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Modeling Volatility in the Nigeria Stock Exchange: The Role of Structural Breaks

Ekejiuba C. Ucheoma^{1,2}

This study models volatility in the daily Nigeria Stock Exchange 30 index from January 30, 2012 to August 31, 2020, comparing the Modified Iterated Cumulative Sums of Squares (MICSS) and the traditional cumulative sum of squares (CUSUMSQ) approaches. Findings reveal that integrating structural breaks reduces volatility persistence, and the MICSS outperformed the CUSUM and CUSUMSQ algorithms. Also, incorporating structural breaks reduces the impact of good and bad news on volatility. The study emphasizes the importance of adopting a robust methodology, such as the MICSS, in detecting structural breaks when estimating conditional variances, as incorporating structural breaks in the estimation is pivotal for model robustness, avoiding biases, and improving the estimation of volatility dynamics in financial time series. For investors and risk managers, the practical implications of these findings are: incorporating these breaks enhances risk assessment, portfolio allocation, and hedging strategies. For policymakers, incorporating structural breaks can lead to a more informed regulatory framework that helps stabilize the financial markets.

JEL Classification: C22, C51, C58, G12

Keywords: GARCH, heteroscedasticity, MICSS, structural break

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1. Introduction

In the dynamic realm of finance, modeling financial time series is indispensable for gaining insights into market dynamics, managing risk, formulating policies, and making informed investment decisions. However, factors such as the data frequency and structural breaks often make it challenging to model financial time series. These complexities manifest in behaviors like heteroscedastic variance, volatility clustering, persistence, and market asymmetric to news (Black, 1976; Mandelbrot, Mandelbrot, 1997). To address this, Bollerslev (1986) proposed the Generalized Autoregressive Conditional Heteroscedastic (GARCH (p, q)) model to account for heteroscedasticity and volatility persistence. Glosten *et al.* (Glosten, 1993) proposed the Threshold Autoregressive Conditional Heteroscedastic (TARCH (p, q)) model to

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account for the asymmetric effect of news on volatility. Financial time series often exhibit time-varying volatility. More so, in modeling volatility, it is apt to recognize the existence of structural breaks and account for them using a robust approach to avoid spurious persistence and misleading empirical inferences (Lamoureux & Lastrapes, 1990; Ewing & Malik, 2005).

Traditional CUSUM tests such as those based on the cumulative sum of squares introduced by Inclán and Tiao 1994) to detect structural breaks in the variance is inappropriate for dependent series because they assume the observations in the series are independently and identically distributed (iid) a condition that may be considered extreme for financial time series with time-varying properties (Sansó *et al.*, 2004). The Modified Iterative Cumulative Sums of Squares (MICSS) algorithm, proposed by Sansó *et al.*, 2004), addressed this limitation by allowing for the detection of multiple structural breaks in the unconditional variance of a dependent series such as those modeled with GARCH processes. This study makes four contributions to the literature, especially in Nigeria. First, it introduces the MICSS algorithm to structural breaks and volatility modeling of the Nigeria Stock Exchange_30 index (NGSE_30). Hence, represents the first attempt in this market. Second, it accounts for structural breaks in the symmetric and asymmetric models, allowing for a comprehensive and robust modeling framework that provides more accurate assessments of volatility dynamics and the leverage effect. Third, compared to some previous studies on volatility modeling, which used weekly or monthly data, this study employs daily data, which offers greater sensitivity in capturing volatility dynamics. Fourth, while the focus is on Nigeria, given the interconnectedness of the global financial markets, especially in understanding regime shift and market volatility, the methodological approach also provides valuable insight for other emerging markets and financial time series with similar characteristics.

By accounting for structural breaks and asymmetries in volatility through a robust modeling framework, the study improves the reliability of volatility estimates and offers practical insights for investors, risk managers, and policymakers. The rest of the paper is structured as follows: Section 2 presents the literature review, Sections 3, 4, and 5 present the data and methodology, results and discussion, and conclusion, respectively.

2. Literature Review

2.1 Theoretical Literature

Modeling structural changes in time series volatility has attracted increasing interest from researchers. This interest is partly driven by stylized facts such as long-range dependence, volatility clustering, the persistence of volatility shocks, the leverage effect, and the presence of risk premiums, among others. These characteristics are often indicative of non-constant unconditional variances (Mikosch & Stărică, 2004; Hillebrand, 2005; Perron & Qu, 2010). One of the most commonly utilized tests for detecting structural breaks in time series is based on the CUSUM of squared test. However, this test is suitable for an independent process. Various studies have proposed and empirically applied different versions of the CUSUM test, including extending the test to make it more robust. Examples include the works of Inclán and Tiao, 1994), Sansó *et al.*, 2004), and Ewing *et al.*, 2005).

Inclán and Tiao (IT) proposed the IT statistic to detect single structural breaks in the variance of a time series. Their approach was designed for independent and identically distributed (iid) processes. Based on the IT statistics, they developed the iterated cumulative sum of squares (ICSS) algorithm, which allows for the detection of multiple structural breaks in variance. Sansó *et al.*, 2004) identified a crucial limitation of the IT statistic when applied to dependent processes like GARCH models. They found that the IT statistic tends to be oversized in such cases, leading to erroneous detection of structural breaks. This limitation arises due to the autocorrelation and volatility clustering inherent in GARCH processes, which violate the independence assumption of the IT statistic. Hence, a need to address the specific characteristics of dependent processes like GARCH when detecting structural breaks. In response to this limitation, Sansó *et al.*, 2004) proposed a modified version of the IT statistic (MIT) that is suitable for application to dependent processes like GARCH. The MIT statistic accounts for the autocorrelation and volatility clustering present in GARCH processes, thereby providing more accurate detection of structural breaks in variance. Kartsonakis-Mademlis and Dritsakis, 2020) applied the ICSS algorithm developed by Inclán and Tiao, 1994) to the MIT statistic proposed by Sansó *et al.*, 2004). This combined approach is referred to as MICSS, and it enables the detection of multiple structural breaks in the unconditional variance of the returns.

2.2 Empirical Literature

Few recent studies have specifically addressed the adoption of methodologies that address structural breaks for dependent processes such as the GARCH model, despite its widespread application in volatility modeling with structural breaks. Instead, much of the existing research has emphasized the significance of incorporating structural breaks into volatility models, often overlooking the methodological nuances involved in detecting these breaks within the framework of GARCH models. Chung and Bravo, (2022) used the GARCH (1,1) model to analyze the impact of structural breaks in forecasting the daily returns volatility of the dollar exchange rate in Peru from March 2010 to September 2021. The MICSS algorithm identified 8 breakpoints, and these breakpoints were incorporated into the GARCH Model estimation, resulting in significant improvements in the parameter estimates across subsamples and a significant gain in predictive ability compared with the benchmark GARCH (1,1) model without the breakpoints. Souffargi and Boubaker, (2022) used symmetric and asymmetric GARCH models to analyze the impact of political uncertainty on the volatility of the Tunisian stock market from November 2010 to February 2016. They applied three test statistics to detect structural breaks in the variance. These include the Inclan and Tiao's ICSS algorithm (IT-ICSS), and the MICSS algorithms (kappa-1 and kappa-2) tests. After incorporating the breaks identified by the IT-ICSS and kappa-1, the behavior of asymmetric volatility was negatively affected by the political uncertainties, indicating the sensitivity of the Tunisian stock market volatility to local and political events. Accounting for these regime shifts reduced the persistence in volatility post-revolutionary periods. Kartsonakis-Madamlis and Dritsakis. (2020) used the MICSS to account for structural breaks in the volatility of the Dow Jones Industrial Average (DJIA), Nikkei Stock Average (Nikkei 225), Financial Times Stock Exchange 100 Index (FTSE100), and Europe Brent crude oil returns. They showed that accounting for structural breaks within the asymmetric GARCH models results in a more accurate estimation of the volatility dynamics and reduces the volatility persistence in these series. More so, integrating structural breaks reveals that good news had less impact on their current volatility, while the effect of bad news on the current volatility only reduces in the Nikkei 225 returns.

Kuhe et al., (2019) used symmetric and asymmetric GARCH models to analyze the impact of volatility shock persistence on the conditional variances of the daily stock returns and volume of trade returns in the Nigerian stock market in the presence

of random level shifts and heavy-tailed distributions from January 1998 to January 2017. The Bai and Perron procedure was adopted to detect structural breaks in the conditional variances. The paper observed a substantial reduction in the volatility persistence, half-life, and long memory after accounting for the random level shifts in the GARCH models. The asymmetric analysis revealed that good news had more impact on the conditional variance than the bad news. Fasanya et al., (2020) employed the Bai and Perron test for the detection of multiple structural breaks in symmetric and asymmetric Autoregressive Distributed Lag (ARDL) in determining the relationship between the oil price and stock returns for selected sectors in Nigeria from January 2007 to December 2016. Findings from the analysis endorsed the robustness of the GARCH model in accounting for structural breaks against other variant competing break models and the relevance of structural breaks to stock returns volatility.

Theoretical insights and empirical evidence collectively acknowledge the estimation biases associated with neglecting structural breaks and emphasize the importance of incorporating structural breaks in volatility models. More importantly, there is a need to employ an appropriate structural breaks test statistic when dealing with dependent series. Hence, this research uses the MICSS algorithm to identify unknown structural breaks in the Unconditional variance of returns.

3. Data and Methodology

3.1 Data

The data employed was derived from the Daily Nigerian Stock Exchange 30 Index (NGSE_30). The index tracks price movements of the 30 most liquid equities in terms of market capitalization listed on its main board. The data spans from January 30, 2012, through August 31, 2020, totaling 2125 daily observations excluding weekends and National holidays.

3.2 Model Specification

A parsimonious model that is vastly used in empirical literature due to its numerous statistical advantages over other symmetric GARCH models in measuring volatility is the GARCH (1, 1) (Chung & Bravo, 2022; Souffargi & Boubaker, 2022). As such, this study adopts the GARCH (1,1) model specified as:

$$\sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \gamma_1 \sigma_{t-1}^2 \quad (1)$$

Where, β_0 is a constant term that represents the average variance, while $\beta_1 \varepsilon_{t-1}^2$ is the ARCH term that provides information about the effects of past squared shocks on the current volatility. $\gamma_1 \sigma_{t-1}^2$ is the GARCH term captures the effects of past volatility on current period volatility. To ensure that the conditional variance remains positive, all parameters must be non-negative. Thus, $\beta_0 > 0, \beta_1 \geq 0, \gamma_1 \geq 0$. The model is covariance stationary when $\beta_1 + \gamma_1 < 1$. If $\beta_1 + \gamma_1 = 1$, volatility shocks persist indefinitely. If $\beta_1 + \gamma_1 > 1$, the conditional variance is non-stationary and explosive. The model assumes the symmetric impact of News on conditional volatility. The positive interaction between the ε_{t-i} and the autoregressive parameter (α_i) of the Y_t sequence explains how the conditional variance (σ_t^2) captures conditional heteroscedasticity in the Y_t sequence. In essence, the autoregressive process (ε_{t-1}^2) of σ_t^2 results in a conditionally heteroscedastic error, consequently affecting the behavior of the Y_t sequence, making it heteroscedastic as well. In a financial context, this model describes the predictive behavior of an asset stakeholder to measure uncertainties by forming a weighted average of the long-run average, which includes the ARCH term and the GARCH term.

An extension of the GARCH (1, 1) model that accounts for the asymmetry in volatility and volatility clustering is the TARARCH (1, 1) model (Zakoian, 1994; Glosten *et al.*, 1993). TARARCH (1, 1) model specifies its conditional variance in the form:

$$\sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \delta_1 A_{t-1} \varepsilon_{t-1}^2 + \gamma_1 \sigma_{t-1}^2 \quad (2)$$

where, β_0 is the average variance, β_1 is the coefficient of the ARCH term that captures the symmetric effect, and $\delta_1 A_{t-1} \varepsilon_{t-1}^2$ measures the asymmetric impact of volatility to negative shocks. If $\delta_1 > 0$ and statistically significant, negative shocks (bad news) increase volatility more than positive shocks (good news) of the same magnitude. Thus, the leverage effect occurs, and volatility tends to increase more following negative returns than positive returns. A_{t-1} is a dummy variable that takes the value 1 if $\varepsilon_{t-1} < 0$ (bad news) and 0 otherwise. Thus, if $\varepsilon_{t-1} < 0$, the effect of an ε_{t-1} shock on σ_t^2 is $(\beta_1 + \delta_1) \varepsilon_{t-1}^2$. This is the bad news impact. Contrarily, if $\varepsilon_{t-1} \geq 0$, the effect of an ε_{t-1} shock on σ_t^2 is $\beta_1 \varepsilon_{t-1}^2$ (Good News impact). $\gamma_1 \sigma_{t-1}^2$ is the GARCH effect term, which accounts for volatility persistence by ensuring that past volatility is carried over into future volatility. To ensure the variance remains positive, $\beta_0 > 0, \beta_1 \geq 0, \delta_1 \geq 0$, and $\gamma_1 \geq 0$. The long-run volatility persistence is mea-

sured by $\beta_1 + \frac{1}{2}\delta_1 + \gamma_1$. The model is covariance stationary if $\beta_1 + \frac{1}{2}\delta_1 + \gamma_1 < 1$. If $\beta_1 + \frac{1}{2}\delta_1 + \gamma_1 = 1$, the model has a unit root in variance, and the closer this value is to 1, the longer the volatility clusters.

3.3 Detecting Structural Breaks

A structural break is an intermittent shock that affects the stability of the regression parameters in time series modeling. Hence, to ensure robustness, structural stability is of the essence. Ignoring structural breaks in financial time series modeling can yield spurious persistence in the conditional volatility parameters. Hence, this paper adopts the MICSS algorithm to detect unknown breaks in the unconditional variance of dependent processes such as the GARCH models, which is one of the core emphases of the research. The MIT statistic is expressed as:

$$MIT = \sup_b \left| T^{-1/2} H_b \right| \quad (3)$$

where the centered and normalized cumulative sum of squares is defined as $H_b = \hat{\varphi}_4^{-1/2} (C_b - \frac{b}{T} C_T)$. $C_b = \sum_{t=1}^b \varepsilon_t^2$ is the cumulative sum of squared residuals up to time b ($b = 1, \dots, T$). C_T is the total sum of squared residuals over the entire sample, and $\varepsilon_t \text{ iid } (0, \sigma^2)$. The estimated long-run variance of ε_t which account for time-varying volatility is;

$$\hat{\varphi}_4 = \frac{1}{T} \sum_{t=1}^T (\varepsilon_t^2 - \hat{\sigma}^2)^2 + \frac{2}{T} \sum_{f=1}^d g(f, d) \sum_{t=f+1}^T (\varepsilon_t^2 - \hat{\sigma}^2) (\varepsilon_{t-f}^2 - \hat{\sigma}^2),$$

where $g(f, d) = 1 - \frac{f}{(d+1)}$ is the lag window (Bartlett). $\hat{\varphi}_4$ depend on d which is the Newey-West bandwidth parameter used to estimate the long-run variance (Sansó *et al.*, 2004). If there is a break in variance, the plot of H_b against b will deviate from the boundaries determined by the asymptotic distribution of H_b , otherwise, H_b will oscillate around zero if there are no breaks in variance. The point of deviation is identified as the estimate of the break date (Inclan & Tiao, 1994). The asymptotic distribution of the MIT statistic is $\max_r |W(r) - rW(1)|$. $W(r) - rW(1)$ represent the Brownian bridge which correct the mean-reverting nature of the variance and the standard Brownian motion is $W(r)$ (Sansó *et al.*, 2004). The MIT statistics is applied to the ICSS algorithm to detect multiple structural breaks in the variance by applying H_b successively to different sample sizes of the series in an iterative manner, and dividing consecutively once a breakpoint is detected (Inclan & Tiao, 1994).

Applying the MIT statistic to the ICSS algorithm corrects all the drawbacks of the IT statistic. As such, the MICSS procedure is apt to identify structural breaks in the

dynamics of stock returns volatility of the NGSE_30 index. The GARCH (1,1) model is modified to account for structural breaks identified by the MICSS algorithm by introducing a dummy variable D_{it} that is equal to 0 before the break event and equal to 1 afterward. As such, equation 2 is modified as:

$$\sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \gamma_1 \sigma_{t-1}^2 + \sum_{i=1}^m \theta_i D_{it} \quad (4)$$

where m is the number of identified structural breaks. If $\theta_i > 0$, then, the break event does affect (increase) the mean of the conditional variance. The introduction of dummy variables to account for breaks in the TARCH (1,1) model of equation 3 is expressed as.

$$\sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \delta_1 \varepsilon_{t-1}^2 A_{t-1} + \gamma_1 \sigma_{t-1}^2 + \sum_{i=1}^m \theta_i D_{it} \quad (5)$$

3.4 Unit Root Tests

Using an autoregressive model, a unit root test determines the non-stationarity of a time series variable. The Augmented Dickey-Fuller Generalized Least Squares (ADF-GLS) is employed in this paper as its testing procedure dominates the standard ADF unit root tests in terms of its statistical power and efficiency. The ADF-GLS test is:

$$\Delta \tilde{y}_t = \varphi \tilde{y}_{t-i} + \sum_{i=1}^q \beta_i \Delta \tilde{y}_{t-i} + u_t \quad (6)$$

here, $\Delta \tilde{y}_t$ is the first difference of the GLS-detrended series, φ is the parameter of interest, q is the number of lags, and u_t is the error term. The null hypothesis of a unit root is rejected in favor of a stationary alternative hypothesis if the ADF-GLS test statistic is less than the critical value. When a unit root is present, the time series is non-stationary. In this case, we fail to reject the null hypothesis (Elliott *et al.*, 1996)

Another useful unit root test employed in this paper is the Phillips-Perron (PP) test. The PP test modified the Dickey-Fuller (DF) test by adjusting for serial correlation

and heteroskedasticity in the error terms. The PP test is based on the regression:

$$\Delta y_t = a + \beta t + \phi y_{t-1} + u_t \quad (7)$$

Here, Δy_t is the first difference of the series and t is a time trend. The null hypothesis of a unit root is rejected in favor of a stationary and trend alternative hypothesis if the PP test statistic is less than the critical value. This indicates a stationary series. We fail to reject the null hypothesis if the PP test statistic is greater than the critical value. As such, the series may have a unit root and is non-stationary (Phillip & Perron, 1988).

3.5 Test for Serial Correlation and ARCH Effects

The Breusch-Godfrey test is a Lagrange multiplier (LM) test for the presence of serial correlation in models where lagged values of the dependent variables are used as independent variables. This test detects serial correlation of any order in the residuals of a regression model, which makes it more general than the Durbin-Watson test, which is only valid for first-order serial correlation. The Breusch-Godfrey LM Statistic is:

$$LM = T * R^2 \quad (8)$$

where T is the number of observations and R^2 is the coefficient of determination from an auxiliary regression obtained by regressing \hat{u}_t on the original regressors and \hat{u}_{t-p} . P is the lag order. The null hypothesis of no serial correlation up to order p is rejected in favor of the alternative hypothesis of a serial correlation if the LM test statistic is greater than the critical value from a chi-square distribution at a chosen significance level. Thus, the residuals are serially correlated (Greene, 1951).

In modelling conditional heteroskedastic variables, the test for ARCH or GARCH effects is crucial. The Test statistic for ARCH effects is the LM test is:

$$LM = N * R^2 \quad (9)$$

where N is the number of observations and R^2 is the coefficient of determination from the auxiliary regression of the estimated squared residuals ($\hat{\epsilon}_{t-1}^2$) on a constant and its lagged squared values. Under the null hypothesis of no ARCH effects, the LM statistic has a Chi-squared distribution with degrees of freedom equal to the number

of lags used in the test. Values of the test statistic larger than the critical value indicate the presence of ARCH (or GARCH) effects (Greene, 1991b; Engle, 1982).

4. Results and Discussion

4.1 Descriptive Statistics

Figure 1 presents the levels and returns plots over the sample period. The returns are preferred as they proved to be stationary. More so, an efficient stock market is measured by the volatility of its returns. The daily returns are functions of the linear transformation of the level data. The linear transformation is the first difference of the log computed as: $RNGSE30_t = 100 * d\log NGSE30_t$, where $RNGSE30_t$ is the current period stock returns of $NGSE30_t$ and $NGSE30_t$ is the current period $NGSE30_t$.

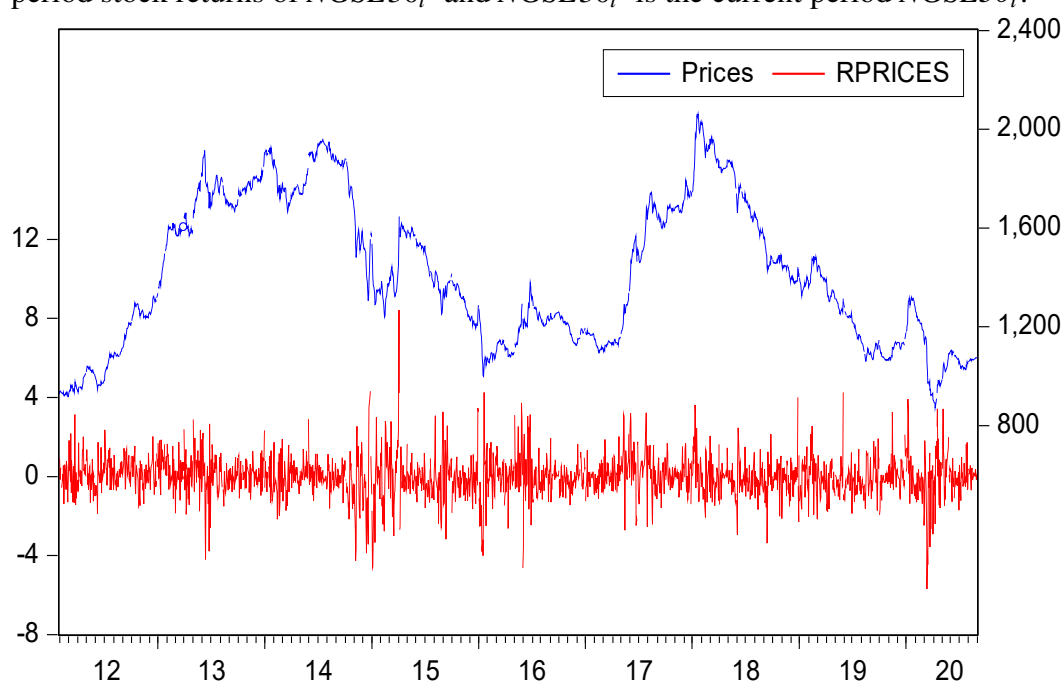


Figure 1: Level and Returns Plot of NGSE_30, 2012-2020

The returns plot highlights periods of volatility clustering, while the level plot reflects notable market fluctuations. The high peak observed in 2013 coincided with the period Nigeria experienced macroeconomic stability and policy coordination, which attracted foreign capital inflow largely driven by the rebasing of Nigeria's GDP, which repositioned the country as Africa's largest economy (IMF, 2013; AFDB, 2014). However, between mid-2014 and 2017, the market experienced a significant trough, influenced by both internal and external shocks, such as the fall in global crude oil prices in 2014, which caused Nigeria's major export revenue to reduce and led to fiscal deficit, and lower the inflow of foreign portfolio significantly (IMF, 2016; EIA, 2015). The market gradually started to recover from 2017 onwards, possibly due to a

series of economic recovery efforts, policy responses, and market reforms. This includes foreign exchange interventions by the Central Bank of Nigeria (CBN, 2017), the Economic Recovery and Growth Plan (ERGP) launched in 2017 by the Government of Nigeria, and an upswing in the global oil prices (Federal Government of Nigeria, 2017; IMF, 2018). These factors combined may have contributed to the improved economic outlook in Nigeria, leading to increased market activities.

Although the market remained vulnerable to both internal and external shocks, such as the fluctuating global oil prices and the domestic policy inconsistencies (IMF, 2018), as reflected in the volatility patterns captured in the returns plot. The presence of these volatility clusters and the fluctuations in market levels during key macroeconomic and political periods suggest strong connections between external shocks and shifts in volatility. These contextual insights reinforce the structural breaks detected in the variance series and justify the use of robust methods such as the MICSS algorithm to account for these regime shifts when modeling the volatility of the NGSE_30 index.

Table 1 presents the summary statistics of RNGSE_30. The positive mean returns imply on average, investors recorded more gains than losses with a substantial standard deviation. The positive skewness is also suggestive of more rise than fall in the market, and a non-normal distribution. The Jarque and Bera statistic p-value is significant at the 5% conventional level. Thus, an error construct with a fat tail distribution is apt.

Table 1: Summary Statistics

Statistic	RNGSE_30
Mean	0.0069
Standard Deviation	1.0456
Skewness	0.1823
Kurtosis	7.9801
Jarque-Bera	2206.6990
Probability	0.0000

4.2 Pre-estimation Tests

4.2.1 Unit Root Test

Table 2: ADF-GLS and Phillip-Perron's Unit Root Test Results

Null Hypothesis	ADF-GLS Test		PP Test	
	Test statistic	5% critical value	Test statistic	5% critical value
$RNGSE30_t$ has a unit root	-4.7490	-2.8900	-32.2973	-3.4119

From the ADF-GLS and Phillip-Perron's unit root tests in Table 2, the test statistic is more negative than the test critical value at the 5% significant level, hence, the null hypothesis is rejected, and the returns series is stationary: when the market is hit by a shock, the effect of this shock is only transient on the price changes, after which there is convergence to the long-run average price. This is also depicted in Figure 1. Following the sharp decline in stock prices experienced in 2014, the market regained momentum and gradually reverted to its initial trend by 2017.

4.3.2 Test for Serial Correlation and ARCH Effects

The Breusch-Godfrey (1978) LM test is used to test for serial correlation, with the null hypothesis of no serial correlation in the residuals up to the specified lag. While the LM ARCH effect test is used to test for heteroscedasticity. LM test to check if the coefficients of the lagged squared residuals are significantly different from zero.

Table 3: Results of Serial Correlation and ARCH Test

Tests	Test Statistics (p-value)
Breusch-Godfrey LM test	1.2721 (0.2595)
LM ARCH Effect Test at lag 1	77.4595 (0.0000)

Note: Values in parentheses are p-values.

From Table 3, the Breusch-Godfrey LM test on the residuals indicates no serial correlation at lag 1. In detecting heteroscedasticity, the LM ARCH effect test at lag 1 is significant. Thus, the ARCH effect test formalizes the presence of heteroscedasticity in squared residuals and the appropriateness of estimating the volatility models.

4.2.3 Test for Structural Breaks

The CUSUM plot is used to visually inspect the returns plot for structural breaks in the mean, while the CUSUM of squared plot is used to visually inspect for structural breaks in the variance. Significant digressions by either of the plots from the 5% bounds is indicative of structural breaks in the series, and the application of the MICSS algorithm to identify the actual break dates.

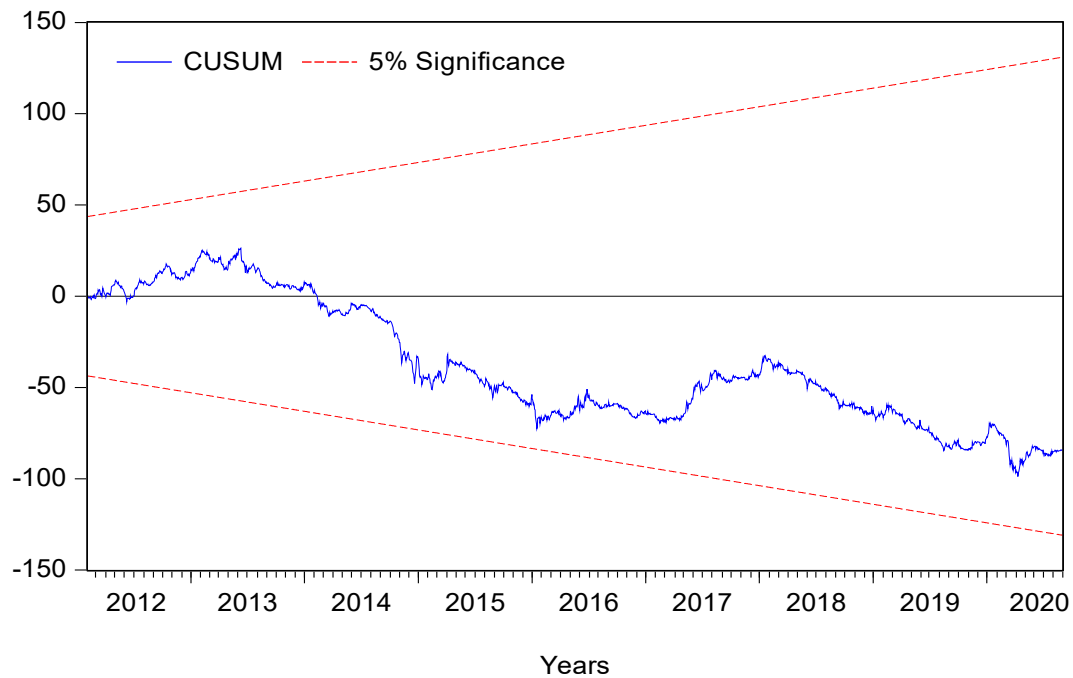


Figure 2: CUSUM for Mean

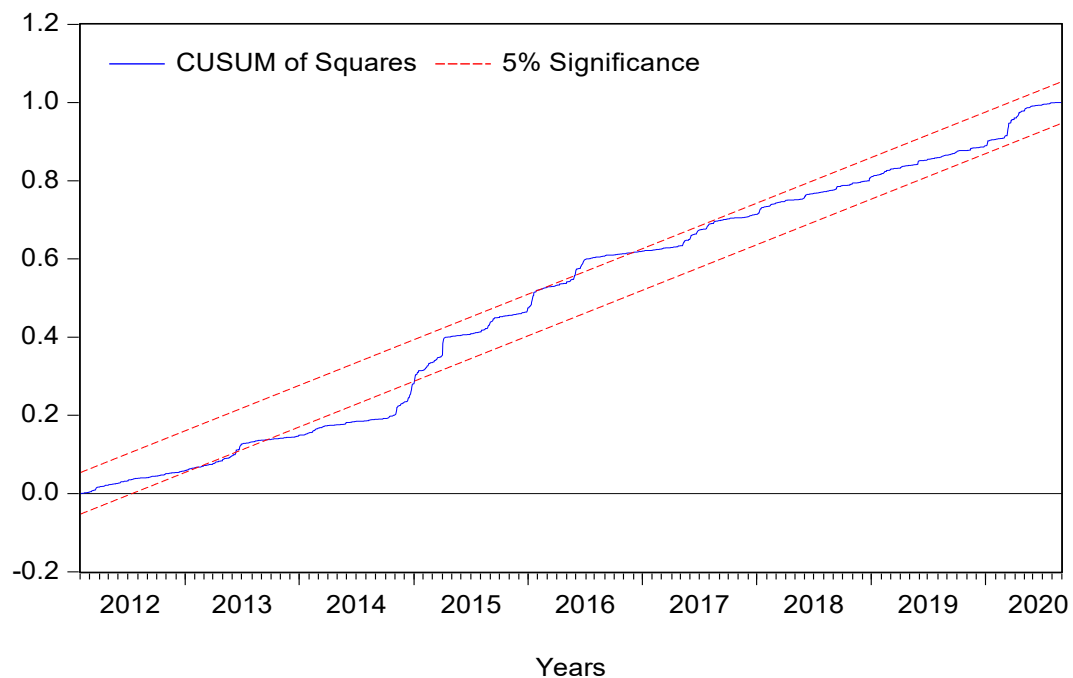


Figure 3: CUSUM of Squares for Variance

From the CUSUM plot in Figure 2, there is no evidence of a break in the mean,

but from the CUSUM of squared plot in Figure 3, there is a digression from the 5% significant boundaries indicating a break in the variance. Thus, the variance of the returns is not stable. Table 4 reports the exact periods the breaks occurred as detected by the MICSS algorithm, while Figure 4 presents the plot of these identified structural breaks in variance.

Table 4: Result of Detected Structural Breaks in Variance by MICSS Algorithm

Year	Detected Break Dates
2013	February 7, May 13
2014	August 14
2015	January 9, September 17
2016	March 16
Total Number of Detected Breaks	6 breaks

Note: Standard Deviation is used as a proxy for unconditional variance. Robust MICSS Algorithm for ± 3 Standard Deviation at 1% level.

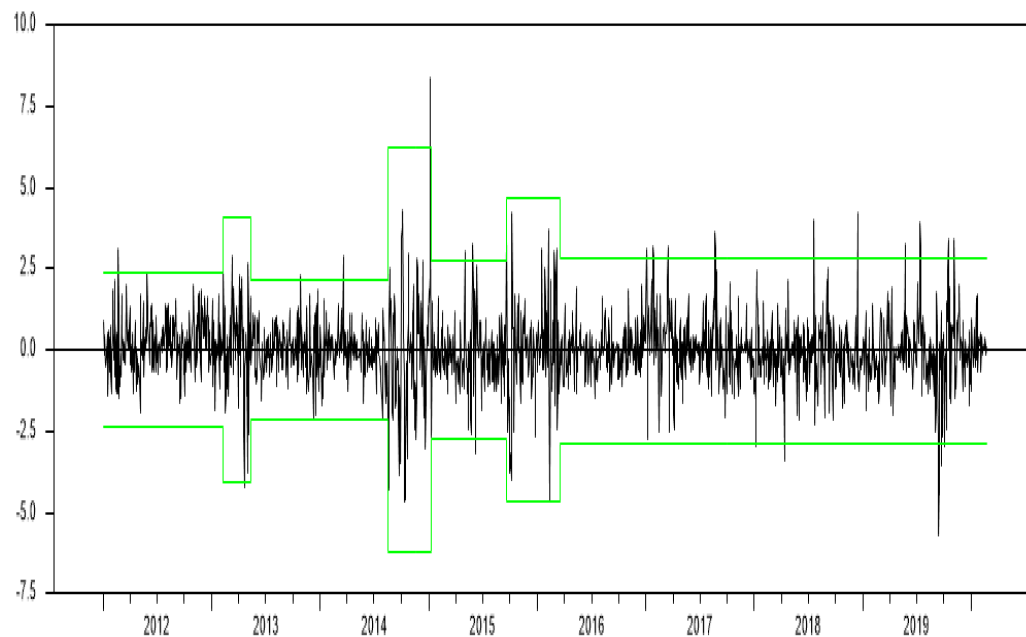


Figure 4: Detected structural Breaks on RNGSE_30

4.3 Main Results

Table 5 presents the results of the estimated GARCH (1, 1) models with and without structural breaks. The model with structural breaks reduces the volatility persistence from 0.919 to 0.907. Thus, when structural breaks are not incorporated into

the conditional variance model, the volatility persistence is overestimated. These results are consistent with those of Souffargi and Boubaker (2022), Kuhe (2019), and Kartsonakis-Madamlis and Dritsakis (2020).

Table 5: Estimated Results for the GARCH (1, 1) Models

Parameters	Without Breaks	With Breaks
Mean Equation		
<i>Constant</i>	-0.017 (0.016)	0.213 (0.068)***
<i>AR(1)</i>	0.240 (0.022)***	0.248 (0.058)***
<i>AR(5)</i>	0.067 (0.019)***	0.061 (0.019)***
Variance Equation		
<i>Constant</i>	0.107 (0.022)***	0.125 (0.042)***
<i>ARCHterm</i>	0.240 (0.040)***	0.229 (0.046)***
<i>GARCHterm</i>	0.679 (0.039)***	0.678 (0.065)***
Diagnostic Tests		
Persistence	0.919	0.907
AIC	2.547	2.538
Log-likelihood	-2696.494	-2667.448

Note: *** significant at the 1% level. Values in parentheses are standard errors.

4.4 Additional Results: Robustness check

Table 6: Estimated Results for the TARCH (1, 1) Models

Parameters	Without Breaks	With Breaks
Mean Equation		
<i>Constant</i>	-0.019 (0.016)	0.209 (0.068)***
<i>AR(1)</i>	0.240 (0.022)***	0.247 (0.058)***
<i>AR(5)</i>	0.067 (0.067)***	0.061 (0.019)***
Variance Equation		
<i>Constant</i>	0.105 (0.021)***	0.123 (0.042)***
<i>ARCHterm</i>	0.213 (0.041)***	0.207 (0.048)***
<i>Asymmetricterm</i>	0.056 (0.054)	0.047 (0.045)
<i>GARCHterm</i>	0.682 (0.039)***	0.681 (0.066)***
Diagnostic Tests		
Persistence	0.923	0.911
AIC	2.548	2.539
Log-likelihood	-2691.799	-2666.970

Note: *** significant at the 1% level. Values in parentheses are standard errors.

Table 6 reports the estimated asymmetric TARCH (1, 1) model. By incorporating structural breaks, the impact of good news on volatility was reduced from 0.213 to 0.207, and bad news was reduced from 0.269 to 0.254. Volatility persistence also re-

duces. The leverage effect is statistically not significant at the 5% level, implying that negative shock does not inflate volatility more than an equal magnitude of positive shock. Hence, there is no asymmetry in the news. The results of the good news, bad news, and volatility persistence show that when structural breaks are not incorporated into the conditional variance model, these parameters are overestimated. These results are consistent with those of Kartsonakis-Mademlis and Dritsakis (2020).

4.5 Residual Diagnostic

The Breusch-Godfrey and the LM ARCH tests were performed on the standardized squared residuals for the estimated models with breaks to inspect for serial correlation and remaining GARCH effects at lag 1, 2, and 5 respectively. The results of the residual diagnostic test are reported in Table 7.

Table 7: Results of Serial Correlation and ARCH LM Tests on Break Models

Test Statistics	GARCH (1, 1)			TARCH (1, 1)		
	Lag 1	Lag 2	Lag 5	Lag 1	Lag 2	Lag 5
Serial Correlation						
Breusch-Godfrey F-stat	3.1090 (0.0780)*	2.9077 (0.1487)*	1.1162 (0.3495)*	3.2985 (0.0695)*	2.0083 (0.1345)*	1.1731 (0.3199)*
ARCH Test						
ARCH Effect F-stat	0.3156 (0.5742)*	0.6496 (0.5224)*	0.7137 (0.6131)*	0.2351 (0.6278)*	0.5730 (0.5639)*	0.6181 (0.6860)*

Note: Values in parentheses are p-values. * indicates significance at 10% level.

At the 5% level, the p-values of the Breusch-Godfrey test statistics are not significant, as such, there is no evidence of serial correlation in these models. Also, the ARCH test revealed no evidence of GARCH effects. Figures 5 and 6 present the stability test via the CUSUM and CUSUM of squares plots applied to check if the estimated break models accurately account for the detected breaks.

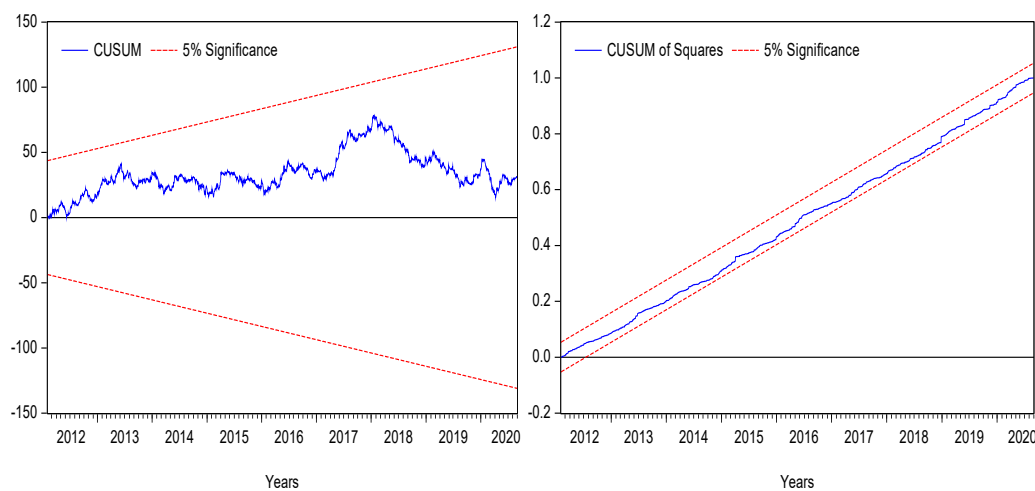


Figure 5: CUSUM and CUSUM of squares plots for GARCH (1, 1)

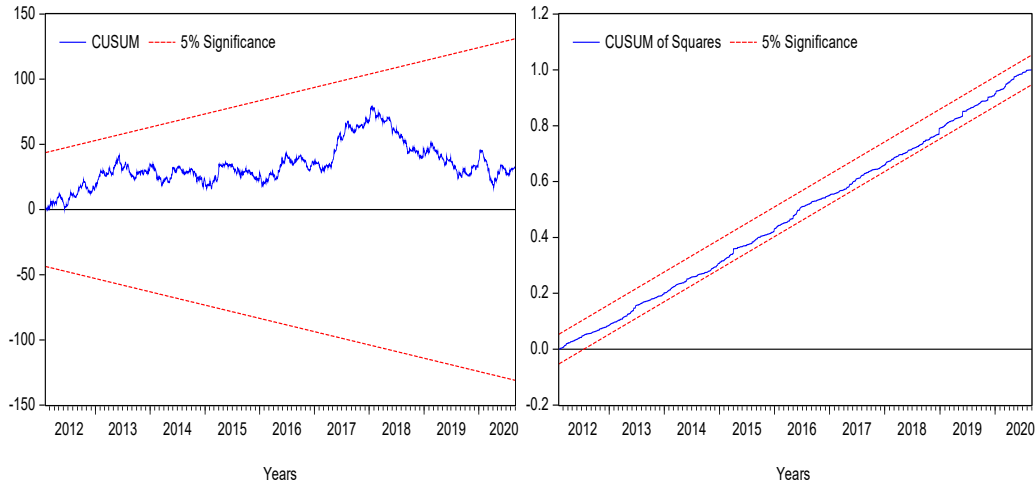


Figure 6: CUSUM and CUSUM of squares plots for TARCH (1, 1)

From both the CUSUM and CUSUM of squared residual plots in Figures 5 and 6, there are no digressions from the 5% significant boundaries: observations lie within the error bands, implying the estimated break models sufficiently accounted for breaks in the variance models. The serial correlation test, heteroscedasticity test, and stability test indicate that conditional heteroscedasticity was correctly accounted for by the estimated break models.

5. Conclusion and Policy Implications

This study discusses the use of the MICSS algorithm to detect and account for multiple structural breaks when modeling conditional heteroscedasticity in the Nigerian Stock Market Exchange_30 index (NGSE_30). The CUSUM of squared showed that the variance process deviated from the 5% stationary bounds, indicating multiple breaks in the variance. The MICSS was used to detect these breaks, and the identified breaks were accounted for within the volatility models. The estimated models improved when structural breaks were accounted for, as it significantly reduced volatility persistence and the effects of good news and bad news on volatility. Accounting for structural breaks helps in assessing the potential impact of changes in model parameters on market volatility, which is important for risk management. The analysis of asymmetric effects from the TARCH (1, 1) model revealed that bad news does not overinflate volatility compared to good news, providing valuable insights into the asymmetric responses of financial markets to different types of information and events. Neglecting structural breaks in volatility modeling can lead to overesti-

mation of the degree of shock persistence and misspecification of the model, resulting in biased parameter estimates, which can lead to inaccurate assessments of risk and market dynamics. The conclusion drawn from the research underscores the critical importance of incorporating structural breaks in financial time series modeling, particularly when analyzing volatility dynamics in stock returns.

While the MICSS algorithm improves structural break detection in dependent time series compared to traditional CUSUMSQ methods, it has some limitations. First, it can be computationally intensive, especially with large datasets or multiple breaks, thereby limiting its scalability. Second, while MICSS accounts for volatility clustering and autocorrelation, it assumes piecewise constant variance between breaks, which may not capture smoothly changing volatility well. This limitation further restricts its broader applicability, particularly in real-time or large-scale financial modeling. Third, the method is sensitive to initial assumptions like bandwidth and kernel choices, which may affect the accuracy of break detection if they are incorrectly selected.

To maintain result consistency and conclusions, this study could not extend the sample size beyond 2021, representing a research limitation. Policy implications stemming from this research suggest that incorporating structural breaks in financial modeling can lead to more reliable analyses, improved risk management strategies, and enhanced decision-making processes in the realm of stock market investments and commodity trading. Thus, stakeholders can better navigate market uncertainties and mitigate potential risks associated with volatile financial data.

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