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Article

Differential Game-Theoretic Models of Cournot Oligopoly with Consideration of the Green Effect

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Abstract: We built and investigated analytically and numerically a differential game model of Cournot oligopoly with consideration of pollution for the general case and the case of symmetrical agents. We conducted a comparative analysis of selfish agents' behavior (a differential game in normal form), their hierarchical organization (differential Stackelberg games), and cooperation (optimal control problem) using individual and collective indices of relative efficiency. The same analysis was performed for the models with the green effect when players chose both output volumes and environmental protection efforts. We used the Pontryagin maximum principle for analytical investigation and the method of qualitatively representative scenarios in simulation modeling for numerical calculations. This method allows for reducing the number of computer simulations, providing sufficient precision. As a result of the comparative analysis, systems of collective and individual preferences were obtained.

Keywords: differential game theory; Cournot oligopoly; green effect; simulation modeling



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1. Introduction

A widespread approach to the analysis of game-theoretic models (particularly, differential games) consists in the comparison of results obtained in the cases of selfish players' behavior (differential games in normal form), their hierarchical organization (Stackelberg games), and cooperation (a differential game is reduced to the optimal control problem). For example, a successful illustration of this approach is presented by Zhang et al. [1]. A variant of the approach is described in Ougolnitsky [2]. Basing on Basar and Zhu [3], Cairns and Martinet [4], and some other papers, we proposed in [2] a system of individual and collective indices of the comparative (relative) efficiency for quantitative evaluation of the different ways of organization of economic agents.

A convenient model for the comparative analysis of the different ways of organization of economic agents is Cournot oligopoly (see Maskin and Tirole [5], Geras'kin [6,7], and Algazin and Algazina [8,9]). For example, Xiao et al. [10,11] studied Cournot duopoly with bounded rationality and investigated the equilibria. Raoufinia et al. [12] analyzed open-loop and closed-loop solutions in a Cournot duopoly game with advertising. Al-Khedhairi [13] considered non-trivial Cournot duopoly based on fractal differential equations. Julien [14] investigated Cournot oligopoly with several Stackelberg leaders and followers. A comparison of Cournot and Stackelberg equilibria performed by Zouhar and Zouharova [15].

Together with a standard setup of the oligopoly model that describes a competition of several firms in a market of homogeneous goods, it is interesting to consider the so-called green effect. Usually, the green effect is concerned with supply chains (see Azevedo et al. [16], Fahimnia et al. [17], and Sharma and Jain [18]). It is assumed in this case that the participants of a supply chain invest in the environmental protection in the processes of production and transportation. The incurred costs are compensated by the willingness of environmentally minded consumers to pay more for products with a green label.

However, the studies of Cournot oligopoly do not include a systematic comparative analysis of the relative profit from the point of view of the whole society and different firms.

Besides, the environmental externalities of economic activity and possible environmental protection efforts are not considered.

Thus, the main idea of this paper consists in a comparison of the different ways of organization of economic agents, such as competition, cooperation, and hierarchical control, for differential Cournot oligopoly with pollution dynamics and differential Cournot oligopoly with the green effect. The specific aim consists in building preference systems based on individual and collective indices of relative efficiency.

The contribution of this paper and its novelty areas follows:

- A differential game-theoretic model of Cournot oligopoly with consideration of pollution for the general case and the case of symmetrical agents is built and investigated analytically and numerically.
- A comparative analysis of selfish agents' behavior (a differential game in normal form), their hierarchical organization (differential Stackelberg games), and cooperation (optimal control problem) using individual and collective indices of relative efficiency is conducted.
- We performed the same analysis for the models with the green effect when players chose both output volumes and environmental protection efforts.
- We constructed systems of collective and individual preferences.

In Section 2, we characterize the materials and methods of the investigation. In Section 3, we build and investigate a differential game model of Cournot oligopoly for the selfish behavior of players, their hierarchical organization, and cooperation. The case of symmetrical players and the general case are considered. We applied the Pontryagin maximum principle for analytical investigation and simulation modeling for numerical calculations. We used a system of individual and collective indices of relative efficiency for the quantitative comparison of the obtained results. In Section 4, similar work is performed for the models with the green effect when players invest in environmental protection. Section 5 concludes the paper.

2. Materials and Methods

The main analytical instrument of the investigation is the well-known Pontryagin maximum principle [19]. For numerical calculations, we used an original method of qualitatively representative scenarios in simulation modeling [20]. The idea consists in choosing a relatively small number of control scenarios that provide a sufficiently precise description of the dynamics of the controlled system. For the substantiation of sufficiency, two conditions are used, namely the conditions of internal and external stability. Suppose an initial set of scenarios is chosen. It is internally stable if for any two scenarios from this set, the respective payoffs of the players differ essentially. It is externally stable if for any feasible scenario that does not belong to this set, we can find a scenario from this set such that the respective payoffs of the players are close. The value of precision of such approximations is chosen empirically and should not exceed 10% from the typical values of payoffs.

For quantitative comparative evaluation of the different ways of organization of economic agents (information structures of the respective game-theoretic models) from the point of view of both the whole society and separate agents, we introduced a system of relative efficiency indices [2], namely:

- collective indices of relative efficiency;

$$SCI^{NE} = \frac{\sum_{i=1}^n J_i^{NE}}{J^C}; \quad SCI^{ST} = \frac{\sum_{i=1}^n J_i^{ST}}{J^C}; \quad SCI^{iST} = \frac{\sum_{i=1}^n J_i^{iST}}{J^C}$$

The values J_i^{ST} and J_i^{iST} determine the payoffs of the i -th player in the Stackelberg and inverse Stackelberg games, respectively, when the i -th player is the leader. Any player can become the leader; in our examples, it is the first player.

- individual indices of relative efficiency.

$$K_i^{NE} = \frac{\min_{x \in NE} J_i^{NE}(x)}{J^C/n}; K_i^{ST} = \frac{J_i^{ST}}{J^C/n}; K_i^{IST} = \frac{J_i^{IST}}{J^C/n}$$

The payoffs are supposed to be non-negative.

We proposed a mathematical model that is a dynamic version of Cournot oligopoly with consideration of environmental pollution. The main approach to its identification is an expert estimate using available real data. There were five main parameters in the model: (1) the concentration of pollutants in the environment in the initial moment of time, (2) a coefficient of the pollutants' emission during production, (3) a coefficient of decay of the pollutants in the environment, (4) a maximal output for any firm, and (5) a cost coefficient for each firm. For the numerical identification of their values, we used the following reasonings. As a pollutant, we can consider carbon monoxide (CO). It is toxic, and its admissible concentration in production premises is 20 mg/m³ during a working day or 50 mg/m³ during an hour or 100 mg/m³ during 30 min. Based on this, the concentration of pollutants in the environment in the initial moment of time varied from 1 to 50 mg/m³. The coefficient of the pollutants' emission during production depends on the production volume. For example, a coke chemical plant emits annually about 2000 tons of carbon monoxide. Based on this, the value of pollutants' emission varied from 0.1 to 30 tons per year; the maximal output for any firm varied from 5 to 70,000 tons per year, and the cost coefficient for each firm varied from 1 to 50. The decay of many pollutants is slow; for example, carbon monoxide decays only in the presence of a catalyst. Thus, we varied the coefficient of decay of the pollutants from 0.1 to 30 kg per year. In addition, discounting was considering in the model, and a discount factor was taken to be equal to 0.004 that corresponds to moderate inflation. The modeling was conducted at an interval of 1200 days.

3. Differential Game Model of Cournot Oligopoly with Consideration of Pollution

Let us consider a dynamic version of Cournot oligopoly with consideration of environmental pollution and the linear equation of dynamics:

$$J_i = \int_0^T e^{-rt} \alpha_1 (a - c_i - \bar{x}(t)) x_i(t) dt - e^{-rT} \alpha_2 y(T) \rightarrow \max \tag{1}$$

$$0 \leq x_i(t), \forall t \in [0, T], i = 1, \dots, n; \tag{2}$$

$$\frac{dy}{dt} = \sum_{i=1}^n k_i x_i(t) - my(t), y(0) = y_0. \tag{3}$$

Here, $\{1, 2, \dots, n\}$ is a set of firms (agents, players) competing in the manner of Cournot oligopoly in a market of homogeneous goods; J_i is the i -th player's profit in time T ; $\bar{x}(t) = \sum_{i=1}^n x_i(t)$; x_i is the output volume of the i -th firm (its strategy); the expression in parentheses in Formula (1) determines the price for the produced good, depending on the demand that is conversely proportional to the total output volume; α_1 and α_2 are dimensioned coefficients that provide the fitness of dimension (for simplicity, they are assumed to be equal to 1); $y(t)$ is the volume of pollutants in the environment (a state variable); k_i is the coefficient of emission in the production of the i -th firm; a is the demand parameter; c_i is the cost coefficient of the i -th firm; m is the coefficient of pollution decay; r is the discount factor; and T is the length of the game. The agents' interaction is described by their strategies and the final value of the state variable in the moment of time T .

In the case of symmetrical agent s ($k_i = k; c_i = c; x_i = x$), the model in Formulas (1)–(3) takes the form

$$J = \int_0^T e^{-rt}(a - c - nx(t))x(t)dt - e^{-rT}y(T) \rightarrow \max \tag{4}$$

$$0 \leq x(t), \forall t \in [0, T], \tag{5}$$

$$\frac{dy}{dt} = knx - my(t), \quad y(0) = y_0. \tag{6}$$

We supposed that all players use open-loop piecewise continuous strategies. The agents may be selfish, so we have a differential game in normal form with Nash equilibrium as its solution. In addition, the agents may cooperate, so the game is reduced to the optimal control problem. Finally, a hierarchical organization is possible that is formalized by differential Stackelberg and inverse Stackelberg games [21,22].

Let us first consider a selfish behavior of the agents and investigate the symmetrical model in Formulas (4)–(6) using the Pontryagin maximum principle [19]. The Hamilton function for each player has the form

$$H(x, u, \lambda) = (a - c - nx(t))x(t) + \lambda(knx(t) - my(t)),$$

where $\lambda(t)$ is a conjugate variable. We obtain

$$\frac{\partial H}{\partial x} = a - c - 2nx + kn\lambda = 0$$

and

$$x^{NE} = \frac{a - c + kn\lambda}{2n}. \tag{7}$$

Here, $\frac{\partial^2 H}{\partial x^2} = -2n < 0$. Therefore, the found value x^{NE} maximizes the Hamilton function if $a - c + kn\lambda > 0$, or the value belongs to the domain of feasible strategies, Formula (5). Otherwise, the point of maximum coincides with the lower bound of the set of feasible strategies of the agent. A conjugate variable is determined from the boundary value problem

$$\frac{\partial \lambda}{\partial t} = r\lambda(t) - \frac{\partial H}{\partial y} = (r + m)\lambda(t); \quad \lambda(T) = -1.$$

Then,

$$\lambda_i^{NE} = \lambda^{NE}(t) = e^{-(r+m)(T-t)}, \tag{8}$$

and Formula (7) is a maximizer of the Hamilton function if $a - c - kne^{-(r+m)(T-t)} > 0$.

Thus, we obtain

$$x^{NE} = \max\left(0, \left(\frac{-ke^{-(r+m)(T-t)}}{2} + (a - c)/(2n)\right)\right) \tag{9}$$

If $a - c - kne^{-(r+m)(T-t)} > 0$ for $\forall t > 0$, then $x^{NE}(t) = \frac{a-c}{2n} - \frac{k}{2}e^{-(r+m)(T-t)}$

$$y^{NE}(t) = y_0e^{-mt} + \frac{k(a-c)}{2m}(1 - e^{-mt}) - \frac{k^2n}{2(r+2m)}(e^{-(r+m)(T-t)} - e^{-mt-(r+m)T})$$

$$J^{NE} = \frac{(a-c)^2}{4nr}(1 - e^{-rT}) - \frac{k^2n}{4(r+2m)}(e^{-rT} - e^{-2(r+m)T})$$

These calculations show that Nash equilibrium exists and is unique.

Using Formulas (8) and (9), we conducted numerical calculations for different input data sets in the case of symmetrical agents. We realized about 100 numerical calculations. We varied the following parameters: n from 2 to 40, a from 5 to 70, c from 1 to 50, m from

0.1 to 30, k from 0.1 to 30, and y_0 from 1 to 50. The input data are presented in Table 1, and the agents' payoffs for the input data from Table 1 for $T = 1200$ and $r = 0.001$ are presented in Table 2. Here and elsewhere, NE stands for Nash equilibrium, C for cooperative solution, and ST and IST for Stackelberg and inverse Stackelberg games, respectively.

Table 1. Input data in the case of symmetrical agents.

Example	n	a	c	k	m	y_0
1	5	30	15	1	2	10
2	5	20	15	1	2	10
3	5	17	15	1	2	10
4	5	10	15	1	2	10
5	5	40	15	1	2	10
6	10	20	15	1	2	10
7	5	20	5	1	2	10
8	5	20	15	5	2	10
9	5	20	15	0.2	2	10
10	5	20	15	1	1	10
11	5	20	15	1	5	10
12	5	20	15	1	2	30
13	5	20	15	1	2	2
14	5	20	15	1	10	10
15	5	20	15	5	2	2
16	5	20	15	5	5	10
17	5	20	15	1	1	5
18	2	20	15	1	2	10
19	20	20	15	1	2	10
20	2	30	15	1	2	10
21	2	20	5	1	2	10
22	2	20	15	1	10	10
23	2	20	15	1	2	1
24	2	20	15	1	2	20
25	2	20	15	1	2	5
26	10	20	5	1	2	10
27	10	10	5	1	2	10
28	10	30	5	1	2	10
29	10	20	5	10	2	10
30	10	20	5	0.2	2	10
31	10	20	5	1	2	1
32	10	20	5	1	2	20
33	10	20	5	1	2	3
34	10	30	15	1	2	10
35	10	20	15	3	2	10
36	10	20	5	1	6	15
37	10	20	1	5	2	10
38	10	20	10	10	2	10

In the case of arbitrary agents, the Hamilton function for the i -th player is

$$H_i(x_i(t), u_i(t), \lambda_i(t)) = (a - c_i - \bar{x}(t))x_i(t) + \lambda_i(t) \left(\sum_{i=1}^n k_i x_i(t) - m(t)y(t) \right)$$

Then,

$$\frac{\partial H_i}{\partial x_i} = a - c_i - \sum_{j=1}^n x_j - x_i + k\lambda_i = 0; \quad i = 1, 2, \dots, n \tag{10}$$

and $\frac{\partial^2 H_i}{\partial x^2} = -2n < 0$. Therefore, the solutions of Formula (10) maximize the Hamilton function if they belong to the sets of feasible strategies. For conjugate variables, Formula (8) is applied.

Table 2. Payoffs for different information structures for symmetrical agents.

No.	J_*	NE	$y(T)$	J^c	C	$y(T)$
1	13,403		3.85	67,015		3.85
2	1476		0	7380		0
3	223		0	1115		0
4	1476		0	7380		0
5	37,274		5.6	186,372		5.6
6	745		3.12	7450		3.12
7	13,418		3.12	67,090		3.12
8	1483		0	7415		0
9	1491		0.22	7455		0.22
10	1490		1.25	7450		1.25
11	1491		0.25	7455		0.25
12	1491		0.63	7455		0.63
13	1491		0.63	7455		0.63
14	1491		0.13	7455		0.13
15	1483		0	7415		0
16	1488		0	7440		0
17	1490		1.25	7450		1.25
18	3727		1	7454		1
19	371.5		0	7430		0
20	33,547		3.5	67,094		3.5
21	33,547		3.5	67,094		3.5
22	3728		0.2	7456		0.2
23	3727		1	7454		1
24	3727		1	7454		1
25	3727		1	7454		1
26	6709		2.5	67,090		2.5
27	744.9		3.1	7449		3.1
28	18,637		6.1	186,370		6.1
29	6648		0	66,480		0
30	6710		0.7	67,100		0.7
31	6709		2.5	67,090		2.5
32	6709		2.5	67,090		2.5
33	6709		2.5	67,090		2.5
34	6709		2.5	67,090		2.5
35	740		0	7400		0
36	6709		0.83	67,090		0.83
37	10,750		0	107,500		0
38	2920		0	29,200		0

Solving the system of equations in Formula (10), we obtain

$$x_i^{NE} = \max \left(0, \left(-k_i e^{-(r+m)(T-t)} + a - nc_i + \sum_{j=1; j \neq i}^n c_j \right) / (n+1) \right); \quad i = 1, 2, \dots, n. \tag{11}$$

Let us consider the case $-k_i e^{-(r+m)(T-t)} + a - nc_i + \sum_{j=1; j \neq i}^n c_j > 0; \forall t > 0$.

Substitute Formula (11) in the equation of dynamics and solve it by the method of variation of parameters:

$$y^{NE} = y_0 e^{-mt} + \frac{1 - e^{-mt}}{(n+1)m} \sum_{i=1}^n k_i \left(a - nc_i + \sum_{j=1; j \neq i}^n c_j \right) - \frac{k_i^2 n}{(n+1)(r+2m)} \left(e^{-(r+m)(T-t)} - e^{-mt-(r+m)T} \right)$$

Then

$$\begin{aligned}
 J_i^{NE} = & \left(a - nc_i + \sum_{j=1; j \neq i}^n c_j \right) \frac{a - c_i}{(n+1)r} (1 - e^{-rT}) - \frac{k_i(a - c_i)}{(n+1)m} (e^{-rT} - e^{-(r+m)T}) + \\
 & \frac{k_i}{(n+1)^2 m} \sum_{j=1}^n \left(a - nc_i + \sum_{k=1; k \neq j}^n c_k \right) (e^{-rT} - e^{-(r+m)T}) + \\
 & \frac{1}{(n+1)^2} (e^{-2(r+m)T} - e^{-rT}) \sum_{j=1}^n k_j \left(\frac{k_j}{r+2m} - \frac{1}{m} \left(a - nc_i + \sum_{k=1; k \neq j}^n c_k \right) \right) - \\
 & \left(a - nc_i + \sum_{k=1; k \neq i}^n c_k \right) \frac{1}{(n+1)^2 r} (1 - e^{-rT}) \sum_{j=1}^n \left(a - nc_j + \sum_{k=1; k \neq j}^n c_k \right) - e^{-rT} y^{NE}(T).
 \end{aligned}$$

The input data are presented in Table 3, and the results for three arbitrary agents at $T = 1200$ and $r = 0.001$ for the input data from the Table 3 are presented in Table 4.

Table 3. Input data in the case of arbitrary agents ($n = 3$).

Example	a	c_1	c_2	c_3	k_1	k_2	k_3	m	y_0
1	25	10	15	5	1	2	3	2	10
2	25	10	15	5	1	0.5	3	2	10
3	25	1	15	3	1	1	3	2	10
4	25	1	15	3	1	5	3	2	10
5	25	20	15	5	1	1	5	2	10
6	30	5	15	20	1	1	5	2	10
7	25	10	5	15	1	2	3	2	10
8	25	5	15	10	5	4	1	2	10
9	25	5	15	10	0.2	1	2	2	10
10	25	1	15	5	1	0.1	0.3	1	10
11	25	20	15	10	1	0.5	0.5	0.5	10
12	25	5	15	1	1	0.1	1	2	30
13	25	5	15	1	1	1	3	2	2
14	25	1	15	10	1	0.5	0.1	0.1	10
15	25	5	15	1	5	0.2	1	2	2
16	25	5	15	1	5	0.5	1	5	10
17	25	5	15	1	1	4	2	1	5
18	18	10	15	5	1	4	3	2	10
19	30	10	1	5	1	4	4	2	10
20	25	1	15	5	1	0.1	0.8	2	10
21	35	1	5	30	1	0.1	0.5	2	10
22	35	10	15	30	1	0.1	1	0.1	10
23	45	30	15	20	1	2	3	2	1
24	45	30	15	20	1	2	0.5	2	20
25	45	30	15	15	1	2	1	2	5
26	30	10	5	2	1	3	2	2	10
27	30	15	5	23	1	3	5	2	10
28	30	15	5	28	1	3	3	2	10
29	30	15	5	12	10	1	0.2	2	10
30	30	15	5	7	0.2	1	0.5	2	10
31	30	20	5	3	1	2	0.5	2	1
32	30	20	5	3	1	2	0.1	2	20
33	30	10	5	2	1	0.5	3	2	3
34	30	1	15	18	1	0.5	0.2	2	10
35	30	1	15	20	3	0.5	0.8	2	10
36	30	1	5	20	1	0.5	1	6	15
37	30	5	1	20	5	2	1	2	10

Table 4. Payoffs of arbitrary agents for different information structures.

No.	NE		ST		IST		C J^C
	J_1	J_2	J_1	J_2	J_1	J_2	
1	16,769	1856	25,342	1025	29,855	854	110,567
2	16,765	1852	24,878	1032	27,856	987	110,211
3	119,277	19,073	136,565	15,456	142,878	12,378	217,345
4	119,292	19,082	136,545	15,467	143,111	12,176	217,431
5	16,753	1847	19,654	1187	21,878	1083	33,962
6	186,388	7456	192,321	4231	195,276	3765	203,477
7	16,777	91,332	22,321	85,344	24,123	80,433	110,367
8	91,328	1859	98,788	1098	103,287	944	111,213
9	91,327	1861	97,878	1112	100,433	952	111,214
10	131,499	14,599	138,766	11,232	145,388	9321	198,376
11	7451	7454	9233	6987	11,653	6012	82,943
12	50,390	14,604	56,578	12,343	59,721	11,488	199,432
13	50,388	14,598	56,699	12,511	61,245	11,298	199,173
14	164,603	5934	171,234	3767	174,832	3077	181,326
15	50,383	14,596	57,655	10,767	59,727	8999	198,234
16	50,392	14,606	57,688	10,822	59,211	9122	198,321
17	50,406	14,608	58,022	10,824	59,344	9211	198,411
18	4773	10,733	6231	9356	7113	8733	74,987
19	2666	131,496	3457	125,676	5234	120,924	186,333
20	131,503	14,604	147,344	11,288	156,344	7455	198,211
21	334,659	193,906	348,656	180,344	356,745	175,901	711,548
22	186,372	67,093	202,378	55,344	213,488	53,977	322,121
23	7438	186,371	8433	180,433	9123	177,843	262,321
24	7443	186,371	8511	180,433	9211	177,855	262,377
25	16,761	150,958	19,431	144,877	21,322	140,234	320,432
26	3640	54,336	5244	47,865	5867	46,904	173,231
27	12,602	209,425	17,355	195,877	19,742	192,511	250,375
28	24,156	250,797	31,866	241,822	35,805	238,091	363,543
29	296	131,522	2033	125,649	5386	124,034	147,786
30	665	102,056	2345	94,545	4129	93,532	169,111
31	36,074	107,647	45,233	94,565	54,005	91,714	306,342
32	36,076	107,648	44,211	100,456	50,056	98,455	306,435
33	3640	54,338	8343	48,234	10,273	47,431	172,768
34	268,380	1186	272,345	756	274,611	690	276,356
35	286,560	2666	294,344	1213	298,053	1054	305,421
36	201,587	96,618	223,187	83,423	228,769	80,521	343,221
37	96,601	201,577	103,421	192,344	113,591	185,732	344,234

When all agents cooperate we obtain an optimal control problem:

$$J^c = \sum_{i=1}^n \int_0^T e^{-rt} \left(a - c_i - \sum_{j=1}^n x_j(t) \right) x_i(t) dt - ne^{-rT} y(T) \rightarrow \max \tag{12}$$

$$0 \leq x_i(t), \forall t \in [0, T], i = 1, \dots, n; \quad \frac{dy^c}{dt} = \sum_{i=1}^n k_i x_i(t) - my^c(t), \quad y^c(0) = y_0$$

In the symmetrical case ($k_i = k; c_i = c; i = 1, 2, \dots, n$), the problem takes the form

$$J^c = \int_0^T e^{-rt} (a - c - x(t)) x(t) dt - ne^{-rT} y(T) \rightarrow \max$$

$$0 \leq x(t), \forall t \in [0, T],$$

$$\frac{dy^c}{dt} = kx(t) - my^c(t); \quad y^c(0) = y_0; \quad x(t) = \sum_{i=1}^n x_i(t).$$

Similar to the case considered earlier, we obtain $\lambda^C(t) = -ne^{-(r+m)(T-t)}$:

$$x^c(t) = \max\left(0, \left(\frac{-kne^{-(r+m)(T-t)}}{2} + \frac{a-c}{2}\right)\right)$$

If $-ke^{-(r+m)(T-t)} + a - c > 0; \forall t > 0$, then

$$\begin{aligned} x^C(t) &= \frac{a-c}{2} - \frac{kn}{2}e^{-(r+m)(T-t)} \\ y^C(t) &= y_0e^{-mt} + \frac{k(a-c)}{2m}(1 - e^{-mt}) - \frac{k^2n}{2(r+2m)}\left(e^{-(r+m)(T-t)} - e^{-mt-(r+m)T}\right) \\ J^C &= \frac{(a-c)^2}{4r}(1 - e^{-rT}) - \frac{k^2n^2}{4(r+2m)}\left(e^{-rT} - e^{-2(r+m)T}\right) - y(T)e^{-rT} \end{aligned}$$

In Table 2, in the third and fourth columns, the results of calculations in the case of cooperation for the input data from Table 1 are presented.

When arbitrary agents cooperated, the solution was found numerically [23,24] using the method of qualitatively representative scenarios in simulation modeling [20]. The initial sets of qualitatively representative scenarios were taken as sets that consisted of three elements: 0, a big number (10,000 as a specific example), and their average value. All elements of the initial set were checked for completeness and redundancy [20], and it was reduced or extended with new elements by necessity. The calculation results are presented in Table 4.

Now, let us consider the case of hierarchical relations between agents in two versions of the information structure. In a Stackelberg game, one of the agents (e.g., the first one) becomes the leader (she). She chooses and reports to the other agents (followers) her open-loop strategy $x_1(t)$.

The followers play a differential game in normal form. The best response of the followers to the leader’s strategy is defined as Nash equilibrium in this game. We solved $n - 1$ optimal control problems (1)–(3) for $i = 1, 2, \dots, n$. A solution of each problem was found using the Pontryagin maximum principle, similar to Formulas (10) and (11), and had the form

$$x_i^*(t) = \max\left(0, x_i^0 - \frac{x_1}{n}\right); \quad i = 2, 3, \dots, n \tag{13}$$

where $i = 2, 3, \dots, n$;

$$x_i^0(t) = \frac{1}{n}\left(a + \sum_{j=2; j \neq i}^n c_j(n-1)k_i - (n-1)c_i\right) + \frac{1}{n}\left(-\sum_{j=2; j \neq i}^n k_j + (n-1)k_i\right)e^{-(r+m)(T-t)}.$$

Substitute Formula (13) into Formulas (1) and (3) and solve the problem in Formulas (1) and (3) using the Pontryagin maximum principle for $i = 1$. An optimal strategy of the first player has the form

$$x_1^*(t) = \max\left(0, x_1^0\right), \tag{14}$$

where

$$\begin{aligned} x_1^0(t) &= \frac{a}{2} - \frac{n}{2}c_1 + \frac{1}{2}e^{-(r+m)(T-t)}\sum_{i=2}^n\left(\sum_{j=2; j \neq i}^n k_j - (n-1)k_i\right) - \\ &\frac{1}{2}\sum_{i=2}^n\left(\sum_{j=2; j \neq i}^n c_j - (n-1)c_i\right) - \frac{k_1n}{2}e^{-(r+m)(T-t)} \end{aligned}$$

Thus, in Stackelberg equilibrium, the first player (leader) chooses her strategy, Formula (14). Given the leader’s strategy, other players choose their strategies according to Formula (13). Given all players’ strategies, the state variable is determined using the solution of Formula (3) and the payoffs are determined using Formula (1).

In an inverse Stackelberg game [21,22] based on the model in Formulas (1)–(3), the leader reports to each follower her strategy, with feedback on their control

$$x_{1i}^0(t) = \begin{cases} x_{1i}^R(t), & \text{if } x_i(t) = x_i^R(t), \quad i = 2, 3, \dots, n \\ x_{1i}^P(t), & \text{otherwise} \end{cases}$$

If a follower refuses to cooperate with the leader, then she punishes the follower using the punishment strategy $x_{1i}^P = (x_{12}^P, x_{13}^P, \dots, x_{1n}^P)$, which according to Formula (13) has the form ($i = 2, 3, \dots, n$).

$$x_{1i}^P(t) = \arg \min_{x_{1i} \geq 0} \left(\left((n-1)k_i - \sum_{j=2; j \neq i}^n k_j \right) e^{-(r+m)(T-t)} - x_{1i} + a - \sum_{j=2; j \neq i}^n c_j - (n-1)c_{1i} \right)$$

Then, a guaranteed result of the i -th follower is equal to

$$L_i = \max_{\{x_j^{NE}\}_{j=2}^n} J_i(\{x_{1k}^P\}_{k=2}^n, x_2, x_3, \dots, x_n) = J_i(\{x_{1k}^P\}_{k=2}^n, 0, 0, \dots, 0) = e^{-rT} y(T) = e^{-rT} (y_0 + k_1 T x_{1i}^P(T))$$

If the followers cooperate with the leader, then she chooses a reward strategy $x_{1i}^R = (x_{12}^R, x_{13}^R, \dots, x_{1n}^R)$. The reward strategies $(x_{12}^R, x_{13}^R, \dots, x_{1n}^R)$ are found as solutions of the optimal control problem

$$J_1 = \int_0^T e^{-rt} (a - c_1 - \bar{x}(t)) x_1(t) dt - e^{-rT} y(T) \rightarrow \max_{\{x_i(t)\}_{i=1}^n} \tag{15}$$

$$0 \leq x_i(t), \quad \forall t \in [0, T], \quad i = 1, 2, \dots, n;$$

$$\frac{dy}{dt} = \sum_{i=1}^n k_i x_i(t) - m y(t), \quad y(0) = y_0$$

$$J_i = \int_0^T e^{-rt} (a - c_i - \bar{x}(t)) x_i(t) dt - e^{-rT} y(T) > L_i; \quad i = 2, 3, \dots, n. \tag{16}$$

A solution of the problem in Formulas (15) and (16) was found numerically with computer simulation. The condition in Formula (16) provides that a reward is always more profitable for the followers than punishment. The payoffs of all players in the Stackelberg and inverse Stackelberg games are presented in Table 4.

The values of individual and collective indices of relative efficiency for different information structures are given in Table 5. In the last row of Table 5, the average values of the collective and individual efficiency indices on the set of simulation experiments are presented. Thus, we obtained the following preference systems:

society: $C \succ NE \succ IST \succ ST$;

agent-leader: $IST \succ ST \succ C \succ NE$;

agent-follower: $C \succ NE \succ ST \succ IST$.

As expected, cooperation is always preferable for the whole society and for followers. However, for the leader, the information structure of the inverse Stackelberg game is the most profitable as a rule. That is why the struggle for leadership arises often.

Table 5. Indices of relative efficiency for different information structures.

Example	NE		ST		IST	
	SCI	$K_1^{NE}/K_2^{NE}/K_3^{NE}$	SCI	$K_1^{ST}/K_2^{ST}/K_3^{ST}$	SCI	$K_1^{IST}/K_2^{IST}/K_3^{IST}$
1	0.99	0.45/0.05/2.48	0.94	0.69/0.03/2.1	0.91	0.81/0.02/1.9
2	0.99	0.45/0.05/2.48	0.93	0.68/0.03/2.09	0.91	0.76/0.03/1.95
3	0.98	1.64/0.26/1.05	0.98	1.89/0.21/0.85	0.97	1.97/0.17/0.78

Table 5. Cont.

Example	NE		ST		IST	
	SCI	$K_1^{NE}/K_2^{NE}/K_3^{NE}$	SCI	$K_1^{ST}/K_2^{ST}/K_3^{ST}$	SCI	$K_1^{IST}/K_2^{IST}/K_3^{IST}$
4	0.98	1.64/0.26/1.05	0.98	1.88/0.21/0.84	0.97	1.97/0.17/0.76
5	0.99	1.48/0.16/1.33	0.95	1.73/0.1/1.0	0.95	1.93/0.1/0.81
6	0.98	2.74/0.1/0.1	0.99	2.84/0.06/0.06	0.99	2.88/0.06/0.06
7	0.99	0.46/2.48/0.05	0.98	0.6/2.32/0.03	0.96	0.7/2.19/0.03
8	0.98	2.48/0.05/0.46	0.99	2.67/0.03/0.03	0.99	2.79/0.03/0.03
9	0.98	2.48/0.05/0.46	0.99	2.67/0.03/0.03	0.99	2.79/0.03/0.03
10	0.99	1.99/0.22/0.76	0.98	2.1/0.17/0.69	0.98	2.2/0.14/0.58
11	0.98	0.27/0.27/2.43	0.92	0.33/0.25/2.18	0.89	0.42/0.22/2.04
12	0.98	0.76/0.22/1.99	0.96	0.83/0.19/1.86	0.93	0.84/0.11/1.24
13	0.98	0.76/0.22/1.99	0.96	0.83/0.19/1.86	0.93	0.84/0.11/1.24
14	0.98	2.72/0.1/0.15	0.99	2.83/0.06/0.1	0.99	2.79/0.05/0.07
15	0.99	0.76/0.22/1.99	0.98	0.87/0.16/1.9	0.96	0.9/0.14/1.84
16	0.99	0.76/0.22/1.99	0.98	0.87/0.16/1.9	0.96	0.9/0.14/1.84
17	0.99	0.76/0.22/1.99	0.98	0.87/0.16/1.9	0.96	0.9/0.14/1.84
18	0.98	0.19/0.43/2.3	0.94	0.25/0.37/2.19	0.89	0.28/0.35/2.02
19	0.99	0.04/2.11/0.81	0.95	0.06/2.02/0.76	0.92	0.08/1.95/0.73
20	0.99	1.99/0.22/0.75	0.97	2.22/0.17/0.53	0.98	2.36/0.13/0.47
21	0.99	1.41/0.82/0.75	0.98	1.47/0.76/0.7	0.97	1.5/0.76/0.67
22	0.99	1.74/0.62/0.62	0.98	1.89/0.52/0.53	0.99	1.99/0.5/0.49
23	0.98	0.09/2.13/0.77	0.95	0.1/2.1/0.7	0.94	0.1/2.0/0.68
24	0.98	0.09/2.13/0.77	0.95	0.1/2.1/0.7	0.94	0.1/2.0/0.68
25	0.99	0.16/1.41/1.41	0.97	0.18/1.36/1.37	0.94	0.2/1.31/1.32
26	0.99	0.06/0.94/1.96	0.9	0.09/0.83/1.78	0.87	0.1/0.81/1.71
27	0.94	0.15/2.51/0.32	0.94	0.2/2.35/0.27	0.94	0.24/2.3/0.26
28	0.99	0.2/2.06/0.71	0.97	0.26/2.0/0.66	0.97	0.3/1.97/0.65
29	0.98	0.01/2.67/0.29	0.94	0.04/2.55/0.24	0.95	0.11/2.52/0.22
30	0.98	0.01/1.81/1.11	0.91	0.04/1.68/1.02	0.91	0.07/1.66/1.00
31	0.98	0.35/1.01/1.54	0.94	0.44/0.92/1.47	0.95	0.53/0.9/1.42
32	0.99	0.35/1.05/1.54	0.94	0.43/0.98/1.42	0.95	0.49/0.96/1.4
33	0.99	0.06/0.94/1.96	0.94	0.14/0.84/1.85	0.94	0.18/0.82/1.81
34	0.99	2.91/0.01/0.05	0.99	2.96/0.01/0.01	0.99	2.98/0.01/0.01
35	0.98	2.87/0.02/0.14	0.99	2.89/0.01/0.06	0.99	2.93/0.01/0.05
	0.99	1/0.87/1.09	0.96	1.11/0.8/0.96	0.97	1.17/0.77/0.96

4. A Model with Consideration of the Green Effect

Now, the model takes the form

$$J_i = \int_0^T \left(e^{-rt} (a - c_i - \bar{x}(t) + \alpha \bar{g}(t)) x_i(t) - \beta_i g_i^2(t) \right) dt - e^{-rT} y(T) \rightarrow \max \quad (17)$$

$$0 \leq x_i(t), 0 \leq g_i(t), \forall t \in [0, T], i = 1, \dots, n; \quad (18)$$

$$\frac{dy}{dt} = \sum_{i=1}^n (k_i x_i(t) - \gamma_i g_i(t)) - my(t), y(0) = y_0 \quad (19)$$

where $g_i(t)$ characterizes green efforts of the i -th firm, α is the coefficient of demand increasing due to the green effect, β_i is the green effort coefficient, and γ_i is the coefficient of additional decreasing of the pollution due to green efforts.

In the case of symmetrical agents, the model takes the form

$$J = \int_0^T \left(e^{-rt} (a - c - nx(t) + \alpha ng(t)) x(t) - \beta g^2(t) \right) dt - e^{-rT} y(T) \rightarrow \max \quad (20)$$

$$0 \leq x(t), 0 \leq g(t), \forall t \in [0, T], \quad (21)$$

$$\frac{dy}{dt} = n(kx(t) - \gamma g(t) - my(t)), \quad y(0) = y_0 \tag{22}$$

The agents' strategies contain two control actions (functions $x_i(t)$ and $g_i(t)$). The Hamilton function for each player in the symmetrical model in Formulas (20)–(22) has the form

$$H(x, g, y, \lambda) = (a - c - nx(t) + \alpha ng(t))x(t) - \beta g^2(t) + \lambda(n(kx(t) - \gamma g(t)) - my(t)).$$

We obtain

$$\frac{\partial H}{\partial x} = a - c - 2nx + \alpha ng + \lambda nk = 0; \quad \frac{\partial H}{\partial g} = \alpha nx - 2\beta g - \gamma n\lambda = 0.$$

So, if $4n\beta - \alpha^2 n^2 \neq 0$, then

$$g^0(t) = \frac{\alpha nx^0 - \gamma n\lambda}{2\beta}; \quad x^0(t) = \frac{(a - c)2\beta - \gamma n^2 \alpha \lambda + 2\beta \lambda kn}{4n\beta - \alpha^2 n^2} \tag{23}$$

If $4n\beta - \alpha^2 n^2 = 0$ and $\gamma n\lambda \neq 2\beta(a - c + \lambda kn)$, then the maximum is attained on the bound of the set of feasible controls (at least one of the optimal controls is not internal).

If $4n\beta - \alpha^2 n^2 = 0$ and $\gamma n\lambda = 2\beta(a - c + \lambda kn)$, then $g^0(t) = \frac{\alpha nx^0 - \gamma n\lambda}{2\beta}$; $x^0(t)$ is an arbitrary function that belongs to the set of feasible strategies, for example, $x^0(t) \equiv 0 \forall t \geq 0$. Denote

$$A = \frac{(a - c)2\beta}{4n\beta - \alpha^2 n^2}; \quad B = \frac{-\gamma n^2 \alpha \lambda + 2\beta \lambda kn}{4n\beta - \alpha^2 n^2}; \quad C = \frac{\alpha nA}{2\beta}; \quad D = \frac{\alpha n\beta - \gamma n}{2\beta}$$

Given $\lambda(t) = -e^{-(r+m)(T-t)}$, if $4n\beta - \alpha^2 n^2 \neq 0$, then

$$x^0(t) = A - e^{-(r+m)(T-t)} B; \quad g^0(t) = C - e^{-(r+m)(T-t)} D.$$

Notice that $\frac{\partial^2 H}{\partial x^2} = -2n$; $\frac{\partial^2 H}{\partial g^2} = -2\beta$; $\frac{\partial^2 H}{\partial x \partial g} = \alpha n$.

Therefore, the found value is a maximizer if a sufficient condition $\Delta = 4n\beta - \alpha^2 n^2 > 0$ is true and the value belongs to the set of feasible strategies, i.e., for $\forall t \geq 0$; $A \geq e^{-(r+m)(T-t)} B$; $C \geq e^{-(r+m)(T-t)} D$. If at least one of the inequalities

$$A \geq e^{-(r+m)(T-t)} B; \quad C \geq e^{-(r+m)(T-t)} D; \quad 4n\beta - \alpha^2 n^2 > 0 \tag{24}$$

is false, then in dependence on the input model parameters, the maximum is obtained in one of the boundary points

$$\left(0, \frac{\gamma m}{2\beta} e^{-(r+m)(T-t)}\right) \text{ or } \left(\frac{a - c - nke^{-(r+m)(T-t)}}{2n}, 0\right)$$

Thus,

$$(x^*, g^*) = \begin{cases} (x^0, g^0) & \text{if } A \geq e^{-(r+m)(T-t)} B; \quad C \geq e^{-(r+m)(T-t)} D; \quad 4n\beta - \alpha^2 n^2 > 0 \\ \left(0, \frac{\gamma m}{2\beta} e^{-(r+m)(T-t)}\right) \text{ or } \left(\frac{a - c - nke^{-(r+m)(T-t)}}{2n}, 0\right) & \text{otherwise.} \end{cases} \tag{25}$$

The state variable was calculated using the method of parameter variation. Given the inequalities in Formula (18), it is explained by the formula

$$y(t) = y_0 e^{-mt} + \frac{E}{m} (1 - e^{-mt}) - \frac{F}{r + 2m} (e^{-(r+m)(T-t)} - e^{-mt - (r+m)T})$$

when $E = nkA - \gamma nC; F = nkB - \gamma nD$. The payoffs are equal to

$$J = \frac{G}{r} (1 - e^{-rT}) - \frac{H}{m} (e^{-rT} - e^{-(r+m)T}) + \frac{I}{r + 2m} (e^{-rT} - e^{-2(r+m)T}) + y(T)e^{-rT},$$

where

$$G = (a - c - nA + \alpha nC) - \beta D^2; H = (a - c - nA + \alpha nC)B + A(\alpha nD - nB) - 2\beta CD;$$

$$I = B(-nB + \alpha nD) - \beta D^2.$$

In the case of cooperation, the model takes the form

$$J = \sum_{i=1}^n J_i = \sum_{i=1}^n \int_0^T ((a - c_i - \bar{x}(t) + \alpha \bar{g}(t))x_i(t) - \beta g_i^2(t))dt - ne^{-rT}y(T) \rightarrow \max$$

$$0 \leq x_i(t), 0 \leq g_i(t), \forall t \in [0, T], i = 1, 2, \dots, n.$$

$$\frac{dy}{dt} = \sum_{i=1}^n (k_i x_i(t) - \gamma_i g_i(t)) - my(t), \quad y(0) = y_0$$

The maximum is obtained by the values

$$x(t) = (x_1(t), x_2(t), \dots, x_n(t)); g(t) = (g_1(t), g_2(t), \dots, g_n(t))$$

Nash equilibrium was calculated numerically using computer simulation [22,23]. The input data are given in Table 6, and the results for symmetrical agents with consideration of the green effect are presented in Table 7 for the input data from Table 6.

Table 6. Input data for symmetrical agents with consideration of the green effect.

Example	<i>n</i>	<i>a</i>	<i>c</i>	<i>k</i>	<i>m</i>	<i>y</i> ₀	α	β	γ
1	5	30	15	1	2	10	0.01	1	0.1
2	5	20	15	1	2	10	0.01	1	0.1
3	5	17	15	1	2	10	0.01	1	0.1
4	5	10	15	1	2	10	0.01	1	0.1
5	5	40	15	1	2	10	0.01	1	0.1
6	10	20	15	1	2	10	0.01	1	0.1
7	5	20	5	1	2	10	0.01	1	0.1
8	5	30	15	1	2	10	0.001	1	0.1
9	5	30	15	1	2	10	0.5	1	0.1
10	5	30	15	1	2	10	0.5	2	0.1
11	5	30	15	1	2	10	0.5	10	0.1
12	5	30	15	1	2	10	0.5	5	0.01
13	5	30	15	1	2	10	0.5	5	0.5
14	5	30	15	1	2	10	0.5	1	0.5
15	5	20	15	5	2	2	0.5	1	0.5
16	5	20	15	5	2	2	0.5	0.5	0.5
17	5	20	15	5	2	2	0.5	1	0.5
18	5	20	15	5	2	2	0.1	1	0.5
19	20	20	15	1	2	10	0.1	1	0.1
20	2	30	15	1	2	10	0.1	1	0.1
21	2	20	5	1	2	10	0.1	1	0.1
22	2	20	15	1	10	10	0.1	1	0.1
23	2	20	15	1	2	1	0.1	1	0.1
24	2	20	15	1	2	20	0.1	1	0.1
25	2	20	15	1	2	20	0.5	1	0.1
26	2	20	15	1	2	20	0.1	0.5	0.1
27	2	20	15	1	2	20	0.1	1	0.5
28	2	20	15	1	2	20	0.5	1	0.5
29	2	20	15	1	2	20	0.3	0.5	0.1
30	2	20	15	1	2	20	0.8	1	0.3
31	10	20	5	1	2	1	0.1	1	0.1
32	10	20	5	1	2	20	0.1	1	0.1
33	10	20	5	1	2	3	0.1	1	0.1
34	10	30	15	1	2	10	0.1	1	0.1
35	10	20	15	3	2	10	0.1	1	0.1
36	10	20	5	1	6	15	0.4	1	0.3

Table 7. Payoffs for different information structures for symmetrical agents with consideration of the green effect.

No.	NE			C		
	<i>J</i>	<i>y(T)</i>	<i>J</i>	<i>J</i>	<i>y(T)</i>	<i>J</i>
1	13,417	3.1	69,012	69,012	3.3	69,012
2	1490	0.6	7611	7611	1	7611
3	238	0	1195	1195	0	1195
4	1491	0	7567	7567	0	7567
5	37,275	5.6	189,455	189,455	5.6	189,455
6	745	0	7570	7570	0	7570
7	13,418	3.1	670,845	670,845	3.1	670,845
8	13,416	3.1	670,922	670,922	3.1	670,922
9	19,515	4	99,011	99,011	4.5	99,011
10	15,901	3.5	80,112	80,112	3.7	80,112
11	13,849	3.2	70,238	70,238	2.8	70,238
12	14,310	3.3	71,987	71,987	3.7	71,987
13	14,310	2.8	71,987	71,987	3.4	71,987
14	19,517	1.1	98,389	98,389	1.5	98,389
15	2160	0	11,100	11,100	0	11,100
16	3964	0	19,910	19,910	0	19,910
17	2160	0	11,100	11,100	0	11,100
18	1502	0	7732	7732	0	7732
19	391	0	7911	7911	0	7911
20	33,713	3.5	68,322	68,322	3.5	68,322
21	33,716	0.7	68,324	68,324	1.1	68,324
22	3746	0.2	7623	7623	0.4	7623
23	3745	1	7623	7623	1.4	7623
24	3745	1	7623	7623	1.3	7623
25	1459	1.1	3027	3027	1.4	3027
26	3764	1	7655	7655	1.2	7655
27	3745	0.8	7623	7623	1.3	7623
28	4259	0.8	8915	8915	1.3	8915
29	4095	1	8312	8312	1.3	8312
30	5480	1.1	11,011	11,011	1.2	11,011
31	6879	2.4	69,114	69,114	2.6	69,114
32	6879	2.4	69,114	69,114	2.6	69,114
33	6879	2.4	69,114	69,114	2.6	69,114
34	6879	2.4	69,114	69,114	2.6	69,114
35	759	0	7701	7701	0	7701
36	11,182	0.3	113,532	113,532	0.5	113,532

In the case of arbitrary agents (model in Formulas (17)–(19)), the Nash equilibria for selfish behavior and cooperative solutions were calculated numerically using computer simulation. In Table 8 the input data are given, and in Table 9 the results for three arbitrary agents at $T = 1200$ and $r = 0.001$ for the input data from Table 8 are presented.

Table 8. Input data for three arbitrary agents with consideration of the green effect.

No.	<i>a</i>	<i>c</i> ₁	<i>c</i> ₂	<i>c</i> ₃	<i>k</i> ₁	<i>k</i> ₂	<i>k</i> ₃	<i>m</i>	<i>y</i> ₀	β_1	β_2	β_3	γ_1	γ_2	γ_3	α
1	30	12	5	10	1	1	5	2	10	1	5	2	0.01	0.5	0.1	0.01
2	20	3	7	5	5	1	2	2	10	5	1	2	0.1	0.5	0.01	0.01
3	17	3	5	5	1	2	4	2	10	2	1	3	0.2	0.1	0.05	0.01
4	10	1	2	1	1	2	5	2	10	1	5	1	0.5	0.1	0.2	0.01
5	40	10	2	5	1	5	1	2	10	1	3	2	0.2	0.1	0.3	0.01
6	20	5	3	5	1	2	4	2	10	2	5	1	0.3	0.1	0.01	0.01
7	20	5	5	2	1	3	5	2	10	1	2	5	0.2	0.1	0.05	0.01

Table 8. Cont.

No.	a	c_1	c_2	c_3	k_1	k_2	k_3	m	y_0	β_1	β_2	β_3	γ_1	γ_2	γ_3	α
8	30	10	7	5	1	5	2	2	10	3	2	1	0.5	0.1	0.7	0.001
9	30	10	5	5	1	2	4	2	10	1	3	4	0.2	0.1	0.01	0.5
10	30	8	7	10	1	1	5	2	10	2	1	5	0.3	0.1	0.2	0.5
11	30	6	10	10	1	5	3	2	10	10	4	5	0.5	0.1	0.1	0.5
12	30	10	5	10	1	4	2	2	10	5	1	1	0.1	0.01	0.05	0.5
13	30	10	10	5	1	2	6	2	10	5	2	3	0.2	0.5	0.1	0.5
14	30	3	5	10	1	5	1	2	10	1	5	1	0.3	0.5	0.05	0.5
15	20	3	2	4	5	2	5	2	2	2	1	8	0.2	0.5	0.1	0.5
16	40	2	5	5	5	2	4	2	2	0.5	1	2	0.05	0.5	0.7	0.5
17	40	4	5	10	5	3	3	2	2	1	2	0.5	0.05	0.5	0.8	0.5
18	40	7	10	10	5	2	1	2	2	2	1	2	0.2	0.5	0.1	0.1
19	40	10	5	5	1	5	2	2	10	1	2	5	0.5	0.1	0.2	0.1
20	30	5	10	2	1	2	4	2	10	2	1	1	0.3	0.1	0.3	0.1
21	20	5	3	6	1	1	5	2	10	3	1	5	0.2	0.1	0.5	0.1
22	20	2	8	1	1	3	4	10	10	1	5	3	0.2	0.1	0.5	0.1
23	20	5	1	4	1	2	5	2	1	1	7	2	0.5	0.1	0.05	0.1
24	20	5	6	2	1	1	4	2	20	2	1	2	0.3	0.1	0.2	0.1
25	20	3	5	2	1	2	1	2	20	1	1	1	0.5	0.1	0.2	0.5
26	20	5	8	5	1	5	5	2	20	0.5	0.5	2	0.5	0.1	0.3	0.1
27	20	5	1	2	1	5	2	2	20	1	5	1	0.1	0.5	0.05	0.1
28	20	5	2	8	1	3	4	2	20	1	1	5	0.05	0.1	0.5	0.5
29	40	5	5	6	1	1	4	2	20	0.5	5	1	0.2	0.05	0.1	0.3
30	40	9	5	4	1	2	5	2	20	2	1	3	0.1	0.3	0.3	0.8
31	40	5	11	6	1	5	3	2	1	1	2	3	0.5	0.1	0.2	0.1
32	40	5	1	10	1	2	4	2	20	1	3	1	0.3	0.1	0.5	0.1
33	25	5	10	5	1	3	5	2	3	5	1	3	0.2	0.1	0.05	0.1
34	50	15	5	5	1	5	2	2	10	1	3	2	0.3	0.1	0.2	0.1

Table 9. Payoffs for different information structures with the green effect.

No.	NE			ST/IST			
	J_1	J_2	J_3	J_0	J_1	J_2	J_3
1	334	3274	550	2617/2911	150/144	2067/1715	400/350
2	726	314	954	1204/1813	467/404	127/114	609/538
3	542	277	581	848/1217	313/266	174/159	360/237
4	147	203	147	427/852	127/112	172/143	127/112
5	1043	2851	3988	6890/8122	861/622	2381/1765	3648/3288
6	342	652	834	1370/3282	296/211	466/352	608/388
7	432	432	1331	1557/2136	249/177	249/177	1059/899
8	572	1863	1467	2960/3172	390/311	1556/1186	1015/886
9	534	1237	2311	3214/3542	306/176	961/721	1947/1783
10	834	987	1237	2471/2712	701/524	821/604	949/745
11	2323	672	672	2724/3006	1862/1562	431/378	431/378
12	525	2631	525	2622/2879	325/256	1972/1568	325/256
13	495	495	2648	2596/2870	316/278	316/278	1963/1672
14	1554	2167	372	3029/3765	1136/756	1739/1453	154/144
15	566	783	822	1567/1872	408/278	500/389	658/465
16	2845	3456	1869	6401/7021	2061/1765	2909/2753	1432/1299
17	2311	3892	883	5833/6023	1844/1567	3337/3098	652/525
18	2391	2981	1456	5659/5762	1781/1566	2595/2385	1284/1098
19	2656	2431	2431	6322/6544	2298/1987	2012/1877	2012/1877
20	2254	411	1663	3183/3211	1742/1567	159/154	1283/1076
21	809	671	322	1294/1277	610/577	470/455	214/197
22	1145	188	1271	2303/2534	1008/899	110/102	1198/934
23	372	824	739	1391/1512	200/177	580/562	611/578

Table 9. Cont.

No.	NE			ST/IST			
	J_1	J_2	J_3	J_0	J_1	J_2	J_3
24	324	242	1302	1474/1634	252/213	162/138	1061/812
25	740	569	824	1836/2195	649/587	427/397	760/674
26	451	178	842	1208/1411	354/302	128/98	727/621
27	415	762	1253	1785/1784	204/199	585/568	995/982
28	551	1675	131	2233/2451	517/462	1607/1523	114/94
29	2231	2231	3912	7608/7923	1983/1642	1983/1765	3642/3277
30	652	1598	3426	4942/5342	474/414	1331/1189	3137/2873
31	2434	1253	3954	6637/7012	2001/1763	949/821	3687/3456
32	3962	2811	942	6712/6712	3467/3434	2502/2488	743/727
33	721	189	1542	2092/2112	642/623	141/128	1307/1198
34	1621	7234	3761	11,063/11,651	1062/821	6684/6431	3318/3176

Now consider a hierarchical setup with consideration of the green effect. Let a specific agent (principal) maximize the functional

$$J_0 = \int_0^T e^{-rt} \left((a - \bar{x}(t) + \alpha \bar{g}(t)) \bar{x}(t) - \sum_{i=1}^n (c_i x_i(t) + \beta_i g_i^2(t)) \right) dt - ne^{-rT} y(T) \rightarrow max$$

by controls $g_i(t); i = 1, 2, \dots, n$.

The other agents' payoff functionals retain the form Formula (17), but now, the maximization is conducted only by the controls $x_i(t)$. The equation of dynamics has the form Formula (19). Control constraints are of the form Formula (18) again.

In the Stackelberg game, similar to the preceding case, we obtained the solution in the game of agents in the form of Formula (25).

In Table 10, the values of the indices of collective and individual relative efficiency with consideration of the green effect are presented. The last row of Table 10 contains the average values of the respective indices. The indices of individual efficiency were calculated only for the case of cooperation.

Table 10. Indices of relative efficiency of the players for different information structures with consideration of the green effect.

Example	SCI	NE		SCI	ST/IST	
		$K_1/K_2/K_3$			$K_1^L/K_2^F/K_3^F$	
1	0.89	0.21/2.1/0.35		0.56/0.47	0.1/1.33/0.26	0.09/1.1/0.23
2	0.95	1.04/0.45/1.36		0.57/0.5	0.67/0.18/0.87	0.58/0.16/0.77
3	0.96	1.11/0.57/1.19		0.58/0.45	0.64/0.36/0.74	0.55/0.33/0.49
4	0.83	0.73/1.01/0.73		0.71/0.61	0.63/0.86/0.63	0.56/0.71/0.56
5	0.74	0.29/0.8/1.12		0.64/0.53	0.24/0.67/1.02	0.18/0.5/0.92
6	0.85	0.48/0.91/1.17		0.64/0.44	0.41/0.65/0.85	0.3/0.5/0.54
7	0.92	0.55/0.55/1.68		0.66/0.44	0.31/0.31/1.34	0.22/0.22/1.13
8	0.94	0.37/1.2/0.95		0.64/0.49	0.25/1.0/0.65	0.2/0.77/0.57
9	0.88	0.35/0.8/1.5		0.69/0.58	0.2/0.62/1.26	0.11/0.47/1.15
10	0.78	0.63/0.75/0.94		0.63/0.47	0.53/0.62/0.72	0.4/0.46/0.57
11	0.85	1.62/0.43/0.47		0.63/0.54	1.3/0.3/0.29	1.01/0.26/0.26
12	0.8	0.34/1.72/0.34		0.57/0.45	0.21/1.29/0.21	0.17/1.02/0.17
13	0.8	0.33/0.33/1.74		0.57/0.47	0.2/0.2/1.29	0.18/0.18/1.10
14	0.75	0.86/1.2/0.21		0.56/0.43	0.49/0.96/0.08	0.42/0.8/0.08
15	0.9	0.71/0.98/1.03		0.65/0.45	0.45/0.62/0.82	0.35/0.49/0.58
16	0.81	0.81/0.98/0.53		0.61/0.55	0.62/0.83/0.41	0.5/0.78/0.37
17	0.75	0.73/1.23/0.28		0.62/0.54	0.61/1.06/0.21	0.5/0.98/0.17

Table 10. Cont.

Example	NE		ST/IST	
	SCI	$K_1/K_2/K_3$	SCI	$K_1^I/K_2^F/K_3^E$
18	0.85	0.89/1.11/0.54	0.7/0.63	0.65/0.97/0.48/0.58/0.89/0.41
19	0.82	0.87/0.8/0.77	0.69/0.63	0.75/0.66/0.66/0.65/0.62/0.62
20	0.75	1.17/0.21/0.86	0.55/0.48	0.9/0.08/0.66/0.81/0.08/0.56
21	0.84	1.13/0.93/0.45	0.6/0.57	0.85/0.65/0.3 /0.8/0.63/0.27
22	0.97	1.27/0.21/1.41	0.86/0.72	1.12/0.12/1.33/1.0/0.11/1.04
23	0.72	0.42/0.92/0.82	0.52/0.49	0.22/0.65/0.68/0.2/0.63/0.65
24	0.79	0.41/0.31/1.64	0.62/0.49	0.32/0.2/1.34/0.27/0.17/1.03
25	0.91	0.95/0.73/1.06	0.78/0.69	0.83/0.55/0.97/0.75/0.51/0.86
26	0.88	0.81/0.32/1.51	0.72/0.61	0.63/0.23/1.3/0.54/0.18/1.11
27	0.92	0.47/0.87/1.43	0.68/0.66	0.23/0.67/1.13/0.23/0.65/1.12
28	0.99	0.69/2.1/0.16	0.94/0.87	0.65/2.02/0.14/0.58/1.92/0.12
29	0.91	0.73/0.73/1.28	0.83/0.73	0.65/0.65/1.19/0.54/0.58/1.07
30	0.79	0.27/0.67/1.43	0.59/0.62	0.19/0.55/1.31/0.17/0.49/1.21
31	0.83	0.8/0.41/1.29	0.72/0.66	0.66/0.31/1.21/0.58/0.27/1.13
32	0.68	1.04/0.74/0.25	0.59/0.59	0.92/0.66/0.2/0.91/0.66/0.19
33	0.82	0.72/0.19/1.55	0.7/0.65	0.65/0.14/1.31/0.63/0.13/1.2
34	0.83	0.32/1.44/0.75	0.73/0.69	0.21/1.32/0.66/0.16/1.28/0.63
	0.84	0.71/0.85/0.96	0.66/0.57	0.58/0.69/0.95/0.49/0.58/0.72

In this case, we obtained the same preference system for the whole society and the followers:

$$C \succ NE \succ ST \succ IST.$$

This preference system remains the same as the system without consideration of the green effect. However, the consideration of the green effect makes the agents' interests more diverse, and the whole economic system becomes less (for some input data sets, essentially less) compatible. In this case, for the whole society, cooperation is much better than other ways of organization.

5. Conclusions

The proposed system of individual and collective indices of the relative efficiency of the ways of organization [2] was used for the analysis of differential game-theoretic models of Cournot oligopoly with consideration of the green effect and without it. For calculation of the indices, we applied averaging on the set of simulation experiments. The numerical calculations showed that the introduced indices allow for evaluation of the average efficiency of different ways of organization (information structures) and for practical recommendations on improving system compatibility.

The preference systems for the whole society and separate agents are contradictory. Cooperation is more profitable for the society and followers. However, the leader prefers a hierarchy in the form of an inverse Stackelberg game that advocates for struggle for leadership. Moreover, for non-symmetrical agents, cooperation may be either more profitable than selfish behavior or vice versa (Table 10, column 3). Notice also that the consideration of the green effect makes cooperation much more profitable (up to 50%) for the whole society and followers.

In the future, we plan to study static and dynamic game-theoretic models of Cournot oligopoly in the form of characteristic function. In addition, the models with a network structure both in normal form and in the form of a characteristic function will be considered. At last, model identification will be precisely performed.

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References

- Zhang, W.; Zhao, S.; Wan, X. Research on Industrial Digital Transformation Strategies Based on Differential Games. *Appl. Math. Model.* **2021**, *98*, 90–108. [[CrossRef](#)]
- Ougolnitsky, G.A. An Approach to Compare Organizational Modes of Active Agents and Control Methods. *Control Sci.* **2022**, *3*, 24–33.
- Basar, T.; Zhu, Q. Prices of Anarchy, Information, and Cooperation in Differential Games. *J. Dyn. Games Appl.* **2011**, *1*, 50–73. [[CrossRef](#)]
- Cairns, R.D.; Martinet, V. An environmental-economic measure of sustainable development. *Eur. Econ. Rev.* **2014**, *69*, 4–17. [[CrossRef](#)]
- Maskin, E.; Tirole, J. A Theory of Dynamic Oligopoly, III. Cournot Competition. *Eur. Econ. Rev.* **1987**, *31*, 947–968. [[CrossRef](#)]
- Geras'kin, M.I. The Properties of Conjectural Variations in the Nonlinear Stackelberg Oligopoly Model. *Autom. Remote Control* **2020**, *6*, 1051–1072. [[CrossRef](#)]
- Geras'kin, M.I. Approximate Calculation of Equilibria in the Nonlinear Stackelberg Oligopoly Model: A Linearization Based Approach. *Autom. Remote Control* **2020**, *9*, 1659–1678. [[CrossRef](#)]
- Algazin, G.I.; Algazina, Y.G. Reflexion Reflexive Dynamics in the Cournot Oligopoly under Uncertainty. *Autom. Remote Control* **2020**, *81*, 287–301. [[CrossRef](#)]
- Algazin, G.I.; Algazina, Y.G. Reflexion Processes and Equilibrium in an Oligopoly Model with a Leader. *Autom. Remote Control* **2020**, *7*, 1258–1270. [[CrossRef](#)]
- Xiao, Y.; Peng, Y.; Lu, Q.; Wu, X. Dynamic investigations in a Stackelberg model with differentiated products and bounded rationality. *J. Comput. Appl. Math.* **2022**, *414*, 114409. [[CrossRef](#)]
- Peng, Y.; Xiao, Y.; Lu, Q.; Wu, X.; Zhao, Y. Chaotic dynamics in Cournot duopoly model with bounded rationality based on relative profit delegation maximization. *Phys. A Stat. Mech. Its Appl.* **2020**, *560*, 125174. [[CrossRef](#)]
- Raoufinia, M.; Baradaran, V.; Shahjerdi, R. A dynamic differential duopoly game with sticky price and advertizing: Open-loop and closed-loop solutions. *Kybernetes* **2019**, *48*, 586–611. [[CrossRef](#)]
- Al-Khedhairi, A. Dynamical Study of Competition Cournot-like Duopoly Games Incorporating Fractional Order Derivatives and Seasonal Influences. *Int. J. Nonlinear Sci. Numer. Simul.* **2020**, *21*, 339–359. [[CrossRef](#)]
- Julien, L.A. On noncooperative oligopoly equilibrium in the multiple leader-follower game. *Eur. J. Oper. Res.* **2017**, *256*, 650–662. [[CrossRef](#)]
- Zouhar, J.; Zouharova, M. Stackelberg versus Cournot duopoly with asymmetric costs: Primary markups, entry deterrence, and a comparison of social welfare and industry profits. *Econ. Theory Bull.* **2020**, *8*, 89–96. [[CrossRef](#)]
- Azevedo, S.G.; Carvalho, H.; Machado, V.C. The influence of green practices on supply chain performance: A case study approach. *Transp. Res. Part E Logist. Transp. Rev.* **2011**, *47*, 850–871. [[CrossRef](#)]
- Fahimnia, B.; Sarkis, J.; Davarzani, H. Green supply chain management: A review and bibliometric analysis. *Int. J. Prod. Econ.* **2015**, *162*, 101–114. [[CrossRef](#)]
- Sharma, A.; Jain, D. Game-Theoretic Analysis of Green Supply Chain Under Cost-Sharing Contract with Fairness Concern. *Int. Game Theory Rev.* **2021**, *2*, 205–217. [[CrossRef](#)]
- Pontryagin, L.S.; Boltayanskii, V.G.; Gamkrelidze, R.V.; Mishchenko, E.F. *The Mathematical Theory of Optimal Processes*; Wiley: Hoboken, NJ, USA, 1962.
- Ougolnitsky, G.A.; Usov, A.B. Computer Simulations as a Solution Method for Differential Games. In *Computer Simulations: Advances in Research and Applications*; Pfeffer, M.D., Bachmaier, E., Eds.; Nova Science Publishers: Hauppauge, NY, USA, 2018; pp. 63–106.
- Olzder, G.Y. Phenomena in Inverse Stackelberg Games, Part 2: Dynamic Problems. *J. Optim. Theory Appl.* **2009**, *143*, 601–618. [[CrossRef](#)]
- Gorelov, M.A.; Kononenko, A.F. Dynamic models of conflicts. III. Hierarchical games. *Autom. Remote Control* **2015**, *76*, 264–277. [[CrossRef](#)]
- Ugol'nitskii, G.A.; Usov, A.B. Equilibria in models of hierarchically organized dynamic systems with regard to sustainable development conditions. *Autom. Remote Control* **2014**, *6*, 1055–1068. [[CrossRef](#)]
- Ougolnitsky, G.A.; Usov, A.B. Solution algorithms for differential models of hierarchical control systems. *Autom. Remote Control* **2016**, *5*, 872–880. [[CrossRef](#)]

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