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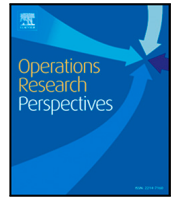
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# The berth allocation problem in bulk terminals under uncertainty

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## ABSTRACT

Uncertainty is critical in bulk terminals because it is inherent to many operations. In particular, the berth allocation problem (BAP) is greatly affected by the uncertain arrival times of the vessels. In this paper, we propose the first distributionally robust optimization (DRO) model for the BAP in bulk terminals, where the probability distribution of the arrival times is assumed to be unknown but belongs to an ambiguity set. To solve the model, we use an exact decomposition algorithm (DA) in which the probability distribution information is iteratively included in the master problem through optimal dual cuts. The DA is then enhanced with two improvement strategies to reduce the associated computational time; however, with these strategies, the DA may no longer be exact and is still inefficient for solving large-scale instances. To overcome these issues, we propose a modified exact DA where the dual cuts used in the original DA are replaced by powerful primal cuts that drastically reduce the time required to solve the DRO model, making it possible to handle large-scale instances. The reported computational experiments also show clear benefits of using DRO to tackle uncertainty compared to stochastic programming and robust optimization.

## 1. Introduction

Port terminals are essential in the global trade market because they are the main structures responsible for connecting maritime and land transportation. Due to recourse limitations, it is crucial to optimize port activities to make ports competitive in the market. For surveys on port activities and related optimization problems, we refer to [1–4]. The berth allocation problem (BAP) is one of the most important problems in port terminals and has been studied for decades. According to Xu et al. [5], the berth plan is the very first level of terminal planning and is used as a key input to yard storage and personnel/equipment deployment planning. This means that poor berth allocation decisions may also lead to poor decisions in subsequent stages. Hence, it is crucial to consider uncertainty when the berthing plan is made to obtain solutions with some protection against unpredictable events and thus avoid readjustment actions that are usually costly and cause a lot of disturbance in the port. As indicated in the recent survey [6], the amount of research on berth allocation problems under uncertainty has increased significantly in recent years; however, almost all the papers covered in that survey focus on container terminals and not bulk terminals.

Bulk terminals differ from container terminals because they usually handle multiple cargo types requiring different equipment facilities, such as mobile cranes, conveyors, and pipelines. This equipment may not exist in all berthing sections of the wharf, which blocks the allocation of some vessels in some sections. Additionally, each cargo type

is stored in a specific area of the yard, so it is necessary to model the interaction between the storage area of specific cargo types in the yard and the berthing position of the vessels in the wharf. This means that the handling time of a vessel depends on both the time taken to move the cargo between its storage area and the quayside and the time taken to move the cargo between the quayside and the vessel.

Umang et al. [7,8] were the first to clearly distinguish the BAP in bulk terminals from the BAP in container terminals. Since then, many researchers have studied the BAP in bulk terminals under deterministic assumptions. For a recent review on BAPs in bulk terminals, see [9]. However, to the best of our knowledge, only [10–13] study BAPs under uncertainty in the context of bulk terminals.

This paper proposes the first distributionally robust optimization (DRO) approach to handle uncertainty in the BAP in bulk terminals. Unlike stochastic programming, where uncertain parameters are assumed to be governed by a known probability distribution, in DRO such a distribution is unknown but belongs to a given ambiguity set [14]. Hence, DRO does not require exact knowledge of the distribution of uncertain parameters, which is the main drawback of stochastic programming. In addition, DRO makes it possible to reduce the overconservatism of robust optimization.

In our DRO model, we consider that the arrival times of the vessels are uncertain parameters following an unknown probability distribution. An empirical distribution of the arrival times can usually be

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estimated from historical data, as often happens in stochastic programming; however, this empirical distribution may not be a reliable approximation of the *true* probability distribution. Hence, in our DRO model, we consider an ambiguity set composed of all probability distributions whose distance to that empirical distribution is lower than a given value  $\epsilon$ . This set is the well-known Kantorovich/Wasserstein ambiguity set [15].

To solve the resulting DRO model, we start by using the decomposition algorithm (DA) proposed in [16], where the information on the probability distributions covered by the ambiguity set is included in the master problem through optimal dual cuts. Those cuts were revealed to be very weak for the BAP in bulk terminals, resulting in a slow convergence of the algorithm. Thus, two improvement strategies are proposed to strengthen the dual cuts. These strategies speed up the algorithm's convergence a lot; however, even with them, the algorithm is still incapable of solving large-scale instances. Hence, we propose a new exact DA that replaces the original dual cuts with powerful primal cuts. Using primal cuts instead of dual ones requires increasing the number of variables and constraints of the master problem, but it clearly pays off. The new DA can solve DRO models in seconds that the original decomposition algorithm solved in hours, even when the latter was combined with the proposed improvement strategies.

Our contributions can be summarized as follows.

- (i) We propose the first DRO approach for handling uncertainty in BAPs in bulk terminals;
- (ii) We use a DA based on dual cuts to solve the DRO model and propose two improvement heuristic strategies to make it more efficient;
- (iii) We propose a new modified exact DA for solving the DRO model where the dual cuts in the original DA are replaced by new powerful primal cuts that drastically reduce the computational time;
- (iv) We report computational experiments that clearly show the benefits of DRO in the BAP in bulk terminals compared to stochastic programming and robust optimization.

The rest of this paper is organized as follows. The relevant literature related to our work is reviewed in Section 2. The deterministic and DRO formulations for the BAP in bulk terminals are introduced in Section 3. The original and modified DAs used to solve the DRO model are presented in Section 4. The improvement strategies used to enhance the original DA are also presented in this section. Section 5 reports the computational experiments carried out to understand the benefits of using DRO optimization in the BAP and evaluate the performance of both DAs and improvement strategies proposed. This section also extends the proposed DRO model and DAs to handle uncertain cargo quantities. Conclusions, limitations, and future research direction are drawn in Section 6.

## 2. Literature review

In this section, we review the relevant literature related to our work. The focus of our paper is the BAP in bulk terminals under uncertainty, so this section is restricted to that subject. For interested readers, a recent survey on (deterministic) BAPs in bulk terminals can be found in [9], while reviews on berth allocation problems in container terminals are provided in [6,17].

The literature on BAPs in container terminals is extensive and the study of these problems started decades ago. This is not the case for bulk terminals whose first studies date from 2011 [7,18]. The first studies on BAPs in bulk and container terminals rely on deterministic assumptions, that is, they assume that all relevant parameters of the problem are exactly known. Since 2006, with the work of Moorthy and Teo [19], the research community has also started to focus on the uncertainty surrounding BAPs, and many important contributions have appeared in the last ten years [6].

The uncertainty in BAPs is mainly related to port congestion, mechanical failures, weather conditions, human behavior, etc. According to [6], the approaches used to deal with uncertainty can be divided into three main categories: proactive, reactive, and proactive-reactive. In proactive approaches [5,19–21], the full berthing plan is made in advance, that is, before the beginning of the planning horizon, and it takes uncertainty into account. In reactive approaches [22,23], a baseline plan is given and then recourse actions are applied to deal with the observed uncertainty. Finally, in proactive-reactive approaches [24–28], a complete or partial baseline plan is derived taking into account some uncertain information, and then recourse actions are applied when the real uncertainty is observed.

All the papers mentioned in the previous paragraph refer to container terminals since the literature on BAPs in bulk terminals under uncertainty is scarce. To the best of our knowledge, only [10–13] studied BAPs under uncertainty in the context of bulk terminals. Umang et al. [13] followed a reactive approach to deal with the uncertain arrival and handling times of vessels in the BAP in bulk terminals. The goal was to minimize the total cost of the updated berthing schedule when the data was revealed in real time. Given a baseline schedule, the authors used an optimization-based recovery algorithm and a smart greedy recovery algorithm on a rolling horizon to reschedule the vessels when disruptions occur.

León et al. [12] studied the BAP in bulk terminals considering stochastic arrival and handling times from a proactive perspective. The authors proposed a simheuristic framework to obtain a Pareto front estimation of the proposed multi-objective model, aiming to minimize the total completion time of the vessels and the estimated penalty cost resulting from delays in the vessel's departure. In that framework, new solutions were obtained by a multi-objective NSGA-II and evaluated through simulation. The proposed framework also included a data-driven heuristic that made it possible to define buffers for each vessel to mitigate the impact of uncertainty. The same problem was studied in [11] from a proactive-reactive perspective to reduce the cost of changes in the baseline berthing plan resulting from uncertain events. In the proactive phase, a baseline solution is obtained using the approach proposed in [12], while in the reactive phase, readjustment actions are applied to deal with the observed uncertainty to minimize deviations from the baseline schedule.

Dai et al. [10] studied the BAP in bulk terminals with quay crane setup times in both offline and online settings. The offline setting assumes that all relevant information is known, while in the online setting the arrival times, types of cargo, and handling times of the vessels are unavailable until they arrive at the port. The online case follows a reactive strategy where the BAP is formulated as a Markov decision process and then, a reinforcement learning algorithm is used to quickly readjust the berthing plan in real time.

The approach followed in this paper is proactive-reactive, and the uncertainty is handled by using DRO. Since its appearance, DRO has become one of the main approaches for handling optimization problems under uncertainty due to its flexibility [14]. That is, it does not require full knowledge of the probability distribution of the uncertain parameters as in stochastic programming. Instead, it assumes that the probability distribution belongs to an ambiguity set. For a detailed description of the different ambiguity sets, see [29].

When deciding on the ambiguity set to use, it is important to consider its tractability and practical application, namely, its adjustment to historical data [30]. Two classical approaches for constructing the ambiguity set are the moment-based and distance-based approaches. In moment-based approaches, the ambiguity set is defined through conditions on the distribution's moments such as mean and covariance. Distance-based approaches consider probability distributions within a given distance of a reference distribution. According to Gao and Kleywegt [15], moment-based ambiguity sets have the drawback of assuming that only information on the moments can be extracted from the data. Keeping this in mind, we use a Wasserstein ambiguity set

to represent the uncertainty. This set depends on a risk parameter that controls its size, and by varying such a parameter, it may be possible to obtain solutions with different degrees of protection against uncertainty. In addition, this ambiguity set is defined through a set of scenarios that can easily be estimated in practice from historical data. It is important to note that scenarios are the most common way to represent uncertainty in BAPs [6], which supports our choice.

DRO has been used to solve different complex optimization problems such as production planning [31], healthcare supply chain [32], surgery planning [33], hub location [34], group decision-making [35], quay crane scheduling [36], among others. However, the application of DRO to berth allocation-related problems is still scarce. Applications of DRO to integrated berth allocation and quay crane assignment problems can be found in [37] and [38]. In both works, the authors consider a mean-variance ambiguity set. The former explores a distributionally robust chance-constrained optimization approach where a chance constraint is used to impose that the service time of the vessels must start after their uncertain arrival time with a given probability to avoid rescheduling actions. The latter explores a two-stage DRO model that aims to minimize the worst-case expectation of the adjustment cost when disruptions occur. A DRO approach was also followed in [39] to solve a two-stage BAP with uncertain handling times in container terminals.

### 2.1. Positioning this work in the literature

This paper differs from the existing literature by providing the first DRO model for the BAP in bulk terminals. The proposed model is solved using two DAs: one based on dual cuts and the other based on primal cuts. The former was originally proposed in [16] and later used in [39] to solve a DRO model for the BAP with uncertain handling times in container terminals, while the latter is new and clearly exhibits better performance. Probably, [39] is the closest paper to this research, but significant differences still exist. First, this paper studies the BAP in bulk terminals, while [39] considers the traditional BAP in container terminals. Since these problems are very different, as clearly explained in [7], applying the DA used in [39] to the BAP addressed in this paper is not straightforward. Second, the uncertain parameters in [39] are the handling times, while here the main uncertain parameters are the arrival times. Third, the problem studied in [39] assumes the existence of a hard deadline to complete the vessel's operations, which made it possible to design an exact improvement strategy to speed up the DA used. Here, it is assumed that such a hard deadline is not provided, which leads to the need to design an alternative improvement strategy. Fourth, the DA used in [39] uses dual cuts to include the information of probability distributions in the master problem, while the new DA proposed here uses more powerful primal cuts. It is important to note that the DA with dual cuts is just used here as a baseline and the new proposed DA with primal cuts drastically reduces the computational time needed to obtain optimal solutions.

### 3. The berth allocation problem in bulk terminals

A bulk terminal handles several cargo types requiring different equipment facilities such as harbor cranes, pipelines, and conveyors. This equipment is not installed in all berthing sections of the wharf, and each cargo type has a specific storage location in the yard. Fig. 1 illustrates a bulk terminal with five berthing sections having different equipment installed. In particular, conveyors are installed in the first two sections, harbor cranes are mounted in sections 2, 3, and 4, and pipelines are accessible in the last two sections. We can also see storage locations for different cargo types.

The BAP in bulk terminals consists of determining the arrival time and berthing section for each vessel arriving at the port in a given time horizon. The goal is to minimize the total completion time of the vessels, that is, the total time they spend in the port.

After this brief overview, we formally introduce the deterministic model for the BAP in bulk terminals where it is assumed that the arrival times of the vessels are known. This model is the basis for the DRO model, also presented in this section, where the vessel arrival times are assumed to be uncertain.

#### 3.1. Deterministic formulation

The deterministic model presented here is the same as that introduced in [8]. Consider a bulk terminal where a set  $W = \{1, \dots, |W|\}$  of different types of cargo are loaded/unloaded by a set  $N = \{1, \dots, |N|\}$  of vessels arriving at the terminal over a time horizon  $T^{max}$ . Each vessel  $i \in N$  has an expected arrival time  $a_i$  and a fixed length  $l_i$ . We assume that each vessel  $i \in N$  can carry a single cargo type  $w_i$  and the cargo quantity to handle is  $q_i$ . The bulk terminal has a hybrid quay with length  $L^{max}$  discretized into a set  $M = \{1, \dots, |M|\}$  of berthing sections. Each section  $k \in M$  has a length  $L_k$  and a starting coordinate  $b_k$ . The handling time of a vessel depends on two factors: i) the time taken to transfer one unit of cargo between the cargo location in the yard and the quay section in which the vessel berths and ii) the time taken to load/unload one unit of cargo from the quayside to the vessel. Therefore, the time required for loading/unloading one unit of cargo type  $w_i$  in vessel  $i \in N$  at section  $k \in M$  is denoted by  $h_k^{w_i}$ . As explained in [8], it is assumed that all quay sections occupied by a vessel are operated simultaneously and that each section handles an amount of cargo proportional to its length. The percentage of cargo handled in section  $j \in M$  if the vessel  $i \in N$  berths in section  $k \in M$  is denoted by  $p_{ikj}$  and can be calculated considering the parameters defined before.

The main decision variables of the problem are defined as follows.

- $m_i$  - the starting handling time for vessel  $i \in N$ ;
- $c_i$  - the total handling time of vessel  $i \in N$ ;
- $s_{ik}$  - 1 if section  $k \in M$  is the starting section of vessel  $i \in N$ ; 0 otherwise;
- $y_{ij}$  - 1 if vessel  $i \in N$  berths to the left of vessel  $j \in N$ ; 0 otherwise;
- $z_{ij}$  - 1 if vessel  $i \in N$  finishes its operations before the handling of vessel  $j \in N$  starts; 0 otherwise.

The deterministic berth allocation problem in bulk terminals can then be formulated as follows.

$$\min \sum_{i \in N} (m_i + c_i - a_i) \quad (1)$$

$$s.t. \sum_{k \in M} s_{jk} b_k + L^{max}(1 - y_{ij}) \geq \sum_{k \in M} s_{ik} b_k + l_i, \quad i, j \in N, i \neq j, \quad (2)$$

$$y_{ij} + y_{ji} + z_{ij} + z_{ji} \geq 1, \quad i, j \in N, i \neq j, \quad (3)$$

$$\sum_{k \in M} s_{ik} = 1, \quad i \in N, \quad (4)$$

$$\sum_{k \in M} s_{ik} b_k + l_i \leq L^{max}, \quad i \in N, \quad (5)$$

$$c_i \geq h_j^{w_i} p_{ikj} q_i s_{ik}, \quad i \in N, k, j \in M, \quad (6)$$

$$m_i \geq a_i, \quad i \in N, \quad (7)$$

$$m_j + T^{max}(1 - z_{ij}) \geq m_i + c_i, \quad i, j \in N, i \neq j, \quad (8)$$

$$m_i \geq 0, \quad i \in N, \quad (9)$$

$$c_i \geq 0, \quad i \in N, \quad (10)$$

$$s_{ik} \in \{0, 1\}, \quad i \in N, k \in M, \quad (11)$$

$$y_{ij}, z_{ij} \in \{0, 1\}, \quad i, j \in N. \quad (12)$$

The objective function (1) minimizes the total completion time of the vessels, corresponding to the difference between their finishing time and arrival time. Constraints (2) and (8) establish spatial and temporal relations between vessels, while constraints (3) ensure no overlaps between vessels neither in time nor in space. Constraints (4) impose that

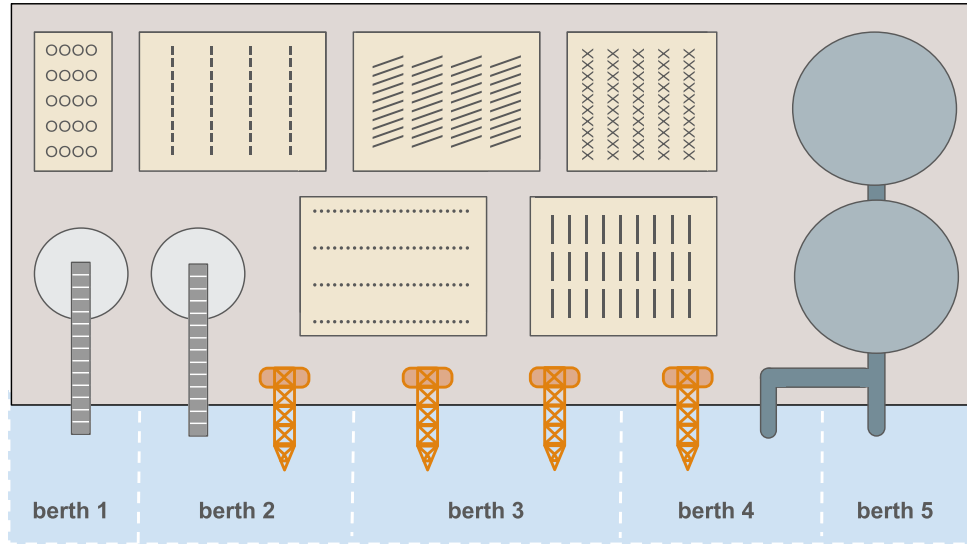


Fig. 1. Illustration of a bulk terminal.

each vessel has exactly a single starting berthing section. Constraints (5) guarantee that the allocation of a vessel does not exceed the length of the quay. Since it is assumed that the amount of cargo handled at a given quay section is proportional to its length, the total handling time of a vessel corresponds to the time taken for loading/unloading the vessel in the section whose operation finishes last, as stated by constraints (6). Constraints (7) ensure that each vessel does not start to be served before its arrival. Constraints (9)–(12) define the domain of the decision variables.

### 3.2. Distributionally robust formulation

The deterministic model presented in the previous section assumes that the arrival times of the vessels are exactly known when the planning is made. However, this is an unrealistic assumption because these parameters are greatly affected by several uncertain factors like mechanical failures, weather conditions, and delays in other ports [6]. Recognizing the existence of uncertainty, this section presents a DRO model for the BAP in bulk terminals where the arrival time of each vessel  $i \in N$  is a random variable governed by a probability distribution  $\mathcal{P}_i$ . Let us denote by  $\mathcal{P} = (\mathcal{P}_1, \dots, \mathcal{P}_{|N|})$  the joint probability distribution of the arrival times of the set of vessels considered. Contrary to stochastic programming where it is assumed that distribution  $\mathcal{P}$  is known, in the DRO, such a distribution is uncertain and belongs to an ambiguity set  $\mathbb{A}$ . Our DRO model assumes that the probability distribution of the arrival times has finite support given by a set of scenarios  $\Omega = \{a^1, \dots, a^{|\Omega|}\}$ , where  $a^\omega = (a_1^\omega, \dots, a_{|N|}^\omega)$  and  $a_i^\omega$  represents the arrival time of vessel  $i \in N$  in scenario  $\omega \in \Omega$ . We note that discrete supports may be used not only to represent discrete probability distributions but also to approximate continuous distributions by discrete distributions, see for instance, [40,41]. Additionally, as highlighted in [6], scenarios are the most common way to represent uncertainty in BAPs because they can be easily obtained from historical data.

The proposed DRO model is a two-stage model in which most of the berthing decisions  $(s, y, z)$  - and consequently the handling times of the vessels  $(c)$  - are first-stage decisions and the berthing times of the vessels  $(m)$  are second-stage decisions adjusted to uncertainty. It aims to optimize the expected value of the sum of the completion times of the vessels for the worst probability distribution in the ambiguity set  $\mathbb{A}$ , that is,

$$\min \left\{ \max_{\mathcal{P} \in \mathbb{A}} \sum_{\omega \in \Omega} \pi_\omega^\omega R(s, y, z, c, \omega) \mid (2) - (6), (10) - (12) \right\}, \quad (13)$$

where  $\pi_\omega^\omega$  is the probability of scenario  $\omega$  according to the probability distribution  $\mathcal{P}$  and  $R(s, y, z, c, \omega)$  provides the minimum sum of completion times for first-stage decisions  $s, y, z$ , and  $c$  in scenario  $\omega \in \Omega$ . Computing the completion times for a given first-stage solution only requires knowledge of the value of the variables  $z$  and  $c$ , thus, for each scenario  $\omega \in \Omega$ ,  $R(s, y, z, c, \omega) = R(z, c, \omega)$ . Denoting by  $m_i^\omega$  the starting time of handling vessel  $i$  in scenario  $\omega$ , we have

$$R(z, c, \omega) = \min_{m^\omega} \sum_{i \in N} (m_i^\omega + c_i - a_i^\omega) \quad (14)$$

$$s.t. \quad m_j^\omega + T^{\max}(1 - z_{ij}) \geq m_i^\omega + c_i, \quad i, j \in N, i \neq j, \quad (15)$$

$$m_i^\omega \geq a_i^\omega, \quad i \in N, \quad (16)$$

$$m_i^\omega \geq 0, \quad i \in N. \quad (17)$$

The DRO model (13) is a minimization problem containing an inner maximization problem based on an infinite ambiguity set, which makes it impossible to solve it as a single model. Therefore, in the next section, we introduce an exact decomposition algorithm to solve it.

## 4. Solution methods

The DRO model presented in the previous section can be solved using the exact DA proposed in [16]. However, as we shall see in the computational experience, the algorithm's convergence is slow, which makes it impractical for large-scale instances. Thus, we propose several (heuristic) improvement strategies for enhancing it. Although drastically reducing the computational time, the DA combined with the improvement strategies is still incapable of solving large instances efficiently. To overcome this issue, we finish this section by presenting a new DA based on new optimality cuts that clearly outperform the original ones.

### 4.1. An exact decomposition algorithm

In this section, we introduce the exact DA proposed in [16]. To do this, we start by rewriting the DRO model (13) by using the epigraph reformulation as follows:

$$\min \quad \theta \quad (18)$$

$$s.t. \quad \theta \geq \sum_{\omega \in \Omega} \pi_\omega^\omega R(z, c, \omega), \quad \forall \pi_\omega \in \mathbb{A}, \quad (19)$$

$$(2) - (6), (10) - (12). \quad (20)$$



When the ambiguity set  $\mathbb{A}$  is not finite, the resulting DRO model is intractable. Hence, the idea of the DA is to replace constraints (19) by a finite number of optimality cuts. To determine those cuts, we start by writing the dual  $R^{dual}(z, c, \omega)$  of the recourse problem (14)–(17), that is,

$$R^{dual}(z, c, \omega) = \max_{u^\omega, v^\omega} \sum_{i \in N} (c_i - a_i^\omega) + \sum_{i, j \in N: i \neq j} u_{ij}^\omega (c_i - T^{max}(1 - z_{ij})) + \sum_{i \in N} a_i^\omega v_i^\omega \quad (21)$$

$$s.t. \quad \sum_{j \in N: j \neq i} (u_{ji}^\omega - u_{ij}^\omega) + v_i^\omega \leq 1, \quad i \in N, \quad (22)$$

$$u_{ij}^\omega \geq 0, \quad i, j \in N: i \neq j, \quad (23)$$

$$v_i^\omega \geq 0, \quad i \in N. \quad (24)$$

If the recourse problem  $R(z, c, \omega)$  has an optimal solution, then, by the strong duality theory,  $R^{dual}(z, c, \omega)$  also has an optimal solution, and the optimal values coincide. Hence, denoting by  $(\bar{u}_{ij}^\omega, \bar{v}_i^\omega)$  the optimal dual solution and using the asterisk symbol to denote optimal values, the following equalities hold

$$R^*(z, c, \omega) = R^{dual*}(z, c, \omega) = \sum_{i \in N} (c_i - a_i^\omega) + \sum_{i, j \in N: i \neq j} \bar{u}_{ij}^\omega (c_i - T^{max}(1 - z_{ij})) + \sum_{i \in N} a_i^\omega \bar{v}_i^\omega$$

leading to the following optimality cut

$$\theta \geq \sum_{\omega \in \Omega} \pi_\omega^\omega \left( \sum_{i \in N} (c_i - a_i^\omega) + \sum_{i, j \in N: i \neq j} \bar{u}_{ij}^\omega (c_i - T^{max}(1 - z_{ij})) + \sum_{i \in N} a_i^\omega \bar{v}_i^\omega \right) \quad (25)$$

where the probabilities  $\pi_\omega^\omega$  are obtained by solving the distribution separation problem (DSP):

$$DSP = \max_{\pi_\omega^\omega \in \mathbb{A}} \sum_{\omega \in \Omega} \pi_\omega^\omega R^*(z, c, \omega). \quad (26)$$

The DSP determines the probability distribution in the ambiguity set leading to the worst expected sum of the completion times. This distribution is determined taking into account the total completion times of all scenarios  $\omega \in \Omega$ , that is, the values of the recourse function  $R^*(z, c, \omega)$  for all  $\omega \in \Omega$ . The way of solving the DSP depends on the ambiguity set considered. In this paper, we use the well-known Kantorovich ambiguity set because it has several advantages (see [15]) and has been widely used in the literature, see, for instance, [39, 42, 43]. Additionally, the Kantorovich ambiguity set depends on a risk parameter  $\epsilon$  that controls its size. Roughly speaking, given a reference probability distribution  $\bar{\pi}_\omega$ , the Kantorovich ambiguity set  $\mathbb{K}_\epsilon$  defines the space of probability distributions whose distance to that reference distribution is at most  $\epsilon$ . That is,

$$\mathbb{K}_\epsilon = \{ \pi_\omega \in \mathbb{R}^{|\Omega|} : d(\pi_\omega, \bar{\pi}_\omega) \leq \epsilon \}. \quad (27)$$

Using this ambiguity set makes it possible to write the DSP as a linear model easy to solve. If the distance  $d(\pi_\omega, \bar{\pi}_\omega)$  is measured by a Wasserstein metric of order 1, then the DSP can be written as follows.

$$DSP = \max_{\pi_\omega} \sum_{\omega \in \Omega} \pi_\omega^\omega R^*(z, c, \omega) \quad (28)$$

$$s.t. \quad \sum_{\omega, \omega' \in \Omega} \| \omega - \omega' \|_1 t_{\omega\omega'} \leq \epsilon, \quad (29)$$

$$\sum_{\omega' \in \Omega} t_{\omega\omega'} = \pi_\omega^\omega, \quad \omega \in \Omega, \quad (30)$$

$$\sum_{\omega \in \Omega} t_{\omega\omega'} = \bar{\pi}_{\omega'}^\omega, \quad \omega' \in \Omega, \quad (31)$$

$$\sum_{\omega \in \Omega} \pi_\omega^\omega = 1, \quad (32)$$

$$\pi_\omega^\omega \geq 0, \quad \omega \in \Omega, \quad (33)$$

$$t_{\omega\omega'} \geq 0, \quad \omega, \omega' \in \Omega. \quad (34)$$

Note that  $R^*(z, c, \omega)$  corresponds to the optimal value of the recourse problem associated with scenarios  $\omega$ , so it is a fixed value. The Wasserstein metric of order 1 appears in constraint (29).

The exact DA proposed in [16] iterates between a master problem (MP), an adversarial problem (AP), and a distribution separation problem (DSP). At each iteration  $k$ , the MP determines a first-stage solution  $(s^k, y^k, z^k, c^k)$ . Then, the partial first-stage solution  $(z^k, c^k)$  is used in the AP to determine the second-stage solution for each scenario. The optimal value associated with each scenario determined in the AP ( $R^*(z^k, c^k, \omega)$ ) is then used in the DSP to determine the probability distribution  $\pi_\omega$  in the ambiguity set leading to the worst expected sum of the completion times. The optimal value of the master problem ( $MP^*$ ) provides a lower bound for the problem, while the upper bound is given by the optimal value of the DSP ( $DSP^*$ ). The algorithm runs while the current lower bound is lower than the current upper bound. At each iteration, a new cut is generated considering the optimal dual solutions from the AP and the probability distribution determined by the DSP. The complete algorithm is illustrated in Fig. 2.

#### 4.2. Improvement strategies

As we shall see in the computational experience, the convergence of the DA introduced in the previous section is slow in the BAP, which makes it impractical for real-size instances. Hence, in this section, we propose two improvement strategies, inspired by [39], to speed up the algorithm; however, it is important to highlight that they may eliminate some optimal solutions, which makes them heuristic improvements.

To understand why the convergence of the DA is slow, we need to look at the structure of the generated cuts (25) carefully and focus on the term

$$\sum_{i, j \in N: i \neq j} \bar{u}_{ij}^\omega (c_i - T^{max}(1 - z_{ij})). \quad (35)$$

The value  $T^{max}$  represents the time horizon and is used here as an upper bound for the completion time of the operations on each vessel  $i \in N$ . When  $z_{ij} = 0$  and  $\bar{u}_{ij}^\omega$  is not zero, the term (35) is a negative value, usually large. Since this term appears on the right-hand side of the cut (25), which is a  $\sum \geq \epsilon$  constraint, it will slow down the increase of  $\theta$  and consequently the increase of the lower bound of the DRO model. A similar situation was identified for the BAP in container terminals studied in [39]. In that problem, it was assumed the existence of a hard deadline for the completion time of each vessel, which is not the case for the problem studied in this paper. The existence of such a deadline allowed to decrease the value of  $T^{max}$  in each constraint, leading to better results and keeping the optimality of the DA.

In this problem, such a deadline does not exist, which makes using the same strategy forbidden. Hence, the first improvement strategy proposed in this paper consists of producing a *reliable* estimation of an upper bound on the completion time of each vessel to reduce the value of  $T^{max}$  used in (35). To that end, we start by solving the deterministic BAP using the formulation (1)–(12) to determine the completion time of each vessel in the deterministic scenario, that is,  $m_i^{det} + c_i^{det}$ ,  $i \in N$ . Then, we define the value  $T_i^{max}$  for each vessel  $i \in N$  as

$$T_i^{max} = \min\{T^{max}, m_i^{det} + c_i^{det} + \delta_1\}, \quad (36)$$

where  $\delta_1$  is a predefined value, and replace  $T^{max}$  by  $T_i^{max}$  in (35). With Eq. (36) we are imposing that the completion time of each vessel cannot be higher than its completion time in the deterministic scenario plus a given tolerance  $\delta_1$ .

The second improvement strategy aims to reduce the number of alternative solutions by fixing some variables to zero that are *likely* to be zero in the optimal solution. In particular, we impose that if in the set of scenarios considered the latest arrival of vessel  $i \in N$  plus a predefined tolerance  $\delta_2$  is lower than the earliest possible arrival of vessel  $j \in N$ , then vessel  $i$  cannot begin to be served after the

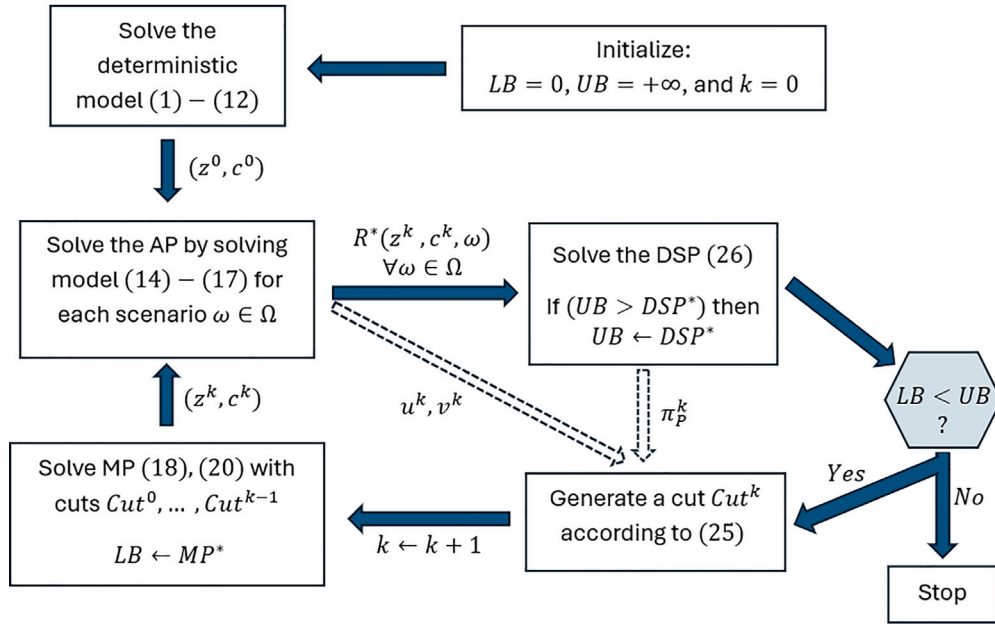


Fig. 2. Decomposition algorithm of Bansal et al. [16].

completion time of vessel  $j$ , that is,

$$\text{If } \max_{\omega \in \Omega} \{a_i^\omega\} + \delta_2 < \min_{\omega \in \Omega} \{a_j^\omega\} \text{ then } z_{ji} = 0$$

This improvement is expected to reduce the number of cuts added to the master problem, that is, the number of iterations of the algorithm, and therefore to reduce the total computational time. Once again, it is important to highlight that both improvement strategies introduced here may cut some optimal solutions, which means that the DA acts as a heuristic when combined with them.

#### 4.3. The modified decomposition algorithm

The DA presented in Section 4.1 successively generates cuts based on the dual solutions provided by the adversarial problem. This avoids using the second-stage variables  $m^\omega$  and corresponding constraints in the master problem, reducing its dimension. However, as explained in the previous section and observed from the computational experience, the optimality cuts (25) are weak, indicating that a large number of iterations may be required until the lower and upper bounds determined by the algorithm coincide.

In an attempt to strengthen the cuts added to the master problem and reduce the global number of iterations performed, we propose a new DA where the master problem contains all the second-stage variables and constraints (15)–(17) associated with each of the scenarios and the optimality cuts are defined in terms of those variables. If all second-stage variables and corresponding constraints are included in the master problem, the following constraint is an optimality cut for the problem:

$$\theta \geq \sum_{\omega \in \Omega} \pi_p^\omega \left( \sum_{i \in N} (m_i^\omega + c_i - a_i^\omega) \right) \quad (37)$$

where the probabilities  $\pi_p^\omega$  are obtained by solving the distribution separation problem (26), as before. The modified master problem solved at iteration  $k$  can then be written as follows:

$$\min \theta \quad (38)$$

$$\text{s.t. } \text{Cut}^\ell, \quad \ell = 0, \dots, k-1, \quad (39)$$

$$(2) - (6), (10) - (12), \quad (40)$$

$$m_j^\omega + T^{\max}(1 - z_{ij}) \geq m_i^\omega + c_i, \quad i, j \in N, i \neq j, \omega \in \Omega, \quad (41)$$

$$m_i^\omega \geq a_i^\omega, \quad i \in N, \omega \in \Omega, \quad (42)$$

$$m_i^\omega \geq 0, \quad i \in N, \omega \in \Omega. \quad (43)$$

where  $\text{Cut}^\ell$  is the cut (37) generated in iteration  $\ell$ . Since the second-stage variables and corresponding constraints are included in the master problem for all scenarios considered, the value of the recourse function  $R(z, c, \omega)$  for scenario  $\omega \in \Omega$  – used in the DSP – can be directly obtained from the master problem, avoiding to solve the adversarial problem. The proposed modified DA used to solve the DRO BAP in bulk terminals is presented in Fig. 3.

Including all second-stage variables and corresponding constraints in the master problem increases its size; however, as we shall see in the next section, this increase is completely compensated by the strength of the new cuts added. It is also important to note that the modified DA is an exact algorithm for the DRO BAP.

## 5. Computational results

In this section, we report the computational experiments carried out to test the proposed DRO model and algorithms. The experiments were run on a computer with a CPU Intel(R) Core i9 with 32 GB RAM by using the Xpress Optimizer Version 9.5.2 with the default options.

### 5.1. Instances used

All computational experiments were conducted on instances based on those used in [8], which correspond to real data gathered from the SAQR port in Ras Al Khaimah, United Arab Emirates. In particular, we consider a bulk terminal that handles six cargo types needing different specialized equipment facilities. Four cargo types need to be handled by quay cranes, one cargo requires pipelines, and the other requires a conveyor. The port has a quay area of  $L^{\max} = 1600$  m divided into 10 berthing sections with lengths {150, 50, 200, 150, 125, 250, 250, 75, 150, 200}. There is a conveyor in sections 1, 2, 4, and 5 and a pipeline in sections 3, 4, 5, 8, and 9. The number of cranes in each berthing section is proportional to its length because it is assumed the existence of a quay crane by each 50 m.

The storage location for each cargo type in the port is shown in Fig. 6 presented in [8]. It is assumed that all sections occupied by a berthed vessel are operated simultaneously, with each section handling

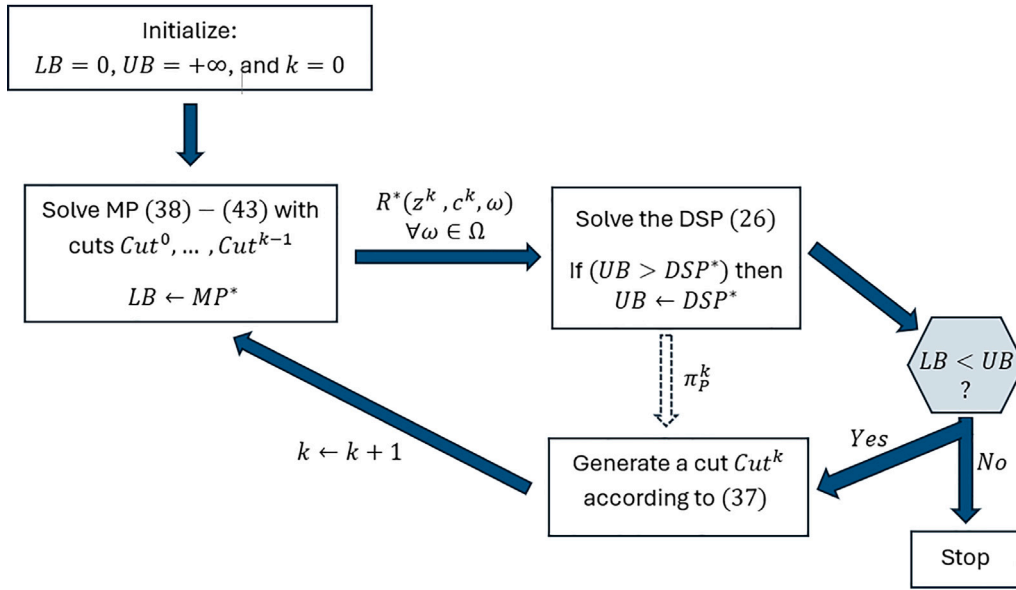


Fig. 3. Modified exact decomposition algorithm.

the amount of cargo proportionally to its length. As in [8], the unit handling time of a vessel with cargo type  $w \in W$  in section  $k \in M$  is given by  $h_k^w = \alpha_k^w + \beta_k^w$ , where  $\beta_k^w$  is the time taken to transfer one unit of cargo  $w$  between its storage location on the yard and the berthing section  $k$ , and  $\alpha_k^w$  is the time taken to load/unload one unit of cargo  $w$  from section  $k$  to the vessel. Denoting by  $R$  the time taken by a single crane to load/unload a unit of cargo and by  $n_k^w$  the number of cranes operating in section  $k$  for cargo type  $w$ , we have  $\alpha_k^w = R/n_k^w$ . Additionally, denoting by  $d_k^w$  the distance between the storage area of cargo  $w$  and section  $k$  and by  $v^w$  the speed at which the cargo type  $w$  is transferred, we have  $\beta_k^w = v^w d_k^w$ .

Parameter  $d_k^w$  is the Euclidean distance between the midpoint of section  $k$  and the storage location of cargo  $w$ . According to [8], the parameter  $v^w$  is defined as  $v^1 = 1/1200$  h/m (conveyor),  $v^6 = 1/3600$  h/m (pipeline), and  $v^2 = v^3 = v^4 = v^5 = 1/600$  h/m (other cargo types). Furthermore,  $R = 1000$  tons per hour and  $n_k^w$  depends on the length of section  $k$ , as explained before.

We consider two sets of instances, S1 and S2. Set S1 is composed of 40 instances with  $|N| = 6, 8, 10, 12$  vessels, having 10 instances for each value of  $|N|$ . Set S2 is composed of 40 instances with  $|N| = 15, 20, 25, 30$  vessels. In both sets, the length of the vessels ( $l_i$ ) ranges between 80 and 260 m, and the total cargo ( $q_i$ ) ranges between 1 and 15 for the instances with more than eight vessels and between 1 and 25 for the instances with six and eight vessels. The arrival times ( $a_i$ ) follow a Poisson process with parameter  $T^{\max}/|N|$ , where  $T^{\max} = 120$  h is the time horizon considered. The instances used are available at <https://github.com/frodriguesISEG/Instances-BAP-bulk-terminals-under-uncertainty>.

## 5.2. Parameters for the DRO model

The DRO model considered is defined in terms of the Kantorovich ambiguity set. This ambiguity set depends on a risk parameter  $\epsilon$  and is based on a set of scenarios of a reference probability distribution. Here, we consider a set  $\Omega$  with 100 scenarios. In each scenario, each vessel has the probability of 30% of being delayed, that is, for each scenario  $\omega \in \Omega$ , and vessel  $i \in N$ ,  $\mathbb{P}(a_i^\omega > a_i) = 0.3$ , where  $a_i$  is the nominal/expected arrival time. If a vessel is delayed in a certain scenario, then its delay is randomly generated between 1 and 10 h. The reference probability distribution considered is the empirical probability distribution where each scenario has the same probability of occurrence.

To determine the value of the risk parameter  $\epsilon$  to use, we start by computing the highest theoretical value worth considering. It is well known that the DRO model coincides with the stochastic model for  $\epsilon = 0$  and with the robust optimization model for values  $\epsilon$  greater than a certain threshold  $\epsilon^{\max}$  value, see [16,39]. Therefore, we can use  $\epsilon \in [0, \epsilon^{\max}]$ . Indeed, for a sufficiently large value  $\epsilon$ , the DRO model coincides with the robust optimization model because the worst probability distribution in the ambiguity set becomes a degenerated probability distribution in which one of the scenarios has probability one and the others have probability zero. There are  $|\Omega|$  possible degenerated probability distributions. Each of them is associated with a specific scenario  $\bar{\omega}$  and is only included in the Kantorovich ambiguity set for a value of  $\epsilon$  greater than a specific value  $\epsilon^{\bar{\omega}}$ . The value  $\epsilon^{\bar{\omega}}$  can be determined by solving the modified distribution separation problem

$$\epsilon^{\bar{\omega}} = \min\{\epsilon \mid (29) - (34)\}$$

with  $t_{\omega\omega'} = \bar{\pi}_{\mathcal{P}}^{\omega'} = 1/|\Omega|$  for all  $\omega' \in \Omega$  and  $t_{\omega\omega'} = 0$  for all  $\omega, \omega' \in \Omega : \omega \neq \bar{\omega}$ . However, since all variables  $t_{\omega\omega'}$  are fixed, the value of  $\epsilon^{\bar{\omega}}$  can be determined by the expression

$$\epsilon^{\bar{\omega}} = \sum_{\omega' \in \Omega} \|\bar{\omega} - \omega'\|_1 t_{\omega\omega'} = \sum_{\omega' \in \Omega} \|\bar{\omega} - \omega'\|_1 \bar{\pi}_{\mathcal{P}}^{\omega'} = \sum_{\omega' \in \Omega} \|\bar{\omega} - \omega'\|_1 \frac{1}{|\Omega|}.$$

The value  $\epsilon^{\bar{\omega}}$  is the minimum value ensuring that the degenerated probability distribution associated with scenario  $\bar{\omega}$  belongs to the ambiguity set  $\mathbb{K}_\epsilon$ . Therefore, to guarantee that the Kantorovich ambiguity set covers all degenerated probability distributions it is enough to consider  $\epsilon^{\max} = \max_{\bar{\omega} \in \Omega} \{\epsilon^{\bar{\omega}}\}$ .

The value of  $\epsilon^{\max}$  depends on the instance considered. Hence, rather than considering a fixed value  $\epsilon$ , we define it as a given percentage of  $\epsilon^{\max}$ , that is,  $\epsilon = \sigma \epsilon^{\max}$ , with  $\sigma \in ]0, 1[$ . The value  $\epsilon^{\max}$  is the smallest theoretical value for the risk parameter ensuring that the DRO model coincides with the robust optimization model; however, it has been observed that the DRO solution often coincides with the robust optimization solution for values of  $\epsilon$  significantly lower than  $\epsilon^{\max}$ . This means that it is not worth considering values of  $\epsilon$  closer to  $\epsilon^{\max}$ . Additionally, as we will see later, the computational time of the DA tends to be lower for smaller values of  $\epsilon$ . Thus, in what follows, we consider only two smaller values of  $\epsilon$ , corresponding to  $\sigma = 0.15$  and  $\sigma = 0.3$ .



### 5.3. Results for the instances of set S1

This section reports the computational results for the instances in set S1, the smallest ones considered. The main goal is to test the improvement strategies and the proposed modified DA. All the instances considered were solved to optimality by all versions of the DA, thus only computational times and the number of iterations are analyzed. Table 1 presents the results obtained by:

- i. the exact DA presented in Section 4.1 (DA);
- ii. the DA combined with the first improvement strategy introduced in Section 4.2 that consists of replacing the value of  $T^{max}$  by a smaller value (DA\_T). We consider  $\delta_1 = 20$ ;
- iii. the DA combined with the second improvement strategy introduced in Section 4.2 that consists of fixing some variables  $z$  equal to zero (DA\_z). We consider  $\delta_2 = 10$ ;
- iv. the DA combined with the first and second improvement strategies (DA\_T\_z);
- v. the modified exact DA introduced in Section 4.3 (DA\_modified);

We also tested the combination of the improvement strategies with the modified DA; however, no significant differences were observed, thus those results are omitted. We note that the new cuts used in the modified DA do not involve either parameter  $T^{max}$  or variables  $z$ , which may explain why no differences were observed by using or not these strategies.

The first column of Table 1 indicates the number of vessels considered, while the second indicates the value of  $\sigma$  that determines the value of the risk parameter  $\epsilon$  in the DRO model, that is,  $\epsilon = \sigma\epsilon^{max}$ , where  $\epsilon^{max}$  vary from instance to instance and is determined as explained in Section 5.2. Columns  $T_{AP}$ ,  $T_{DSP}$ , and  $T_T$  report the average total computational time (in seconds) associated with the adversarial problem, DSP, and full algorithm, respectively, for the corresponding set of 10 instances used. Columns  $\#it$  indicate the average total number of iterations performed by the algorithm, while columns  $\#it^{imp}$  display the average total number of iterations leading to an improvement of the current upper bound of the model. All values in Table 1 were rounded to the nearest integer to facilitate reading, and the best results were marked in bold.

The results reported in column DA show the weak performance of the original DA when solving the DRO BAP under uncertainty because the total computational time increases drastically as the number of vessels increases. Moreover, as reported in column  $\#it$ , the average total number of iterations performed is huge, which empirically shows the weakness of the optimality cuts (25).

The efficiency of the DA is strongly improved with any of the improvement strategies, and the best results – total time and iterations required – are obtained when both of them are used together (column DA\_T\_z). It is important to recall that both improvement strategies may eliminate some optimal solutions; however, this is not the case for this set of instances. Indeed, the heuristic algorithms DA\_z, DA\_T, and DA\_T\_z obtained optimal solutions for all 40 instances considered. For this reason, the objective function values of the obtained solutions are not reported in Table 1.

The final – and maybe the most important conclusion from Table 1 – is the impressive performance of the proposed modified DA. The average total computational time for all subsets of instances in S1 is less than 10 s, and the maximum computational time registered is 20 s. It is important to note that the maximum observed computational time associated with the original DA for the instances of set S1 is almost two hours. The master problem in the modified DA includes all the second-stage variables and corresponding constraints for all the scenarios considered, which makes each master problem harder to solve; however, the presence of such variables and constraints makes it possible to work with optimality cuts based on second-stage variables (primal cuts). These primal cuts are stronger than the dual ones for the

BAP in bulk terminals, and they drastically accelerate the convergence of the algorithm, as demonstrated by the lower number of iterations required.

After analyzing the performance of the proposed modified DA and improvement strategies, we analyze the DRO solutions and compare them with the deterministic ones (where the uncertainty is not considered) and with those obtained by stochastic programming (SP) and robust optimization (RO). As mentioned before, the stochastic and robust solutions can be obtained through the DRO model by considering  $\epsilon = 0$  and  $\epsilon \geq \epsilon^{max}$ , respectively. However, these solutions can also be directly obtained by solving, respectively, the models

$$SP = \{\min \frac{1}{|\Omega|} \sum_{\omega \in \Omega} \sum_{i \in N} (m_i^{\omega} + c_i - a_i^{\omega}) \mid (55) - (43)\} \quad (44)$$

$$RO = \{\min \tau \mid \tau \geq \sum_{i \in N} (m_i^{\omega} + c_i - a_i^{\omega}), \omega \in \Omega, (55) - (43)\}. \quad (45)$$

For all instances in set S1, the maximum time required for solving each one of the models (44) and (45) was 3 s. To compare the different approaches, we i) solve the DA with  $\epsilon = 0.15\epsilon^{max}$  and  $\epsilon = 0.3\epsilon^{max}$  to obtain two DRO solutions for each instance, ii) solve model (44) to obtain the SP solution, iii) solve model (45) to obtain the RO solution, and (iv) solve the deterministic model presented in Section 3.1 to obtain the deterministic solution. Then, we evaluate all the obtained solutions in all the Kantorovich ambiguity sets  $\mathbb{K}_{\epsilon}$  with  $\epsilon \in [0, 50]$  and compute the worst expected total completion time. The obtained results were grouped for each set of instances and shown in Fig. 4. These experiments aim to understand how the different solutions behave when different degrees of uncertainty are considered. The horizontal axis indicates the values of  $\epsilon$ , while the vertical axis indicates the worst expected completion time in the ambiguity set  $\mathbb{K}_{\epsilon}$ . For  $|N| = 6$  and 8 the DRO solution with  $\epsilon = 0.3\epsilon^{max}$  coincides with the RO solution for all instances, and therefore, the corresponding lines overlap in the first two plots.

Fig. 4 clearly shows the importance of considering uncertainty in the BAP in bulk terminals because the quality of the deterministic solutions drastically deteriorates as the uncertainty increases. When the uncertainty is low ( $\epsilon < 1.5$ ), SP leads to the best results; however, the quality of the solutions obtained by this approach deteriorates a lot when the uncertainty increases, as also observed for the deterministic approach. There are no significant differences between the  $DRO_{30\%}$  and RO solutions. Contrary to SP, the solutions of these two approaches perform well when the uncertainty is high and badly when it is low. The DRO model makes it possible to obtain solutions more protected against uncertainty than those obtained by SP and less conservative than those obtained by RO. This is clearly shown by the  $DRO_{15\%}$  solutions. These solutions are very competitive with those obtained by SP when the uncertainty is very low and are better than them when the uncertainty increases. Additionally, the  $DRO_{15\%}$  solutions perform much better than the RO solutions when the uncertainty is low and the performance of these approaches does not deviate too much when the uncertainty increases.

To better analyze the behavior of the approaches, we start by computing, for each  $\epsilon \in [0, 50]$ , the minimum worst expected value among all the five approaches considered. Let us call the line associated with that set of values the *artificial best line*. Table 2 represents the highest difference between the line associated with each approach presented in Fig. 4 and the artificial best line. That is, the difference between the worst average total completion time in the solutions obtained by a certain approach and the lowest worst average total completion time observed.

To analyze the results obtained, let us recall that we are considering a time horizon of 120 h (5 days), and therefore, the values reported in Table 2 are measured in hours. Table 2 indicates that ignoring the uncertainty (deterministic approach) can lead to very poor solutions with a worst average total completion time up to 10 h higher than the one in the best approach. The worst performance for SP is 4.3 h

**Table 1**  
Results for the instances of set S1.

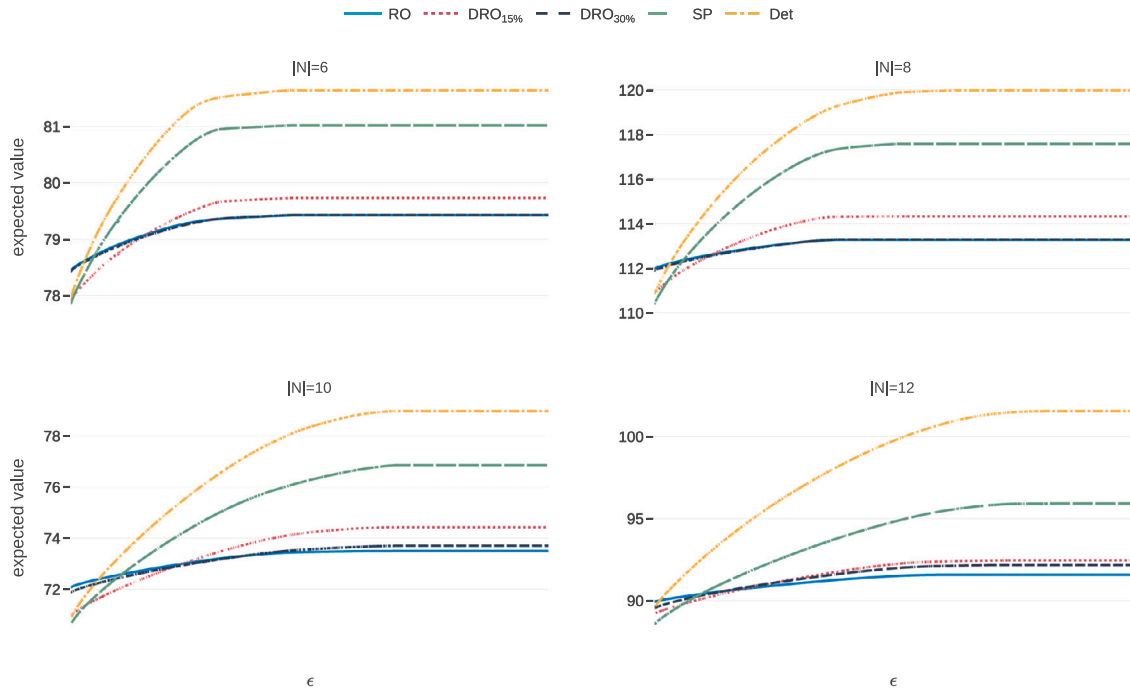
N	$\sigma$	DA					DA <sub>z</sub>				
		T <sub>AP</sub>	T <sub>DSP</sub>	T <sub>T</sub>	#it	#it <sup>imp</sup>	T <sub>AP</sub>	T <sub>DSP</sub>	T <sub>T</sub>	#it	#it <sup>imp</sup>
6	0.15	0	0	2	24	2	0	0	2	18	2
	0.3	0	0	4	33	2	0	0	1	16	2
8	0.15	5	3	67	196	3	4	3	45	151	3
	0.3	4	3	50	173	3	2	2	22	100	3
10	0.15	20	6	267	370	3	4	2	49	145	3
	0.3	9	5	129	257	3	10	4	165	201	3
12	0.15	26	22	2972	1099	4	22	5	230	299	4
	0.3	25	24	2461	1183	5	24	6	285	316	5

N	$\sigma$	DA <sub>T</sub>					DA <sub>T_z</sub>				
		T <sub>AP</sub>	T <sub>DSP</sub>	T <sub>T</sub>	#it	#it <sup>imp</sup>	T <sub>AP</sub>	T <sub>DSP</sub>	T <sub>T</sub>	#it	#it <sup>imp</sup>
6	0.15	0	0	2	17	2	0	0	1	14	2
	0.3	0	0	2	17	2	0	0	1	12	2
8	0.15	2	1	20	92	3	1	1	12	72	3
	0.3	2	2	22	91	3	1	1	9	59	3
10	0.15	6	3	93	181	3	2	2	30	98	3
	0.3	6	4	177	204	3	2	2	36	99	3
12	0.15	30	6	392	366	4	9	3	100	171	4
	0.3	20	5	273	300	4	14	4	158	209	5

N	$\sigma$	DA <sub>modified</sub>				
		T <sub>AP</sub>	T <sub>DSP</sub>	T <sub>T</sub>	#it	#it <sup>imp</sup>
6	0.15	0	0	1	2	2
	0.3	0	0	1	2	2
8	0.15	0	0	2	3	2
	0.3	0	0	2	3	2
10	0.15	0	0	3	3	2
	0.3	0	0	4	3	2
12	0.15	0	0	4	3	3
	0.3	0	0	5	4	3



**Fig. 4.** Comparison of the solutions obtained for instances of set S1 for  $\epsilon \in [0, 50]$ .

more than the best approach, while for RO is 1.6 h. In the four sets of instances, the best results were obtained by the  $DRO_{15\%}$  approach.

The results indicate that when the  $DRO_{15\%}$  approach does not lead to the best solution, the maximum deviation from it is not higher than

**Table 2**  
Worst observed performance in set S1.

Appr.	$ N $			
	6	8	10	12
<i>Det</i>	2.2	6.7	5.5	10.0
<i>SP</i>	1.6	4.3	3.4	4.3
<i>DRO</i> <sub>15%</sub>	0.3	1.1	0.9	0.9
<i>DRO</i> <sub>30%</sub>	0.6	1.6	1.3	1.0
<i>RO</i>	0.6	1.6	1.5	1.4

**Table 3**  
Results for the instances of set S2.

$ N $	Appr.	Seconds	Seconds <sub>opt</sub>	#it	#it <sub>opt</sub>
15	<i>Det</i>	1	–	–	–
	<i>SP</i>	4	–	–	–
	<i>DRO</i> <sub>15%</sub>	12	7	3	3
	<i>DRO</i> <sub>30%</sub>	17	8	4	4
	<i>RO</i>	5	–	–	–
20	<i>Det</i>	2	–	–	–
	<i>SP</i>	15	–	–	–
	<i>DRO</i> <sub>15%</sub>	154	36	5	3
	<i>DRO</i> <sub>30%</sub>	381	168	7	5
	<i>RO</i>	98	–	–	–
25	<i>Det</i>	3	–	–	–
	<i>SP</i>	121	–	–	–
	<i>DRO</i> <sub>15%</sub>	2881	1308	6	4
	<i>DRO</i> <sub>30%</sub>	4058	1563	8	4
	<i>RO</i>	632	–	–	–
30	<i>Det</i>	4	–	–	–
	<i>SP</i>	285	–	–	–
	<i>DRO</i> <sub>15%</sub>	1649	897	6	4
	<i>DRO</i> <sub>30%</sub>	4931	2415	10	5
	<i>RO</i>	709	–	–	–

1.1 h. It is also important to note that none of the approaches *Det*, *SP*, and *RO* outperform the *DRO*<sub>15%</sub> and *DRO*<sub>30%</sub> approaches regarding this indicator. This clearly shows the advantage of using DRO to deal with uncertainty in BAPs in bulk terminals.

#### 5.4. Results for the instances of set S2

This section reports the computational results for the instances in set S2, the largest ones considered. From the results reported in the previous section, we concluded that the modified DA – based on the primal cuts – clearly outperforms the original one, even when the latter is combined with the proposed improvement strategies. Therefore, in this section, we only consider the modified decomposition algorithm proposed in Section 4.3.

Table 3 reports the average results obtained for each group of 10 instances with 15, 20, 25, and 30 vessels. The third column indicates the average total computational time required to obtain the solution of each approach, while the fifth column reports the average number of iterations performed. All instances were solved to optimality by all the approaches, and in most of the instances, the last iterations of the modified DA are just used to prove the optimality of the current solution. Hence, in the fourth and sixth columns, we display the average total time and the average number of iterations required to find the optimal solution. It is important to note that the solutions of *SP* and *RO* were determined using the models (44) and (45), respectively. For that reason, no information on these approaches is reported in the fourth, fifth, and sixth columns.

These results clearly show how powerful are the new cuts used in the modified DA, since optimal solutions are obtained with a small number of iterations. The modified DA solves instances with 20 vessels in less than 10 min and instances with 30 in less than 2 h, which are acceptable times.

**Table 4**  
Worst observed performance in set S2.

Appr.	$ N $			
	15	20	25	30
<i>Det</i>	9.0	16.9	19.9	25.0
<i>SP</i>	4.0	5.6	5.8	6.8
<i>DRO</i> <sub>15%</sub>	1.4	2.4	1.8	1.9
<i>DRO</i> <sub>30%</sub>	1.1	1.0	1.0	1.0
<i>RO</i>	2.3	2.9	1.5	3.2

Next, we compare the performance of the solutions obtained by the different approaches for the instances of set S2, as done for those of set S1. Since the number of vessels in the instances of set S2 is higher than in S1, the Wasserstein distance between any two scenarios in S2 tends to be higher than in S1. This means that the threshold value  $\epsilon^{max}$  after which DRO coincides with RO in S2 is also higher than in S1. Hence, we consider now  $\epsilon \in [0, 100]$  instead of  $\epsilon \in [0, 50]$ .

The results reported in Fig. 5 and Table 4 are aligned with those reported in the previous section; however, the poor performance of the deterministic approach is even more pronounced here. Again, the best results in terms of the worst performance (Table 4) correspond to both DRO approaches. In particular, when the solutions of *DRO*<sub>15%</sub> and *DRO*<sub>30%</sub> approaches are not the best, the maximum deviation to the best ones is not higher than 2.4 and 1.1 h, respectively.

In summary, the results reported in this section reveal that the proposed modified decomposition algorithm can solve real-size instances in an acceptable time and that compared to the deterministic, stochastic, and robust solutions, the DRO solutions are either the best ones or do not deviate too much from them.

The results reported in Table 3 suggest that the computational times of the DA can vary a lot for different values of  $\sigma$ . To analyze that, we started by using the modified DA to solve the DRO model with all values of  $\sigma$  in  $\{0.1, 0.2, 0.3, \dots, 0.8, 0.9\}$  for the instances with 15, 20, and 25 vessels. Then, the computational time of the DRO model associated with each value of  $\sigma$  for each instance was divided by the computational time of the corresponding *DRO*<sub>0.1%</sub> model. Fig. 6 shows the average of those normalized values. The vertical axis indicates the ratio between the computational time of the *DRO* <sub>$\sigma\%$</sub>  and *DRO*<sub>0.1%</sub> models. It indicates that the computational time of the DRO model tends to increase with the value of  $\sigma$  until a certain point, and then starts to decrease. This behavior is explained by the number of iterations performed by the DA, which increases in the first moment and then starts to decrease.

#### 5.5. A DRO model considering uncertain cargo quantities

The DRO model presented before only assumes the arrival times of the vessels as uncertain. In this section, we extend that model to incorporate both uncertain arrival times and cargo quantities. First, we explain how to modify the model and algorithms to accommodate such uncertainty, and then we present new computational results.

Let us assume that the arrival time and cargo quantity of each vessel  $i \in N$  are random variables governed by probability distributions  $\mathcal{P}_i$  and  $\mathcal{P}_{|N|+i}$ , respectively. Let us denote by  $\mathcal{P} = (\mathcal{P}_1, \dots, \mathcal{P}_{|N|}, \dots, \mathcal{P}_{2|N|})$  the joint probability distribution of the arrival times and cargo quantities of the set of vessels considered. As before, we assume that such a distribution has finite support given by a set of scenarios  $\Omega = \{\xi^1, \dots, \xi^{|\Omega|}\}$ . However, in this case, each scenario  $\xi^\omega$  is a vector  $(a_1^\omega, \dots, a_{|N|}^\omega, q_1^\omega, \dots, q_{|N|}^\omega)$  where  $a_i^\omega$  and  $q_i^\omega$  represent, respectively, the arrival time and cargo quantity of vessel  $i \in N$  in scenario  $\omega \in \Omega$ .

When the cargo quantities are uncertain, the handling time of the vessels also becomes uncertain. Thus, the decision variable  $c_i$  becomes

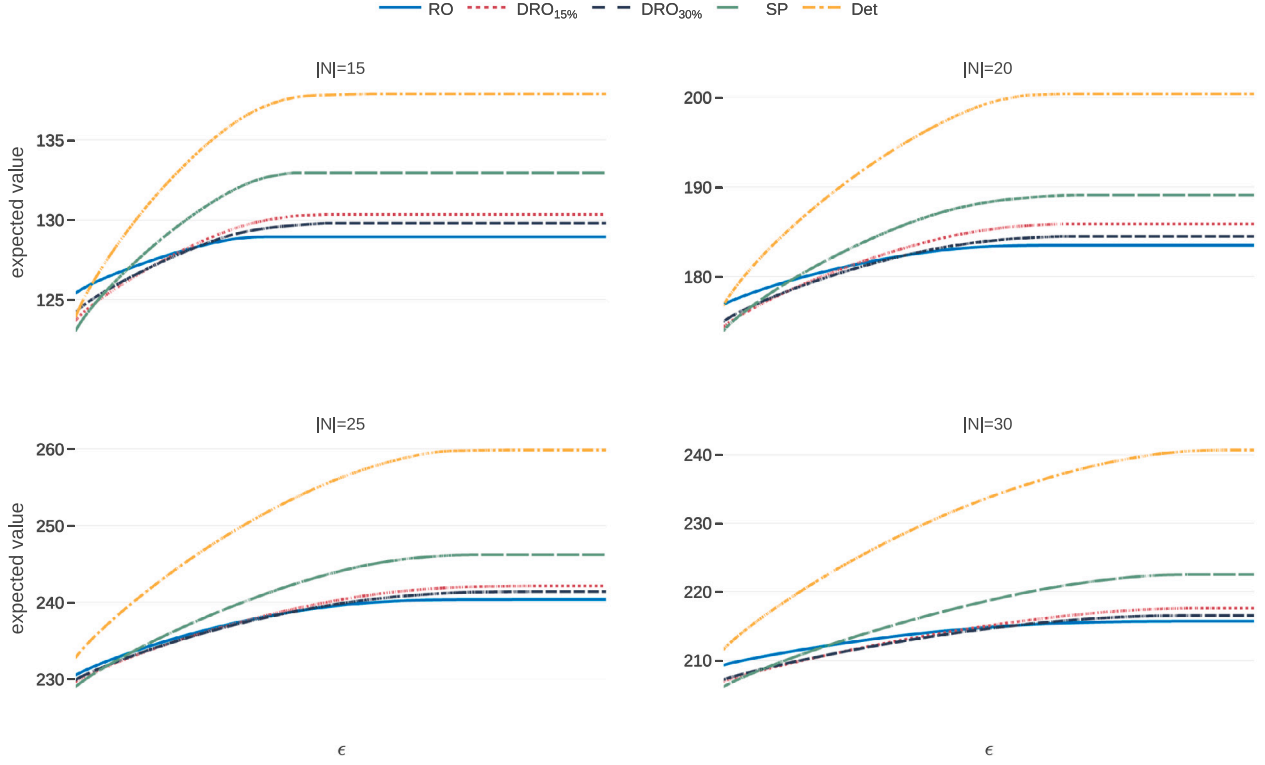


Fig. 5. Comparison of the solutions obtained for the instances of set S2 for  $\epsilon \in [0, 100]$ .

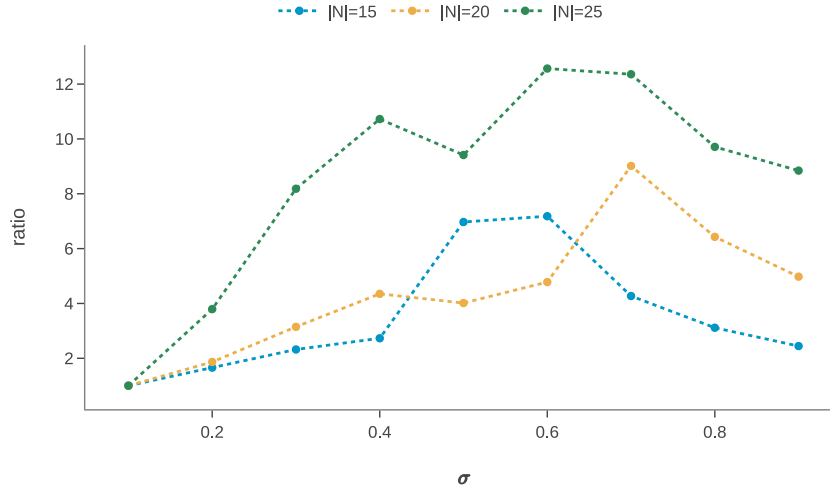


Fig. 6. Comparison of computational times for different values of  $\sigma$ .

a second-stage variable and the resulting DRO model (13) is written as:

$$\min \left\{ \max_{\mathcal{P} \in \mathcal{A}} \sum_{\omega \in \Omega} \pi_{\mathcal{P}}^{\omega} R(s, y, z, \omega) \mid (2) - (5), (11) - (12) \right\}, \quad (46)$$

where the recourse function  $R(s, y, z, \omega) = R(s, z, \omega)$  is given by

$$R(s, z, \omega) = \min_{m_i^{\omega}, c_i^{\omega}} \sum_{i \in N} (m_i^{\omega} + c_i^{\omega} - a_i^{\omega}) \quad (47)$$

$$s.t. \quad m_j^{\omega} + T^{\max}(1 - z_{ij}) \geq m_i^{\omega} + c_i^{\omega}, \quad i, j \in N, i \neq j, \quad (48)$$

$$c_i^{\omega} \geq h_k^w p_{ijk} q_i^{\omega} s_{ij}, \quad i \in N, k, j \in M, \quad (49)$$

$$m_i^{\omega} \geq a_i^{\omega}, \quad i \in N, \quad (50)$$

$$m_i^{\omega}, c_i^{\omega} \geq 0, \quad i \in N. \quad (51)$$

By following the dualization process explained before, we end up with the following optimality dual cut:

$$\begin{aligned} \theta \geq \sum_{\omega \in \Omega} \pi_{\mathcal{P}}^{\omega} \left( \sum_{i, j \in N: i \neq j} \bar{u}_{ij}^{\omega} (T^{\max}(z_{ij} - 1)) \right. \\ \left. + \sum_{i \in N, j, k \in M} \bar{r}_{ijk}^{\omega} (h_k^w p_{ijk} q_i^{\omega} s_{ij}) + \sum_{i \in N} (\bar{v}_i^{\omega} - 1) a_i^{\omega} \right) \end{aligned} \quad (52)$$

The DSP stays the same, and by denoting by  $Cut_D^{\ell}$  the dual cut (52) generated in iteration  $\ell$ , the master problem of the DA with dual cuts at iteration  $k$  becomes

$$\min \quad \theta \quad (53)$$

$$s.t. \quad Cut_D^{\ell}, \quad \ell = 0, \dots, k-1, \quad (54)$$

$$(2) - (5), (11) - (12). \quad (55)$$

The primal cut is simply given by

$$\theta \geq \sum_{\omega \in \Omega} \pi_{\mathcal{P}}^{\omega} \left( \sum_{i \in N} (m_i^{\omega} + c_i^{\omega} - a_i^{\omega}) \right) \quad (56)$$

and therefore, if  $Cut_{\mathcal{P}}^{\ell}$  is the primal cut (56) generated in iteration  $\ell$ , the modified master problem of the DA with primal cuts at iteration  $k$  is as follows:

$$\min \quad \theta \quad (57)$$

$$s.t. \quad Cut_{\mathcal{P}}^{\ell}, \quad \ell = 0, \dots, k-1, \quad (58)$$

$$(2) - (5), (11) - (12), \quad (59)$$

$$c_i^{\omega} \geq h_k^{w_i} p_{ijk} q_i^{\omega} s_{ij}, \quad i \in N, k, j \in M, \omega \in \Omega, \quad (60)$$

$$m_j^{\omega} + T^{max}(1 - z_{ij}) \geq m_i^{\omega} + c_i, \quad i, j \in N, i \neq j, \omega \in \Omega, \quad (61)$$

$$m_i^{\omega} \geq a_i^{\omega}, \quad i \in N, \omega \in \Omega, \quad (62)$$

$$m_i^{\omega}, c_i^{\omega} \geq 0, \quad i \in N, \omega \in \Omega. \quad (63)$$

### 5.5.1. Computational results

This section reports the computational results for the instances of set S2 considering both uncertain arrival times and cargo quantities. We also consider a set  $\Omega$  with 100 scenarios; however, each scenario in this set is now a vector with dimension  $2|N|$  where the first  $|N|$  components correspond to the arrival times of vessels and the second  $|N|$  components correspond to the cargo quantities. In each scenario, each vessel has the probability 30% of being delayed and of requiring a cargo quantity different than the nominal one. That is, for each scenario  $\omega \in \Omega$ , and vessel  $i \in N$ ,  $\mathbb{P}(a_i^{\omega} > a_i) = 0.3$  and  $\mathbb{P}(q_i^{\omega} \neq q_i) = 0.3$ . As before, if a vessel is delayed in a certain scenario, its delay is randomly generated between 1 and 10 h. If the cargo quantity required by a vessel in a certain scenario is not the nominal one, then it is randomly generated between  $-2$  and  $2$ .

The obtained results revealed that the original DA with dual cuts performs even worse in the DRO model with uncertain arrival times and cargo quantities than when only uncertain arrival times are considered. For example, the first instance with 6 vessels was solved by that algorithm in 26415 s after 3295 iterations. Thus, in this section, we only present the results for the new DA with primal cuts. Table 5 reports the computational times for the instances of set S2. Table 6 indicates the worst average performance of each approach for all possible values of  $\epsilon \in [0, 100]$ . The layout of Tables 5 and 6 is the same as that of Tables 3 and 4, respectively.

The results reported in Tables 5 and 6 are not very different from those presented for the DRO considering only uncertain arrival times. On average, the new DA proposed can solve instances with 30 vessels in less than 1.5 h. It is also important to note that the average time required for finding an optimal solution for the instances with 30 vessels is, on average, lower than half an hour, as reported in column  $Seconds_{opt}$ . Additionally, the DRO approaches are still the ones performing better regarding the worst performance indicator reported in Table 6.

## 6. Conclusion, limitations, and research direction

In this paper, we follow a DRO approach where the arrival times of the vessels are governed by an unknown probability distribution belonging to an ambiguity set. After introducing the full DRO model for the BAP in bulk terminals under uncertainty, we used an existing DA to solve it; however, this procedure was revealed to be very time-consuming, even for small-scale instances. To overcome this issue, we proposed two heuristic improvement strategies that decreased the running time of the DA. However, despite the positive impact of these strategies in the DA, it was not possible yet to solve instances with at

**Table 5**

Results for the instances of set S2 considering uncertain arrival times and cargo quantities.

$ N $	Appr.	Seconds	Seconds <sub>opt</sub>	#it	#it <sub>opt</sub>
15	Det	7	–	–	–
	SP	5	–	–	–
	DRO <sub>15%</sub>	11	8	3	3
	DRO <sub>30%</sub>	13	8	3	3
	RO	6	–	–	–
20	Det	16	–	–	–
	SP	18	–	–	–
	DRO <sub>15%</sub>	99	40	4	3
	DRO <sub>30%</sub>	171	93	6	4
	RO	46	–	–	–
25	Det	23	–	–	–
	SP	175	–	–	–
	DRO <sub>15%</sub>	2405	1359	6	4
	DRO <sub>30%</sub>	2932	1274	5	3
	RO	779	–	–	–
30	Det	39	–	–	–
	SP	128	–	–	–
	DRO <sub>15%</sub>	1926	1067	6	4
	DRO <sub>30%</sub>	4587	1799	7	4
	RO	598	–	–	–

**Table 6**

Worst performance in set S2 considering uncertain arrival times and cargo quantities.

	$ N $			
Appr.	15	20	25	30
Det	8.3	13.7	23.2	27.6
SP	1.9	4.8	7.7	5.1
DRO <sub>15%</sub>	1.4	3.1	2.5	2.4
DRO <sub>30%</sub>	1.0	2.0	3.0	1.6
RO	2.6	3.7	4.9	2.6

least 15 vessels in less than 2 h. Thus, we developed an exact modified DA that clearly outperformed the original one and made it possible to solve large-scale instances efficiently. The new DA solves instances with 20 vessels in less than 10 min and instances with 30 vessels in less than 1.5 h.

To show the benefits of using DRO, we compare the DRO solutions with the deterministic ones and with those obtained by two of the most popular approaches to deal with uncertainty: stochastic programming and robust optimization. The comparison was done in different uncertain environments, leading to very interesting conclusions. First, it became clear how bad it can be not to consider uncertainty when the berth allocation plan is made. Second, the results showed that DRO leads to solutions more protected against uncertainty than those provided by stochastic programming and less conservative than those obtained with robust optimization. In fact, when the DRO solutions are not the best ones – compared to the deterministic, stochastic, and robust ones – the maximum observed deviation regarding the best solution was at most 3 h. This value is significantly lower than those obtained for the deterministic, stochastic, and robust approaches (27.6, 7.7, and 4.9, respectively). This is a clear indicator of the benefits of using DRO in BAPs in bulk terminals.

The proposed approach can be directly applied in practice because it only requires a sample of scenarios of uncertainty that can easily be obtained from historical data available at the ports. The proposed DRO model depends on a risk parameter  $\epsilon$  that controls the size of the ambiguity set. By varying this parameter, it becomes possible to obtain structurally different and good-quality solutions. Hence, the proposed approach can be used by port managers as a tool to generate a set of good quality solutions with different degrees of protection against uncertainty. Such solutions can then be easily evaluated by simulation techniques or out-of-sample approaches to choose the best one according to the port manager's preference.



This paper represents the first step on DRO in BAPs in bulk terminals and despite its significant contributions, the proposed approach also has some limitations that can lead to promising future research directions. The new DA proposed makes it possible to solve instances with up to 30 vessels in a reasonable time; however, the computational time tends to increase a lot when more vessels are considered. We tried to use our algorithm to solve instances with 40 vessels, but only one of the 10 instances considered was solved in less than two hours. All the other instances were not solved in one day. This is mainly related to the hardness of solving the resulting master problems to optimality, which is required to ensure the algorithm's convergence. This means that the proposed DA cannot be applied directly to instances with more than 30 vessels; however, it can be embedded in a rolling horizon heuristic (RHH) as usually done in other complex optimization problems [44]. RHHs can be applied to the BAP in bulk terminals by partitioning the set of vessels into  $k$  groups according to their expected arrivals and defining an ambiguity set for each of those groups. The proposed DA can then be used to solve the resulting DRO subproblems independently and sequentially, taking into account the solutions of the previous subproblems. Notice that our DA can solve instances with 15 vessels in less than 20 s, which makes it possible to embed it in the RHH to (heuristically) solve very large instances efficiently.

The number of scenarios considered in the ambiguity set can impact the computational time of the DA because the number of second-stage variables and constraints in each master problem depends on the number of scenarios considered. Thus, large scenario samples may be an obstacle to the efficiency of the DA. In this case, scenario reduction/aggregation strategies may be applied to reduce the size of the resulting sample. This is a promising future research direction. Another possible path to follow when large samples are available is to use them to estimate the moments of the *true* probability distribution of the uncertain parameters and then use moment-based ambiguity sets instead of discrepancy-based sets.

### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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### Data availability

Data will be made available on request.

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