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## Article

# Training, abilities and the structure of teams

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## Article

# Training, Abilities and the Structure of Teams

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**Abstract:** Training in firms has an effect on the productivity of employees who receive the training, and depending on the production technology, on the other employees as well. Meanwhile, changing the remuneration structure within a team can change the stability of a team. In this paper, we apply the production games approach of cooperative game theory to analyze how training employees affects the stability of team structures and employee wages. Concretely, we apply coalition structures and the  $\chi$  value. Our results are in line with the literature and numerous further research questions can be addressed based on our approach.

**Keywords:** production games; abilities; structure of teams; training

**JEL Classification:** C71; D21; J24

## 1. Introduction

The productivity of employees who undergo training in firms increases, and this effect may extend to other employees depending on the production technology used. This may result in changes to wages. Meanwhile, changing the remuneration structure within a team can change the stability of a team.

With respect to the wages of employees after training, Becker [1] introduced a simple model to answer the question of who benefits from human capital investments: the employee or the firm. He distinguished between two types of human capital: specific and general. General human capital is equally productive in all firms, i.e., the employee alone benefits from the investment through a higher wage. In comparison, investments in specific human capital increase only the employee's productivity in the current firm. Hence, the firm benefits most from these investments. The articles by [2–9] enhance the analysis of the allocation of returns from investments in specific human capital. The effect of informational asymmetry between a firm and employee with respect to general human capital on the allocation of returns from investments in general human capital is analyzed in [10–18] offered models that analyzed how imperfect labor markets influence the allocation of returns from investments in general human capital. One result of these articles is that due to imperfections, firms can capture some amount of the returns from investments in general human capital. Literature reviews on this topic are presented by [19–23].<sup>1</sup>

With respect to the structure of teams, one line of related literature are hedonic games.<sup>2</sup> In these games, the preferences of players for one group over others are used to model the formation of coalitions (groups) [26,27]. In Barber et al. [28], hedonic games and employees with different level of ability were used. In their article, conditions under which stable structures consists of non-segregated teams are identified. Another approach for this analysis has been introduced by Piccione and Razin [29]. They apply exogenous power relations over the set of coalitions of players to obtain stable orders (groups). However, this article does not consider explicit how (the heterogeneity of) abilities influence the set of stable groups. Damiano et al. [30] considers this aspect. Their model takes into account two effects: peer effect and pecking order. The first one means that players/employees prefer to cooperate with more capable peers in the same organization or firm. The pecking order effect models the opposite—the wish of players to be in a good rank position according



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to their ability within the firm. In equilibrium segregation occurs for the more capable and less capable players whereas the intermediate able players are in mixed teams. Very similar results achieve Watts [31] considering two payoff functions—global status and local status (“big fish”). Herings et al. [32] model two types of societies—egalitarian and individualistic—to analyze the segregation of groups. If some persons are not part of a group, then these are the least productive ones for egalitarian societies and the most productive ones for individualistic societies.

In our article, we use a model in which both aspects can be analyzed—the influence of training on wages and the influence of training on team stability. Therefore, we apply cooperative game theory. In a recent paper, Hiller [33] introduced production games within the framework of cooperative game theory. Hiller [33] is based on ideas by Hernández-Lamóneda and Sánchez-Sánchez [34], Morelli and Park [35], Hiller [36]. In Hiller [33], every employee  $i$  in the firm has an ability  $a_i$ . These abilities of employees determine the economic worth of groups/coalitions of employees. In addition, two other factors influence in Hiller [33] the value of a coalition. First, the worth of a coalition has a graduality with respect to the number of employees. Assume a firm has a task for which a certain number of employees is required, e.g., three employees are necessary for the task. For example, when adding a fourth employee, the firm will assign the best three of the four employees in a team to accomplish the task. If the firm has six employees, maybe two teams of three will be formed. The second factor that influences the worth of a coalition is a minimum level of abilities. For achieving a task, a certain sum of abilities is necessary. After this threshold is crossed, the worth of a team increases as described above.

In our paper, we simplify the approach in Hiller [33] by removing from the model the need for the sum of abilities. Afterwards, we analyze two questions. First, how does training (an increase in  $a_i$ ) influences employee wages. The second question deals with the influence of training on team structure stability. One could interpret stable structures as enforceable from the point of view of the firm. A higher range of stable structures is associated with a higher degree of freedom for the firm when deciding on the team structure within the firm and the scope of training.

In our article, we use the transferable utility (TU) approach of cooperative game theory. To model structures of teams, firm structures (FS) or coalition structures are used. An FS divides employees into disjointed teams (components). To determine the compensation of employees, a reward function for games with a FS is used. The most popular FS reward function was introduced by Casajus [37]. This function is team-efficient, meaning that the worth of the team is divided among the team members<sup>3</sup>, and reflects the outside options of the employees. The better an employee’s outside options, the higher the employee’s share within the team.<sup>4</sup> Another application of cooperative game theory in the research area of training was performed by Hiller [43]. He uses FS games to model the bargaining explanation for quantitative overeducation in the labor market. The main idea is that the employer trains more employees than necessary and uses employees outside the firm after training to raise the bargaining power when they negotiate with the employees within the firm on how to share the profit from a specific human capital investment.

The remainder of this paper is structured as follows. Basic notations of cooperative game theory are given in Section 2. Section 3 presents the results and concludes.

## 2. Preliminaries

A TU (transferable utility) game is a pair  $(N, v)$ .  $N = \{1, 2, \dots, n\}$  is the set of players. The coalition function  $v$  specifies for every subset  $K \subseteq N$  a certain worth  $v(K)$  reflecting the economic abilities of  $K$ , i.e.,  $v : 2^N \rightarrow \mathbb{R}$  with  $v(\emptyset) = 0$ . If  $v$  is symmetric, there is a function  $f : N \rightarrow \mathbb{R}$  with  $v(K) = f(|K|)$  for all non-empty sets  $K \subseteq N$ .  $v$  is called monotone if  $v(K) \geq v(S) \forall S \subseteq K \subseteq N$ .

In our model, besides the set  $K$ , the ability of employees influence the worth of a coalition.<sup>5</sup> The ability of an employee  $i$  is denoted by  $a_i$ ,  $a_i \in \mathbb{R}^+$ . The vector of abilities for all employees is  $a$ . The vector restricted to employees  $K \subseteq N$  is denoted by  $a|_K$ . In addition,

the number of employees that is necessary to accomplish the task is  $t \in \mathbb{N}^+$ . Using this information, we introduce the production function  $p$ :

$$p(K, a, t) = \begin{cases} \max \sum_{i=1}^{d \cdot t} a_i & \text{with } i \in K, \quad |K| \geq t \\ 0, & \text{else} \end{cases} \quad (1)$$

whereby we have  $d = \max \left\{ d \in \mathbb{N}^+ \mid d \leq \frac{|K|}{t}, |K| \geq t \right\}$ . Considering a coalition  $K$ , the task is accomplished  $d$  times. In the first line of Equation (1), the best  $d \cdot t$  abilities of employees in  $K$  are summarized to determine the economic worth of coalition  $K$ . With these definitions, a production game is denoted by  $(N, p)$ .

**Example 1.** We have three employees with abilities  $a_1 = 2, a_2 = 8$  and  $a_3 = 6$ . The number of employees that are necessary for the task is  $t = 2$ . We have

$$p(K, a, t) = \begin{cases} 10, & K = \{1, 2\} \\ 8, & K = \{1, 3\} \\ 14, & K = \{2, 3\} \\ 14, & K = N \\ 0, & \text{else.} \end{cases} \quad (2)$$

A value is an operator  $\phi$  that assigns (unique) payoff vectors to all games  $(N, v)$  and  $(N, p)$  (i.e., uniquely determines a payoff for every player in every TU game).<sup>6</sup> The most important value is the Shapley value [44]. To calculate a player's payoff, rank orders  $\rho = (\rho_1, \dots, \rho_n)$  on  $N$  are used where  $\rho_1$  is the first player in the order, etc. The set of rank orders is  $RO(N)$ ;  $n!$  rank orders exist. The set of players before  $i$  in rank order  $\rho$  including  $i$  is called  $K_i(\rho)$ . For  $i$ , the Shapley payoff is:

$$\text{Sh}_i(N, p) = \frac{1}{n!} \sum_{\rho \in RO(N)} p(K_i(\rho)) - p(K_i(\rho) \setminus \{i\}). \quad (3)$$

We interpret the Shapley value as a reward function in the firm and the payoff vector represents the remuneration for each employee.

The reward function by Shapley assumes that all employees work together without a structure and  $p(N, a, t)$  is distributed among them. As mentioned in the introduction, we assume a firm structure with teams producing a worth in our model. Hence, the reward function of the firm should take this structure into account, i.e., the employees of each team of the firm should be remunerated by the worth produced by this team.

A firm structure (FS) is a partition  $\mathcal{P}$  of  $N$  into non-empty teams  $G_1, \dots, G_m$ ,  $\mathcal{P} = \{G_1, \dots, G_m\}$ , with  $G_i \cap G_j = \emptyset$ ,  $i \neq j$  and  $N = \bigcup G_j$ . The team containing employee  $i$  is denoted by  $\mathcal{P}(i)$ . The set of partitions of  $N$  is  $\mathfrak{P}(N)$ . A rank order  $\rho$  is called consistent with  $\mathcal{P}$  if for each  $G \in \mathcal{P}$  exist an index  $j$  and a number  $l \in \{0, \dots, n - j\}$  fulfilling  $G = \{\rho_j, \rho_{j+1}, \dots, \rho_{j+l}\}$  [45]. The set of these orders is denoted by  $RO^{\mathcal{P}}(N)$ .

**Example 2.** For partition  $\mathcal{P} = \{\{1\}, \{2\}, \{3, 4\}, \{5, 6, 7\}\}$  rank order  $\rho = (1, 2, 6, 5, 7, 4, 3)$  is consistent whereas  $\rho = (1, 2, 3, 5, 6, 7, 4)$  is not consistent.

An FS game is a game with a firm structure,  $(N, v, \mathcal{P})$ . A tuple  $(N, p, \mathcal{P})$  characterizes an FS production game. An FS reward function is an operator  $\varphi$  that assigns wage vectors to all games  $(N, v, \mathcal{P})$  and  $(N, p, \mathcal{P})$ . The  $\chi$  reward function [37] is one FS reward function. It divides the worth of a team,  $p(\mathcal{P}(i), a, t)$ , among its members,  $j \in \mathcal{P}(i)$ . In contrast to the function introduced by Aumann-Drèze [38], the  $\chi$  reward function accounts for outside options. The  $\chi$  wage of employee  $i \in N$  is calculated by:

$$\chi_i(N, p, \mathcal{P}) = \text{Sh}_i(N, p) + \frac{p(\mathcal{P}(i), a, t) - \sum_{j \in \mathcal{P}(i)} \text{Sh}_j(N, p)}{|\mathcal{P}(i)|}. \quad (4)$$

Hence, the employees are rewarded solely on the basis of performance.

Later on, we use the concept of stability. A firm structure  $\mathcal{P}$  for  $(N, p)$  is  $\chi$  stable iff for all  $\emptyset \neq K \subseteq N$  there is some  $i \in K$  such that [37,40,46]<sup>7</sup>

$$\chi_i(N, p, \mathcal{P}) \geq \chi_i(N, p, \{K, N \setminus K\}). \quad (5)$$

Hence, starting from a firm structure  $\mathcal{P}$  it is not possible to raise the wages of all  $i \in K$ , if  $K$  is separated in one team. In other words, the structure of teams is stable, if no other team structure increases the wage of all employees in one new team at least. We denote the set of all  $\chi$  stable firm structures for  $(N, p)$  by  $\mathfrak{S}(N, p)$ .

### 3. Results

In this section, we present our results on how an increase of  $a_i$  influences the stability of team structures in the firm and employee wages. A training is modeled by a vector  $\hat{a}$  that represents for every employee  $i \in N$  the additional ability created by training. Analogously, we denote the production function after the training with  $\hat{p}$ .

#### 3.1. The Firm's Perspective

First, we analyze which trainings do not influence the stability of existing structures. This question could be motivated from the firm's perspective; the training should improve the firm's profit, but structural changes are not intended.<sup>8</sup> We assume a firm with symmetric employees before training. This means that all employees have the same ability  $\bar{a}$ . The production function of a symmetric production game is:

$$p(K, a, t) = \begin{cases} d \cdot t \cdot \bar{a}, & |K| \geq t \\ 0, & \text{else.} \end{cases} \quad (6)$$

Since outside options of employees are the same in symmetric production games, the employees obtain equal Shapley wages. Hence, the employees  $\chi$  wage is the average worth of their team  $G$ . In our analysis, firstly we assume a  $\chi$  stable monotone symmetric FS production game  $(N, p, \mathcal{P})$ . We have:

**Theorem 1.** In  $\chi$  stable monotone symmetric FS production games  $(N, p, \mathcal{P})$  with  $\mathcal{P} = \{G_1, G_2, \dots, G_m\}$  and

- $|G_1| < t$  and
  - $|G_h| = t \ \forall G_h \in \{G_2, \dots, G_m\}$
- only a training vector  $\hat{a}$  with
- $\hat{a}_i \leq \hat{a}_j \ \forall i \in G_1, j \in \{G_2, \dots, G_m\}$  and
  - $\hat{a}_l, l \in \{G_2, \dots, G_m\}$ , with the possibility of a rank order  $\rho \in RO^{\mathcal{P}}(\{G_2, \dots, G_m\})$  with  $\hat{a}_{\rho(i)} \leq \hat{a}_{\rho(i+1)}, i \in \{1, \dots, |N| - |G_1| - 1\}$  does not affect the stability of  $\mathcal{P}$ .

The proof is in the Appendix A. The third point of the theorem states that employees in  $G_1$  receive less or the same amount of training as employees in other teams. The last point of the theorem requires that if the employees in teams  $G_2$  to  $G_m$  are lined up according to their additional training, the result is an order consistent with  $\mathcal{P} \setminus G_1$ , i.e., by ranking the players according to the additional training, no teams will be torn apart.

For our next result, we modify the firm structure and assume teams  $K$  with size  $|K| > t$ . Again, the firm structure is  $\chi$  stable initially. From Theorem 1, we deduce:

**Corollary 1.** In  $\chi$  stable monotone symmetric FS production games  $(N, p, \mathcal{P})$  with  $\mathcal{P} = \{G_1, G_2, \dots, G_m\}$  and

- $|G_1| < t$  and
- $|G_h| = b \cdot t, b \in \mathbb{N}$ , with  $b > 1 \forall G_h \in \{G_2, \dots, G_m\}$

only a training vector  $\hat{a}$  with

- $\hat{a}_i \leq \hat{a}_j \forall i \in G_1, j \in \{G_2, \dots, G_m\}$  and
  - $\hat{a}_l = \hat{a}_k, \forall l, k \in N \setminus G_1$  with  $\mathcal{P}(l) = \mathcal{P}(k)$
- does not affect the stability of  $\mathcal{P}$ .

Some explanations are in Appendix A.

From both results, we see that the firm does not need to train all employees of the firm to the same extent to ensure persistence of firm structure  $\mathcal{P}$ . It is crucial in the case of team size  $|G_h| > t$  that all employees in one team are equally trained. In the case of teams with size  $t$ , it is also possible to train employees of a team in a different way if no other employee of the firm has an ability that is between the lowest and the highest ability of the team.

Seen from another perspective, training could increase the range of stable structures for the employer, i.e., the firm has a higher degree of freedom in deciding on their structure.

**Corollary 2.** For FS production games  $(N, p, \mathcal{P})$  with  $a_i \neq a_j \forall i, j \in N$  and  $n > t$  there is a least one training vector  $\hat{a}$  ensuring  $|\mathfrak{S}(N, p)| > |\mathfrak{S}(N, p)|$ .

One approach to determine a vector  $\hat{a}$  could be to train every employee  $i$  with the difference between their initial ability and the highest ability of all employees.

### 3.2. The Employee's Perspective

In this subsection, we analyze how trainings influence the employee's wage. First, we see that  $\text{Sh}_i(N, p)$  is increasing in  $a_i$ . In addition, we know from Equation (1) that the Shapley wage of  $i$  is increasing more slowly than  $a_i$  since the marginal contribution of  $i$  is lower than  $a_i$  in rank orders in which the employee substitutes only an employee with lower ability in the determination of  $p(K_i(\rho))$ . From Equations (3) and (4) it is easy to see that an increase in  $a_i$  does not reduce the  $\chi$  wage of employee  $i$  if  $\mathcal{P}$  and the abilities of the other employees are unchanged. In addition, we can deduce from Equation (4):

**Corollary 3.** In monotone symmetric FS production games  $(N, p, \mathcal{P})$ ,  $\mathcal{P} = \{G_1, \dots, G_s, G_{s+1}, \dots, G_m\}$ , and a training vector  $\hat{a}$  with

- $|\bigcup G_1, \dots, G_s| < t$
- $|G_{s+1}|, \dots, |G_m| \in \{l | l = b \cdot t\}$  with  $b \in \mathbb{N}^+$
- $\hat{a}_i = \hat{a}_j > 0, i \neq j, i, j \in \{G_{s+1}, \dots, G_m\}$  and  $\hat{a}_l = 0 \forall l \in N \setminus \{i, j\}$

we have

- $\chi_i(N, \hat{p}, \mathcal{P}) \geq \chi_j(N, \hat{p}, \mathcal{P})$  iff  $|\mathcal{P}(i)| \leq |\mathcal{P}(j)|$ .

Hence, employees in smaller teams have a higher motivation to participate in trainings than employees in larger teams. The attitude of an employee on the structure of the firm can be deduced from Theorem 1 and Corollary 1. For example, an employee in teams  $K$  with size  $|K| > t$ , that is trained more than the other employees in their team would prefer to be separated in a smaller team with size  $t$ .

### 3.3. Summary and Outlook

In our paper, we use the firm structure approach of cooperative game theory to analyze how training influences employee wages and the stability of team structures. Assuming an unchanged  $\mathcal{P}$  and unchanged abilities of the other employees, an increase in  $a_i$  does not reduce the  $\chi$  wage of employee  $i$ . This result is in line with the literature on training. We



cannot make statements about the distribution of revenues of human capital investments between a firm and employee because the firm and its owners are not modeled in our approach. This question is open for further research. Additionally, we show for monotone symmetric FS production games that employees in smaller teams benefit more from training than employees in larger teams. With respect to team stability, we show that a firm does not need to train all employees to the same extent to ensure the stability of  $\mathcal{P}$ . Rather, training can increase the range of stable structures for the employer. This is an important result. Our model allows, in principle, for training to change the  $\chi$ -wages of employees in such a way that the team is no longer stable, i.e., employees desire to work with employees without their team. However, this can be avoided by carefully planning the amount of training. Our results are in line with Damiano et al. [30]. In their article, segregation occurs for the more capable and less capable individuals whereas the intermediate able employees are in mixed teams. Our training vectors  $\hat{a}$  in Theorem 1 and Corollary 1 ensure that teams only become heterogeneous to a limited extent through training. With respect to Herings et al. [32], our results are in line with an egalitarian society. This result comes from the application of the  $\chi$  wage, which divides the surplus of a group equally among the group members.

Our analysis is only one step and many questions remain unanswered. These can be future research tasks. For example, we do not consider the mutual interrelation between the abilities of the employees. In addition, we do not consider different  $t$ s for the teams in the firm. Another starting point for enhancing the model is the integration of interrelationships among the teams. With this integration, it is possible to model the existence of the firm as a coordinating element between different teams.

From a theoretical perspective, our analysis can be repeated with other FS values [40–42] to check whether the results differ from ours. In addition, it is possible to examine the decision on trainings using non-cooperative models of game theory with FS payoffs of employees as possible outcomes of the training's decision. Finally, to model development of abilities over time, dynamic/evolutionary cooperative game theory (see Newton [48] for an overview and Casajus et al. [49] for some new insights) could be applied.

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## Appendix A

**Theorem A1.** A part of the proof is based on Theorems 1 and 3 in Hiller [33]. Theorem 3 in Hiller [33] states that in monotone FS production games  $(N, p, \mathcal{P})$  with  $a_i \neq a_j \forall i, j \in N, i \neq j$ , only partitions  $\mathcal{P} = \{G_1, \dots, G_q, G_{q+1}, \dots, G_m\}$  with

- $\sum_{i \in \{G_1, \dots, G_q\}} a_i < z, a_i < a_j \forall i \in \{G_1, \dots, G_q\}$  and  $j \in \{G_{q+1}, \dots, G_m\}$
- $|G_{q+1}|, \dots, |G_s| = b \cdot t$ , with  $b \in \mathbb{N}^+, \sum_{i \in G_h} a_i > z, \sum_{i \in G_h} a_i - a_j < z \forall j \in G_h, G_h \in \{G_{q+1}, \dots, G_s\}$  and  $a_i > a_j$  for  $i \in G_l, j \in G_{l-1}, l = m, \dots, q+1$ .

These are  $\chi$  stable where  $z$  is the sum of abilities that a team must exceed to produce a worth. First, we analyze the case  $\hat{a}_i = \hat{a}_j \forall i, j \in N$ . In this case, it is obvious that  $\mathcal{P}$  stays  $\chi$  stable. The employees in  $G_1$  obtain the  $\chi$  wages zero before training. After training, it is not possible to form a partition  $\mathcal{P}' = \{K, N \setminus K\}$  with  $p(K) > 0$  and  $G_1 \cap K \neq \emptyset$  without at least one employee from  $G_2, \dots, G_m$ . The  $\chi$  wage of this employee is unchanged/reduced. The employees in  $G_2, \dots, G_m$  obtain  $\chi$  wages  $\bar{a}$  before training. In the case of different levels of training, we know from Theorem 3 in Hiller [33] that employees of a team should be homogenous as possible. With training vector  $\hat{a}$  this condition is fulfilled.

**Corollary A1.** For this FS, all employees of a team must be trained equally. Otherwise, the  $t$  employees with highest abilities form a new team and raise their wages (see Theorem 3 in Hiller [33]). If all employees of a team  $i \in G_h$  are trained equally, the employees of a team obtain the average of the team's worth  $p(G_h)$ . It is not possible to form a  $\mathcal{P}' = \{G'_h, N \setminus G'_h\}$  with  $p(G'_h) > p(G_h)$  (and hence  $\chi_i(N, p, \mathcal{P}') > \chi_i(N, p, \mathcal{P})$ ,  $i \in G'_h \cap G_h$ ) without at least one more trained employee from  $N \setminus G_h$ . The  $\chi$  remuneration of this employee is unchanged or reduced.

## Notes

- <sup>1</sup> Extensive empirical literature on returns from human capital investments were inspired by [24].
- <sup>2</sup> Casajus [25] outlined the relation of hedonic games to the TU games, which our article belongs to.
- <sup>3</sup> According to Aumann and Drèze [38], components are active groups as in our understanding. In contrast, the Owen [39] value interprets components as bargaining unions.
- <sup>4</sup> Other FS reward functions being team-efficient and reflecting the outside options of employees are introduced by Wiese [40], Alonso-Mejide et al. [41], for example. One other value with respect to the Owen interpretation of components is introduced by Kamijo [42], for example.
- <sup>5</sup> The notation is partly based on Hiller [33].
- <sup>6</sup> In the following, only the application to games  $(N, p)$  is relevant.
- <sup>7</sup> Abe [47] studied the relationship between an FS value and stable coalition structures.
- <sup>8</sup> As noted in the previous section, one could assume that instead of  $v(\mathcal{P}(i))$  only a fraction  $c$ ,  $0 < c < 1$  is distributed among the team's members by  $\chi$ .

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