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Unboxing Okun's relation between economic growth and unemployment rate: Evidence from the United States, 1948-2024

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Article

Unboxing Okun's Relation Between Economic Growth and Unemployment Rate: Evidence from the United States, 1948-2024

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Abstract: We develop the first disaggregation of Okun's law that quantifies all of the information that is subsumed within its coefficients. The proposed method decomposes the coefficients into the sum of the direct effect of the change in output upon the unemployment rate, plus the indirect effects of the variations in the output per hour worked, the hours worked per employed person, the participation rate, and the size of the working-age population. With quarterly data for the United States from 1948 to 2024, we found that the value of the intercept in Okun's relation is determined by the increases in working-age population and output per hour of work, along with the decrease in the number of hours worked per employed person, plus the growth of the participation rate until the 1990s and its subsequent decline. For its part, the slope, that is, the value of Okun's coefficient, depends mainly upon the variations in output per hour of work and the hours per employed person. The other factors were scarcely relevant. Changes in these components caused the Okun's relation to vary over time, showing a greater sensitivity of the unemployment rate to variations in production since the 2008 crisis.

Keywords: Okun's law; factor decomposition; unemployment rate; labor productivity; hours worked; participation rate; working age population



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1. Introduction

Early in the 1960s, Arthur Okun described the relation between unemployment and economic growth as simply as possible. His purpose was, indeed, to provide a "simple and direct" formula (Okun, 1962, p. 2). The result, which he obtained by examining quarterly data from the United States for the years 1947–1960, was that "on the average, each extra percentage point in the unemployment rate above four percent has been associated with about a three percent decrement in real GNP [gross national product]" (Okun, 1962, p. 2). Thirlwall (1969) soon gave the name "Okun's law" to this relationship of interchange of three percent of real GNP for each additional percentage point in the unemployment rate.

Gordon (1984, p. 539) argues that Okun's law owes its popularity both to the stability of that relationship during the space of two decades, and to the fact that "it short-circuits the rather complex identity that links output and unemployment". Similarly, Anderton et al. (2014, p. 6) noted that the "elegance and simplicity" of Okun's approach is an important reason why Okun's law is still a recurrent theme half a century after its first formulation.

In recent years, several authors have discussed the precision and temporal stability of Okun's rule of thumb. For example, Ball et al. (2017, p. 1439) conclude that Okun's law "is strong and stable by the standards of macroeconomics". Similarly, Daly et al. (2017, p. 18) advise us that in the United States, "the Okun coefficient is relatively stable over time". However, Huang and Lin (2008) observe that a fixed coefficient may be inappropriate, Economies 2025, 13, 59 2 of 21

and Grant (2018) finds that the coefficient changed after the 1980s. Other authors who have examined the temporal stability of Okun's law include Aguiar-Conraria et al. (2020), Karlsson and Österholm (2020), and Krüger and Neugart (2021). Also, some analyses of Okun's law compare various countries (Furceri et al., 2020; Fontanari et al., 2022; Porras-Arena & Martín-Román, 2023) or regions (Guisinger et al., 2018; Maza, 2022; Porras-Arena et al., 2024). In addition, the variation in Okun's coefficient has been analyzed according to the phase of the economic cycle (Nebot et al., 2019; Benos & Stavrakoudis, 2022; Donayre, 2022) and the type of shock that produced each economic crisis (Anderton et al., 2014; Gelfer, 2020; Ziegenbein, 2021).

In general, such investigations of Okun's law have modified it by including additional or lagged variables, or by using other functional forms and techniques for estimation. These modifications were attempts to improve fits to the cases that were studied, but have moved away from the essence of Okun's problem and do not take advantage of information that is contained in Okun's original formulation.

To understand this point, please recall that Okun (1962) described the relation between the unemployment rate and increase in production via "reduced form" equations (Mairesse, 1984). Okun was aware that other factors affect this relation, but assumed that the unemployment rate is a satisfactory proxy for them. In the last pages of his 1962 article, Okun attempted to separate the effects of each factor, but described only their influence, in hopes that future advances in analytical techniques and availability of data would allow a more systematic study. Along those lines, later authors (e.g., Hamada & Kurosaka, 1984; Sögner & Stiassny, 2002; Ismihan, 2016) attempted to disaggregate Okun's coefficients into structural parameters that complete the output–unemployment relation. However, the results were only approximate.

The principal contribution of the present article is the development of a procedure that disaggregates the slope and intercept of Okun's law into an exact sum of the direct effect of variations in production upon the unemployment rate, and the indirect effects of variations in output per hour worked, hours worked per employed person, participation rate, and working-age population. This procedure recovers the information that is implicit in Okun's equation and quantifies the contribution of each factor to the value of Okun's coefficients. The present article will limit itself to the case of the United States during 1948–2024. However, the proposed disaggregation enables investigators to study how each factor's contribution varies with time, as well as among countries and regions. This markedly enriches the possibilities for performing systematic multivariate analyses based on Okun's law. An initial and much shorter version of this article, published as a letter (Peláez-Herreros, 2025), only decomposes the slope of the Okun relationship from a different conceptual basis. That earlier version does not analyze the intercept, nor the inverse relationship, nor does it use quarterly data or rolling regressions; it only divides the period 1970–2021 into two subperiods and notes that there are differences between them, but it does not analyze the evolution of the parameters over time, which will be performed in the following pages.

The causes of geographic and temporal variations in Okun's coefficients have been analyzed previously by authors such as Blackley (1991), Herwartz and Niebuhr (2011), and Kraim et al. (2023). These authors used a two-step analysis. First, they estimated Okun's coefficients for the time periods, countries, or regions that were being compared. Then, the authors explained the differences via cross-sectional or panel data regression models. They included regressor variables such as the industrial structure (Gonzalez-Prieto et al., 2018), the percentage of public employment (Binet & Facchini, 2013; Furceri et al., 2020), or the percentage of women and youths in the employed population (Maza, 2022), among other variables that can affect the Okun's coefficient.

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Those earlier investigations analyzed a wide range of secondary variables. Here, we analyze only those which, by definition, generate the value of the coefficients in Okun's law. Our objective is to estimate the Okun's law coefficients for the United States during distinct time periods, so that we can determine not only whether the values of these coefficients are stable over time, but also identify, precisely, which intrinsic factors give rise to those values and associated dynamics. Our analyses include data from quarters that fall within the period of the COVID-19 pandemic, so that we may examine whether the associated variation in production affected the employment rate differently during that time, as has been indicated by Barisic and Kovac (2022), and Porras-Arena et al. (2024). The hypothesis is that the intensity and composition of the relation between economic growth and unemployment rate changed over time. The proposed method allows us to detect and analyze the intrinsic causes of these changes.

The structure of this article is as follows. Section 2 reviews Okun's approach, including its bases and implications. Section 3 develops the procedure for decomposing the Okun's law coefficients into their intrinsic factors. Section 4 shows the results of analyses for the United States, after which Section 5 presents conclusions.

2. The Essence of Okun's Proposal

Okun (1962, p. 1) sought to answer the question, "How much output can the economy produce under conditions of full employment?" To that end, he analyzed the relation between variations in production and unemployment. Although Okun recognized that other factors affect this relation, for simplicity, he assumed the following:

whatever the influence of slack economic activity on average hours, labor force participation, and manhour productivity, the magnitudes of all these effects are related to the unemployment rate. With this assumption, the unemployment rate can be viewed as a proxy variable for all the ways in which output is affected by idle resources. (Okun, 1962, p. 2)

This simplification allowed Okun to model the relation between the two principal variables (production and unemployment) as follows:

$$\Delta U_t = \alpha_0 + \alpha_1 y_t + \varepsilon_0 \tag{1}$$

where U_t is the unemployment rate for the year t, and Y_t is the real gross domestic product (GDP). Thus, $\Delta U_t = U_t - U_{t-1}$ is the change in unemployment rate between t-1 and t, and $y_t = \Delta Y_t / Y_{t-1}$ is the percentage change in real GDP. The symbols α_0 and α_1 are model parameters, and ε_0 is the error term.

Okun (1962) estimated that for 55 quarterly variations from 1947 to 1960, $\alpha_0 = 0.30$ and $\alpha_1 = -0.30$, with a correlation of r = 0.79. Okun confirmed these results by analyzing the same data according to two other models.

Okun's second model explained the unemployment rate by means of a constant term (which represents the natural unemployment rate) and the gap between observed and potential output. Okun (1962) found that the constant term was equal to 3.72, and the slope of the model was 0.36. The latter value indicated that an increase of 1% in the difference between the real and potential output is associated with an increase of 0.36 points in the unemployment rate. Ball et al. (2017) explain that this second model is more general than the first. When the natural unemployment rate is constant and the potential output grows at a constant rate, this second model expressed in differences yields Equation (1). Therefore, Equation (1) assumes the constancy of these elements.

The third model related the logarithm of the employment rate to the logarithm of output, a time trend, and a constant. Okun (1962, p. 3) found that, depending upon the

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selected time period, "the estimated elasticity coefficient ran 0.35 to 0.40, suggesting that each one percentage point reduction in unemployment means slightly less than a 3 percent increment in output".

As Okun (1962, p. 3) notes, "The uniformity that emerged from these various techniques was the approximate 3–to–1 link between output and the unemployment rate". This three-to-one ratio (Okun's law) had a great impact upon national politics and economics because it showed that the link between the growth in production and decrease in unemployment was stronger than was commonly accepted.

Okun (1962, p. 5) explains that "the simple addition of one percent of a given labor force to the ranks of the employed would increase employment by only slightly more than one percent: 100/100–*U* percent to be exact. If the workweek and productivity were unchanged, the increment to output would be only that one + percent. The three percent result implies that considerable output gains in a period of rising utilization rates stem from some or all of the following: induced increases in the size of the labor force; longer average weekly hours; and greater productivity".

Okun cautions that the coefficients of approximately one-third that derive from his models do not reflect a ceteris paribus relation between changes in output and the unemployment rate. The ceteris paribus Okun coefficient for an unemployment rate of (for example) 6% would be approximately $(1 + 0.06)^{-1} = 0.94$. As Okun was aware, his coefficients capture not only this direct effect, but also the indirect effects of other variables that do not appear explicitly in his model, but do affect ΔU through their correlations with y. The value of Okun's coefficient, α_1 , indicates the amount by which ΔU will change for each unit change in y, when all of the implicit variables co-evolve as they did, on the average, during the period analyzed. In contrast, α_1 will change, and cease to be stable, if the correlations between the implicit variables and y change; for example, if the decrease in production that occurs during one recession is due more to the reduction in hours worked, than was the case during other recessions.²

The technique that will be developed in the next section allows an analysis of the changes in the values and content of Okun's coefficients.

3. Analysis Techniques and Data Sources

Okun proposed Equation (1) in order to summarize as simply as possible the relation between changes in output and the unemployment rate. For that reason, Okun omitted some variables that affect that relation. However, these variables are known. Okun himself mentions them, and Gordon (1973)—for example—has used them to define a complete relation between changes in output and the employment rate.

Gordon's most recent attempt along those lines (Gordon, 2010a) decomposes the real GDP into the product of five factors:

$$Y_t \equiv \frac{Y_t}{H_t} \cdot \frac{H_t}{E_t} \cdot \frac{E_t}{A_t} \cdot \frac{A_t}{N_t} \cdot N_t \tag{2}$$

where H_t is the number of hours worked during the period t, E_t is the number of workers, A_t is the number of active persons (employed and unemployed), and N_t is the working-age population. Each ratio has a specific interpretation: Y_t/H_t represents the output per hour worked (productivity), H_t/E_t is the number of hours worked per employed person, E_t/A_t is the proportion of workers in the active population (the employment rate), and A_t/N_t is the proportion of active persons in that population (the participation rate).

Gordon (2010a) takes logarithms in Equation (2), proposes that the tendencies of the variables have the same structural decomposition as the observed values, and analyzes the responses of the components to variations in the output gap. However, Gordon (2010a)

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does not directly link changes in output and the unemployment rate, as Okun (1962) had. Perhaps that is why few subsequent analyses have incorporated Gordon's (2010a) methodological approach.

Here, we will show how Equation (2) can indeed be transformed to relate ΔU to y, and to express Gordon's identity in terms of Okun's (1962) equations, thus allowing analysts to decompose the coefficients α_0 and α_1 . The transformation consists of several steps.

First, the notation in Equation (2) is simplified by letting $Y_t/H_t = X_{2t}$, $H_t/E_t = X_{3t}$, $E_t/A_t = L_t$, $A_t/N_t = X_{4t}$, and $N_t = X_{5t}$. Then, we divide Equation (2) by itself, evaluated at t-1. (For example, the left-hand side becomes Y_t/Y_{t-1}). After this, we take logarithms and replace the logarithm of each quotient by the percentage change in the corresponding variable. Rearranging terms, the result is written as follows:

$$y_t \cong l_t + \sum_{i=2}^5 x_{jt} \tag{3}$$

where lower-case letters denote percentage changes. Equation (3) indicates that the percentage change in the real GDP (y) is approximately equal to the sum of the percentage changes in the employment rate (l), the output per hour worked (x_2), the hours worked per employed person (x_3), the participation rate (x_4), and the working-age population (x_5). This identity is not exact because the percentage changes have been approximated by logarithms.

Because Equation (3) does not include the unemployment rate explicitly, the next step in the transformation is to recognize that the unemployment rate is the complement of the employment rate: $U_t = 1 - L_t$. Therefore, the relation between the variations of these two rates is $\Delta U_t = -\Delta L_t$. Using this result, and recognizing that the percentage change in the employment rate is $l_t = \Delta L_t/L_{t-1}$, we see that $l_t = -\Delta U_t/L_{t-1}$. Now, after substituting that expression in Equation (3) and solving for ΔU_t , we obtain the following:

$$\Delta U_t \cong L_{t-1} \left(-y_t + \sum_{j=2}^5 x_{jt} \right) \tag{4}$$

Like Equation (1) (Okun's first differences model), Equation (4) relates ΔU to y directly, but also relates ΔU to all of the other factors that affect that relation. If we include these other factors in Equation (1), we obtain a relation that is almost deterministic because Equation (4) is an approximate identity. Concretely, if we estimate the regression

$$\Delta U_t = \beta_0 + \beta_1 y_t + \sum_{j=2}^5 \beta_j x_{jt} + \varepsilon_1 \tag{5}$$

we would expect that $\beta_0 \approx 0$, $\beta_1 \approx -L_{t-1}$, and $\beta_j \approx L_{t-1}$, with $R^2 \approx 1$. Thus, these parameters are practically predetermined. For that reason, Equation (5) provides little information. However, that information is important because it allows the analyst to estimate the parameters by ordinary least squares (OLS), thus avoiding the imprecision that arises in Ismihan's (2016) decomposition of Okun's coefficient.³

As the next step, we follow Johnston (1984, p. 260) to extract the information that is available via Equation (5). According to the properties of OLS estimators, the coefficients α_0 and α_1 in Equation (1) (a reduced-form relation between ΔU and y) are linked to the parameters in Equation (5) (an extended version) as follows:

$$\alpha_0 = \beta_0 + \sum_{j=2}^5 \beta_j \lambda_j \tag{6}$$

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$$\alpha_1 = \beta_1 + \sum_{j=2}^5 \beta_j \gamma_j \tag{7}$$

where β_0 through β_5 are the parameters in Equation (5), and λ_j and γ_j are the parameters of the auxiliary regressions:

$$x_{it} = \lambda_i + \gamma_i y_t + \varepsilon_i \tag{8}$$

The ε_i terms are error terms, and j = 2, ..., 5.

Kmenta (1997, p. 443) explains that these auxiliary regressions are not regression equations in the usual sense; instead, they are used as an artifice to facilitate interpretation. Ismihan (2016, p. 182) interprets these auxiliary parameters within the framework of Okun's law, pointing out that γ_j "represent the effect of a one–percentage–point change in output growth on $[x_j]$ ", while λ_j "represent the growth rate of $[X_j]$ independent of the growth rate of output demand". These effects are implicit in Okun's reduced-form equation, as Gordon (1984) and Prachowny (1993) recognized.

It should be pointed out that Gordon (1984) used similar auxiliary equations to explain the employment rate (rather than the unemployment rate) as a function of output. Daly et al. (2017), for their part, disaggregated Okun's coefficient for the inverse relation by decomposing the covariance through the use of accounting identities. Kaufman (1988) decomposed the elasticity of the employment rate with respect to real production (i.e., the coefficient in Okun's third model). And, from the perspective of production functions, Prachowny (1993) as well as Huang and Lin (2006, 2008) disaggregated Okun's coefficient, but did not specify completely the relationship between growth and unemployment. Other partial disaggregations of Okun's coefficient were derived by Favereau and Mouillart (1981), Hamada and Kurosaka (1984), and Sögner and Stiassny (2002). The most complete disaggregation was that of Ismihan (2016).

Equations (6) and (7) completely disaggregate the intercept and slope of Equation (1). Specifically, Equation (7) decomposes the original Okun coefficient, α_1 , into the following:

- The direct effect of y upon ΔU . This effect is captured by the coefficient β_1 , which is interpreted in ceteris paribus terms because it no longer includes other influences.
- The indirect effect of other factors (not made explicit in Equation (1)) whose variations affect ΔU via y. These factors are as follows:
 - \bigcirc the output per hour of work, $\beta_2 \gamma_2$;
 - \bigcirc the hours worked per employed person, $\beta_3 \gamma_3$;
 - \bigcirc the participation rate, $\beta_4 \gamma_4$;
 - \bigcirc and the working-age population, $\beta_5 \gamma_5$.

The decomposition of the intercept, α_0 , is analogous to that for Okun's coefficient: Equation (6) disaggregates the constant contribution of each factor to ΔU .

Alternatively, the parameters in Equation (1) tend to be estimated via the inversion of causality. Okun (1962) estimated his equations with unemployment as the dependent variable, although he interpreted the reciprocals of the coefficients to explain the evolution of GDP as a function of unemployment. The detail is that Equation (1) is "a reduced–form relationship between two endogenous variables" (Ball et al., 2017, p. 1416), and the cause–effect relationship to be specified depends on the question to be answered. Therefore, it is also possible to write

$$y_t = \alpha_0' + \alpha_1' \Delta U_t + \varepsilon_0' \tag{9}$$

In this case, the equation that includes all of the factors is

$$y_t = \beta_0' + \beta_1' \Delta U_t + \sum_{i=2}^{5} \beta_j' x_{jt} + \varepsilon_1'$$
 (10)

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From Equation (4), it would be expected that the parameters in Equation (10) take the values $\beta'_0 \approx 0$, $\beta'_1 \approx -1/L_{t-1}$, $\beta'_i \approx 1$, and $R^2 \approx 1$.

The parameter β'_1 indicates the percentage increase in production that occurs when the unemployment rate falls by one percentage point, ceteris paribus. This is because the value of L_{t-1} tends to be about 0.94, $\beta'_1 \approx -1.06$, which is the figure posited by Okun (1962, p. 5): "slightly more than one percent: 100/100-U percent to be exact [. . .] the increment to output would be only that 1 + percent". Moreover, this case is not susceptible to the criticism that various authors (Plosser & Schwert, 1979; Barreto & Howland, 1993) directed at the way in which Okun (1962) interpreted the reciprocals of his estimations. Specifically, because $\beta_1\beta'_1 \approx 1$ in the extended equations, the reciprocal of β_1 effectively approaches β'_1 . In contrast, $1/\alpha_1 \neq \alpha'_1$ in Okun's equations, because $r^2 \neq 1$, as Okun himself noted in the "general discussion" of Perry (1971, p. 578).

The relation between the coefficients in Equations (9) and (10) can be understood by following the same logic as was described earlier for Equations (1) and (5). The intercept and slope are disaggregated into the coefficient of the pure direct relation, plus the indirect effects of the variables that are not specified:

$$\alpha_0' = \beta_0' + \sum_{j=2}^5 \beta_j' \lambda_j' \tag{11}$$

$$\alpha_1' = \beta_1' + \sum_{j=2}^{5} \beta_j' \gamma_j' \tag{12}$$

The auxiliary regressions are as follows:

$$x_{jt} = \lambda_j' + \gamma_j' \Delta U_t + \varepsilon_j' \tag{13}$$

In this case, because ${\beta'}_0 \approx 0$ and ${\beta'}_j \approx 1$, the ${\alpha'}_0$ and ${\alpha'}_1$ in Equations (11) and (12) can be approximated as

$$\alpha_0' \approx \sum_{j=2}^5 \lambda_j' \tag{14}$$

$$\alpha_1' \approx \beta_1' + \sum_{i=2}^5 \gamma_j' \tag{15}$$

Next, we apply the resulting equations to a data set that is basically the same as used by Gordon (2010b), but updated to include the latest available figures. Data for real GDP of the United States are from the Bureau of Economic Analysis (BEA, 2024), and span the period from the first quarter of 1947 to the first quarter of 2024 (1947Q1–2024Q1). Values of H_t , E_t , A_t and N_t are from the Bureau of Labor Statistics (BLS, 2024) for the period 1948Q1 to 2024Q1. This data set limits the analysis to 1948Q1–2024Q1, for which a total of 305 quarterly observations are available for each variable.

These data allow us to calculate the ratios that appear in Equation (2), as well as the unemployment rates and their changes, plus the percentage changes of the ratios, Y_t , and N_t , as can be seen in the Supplementary Materials. We did not use quarterly variations; instead, we followed Zanin and Marra (2012, p. 97) and used interannual variations "in order to reduce the variability and autocorrelation in the data". From the 301 interannual variations that were available for each series, we calculated Regressions (1), (5), and (8), which provide coefficients for performing the disaggregations in Equations (6) and (7). Similarly, we calculated Regressions (9), (10), and (13) (which are for the inverse relation) in order to perform the disaggregations that are proposed in Equations (11) and (12).

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We calculated the estimates for the entire 1948Q1–2024Q1 period and various subperiods, as well as via rolling regressions. For the latter, we followed Knotek (2007) in using a window size of 13 years. This size ensured that each regression contained at least one recession, as was also the case for the analyses published by Owyang and Sekhposyan (2012) and Mussida and Zanin (2023). The first of the rolling regressions was for the window of 52 interannual variations from 1949Q1 to 1961Q4—practically the same period that Okun treated in his seminal 1962 article. The second regression omitted the 1949Q1 variations and added those of 1962Q1. This pattern of advancing the window one quarter at a time continued until the last regression, which treated the variations from 2011Q2 to 2024Q1. In the section that follows, the estimates for each window are associated with the last date of each window. In total, we estimated 250 rolling regressions for the principal and auxiliary equations. From these results, it was possible to discern how the values of Okun's coefficients and their components evolved through time, and thus to note the changes that have occurred over the last 76 years.

4. Results and Discussion

4.1. Correlations and Evolutions

The strongest correlation was the one between y and ΔU (= -0.789; Table 1). These are the only two variables that Okun (1962) specified in his models. The second strongest (= -0.501) was between ΔU and the hours worked per employed person (x_3). These results indicate that although the variables share a common evolution, they also provide differentiated information.

	Change in the	Percentage Change							
	Unemployment Rate ΔU	Real GDP	Output per Hour Worked	Hours Worked per Employed Person	Participation Rate	Working-Age Population			
		\boldsymbol{y}	x_2	x_3	x_4	x_5			
y	-0.789	1							
x_2	0.171	0.291	1						
x_3	-0.501	0.455	-0.471	1					
x_4	-0.371	0.279	-0.452	0.192	1				
x_5	0.083	0.030	-0.163	-0.099	0.236	1			

Table 1. Coefficients of the correlation among variables in Equation (5) during 1948Q1–2024Q1.

Source: prepared by authors with data from BEA (2024) and BLS (2024).

Figure 1 shows the evolution of the six variables that are involved in the relation between output and unemployment rate. All six of these vary, but around distinct levels and with distinct amplitudes. The variations in unemployment rate and participation rate oscillate around 0, as would be expected because neither can increase or decrease indefinitely: both rates are bounded by 0 and 1. For both variables, the positive changes tend to be compensated by the negative ones.

The same is not true of the other variables. For the GDP, productivity, and working-age population, the positive variations outweigh the negative ones. Therefore, all three factors increased over the long run. This behavior is reflected in the fact that the values of these variables fluctuated around positive numbers (3.1%, 1.8%, and 1.3%, respectively). For that reason, these variables contributed to making the Okun equation's intercept positive, as we will see in the next section. However, the output per hour worked also has an important impact on the slope due to its wide variation around its average.

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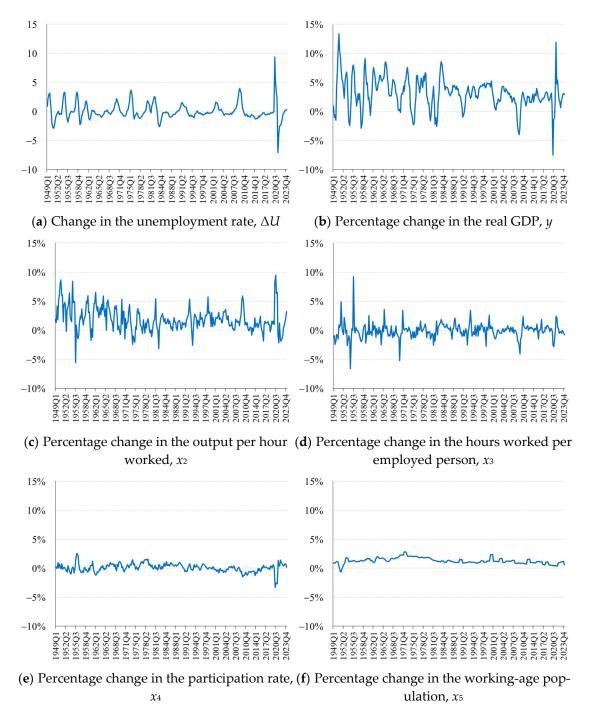


Figure 1. Evolutions of the variables in Equation (5), 1948Q1–2024Q1.

Although the number of hours worked per employed person does not have a strict upper limit, it can neither increase nor decrease indefinitely. Over the long term (1948Q1–2024Q1), it tended to decrease from 41.9 to 38.1 h per week per employee (BLS, 2024). That decrease was a negative contribution to the intercept of Okun's relation. Moreover, the variability in the hours worked influenced the slope. As explained by Monhollon and Cullison (1970), all of the factors affect every parameter of the regression, but Thirlwall (1969) was correct in predicting that some would have a greater effect upon the intercept, while others would have a greater effect upon the slope.

4.2. Estimates

For the period 1948Q1–2024Q1, the estimation of Equation (1) gave an intercept (α_0) of 1.31, and a slope (α_1) of - 0.42. Both values are significant at the 0.1% level (Table 2).

Table 2. Estimations of Equation (1) for various time periods.

Period	α_0	α_1	r^2
1948Q1-2024Q1	1.3133 (0.0776) ***	-0.4200 (0.0189) ***	0.6230
1948Q1-2001Q2	1.3532 (0.0698) ***	-0.3752 (0.0155) ***	0.7379
2001Q3-2020Q1	1.1156 (0.1284) ***	-0.5558 (0.0498) ***	0.6304
2001Q3-2024Q1	1.5102 (0.1338) ***	-0.7386 (0.0449) ***	0.7528

Notes: standard errors are in parenthesis. *** denotes statistical significance at the 0.1% level. Source: prepared by authors with data from BEA (2024) and BLS (2024).

The intercept indicates that the unemployment rate increases by 1.31 percentage points in a year if the GDP does not grow, and if all of the other variables evolve as they did, on the average, during the last 76 years. The advantage of the decomposition that we have proposed is that it allows analysts to determine, with precision, how much each variable contributes to the value of α_0 .

These contributions are presented in Table 3 for 1948Q1–2024Q1 and various subperiods. Together, the long-term evolution of the working-age population and the production per hour of work contributed + 2.18 points to the value of α_0 during 1948Q1–2024Q1. In contrast, the long-term evolution in hours per employed person and the rate of participation reduced it by 0.85 points. The direct effect of the percentage variation in production was practically nil, as expected from Equation (4). The value of the intercept is exactly the sum of this direct effect and the four indirect ones.

Table 3. Values of the intercept (α_0) in Equation (1), and of α_0 's component factors.

	01. /		Indirect Effects Through the Indicated Variations				
Period	Okun's Intercept α ₀	Direct Effect β_0	Output per Hour Worked $\beta_2 \lambda_2$	Hours per Employed Person $\beta_3\lambda_3$	Participation Rate $eta_4\lambda_4$	Working-Age Population $eta_5\lambda_5$	
1948Q1-2024Q1	1.3133	-0.0131	1.0310	-0.7178	-0.1352	1.1485	
1948Q1-2001Q2	1.3532	-0.0055	0.5951	-0.7583	0.1679	1.3540	
2001Q3-2020Q1	1.1156	0.0087	1.4605	-0.8600	-0.3892	0.8956	
2001Q3-2024Q1	1.5102	-0.0023	2.0433	-0.7576	-0.6250	0.8518	

Note: α_0 is from Equation (1) (Table 2), β_0 to β_5 are from Equation (5) (Table A1 in the Appendix A), and λ_2 to λ_5 are from Equation (8), for j = 2, ..., 5 (Table A2). Source: prepared by authors with data from BEA (2024) and BLS (2024).

The value of Okun's coefficient (α_1) indicates that an additional 1% increase in real GDP reduces the unemployment rate by 0.42 percentage points when the other variables evolve as they did during 1948–2024 (Table 4). That value is lower than the real GDP's ceteris paribus direct effect (0.8984 percentage points) because of the compensating indirect effects of GDP variation on the unemployment rate through variations in output per hour, hours per employed person, and (to a lesser degree) the participation rate and working-age

population (Figure 2). In the period 1948–2024, indirect effects offset half of the direct effect. In other periods, the figures change.

Table 4. Values of the slope (α_1) in Equation (1), and of $lpha_1$'s component factors.
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	01		Indirect Effects Through the Indicated Variations				
Period	Okun's Coefficient α ₁	Direct Effect β_1	Output per Hour Worked $eta_2\gamma_2$	Hours per Employed Person $\beta_3 \gamma_3$	Participation Rate $eta_4\gamma_4$	Working-Age Population $eta_5\gamma_5$	
1948Q1-2024Q1	-0.4200	-0.8984	0.2038	0.2011	0.0684	0.0051	
1948Q1-2001Q2	-0.3752	-0.9019	0.3422	0.1884	0.0164	-0.0203	
2001Q3-2020Q1	-0.5558	-0.9202	-0.1021	0.3852	0.0465	0.0349	
2001Q3-2024Q1	-0.7386	-0.8959	-0.3454	0.3158	0.1725	0.0144	

Note: α_1 is from Equation (1) (Table 2), β_1 to β_5 are from Equation (5) (Table A1), and γ_2 to γ_5 are from Equation (8), for j = 2, ..., 5 (Table A2). Source: prepared by authors with data from BEA (2024) and BLS (2024).

The estimation of Equations (9) and (10) (Table 5) made it possible to verify that the parameters in these equations had the expected values ($\beta'_0 \approx 0$, $\beta'_1 \approx -1/L_{t-1}$, $\beta'_j \approx 1$, $R^2 \approx 1$). In addition, the reciprocal of the coefficient β_1 that appears in Equation (5) (1/- 0.898 = -1.113) is almost equal to the β'_1 in Equation (10) (-1.109). This is not the case for the Okun coefficients in Equations (1) and (9): the reciprocal of α_1 (1/-0.42 = -2.38) is much different than the coefficient α'_1 that appears in Equation (9) (-1.48). Okun (1962) proposed this simplification (1/ $\alpha_1 \approx \alpha'_1$) for his reduced model, but recognized later that it had turned out to be quite imprecise. As Okun correctly explained, "The difference comes because the correlations are not perfect" (Perry, 1971, p. 578). In contrast, the expanded model's approximation by means of the reciprocal is good because the correlation is very strong: R^2 is practically equal to 1. In addition, the value of β'_1 is that mentioned by Okun (1962, p. 5): "1 + percent".

Table 5. Estimations of Equations (9), (10) and (13) for the period 1948Q1–2024Q1.

Ezzation	α' ₀	α'_1	r^2				
Equation (9)	3.128 (0.094) ***	-1.483 (0.067) ***	0.6230	-			
Equation	eta'_0	eta'_1	eta'_2	eta'_3	eta'_4	eta'_5	R^2
Equation (10)	-0.015 (0.014)	-1.109 (0.004) ***	1.010 (0.003) ***	1.018 (0.004) ***	1.027 (0.008) ***	1.015 (0.009) ***	0.999
	x_{2t}	$=\lambda_2'+\gamma_2'\Delta U_t$	$+ \varepsilon_2'$	$x_{3t} = \lambda_3' + \gamma_3' \Delta U_t + \varepsilon_3'$			
	λ'_2	γ'_2	r^2	λ'3	γ'_3	r^2	
Equation	1.840 (0.117) ***	0.248 (0.083) **	0.029	-0.097 (0.064)	-0.456 (0.046) ***	0.251	
(13)	$x_{4t} = \lambda_4' + \gamma_4' \Delta U_t + arepsilon_4'$			x_{5t}	$=\lambda_5'+\gamma_5'\Delta U_t$	$+ \varepsilon_5'$	
	λ'_4	γ'_4	r^2	λ'_5	γ'_5	r^2	
	0.086 (0.038) *	-0.186 (0.027) ***	0.137	1.276 (0.029) ***	0.029 (0.020)	0.007	

Notes: standard errors are in parenthesis. The symbols *, **, and *** denote statistical significance at the 5%, 1%, and 0.1% levels, respectively. Source: prepared by authors with data from BEA (2024) and BLS (2024).

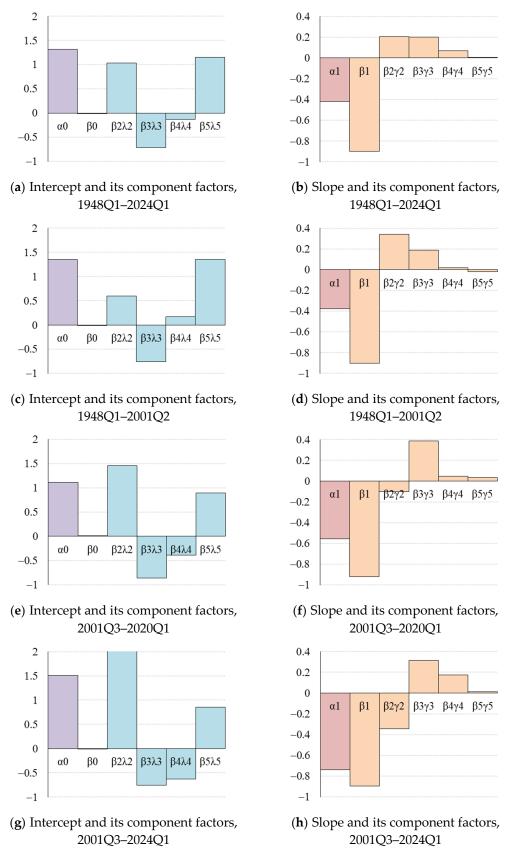


Figure 2. Values of the intercept and the slope in Equation (1), and of their component factors. Notes: α_0 is the intercept, α_1 is the Okun's coefficient, β_0 and β_1 are the direct effects, $\beta_j \lambda_j$ and $\beta_j \gamma_j$ are the indirect effects through the variations in output per hour of work ($\beta_2 \lambda_2$ and $\beta_2 \gamma_2$), hours worked per employed person ($\beta_3 \lambda_3$ and $\beta_3 \gamma_3$), the participation rate ($\beta_4 \lambda_4$ and $\beta_4 \gamma_4$), and the working-age population ($\beta_5 \lambda_5$ and $\beta_5 \gamma_5$).

The Okun coefficient ($\alpha'_1 = -1.483$) indicates the amount by which the rate of change in real GDP decreases for each additional percentage point of the unemployment rate, given the average conditions of the analyzed period. The value of α'_1 is the sum of the direct effect, upon production, of the increase in the unemployment rate ($\beta'_1 = -1.109$), plus the indirect effects that operate through the percentage changes in the output per hour of work ($\beta'_2\gamma'_2 = 0.251$), the hours per working person ($\beta'_3\gamma'_3 = -0.464$), the participation rate ($\beta'_4\gamma'_4 = -0.190$), and the working-age population ($\beta'_5\gamma'_5 = 0.030$). The variations that correlate negatively with the variation in unemployment (Table 1) reinforce its negative effect upon production. This is because an increase in the unemployment rate is associated with reductions in the participation rate as well as in the hours worked per employed person. Both of these reductions decrease production. The reverse is true for productivity (output per hour worked), which increases with unemployment, thereby partially compensating for some of the negative direct effect of unemployment upon production.

The intercept ($\alpha'_0 = 3.128$) represents the annual growth rate of real GDP when the unemployment rate remains stable. In the words of Mankiw (1994, p. 37–8), "If the unemployment rate remains the same, real GDP grows by about 3 percent; this normal growth is due to population growth, capital accumulation, and technological progress". From the decomposition that we propose, we know precisely that this growth comes principally from the secular increase in output per hour worked ($\beta'_2\lambda'_2=1.859$), and in the working-age population ($\beta'_5\lambda'_5=1.295$). Other factors scarcely modify the contributions of these two.

Thus far in the article, we have examined only the average tendencies for the entire period 1948–2024. However, the averages do not capture changes that occurred in the underlying dynamics during these years, and thus modified the Okun coefficients and their components. These modifications are revealed by a study of the estimates for specific sub-periods (Tables 2–4). Table 4 shows that the Okun coefficient became increasingly negative: from -0.3752 for the period 1948Q1–2001Q2, to -0.5558 for 2001Q3–2020Q1, and to -0.7386 for 2001Q3–2024Q1 (during which the coefficient was affected by the COVID-19 pandemic). The dominant factor in this evolution is the variation in output per hour worked, which was a positive contribution (0.3422 points) for the first 50 years, but a negative one (-0.1021) during the ensuing 20. These results are related to other authors' findings that productivity changed from being pro-cyclical before the 80s to counter-cyclical in the following decades (Galí & Gambetti, 2009; Barnichon, 2010; Gordon, 2010a, 2010b; Daly et al., 2017). The rolling regressions that are discussed in the next section allow these dynamics to be analyzed in greater detail.

4.3. Rolling Coefficients

As Owyang and Sekhposyan (2012, p. 406) have noted, "the rolling coefficients capture the changes in correlation and the time of their occurrence in a smooth manner; thus, the magnitude and the timings of shifts in the mean and slope coefficients should be interpreted in that light". Here, we analyze the shifts that can be seen in Figure 3, which shows the value of Okun's coefficient from Equation (1), along with the coefficient's components, for every possible set of 52 consecutive quarters. Okun's coefficient remained relatively stable at around -0.35 until the 2000s, when the relationship between growth and unemployment began to intensify. In 2008Q2, α_1 was -0.305, but it descended to -0.464 in 2009Q4, after which it gradually fell to -0.647 in 2019Q2. The COVID-19 crisis provoked a new break in this tendency: α_1 fell from -0.637 in 2020Q1 to -0.816 in 2020Q3, then remained at around -0.84. However, if the quarters that were affected by the COVID-19 pandemic are excluded (2020Q2 a 2022Q1), α_1 is close to -0.60.

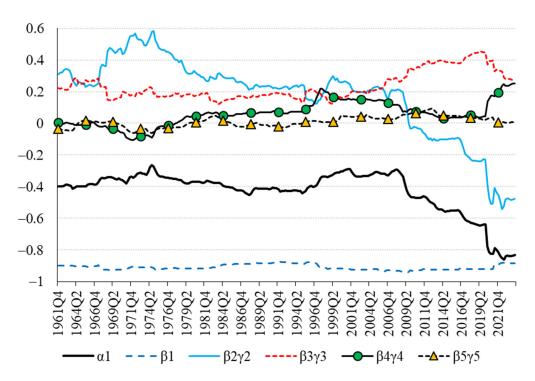


Figure 3. Okun's coefficient and its component factors: rolling regression estimates. Notes: α_1 is the Okun's coefficient, β_1 is the direct effect of y upon ΔU , and the $\beta_j \gamma_j$ are the indirect effects through the percentage changes in the output per hour of work ($\beta_2 \gamma_2$), hours worked per employed person ($\beta_3 \gamma_3$), the participation rate ($\beta_4 \gamma_4$), and the working-age population ($\beta_5 \gamma_5$). The sum of these five factors is exactly equal to α_1 .

Other authors describe similar dynamics of Okun's coefficient, even though those authors used different techniques and studied shorter periods. For example, Moosa (1997, p. 349) found that Okun's coefficient was stable from 1960 to 1995. Using Bayesian approximation for the period 1948Q1–2006Q1, Huang and Lin (2008, p. 370) found that the slope and intercept in Okun's relation evolved similarly to what we show in Figures 3 and 4. Similarities are also found in Grant (2018, p. 294) and Aguiar-Conraria et al. (2020, p. 9), who noted that the absolute value of the Okun's coefficient increased during recent decades. The greatest discrepancy between our results and Grant's (2018) estimates is the large change in Okun's coefficient that Grant reported for the beginning of the 1980s. Valadkhani and Smyth (2015) noted the same large change, while our own analysis showed that absolute value of the coefficient increased only slightly, from 0.363 in 1982Q1 to 0.455 in 1987Q4. The large magnitude of change that is reported by Grant (2018) and Valadkhani and Smyth (2015) may be attributable to their use of the gap version of Okun's law, with unemployment rate as the explanatory variable.

The principal advantage of our proposal is that it identifies the factors that cause the above-mentioned dynamics by disaggregating the coefficients that appear in Equation (1) into their components. As shown in Figure 3, the direct effect of output upon the unemployment rate (β_1) remained stable at between -0.875 and -0.944 throughout the 76-year period. Nearly the same is true for the indirect effect of the working-age population ($\beta_5\gamma_5$), which varied little (between 0.094 and -0.061). The other factors were more unstable, causing changes in Okun's coefficient when their dynamics did not compensate for each other.

Of those other factors, the production per hour of work is the one whose contribution varied the most. During the first decades, the indirect effect of output growth on the unemployment rate via productivity ($\beta_2 \gamma_2$) compensated the direct effect (β_1) by 0.25 points. This indirect effect increased at the end of the 1960s, reaching a maximum $\beta_2 \gamma_2$ value of

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0.583 in 1974Q4. Since then, $\beta_2\gamma_2$ has decreased, becoming negative as a result of the 2008 crisis, and reaching its historic minimum of - 0.542 in 2022Q2 after falling abruptly during the COVID-19 pandemic. In the most recent decades, reductions in production correspond to increases in productivity, which has now become counter-cyclical. For that reason, a larger part of the adjustment is achieved via the unemployment rate, and the absolute value of Okun's coefficient is greater. In fact, the α_1 coefficients for periods that include quarters during which COVID-19 was most widespread are almost equal to the direct effect β_1 . For such periods, the indirect effects $\beta_j\gamma_j$ compensated each other. Concretely, the effect of the variation in output per hour of work ($\beta_2\gamma_2$) compensated for the variations in the participation rate ($\beta_4\gamma_4$) and the hours worked per employed person ($\beta_3\gamma_3$).

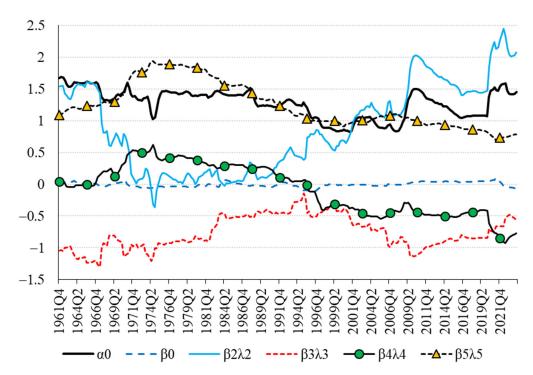


Figure 4. Values of the intercept and its component factors: rolling regression estimates. Notes: α_0 is the intercept from Equation (1), β_0 is the direct effect, and the $\beta_j \lambda_j$ are the indirect effects through the variations in output per hour of work ($\beta_2 \lambda_2$), hours worked per employed person ($\beta_3 \lambda_3$), the participation rate ($\beta_4 \lambda_4$), and the working-age population ($\beta_5 \lambda_5$). The sum of these five factors is exactly equal to α_0 .

The value of $\beta_3\gamma_3$ was quite stable (between 0.119 and 0.288) until 2007, when it began a progressive rise, reaching a maximum of 0.454 in 2019Q2. That increase compensated in part for the evolution in the production per hour of work. The quarters of the COVID-19 pandemic, during which the decline in output mainly affected unemployment and participation rates, reduced the contribution of this factor to 0.270 in 2024Q1.

The variation in the participation rate had little effect until the end of the 1970s, after which it became another pro-cyclical production–adjustment mechanism. The predominance of the discouraged worker effect over the added worker effect meant that the variations in the participation rate left less room for adjustment via the unemployment rate (Lee & Parasnis, 2014). In times of economic crisis, some potential workers dropped out of the labor market rather than remain unemployed. The COVID-19 pandemic was an extreme case, in which the impossibility of seeking work made many people inactive. The effect of this factor ($\beta_4 \gamma_4$) upon the Okun coefficient rose from 0.042 in 2020Q1 to 0.138 in 2020Q2, then rose further to 0.250 in 2024Q1.

As pointed out previously by Barisic and Kovac (2022) and Porras-Arena et al. (2024), estimates of Okun's coefficients change substantially when the analyses include quarters in which production fell because of COVID-19. During the pandemic, the relation between the productive dynamic and the labor market was distinct; compared to earlier crisis periods, the rates of unemployment and participation overreacted. However, that atypical period should not cause us to overlook the changes that took place during previous decades. In 2001Q2, the absolute value of the Okun coefficient for the rolling regressions reached a minimum of 0.289. The coefficient then remained stable for a few years before beginning to rise sharply after the 2008 crisis. In 2020Q1, just before the emergence of COVID-19, the coefficient had already reached -0.637.

The intercept (which represents the long-term effect of all of the other factors) took positive values between its minimum of 0.827 (2001Q2; Figure 4) and its maximum of 1.687 (1962Q1). The elements that contributed to increasing the intercept were the variations in working-age population, the output per hour of work, and (until 1995Q2) the participation rate. The value of the intercept was decreased by variations in the participation rate (after 1995Q3) and the hours worked per employed person.

Figure 4 reveals that these factors did not remain stable. Instead, the effects of their intensities changed, thus causing a tendency for the intercept to decrease from 1.5 during the first decades to values nearer to 1 in the recent ones. Estimates from the rolling regressions show that during the crises of 2008 and 2020, the intercept rose again to 1.5. However, these increases could be due to the fact that we are confounding part of the cycle with the trend, because each regression covers only 52 quarters. In that regard, it should be noted that Huang and Lin (2008), as well as other regressions that used longer periods, such as those in Table 2, also show an evolution towards a decreasing intercept.

The key factor in this dynamic was the slower growth of the working-age population. The impact of that factor was strengthened by the decline in the participation rate, but compensated by the positive (and rising) contribution of the output per hour of work. The contribution of the change in working-age population to the value of the intercept increased (reaching a maximum of 1.939 in 1974Q3) as the baby-boom generation entered the workforce, but then decreased slowly until the beginning of the 1980s, after which it fell more rapidly to about 1 during 1995–2010. It has continued to fall during recent years, reaching 0.784 in 2024Q1.

Similar dynamics, but weaker ones, can be seen in the effect of the participation rate ($\beta_4\lambda_4$). Its value increased during the first years, reaching a maximum of 0.616 in 1974Q3. It then entered a decline that has lasted to the present. It has been negative since 1995Q3, and fell to its minimum of -0.930 in 2022Q3.

The number of hours worked per employed person declined throughout 1948Q1–2024Q1. Therefore, the contribution of this factor to the intercept's value is negative. Until 1982, and from 2006 to 2020, $\beta_3\lambda_3$ was close to -1, while the rest of the time it was nearer to -0.5.

In contrast, the output per hour of work tended to increase. Therefore, $\beta_2\lambda_2$ made a positive contribution to the value of the intercept. Until 1967Q2, and after 2002Q1, $\beta_2\lambda_2$ was greater than 1. However, it was near zero during the period 1971Q3–1991Q1, which coincides with the years when the contributions of participation rate and working-age population were greatest.

5. Conclusions

Okun (1962) summarized the relation between economic growth and the labor market in a single number: the ratio of exchange between real GNP and the unemployment rate. In doing so, Okun noted that this coefficient subsumes the effects of other variables that are not

explicit in his model, but do affect the output–unemployment relation. Several subsequent investigations attempted to recover information about effects of those other variables in order to offer a more complete interpretation of the relation, but did so only approximately.

The disaggregation proposed in the present article allows economists to identify all of the factors that affect Okun's relation, and to quantify their effects with precision. This is achieved by unpacking the contents of Okun's relation, specifically the structure of long-term growth and its cyclical variations. By applying this method to the case of the United States during the period 1948Q1–2024Q1, we found that the value of the intercept in Okun's relation is determined by the increases in working-age population and output per hour of work, along with the decrease in the number of hours worked per employed person, plus the growth of the participation rate until the 1990s and its subsequent decline. For its part, the value of Okun's coefficient depends mainly upon the variations in output per hour of work and the hours per employed person, which together reduce by half the direct effect of the variation in production upon the unemployment rate. The output per hour of work had the greatest impact during the first decades of the analyzed period, while hours per employed person were more important in recent decades. The other factors were scarcely relevant.

Changes in the above-mentioned components caused Okun's relation to vary over time: the intercept tended to decrease, and the slope increased. These trends indicate a greater sensitivity of the unemployment rate to variations in production, even before the COVID-19 pandemic. Compared to their behavior during previous economic crises, the post-2020Q2 rates of unemployment and participation reacted more strongly to adjustments in production.

In essence, the evolution of Okun's coefficient that we report here is similar to that described by Huang and Lin (2008), Grant (2018), and Aguiar-Conraria et al. (2020), even though those researchers used other estimation techniques. In this article, we have provided the inter-relations and dynamics of all of the factors that contribute to that evolution, thereby markedly enriching the possibilities for performing systematic multivariate analyses of the bases of Okun's law.

Among the limitations of the analysis developed, it can be mentioned that the most recent data are still preliminary and could change. In addition, we have only used a window size of 13 years in the rolling regressions. Other window sizes would yield somewhat different results. Moreover, the disaggregation of the coefficients has only been formulated for Okun's first differences model. Ismihan (2016) extends his approach to the gap version, so a similar adaptation could be attempted.

Other future work could include integrating this article's decompositions into other developments that have been taking place, such as the differentiated analysis of expansive and recessive phases of production, plus comparisons among countries and regions.

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Abbreviations

The following abbreviations are used in this manuscript:

BEA Bureau of Economic Analysis

BLS Bureau of Labor Statistics

GDP Gross domestic product

GNP Gross national product

OLS Ordinary least squares

Appendix A

Table A1. Estimations of Equation (5) for various time periods.

Period	eta_0	eta_1	eta_2	eta_3	eta_4	$oldsymbol{eta_5}$	R^2
1948Q1-2024Q1	-0.013 (0.012)	-0.898 (0.003) ***	0.907 (0.004) ***	0.912 (0.006) ***	0.920 (0.009) ***	0.913 (0.009) ***	0.998
1948Q1-2001Q2	-0.005 (0.017)	-0.902 (0.005) ***	0.911 (0.007) ***	0.916 (0.009) ***	0.914 (0.012) ***	0.914 (0.011) ***	0.997
2001Q3-2020Q1	0.009 (0.015)	-0.920 (0.005) ***	0.911 (0.006) ***	0.927 (0.010) ***	0.936 (0.010) ***	0.932 (0.014) ***	0.999
2001Q3-2024Q1	-0.002 (0.029)	-0.896 (0.007) ***	0.906 (0.009) ***	0.910 (0.017) ***	0.925 (0.020) ***	0.893 (0.026) ***	0.999

Notes: standard errors are in parenthesis. *** denotes statistical significance at the 0.1% level. Source: prepared by authors with data from BEA (2024) and BLS (2024).

Table A2. Estimations of Equation (8) for various time periods.

	Equation $x_{2t} = \lambda_2 + \gamma_2 \Delta U_t + \varepsilon_2$			Equation $x_{3t} = \lambda_3 + \gamma_3 \Delta U_t + \varepsilon_3$				
Period	λ_2 γ_2		r ²	λ_3	γ3	r^2		
1948Q1-2024Q1	1.1365 (0.1752) ***	0.2246 (0.0427) ***	0.085	-0.7870 (0.1025) ***	0.2205 (0.0250) ***	0.207		
1948Q1-2001Q2	0.6535 (0.2068) **	0.3757 (0.0459)	0.244	-0.8275 (0.1470) ***	0.2056 (0.0326) ***	0.160		
2001Q3-2020Q1	1.6034 (0.2466) ***	-0.1121 (0.0957) ***	0.018	-0.9277 (0.1200) ***	0.4156 (0.0466) ***	0.522		
2001Q3-2024Q1	2.2554 (0.2592) ***	-0.3813 (0.0869) ***	0.178	-0.8324 (0.0949) ***	0.3470 (0.0318) ***	0.572		
	Equation :	Equation $x_{4t} = \lambda_4 + \gamma_4 \Delta U_t + \varepsilon_4$			Equation $x_{5t} = \lambda_5 + \gamma_5 \Delta U_t + \varepsilon_5$			
Period	λ_4	γ_4	r^2	λ_5	γ5	r ²		
1948Q1-2024Q1	-0.1470 (0.0607) *	0.0744 (0.0148) ***	0.078	1.2583 (0.0445) ***	0.0056 (0.0108)	0.001		
1948Q1-2001Q2	0.1838 (0.0690) **	0.0180 (0.0153)	0.007	1.4807 (0.0583) ***	-0.0222 (0.0129)	0.014		
2001Q3-2020Q1	-0.4157 (0.0906) ***	0.0496 (0.0352)	0.027	0.9611 (0.0544) ***	0.0374 (0.0211)	0.041		
2001Q3-2024Q1	-0.6757 (0.0993) ***	0.1865 (0.0333) ***	0.261	0.9536 (0.0454) ***	0.0162 (0.0152)	0.012		

Notes: standard errors are in parenthesis. The symbols *, **, and *** denote statistical significance at the 5%, 1%, and 0.1% levels, respectively. Source: prepared by authors with data from BEA (2024) and BLS (2024).

Notes

- Okun (1962) used the GNP, but the GDP has been used in these analyses since the 1990s (Palley, 1993; Guisinger et al., 2018).
- Similar reasoning applies to comparisons among countries. For example, in Japan, the changes in output are related more closely to changes in output per hour and to hours worked per employed person, than to the evolution in the unemployment rate

(Hamada & Kurosaka, 1984; Ismihan, 2016). The reverse is true in the United States. For that reason, Japan's Okun coefficient is different from that of the US. Specifically, the absolute value of Japan's coefficient is smaller (Ball et al., 2019; Furceri et al., 2020; Fontanari et al., 2022).

- Ismihan (2016) does not estimate the parameters β_1 through β_5 via regression, as in Equation (5). Instead, Ismihan assigns these parameters "the average value of the employment rate over the time period under consideration" (p. 179). In this way, Ismihan obtains a decomposition of Okun's coefficient that is very approximate, rather than exact.
- Equations (6) and (7) can be derived by substituting Equation (8) in Equation (5), reordering the terms, and comparing the intercept and slope to those in Equation (1).
- Prachowny (1993, p. 332) explains that "Okun's coefficient of three is derived from a complicated weighted sum of all other changes". Similarly, our Equation (12) disaggregates Okun's coefficient (in the relation $y = f(\Delta U)$) into a "weighted sum of all other changes".
- Specifically, the BEA (2024) data correspond to the GDP series in chained millions of dollars (2017), seasonally adjusted at annual rates. The series obtained from BLS (2024) were as follows: LNU02033120Q for H_t/E_t , LNS12000000Q for E_t , LNS11000000Q for A_t , and LNU00000000Q for N_t .

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