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Article

Evasion Differential Games in the Space of Square Summable Sequences

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Abstract: In this article, we consider simple-motion pursuit–evasion differential games in the Hilbert space of square summable sequences. We show that when the players have the same dynamic capabilities, evasion is possible under some assumptions about the initial positions of the players.

Keywords: Hilbert spaces; basis; pursuer; evader; pursuit–evasion games; evasion strategy

1. Introduction

Differential games were initiated with the pioneering works of Isaacs [1] and Pontryagin [2] in the 1960s and have since then been studied extensively. For example, see [3–9] and the references therein for an extensive historical account of the topic. Differential games are often divided into two problems: the problem of pursuit and the problem of evasion. In considering these games, the most common dynamics are the ones given by simple motion. Often, geometric or integral constraints are imposed on the control parameters of the players. In [10], it is shown that in the n -dimensional Euclidean ball, n lions can catch the man, while the man can escape from $n - 1$ lions when the control of the players is subject to geometric constraints. A similar game problem was studied by Ivanov [11] on any convex compact set, and an estimate from the above scenario was obtained for guaranteed pursuit time. The context in which differential games are studied is very wide. We refer to [12–17] for various interesting results for games in unbounded regions as well as on graphs.

The evasion problem on an infinite time interval was introduced and studied in [18]. Later, in [19], the authors suggested a new type of manoeuvre for evasion in the game with many pursuers. A striking strategy was suggested in [20] to study an evasion game of one evader and several pursuers with a state constraint, where the evader was supposed to move in a small neighborhood in a given direction during the game. The author proved that if the evader moves faster than the pursuers, then evasion is possible. This result was extended in a series of works [21–24] to more general differential situations. Related problems of evasion from a group of pursuers were studied in [25,26].

In [27], Pshenichnii considered a simple-motion differential game with many pursuers and one evader in \mathbb{R}^n , where all players had the same maximal speed. He proved that if the initial state of the evader is in the interior of the convex hull of the pursuers' initial states, then the pursuit can be completed; otherwise, evasion is possible. Based on this work, Pshenichnii et al. [28] developed a method of resolving functions for solving linear pursuit problems with many pursuers. It should be noted that the results of [27] have been extended by many researchers to cover various cases in finite-dimensional spaces [29–31].

Recently, control problems and differential games in infinite-dimensional spaces have been studied actively due to their connection to processes described by partial differential equations after decomposing the control problem [32], which has also shown some phenomena [33,34] intrinsic to infinite-dimensional spaces. It is worth noting that infinite-dimensional spaces have qualitatively different properties [35]. Pursuit–evasion games in



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infinite-dimensional spaces are considered in [36–38]. For recent results in this area, we refer to [39–42]

The setting of the problem in [27] is also natural to consider in infinite-dimensional spaces. But, the pursuit problem seems to be challenging in this situation. This is connected to the fact that the set of coordinate octants in infinite-dimensional spaces has continuum cardinality. Using this idea, it was shown in [43] that evasion is possible from any initial condition when integral constraints are imposed on the control parameters of the players. However, a natural generalization of [27] would be to consider geometric constraints. This work is an attempt to show that in infinite-dimensional spaces, the analogs of the theorems given in [27] are false in general. Here, we prove that under relatively mild assumptions on the dynamics of the system and the initial positions of the players, evasion is possible when the evader and the pursuers of the players have the same energy. The rest of this paper is organized as follows: Section 2 is devoted to preliminaries and contains standard results from functional analysis, and Section 3 is devoted to the main results.

2. Preliminaries

Let $(E, \|\cdot\|_E)$ be an infinite-dimensional Banach space over real numbers, \mathbb{R} , with the zero element $\mathbf{0}$. A system of elements, $\{x_k\}_{k \in F}$ ($\text{card}(F) < \infty$ or $\text{card}(F) = \aleph_0$), is called *linearly independent* ([44], p. 142) if, for any $n \in \mathbb{N}$ and $\{\alpha_k\}_{k=1}^n \subset \mathbb{R}$ such that $\sum_{k=1}^n \alpha_k x_k = 0$, it follows that $\alpha_k = 0$ for every $k \in \overline{1, n}$. The sequence $\{e_n\}_{n=1}^\infty \subset E$ is called a (Schauder) *basis* if, for any $x \in E$, there is a unique sequence, $\{\alpha_n\}_{n=1}^\infty$, of real numbers such that $x = \sum_{n=1}^\infty \alpha_n e_n$ (i.e., such that $\lim_{n \rightarrow \infty} \|x - \sum_{k=1}^n \alpha_k e_k\|_E = 0$). It is obvious that every basis of E must be linearly independent. Recall that every non-zero subspace, L , of a Hilbert space, \mathcal{H} , has the following property: for each $z \in \mathcal{H}$, there are elements $x \in L$ and $y \in \mathcal{H} \setminus L$ such that $z = x + y$. This fact we write as $\mathcal{H} = L \oplus L^\perp$.

Let f be an integrable function on a segment $[a, b]$. Using the Cauchy–Schwarz inequality— $\|f \cdot g\|_2 \leq \|f\|_2 \cdot \|g\|_2$ —for the functions $f(x)$ and $g(x) \equiv 1$, we obtain that

$$\left(\int_a^b f(x) dx \right)^2 \leq (b - a) \int_a^b (f(x))^2 dx. \tag{1}$$

Also, from Cauchy–Schwarz inequality, it follows that if

$$\sum_{j=1}^\infty (\alpha_j + \beta_j)^2 < \infty, \quad \sum_{j=1}^\infty \alpha_j^2 < \infty \quad \text{and} \quad \sum_{j=1}^\infty \beta_j^2 < \infty,$$

then

$$\sum_{j=1}^\infty (\alpha_j + \beta_j)^2 = \sum_{j=1}^\infty \alpha_j^2 + 2 \sum_{j=1}^\infty \alpha_j \beta_j + \sum_{j=1}^\infty \beta_j^2, \tag{2}$$

where $\{\alpha_j\}_{j=1}^\infty, \{\beta_j\}_{j=1}^\infty \subset \mathbb{R}$.

3. Results

Let $(\ell_2, \|\cdot\|)$ be a separable Hilbert space over real numbers, \mathbb{R} , with the usual scalar product $\langle \cdot, \cdot \rangle$. Let F be a finite or countable subset of \mathbb{N} . In the space ℓ_2 , consider a differential game with F denoting pursuers whose coordinates at time t are given by $\mathbf{x}_k(t) = (x_{k1}(t), x_{k2}(t), \dots)$, $k \in F$, and one evader whose coordinates at this time are given by $\mathbf{y}(t) = (y_1(t), y_2(t), \dots)$. Let $\dot{\mathbf{y}}(t) = \mathbf{v}(t)$ and $\dot{\mathbf{x}}_k(t) = \mathbf{u}_k(t)$ for all $k \in F$. For $t > 0$, we assume that $\|\mathbf{v}(t)\| \leq 1$ and $\|\mathbf{u}_k(t)\| \leq 1$ for all $k \in F$. As usual, $\mathbf{y}(0) \neq \mathbf{x}_k(0)$ for every $k \in F$; i.e., the initial position of the evader does not coincide with initial position of any pursuer.

Lemma 1. *If there exists a non-zero vector $\mathbf{s} \in \ell_2$ such that $\langle \mathbf{s}, \mathbf{y}(0) - \mathbf{x}_k(0) \rangle \geq 0$ for all $k \in F$, then avoidance of contact is possible.*

Proof. Consider a unit vector $\mathbf{w} = \frac{\mathbf{s}}{\|\mathbf{s}\|}$, which has the same direction as \mathbf{s} . Set $\mathbf{v}(t) = w$ for $t > 0$. Hence, $\mathbf{y}(t) = \mathbf{y}(0) + \int_0^t \mathbf{v}(s)ds = (y_1(0) + w_1t, y_2(0) + w_2t, \dots) \in \ell_2$. We will show that $\mathbf{x}_k(t) \neq \mathbf{y}(t)$ for all $k \in F$ and $t > 0$. Let us assume the opposite; i.e., suppose that there exist $n \in F$ and $\tau > 0$ such that $\mathbf{y}(\tau) = \mathbf{x}_n(\tau)$. Since

$$\mathbf{x}_n(\tau) = (x_{n1}(0) + \int_0^\tau u_{n1}(s)ds, x_{n2}(0) + \int_0^\tau u_{n2}(s)ds, \dots)$$

and

$$\mathbf{y}(\tau) = \mathbf{y}(0) + \int_0^\tau \mathbf{v}(s)ds = (y_1(0) + w_1\tau, y_2(0) + w_2\tau, \dots),$$

$x_{nk}(0) + \int_0^\tau u_{nk}(s)ds = y_k(0) + w_k\tau$ for every $k \in \mathbb{N}$. Therefore,

$$A := \left(\int_0^\tau u_{n1}(s)ds \right)^2 + \left(\int_0^\tau u_{n2}(s)ds \right)^2 + \dots = (y_1(0) - x_{n1}(0) + w_1\tau)^2 + (y_2(0) - x_{n2}(0) + w_2\tau)^2 + \dots =$$

(Using Equality (2), we obtain)

$$= \sum_{j=1}^\infty (y_j(0) - x_{nj}(0))^2 + 2\tau \sum_{j=1}^\infty (y_j(0) - x_{nj}(0))w_j + \tau^2 \sum_{j=1}^\infty w_j^2 = \sum_{j=1}^\infty (y_j(0) - x_{nj}(0))^2 + \frac{2\tau}{\|\mathbf{s}\|} \langle \mathbf{y}(0) - \mathbf{x}_n(0), \mathbf{s} \rangle + \tau^2 \|\mathbf{w}\|^2$$

Since $\sum_{j=1}^\infty (y_j(0) - x_{nj}(0))^2 > 0$, $\tau > 0$, $\|\mathbf{w}\| = 1$ and $\langle \mathbf{y}(0) - \mathbf{x}_n(0), \mathbf{s} \rangle \geq 0$, $A > \tau^2$.

On the other hand, taking into account that $\|\mathbf{u}(t)\| \leq 1$ for every $t > 0$ and applying (1) to every term, we have

$$A = \left(\int_0^\tau u_{n1}(s)ds \right)^2 + \left(\int_0^\tau u_{n2}(s)ds \right)^2 + \dots \leq \tau \int_0^\tau u_{n1}^2(s)ds + \tau \int_0^\tau u_{n2}^2(s)ds + \dots = \tau \int_0^\tau \sum_{j=1}^\infty u_{nj}^2(s)ds \leq \tau \int_0^\tau 1ds = \tau^2$$

Hence, $A \leq \tau^2$. The obtained contradiction shows that $\mathbf{x}_k(t) \neq \mathbf{y}(t)$ for all $k \in F$ and $t > 0$. \square

Theorem 1. *Let $\{\mathbf{x}_k(0)\}_{k=1}^\infty$ be a basis of ℓ_2 and $\mathbf{y}(0) \neq \sum_{k=1}^\infty \beta_k \mathbf{x}_k(0)$, where $\sum_{j=1}^\infty \beta_j = 1$. Then, avoidance of contact is possible.*

Proof. Let $\mathbf{z}_k = \mathbf{y}(0) - \mathbf{x}_k(0)$ for every $k \in \mathbb{N}$. There are two cases:

Case 1. The sequence $\{\mathbf{z}_k\}_{k=1}^\infty$ is a basis of ℓ_2 .

Let L be the closed linear span of the vectors $\{\mathbf{z}_k\}_{k=2}^\infty$; hence, $L \neq \ell_2$. Since ℓ_2 is a Hilbert space, $\ell_2 = L \oplus L^\perp$, where L^\perp is the orthogonal complement of L and $L^\perp \neq \{\mathbf{0}\}$. Let $\tilde{\mathbf{w}} \in L^\perp$ be a non-zero vector. We define the vector $\mathbf{w} \in \ell_2$ in the following way:

$$\mathbf{w} = \begin{cases} \tilde{\mathbf{w}} & \text{if } \langle \tilde{\mathbf{w}}, \mathbf{z}_1 \rangle \geq 0 \\ -\tilde{\mathbf{w}} & \text{if } \langle \tilde{\mathbf{w}}, \mathbf{z}_1 \rangle < 0 \end{cases}$$

Therefore, $\langle \mathbf{w}, \mathbf{z}_k \rangle \geq 0$ for every $k \in \mathbb{N}$, and hence, due to lemma 1, the avoidance of contact is possible.

Case 2. The sequence $\{\mathbf{z}_k\}_{k=1}^\infty$ is not a basis of ℓ_2 .

Suppose that $L = \ell_2$. Since the sequence $\{\mathbf{z}_k\}_{k=1}^\infty$ is not a basis of ℓ_2 , there exists a vector $\mathbf{r} \in \ell_2$ such that $\mathbf{r} = \sum_{k=1}^\infty \alpha_k \mathbf{z}_k = \sum_{k=1}^\infty \beta_k \mathbf{z}_k$, where $\{\alpha_j\}_{j=1}^\infty, \{\beta_j\}_{j=1}^\infty \subset \mathbb{R}$ and $\alpha_j \neq \beta_j$ for at least one $j \in \mathbb{N}$. Hence, $\sum_{k=1}^\infty \gamma_k \mathbf{z}_k = \mathbf{0}$, where $\gamma_k = \alpha_k - \beta_k (\forall k \in \mathbb{N})$ and $\sum_{j=1}^\infty \gamma_j^2 \neq 0$. Taking into account that $\mathbf{z}_k = \mathbf{y}(0) - \mathbf{x}_k(0) (\forall k \in \mathbb{N})$, we obtain that $\sum_{k=1}^\infty \gamma_k \mathbf{y}(0) = \sum_{k=1}^\infty \gamma_k \mathbf{x}_k(0)$. If $\sum_{j=1}^\infty \gamma_j = 0$, then $\sum_{k=1}^\infty \gamma_k \mathbf{x}_k(0) = \mathbf{0}$, which contradicts the assumption that $\{\mathbf{x}_k(0)\}_{k=1}^\infty$ is a basis of ℓ_2 . Therefore, $\sum_{j=1}^\infty \gamma_j \neq 0$. If for each $j \in \mathbb{N}$, we define $\tilde{\gamma}_j = \frac{\gamma_j}{\sum_{j=1}^\infty \gamma_j}$, then $\sum_{j=1}^\infty \tilde{\gamma}_j = 1$ and

$\mathbf{y}(0) = \sum_{k=1}^\infty \tilde{\gamma}_k \mathbf{x}_k(0)$, which also contradict the condition of the theorem. Thus, the equality $L = \ell_2$ does not hold. Let L be the closed linear span of the vectors $\{\mathbf{z}_k\}_{k=1}^\infty$. If $L \neq \ell_2$, then, according to the above argument, there exists a non-zero vector $\tilde{\mathbf{w}} \in \ell_2$ such that $\langle \tilde{\mathbf{w}}, \mathbf{z}_k \rangle = 0$, for every $k \in \mathbb{N}$, and hence, due to Lemma 1, the avoidance of contact is possible. In order to finish the proof, it is enough to show that the equality $L = \ell_2$ does not hold. \square

Since each basis of a Banach space zero element may have 0 coefficients, from the above theorem, we obtain the following:

Corollary 1. Let $\{\mathbf{x}_k(0)\}_{k=1}^\infty$ be a basis of ℓ_2 and $\mathbf{y}(0) = \mathbf{0}$. Then, avoidance of contact is possible.

The following proposition shows that the conditions of Theorem 1 are sufficient but not necessary.

Let $\mathcal{E}(t)$ be the set of points of ℓ_2 that evader E can reach during the time $t > 0$ from the starting position. The set $\mathcal{P}_k(t)$ has a similar meaning for each $k \in \mathbb{N}$.

Proposition 1. Let $\mathbf{y}(0) = \mathbf{0}$ and $\mathbf{x}_{2k}(0) = \mathbf{e}_k, \mathbf{x}_{2k-1}(0) = -\mathbf{e}_k$ for each $k \in \mathbb{N}$. Then, the set $\mathcal{E}(t)$ is not a subset of the set $\bigcup_{k \in \mathbb{N}} \mathcal{P}_k(t)$.

Proof. Fix any $\tau > 0$. Since $\|\dot{\mathbf{y}}(t)\| \leq 1, \mathcal{E}(\tau) = B(\mathbf{0}, \tau)$. For the same reason, $\mathcal{P}_k(\tau) = B(\mathbf{x}_k(0), \tau)$ for each $k \in \mathbb{N}$. We will show that $B(\mathbf{0}, \tau) \not\subset \bigcup_{k \in \mathbb{N}} B(\mathbf{x}_k(0), \tau)$. Let $n \in \mathbb{N}$ be such that $n > 4\tau^2$. Construct the vector

$$\mathbf{z}_\tau = \left(\frac{\tau}{\sqrt{n}}, \frac{\tau}{\sqrt{n}}, \dots, \frac{\tau}{\sqrt{n}}, 0, 0, \dots \right) \in \ell_2$$

where 0 starts from the $(n + 1)$ -th coordinate. It is obvious that $\|\mathbf{z}_\tau\| = \tau$, and hence, $\mathbf{z}_\tau \in B(\mathbf{0}, \tau)$. If $k \geq n + 1$, then

$$\|\mathbf{z}_\tau \pm \mathbf{e}_k(0)\| = \sqrt{\tau^2 + 1} > \tau;$$

i.e., $\mathbf{z}_\tau \notin B(\pm \mathbf{e}_k(0), \tau)$ for any $k \geq n + 1$. If $k \leq n$, then

$$\|\mathbf{z}_\tau \pm \mathbf{e}_k(0)\| = \sqrt{n \cdot \frac{\tau^2}{n} \pm \frac{2\tau}{n} + 1}.$$

Since $\tau > 0$ and $n > 4\tau^2$, $\pm \frac{2\tau}{n} + 1 > 0$. Therefore,

$$\sqrt{n \cdot \frac{\tau^2}{n} \pm \frac{2\tau}{n} + 1} > \tau,$$

i.e., $\mathbf{z}_\tau \notin B(\pm \mathbf{e}_k(0), \tau)$ for any $k \leq n$. Hence, $\mathbf{z}_\tau \notin \bigcup_{k \in \mathbb{N}} B(\mathbf{x}_k(0), \tau)$. Therefore,

$$B(\mathbf{0}, \tau) \setminus \bigcup_{k \in \mathbb{N}} B(\mathbf{x}_k(0), \tau) \neq \emptyset,$$

and this directly implies that $\mathcal{E}(t) \not\subset \bigcup_{k \in \mathbb{N}} \mathcal{P}_k(t)$. \square

4. Discussion

In this work, we considered simple-motion differential games in ℓ^2 where all the players have the same dynamic capabilities. Under some assumptions on the initial positions of the players, we showed that evasion is possible from any initial position. For the moment, it is not clear to us how to generalize the results of this paper to ℓ^p spaces with $1 \leq p \leq +\infty$. It should be possible with suitable changes to Lemma 1, which uses the scalar product in an essential way. This work is an attempt to show that the results of [27] are false in infinite-dimensional spaces. We constructed a piecewise-constant strategy for the evader, which guarantees evasion. To strengthen our results, one has to resort to a more wider class of strategies. It would also be interesting to consider a similar problem in an arbitrary Banach space. It would also be interesting to consider the problem when the control parameters of the players satisfy different types of constraints.

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