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A citizen-candidate model of party formation

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Abstract

There are multiple models of party formation in the political economy literature. However, most of these works consider individuals as parties and do not model parties as a group of candidates. In this paper, we follow the latter approach and assume that parties form as a result of *mutually agreeable* links between candidates. We consider a citizen-candidate model of electoral competition where candidates decide whether they want to participate in election alone or offer *links* to *adjacently* placed candidates on a one-dimensional policy space. We characterize one-party and two-party equilibrium and show that no multi-party equilibrium exists with three or more parties. We provide conditions on the rents of winning with respect to the cost of participating in election which also depend on the number of candidates. We provide new insights which explain party formation from the perspective of group formation. Our results confirm the Duverger's law and are consistent with empirical evidence on plurality voting systems.

Keywords Electoral competition · Party formation · Citizen-candidate model

JEL Classification D70 · D72

1 Introduction

There are two types of models of party formation in the positive political theory literature: one, where parties are treated as unitary players and no distinction is made between a party and its members (Jackson and Moselle 2002; Levy 2004; Callander 2005; Riviere 1999), and the other, where a party is considered to be a set of political

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candidates who commit to a contract in order to share the costs of standing in election or to credibly commit to a wider set of policies. As a result, it is implicitly assumed that the individuals find it optimal to remain within the party given the equilibrium conditions.¹ However, these papers do not model party formation as a result of interaction between candidates agreeing to share rents and costs. We model party formation using the concept of pairwise acceptable links as introduced by Jackson and Wolinsky (1996) for networks.

The importance of social and economic networks in party formation has been noted in the literature on origin of political parties. Parigi and Sartori (2014) finds evidence supporting the interpretation of parties as a network of interconnected individuals for party formation in Italy and Spain. They note the importance of linkages between party members in parliament on the basis of national cleavages. Hopkin and Paolucci (1999) studies the role of social networks in parties like Forza Italia in Italy and Unión de Centro Democrático (UCD) in Spain. Their findings indicate that it is imperative to design a model of electoral competition based on interactions and mutual agreement between party members. However, most of the literature on party formation till now has focused only on inter-party competition without modeling intra-party interaction. Our goal in this paper is to address this lacuna in the literature to some extent.

We assume that voters are potential candidates as in the standard citizen-candidate model by Osborne and Slivinsky (1996). Voters can participate in election on their own or by offering links to other ideologically close voters who may also want to participate in elections.² We use this to analyze the number of parties and the policy outcomes that can exist in equilibrium. This also allows us to test the Duverger's law, which states that in the first-past-the-post plurality voting system, at most two parties can exist in equilibrium.

The notion of pairwise stability is ideal for such settings where a pair of agents may coordinate and deviate when it is profitable. It is not as general as *strong stability* which allows for deviations by any possible group on a network but is more tractable and usually does not suffer from non-existence of equilibrium (Jackson 2008). Moreover, it is realistic since it is much easier to achieve coordination between two agents rather than a group of multiple agents. We briefly describe the features of our model.

We consider a voting model where the policy space is the unit interval, $[0, 1]$. There is a finite set of citizen-candidates who are voters but can also decide to participate in elections. Candidates decide whether or not to participate in elections and also propose 'links' to adjacently placed candidates in order to form parties. If two candidates offer each other a link, then that link forms, i.e., every link must be mutually agreeable. If a candidate chooses not to stand, she remains a voter.³ A candidate may also choose to stand as an independent. A profile of actions leads to a party structure. If a candidate wins the election by obtaining the most number of votes, she receives rents of participation, R . These are shared equally among the winning party members.

¹ Dhillon (2005) provides an extensive survey of the literature on party/coalition formation. Laslier (2005) considers another model of electoral competition with parties as organizations whose members have different goals.

² Isotalo et al. (2020) studies the relationship between ideological positions of members of a party and their electoral preference within the party for Finnish parliamentary elections.

³ In our model, we allow candidates to vote in elections along with non-candidates.

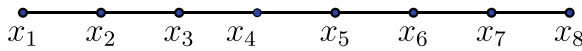


Fig. 1 Distribution of citizen-candidates' policy positions

However, standing in election requires a cost of participation, C . We assume that these are also shared among the party members.⁴ The motivation for participation is strong when $R > C$, but candidates may still want to participate when $R < C$ if winning in election obtains a strictly beneficial policy. Our framework will allow us to study the implications of difference in motivation on party formation.

A party is a set of mutually interlinked candidates who have formed mutually agreeable links. Each party adopts a policy position from the *range* of the party which is the set of policy positions lying between the ideal policy positions of the left and right extreme members of the party. Voting takes place after the policies have been announced and we allow voters to be strategic. The winner of the election is the party whose policy wins the most number of votes and all its members share the rents and costs equally. If there are multiple parties in equilibrium each policy is implemented with equal probability. If no parties form, then the median voter's policy position is implemented.

Another feature of our model is the Party Policy Protocol (PPP). These protocols allow us to define the model generally and get more robust results. They also help in limiting the possible changes in the party's policy position if a member or a pair of members leave the party, or if new members join the party. There are three conditions in the PPP. The first condition of PPP states that the policy position of a party can be any policy in the range of the policy positions of its party members as long as it maximizes the win probability of the party. For example, if $x_1 = 0$ is the policy of the leftmost extreme party member and $x_k = 0.6$ is the ideal policy of the rightmost party member, then the policy position of the party can be any policy in the range of the policy positions of the party, $[x_1, x_k] = [0, 0.6]$ which maximizes the win probability of the party. The second condition pertains to the case where a party member leaves the party by breaking *both her links*. Part (i) of this condition applies to the case when a leaving member of the party leaves the party without breaking the party into two or three new parties. In this case, remaining party's policy position is the closest available policy to the earlier policy position which maximizes the win probability of the party.

In the above case, when one or two members leave the party, it may break the party into two or more connected set of candidates. In this case, PPP allows the two adjacent members to the agent i (who belonged to the same party) to offer links to each other to keep the party connected. For example, if i leaves, and party members, $i - 1$ and $i + 1$ rejoin the party by offering each other links, then the range of the new party continues to be the same and the party can choose the same policy if it maximizes the win probability. We provide an illustrative example (Fig. 1).

Example 1 The policy space is $X = [0, 1]$, and the set of candidates is $N = \{1, 2, \dots, 8\}$. Policy positions $x_i = \frac{i-1}{n-1}$ for any candidate $i \in N$ as shown in Fig. 1. We provide some features of our model:

⁴ Note that the cost is shared irrespective of whether the party wins or not.

- Link formation: Suppose all individuals from 2 to 7 offer links to adjacent candidates on their left and right. Candidate 1 offers a link to 2 and candidate 8 offers a link to candidate 7.
- Party structure: The only party that forms is $P_1 = \{1, 2, \dots, 8\}$. Here, as per the PPP any policy $x \in [0, 1]$ can be assigned as the party's position.
- If 4 leaves the party, PPP allows 3 and 5 to rejoin (as an exception to the rule allowing only adjacent candidates to offer links). The range of the party, therefore, continues to be the same. This prevents some of the deviations in equilibrium.

Part (ii) of the condition is applicable to the case when the party does not break into two or more parties. In such cases as well, the remaining party must continue to select the closest policy position to the older one which maximizes the win probability. Similar conditions apply when one or two members join a party; any policy which maximizes the win probability may be chosen. However, if the old policy is available and still maximizes win probability, then it must continue to be chosen.

We discuss the relevance of PPP for our model. When a party is formed, it comprises of many members. Each member would prefer to have her own policy position as the party position if it can guarantee winning the election. However, we do not explicitly model the bargaining process which would take place within the party to decide the party policy. Our goal in this paper is to characterize the number of parties that can form in equilibrium for a given PPP. In the above example, suppose the party position is $x_{P_1} = x_4$. If party member 5 leaves or breaks the party by breaking her links with 4 and 6, the PPP will allow agents 4 and 6 to rejoin. Suppose they rejoin by offering each other links. Then clearly, by choosing the new party's position as x_4 (since it is in the range of the party's policy positions), it will continue to win the election, irrespective of whether or not agent 5 continues to run in the election alone. This is intuitive and captures what one would expect in real life where the remaining party members have the option of consolidating the party.⁵

To check for equilibrium, we check for all such deviations by individuals or pairs of individuals. If no such deviations are possible, then the given party structure, along with the above mentioned PPP and voting behavior, will be called a political equilibrium. Therefore, there may be multiple PPP which are consistent with the above explanation. Our results will be able to capture all such situations for a party equilibrium.

There are two reasons why parties emerge in this model. The first is that it allows candidates to commit to a policy position which is not their own. In addition to this, each candidate standing in an election has to pay a cost of standing in election. These costs are shared equally in a party. There is a fixed benefit from winning which has to be shared equally among all party members. In the literature, different conditions are obtained on the relationship between these costs and benefits which sustain different party equilibria. However, most of the conditions are obtained as a result of Nash's notion of equilibrium. We will take a more general notion of equilibrium in this paper. It is well known that Nash equilibrium is an unsatisfactory equilibrium notion in network formation models since it only takes into account unilateral deviations by players (Jackson 2008). We, therefore, use a notion, *pairwise stability*, introduced by

⁵ The obvious exception being the case where a party leader breaks the party and takes all her loyalists with her. However, we do not model leadership in this paper.

Jackson and Wolinsky (1996) for networks, modified for our setting. According to this notion, a set of strategies (given a party policy platform vector and an undominated voting vector⁶) is a *political equilibrium* if the following three conditions are met: (i) No individual can unilaterally deviate beneficially by breaking links or by (not) participating, (ii) no pair of candidates can jointly deviate beneficially by forming a new link (thus consolidating a party) and (or) by breaking old links, and (iii) if no other policy position in the range of the party policies can strictly increase the probability of winning. These three conditions must hold for any given undominated voting set of strategies, i.e., if there is one such deviation which, for *some* undominated vote vector, gives the deviating members a strictly higher payoff to the deviating members, then the initial set of strategies is not an equilibrium. This formulation allows for strategic voting which is usually intractable in multi-party settings. This is a crucial aspect for (non-) existence of multi-party equilibria in our model since it often discourages party structures with more than two parties. This is an aspect that has been explained as the ‘psychological effect’ that voters face when anticipating the ‘mechanical effect’ of the plurality rule (Van Der Straeten et al. 2013). This refers to the advantage that bigger parties face in elections. In our model, this translates to ‘more centrally’ located parties, since voters being risk-averse would rather vote strategically for more centrist party positions than vote for their favorite policy positions which have a lower chance of winning. These factors influence the type of equilibria that can exist in our model and play a crucial role while assessing the relevance of Duverger’s law. In our model, since voters are candidates, they internalize these factors while forming parties and this translates to non-existence of multi-party equilibria as predicted by Duverger. We briefly describe our results.

In the one-party equilibrium, the single party consists of the unique median voter when the number of voters is odd and consists of only the two median voters when the number of voters is even. The intuition for this equilibrium relies on the fact that no other voter can defeat the party consisting of the median(s) by entering independently or in pairs since the median party will obtain a strict majority from one side of the median policy position. Moreover, if a party consists of candidates other than the median voter, the last but one extreme member of the party can break the link with the extreme adjacent party member and continue to win the election. This increases the overall payoff and is a strictly beneficial deviation. This result is consistent with the result obtained in the coalition formation literature where the median voter plays a crucial role (Bandyopadhyay and Oak 2022 and Dhillon 2005).

Our next result shows that no two-party equilibrium can exist in general, unless a specific party policy protocol is declared by the party concerned, in which case both the medians participating independently is an equilibrium when the number of voters is even and the cost is less than the distance between adjacent ideal policy positions. To prove this, we first show that a two-party equilibrium is only possible if the number of voters is even. Moreover, in such an equilibrium the two median voters stand alone in equilibrium. The rationale for this equilibrium is that if any party consists of more members on the left of the left-median candidate, the second to one extreme left

⁶ A voting strategy s_i is undominated if it always gives a weakly higher payoff compared to any other strategy.

candidate will sever her link with the extreme candidate. There will be undominated voting strategies where the party whose member severed the link continues to win, thus making the deviation beneficial. Further arguments show that the two medians would want to join to form a single party without the specified party protocol. A basic intuition is that if they join, their rents would not go down but costs would. Moreover, the concavity of the utility function over policy outcomes ensures that sharing a certainty mean policy is better than a lottery over each favorite policy. Therefore, the only possible protocol which prevents the deviation is where one or both of the median voters do not want to change their announced party policy position. This is allowed in our definition of equilibrium but goes against standard rationality. The rationale for this could be behavioral: Since both the medians win half the votes, they do not want to compromise and would prefer to stick to their policy positions even though choosing the mean policy would be beneficial for both.⁷ Our final result extends this and shows that no multi-party equilibria can exist. Therefore, we are able to verify Duverger's law.

Our results are also consistent with the empirical evidence on the effective number of parties at the district or constituency level in various countries like Canada, Great Britain, India and the USA with the plurality system of voting. Chhibber and Kollman (2009) finds that the effective number of parties at the constituency level is around two for most countries with first-past-the-post voting rules. However, as observed in India, this translates to a greater number of parties at the country level. This empirical evidence is also seen as a validation of Duverger's law.

Our results point to a stronger tendency to form groups when individuals are strategic and only a few players have the bargaining power (here, the median voters) as predicted by the literature. In most cases, the median alone can win the election, so it is optimal for her to not share the rents with anyone else. If a bigger party forms, then the second most leftmost or rightmost extreme party members have incentives to sever links with the adjacently located more extreme party members since it does not affect their probability of winning (under strategic voting). We also find that individuals other than the medians can stand alone in equilibrium when the costs of standing in election are strictly higher than the rents of winning. This is a novel result which has not been captured in any of the works mentioned above. An intuition for why this equilibrium exists is that the median voter does not find it worthwhile to enter the election and win. For high enough costs, the additional benefit of obtaining a policy outcome closer to her own preferred policy is not worth the additional cost.

Levy (2004) finds that parties are not effective in one-dimensional models of electoral competition, where effectiveness may be defined as the ability to choose non-median policies in equilibrium. We find exceptions to this result in our model when the costs of participation are strictly greater than the rents. Specifically, we find that in the one-party equilibrium, there are cases where a 'moderately' extreme candidate away from the median voter may be the only candidate standing in election. In such cases, the median voter may decide to stay out of electoral competition since the policy difference is not

⁷ Another situation such a two-party equilibrium could have been sustained would be when $C = 0$ and candidates do not care about policy. There are numerous works which model electoral competition with *policy-motivated* candidates/parties—Wittman (1983), Duggan and Fey (2005), Peress (2010) and Casamatta and De Donder (2005), etc.

worth the additional cost. In such cases, we find that non-median policy outcomes are implemented. This is a novel result in the party formation literature and is also consistent with empirical evidence in developing countries where costs and risks of political participation are high and policy outcomes are more extreme (Chauvet and Collier 2009).

Osborne and Tourky (2008) provides another model of party formation in the one-dimensional policy space. However, there are some crucial differences between our model and theirs. Firstly, in our model the policy positions of the candidates are fixed and candidates form parties to commit to a wider set of policies, while in their model candidates can choose any policy position. Secondly, in our model parties are formed as a result of mutually agreeable links while in Osborne and Tourky (2008) parties are seen as a set of individuals opting for the same policy position to save costs.

Morelli (2004) considers a model of party formation across multiple districts with a maximum of three possible parties. It compares the electoral outcomes between plurality rule and proportional representation and finds that under certain symmetry assumptions on the distribution of voter preferences the number of effective parties is larger under proportional representation.⁸ However, they find that under plurality voting only the median candidate stands in election. In our model, we are able to capture a wider set of outcomes in equilibrium, i.e., when the costs of participation are high, a moderately extreme candidate may be the only candidate standing in election. There are many works on coalition formation which take a cooperative game theoretic approach. In most of these papers, the payoff functions are given as a function of different groups that can form (see Ray 2007 for an authoritative exposition of the coalition formation literature). Shenoy (1979) considers an n -person cooperative game theoretic model of coalition formation and provides conditions for the existence of the core. Bhattacharya (2014) considers a game theoretic model of group formation with information sharing. This paper considers a model where agents form groups in order to share private information but face a trade-off in the ability to take individual actions. However, our approach in this paper is to model party formation as a non-cooperative game. This allows us to study party formation from the perspective of the strategic agent in a Hotelling–Downs game.

Our paper adopts the citizen-candidate model of Osborne and Slivinsky (1996) (henceforth, OS) to examine the features of electoral competition in a model of party formation. Our model can also be thought as party (network) formation in the background of the OS model. However, none of the papers mentioned above model the interactions between potential members of a party in the process of party formation. Therefore, we obtain different results for policy outcomes and the number of parties in equilibrium. Table 1 provides the comparative results on party formation between our paper and OS. The most crucial difference that we observe in our model compared to their paper is the non-existence of multi-party equilibrium. This occurs in our model due to the generalization of the party formation process and the fact that candidates are

⁸ This is the Duverger's hypothesis (Duverger 1954). Duverger's law states that under the plurality rule the number of effective parties must be at most two. The Duverger's hypothesis states that proportional representation tends to favor multi-party systems.

Table 1 Comparison of the results between our model and Osborne and Slivinsky (1996)

	Our party formation model	Osborne and Slivinsky (1996)'s citizen-candidate model
<i>One-party</i>		
Existence	Yes	Yes
$R \geq C$	Party consists of only the median(s), $C \geq \frac{1}{(n-1)^2}$	$R \leq 2C$, participating candidate is the median
$R < C$	Party can consist of single non-median candidates, or two members including the median (n odd), or the two medians in one party (n even), additional conditions required	Participating candidate within $(C - R)/2$ distance of the median
<i>Two-party</i>		
Existence	Yes	Yes
$R \geq C$	Only if n is even, additional conditions required	Yes. $R \geq 2(C - e_p)$, where $e_p > 0$ is the critical distance around the median within which every entering candidate loses
$R < C$	Only if n is even, additional conditions required	Yes, $R \geq 2(C - e_p)$, where $e_p > 0$ is the critical distance around the median within which every entering candidate loses
<i>Three-party</i>		
Existence	No	Yes, many multi-party equilibria

R and C denote the rents of winning the election and the cost of participation, respectively

allowed to form links. Therefore, our results can also be seen as a proof of Duverger's law.⁹ We provide a discussion in Table 1.

The conditions for existence of one-party results are different for the two models: In our model, the cost of participation depends on the number of citizens in the model and must be in the required range of values. In the OS model, the distribution of voters is continuous, but the cost must be greater than half the rents of participation, R . In both the models, the cost of participation, C , must be high enough to deter other candidates from entering. There are similarities in the two models, in the sense that when $R \geq C$, then the participating candidate(s) is (are) the median(s), and if $R \leq C$, then there exist one-candidate or one-party equilibrium where non-median candidates may participate in equilibrium. If $C \leq R \leq 2C$, then the median candidate participates in equilibrium, while if $R < C$, then a candidate within the distance $(C - R)/2$ of the median stands in election. For the two-party equilibrium, two party can only form in our model when the number of voters is even. Moreover, the cost of participation, C , must be in a specific range for the two median voters participating separately to be an equilibrium in our model. In the OS model, similar conditions are required for the two candidates on either side of the median to be participating. However, since

⁹ Palfrey (1988) also provides a proof of Duverger's law in an electoral competition setting with three candidates.

the distribution of candidates is continuous, the distance between the two candidates cannot be too large. Therefore, the variable e_p denotes the critical distance from the median within which any entering candidate will lose the election. Further, multi-party equilibria are possible in the OS model, whereas no such equilibria exist in our model. We therefore skip the details of such equilibria in the OS model.

This paper is organized as follows. Section 2 sets out the model and the equilibrium concept. Section 3 contains the results for different relationships between R and C . In Sect. 3.1, we look at the case when $R < C$. We provide a short discussion on the results and policy outcomes in our model in Sect. 3.2. Section 4 provides the conclusion.

2 The model

The policy space $X = [0, 1]$ is an interval of the real line \mathbb{R} . The set of finite citizen-candidates is $N = \{1, 2, \dots, n\}$. Candidates' ideal policy positions or *ideal points* are ordered, $x_1 < x_2 < \dots < x_n$, and are uniformly distributed over X with $x_i = \frac{i-1}{n-1}$ for all $i \in \{1, 2, \dots, n\}$.¹⁰ Note that $|x_{i+1} - x_i| = \frac{1}{n-1}$ for all $i \in \{1, \dots, n-1\}$.

Candidates have *single-peaked preferences* on X represented by utility functions $u_i : X \rightarrow \mathbb{R}$ for $i \in N$. We assume that $u_i(x) = -|x - x_i|^2$ for all $x \in X$. Two distinct candidates i and j are *adjacent* if either $j = i + 1$ or $j = i - 1$. If the former holds, we will say that j is the right-adjacent to i , and if the latter holds, then j is left-adjacent to i .

Strategies of the players: There are two types of strategies that the candidates have: (i) to offer links or decide how to run for election (if at all) as candidates and (ii) how to vote (as citizens). Note that candidates continue to vote as citizens even if they belong to a party. In this model, we restrict attention to the case where candidates can offer links to only adjacent candidates. This keeps the model tractable and also allows for a wide range of equilibria. We list the possible set of actions separately:

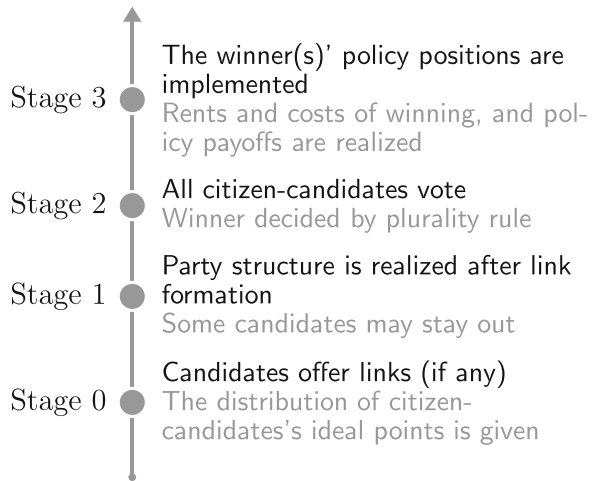
Offering links: There are five types of participatory actions that a citizen-candidate i can take:

- (i) $s_i = a^\pm$, i.e., offering links to both left-adjacent and right-adjacent candidates (if they exist), $i - 1$ and $i + 1$, respectively,
- (ii) $s_i = a^+$, i.e., offering a link to only the right-adjacent candidate (if it exists), $i + 1$,
- (iii) $s_i = a^-$, i.e., offering a link to only the left-adjacent candidate (if it exists), $i - 1$,
- (iv) $s_i = a^0$, i.e., standing for election as an independent candidate/party,
- (v) $s_i = a^\emptyset$, i.e., deciding not to stand for election.

Therefore, every voter $i \in N$ has a strategy space $S_i = \{a^\pm, a^+, a^-, a^0, a^\emptyset\}$. Let $\mathcal{S} = \prod_{i \in N} S_i$ and let $s = (s_1, \dots, s_n)$ denote a link strategy profile in \mathcal{S} . Each citizen-candidate must choose one of the above actions. Given a tuple, (s_1, \dots, s_n) , of actions, we can define a party structure as follows.

Party structure: Links are assumed to be *mutually acceptable*, i.e., link $\{i, i + 1\}$ is formed if and only if $s_{i+1} \in \{a^\pm, a^-\}$ and $s_i \in \{a^\pm, a^+\}$ for all $i \in \{1, 2, \dots, n - 1\}$. In other words, a link is formed between two adjacent candidates only if both

¹⁰ This is a simplifying assumption, and it does not affect the qualitative results on the number of parties that can be supported in equilibrium.

Fig. 2 Timeline of the game

offer links to each other.¹¹ Let $L(s)$ be the set of all links formed under $s \in \mathcal{S}$. A set of candidates $\{i^1, \dots, i^q\}$ are called *path-connected* if the sequence of candidates (i^1, \dots, i^q) is connected by the set of links $\{j, j+1\}_{j=1}^{q-1} \subseteq L(s)$. Note that it is possible that some of the offers to form links may not be reciprocated. If all the offers of link formation are not reciprocated, we assume that the candidate will participate independently in election.¹²

A **party**, $P_k(s)$, is the set of all path-connected candidates in $s \in \mathcal{S}$, i.e., $P_k(s) = \{i \in N \mid [i, j] \in L(s) \forall j \in P_k(s), j \neq i\}$ for $k \in \{1, 2, \dots, |\mathcal{P}(s)|\}$ where $\mathcal{P}(s)$ is the set of all parties in the given link profile $s \in \mathcal{S}$. Note that independent candidates are treated as a party with only one member.

Winning set: The winning set is the set of all parties that get the most votes in $v(s)$ denoted by $W(v(s)) = \{P_1(s), \dots, P_{\bar{k}}(s)\}$ where $\bar{k} = |W(v(s))|$. Therefore, the winner is decided by plurality rule as in the first-past-the-post voting systems. Each winning party's policy position is implemented with probability $\frac{1}{|W(v(s))|}$. If no party forms, we assume that the median policy x_M is implemented if n is odd and M is the unique median voter, and $\frac{x_{M_1} + x_{M_2}}{2}$ is implemented if the number of voters is even, where M_1 and M_2 are the two medians.

Timeline of the game: The game proceeds in four stages:

Stage 0: Given the distribution of ideal points of the citizen-candidates, they decide whether to offer any links or stay out.

Stage 1: Mutually acceptable links are formed which leads to party formation and a party structure. Party policies are decided as per the Party Policy Protocol (PPP) as described below.¹³

¹¹ Note that $a^\pm \in s_{i+1}$ indicates that individual $i+1$ has offered a link to i and $i+2$.

¹² This is without loss of generality since in equilibrium that candidate may decide to stay out.

¹³ Party policy positions of this nature have been considered in Jackson et al. (2007) and Teorell (1999).

Stage 2: All citizen-candidates including those who are participating in elections vote for one of the policies of the parties formed in the previous stage. Note that we do not assume that party members will vote for their own party.

Stage 3: The winner is decided according to the plurality rule. If there are multiple winners, the rents are shared equally among the parties and each policy is implemented with equal probabilities. The timeline of the electoral competition game is illustrated in Fig. 2.

When a party is formed it comprises of many members. Each member would prefer to have her own policy position as the party position if it can guarantee winning the election. However, we do not explicitly model the bargaining process which would take place within the party to decide the party policy. We, therefore, formalize the possible policies a party can set after its formation through the Party Policy Protocol which is described below.

Party Policy Protocol (PPP):

(1) The policy of any party can be any policy which is in the **range** of the party which is defined as follows. The range of a party $P_k(s) \in \mathcal{P}(s)$ for any given link profile $s \in \mathcal{S}$, denoted as $Range(P_k(s))$, is the interval $[x_l^k, x_r^k]$, where x_l^k and x_r^k are the leftmost and rightmost policies of members in $P_k(s)$. The policy position of a party $P_k(s)$ is denoted by $x_{P_k(s)}$ under strategy s . The party is, therefore, allowed to choose any policy in the range of the policy positions of its members. However, equilibrium conditions will ensure that only those policies which maximize the probability of winning be selected.

(2) If a party member i leaves a party $P_k(s)$, then there are two possibilities: (i) The remaining members continue to form a party, i.e., $P_k(s) \setminus \{i\}$ is a well-defined party, or (ii) she breaks the party into two other parties by breaking one or both the links. In both these conditions, the protocol needs to specify how policies can be chosen. We consider these two cases separately. (i) Suppose agent i 's leaving the party does not break the party into two new parties. This can happen either when her ideal policy is at the left or right extreme within the party. In such a case, the new party's policy position $x_{P_{k'}(s')}$ will remain the same if it is still available and maximizes win probability, i.e., if $x_{P_k(s)} \in Range(P_{k'}(s'))$, then $x_{P_k(s)} = x_{P_{k'}(s')}$ where s' is the new strategy after member i breaks away from the party. On the other hand if $x_{P_k(s)} \notin Range(P_{k'}(s'))$, then the new party's policy is the closest policy available to the earlier party position, i.e., $x_{P_{k'}(s')} = \arg \min_{x \in Range(P_{k'}(s'))} |x - x_{P_k(s)}|$. Moreover, such a policy should also maximize the win probability for the resulting party.

(ii) If a party member i leaves a party $P_k(s)$ and breaks the party into two parties, there are two possibilities:

(a) Breaking both links: When i leaves the party by breaking both her links, we allow adjacent members of i in the remaining party to connect with each other. For example, suppose $i, i+1, i+2$ belong to the party $P_1(s)$ and the party member $i+1$ breaks all her links. This breaks the party into two parties: one to which i belongs, and the other, to which $i+2$ belongs. In this case, we allow i to send a link to $i+2$ and $i+2$ to send a link to i . If the link is mutually agreeable, it gets formed. In such a case, the PPP states the policy position of the new party is the closest policy available to the earlier party position that maximizes win probability. Note that the range of the new party is

the same due to the consolidation. Therefore, the party policy does not change if the earlier policy maximizes the win probability of the party.

(b) By breaking only one link: If part (a) above does not take place and two parties $P_{k'}(s')$ and $P_{k''}(s')$ are formed, the new parties' respective policies are the closest policies available to the earlier party's position, i.e., $x_{P_{k'}(s')} = \arg \min_{x \in \text{Range}(P_{k'}(s'))} |x - x_{P_k(s)}|$ and $x_{P_{k''}(s')} = \arg \min_{x \in \text{Range}(P_{k''}(s'))} |x - x_{P_k(s)}|$. Moreover, such policies must also maximize the probability of winning for the respective parties.

(3) If a new party member j (and/or j') joins the party, the party may change its policy in accordance with the goal of maximizing win probability. However, if the same winning policy is available, it should continue to be chosen. If two members from different parties which were both in the winning set join, they may offer any new policy which maximizes the probability of winning.

Note that PPP does not describe equilibrium policy positions. It restricts the set of possible party policies that can be chosen in the range of the party, following a change in the membership of the party. One of the main implications of the PPP is that whenever a member of the party leaves or a new member joins the party, the same policy must continue to be chosen if it is still available and also maximizes the win probability. If such a policy is not unique, any new winning policy can be offered which is beneficial to the deviating members. These protocols also make the model more tractable by allowing broken parties to rejoin again without much loss of generality in the party formation process. If a party member i breaks all her links with the party members, we assume that she continues to run for election as an independent.¹⁴

Remark 1 Note that our approach in defining PPP is very general. We do not model the process of policy formation within the party. As will be clear in the next subsection on equilibrium, a party policy position must be one that maximizes the win probability of a party. PPP, therefore, restricts the set of winning policies that can be chosen by a party when it goes through a change in its membership.

Voting for a party: The second type of strategy of an agent is as a citizen, i.e., that of voting. For a given link profile $s \in \mathcal{S}$ and policy vector $x(s)$ which generates a party structure $\mathcal{P}(s)$ with the respective policies, each voter i votes for a party $v_i(x(s)) \in \mathcal{P}(s)$. For simplicity of notation, we will denote a vote vector $v(x(s))$ as $v(s)$ since the policies will be assumed to be given. A vote profile is denoted by $v(s) = (v_1(s), v_2(s), \dots, v_n(s))$. Let $\mathcal{V}(s) = \prod_{i \in N} \mathcal{V}_i(s)$ denote the set of all vote profiles given the link profile $s \in \mathcal{S}$. For simplicity in notation, we will write v_i in place of $v_i(s)$ and v'_i in place of $v'_i(s)$.

We do not assume sincere voting strategies and members of a party are allowed to vote for other parties.¹⁵ To allow for strategic voting in our model, we use the notion of *undominated strategies*. For any given party structure $\mathcal{P}(s)$ given a link profile s , the voting strategy $v_i(s)$ is undominated for an agent $i \in N$ if either (i) $u_i(x(v_i, v_{-i})) \geq u_i(x(v'_i, v_{-i}))$ for all $v'_i \in \mathcal{V}_i(s)$ for all $v_{-i} \in \prod_{j \neq i} \mathcal{V}_j(s)$ or (ii) there exists a strategy $v'_i(s)$ and another strategy $v_{-i}(s)$ of the other voters such that

¹⁴ Removing this assumption does not have any effect on our results since the leaving candidate will never be able to win by staying in. This is a direct consequence of the PPP.

¹⁵ However, in equilibrium it is always beneficial to vote for her own party.

$u_i(x(v_i, v_{-i})) > u_i(x(v'_i, v_{-i}))$. In other words, voting for a party $v_i(s)$, given a party structure $\mathcal{P}(s)$ is undominated if either of the following conditions are true: (i) It always gives a weakly higher payoff or (ii) gives a strictly higher payoff compared to at least one strategy v'_i of voter i for a given strategy v_{-i} of the other voters.

Let $\mathcal{V}^d(v_{-i})$ denote the set of all undominated strategies of agent i given $v_{-i}(s)$. We say that a voting vector $v(s) \in \mathcal{V}^d$ is a vector of undominated strategies if $v_i(s) \in \mathcal{V}^d(v_{-i})$ for all $i \in N$. In the rest of the paper, we will only consider undominated strategies in equilibrium. It is also useful to note that when the number of parties is less than or equal to two, voting sincerely is the only undominated strategy for each voter. This is due single-peakedness of preferences. If a voter votes for a policy further away, then that policy is implemented with a weakly higher probability. However, due to single-peakedness, this does not make her strictly better-off and makes her strictly worse-off if the probability strictly increases. We now define the payoffs of citizen-candidates.

Payoffs: We assume that each party has to pay a fixed cost $C > 0$ for participating in elections, which are shared equally among the members of a party. The party that gains the highest vote share wins the election. A party which wins receives rents R which are also shared equally. We assume that $R \geq C$.¹⁶

Payoff function $\pi_i(v(s)) : \mathcal{V}(s) \rightarrow \mathbb{R}$ of candidate i for a given vote profile $v(s)$ is given by,

$$\pi_i(v(s)) = \begin{cases} \frac{1}{|P_k^*(s)|} \left(\frac{R}{|W(v(s))|} - C \right) + \frac{\sum_{k=1}^{\bar{k}} u_i(x_{P_k}(s))}{|W(v(s))|} & \text{if } i \in P_k^*(s) \in W(v(s)) \\ \frac{\sum_{k=1}^{\bar{k}} u_i(x_{P_k}(s))}{|W(v(s))|} - \frac{C}{|P_k^*(s)|} & \text{if } i \in P_k^*(s) \notin W(v(s)) \\ \frac{\sum_{k=1}^{\bar{k}} u_i(x_{P_k}(s))}{|W(v(s))|} & \text{otherwise.} \end{cases}$$

Therefore, if voter i is in a winning party she splits the spoils of victory R with her party members weighted by the probability of winning when there are $|W(v(s))|$ parties in the winning set. She also splits the costs of standing in election, but these are not dependent on winning. Therefore, in the first expression, cost C is shared among the party members irrespective of whether the party wins or not. Finally, she also derives expected utility from the winning policies where each policy has a probability $\frac{1}{|W(v(s))|}$ of being implemented.

If voter i does not belong to a winning party, then she does not enjoy rents of winning but the other components of her payoff remain as above. Finally, if voter i does not participate in election, then her payoff is the expected utility over the set of winning policies.

The main features of the game are summarized. (i) The distribution of voters' policy position is fixed and common knowledge. (ii) Candidates decide whether or not to participate in electoral competition by offering links or standing alone, and (iii) this leads to party formation and party policy platforms. (iv) Voters vote for parties based on the party policy platforms. (v) The parties that win the election implement their policy

¹⁶ We will relax this assumption in a later section.

platforms with equal probabilities. (vi) Winning political parties enjoy added rents and pay costs that are equally shared among its members. In addition to this, payoffs from policy are realized for all participating and non-participating citizen-candidates. We provide an illustrative example to show the different stages of the game before proceeding to the equilibrium notion.

Example 2 The policy space is $X = [0, 1]$ and the set of candidates is $N = \{1, 2, \dots, 8\}$. Policy positions $\{x_i\}_{i=1}^8$ with $x_i = \frac{i-1}{n-1}$ for all $i \in N$ are at equal distances from each other as shown in Fig. 1.

Consider the following set of participatory actions:

$s_i = a^\pm$ for all $i \in \{2, \dots, 7\}$, $s_1 = a^+$, and $s_n = a^-$. Then, $\mathcal{P}(s) = \{P_1(s)\} = \{\{1, 2, \dots, 8\}\}$, i.e., there is only one party which forms. Here, as per the PPP any policy $x \in [0, 1]$ can be assigned as the party's position. Suppose x_4 is the party's position. Then, voter 4's payoff is $\frac{R-C}{8}$ since there are 8 members in the party so rents and costs are shared equally. Moreover, she gets a maximum utility of 0 from the implemented policy since her own policy position is implemented. Payoffs for the other agents can be calculated similarly. We now define the equilibrium notion.

2.1 The equilibrium notion

It is well known that the notion of Nash equilibrium applied to network formation is very weak and leads to unsatisfactory predictions (Jackson 2008). This is due to the fact that it fails to capture some possible deviations in strategic interactions. Consider the situation where two parties have formed. Nash equilibrium does not consider the deviation where two adjacent candidates from each party join by forming a mutually agreeable link and consolidating the two parties. Therefore, following (Jackson and Wolinsky 1996), we use a modified version of *pairwise stability* adapted to our model. According to this notion, a given party structure is a political equilibrium if (a) no individual agent can deviate beneficially, and (b) no pair of agents i and j can jointly deviate and benefit by forming a new link between themselves. This notion is well suited to our model since links can only be formed if both agents agree; therefore, in most cases the agents will need to deviate jointly.¹⁷ Let $s' = (s'_i, s_{-i})$ denote the set of strategies where i individually deviates from $s = (s_i, s_{-i})$, and let $s'_{ij} = (s'_i, s'_j, s_{-ij})$ denote the set of strategies where i and j jointly deviate from the strategy $s = (s_i, s_j, s_{-ij})$. We define a political equilibrium where each voter plays undominated strategies.

Political equilibrium: A given link profile $s \in \mathcal{S}$, a party policy vector $\{x_{P_k(s)}\}_{k=1}^{|\mathcal{P}(s)|}$ and a vote profile $v(s) \in \mathcal{V}^d(s)$ is an equilibrium if,

- (i) For all $i \in N$, $\pi_i(v(s)) \geq \pi_i(v(s'_i, s_{-i}))$, for any given party policy vector $\{x_{P_k(s')}\}_{k=1}^{|\mathcal{P}(s')|}$ for all $v(s') \in \mathcal{V}^d(s')$,
- (ii) For all adjacent candidates $i, j \in N$, if $\pi_i(v(s'_{ij})) > \pi_i(v(s))$ for some $v(s') \in \mathcal{V}^d(s')$ and some party policy vector $\{x_{P_k(s')}\}_{k=1}^{|\mathcal{P}(s')|}$, then $\pi_j(v(s)) > \pi_j(v(s'_{ij}))$, and

¹⁷ Except in cases where one of the agents had already offered a link in the initial set of strategies which was not reciprocated.

(iii) For every party $P_k \in \mathcal{P}(s)$ there is no $x_{P_k}(s)'$ in the range of the party policies which strictly increases the winning probability of the party P_k compared to the chosen policy $x_{P_k}(s)$ for any vote profile $v'(s) \in \mathcal{V}^d(s)$.

Any vector of link strategies $s \in \mathcal{S}$, a party policy vector $(x_{P_1(s)}, \dots, x_{P_k(s)})$ where $k = |\mathcal{P}(s)|$ and a vector of undominated voting strategies $s \in \mathcal{V}^d(s)$ is a political equilibrium if three conditions hold: (i) Any individual party member should not be able to deviate beneficially for any given party policy vector given any undominated vector over the new party policies vector, and (ii) no pair of agents, i, j should find it strictly beneficial to deviate. In other words, if i strictly prefers to form the link $\{i, j\}$ for some vote vector $v(s'_{ij})$ in the new party structure, then it must be the case that j is strictly worse-off with respect to $v(s'_{ij})$ after the deviation s'_{ij} .

We will say that a vote vector $v(s) \in \mathcal{V}^d(s)$ supports an equilibrium party structure $\mathcal{P}(s)$ with the policy positions $\{x_{P_k(s)}\}$ if it maximizes voters' expected utilities over the set of winning party policies, i.e., is an equilibrium strategy for voters as well. We will mostly focus on the party structure and policy positions of the parties in equilibrium since voters play a passive role in our model. We provide an example.

Example 3 Consider the same set of ideal points distribution as in Example 1. Suppose the same links form as earlier with $\mathcal{P}(s) = \{P_1(s)\} = \{\{1, 2, \dots, .8\}\}$. Suppose the party policy is $x_{P_1} = x_8$. Consider the payoffs of candidates 7. Note that $v(s) = (P_1, \dots, P_1)$ since there is only one party.

$$\pi_7(v(s)) = \frac{R - C}{8} - (|x_7 - x_8|)^2 = \frac{R - C}{8} - (|0.857 - 1|)^2.$$

We show that candidate 7 can break the link $\{7, 8\}$ and be better-off. After deviating to $s'_7 = a^-$, and considering $s'_i = s_i$ for all $N \setminus \{7\}$ we get $\mathcal{P}(s') = \{P_1(s'), P_2(s')\}$ where $P_1(s') = \{1, 2, \dots, 7\}$ and $P_2(s') = \{8\}$ since candidate 8 is now independent. Let $x_{P_1}(s') = 0.857$ and $x_{P_2}(s') = 1$. Clearly, if we take $v(s') = (P_1, \dots, P_1, P_2)$, this is optimal for each player (deviating to the other party does not benefit any voter). Also, according to the PPP, a policy closest to the previous policy which is win maximizing must be chosen. Moreover, $v(s')$ is an undominated vote vector. The winning party is $P_1(s')$, and $x_{P_1}(s') = x_7$ is implemented. Therefore,

$$\pi_7(v(s')) = \frac{R - C}{7} - (|0.857 - 0.857|)^2 = \frac{R - C}{7} > \pi_7(v(s)).$$

Similar deviations would be possible for some undominated voting vector unless $x_{P_1}(s) \in [x_4, x_5]$. If voter 6 is contemplating standing alone in election, she will take into consideration all undominated voter strategies. If the other party's policy is between her policy and the median voter's policy, then in any undominated vote vector she will lose the election with certainty. We will use this reasoning to check for possible deviations from the proposed equilibrium party structure and link profile s . Our first result characterizes the one-party equilibria and specifies the PPP that supports those equilibria.

3 Results

Theorem 1 (One-party equilibrium ($R \geq C$)) *Any link profile $s \in \mathcal{S}$ such that $\mathcal{P}(s) = \{P_1(s)\}$ and any $v(s) \in \mathcal{V}^d(s)$ which supports this is a one-party equilibrium if and only if*

(i) *For n odd: $\mathcal{P}(s) = \{P_1(s)\} = \{\{M\}\}$ and $x_{P_1(s)} = x_M$.*

(ii) *For n even: $\mathcal{P}(s) = \{P_1(s)\} = \{\{M_1, M_2\}\}$. Moreover, any policy $x_{P_1(s)} \in [x_{M_1}, x_{M_2}]$ can be supported in equilibrium if $C \geq \frac{1}{(n-1)^2}$.*

Theorem 1 states that the only equilibrium with one party (with one or more members) is where only the median citizen-candidate(s) is (are) in the only party, i.e., every other citizen-candidate stays out. Moreover, the party policy is the unique median when n is odd or belongs to the interval of the two medians when n is even.

Proof (\Leftarrow) We first show that the mentioned party structure and the voting strategy is a PE. Suppose $s \in \mathcal{S}$ is any link profile which leads to $P_1(s) = \{M\}$. Clearly, no party can enter the electoral competition and win since the median voter M will always get strictly $n + 1$ votes with the party policy x_M . We show that the median voter M will also not accept any links from $M - 1$ or $M + 1$. Suppose $s' \in \mathcal{S}$ is a profile where $s'_{M-1} \in \{a^\pm, a^+\}$ and $s'_M \in \{a^\pm, a^-\}$. Let $P_1(s') = \{M - 1, M\}$. Then comparing payoffs, we get,

$$\pi_M(v(s)) = R - C > \pi_M(v(s')) = \frac{R - C}{2} + u_M(x_{P_1(s')})$$

for any $x_{P_1(s')} \in [x_{M-1}, x_M]$.

Therefore, M will not accept the link. Similarly, we can show that M will also not accept the link from $M + 1$. Suppose n is even, and $P_1(s) = \{M_1, M_2\}$ where M_1 and M_2 are the two median voters. Suppose $x_{P_1(s)} = x_{M_2}$. This is the worst policy in the interval $[x_{M_1}, x_{M_2}]$ for candidate M_1 . Clearly, M_1 would not prefer to form a link with $M_1 - 1$ since the policy cannot be improved by PPP. Therefore, their win probability is not affected. Moreover, M_1 would not want to share the additional benefits with a larger set of individuals.

Finally, we need to verify that neither M_1 nor M_2 would prefer to break the link with the other member and participate alone in the election. To prevent such a deviation by M_1 , we again assume w.l.o.g. that $x_{P_1(s)} = x_{M_2}$. A similar argument would prevent deviation from M_2 as well. For the deviation to not be beneficial, we must have,

$$\frac{R - C}{2} - \frac{1}{(n-1)^2} \geq \frac{R}{2} - C - \frac{1}{2(n-1)^2}.$$

Note that the LHS is the current payoff when the policy is x_{M_2} and the RHS is the deviation payoff when the two policies x_{M_1} and x_{M_2} are implemented with equal probabilities since the two parties $\{M_1\}$ and $\{M_2\}$ will be tied under any undominated voting vector. Simplifying the above inequality, we get $C \geq \frac{1}{(n-1)^2}$.

(\Rightarrow) We first prove necessity of the conditions for equilibrium when n is odd. Suppose $s \in \mathcal{S}$ with $\mathcal{P}(s) = \{P_1(s)\}$, $x(P_1(s)) \in \text{Range}(P_1(s))$ and $v(s) = (P_1, \dots, P_1)$ is a

one-party equilibrium. We show that $\mathcal{P}(s) = \{P_1(s)\} = \{\{M\}\}$, $x(P_1(s)) = x_M$ and $v(s) = (P_1, \dots, P_1) \in \mathcal{V}^d(s)$ is the only equilibrium.

We show that only M standing for election is the unique equilibrium. Clearly, M standing in election is an equilibrium, since no other candidate can enter and win the election with positive probability. Any other candidate or pair of candidates who enter with a party policy $x_{P_2(s')}$ will get less than a strict majority of votes in any equilibrium. Similarly, M will also reject links from any other candidate $j \neq M$ since her payoff falls even if she gets her most preferred policy position after the change, i.e.,

$$\pi_M(v(s)) = R - C > \pi_M(v(s')) = \frac{R - C}{2}$$

for all $v(s') \in \mathcal{V}^d(s')$. We have assumed without loss of generality that the new party that has formed with $M - 1$ (or $M + 1$) has set the party policy to x_M . Her payoff decreases even with the best possible party policy (her ideal point) since she has to share the net benefits $R - C$ with another candidate.

We now argue that if $s \in \mathcal{S}$ is such that $\mathcal{P}(s) = \{P_1(s)\}$ such that $|P_1(s)| = K > 1$, is not an equilibrium. Firstly, note that in any one-party equilibrium, M must be a member of that party. If not, then by construction, any party that does not include M cannot offer x_M as the policy position. This implies that M can enter the election and win a majority of votes since all the members of the existing party will be on one side of x_M . Therefore, $M \in P_1(s)$. Secondly, note that since M is unique (n is odd), her policy position must be selected as the party policy. If not, then she can break one of the links $\{M - 1, M\}$ or $\{M, M + 1\}$ but not both. Suppose she breaks the link $\{M, M + 1\}$ and the remaining party is $P_2(s') = \{M + 1, \dots, M + k\}$ for some $k \in \{1, \dots, n - \frac{n-1}{2}\}$. Similarly, the other party consisting of M is $P_1(s') = \{M - k', \dots, M - 1, M\}$ for some $k' \in \{0, \dots, n - \frac{n-1}{2}\}$. By single-peakedness, all the voters $1, \dots, M$ will prefer to vote for the policy $x_{P_1(s')} = x_M$ and the party will win. M is better-off after the deviation as the net benefits $R - C$ are shared with a smaller party, and the party policy is her own ideal point. Therefore, M must belong to the winning party. However, we show that M will prefer to delete the link $\{M, M + 1\}$, i.e.,

$$\pi_M(v(s)) = \frac{R - C}{K} > \pi_M(v(s')) = \frac{R - C}{k'}.$$

Therefore, M standing alone for the election is the only one-party equilibrium if n is odd. Similarly, if n is even, any link $\{M_1 - 1, M_1\}$ (or $\{M_2, M_2 + 1\}$) will be broken by M_1 (or M_2) since after the break with no change in party policy, her party will continue to win (as long as both M_1 and M_2 are in the party) and the net rents $R - C$ are shared with a smaller set of individuals. Therefore, the only one-party equilibrium is where M_1 and M_2 are the only members of the party. Moreover, as shown above, for $C \geq \frac{1}{(n-1)^2}$, the link $\{M_1, M_2\}$ is not broken. No new link is formed for any party policy $x_{P_1(s)} \in [x_{M_1}, x_{M_2}]$ since it does not benefit the existing party members. PPP ensures this by restricting the possible deviations in party policy positions while deviating. Therefore, $\{M_1, M_2\}$ is the only one-party equilibrium when n is even. \square

Theorem 1 also characterizes the set of policies that can result in a one-party equilibrium. In every such equilibria, the policy of the median voter (when n is odd) or any policy in the region between the two ideal points of the median voters (when n is even) is implemented. We now focus on two-party equilibrium. Our first observation is that in any multi-party equilibria, all the parties must receive the same number of votes.

Claim 1 *In any multi-party equilibrium $\mathcal{P}(s)$ with $|\mathcal{P}(s)| \geq 2$ for some $s \in \mathcal{S}$, party policy vector $\{x_{P_k}(s)\}$ and voting vector $v(s) \in \mathcal{V}^d(s)$, each party receives the same number of votes.*

The above claim is easy to prove. If any one party receives strictly less votes, then this party is not in the winning set. Therefore, not only does it not affect the policy outcome, none of the party members receive any portion of the rents R . Moreover, they incur the costs of participation. Therefore, any member of such a party would be strictly better-off by deviating to ‘not participating.’¹⁸ Our next result characterizes the two-party equilibria.

Theorem 2 (Two-party equilibrium ($R \geq C$)) (i) *There is no two-party political equilibrium when the number of voters is odd.*

(ii) *There is a two-party political equilibrium, $s \in \mathcal{S}$, where $P_1(s) = \{M_1\}$ and $P_2(s) = \{M_2\}$ only if the following protocol is specified: If M_1 and M_2 join to form a new party, then the party policy be within at least $k = \frac{1}{\sqrt{2(n-1)}}$ distance away from either x_{M_1} or x_{M_2} and $2k^2 - \frac{1}{(n-1)^2} \leq C \leq \frac{1}{2} \left(R + \frac{1}{(n-1)^2} \right)$. In equilibrium, voters to the left of M_1 vote for M_1 , while those on the right of M_2 vote for M_2 .*

Proof (i) Suppose n is odd. We show that there is no two-party equilibrium. Suppose for contradiction that there is a two party equilibrium with $P_1(s)$ and $P_2(s)$ as the two parties. We first show that in any two party equilibrium one of the parties must consist of the median voter M . Suppose w.l.o.g. that $M \notin P_1(s) \cup P_2(s)$ where $x_{P_1(s)} < x_{P_2(s)}$. Suppose the left-extreme member of the party $P_2(s)$ offers a link to M (or the left-adjacent candidate who is not a member of the party) and offers to set the new policy position same as earlier. Note that this is required under part (iv) of PPP since the new party will continue to win under any undominated voter strategy. As a result of this, M benefits from the additional $\frac{1}{|P_1(s)|+1}$ fraction of $R - C$ if party $P_1(s)$ wins the election for some undominated vote vector. Both members are strictly better-off in the case where every voter i such that $x_i \geq M$ votes for party $P_1(s)$. Their party becomes the sole winner, and M earns the additional revenue and benefits from the policy position as well.

Suppose $M \in P_1(s)$. By setting $x_{P_1(s)} = x_M$ party $P_1(s)$ wins a strict majority of votes and the other party loses. This cannot be an equilibrium by Claim 1. Hence, no two-party equilibrium can exist with an odd number of voters.

(ii) Suppose n is even. We first show that M_1 and M_2 must be participating in election, and that they must belong to different parties. Firstly, note that if neither of the two parties consist of the two medians, w.l.o.g. we can assume that the party positions

¹⁸ Note that this observation is true for $R \leq C$ as well.

are $x_{P_1(s)} < x_{M_1} < x_{M_2} < x_{P_2(s)}$ where $P_1(s)$ and $P_2(s)$ are the only two parties participating for some $s \in \mathcal{S}$. However, the rightmost party member in $P_1(s)$ can deviate with the right-adjacent candidate by forming a link and move the party policy closer to the other party's position. This would ensure at least half a vote more (it gains half the votes of individual(s) with policy positions between the two party positions since they are indifferent) than earlier. Moreover, party member i from $P_1(s)$ would obtain,

$$\pi_i(s') = \frac{R - C}{|P_k(s)| + 1} - u_i(x_{P_1(s)+1}) >$$

$$\pi_i(s) = \frac{R}{2|P_k(s)|} - \frac{C}{|P_k(s)|} - \frac{u_i(x_{P_1(s)}) + u_i(x_{P_2(s)})}{2},$$

where $x_{P_1(s)+1}$ is the policy right-adjacent to $x_{P_1(s)}$. Therefore, M_1 and M_2 must belong to the parties $P_1(s)$ and $P_2(s)$, respectively, since that is the only case where a party cannot improve its winning chances. If they were in the same party, then the other party would not be able to win, and that would not be a two-party equilibrium. Now, we show that in any two-party equilibrium, $M_1 \in P_1(s)$ and $M_2 \in P_2(s)$. We first prove that $|P_1(s)| = |P_2(s)| = \frac{n}{2}$. If there are any other members in the party, the second-to-left-extreme member $\bar{i} + 1$ of $P_1(s)$ can delete her link with \bar{i} , the left-extreme candidate in the left party $P_1(s)$. By single-peakedness of preferences, there is a voting strategy $v(s) \in \mathcal{V}^d(s)$, where all the voters to the left of M_1 will continue to vote for $x_{P_1(s')} = x_{M_1}$. This implies that party member $\bar{i} + 1$ will be strictly better-off after the deviation. This argument can be made for any set of party members $P_1(s) = \{\bar{i}, \bar{i} + 1, \dots, M_1\}$ for any $\bar{i} \in \{1, \dots, M_1 - 1\}$. Similar arguments imply that $P_1(s) = \{M_1\}$ and $P_2(s) = \{M_2\}$. Suppose $k_1 = |P_1(s)|$. Formally, having one member less is better if there is no change in policy outcomes since,

$$\frac{R}{2k_1} - \frac{C}{k_1} \geq \frac{R}{2(k_1 + 1)} - \frac{C}{k_1 + 1}.$$

Rewriting the above inequality,

$$\frac{R - 2C}{2k_1} \geq \frac{R - 2C}{2(k_1 + 1)} \iff \frac{1}{2k_1} \geq \frac{1}{2(k_1 + 1)} \iff k_1 + 1 \geq k_1.$$

We only have to show that M_1 (or M_2) does not want to deviate. Clearly, not participating is not beneficial since the policy will be further away and the net rents will be lost. Forming new links on the left are also not beneficial since they do not increase the probability of winning and also reduce the net benefits. Forming a link with M_2 is not beneficial if the following condition holds,

$$\frac{R}{2} - C - \frac{1}{2(n-1)^2} \geq \frac{R - C}{2} - k^2$$

$$\iff \frac{C}{2} \geq k^2 - \frac{1}{2(n-1)^2},$$

where k is the distance between x_{M_1} and the new policy. This is only possible if $k^2 \geq \frac{1}{2(n-1)^2}$ or if $k \geq \frac{1}{\sqrt{2(n-1)}}$. Similarly, M_1 will not drop out of election if $-\frac{1}{2(n-1)^2} + \frac{R}{2} - C \geq -\frac{1}{(n-1)^2}$, i.e., if $C \leq \frac{1}{2} \left(R + \frac{1}{(n-1)^2} \right)$. Therefore, $2k^2 - \frac{1}{(n-1)^2} \leq C \leq \frac{1}{2} \left(R + \frac{1}{(n-1)^2} \right)$ where k is the distance between M_1 's (or alternatively, M_2 's) ideal policy position that would be chosen if M_1 and M_2 decide to join. \square

The formation of two parties in equilibrium is a median-based party equilibrium. Our result can also be seen as an alternative generalization of a similar result in Osborne and Tourky (2008). A two-party equilibrium is possible only when the number of voters is even and only the two medians are participating independently. Only a restriction on the PPP can prevent such a deviation: the declaration that the new party policy will be within $\frac{1}{\sqrt{2(n-1)}}$ distance from the other median's policy position. This will prevent the other median voter from joining unless the costs are greater than the prescribed threshold. The rationale for this can be behavioral or institutional. An example of the latter is the *self-enhancement bias* where individuals value their own contribution to the success of the party more than that of the others. In this case, the two medians consider themselves more worthy of winning with their own policy position even though a compromise would be beneficial. The institutional reason would be external funding constraints which restrict them from deviating from a comparatively left- or right-of-center policy. A restriction on the PPP as described is able to sustain the two-party equilibrium characterized above. Our next result shows that there is no multi-party equilibria with three or more parties.

Theorem 3 (Multi-party equilibrium ($R \geq C$)) *If there are no additional restrictions on the PPP, then there is no equilibrium where the number of parties, k , is greater than or equal to 3.*

Proof Suppose for contradiction that there is a strategy profile $s \in \mathcal{S}$, party structure $\mathcal{P}(s)$ with $|\mathcal{P}(s)| \geq 3$ with party policies $\{x_{P_k}(s)\}$ and $v(s) \in \mathcal{V}^d(s)$ which is a political equilibrium. Let the extreme right party be denoted as $P_{\bar{k}}$ where $\bar{k} = |\mathcal{P}(s)|$, and let $\bar{k} - 1$ be the second extreme party from the right. We first argue that every member in the party $P_{\bar{k}}$ will vote for her own party in any voting equilibrium $v(s) \in \mathcal{V}^d(s)$. Suppose that any party member is not voting for her own party. Since this is the extreme right party, the party policy $x_{P_{\bar{k}}}(s)$ would not be an equilibrium policy if that were the case. Consider the policy $x_{P_{\bar{k}}}(s) = x_{l^k}^k$ where l^k is the left-extreme policy in the party $P_{\bar{k}}$. By single-peakedness every member in $P_{\bar{k}}$ would strictly prefer this policy to any other party's policy. Therefore, by voting for her own party she obtains a strictly higher expected utility over the set of party policies and also ensures that her party wins (since every party must be tied in equilibrium). This deviation will make any party member of $P_{\bar{k}}$ who was not voting for her party strictly better-off as a result of the deviation.

Let l^k be the left-extreme member in the party $P_{\bar{k}}(s)$ as mentioned above. We show that there will be no non-participating candidates to the immediate left. If there is a voter $l^k - 1$ on the left who is not participating, then that voter and l^k would prefer to form a link and offer a new party policy $x'_{P_{\bar{k}}}(s) = x_{l^k-1}$ which would get at least

one or half more vote from voters who were voting for the left-adjacent party $P_{\bar{k}-1}(s)$ or were indifferent. However, this is not possible in equilibrium. Therefore, the voter $l^k - 1$ must be a member of the left-adjacent party $P_{\bar{k}-1}(s)$.

Finally, we show that $l^k - 1$ and l^k will prefer to form the link $\{l^k - 1, l^k\}$ thus consolidating the parties $P_{\bar{k}-1}(s)$ and $P_{\bar{k}}(s)$. As argued above, we can assume w.l.o.g. that the party policy of $P_{\bar{k}}$ in equilibrium is $x_{P_{\bar{k}}}(s) = x_{l^k}$. As argued in Claim 1, each party obtains the same number of votes in equilibrium. Since $P_{\bar{k}}(s)$ gets $n_{\bar{k}}$ number of votes, there must be at least as many party members in $P_{\bar{k}-1}(s)$. This implies that all the other policies of winning parties are at least $\frac{\bar{k}-1}{n-1}$ distance away. We can show that forming the link will be better for both agents in terms of policy. Clearly, candidate l^k will be better off in terms of policy since all other policies are further away. We show that the same holds true for the candidate $l^k - 1$. For simplicity of notation, we will denote $x_{\bar{k}-1}$ and $x_{\bar{k}}$ as the policies of the two parties. We set the policy of the new party $P'_k(s') = P_{\bar{k}-1}(s) \cup P_{\bar{k}}(s)$ as $x_{k'} = \frac{x_{\bar{k}-1} + x_{\bar{k}}}{2}$ (this is allowed under the PPP). We can show that the policy outcome is better for $i = l^k - 1$ after the deviation, i.e.,

$$\begin{aligned} u_i(x_{k'}) - \frac{\sum_{k=1}^{|\mathcal{P}(s)|} u_i(x_{P_k}(s))}{|W(s)|} &\geq u_i(x_{k'}) - \frac{u_i(x_{\bar{k}-1}) + u_i(x_{\bar{k}})}{|\bar{k}|} \\ &\geq -\frac{1}{4(n-1)^2} + \frac{(\bar{k}-1)^2 + 1}{\bar{k}} \geq \frac{4+1}{3} > 0. \end{aligned}$$

We have used the fact that the smallest value of $-\frac{1}{4(n-1)^2}$ is when $n = 2$, i.e., $-\frac{1}{4(2-1)^2} = -\frac{1}{4}$. Similarly, $\frac{(\bar{k}-1)^2 + 1}{\bar{k}} \geq \frac{5}{3}$ since $\bar{k} \geq 3$. Therefore, utility from the new policy outcome is greater than the lottery over the previous set of party policies. Note that the inequality uses the fact that having even three parties is not optimal for player $l^k - 1$ which implies that having more parties will be worse since the some party policies will be further away and also have positive probability of being chosen. We now show that the rents and costs after the deviation are also strictly higher. Since costs are shared with a bigger set of candidates, it is lower than before. We only need to show that the shared rents after the deviation are higher than before. Let the number of party members in $P_{\bar{k}-1}$ and $P_{\bar{k}}(s)$ be denoted by k and k' , respectively. As noted earlier every member of $P_{\bar{k}}(s)$ votes for her own party. Therefore, $k' \leq k$. Since every party must get exactly k' votes, there must be at least $|W(s)|k'$ voters. Moreover, $n \geq |W(s)|k' > 2k' \geq k' + k$, since the total number of voters will exceed the total number of party members in both the parties. The second inequality is due the above observation that every member of party $P_{\bar{k}}(s)$ votes for that party and that in equilibrium every party must be k' votes. Let $\alpha = \frac{k'+k}{k}$. Then, an implication of the above two observations is that $|W(s)| > \frac{k'+k}{k'} \implies |W(s)| > \alpha \implies \frac{\alpha}{|W(s)|} < 1$. We will use this to show that the rents after the deviation will be greater,

i.e.,

$$\begin{aligned} \left[\frac{R}{k+k'} > \frac{R}{|W(s)|k} \right] &\iff \left[\frac{R}{\alpha k} > \frac{R}{|W(s)|k} \right] \\ &\iff R \left(1 - \frac{\alpha}{|W(s)|} \right) > 0 \end{aligned}$$

since $0 < \frac{\alpha}{|W(s)|} < 1$. Similarly, for party member l^k , the policy outcome gets strictly better after the deviation, rents improve since $\frac{R}{k+k'} > \frac{R}{|W(s)|k'}$ and $|W(s)|k' > k+k'$ as observed above. The costs are strictly less since the party size has increased. Therefore, both $l^k - 1$ and l^k are strictly better-off after the deviation to form the link $\{l^k - 1, l^k\}$. \square

Theorem 3 states that without further restrictions on PPP, no multi-party equilibria is possible. There will be candidates in the right or left two extreme parties who will prefer to form a link to obtain a strictly better policy with certainty rather than a lottery over policies, some of which are located further away. If the number of parties is greater than 3, then policies of some parties will be further away since parties must be tied in equilibrium. In such cases, the incentive for two adjacent candidates in two extreme parties to jointly deviate is even stronger.

An advantage of our model and results is that since the distance between any two adjacent candidates is $\frac{1}{n-1}$, the conditions on rents and costs are intuitive. In the one-party case, costs need to be greater than or equal to $\frac{1}{(n-1)^2}$ when the number of voters is even. However, in the two party case, an equilibrium is only possible if the number of citizen-candidates is even. Additional PPP restrictions need to be imposed to ensure that the two median parties do not join to consolidate the parties. Further conditions on the range of the cost of participation are required. Our final theorem is consistent with what is observed in the literature: Multi-party equilibria does not exist in a one-dimensional policy space unless further restrictions are imposed on the process of choosing party policies internally (Dhillon 2005).

3.1 $R < C$ (high cost of participation)

We look at the case where $R < C$, i.e., it is not beneficial to win the elections unless the policy outcome is worth the additional cost. The latter case is more applicable for election settings with significant obstacles to competing in elections or where participation in elections is strongly discouraged, for example, in conflict-prone regions. In such situations, an individual or pair of candidates would not participate in election unless they can significantly influence the outcome. We look at different equilibria separately.

Theorem 4 (One-party equilibrium($R < C$)) (i) If the number of voters is odd, there are two types of one-party equilibrium: (a) $P_1(s) = \{i\}$ for any $i \in \{M-1, M+1\}$, $C = R + \frac{1}{(n-1)^2}$ and $R \geq \frac{2}{(n-1)^2}$. (b) A two-member one-party equilibrium forms, where either $P_1(s) = \{M-1, M\}$ or $P_1(s) = \{M, M+1\}$ where $C \geq \max\{R - k^2 +$

$\frac{2}{(n-1)^2}, R + 2k^2]$ where $k \in (0, \frac{1}{n-1})$ is the minimum distance of party's policy from x_M and x_{M+1} .

(ii) If the number of voters is even, then the only party is $P_1(s) = \{M_1, M_2\}$ and the party position is $k \in (0, 1)$ distance away from x_{M_1} or x_{M_2} such that $\frac{1}{(n-1)^2} \leq C \leq 2\left(\frac{1}{(n-1)^2} - k^2\right) + R$.

Proof Suppose the number of voters is odd. In any party $P_k(s)$, if a member can leave the party and the party remains consolidated by the PPP, then the party is required to choose the same policy position. Therefore, any such member who can leave without changing the policy would leave and same the net cost $\frac{R-C}{k} < 0$ where k is the number of members in the party $P_k(s)$. Therefore, no three-member party can form since the 'centrist' member can leave the party without affecting the outcome.

(a) A single individual i joining in equilibrium can be sustained if $R - C \geq -k^2$, where k is the distance from the median policy. Note that if i drops out, by the status quo policy, the median policy x_M is implemented. Therefore, individual i only stands in equilibrium if $C \leq R + k^2$. The median will not join the election if $-k^2 \geq R - C$, i.e., $C \geq R + k^2$. Therefore, in equilibrium it must be that case that $C = R + k^2$. Clearly, M participating alone is not an equilibrium since she can drop out of the election and obtain the same policy. Moreover, she saves the net costs $R - C < 0$ as a result of this deviation. We argue that only $M - 1$ or $M + 1$ participating alone is the only equilibrium. If $M - 2$ participates alone, then as shown above, the condition $C = R + k^2 = R + \frac{4}{(n-1)^2}$ must hold. However, if $M + 1$ participates in election, then she wins and obtains a payoff of $R - C$. This is beneficial if $R - C > -\frac{9}{(n-1)^2}$, i.e., if $C < R + \frac{9}{(n-1)^2}$ which is indeed the case. Similar arguments can be made to show that any voter strictly to the left of $M - 1$ or strictly to the right of $M + 1$ participating alone is not an equilibrium. If $M - 1$ participates alone, the condition to sustain that equilibrium is $C = R + \frac{1}{(n-1)^2}$. In this case, the median voter is the only one who can defeat her in election, but she will stay out if $-\frac{1}{(n-1)^2} \geq R - C$, i.e., $C \geq R + \frac{1}{(n-1)^2}$. Therefore, she will not participate. If $M + 1$ enters the election she will be tied for the first position. However, she will stay out if $-\frac{4}{(n-1)^2} \geq -\frac{4}{2(n-1)^2} + \frac{R}{2} - C$, i.e., $C \geq \frac{R}{2} + \frac{2}{(n-1)^2}$. However, since $C = R + \frac{1}{(n-1)^2}$, the former condition can only hold if $R + \frac{1}{(n-1)^2} \geq \frac{R}{2} + \frac{2}{(n-1)^2}$, i.e., $R \geq \frac{2}{(n-1)^2}$.

(b) In any two-member party equilibrium will require that the mid-point of the policy positions be chosen as an outcome. If not, then by above arguments, one of the candidates can leave without changing the outcome. As argued above, $\{M - 1, M, M + 1\}$ is not an equilibrium party structure since one of the members can drop out without affecting the policy (depending on what policy is chosen) and save the net costs $\frac{R-C}{3} < 0$. Similar arguments show that any other two-member party $\{i - 1, i\}$ cannot be an equilibrium if $i < M$. Suppose $\{M - 2, M - 1\}$ is a one-party equilibrium. Then, by previous arguments, $C \in [R + \frac{1}{2(n-1)^2}, R + \frac{3}{2(n-1)^2}]$. However, M will enter and participate alone if $R - C > -\frac{9}{4(n-1)^2}$ or if $C < R + \frac{9}{4(n-1)^2}$. Since this is indeed the case, $M - 2$ and $M - 1$ cannot form a party in equilibrium. Similar arguments can be made to show that no other one-party equilibrium is possible.

The only two-member one-party equilibrium to consider is the one where the only party is $\{M - 1, M\}$ or $\{M, M + 1\}$. Suppose M and $M + 1$ form a party a policy is chosen from k distance away from x_M . Then, M does not leave if $-k^2 + \frac{R-C}{2} \leq -\frac{1}{(n-1)^2}$, i.e., $C \geq R - k^2 + \frac{2}{(n-1)^2}$. The condition which ensures that M does not break the link is $-k^2 + \frac{R-C}{2} \geq R - C$, i.e., $C \geq R + 2k^2$. Therefore, $C \geq \max\{R - k^2 + \frac{2}{(n-1)^2}, R + 2k^2\}$ for $k \in (0, \frac{1}{n-1})$.

Suppose n is even. Then by arguments made in the proof of Theorem 2, we can show that the strategy profile where M_1 and M_2 participate together is the only possible equilibrium. Here, unlike the previous cases when $R \geq 0$, M_1 will drop out of election if the other member's policy position is chosen as the party position. Therefore, a policy can be k distance away from her own ideal point, in which case the following condition will prevent the deviation,

$$-k^2 + \frac{R - C}{2} \geq -\frac{1}{(n-1)^2} \iff C \leq 2\left(\frac{1}{(n-1)^2} - k^2\right) + R.$$

Therefore, $\frac{1}{(n-1)^2} \leq C \leq 2\left(\frac{1}{(n-1)^2} - k^2\right) + R$, in equilibrium. Note that the first inequality ensures that neither of the two candidates drops out of election. \square

Theorem 5 (Two-party equilibrium($R < C$)) (i) *There is no two-party equilibrium when the number of voters is odd.*

(ii) *There is a two-party equilibrium $s \in S$, where $P_1(s) = \{M_1\}$ and $P_2(s) = \{M_2\}$ only if the following protocol is specified: If M_1 and M_2 join to form a new party, then the party policy be within at least $k = \frac{1}{\sqrt{2(n-1)}}$ distance away from either x_{M_1} or x_{M_2} and $2k^2 - \frac{1}{(n-1)^2} \leq C \leq \frac{1}{2}\left(R + \frac{1}{(n-1)^2}\right)$. In equilibrium, voters to the left of M_1 vote for M_1 , while those on the right of M_2 vote for M_2 .*

This can be proved using similar arguments as the ones made for Theorem 2.

Theorem 6 (Multi-party equilibrium($R < C$)) *There is no equilibrium where the number of parties is greater than or equal to 3.*

The proof of this is very similar to the one for $R > C$. Suppose n is odd. By similar arguments as the ones made earlier, we can show that the outcome of the election must be the median voter's policy position. Otherwise party members from different parties on one side of the median policy can consolidate their parties. By doing so, not only do they improve their outcomes but reduce their share of the total net costs $R - C < 0$. Similar arguments can be made when n is even.

3.2 Discussion

The results in the previous sections throw light on the following question: *Do parties affect policy outcomes?* In particular, the literature on electoral competition has often studied the effect (if any) of institutional and party structures on policy outcomes. A measure of effectiveness in these models is the distance between the implemented

policy outcomes and the median's ideal policy position. Since the benchmark output is the median voter's policy, under positive net benefits ($R > C$), one may expect a group to form to obtain a different policy outcome. However, as pointed out by the results the median voter has a monopoly power in such settings. Levy (2004)'s results also indicate that parties are ineffective in this regard when the policy dimension is one. Our results are consistent with this observation except the case when $R < C$. In the latter case, there is an additional one-party equilibrium where the outcome is not equal to the median voter's policy outcome when the number of voters is odd. This is due to the high cost of participation which results in a trade-off between participating in election versus a better policy outcome. For a high cost of participation, the median voter prefers to stay out, even though she would win the election. Therefore, for a given range of the costs, non-median outcomes can be implemented. This seems to reflect the situation in some developing countries which often experience non-centrist policy outcomes in elections. Chauvet and Collier (2009) finds evidence of extreme policy outcomes in many developing countries in Asia and Africa. Factors like the high likelihood of military coups and unstable democratic processes contribute to this. These are often accompanied by high costs of political participation. This affects the decisions of potential winning candidates and prevents them from participating in elections. This in turn leads to more extreme policy outcomes since only the extremely ideological candidates find it worthwhile to participate.

In all the other cases ($R \geq C$), the median voter comes out strong as in most works on coalition formation (Osborne and Tourky 2008; Levy 2004; Dhillon and Lockwood 2002). Morelli (2004) also find a strong policy bias in favor of the median outcomes in a multi-district model of party formation. Therefore, our results validate the empirical and theoretical observations on policy outcomes in developing countries which has not been captured before. Our result on the number of parties in equilibrium are also consistent with the Duverger's law: At most two parties exist in a plurality system of voting. There is divergence from the median outcomes in the one-party equilibrium when $C > R$. However, the set of two-party equilibria remains the same throughout the analysis.¹⁹

Our results can also be obtained through an alternative approach to study party formation. Consider an electoral competition model where a continuum of voters are uniformly distributed on the unit interval and a set of candidates where only the latter can participate in elections. In such a setting, we conjecture that similar results as the ones in this paper can be obtained. It would be important to obtain the qualitative result validating the Duverger's law regarding the non-existence of multi-party equilibrium. However, such an analysis would require a different set of arguments using the distribution of voters' ideal points.²⁰ The following two assumptions would be crucial for such a conjecture to be valid: (i) Only adjacently placed candidates are allowed to form links, and (ii) the equilibrium notion is the same, i.e., pairwise stability. The intuition behind this is the following: For multi-party equilibria with three or more parties, the extreme two parties would want to join (through adjacently placed candidates in different parties) and win the election. This would be strictly beneficial for the

¹⁹ Recall that two-party equilibrium only exists when n is even.

²⁰ I am thankful to an anonymous reviewer for pointing out the comparison between the two models.

deviating pair of individuals. Similar deviations between adjacently placed candidates can be replicated in the alternative benchmark model since the distribution of voter ideal points is uniform and continuous. However, relaxing either of the above two assumptions may render the model intractable.

4 Conclusion

We provide a framework for party formation with mutually agreeable links. This allows us to characterize different types of party equilibria in the one-dimensional policy space. We find that in most equilibria, only the median voter(s) participate in election but when costs are higher than rents, non-median candidates may stand alone in election. In the latter cases, non-median outcomes can also be implemented in equilibrium. There is no multi-party equilibria in our model, since members in extreme parties would either prefer to collaborate to improve outcomes or extreme candidates would rather drop out of election. We are also able to validate the Duverger's law for plurality voting systems.

It remains an open question if our results would continue to hold for a more general link formation model where non-adjacent candidates can offer links to each other. One may also consider a more general notion of equilibrium like strong stability. However, additional assumptions on party policy protocols may be needed to make them tractable. Results from network theory as provided by Jackson and Moselle (2002) and Goyal (2012) could also prove useful in such contexts.

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