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Working Paper

Product quality, product price, and share dynamics in the German compact car market

Jena economic research papers, No. 2010,024

Provided in Cooperation with:
Max Planck Institute of Economics

Suggested Citation: Cantner, Uwe; Krüger, Jens J.; Söllner, René (2010) : Product quality, product price, and share dynamics in the German compact car market, Jena economic research papers, No. 2010,024, Univ. [u.a.], Jena

This Version is available at:
http://hdl.handle.net/10419/32632

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www.jenecon.de

ISSN 1864-7057

The JENA ECONOMIC RESEARCH PAPERS is a joint publication of the Friedrich Schiller University and the Max Planck Institute of Economics, Jena, Germany. For editorial correspondence please contact markus.pasche@uni-jena.de.

Impressum:

Friedrich Schiller University Jena
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D-07743 Jena
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Uwe Cantner* Jens J. Krüger † René Söllner ‡

April 7, 2010

Abstract

The present paper examines one of the central elements of evolutionary thinking – competition formalized by the replicator dynamics mechanism. Using data on product characteristics of automobiles sold on the German domestic market over the period 2001-2006, we construct a competitiveness or fitness variable for each car model applying non-parametric efficiency measurement techniques. The basic question we intend to answer is whether cars providing a higher quality-price ratio for consumers tend to increase their market share compared to variants with lower quality-price ratios. The relationship between a car models' fitness and its market performance is empirically tested in a regression framework. The results show that the principle of ‘growth of the fitter’ is working as suggested by evolutionary theory. In particular, we find that car models with considerably lower fitness than the market average lose, whereas models with above-average fitness gain additional market shares.

JEL classification: O33, D12, L15

Keywords: Replicator Dynamics, Product Characteristics, Data Envelopment Analysis

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1 Introduction

Since the seminal work of Nelson and Winter (1982) a multitude of methodological advancements have deepened and extended the theoretical understanding of the mechanisms of economic evolution (e.g. Dopfer, 2001; Foster and Metcalfe, 2001). One of the fundamental principles underlying evolutionary modelling is competition in general and the principle of ‘growth of the fitter’ in particular (see Nelson and Winter, 1982; Metcalfe, 1994, 1998; Winter et al., 2000, 2003). The principle dates back to Fisher (1930) and can formally be described by the mechanism of replicator dynamics, which states that:

\[ \dot{s}_i = \frac{ds_i}{dt} = \lambda s_i \left( f_{it} - \bar{f}_t \right) \quad i = 1, \ldots, n \quad t = 1, \ldots, T \]

where \( \lambda > 0 \) is a parameter controlling the speed of selection and \( s_{it} \) denotes the period \( t \) market share of an firm \( i \) within a population of \( n \) competing firms. \( f_{it} \) describes the fitness of firm \( i \) in period \( t \) and \( \bar{f}_t = \sum s_{it} f_{it} \) is the average weighted fitness in the population. Replicator dynamics implies that firms tend to grow or decline in terms of market shares depending on whether their fitness is above or below the average fitness of all other competing firms in the market. Despite its simplicity and elegance, when the basic mechanism is applied in an effort to explain the development of certain sectors or entire economies, models of high complexity, which do not result in analytical solutions, are frequently obtained (?). As a consequence, agent-based simulation modelling has become the main tool in the evolutionary literature (e.g. Kwasnicki and Kwasnicka, 1992; Saviotti and Mani, 1995; Dosi et al., 1995; Marsili, 2001). Empirical attempts trying to answer the question of whether market selection is operating as proposed by evolutionary theory are few and far between. This is quite astonishing given the central position of replicator dynamics in evolutionary economics. Although it appears to be trivial, in practice such an analysis is not easily accomplished (Andersen, 2004), since the data requirements are tremendous. An exception in this respect is a study by Metcalfe and Calderini (2000), who compute the selection parameter (measuring the speed of selection) for a dataset on the Italian steel industry. However, partly due to data limitations, Metcalfe and Calderini cannot convincingly show that an evolutionary process according to replicator dynamics is at work. Recently, other scholars have concentrated on the empirical analysis of evolutionary principles. Using a database of Italian manufac-
turing firms, Bottazzi et al. (2008) investigate how profitability and productivity is related to firm growth. Their results show that the overall selection process is only weakly operating in the expected way. In fact, they do not find a considerable relationship between profitability (respectively productivity) and firm growth (see also Dosi, 2005). In a related study on French manufacturing firms, Coad (2007) raises doubts about the validity of the principle of ‘growth of the fitter’. He finds only a minor influence of profits on sales growth and concludes that evolutionary models should rather abandon the assumption of a direct relationship between them.

Another branch of empirical studies investigates the formal mechanism of replicator dynamics by linking it to the dynamics of aggregate productivity development (Cantner and Krüger, 2008; Krüger, 2008). By a decomposition of aggregate productivity change at an industry-level using a dataset of German manufacturing firms over the period 1981-1998, Cantner and Krüger (2008) find a weak tendency that above-average productivity firms are selected in favour of below-average productivity firms. This gives support to a market selection process with respect to the replicator dynamics.1 Note, however, that the results need to be interpreted with caution since Krüger (2008), in follow-up study, could not confirm a consistent statistical significance.

The main purpose of this paper is to contribute to the very few studies that deal with empirical tests of the replicator dynamics mechanism. The current paper is exceptional in this respect, since we consider products rather than firms to be the primary entity of selection on markets. We put forward the idea that the competitiveness of a product depends on the values of its characteristics and its price. In fact, we assume that products with better characteristics and lower prices will be preferred by consumers.2 If our conjecture holds, products offering a higher value to consumers exhibit a competitive advantage. This should come along with an increasing market share within a population of competing products. The crucial part of the research project is to test this relationship empirically. In order to do so, we construct a fitness variable for each product model offered on the market. The proposed fitness variable is based on the characteristics of a product and will

1 The decomposition of aggregate productivity change was conducted using the formula proposed by Foster et al. (1998). An alternative decomposition formula can be found in Griliches and Regev (1995).
2 For a number of reasons this might not be case. Consumers may not have the ability to distinguish the quality of goods or factors such as brand recognition circumvent the selection by consumers of ‘objectively’ best products.
be interpreted as the ratio of product quality\footnote{Product quality is determined by the intrinsic characteristics of a product. A detailed description is given in section 3.} to product price. The computation of a product’s fitness is carried out by using non-parametric techniques, which we borrow from efficiency analysis.

There is some research examining the competitiveness of products by comparing price jointly with quality (e.g. Papahristodoulou, 1997; Fernandez-Castro and Smith, 2002; Lee et al., 2005; among others). These studies describe a product as a point in the price-quality space and construct a frontier which is shaped by the products with lowest price and highest quality. The competitiveness of a product is measured by the distance to the frontier and specified by a single index number called product efficiency. The present paper is related to these studies with two major differences: Firstly, we employ robust non-parametric methods to compute the efficiency of products. Robust techniques seem better suited in this framework for their convenient property of not being affected by measurement errors and/or outliers in the data. Second, to the best of our knowledge, the product efficiency concept is applied for the first time to test the replicator dynamics mechanism econometrically. This is done by treating the computed efficiency index as a fitness indicator. Subsequently, the fitness indicator is employed as an explanatory variable in a regression to estimate the parameters of the replicator dynamics equation. The proposed methodology is applied to a specific segment of the German automobile market.

The paper is structured as follows. After this short introduction, we will introduce our multi-dimensional measure of fitness in section 2. This is followed by a discussion of non-parametric techniques used to assess the competitiveness of products in section 3. Section 4 reports the results of the empirical analysis. A discussion of the main limitations of our methodology is offered in section 5. Section 6 summarizes the findings.

## 2 A Multi-dimensional Measure of Fitness

In most analyses following evolutionary principles, the entity that is selected during the process of competition is a firm. Accordingly, a reallocation of market shares is explained by a market selection process operating on firm-specific characteris-
tics. These firm-specific characteristics are assumed to represent the fundamental sources of firms’ differential competitiveness, namely the fitness within a population of heterogeneous economic agents. In evolutionary economics (including theoretical analysis and empirical studies), profit rates, productivity measures, unit costs of production and product price are most frequently used as proxies for firm competitiveness (see e.g. Nelson and Winter, 1982; Metcalfe, 1994; Mazzucato, 1998; Bottazzi et al., 2008).

Even though firms are affected by market selection, we claim that firms are not selected directly. In consumer goods markets it is a firm’s output – namely its products – that is selected through the market process. As a consequence, we consider products to be the primary entity of selection, which leads to an indirect selection of the producing firm.

However, according to Lancaster (1966), consumers do not seek a unique commodity of constant quality, but rather try to satisfy a number of wants through the consumption of a good. These multiple wants are satisfied by different product characteristics, and it is these characteristics, not goods themselves, from which the consumers derive utility. As a result, any fitness variable constructed in this kind of evolutionary framework is required to take the characteristics of products explicitly into account.

Based upon Lancaster’s work, Saviozzi and Metcalfe (1984) introduced the twin characteristics representation of a product technology. Accordingly, a product can be identified by two sets of characteristics. The technical characteristics describe the internal structure of a product, while service characteristics determine the utility for the users during the process of consumption. Since service characteristics cannot be ‘produced’ directly, there is a pattern of mapping between them. The characteristics approach has been used in various applications. Most frequently, it is applied to measure the degree of technological progress (e.g. Grupp, 1994; Grupp and Hohmeyer, 1986; Grupp and Maital, 2001; Dodson, 1985; Saviozzi et al., 1982; Saviozzi, 1985; Gibbons et al., 1982) and to identify the emergence of product niches and dominant product designs at the industry level (Frenken and Leydesdorff, 2000; Frenken et al., 1999).

In this paper, the characteristics approach is the basis upon which to assess the competitiveness of products. Specifically, we measure a product’s competitiveness by
computing its distance from a frontier that is spanned by those products that attain a maximum level of competitiveness in a multi-dimensional product characteristics space. In the empirical analysis, this distance from the frontier is used as a proxy for the fitness of a product model. The next section discusses how to derive such a distance measure.

3 DEA as a Method to Assess the Performance of Products

Data Envelopment Analysis (DEA) is a linear programming procedure to measure the efficiency of observations on the basis of multiple inputs and multiple outputs. The efficiency level of an observation indicates its relative performance and is obtained by comparing an observation to a set of best practice units which shape a so-called efficiency frontier (Cooper et al., 2007).

Another concept to efficiency measurement frequently applied in the literature is the stochastic frontier approach (SFA). The SFA is an econometric estimation technique introduced independently by Aigner et al. (1977) and Meesen and van den Broeck (1977). Compared to most nonparametric approaches the SFA has the advantage in handling measurement errors and random influences on efficiency. Due to its parametric nature, however, an a priori assumption about the shape of the efficiency frontier is required.

The DEA has its origin in the seminal work of Charnes et al. (1978) and Banker et al. (1984). The basic idea of the DEA is to compare input-output combinations of Decision Making Units (DMUs) and to assess their relative performance. Relative performance in terms of efficiency is measured by the distance of DMUs (e.g. firms, products, etc.) to a piecewise empirical extremal production surface which represents the best practice production function. DEA models have a number of attractive properties. DEA approaches, for instance, do not assume that all DMUs have an identical production function. The parameterization of the aggregation functions and thus the aggregation weights are determined endogenously. Moreover, see Kalirajan and Shand (1999) for a detailed comparison between SFA and DEA techniques.

See Charnes et al. (2000) and Cooper et al. (2007) for an overview about various applications of the DEA concept.
the possibility of using multiple inputs and outputs at the same time is a major advantage of DEA over SFA.

The central idea of the current paper is to employ the concept of nonparametric efficiency measurement to assess the competitiveness of products. In fact, we assume that consumers do not search for products with maximum quality or minimum price, but seek to optimize on the quality-price-ratio. If we perceive the quality of a product \( i \) at time \( t \) as a linear combination of \( J \) product characteristics \( q_{itj} \) \((j = 1, ..., J)\), collected together in a vector \( q_{it} = (q_{it1}, ..., q_{itJ})' \), and denote the product price \( p_{it} \), the ratio between product quality and product price can be formulized as

\[
e_{it} = \frac{a_1q_{it1} + ... + a_Jq_{itJ}}{p_{it}} = \frac{a'q_{it}}{b'p_{it}}
\]

where the vector \( a \) contains the weights for aggregating the product characteristics into the scalar product quality measure. \( b \) serves as a normalizing constant.

The fitness measure \( e_{it} \) is larger if one or several of the measures for the product characteristics are larger at a given price or if the price is smaller for a given bundle of product characteristics. Thus, the fitness measure is analogous to a productivity index, generally defined as a ratio of an output aggregate to an input aggregate. Here, the output is what the consumer receives in terms of services from buying the product and the single input is the price he has to pay. This close resemblance justifies the application of methods for productivity analysis to derive a fitness variable which captures the competitiveness of a product in price-quality-space. In Appendix A.1, an output oriented variant of the DEA approach is used to describe the way in which to construct such a fitness measure.

The idea of using nonparametric concepts to quantify the performance of products has already gained interest in the literature. In business economics, DEA is frequently applied to derive market segmentations and to reveal competitive relationships among producers (Bauer et al., 2003; Despotis et al., 2001; Staat and Hammerschmidt, 2005). In engineering, DEA is used as a tool to measure the performance of machines and devices (e.g. Khouja, 1995; Sun, 2002; Triantis, 2003). Also, scholars of the economics of innovation and industrial economics recently employed nonparametric concepts for their purposes (Bernard et al., 1996; Haller and
The method presented in Appendix A.1 exhibits a severe drawback which is common to all standard DEA models. In fact, every deviation from the frontier is considered as inefficiency. Statistical noise or measurement errors are not accounted for. This makes the approach very sensitive to extreme data points and outliers. In order to overcome these limitations, the order-\( m \) approach to robust stochastic non-parametric efficiency measurement is applied here. The basic idea of order-\( m \) has been proposed by Cazals et al. (2002) and was developed further and applied to real data by Daraio and Simar (2005, 2007a,b), Simar (2007), Simar and Zelenyuk (2008) and Wheelock and Wilson (2004). A discussion of fundamentals of the order-\( m \) approach is provided in Appendix A.2. The application of the order-\( m \) method on a product dataset implies that the efficiency of each product is evaluated repeatedly against a partial product-efficiency frontier spanned by \( m \) of the sample products. This gives an estimator of our fitness variable which is quite robust to outliers and measurement errors.

4 Empirical Analysis

4.1 Data Description and Sample Selection

The subject of the empirical analysis is the segment of compact cars in the German market. We employ two distinct data sources to obtain the required information. Sales data is available from the Kraftfahrtbundesamt (KBA), Germany’s national road vehicle registration authority. The KBA annually publishes data on the frequency of sales of specific car models in the “New registrations for motor vehicle and vehicle trailer by type, size class, producer and federal state”\(^6\) statistics. The collected data covers the period 2001 to 2006. To ensure a rather homogeneous dataset and in order to avoid a comparison of “apples and oranges”, we restrict our analysis to a particular segment of the entire car market, namely the market for compact cars. To distinguish compact cars from non-compact cars, standard classifications

\(^6\) Statistics for Neuzulassungen von Kraftfahrzeugen und Kraftfahrzeuganhängern nach Fahrzeugarten, Größenklassen Herstellern, Typen und Bundesländern.
offered by the KBA are used.\footnote{Note that the KBA does not provide detailed information as to how such a classification is derived, i.e. what kind of technical specification [size, engine power etc.] is required to identify a car as a compact.}

Information on prices and quality attributes for each car model come from the ADAC, Germany’s largest automobile club. The ADAC annually publishes electronic catalogues containing data on prices, and technical and non-technical features of new cars. These electronic databases are used to collect data on quality attributes of cars. Altogether, information on 41 quality attributes for each variant of the numerous car models was accessible.\footnote{A model variant is a specific version of a car model that differs from another version of the same model by a few attributes such as the availability of optional items [supplementary equipment]. The main characteristics such as type of engine, horsepower or cylinder capacity for each variant of the same model are the same. See Appendix A.3 for a description of the terminology.} Note that the price information for new cars does not incorporate any sales returns or rebates which are quite often used in car purchasing. However, in the absence of more detailed price information, we assume that the list price is the most reliable proxy variable available.

Information on sales frequency is provided on the level of car models (e.g. VW Golf 1.6). Since the ADAC database offers data on prices and quality features for specific variants (e.g. VW Golf 1.6 Trendline), we proceed by aggregating price and quality data for the various variants of the same model. In fact, we calculate the arithmetic mean for each attribute over all variants of the same car model. This yields a vector of average quality attributes (including price) for each car model. After matching sales and product characteristics, we end up with a sample of 635 distinct car models over the period 2001 to 2006.

\section{4.2 The German Car Market}

In 2007, Germany accounted for almost 11\% of the worldwide automobile production (OICA, 2008). The German automotive industry is one of the major backbones of the German economy. With a strong labor force of around 745,000 the industry is one of the most important employers in the country (VDA, 2008). Germany is the largest national car market in Europe (ACEA, 2008). Between 1998 and 2007, on average, 3.43 million passenger cars per year were sold (KBA, 2008a). Currently, the total vehicle population is of about 41 million passenger cars (KBA, 2008b). In their buying decisions, German consumers tend to prefer domestic brands over the
products of foreign producers. According to statistics of the KBA (KBA (2008a)), in 2007, 64% of new registered cars were produced by German manufacturers. Figure 1 illustrates the dominance of domestic brands in Germany. We can see that VW (Volkswagen) is the undisputed market leader, followed by Mercedes, Opel, BMW and Audi. Ford, ranked at the sixth place, is the first foreign brand among the leading automobile manufacturers.

![Figure 1: Market share development of the 10 largest brands in Germany between 2001 and 2007.](image)

Source: KBA.

According to the KBA, the entire car market is divided into ten segments. The segments are defined in terms of horsepower, cylinder capacity, size, design and price. During the period under observation, the demand structure changed considerably (Figure 2). In 2001, cars of the middle class attracted 25% of new car buyers; small cars realized a market share of 17%. In subsequent years, small cars increased their market share to 19%, while middle class cars lost market share considerably. In 2007, the middle class accounted for only 16% of the entire market. Also the demand for vans exhibited a remarkable change. This segment increased its market penetration from 7% in 2001 to 12% in 2007. The market share of all other segments remained fairly stable over the time span. Figure 2 clearly indicates that the compact class is the largest segment in the German car market. Approximately 26% of all newly registered cars in Germany were compact cars.
4.3 Car Efficiency Estimates

In order to compute the efficiency of a car model – indicating its fitness – the order-m approach is applied to the data. The first step required is a selection of inputs and outputs. The choice of the 'right' characteristics is a crucial task, as it determines the accuracy of later statistical analyses. We have already pointed out that consumers are primarily interested in the services delivered by a product. However, since service characteristics cannot be produced 'directly', producers need to modify technical characteristics in order to enhance a product's service characteristics. In the case of a car, typical service characteristics influencing the evaluation of potential car buyers are speed of transportation, ecological efficiency, safety, space, convenience etc. Corresponding technical characteristics are engine power, fuel consumption, number of airbags, dimension, and special equipment available, to mention just a few.

Since cars are highly differentiated products, the full range of characteristics to distinguish one car model from another is very large (Papahlis, 1997). To guarantee a correct efficiency evaluation, ideally all of them should be taken into account. However, various quality attributes are not measurable in an objective way.
(e.g. style) or necessary data on specific characteristics are not accessible. Faced with that problem, we restrict our analysis to a small subset of the possible characteristics. In order to ensure that the selected characteristics are relevant for the buying decision of car buyers, we used expert judgments gathered by means of interviews, questionnaires and other types of corresponding publications. In particular, we applied only those characteristics that are frequently regarded as important by consumer reports or related studies (e.g. ADAC, 2007; DAT, 2006; Oh et al., 2005; Staat et al., 2002). To avoid the use of redundant information, we conducted a correlation test among the relevant characteristics (see Appendix A.4).

Finally, the following technical characteristics were incorporated in the efficiency measurement of a car model: The maximum engine power in kilowatts is used as a proxy for the performance of a car. Fuel efficiency, defined as the amount of covered distance (in kilometres) per litre of petrol, indicates the environmental friendliness of a car model and is obtained by calculating the reciprocal of fuel consumption. As an indicator for the loading capacity, we utilize the luggage space (in litres), and as a proxy for safety, we employ the dimension (length\times width\times height) of a car in cubic meters.\footnote{As an alternative to the dimension as a proxy for safety the number of airbags can be applied. However, until the year 2004, we had only information about the endowment with driver-, passenger- and side-airbag. Since airbags became more and more a standard feature during the period of investigation, the variation in the number of airbags declined sharply. To avoid problems caused by this low variation, we follow Papapristodoulou (1997) and rely on the dimension as a proxy for safety. Note, however, that the efficiency scores calculated using the number of airbags did not differ substantially.} As a cost parameter the list price for each car model is utilized. Basic descriptive statistics of the characteristics incorporated in the yearwise order-\textit{m} estimation are reported in Table 1.

The second step of the empirical analysis is the computation of order-\textit{m} efficiency estimates. The four technical performance characteristics serve as outputs in the nonparametric frontier estimation approach. As sole input variable, the list price of a car model is used. For the purpose of this study, efficiency is measured in output orientation. The order-\textit{m} estimates are computed using the package “FEAR” for R, supplied by Paul Wilson on his web page (see Wilson, 2008). Note that, in contrast to standard DEA approaches, the order-\textit{m} estimates are not bounded by 1.\footnote{The reason for this is the allowance for random noise.} As the number of car models is very large, Table 2 illustrates the summary statistics of efficiency estimates for different years.

\noindent Table 2 reveals a remarkable degree of stationarity. Minimum, maximum, median
Table 1: Descriptive statistics of characteristics incorporated in the order-$m$ estimation (by year)

<table>
<thead>
<tr>
<th></th>
<th>Kilo watt</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>80</td>
<td>21</td>
<td>44</td>
<td>160</td>
</tr>
<tr>
<td></td>
<td>14.90</td>
<td>2.94</td>
<td>9.09</td>
<td>22.22</td>
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<tr>
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<td>400</td>
<td>8.1</td>
<td>177</td>
<td>550</td>
</tr>
<tr>
<td></td>
<td>10.54</td>
<td>0.61</td>
<td>9.02</td>
<td>12.26</td>
</tr>
<tr>
<td></td>
<td>18292</td>
<td>3825</td>
<td>9965</td>
<td>30550</td>
</tr>
<tr>
<td>2002</td>
<td>84</td>
<td>23</td>
<td>44</td>
<td>195</td>
</tr>
<tr>
<td></td>
<td>15.03</td>
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<td>8.3</td>
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<td></td>
<td>10.72</td>
<td>0.57</td>
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<td>12.26</td>
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<td></td>
<td>19475</td>
<td>3918</td>
<td>10890</td>
<td>37340</td>
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<tr>
<td>2003</td>
<td>86</td>
<td>25</td>
<td>50</td>
<td>195</td>
</tr>
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<td></td>
<td>15.06</td>
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<td>25</td>
<td>50</td>
<td>195</td>
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<td></td>
<td>19788</td>
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<td>20008</td>
<td>3706</td>
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Table 2: Summary statistics of efficiency estimates

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<tr>
<th>Year</th>
<th>Min.</th>
<th>1st Quantile</th>
<th>Median</th>
<th>Mean</th>
<th>3rd Quantile</th>
<th>Max.</th>
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</thead>
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<td>0.919</td>
<td>1.017</td>
<td>1.059</td>
<td>1.068</td>
<td>1.107</td>
<td>1.323</td>
</tr>
<tr>
<td>2002</td>
<td>0.941</td>
<td>1.022</td>
<td>1.062</td>
<td>1.074</td>
<td>1.113</td>
<td>1.401</td>
</tr>
<tr>
<td>2003</td>
<td>0.947</td>
<td>1.020</td>
<td>1.061</td>
<td>1.074</td>
<td>1.110</td>
<td>1.405</td>
</tr>
<tr>
<td>2004</td>
<td>0.938</td>
<td>1.016</td>
<td>1.061</td>
<td>1.073</td>
<td>1.105</td>
<td>1.364</td>
</tr>
<tr>
<td>2005</td>
<td>0.913</td>
<td>1.011</td>
<td>1.048</td>
<td>1.066</td>
<td>1.095</td>
<td>1.375</td>
</tr>
</tbody>
</table>
and mean efficiency do not exhibit remarkable changes over time. Additionally, we provide Kernel density estimates to display the distribution of the efficiency measures over time (Figure 3). Visual inspection of this plot displays two distinct features: Firstly, the distribution of efficiency scores remains nearly constant over time. Second, the right tails of the efficiency distributions in Figure 3 indicate a number of car models with relatively high efficiency estimates compared to the rest of the market.

### 4.4 Car Efficiency and Market Success

In this section, we analyze the relationship between performance superiority and market success. As previously described, efficiency is assumed to indicate the competitiveness of a car in quality-price space. Now, we employ this competitive measure as a fitness indicator in the replicator dynamics equation. According to the ‘growth of the fitter’ principle, we suspect above-average efficiency to be associated with growth in terms of market shares at the product level.
The replicator dynamics mechanism in continuous time is exhaustively described by

$$\dot{s}_i = \frac{ds_i}{dt} = \lambda s_{it}(e_{it} - \bar{e}_t), \quad \forall i = 1, ..., n$$

(3)

where $s_{it}$ stands for the market share of car model $i$ within a population of $n$ competing car models, $e_{it}$ denotes the efficiency of a certain model and $\bar{e}_t = \sum s_{it}e_{it}$ is the average (weighted) efficiency on the market. Conversion of (3) into a model of discrete time leads to

$$\Delta s_{i,t:t+k} = s_{i,t+k} - s_{it} = \lambda s_{it}(e_{it} - \bar{e}_t).$$

(4)

In order to estimate this equation, we transform (4) into the following econometric model:

$$\Delta s_{i,t:t+k} = \beta_0 + \beta_1 \text{DevAvg}_{it} + \zeta_{it}, \quad t = 2001, ..., 2005 \quad k = 1, ..., 5$$

(5)

where $\text{DevAvg}_{it} = s_{it}(e_{it} - \bar{e}_t)$ is the sole explanatory variable. $\beta_1$ is the parameter to be estimated, and $\zeta_{it}$ is an i.i.d. error term. The dependent variable, $\Delta s_{i,t:t+k}$, denotes the change in market share of car model $i$ between period $t$ and $t+k$ within a cohort of competing car models on the market at time $t$. The term $\text{DevAvg}_{it}$ is the relative distance of a car model from the (weighted) average efficiency of the market. From the theoretical considerations described above, we expect a positive sign of the estimated coefficient $\beta_1$. A positive coefficient implies that car models with (share-weighted) above-average efficiency at time $t$ tend to increase their market share between $t$ and $t+k$, while models with below-average efficiency scores lose in terms of market shares.

The first step in our econometric analysis is to estimate equation (5) using ordinary least squares regressions (OLS). Standard diagnostics for linear regressions revealed a heteroscedastic nature of the data. A potential source for heteroscedasticity that

$^11\beta_1$ can be interpreted as a parameter that accounts for the speed of selection and is equivalent to the parameter $\lambda$ in equation 4.
violates the i.i.d assumption of the OLS estimator could be the correlation of market shares of car models within the same product line. One way to rule out misleading statistical inference would be to employ clustered robust standard errors. However, since the application of clustered standard errors requires a large number of clusters (i.e., product lines) and/or balanced cluster sizes (Wooldridge, 2003), we instead use heteroscedasticity-consistent standard errors, as proposed by MacKinnon and White (1985).

Table 3 presents the regression results estimating equation (5) for various years and changing time lag $k$ of the dependent variable.

<table>
<thead>
<tr>
<th>$k=1$</th>
<th>$t=2001$</th>
<th>$t=2002$</th>
<th>$t=2003$</th>
<th>$t=2004$</th>
<th>$t=2005$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DevAvg</td>
<td>-0.3813 (0.8601)</td>
<td>-0.1921 (0.72964)</td>
<td>10.1254** (4.2198)</td>
<td>-3.0383 (2.7386)</td>
<td>-1.526 (3.3214)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.01864</td>
<td>0.3892</td>
<td>0.3255</td>
<td>0.06967</td>
<td>0.01718</td>
</tr>
<tr>
<td>Obs.</td>
<td>326</td>
<td>286</td>
<td>300</td>
<td>313</td>
<td>336</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$k=2$</th>
<th>$t=2001$</th>
<th>$t=2002$</th>
<th>$t=2003$</th>
<th>$t=2004$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DevAvg</td>
<td>-2.6381** (1.1093)</td>
<td>10.6850** (5.3073)</td>
<td>13.4381*** (5.0123)</td>
<td>9.2037** (3.7434)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.03794</td>
<td>0.2551</td>
<td>0.1866</td>
<td>0.06857</td>
</tr>
<tr>
<td>Obs.</td>
<td>326</td>
<td>286</td>
<td>300</td>
<td>313</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$k=3$</th>
<th>$t=2001$</th>
<th>$t=2002$</th>
<th>$t=2003$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DevAvg</td>
<td>6.4514 (6.4364)</td>
<td>12.8320** (5.1065)</td>
<td>10.6517* (5.5161)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.08511</td>
<td>0.1642</td>
<td>0.06322</td>
</tr>
<tr>
<td>Obs.</td>
<td>326</td>
<td>286</td>
<td>300</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$k=4$</th>
<th>$t=2001$</th>
<th>$t=2002$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DevAvg</td>
<td>9.7523* (5.3163)</td>
<td>14.5606** (7.3995)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.06845</td>
<td>0.08308</td>
</tr>
<tr>
<td>Obs.</td>
<td>326</td>
<td>286</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$k=5$</th>
<th>$t=2001$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DevAvg</td>
<td>8.4041 (5.7581)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.0195</td>
</tr>
<tr>
<td>Obs.</td>
<td>326</td>
</tr>
</tbody>
</table>

Note: Dependent variable is the market share change between $t$ and $t+k$. Robust standard errors in parentheses. *** 1% level of significance, ** 5% level of significance, * 10% level of significance.

The initial model incorporates the deviation from mean efficiency of model $i$ at time $t$. Other control variables have not been included since we want to estimate the replicator dynamics equation in its pure form. A closer look at the regression results in
Table 3 reveals that efficiency levels of car models in a base year only partly explain the market share development in subsequent years. The estimates for the group of regressions with \( k = 1 \), i.e. when the independent variable is the change in market share between \( t \) and \( t + 1 \), do not point to a significant effect of product efficiency on the market share development. Only for \( t = 2003 \) is significantly positive coefficient obtained. However, increasing the parameter \( k \) makes us more confident that our fitness variable works in the expected way. Except for the parameter constellation \( k = 2 \) and \( t = 2001 \), the sign of the estimated coefficients is always positive. This suggests that car models providing above-average value to consumers were able to increase their market share compared to models with performance-price ratios below the market average. However, even if most of the estimates turn out to be positive, not all of corresponding coefficients are significant. Moreover, an unstable and sometimes fairly small R-squared does not reflect a good fit of this simple model. Obviously, other unobserved factors heavily influence the market performance of automobiles. For instance, we could think of brand effects that impinge on sales. The impact of brand image in the automobile market is subject of a number of studies (De Pelsmacker, 1988; Manering and Winston, 1991; Nichols, 1998; among others). This literature stresses that the brand image is a key element for the long-term success on the market. Swait (1994) argues that the impact of brand image on the buying decision becomes even stronger when costumers imperfectly observe the attributes of products. With respect to automobiles, this is certainly the case. Nevertheless, some preliminary conclusions from the initial model can already be drawn: (i) in the short run, i.e. with a lag of one year in the dependent variable, no clear effect of a car model’s efficiency on its market performance can be monitored; (ii) in the longer run, product efficiency positively affects market share growth; (iii) factors other than efficiency apparently determine the economic performance of car models. In the following, we check whether these preliminary findings turn out to be robust across alternative specifications of the basic model.

In the next step of our analysis, we move from yearly to pooled OLS regressions. Pooling yearly cross-sections increases the sample size and provides more powerful test statistics with respect to statistical inference. In order to account for brand specific factors we include dummy variables for the ten largest compact car producers in all our regressions.\(^{12}\) Further, year dummies enter the estimations (baseline year = 2001). The pooled OLS estimates are displayed in Table 4.

\(^{12}\) All other brands in the market serve as a reference group.
The results of Table 4 suggest that brand-specific factors entail an impact on the market success of car models. The statistical significance of the manufacturer dummies indicates that market share changes might partly be due to idiosyncratic effects related to the car producer. Over the observed time span, in particular, automobiles from Opel and Renault performed poorly in economic terms. To a minor extent the same holds for Citroen and Audi. Quite the contrary is found for Toyota. Evidently, Toyota was able to meet the taste of German consumers, which kept the market shares of its car models growing. Notably, the inclusion of additional
control variables for brand-specific impacts does not substantially change the basic findings. With regard to the relationship between car model efficiency and market share changes, we obtain positive signs coefficients. However, as in the case of the yearly OLS, a positive and significant impact of efficiency on market shares cannot be observed in the short run (i.e. with lag of one year). Increasing the time lag of the response variable makes us more confident that car efficiency works in the expected way. For \( k \in \{2, 4\} \), the estimated coefficients are statistically significant, which indicates a positive impact of efficiency on changes in market shares. Hence, producing car models that offer a high value for customers does not seem to pay off in the short but rather in the longer run.

Next, a variable \( Age \) accounting for the number of years since market introduction of a car model is incorporated in the estimation. Since car buyers might prefer car models that are more up to date, the variable reflects the valuation of consumers for modern cars. Another rationale behind the inclusion of this variable is that car models might exhibit a negative growth in market shares due to market exit. This market exit can be the result of a bad economic performance but it can also be induced by the decision of a manufacturer to stop a model’s production following the introduction of a successor model. We assume this is more likely to occur for cars which had already been a considerable time on the market. The variable \( Age \) controls for these effects. The empirical results are illustrated in Table 5.
Table 5: Pooled OLS regression results

<table>
<thead>
<tr>
<th></th>
<th>(k=1)</th>
<th>(k=2)</th>
<th>(k=3)</th>
<th>(k=-4)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DevAvg</strong></td>
<td>0.6304**</td>
<td>6.7680***</td>
<td>9.6829***</td>
<td>11.4473***</td>
</tr>
<tr>
<td></td>
<td>(1.5444)</td>
<td>(2.3114)</td>
<td>(2.9519)</td>
<td>(1.7442)</td>
</tr>
<tr>
<td><strong>Age</strong></td>
<td>-0.0005***</td>
<td>-0.0004**</td>
<td>-0.0006**</td>
<td>-0.0009***</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0002)</td>
<td>(0.0002)</td>
<td>(0.0005)</td>
</tr>
<tr>
<td><strong>VW</strong></td>
<td>0.0003***</td>
<td>-0.0009</td>
<td>-0.0010</td>
<td>0.0003</td>
</tr>
<tr>
<td></td>
<td>(0.0005)</td>
<td>(0.0008)</td>
<td>(0.0014)</td>
<td>(0.0019)</td>
</tr>
<tr>
<td><strong>Opel</strong></td>
<td>-0.0011</td>
<td>-0.0028***</td>
<td>-0.0044***</td>
<td>-0.0067***</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0005)</td>
<td>(0.0009)</td>
<td>(0.0029)</td>
</tr>
<tr>
<td><strong>Ford</strong></td>
<td>0.0005</td>
<td>-0.0012</td>
<td>-0.0020</td>
<td>-0.0068***</td>
</tr>
<tr>
<td></td>
<td>(0.0004)</td>
<td>(0.0008)</td>
<td>(0.0014)</td>
<td>(0.0030)</td>
</tr>
<tr>
<td><strong>Mercedes</strong></td>
<td>-0.0009</td>
<td>0.0007</td>
<td>-0.0011</td>
<td>-0.0083</td>
</tr>
<tr>
<td></td>
<td>(0.0012)</td>
<td>(0.0032)</td>
<td>(0.0055)</td>
<td>(0.0060)</td>
</tr>
<tr>
<td><strong>Audi</strong></td>
<td>0.0009**</td>
<td>0.0024</td>
<td>-0.0041***</td>
<td>-0.0060***</td>
</tr>
<tr>
<td></td>
<td>(0.0012)</td>
<td>(0.0037)</td>
<td>(0.0014)</td>
<td>(0.0040)</td>
</tr>
<tr>
<td><strong>Toyota</strong></td>
<td>-0.0000</td>
<td>0.0042**</td>
<td>0.0075*</td>
<td>0.0068</td>
</tr>
<tr>
<td></td>
<td>(0.0004)</td>
<td>(0.0020)</td>
<td>(0.0041)</td>
<td>(0.0035)</td>
</tr>
<tr>
<td><strong>Skoda</strong></td>
<td>0.0011***</td>
<td>0.0000</td>
<td>0.0008</td>
<td>0.0017</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0006)</td>
<td>(0.0013)</td>
<td>(0.0033)</td>
</tr>
<tr>
<td><strong>Citroen</strong></td>
<td>0.0001</td>
<td>-0.0004</td>
<td>-0.0011</td>
<td>-0.0011</td>
</tr>
<tr>
<td></td>
<td>(0.0002)</td>
<td>(0.0004)</td>
<td>(0.0007)</td>
<td>(0.0015)</td>
</tr>
<tr>
<td><strong>Renault</strong></td>
<td>-0.0008***</td>
<td>-0.0023***</td>
<td>-0.0029***</td>
<td>-0.0040***</td>
</tr>
<tr>
<td></td>
<td>(0.0002)</td>
<td>(0.0004)</td>
<td>(0.0007)</td>
<td>(0.0022)</td>
</tr>
<tr>
<td><strong>Peugeot</strong></td>
<td>0.0003</td>
<td>0.0017</td>
<td>0.0015</td>
<td>0.0033</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0013)</td>
<td>(0.0020)</td>
<td>(0.0033)</td>
</tr>
<tr>
<td><strong>Constant</strong></td>
<td>0.0013***</td>
<td>0.0015***</td>
<td>0.0021**</td>
<td>0.0039***</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0006)</td>
<td>(0.0009)</td>
<td>(0.0020)</td>
</tr>
<tr>
<td><strong>Year Dummies</strong></td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td><strong>R-squared</strong></td>
<td>0.0532</td>
<td>0.114</td>
<td>0.127</td>
<td>0.118</td>
</tr>
<tr>
<td><strong>F-statistic</strong></td>
<td>5.418</td>
<td>10.280</td>
<td>9.340</td>
<td>6.155</td>
</tr>
<tr>
<td><strong>p-value</strong></td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td><strong>Obs.</strong></td>
<td>1558</td>
<td>1221</td>
<td>906</td>
<td>607</td>
</tr>
</tbody>
</table>

Note: Dependent variable is the market share change between \(t\) and \(t+k\). Robust standard errors in parentheses. *** 1% level of significance, ** 5% level of significance, * 10% level of significance.

What we find is fully in line with our expectation. The coefficients for the variable Age are all negative and significant, revealing a negative impact of time since market introduction on share development. As previously described, this might reflect the fact that newer car models are more attractive for potential buyers. It can also be, however, that older car models are more likely to be substituted by manufacturers. Unfortunately, the dataset does not allow us to disentangle both effects. Turning to
the relationship between efficiency and market share changes, the estimated coefficients keep their sign and magnitude compared to previous model specifications. In fact, except for a lag of one year, (share weighted) above-average efficiency scores tend to increase market shares in subsequent years. Further, significant coefficients for a number of producer dummies suggest that brand-specific factors matter.

Instead of using the (share weighted) deviation from average efficiency on the market, all model specifications were also estimated, employing merely the efficiency scores of automobiles as an explanatory variable. Quite interestingly, the results reveal a positive but never a significant effect of efficiency on changes in market shares.

5 Discussion

A shortcoming of this study might be that a dynamic perspective is not yet fully developed. In particular, we measure the competitive relation among products at a specific point in time by computing the corresponding efficiency values for each product. Then we explore the market share development over the subsequent years in order to answer the question whether a car model’s (share weighted) deviation from average efficiency in the base year $t$ exerts an influence on changes in market shares between $t$ and a certain point in time $t+k$. By doing so, we implicitly assume that the efficiency of a car model remains the same over the whole time span between $t$ and $t+k$. This is a strong assumption. For instance, one could think of car producers reacting to the market performance of their products by changing the price or the quality characteristics, e.g. by offering supplementary equipment. We checked our data for price changes during the time spent on the market. We find that the price of car models remains fairly stable. However, as we discussed earlier, our pricing information for new cars reflects list prices which do not incorporate temporary rebates or other price promoting methods.\footnote{However, in the absence of more detailed price information, we assume that the list price is the most reliable proxy variable available.} Concerning changes in quality attributes, we point out that our efficiency measure is based on the characteristics which are mostly purchasing relevant. We can rule out that these characteristics undergo a fundamental change during the lifetime of a product.\footnote{In the case of fundamental modifications concerning the engine or the dimensions of a car, producers have to apply for a registration approval by the KBA. In such a case, the KBA records the modified car as a new model.}
ment, this might do not hold. We cannot deny that luxury or convenience features impinge on the choice of consumers. However, if we conceive the consumer choice as a hierarchical elimination process as outlined by Devetag (1999), the reliance on the key characteristics can be justified. Nevertheless, implementing supplementary quality features in our analysis remains an important point on our research agenda.

6 Conclusions

The present paper aimed at shedding light on the empirical validation of the principle of ‘growth of the fitter’ as a central element of evolutionary thinking. In particular, we explored the relevance of the replicator dynamics mechanism in the German compact car market. Unlike most empirical analyses in an evolutionary framework, our approach considers products, namely car models, to be the primary units of selection on markets. Based on product characteristics, we calculated the fitness for each automobile on the market by employing a stochastic version of a non-parametric efficiency measurement approach. The fitness indicator was used to estimate the replicator dynamics equation econometrically. Our results provide preliminary evidence for the existence of a market selection process according to the replicator dynamics mechanism. Indeed, we find that, in the long run, car models with considerably lower fitness than the market average lose while models with above-average fitness gain additional market shares.

Future work should aim to expand the findings of this paper in at least three directions. First, a more dynamic perspective across longer time spans will certainly provide a more accurate representation of the observed phenomena. Second, many additional insights might be gained by looking at different industries and broader datasets. Third, in order to validate our results and to be able to derive more general conclusions we have to uncover additional factors explaining the economic success of products and firms in an industry. The fairly small explanatory power of our model reveals that there is still room for improvement in this respect. Nevertheless, the results obtained so far leave us with the strong belief that it is worth the effort to look at demand side factors and product characteristics in order to explore the patterns of competitive dynamics on consumer goods markets.
Acknowledgements

We would like to thank Marco Guerzoni, Tatiana Plotnikova, Viktor Slavtchev, Simon Wiederhold, and seminar participants at the GSBC-EIC, Jena, for their helpful comments on this draft. Their generous help is highly appreciated. We are thankful to Serguey Braguinsky and other participants of the European Meeting of Applied Evolutionary Economics (EMAEE) 2009 in Jena. The authors also wish to acknowledge the comments received from participants of the EEFS 2009 conference 'Current Challenges in the Global Economy: Prospects and Policy Reforms' in Warsaw and of the thematic meeting of the French Economic Association (AFSE) 2009 in Sophia Antipolis.
References


A Appendix

A.1 Product Evaluation Using DEA

Assume product quality of product $i$ at time $t$ is determined by a linear combination of $J$ product characteristics $q_{itj}$ ($j = 1, ..., J$), collected together in a vector $\mathbf{q}_{it} = (q_{it1}, ..., q_{itJ})'$, and denote the product price $p_{it}$, the quality-price-ratio, $e_{it}$, can be stated as

$$e_{it} = \frac{a_1 q_{it1} + ... + a_J q_{itJ}}{p_{it}} = \frac{\mathbf{a} \cdot \mathbf{q}_{it}}{p_{it}}.$$

where the vector $\mathbf{a}$ contains the weights for aggregating the product characteristics into the scalar product quality measure. The basic task is to compute the weights $\mathbf{a}$ in order to minimize the inverse fitness of product $i$, subject to a set of normalization restrictions, by solving the following fractional programming problem

$$\min_{\mathbf{a}, b} \frac{1}{e_{it}} = \frac{b p_{it}}{\mathbf{a} \cdot \mathbf{q}_{it}}$$

s.t. $\frac{b p_{it}}{\mathbf{a} \cdot \mathbf{q}_{it}} \geq 1$ \quad $\forall l = 1, ..., n$

$\mathbf{a} > 0$

$\mathbf{b} > 0$

The weight $b$ in that program just serves as a normalizing constant and has no effect on the ability of the approach to compute suitable aggregation weights for the product characteristics (in effect the aggregation weights will just be equal to $\mathbf{a}/b$). Charnes and Cooper (1962) have proposed a transformation into an ordinary linear programming problem which is straightforwardly solvable by the standard simplex algorithm. Performing the Charnes-Cooper transform gives the linear programming problem
The solution of the above linear program for each product and each time period gives a set of fitness variables $e_{it}$ which result from a multilateral benchmarking performed by DEA. The inverse fitness $1/e_{it}$ can be interpreted as the factor by which all characteristics of a product have to be increased in order to reach the fitness level of the fittest products in the sample (which get assigned a normalized fitness value of unity).

For the actual computation of the fitness variable, one can take the dual of the above linear programming problem

$$
\max_{\varphi, \lambda} \varphi
\text{s.t. } p_t^\prime \lambda \leq p_{it}
\varphi q_{it} - Q_t \lambda \leq 0
\lambda \geq 0
$$

where $p_t = (p_{1t}, ..., p_{nt})^\prime$ is the vector of prices in period $t$ and the quality vectors of the $n$ products are collected together in the $J \times n$ matrix $Q_t = (q_{1t}, ..., q_{nt})$. The solution values for $\lambda = (\lambda_1, ..., \lambda_n)^\prime$ give the weights for the observations that serve as the benchmarks against which the fitness is evaluated. The crucial feature of the duality theorem of linear programming which we exploit here is that the value of
the target function at the optimum is unchanged. Thus, at the optimum, it holds that the inverse of the solution value for $\varphi$ in the case of product $i$ at time $t$, $1/\varphi_{it}$, is equal to the fitness variable $e_{it}$.

All the above reasoning implicitly rests on a restriction that is related to the assumption of constant returns to scale in an efficiency measurement application. To gain a more flexible benchmark, we have to introduce the additional constraint that the $\lambda$-values sum to unity, $\sum_{i=1}^{n} \lambda_{i} = 1$, which is analogous to the variables-returns-to-scale property in a production context.
A.2 Order-\( m \) Approach to Robust Stochastic Nonparametric Efficiency Measurement

Consider a production technology, where the activity of decision making units (DMUs)\(^{15}\) is characterized by a set of inputs \( x \in \mathbb{R}^p_+ \) used to produce a set of outputs \( y \in \mathbb{R}^q_+ \).

The production set of technically feasible combinations \((x, y)\) is defined as:

\[
\Psi = \{(x, y) \in \mathbb{R}^{p+q}_+ | x \text{ can produce } y \}.
\]

The Farrell-Debreu measure of efficiency in input direction for a unit operating at the level \((x, y)\) would be:

\[
\theta(x, y) = \inf \{ \theta | \theta x, y \in \Psi \}.
\]

Here \( \theta(x, y) \leq 1 \) indicates the proportionate reduction of all inputs a DMU should attain to be considered as efficient. Daraio and Simar (2005) propose a probabilistic formulation of the production process. Accordingly, let us assume the probability for a DMU, operating at level \((x, y)\), to be dominated is given by \( H_{XY}(x, y) = \Pr(X \leq x | Y \geq y) \). The joint probability \( H_{XY}(x, y) \) can be decomposed as follows:

\[
H_{XY}(x, y) = \Pr(X \leq x | Y \geq y) \cdot \Pr(Y \geq x) = F_{X|Y}(x | y) \cdot S_Y(y).
\]

Supposing \( S_Y(y) > 0 \), in a stochastic framework the input oriented radial efficiency measure\(^{16}\), \( \theta(x, y) \), is defined as:

\[
\theta(x, y) = \inf \{ \theta | F_{X|Y}(\theta x | y) > 0 \} = \inf \{ \theta | H_{XY}(\theta x, y) > 0 \}.
\]

The conditional distribution function \( F_{X|Y}(\cdot | y) > 0 \) defines the attainable set of inputs for a given level of output. In practice, the efficiency measure, \( \theta(x, y) \), could be computed by estimating \( F_{X|Y}(\cdot | y) \) with the corresponding empirical distribution function.

Instead of taking the boundary of the feasible production set, the order-\( m \) approach defines as a benchmark "the average of the minimal value of inputs for \( m \) units ran-

---

\(^{15}\) In our case the DMUs are products.

\(^{16}\) Here we refer to the input oriented framework. See Daraio and Simar (2005) for a description of the analogous output oriented approach which is the conceptual basis for the efficiency estimates in this paper.
domly drawn according to $F_{X|Y} (\cdot | y)^n$ (Daraio and Simar, 2007a, p.16). Therefore, in order to estimate the efficiency of a specific DMU, it is compared to $m$ DMUs randomly drawn from the entire population of units. These reference units are required to produce at least the output level of the DMU under evaluation. This is undoubtedly a less extreme benchmark than the “absolute” minimal achievable level of inputs given a certain level of output. The resulting partial production set of order-$m$ is given by:

$$\Psi_m(y) = \{ (x, y') \in \mathbb{R}^{p+q}_+ \mid x_j \leq x, y'_j \geq y, j = 1, \ldots, m \}.$$  

The partial production set is used to define the input oriented radial efficiency measure:

$$\hat{\theta}_m(x, y) = \inf \{ \theta \mid (\theta x, y) \in \Psi_m(y) \}.$$  

According to Cazals et al. (2002), the order-$m$ efficiency score is simply the expected value of $\hat{\theta}_m(x, y)$ with respect to the distribution $F_{X|Y} (\cdot | y)$, i.e.

$$\theta_m(x, y) = E_{x|y} \left( \hat{\theta}_m(x, y) \mid Y \geq y \right).$$  

In practice, the easiest way to compute the order-$m$ efficiency measure is to conduct a Monte Carlo algorithm. For a sample of $n$ DMUs with input-output combinations $(x_i, y_i), i = 1, \ldots, n$ the algorithm proceeds in the following way: For any chosen DMU $i$ under investigation with output vector $y_i$, draw $m$ ($< n$) DMUs $(x_j, y_j), j = 1, \ldots, m$ from the sample with $y_j \geq y_i$ and compute the non-parametric efficiency measure with respect to this technology set. Denote the result by $\hat{\theta}_m^b$. Repeating these steps $B$ times, results in $B$ different efficiency measures $\hat{\theta}_m^1, \ldots, \hat{\theta}_m^B$ from which the order-$m$ efficiency measure of a DMU $i$ is finally computed as the average $\hat{\theta}_m = \sum_{b=1}^{B} \hat{\theta}_m^b$. 

Since each DMU is repeatedly evaluated against a partial production frontier, it is not required that the entire sample of observations has to be enveloped by the estimated frontier. This fact makes the resulting efficiency estimator very robust to extreme values and outliers. Moreover, the estimator does not suffer the so called curse

\footnote{For finite samples, in practise, several values $m$ and $B$ can be chosen. Typical default values are $m=25$ and $B=200$ (Daraio and Simar, 2007b).}
of dimensionality\textsuperscript{18} characterizing most non-parametric estimators.\textsuperscript{19} As Cazals et al. (2002) have shown, the order-\textit{m} efficiency measure is a consistent estimator and converges at rate $\sqrt{n}$ irrespective of the number of inputs and outputs. This is rather exceptional for non-parametric estimators, where the rate of convergence usually declines with dimensionality (the dimension $p + q$) of the problem. To sum up, the order-\textit{m} approach combines the best properties of both non-parametric and stochastic methods. Keeping its non-parametric nature allows modelling multiple-inputs-multiple-outputs relations without imposing functional specifications. Simultaneously, being stochastic lets the frontier estimates be robust to extreme values, noise or outliers. Further, the estimates are consistent and converge at rate root-$n$, thus avoiding the curse of dimensionality that plagues traditional data envelopment analysis estimators.

---

\textsuperscript{18} The number of observations required to obtain meaningful estimates of efficiency increases dramatically with the number of production inputs and outputs.

\textsuperscript{19} See Simar and Wilson, 2000 for a discussion of the ‘curse of dimensionality’.
### A.3 Description of terminology

<table>
<thead>
<tr>
<th>Category</th>
<th>Example</th>
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<tbody>
<tr>
<td>Brand</td>
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<tr>
<td>Product line</td>
<td>VW Golf</td>
</tr>
<tr>
<td>Model</td>
<td>VW Golf 1.6</td>
</tr>
<tr>
<td>Variant</td>
<td>VW Golf 1.6 Trendline</td>
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### A.4 Correlation Matrix for a Subset of Characteristics

<table>
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<tbody>
<tr>
<td>1) Torque</td>
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<td>0.35*</td>
<td>0.36*</td>
<td>0.06*</td>
<td>-0.43*</td>
<td>0.41*</td>
<td>-0.27*</td>
<td>0.21*</td>
<td>0.34*</td>
<td>0.31*</td>
<td>0.20*</td>
<td>0.30*</td>
<td>0.18*</td>
</tr>
<tr>
<td>2) Kilowatt</td>
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<td>1</td>
<td>0.60*</td>
<td>0.06*</td>
<td>-0.18*</td>
<td>0.41*</td>
<td>-0.24*</td>
<td>0.32*</td>
<td>0.31*</td>
<td>0.20*</td>
<td>0.30*</td>
<td>0.30*</td>
<td>0.18*</td>
</tr>
<tr>
<td>3) Engine Size</td>
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<td>0.14*</td>
<td>0.14*</td>
<td>0.04*</td>
<td>0.20*</td>
<td>0.32*</td>
<td>0.32*</td>
<td>0.14*</td>
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<tr>
<td>4) Revolutions</td>
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<td>-0.18*</td>
<td>0.41*</td>
<td>1</td>
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<td>0.14*</td>
<td>0.14*</td>
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<td>0.20*</td>
<td>0.32*</td>
<td>0.32*</td>
<td>0.14*</td>
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<tr>
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<td>0.14*</td>
<td>1</td>
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<td>0.24*</td>
<td>0.32*</td>
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<td>0.32*</td>
<td>0.32*</td>
<td>0.14*</td>
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<td>0.24*</td>
<td>0.14*</td>
<td>-0.27*</td>
<td>1</td>
<td>0.24*</td>
<td>0.32*</td>
<td>0.04*</td>
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<td>0.32*</td>
<td>0.32*</td>
<td>0.14*</td>
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<tr>
<td>7) Fuel Consumption</td>
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<td>0.32*</td>
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<td>0.18*</td>
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<td>0.30*</td>
<td>0.30*</td>
<td>0.18*</td>
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<tr>
<td>11) Airbag</td>
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<td>0.04*</td>
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<td>0.31*</td>
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<td>0.30*</td>
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