of our knowledge, we are the first to apply the theory of supermodular games to a model of commercial media bias. This approach allows us to obtain fairly general results in a model with many asymmetric media outlets. Specifically, we show that in our model of commercial media bias, program qualities are strategic complements rather than strategic substitutes.

Fourth, as a consequence of strategic complementarities, public investments into program quality induce commercial media to provide high quality, too. Hence, our results support media policies that advocate financially strong PSM. In this way, we also contribute to the economics literature on PSM (see (Armstrong & Weeds, 2007b; Strömberg, 2015; Weeds, 2020) for surveys). To the best of our knowledge, the issue how PSM affect the program of commercial media has not been studied yet in the literature on commercial media bias. Other aspects of this debate have, of course, been analyzed; in addition to the empirical literature referenced above, several theoretical studies on the market impact of PSM exist. (Armstrong & Weeds, 2007a) study investments in a vertical quality dimension. (Richardson, 2006) investigates how a publicly-provided radio station offering local programs affects the provision of local programs by commercial stations. (Garcia Pires, 2016) compares program diversity in commercial versus mixed public and private duopolies. Our paper complements this line of research by studying commercial media bias. E.g., neither (Armstrong & Weeds, 2007a) nor (Richardson, 2006) consider advertisers who value program qualities in terms of (un-)biased reporting. In (Armstrong & Weeds, 2007a), viewers are ad averse and higher advertising quantities reduce viewers' utility. PSM maximize viewer welfare, whereby viewers are better off than in a purely commercial market. However, in contrast to our paper, this is because PSM partially crowd out commercial media, whereby subscription prices and advertising quantities decrease. Similarly, (Richardson, 2006) shows that in a Hotelling model with ad averse viewers, PSM reduce profits of commercial media, but increase viewer welfare. Thus, in both models, viewers are better off because audience-friendly PSM displace commercial media. Our paper considers a different mechanism: financially strong PSM enhance viewers' utility because they crowd in program qualities by commercial media.

The remainder of this paper is structured as follows. Section 2 introduces our theoretical framework. In Section 3, we demonstrate that program qualities in terms of

unbiased reporting are strategic complements, which is our main finding, and describe the implications for *crowding in* effects of PSM. Section 4 considers the case where some commercial media are pay media. Section 5 discusses several extensions of our model. Section 6 concludes.

2. Model

This section introduces a fairly standard model of commercial media bias (Ellman & Germano, 2009; Germano & Meier, 2013; Kerkhof & Münster, 2015), see (Blasco et al., 2012) for a survey). Consider a model with n commercial media denoted by $1, \dots, n$ and m PSM denoted $n + 1, \dots, n + m$. The set of commercial media is denoted by $C = \{1, \dots, n\}$, the set of PSM is $P = \{n + 1, \dots, n + m\}$. Each media outlet $i \in C \cup P$ chooses a program quality $v_i \in V_i \subset \mathbb{R}_+$. (An extension to multidimensional strategy spaces is considered in Section 5). Program quality v_i is about unbiased reporting, i.e., about fully and truthfully reporting facts, as opposed to withholding information.⁶ The audience prefers high program quality, while advertisers prefer the opposite. We assume that the strategy sets V_i are compact and contain $v_i = 0$.

A consumer's utility from consuming outlet i is $u_i = f_i(v_i)$, where f_i is continuous, strictly increasing, and satisfies $f_i(0) = 0$. Unless otherwise noted, we simply assume $f_i(v_i) = v_i$. In Sections 2 and 3, nothing is lost in setting $u_i = v_i$; the distinction between utility u_i and program quality v_i becomes important when considering pay media or multidimensional strategies. For a commercial outlet $i \in C$, let $u_{-i}^C = (u_1, \dots, u_{i-1}, u_{i+1}, \dots, u_n)$ denote the vector of the utilities of i 's commercial competitors, $u^P = (u_{n+1}, \dots, u_{n+m})$ the vector of utilities of the PSM, and $u_{-i} = (u_{-i}^C, u^P)$.⁷

The size of the audience of a media outlet is denoted by s_i . We impose the following assumptions.⁸

Assumption 1. For all $i \in C$, s_i is positive, continuous, weakly increasing in u_i , and weakly decreasing in u_j for all $j \in P \cup C \setminus \{i\}$.

Assumption 1 is reasonable if consumers care about quality, and the media outlets are substitutes for the consumers.

Assumption 2. For all $i \in C$, s_i has weakly increasing differences in (u_i, u_{-i}) .

If s_i is twice continuously differentiable, and the strategy spaces are intervals, Assumption 2 means that

$$\frac{\partial^2}{\partial u_j \partial u_i} s_i \geq 0$$

for all $j \neq i$.⁹ Note that Assumption 2 only assumes that the differences are *weakly* increasing. In particular, it is fulfilled in the case of constant differences, where the above inequality holds with equality.

As we discuss in detail in Appendix A, Assumptions 1 and 2 are satisfied by many—but not all—models for audience demand that are frequently used in media economics. For example, any model where s_i is linear in u_i and u_j for all $j \neq i$, and does not include any interaction terms, satisfies Assumption 2 because s_i has constant differences in (u_i, u_{-i}) . This class of models comprises the Hotelling duopoly model of horizontally differentiated goods enriched by a vertical quality differentiation, and generalizations of the Hotelling model to more than two outlets such as the Spokes model and the Salop circle model. Similarly, s_i has constant differences in (u_i, u_{-i}) in representative consumer models with quadratic utility functions. See Appendix A for the functional forms of s_i in these models, and for references to publications in media economics using these specifications.

Note, however, that our assumptions are far more general than simply assuming linear demand functions. For example, when the audience demand functions include linear and quadratic terms,

$$s_i(u_i, u_{-i}) = a_i + b_i u_i - \sum_{j \neq i} b_{ij} u_j + \sum_k c_{ik} u_k u_i,$$

Assumption 2 holds as long as $c_{ik} \geq 0$ for all $i \in C$ and all $k \neq i$.

On the other hand, the logit model violates Assumption 2 whenever there are $n \geq 2$ commercial outlets. We deal with possible violations of Assumption 2 in two distinct ways. First, in Appendix F we give a sufficient condition for our main results to hold when Assumption 2 is violated, and thereby show the robustness of our results to sufficiently small violations of Assumption 2. Second, we explore our model with a weaker version of Assumption 2:

Assumption (2 log) For all $i \in C$, $\ln(s_i)$ has weakly increasing differences in (u_i, u_{-i}) .

Given Assumption 1, Assumption 2 implies Assumption (2 log), but not vice versa. For example, the logit model violates Assumption 2, but satisfies assumption (2 log) (see Appendix F). Moreover, the nested logit model—which is often used in empirical studies of audience demand in media economics (see [Berry & Waldfogel, 2015](#) for a survey)—satisfies Assumption (2 log) as well (see Appendix F).¹⁰ For some of our results, Assumption (2 log) is sufficient. Unless otherwise noted, below we will assume that Assumption 2 holds; we will explicitly state when we weaken it to Assumption (2 log).

Denote the advertising revenue of outlet i , per member of the audience, by R_i . A crucial assumption in models of advertiser bias is that, for a given audience, ad revenue depends negatively on program quality:

Assumption 3. For all $i \in C$, R_i is positive, continuous, weakly decreasing in v_i , and independent of v_j for all $j \neq i$.

By Assumption 3, R_i is independent from the program quality of other outlets as in ([Ellman & Germano, 2009](#)) and ([Kerkhof & Münster, 2015](#)). ([Germano & Meier, 2013](#)) model spillover effects of program quality on the advertising revenue of other outlets; we will discuss spillover effects in Section 5.¹¹

Each media outlet i has a cost $c_i(v_i)$ that may depend on its program quality.¹²

Assumption 4. For all media outlets $i \in C \cup P$, c_i is continuous, weakly increasing in v_i , and zero at $v_i = 0$.

We distinguish between two cases. First, program quality could be about fully and truthfully reporting facts that the media already have. In this case, the only cost of quality is lower advertising revenue, but there is no additional direct cost of obtaining the information in the first case. Formally, in the current model it means that $c_i(v_i) = 0$ for all $v_i \in V_i$. We refer to this case as “withholding facts”. Second, program quality could also be about investigative journalism, about establishing new facts and information. Then it seems plausible that $c_i(v_i)$ is strictly increasing in v_i . For example, the media might have to hire more journalists to increase program quality (see ([Hamilton, 2016](#)) for a detailed description of the economics of investigative journalism). We refer to this case as “investigating facts”.

Arguably, the case of withholding facts is highly relevant for the study of commercial media bias; indeed several papers in the literature focus on this case ([Blasco et al., 2016](#); [Ellman & Germano, 2009](#); [Germano & Meier, 2013](#); [Kerkhof & Münster, 2015](#)). For instance, advertising has been known to influence editors on crucial topics such as the health risks of smoking (e.g., ([Bagdikian, 2008](#)) Chapter 12) and climate change ([Beattie, 2020](#); [Boykoff & Boykoff, 2004](#)). The scientific facts about these topics were long well established and easily

accessible, but media coverage and public perception significantly lagged in time behind the scientific consensus. Moreover, as shown in (Beattie, 2024), commercial media bias in the *tone* of coverage about climate change, measured based on comparisons of environmental and skeptical texts, can have important behavioral consequences, and merely changing the tone of coverage does not impact its cost.

On the other hand, pressure from advertisers may also deter media from investigating facts. Our main results on freely available media do not depend on whether we study withholding or investigating facts. For pay media, we show that the distinction matters.

Commercial media in our main model are funded by advertising, and their program is freely available for consumers; pay media will be considered in Section 4. The profit of a commercial media outlet $i = 1, \dots, n$ is (substituting $v_i = u_i$ and $v_{-i} = u_{-i}$ into s_i)

$$\pi_i(v_i, v_{-i}) = s_i(v_i, v_{-i})R_i(v_i) - c_i(v_i).$$

Commercial outlet i maximizes $\pi_i(v_i, v_{-i})$ by choosing $v_i \in V_i$. Note that we disregard fixed costs which could be saved by going out of business; we defer a discussion of exit and entry to Section 5.

The PSM in our model are not-for-profit and financed independent of advertising. Their program is freely available to all consumers.¹³ The budget of PSM $i \in P$ is b_i . We assume the PSM spend their budget to maximize consumer utility by choosing $v_i \in V_i$ subject to $c_i(v_i) \leq b_i$.¹⁴ The feasible sets $V_i \subset \mathbb{R}_+$ are compact, contain $v_i = 0$, and may depend on the budget b_i . We assume that a larger budget enlarges the feasible set: if $b_i < b'_i$, then $V_i(b_i) \subseteq V_i(b'_i)$. The model allows for inefficiencies of PSM, since different media outlets can have different cost functions and different feasible sets of program quality. We discuss potential biases of PSM in Section 5.

Some (but not all) of our considerations below impose the additional assumption that a sufficiently high program quality is necessary for a PSM to attract an audience. To express this formally, for $i \in P$ let $v_{-i}^P = (v_{n+1}, \dots, v_{i-1}, v_{i+1}, \dots, v_{n+m})$ denote the vector of program qualities of the other PSM.

Assumption 5. A PSM outlet with zero program quality ($v_i = 0$) attracts no audience: $s_i(0, v_{-i}^C, v_{-i}^P) = 0$ for all $i \in P$ and (v^C, v_{-i}^P) , and demand for the other media outlets is as if outlet i did not exist.

Assumption 5 seems reasonable when the audience has a sufficiently attractive outside option not to consume any media. Note that under Assumption 5, a PSM with an insufficient budget cannot produce a program that attracts any audience; then the game reduces to a game between the remaining media outlets only. We will explicitly indicate where we use Assumption 5.

3. Main Results

Consider the PSM first.

Lemma 1. A PSM i chooses

$$v_i = \bar{v}_i(b_i) := \max_{v_i \in V_i(b_i)} \{v_i | c_i(v_i) \leq b_i\}.$$

Moreover, $\bar{v}_i(b_i)$ is weakly increasing in b_i , and independent of the strategies of the other media outlets.

Proof. Outlet $i \in P$ solves

$$\max_{v_i \in V_i} v_i \text{ s.t. } c_i(v_i) \leq b_i.$$

An increase of b_i relaxes the PSM's budget constraint and enlarges the feasible set V_i , hence \bar{v}_i is weakly increasing in b_i . Moreover \bar{v}_i is unique and independent of the strategies chosen by the other media outlets. \square

We now turn to the commercial media. Lemma 1 allows us to view the game between the commercial media as parameterized by the budgets of the PSM $b := (b_{n+1}, \dots, b_{n+m})$. Let $\bar{v}^P(b) = (\bar{v}_i(b_i))_{i=n+1}^m$ denote the vector of program qualities chosen by the PSMs. For $i \in C$, let

$$\tilde{\pi}_i(v_i, v_{-i}^C, b) := \pi_i(v_i, v_{-i}^C, \bar{v}^P(b))$$

and let $\Gamma_b = (C, (\tilde{\pi}_i)_{i=1}^n, \Pi_{i=1}^n V_i)$ denote the resulting game between the commercial media outlets: the set of players is C , payoff functions are $\tilde{\pi}_i$, and strategy spaces are V_i .

To state our main result, we need the concept of a parameterized supermodular game.¹⁵ Consider a family of games with set of players N , strategy spaces X_i , and payoff functions u_i parameterized by t in some partially ordered set of parameters values T . The game $(N, (u_i, (X_i)_{i \in N})_{i \in N}, T)$ is a parameterized supermodular game if, for each $i \in N$: (i) $X_i \subseteq \mathbb{R}^{m_i}$ is a lattice and is compact, (ii) $u_i(x_i, x_{-i}, t)$ is continuous in x_i for fixed x_{-i} and t , (iii) $u_i(x_i, x_{-i}, t)$ is supermodular in x_i and has weakly increasing differences in (x_i, x_{-i}, t) .

Proposition 1. Γ_b is a parameterized supermodular game.

Proof. The strategy spaces $V_i \subset \mathbb{R}_+$ are compact by assumption, hence compact lattices, and the objective functions $\tilde{\pi}_i$ are continuous in v_i for fixed v_{-i} and b .

Next, we show that π_i has weakly increasing differences in (v_i, v_{-i}) . For simplicity of the exposition, we will assume here that the functions R_i , s_i and c_i are differentiable; Appendix B gives the proof without assuming differentiability. From

$$\pi_i(v_i, v_{-i}) = s_i(v_i, v_{-i})R_i(v_i) - c_i(v_i)$$

we obtain

$$\frac{\partial \pi_i}{\partial v_i} = \frac{\partial s_i(v_i, v_{-i})}{\partial v_i} R_i(v_i) + s_i(v_i, v_{-i}) R'_i(v_i) - c'_i(v_i)$$

and

$$\frac{\partial^2 \pi_i}{\partial v_j \partial v_i} = \frac{\partial^2 s_i(v_i, v_{-i})}{\partial v_j \partial v_i} R_i(v_i) + \frac{\partial s_i(v_i, v_{-i})}{\partial v_j} R'_i(v_i) \geq 0, \quad j \neq i \quad (1)$$

where the inequality follows because of $\frac{\partial^2 s_i(v_i, v_{-i})}{\partial v_j \partial v_i} \geq 0$ by Assumption 2, $\frac{\partial s_i(v_i, v_{-i})}{\partial v_j} \leq 0$ by Assumption 1, and $R'_i(v_i) \leq 0$ by Assumption 3.

We have shown that π_i has weakly increasing differences in (v_i, v_{-i}) . Therefore, $\tilde{\pi}_i$ has weakly increasing differences in (v_i, v_{-i}^C) . Moreover, π_i has weakly increasing differences in (v_i, v^P) . It remains to show that $\tilde{\pi}_i$ has weakly increasing differences in (v_i, b) . By Lemma 1, $\bar{v}_k(b_k)$ is increasing in b_k , while $\bar{v}_{k'}$ does not depend on b_k for $k' \neq k$. Since π_i has weakly increasing differences in (v_i, v^P) , it follows that $\tilde{\pi}_i$ has weakly increasing differences in (v_i, b) . \square

Proposition 1 shows that the program qualities are strategic complements. The economics of the result is straightforward. The fundamental trade-off for a commercial outlet in a model of commercial media bias is as follows: providing a program in line with the preferences of the audience attracts a bigger audience, but leads to lower advertising revenue per consumer. If the program qualities of competing media increase, the audience of

a given outlet is smaller, hence also the implicit cost of increasing its own program quality. The logic is closely related to the finding in (Germano & Meier, 2013) that withholding facts typically increases with the concentration of ownership on the media market, which has found empirical support in (Beattie et al., 2021).

Note that violations of Assumption 2 do not necessarily overturn Proposition 1. In Appendix F, we give a sufficient condition for Proposition 1 to hold if Assumption 2 is violated. Moreover, a different sufficient condition is available in the case of hiding information:

Corollary 1. *Suppose that Assumptions 1, (2 log), and 3 hold. Consider the case of hiding information, i.e., $c_i(v_i) = 0$ for all $v_i \in V_i$. Suppose that s_i and R_i are strictly positive for all $(v_i, v_{-i}) \in V_i \times V_{-i}$. Then Γ_b is a parameterized supermodular game.*

Proof. We only give the proof assuming differentiability for simplicity. Commercial outlet i maximizes $\pi_i(v_i, v_{-i}) = s_i(v_i, v_{-i})R_i(v_i)$. By assumption, $s_i(v_i, v_{-i})R_i(v_i) > 0$. Thus the profit maximizing quality of firm i also has to maximize

$$\ln \pi_i = \ln s_i + \ln R_i.$$

Since $\ln s_i$ has weakly increasing differences by Assumption (2 log), and R_i is independent of v_j for $j \neq i$, it follows that $\ln \pi_i$ has weakly increasing differences in (v_i, v_{-i}) . The rest of the proof is as in the proof of Proposition 1. \square

The logit model, and the nested logit model, are two frequently used audience demand functions in media economics. As mentioned above, they satisfy Assumption (2 log). Corollary 1 thus implies that in the case of hiding information our main results apply to the (nested) logit model.

Leveraging the theory of supermodular games (see (Vives, 2005a), (Vives, 2005b) or (Sarver, 2023) for expositions) allows us to generate fairly general results in our model featuring many asymmetric media outlets. Denote a strategy profile in game Γ_b by $v^C = (v_1, \dots, v_n)$. The following corollary collects standard results for parametrized supermodular games that are useful in our context:

Corollary 2. *If Γ_b is a parameterized supermodular game, then Γ_b has, for any b , a lowest equilibrium $v^{C,low}$ and a highest equilibrium $v^{C,high}$, such that any equilibrium v^C satisfies $v^{C,low} \leq v^C \leq v^{C,high}$. Moreover, the equilibria $v^{C,low}$ and $v^{C,high}$ are weakly monotone increasing in b .*

In particular, an equilibrium exists; the standard proof for equilibrium existence in supermodular games uses the Tarski fixed point theorem. Turning to the comparative statics, note Corollary 2 does not imply that all equilibria are monotone increasing in b : only the highest and lowest equilibria are guaranteed to be weakly monotone increasing in b (see (Sarver, 2023) for a general elaboration of this point for supermodular games).¹⁶ When the equilibrium is unique, a stronger monotone comparative static result is available, which we highlight in the following Proposition 2.

Proposition 2. *Suppose Γ_b is a parameterized supermodular game and has a unique equilibrium. Then the equilibrium program quality of each commercial media outlet $i \in C$ is weakly increasing in the budget b_j of any PSM $j \in P$.*

Proposition 2 states that PSM do not crowd out, and may even crowd in program quality, in line with the idea that PSM engage commercial media in a race to the top. To

illustrate the result, we compare a “dual” (or mixed public and commercial) media market, featuring both PSM and n commercial media outlets, with a purely commercial media market consisting only of the same n commercial outlets, postponing considerations about entry to Section 5. Under Assumption 5, Proposition 2 implies that in a “dual” media market, the program of the commercial media outlets is weakly more audience-friendly and weakly less advertiser-friendly than when there are only the n commercial media. To see this, recall that when b_i is insufficient, the PSM i cannot attract any audience in our model by Assumption 5, and then the resulting competition between the remaining media is as if PSM i was not on the market. Applying Proposition 2 shows that the program qualities of the commercial media will be weakly lower in this situation than when there are viable PSM.

A sufficient condition for uniqueness of the equilibrium can be given from the contraction approach: If the profit functions $\tilde{\pi}_i$ are smooth and strictly quasiconcave in v_i , a sufficient condition for equilibrium uniqueness is that

$$\frac{\partial^2 \tilde{\pi}_i}{\partial v_i^2} + \sum_{\substack{j \neq i \\ j \in C}} \left| \frac{\partial^2 \tilde{\pi}_i}{\partial v_i \partial v_j} \right| < 0$$

for all (v_i, v_{-i}) (Vives, 1999, Chapter 2.5). In Appendix I, we use the contraction approach to spell out sufficient conditions for equilibrium uniqueness with a linear audience function s_i , and in the case of withholding information with a logit audience function.

The monotone comparative statics results provided so far do not rule out the possibility that the quality of a commercial outlet remains constant when the budget of a PSM increases, for example because the set of possible qualities V_i could be discrete, or because of corner solutions or kinks in the profit function. Moreover, so far nothing in our assumptions guarantees that a larger budget of a PSM translates into *strictly* higher quality.¹⁷ Following (Edlin & Shannon, 1998), we now provide a strict monotone comparative statics result by adding assumptions on interior solutions and differentiability and employing “strict” versions of Assumptions 1 and 3 above. Note we do *not* assume that s_i has strictly increasing differences, however.

Corollary 3. Suppose that $V_i = [0, v_i^{high}]$ with $v_i^{high} > 0$ for all media outlets i . Moreover, suppose that for all $j \in P$, a higher budget b_j strictly increases v_j^{high} . For each commercial outlet $i \in C$, assume that $\pi_i(v_i, v_{-i})$ is differentiable, $s_i(v_i, v_{-i})$ is strictly decreasing in v_j for each $j \neq i$ whenever $s_i(v_i, v_{-i}) > 0$, and $R_i(v_i)$ is strictly decreasing in v_i . Suppose Γ_b is a parameterized supermodular game and has a unique equilibrium, which is interior in the sense that $0 < v_i < v_i^{high}$ and $0 < s_i < 1$ for all $i \in C$. Then the equilibrium program quality of each commercial media outlet $i \in C$ is strictly increasing in the budget b_j of any PSM $j \in P$.

Proof. Outlet $j \in P$ solves

$$\max_{v_j \in V_j} v_j \text{ s.t. } c_j(v_j) \leq b_j.$$

An increase of b_j relaxes the PSM’s budget constraint and strictly enlarges the feasible set $V_j = [0, v_j^{high}]$ by strictly increasing v_j^{high} . Hence \bar{v}_j is strictly increasing in b_j .

Moreover, under the current assumptions, inequality (1) in the proof of Proposition 1 holds with strict inequality because $\frac{\partial s_i(\bar{v}_i, v_{-i})}{\partial v_j} < 0$ and $R_i'(v_i) < 0$ by the current assumptions. That is, π_i has strictly increasing marginal returns in (v_i, v_j) , holding constant all other $v_k, k \neq i, j$, and hence (by the strict monotone comparative statics results of (Edlin & Shannon, 1998)) the best reply of each commercial outlet i strictly increases in b_j . The new equilibrium must therefore involve strictly higher qualities of all commercial outlets. \square

4. Pay Media

This section studies an extension to the model where some commercial media outlets are pay media, i.e., they earn revenue from direct payments from consumers. Suppose that media outlets $i \in C^f = \{1, \dots, n_f\}$ are freely available: they are funded solely by advertising and “free” in the sense that consumers do not pay a monetary price for consumption. Outlets $i \in C^{pay} \in \{n_f + 1, \dots, n\}$ are pay media. The set of all commercial media is $C = C^f \cup C^{pay}$.¹⁸ As above, outlets $i \in P = \{n + 1, \dots, m\}$ are PSM.

A pay media outlet $i \in C^{pay}$ chooses program quality $v_i \in V_i$ and price $p_i \in P_i$. In this section, we focus on the case where $V_i = [0, v_i^{high}]$ and $P_i = [0, p_i^{high}]$, with $v_i^{high} > 0$ and $p_i^{high} > 0$, and assume that functions are smooth and solutions are interior¹⁹, and that there is a unique $v_i^* \in V_i$ such that $R'_i(v_i^*) = -1$. To obtain a strict monotone comparative static result, we also assume that for all $i \in C$, s_i is strictly decreasing in u_j for $j \neq i$ as long as $s_i > 0$.

Utility from consuming outlet i is $u_i = v_i - p_i$, utility from consuming an outlet $j \in C^f \cup P$ is simply $u_j = v_j$.

The profit of a pay media outlet $i \in C^{pay}$ is

$$\pi_i(v_i, p_i, u_{-i}) = s_i(u_i, u_{-i})(R_i(v_i) + p_i) - c_i(v_i),$$

where u_{-i} is the vector of utilities offered by the other outlets. The profit of a freely available media outlet $i \in C^f$ is

$$\pi_i(u_i, u_{-i}) = s_i(u_i, u_{-i})R_i(v_i) - c_i(v_i).$$

In all other respects, the model is as in Section 2 above.

For pay media, results depend on whether we consider the case of withholding or investigating facts (see the discussion after Assumption 4 above). Results are clear cut in the case of withholding facts, where $c_i(v_i) = 0$ for all v_i . We show that in this case, when some competitor $j \neq i$ increases the utility u_j , outlet i will also offer a higher utility: pay media keep their program quality constant but lower their price, while freely available media increase their program quality. We briefly discuss the case of investigating facts towards the end of this section.

Lemma 2. *Consider the case of withholding information where $c_i(v_i) = 0$ for all $v_i \in V_i$ and all $i \in C^{pay}$. Then outlet $i \in C^{pay}$ will choose program quality $v_i = v_i^*$, independent of the strategies of the other media outlets. Moreover, the price p_i is strictly decreasing in u_j for $j \neq i$, holding constant the strategies of the remaining outlets $k \neq i, j$.*

Proof. The first order conditions for an interior solution are

$$\begin{aligned} \frac{\partial \pi_i}{\partial v_i} &= \frac{\partial s_i(v_i - p_i, u_{-i})}{\partial u_i} (R_i(v_i) + p_i) + s_i(v_i - p_i, u_{-i}) R'_i(v_i) = 0, \\ \frac{\partial \pi_i}{\partial p_i} &= -\frac{\partial s_i(v_i - p_i, u_{-i})}{\partial u_i} (R_i(v_i) + p_i) + s_i(v_i - p_i, u_{-i}) = 0. \end{aligned}$$

Rearranging the second line and inserting it into the first gives, after dividing by $s_i > 0$,

$$1 + R'_i(v_i) = 0.$$

Therefore, firm i chooses $v_i = v_i^*$ defined by $R'_i(v_i^*) = -1$.

Plugging $v_i = v_i^*$ into the first order condition for p_i gives

$$-\frac{\partial s_i(v_i^* - p_i, u_{-i})}{\partial u_i} (R_i(v_i^*) + p_i) + s_i(v_i^* - p_i, u_{-i}) = 0.$$

By the implicit function rule, the sign of the reaction of p_i on u_j ($j \neq i$) is equal to

$$-\frac{\partial^2 s_i(v_i^* - p_i, u_{-i})}{\partial u_j \partial u_i} (R_i(v_i^*) + p_i) + \frac{\partial s_i(v_i^* - p_i, u_{-i})}{\partial u_j} > 0,$$

where the inequality follows because of $\frac{\partial^2 s_i(u_i, u_{-i})}{\partial u_j \partial u_i} \geq 0$ by Assumption (2), $R_i(v_i^*) \geq 0$ and $p_i > 0$, and $\frac{\partial s_i(u_i, u_{-i})}{\partial u_j} < 0$ by assumption. \square

Lemma 2 allows us to consider the game as one where the pay media have only one choice variable: the price. To find the profit maximizing prices, substitute $v_i = v_i^*$ into the profit function of outlet $i \in C^{pay}$. Instead of maximizing profits by choosing p_i , we can equivalently think of firm i as choosing utility u_i , taking into account that $u_i = v_i^* - p_i$.

The set of utilities that outlet i can choose from is $U_i := \{u_i | u_i = v_i^* - p_i, p_i \in P_i\}$. The objective function is

$$\hat{\pi}_i(u_i, u_{-i}) := s_i(u_i, u_{-i})(R_i(v_i^*) + v_i^* - u_i).$$

Mirroring our definitions leading to Proposition 1 above, let

$$\tilde{\pi}_i(u_i, u_{-i}^C, b) = \begin{cases} \pi_i(u_i, u_{-i}^C, \bar{v}^P(b)), & \text{if } i \in C^f, \\ \hat{\pi}_i(u_i, u_{-i}^C, \bar{v}^P(b)), & \text{if } i \in C^{pay}. \end{cases}$$

Let $\Gamma_b^{pay} = (C, (\tilde{\pi}_i)_{i=1}^n, \Pi_{i=1}^{n_f} V_i \times \Pi_{i=n_f+1}^n U_i)$ denote the resulting game between the commercial media outlets: the set of players is $C = C^f \cup C^{pay}$, payoff functions are $\tilde{\pi}_i$, outlets $i \in C^f$ chooses $u_i \in V_i$, outlets $i \in C^{pay}$ choose $u_i \in U_i$ while their program quality is fixed at v_i^* by Lemma 2, and as above the utilities offered by the PSM are given by $\bar{v}^P(b)$.

Proposition 3. *In the case of withholding information where $c_i(v_i) = 0$ for all $v_i \in V_i$ and all $i \in C^{pay}$, Γ_b^{pay} is a parameterized supermodular game.*

Proof. To begin with, since P_i is compact, the set of feasible utilities U_i is compact as well. The rest of the proof follows similar lines as the proofs of Proposition 1 and Lemma 2. For $i \in C^{pay}$ and $j \neq i$,

$$\begin{aligned} \frac{\partial^2 \hat{\pi}_i}{\partial u_j \partial u_i} &= \frac{\partial}{\partial u_j} \left(\frac{\partial s_i(u_i, u_{-i})}{\partial u_i} (R_i(v_i^*) + v_i^* - u_i) - s_i(u_i, u_{-i}) \right) \\ &= \frac{\partial^2 s_i(u_i, u_{-i})}{\partial u_j \partial u_i} (R_i(v_i^*) + v_i^* - u_i) - \frac{\partial s_i(u_i, u_{-i})}{\partial u_j} > 0 \end{aligned}$$

where the inequality follows because of $\frac{\partial^2 s_i(u_i, u_{-i})}{\partial u_j \partial u_i} \geq 0$ by Assumption (2), $R_i(v_i^*) \geq 0$, $v_i^* - u_i = p_i \geq 0$, and $\frac{\partial s_i(u_i, u_{-i})}{\partial u_j} < 0$ by assumption. The remainder of the proof is similar to the proof of Proposition 1. \square

Proposition 3 shows that Γ_b^{pay} has weakly increasing reaction functions. That is, when some competitor of a commercial media outlet i increases the utility it offers, ceteris paribus

outlet i will also offer a higher utility. For a pay media outlet $i \in C^{pay}$, program quality is constant by Lemma 2, but i strictly lowers its price. The economics behind the result is straightforward: tougher competitors reduce residual demand, and as a reaction firm i charges a lower price. Proposition 3 also shows that the freely available media will, as in Section 3 above, ceteris paribus react to increases in the utility offered by a competitor by weakly increasing their program quality.

As above, we can leverage the theory of supermodular games to obtain results on Γ_b^{pay} . In particular, Proposition 2 generalizes in the following way:²⁰

Proposition 4. *Consider the case of withholding information where $c_i(v_i) = 0$ for all $v_i \in V_i$ and all $i \in C^{pay}$, and suppose that Γ_b^{pay} has a unique equilibrium. Then an increase of the utility u_j of consuming a PSM $j \in P$ will weakly increase the utilities u_i offered by all commercial outlets: the program qualities of freely available media weakly increase, the program qualities of pay media stay constant but their prices decline.*

A sufficient condition for uniqueness of the equilibrium can be given by applying the contraction approach to the game Γ_b^{pay} .

To conclude this section, we briefly consider to the case of investigating facts. We point out that the assumption that $c_i(v_i) = 0$ is crucial for our above results. If c_i is strictly increasing in v_i , then the equilibrium program quality of firm i can be strictly decreasing in u_{-i} , and the total effect of u_{-i} on u_i is ambiguous, as we show in example in Appendix C below. We leave a full exploration of pay media in the case of investigating facts for future research.

5. Discussion

Of course, our model abstracts away from several issues that could potentially be relevant. In this section, we discuss multidimensional strategy spaces, spillover effects of program quality on the advertising revenue of other media outlets, entry, and potential biases in PSM. We focus on freely available media, and assume differentiability wherever convenient for expositional simplicity. Formal analyses are deferred to the Appendix.

5.1. Multidimensional Strategy Spaces

Media outlets may report differently on different topics, and may choose other dimensions of program quality that are less of a concern for advertisers. E.g., media outlet i could report on current events such as climate change, economic developments like changes in bank rates, and sports events like the football world cup. Advertisers from carbon-emitting industries might be concerned about high program quality on the topic of climate change, and indifferent about program qualities of the remaining topics. Advertisers from the sportswear industry might be concerned about full reporting on the production conditions of football gear, and indifferent about full reporting on tax evasion by football players. Media outlet i could choose high program quality on all, several, or none of these topics. How does this affect our results?

Suppose that outlet i reports about k_i topics, and let $v_{i,k}$ denote program quality of topic k .²¹ We assume that $v_{i,k} \in [0, v_{i,k}^{high}]$ with $v_{i,k}^{high} > 0$. Outlet i thus chooses a vector $v_i \in V_i = \times_{k=1}^{k_i} [0, v_{i,k}^{high}]$. Consumer utility from consuming outlet i is $u_i = f_i(v_i)$, where f_i is continuous, strictly increasing with respect to each argument $v_{i,k}$, and satisfies $f_i(0) = 0$. Advertising revenue per consumer is $R_i(v_i)$, where $R_i : V_i \rightarrow \mathbb{R}_+$ is positive, continuous, weakly decreasing in each argument $v_{i,k}$, and independent of v_{-i} . The profit of $i \in C$ is $\pi_i = s_i(u_i, u_{-i})R_i(v_i) - c_i(v_i)$. Turning to the PSM, suppose that $i \in P$ chooses $v_i \in V_i(b_i)$

to maximize $u_i = f_i(v_i)$ subject to $c_i(v_i) \leq b_i$. As above, a higher budget may enlarge the feasible set V_i .

5.1.1. Withholding Facts

In the following, it is important to distinguish between withholding facts and investigating facts. In the case of withholding facts, there are no direct costs of raising program quality. Thus, $c_i(v_i) = 0$ for all $v_i \in V_i$ and all $i \in C$, and our results generalize. See Appendix D.1 for a formal analysis.

5.1.2. Investigating Facts

In the case of investigating facts, $c_i(v_i)$ is strictly increasing in v_i for all $i \in C$. Trivially, as long as the game remains supermodular, our results generalize. A relevant concern is, however, whether profit π_i will be supermodular in its own choice variables.

We illustrate this with a two-dimensional case inspired by (Germano & Meier, 2013). Suppose that outlet i chooses program quality $v_i \in V_i = [0, v_i^{high}]$ and general quality $y_i \in Y_i = [0, y_i^{high}]$. The general quality y_i captures all dimensions of quality except (un-)biased reporting (e.g., quality of exposition or pleasing visuals) and does not affect R_i .

Appendix D.2 demonstrates that π_i has weakly increasing differences in $((v_i, y_i), (v_{-i}, y_{-i}))$. For a supermodular game, however, π_i must be supermodular in (v_i, y_i) , too. Whether or not this is the case depends on the cost function c_i and the utility function f_i .

If there are sufficiently strong economies of scope in producing (v_i, y_i) , or if there are sufficiently strong complementarities between v_i and y_i for the consumers, then π_i is supermodular in (v_i, y_i) . If this is the case for all commercial outlets $i \in C$, our main results generalize. In contrast, if there are no complementarities between v_i and y_i stemming from c_i or f_i , and s_i is concave in u_i , profit π_i will be submodular in (v_i, y_i) . In this case, the effect of a higher budget for the PSM on the utility of the audience from consuming commercial media is ambiguous, as we show by example in Appendix D.3.

5.2. Spillover Effects of Program Qualities on Advertising Revenue of Other Media Outlets

Our main model assumes that the advertising revenue of a commercial media outlet depends on its own program quality, but not on the program qualities of other media outlets. Arguably, advertising revenue of all outlets might be negatively affected when some outlets report about deficiencies of a product, hence there may be spillover effects as in (Germano & Meier, 2013). We show in Appendix E.1 that our results are robust when these spillover effects are small compared to the direct effect of an outlet's own program quality on its advertising revenue. If spillover effects were as strong as the direct effects, however, the strategic complementarities between the outlets' program qualities could diminish, resulting in program qualities becoming independent of each other (see Appendix E.2).

5.3. Entry and Exit

Our results on strategic complementarities apply to situations where the PSM do not induce any of the commercial outlets to exit the market. In reality, when the PSM budgets are sufficiently increased, commercial outlets may be driven out of business. By the same token, if PSM were scaled back, this could trigger entry of additional commercial outlets. The new entrants would have to provide sufficiently high quality in order to overturn the results in Proposition 2, however.

To illustrate, suppose the PSM were abolished in favor of a purely commercial media market. Without additional entry, our results above predict that program quality of the commercial outlets would decline. Entry of m additional commercial outlets would keep

the number of media outlets constant. If these entrants offer lower program quality than the PSM used to, however, incumbent commercial media will still provide lower program quality than before the commercialization of the media market. Consequently, all consumers are negatively affected by lower program qualities. Whether there will be enough entrants with sufficiently high program quality to overcome this negative effect on consumers will depend on various factors, including barriers to entry, the revenue potential of the market, the PSMs' budgets and hence quality, and potentially cost advantages or disadvantages of new entrants.

In Appendix G, we study a Hotelling example with one PSM and one potential commercial entrant, and compare the equilibrium qualities with those in a purely commercial duopoly. The example illustrates the importance of the quality of the PSM in comparison to the equilibrium qualities in a purely commercial market: If the PSM quality is lower, then because of the strategic complementarities the commercial outlet will also have a lower quality than in a purely commercial market. Similarly, if the PSM quality is higher but not too high, the quality of the private outlet will also be higher than in a purely private market. On the other hand, if the quality of the PSM is too high, it crowds out the commercial media outlet.

5.4. Biases in PSM

Our model allows PSM to be cost-inefficient but assumes them to be commercially unbiased, i.e., they choose a program that is as audience-friendly as possible, given their budget. Of course, PSM may also be biased (e.g., Crawford & Levonyan, 2018). In particular, PSM also engage in advertising and product placement, and therefore their content may be influenced by advertisers. Moreover, in some countries, governments are major advertisers themselves, making PSM susceptible to government influence (Tella & Franceschelli, 2011). E.g., (Szeidl & Szucs, 2021) and (Bátorfy & Urbán, 2020) document large advertising favors from the Hungarian government to national media outlets, and large corruption coverage favors from the privileged media to the government in return. Similarly, (Yanatma, 2021) reports that the Turkish government has recently emerged as the media's largest advertiser and has used its power to shape the media's reporting.²²

While such considerations are clearly important, we point out that our results can allow for some biases in PSM. The strategic complementarities between the program qualities of PSM and commercial media do not depend on assumptions about the PSM at all. The key issue is whether a higher budget of a PSM will translate into more or less severe biases of this outlet. As long as the program qualities of PSM are increasing in their budgets—i.e., as long as the programs become more audience-friendly when funding of PSMs increases—, Propositions 1 and 2 are robust, no matter if the advertiser is a political player or not. If more government spending induces PSM to withhold more critical information about political deficits, however, the PSMs' program quality would decrease, and thereby also the program quality of their commercial competitors.²³

As in our discussion of entry above, any evaluation of the impact of potentially biased PSM on the program of commercial media outlets crucially depends on what the alternatives to PSM are. PSM are typically not for profit, and they often face tighter limitations on advertising than commercial outlets (see e.g., (Crawford et al., 2017)), which may counteract commercial media bias (Kerkhof & Münster, 2015). Indeed, PSM typically have a higher share of hard news and socially relevant topics in their program, so their commercial biases may be lower (see (Cushion, 2017) for a wide ranging review). On the other hand, advertising revenue helps against other sources of biases in media content (see for example (Besley & Prat, 2006; Petrova, 2011), and (Szeidl & Szucs, 2021)).

6. Conclusions

In this paper, we show that in a model of commercial media bias, program qualities in terms of unbiased reporting are strategic complements rather than strategic substitutes. The strategic complementarity stems from the media's fundamental trade-off in these models: Increasing program quality increases the value of the program for the audience but decreases the willingness to pay of the advertisers to reach consumers. The latter effect becomes less important when a media company has a smaller audience; hence, its incentives to increase program qualities are higher. Thus, in a media market with both PSM and commercial media, PSMs' with high program qualities give commercial media incentives to provide high quality themselves, too. As a result, the PSM *crowd in* quality and engage the commercial media in a race to the top. This is in line with recent empirical evidence on public and private investments into program quality (Sehl et al., 2020; Simon, 2013).

Our results hold under fairly general conditions. One important assumption is that audience demand functions have weakly increasing differences, a condition met by various standard demand functions. These include linear demand functions, quadratic demand functions with positive coefficients for the interaction terms, as well as models such as Hotelling, Salop, and Spokes. We also give sufficient conditions for our results to hold for audience demand functions that do not have weakly increasing differences, such as the logit model. While our main analysis considers media outlets who offer their programs for free, our results also extend to pay media when commercial media bias is about withholding facts that the media already have. Similarly, our results hold for multidimensional strategy spaces in the case of withholding facts; for the case of investigating facts, we provide conditions under which our main results generalize. Arguably, advertising revenue of all outlets might be negatively affected when some outlets report about deficiencies of a product, hence there may be spillover effects. However, we show that our results are robust when these spillover effects are small compared to the direct effect of an outlet's own program quality on its advertising revenue. If entry or exit of commercial media was possible, commercial entrants would have to provide sufficiently high quality in order to overturn our main results. Finally, we point out that our results can allow for some biases in PSM as long as a higher budget of a PSM will translate into less severe biases of this outlet.

We have also show, however, that program qualities are not always strategic complements. We give examples that show that, for pay media (or multidimensional strategy spaces) in the case of costly investigative journalism, higher quality PSMs can increase or decrease the quality of commercial media outlets, depending on details of the model. Moreover, strong PSM may prevent entry of commercial outlets in the first place. Furthermore, higher budgets for PSMs could, in case of politically captured PSMs, increase their biases, and by strategic complementarities the biases of commercial media would increase as a consequence.

The paper contributes to recurrent media policy debates about the proper role and scope of PSM. While several regulation authorities fear that raising the program quality of PSM could crowd out private investments into program quality, our results support policies that advocate strong and financially well-equipped PSM. Reductions in the funding of PSM might result in a worse media landscape altogether.

Our insights might be especially important for modern media markets like social media, where systematic quality controls are missing and quality standards are often claimed to be low (Zhuravskaya et al., 2020). While PSM have typically played a minor role here, our results encourage PSM to develop a stronger presence and provide high-quality content on social media, too. This reasoning is in line with recent scholarly advances calling on PSM to become "Public Service Internet platforms" with the objective to provide

opportunities for public debate, participation, and the advancement of social cohesion (Unterberger & Fuchs, 2021).

An interesting avenue for future research would be to test our predictions empirically. In particular, we hypothesize that an increase in PSMs' budget would *ceteris paribus* translate both into higher program qualities of the PSM themselves as well as into higher program qualities of the PSMs' commercial competitors. Similarly, reductions in PSM budgets would lead to lower program qualities of both PSM and commercial media. However, the identification of a causal relationship between PSM budgets and program qualities would require some exogenous variation in PSM budgets.

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Appendix A. Assumptions 1 and 2 Are Satisfied by Many Frequently Used Models of Audience Size in Media Economics

In the media economics literature, there are four commonly used ways to model audience size when media quality matters (see the surveys by (Anderson & Gabszewicz, 2006; Anderson & Jullien, 2015)). These models are also widely used in other fields, see (Huang et al., 2013) for a survey. This appendix discusses which of these models satisfy Assumptions 1 and 2, and gives references to publications in media economics that employ these models.

First, Hotelling model of horizontally differentiated goods, enriched by a vertical quality differentiation. In these models, there are two competing outlets. When all market shares are positive,

$$s_i(u_i, u_j) = \frac{1}{2} + \frac{u_i - u_j}{2\tau},$$

where $\tau > 0$ is a parameter for the degree of product differentiation. This specification is used (with $u_i = v_i - p_i$ where p_i is the price) for example, in (Armstrong & Weeds, 2007a, 2007b; Bisceglia, 2023; D'Annunzio, 2017; Gonzalez-Maestre & Martínez-Sánchez, 2015; Li et al., 2023; Li & Zhang, 2016; Liu et al., 2004; Stennek, 2014). Generalizations to more than two competing outlets include the Spokes model (Chen & Riordan, 2007) used by (Germano & Meier, 2013), and the frequently used Salop circle model (used by (Kerkhof & Münster, 2015) to study commercial media bias²⁴). Assuming all market shares to be strictly positive, these models satisfy Assumptions 1 and 2; in particular, s_i has constant differences in (u_i, u_{-i}) .

Second, representative consumer models with a quadratic utility function are used in media economics by several papers, including (Dewenter et al., 2011; Godes et al., 2009; Kind et al., 2007, 2009, 2016; Motta & Polo, 2003). The model can allow for firm specific demand intercepts (see (Choné & Linnemer, 2020)) and in this way be used to study

competition in qualities (as in (Banker et al., 1998) and (Motta & Polo, 2003)). The resulting audience demand is

$$s_i(u_i, u_{-i}) = a_i + b_i u_i - \sum_{j \neq i} b_{ij} u_j,$$

where $a_i, b_i > 0$ and $b_{ij} \geq 0$. In this class of models, Assumptions 1 and 2 hold; in particular, s_i has constant differences in (u_i, u_{-i}) .

A third type of model of audience size is the random utility model of discrete choice. The utility of choosing outlet i is composed of $u_i + \varepsilon_i$, the ε_i are assumed to be i.i.d. distributed, and the consumer chooses the outlet offering the highest utility. Assuming the ε_i to be Gumbel (or extreme value type I) distributed, this results in the logit model

$$s_i = \frac{\exp(\mu u_i)}{\sum_{j=0}^{n+m} \exp(\mu u_j)},$$

where $j = 0$ corresponds to the outside option, and $\mu > 0$ is a parameter related to the variance of the ε_i . The logit model violates Assumption 2 whenever there are $n \geq 2$ commercial outlets, but it satisfies Assumption (log 2). In addition to the logit model, the nested logit model—which is often used in empirical studies of audience demand in media economics (see (Berry & Waldfogel, 2015) for a survey)—satisfies Assumption (2 log) as well (see Appendix F). The log-separable model ((Bernstein & Federgruen, 2004), see also (Huang et al., 2013)) also satisfies Assumption (2 log) when all prices are zero.

Fourth, models of vertical quality differentiation in the tradition of (Mussa & Rosen, 1978); see for example (Roger, 2017) for an application to media economics. These models have the property that, if all goods are available for free, then all consumers choose the good with the highest quality. Therefore, the resulting audience functions for freely available media are discontinuous in quality at the highest quality of the competitors, violating our assumptions. (With positive prices, the discontinuities are smoothed out because consumers differ in their willingness to pay for quality.) While these are valuable models for pay media, we do not consider them in this paper, where our main focus is on free media.

Appendix B. Weakly Increasing Differences Without Differentiability

In the proof of Proposition 1, we assumed that the functions s_i , R_i , and c_i are differentiable in order to prove that π_i has weakly increasing differences in (v_i, v_{-i}) . In this appendix we give the proof without assuming differentiability. Consider one outlet $j \neq i$, hold all other v_k ($k \neq i, j$) constant and suppress them in the formulas to avoid notational clutter. Then

$$\pi_i(v_i, v_j) = s_i(v_i, v_j) R_i(v_i) - c_i(v_i).$$

Suppose that $v_i^h > v_i^l$ and $v_j^h > v_j^l$. Then

$$\pi_i(v_i, v_j^h) - \pi_i(v_i, v_j^l) = (s_i(v_i, v_j^h) - s_i(v_i, v_j^l)) R_i(v_i)$$

and

$$\begin{aligned} & \pi_i(v_i^h, v_j^h) - \pi_i(v_i^h, v_j^l) - (\pi_i(v_i^l, v_j^h) - \pi_i(v_i^l, v_j^l)) \\ &= (s_i(v_i^h, v_j^h) - s_i(v_i^h, v_j^l)) R_i(v_i^h) - (s_i(v_i^l, v_j^h) - s_i(v_i^l, v_j^l)) R_i(v_i^l) \\ &= (s_i(v_i^h, v_j^h) - s_i(v_i^h, v_j^l) - (s_i(v_i^l, v_j^h) - s_i(v_i^l, v_j^l))) R_i(v_i^h) \\ &+ (s_i(v_i^l, v_j^h) - s_i(v_i^l, v_j^l)) (R_i(v_i^h) - R_i(v_i^l)) \end{aligned}$$

By Assumption 2, s_i has weakly increasing differences in (v_i, v_j) , i.e.,

$$s_i(v_i^h, v_j^h) - s_i(v_i^l, v_j^h) \geq s_i(v_i^h, v_j^l) - s_i(v_i^l, v_j^l)$$

or equivalently

$$s_i(v_i^h, v_j^h) - s_i(v_i^h, v_j^l) \geq s_i(v_i^l, v_j^h) - s_i(v_i^l, v_j^l).$$

Since $R_i(v_i^h) \geq 0$, it follows that

$$(s_i(v_i^h, v_j^h) - s_i(v_i^h, v_j^l) - (s_i(v_i^l, v_j^h) - s_i(v_i^l, v_j^l)))R_i(v_i^h) \geq 0.$$

Moreover, by (2) $s_i(v_i^l, v_j^h) \leq s_i(v_i^l, v_j^l)$ and by (3), $R_i(v_i^h) \leq R_i(v_i^l)$, thus

$$(s_i(v_i^l, v_j^h) - s_i(v_i^l, v_j^l))(R_i(v_i^h) - R_i(v_i^l)) \geq 0.$$

It follows that

$$\pi_i(v_i^h, v_j^h) - \pi_i(v_i^h, v_j^l) \geq \pi_i(v_i^l, v_j^h) - \pi_i(v_i^l, v_j^l)$$

or equivalently

$$\pi_i(v_i^h, v_j^h) - \pi_i(v_i^l, v_j^h) \geq \pi_i(v_i^h, v_j^l) - \pi_i(v_i^l, v_j^l)$$

i.e., π_i has weakly increasing differences in (v_i^h, v_j^h) .

Appendix C. Pay Media and Investigative Reporting: An Example

In this appendix we consider an example of a pay media outlet in the case of investigating facts. Consider a Hotelling duopoly. Outlet 1 is a pay media outlet. We investigate the comparative statics of the profit maximizing choices of outlet 1 with respect to u_2 ; for this exercise it does not matter whether outlet 2 is another commercial (pay or freely available) media outlet or a PSM.

Example A1. Suppose that $V_1 = [0, \bar{v}]$ with $0 < \bar{v} < 1/\beta$, $c_1(v_1) = kv_1^2/2$ where $k > 0$ is a parameter, and $R_1(v_1) = 1 - \beta v_1$ with $0 < \beta < 1$. The total audience has a fixed size normalized to 1. The market share of outlet 1 is given by the Hotelling specification

$$s_1(v_1, p_1, u_2) = \begin{cases} 0, & \text{if } \frac{1}{2} + \frac{v_1 - p_1 - u_2}{2\tau} \leq 0, \\ \frac{1}{2} + \frac{v_1 - p_1 - u_2}{2\tau}, & \text{if } 0 < \frac{1}{2} + \frac{v_1 - p_1 - u_2}{2\tau} < 1, \\ 1, & \text{otherwise,} \end{cases}$$

where $\tau > 0$ is a parameter for the degree of product differentiation. The profit of outlet 1 is

$$\pi_1(v_1, p_1, u_2) = s_1(v_1, p_1, u_2)(R_1(v_1) + p_1) - \frac{kv_1^2}{2}.$$

Assume that

$$4k\tau > (1 - \beta)^2 \quad (\text{A1})$$

in order that π_1 is strictly concave in (v_1, p_1) in the relevant range.

Note that in this example k has to be sufficiently high for the second order condition to hold, hence the case of withholding facts is not a limit case of this example. In order to have a unified treatment of the cases of investigating facts (where $k > 0$) and withholding

facts (where $k = 0$), we will also comment on the case where inequality A1 is reversed in Appendix C.2.

Before we proceed, we point out that Example A1 is strategically equivalent to a model where u_1 is strictly concave in quality, and quality has linear costs. To see this, think of outlet 1 choosing $w_1 := kv_1/2$. Then $u_1 = v_1 - p_1 = \sqrt{2w_1/k} - p_1$ is strictly concave in w_1 , and w_1 has constant marginal costs of 1. A model with linear costs and concave consumer utility could result, for example, if quality represents the number of realizations from a noisy signal acquired by media firms, or the time spend on investigation before reporting. The concavity of consumer utility then represents decreasing marginal utility of new signal realizations or more time spend on investigating the same issue.

Appendix C.1. Analysis of Example A1

Remark A1. In Example A1, suppose that the profit maximization problem of 1 has an interior solution where $0 < v_1 < \bar{v}$, $p_1 > 0$, $R_1 > 0$ and $0 < s_1 < 1$.²⁵ Then v_1 and p_1 are strictly decreasing in u_2 . Moreover, $u_1 = v_1 - p_1$ is strictly increasing in u_2 if $2k\tau > (1 - \beta)^2$, and strictly decreasing if $2k\tau < (1 - \beta)^2$.

Proof. In the relevant range,

$$\pi_1(v_1, p_1, u_2) = \left(\frac{1}{2} + \frac{v_1 - p_1 - u_2}{2\tau} \right) (1 - \beta v_1 + p_1) - \frac{kv_1^2}{2}.$$

The partial derivatives are

$$\begin{aligned} \frac{\partial \pi_1}{\partial p_1} &= -\frac{1}{2\tau} (1 - \beta v_1 + p_1) + \frac{1}{2} + \frac{v_1 - p_1 - u_2}{2\tau}, \\ \frac{\partial \pi_1}{\partial v_1} &= \frac{1}{2\tau} (1 - \beta v_1 + p_1) - \beta \left(\frac{1}{2} + \frac{v_1 - p_1 - u_2}{2\tau} \right) - kv_1. \end{aligned}$$

Moreover,

$$\begin{aligned} \frac{\partial^2 \pi_1}{\partial p_1^2} &= -\frac{1}{\tau} < 0, \\ \frac{\partial^2 \pi_1}{\partial v_1^2} &= -\frac{\beta}{\tau} - k < 0, \\ \frac{\partial^2 \pi_1}{\partial p_1 \partial v_1} &= \frac{1 + \beta}{2\tau}. \end{aligned}$$

Hence the determinant of the Hessian is

$$\frac{1}{\tau} \left(\frac{\beta}{\tau} + k \right) - \left(\frac{1 + \beta}{2\tau} \right)^2 > 0$$

iff $4k\tau > (1 - \beta)^2$. This shows π_1 is strictly concave in the relevant range if inequality (A1) holds.

The first order conditions for an interior solution are

$$\begin{aligned} \frac{1}{2\tau} (1 - \beta v_1 + p_1) &= \frac{1}{2} + \frac{v_1 - p_1 - u_2}{2\tau}, \\ \frac{1}{2\tau} (1 - \beta v_1 + p_1) &= \beta \left(\frac{1}{2} + \frac{v_1 - p_1 - u_2}{2\tau} \right) + kv_1. \end{aligned}$$

Solving the first order conditions gives

$$v_1^*(u_2) = \frac{(\tau + 1 - u_2)(1 - \beta)}{4k\tau - (1 - \beta)^2},$$

$$p_1^*(u_2) = \frac{(\beta(1 - \beta) + 2k\tau)(\tau - u_2) + 1 - \beta - 2k\tau}{4k\tau - (1 - \beta)^2}.$$

Differentiate

$$\frac{\partial v_1^*(u_2)}{\partial u_2} = -\frac{1 - \beta}{4k\tau - (1 - \beta)^2} < 0,$$

$$\frac{\partial p_1^*(u_2)}{\partial u_2} = -\frac{\beta(1 - \beta) + 2k\tau}{4k\tau - (1 - \beta)^2} < 0.$$

Moreover, from $u_1^*(u_2) = v_1^*(u_2) - p_1^*(u_2)$,

$$\frac{\partial u_1^*(u_2)}{\partial u_2} = \frac{2k\tau - (1 - \beta)^2}{4k\tau - (1 - \beta)^2}.$$

Therefore, $u_1^*(u_2)$ is strictly increasing in u_2 if $2k\tau > (1 - \beta)^2$, and $u_1^*(u_2)$ is strictly decreasing in u_2 if $2k\tau < (1 - \beta)^2$.

It remains to check under which parameter constellations an interior solution exists. Note that $v_1^*(u_2) > 0$ iff

$$\tau + 1 > u_2, \quad (\text{A2})$$

and $p_1^*(u_2) > 0$ iff

$$\tau + \frac{1 - \beta - 2k\tau}{(\beta(1 - \beta) + 2k\tau)} > u_2. \quad (\text{A3})$$

Note that

$$\frac{1 - \beta - 2k\tau}{(\beta(1 - \beta) + 2k\tau)} < 1$$

by inequality (A1). Thus inequality (A2) is implied by inequality (A3).

We turn to advertising revenue next. Note that

$$R_1(v_1^*(u_2)) = 1 - \beta \frac{(\tau + 1 - u_2)(1 - \beta)}{4k\tau - (1 - \beta)^2}$$

is strictly positive iff

$$u_2 > \tau + 1 - \frac{4k\tau - (1 - \beta)^2}{\beta(1 - \beta)}. \quad (\text{A4})$$

Inequalities (A3) and (A4) hold simultaneously iff

$$\tau + \frac{1 - \beta - 2k\tau}{(\beta(1 - \beta) + 2k\tau)} > u_2 > \tau + 1 - \frac{4k\tau - (1 - \beta)^2}{\beta(1 - \beta)}. \quad (\text{A5})$$

By inequality (A1), the right hand side is strictly smaller than the left hand side; therefore (A5) is satisfied in a non-empty open set of values for u_2 .

The requirement $v_1 < \bar{v}$ is satisfied whenever \bar{v} is sufficiently high.

Finally, we need to make sure that $0 < s_1(u_1^*(u_2), u_2) < 1$. This is the case iff

$$0 < \frac{1}{2} + \frac{v_1^*(u_2) - p_1^*(u_2) - u_2}{2\tau} < 1,$$

or equivalently

$$-\tau < v_1^*(u_2) - p_1^*(u_2) - u_2 < \tau.$$

We have

$$\begin{aligned} & v_1^*(u_2) - p_1^*(u_2) - u_2 \\ &= \frac{(\tau + 1 - u_2)(1 - \beta)}{4k\tau - (1 - \beta)^2} - \frac{(\beta(1 - \beta) + 2k\tau)(\tau - u_2) + 1 - \beta - 2k\tau}{4k\tau - (1 - \beta)^2} - u_2 \\ &= \tau \frac{2k - 2k\tau + (1 - \beta)^2 - 2ku_2}{4k\tau - (1 - \beta)^2}. \end{aligned}$$

Thus $0 < s_1(u_1^*(u_2), u_2) < 1$ iff

$$-1 < \frac{2k - 2k\tau + (1 - \beta)^2 - 2ku_2}{4k\tau - (1 - \beta)^2} < 1,$$

or equivalently

$$-(4k\tau - (1 - \beta)^2) < 2k - 2k\tau + (1 - \beta)^2 - 2ku_2 < 4k\tau - (1 - \beta)^2. \quad (\text{A6})$$

The expression in the middle is a strictly decreasing function of u_2 .

Since $u_2 < \tau + 1$ by (A2),

$$\begin{aligned} 2k - 2k\tau + (1 - \beta)^2 - 2ku_2 &> 2k - 2k\tau + (1 - \beta)^2 - 2k(\tau + 1) \\ &= -(4k\tau - (1 - \beta)^2), \end{aligned}$$

thus the first inequality in (A6) holds.

Similarly, by (A4),

$$\begin{aligned} & 2k - 2k\tau + (1 - \beta)^2 - 2ku_2 \\ &< 2k - 2k\tau + (1 - \beta)^2 - 2k\left(\tau + 1 - \frac{4k\tau - (1 - \beta)^2}{\beta(1 - \beta)}\right) \\ &= \frac{1}{\beta(1 - \beta)}(\beta^2 - \beta + 2k)(-\beta^2 + 2\beta + 4k\tau - 1). \end{aligned}$$

Therefore, a sufficient condition for the second inequality in (A6) is that

$$\begin{aligned} & (4k\tau - (1 - \beta)^2) - \left(\frac{1}{\beta(1 - \beta)}(\beta^2 - \beta + 2k)(-\beta^2 + 2\beta + 4k\tau - 1)\right) \\ &= 2(\beta(1 - \beta) - k) \frac{4k\tau - (1 - \beta)^2}{\beta(1 - \beta)} > 0, \end{aligned}$$

which is true iff

$$\beta(1 - \beta) > k. \quad (\text{A7})$$

We have established that the problem has an interior solution under the conditions (A7), (A1), and (A5), which we repeat here for convenience:

$$\begin{aligned} & \beta(1 - \beta) > k, \\ & 4k\tau > (1 - \beta)^2, \\ & \tau + \frac{1 - \beta - 2k\tau}{(\beta(1 - \beta) + 2k\tau)} > u_2 > \tau + 1 - \frac{4k\tau - (1 - \beta)^2}{\beta(1 - \beta)}. \end{aligned}$$

To see they can be satisfied simultaneously, first choose β and k such that the first line holds. Then choose τ such that the second line holds; note that depending on how you choose τ , either $2k\tau > (1 - \beta)^2$ or $2k\tau < (1 - \beta)^2$. Finally, choose u_2 for the last line. \square

A numerical example that satisfies all the constraints may be reassuring. Let $\beta = 0.5$, $\tau = 1.25$, and $u_2 = 1.5$. For $k = 0.11$, $2k\tau = 2 * 0.11 * 1.25 = 0.275 > (1 - \beta)^2 = 0.25$ and $u_1^*(u_2)$ is strictly increasing in u_2 . For $k = 0.09$, $2k\tau = 2 * 0.09 * 1.25 = 0.225 < 0.25 < 4k\tau = 0.45$, and $u_1^*(u_2)$ is strictly decreasing.

Within our parameter restrictions, $u_1^*(u_2)$ is strictly increasing in u_2 if k is large. An economic intuition is that the marginal costs of v_1 are rapidly increasing if k is large, and hence then the falling price dominates the decrease in program quality. To give more details, recall that $v_1^*(u_2)$ and $p_1^*(u_2)$ are strictly decreasing in u_2 . If k is large, the effect of u_2 on $v_1^*(u_2)$ becomes less important (smaller in absolute value):

$$\frac{\partial}{\partial k} \frac{\partial v_1^*(u_2)}{\partial u_2} = \frac{4\tau(1 - \beta)}{(4k\tau - (1 - \beta)^2)^2} > 0.$$

On the other hand, the effect of u_2 on $p_1^*(u_2)$ also becomes less important:

$$\begin{aligned} \frac{\partial}{\partial k} \frac{\partial p_1^*(u_2)}{\partial u_2} &= \frac{\partial}{\partial k} \left(-\frac{\beta(1 - \beta) + 2k\tau}{4k\tau - (1 - \beta)^2} \right) \\ &= \frac{2\tau(1 - \beta^2)}{(4k\tau - (1 - \beta)^2)^2} > 0 \end{aligned}$$

But note that $4\tau(1 - \beta) - 2\tau(1 - \beta^2) = 2\tau(1 - \beta)^2 > 0$, thus

$$\frac{\partial}{\partial k} \frac{\partial v_1^*(u_2)}{\partial u_2} > \frac{\partial}{\partial k} \frac{\partial p_1^*(u_2)}{\partial u_2}.$$

That is, if k increases, the change of $v_1^*(u_2)$ in u_2 is vanishing quicker than the change of $p_1^*(u_2)$ in u_2 . For large enough k , $u_1^*(u_2)$ increases in u_2 because the falling price overcompensates for the falling quality.

Appendix C.2. The Case Where Inequality (A1) Is Reversed

This subsection considers the case where $4k\tau < (1 - \beta)^2$. This is of particular interest in order to have a unified treatment with the case of hiding information where $k = 0$. In this case, the profit maximization problem of 1 cannot have an interior solution where $0 < v_1 < \bar{v}$, $p_1 > 0$, $R_1 > 0$ and $0 < s_1 < 1$, since the necessary second order condition for a maximum would be violated.

We continue to study pay media, i.e., we will focus on constellations where $p_1 > 0$ in the solution to the profit maximization problem; as we will show, a sufficient condition for this to be the case is that the competitor's quality is not too high.

Remark A2. Consider Example A1 but suppose that $4k\tau < (1 - \beta)^2$, and the solution to the profit maximization problem of firm 1 involves $p_1 > 0$, $R_1 > 0$ and $0 < s_1 < 1$.²⁶ If $u_2 < \tau - 1$, then $v_1 = \bar{v}$ and p_1 is strictly decreasing in u_2 .

Proof. In the relevant range,

$$\pi_1(v_1, p_1) = \left(\frac{1}{2} + \frac{v_1 - p_1 - u_2}{2\tau} \right) (1 - \beta v_1 + p_1) - \frac{kv_1^2}{2}.$$

Since in the profit maximum $p_1 > 0$, the following first order condition has to hold:

$$\frac{\partial \pi_1(v_1, p_1)}{\partial p_1} = -\frac{1}{2\tau}(1 - \beta v_1 + p_1) + \frac{1}{2} + \frac{v_1 - p_1 - u_2}{2\tau} = 0.$$

Solving for p_1 gives

$$p_1 = p_1^*(v_1) := \frac{1}{2}(\tau - u_2 + (1 + \beta)v_1 - 1).$$

Note that the assumption that $u_2 < \tau - 1$ implies that $p_1^*(v_1) > 0$ for all $v_1 \in [0, \bar{v}]$.
Define

$$\begin{aligned} \pi_1(v_1) &:= \pi_1(v_1, p_1^*(v_1)) \\ &= \left(\frac{1}{2} + \frac{v_1 - \frac{1}{2}(\tau - u_2 + (1 + \beta)v_1 - 1) - u_2}{2\tau} \right) \left(1 - \beta v_1 + \frac{1}{2}(\tau - u_2 + (1 + \beta)v_1 - 1) \right) \\ &\quad - \frac{kv_1^2}{2}. \end{aligned}$$

Differentiating $\pi_1(v_1)$ shows that

$$\pi_1'(v_1) = \frac{1}{4\tau}(1 - \beta)(\tau - u_2 + v_1 - \beta v_1 + 1) - kv_1.$$

Evaluated at $v_1 = 0$, this is

$$\pi_1'(0) = \frac{1}{4\tau}(1 - \beta)(\tau - u_2 + 1) > 0.$$

Moreover,

$$\pi_1''(v_1) = \frac{1}{4\tau}(1 - \beta)^2 - k > 0.$$

It follows that in the profit maximum, $v_1 = \bar{v}$, and

$$p_1 = \frac{1}{2}(\tau - u_2 + (1 + \beta)\bar{v} - 1),$$

which is strictly decreasing in u_2 .

It remains to check there are parameter constellations where $R_1 > 0$ and $0 < s_1 < 1$ in this solution. Note $R_1 = 1 - \beta\bar{v} > 0$ since $\bar{v} < 1/\beta$. Moreover, the market share of outlet 1 is

$$\begin{aligned} &\left(\frac{1}{2} + \frac{\bar{v} - \frac{1}{2}(\tau - u_2 + (1 + \beta)\bar{v} - 1) - u_2}{2\tau} \right) \\ &= \frac{1}{4\tau}(\bar{v}(1 - \beta) + \tau - u_2 + 1) > 0 \end{aligned}$$

since by assumption $u_2 < \tau - 1$. Finally, the market share of outlet 1 is smaller than 100% iff

$$\bar{v}(1 - \beta) - u_2 + 1 < 3\tau,$$

which is true if τ is sufficiently high. \square

To relate Remark A2 to Lemma 2, note that here $R_1'(v_1) = -\beta > -1$ for all v_1 , therefore choosing the highest possible quality is optimal.

Appendix C.3. Summary

To summarize, we found that in all cases considered in this Appendix, the price p_1 is strictly decreasing in u_2 . Moreover, the quality v_1 is strictly decreasing in u_2 unless k is small in which case v_1 is constant in u_2 . Finally, the utility of consuming outlet 1, $u_1 = v_1 - p_1$, is strictly increasing in u_2 if k is small (see Remark A2) or k is large, but u_1 is strictly decreasing in u_2 if k is in an intermediate range (see Example A1). These predictions could, in principle, be tested empirically.

Appendix D. Multidimensional Strategy Spaces

This appendix provides a formal analysis of multidimensional strategy spaces. As in the main text, suppose that outlet i reports about k_i topics, and let $v_{i,k}$ denote program quality of topic k . We assume that $v_{i,k} \in [0, v_{i,k}^{high}]$ with $v_{i,k}^{high} > 0$. Outlet i chooses a vector $v_i \in V_i = \times_{k=1}^{k_i} [0, v_{i,k}^{high}]$. Consumer utility from consuming outlet i is $u_i = f_i(v_i)$, where f_i is continuous, strictly increasing with respect to each argument $v_{i,k}$, and satisfies $f_i(0) = 0$. Advertising revenue per consumer is $R_i(v_i)$, where $R_i : V_i \rightarrow \mathbb{R}_+$ is positive, continuous, weakly decreasing in each argument $v_{i,k}$, and independent of v_{-i} . The profit of $i \in C$ is $\pi_i = s_i(u_i, u_{-i})R_i(v_i) - c_i(v_i)$. Turning to the PSM, suppose that $i \in P$ chooses $v_i \in V_i(b_i)$ to maximize $u_i = f_i(v_i)$ subject to $c_i(v_i) \leq b_i$. As above, a higher budget may enlarge the feasible set V_i .

Appendix D.1. Withholding Facts

In the case of withholding facts, there are no direct costs of raising program quality. Thus, $c_i(v_i) = 0$ for all $i \in C$. We show that our main results generalize.

In our analysis, we make use of the fact that if the qualities v_i of a commercial outlet i maximize its profit and generate utility u_i for a consumer, then v_i must maximize advertising revenue per consumer R_i subject to the constraint that the utility for the consumer is at least u_i . (If not, then there exists a feasible \hat{v}_i that generates weakly higher utility, hence weakly higher demand, and at the same time generates strictly higher advertising revenue R_i , contradicting the optimality of the v_i .) We can therefore decompose the problem of commercial media outlet i into two steps: The first step asks which v_i maximizes advertising revenue subject to the constraint that the utility of the consumer is at least equal to some given u_i . The second stage then optimizes over u_i .²⁷

Step 1

In the first step, the vector of program qualities is chosen to maximize advertising revenue per consumer, subject to the constraint that the utility of the consumer is at least equal to some given u_i . The maximal value of advertising revenue under this constraint is

$$R_i^*(u_i) = \max_{v_i \in V_i} \{R_i(v_i) | f_i(v_i) \geq u_i\}.$$

We show that R_i^* has all the features assumed about R_i in our main model (see Assumption 3): R_i^* is positive, continuous, weakly decreasing in u_i and independent of u_{-i} .

First, R_i^* is positive since R_i is positive by assumption.

Second, we use the Maximum Theorem to show that R_i^* is continuous. R_i is continuous by assumption. Let

$$U_i := \{u_i \in \mathbb{R}_+ | \exists v_i \in V_i : f_i(v_i) \geq u_i\}.$$

Since $0 \in V_i$ and $f_i(0) = 0$, $0 \in U_i$. Moreover, since V_i is compact and f_i is continuous, by the Weierstrass Theorem a maximum achievable utility exists, thus $U_i = [0, \max_{v_i \in V_i} f_i(v_i)]$.

To prove that R_i^* is continuous, it remains to show that the constraint correspondence, which gives for any $u_i \in U_i$ the set of program qualities that achieve utility at least equal u_i , is continuous. Let $g_i : U_i \rightarrow V_i$, $g_i(u_i) = \{v_i \in V_i \mid f_i(v_i) \geq u_i\}$, denote the constraint correspondence. The range of g_i is V_i , which is compact by assumption. Moreover, g_i is upper hemicontinuous by continuity of f_i .²⁸

We now show that g_i is lower hemicontinuous, using a standard argument for establishing continuity of the expenditure function via the Maximum Theorem with a minor modification to take care of the upper bounds on the feasible qualities. The constraint correspondence g_i is lower hemicontinuous if, for all sequences $u_i^m \rightarrow u_i$ such that $u_i^m \in U_i$ and $u_i \in U_i$, and for all $v_i \in g_i(u_i)$, there exists a sequence $v_i^m \rightarrow v_i$ such that $v_i^m \in g_i(u_i^m)$ for all m sufficiently high. So suppose that $u_i^m \rightarrow u_i$ such that $u_i^m \in U_i$ and $u_i \in U_i$. Moreover, suppose that $v_i \in g_i(u_i)$. We want to show that there exists a sequence v_i^n , with $v_i^n \in V_i$ for all n , such that $v_i^n \rightarrow v_i$ and $v_i^n \in g_i(u_i^n)$ (i.e., $v_i \in V_i$ and $f_i(v_i^n) \geq u_i^n$) for all n .

If v_i containst the highest possible quality in each dimension, i.e., $v_{i,k} = v_{i,k}^{high}$ for all quality dimensions $k = 1, \dots, k_i$, then $f_i(v_i) = \max U_i$. In this case, the constant series $v_i^n = v_i$ satisfies $f_i(v_i^n) \geq u_i^n$, and there is nothing left to prove. If $v_{i,k} < v_{i,k}^{high}$ for at least one quality dimension k , then because f_i is strictly increasing with respect to $v_{i,k}$, for each m there exists $y_i^m \in V_i$ such that $f_i(y_i^m) > f_i(v_i)$ and $\|y_i^m - v_i\| \leq \frac{1}{m}$. Clearly $y_i^m \rightarrow v_i$. By standard arguments (see (Kreps, 2013), p. 237), the sequence y_i^m can be used to define a sequence v_i^n with the required properties. This completes the proof that g_i is lower hemicontinuous.

Since g_i is both upper and lower hemicontinuous, it is continuous. By the Maximum Theorem, it follows that the function R_i^* is continuous.

Third, R_i^* is weakly decreasing in u_i . To see this, suppose to the contrary that $u_i^1 \geq u_i^0$ but $R_i^*(u_i^1) > R_i^*(u_i^0)$. Then there exists $v_i^1 \in V_i$ such that $f_i(v_i^1) \geq u_i^1$ and $R_i^*(u_i^1) = R_i(v_i^1)$. But since $u_i^1 \geq u_i^0$, it is also true that $f_i(v_i^1) \geq u_i^0$, and therefore

$$R_i^*(u_i^0) = \max_{v_i \in V_i} \{R_i(v_i) \mid f_i(v_i) \geq u_i^0\} \geq R_i(v_i^1) = R_i^*(u_i^1),$$

contradicting the assumption that $R_i^*(u_i^1) > R_i^*(u_i^0)$.

Fourth, R_i^* is obviously independent of v_{-i} .

Step 2

The second step then optimizes by choosing $u_i \in U_i$. Note that $U_i = [0, \max_{v_i \in V_i} f_i(v_i)]$ has the properties assumed about the choice set V_i in our main model, i.e., U_i is compact and contains zero.

This two-step procedure allows us to consider the interaction between the commercial outlets as a game where each outlet $i \in C$ has one decision variable u_i , choice set U_i , and payoff function

$$\pi_i^*(u_i, u_{-i}) = s_i(u_i, u_{-i})R_i^*(u_i).$$

From here, the analysis is as in our main model above. In particular, it follows that if the equilibrium of the game $\pi_i^*(u_i, u_{-i})$ is unique,²⁹ an increase of the budget of a PSM weakly increases the utilities offered by all commercial outlets. A sufficient condition for the uniqueness of the equilibrium of the game $\pi_i^*(u_i, u_{-i})$ can be given from the contraction approach.

Appendix D.2. Investigating Facts: Conditions for a Supermodular Game

Consider next the case of investigating facts. Trivially, as long as the game remains supermodular, our results generalize. A relevant concern is, however, whether the profit of an outlet will be supermodular in its own choice variables.

We illustrate this with a two-dimensional case inspired by (Germano & Meier, 2013). Instead of denoting the choice variable of outlet i by (v_{i1}, v_{i2}) , we denote it by (v_i, y_i) to avoid notational clutter, slightly abusing notation. Suppose that outlet i chooses program quality $v_i \in V_i \subseteq \mathbb{R}_+$ and general quality $y_i \in Y_i \subseteq \mathbb{R}_+$, where V_i and Y_i are compact, convex, and contain 0. Consumer utility from consuming outlet i is $u_i = f_i(v_i, y_i)$, where f_i is a continuous and strictly increasing function with $f_i(0, 0) = 0$. The profit of $i \in C$ is

$$\pi_i = s_i(u_i, u_{-i})R_i(v_i) - c_i(v_i, y_i).$$

We assume π_i is smooth.

PSM $i \in P$ maximizes $u_i = f_i(v_i, y_i)$ subject to $c_i(v_i, y_i) \leq b_i$ by choosing $v_i \in V_i$ and $y_i \in Y_i$. As above, a higher budget may enlarge the feasible sets V_i and Y_i .

First, we show that π_i has weakly increasing differences in $((v_i, y_i), (v_{-i}, y_{-i}))$:

$$\begin{aligned} \frac{\partial \pi_i}{\partial v_i} &= \frac{\partial s_i(u_i, u_{-i})}{\partial u_i} \frac{\partial f_i(v_i, y_i)}{\partial v_i} R_i(v_i) + s_i(u_i, u_{-i}) R'_i(v_i) - \frac{\partial c_i(v_i, y_i)}{\partial v_i}, \\ \frac{\partial \pi_i}{\partial y_i} &= \frac{\partial s_i(u_i, u_{-i})}{\partial u_i} \frac{\partial f_i(v_i, y_i)}{\partial y_i} R_i(v_i) - \frac{\partial c_i(v_i, y_i)}{\partial y_i}. \end{aligned}$$

Note that for all $j \neq i$ and $x_j \in \{v_j, y_j\}$,

$$\frac{\partial}{\partial x_j} \frac{\partial \pi_i}{\partial v_i} = \left(\frac{\partial^2 s_i(u_i, u_{-i})}{\partial u_j \partial u_i} \frac{\partial f_i(v_i, y_i)}{\partial v_i} R_i(v_i) + \frac{\partial s_i(u_i, u_{-i})}{\partial u_j} R'_i(v_i) \right) \frac{\partial f_j(v_j, y_j)}{\partial x_j} \geq 0$$

and

$$\frac{\partial}{\partial x_j} \frac{\partial \pi_i}{\partial y_i} = \frac{\partial^2 s_i(f_i(v_i, y_i), u_{-i})}{\partial u_j \partial u_i} \frac{\partial f_j(v_j, y_j)}{\partial x_j} \frac{\partial f_i(v_i, y_i)}{\partial y_i} R_i(v_i) \geq 0,$$

so π_i has weakly increasing differences in $((v_i, y_i), (v_{-i}, y_{-i}))$.

For a supermodular game, however, π_i also needs to be supermodular in (v_i, y_i) . To study when this is the case, calculate the cross-partial

$$\begin{aligned} \frac{\partial^2 \pi_i}{\partial y_i \partial v_i} &= \frac{\partial^2 s_i(u_i, u_{-i})}{\partial u_i^2} \frac{\partial f_i(v_i, y_i)}{\partial y_i} \frac{\partial f_i(v_i, y_i)}{\partial v_i} R_i(v_i) + \frac{\partial s_i(u_i, u_{-i})}{\partial u_i} \frac{\partial^2 f_i(v_i, y_i)}{\partial v_i \partial y_i} R_i(v_i) \\ &\quad + \frac{\partial s_i(u_i, u_{-i})}{\partial u_i} \frac{\partial f_i(v_i, y_i)}{\partial y_i} R'_i(v_i) - \frac{\partial^2 c_i(v_i, y_i)}{\partial y_i \partial v_i}. \end{aligned}$$

Thus π_i will be supermodular in (v_i, y_i) if there are pronounced economies of scope in producing (v_i, y_i) so that $\frac{\partial^2 c_i(v_i, y_i)}{\partial y_i \partial v_i}$ is sufficiently negative, or if v_i and y_i are strong complements for the consumers so that $\frac{\partial^2 f_i(v_i, y_i)}{\partial v_i \partial y_i}$ is sufficiently positive.

But note that the third term in the above formula for the cross-partial of π_i ,

$$\frac{\partial s_i(u_i, u_{-i})}{\partial u_i} \frac{\partial f_i(v_i, y_i)}{\partial y_i} R'_i(v_i),$$

is negative. Moreover the first term,

$$\frac{\partial^2 s_i(u_i, u_{-i})}{\partial u_i^2} \frac{\partial f_i(v_i, y_i)}{\partial y_i} \frac{\partial f_i(v_i, y_i)}{\partial v_i} R_i(v_i),$$

is negative if s_i is concave in u_i . Therefore, π_i will be submodular in (y_i, v_i) if there are no complementarities stemming from c_i and f_i , and s_i is concave in u_i . In this case, the effect of a higher budget for the PSM on the utility of the audience of consuming commercial media is ambiguous, as we show in an example in Appendix D.3 below.

Appendix D.3. Investigating Facts: An Example with a Submodular Profit Function

In this appendix, we show by example that π_i may be strictly submodular in the choice variables of outlet i , and that in this case the effect of PSM on the utility of consuming the product offered by outlet i is ambiguous. We consider a Hotelling duopoly. Outlet 1 is a commercial outlet. We investigate the comparative statics of the profit maximizing choices of outlet 1 with respect to u_2 ; for this exercise it does not matter whether outlet 2 is another commercial (pay or free) media outlet or a PSM.

Example A2. Consider a duopoly where outlet 1 is a commercial outlet. Suppose $V_1 = Y_1 = \mathbb{R}_+$. The audience has a fixed total size of 1. The market share of outlet 1 is given by the Hotelling demand specification

$$s_1(u_1, u_2) = \begin{cases} 0, & \text{if } \frac{1}{2} + \frac{u_1 - u_2}{2\tau} \leq 0, \\ \frac{1}{2} + \frac{u_1 - u_2}{2\tau}, & \text{if } 0 < \frac{1}{2} + \frac{u_1 - u_2}{2\tau} < 1, \\ 1, & \text{otherwise,} \end{cases}$$

for $i = 1, 2$, where $\tau > 0$ is a parameter for degree of product differentiation. Consumer utility from consuming the commercial media outlet is $u_1 = f_1(v_1, y_1) = v_1 + y_1$ and the cost function is $c_1(v_1, y_1) = ky_1^2/2$. Advertising revenue per consumer is $R_1(v_1) = \max\{1 - \beta v_1, 0\}$ where $\beta > 0$ is an exogenous parameter. We assume that

$$4k\tau > \beta \quad (\text{A8})$$

in order that the π_1 is strictly concave in (v_1, y_1) in the relevant range. Moreover, suppose that the profit maximization problem of 1 has an interior solution where $v_1 > 0$, $y_1 > 0$, $R_1 > 0$ and $0 < s_1 < 1$.³⁰

Note that in Example A2, there are no complementarities between v_1 and y_1 stemming from the cost function c_1 or the utility function f_1 . Moreover, the demand function is linear in u_1 in the relevant range. As a consequence, π_1 is submodular in (v_1, y_1) in the relevant range.

Remark A3. In Example A2, v_1 is strictly increasing in u_2 , and y_1 is strictly decreasing in u_2 . The utility offered by the commercial outlet, u_1 , is strictly increasing in u_2 if $2k\tau > \beta$, and strictly decreasing in u_2 if $2k\tau < \beta$.

Proof. In the relevant range,

$$\pi_1 = \left(\frac{1}{2} + \frac{v_1 + y_1 - u_2}{2\tau} \right) (1 - \beta v_1) - \frac{k}{2} y_1^2.$$

The first order conditions are

$$\begin{aligned} \frac{\partial \pi_1}{\partial v_1} &= \frac{1}{2\tau} (1 - \beta v_1) - \beta \left(\frac{1}{2} + \frac{v_1 + y_1 - u_2}{2\tau} \right) = 0, \\ \frac{\partial \pi_1}{\partial y_1} &= \frac{1}{2\tau} (1 - \beta v_1) - ky_1 = 0. \end{aligned}$$

The second derivatives are

$$\begin{aligned}\frac{\partial^2 \pi_1}{\partial v_1^2} &= -\frac{\beta}{\tau} < 0, \\ \frac{\partial^2 \pi_1}{\partial y_1^2} &= -k < 0, \\ \frac{\partial^2 \pi_1}{\partial v_1 \partial y_1} &= -\frac{\beta}{2\tau} < 0.\end{aligned}$$

The last inequality shows that π_1 is strictly submodular in (v_1, y_1) in the relevant range. The determinant of the Hessian matrix is

$$\frac{k\beta}{\tau} - \frac{\beta^2}{4\tau^2} = \frac{\beta}{\tau} \left(k - \frac{\beta}{4\tau} \right) > 0$$

iff $4k\tau > \beta$; i.e., π_1 is strictly concave in the relevant range if inequality (A8) holds.

Assuming an interior solution, the best reply function is

$$\begin{aligned}v_1^*(u_2) &= \frac{1}{\beta(4k\tau - \beta)} \left(-\beta + 2k\tau - 2k\beta\tau^2 + 2k\beta\tau u_2 \right), \\ y_1^*(u_2) &= \frac{1}{4k\tau - \beta} (\beta\tau - \beta u_2 + 1).\end{aligned}$$

Note that, since $4k\tau > \beta$ by assumption (A8),

$$\begin{aligned}\frac{\partial v_1^*(u_2)}{\partial u_2} &= \frac{2k\tau}{4k\tau - \beta} > 0, \\ \frac{\partial y_1^*(u_2)}{\partial u_2} &= \frac{-\beta}{4k\tau - \beta} < 0.\end{aligned}$$

The utility offered by 1 is $u_1^*(u_2) = v_1^*(u_2) + y_1^*(u_2)$. Thus

$$\frac{\partial u_1^*(u_2)}{\partial u_2} = \frac{2k\tau - \beta}{4k\tau - \beta}$$

which is strictly positive if $2k\tau > \beta$, but strictly negative if $2k\tau < \beta$.

It remains to establish conditions on the fundamentals such that the solution is interior. Note that $v_1^*(u_2) > 0$ if u_2 is sufficiently large, and $y_1^*(u_2) > 0$ when u_2 is sufficiently small. We show that there exists a non-empty open interval of values for u_2 such that both $v_1^*(u_2) > 0$ and $y_1^*(u_2) > 0$. We have $v_1^*(u_2) > 0$ iff

$$u_2 > \frac{1}{2k\beta\tau} (2k\beta\tau^2 - 2k\tau + \beta)$$

and $y_1^*(u_2) > 0$ iff

$$u_2 < \frac{\beta\tau + 1}{\beta}.$$

Moreover,

$$\frac{\beta\tau + 1}{\beta} > \frac{1}{2k\beta\tau} (2k\beta\tau^2 - 2k\tau + \beta),$$

since by inequality (A8)

$$\frac{\beta\tau + 1}{\beta} - \frac{1}{2k\beta\tau} (2k\beta\tau^2 - 2k\tau + \beta) = \frac{1}{2k\beta\tau} (4k\tau - \beta) > 0.$$

Therefore, whenever

$$\frac{1}{2k\beta\tau}(2k\beta\tau^2 - 2k\tau + \beta) < u_2 < \frac{\beta\tau + 1}{\beta}, \quad (\text{A9})$$

we have both $v_1^*(u_2) > 0$ and $y_1^*(u_2) > 0$.

We also need to make sure that $R_1(v_1^*(u_2)) > 0$ and $s_1(u_1^*(u_2), u_2) \in (0, 1)$.

$$\begin{aligned} R_1(v_1^*(u_2)) &= 1 - \frac{1}{(4k\tau - \beta)}(-\beta + 2k\tau - 2k\beta\tau^2 + 2k\beta\tau u_2) \\ &= \frac{2k\tau}{4k\tau - \beta}(\beta\tau + 1 - \beta u_2) > 0, \end{aligned}$$

which is strictly positive because $\beta\tau + 1 > \beta u_2$ by (A9).

Moreover,

$$s_1(u_1^*(u_2), u_2) = \frac{k(\beta\tau + 1 - \beta u_2)}{\beta(4k\tau - \beta)} > 0$$

by inequality (A9). It remains to check whether $s_1(u_1^*(u_2), u_2) < 1$. Note that $s_1(u_1^*(u_2), u_2)$ is strictly decreasing in u_2 . By inequality (A9),

$$\frac{k(\beta\tau + 1 - \beta u_2)}{\beta(4k\tau - \beta)} < \frac{k\left(\beta\tau + 1 - \beta\left(\frac{1}{2k\beta\tau}(2k\beta\tau^2 - 2k\tau + \beta)\right)\right)}{\beta(4k\tau - \beta)} = \frac{1}{2\beta\tau},$$

so a sufficient condition for $s_1(u_1^*(u_2), u_2) < 1$ is that

$$2\beta\tau > 1. \quad (\text{A10})$$

We have shown that the maximization problem has an interior solution if inequalities (A8)–(A10) hold, which we repeat here for convenience:

$$\begin{aligned} 2\beta\tau &> 1, \\ 4k\tau &> \beta, \\ \frac{1}{2k\beta\tau}(2k\beta\tau^2 - 2k\tau + \beta) &< u_2 < \frac{\beta\tau + 1}{\beta}. \end{aligned}$$

To see they can be simultaneously satisfied, first choose β and τ to satisfy the first inequality. Then choose k to satisfy the second inequality; note that depending on how you choose k , you can have either $2k\tau > \beta$ or $2k\tau < \beta$. Finally, choose u_2 to satisfy the third inequality. \square

A numerical example that satisfies all the constraints may be reassuring. Let $\beta = 2/3$, $\tau = 1$, and $u_2 = 2.2$. For $k = 1$, $2k\tau = 2 > \beta$ so u_1 is strictly increasing in u_2 . For $k = \frac{2}{10}$, $4k\tau = \frac{8}{10} > \frac{2}{3} = \beta > \frac{4}{10} = 2k\tau$ so u_1 is strictly decreasing.

Within our parameter restrictions, u_1 is strictly increasing in u_2 if k is sufficiently large. An economic intuition for this is that, if k is large, the marginal costs of y_1 are rapidly increasing, hence the positive impact of u_2 on v_1 overcompensates the negative impact of u_2 on y_1 . In more detail, the reaction of v_1^* and y_1^* to changes in u_2 both become smaller in absolute value when k increases:

$$\begin{aligned} \frac{\partial}{\partial k} \frac{\partial v_1^*(u_2)}{\partial u_2} &= \frac{-2\beta\tau}{(4k\tau - \beta)^2} < 0, \\ \frac{\partial}{\partial k} \frac{\partial y_1^*(u_2)}{\partial u_2} &= \frac{4\beta\tau}{(4k\tau - \beta)^2} > 0. \end{aligned}$$

But k affects the reaction of y_1^* to changes in u_2 more than it affects the reaction of v_1^* , so for large enough values of k , the reaction of v_1^* dominates the effect of u_2 on $u_1^*(u_2)$.

Appendix E. Spillover Effects of Program Qualities on Advertising Revenue of Other Media Outlets

As argued in the main text, advertising revenue of all outlets might be negatively affected when some outlets report about deficiencies of a product, hence there may be spillover effects. In this appendix we show that our results are robust when these spillover effects are small compared to the direct effect of an outlet's own program quality on its advertising revenue.

To make this precise, replace Assumption 3 by

Assumption A1. For all $i \in C$, R_i is positive, continuous, weakly decreasing in (v_i, v_{-i}) , and has weakly increasing differences in (v_i, v_{-i}) .

Note that, since R_i is weakly decreasing in v_i , weakly increasing differences here mean that advertising revenue is weakly not as severely affected by an increase in v_i when other outlets have a high program quality, which seems a reasonable assumption. We show in Appendix E.1 that, under Assumptions 1, 2, A1, and 4, a sufficient condition for Γ_b to be a parameterized supermodular game is that spillover effects are small in the sense that

$$\frac{\left| \frac{\partial R_i}{\partial v_j} \right|}{\left| \frac{\partial s_i}{\partial v_j} \right|} \leq \frac{\left| \frac{\partial R_i}{\partial v_i} \right|}{\frac{\partial s_i}{\partial v_i}}$$

for all $i \in C$ and all $j \neq i$.

Appendix E.1. Small Spillover Effects

This appendix proves the claim that under Assumptions 1, 2, A1, and 4, Γ_b is a parameterized supermodular game if spillover effects are small in the sense that

$$\frac{\left| \frac{\partial R_i}{\partial v_j} \right|}{\left| \frac{\partial s_i}{\partial v_j} \right|} \leq \frac{\left| \frac{\partial R_i}{\partial v_i} \right|}{\frac{\partial s_i}{\partial v_i}}.$$

Differentiate

$$\pi_i = s_i(v_i, v_{-i})R_i(v_i, v_{-i}) - c_i(v_i)$$

to obtain

$$\begin{aligned} \frac{\partial \pi_i}{\partial v_i} &= \frac{\partial s_i}{\partial v_i} R_i + s_i \frac{\partial R_i}{\partial v_i} - \frac{\partial c_i}{\partial v_i}, \\ \frac{\partial^2 \pi_i}{\partial v_j \partial v_i} &= \frac{\partial^2 s_i}{\partial v_j \partial v_i} R_i + \frac{\partial s_i}{\partial v_i} \frac{\partial R_i}{\partial v_j} + \frac{\partial s_i}{\partial v_j} \frac{\partial R_i}{\partial v_i} + s_i \frac{\partial^2 R_i}{\partial v_j \partial v_i}. \end{aligned}$$

We have $\frac{\partial^2 s_i}{\partial v_j \partial v_i} \geq 0$ by Assumption (2) and $\frac{\partial^2 R_i}{\partial v_j \partial v_i} \geq 0$ by Assumption A1. Therefore, $\frac{\partial^2 \pi_i}{\partial v_j \partial v_i} \geq 0$ holds if

$$\frac{\partial s_i}{\partial v_i} \frac{\partial R_i}{\partial v_j} + \frac{\partial s_i}{\partial v_j} \frac{\partial R_i}{\partial v_i} \geq 0.$$

Rearranging completes the proof.

Appendix E.2. Large Spillover Effects: An Example

In this appendix, we show by example that π_i may have constant differences in (v_i, v_{-i}) if spillover effects are large. We consider a Hotelling duopoly. Outlet 1 is a commercial outlet. We investigate the comparative statics of the profit maximizing choices of outlet 1 with respect to v_2 ; for this exercise it does not matter whether outlet 2 is another freely available commercial media outlet or a PSM.

Example A3. Suppose that $V_1 = \mathbb{R}_+$,

$$R_1(v_1, v_2) = \max\{1 - (\alpha v_1 + \beta v_2), 0\}$$

where $\alpha > 0$ and $\beta > 0$ are exogenous parameters, s_1 is given by a Hotelling specification. Moreover, suppose that the profit maximization problem of 1 has an interior solution where $v_1 > 0$, $R_1 > 0$, and $0 < s_1 < 1$.³¹

Note that R_1 in Example A3 satisfies Assumption A1.

Remark A4. Consider Example A3. If $\alpha > \beta$, then π_1 has strictly increasing differences in (v_1, v_2) in the relevant range. If $\alpha = \beta$, then π_1 has constant differences in (v_1, v_2) in the relevant range.

Proof. In the relevant range,

$$s_1(v_1, v_2) = \frac{1}{2} + \frac{v_1 - v_2}{2\tau},$$

hence

$$\frac{\partial^2 s_1(v_1, v_2)}{\partial v_2 \partial v_1} = 0$$

and

$$\frac{\partial s_1(v_1, v_2)}{\partial v_1} = -\frac{\partial s_1(v_1, v_2)}{\partial v_2} = \frac{1}{2\tau}.$$

The profit of commercial outlet 1 is

$$\pi_1(v_1, v_2) = s_1(v_1, v_2)(1 - (\alpha v_1 + \beta v_2)) - c_1(v_1).$$

Hence

$$\frac{\partial \pi_1}{\partial v_1} = \frac{1}{2\tau}(1 - (\alpha v_1 + \beta v_2)) - \alpha s_1(v_1, v_2) - \frac{\partial c_1(v_1)}{\partial v_1}$$

and

$$\frac{\partial^2 \pi_1}{\partial v_1 \partial v_2} = \frac{\alpha - \beta}{2\tau}.$$

Therefore, if $\alpha > \beta$, π_1 has strictly increasing differences in (v_1, v_2) . On the other hand, if $\alpha = \beta$, then π_1 has constant differences in (v_1, v_2) . \square

An implication of Remark A4 is that, if the cost function c_1 is strictly convex and twice differentiable, the profit maximizing program quality $v_1^*(v_2)$ is strictly increasing in v_2 if $\alpha > \beta$, and $v_1^*(v_2)$ is constant in v_2 if $\alpha = \beta$.³²

To conclude this appendix, we assume a quadratic cost function to illustrate that all the assumptions in Example A3 are consistent with each other. Suppose that $c_1(v_1) = kv_1^2/2$, $k > 0$. Then the best reply function is

$$v_1^*(v_2) = \frac{\frac{1}{2\tau} - \frac{\alpha}{2} + \frac{\alpha - \beta}{2\tau}v_2}{\left(\frac{\alpha}{\tau} + k\right)}.$$

Example A3 assumed an interior solution with $v_1 > 0$, $R_1 > 0$ and $s_1 \in (0, 1)$. To see these assumptions are consistent with each other, consider the symmetric case where both firms are commercial media and have the same cost and advertising revenue functions. In a symmetric equilibrium,

$$v_1 = v_2 = \frac{1 - \alpha\tau}{\alpha + \beta + 2k\tau} > 0$$

iff $\alpha\tau < 1$. Moreover, for $i = 1, 2$,

$$R_i(v_i) = 1 - \frac{(\alpha + \beta)(1 - \alpha\tau)}{\alpha + \beta + 2k\tau} = \frac{\tau(\alpha^2 + \alpha\beta + 2k)}{\alpha + \beta + 2k\tau} > 0.$$

Finally, by symmetry $s_1(v_1, v_2) = s_2(v_1, v_2) = 1/2$.

Appendix F. Audience Functions with Decreasing Differences, the Logit and Nested Logit Models

Appendix F.1. Audience Functions with Decreasing Differences: A Sufficient Condition for Proposition 1

Violations of Assumption 2 do not overturn our results when the elasticity of advertising revenue with respect to program quality is sufficiently high. To see this, note that the crucial inequality (1) in the proof of Proposition 1 holds if

$$\frac{|R'_i(v_i)|}{R_i(v_i)} \geq \frac{\frac{\partial^2 s_i(v_i, v_{-i})}{\partial v_j \partial v_i}}{\frac{\partial s_i(v_i, v_{-i})}{\partial v_j}}.$$

Under Assumption 2, the right hand side is negative hence the above inequality is always satisfied; when Assumption 2 is violated advertising revenue must react sufficiently strong to program quality for the inequality to hold.

Appendix F.2. The Logit Model

To illustrate, we consider a generalization of the logit model. Suppose there is mass of consumers normalized to one, and the market share of outlet i is

$$s_i(v_i, v_{-i}) = \frac{f_i(v_i)}{\sum_{j=0}^{n+m} f_j(v_j)},$$

where the functions $f_i(v_i)$ are strictly positive and strictly increasing, and v_0 is the utility of the outside option. We allow (but do not require) the functions f_i to differ across media outlets. The logit model is a special case where $f_i(v_i) = \exp(\mu v_i)$ for some exogenous parameter $\mu > 0$.

Note that this audience function satisfies Assumption 1, but in general violates Assumption 2. In particular, if there are two or more commercial media outlets, s_i cannot have increasing differences for all $i \in C$, as we show below. We also prove, however, that a sufficient condition for Γ_b to be a supermodular game is that

$$\frac{|R'_i(v_i)|}{R_i(v_i)} \geq \frac{f'_i(v_i)}{f_i(v_i)}$$

for all $i \in C$. In the logit model, this sufficient condition reduces to $|R'_i(v_i)|/R_i(v_i) \geq \mu$ for all $i \in C$.

This illustration shows that, while Assumption 2 is restrictive, decreasing differences in the demand functions do not necessarily overturn our results when advertising revenue reacts strongly on program quality.

Appendix F.3. The Logit Model Violates Assumption 2

As in the last subsection, suppose that

$$s_i(v_i, v_{-i}) = \frac{f_i(v_i)}{\sum_{j=0}^{n+m} f_j(v_j)},$$

where the functions $f_i(v_i)$ are strictly positive and strictly increasing, and v_0 is the utility of the outside option. For $k \neq i$,

$$\begin{aligned} \frac{\partial s_i}{\partial v_k} &= \frac{-f_i(v_i)f'_k(v_k)}{\left(\sum_{j=1}^{n+m} f_j(v_j)\right)^2}, \\ \frac{\partial^2 s_i}{\partial v_i \partial v_k} &= \frac{f'_k(v_k)f'_i(v_i)\left(f_i(v_i) - \sum_{j \neq i} f_j(v_j)\right)}{\left(\sum_{j=1}^{n+m} f_j(v_j)\right)^3}. \end{aligned}$$

This implies that, if there are two or more commercial outlets, s_i cannot have weakly increasing differences for all $i \in C$, so Assumption 2 is violated.

Appendix F.4. A Sufficient Condition for Proposition 1 in the Logit Model

Note that

$$\begin{aligned} \frac{\partial^2 \pi_i}{\partial v_i \partial v_k} &= \frac{f'_k(v_k)f'_i(v_i)\left(f_i(v_i) - \sum_{j \neq i} f_j(v_j)\right)}{\left(\sum_{j=1}^{n+m} f_j(v_j)\right)^3} R_i(v_i) - \frac{f_i(v_i)f'_k(v_k)}{\left(\sum_{j=1}^{n+m} f_j(v_j)\right)^2} R'_i(v_i) \\ &= \frac{f'_k(v_k)}{\left(\sum_{j=1}^{n+m} f_j(v_j)\right)^2} \left(\frac{f'_i(v_i)\left(f_i(v_i) - \sum_{j \neq i} f_j(v_j)\right)}{\left(\sum_{j=1}^{n+m} f_j(v_j)\right)} R_i(v_i) - f_i(v_i) R'_i(v_i) \right) \\ &> \frac{f'_k(v_k)}{\left(\sum_{j=1}^{n+m} f_j(v_j)\right)^2} \left(f'_i(v_i) \frac{\left(-\sum_{j \neq i} f_j(v_j)\right)}{\left(\sum_{j=1}^{n+m} f_j(v_j)\right)} R_i(v_i) - f_i(v_i) R'_i(v_i) \right) \\ &> \frac{f'_k(v_k)}{\left(\sum_{j=1}^{n+m} f_j(v_j)\right)^2} \left(-f'_i(v_i) R_i(v_i) - f_i(v_i) R'_i(v_i) \right) \end{aligned}$$

so a sufficient condition for π_i to have weakly increasing differences in (v_i, v_{-i}) is that

$$-f_i(v_i) R'_i(v_i) \geq f'_i(v_i) R_i(v_i)$$

or equivalently

$$\frac{|R'_i(v_i)|}{R_i(v_i)} \geq \frac{f'_i(v_i)}{f_i(v_i)}.$$

In the logit model, $f_i(v_i) = \exp(\mu v_i)$ and hence $f'_i(v_i)/f_i(v_i) = \mu$. Therefore, our main results hold in the logit model whenever $|R'_i(v_i)|/R_i(v_i) \geq \mu$ for all $i \in C$.

Appendix F.5. The Logit Model Satisfies Assumption (2 log)

Here we establish that the logit model satisfies Assumption (2 log). Suppose there is mass of consumers normalized to one, and the market share of outlet i is

$$s_i(v_i, v_{-i}) = \frac{f_i(v_i)}{\sum_{j=0}^{n+m} f_j(v_j)},$$

where the functions $f_i(v_i)$ are strictly positive and strictly increasing, and v_0 is the utility of the outside option. We allow (but do not require) the functions f_i to differ across media outlets. The logit model is a special case where $f_i(v_i) = \exp(\mu v_i)$ for some exogenous parameter $\mu > 0$.

Then

$$\begin{aligned} \ln s_i &= \ln(f_i(v_i)) - \ln\left(\sum_{j=0}^{n+m} f_j(v_j)\right) \\ \frac{\partial \ln s_i}{\partial v_i} &= \frac{f'_i(v_i)}{f_i(v_i)} - \frac{f'_i(v_i)}{\sum_{j=0}^{n+m} f_j(v_j)} \end{aligned}$$

for $j \neq i$, this is strictly increasing in v_j , thus

$$\frac{\partial}{\partial v_j} \frac{\partial \ln s_i}{\partial v_i} > 0.$$

Therefore, $\ln s_i$ has strictly increasing differences in (s_i, s_{-i}) .

Appendix F.6. The Nested Logit Model Satisfies Assumption (2 log)

This subsection considers the nested logit model, which is often used in empirical studies of media demand (see (Berry & Waldfogel, 2015) for a survey). Our exposition of the nested logit model follows (Berry, 1994). Firms are partitioned into groups. Suppose firm i belongs to group g . The market share of a firm i is given by $s_i = s_{i|g} s_g$, where $s_{i|g}$ is the share of firm i within its group g , and s_g is the market share of group g . Moreover,

$$s_{i|g} = \frac{\exp(\frac{u_i}{1-\sigma})}{\sum_{k \in g} \exp(\frac{u_k}{1-\sigma})}$$

and

$$s_g = \frac{\left(\sum_{k \in g} \exp(\frac{u_k}{1-\sigma})\right)^{1-\sigma}}{\sum_{g'} \left(\sum_{k \in g'} \exp(\frac{u_k}{1-\sigma})\right)^{1-\sigma}},$$

where σ is a parameter with $0 \leq \sigma < 1$.

We now show that the nested logit model satisfies Assumption (2 log).

Remark A5. In the nested logit model,

$$\frac{\partial^2}{\partial u_j \partial u_i} \ln s_i > 0.$$

Proof.

$$\begin{aligned}
 \ln s_i &= \frac{u_i}{1-\sigma} - \ln \left(\sum_{k \in g} \exp \left(\frac{u_k}{1-\sigma} \right) \right) \\
 &\quad + (1-\sigma) \ln \left(\sum_{k \in g} \exp \left(\frac{u_k}{1-\sigma} \right) \right) \\
 &\quad - \ln \left(\sum_{g'} \left(\sum_{k \in g'} \exp \left(\frac{u_k}{1-\sigma} \right) \right)^{1-\sigma} \right) \\
 &= \frac{u_i}{1-\sigma} - \sigma \ln \left(\sum_{k \in g} \exp \left(\frac{u_k}{1-\sigma} \right) \right) \\
 &\quad - \ln \left(\sum_{g'} \left(\sum_{k \in g'} \exp \left(\frac{u_k}{1-\sigma} \right) \right)^{1-\sigma} \right).
 \end{aligned}$$

Differentiate with respect to u_i , keeping in mind firm i belongs to group g

$$\begin{aligned}
 \frac{\partial \ln s_i}{\partial u_i} &= \frac{1}{1-\sigma} - \frac{\sigma}{1-\sigma} \frac{\exp \left(\frac{u_i}{1-\sigma} \right)}{\sum_{k \in g} \exp \left(\frac{u_k}{1-\sigma} \right)} \\
 &\quad - \frac{\left(\sum_{k \in g} \exp \left(\frac{u_k}{1-\sigma} \right) \right)^{-\sigma} \exp \left(\frac{u_i}{1-\sigma} \right)}{\sum_{g'} \left(\sum_{k \in g'} \exp \left(\frac{u_k}{1-\sigma} \right) \right)^{1-\sigma}}.
 \end{aligned}$$

The sign of the crosspartial $\frac{\partial^2 \ln s_i}{\partial u_j \partial u_i}$ can be determined by considering how the terms in this sum depend on u_j .

Suppose j belongs to a different group than i . Then the first two terms are independent of u_j , and the numerator of the third term is independent of u_j as well. The sign of crosspartial is equal to the sign of

$$\frac{\partial}{\partial u_j} \left(- \frac{1}{\sum_{g'} \left(\sum_{k \in g'} \exp \left(\frac{u_k}{1-\sigma} \right) \right)^{1-\sigma}} \right) > 0.$$

Suppose i and j belong to the same group g . Consider the terms one by one. The first term $1/(1-\sigma)$ does not depend on u_j . The second term is nondecreasing in u_j . Consider the third term. Since $\sigma \geq 0$, $\left(\sum_{k \in g} \exp \left(\frac{u_k}{1-\sigma} \right) \right)^{-\sigma}$ is nonincreasing in u_j . Moreover, $\sum_{g'} \left(\sum_{k \in g'} \exp \left(\frac{u_k}{1-\sigma} \right) \right)^{1-\sigma}$ is strictly increasing in u_j . This implies the third term is strictly increasing in u_j .

We conclude that $\frac{\partial^2 \ln s_i}{\partial u_j \partial u_i} > 0$. \square

Appendix G. Entry: A Hotelling Example

In this Appendix, we illustrate the our considerations on entry in a Hotelling model.

Example A4. Assume that there are at most two media outlets active in the market. Conditional on entry and assuming that $0 < s_i < 1$ and $v_i \in \left[0, \frac{1}{\beta} \right]$, the profit of a commercial outlet i is

$$\pi_i = \left(\frac{1}{2} + \frac{v_i - v_j}{2\tau} \right) (1 - \beta v_i) - F.$$

The fixed costs $F > 0$ can be saved by staying out of the market. Let $3\beta\tau > 1 > \beta\tau$ and $F < \beta\tau/2$.

We compare a mixed commercial and public media market (dual media market) with a purely commercial media market. In the dual market, there is one PSM with an exogenously given quality v_P , and one commercial outlet decides whether to enter. In a purely commercial market, there is no PSM, and up to two commercial outlets may enter. We model entry by a standard two stage game, where in the first stage entry decisions are taken, and in the second stage qualities are chosen. The solution concept is subgame perfect equilibrium.

The assumptions on parameters β , τ , and F allow us to focus on the most interesting cases. Specifically, the assumption $1 > \beta\tau$ rules out situations where the profit maximizing quality equals zero. The assumption $3\beta\tau > 1$ rules out situations where the best reply of a commercial outlet to a PSM with low quality is such that the commercial outlet has 100% market share. The assumption that $F < \beta\tau/2$ ensures that in a purely commercial media market two media outlets enter.

In the dual market, the commercial outlet will enter if the PSM's quality is not too high. Define

$$\hat{v} := \frac{1 + \beta\tau - \sqrt{8\beta\tau F}}{\beta}.$$

We will show below that \hat{v} is the relevant cutoff for the PSM's quality, below which the commercial outlet enters.

Remark A6. Consider Example A4. In a purely commercial market, both commercial media outlets enter. (i) In a dual market where the PSM's quality is $v_P < (1 - \beta\tau)/\beta$, the commercial outlet enters. The quality of the PSM and the quality of the commercial outlet are both strictly lower than the equilibrium qualities in a purely commercial duopoly. (ii) In a dual market where the PSM's quality satisfies $(1 - \beta\tau)/\beta < v_P < \hat{v}$, the private outlet enters. The qualities of the PSM and of the commercial outlet are both strictly higher than the equilibrium qualities in a purely commercial duopoly. (iii) In a dual market where the PSM's quality is $v_P > \hat{v}$, the private outlet will not enter.

Proof. We solve the game by backward induction. Suppose there are two firms in the market. Firm i is a commercial outlet. Firm j may be a commercial outlet or a PSM. The first order condition of outlet i is

$$\frac{\partial}{\partial v_i} \left(\left(\frac{1}{2} + \frac{v_i - v_j}{2\tau} \right) (1 - \beta v_i) \right) = \frac{1}{2\tau} (1 - \beta\tau - 2\beta v_i + \beta v_j) = 0.$$

The reaction function of i is

$$v_i^*(v_j) = \frac{1 - \beta\tau + \beta v_j}{2\beta} > 0$$

where the inequality follows from the assumption $1 > \beta\tau$.³³

Note that $v_i^*(v_j)$ is strictly increasing in v_j .

Consider the purely commercial market. There are two symmetric potential entrants. If both of them enter, in equilibrium of the resulting subgame $v_i^*(v_j) = v_j$ for $i, j = 1, 2$, thus

$$v_1 = v_2 = \frac{1 - \beta\tau}{\beta},$$

and profits are

$$\pi_i = \frac{1}{2} \left(1 - \beta \frac{1 - \beta\tau}{\beta} \right) - F = \frac{\beta\tau}{2} - F.$$

Since by assumption $F < \beta\tau/2$, in equilibrium both commercial firms enter. Now consider the dual market. Note that

$$\begin{aligned} R_i(v_i^*(v_P)) &= 1 - \beta \frac{1 + \beta v_P - \beta\tau}{2\beta} \\ &= \frac{1 + \beta\tau - \beta v_P}{2} \end{aligned}$$

is strictly positive iff $v_P < (1 + \beta\tau)/\beta$. In case that $v_P \geq (1 + \beta\tau)/\beta$, i cannot generate a strictly positive revenue, and hence will not enter.³⁴

For the rest of the proof, consider the case where $v_P < (1 + \beta\tau)/\beta$, unless otherwise noted. The market share of firm i is

$$\begin{aligned} s_i(v_i^*(v_P), v_P) &= \frac{1}{2} + \frac{\frac{1 + \beta v_P - \beta\tau}{2\beta} - v_P}{2\tau} \\ &= \frac{1 + \beta\tau - \beta v_P}{4\beta\tau}. \end{aligned}$$

Note this is positive when $v_P < (1 + \beta\tau)/\beta$; moreover $s_i(v_i^*(v_P), v_P)$ is smaller than one (for all $v_P \geq 0$) if $1 + \beta\tau < 4\beta\tau$ or $1 < 3\beta\tau$ which we assume to be the case. The profit of firm i is

$$\pi_i(v_i^*(v_P), v_P) = \frac{(1 + \beta\tau - \beta v_P)^2}{8\beta\tau} - F.$$

The commercial outlet will enter if

$$\frac{(1 + \beta\tau - \beta v_P)^2}{8\beta\tau} > F,$$

or equivalently

$$v_P < \hat{v} := \frac{1 + \beta\tau - \sqrt{8\beta\tau F}}{\beta}.$$

Note that the assumption $F < \beta\tau/2$ implies

$$\hat{v} = \frac{1 + \beta\tau - \sqrt{8\beta\tau F}}{\beta} > \frac{1 + \beta\tau - \sqrt{8\beta\tau \frac{\beta\tau}{2}}}{\beta} = \frac{1 - \beta\tau}{\beta}.$$

We are now in a position to complete the proof.

Case (i): $v_P < (1 - \beta\tau)/\beta$. Then $v_P < \hat{v}$, thus the commercial outlet enters. Moreover, v_P is by assumption strictly lower than the equilibrium quality in a purely commercial duopoly $(1 - \beta\tau)/\beta$. Because of the strategic complementarities, v_i is also strictly lower than the equilibrium qualities in a purely commercial duopoly.

Case (ii): $(1 - \beta\tau)/\beta < v_P < \hat{v}$. Since $v_P < \hat{v}$, the private outlet enters. Moreover, v_P is by assumption strictly higher than the equilibrium quality in a purely commercial duopoly $(1 - \beta\tau)/\beta$. Because of the strategic complementarities, v_i is also strictly higher than the equilibrium qualities in a purely commercial duopoly.

Case (iii): If $\hat{v} < v_P < (1 + \beta\tau)/\beta$, the commercial outlet will not enter because the revenue it can generate does not cover the fixed costs F . Moreover, as argued above, if $v_P \geq (1 + \beta\tau)/\beta$ the commercial outlet cannot generate any strictly positive revenue and hence will not enter. \square

Appendix H. Income Effects

Consumers in our model have to pay taxes or licence fees to cover the budgets of the PSM. These payments are independent of individual media consumption. They could,

however, affect demand via income effects. Such income effects can strengthen our main results, however, when the media are normal goods, i.e., demand increases in income.

Suppose that for $i \in C$, $s_i(v_i, v_{-i}, b)$ is weakly decreasing in b (the higher b , the lower consumers' remaining income; if media are normal goods, demand is lower). Moreover, suppose that s_i has weakly increasing differences in (v_i, b) , i.e., demand reacts more on quality differences when income is lower. The strategic complementarities between the commercial media are not affected by the income effects. For $i \in C$ and $j \in P$, consider the cross-partial

$$\begin{aligned} \frac{\partial^2 \tilde{\pi}_i}{\partial b_j \partial v_i} = & \left(\frac{\partial^2 s_i(v_i, v_{-i}, b)}{\partial v_j \partial v_i} R_i(v_i) + \frac{\partial s_i(v_i, v_{-i}, b)}{\partial v_j} R'_i(v_i) \right) \bar{v}'_j(b_j) \\ & + \frac{\partial^2 s_i(v_i, v_{-i}, b)}{\partial b_j \partial v_i} R_i(v_i) + \frac{\partial s_i(v_i, v_{-i}, b)}{\partial b_j} R'_i(v_i). \end{aligned}$$

The first line describes the effects studied in our main model above: an increase of b_j increases \bar{v}_j and this has the effects studied above (the terms in the bracket are the same as in inequality (1) in the proof of Proposition 1). The second line stems from the income effect. Note that $\frac{\partial^2 s_i(v_i, v_{-i}, b)}{\partial b_j \partial v_i} \geq 0$ because s_i has weakly increasing differences in (v_i, b) , and $\frac{\partial s_i(v_i, v_{-i}, b)}{\partial b_j} \leq 0$ because good i is normal; hence the second line is positive. This shows that income effects strengthen the strategic complementarities that drive our results.

On the other hand, PSM might lead commercial media to exit the market. Income effects can strengthen this type of crowding out: the PSM do not only offer competing products, but also lower demand for commercial media via income effects.

Appendix I. Equilibrium Uniqueness

In this Appendix, we use the contraction approach to give sufficient conditions for equilibrium uniqueness in the main model with a linear audience function s_i , and in the case of withholding information with a logit audience function. Consider the case of a linear audience function first:

Remark A7. Consider the case of a linear audience function

$$s_i(v_i, v_{-i}) = a_i + b_i v_i - \sum_{\substack{j \neq i \\ j \in C \cup P}} c_{ij} v_j.$$

Assume that

$$\begin{aligned} a_i &\geq \sum_{j \neq i} c_{ij} \max_{v_j \in V_j} v_j, \\ b_i &> 0, \\ c_{ij} &> 0 \text{ for all } j \neq i. \end{aligned}$$

Moreover, assume that R_i is smooth with $R'_i(v_i) < 0$ for all $v_i \in V_i$. A sufficient condition for a unique equilibrium is that

$$\begin{aligned} 2b_i &\geq \sum_{j \neq i} c_{ij}, \\ R''_i(v_i) &\leq 0, \text{ and } c''_i(v_i) \geq 0 \text{ for all } v_i \in V_i \end{aligned}$$

with at least one of these inequalities strict.

The assumption $a_i \geq \sum_{j \neq i} c_{ij} \max_{v_j \in V_j} v_j$ ensures that $s_i \geq 0$ for all $(v_i, v_{-i}) \in V_i \times V_{-i}$; $b_i > 0$ ensures that s_i is strictly increasing in v_i ; $c_{ij} > 0$ ensures s_i is strictly decreasing in v_j for $j \neq i$.

Proof. With the linear audience function,

$$\frac{\partial^2 s_i}{\partial v_i^2} = \frac{\partial^2 s_i}{\partial v_j \partial v_i} = 0,$$

so s_i has constant differences. It follows that $\frac{\partial^2 \pi_i}{\partial v_i \partial v_j} > 0$.

Note that

$$\frac{\partial \pi_i}{\partial v_i} = bR_i + s_i R'_i - c'_i$$

and

$$\frac{\partial^2 \pi_i}{\partial v_i^2} = 2bR'_i + s_i R''_i - c''_i < 0$$

where the strict inequality follows because $b > 0 > R'_i$ by assumption.

The sufficient condition for a contraction is

$$\frac{\partial^2 \pi_i}{\partial v_i^2} + \sum_{j \neq i} \frac{\partial^2 \pi_i}{\partial v_i \partial v_j} < 0.$$

Here,

$$\begin{aligned} & \frac{\partial^2 \pi_i}{\partial v_i^2} + \sum_{j \neq i} \frac{\partial^2 \pi_i}{\partial v_i \partial v_j} \\ &= \left(2b_i - \sum_{j \neq i} c_{ij} \right) R'_i + s_i R''_i - c''_i \end{aligned}$$

Therefore, a sufficient for uniqueness is that

$$\begin{aligned} 2b_i &\geq \sum_{j \neq i} c_{ij}, \\ R''_i(v_i) &\leq 0, \text{ and } c''_i(v_i) \geq 0 \text{ for all } v_i \in V_i \end{aligned}$$

with at least one of these inequalities strict. \square

In the Hotelling or Spokes model in the relevant range where all market shares are interior, $b_i = \sum_{j \neq i} c_{ij}$, so $2b_i > \sum_{j \neq i} c_{ij}$ holds automatically.

Remark A8. Consider the logit model

$$s_i(v_i, v_{-i}) = \frac{\exp(\mu v_i)}{\sum_{k=0}^{n+m} \exp(\mu v_k)}$$

in the case of withholding information where $c_i(v_i) = 0$ for all i . Moreover, assume that R_i is smooth, $R_i(v_i) > 0$ for all $v_i \in V_i$, and R_i is strictly log concave in v_i . Then the game Γ_b is a parameterized supermodular game. Moreover, the equilibrium is unique.

Proof. Corollary 1 implies that Γ_b is a parameterized supermodular game. Note that $\pi_i > 0$ for all (v_i, v_{-i}) . Therefore, we can think of the commercial media outlets as maximizing

$$\ln \pi_i = \ln s_i(v_i, v_{-i}) + \ln(R_i(v_i)).$$

Consider

$$\ln s_i = \mu v_i - \ln \left(\sum_j \exp(\mu v_j) \right).$$

Differentiate to obtain

$$\frac{\partial \ln s_i}{\partial v_i} = \mu - \frac{\mu \exp(\mu v_i)}{\sum_k \exp(\mu v_k)}.$$

For $j \neq i$, the term $\frac{\mu \exp(\mu v_i)}{\sum_k \exp(\mu v_k)}$ is strictly decreasing in v_j , thus

$$\frac{\partial}{\partial v_j} \frac{\partial \ln \pi_i}{\partial v_i} = \frac{\partial}{\partial v_j} \frac{\partial \ln s_i}{\partial v_i} > 0.$$

Therefore, $\ln \pi_i$ has strictly increasing differences in (v_i, v_{-i}) .

Moreover, $\ln \pi_i$ is strictly concave in v_i . Since $\ln(R_i(v_i))$ is strictly concave in v_i by assumption, this result follows from

$$\begin{aligned} \frac{\partial^2 \ln s_i}{\partial v_i^2} &= - \frac{\left(\sum_j \exp(\mu v_j) \right) \exp(\mu v_i) \mu^2 - \exp(\mu v_i) \exp(\mu v_i) \mu^2}{\left(\sum_j \exp(\mu v_j) \right)^2} \\ &= - \frac{\left(\sum_{j \neq i} \exp(\mu v_j) \right) \exp(\mu v_i) \mu^2}{\left(\sum_j \exp(\mu v_j) \right)^2} < 0. \end{aligned}$$

To prove uniqueness, we show that $\ln \pi_i$ satisfies the contraction condition

$$\frac{\partial^2 \ln \pi_i}{\partial v_i^2} + \sum_{\substack{j \neq i \\ j \neq 0}} \left| \frac{\partial^2 \ln \pi_i}{\partial v_j \partial v_i} \right| < 0.$$

Here this is equivalent to

$$\frac{\partial^2 \ln s_i}{\partial v_i^2} + \frac{\partial^2 \ln R_i}{\partial v_i^2} + \sum_{j \neq i} \frac{\partial^2 \ln s_i}{\partial v_j \partial v_i} < 0.$$

By strict log concavity of R_i , it is enough to show that

$$\frac{\partial^2 \ln s_i}{\partial v_i^2} + \sum_{j \neq i} \frac{\partial^2 \ln s_i}{\partial v_j \partial v_i} \leq 0.$$

From

$$s_i = \frac{\exp(\mu v_i)}{\sum_k \exp(\mu v_k)}$$

it follows that

$$\ln s_i = \mu v_i - \ln \left(\sum_j \exp(\mu v_j) \right)$$

and

$$\begin{aligned}\frac{\partial \ln s_i(v_i, v_{-i})}{\partial v_i} &= \mu - \frac{\mu \exp(\mu v_i)}{\sum_k \exp(\mu v_k)} \\ &= \mu - \frac{\mu}{\frac{\exp(\mu v_0)}{\exp(\mu v_i)} + \sum_{k=1}^n \frac{\exp(\mu v_k)}{\exp(\mu v_i)}}.\end{aligned}$$

Consider increasing v_i and all $v_k, k \neq i, k = 1, \dots, n + m$ at the same time by the same amount $x > 0$. This leaves $\sum_{k=1}^n \frac{\exp(\mu v_k)}{\exp(\mu v_i)}$ unaffected, and decreases $\frac{\exp(\mu v_0)}{\exp(\mu v_i)}$. Therefore

$$\frac{\partial^2 \ln s_i}{\partial v_i^2} + \sum_{j \neq i} \frac{\partial^2 \ln s_i}{\partial v_j \partial v_i} < 0.$$

□

Notes

- ¹ These correlations might, however, also be driven by unobserved confounding factors such as high preferences for television, and do not allow to infer causality. (Weeds, 2020) reviews the literature on the question whether PSM crowd out or crowd in private programming and concludes that “further research is needed in this area before firm conclusions can be drawn” ((Weeds, 2020), p. 10). Similarly, (Nielsen et al., 2016) review academic publications and studies funded by stakeholders such as government agencies and public or private media organizations. They point out that there is little research on the market impact of PSM, and conclude that “existing studies provide little evidence for a negative market impact of PSM upon domestic private sector media” (p. 17).
- ² Our reasoning generalizes to all types of media content, including television, radio, newspapers, and social media, both on- and offline. For ease of exposition, and since television broadcasting is one of the most prominent examples of PSM, we speak of “programs” and “program quality” throughout the paper.
- ³ Moreover, studies from marketing have shown that advertisers prefer light genres like comedy that put consumers in a more ad-receptive mood, whereas consumers prefer action and news. See (Ellman & Germano, 2009; Kerkhof & Münster, 2015) for extensive discussion.
- ⁴ In the appendix, we also consider demand functions with decreasing differences and income effects of taxes or license fees used to finance the PSM.
- ⁵ (Blasco et al., 2012) provide a survey on competition and commercial media bias. See also (Petrova, 2012) for an interesting discussion of complementarity between media bias and audience size.
- ⁶ To keep the model simple and focused, we implicitly assume that consumers can fully observe v_i , in contrast to forming expectations about program qualities. This implies that consumers can also observe the bias of each media outlet i . Our model can therefore not address questions related to transparency mandates requiring media disclosures about funding and advertising sources.
- ⁷ We implicitly assume that $f_i(v_i)$ captures the utility from consuming program quality v_i given the facts that consumers already have. E.g., consumers might have obtained and combined facts on the health risks of smoking from peers and other media outlets, and thus formed prior beliefs on these issues. While consumers’ utility from consuming a program of quality v_i does not explicitly depend on the overall level of information that they achieve, it is plausible to assume that $f_i(v_i)$ implicitly accounts for consumers’ prior beliefs, too. E.g., (Gentzkow & Shapiro, 2006) argue that media outlets who care about their reputation tend to provide information aligned to consumers’ prior beliefs; moreover, if consumers have access to means of verification, the media outlets’ incentives to misreport information are weakened. While an explicit formal analysis of such considerations is beyond the scope of this paper, we argue that these considerations also can induce strategic complementarities: if an outlet provides low quality, consumers are more likely to realize that the quality is low when the quality of the competing media outlets is high.
- ⁸ In this paper, unless otherwise stated, “positive” means “non-negative”, and “negative” means “non-positive”.
- ⁹ See Appendix B for the formal definition of weakly increasing differences in case that s_i is not differentiable.
- ¹⁰ The log-separable model ((Bernstein & Federgruen, 2004), see also (Huang et al., 2013)) also satisfies Assumption (2 log) when all prices are zero. We are not aware of any paper in media economics using this particular model, however.
- ¹¹ R_i can be interpreted as the outcome of all economic (inter-)actions in the advertiser market. The function can thus represent any such market, irrespective of the number of advertisers, the degree of competition in the market, and whether the advertisers’ preferences with respect to truthful reporting are aligned or not. E.g., some advertisers who provide high quality products might be indifferent about truthful reporting on behalf of the media, while other advertisers prefer biased reporting. In our model, R_i

describes the aggregate of advertisers' preferences. The more advertisers prefer biased reporting, or the stronger their preference for biased reporting, the stronger is the negative relationship between program quality v_i and advertising revenue R_i . If only few advertisers prefer biased reporting, the relationship between v_i and R_i becomes weaker. Our main results hold as long as R_i is weakly decreasing in v_i .

The cost c_i does not depend on audience size; it can be thought of as "first copy costs". In online and broadcast media (radio and TV), once distribution channels are in place, marginal costs of an additional audience is basically zero. In printed newspaper markets the costs for paper, printing and delivery are substantial. Any constant marginal cost of an additional consumer can be thought of as incorporated in the function R_i , which then gives advertising revenue per consumers net of marginal costs per consumer. Note that with this interpretation of R_i , it could become strictly negative for high values of v_i (since advertising revenue might be strictly smaller than marginal costs), in conflict with Assumption 3. Such values of v_i , however, lead to losses and hence are dominated; they can be eliminated from the strategy set V_i , restoring Assumption 3.

Consumers have to pay taxes or licence fees that are used to finance the PSM, but these payments are independent of personal media consumption. These payments could affect media demand via income effects. We discuss income effects in Appendix H. Alternatively, one may assume that the PSM maximize the size of their audience. In our model this also implies that PSM spend their budget to maximize consumer utility, since s_i is increasing in u_i . A limitation of our analysis is that in reality, consumers may misjudge media quality; then maximizing quality and maximizing audience size are different objectives, and there might be demand driven media bias in the PSM. See also Section 5 for a discussion of possible biases of PSM.

See e.g., (Sarver, 2023) Chapter 3 for the definition of parameterized supermodular games.

A further implication is that, since $\tilde{\pi}_i$ is continuous in (v_i, v_{-i}) for all $i \in C$, the set strategy combinations that survive iterated elimination of strictly dominated strategies has smallest and largest elements $v^{C,low}$ and $v^{C,high}$ (Milgrom & Roberts, 1990).

If $c_i(v_i)$ is identically zero for PSM i , and V_i does not depend on b_i , then an increase of b_i has no effect at all in our model.

We assume $n_f < n$. The case $n_f = n$ is the case without pay media studied above. We can allow for the case where all commercial outlets are pay media ($n_f = 0$). The most relevant case is when pay media and freely available media co-exist ($0 < n_f < n$). Note we assume that outlets $i \in C^f$ provide their program for free to consumers. Similarly, we assume that outlets in C^{pay} choose strictly positive prices.

To be precise, we assume that the best reply of a pay media outlet $i \in C^{pay}$ satisfies $0 < v_i < v_i^{high}$, $0 < p_i < p_i^{high}$, and $s_i > 0$ for all strategy combinations of its rivals.

Of course, if $c_j(v_j)$ is identically zero for the public media as well, and V_j does not depend on b_j , then changes in the budgets of PSM have no effects in our model. Even if $c_j(v_j)$ is identically zero for $j \in P$, however, $\bar{v}_j(b_j)$ will be strictly increasing if a larger budget strictly enlarges the feasible set V_j by allowing strictly higher qualities. Moreover, a larger budget may allow the PSM to increase their attractiveness in other ways that are not related to program quality.

To cover the most general case, we allow different outlets i to cover different numbers of topics k_i .

Of course, the government may also exert immediate pressure on PSM (Besley & Prat, 2006), as is documented by (Kus, 2019) for Poland.

Note that our reasoning assumes that all consumers prefer the media to fully and truthfully report about political deficits rather than withholding such information, irrespective of their own political ideology. In other words, we abstract away from cases where some consumers prefer the media to report and others prefer the media to withhold critical information about the government.

(Choi, 2006) and (Crampes et al., 2009) use the Salop model to study entry in media markets, but do not consider a quality dimension.

Conditions on the fundamentals such that the solution is interior will be given in the proof below.

We provide conditions on the fundamentals where this is the case in the proof below.

This approach is conceptually similar to a well known approach in the theory of the firm, where a first step is to find the combination of inputs that is the cheapest way to produce a given output, and in the second stage the optimal output is determined (see for example (Jehle & Reny, 2011), pp. 146–147). Here the inputs correspond to the different quality dimensions of the media, and the output to consumer utility.

Since the range of g_i is compact, g_i is upper hemicontinuous if for any two sequences $u_i^m \rightarrow u_i \in U_i$ and $v_i^m \rightarrow v_i$, with $u_i^m \in U_i$ and $v_i^m \in g_i(u_i^m)$ for all m , we have $v_i \in g_i(u_i)$ (see e.g., (Mas-Colell et al., 1995), Section M.H). Since $v_i^m \in g_i(u_i^m)$ for all m , $f_i(v_i^m) \geq u_i^m$ for all m , hence $f_i(v_i) \geq u_i$ by continuity of f_i . Moreover, $v_i^m \in g_i(u_i^m)$ for all m implies $v_i^m \in V_i$ for all m , and since V_i is compact, $v_i \in V_i$. This completes the proof that $v_i \in g_i(u_i)$.

Note this requires only that the u_i is unique for each commercial outlet, but it does not require that the first step discussed above has a unique solution: there may be different v_i that achieve the optimal u_i and generate the same advertising revenue.

See note 26 above.

We show by example after the proof that there are parameter constellations where the solution is interior.

32 To see this directly, note the first order condition for an interior solution is

$$\frac{\partial \pi_1(v_1, v_2)}{\partial v_1} = 0.$$

The second order condition holds since

$$\frac{\partial^2 \pi_1(v_1, v_2)}{\partial v_1^2} = -c_1''(v_1) < 0.$$

By the implicit function rule,

$$\text{sign}\left(\frac{dv_1^*(v_2)}{dv_2}\right) = \text{sign}\left(\frac{\partial^2 \pi_1}{\partial v_1 \partial v_2}\right) = \text{sign}(\alpha - \beta).$$

33 The profit maximizing v_i is zero if $1 - \beta\tau + \beta v_j \leq 0$, because then

$$\begin{aligned} \frac{\partial}{\partial v_i} \left(\left(\frac{1}{2} + \frac{v_i - v_j}{2\tau} \right) (1 - \beta v_i) \right) \\ = -\frac{1}{2\tau} (1 - \beta\tau - 2\beta v_i + \beta v_j) \leq 0 \end{aligned}$$

for all $v_i \geq 0$. The assumption that $1 > \beta\tau$ serves to rule out this case.

34 To see this, suppose that advertising revenue per consumer is positive, i.e., $1 - \beta v_i > 0$, or equivalently $v_i < 1/\beta$. But then firm i has no audience because

$$s_i = \left(\frac{1}{2} + \frac{v_i - v_p}{2\tau} \right) < \frac{1}{2} + \frac{\frac{1}{\beta} - \frac{1+\beta\tau}{\beta}}{2\tau} = 0.$$

It follows that firm i cannot generate a strictly positive revenue.

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