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Global Warming Policy in a Federation

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Global Warming Policy in a Federation

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Abstract

We explore the efficiency of alternate allocations of responsibility for greenhouse gas emissions pricing policies in a small open economy federation. Emissions result from consumption and production of a tradeable dirty good, and their intensity depends on the emissions technology. National emissions have an imperceptible effect on global warming, so preferences for control are based on social norms which can differ depending on the allocation of policy responsibility. Policies include emissions taxes and permit trading systems. The costs of collecting emission taxes and administering a permit trading system are lower for the regional governments than the federal government because of informational advantages. Unlike the regions, the federal government internalizes the social costs of emissions borne by both regions. Both federal and regional optimal emission pricing policies are variants of Pigovian taxes. When regional governments are responsible for emissions policy and the federal government makes interregional income transfers, the timing of government decisions affects policies.

Key Words: fiscal federalism, global warming, greenhouse gas pricing

JEL Classification: H23, H7, Q54, Q58

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1 Introduction

Policies to address global warming due to greenhouse gas emissions are complex. They include mixtures of emissions pricing and regulation. The regulations can include emissions targets and technology restrictions, such as rules for electrical generation and consumer technologies. They can also include regulation of environmental infrastructure, such as hydroelectric transmission lines, electric vehicle charging stations, wind turbines, and nuclear power facilities. Emissions pricing can use emissions taxes or marketable permits. The revenues raised by emissions pricing schemes are used for a variety of purposes, including rebates to households, reductions in tax rates and investment in environmental infrastructure. Governments may even forgo revenues by giving emissions permits to producers free of charge. Non-pricing schemes, like emissions quantity controls, can also be used despite their inefficiency. For a recent summary of emissions policies, see Kotchen (2024).

The design and implementation of emissions policies lead to special problems in federations. Federal constitutions typically do not assign responsibility for emissions policy to one level of government or another, since environmental legislation was not foreseen when the constitutions were written. There can be considerable discretion about which level of government ought to be responsible for the various components of emissions policy. Consequently, given the diversity of policy instruments, some environmental issues are addressed federally and others by regional governments.¹

In this paper, we explore the federal-regional assignment of responsibility for emissions policy in a highly stylized model of a small open federation designed to capture the main issues and trade-offs. We focus especially on emissions pricing and set aside policies for introducing green technologies for subsequent work. We pay particular attention to features of environmental pricing policy that are unique to greenhouse gas emissions (as opposed to say air or water pollution). First and foremost, global warming is a long-term issue spanning several generations. Greenhouse gases have a long life, and global warming is caused by the cumulation of past greenhouse gas emissions in the atmosphere. The effect of current emissions largely occurs several decades from now. The cost of reducing current emissions is borne by those now alive, while the benefits accrue mainly to future generations. At

¹We refer to subnational governments as regional governments to avoid using country-specific terms like states, provinces and länder.

the same time, current emissions of any one country are a small proportion of total world emissions. From a purely selfish point of view, the incentive for reducing current emissions is minimal so emissions policies must be based on non-selfish or altruistic arguments. The strength of these non-selfish motives, which can be based on social norms or social solidarity, can differ among regions. They can also affect citizen support for federal relative to regional emissions control policies.

Second, the process of emissions generation is complex. Emissions can arise from both consumption and production activities, and the rate of emissions depends on the technology used in the consumption and production activities. For example, the rate of emissions is higher when the energy used in production is based on carbon fuels like coal and oil rather than cleaner sources like natural gas or hydro-electricity. And, emissions by consumers is lower when they use electric vehicles, heat pumps and green heating energy rather than petroleum vehicles and oil home heating.

Third, the collection of emissions pricing revenues and the monitoring of emissions in a marketable permit system is challenging. Emissions themselves are difficult to measure, so pricing applies to the fuels that generate the emissions. These include gasoline and diesel fuel used to drive vehicles, home heating fuel, natural gas used industrially and by consumers, coal, wood, and so on. These substances are heterogeneous and emit greenhouse gases at differing rates. Practical considerations mean that there is a limit to the extent to which emissions prices can vary among greenhouse gas producing substances and activities. Furthermore, governments incur costs of collecting emissions taxes and overseeing emission permit systems, and these costs can differ for regional and federal governments.

Finally, technologies and forms of emissions differ between regions. Some regions are better equipped to generate hydro-electricity, and other regions have advantages at wind or solar power. This means that the viability of energy infrastructure differs between regions, which affects regulatory policies that might be adopted. Optimal emissions prices might also differ over regions. This diversity affects the efficacy of regional versus federal policy-making.

Our model incorporates these features in the simplest of ways. We consider a small open federation consisting of a federal government and two regions. Three goods are consumed and produced in each region: two goods traded at fixed world prices and one non-traded

good. One of the traded goods, called the *dirty good*, is the source of greenhouse gas emissions. Households consume all three goods, which are produced by good-specific firms using labour alone. Firms differ in their emissions intensity. Households within each region are identical, and we characterize them by a representative household who supplies a fixed amount of labour. Population in each region is normalized to unity. Labour is immobile between regions, and the wage rate and non-traded good price in each region are endogenous.

Greenhouse gases emissions are given proportions of the consumption and production of the dirty good. Governments obtain revenues from emissions pricing and return the revenues net of collection costs to households as lump-sum transfers. Which emissions pricing applies and to whom revenues are returned depends upon whether the federal or regional governments are responsible for emissions policy.

Household utility has a private and a social component. Private utility depends on consumption of the three goods and takes a standard form. Utility also includes the disutility due to global warming, which is determined by the cumulation of greenhouse gases in the atmosphere. Since the representative household has an imperceptible cumulated emissions, we take them as given and suppress the disutility they cause. Instead, we assume that the representative household in a given region obtains *social disutility* from their own emissions as well as from the emissions of the other region. This social disutility is based on social or altruistic motives. We assume that private behaviour is based on private utility, while government emissions policy is based on both private and social utility.² As mentioned, social disutility may differ depending on whether the federal government or the regions are responsible for emissions control.

We focus on emissions pricing policies enacted either by the federal government or the regions. These can differ because of differences in the social disutility of emissions and because of differences in the administrative cost of emissions pricing by the federal and regional governments. As well, federal emissions pricing policies internalize external effects of emissions in one region on the social disutility of emissions in the other.

Our model is restrictive in a number of ways. The only roles for government are to implement emissions pricing policies and make lump-sum transfers to households in the

²This is in contrast with van der Ploeg and Venables (2022) and Besley and Persson (2023), who assume that social norms affect individual rather than government behaviour.

two regions. The regions can differ in per capita incomes, so the federal government uses differential lump-sum transfers to the representative individuals to address interpersonal or interregional equity. We also assume a deterministic setting so there are no interregional shocks that would give rise to a federal government insurance role.³ Our model is static and takes no explicit account of the long-term dynamics of climate change and its effect on future generations as emphasized by Stern (2007). This precludes us from comparing the outcomes of our optimal emissions pricing schemes with a social optimum.⁴ The social ordering we use is based on the social utilities of currently alive households.

Our approach follows a long tradition of analyses of optimal environmental policies in second best settings. Examples include Sandmo (1975), Bovenberg and de Mooij (1994) and Cremer, Gahvari and Ladoux (1998). These all adopt the standard approach of treating the cost of externalities as a private cost. There has also been a large literature on environmental federalism. Representative papers include Oates (2002), Goulder and Stavins (2011), Alm and Banzhaf (2012), Banzhaf and Chupp (2012), Shobe and Burtram (2012) and Segerson (2020). These papers also recount the benefits of decentralized responsibility, and they do so using the standard approach to environmental externalities.

In the following section, we outline the details of our model including the behaviour of households and producers. We take as our base case a decentralized federation in which the regional governments are responsible for emissions control policies and transfers to households. In Section 3, we derive the general equilibrium conditions of the model including the determination of equilibrium regional wage rates and prices of nontraded goods. Section 4 derives optimal emissions pricing rules when regional governments are responsible for emissions control. In Section 5, we consider the case in which the federal government implements both emissions control policies and transfers to households. Section 6 considers the mixed case in which the regions are responsible for emissions control policies while the federal government can make interregional transfers. Section 7 concludes with some observations about the efficacy of federal versus regional responsibility for emissions policy and discusses some extensions to the analysis.

³Analyses of interregional insurance can be found in Persson and Tabellini (1996) and Lockwood (1999).

⁴An alternative approach, outlined in Kotchen (2024), is to abstract from the social optimum by focusing on the least cost way of achieving a given level of emissions control. The level we consider is that which is generated by social preferences rather than a social welfare function.

2 The model

We begin by describing the features of our model. This includes preferences and production technologies, the social costs of emissions and objective functions of regional and federal governments. We summarize the behaviour of producers and households, and the supply and demand functions and emissions they generate.

2.1 Assumptions

We consider a federal nation in a small open economy setting. The nation has one federal government and two regional governments, indexed by superscripts $j = A, B$. The population size in each region is normalized to unity, and all households within a region have identical preferences, although they can differ between regions. Households in each region are represented by a representative household who obtains utility from three goods: a nontradable good, x_0^j , and two tradable goods, x_1^j and x_2^j . Both the consumption and production of good 2 — the ‘dirty’ good — can generate greenhouse gas emissions. Neither the nontraded good nor good 1 generate emissions. Labour supply of the representative resident is fixed in region j at L^j , and there is no interregional labour mobility.

Technologies of production of all three goods and the intensity of emissions from good 2 can differ between regions. Production of good i by a representative firm in region j is denoted by y_i^j and is determined by a production function in labour ℓ_i^j

$$y_i^j = f_i^j(\ell_i^j), \quad j = A, B \quad i = 0, 1, 2 \quad (1)$$

where $f_i^j(\ell_i^j)$ is increasing and strictly concave. The number of firms in each industry is normalized to unity. Those producing goods 0 and 1 are identical, so y_0^j and y_1^j are industry outputs. As discussed below, firms producing the dirty good are heterogeneous, so y_2^j differs over firms. We denote aggregate output of the dirty good in region j as \bar{y}_2^j . We assume for simplicity that both regions produce all three goods in equilibrium. Producers are competitive and maximize profits.

Emissions from the use of the dirty good are given by fixed proportions of consumption and production. Let ϕ^j be the quantity of emissions generated per unit of consumption by

all households in region j . Consumption emissions in region j , are then:

$$e_x^j = \phi^j x_2^j. \quad j = A, B \quad (2)$$

Production emissions differ across both firms and regions. Let μ^j denote emissions generated per unit of production of the dirty good in region j where μ^j varies across firms in region j according to the distribution function $G^j(\mu^j)$ with density $g^j(\mu^j) > 0$ on the interval $[\underline{\mu}^j, \bar{\mu}^j]$. Total production emissions generated in region j are then:

$$e_y^j = \int_{\underline{\mu}^j}^{\bar{\mu}^j} \mu^j y_2^j dG^j(\mu^j). \quad j = A, B \quad (3)$$

Total emissions in region j are:

$$e^j = e_x^j + e_y^j, \quad j = A, B \quad (4)$$

while total world emissions are:

$$e = e^R + \sum_{j=A,B} e^j, \quad (5)$$

where e^R are total emissions in the rest of the world and e^j is given by (4).

Since a region's emissions are a negligible part of world contribution to global warming, emissions control would be minimal if regional or federal governments acted on the basis of their residents' self-interest alone. Social norms can rationalize non-negligible emissions controls in a region or nation. Social norms could depend on what other regions or countries are emitting, as well as on obligations under international agreements. International agreements typically take the form of treaties, which are binding for the federal government, but not necessarily for the regions. The social norm associated with choosing to emit less than a 'selfish' amount can be interpreted as analogous to the joy of giving or altruism in the case of charitable contributions (Andreoni, 1990; Andreoni and Payne, 2013; Cremer and Pestieau, 2006; Diamond, 2006). This joy of reducing emissions may be contingent on emissions in the other region or in the rest of the world. Emissions control policies enacted by the government on the households' behalf fulfill what households regard as a social obligation.

It is useful to distinguish between households' individual preferences and their social, or expressive, preferences. The latter refer to the preferences that households have over collective outcomes. Social preferences determine household voting behaviour (Arrow, 1951;

Brennan and Brooks, 2013; Hamlin and Jennings, 2019) or their support for government policies (Ostrom, 2000; Kleven, 2014; Jacobs et al., 2017). Individual preferences determine households' own behaviour, including their demands for goods and emissions abatement. Social preferences refer to households' social or ethical orderings over the emissions control policies that they prefer their governments to undertake to avoid free-riding problems.⁵ We assume that social norms are reflected in social preferences but not individual preferences.

Individual preferences in region j are represented by a strictly concave utility function in the three goods $U^j(x_0^j, x_1^j, x_2^j)$, which is the same for all households within a region, but can differ across regions. There will be a representative household in each region if households also have the same income. If households have different incomes, then to allow us to aggregate households within a given region, we assume preferences take the Stone-Geary form:

$$U^j(x_0^j, x_1^j, x_2^j) = (x_0^j - \underline{x}_0)^{\beta_0^j} (x_1^j - \underline{x}_1)^{\beta_1^j} (x_2^j - \underline{x}_2)^{\beta_2^j}, \quad j = A, B \quad (6)$$

This utility function is convenient because, besides being relatively easy to use, it allows us to aggregate households with different incomes into a single representative household.⁶ We suppress the private disutility that the households obtain from global warming since that can be taken as fixed. We also assume that current emissions have a negligible effect on individual utility since they are a small share of global greenhouse emissions (Stern, 2007).

Social preferences represent the preferences that households expect their governments to act on. They will depend on the level of government that is responsible for emissions control policy. For example, social solidarity may be stronger at the regional than at the national level. Suppose this responsibility lies with the regional governments. The policy of region j is determined by the social preferences of the representative household in region j , which are individual preferences in (6) augmented by a regional social disutility function:

$$W^j(x_0^j, x_1^j, x_2^j, e^j, e^{-j}, e^R) = U^j(x_0^j, x_1^j, x_2^j) - s^j(e^j, e^{-j}; e^R), \quad j = A, B \quad (7)$$

⁵This is in contrast with social norms that affect individual environmental behaviour as in Nyborg (2020), Besley and Persson (2020) and van der Ploeg and Venables (2022). These authors emphasize that government policies can facilitate cooperative environmental behaviour.

⁶We could have instead assumed preferences are additively separable in the traded and non-traded goods and linear in the non-traded good. In this case, the optimal emission pricing rules would be denominated in units of non-traded good rather than income units since the marginal utility of income for the representative household would be one over the price of the non-traded good.

where $s^j(e^j, e^{-j}; e^R)$ is increasing in all its arguments, that is, $s_{e^j}^j, s_{e^{-j}}^j, s_R^j > 0$. Emissions e^j are determined by the policies of region j 's government, while e^{-j} is determined by the other region's government. In what follows, we assume that e^R can be taken as given and we suppress it from the social disutility function.

We assume that the marginal social disutility of regional emissions, $s_{e^j}^j$, is increasing in e^j so $s_{e^j e^j}^j > 0$. The sign of $s_{e^j e^{-j}}^j$ does not affect our general results, but it will affect total emissions in the federation in a Nash equilibrium. For example, if $s_{e^j e^{-j}}^j < 0$, this implies that when emissions in the other region rise, the marginal cost of emissions in region j falls. That is, emissions in region j and the other region are strategic complements so are mutually reinforcing. On the other hand, if $s_{e^j e^{-j}}^j > 0$, regional emissions are strategic substitutes, so mutually offsetting. This would be the case if regional emissions are perfect substitutes so $s^j(e^j, e^{-j}) = s^j(e^j + e^{-j})$. We return to this case when we consider centralized emissions control.

To reiterate, the social disutility of emissions function does not represent the disutility the household actually receives from current emissions. Instead, it represents the view of the household about what the level of emissions should be from an ethical point of view, for example, taking into account damages inflicted by current emissions on future generations, and given that regional governments are responsible for emission pricing policies.

Social preferences differ in two ways if the federal government is responsible for emissions control policy. First, the federal government takes into account the utility of the representative households in both regions so aggregates household preferences into a federal social welfare function. Second, the social utility of emissions function will differ since emissions in both regions are determined by the policies of the federal government, and federal government policy controls aggregate emissions, $e^A + e^B$, rather than emission separately in the two regions. To capture these two effects, we assume that social preferences of the federal government take the form:

$$W(x_0^A, x_1^A, x_2^A, x_0^B, x_1^B, x_2^B, e^A + e^B) = \sum_{j=A,B} U^j(x_0^j, x_1^j, x_2^j) - \sigma(e^A + e^B), \quad (8)$$

where $\sigma(e^A + e^B)$ is the federal social disutility of emissions function, which aggregates preferences for national emissions by the representative households in the two regions. It is increasing and strictly convex in aggregate emissions. Social norms regarding what ag-

aggregate emissions should be when the federal government is responsible might differ across the two regions. For example, the social disutility of aggregate emissions if regional governments are responsible would be $s^j(e^j + e^{-j})$ in which case federal social disutility is $\sigma(e^A + e^B) = \sum_{j=A,B} s^j(e^j + e^{-j})$. Social preferences in (8) imply that the federal government has zero aversion to utility inequality so will aim to equalize marginal utilities of income across regions.

Labour is mobile across production sectors in each region, and therefore regional wages w^j are uniform. Since labour is immobile across regions, w^A can differ from w^B . The producer prices of the three goods are p_0^j , p_1 and p_2 . The price of the nontraded good, p_0^j , is endogenous and can also differ between regions. Producer prices of traded goods, p_1 and p_2 , are the same in both regions and are denominated in the domestic currency. They are related to fixed world prices, r_1 , and r_2 , through the exchange rate ϵ , so $p_i = \epsilon r_i$, $i = 1, 2$.

The government responsible for emissions pricing — the regional government in our base case — imposes a per unit tax of τ_x^j on consumption emissions in region j . The consumer price of the dirty good is therefore $p_2 + \tau_x^j \phi^j$. The price for production emissions is implemented via an emissions permit scheme. The region- j government gives $\bar{\kappa}^j$ emissions permits free of charge to all producers who can then trade them in a competitive permit market. Giving the permits to producers precludes them from facing competitive pressures from foreign firms. The price of a permit is determined by the emissions permit market.⁷ The auction for permits is assumed to be competitive, so producers take as given the equilibrium price of permits, denoted by p_κ^j .

The regional government incurs a cost of collecting the consumption emissions tax of γ_x^j per unit of consumption emissions e_x^j . It also incurs a cost γ_y^j per unit of production emissions e_y^j to administer the emissions permit auction. These costs are partly monitoring costs since households and firms must self-report their emissions. The government obtains revenues from the consumption emissions tax less the costs of collecting the tax and administering the permits. It returns these revenues to the representative households as lump-sum

⁷As discussed further below, the same emissions pricing outcome can be achieved if the government either sells the permits by auction to producers in the region or uses a production emissions tax. An alternative would be to accompany the auction of permits with border tax adjustments (BTA) based on emissions pricing abroad. See Keen and Kotsogiannis (2024) for an analysis of this case. In fact, as we discuss below, given our small open economy assumptions, BTAs are redundant in our model.

transfers, a^j . We assume that emissions collection and administration costs, denoted c^j , are measured in units of good 1 so the value of these costs in region j are:

$$p_1 c^j = \gamma_x^j e_x^j + \gamma_y^j e_y^j. \quad j = A, B \quad (9)$$

This assumption ensures that emissions taxes can be collected and emission cap administered at constant costs since x_1 is tradable so its price is given. These costs of tax collection and permit administration could differ by region. They can also differ depending on which level of government is responsible for emissions control. For example, the regional governments might be better informed than the federal government about emissions from production and therefore better able to administer an emissions permit system, but both levels of government might be equally informed about consumption emissions. This information assumption is simple and tractable, but it has the disadvantage that full observation of emissions is possible, albeit at a cost.

2.2 Producer and household behaviour

We first characterize the behaviour of producers and consumers, and then turn to the determination of equilibrium.

2.2.1 Regional supply functions

Consider the production of good i in region j given by (1). Producers take as given output prices p_0^j , p_1 and p_2 , the regional wage rate w^j , the per producer quantity of emissions permits $\bar{\kappa}^j$, and the permit price p_κ^j . We consider profit maximization by producers of clean and dirty goods in turn.

Production of clean goods

Given the production function (1), the profit-maximizing problem of producers of clean goods y_0^j and y_1^j can be written:

$$\max_{\{\ell_i^j\}} p_i^j f_i^j(\ell_i^j) - w^j \ell_i^j. \quad j = A, B \quad i = 0, 1$$

The first-order condition, $p_i^j f_i^{j'}(\ell_i^j) = w^j$, yields labour demand and good supply functions:

$$\ell_i^j(p_i^j, w^j), \quad y_i^j(p_i^j, w^j) = f_i^j(\ell_i^j(p_i^j, w^j)). \quad j = A, B \quad i = 0, 1 \quad (10)$$

Both are decreasing in w^j , increasing in p_i^j , and homogeneous of degree zero in p_i^j and w^j .

The value function of the producer's problem is the profit function, $\pi_i^j(p_i^j, w^j)$, where

$$\frac{\partial \pi_i^j}{\partial p_i^j} = y_i^j(p_i^j, w^j) > 0, \quad \frac{\partial \pi_i^j}{\partial w^j} = -\ell_i^j(p_i^j, w^j) < 0. \quad j = A, B \quad i = 0, 1 \quad (11)$$

The profit functions, $\pi_0^j(p_0^j, w^j)$ and $\pi_1^j(p_1^j, w^j)$, are homogeneous of degree one in prices.

Production of the dirty good

Consider a producer of y_2^j with the emissions parameter μ^j . The firm receives $\bar{\kappa}^j$ permits for free from the government and takes the price they can buy/sell permits p_κ^j as given. The problem of the producer is:

$$\max_{\{\ell_2^j\}} p_2 f_2^j(\ell_2^j) - w^j \ell_2^j - p_\kappa^j (\mu^j f_2^j(\ell_2^j) - \bar{\kappa}^j). \quad j = A, B$$

The first-order condition is $p_2 f_2^{j'}(\ell_2^j) - w^j - p_\kappa^j \mu^j f_2^{j'}(\ell_2^j) = 0$, which gives labour demand and output supply:

$$\ell_2^j(p_2, w^j, p_\kappa^j; \mu^j), \quad y_2^j(p_2, w^j, p_\kappa^j; \mu^j) = f_2^j(\ell_2^j(p_2, w^j, p_\kappa^j; \mu^j)). \quad j = A, B \quad (12)$$

The profit function is $\pi_2^j(p_2, w^j, p_\kappa^j; \mu^j)$ and aggregate profits are

$$\begin{aligned} \bar{\pi}_2^j(p_2, p_\kappa^j, w^j, \bar{\kappa}^j) &= \int_{\underline{\mu}^j}^{\bar{\mu}^j} \left(p_2 f_2^j(\ell_2^j(p_2, p_\kappa^j, w^j; \mu^j)) - w^j \ell_2^j(p_2, p_\kappa^j, w^j; \mu^j) \right) dG^j \\ &\quad + p_\kappa^j \left(\bar{\kappa}^j - \int_{\underline{\mu}^j}^{\bar{\mu}^j} \mu^j f_2^j(\ell_2^j(p_2, p_\kappa^j, w^j; \mu^j)) dG^j \right) \end{aligned}$$

where

$$\begin{aligned} \frac{\partial \bar{\pi}_2^j}{\partial p_2} &= \int_{\underline{\mu}^j}^{\bar{\mu}^j} y_2^j(p_2, w^j, p_\kappa^j; \mu^j) dG^j = \bar{y}_2^j(p_2, w^j, p_\kappa^j), \quad \frac{\partial \bar{\pi}_2^j}{\partial p_\kappa^j} = \left(\bar{\kappa}^j - \int_{\underline{\mu}^j}^{\bar{\mu}^j} \mu^j f_2^j(\ell_2^j(p_2, p_\kappa^j, w^j; \mu^j)) dG^j \right) \\ \frac{\partial \bar{\pi}_2^j}{\partial w^j} &= - \int_{\underline{\mu}^j}^{\bar{\mu}^j} \ell_2^j(p_2, p_\kappa^j, w^j; \mu^j) dG^j = -\bar{\ell}_2^j(p_2, p_\kappa^j, w^j), \quad \frac{\partial \bar{\pi}_2^j}{\partial \bar{\kappa}^j} = p_\kappa^j. \quad j = A, B \end{aligned} \quad (13)$$

The net demand for permits for each producer of good 2 is

$$\mu^j y_2^j(p_2, w^j, p_\kappa^j; \mu^j) - \bar{\kappa}^j. \quad j = A, B \quad (14)$$

2.2.2 Regional demand functions

Consider the representative household in region j . Given producer prices p_0^j , p_1 , and p_2 , the emissions tax τ_x^j , and income m^j , the household's budget constraint is

$$p_0^j x_0^j + p_1 x_1^j + (p_2 + \tau_x^j \phi^j) x_2^j = m^j. \quad j = A, B \quad (15)$$

The household chooses x_i^j , $i = 0, 1, 2$, to maximize individual utility in (6) subject to the budget constraint (15), given commodity prices, the emission tax and income. The solution yields demand functions

$$x_i^j(p_0^j, p_1, p_2, \tau_x^j, m^j). \quad i = 0, 1, 2 \quad j = A, B \quad (16)$$

These demand functions are homogeneous of degree zero in all arguments.

The value function of the household's utility maximization problem is the indirect utility function for the representative household of region j :

$$V^j(p_0^j, p_1, p_2, \tau_x^j, m^j), \quad j = A, B \quad (17)$$

where by the envelope theorem, $V^j(\cdot)$ is decreasing in p_0^j , p_1 , p_2 , and τ_x^j , and increasing in m^j . Note that (17) is the indirect utility function based on the individual preferences of the household. As discussed below, the government uses social preferences to determine optimal emissions control policy.

Although the representative household takes income m^j as given, m^j includes wage income, producer profits and the government transfer all of which are variable. Using the expressions for profits above, household income is:

$$m^j(p_0^j, p_1, p_2, w^j, a^j, p_\kappa^j, \bar{\kappa}^j) = w^j L^j + a^j + \pi_0^j(p_0^j, w^j) + \pi_1^j(p_1, w^j) + \pi_2^j(p_2, w^j, p_\kappa^j, \bar{\kappa}^j). \quad j = A, B \quad (18)$$

This function is homogeneous of degree one in all its arguments.

Using (18), we can write the indirect utility function (17) as:

$$V^j(p_0^j, p_1, p_2, \tau_x^j, m^j(p_0^j, p_1, p_2, w^j, a^j, p_\kappa^j, \bar{\kappa}^j)). \quad j = A, B \quad (19)$$

Taking total derivatives and using the envelope theorem, we obtain:

$$\begin{aligned} \frac{dV^j}{dp_0^j} &= -V_m^j \cdot (x_0^j - y_0^j), & \frac{dV^j}{dw^j} &= V_m^j \cdot (L^j - \ell_0^j - \ell_1^j - \bar{\ell}_2^j), \\ \frac{dV^j}{da^j} &= V_m^j, & \frac{dV^j}{d\tau_x^j} &= -V_m^j e_x^j, & \frac{dV^j}{dp_\kappa^j} &= V_m^j \cdot (-e_y^j + \bar{\kappa}^j), & \frac{dV^j}{d\bar{\kappa}^j} &= V_m^j p_\kappa^j. \end{aligned} \quad (20)$$

The indirect utility function is also homogeneous of degree zero in all prices, policy variables, and income. More generally, the entire system of demand functions and supply functions are homogeneous of degree zero. That implies that we can normalize prices to fix the price level. It is convenient for analytical purposes to normalize the exchange rate by setting $\epsilon = 1$ and measuring all variables in terms of the world currency.⁸ We also normalize traded goods prices r_1 and r_2 to be unity, so $p_1 = p_2 = 1$ and we can suppress them from our analysis. We are then left with three endogenous prices facing households and firms, p_0^j , w^j and p_κ^j , as well as government policy variables, a^j , τ_x^j and $\bar{\kappa}^j$, for $j = A, B$.

We now turn to a characterization of equilibrium in the federation followed by an analysis of optimal regional government policies.

3 National equilibrium

We characterize national equilibrium when the regional governments impose region-specific policies. There are seven equilibrium conditions that must be satisfied: a) labour market equilibrium in each region, b) equilibrium in the international market for traded goods, c) equilibrium in the demand and supply for nontradable goods in each region, and d) equilibrium in the auction market for emission permits in each region. Together these seven conditions determine the regional wage rates, w^A and w^B , the prices of nontradable goods, p_0^A and p_0^B , and the price of emission permits, p_κ^A and p_κ^B . The exogenous variables include government policies, a^j , τ_x^j and $\bar{\kappa}^j$, and labour supplies L^j , which can vary across regions. Consider equilibrium conditions a)–d) in turn.

Regional labour market equilibrium

The labour market in region j will be in equilibrium when the supply of labour, L^j , equals the demand for labour in the production of the three goods. Using labour demand functions (10) and (12), labour market equilibrium conditions in the two regions are:

$$\ell_0^j(p_0^j, w^j) + \ell_1^j(w^j) + \bar{\ell}_2^j(w^j, p_\kappa^j) = L^j. \quad j = A, B \quad (21)$$

⁸There is a precedent for this in the literature. In their cost-benefit analysis manual, Little and Mirrlees (1968) evaluate the benefits and costs of projects in terms of world prices.

These equations contain all six endogenous variables, p_0^A , p_0^B , w^A , w^B , p_κ^A and p_κ^B .

National trade balance

In equilibrium, the value of imports must equal the value of exports for the nation as a whole. Using the supply and demand functions for traded goods, (10), (12) and (16), and the emissions tax and permit collection costs, (9), national trade balance requires:

$$\sum_{j=A,B} \left(\left(y_1^j(w^j) - x_1^j(p_0^j, m^j) - c^j \right) + \left(\bar{y}_2^j(w^j, p_\kappa^j) - x_2^j(p_0^j, \tau_x^j, m^j) \right) \right) = 0, \quad (22)$$

where m^j is given by (18) with $p_1 = p_2 = 1$:

$$m^j(p_0^j, w^j, a^j, p_\kappa^j, \bar{\kappa}^j) = w^j L^j + a^j + \pi_0^j(p_0^j, w^j) + \pi_1^j(w^j) + \bar{\pi}_2^j(w^j, p_\kappa^j, \bar{\kappa}^j). \quad j = A, B$$

Note that unlike labour market equilibrium conditions (21), the trade balance condition applies nationwide.

Regional nontradable good equilibrium

In equilibrium, the supply of the nontradable good must equal the demand in each region. The supply of the nontradable good in region j is $y_0^j(p_0^j, w^j)$ by (10) and the demand for the nontradable good by consumers is $x_0^j(p_0^j, \tau_x^j, m^j(p_0^j, w^j, a^j, p_\kappa^j, \bar{\kappa}^j))$ by (16). Therefore, the nontradable goods equilibrium conditions are:

$$y_0^j(p_0^j, w^j) = x_0^j(p_0^j, \tau_x^j, m^j(p_0^j, w^j, a^j, p_\kappa^j, \bar{\kappa}^j)). \quad j = A, B \quad (23)$$

Regional emission permit market equilibrium

The sum of net demand for permits by producers of the dirty good must equal zero. Using (14), the permit market-clearing condition that determines the equilibrium price is:

$$e_y^j(w^j, p_\kappa^j) = \int_{\underline{\mu}^j}^{\bar{\mu}^j} \mu^j y_2^j(w^j, p_\kappa^j; \mu^j) dG^j = \bar{\kappa}^j \quad j = A, B \quad (24)$$

Eqs. (21), (22), (23) and (24) constitute seven equations in six unknowns: w^A , w^B , p_0^A , p_0^B , p_κ^A and p_κ^B . However, these seven equations are not independent. If the budget

constraints of households, producers and governments are binding, then if any six of these equations are satisfied, the seventh one will be as well.

To see this when regional governments are responsible for emissions policy, substitute (18) for m^j into the budget constraint for the region- j household, (15), using profits, $\pi_0^j = p_0^j y_0^j - w^j \ell_0^j$, $\pi_1^j = y_1^j - w^j \ell_1^j$, and $\bar{\pi}_2^j = \bar{y}_2^j - w^j \bar{\ell}_2^j + p_\kappa^j (\bar{\kappa}^j - e_y^j)$ to obtain

$$p_0^j(x_0^j - y_0^j) = a^j - \tau_x^j e_x^j + p_\kappa^j \bar{\kappa}^j - p_\kappa^j e_y^j + w^j(L^j - \ell_0^j - \ell_1^j - \bar{\ell}_2^j) + (y_1^j - x_1^j) + (\bar{y}_2^j - x_2^j).$$

Using the auction market equilibrium, (24), the regional budget constraint, $a^j = (\tau_x^j - \gamma_x^j)e_x^j - \gamma_y^j e_y^j$, and (9),

$$p_0^j(x_0^j - y_0^j) = c^j + w^j(L^j - \ell_0^j - \ell_1^j - \bar{\ell}_2^j) + (y_1^j - x_1^j) + (\bar{y}_2^j - x_2^j).$$

Then, summing this over $j = A, B$ yields:

$$\sum_{j=A,B} p_0^j(x_0^j - y_0^j) + \sum_{j=A,B} w^j \left(\sum_{i=0,1} \ell_i^j + \bar{\ell}_2^j - L^j \right) = (y_1^j - x_1^j - c^j) + \sum_{j=A,B} (\bar{y}_2^j - x_2^j). \quad (25)$$

The first summation term is the value of excess demands for the nontraded good in the two regions, the second is the value of excess demands for labour in the two regions, and the third is the value of net exports nationwide. This implies that if any six of the seven equilibrium conditions (21)–(24) are satisfied, then the seventh equilibrium condition is necessarily satisfied.

This is Walras' Law applied nationwide and it also holds below when the federal government is responsible for emissions policy. It implies that we can neglect the national trade balance equilibrium condition (22) in studying the properties of the general equilibrium regardless of the government responsible for emissions policies. We are then left with six independent equilibrium conditions, (21), (23) and (24) for $j = A, B$, and six endogenous regional variables, $p_0^A, p_0^B, w^A, w^B, p_\kappa^A$ and p_κ^B . We can solve these six equations for each region $j = A, B$ separately in terms of the exogenous policy variables.

For analytical purposes, it simplifies matters by segmenting the equilibrium conditions as follows. Consider the labour market and nontraded goods market conditions, (21) and (23). These can be solved for p_0^j and w^j in terms of $a^j, \tau_x^j, p_\kappa^j$ and $\bar{\kappa}^j$:

$$p_0^j(a^j, \tau_x^j, p_\kappa^j, \bar{\kappa}^j), \quad w^j(a^j, \tau_x^j, p_\kappa^j, \bar{\kappa}^j). \quad j = A, B \quad (26)$$

We can then treat p_κ^j as an artificial control variable for the government that must satisfy the permit equilibrium condition, (24). This condition will be added as a constraint in the government's problem.

Using (26), we can express the levels of consumption and production emissions as a function of government policy variables (both actual and artificial) by substituting (16) and (12) into the expressions for e_x^j and e_y^j in (2) and (3) to obtain:

$$\begin{aligned} e_x^j(a^j, \tau_x^j, p_\kappa^j, \bar{\kappa}^j) &= \phi^j x_2^j(p_0^j, \tau_x^j, m^j(p_0^j, w^j, a^j, p_\kappa^j, \bar{\kappa}^j)), \\ e_y^j(a^j, \tau_x^j, p_\kappa^j, \bar{\kappa}^j) &= \int_{\mu^j} \mu^j y_2^j(w^j, p_\kappa^j; \mu^j) dG^j(\mu^j). \end{aligned} \quad (27)$$

where p_0^j and w^j are given by (26). The properties of (27) are derived in the Appendix. Total emissions generated in region j as a function of government policies are:

$$e^j(a^j, \tau_x^j, p_\kappa^j, \bar{\kappa}^j) = e_x^j(a^j, \tau_x^j, p_\kappa^j, \bar{\kappa}^j) + e_y^j(a^j, \tau_x^j, p_\kappa^j, \bar{\kappa}^j). \quad j = A, B \quad (28)$$

These general equilibrium results have implications for the representative household's indirect utility in (19). The non-traded goods and labour market equilibrium conditions (23) and (21) imply that from the envelope results in (20) we obtain:

$$\frac{dV^j}{dp_0^j} = -V_m^j \cdot (x_0^j - y_0^j) = 0, \quad \frac{dV^j}{dw^j} = V_m^j \cdot (L^j - \ell_0^j - \ell_1^j - \bar{\ell}_2^j) = 0. \quad j = A, B \quad (29)$$

That is, the household's indirect utility is not affected by general equilibrium changes in p_0^j or w^j . Therefore, we can write the household indirect utility function as:

$$V^j(a^j, \tau_x^j, p_\kappa^j, \bar{\kappa}^j) \equiv V^j(p_0^j, \tau_x^j, m^j(p_0^j, w^j, a^j, p_\kappa^j, \bar{\kappa}^j)), \quad j = A, B \quad (30)$$

where using (20)

$$V_a^j = V_m^j, \quad V_{\tau_x^j}^j = -V_m^j e_x^j, \quad V_{p_\kappa^j}^j = V_m^j (-e_y^j + \bar{\kappa}^j), \quad V_{\bar{\kappa}^j}^j = V_m^j p_\kappa^j.$$

4 Regional government policies

Regional governments A and B impose per unit taxes τ_x^j on consumption emissions and give emissions permits $\bar{\kappa}^j$ for free to each firm. They return tax revenues net of collection

and permit administration costs to the representative household. The government budget constraint for region j is:

$$(\tau_x^j - \gamma_x^j)e_x^j(a^j, \tau_x^j, p_\kappa^j, \bar{\kappa}^j) - \gamma_y^j \bar{\kappa}^j = a^j, \quad j = A, B \quad (31)$$

where $e_x^j(a^j, \tau_x^j, p_\kappa^j, \bar{\kappa}^j)$ is given by (27).

Regional governments abide by the social preferences of the representative household, given by (7), in choosing emissions control policies. Each region takes as given policies in the other region, but takes account of how its policies affect the price of the nontraded good and the wage rate in the region. Both regions behave in the same way, and since region j 's policies affect the other region through the social disutility function $s^{-j}(e^{-j}, e^j)$, there will be a Nash equilibrium in regional policies.

Consider the policy choices of the region- j government. It chooses policy variables a^j , τ_x^j , $\bar{\kappa}^j$ and the artificial variable p_κ^j to maximize social utility subject to the budget constraint (31) and the permit equilibrium condition (24). The Lagrangian expression can then be written, using (28):

$$\begin{aligned} \mathcal{L}^j(a^j, \tau_x^j, p_\kappa^j, \bar{\kappa}^j) = & V^j(a^j, \tau_x^j, p_\kappa^j, \bar{\kappa}^j) - s^j(e_x^j(a^j, \tau_x^j, p_\kappa^j, \bar{\kappa}^j) + e_y^j(a^j, \tau_x^j, p_\kappa^j, \bar{\kappa}^j), e^{-j}) \\ & + \lambda^j((\tau_x^j - \gamma_x^j)e_x^j(a^j, \tau_x^j, p_\kappa^j, \bar{\kappa}^j) - \gamma_y^j \bar{\kappa}^j - a^j) + \delta^j(e_y^j(a^j, \tau_x^j, p_\kappa^j, \bar{\kappa}^j) - \bar{\kappa}^j). \quad j = A, B \end{aligned} \quad (32)$$

We show in the Appendix that the solution to the first-order conditions from maximizing (32) is the following policy rule:

$$\tau_x^j - \gamma_x^j = p_\kappa^j - \gamma_y^j = \frac{s_{e^j}^j}{\lambda^j} \quad \text{with} \quad \lambda^j = V_a^j. \quad j = A, B \quad (33)$$

The simplicity of the policy rule in (33) reflects the fact that taxes do not impose any inefficiencies in our model: they are used solely to correct for the social disutility of emissions. In a more general setting with variable labour supply when the government must raise revenue to finance public expenditures, emissions taxes can give rise to a double dividend as summarized by Auerbach and Hines (2002). This will affect the optimal emissions tax rules. We can summarize this result in the following proposition.

Proposition 1 *Pigovian regional adjusted emissions pricing rules At the emissions policy optimum of the government of region j , and given the emissions policy of the other region,*

optimal emissions pricing policies satisfy:

$$\tau_x^j - \gamma_x^j = p_\kappa^j - \gamma_y^j = \frac{s_e^j}{V_a^j}. \quad j = A, B \quad (34)$$

Three features of Proposition 1 are notable. First, regional governments adopt adjusted Pigovian pricing policies whereby the emissions prices net of regional collection and administration costs, $\tau_x^j - \gamma_x^j$ and $p_\kappa^j - \gamma_y^j$, are set equal the marginal social disutility of emissions in the region measured in income units. The adjusted Pigovian prices on consumption and production emissions are identical because both forms of emissions are perfect substitutes from a social disutility point of view. Second, no account is taken of the effect of emissions in region j on social disutility in the other region. This is because regional governments only give weight to their own residents' social utility. We return to this below. Third, even though emissions taxes affect the wage rate and the price of the nontraded good, these general equilibrium effects are not relevant for setting optimal emissions taxes.

It follows directly from Proposition 1 that if regions are identical then regional governments will choose the same adjusted Pigovian emissions prices. Furthermore, these emission prices net of collection and administration costs will not account for the impact regional emission prices have on the social disutility of emissions in the other region as shown in the following corollary.

Corollary 1 *Identical regional governments adopt the same adjusted Pigovian emissions pricing given by*

$$\tau_x - \gamma_x = p_\kappa - \gamma_y = \frac{s_e}{V_a}. \quad (35)$$

The above results were derived assuming that the regions priced production emissions by giving $\bar{\kappa}^j$ tradable emissions permits free to all producers. As we show in the Appendix, the analogue of Proposition 1 applies when the regional governments either use production emissions taxes, τ_y^j , or sell $\bar{\kappa}^j$ permits by auction to producers in the region. In the former case, the optimal τ_y^j equals the equilibrium permit price p_κ^j . In the latter case, the price of permits sold by auction is the same as when regional governments give emission permits

free-of-charge to producers and allow permit trading in regional permit markets. This is summarized as follows.

Remark 1 *Alternative regional pricing of production emissions*

1. *Emission pricing rule (34) applies when regions auction off emission permits to producers.*
2. *Emissions pricing when regions tax production emissions satisfy the analogue of (34),*

$$\tau_x^j - \gamma_x^j = \tau_y^j - \gamma_y^j = \frac{s_{e^j}^j}{V_a^j}. \quad j = A, B \quad (36)$$

In either case, the optimal emissions price is the same as in the base case when emissions permits are given to producers for free and then traded in the regional permit market.

4.1 Nash equilibrium

Both regions A and B choose their emissions policies taking as given the level of emissions in the other region. In a Nash equilibrium, the levels of emissions are mutually compatible. For convenience, we assume the production emissions are controlled using an emissions tax τ_y^j rather than tradable permits. This does not affect our results, but simplifies the analysis by allowing regional governments to directly control all emissions prices. We have shown above that each region sets $\tau_x^j - \gamma_x^j = \tau_y^j - \gamma_y^j$ to satisfy (36). We can treat, say, τ_x^j as the regional government's emissions tax choice variable and let τ_y^j be determined endogenously by $\tau_y^j = \tau_x^j - \gamma_x^j + \gamma_y^j$. The emissions tax rate τ_x^j and the lump-sum transfer a^j are then the main strategic variables to be determined in equilibrium, and a^j is determined by τ_x^j through the regional government budget constraint, which now reads:

$$(\tau_x^j - \gamma_x^j)e_x^j(a^j, \tau_x^j, \tau_y^j) + (\tau_y^j - \gamma_y^j)e_y^j(a^j, \tau_x^j, \tau_y^j) = a^j. \quad j = A, B$$

To characterize the Nash equilibrium, we need to know the properties of the regions' emissions tax reaction curves.

Consider region j and for expositional purposes suppress the superscript j . In the Nash equilibrium, the following equations are satisfied based on the budget constraint and the optimal emissions tax expression (36), using $\tau_y = \tau_x - \gamma_x + \gamma_y$:

$$(\tau_x - \gamma_x)e(a, \tau_x, \tau_x - \gamma_x + \gamma_y) - a = 0,$$

$$\tau_x - \gamma_x - \frac{s_e(e(a, \tau_x, \tau_x - \gamma_x + \gamma_y), e^-)}{V_a(a, \tau_x, \tau_x - \gamma_x + \gamma_y)} = 0,$$

where e^- represents emissions in the other region. Differentiating these and suppressing the arguments of functions, we obtain:

$$\begin{aligned} & \left(e + (\tau_x - \gamma_x) \left(\frac{de}{d\tau_x} + \frac{de}{d\tau_y} \right) \right) d\tau_x + \left((\tau_x - \gamma_x) \frac{de}{da} - 1 \right) da = 0, \\ & \left(1 + \frac{s_e}{V_a^2} \left(\frac{dV_a}{d\tau_x} + \frac{dV_a}{d\tau_y} \right) - \frac{s_{ee}}{V_a} \left(\frac{de}{d\tau_x} + \frac{de}{d\tau_y} \right) \right) d\tau_x + \left(\frac{s_e}{V_a^2} \frac{dV_a}{da} - \frac{s_{ee}}{V_a} \frac{de}{da} \right) da = \frac{s_{ee^-}}{V_a} de^-. \end{aligned}$$

These are two equations determining $d\tau_x$ and da in terms of de^- . The solution yields:

$$\frac{d\tau_x}{d\tau_x^-} = \frac{1}{|D|} \frac{s_{ee^-}}{V_a} \left(1 - (\tau_x - \gamma_x) \frac{de}{da} \right) \frac{de^-}{d\tau_x^-}. \quad (37)$$

where $|D|$ as shown in the Appendix is the determinant of the system of coefficients of $d\tau_x$ and da , which we assume to be positive.

If $1 - (\tau_x - \gamma_x)de/da > 0$ (so an increase in a does not create a budget surplus) and $de^-/d\tau_x^- < 0$ (so direct effects of changes in τ_x^- are larger than indirect effects), we have by (37) that $d\tau_x/d\tau_x^- > 0$ if $s_{ee^-} < 0$. This means that the emissions tax reaction curves of each region are upward sloping so τ_x and τ_x^- are strategic complements. On the other hand, if $s_{ee^-} > 0$, emissions taxes are strategic substitutes so reaction curves are downward sloping. Strategic complementarity tends to make the Nash equilibrium value of emissions tax rates higher and emissions lower than under strategic substitutability.

4.2 Regional coordination

Suppose now that the two regional governments are able to coordinate their policy choices. We begin in a Nash equilibrium where both regions, following Remark 1, adopt adjusted Pigovian emissions taxes so both set the same emissions tax net of collection and administration costs on consumption and production emissions. Adjusted emissions taxes can however vary between regions. The question we consider is whether both would agree to a small coordinated change in these emissions tax policies.

Consider one of the regions, whose superscript we suppress for simplicity, and imagine a uniform increase in the emissions tax rates in both regions. That is, starting at $\tau_x - \gamma_x = \tau_y - \gamma_y$ and $\tau_x^- - \gamma_x^- = \tau_y^- - \gamma_y^-$, the regions agree to changes in emissions prices such that

$d\tau_x = d\tau_y = d\tau_x^- = d\tau_y^- > 0$. The effect on the representative households' welfare of a change in emissions taxes enacted by both regions starting in a Nash equilibrium can be found by differentiating the analogue of Lagrangian Eq. (32) by τ_x , τ_y , τ_x^- and τ_y^- , taking into account that the common policy change will affect emissions by the other region:

$$\begin{aligned}\mathcal{L}_{\tau_x} &= V_{\tau_x} - s_e \frac{de}{d\tau_x} - s_{e^-} \frac{de^-}{d\tau_x^-} + \lambda \left(e_x + (\tau_x - \gamma_x) \frac{de_x}{d\tau_x} + (\tau_y - \gamma_y) \frac{de_y}{d\tau_x} \right) \\ \mathcal{L}_{\tau_y} &= V_{\tau_y} - s_e \frac{de}{d\tau_y} - s_{e^-} \frac{de^-}{d\tau_y^-} + \lambda \left(e_y + (\tau_x - \gamma_x) \frac{de_x}{d\tau_y} + (\tau_y - \gamma_y) \frac{de_y}{d\tau_y} \right)\end{aligned}$$

Since we are starting in a Nash equilibrium, the regions' first-order conditions for the choice of τ_x^j and τ_y^j will be satisfied. Using these, it is straightforward to show that this expression reduces to:

$$\mathcal{L}_{\tau_x} = -s_{e^-}(e, e^-) \frac{de^-}{d\tau_x^-}, \quad \mathcal{L}_{p_\kappa} = -s_{e^-}(e, e^-) \frac{de^-}{d\tau_y^-}. \quad (38)$$

Assume, as above, that direct effects of changes in τ_x and p_κ outweigh indirect effects, so $de^-/d\tau_x^- < 0$ and $de^-/d\tau_y^- < 0$, then $\mathcal{L}_{\tau_x} > 0$ and $\mathcal{L}_{\tau_y} > 0$. The same analysis applies in the other region. Starting in a Nash equilibrium, a coordinated increase in both regions' emissions prices will make the representative household in both regions better off. This will be the case regardless of the sign of s_{ee^-} , that is, regardless of whether e and e^- are strategic complements or substitutes. We summarize this in the following proposition.

Proposition 2 *Pareto-improving coordinated emissions tax increases*

Starting from a Nash equilibrium, if both regions increase their emissions tax rates by a common amount, the representative households in both regions will be better off provided $de^j/d\tau_x^j, de^j/d\tau_y^j < 0$ for $j = A, B$.

5 Federal responsibility for emissions control and transfers

Suppose now that policies are fully centralized and that federal emissions control policies are consumption and production emissions taxes. By analogy with Remark 1, the same results would apply if the federal government used emissions permits. Suppose also that the federal government can choose different consumption and production emissions tax rates in the two regions, τ_x^j and τ_y^j . The federal government transfers amounts a^j ($j = A, B$) to

the representative households in the two regions to maximize the social welfare function (8), subject to the federal government budget constraint. Using household indirect utilities (30), we can write (8) as:

$$W = \sum_{j=A,B} V^j(a^j, \tau_x^j, \tau_y^j) - \sigma \left(e^A(a^A, \tau_x^A, \tau_y^A) + e^B(a^B, \tau_x^B, \tau_y^B) \right). \quad (39)$$

Assume that γ_x and γ_y are the costs to the federal government of observing each unit of emissions from consumption and production, respectively. We assume that these are greater than costs to the regional governments of observing emissions, γ_x^j and γ_y^j . The federal budget constraint is:

$$\sum_{j=A,B} (\tau_x^j - \gamma_x) e_x^j(a^j, \tau_x^j, \tau_y^j) + \sum_{j=A,B} (\tau_y^j - \gamma_y) e_y^j(a^j, \tau_x^j, \tau_y^j) = \sum_{j=A,B} a^j.$$

5.1 Federal government policies

The Lagrangian expression for the federal government problem is:

$$\begin{aligned} \mathcal{L}^F(a^A, \tau_x^A, \tau_y^A, a^B, \tau_x^B, \tau_y^B) = & \sum_{j=A,B} V^j(a^j, \tau_x^j, \tau_y^j) - \sigma \left(e^A(a^A, \tau_x^A, \tau_y^A) + e^B(a^B, \tau_x^B, \tau_y^B) \right) \\ & + \lambda^F \sum_{j=A,B} \left((\tau_x^j - \gamma_x) e_x^j(a^j, \tau_x^j, \tau_y^j) + (\tau_y^j - \gamma_y) e_y^j(a^j, \tau_x^j, \tau_y^j) - a^j \right). \end{aligned} \quad (40)$$

There are potentially several differences between the federal and regional responsibility cases. The first is that there are equity weights on the effect of policies on household utility given that household utilities are strictly concave so the federal government is averse to income inequality. This will affect the federal choice of transfers to the regions.

The second is that the federal government internalizes the externalities associated with emissions reduction policies applying to the two regions. That is, emissions taxes τ_x^j and τ_y^j reduce aggregate emissions, and these emissions reductions affect social disutility of emissions nationwide. This gives an efficiency advantage to centralized emissions control, unless the regions can eliminate the externalities by cooperative emissions control policies.

The third difference is that the social disutility of emissions function can differ between the federal and regional governments. We might assume that the disutility that resident households place on national emissions when the federal government sets emissions control policy is a fraction of that if instead their regional government were in charge of emission

pricing policies. That is, households feel less social coherence when represented by the federal government than when represented by their regional government. This difference in the social disutility of emissions at the regional and federal levels of government, and the fact that social disutilities can differ between regions, makes it impossible to specify a unique socially optimal level of emissions.

The final difference is that the federal government may be less able to observe emissions levels than the regions. This disadvantage is reflected in the cost of taxing emissions, γ_x and γ_y , which we assume to be higher than regional emissions tax collection costs, γ_x^j and γ_y^j . This effect works in favour of decentralized decision-making.

Using the same method to solve the first-order conditions of the federal government problem for the optimal federal net taxes in both regions, we show in the Appendix that

$$\tau_x^j - \gamma_x = \tau_y^j - \gamma_y = \frac{\sigma'}{V_a^j}, \quad \text{and} \quad \lambda^F = V_a^j. \quad j = A, B$$

The structure of these two conditions are the identical to the two conditions on the optimal net tax variables in the regional government's problem, except the federal government has a different social disutility function. Therefore, we have the following proposition.

Proposition 3 *Uniform net federal consumption and production emission taxes*

The optimal federal region-specific emissions tax rates net of the cost of collection are the same for both forms of emissions and for both regions and equal to the federal marginal social disutility of emissions.

$$\tau_x^j - \gamma_x = \tau_y^j - \gamma_y = \frac{\sigma'}{V_a^j}. \quad j = A, B \quad (41)$$

Thus, even though the federal government could choose differential consumption and production emissions tax rates, net Pigovian emissions tax rates will be uniform in an optimum. That is because all sources of emissions are perfect substitutes in the federal social disutility function $\sigma(e_x^A + e_y^A + e_x^B + e_y^B)$. Note that Proposition 3 states that federal emissions tax rates net of collection costs are equalized across types of emissions and regions. Actual emissions tax rates will differ from those net of collection costs if the latter differ, that is, $\tau_x^j \neq \tau_y^j$ if $\gamma_x \neq \gamma_y$.

This can be compared with the case of regional emissions policies in the identical region case. In this case, by Corollary 1, we have

$$\tau_x^A - \gamma_x^A = p_\kappa^A - \gamma_y^A = \frac{s_e^A}{V_a^A} = \frac{s_e^B}{V_a^B} = \tau_x^B - \gamma_x^B = p_\kappa^B - \gamma_y^B.$$

Federal tax rates differ from this in three ways. First, federal taxes endogenize the spillover benefits of emission reductions due to the tax and this tends to increase federal tax rates over regional ones. Second, the higher tax collection costs for the federal government relative to the regional governments will also tend to make federal emissions tax rates higher. Third, social damages may be discounted in the federal case because of our presumption that households have less solidarity with decisions taken by the federal government. On balance, federal emissions tax rates could be higher or lower than regional ones. If regional social disutility was $s^j(e^A + e^B)$ and identical across regions, and federal social disutility was the aggregate of regional social disutility, for example, $\sigma(e^A + e^B) = 2s^j(e^A + e^B)$, then it would necessarily be the case that net federal tax rates would be higher than regional net rates.

5.2 Border carbon tax adjustments

When a country imposes emissions taxes on domestic industries, it risks the possibility that its firms will become uncompetitive. Border carbon tax adjustments (BCTAs) using import tariffs or export taxes on dirty goods could be used to mitigate this (Copeland, Shapiro and Taylor, 2022). In our context, BCTAs would be taxes or tariffs on the export or import of the dirty good. Keen and Kotsogiannis (2024) have analyzed BCTAs in a two-country two-good model of the world economy, where one good is a dirty good emitting carbon gases based on production. Each country can implement an emissions tax as well as trade taxes on the dirty good. There is a representative household in each country, and the emphasis of their analysis is on Pareto efficient policy outcomes. The latter are characterized as equilibria in which both countries maximize the utility of their representative household subject to a given level of utility of the household in the other jurisdiction as well as resource constraints and trade equilibrium.

As a special case, they consider behaviour of the home country when it behaves selfishly, that is, when it ignores the utility constraint of the representative household of the foreign country. The result is their Proposition 6 which states that the optimal home country tariff

on the dirty good t , given its carbon price s^* , is as follows:

$$t = M(M_P)^{-1} + \Delta B - (\Delta - s^*)b,$$

where M is net imports of the dirty good in the foreign country, P is its price, Δ is damages to each household per unit of world emissions, and b and B are emissions per unit of production at home and abroad. The first component $M(M_P)^{-1} > 0$ is an optimal tariff due to a terms of trade effect, given that $M_P > 0$. The second term ΔB is the marginal damage to home resident from foreign emissions, so is like a Pigovian tax. It applies only to net imports of the foreign country since the home country can only tax imports and not production in the foreign country. The third term is a term that corrects for any difference between the home carbon tax and marginal damages. That is, the import tax is reduced to the extent that the carbon tax is less than the rate of domestic emissions.

In the context of our model with federal responsibility for emissions control, these three terms would all be zero. The first term is zero given our assumption that the federation faces fixed world prices (so $M_P = \infty$). The second term is also zero, but for more a more subtle reason. This term in Keen and Kotsogiannis captures the fact that when emissions in the foreign country cause worldwide emissions to rise, the home country is adversely affected. This effect does not appear in our model since we take current world emissions damages to be fixed because they are mostly due to the cumulation of greenhouse gas emissions in the past. Current emissions have an imperceptible effect. The third term is zero since the federal government equates the emissions tax to marginal damages when the regions are identical so there is a representative national resident.⁹

The upshot is that there is no case for BCTAs in our model regardless of the policies undertaken by foreign countries.

Proposition 4 *Border carbon tax adjustments*

In a small open economy and taking current world emissions to be fixed, there is no need for border carbon tax adjustments.

The following corollary of Proposition 4 is immediately apparent.

⁹When regions are not identical, the optimal federal emissions tax net of collection costs is uniform by Proposition 3. Since the federal emissions tax is set optimally, the term analogous to the third one is zero. This is stated in Corollary 6.

Corollary 2 *Optimal tariff policy*

In a small open economy and taking current world emissions to be fixed, introducing a tariff on the dirty good will not increase the value of the social objective if the federal government is implementing the optimal emission pricing policies.

To illustrate Corollary 6, suppose the dirty good is being imported into the country, that is, $\sum_j x_2^j > \sum_j y_2^j$ and there is a tariff $t > 0$. Taking world prices, r_1 and r_2 , as given implies that domestic consumers must pay the world price times the domestic exchange rate, ϵ , plus the tariff or import tax on each unit of imported good 2. Domestic producers can then charge the same price on domestic production so both the producer and consumer price of good 2 is now given by $\epsilon r_2 + t$. Consumers are indifferent between importing the dirty good and buying the good domestically. Consumers will be worse off because their price has risen, while producers will be better off. In this case, the tariff is equivalent to a tax on consumption combined with a subsidy on production.¹⁰

The problems of the household and producers are as before except they face this different price for good 2 which depends on t . The conditions determining national equilibrium are also unchanged. Using the binding household and federal governments budget constraints and taking into account the federal tax revenue from tariff, which is given by $t \cdot (x_2^A + x_2^B - y_2^A - y_2^B)$,¹¹ Walras' Law holds. We can again use the regional nontraded good and labour market equilibrium conditions to solve for the four endogenous regional variables, p_0^A , p_0^B , w^A , and w^B as a function of regional tax policies, τ_x^j and τ_y^j , lump-sum transfers a^j , and tariff policy t , where we continue to normalize the exchange rate and world prices such that $\epsilon = r_1 = r_2 = 1$ so $p_1 = p_2 = 1$. Regional emissions will depend on all federal government policies. Indirect household utility in region j will also now be a function of t , where

$$V_t^j = -V_a^j(x_2^j - y_2^j) \geq 0. \quad (42)$$

That is, an increase in t increases the price of x_2^j to the household, but also increases the

¹⁰The subsequent analysis holds if, instead, the dirty good is being exported, that is, $\sum_j y_2^j > \sum_j x_2^j$. The incidence of any tariff falls completely on the representative household given that the world price of the dirty good is fixed. Any revenue raised from the tariffs is subsequently returned to the consumer and since the emissions pricing policies ensure optimal emissions from the point of view of the federal government, there is no role for tariff policy.

¹¹As imports and exports are relatively easy to observe, we assume no collection costs of tariff policy.

profits on the production of y_2^j , which goes to the household.

The Lagrangian expression for the federal government problem is:

$$\begin{aligned} \mathcal{L}^F(a^A, \tau_x^A, \tau_y^A, a^B, \tau_x^B, \tau_y^B, t) = & \sum_{j=A,B} V^j(a^j, \tau_x^j, \tau_y^j, t) - \sigma \left(\sum_{j=A,B} e^j(a^j, \tau_x^j, \tau_y^j, t) \right) \\ & + \lambda^F \sum_{j=A,B} \left((\tau_x^j - \gamma_x) e_x^j(a^j, \tau_x^j, \tau_y^j, t) + (\tau_y^j - \gamma_y) e_y^j(a^j, \tau_x^j, \tau_y^j, t) \right. \\ & \left. + t(x_2^j(a^j, \tau_x^j, \tau_y^j, t) - y_2^j(a^j, \tau_x^j, \tau_y^j, t) - a^j) \right). \end{aligned} \quad (43)$$

To determine whether the federal government should impose tariffs, differentiate (43) with respect to t and evaluate the resulting expression at $t = 0$ to obtain

$$\mathcal{L}_t^F|_{t=0} = \sum_{j=A,B} \left(-V_a^j(x_2^j - y_2^j) - \sigma' \frac{de^j}{dt} + \lambda^F \left(x_2^j - y_2^j + (\tau_x^j - \gamma_x) \frac{de_x^j}{dt} + (\tau_y^j - \gamma_y) \frac{de_y^j}{dt} \right) \right) \quad (44)$$

When there are no tariffs, the federal optimum is given by Proposition 3. Substituting in the optimality conditions, $\tau_x^j - \gamma_x = \tau_y^j - \gamma_y = \sigma' / V_a^j$, and regional emissions, $e^j = e_x^j + e_y^j$, (44) gives

$$\mathcal{L}_t|_{t=0} = \sum_{j=A,B} \left(-V_a^j \left(x_2^j - y_2^j + \frac{\sigma'}{V_a^j} \frac{de^j}{dt} \right) + \lambda^F \left(x_2^j - y_2^j + \frac{\sigma'}{V_a^j} \frac{de^j}{dt} \right) \right) = 0 \quad (45)$$

where the last equality follows from $\lambda^F = V_a^A = V_a^B$ as stated in Proposition 3. Therefore, starting from a situation in which the federal optimal emission pricing policies are in place, introducing a small tariff will not increase federal social welfare.

6 Federal-regional shared policy responsibility

So far we have assumed that both emissions control policies and transfers to households were assigned exclusively either to the regions or to the federal government. We now consider the more realistic case where policy responsibilities are shared. With both levels of government involved, outcomes will depend on the order in which decisions are made. Of particular relevance is the case where decisions are sequential: either the federal government moves first and then the regions, or the reverse.

We focus the case where the regional governments are responsible for emissions control policies using emission taxes while the federal government transfers income from one

region to the other.¹² Federal regional income transfers are purely redistributive and are given by θ^A and θ^B , where $\theta^A + \theta^B = 0$. We assume these transfers are made to the regional governments as opposed to the regional households, although that does not affect equilibrium outcomes. Emissions pricing policies consist of consumption and production emissions taxes, τ_x^j and τ_y^j , although similar results would occur if emissions permits were used instead as discussed in Remark 1 above. We begin with the case where the federal government chooses its policies before the regions and then consider the case where regions move first.

6.1 Federal government moves first

The federal government chooses its interregional transfers to maximize national social welfare anticipating regional government emission policies. Regional governments then choose their emission taxes non-cooperatively, taking as given federal transfers. Finally, households and firms make their consumption and production decisions and national equilibrium is achieved. We derive optimal policy rules by backward induction beginning with private sector decisions.

6.1.1 Consumption and production decisions and market equilibrium

The problems of the representative household and producer in each region are as before, as are the conditions determining national equilibrium. Since all household and government budget constraints (regional and federal) are satisfied, we can again use the regional nontraded good and labour market equilibrium conditions to solve for the four endogenous regional variables, p_0^A , p_0^B , w^A , and w^B as a function of regional tax policies, τ_x^j and τ_y^j , and lump-sum transfers a^j , where we continue to normalize the exchange rate and world prices such that $\epsilon = r_1 = r_2 = 1$ so $p_1 = p_2 = 1$. The prices of non-traded goods and wage rates are given by the analogue of (26):

$$p_0^j(a^j, \tau_x^j, \tau_y^j), \quad w^j(a^j, \tau_x^j, \tau_y^j). \quad j = A, B$$

¹²An alternate policy assignment would have both levels of government responsible for emission control policies, for example, the federal government controlling consumption emission policies and the regions controlling production emission policies or each level of government controlling both types of emission policies. We leave these alternatives for future analysis.

The levels of consumption and production emissions are given by (2) and (3). Using these expressions for p_0^j and w^j and the analogues of (16) and (12), emissions are:

$$\begin{aligned} e_x^j(a^j, \tau_x^j, \tau_y^j) &= \phi^j x_2^j(p_0^j, \tau_x^j, m^j(p_0^j, w^j, a^j, \tau_y^j)), \\ e_y^j(a^j, \tau_x^j, \tau_y^j) &= \int_{\mu^j} \mu^j y_2^j(w^j, \tau_y^j; \mu^j) dG^j(\mu^j). \end{aligned} \quad (46)$$

6.1.2 Regional government problem

Turning to the region governments, their budget constraints become:

$$(\tau_x^j - \gamma_x^j)e_x^j + (\tau_y^j - \gamma_y^j)e_y^j + \theta^j = a^j, \quad j = A, B \quad (47)$$

where θ^j is the given federal transfer. As the regional governments do not provide any public goods, the emissions tax revenue net of collection/monitoring costs plus the federal transfer is returned to the representative household as a lump-sum income transfer, a^j . We treat a^j as a regional choice variable and impose regional budget balance (47) as a constraint.

The region- j government problem is

$$\max_{\{a^j, \tau_x^j, \tau_y^j\}} V^j(a^j, \tau_x^j, \tau_y^j) - s^j \left(e^j(a^j, \tau_x^j, \tau_y^j), e^{-j}(a^{-j}, \tau_x^{-j}, \tau_y^{-j}) \right) \quad \text{s.t.} \quad (47).$$

This is analogous to regional problem (32) in the base case except that now production emissions taxes are used instead of permits and the regions receive a given transfer θ^j from the federal government. In this case, the solution to region's problem will be a set of policies that depend upon the federal transfer:

$$a^j(\theta^j), \tau_x^j(\theta^j), \tau_y^j(\theta^j). \quad j = A, B \quad (48)$$

The analogue to policy choices (33) and Proposition 1 apply, so optimal regional government policy entails $V_a^j = \lambda^j$, where λ^j is the Lagrange multiplier on the budget constraint (47), with net emissions tax rates:

$$\tau_x^j - \gamma_x^j = \tau_y^j - \gamma_y^j = \frac{s_{e^j}^j}{V_a^j}. \quad j = A, B \quad (49)$$

The policies in (48) satisfy this condition.

6.1.3 Federal government problem

The federal government chooses interregional transfers θ^A, θ^B to maximize national social welfare (8) subject to its budget constraint $\theta^A + \theta^B = 0$, anticipating that the regional government will choose policies (48). The federal problem is:

$$\max_{\{\theta^A, \theta^B\}} \sum_{j=A,B} V^j \left(a^j(\theta^j), \tau_x^j(\theta^j), \tau_y^j(\theta^j) \right) - \sigma(e^A(\theta^A) + e^B(\theta^B)) \quad \text{s.t.} \quad \theta^A + \theta^B = 0.$$

The first-order conditions on θ^j are

$$V_a^j \frac{da^j}{d\theta^j} + V_{\tau_x^j}^j \frac{d\tau_x^j}{d\theta^j} + V_{\tau_y^j}^j \frac{d\tau_y^j}{d\theta^j} - \sigma' \frac{de^j}{d\theta^j} - \lambda^F = 0, \quad j = A, B \quad (50)$$

where λ^F is the Lagrange multiplier on the government budget constraint. Eliminating λ^F from these two first-order conditions and using (20),

$$V_a^A \left(\frac{da^A}{d\theta^A} - e_x^A \frac{d\tau_x^A}{d\theta^A} - e_y^A \frac{d\tau_y^A}{d\theta^A} \right) - \sigma' \frac{de^A}{d\theta^A} = V_a^B \left(\frac{da^B}{d\theta^B} - e_x^B \frac{d\tau_x^B}{d\theta^B} - e_y^B \frac{d\tau_y^B}{d\theta^B} \right) - \sigma' \frac{de^B}{d\theta^B}, \quad (51)$$

where $d\theta^A = -d\theta^B$.

The federal choice of transfers equalizes the marginal benefit of an increment of federal transfers to the regional representative households less the social marginal disutility the additional transfer has on regional emissions and therefore federal social disutility. Without a federal motivation for emission control policies ($\sigma' = 0$), there is still an equity motivation for interregional redistribution, and optimal federal transfers equalize the adjusted regional marginal utilities of income.

Note that the regions choose adjusted Pigovian emissions tax rates (49), which are analogous to the case where policies are decentralized for the regions. These policies are non-optimal from a federal perspective for two reasons. First, regional social disutility of emissions differs from the federal social disutility, and second, regions do not take account of the benefit their emissions policies have on the other region.

Further insight about federal government transfer policies can be obtained by taking account of how regional policies in (48) are affected by θ^j . Totally differentiate the regional budget constraint (47) taking account of the fact that a^j , τ_x^j and τ_y^j depend on θ^j :

$$e_x^j \frac{d\tau_x^j}{d\theta^j} + e_y^j \frac{d\tau_y^j}{d\theta^j} + (\tau_x^j - \gamma_x^j) \frac{de_x^j}{d\theta^j} + (\tau_y^j - \gamma_y^j) \frac{de_y^j}{d\theta^j} + 1 - \frac{da^j}{d\theta^j} = 0$$

Substituting this into (51) yields:

$$\begin{aligned} & V_a^A + \left(V_a^A(\tau_x^A - \gamma_x^A) - \sigma' \right) \frac{de_x^A}{d\theta^A} + \left(V_a^A(\tau_y^A - \gamma_y^A) - \sigma' \right) \frac{de_y^A}{d\theta^A} \\ &= V_a^B + \left(V_a^B(\tau_x^B - \gamma_x^B) - \sigma' \right) \frac{de_x^B}{d\theta^B} + \left(V_a^B(\tau_y^B - \gamma_y^B) - \sigma' \right) \frac{de_y^B}{d\theta^B} \end{aligned}$$

Substituting for the net emissions tax rates using (49) and simplifying, we obtain:

$$V_a^A + \left(s_{e^A}^A - \sigma' \right) \frac{de^A}{d\theta^A} = V_a^B + \left(s_{e^B}^B - \sigma' \right) \frac{de^B}{d\theta^B} \quad (52)$$

The optimal regional emissions policies ensure that the effect on the private welfare of the regional representative household from an increase in the federal transfer is the same as the effect of an increase in the transfer on regional social disutility of emissions. Consequently, the optimal federal transfer equates the sum of the marginal utility of income and the difference between the regional and federal social marginal disutilities of emission from an increase in federal transfer across the two regions. Eq. (52) indicates that part of the role for federal transfers is to correct for the fact that regional emissions policies deviate from optimal federal ones because marginal social valuations of emissions differ.

The following proposition summarizes the results of this case.

Proposition 5 *When the federal government chooses its interregional transfers before regions choose emissions taxes, transfers equalize the marginal utility to regional households net of the national social disutility of emissions induced by the transfers. The regions then set emissions tax rates net of costs of collection equal to the regional disutility of emissions.*

6.1.4 Role for federal tariffs

We showed in Section 5 that when the federal government is responsible for emissions policy, there is no role for border carbon tax adjustments or trade taxes. We now revisit that issue when regions when regional governments control emissions policy while the federal government makes transfers to the regions. For concreteness we assume that federal policies are enacted first, followed by regional emissions policies, where the former consist of interregional transfers and a federal tax on trade in the dirty good. We assume that the dirty

good is both imported and produced domestically, so $x_2^j > y_2^j > 0$ for $j = A, B$, although analogous results can be obtained if it is exported. The approach is a simple extension of the analysis earlier in this section.

Assume the federal government imposes a tariff at the rate t on imports of the dirty good, $\sum_{j=A,B}(x_2^j - y_2^j)$. The domestic price of the dirty traded good to both consumers and producers will be $\epsilon r_2 + t$. Given this price, the problems of the household and producers are as before. The conditions determining national equilibrium are unchanged. Walras' Law holds, and we normalize the exchange rate and world prices such that $\epsilon = r_1 = r_2 = 1$ so $p_1 = p_2 = 1$. We can solve the regional nontraded good and labour market equilibrium conditions for the four endogenous regional variables, p_0^A, p_0^B, w^A , and w^B as a function of regional tax policies, τ_x^j and τ_y^j , lump-sum transfers a^j , and the tariff t :

$$p_0^j(a^j, \tau_x^j, \tau_y^j; t), \quad w^j(a^j, \tau_x^j, \tau_y^j; t). \quad j = A, B$$

Regional emissions will then be given by

$$\begin{aligned} e_x^j(a^j, \tau_x^j, \tau_y^j; t) &= \phi^j x_2^j(p_0^j(\cdot), \tau_x^j, m^j(p_0^j(\cdot), w^j(\cdot), a^j, \tau_y^j; t)), \\ e_y^j(a^j, \tau_x^j, \tau_y^j; t) &= \int_{\mu^j} \mu^j y_2^j(w^j(\cdot), \tau_y^j; t) dG^j(\mu^j). \end{aligned} \quad (53)$$

Indirect utility in region j will now be $V^j(a^j, \tau_x^j, \tau_y^j, t)$, where, analogous to (42),

$$V_t^j = -V_a^j(x_2^j - y_2^j) < 0.$$

The federal government moves first mover and chooses regional transfers θ^A and θ^B and the tariff t to maximize the federal social objective subject its revenue constraint

$$\theta^A + \theta^B + \sum_{j=A,B} t(y_2^j - x_2^j) = 0. \quad (54)$$

The regional governments then choose τ_x^j, τ_y^j and a^j taking as given θ^A, θ^B and t . Their budget constraint is

$$(\tau_x^j - \gamma_x^j)e_x^j + (\tau_y^j - \gamma_y^j)e_y^j + \theta^j = a^j. \quad j = A, B \quad (55)$$

Proceeding by backward induction, the solution to region j 's problem is $a^j(\theta^j, t), \tau_x^j(\theta^j, t), \tau_y^j(\theta^j, t)$, ($j = A, B$). The analogue to Proposition 1 holds, so optimal regional government

policy entails $V_a^j = \lambda^j$, where λ^j is the Lagrange multiplier on the region- j budget constraint with net emissions tax rates:

$$\tau_x^j - \gamma_x^j = \tau_y^j - \gamma_y^j = \frac{s_{ej}^j}{V_a^j}. \quad j = A, B \quad (56)$$

Turning to the federal government, focus first on its choice of regional transfers. It anticipates regional government responses and solves:

$$\begin{aligned} \max_{\{\theta^A, \theta^B\}} \quad & \sum_{j=A, B} V^j \left(a^j(\theta^j; t), \tau_x^j(\theta^j; t), \tau_y^j(\theta^j; t); t \right) - \sigma \left(e^A(\theta^A; t) + e^B(\theta^B; t) \right) \\ \text{s.t.} \quad & \theta^A + \theta^B + \sum_{j=A, B} t \left(y_2^j(\theta^j; t) - x_2^j(\theta^j; t) \right) = 0. \end{aligned} \quad (57)$$

The first-order conditions on θ^j are

$$V_a^j \frac{da^j}{d\theta^j} + V_{\tau_x}^j \frac{d\tau_x^j}{d\theta^j} + V_{\tau_y}^j \frac{d\tau_y^j}{d\theta^j} - \sigma' \frac{de^j}{d\theta^j} - \lambda^F \left(1 + t \frac{d(y_2^j - x_2^j)}{d\theta^j} \right) = 0, \quad j = A, B \quad (58)$$

where λ^F is the Lagrange multiplier on the federal budget constraint. The first three terms represent the marginal private benefit to the regional household of the federal transfer taking into account the transfer will affect the regional policy choices. The fourth term reflects the effect of federal transfer on regional emissions and subsequently on the federal social disutility of emissions. The last term reflects the revenue impacts of the federal transfer. Transferring one more dollar incurs a cost of one, but with a positive tariff generates additional tax revenue if the net imports of the dirty traded good by region j goes up with the federal transfer. The solution to (58) yields optimal interregional transfers $\theta^A(t)$ and $\theta^B(t)$.

To determine whether the federal government should impose tariffs, differentiate the Lagrangian to the federal government's problem (57) with respect to t and evaluate the resulting expression at $t = 0$ to obtain¹³

$$\mathcal{L}_t^F|_{t=0} = \sum_{j=A, B} \left(V_a^j \frac{\partial a^j}{\partial t} + V_{\tau_x}^j \frac{\partial \tau_x^j}{\partial t} + V_{\tau_y}^j \frac{\partial \tau_y^j}{\partial t} - V_a^j (x_2^j - y_2^j) - \sigma' \frac{de^j}{dt} - \lambda^F (y_2^j - x_2^j) \right). \quad (59)$$

To interpret this, differentiate the regional budget (55) with respect to t taking θ^j as given:

$$e_x^j \frac{\partial \tau_x^j}{\partial t} + e_y^j \frac{\partial \tau_y^j}{\partial t} + (\tau_x^j - \gamma_x^j) \frac{de_x^j}{dt} + (\tau_y^j - \gamma_y^j) \frac{de_y^j}{dt} - \frac{\partial a^j}{\partial t} = 0.$$

¹³We use the envelope theorem to ignore changes in θ^j induced by changes in t .

Multiplying by V_a^j and using (56), $V_{\tau_x}^j = -V_a^j e_x^j$ and $V_{\tau_y}^j = -V_a^j e_y^j$,

$$-V_{\tau_x}^j \frac{\partial \tau_x^j}{\partial t} - V_{\tau_y}^j \frac{\partial \tau_y^j}{\partial t} + s_e^j \frac{de^j}{dt} - V_a^j \frac{\partial a^j}{\partial t} = 0.$$

Substitute this into (59),

$$\mathcal{L}_t^F|_{t=0} = \sum_{j=A,B} \left((s_e^j - \sigma') \frac{de^j}{dt} + (\lambda^F - V_a^j) (x_2^j - y_2^j) \right). \quad (60)$$

This consists of two terms. The first reflects the fact that the marginal emissions costs to the federal government σ' generally differ from those to the regional government s_e^j . Social norms may be stronger at the regional level, while the national government endogenizes social costs to both regions. For example, if $s_e^j > \sigma'$ and $de^j/dt < 0$, an increase in the tariff rate from $t = 0$ will reduce the value of the national government's objective function. The second term reflects the fact that an increment of income has a different value to the national and regional governments, $\lambda^F \neq V_a^j = \lambda^j$. The change in tariff revenue from an increase in t , $x_2^j - y_2^j$, will increase national social welfare if $\lambda^F > \sum_j V_a^j$.¹⁴ Overall, the sign of (60) is ambiguous, but generally non-zero.

An alternative interpretation of (60) is obtained as follows. Eq. (58) can be simplified by differentiating the regional budget constraint (55) taking account of the fact that a^j , τ_x^j and τ_y^j depend on θ^j (holding t fixed):

$$e_x^j \frac{d\tau_x^j}{d\theta^j} + e_y^j \frac{d\tau_y^j}{d\theta^j} + (\tau_x^j - \gamma_x^j) \frac{de_x^j}{d\theta^j} + (\tau_y^j - \gamma_y^j) \frac{de_y^j}{d\theta^j} + 1 - \frac{da^j}{d\theta^j} = 0.$$

Multiplying this by V_a^j , using $V_{\tau_x}^j = -V_a^j e_x^j$ and $V_{\tau_y}^j = -V_a^j e_y^j$, setting $t = 0$ and substituting into (58),

$$\lambda^F|_{t=0} = \left(s_{e^j}^j - \sigma' \right) \frac{de^j}{d\theta^j} + V_a^j. \quad j = A, B$$

Substituting this into (60)

$$\mathcal{L}_t^F|_{t=0} = \sum_{j=A,B} \left((s_{e^j}^j - \sigma') \left(\frac{de^j}{dt} + (x_2^j - y_2^j) \frac{de^j}{d\theta^j} \right) \right). \quad (61)$$

This indicates that a change in the tariff rate t from $t = 0$ can improve national social welfare if the regional and national marginal social costs of emissions differ, $s_e^j \neq \sigma'$. Whether t should be increased or decreased is ambiguous. Equivalently, if $\sigma' = s_e^A = s_e^B$ then the

¹⁴This assumes for simplicity that both regions are net importers so $x_2^j - y_2^j > 0$ for both.

optimal interregional transfer ensures that the regional marginal utilities of income are equalized across regions ($V_a^A = V_a^B$) in which case (61) reduces to $\mathcal{L}_t^F|_{t=0} = 0$.

We summarize this in the following proposition.

Proposition 6 *If national and regional marginal social costs of emissions differ, $\sigma' \neq s_e^j$, a trade tax on net imports can improve national social welfare. The trade tax could be positive or negative.*

Further insight into the role of tariffs can be obtained by considering the special case alluded to earlier where emissions are perfect substitutes to the regional governments, and national emissions costs are the sum of regional ones. In this case, regional emissions costs are $s^j(e^A + e^B)$ while national emissions costs are $\sigma(e^A + e^B) = \sum_{j=A,B} s^j(e^A + e^B)$. These imply that $s_{e^j}^j = s_{e^{-j}}^j$ and $\sigma' > s_{e^j}^j$, for $j = A, B$. In effect, differences in social norms are suppressed and the national emissions cost function internalizes the spillover benefits of each region reducing emissions. The following corollary to Proposition 6 is apparent.

Corollary 3 *If social costs of national and regional emissions are $\sum_{j=A,B} s^j(e^A + e^B)$ and $s^j(e^A + e^B)$, respectively, an increase in the trade tax on net imports of the dirty good will increase national social welfare if emissions fall with the tax change, and vice versa.*

The upshot of this analysis is that, unlike in the case where the federal government sets emissions policy, when the regions are responsible for emissions their choice of policy reflects regional social disutilities of emissions. To the extent that these differ from federal emissions social disutilities, regional policies will differ from federal ones. Tariff policy serves as a corrective device, albeit a very imperfect one. A more direct approach would be if both the federal and regional governments enacted emissions pricing policies simultaneously. Such an approach applies on Canada where the federal government can augment regional government emissions pricing.

6.2 Federal government moves second

Now suppose the regional governments move first. In this case, regional governments maximize regional social welfare anticipating how the federal government will choose its interregional transfers. The federal government then chooses its interregional transfer, taking as

given regional emission pricing policies and lump-sum household transfers, based on regional budget constraints. Finally the households and firms make their consumption and production decisions, emissions are determined, and national equilibrium is achieved. As above we solve this multi-stage problem by backward induction. Since private sector outcomes are as before, we can focus on federal and regional government problems in turn.

6.2.1 Federal government problem

The federal government takes the regional policies $(\tau_x^j, \tau_y^j, a^j)$ as given and chooses interregional transfers θ^A, θ^B . These policies must satisfy the regional budget constraints:

$$(\tau_x^j - \gamma_x^j)e_x^j(\tau_x^j, \tau_y^j, a^j) + (\tau_y^j - \gamma_y^j)e_y^j(\tau_x^j, \tau_y^j, a^j) + \theta^j = a^j. \quad j = A, B$$

These regional budget constraints determine a^j as a function of θ^j for a given set of emission taxes τ_x^j and τ_y^j , or $a^j(\theta^j; \tau_x^j, \tau_y^j)$, where the properties of this expression are derived in the Appendix.

The federal government problem can then be written as:

$$\begin{aligned} \max_{\{\theta^A, \theta^B\}} \quad & V^A(\tau_x^A, \tau_y^A, a^A(\theta^A; \tau_x^A, \tau_y^A)) + V^B(\tau_x^B, \tau_y^B, a^B(\theta^B; \tau_x^B, \tau_y^B)) \\ & - \sigma \left(e^A(\tau_x^A, \tau_y^A, a^A(\theta^A; \tau_x^A, \tau_y^A)) + e^B(\tau_x^B, \tau_y^B, a^B(\theta^B; \tau_x^B, \tau_y^B)) \right) \quad \text{s.t.} \quad \theta^A + \theta^B = 0 \end{aligned}$$

The first-order conditions are

$$\left(V_a^A - \sigma' \frac{\partial e^A}{\partial a^A} \right) \frac{\partial a^A}{\partial \theta^A} = \lambda^F = \left(V_a^B - \sigma' \frac{\partial e^B}{\partial a^B} \right) \frac{\partial a^B}{\partial \theta^B} \quad (62)$$

which together with the constraint, $\theta^B = -\theta^A$, yields $\theta^A(\tau_x^A, \tau_y^A, \tau_x^B, \tau_y^B)$. Eq. (62) shows that the federal government equalizes the marginal utility from an increment in regional transfers less the social cost of emissions generated.

6.2.2 Regional government problem

Each regional government chooses its emissions taxes taking as given the emission taxes of the other region and anticipating how its choice of emission taxes will affect federal transfers to both regions. The regional government's problem is

$$\max_{\{\tau_x^j, \tau_y^j\}} V^j(\tau_x^j, \tau_y^j, a^j(\cdot)) - s^j \left(e^j(\tau_x^j, \tau_y^j, a^j(\cdot)), e^{-j}(\tau_x^{-j}, \tau_y^{-j}, a^{-j}(\cdot)) \right)$$

where $a^j(\cdot) = a^j(\theta^j(\tau_x^j, \tau_y^j, \tau_x^{-j}, \tau_y^{-j}); \tau_x^j, \tau_y^j)$. The first-order conditions can be written as follows after some rearranging:

$$\begin{aligned} V_{\tau_x}^j - s_{e^j}^j \frac{\partial e^j}{\partial \tau_x^j} + \left(V_a^j - s_{e^j}^j \frac{\partial e^j}{\partial a^j} \right) \frac{\partial a^j}{\partial \tau_x^j} + \left(\left(V_a^j - s_{e^j}^j \frac{\partial e^j}{\partial a^j} \right) \frac{\partial a^j}{\partial \theta^j} + s_{e^{-j}}^j \frac{\partial e^{-j}}{\partial a^{-j}} \frac{\partial a^{-j}}{\partial \theta^{-j}} \right) \frac{\partial \theta^j}{\partial \tau_x^j} &= 0 \\ V_{\tau_y}^j - s_{e^j}^j \frac{\partial e^j}{\partial \tau_y^j} + \left(V_a^j - s_{e^j}^j \frac{\partial e^j}{\partial a^j} \right) \frac{\partial a^j}{\partial \tau_y^j} + \left(\left(V_a^j - s_{e^j}^j \frac{\partial e^j}{\partial a^j} \right) \frac{\partial a^j}{\partial \theta^j} + s_{e^{-j}}^j \frac{\partial e^{-j}}{\partial a^{-j}} \frac{\partial a^{-j}}{\partial \theta^{-j}} \right) \frac{\partial \theta^j}{\partial \tau_y^j} &= 0 \end{aligned} \quad (63)$$

These conditions are much more complicated than those when the regions alone are responsible, and which lead to the simple emissions tax rules in Proposition 1. In general, emissions taxes net of collection costs satisfying (63) will not be equalized between regions, and they can be higher or lower than those in Proposition 1.

Some insight into the direction of deviation of emissions taxes from those in Proposition 1 can be obtained by focusing on the last terms in the conditions in (63). These terms are unique to this problem and reflect the consequences of the regions anticipating the effects of their policies on federal transfers. The first three terms in the conditions (63) correspond to those if the regions were solely responsible for policy. They would be set to zero and would lead to the emissions tax rules in Proposition 1, $\tau_x^j - \gamma_x^j = \tau_y^j - \gamma_y^j = s_e^j / V_a^j$. By substituting these optimal tax rules into the expression for $\partial a^j / \partial \theta^j$, the last terms of (63) can be expressed as follows:¹⁵

$$\left(V_a^j + s_{e^{-j}}^j \frac{\partial e^{-j}}{\partial a^{-j}} \frac{\partial a^{-j}}{\partial \theta^{-j}} \right) \frac{\partial \theta^j}{\partial \tau_x^j} \quad \text{and} \quad \left(V_a^j + s_{e^{-j}}^j \frac{\partial e^{-j}}{\partial a^{-j}} \frac{\partial a^{-j}}{\partial \theta^{-j}} \right) \frac{\partial \theta^j}{\partial \tau_y^j}.$$

A change in the federal transfer to region j impacts both the private utility of the regional representative household as given by the first term in the bracket and the transfer in the other region via the federal government's budget constraint, $d\theta^j = -d\theta^{-j}$. A change in the other region's federal transfer impacts its total regional emissions and this in turn affects region j 's social disutility of emissions as given by the second term in the brackets. Assuming the regional transfer is increasing in the federal transfer, the sign of this second

¹⁵In particular, we have

$$\frac{\partial a^j(\theta^j; \tau_x^j, \tau_y^j)}{\partial \theta^j} = \left(1 - \frac{s_e^j}{V_a^j} \frac{\partial e_x^j}{\partial a^j} - \frac{s_e^j}{V_a^j} \frac{\partial e_y^j}{\partial a^j} \right)^{-1} = \left(1 - \frac{s_e^j}{V_a^j} \frac{\partial e^j}{\partial a^j} \right)^{-1}, \quad j = A, B$$

since $\partial e^j / \partial a^j = \partial e_x^j / \partial a^j + \partial e_y^j / \partial a^j$.

term depends on how total emissions changes with the regional transfer. If $\partial e^{-j}/\partial a^{-j}$ is positive then this second term is also positive and both terms in the brackets will be positive.¹⁶ It follows that regional governments will have an incentive to increase their emissions taxes if by doing so the federal transfer increases. On the other hand, if an increase in the consumption or production emission tax reduces the federal transfer, then regions would have an incentive to reduce the respective emissions tax relative to the case when the federal government does not make federal transfers after regional policies have been chosen. We summarize this in the following Proposition.

Proposition 7 *Suppose the regions move first anticipating the effects of their emissions tax policies on federal transfers. Assume that transfers to residents in each region are increasing in federal transfers to the region, $\partial a^j/\partial \theta^j > 0$, and that emissions are increasing in transfers to households, $\partial e^j/\partial a^j > 0$. Then if transfers to a region are increasing in emissions tax rates, regions will have an incentive to increase emissions tax rates above those set in the absence of federal transfers, and vice versa.*

7 Conclusions: Federal versus regional responsibility

The object of our analysis has been to compare the efficiency of greenhouse gas emissions control policies when the federal government is responsible with those when emissions control is decentralized to regional governments. We have done so in a stripped-down model of a federation in which emissions control is the only policy choice. When regions are responsible for emissions control, revenues raised are returned to their residents by lump-sum transfers. If the federal government controls emissions policies, revenues raised are either transferred to the regions, who in turn can make transfers to their residents, or are transferred directly to residents. In either case, equity considerations affect federal transfer policies since regions and their residents are heterogeneous.

Our model incorporates some key features that distinguish greenhouse gas emissions policies from those used to address other externalities, and that distinguish our model

¹⁶As we show in the Appendix, e_x^j is increasing in a^j and e_y^j is decreasing in a^j , so the overall effect of a^j on e^j is ambiguous.

of federalism with ones conventionally used in the literature.¹⁷ The most important is that greenhouse gas emissions are both global and long-lasting. Global warming currently experienced is due to past worldwide emissions that have cumulated in the atmosphere and take many years to dissipate. The consequence is that current national emissions have only a small effect on worldwide emissions, and that effect is felt mostly by future generations. If government policy were based on the selfish interests of currently alive residents, no emissions controls would be enacted. This is not what we observe in practice. Governments do implement emissions control policies, and we assume that is because residents attach some social or altruistic value to emissions control policies per se. Specifically, we say that they obtain social disutility from emissions emanating both from their own region and from the other region in the federation. The private disutility they incur from global emissions is based mainly on cumulated greenhouse gases and is essentially fixed.

Household choices are determined by their private utility, while those of the government are based on the total utility of resident households, including both private utility and the social disutility of emissions. That is, governments are the agents responsible for acting on the social preferences of households. Optimal government policies include emissions pricing, either by taxes or tradeable permits. These are similar to Pigovian taxes except that they are based on marginal social disutilities of regional emissions rather than the actual disutility caused by global warming. Our concept of optimality is based on social disutility rather than on worldwide private disutility from global warming. This distinguishes our approach from other models of optimal carbon pricing, exemplified by Stern (2007).

Our federation is a small open economy in a larger world in which the price of traded goods is fixed. The prices of non-traded goods and the wage rate are determined endogenously in national general equilibrium and are affected by emissions control policies. Despite these general equilibrium pricing effects, the effect of emissions prices on endogenous prices does not affect optimal emissions pricing rules.

We analyze outcomes when the federal government and the regions assume full control of emissions policies. We also consider mixed cases where the regions control emissions pricing while the federal government implements inter-regional transfers, in which case the timing

¹⁷See, for example, the overviews in Wilson (1986), Boadway and Tremblay (2012), Keen and Konrad (2013), Boadway and Cuff (2017) and Agrawal, Brueckner and Brühlhart (2024).

of decisions becomes relevant. Explicit comparisons between federal and regional emissions pricing are ambiguous: the federal government enjoys some advantages and the regions others. What we can do is identify differences between the two cases and the implications of those differences for policy outcomes.

Differences in the levels and efficiency of federal versus regional emissions control policies arise from a number of sources. Federal government policies endogenize spillover benefits of one region's emissions policies on residents of the other region. While this tends to increase federal emissions taxes relative to those of the regions, that incentive is mitigated by the assumption that social disutility effects are stronger at the regional than the federal level because social solidarity is higher locally. As well, the cost of pricing emissions is higher for the federal government than for the regions because the former is at an informational disadvantage. This tends to reduce optimal federal emissions pricing relative to the regions. Overall, the level of emissions pricing under federal responsibility can be higher or lower than if the regions are responsible. Moreover, regions will generally choose different emissions prices, which may be efficient if marginal social disutility of emissions differs between regions. On the other hand, since the federal (national) social utility of emissions depends upon aggregate regional emissions such that regional emissions are perfect substitutes, optimal federal emissions prices are uniform nationwide. The federal government is able to enforce a uniform emissions price because it can either impose a uniform emissions tax nationwide or it can take advantage of nationwide permit trading systems which yield a uniform price. The federal government can also pursue interregional equity objectives by differentiating the transfer it provides to the regions or their residents.

We derived our results in a relatively simple setting. We conclude with a number of suggestions for extensions. Increasing the number of regions in the federation beyond two would be straightforward. Social disutility would be based on emissions from all regions. Regional emissions taxes would still take a Pigovian form. They would differ among regions and would be inefficient due to inter-regional externalities. Federal emissions taxes would be uniform across regions. They would internalize interregional externalities and incorporate interregional equity into transfers. Optimal federal policies would be based on a federal disutility of emissions function, which might reflect weaker social norms than regional social disutility functions.

More ambitiously, we have assumed that greenhouse gas emissions are proportional to consumption and production of the dirty good. In future analysis we plan to explore policies for changing the intensity of emissions, for example, by replacing gasoline combustion engines with electric vehicles, investing in heat pumps, and adopting green energy sources. Government intervention might be needed because of scale and network effects. These initiatives are typically pursued by regulatory or infrastructure policies which also raise issues of assignment to federal versus regional governments. For example, regions might be responsible for industrial emissions and green production technologies, and the federal government for consumption emissions and green consumption technologies. An important consideration would then be the order of government decision-making. Both levels of government could choose policies simultaneously in which case there would be a federal-regional Nash equilibrium in policies (e.g., Dahlby, 1996; Agrawal, 2023). Alternatively, the federal government could move first followed by the regions (e.g., Persson and Tabellini, 1996), or vice versa (e.g., Caplan, Cornes and Silva, 2000).

These policies focus on reducing future emissions. However, there is already a substantial stock of greenhouse gases accumulated in the atmosphere that are responsible for elevated global temperatures and the costs they impose on society. These costs can be mitigated by adaptation measures such as flood and forest fire mitigation, pest control, housing relocation and reinforcement of buildings. As well, investment could be made in future technologies that could potentially remove greenhouse gases from the atmosphere. These policies are complementary with emissions reduction measures.

Finally, a more ambitious extension would be to incorporate the dynamics of global warming into the analysis, as in van der Ploeg and Venables (2022) and Besley and Persson (2023) who emphasize the policy challenges of a transition to green technologies. Even more ambitious would be to follow Stern (2007) by taking into consideration the fact that global warming is a very long-term phenomenon that has an intergenerational dimension. This can result in a rising carbon tax price over time which can lead to seemingly perverse changes in the path of emissions as Sinn (2012) has argued.

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Appendix

Section 2.2.2: Regional demand functions

Suppressing the regional superscript j , the first-order conditions of the regional household's problem are:

$$\frac{\partial U(\cdot)}{\partial x_0} - \alpha p_0 = 0, \quad \frac{\partial U(\cdot)}{\partial x_1} - \alpha p_1 = 0, \quad \frac{\partial U(\cdot)}{\partial x_2} - \alpha(p_2 + \tau_x \phi) = 0,$$

where α is the multiplier on the household's budget constraint and together with the binding budget constraint yields the optimal demands:

$$\begin{aligned} x_0(p_0, p_1, p_2, \tau_x, m) &= \underline{x}_0 + \frac{\beta_0}{p_0} (m - p_0 \underline{x}_0 - p_1 \underline{x}_1 - (p_2 + \tau_x \phi) \underline{x}_2) \\ x_1(p_0, p_1, p_2, \tau_x, m) &= \underline{x}_1 + \frac{\beta_1}{p_1} (m - p_0 \underline{x}_0 - p_1 \underline{x}_1 - (p_2 + \tau_x \phi) \underline{x}_2) \\ x_2(p_0, p_1, p_2, \tau_x, m) &= \underline{x}_2 + \frac{\beta_2}{p_2 + \tau_x \phi} (m - p_0 \underline{x}_0 - p_1 \underline{x}_1 - (p_2 + \tau_x \phi) \underline{x}_2). \end{aligned}$$

Indirect utility is then given by $V(p_0, p_1, p_2, \tau_x, m)$ where

$$\begin{aligned} V_{p_0} &= -\alpha^* x_0 < 0, \quad V_{p_1} = -\alpha^* x_1 < 0, \quad V_{p_2} = -\alpha^* x_2 < 0, \\ V_{\tau_x} &= -\alpha^* \phi x_2 < 0, \quad V_m = \alpha^* > 0, \end{aligned}$$

and the optimal value of α is

$$\alpha^* = \left(\frac{\beta_0}{p_0} \right)^{\beta_0} \left(\frac{\beta_1}{p_1} \right)^{\beta_1} \left(\frac{\beta_2}{p_2 + \tau_x \phi} \right)^{\beta_2} (m - p_0 \underline{x}_0 - p_1 \underline{x}_1 - (p_2 + \tau_x \phi) \underline{x}_2).$$

Suppressing world prices, household income will be given by

$$m(p_0, w, a, p_\kappa, \bar{\kappa}) = wL + a + \pi_0(p_0, w) + \pi_1(w) + \bar{\pi}_2(w, p_\kappa, \bar{\kappa}),$$

where

$$\begin{aligned} \frac{\partial m(\cdot)}{\partial p_0} &= \frac{\partial \pi_0(\cdot)}{\partial p_0} = y_0 > 0, \quad \frac{\partial m(\cdot)}{\partial w} = L + \frac{\partial \pi_0(\cdot)}{\partial w} + \frac{\partial \pi_1(\cdot)}{\partial w} + \frac{\partial \bar{\pi}_2(\cdot)}{\partial w} = L - \ell_0 - \ell_1 - \bar{\ell}_2 \\ \frac{\partial m}{\partial a} &= 1, \quad \frac{\partial m}{\partial p_\kappa} = \frac{\partial \bar{\pi}_2}{\partial p_\kappa} = -e_y + \bar{\kappa}, \quad \frac{\partial m}{\partial \bar{\kappa}} = \frac{\partial \bar{\pi}_2}{\partial \bar{\kappa}} = p_\kappa. \end{aligned}$$

The second and fourth expression will be zero when the labour and permit markets clear, respectively. Household demand for the non-traded good is

$$x_0(p_0, \tau_x, m(p_0, w, a, p_\kappa, \bar{\kappa})) = \underline{x}_0 + \frac{\beta_0}{p_0} (m(p_0, w, a, p_\kappa, \bar{\kappa}) - p_0 \underline{x}_0 - p_1 \underline{x}_1 - (p_2 + \tau_x \phi) \underline{x}_2)$$

and totally differentiating yields (in equilibrium):

$$\begin{aligned}\frac{dx_0}{dp_0} &= \frac{\partial x_0(\cdot)}{\partial p_0} + \frac{\partial x_0(\cdot)}{\partial m} \frac{\partial m(\cdot)}{\partial p_0} = -\frac{\beta_0}{(p_0)^2} (p_1(x_1 - \underline{x}_1) + (p_2 + \tau_x \phi)(x_2 - \underline{x}_2)) < 0, \\ \frac{dx_0}{dw} &= \frac{\partial x_0(\cdot)}{\partial m} \frac{\partial m(\cdot)}{\partial w} = 0, \quad \frac{dx_0}{d\tau_x} = \frac{\partial x_0(\cdot)}{\partial \tau_x} = -\frac{\beta_0}{p_0} \phi \underline{x}_2 < 0, \\ \frac{dx_0}{da} &= \frac{\partial x_0(\cdot)}{\partial m} \frac{\partial m(\cdot)}{\partial a} = \frac{\beta_0}{p_0} > 0, \quad \frac{dx_0}{dp_\kappa} = \frac{\partial x_0(\cdot)}{\partial m} \frac{\partial m(\cdot)}{\partial p_\kappa} = 0, \quad \frac{dx_0}{d\bar{\kappa}} = \frac{\partial x_0(\cdot)}{\partial m} \frac{\partial m(\cdot)}{\partial \bar{\kappa}} = \frac{\beta_0}{p_0} p_\kappa > 0.\end{aligned}$$

Household demand for the dirty traded good is

$$x_2(p_0, \tau_x, m(p_0, w, a, p_\kappa, \bar{\kappa})) = \underline{x}_2 + \frac{\beta_2}{p_2 + \tau_x \phi} (m(p_0, w, a, p_\kappa, \bar{\kappa}) - p_0 \underline{x}_0 - p_1 \underline{x}_1 - (p_2 + \tau_x \phi) \underline{x}_2),$$

and totally differentiating yields (in equilibrium):

$$\begin{aligned}\frac{dx_2}{dp_0} &= \frac{\partial x_2(\cdot)}{\partial p_0} + \frac{\partial x_2(\cdot)}{\partial m} \frac{\partial m(\cdot)}{\partial p_0} = \frac{\beta_2}{p_2 + \tau_x \phi} (y_0 - \underline{x}_0) > 0, \quad \frac{dx_2}{da} = \frac{\partial x_2(\cdot)}{\partial m} \frac{\partial m(\cdot)}{\partial a} = \frac{\beta_2}{p_2 + \tau_x \phi} > 0, \\ \frac{dx_2}{d\tau_x} &= \frac{\partial x_2(\cdot)}{\partial \tau_x} = -\frac{\beta_2}{p_2 + \tau_x \phi} \phi \underline{x}_2 < 0, \quad \frac{dx_2}{dw} = \frac{\partial x_2(\cdot)}{\partial m} \frac{\partial m(\cdot)}{\partial w} = 0, \\ \frac{dx_2}{dp_\kappa} &= \frac{\partial x_2(\cdot)}{\partial m} \frac{\partial m(\cdot)}{\partial p_\kappa} = 0, \quad \frac{dx_2}{d\bar{\kappa}} = \frac{\partial x_2(\cdot)}{\partial m} \frac{\partial m(\cdot)}{\partial \bar{\kappa}} = \frac{\beta_2}{p_2 + \tau_x \phi} p_\kappa > 0\end{aligned}$$

Section 3: National equilibrium

Regional price of the nontraded good and wage rate

Focusing on region j and suppressing the regional superscript, the equilibrium conditions can be written as

$$D(p_0, w, p_\kappa) \equiv \ell_0(p_0, w) + \ell_1(w) + \bar{\ell}_2(w, p_\kappa) - L = 0$$

$$X(p_0, w, a, \tau_x, p_\kappa, \bar{\kappa}) \equiv x_0(p_0, \tau_x, m(p_0, w, a, p_\kappa, \bar{\kappa})) - y_0(p_0, w) = 0,$$

where by differentiation

$$\begin{aligned}D_{p_0} &= \frac{\partial \ell_0}{\partial p_0} > 0, \quad D_w = \frac{\partial \ell_0}{\partial w} + \frac{\partial \ell_1}{\partial w} + \frac{\partial \bar{\ell}_2}{\partial w} < 0, \quad D_{p_\kappa} = \frac{\partial \bar{\ell}_2}{\partial p_\kappa} < 0, \\ X_{p_0} &= \frac{dx_0}{dp_0} - \frac{\partial y_0}{\partial p_0} = -\frac{\beta_0}{p_0^2} (p_1(x_1 - \underline{x}_1) + (p_2 + \tau_x \phi)(x_2 - \underline{x}_2)) - \frac{\partial y_0}{\partial p_0} < 0, \\ X_w &= -\frac{\partial y_0}{\partial w} < 0, \quad X_a = \frac{dx_0}{da} = \frac{\beta_0}{p_0} > 0,\end{aligned}$$

$$X_{\tau_x} = \frac{dx_0}{d\tau_x} = -\frac{\beta_0}{p_0}\phi\bar{x}_2 < 0, \quad X_{p_\kappa} = \frac{dx_0}{dp_\kappa} = 0, \quad X_{\bar{\kappa}} = \frac{dx_0}{d\bar{\kappa}} = \frac{\beta_0}{p_0}p_\kappa > 0.$$

The two market-clearing conditions implicitly define $p_0(a, \tau_x, p_\kappa, \bar{\kappa})$ and $w(a, \tau_x, p_\kappa, \bar{\kappa})$.

Totally differentiating $D(p_0, w, p_\kappa)$ and $X(p_0, w, a, \tau_x, p_\kappa, \bar{\kappa})$ yield

$$\begin{bmatrix} D_{p_0} & D_w \\ X_{p_0} & X_w \end{bmatrix} \begin{bmatrix} dp_0 \\ dw \end{bmatrix} = \begin{bmatrix} 0 & 0 & -D_{p_\kappa} & 0 \\ -X_a & -X_{\tau_x} & 0 & -X_{\bar{\kappa}} \end{bmatrix} \begin{bmatrix} da \\ d\tau_x \\ dp_\kappa \\ d\bar{\kappa} \end{bmatrix}$$

Define

$$\begin{aligned} |Det| &= D_{p_0}X_w - D_wX_{p_0} \\ &= -\frac{\partial \ell_0}{\partial p_0} \frac{\partial y_0}{\partial w} - \left[\frac{\partial \ell_0}{\partial w} + \frac{\partial \ell_1}{\partial w} + \frac{\partial \bar{\ell}_2}{\partial w} \right] \left(-\frac{\beta_0}{p_0^2} (p_1(x_1 - \bar{x}_1) + (p_2 + \tau_x \phi)(x_2 - \bar{x}_2)) \right) \\ &\quad + \left[\frac{\partial \ell_0}{\partial w} + \frac{\partial \ell_1}{\partial w} + \frac{\partial \bar{\ell}_2}{\partial w} \right] \frac{\partial y_0}{\partial p_0} \\ &= \left[\frac{\partial \ell_0}{\partial w} + \frac{\partial \ell_1}{\partial w} + \frac{\partial \bar{\ell}_2}{\partial w} \right] \left(\frac{\beta_0}{p_0^2} (p_1(x_1 - \bar{x}_1) + (p_2 + \tau_x \phi)(x_2 - \bar{x}_2)) \right) + \left[\frac{\partial \ell_1}{\partial w} + \frac{\partial \bar{\ell}_2}{\partial w} \right] \frac{\partial y_0}{\partial p_0} < 0 \end{aligned}$$

where the second equality follows from

$$\frac{\partial \ell_0}{\partial p_0} \frac{\partial y_0}{\partial w} = f_0'(\ell_0) \frac{\partial \ell_0}{\partial p_0} \frac{\partial \ell_0}{\partial w} = \frac{\partial \ell_0}{\partial w} \frac{\partial y_0}{\partial p_0}.$$

Use Cramer's Rule to obtain

$$\begin{aligned} \frac{dp_0}{da} &= \frac{\begin{vmatrix} 0 & D_w \\ -X_a & X_w \end{vmatrix}}{|Det|} = \frac{D_w X_a}{|Det|} > 0, & \frac{dp_0}{d\tau_x} &= \frac{\begin{vmatrix} 0 & D_w \\ -X_{\tau_x} & X_w \end{vmatrix}}{|Det|} = \frac{D_w X_{\tau_x}}{|Det|} < 0 \\ \frac{dp_0}{dp_\kappa} &= \frac{\begin{vmatrix} -D_{p_\kappa} & D_w \\ 0 & X_w \end{vmatrix}}{|Det|} = \frac{-D_{p_\kappa} X_w}{|Det|} > 0, & \frac{dp_0}{d\bar{\kappa}} &= \frac{\begin{vmatrix} -0 & D_w \\ -X_{\bar{\kappa}} & X_w \end{vmatrix}}{|Det|} = \frac{D_w X_{\bar{\kappa}}}{|Det|} > 0 \\ \frac{dw}{da} &= \frac{\begin{vmatrix} D_{p_0} & 0 \\ X_{p_0} & -X_a \end{vmatrix}}{|Det|} = \frac{-D_{p_0} X_a}{|Det|} > 0, & \frac{dw}{d\tau_x} &= \frac{\begin{vmatrix} D_{p_0} & 0 \\ X_{p_0} & -X_{\tau_x} \end{vmatrix}}{|Det|} = \frac{-D_{p_0} X_{\tau_x}}{|Det|} < 0 \\ \frac{dw}{dp_\kappa} &= \frac{\begin{vmatrix} D_{p_0} & -D_{p_\kappa} \\ X_{p_0} & 0 \end{vmatrix}}{|Det|} = \frac{D_{p_\kappa} X_{p_0}}{|Det|} < 0, & \frac{dw}{d\bar{\kappa}} &= \frac{\begin{vmatrix} D_{p_0} & 0 \\ X_{p_0} & -X_{\bar{\kappa}} \end{vmatrix}}{|Det|} = \frac{-D_{p_0} X_{\bar{\kappa}}}{|Det|} < 0. \end{aligned}$$

Emissions

The first-order condition from the problem of the dirty traded good producer with emissions parameter μ is $p_2 f_2'(\ell_2) - w - p_\kappa \mu f_2'(\ell_2) = 0$, which again suppressing the world prices yields labour demand and output supply $\ell_2(w, p_\kappa; \mu)$ and $y_2(w, p_\kappa; \mu) = f_2(\ell_2(w, p_\kappa; \mu))$, respectively. By differentiation

$$\frac{\partial \ell_2(w, p_\kappa; \mu)}{\partial w} = \frac{1}{f_2''(\ell_2)(p_2 - p_\kappa \mu)} < 0, \quad \frac{\partial \ell_2(w, p_\kappa; \mu)}{\partial p_\kappa} = \frac{f_2'(\ell_2)\mu}{f_2''(\ell_2)(p_2 - p_\kappa \mu)} < 0,$$

and

$$\frac{\partial y_2(w, p_\kappa; \mu)}{\partial w} = f_2'(\ell_2) \frac{\partial \ell_2(w, p_\kappa; \mu)}{\partial w} < 0, \quad \frac{\partial y_2(w, p_\kappa; \mu)}{\partial p_\kappa} = f_2'(\ell_2) \frac{\partial \ell_2(w, p_\kappa; \mu)}{\partial p_\kappa} < 0.$$

Given $p_0(a, \tau_x, p_\kappa, \bar{\kappa})$ and $w(a, \tau_x, p_\kappa, \bar{\kappa})$, we can determine the amount of production emissions in equilibrium as a function of the regional government policies:

$$e_y(a, \tau_x, p_\kappa, \bar{\kappa}) = \int_{\underline{\mu}}^{\bar{\mu}} \mu y_2(w(a, \tau_x, p_\kappa, \bar{\kappa}), p_\kappa; \mu) dG(\mu)$$

where

$$\begin{aligned} \frac{de_y(\cdot)}{da} &= \underbrace{\left(\int_{\underline{\mu}}^{\bar{\mu}} \mu \frac{\partial y_2(w, p_\kappa; \mu)}{\partial w} dG \right)}_{(-)} \underbrace{\frac{dw}{da}}_{(+)} < 0, \quad \frac{de_y(\cdot)}{d\tau_x} = \underbrace{\left(\int_{\underline{\mu}}^{\bar{\mu}} \mu \frac{\partial y_2(w, p_\kappa; \mu)}{\partial w} dG \right)}_{(-)} \underbrace{\frac{dw}{d\tau_x}}_{(-)} > 0, \\ \frac{de_y(\cdot)}{dp_\kappa} &= \underbrace{\left(\int_{\underline{\mu}}^{\bar{\mu}} \mu \frac{\partial y_2(w, p_\kappa; \mu)}{\partial w} dG \right)}_{(-)} \underbrace{\frac{dw}{dp_\kappa}}_{(-)} + \underbrace{\left(\int_{\underline{\mu}}^{\bar{\mu}} \mu \frac{\partial y_2(w, p_\kappa; \mu)}{\partial p_\kappa} dG \right)}_{(-)} \geq 0, \\ \frac{de_y(\cdot)}{d\bar{\kappa}} &= \underbrace{\left(\int_{\underline{\mu}}^{\bar{\mu}} \mu \frac{\partial y_2(w, p_\kappa; \mu)}{\partial w} dG \right)}_{(-)} \underbrace{\frac{dw}{d\bar{\kappa}}}_{(-)} > 0, \end{aligned}$$

Given $p_0(a, \tau_x, p_\kappa, \bar{\kappa})$ and $w(a, \tau_x, p_\kappa, \bar{\kappa})$, consumption emissions in equilibrium as a function of the regional government policies are:

$$e_x(a, \tau_x, p_\kappa, \bar{\kappa}) = \phi x_2(p_0(a, \tau_x, p_\kappa, \bar{\kappa}), \tau_x, m(p_0(a, \tau_x, p_\kappa, \bar{\kappa}), w(a, \tau_x, p_\kappa, \bar{\kappa}), a, p_\kappa, \bar{\kappa})),$$

where

$$\frac{de_x(\cdot)}{da} = \underbrace{\phi \frac{dx_2}{da}}_{(+)} + \underbrace{\phi \frac{dx_2}{dp_0}}_{(+)} \underbrace{\frac{dp_0}{da}}_{(+)} > 0, \quad \frac{de_x(\cdot)}{d\tau_x} = \underbrace{\phi \frac{dx_2}{dp_0}}_{(+)} \underbrace{\frac{dp_0}{d\tau_x}}_{(-)} + \underbrace{\phi \frac{dx_2}{d\tau_x}}_{(-)} < 0,$$

$$\frac{de_x(\cdot)}{dp_\kappa} = \underbrace{\phi \frac{dx_2}{dp_0}}_{(+)} \underbrace{\frac{dp_0}{dp_\kappa}}_{(-)} < 0, \quad \frac{de_x(\cdot)}{d\bar{\kappa}} = \underbrace{\phi \frac{dx_2}{d\bar{\kappa}}}_{(+)} + \underbrace{\phi \frac{dx_2}{dp_0}}_{(+)} \underbrace{\frac{dp_0}{d\bar{\kappa}}}_{(-)} \geq 0.$$

Household welfare

In equilibrium, household welfare is

$$V(\tau_x, a, p_\kappa, \bar{\kappa}) = V(p_0(\cdot), \tau_x, m(p_0(\cdot), w(\cdot), a, p_\kappa, \bar{\kappa}))$$

where given that the permit, non-traded good and labour markets all clear,

$$\frac{dV}{d\tau_x} = V_{\tau_x}, \quad \frac{dV}{da} = V_a, \quad \frac{dV}{dp_\kappa} = V_{p_\kappa} = 0, \quad \frac{dV}{d\bar{\kappa}} = V_a p_\kappa.$$

Section 4: Regional government policies

The regional government's problem is:

$$\begin{aligned} \max_{\{\tau_x, a, p_\kappa, \bar{\kappa}\}} & V(\tau_x, a, p_\kappa, \bar{\kappa}) - s(e(\tau_x, a, p_\kappa, \bar{\kappa}), e^-) \\ & + \lambda((\tau_x - \gamma_x)e_x(\tau_x, a, p_\kappa, \bar{\kappa}) - \gamma_y e_y(\tau_x, a, p_\kappa, \bar{\kappa}) - a) + \delta(e_y(\tau_x, a, p_\kappa, \bar{\kappa}) - \bar{\kappa}). \end{aligned}$$

The first-order conditions are

$$\begin{aligned} V_{\tau_x} - s_e \frac{de}{d\tau_x} + \lambda\left(e_x + (\tau_x - \gamma_x) \frac{de_x}{d\tau_x} - \gamma_y \frac{de_y}{d\tau_x}\right) + \delta \frac{de_y}{d\tau_x} &= 0 \\ V_a - s_e \frac{de}{da} + \lambda\left((\tau_x - \gamma_x) \frac{de_x}{da} - \gamma_y \frac{de_y}{da} - 1\right) + \delta \frac{de_y}{da} &= 0 \\ V_{p_\kappa} - s_e \frac{de}{dp_\kappa} + \lambda\left((\tau_x - \gamma_x) \frac{de_x}{dp_\kappa} - \gamma_y \frac{de_y}{dp_\kappa}\right) + \delta \frac{de_y}{dp_\kappa} &= 0 \\ V_{\bar{\kappa}} - s_e \frac{de}{d\bar{\kappa}} + \lambda\left((\tau_x - \gamma_x) \frac{de_x}{d\bar{\kappa}} - \gamma_y \frac{de_y}{d\bar{\kappa}}\right) + \delta \left(\frac{de_y}{d\bar{\kappa}} - 1\right) &= 0. \end{aligned}$$

Multiplying the first-order condition on a by p_κ and using the expressions for $V_{\bar{\kappa}} = p_\kappa V_m = p_\kappa V_a$, $de_x/d\bar{\kappa} = p_\kappa de_x/da$ and $de_y/d\bar{\kappa} = p_\kappa de_y/da$ yields,

$$V_{\bar{\kappa}} - s_e \frac{de}{d\bar{\kappa}} + \lambda\left((\tau_x - \gamma_x) \frac{de_x}{d\bar{\kappa}} - \gamma_y \frac{de_y}{d\bar{\kappa}} - p_\kappa\right) + \delta \frac{de_y}{d\bar{\kappa}} = 0.$$

Combining this with the first-order condition on $\bar{\kappa}$, we obtain $\lambda p_\kappa = \delta$. Substituting $\lambda p_\kappa = \delta$ into the first-order conditions for a , τ_x and p_κ and using $V_{p_\kappa} = 0$ and $V_{\tau_x} = -V_a e_x$,

$$V_a - s_e \frac{de}{da} + \lambda\left((\tau_x - \gamma_x) \frac{de_x}{da} + (p_\kappa - \gamma_y) \frac{de_y}{da} - 1\right) = 0 \quad (a)$$

$$-V_a e_x - s_e \frac{de}{d\tau_x} + \lambda \left(e_x + (\tau_x - \gamma_x) \frac{de_x}{d\tau_x} + (p_\kappa - \gamma_y) \frac{de_y}{d\tau_x} \right) = 0 \quad (\tau_x)$$

$$-s_e \frac{de}{dp_\kappa} + \lambda \left((\tau_x - \gamma_x) \frac{de_x}{dp_\kappa} + (p_\kappa - \gamma_y) \frac{de_y}{dp_\kappa} \right) = 0. \quad (p_\kappa)$$

Solving for λ from (a) and substituting into (τ_x) and (p_κ) yields

$$-V_a e_x - s_e \frac{de}{d\tau_x} + \frac{V_a - s_e \frac{de}{da}}{1 - (\tau_x - \gamma_x) \frac{de_x}{da} - (p_\kappa - \gamma_y) \frac{de_y}{da}} \left(e_x + (\tau_x - \gamma_x) \frac{de_x}{d\tau_x} + (p_\kappa - \gamma_y) \frac{de_y}{d\tau_x} \right) = 0$$

$$-s_e \frac{de}{dp_\kappa} + \frac{V_a - s_e \frac{de}{da}}{1 - (\tau_x - \gamma_x) \frac{de_x}{da} - (p_\kappa - \gamma_y) \frac{de_y}{da}} \left((\tau_x - \gamma_x) \frac{de_x}{dp_\kappa} + (p_\kappa - \gamma_y) \frac{de_y}{dp_\kappa} \right) = 0.$$

Cross-multiplying and simplifying the two conditions can be written as:

$$\begin{aligned} & (V_a(\tau_x - \gamma_x) - s_e) \left(e_x \frac{de_x}{da} + \frac{de_x}{d\tau_x} \right) + (V_a(p_\kappa - \gamma_y) - s_e) \left(e_x \frac{de_y}{da} + \frac{de_y}{d\tau_x} \right) \\ & + s_e(p_\kappa - \gamma_y - (\tau_x - \gamma_x)) \left(\frac{de_x}{d\tau_x} \frac{de_y}{da} - \frac{de_y}{d\tau_x} \frac{de_x}{da} \right) = 0, \\ & (V_a(p_\kappa - \gamma_y) - s_e) \frac{de_y}{dp_\kappa} + (V_a(\tau_x - \gamma_x) - s_e) \frac{de_x}{dp_\kappa} \\ & + s_e(\tau_x - \gamma_x - (p_\kappa - \gamma_y)) \left(\frac{de_y}{dp_\kappa} \frac{de_x}{da} - \frac{de_y}{da} \frac{de_x}{dp_\kappa} \right) = 0, \end{aligned}$$

which can then be written in the following form:

$$c_x(\tau_x - \gamma_x) + c_y(p_\kappa - \gamma_y) = c,$$

$$d_x(\tau_x - \gamma_x) + d_y(p_\kappa - \gamma_y) = d.$$

Applying Cramer's rule yields:

$$\tau_x - \gamma_x = \frac{cd_y - c_yd}{c_xd_y - c_yd_x},$$

$$p_\kappa - \gamma_y = \frac{c_xd - cd_x}{c_xd_y - c_yd_x}.$$

It is straightforward to show that $cd_y - c_yd = c_xd - cd_x$ and there is unique solution to the two first-order conditions in which $\tau_x - \gamma_x = p_\kappa - \gamma_y$. Substituting this equality into the first-order conditions on a , it follows that at the optimum, $\lambda = V_a$ and we obtain the following result

$$\tau_x^j - \gamma_x^j = p_\kappa^j - \gamma_y^j = \frac{s_{ej}^j}{V_a^j}. \quad j = A, B$$

Remark 1.1: Regional governments auction production emission permits

Suppose instead of giving $\bar{\kappa}^j$ emissions permits per producer for free the region- j government chooses $\bar{\kappa}^j$ and sells the permits by auction to producers in the region. The auction for permits is assumed to be competitive, so producers take the equilibrium price of permits, denoted by p_κ^j , as given. The revenues from the sale of permits are returned as a lump sum transfer to the representative household. The household problem and the problems of the clean goods' producers are unchanged. Walras' law continues to hold nationwide.

The problem of the dirty good producer in region j is:

$$\max_{\{\ell_2^j, \kappa^j\}} p_2 f_2^j(\ell_2^j) - w^j \ell_2^j - p_\kappa^j \kappa^j \quad \text{s.t.} \quad \kappa^j \geq \mu^j f_2^j(\ell_2^j). \quad j = A, B$$

The firm's constraint will necessarily be binding given that permits are costly. The first-order conditions reduce to $p_2 f_2^{j'}(\ell_2^j) = w^j - p_\kappa^j \mu^j f_2^{j'}(\ell_2^j)$, which yields, suppressing world prices, $\ell_2^j(w^j, p_\kappa^j; \mu^j)$ where

$$\frac{\partial \ell_2^j(\cdot)}{\partial w^j} = \frac{1}{f_2^{j''}(\ell_2^j)(p_2 - p_\kappa^j \mu^j)} < 0, \quad \frac{\partial \ell_2^j(\cdot)}{\partial p_\kappa^j} = \frac{f_2^{j'}(\ell_2^j) \mu^j}{f_2^{j''}(\ell_2^j)(p_2 - p_\kappa^j \mu^j)} < 0.$$

Aggregate labour demand by producers of good 2 is given by:

$$\bar{\ell}_2^j(w^j, p_\kappa^j) = \int_{\underline{\mu}^j}^{\bar{\mu}^j} \ell_2^j(w^j, p_\kappa^j; \mu^j) dG^j$$

where

$$\frac{\partial \bar{\ell}_2^j(\cdot)}{\partial w^j} = \int_{\underline{\mu}^j}^{\bar{\mu}^j} \frac{\partial \ell_2^j(w^j, p_2, p_\kappa^j; \mu^j)}{\partial w^j} dG^j < 0, \quad \frac{\partial \bar{\ell}_2^j(\cdot)}{\partial p_\kappa^j} = \int_{\underline{\mu}^j}^{\bar{\mu}^j} \frac{\partial \ell_2^j(w^j, p_2, p_\kappa^j; \mu^j)}{\partial p_\kappa^j} dG^j < 0.$$

Firm output is given by

$$y_2^j(w^j, p_\kappa^j; \mu^j) = f_2^j(\ell_2^j(w^j, p_\kappa^j; \mu^j))$$

where

$$\frac{\partial y_2^j(\cdot)}{\partial w^j} = f_2^{j'}(\ell_2^j) \frac{\partial \ell_2^j(\cdot)}{\partial w^j} < 0, \quad \frac{\partial y_2^j(\cdot)}{\partial p_\kappa^j} = f_2^{j'}(\ell_2^j) \frac{\partial \ell_2^j(\cdot)}{\partial p_\kappa^j} < 0.$$

Firm profit is given by

$$\pi_2^j(w^j, p_\kappa^j; \mu^j) = p_2 f_2^j(\ell_2^j(w^j, p_\kappa^j; \mu^j)) - w^j \ell_2^j(w^j, p_\kappa^j; \mu^j) - p_\kappa^j \mu^j f_2^j(\ell_2^j(w^j, p_\kappa^j; \mu^j))$$

where

$$\frac{\partial \pi_2^j(\cdot)}{\partial w^j} = -\ell_2^j < 0, \quad \frac{\partial \pi_2^j(\cdot)}{\partial p_\kappa^j} = -\mu^j y_2^j < 0, \quad \frac{\partial \pi_2^j(\cdot)}{\partial \mu^j} = -\tau_y^j y_2^j < 0.$$

Aggregate profits are given by

$$\bar{\pi}(w^j, p_\kappa^j) = \int_{\underline{\mu}^j}^{\bar{\mu}^j} \pi_2^j(w^j, p_\kappa^j; \mu^j) dG^j$$

where

$$\frac{\partial \bar{\pi}_2^j(\cdot)}{\partial w^j} = - \int_{\underline{\mu}^j}^{\bar{\mu}^j} \ell_2^j(w^j, p_\kappa^j; \mu^j) dG^j < 0, \quad \frac{\partial \bar{\pi}_2^j(\cdot)}{\partial p_\kappa^j} = - \int_{\underline{\mu}^j}^{\bar{\mu}^j} \mu^j y_2^j(w^j, p_\kappa^j; \mu^j) dG^j < 0.$$

The firm's optimal demand for permits is given by:

$$\kappa^j(w^j, p_\kappa^j; \mu^j) = \mu^j y_2^j(w^j, p_\kappa^j; \mu^j).$$

Aggregate demand for permits is given by:

$$\int_{\underline{\mu}^j}^{\bar{\mu}^j} \kappa^j(w^j, p_\kappa^j; \mu^j) dG^j = e_y^j(p_\kappa^j, w^j).$$

Household income is given by

$$m^j(p_0^j, w^j, a^j, p_\kappa^j) = w^j L^j + a^j + \pi_0^j(p_0^j, w^j) + \pi_1^j(w^j) + \bar{\pi}_2^j(w^j, p_\kappa^j) \quad j = A, B$$

where

$$\begin{aligned} \frac{\partial m^j(\cdot)}{\partial p_0^j} &= \frac{\partial \pi_0^j(\cdot)}{\partial p_0^j} > 0, & \frac{\partial m^j(\cdot)}{\partial w^j} &= L^j + \frac{\partial \pi_0^j(\cdot)}{\partial w^j} + \frac{\partial \pi_1^j(\cdot)}{\partial w^j} + \frac{\partial \bar{\pi}_2^j(\cdot)}{\partial w^j} = L^j - \ell_0^j - \bar{\ell}_1^j - \ell_2^j, \\ \frac{\partial m^j(\cdot)}{\partial a^j} &= 1, & \frac{\partial m^j(\cdot)}{\partial p_\kappa^j} &= \frac{\partial \bar{\pi}_2^j(\cdot)}{\partial p_\kappa^j} < 0, \end{aligned}$$

and household demands are

$$x_i^j(p_0^j, w^j, \tau_x^j, p_\kappa^j, a^j) = x_i^j(p_0^j, \tau_x^j, m^j(p_0^j, w^j, a^j, p_\kappa^j)). \quad i = 0, 1, 2$$

Totally differentiating household demand for the non-traded good,

$$x_0^j(p_0^j, \tau_x^j, m^j(p_0^j, w^j, a^j, p_\kappa^j)) = \underline{x}_0 + \frac{\beta_0^j}{p_0^j} (m^j(p_0^j, w^j, a^j, p_\kappa^j) - p_0^j \underline{x}_0 - p_1 \underline{x}_1 - (p_2 + \tau_x^j \phi^j) \underline{x}_2),$$

yields (in equilibrium)

$$\begin{aligned} \frac{dx_0^j}{dp_0^j} &= \frac{\partial x_0^j(\cdot)}{\partial p_0^j} + \frac{\partial x_0^j(\cdot)}{\partial m^j} \frac{\partial m^j(\cdot)}{\partial p_0^j} = - \frac{\beta_0^j}{(p_0^j)^2} \left(p_1 (x_1^j - \underline{x}_1) + (p_2 + \tau_x^j \phi^j) (x_2^j - \underline{x}_2) \right) < 0, \\ \frac{dx_0^j}{dw^j} &= \frac{\partial x_0^j(\cdot)}{\partial m^j} \frac{\partial m^j(\cdot)}{\partial w^j} = 0, & \frac{dx_0^j}{d\tau_x^j} &= \frac{\partial x_0^j(\cdot)}{\partial \tau_x^j} = - \frac{\beta_0^j}{p_0^j} \phi^j \underline{x}_2 < 0, \end{aligned}$$

$$\frac{dx_0^j}{da^j} = \frac{\partial x_0^j(\cdot)}{\partial m^j} \frac{\partial m^j(\cdot)}{\partial a^j} = \frac{\beta_0^j}{p_0^j} > 0, \quad \frac{dx_0^j}{dp_\kappa^j} = \frac{\partial x_0^j(\cdot)}{\partial m^j} \frac{\partial m^j(\cdot)}{\partial p_\kappa^j} = -\frac{\beta_0^j}{p_0^j} e_y^j < 0.$$

The price of the emission permits is treated as an artificial variable in the regional government's problem subject to the permit-marketing clearing condition. Therefore, in determining the equilibrium price of the nontraded good and the wage rate, the emission permit price is taken as given. Focusing on region j and suppressing the regional superscript, the market-clearing conditions are

$$D(p_0, w, p_\kappa) \equiv \ell_0(p_0, w) + \ell_1(w) + \bar{\ell}_2(w, p_\kappa) - L = 0$$

$$X(p_0, w, a, \tau_x, p_\kappa) \equiv x_0(p_0, \tau_x, m(p_0, w, a, p_\kappa)) - y_0(p_0, w) = 0$$

where

$$\begin{aligned} D_{p_0} &= \frac{\partial \ell_0}{\partial p_0} > 0, \quad D_w = \frac{\partial \ell_0}{\partial w} + \frac{\partial \ell}{\partial w} + \frac{\partial \bar{\ell}_2}{\partial w} < 0, \quad D_{p_\kappa} = \frac{\partial \bar{\ell}_2}{\partial p_\kappa} < 0, \\ X_{p_0} &= \frac{dx_0}{dp_0} - \frac{\partial y_0}{\partial p_0} = -\frac{\beta_0}{p_0^2} (p_1(x_1 - \underline{x}_1) + (p_2 + \tau_x \phi)(x_2 - \underline{x}_2)) - \frac{\partial y_0}{\partial p_0} < 0, \\ X_w &= -\frac{\partial y_0}{\partial w} > 0, \quad X_a = \frac{dx_0}{da} = \frac{\beta_0}{p_0} > 0, \\ X_{\tau_x} &= \frac{dx_0}{d\tau_x} = -\frac{\beta_0}{p_0} \phi \underline{x}_2 < 0, \quad X_{p_\kappa} = \frac{dx_0}{dp_\kappa} = -\frac{\beta_0}{p_0} e_y < 0. \end{aligned}$$

The market-clearing conditions implicitly define $w(a, \tau_x, p_\kappa)$ and $p_0(a, \tau_x, p_\kappa)$ and total differentiation yields

$$\begin{bmatrix} D_{p_0} & D_w \\ X_{p_0} & X_w \end{bmatrix} \begin{bmatrix} dp_0 \\ dw \end{bmatrix} = \begin{bmatrix} 0 & 0 & -D_{p_\kappa} \\ -X_a & -X_{\tau_x} & -X_{p_\kappa} \end{bmatrix} \begin{bmatrix} db \\ d\tau_x \\ dp_\kappa \end{bmatrix}$$

Define

$$\begin{aligned} |Det| &= D_{p_0} X_w - D_w X_{p_0} \\ &= -\frac{\partial \ell_0}{\partial p_0} \frac{\partial y_0}{\partial w} - \left[\frac{\partial \ell_0}{\partial w} + \frac{\partial \ell_1}{\partial w} + \frac{\partial \bar{\ell}_2}{\partial w} \right] \left(-\frac{\beta_0}{p_0^2} (p_1(x_1 - \underline{x}_1) + (p_2 + \tau_x \phi)(x_2 - \underline{x}_2)) \right) \\ &\quad + \left[\frac{\partial \ell_0}{\partial w} + \frac{\partial \ell_1}{\partial w} + \frac{\partial \bar{\ell}_2}{\partial w} \right] \frac{\partial y_0}{\partial p_0} \\ &= \left[\frac{\partial \ell_0}{\partial w} + \frac{\partial \ell_1}{\partial w} + \frac{\partial \bar{\ell}_2}{\partial w} \right] \left(\frac{\beta_0}{p_0^2} (p_1(x_1 - \underline{x}_1) + (p_2 + \tau_x \phi)(x_2 - \underline{x}_2)) \right) + \left[\frac{\partial \ell_1}{\partial w} + \frac{\partial \bar{\ell}_2}{\partial w} \right] \frac{\partial y_0}{\partial p_0} < 0 \end{aligned}$$

where the second equality follows from

$$\frac{\partial \ell_0}{\partial p_0} \frac{\partial y_0}{\partial w} = f_0'(\ell_0) \frac{\partial \ell_0}{\partial p_0} \frac{\partial \ell_0}{\partial w} = \frac{\partial \ell_0}{\partial w} \frac{\partial y_0}{\partial p_0}.$$

Use Cramer's Rule to obtain

$$\begin{aligned} \frac{dp_0}{da} &= \frac{\begin{vmatrix} 0 & D_w \\ -X_a & X_w \end{vmatrix}}{|Det|} = \frac{D_w X_a}{|Det|} > 0, \\ \frac{dp_0}{d\tau_x} &= \frac{\begin{vmatrix} 0 & D_w \\ -X_{\tau_x} & X_w \end{vmatrix}}{|Det|} = \frac{D_w X_{\tau_x}}{|Det|} < 0, \quad \frac{dp_0}{dp_\kappa} = \frac{\begin{vmatrix} -D_{p_\kappa} & D_w \\ -X_{p_\kappa} & X_w \end{vmatrix}}{|Det|} = \frac{-D_{p_\kappa} X_w + D_w X_{p_\kappa}}{|Det|} < 0, \\ \frac{dw}{da} &= \frac{\begin{vmatrix} D_{p_0} & 0 \\ X_{p_0} & -X_a \end{vmatrix}}{|Det|} = \frac{-D_{p_0} X_a}{|Det|} > 0, \\ \frac{dw}{d\tau_x} &= \frac{\begin{vmatrix} D_{p_0} & 0 \\ X_{p_0} & -X_{\tau_x} \end{vmatrix}}{|Det|} = \frac{-D_{p_0} X_{\tau_x}}{|Det|} < 0, \quad \frac{dw}{dp_\kappa} = \frac{\begin{vmatrix} D_{p_0} & -D_{p_\kappa} \\ X_{p_0} & -X_{p_\kappa} \end{vmatrix}}{|Det|} = \frac{-D_{p_0} X_{p_\kappa} + D_{\tau_y} X_{p_0}}{|Det|} < 0, \end{aligned}$$

Given $p_0(a, \tau_x, p_\kappa)$ and $w(a, \tau_x, p_\kappa)$, we can determine the amount of emissions in equilibrium as a function of the regional government policies:

$$\begin{aligned} e_y(w(a, \tau_x, p_\kappa), p_\kappa) &= \int_{\underline{\mu}}^{\bar{\mu}} \mu y_2(w(a, \tau_x, p_\kappa), p_\kappa; \mu) dG \\ e_x(p_0(a, \tau_x, p_\kappa), \tau_x) &= \phi x_2(p_0(a, \tau_x, p_\kappa), \tau_x) \end{aligned}$$

where

$$\begin{aligned} \frac{de_y(\cdot)}{da} &= \underbrace{\left(\int_{\underline{\mu}}^{\bar{\mu}} \mu \frac{\partial y_1(w, p_\kappa; \mu)}{\partial w} dG \right)}_{(-)} \underbrace{\frac{dw}{da}}_{(+)} < 0, \quad \frac{de_y(\cdot)}{d\tau_x} = \underbrace{\left(\int_{\underline{\mu}}^{\bar{\mu}} \mu \frac{\partial y_1(w, p_\kappa; \mu)}{\partial w} dG \right)}_{(-)} \underbrace{\frac{dw}{d\tau_x}}_{(-)} > 0, \\ \frac{de_y(\cdot)}{dp_\kappa} &= \underbrace{\left(\int_{\underline{\mu}}^{\bar{\mu}} \mu \frac{\partial y_1(w, p_\kappa; \mu)}{\partial w} dG \right)}_{(-)} \underbrace{\frac{dw}{dp_\kappa}}_{(-)} + \underbrace{\left(\int_{\underline{\mu}}^{\bar{\mu}} \mu \frac{\partial y_1(w, p_\kappa; \mu)}{\partial p_\kappa} dG \right)}_{(-)} \geq 0, \\ \frac{de_x(\cdot)}{da} &= \underbrace{\phi \frac{\partial x_1}{\partial p_0}}_{(+)} \underbrace{\frac{dp_0}{da}}_{(+)} > 0, \quad \frac{de_x(\cdot)}{d\tau_x} = \underbrace{\phi \frac{\partial x_1}{\partial p_0}}_{(+)} \underbrace{\frac{dp_0}{d\tau_x}}_{(-)} + \underbrace{\phi \frac{\partial x_1}{\partial \tau_x}}_{(-)} < 0, \quad \frac{de_x(\cdot)}{dp_\kappa} = \underbrace{\phi \frac{\partial x_1}{\partial p_0}}_{(+)} \underbrace{\frac{dp_0}{dp_\kappa}}_{(-)} < 0. \end{aligned}$$

In equilibrium, the indirect utility is $V(a, \tau_x, p_\kappa) \equiv V(p_0(\cdot), \tau_x, m(p_0(\cdot), w(\cdot), a, p_\kappa))$, where

$$V_a = V_m, \quad V_{\tau_x} = -V_m e_x, \quad V_{p_\kappa} = -V_m e_y$$

since there are no general equilibrium effects on household's maximized utility in equilibrium, that is,

$$\begin{aligned} \frac{dV(\cdot)}{dp_0} &= \frac{\partial V(\cdot)}{\partial p_0} + \frac{\partial V(\cdot)}{\partial m} \frac{\partial m(\cdot)}{\partial p_0} = -V_m x_0 + V_m y_0 = 0, \\ \frac{dV(\cdot)}{dw} &= \frac{\partial V(\cdot)}{\partial m} \frac{\partial m(\cdot)}{\partial w} = V_m (L - \ell_0 - \ell_1 - \bar{\ell}_2) = 0. \end{aligned}$$

The regional government's problem is

$$\begin{aligned} \max_{a, \tau_x, p_\kappa, \bar{\kappa}} \quad & V(a, \tau_x, p_\kappa) - s \left(e_x(a, \tau_x, p_\kappa) + e_y(a, \tau_x, p_\kappa), e^{-j} \right) \\ & + \lambda \left((\tau_x - \gamma_x) e_x(a, \tau_x, p_\kappa) + (p_\kappa - \gamma_y) \bar{\kappa} - a \right) + \delta (e_y(a, \tau_x, p_\kappa) - \bar{\kappa}). \end{aligned}$$

The first-order conditions are:

$$\begin{aligned} V_a - s_e \frac{de}{da} + \lambda \left((\tau_x - \gamma_x) \frac{de_x}{da} - 1 \right) + \delta \frac{de_y}{da} &= 0, \\ V_{\tau_x} - s_e \frac{de}{d\tau_x} + \lambda \left(e_x + (\tau_x - \gamma_x) \frac{de_x}{d\tau_x} \right) + \delta \frac{de_y}{d\tau_x} &= 0, \\ V_{p_\kappa} - s_e \frac{de}{dp_\kappa} + \lambda \left((\tau_x - \gamma_x) \frac{de_x}{dp_\kappa} + \bar{\kappa} \right) + \delta \frac{de_y}{dp_\kappa} &= 0, \\ \lambda (p_\kappa - \gamma_y) - \delta &= 0. \end{aligned}$$

Substituting out δ using the first-order condition on $\bar{\kappa}$, setting $\bar{\kappa} = e_y$, and using $V_{\tau_x} = -V_a e_x$ and $V_{p_\kappa} = -V_a e_y$, the first-order conditions on (a, τ_x, p_κ) reduce to:

$$\begin{aligned} V_a - s_e \frac{de}{da} + \lambda \left((\tau_x - \gamma_x) \frac{de_x}{da} + (p_\kappa - \gamma_y) \frac{de_y}{da} - 1 \right) &= 0, \\ -V_a e_x - s_e \frac{de}{d\tau_x} + \lambda \left(e_x + (\tau_x - \gamma_x) \frac{de_x}{d\tau_x} + (p_\kappa - \gamma_y) \frac{de_y}{d\tau_x} \right) &= 0, \\ -V_a e_y - s_e \frac{de}{dp_\kappa} + \lambda \left((\tau_x - \gamma_x) \frac{de_x}{dp_\kappa} + (p_\kappa - \gamma_y) \frac{de_y}{dp_\kappa} + e_y \right) &= 0. \end{aligned}$$

Next, substituting the expression for λ from the first-order condition on a into the first-order conditions on τ_x and p_κ cross-multiplying and simplifying the resulting expressions, the first-order conditions on τ_x and p_κ can be written as:

$$(V_a(\tau_x - \gamma_x) - s_e) \left(e_x \frac{de_x}{da} + \frac{de_x}{d\tau_x} \right) + (V_a(p_\kappa - \gamma_y) - s_e) \left(e_x \frac{de_y}{da} + \frac{de_y}{d\tau_x} \right)$$

$$\begin{aligned}
& +s_e(p_\kappa - \gamma_y - (\tau_x - \gamma_x)) \left(\frac{de_x}{d\tau_x} \frac{de_y}{da} - \frac{de_y}{d\tau_x} \frac{de_x}{da} \right) = 0, \\
& (V_a(p_\kappa - \gamma_y) - s_e) \left(e_y \frac{de_y}{da} + \frac{de_y}{dp_\kappa} \right) + (V_a(\tau_x - \gamma_x) - s_e) \left(e_y \frac{de_x}{da} + \frac{de_x}{dp_\kappa} \right) \\
& +s_e(\tau_x - \gamma_x - (p_\kappa - \gamma_y)) \left(\frac{de_y}{dp_\kappa} \frac{de_x}{da} - \frac{de_y}{da} \frac{de_x}{dp_\kappa} \right) = 0.
\end{aligned}$$

These two first-order conditions can then be written in the following form:

$$\alpha_x(\tau_x - \gamma_x) + \alpha_y(p_\kappa - \gamma_y) = \alpha,$$

$$\beta_x(\tau_x - \gamma_x) + \beta_y(p_\kappa - \gamma_y) = \beta.$$

Applying Cramer's rule yields:

$$\begin{aligned}
\tau_x - \gamma_x &= \frac{\alpha\beta_y - \alpha_y\beta}{\alpha_x\beta_y - \alpha_y\beta_x}, \\
p_\kappa - \gamma_y &= \frac{\alpha_x\beta - \alpha\beta_x}{\alpha_x\beta_y - \alpha_y\beta_x}.
\end{aligned}$$

It is straightforward to see that $\alpha\beta_y - \alpha_y\beta = \alpha_x\beta - \alpha\beta_x$ and there is unique solution to the two first-order conditions in which $\tau_x - \gamma_x = p_\kappa - \gamma_y$. Substituting this into the first-order conditions on τ_x , p_κ and a , it follows that at the optimum,

$$\tau_x - \gamma_x = p_\kappa - \gamma_y = \frac{s_e}{\lambda}, \quad V_a = \lambda.$$

Remark 1.2: Regional governments tax production emissions

Rather than selling production emission permits to firms suppose the region j government imposes a unit production emission tax τ_y^j . The producer problems for the non-traded and clean traded good and the household's problem are unchanged. The first-order condition of the regional good 2 producers will now be $f_2^{j'}(\ell_2^j)(p_2 - \tau_y^j \mu^j) - w^j = 0$ which determines labour demand as a function of the production emissions tax. Consequently, aggregate labour demand, aggregate output, aggregate profits and total emissions of domestic producers of good 2 will depend on the emissions tax in the same way they depended on the price of emission permits in the above analysis. Further Walras' law continues to hold nationwide. Therefore, the national equilibrium will be the same with p_κ^j replaced by τ_y^j , and the regional government's problem objective can be written as

$$\mathcal{L}^j(a^j, \tau_x^j, \tau_y^j) = V^j(a^j, \tau_x^j, \tau_y^j) - s^j \left(e_x^j(a^j, \tau_x^j, \tau_y^j) + e_y^j(a^j, \tau_x^j, \tau_y^j), e^{-j} \right)$$

$$+\lambda^j \left((\tau_x^j - \gamma_x^j) e_x^j(a^j, \tau_x^j, \tau_y^j) + (\tau_y^j - \gamma_y^j) e_y^j(a^j, \tau_x^j, \tau_y^j) - a^j \right). \quad j = A, B$$

The first-order conditions of this problem are identical to the conditions above with p_κ^j replaced by τ_y^j , and as shown above the unique solution is

$$\tau_x^j - \gamma_x^j = \tau_y^j - \gamma_y^j = \frac{s_e^j}{\lambda^j}, \quad V_a^j = \lambda^j.$$

Section 4.1: Nash equilibrium

The expression for the determinant is given by

$$\begin{aligned} |D| = & - \left(e + (\tau_x - \gamma_x) \frac{de_x}{d\tau_x} + (\tau_y - \gamma_y) \frac{de_y}{d\tau_x} \right) \left(s_e \frac{dp_0}{db} + p_0 s_{ee} \frac{de}{db} \right) \\ & - \left((\tau_x - \gamma_x) \frac{de_x}{db} + (\tau_y - \gamma_y) \frac{de_y}{db} - 1 \right) \left(1 - s_e \frac{dp_0}{d\tau_x} - p_0 s_{ee} \left(\frac{de_x}{d\tau_x} + \frac{de_y}{d\tau_x} \right) \right). \end{aligned}$$

Given $\tau_x - \gamma_x = \tau_y - \gamma_y$ the above expression reduces to

$$|D| = - \left(e + (\tau_x - \gamma_x) \frac{de}{d\tau_x} \right) s_e \frac{dp_0}{db} - p_0 s_{ee} \frac{de}{db} + \left(1 - (\tau_x - \gamma_x) \frac{de}{db} \right) \left(1 - s_e \frac{dp_0}{d\tau_x} \right) - p_0 s_{ee} \frac{de}{d\tau_x}.$$

Section 5.1: Federal government policies

The first-order conditions with respect to a^j , τ_x^j and τ_y^j are

$$\begin{aligned} V_a^j - \sigma' \frac{de^j}{da^j} + \lambda^F \left((\tau_x^j - \gamma_x) \frac{de_x^j}{da^j} + (\tau_y^j - \gamma_y) \frac{de_y^j}{da^j} - 1 \right) &= 0, \quad j = A, B \\ V_{\tau_x}^j - \sigma' \frac{de^j}{d\tau_x^j} + \lambda^F \left(e_x^j + (\tau_x^j - \gamma_x) \frac{de_x^j}{d\tau_x^j} + (\tau_y^j - \gamma_y) \frac{de_y^j}{d\tau_x^j} \right) &= 0, \quad j = A, B \\ V_{\tau_y}^j - \sigma' \frac{de^j}{d\tau_y^j} + \lambda^F \left(e_y^j + (\tau_x^j - \gamma_x) \frac{de_x^j}{d\tau_y^j} + (\tau_y^j - \gamma_y) \frac{de_y^j}{d\tau_y^j} \right) &= 0. \quad j = A, B \end{aligned} \quad (64)$$

To determine optimal federal emissions taxes, we follow the same procedure as in the regional government case. Solving for λ^F from the first-order conditions on a^j :

$$\lambda^F = \frac{V_a^j - \sigma'(e^A + e^B) \frac{de^j}{da^j}}{1 - (\tau_x - \gamma_x) \frac{de_x^j}{da^j} - (\tau_y - \gamma_y) \frac{de_y^j}{da^j}}. \quad j = A, B$$

Substituting the expression for λ^F into the first-order conditions on τ_x^j and τ_y^j in (64), and the cross-multiplying and simplifying yields:

$$(V_m^j(\tau_x^j - \gamma_x) - \sigma') \left(e_x^j \frac{de_x^j}{da^j} + \frac{de_x^j}{d\tau_x^j} \right) + (V_m^j(\tau_y^j - \gamma_y) - \sigma') \left(e_x^j \frac{de_y^j}{da^j} + \frac{de_y^j}{d\tau_x^j} \right)$$

$$\begin{aligned}
& +\sigma'(\tau_y^j - \gamma_y - (\tau_x^j - \gamma_x)) \left(\frac{de_x^j}{d\tau_x^j} \frac{de_y^j}{da^j} - \frac{de_y^j}{d\tau_x^j} \frac{dev_x}{da^j} \right) = 0, \\
& (V_m^j(\tau_y^j - \gamma_y) - \sigma') \left(e_y^j \frac{de_y^j}{da^j} + \frac{de_y^j}{d\tau_y^j} \right) + (V_m^j(\tau_x^j - \gamma_x) - \sigma') \left(e_y^j \frac{de_x^j}{da^j} + \frac{de_x^j}{d\tau_y^j} \right) \\
& +\sigma'(\tau_x^j - \gamma_x - (\tau_y^j - \gamma_y)) \left(\frac{de_y^j}{d\tau_y^j} \frac{de_x^j}{da^j} - \frac{de_y^j}{da^j} \frac{de_x^j}{d\tau_y^j} \right) = 0.
\end{aligned}$$

These two first-order conditions can then be written in the following form:

$$\alpha_x(\tau_x^j - \gamma_x) + \alpha_y(\tau_y^j - \gamma_y) = \alpha,$$

$$\beta_x(\tau_x^j - \gamma_x) + \beta_y(\tau_y^j - \gamma_y) = \beta.$$

Applying Cramer's rule obtains:

$$\tau_x^j - \gamma_x = \frac{\alpha\beta_y - \alpha_y\beta}{\alpha_x\beta_y - \alpha_y\beta_x},$$

$$\tau_y^j - \gamma_y = \frac{\alpha_x\beta - \alpha\beta_x}{\alpha_x\beta_y - \alpha_y\beta_x}.$$

It is straightforward to see that $\alpha\beta_y - \alpha_y\beta = \alpha_x\beta - \alpha\beta_x$ and there is unique solution to the two first-order conditions in which $\tau_x^j - \gamma_x = \tau_y^j - \gamma_y$. Substituting this into the first-order conditions on τ_x^j , τ_y^j and a^j , it follows that at the optimum,

$$\tau_x^j - \gamma_x = \tau_y^j - \gamma_y = \frac{\sigma'}{\lambda^F}, \quad \lambda^F = V_a^j. \quad j = A, B$$

Section 6.2: Federal government moves second

The regional budget constraints determine a^j as a function of t^j for a given set of emission taxes τ_x^j and τ_y^j , or $a^j(t^j; \tau_x^j, \tau_y^j)$, where for $j = A, B$:

$$\begin{aligned}
\frac{\partial a^j(\theta^j; \tau_x^j, \tau_y^j)}{\partial \theta^j} &= \frac{1}{1 - (\tau_x^j - \gamma_x^j) \frac{\partial e_x^j}{\partial a^j} - (\tau_y^j - \gamma_y^j) \frac{\partial e_y^j}{\partial a^j}}, \\
\frac{\partial a^j(\theta^j; \tau_x^j, \tau_y^j)}{\partial \tau_x^j} &= \frac{e_x^j + (\tau_x^j - \gamma_x^j) \frac{\partial e_x^j}{\partial \tau_x^j} + (\tau_y^j - \gamma_y^j) \frac{\partial e_y^j}{\partial \tau_x^j}}{1 - (\tau_x^j - \gamma_x^j) \frac{\partial e_x^j}{\partial a^j} - (\tau_y^j - \gamma_y^j) \frac{\partial e_y^j}{\partial a^j}}, \\
\frac{\partial a^j(\theta^j; \tau_x^j, \tau_y^j)}{\partial \tau_y^j} &= \frac{e_y^j + (\tau_x^j - \gamma_x^j) \frac{\partial e_x^j}{\partial \tau_y^j} + (\tau_y^j - \gamma_y^j) \frac{\partial e_y^j}{\partial \tau_y^j}}{1 - (\tau_x^j - \gamma_x^j) \frac{\partial e_x^j}{\partial a^j} - (\tau_y^j - \gamma_y^j) \frac{\partial e_y^j}{\partial a^j}}.
\end{aligned}$$