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Article

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Evaluating metaheuristic solution quality for a hierarchical vehicle routing problem by strong lower bounding

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ABSTRACT

We study a vehicle routing problem that originates from a Nordic distribution company and includes the essential decision-making components of the company's logistics operations. The problem considers customer deliveries from a depot using heavy depot vehicles, swap bodies, optional switch points, and lighter local vehicles; a feature is that deliveries are made by both depot and local vehicles. The problem has earlier been solved by a fast metaheuristic, which does however not give any quality guarantee. To assess the solution quality, two strong formulations of the problem based on the column generation approach are developed. In both of these the computational complexity is mitigated through an enumeration of the switch point options. The formulations are evaluated with respect to the quality of the linear programming lower bounds in relation to the bounds obtained from a compact formulation. The strong lower bounding quality enables a significant reduction of the optimality gap compared to the compact formulation. Further, the bounds verify the high quality of the metaheuristic solutions, and for several problem instances the optimality gap is even closed.

1. Introduction

The Nordic countries are sparsely populated, especially in the northern parts, and the need for cost-effective and environmentally friendly goods transportation has necessitated the adaptation of longer and heavier vehicles.

Such vehicles consume more energy than conventional vehicles, but fewer vehicles are needed to deliver the same amount of goods, and the cost and emission caused are lower per transported unit. Finland was the first European country to allow vehicles up to 34.5 meters long, carrying a load of 74 tonnes. In Sweden vehicles carrying up to 74 tonnes have been allowed since 2018. From December 2023 vehicles up to 34.5 meters long are allowed on parts of the road transportation network.

The new vehicle routing problem considered in this work, the Hierarchical Multi-Switch Multi-Echelon Vehicle Routing Problem (HMSME-VRP), involves high-capacity vehicles (HCV) which are up to 34.5 meters long and can carry a load of up to 74 tonnes. The problem is derived from the real-world logistics operations of a Nordic distribution company, and it condenses the essential components of its decision-making into a manageable framework. All important structural decisions, restrictions, and goals are included, while some non-essential elements are disregarded.

The HMSME-VRP takes into consideration the fact that distribution companies are interested in covering large geographical areas without significant infrastructure investments. This is possible since some roads allow up to three conventional distribution vehicles to be replaced with one heavy vehicle, thereby enabling an environmentally friendlier and more efficient distribution of goods. This problem has been little studied. It was introduced by Tadaros et al. [1] and has shown to be computationally hard to solve to optimality by using a commercial solver, even for instances of small sizes. In order to solve instances of realistic sizes approximately, Tadaros et al. [2] developed General Variable Neighborhood Search procedures. Subsequently, Tadaros and Kyriakakis [3] extended the problem by introducing service and maximum route times, and developed a mixed-integer formulation for the extended model, called HMSME-VRP-ST. Given the additional complexity that the service and route times entail, they devised a Hybrid Clustered Ant Colony Optimization approach for the approximate solution of the problem.

The metaheuristic presented in Tadaros and Kyriakakis [3] can in short computation times find feasible solutions, and corresponding upper bounds for the optimal values, that are better than what can be obtained from an off-the-shelf branch-and-cut algorithm after long computation times, and this holds even for smaller instances. It is

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however unknown how much the metaheuristically found solutions deviate from exact optimality.

The objective of this work is to further assess the performance of this metaheuristic. We specifically aim at evaluating the quality of the previously reported metaheuristic solutions by computing tight lower bounds for the optimal values. To this end we propose two strong problem formulations based on the general column generation principle, but heavily adapted to the problem at hand.

The first formulation employs rich columns, which include choices of both vehicles and routes, and a master problem with a simple structure. In contrast, the second formulation employs columns that correspond to routes, while the master problem is more complex and includes the choices of vehicles. The foundation for both approaches is an explicit enumeration of certain key decisions, which enables us to construct a number of different column categories and column generation problems. The latter are capacitated prize-collecting traveling salesperson problems, or generalizations thereof.

1.1. Problem description

This paper deals with the problem HMSME-VRP-ST [3], henceforth simply called HME-VRP, where heavy distribution vehicles carry up to three interchangeable containers, called *swap bodies*, through a non-predetermined distribution network hierarchy. The swap bodies can be detached from the vehicle and stand on fold-out support legs, and be attached to lighter distribution vehicles. Customers are geographically distributed with known demands which must be met. Customer deliveries are managed by routing vehicles directly from a *central depot* or from given, optional intermediate facilities called *switch points*. These are given locations with enough space for temporarily storing swap bodies that are detached from one vehicle in order to be attached to another.

The problem includes two types of vehicles, which are heavy *depot vehicles* (DV) and lighter *local vehicles* (LV). Both types of vehicles make deliveries to customers along routes. Depot vehicles start at the central depot and carry up to three swap bodies. Local vehicles start at switch points and carry single swap bodies. If a depot vehicle carry two or three swap bodies, then one or two of these, respectively, are first detached at switch points before it makes customer deliveries from the remaining swap body. If two swap bodies are detached from a depot vehicle, then it can be at the same switch point or at a first and a second switch point. Any detached swap body at a switch point is picked up by a local vehicle, which makes customer deliveries from the swap body. The local vehicle returns the emptied swap body to the switch point, and it is then picked up by the same depot vehicles as carried the swap body to the switch point. A depot vehicle hence revisits switch points to re-attach earlier detached swap bodies before returning to the central depot, within a given time limit.

Problem HME-VRP takes into consideration the flexibility for depot vehicles to visit customers directly from the depot, with one swap body attached, or to transfer swap bodies at optional switch points to local vehicles that perform customer deliveries. Each customers is served from the depot or from a first-level or second-level switch point, and each vehicle, depot as local, can perform only one customer route, which is constrained by swap body capacity and a time limit for transportation and activities at switch points. The goal is to minimize total routing cost, comprising fixed costs for the vehicle fleet and the swap bodies, and variable transportation costs. A solution encompasses a set of depot and local vehicle routes, the number of utilized swap bodies, and the allocation of customers to vehicle tours from either the depot or a switch point. Notice that the hierarchy of the switch points, into first- or second-level points, holds for a particular depot vehicle and that it is not determined a priori, but given by an actual solution.

Fig. 1 depicts a feasible solution for an instance with 16 customers and three switch points. Here, three depot vehicles, three local vehicles, and six swap bodies are used to serve the customers. The first depot

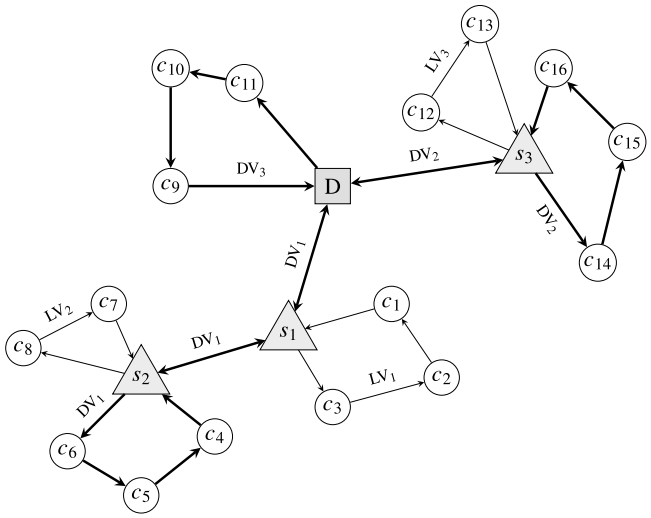

Fig. 1. A feasible solution to the HME-VRP for an instance with 16 customers and three switch points.

vehicle (DV_1) leaves the depot with three swap bodies, first visits switch point s_1 to detach one swap body, and then continues to switch point s_2 to detach a second swap body. As soon as the swap bodies have been detached from DV_1, local vehicles attach them and perform customer tours independently, with LV_1 starting from s_1 and LV_2 starting from s_2. At the same time, DV_1 continues with a tour that serves three customers, after which it returns first to s_2 and then to s_1 to re-attach the swap bodies before returning to the depot. Depot vehicle DV_2 only visits s_3 where it detaches a swap body, which is assigned to local vehicles LV_3. Lastly, DV_3 serves customers directly from the depot with only one swap body attached.

The remainder of this paper is organized as follows. In Section 1.2 we review related problems in the literature and some central contributions with regard to exact methods, in particular decomposition based approaches. Section 2 provides a formal description of our problem and two alternative column generation formulations. (The reader is assumed to be familiar with the column generation principle.) Numerical results based on existing and newly generated instances are reported in Section 4. The results are discussed in Section 5 before we conclude the paper in Section 6.

1.2. Related problems

The HME-VRP is little studied but related to and shares some common characteristics with other extensions of the classical Vehicle Routing Problem (VRP), namely the Two-Echelon VRP (2E-VRP), the Truck and Trailer Routing Problem (TTRP), and its extension the Swap-Body VRP (SB-VRP). In the 2E-VRP, two fleets of vehicles are considered. Goods are delivered to intermediate facilities, where they are stored, consolidated, and transferred to other vehicles before being shipped to their final destinations. The two vehicle fleets handle specific echelons of the network and are referred to as first- and second-echelon vehicles. In contrast to HME-VRP, first-echelon vehicles in the 2E-VRP are not allowed to continue from an intermediate facility to serve customers, and customers cannot be reached directly from the central depot. In the HME-VRP the depot vehicles may serve customers and can visit up to two switch points, thus imposing a distribution hierarchy between them. Moreover, the nature of the intermediate facilities is different. In the 2E-VRP goods are stored, consolidated, and transferred between vehicles, while in HME-VRP complete carriers are transferred between vehicles.

Table 1, adapted from Tadaros and Kyriakakis [3], provides an overview of the characteristics of the different problems.

Table 1
Comparison of the characteristics between the HME-VRP and related problems.

	HME-VRP	2E-VRP	TTRP	SB-VRP
Network	Non-predetermined hierarchy	Predetermined hierarchy	Non-hierarchical	Non-hierarchical
Customers	No differentiation	No differentiation	Truck customers, Vehicle Customers	Truck customers, Train Customers, Mandatory Train Customers
Vehicle fleet(s)	Heterogeneous	Heterogeneous	Homogeneous	Homogeneous
Use of loading carriers	Swap bodies	-	Trailers	Swap bodies
Operations at intermediate facilities	Swap body transfer	Unloading/loading	Park, Pick-up	Park, Pick-up, Exchange, Re-order
See for example	Tadaros and Kyriakakis [3]	Sluijk et al. [4]	Drexl [5]	Toffolo et al. [6]

Due to the computational complexity of the abovementioned problems, many contributions in the field are concerned with the design of (meta-)heuristics for solving instances of larger size, see for e.g.[6–11]. Some exact methods, especially decomposition-based approaches, have also been proposed, see e.g.[5,12–18], as outlined below. For a comprehensive discussion about the mentioned problems, their properties, extensions and proposed solution methods, the interested reader is referred to Sluijk et al. [4] and Cuda et al. [19] and references therein.

1.2.1. TTRP and SB-VRP

The TTRP does not consider multiple echelons and involves a fleet of trucks and trailers. A truck can be used alone or it can pull an additional trailer, which is then referred to as a complete vehicle. Further, customers are grouped into two categories: those that either vehicle configuration can serve and those that can only be served by a truck alone. A feature of the TTRP is that a complete vehicle can detach its trailer at specific customer locations and then continue to serve truck customers before returning to the trailer and re-attach it. The SB-VRP is an extension of the TTRP. Here the trucks and trailers carry swap bodies that can be coupled and decoupled at intermediate facilities, called swap locations. Furthermore, a third group of customers are introduced, which has to be served by a complete vehicle. In contrast to HME-VRP, the swap bodies cannot change vehicle assignments during service.

A generalized TTRP was studied by Drexl [5]. Compared to the basic version of the TTRP the essential difference is that the locations at which trailers can be parked and load transferred are independent of customer locations. The author presents a mixed-integer programming formulation and proposes a branch-and-price algorithm to solve the problem. The algorithm incorporates both exact and heuristic elements. The pricing problem is a shortest-path problem with resource constraints which is solved heuristically as long as it yields negative reduced cost, and otherwise it is solved exactly. Based on the computational experiments conducted, the author concludes that with a heuristic column generation approach, high-quality solution to instances of real-world size can be found in short time.

1.2.2. 2E-VRP

The 2E-VRP has received more attention in the literature concerning exact methods. In order to find lower bounds for the problem, Gonzalez-Feliu [12] proposed a column generation method. Contardo et al. [13] derived both upper and lower bounds by combining a branch-and-cut algorithm based on a new two-indexed formulation with an Adaptive Large Neighborhood Search algorithm proposed by Hemmelmayr et al. [9]. Baldacci et al. [14] proposed a new mathematical formulation of the problem with the aim to derive lower bounds. They further propose a decomposition-based algorithm in which all feasible first-echelon routes are enumerated, lower and upper bounds are computed, and a set of first-echelon solutions are generated. For each promising first-echelon solution, an upper bound to the problem is obtained by solving a resulting multi-depot VRP with side constraints. A branch-and-cut-and-price algorithm for the 2E-VRP was proposed by Marques et al. [15], based on a new route-based formulation of the problem. Further, they proposed a new family of valid inequalities and a new branching strategy, which is able to reduce the size of the branch-and-bound tree. The authors were able to solve instances including as many as 200 customers and 10 intermediate facilities to optimality.

Li et al. [20] study the 2E-VRP with grouping constraints and simultaneous pickup and delivery. The authors developed a path-based model, which is solved by a branch-and-cut-and-price algorithm. Furthermore, the pricing problem was solved using a labeling algorithm that incorporates a dominance rule. Along with several valid inequalities, the solution process could be improved by leveraging specific characteristics of the problem.

For the 2E-VRP with hard time windows, Dellaert et al. [18] proposed an arc-based and two path-based formulations of the problem. The two path-formulations differ in the definitions of the paths; one defines the paths over both first- and second-echelon tours, and in the other the first- and second-echelon tours are decomposed. The formulations are solved by branch-and-price procedures, with the one based on decomposed first- and second-echelon routes performing best. For this formulation, the procedure first enumerates the first-echelon routes and then the second-echelon part of the problem is solved by column generation with respect to second-echelon routes. In addition to time windows, Dellaert et al. [16] consider customer-specific demands, that is, non-substitutable demands, and propose multi-commodity formulations which are both arc- and path-based. The authors extend the solution method proposed by Dellaert et al. [18] by introducing new types of capacity constraints and better lower bound estimates regarding the first echelon of the two-path formulation.

More recently, Sluijk et al. [17] considered stochastic demands in the 2E-VRP and modeled the problem as a chance-constrained stochastic optimization problem. The authors proposed two solution procedures based on column generation to compute lower bounds. Further, multi-labeling algorithms are used, one where second-echelon routes are constructed simultaneously and one where they are constructed sequentially.

2. Column generation based formulations

In this section a formal statement of the problem is provided followed by the formulation of two alternative models, which are both based on the column generation concept. The models differ with respect to how a column is defined.

2.1. Basic notations

The distribution network is represented mathematically by a directed graph $G = (N, A)$. The node set $N = \{0\} \cup P \cup I$, where node 0 represents the depot, node set P represents the switch points, and node set I represents the customers. The arc set A represents the possible ways to travel between the depot, the switch points, and the customers.

The depot vehicles, local vehicles, and the swap bodies have fixed costs of F_{DV}, F_{LV}, and F_{SB}, respectively. For each arc $(i, j) \in A$ there is variable travel cost c_{ij}, for both a depot vehicle and a local vehicle. The capacity of a swap body is C. The demand of a customer is $d_i \leq C$ and it must be satisfied entirely from a single vehicle and swap body. For each arc $(i, j) \in A$ there is a travel time T_{ij}, for both types of vehicles. The transfer of swap bodies between vehicles at switch points and the unloading at customers are subject to service times s_i, $i \in P \cup I$.

A depot vehicle must return to the depot with the same swap bodies as it left with, and within the time limit T^{max}. We assume that the triangle inequality holds for both travel costs and travel times.

A compact mixed-integer formulation of the problem is given in Tadaros and Kyriakakis [3], as well as in Appendix. This formulation has a linear programming (LP) relaxation that is in general very weak and it is therefore intractable for standard mixed-integer solvers based on branch-and-bound. We here suggest two stronger formulations based on column generation.

2.2. Decomposition of the HME-VRP

A solution to HME-VRP is composed of a number of *customer tours*, each of which comprises a single swap body on a vehicle. The reader should note that there will therefore be a one-to-one-to-one relationship between customer tours, swap bodies, and vehicles.

A customer tour originates from the central depot or from a switch point, it serves a number of customers before it returns to its starting point, and the total demand of the customers along the tour can of course not exceed the swap body capacity. It is assumed that these tours are always routed in an optimal way. (The proposed column generation approaches guarantee optimal routing.)

If the customer tour originates from the depot, then it is performed by a depot vehicle. If the tour starts at a switch point, then the swap body has first been carried to the switch point by a depot vehicle and then the tour is performed by the depot vehicle itself or by a local vehicle.

The foundation for the proposed column generation approaches is the observation that there are four distinct *configurations* for routing swap bodies from the central depot to customers and back to the depot, by using depot vehicles and local vehicles, as illustrated in Fig. 2. Here loops depict customer tours and thicker arcs correspond to depot vehicles. The maximal allowed time for a customer tour depends on the configuration considered.

Configuration 1 comprises a depot vehicle with a single swap body which performs a customer tour. The maximal available tour time is T^{max}.

Configuration 2 comprises a depot vehicle with two swap bodies which travels from the depot to a first-level switch point $p \in P$, at which the depot vehicle and a local vehicle perform customer tours, with one swap body each. Configuration 3 is similar, but here the depot vehicle carries three swap bodies from the depot and two local vehicles are used. For these two configurations the available tour time is $T^{max} - 2(T_{0p} + s_p)$.

Configuration 4 comprises one depot vehicle with three swap bodies. It first delivers one swap body to a first-level switch point $p \in P$ and then continues to a second-level switch point q. We denote by $Q_p = P \setminus \{p\}$ the switch points that can be visited after switch point p. The depot vehicle makes a tour from point q and local vehicles make tours from each of the points p and q. The available tour times from switch points p and $q \in Q_p$ are $T^{max} - 2(T_{0p} + s_p)$ and $T^{max} - 2(T_{0p} + s_p) - 2(T_{pq} + s_q)$, respectively.

Whenever a depot vehicle has detached a swap body at a switch point, this swap body is also re-attached to the depot vehicle on it route back to the depot.

The four configurations cover all possible distribution structures included in an optimal solution. If travel costs and times fulfill the triangle inequality, then it is never better to distribute a swap body via a switch point unless this is needed for transferring the swap body, or another swap body, to a local vehicle. Hence, if there is an empty customer tour from switch point p in Configuration 4 then it can be reduced to Configuration 2. Likewise, if there is an empty tour from switch point p in Configuration 2 then it can be reduced to Configuration 1.

All possible customer tours in accordance with Configuration 1 start at the depot. There is however one Configuration 2 and one Configuration 3 for each of the $|P|$ switch points. Further, there is one Configuration 4 for each ordered pair of distinct switch points. Hence there is in total $1 + 2|P| + |P|(|P| - 1)$ sub-configurations for depot vehicles and local vehicles, and within each sub-configuration the customer tours can differ. A specific sub-configuration with a choice of customer tours is called a *distribution pattern*. The above made explicit enumeration of configurations and sub-configurations is a key component in our approach. In practice the number $|P|$ of optional switch points is quite limited, and the enumeration is therefore computationally feasible.

We give two column-oriented formulations of the HME-VRP that are based the configurations. In the first formulation, each sub-configuration gives rise to a subset of columns and each column in the subset corresponds to a distribution pattern for the sub-configuration. In the second, each column instead corresponds to a single customer tour.

2.3. A lower bound on customer tours

A trivial lower bound for the number of swap bodies needed for the deliveries to the customers is given by $\lceil \sum_{i \in I} d_i / C \rceil$.

A lower bound that is at least as strong can be found by solving a bin packing problem. Given a set J of swap bodies that can be used, this bin packing problem is stated as

$$[\text{BIN}] \quad SB^{min} = \min \sum_{j \in J} y_j \tag{1}$$

$$\text{s.t.} \sum_{i \in I} d_i x_{ij} \leq C y_j, j \in J \tag{2}$$

$$\sum_{j \in J} x_{ij} = 1, i \in I \tag{3}$$

$$x_{ij} \in \{0,1\}, i \in I,\ j \in J \tag{4}$$

$$y_j \in \{0,1\}, j \in J, \tag{5}$$

where y_j is 1 if swap body j is used and 0 otherwise, and x_{ij} is 1 if customer i is served from swap body j and 0 otherwise. The objective function (1) minimizes the number of swap bodies used, constraint (2) prevents violation of swap body capacity and enforces the correct logical relationship between any y_j and any x_{ij}, and constraint (3) ensures that each customer is served from a single swap body. (Clearly, $|J|$ must be large enough to admit a feasible solution.)

Notice that no routing aspects of the HME-VRP are included in the bin packing problem. Since each swap body is used for deliveries on one customer tour made by one vehicle (depot or local), the number SB^{min} is also a lower bound for the number of customer tours and the number of vehicles used.

The lower bound SB^{min} is used to construct a valid inequality that is included in the two formulations of HME-VRP presented below; this inequality mostly strengthens their LP relaxations.

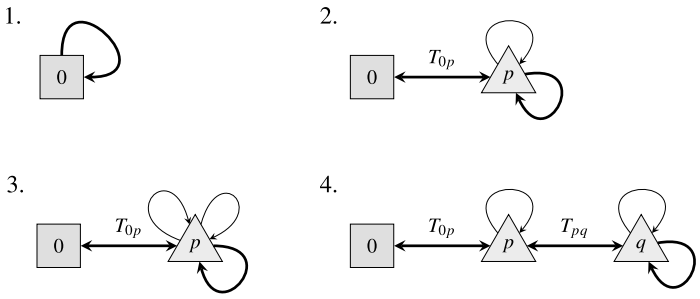

Fig. 2. The four possible configurations for a depot vehicle, local vehicles, and their swap bodies.

2.4. Column generation based on configurations

In the first formulation each column corresponds to a distribution pattern for one of the sub-configurations described in Section 2.2. The master problem is a set partitioning problem with a single side constraint on the minimal number of swap bodies to be used. With $\mathcal{K}_{\mathsf{CG1}}$ denoting the set of all feasible distribution patterns, the master problem is

$$[\,\mathsf{MP\,1}\,] \quad \min \; z = \sum_{k\in\mathcal{K}_{\mathsf{CG1}}} c_k u_k \tag{6}$$

$$\text{s.t.} \sum_{k\in\mathcal{K}_{\mathsf{CG1}}} a_{ik} u_k = 1, \qquad i \in I \tag{7}$$

$$\sum_{k\in\mathcal{K}_{\mathsf{CG1}}} n_k u_k \geq \mathrm{SB}^{\min}, \tag{8}$$

$$u_k \in \{0,1\}, \qquad k \in \mathcal{K}_{\mathsf{CG1}}. \tag{9}$$

The variable u_k indicates whether distribution pattern k is used. Its cost is

$$c_k = f_k + \sum_{(i,j)\in A_k} c_{ij}, \tag{10}$$

where f_k is the total fixed cost for the vehicles and swap bodies that are used in the distribution pattern (as described below), and the set A_k contains the arcs traversed in the customer tours of the pattern. The parameter $a_{ik} \in \{0,1\}$ indicates whether customer i is included in some customer tour in distribution pattern k, and the integer parameter n_k specifies the number of swap bodies used in the pattern.

The total fixed costs depend on the configuration for the distribution pattern. Contributions come from (*i*) the cost of one depot vehicle ($\mathrm{F_{OV}}$), (*ii*) the cost of swap bodies ($\mathrm{F_{SB}}$), (*iii*) the cost of local vehicles ($\mathrm{F_{LV}}$), and (*iv*) the cost of going back and forth between the depot and a switch point (c_{0p}), and the cost of going back and forth between two switch point (c_{pq}). For the four configurations, the total fixed costs are as follows.

$$f_k = \begin{cases} f_0^1 = \mathrm{F_{DV}} + 1\mathrm{F_{SB}} \\ f_p^2 = \mathrm{F_{DV}} + 2\mathrm{F_{SB}} + 1\mathrm{F_{LV}} + 2c_{0p} \\ f_p^3 = \mathrm{F_{DV}} + 3\mathrm{F_{SB}} + 2\mathrm{F_{LV}} + 2c_{0p} \\ f_{pq}^4 = \mathrm{F_{DV}} + 3\mathrm{F_{SB}} + 2\mathrm{F_{LV}} + 2c_{0p} + 2c_{pq} \end{cases} \tag{11}$$

Moving forward, we assume that a subset $K_{\mathsf{CG1}} \subset \mathcal{K}_{\mathsf{CG1}}$ of columns is available, and that an optimal solution to the LP relaxation of the resulting restricted master problem, called RMP 1, has been found. Let the dual variables for constraints (7) and (8) be denoted by λ_i, $i \in I$, and π, respectively.

There is a column generation problem for each of the $1 + 2|P| + |P|(|P| - 1)$ sub-configurations for depot vehicles and local vehicles. For each of these the column generation finds one to three customer tours for the vehicles used, depending on the configuration considered. For Configuration 1 the column generation problem is a capacitated prize-collecting traveling salesperson problem for a depot vehicle; for the other three configurations the column generation is a generalization of that problem, since it then involves routing two or three vehicles on non-overlapping customer tours from one or two switch points.

Let R be the set of customer tours to be constructed for a certain sub-configuration, and let i_r denote the starting node for tour $r \in R$. Each tour r has a maximal possible time, which is denoted Γ_r; it is described in Section 2.2 how it is derived for each of the configurations. Let the binary variable x_{ij}^r be 1 if tour r follows arc (i,j) and 0 otherwise, and let the binary variable y_i^r be 1 if tour r visits customer i and 0 otherwise. The continuous variable t_i^r is the visiting time of tour r at customer i, and τ_r is the total time for tour r. Both these variables count the time relative to the starting time for tour r. A generic column generation problem for all sub-configurations can then be formulated as follows.

$$[\,\mathsf{CG\,1}\,] \quad \min \; \mathrm{RC} = f + \sum_{r\in R}\left[\sum_{(i,j)\in A} c_{ij} x_{ij}^r - \sum_{i\in I} \lambda_i y_i^r\right] - |R|\pi \tag{12}$$

$$\text{s.t.} \sum_{(i_r,j)\in A} x_{i_r j}^r = 1, \qquad r \in R \tag{13}$$

$$\sum_{(i,j)\in A} x_{ij}^r = y_i^r, \qquad i \in I \setminus \{i_r\},\ r \in R, \tag{14}$$

$$\sum_{(i,j)\in A} x_{ij}^r = y_j^r, \qquad j \in I \setminus \{i_r\},\ r \in R, \tag{15}$$

$$\sum_{i\in I} d_i y_i^r \leq C, \qquad r \in R \tag{16}$$

$$t_i^r + (s_i + T_{ij}) x_{ij}^r \leq t_j^r + \Gamma_r(1 - x_{ij}^r), \qquad (i,j) \in A : j \neq i_r,\ r \in R, \tag{17}$$

$$t_i^r + (s_i + T_{ii_r}) x_{ii_r}^r \leq \tau_r + \Gamma_r(1 - x_{ii_r}^r), \qquad i \in I \setminus \{i_r\},\ r \in R \tag{18}$$

$$\sum_{r\in R} y_i^r \leq 1, \qquad i \in I \tag{19}$$

$$0 \leq t_i^r \leq \Gamma_r, \qquad i \in I \tag{20}$$

$$0 \leq \tau_r \leq \Gamma_r, \qquad r \in R \tag{21}$$

$$x_{ij}^r \in \{0,1\}, \qquad (i,j) \in A,\ r \in R, \tag{22}$$

$$y_i^r \in \{0,1\}, \qquad i \in I,\ r \in R \tag{23}$$

Here RC is the reduced cost of a column in MP 1, which corresponds to a distribution pattern. Constraint (13) states that customer tour r should leave its starting node, while constraints (14) and (15) state that if the tour visits customer i then it must arrive to and leave the customer, respectively. Constraint (16) states that the total demand of the customers along a tour may not exceed the swap body capacity. Constraints (17), (18), (20), and (21) eliminate subtours, by using the Miller–Tucker–Zemlin principle, and ensure that the maximal tour time Γ_r is respected. Constraint (19) states that each customer is visited by at most one tour.

The parameters in the model depend on the sub-configuration considered, as stated below. Here the enumeration of the sub-configurations is again used.

Configuration 1:
$R = \{1\}$, $i_1 = 0$, $\Gamma_1 = \mathrm{T}^{\max}$, and $f = f_0^1$.
Configuration 2, for each $p \in P$:
$R = \{1, 2\}$, $i_r = p$ and $\Gamma_r = \mathrm{T}^{\max} - 2(T_{0p} + s_p)$, $r \in R$, and $f = f_p^2$.
Configuration 3, for each $p \in P$:
$R = \{1, 2, 3\}$, $i_r = p$ and $\Gamma_r = \mathrm{T}^{\max} - 2(T_{0p} + s_p)$, $r \in R$, and $f = f_p^3$.
Configuration 4, for each $p \in P$ **and each** $q \in Q_p$:
$R = \{1, 2, 3\}$, $i_1 = p$ and $\Gamma_1 = \mathrm{T}^{\max} - 2(T_{0p} + s_p)$, $i_2 = i_3 = q$
and $\Gamma_2 = \Gamma_3 = \mathrm{T}^{\max} - 2(T_{0p} + s_p) - 2(T_{pq} + s_q)$, and $f = f_{pq}^4$.

We note that all customer tours obtained will always be optimally routed, given that the column generation problem CG 1 is solved to optimality. This column generation scheme, which is based on the four configurations and comprises the restricted master problem RMP 1 and column generation CG 1 is henceforth referred to as $\mathsf{CG}_{\text{conf}}$.

The scheme $\mathsf{CG}_{\text{conf}}$ terminates when CG 1 does not find a negative reduced cost for any sub-configuration. Then the restricted problem RMP 1 contains columns that gives an optimum to the LP relaxation of MP 1. The optimal value of RMP 1 is therefore a lower bound for the optimal value of MP 1, and for HME-VRP. The generated columns are typically not sufficient to find an optimum to the integer problem MP 1, but by solving the integer version of the final RMP 1 one can however find an integer feasible solution to MP 1, which gives a feasible solution and an upper bound for the HME-VRP.

2.5. Column generation based on customer tours

In the second formulation each column corresponds to a customer tour from the depot or from a switch point. The master problem is a set partitioning problem with additional variables and constraints which ensure that the customer tours used can be realized as sub-configurations for depot vehicles and local vehicles, as defined in Section 2.2. Let $\mathcal{K}_0$, $\mathcal{K}_p$, $p \in P$, and $\mathcal{K}_{pq}$, $q \in Q_p$, $p \in P$, be the sets all feasible customer tours from the depot, from first-level switch points, and from second-level switch points, respectively. The available times for these tours are $\mathrm{T}^{\max}$, $\mathrm{T}^{\max} - 2(T_{0p} + s_p)$, and $\mathrm{T}^{\max} - 2(T_{0p} + s_p) - 2(T_{pq} + s_q)$, respectively. The set of all feasible tours is

$$\mathcal{K}_{\text{CG2}} = \mathcal{K}_0 \cup \left(\cup_{p \in P} \mathcal{K}_p\right) \cup \left(\cup_{p \in P} \cup_{q \in Q_p} \mathcal{K}_{pq}\right),$$

and the master problem is

$$[\,\text{MP 2}\,] \quad \min z = \sum_{k \in \mathcal{K}_{\text{CG2}}} c_k v_k + f_0^1 w^1 + \sum_{p \in P} \left[f_p^2 w_p^2 + f_p^3 w_p^3 + \sum_{q \in Q_p} f_{pq}^4 w_{pq}^4 \right] \tag{24}$$

$$\text{s.t.} \sum_{k \in \mathcal{K}_{\text{CG2}}} a_{ik} v_k = 1, \qquad i \in I \tag{25}$$

$$\sum_{k \in \mathcal{K}_0} v_k = w^1, \tag{26}$$

$$\sum_{k \in \mathcal{K}_p} v_k = 2w_p^2 + 3w_p^3 + \sum_{q \in Q_p} w_{pq}^4, \qquad p \in P \tag{27}$$

$$\sum_{k \in \mathcal{K}_{pq}} v_k = 2w_{pq}^4, \qquad q \in Q_p, p \in P \tag{28}$$

$$\sum_{k \in \mathcal{K}_{\text{CG2}}} v_k \geq \mathrm{SB}^{\min}, \tag{29}$$

$$v_k \in \{0, 1\}, \qquad k \in \mathcal{K}_{\text{CG2}} \tag{30}$$

$$w^1 \in \mathbb{N}, \tag{31}$$

$$w_p^2,\ w_p^3 \in \mathbb{N}, \qquad p \in P \tag{32}$$

$$w_{pq}^4 \in \mathbb{N}, \qquad q \in Q_p, p \in P. \tag{33}$$

The variable v_k indicates whether customer tour k is used. The integer variables w^1, w_p^2, w_p^3, and w_{pq}^4, describe how many of each of the $1 + 2|P| + |P|(|P| - 1)$ sub-configurations for depot vehicles and local vehicles that should be used. The parameter $a_{ik} \in \{0, 1\}$ indicates whether customer i is included in tour k, and c_k is the cost of an optimal routing of the tour. The fixed costs for vehicles and swap bodies in the sub-configurations are as defined in (11).

Assume that a subset $K_{\text{CG2}} \subset \mathcal{K}_{\text{CG2}}$ of columns is available, and that an optimal solution to the LP relaxation of the resulting restricted master problem, called RMP 2, has been found. Let the dual variables for constraints (25)–(29) be denoted by λ_i, $i \in I$, α_0, α_p, $p \in P$, β_{pq}, $p \in P$, $q \in Q_p$, and π, respectively.

The column generation problem is to find a single customer tour, which can begin at the depot, a first-level switch point, or a second-level switch point. Hence, there are $1 + |P| + |P|(|P| - 1)$ column generation problems.

Let k be the starting node for the tour to be found. The tour has a maximal possible time, which is denoted Γ. It depends on if k is the depot, a first-level switch point, or a second-level switch point, see above and Section 2.2. Let the binary variable x_{ij} be 1 if the tour includes arc (i, j) and 0 otherwise, and let the binary variable y_i be 1 if the tour visits customer i and 0 otherwise. The continuous variable t_i is the visiting time of the tour at customer i, and τ is the time when the tour returns to the starting point. The column generation problem can for any starting point for the tour be stated as follows.

$$[\,\text{CG 2}\,] \quad \min \mathrm{RC} = \sum_{(i,j) \in A} c_{ij} x_{ij} - \sum_{i \in I} \lambda_i y_i - \Omega - \pi \tag{34}$$

$$\text{s.t.} \sum_{j \in I} x_{kj} = 1, \tag{35}$$

$$\sum_{(i,j) \in A} x_{ij} = y_i, \qquad i \in I \setminus \{k\} \tag{36}$$

$$\sum_{(i,j) \in A} x_{ij} = y_j, \qquad j \in I \setminus \{k\} \tag{37}$$

$$\sum_{i \in I} d_i y_i \leq C, \tag{38}$$

$$t_i + (s_i + T_{ij}) x_{ij} \leq t_j + \Gamma(1 - x_{ij}), \qquad (i, j) \in A : j \neq k \tag{39}$$

$$t_i + (s_i + T_{ik}) x_{ik} \leq \tau + \Gamma(1 - x_{ik}), \qquad i \in I \setminus \{k\} \tag{40}$$

$$0 \leq t_i \leq \Gamma, \qquad i \in I \tag{41}$$

$$0 \leq \tau \leq \Gamma, \tag{42}$$

$$x_{ij} \in \{0, 1\}, \qquad (i, j) \in A \tag{43}$$

$$y_i \in \{0, 1\}, \qquad i \in I \tag{44}$$

Here RC is the reduced cost of a column in MP 2, which corresponds to a customer tour. Further, Ω is the contribution to the reduced cost from the dual variables for the constraints (26)–(28), which depends on the starting point of the tour. If k is the depot then $\Omega = \alpha_0$, and if k is a first-level switch point $p \in P$ then $\Omega = \alpha_p$, and if k is a second-level switch point, that is $q \in Q_p$ with $p \in P$, then $\Omega = \beta_{pq}$.

The tour found by CG 2 will of course be optimally routed, if the problem is solved to optimality. This column generation scheme, which is based on customer tours and comprises the restricted master problem RMP 2 and column generation CG 2 is henceforth be referred to as $\mathsf{CG}_{\text{tour}}$.

The scheme $\mathsf{CG}_{\text{tour}}$ terminates when CG 2 cannot find any customer tour with negative reduced cost, and the restricted problem RMP 2 then gives an optimum to the LP relaxation of MP 2. The optimal value of RMP 2 is a hence a lower bound for MP 2 and for HME-VRP. The columns in the final problem RMP 2 are typically not enough to find an optimum to the integer problem MP 2, but by solving the integer version of the final RMP 2 one can find an integer feasible solution to MP 2, which gives a feasible solution and an upper bound for the HME-VRP.

A difference between the two column models is that MP 1 includes more columns than MP 2. Further, comparing the LP relaxation of MP 1 and MP 2, the former might provide a stronger lower bound for the HME-VRP. Assume, for example, that we have a complete set of columns from both models. Then any linear programming feasible solution to MP 1, which considers complete configurations, can be mapped

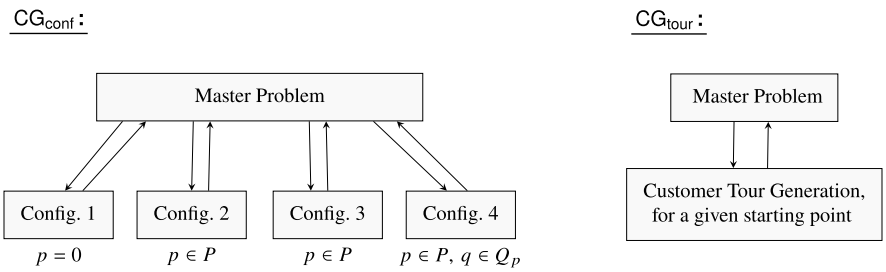

Fig. 3. To the left, the column generation scheme CG_{conf}, which is based on configurations. To the right, the column generation scheme CG_{tour}, which is based on customer tours.

into a linear programming feasible solution of MP 2. Feasible solutions to MP 2, which are based on customer tours, can however generally not be mapped into feasible solutions to MP 1. The advantage of CG_{tour} is that the problem is decomposed further than in CG_{conf}, and therefore CG_{tour} is able to tackle and provide lower bounds for larger instances, although with a lower bounding quality.

3. Overview of the column generation schemes

This section gives an outline of how the two column generation approaches have been implemented. In both schemes the restricted master problem is resolved whenever any new column has been generated, from CG 1 and CG 2, respectively. The dual information used in the column generation problems is therefore updated as frequently as possible, which improves the convergence behavior. This strategy is computationally feasible since the restricted master problems are computationally very cheap and solved almost instantly.

A non-standard implementation detail is the use of a time limit when solving the column generation problems, defined by the parameter `max_time` which is initially set to 2 seconds. When no further progress is made within the time limit and it is active for some column generation problem, then the time limit parameter is multiplied by 5 and the column generation progresses. When no progress is made and the time limit is never active, then LP optimality has been reached and verified. To further save computations, if a certain column generation problem does not find a negative reduced cost, then this column generation problem is put on hold until all column generation problems have failed to find a negative reduced cost within the current time limit. When the time limit is increased, all column generation problems are again considered.

Fig. 3 gives overviews of the computations made in CG_{conf} and CG_{tour}. Both column generation schemes have the same general structure, given in Algorithm 1.

In order to accelerate the convergence of CG_{tour} toward an LP optimum of MP 2, we convert every generated column into several similar, eligible columns. The conversion is based on the observation that the set of customers included in a tour can be re-routed from another starting point than the original, that is, from another switch point or from the depot. This is done by, for each possible starting point for the tour, solving a small traveling salesperson problem (TSP) and checking if the time required by the TSP tour found is feasible. This procedure is described in Algorithm 2, where $K_p \subset \mathcal{K}_p$ and $K_{pq} \subset \mathcal{K}_{pq}$ are subsets of generated columns.

The primary goal of using the schemes CG_{conf} or CG_{tour} is to find strong lower bounds for the HME-VRP. The final problems RMP 1 and RMP 2 give optima to the LP relaxations of MP 1 and MP 2, respectively, and thus lower bounds for HME-VRP. Even though LP optima are obtained, the columns included in the final RMP 1 and RMP 2 are usually not sufficient to provide optima to the HME-VRP, but by solving their integer versions one can at least find integer feasible solutions and upper bounds for HME-VRP.

Algorithm 1 Computational scheme with k column generation problems

Input: Set of column generation problems, SP
Initialize: $A := SP$, `progress` := *true*
while `progress` **do**
  `progress` := false
  for $k \in A$ **do**
    Solve column generation problem k with time limit `max_time`
    if *column with reduced cost* < 0 *found* **then**
      `progress` := true
      Update column set
      Solve master problem
    else
      Let $A := A \setminus \{k\}$
  if $A = \emptyset$ **then**
    if *all SP problems were solved within current time limit* **then**
      break
    else
      increase `max_time`
    Let $A := SP$
    `progress` := true
Result: An LP optimum

Algorithm 2 TSP re-routing procedure

Input: Subset of customers, S
for $p \in \{0\} \cup P$ **do**
  Solve a TSP for the set $S \cup \{p\}$
  if *feasible TSP tour and its route time is less than* $T^{\max} - 2(T_{0p} + s_p)$ **then**
    Add tour to column set K_p
    for $q \in Q_p$ **do**
      Solve a TSP for the set $S \cup \{q\}$
      if *feasible TSP tour and its route time is less than* $T^{\max} - 2(T_{0p} + s_p) - 2(T_{pq} + s_q)$ **then**
        Add tour to column set K_{pq}
Result: Updated column sets K_p and K_{pq}

We improve upon these upper bounding capabilities by using all the customer tours found in the course of CG_{conf} to build an additional integer problem. This problem has the same structure as RMP 2 and it is henceforth referred to as IP. The rationale for disaggregating distribution patterns from CG_{conf} into customer tours and using them in IP is that this makes it possible to combine the tours from CG_{conf} into new, and possibly better, distribution patterns. Further, in order to provide even more columns and options to IP, every customer tour is re-routed by using Algorithm 2. The upper bound for HME-VRP obtained from the resulting problem IP is clearly always at least as good as that

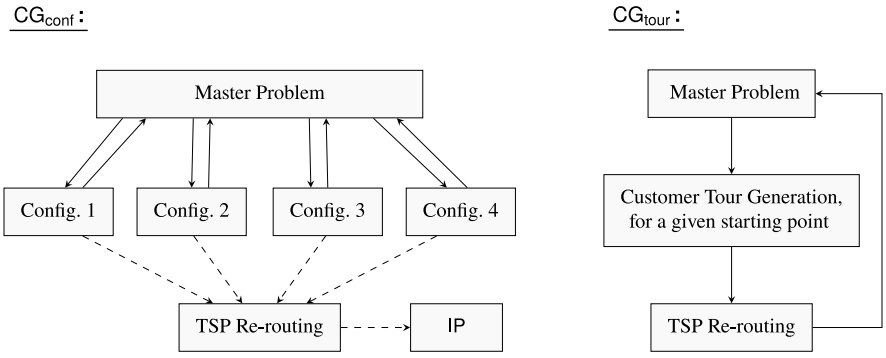

Fig. 4. To the left, the enhanced column generation scheme for CG_{conf}. To the right, the enhanced column generation scheme for CG_{tour}.

obtained from RMP1. Note that problem IP is solved only once, when CG_{conf} has reached LP optimality.

Fig. 4 gives overviews of the computations made in the enhanced versions of CG_{conf} and CG_{tour}.

4. Numerical experiments

This section begins with a description of the studied problem instances and their properties. Then the performance of the two column generation approaches on the instances, compared to both a commercial solver and metaheuristic algorithms, is presented and discussed.

4.1. Problem instances

The problem HME-VRP was introduced by Tadaros and Kyriakakis [3], but there referred to as HMSME-VRP-ST. Further, 36 instances with diverse sizes and characteristics were given. There are three subsets of instances based on the method used to generate customer locations: clustered (C), random (R), and a combination of clustered and random (CR). Notably, the smallest instances, including 25 customers and up to 4 switch points, were the only ones for which the mixed-integer solver Gurobi v10.0 could provide a feasible solution within a time limit of eight hours.

We here use 18 instances from Tadaros and Kyriakakis [3]. They have 25 or 50 customers and 2 – 4 switch points. We also use 18 new instances, generated with the same methodology as outlined in Tadaros and Kyriakakis [3]. The new instances vary in size, and have 10, 15 or 20 customers and 2 or 3 switch points. Each instance is denoted by a name in the format X-Y-Z, where X indicates the customer locations (C, R, CR), Y is the number of customers, and Z is the number of switch points. The complete set of instances is available upon request.

4.2. Performance of the compact formulation and metaheuristic algorithms

Table 2 presents the performance of the compact formulation of HME-VRP (see Appendix) using Gurobi v10.0 and of the metaheuristic scheme proposed in Tadaros and Kyriakakis [3]. Here a time limit of eight hours of CPU time was imposed on Gurobi. The metaheuristic scheme is randomized and is run 10 times, each time for 300 seconds. The first column in the table states the instance, whereas the following columns report the results obtained by the Gurobi solver; these are the lower bound (LBD_g) and the upper bound (UBD_g), while time is the wall time (in seconds). A dash (–) indicates that Gurobi could not solve the instance to verified optimality within the time limit. The column UBD_h is the objective value of the best feasible solution found by the metaheuristic in the 10 runs. The column Average is the average deviation in objective value versus the best found objective value, over the 10 runs. In the last two columns, gap and mean, the relative gap (in percent) between the best found feasible solution (the best of UBD_g and UBD_h) and LBD_g are reported, together with the mean gap for each set of problem instances with the same number of customers.

As seen in Table 2, Gurobi solved only three of the instances to verified optimality within the time limit, and all of these are among the smallest. Further, Gurobi fails to report any feasible solutions for the instances with 50 customers. For all instances but one the upper bounds from Gurobi are matched or outperformed by the upper bounds found by the metaheuristic. However, given that the gaps are large and that the lower bounds from Gurobi are most likely poor, it is hard to draw any definitive conclusion on the performance of the metaheuristic.

4.3. Performance of column generation based on configurations

In Table 3 we report the results for instances including up to 20 customers for CG_{conf}. (Beyond that instance size the computation times become exceedingly long.) The lower bound from Gurobi (LBD_g), the upper bound from the metaheuristic (UBD_h), and the relative gap from Table 2 are repeated, to simplify comparison. The results for CG_{conf} are the final LP objective value (LBD_1), the objective value of an optimal integer solution to the final RMP1 (UBD_1), the objective value of the optimal integer solution to problem IP (UBD_{IP}), the relative gap (gap) between UBD_{IP} and LBD_1, and the wall time (time) in seconds for CG_{conf}. (The time needed for solving IP once is negligible.) In the gap columns, an "*" indicates verified optimality. The values for UBD_1 are given relative to UBD_{IP}, since the latter are always at least as good as the former.

In Table 3 it can be observed that the lower bounding capability of CG_{conf} is strong and that computation times needed to reach LP optimality in MP1 are relatively short. Further, CG_{conf} can find verified optimal solutions to nine instances of the HME-VRP. When verified optimality was not reached, the remaining gap is only between 1 and 8 percent.

The upper bounds UBD_{IP} all match the solutions found by the metaheuristic, except for instance C-15-2 where UBD_{IP} finds a better solutions than the metaheuristic, and instance C-15-3 where the metaheuristic finds a better.

Despite the good bounding performance of CG_{conf}, it encounters difficulties when used on larger instances, and it is not computationally

Table 2
Results obtained by Gurobi on compact formulation and metaheuristic algorithm.

Instance	Full model (Gurobi)			Metaheuristic (Python)		gap	mean
	LBD_g	UBD_g	time	UBD_h	Average		
C-10-2	456.681	621.220	–	621.220	+0	36	
C-10-3	332.489	463.611	–	463.611	+0	39	
CR-10-2	407.114	407.114	1299	407.114	+0	*	
CR-10-3	493.595	493.595	15 393	493.595	+0	*	
R-10-2	746.644	838.355	–	838.356	+0	12	
R-10-3	668.554	668.554	7881	668.554	+0	*	14.5
C-15-2	278.347	751.903	–	762.027	+0	174	
C-15-3	217.366	625.168	–	621.278	+4.841	186	
CR-15-2	322.050	610.094	–	610.094	+0	89	
CR-15-3	305.497	745.045	–	745.045	+0	144	
R-15-2	527.944	780.826	–	780.826	+0	48	
R-15-3	536.039	1021.923	–	1021.923	+0	91	122
C-20-2	201.211	719.966	–	719.966	+0	258	
C-20-3	242.995	971.807	–	971.808	+0	300	
CR-20-2	275.615	720.794	–	720.794	+0	162	
CR-20-3	264.541	912.267	–	895.715	+23.936	239	
R-20-2	453.663	1203.586	–	1203.586	+0	165	
R-20-3	444.405	1235.596	–	1235.596	+8.099	178	217
C-25-2	206.030	829.680	–	826.345	+0.51	301	
C-25-3	231.780	738.590	–	737.319	+0	218	
C-25-4	272.880	795.630	–	792.982	+0	191	
CR-25-2	243.500	967.190	–	950.380	+0.06	290	
CR-25-3	229.730	777.880	–	774.870	+1.86	237	
CR-25-4	233.860	862.780	–	844.495	+0.02	261	
R-25-2	429.090	1287.360	–	1254.520	+0	192	
R-25-3	413.250	1346.560	–	1288.570	+0.01	212	
R-25-4	426.360	1193.640	–	1170.570	+0	175	230.8
C-50-2	.	.	.	867.010	+5.820	.	
C-50-3	.	.	.	911.410	+42.16	.	
C-50-4	.	.	.	1015.600	+15	.	
CR-50-2	.	.	.	1262.480	+6.16	.	
CR-50-3	.	.	.	1139.470	+10.84	.	
CR-50-4	.	.	.	1248.380	+12.72	.	
R-50-2	.	.	.	1355.370	+40.050	.	
R-50-3	.	.	.	1543.740	+40.680	.	
R-50-4	.	.	.	1274.020	+25.570	.	

Table 3
Results for column generation based on configurations.

Instance	Gurobi	Heuristic		Column Generation (CG_{conf})				
	LBD_g	UBD_h	gap	LBD_1	UBD_1	UBD_{IP}	gap	time
C-10-2	456.681	621.220	36	602.184	+0	621.220	3	38
C-10-3	332.489	463.611	39	463.611	+0	463.611	*	63
CR-10-2	407.114	407.114	*	407.114	+0	407.114	*	27
CR-10-3	493.595	493.595	*	493.595	+0	493.595	*	71
R-10-2	746.644	838.356	12	838.356	+0	838.356	*	29
R-10-3	668.554	668.554	*	668.554	+0	668.554	*	70
C-15-2	278.347	762.027	174	702.316	+13.797	751.903	7	208
C-15-3	217.366	621.278	186	578.431	+316.633	622.338	8	371
CR-15-2	322.050	610.094	89	610.094	+0	610.094	*	615
CR-15-3	305.497	745.045	144	745.045	+0	745.045	*	481
R-15-2	527.944	780.826	48	776.784	+134.221	780.826	2	202
R-15-3	536.039	1021.923	91	1021.923	+0	1021.923	*	257
C-20-2	201.211	719.966	258	677.043	+432.452	719.966	6	15 613
C-20-3	242.995	971.808	300	955.644	+125.905	971.808	2	14 453
CR-20-2	275.615	720.794	162	706.411	+134.034	720.794	2	1847
CR-20-3	264.541	895.715	239	889.005	+0	895.715	1	1072
R-20-2	453.663	1203.586	165	1185.429	+12.073	1203.586	2	515
R-20-3	444.405	1235.596	178	1235.596	+0	1235.596	*	585

viable for instances with more than 20 customers. This is due to the complexity of its column generation problem CG 1.

4.4. Performance of column generation based on customer tours

The results of the performance of CG_{tour} are reported in Table 4. Here columns LBD_2 and UBD_2 are the final LP objective value and the objective value of an optimal integer solution of the final problem RMP 2, respectively. The other notations are as introduced above.

As seen in Table 4, CG_{tour} converges to LP optimality in MP 2 much faster than CG_{conf} finds an optimum to MP 1, although this is at the expense of somewhat weaker lower bounds obtained. For instances with more that 15 customers the lower bounds are however still significantly stronger than those from Gurobi. Interestingly, when

Table 4
Results for column generation based on customer tours.

Instance	Gurobi	Heuristic		Column Generation (CG_{tour})			
	LBD_g	UBD_h	gap	LBD_2	UBD_2	gap	time
C-10-2	456.681	621.220	36	571.220	662.939	16	6
C-10-3	332.489	463.611	39	459.893	463.611	1	16
CR-10-2	407.114	407.114	*	367.189	407.114	11	14
CR-10-3	493.595	493.595	*	482.583	493.595	2	17
R-10-2	746.644	838.356	12	760.873	838.356	10	7
R-10-3	668.554	668.554	*	608.181	668.554	10	16
C-15-2	278.347	762.027	174	653.751	755.157	16	17
C-15-3	217.366	621.278	186	530.652	639.396	20	24
CR-15-2	322.050	610.094	89	554.682	642.776	16	38
CR-15-3	305.497	745.045	144	704.424	745.045	6	38
R-15-2	527.944	780.826	48	741.645	803.203	8	26
R-15-3	536.039	1021.923	91	985.478	1226.966	25	31
C-20-2	201.211	719.966	258	643.018	1003.271	56	59
C-20-3	242.995	971.808	300	860.255	990.484	15	53
CR-20-2	275.615	720.794	162	665.083	722.969	9	58
CR-20-3	264.541	895.715	239	836.805	1012.820	21	39
R-20-2	453.663	1203.586	165	1156.716	1234.063	7	34
R-20-3	444.405	1235.596	178	1184.155	1246.959	5	35
C-25-2	206.030	826.345	301	748.786	1204.146	61	144
C-25-3	231.780	737.319	218	636.941	931.872	46	230
C-25-4	272.880	792.982	191	759.333	799.892	5	181
CR-25-2	243.500	950.380	290	886.999	1294.919	46	154
CR-25-3	229.730	774.870	237	726.556	886.000	22	267
CR-25-4	233.860	844.495	261	791.301	1066.550	35	252
R-25-2	429.090	1254.520	192	1183.624	1383.192	17	68
R-25-3	413.250	1288.570	212	1269.528	1317.552	4	111
R-25-4	426.360	1170.570	175	1144.033	1222.732	7	122
C-50-2	.	867.010	.	813.548	948.102	17	6578
C-50-3	.	911.410	.	861.904	909.033	5	13 228
C-50-4	.	1015.600	.	966.377	1514.249	57	23 221
CR-50-2	.	1262.480	.	1128.833	2271.496	101	27 356
CR-50-3	.	1139.470	.	1055.669	2021.827	92	31 032
CR-50-4	.	1248.380	.	1211.815	1444.912	19	3760
R-50-2	.	1355.370	.	1166.211	1659.133	42	3743
R-50-3	.	1543.740	.	1430.339	1603.431	12	50 968
R-50-4	.	1274.020	.	1169.333	1330.110	14	80 325

comparing the results of CG_{tour} and Gurobi, there is no consistency in the results for the smallest instances of 10 customers. In three out of six the instances, Gurobi is able to find a higher lower bound than CG_{tour}, and Gurobi also finds optimal solutions to these three instances. Further, CG_{tour} finds optimal integer solution to four instances, but cannot verify it because its lower bound is not strong enough.

As described above Algorithm 2 is used in CG_{tour} to accelerate the column generation to find the LP optimum of MP 2 faster. The inclusion of this acceleration decreased the computation time needed to find an LP optimum by 36% on average.

4.5. Comparison of the column generation approaches

Table 5 gives a summary of bounds found for the studied instances of the HME-VRP. Column Str. gives the relative strength of LBD_2 compared to LBD_1 (in percent). Column LBD shows the highest lower bound found, and column UBD shows the best known upper bound, from any solution approach considered. The relative gap between LBD and UBD is given in the column gap, while the column mean is the mean gap for each set of problem instances with the same number of customers.

We have found verified optimal solutions to nine of the instances, and for the other instances the gap between the bounds is below 10 percent for all but four instances. Worth noticing is that the lower bound provided by CG_{tour} is, on average, just above 94% compared to the lower bound found by CG_{conf}. Further, the relative strength of the two lower bounds does not seem to change with the size of the instance. The advantage of CG_{tour} is that it can provide good lower bounds for larger instances within a reasonable time frame. It can even run on instances with 50 customers, but with run times of several hours on a computer cluster with one compute node and 32 threads.

5. Discussion

We have studied HME-VRP, which is a hierarchical multi-switch multi-echelon vehicle routing problem with service and maximum route times. The problem is a newly introduced variant of a multi-echelon VRP which has been shown to be very challenging to solve using a compact mixed-integer formulation and state-of-the-art exact solvers. Solvers can often produce good upper bounds, at least for smaller instances. For realistic-sized instances the lower bounds are however weak even after extensive computation times. A metaheuristic has therefore been proposed Tadaros and Kyriakakis [3] to solve realistic-sized instances. This metaheuristic showed promise, but because of the lack of strong lower bounds, its performance in terms of deviations from optimality was unknown.

We have proposed two strong formulations of the HME-VRP which are based on column generation, primarily to generate good lower bounds and evaluate the performance of a previously proposed metaheuristic. The difference between the two formulations is how the problem is decomposed, that is, the interpretation of a column and how the column generation is made. The first column generation approach, CG_{conf}, is based on distribution patterns that include one to three swap bodies and customer tours for each of them, together with vehicles

Table 5
Summary of bounds for the HME-VRP.

Instance	Lower bounds				Best found bounds			
	LBD_g	LBD_1	LBD_2	Str.	LBD	UBD	gap	mean
C-10-2	456.681	602.184	571.220	94.9	602.184	621.220	3	
C-10-3	332.489	463.611	459.893	99.2	463.611	463.611	*	
CR-10-2	407.114	407.114	367.189	90.2	407.114	407.114	*	
CR-10-3	493.595	493.595	482.583	97.8	493.595	493.595	*	
R-10-2	746.644	838.356	760.873	90.8	838.356	838.355	*	
R-10-3	668.554	668.554	608.181	91.0	668.554	668.554	*	0.5
C-15-2	278.347	702.316	653.751	93.1	702.316	751.903	7	
C-15-3	217.366	578.431	530.652	91.7	578.431	622.338	8	
CR-15-2	322.050	610.094	554.682	90.9	610.094	610.094	*	
CR-15-3	305.497	745.045	704.424	94.5	745.045	745.045	*	
R-15-2	527.944	776.784	741.645	95.5	776.784	780.826	1	
R-15-3	536.039	1021.923	985.478	96.4	1021.923	1021.923	*	2.7
C-20-2	201.211	677.043	643.018	95.0	677.043	719.966	6	
C-20-3	242.995	955.645	860.255	90.0	955.645	971.807	2	
CR-20-2	275.615	706.411	665.083	94.1	706.411	720.794	2	
CR-20-3	264.541	889.005	836.805	94.1	889.005	895.715	1	
R-20-2	453.663	1185.429	1156.716	97.6	1185.429	1203.586	2	
R-20-3	444.405	1235.596	1184.155	95.8	1235.596	1235.596	*	2.2
C-25-2	206.030	.	748.786	.	748.786	826.345	10	
C-25-3	231.780	.	636.941	.	636.941	737.319	16	
C-25-4	272.880	.	759.333	.	759.333	792.982	4	
CR-25-2	243.500	.	886.999	.	886.999	950.378	7	
CR-25-3	229.730	.	726.556	.	726.556	774.870	7	
CR-25-4	233.860	.	791.301	.	791.301	844.495	7	
R-25-2	429.090	.	1183.624	.	1183.624	1254.520	6	
R-25-3	413.250	.	1269.528	.	1269.528	1288.568	1	
R-25-4	426.360	.	1144.033	.	1144.033	1170.570	2	6.7
C-50-2	.	.	813.548	.	813.548	867.010	7	
C-50-3	.	.	861.904	.	861.904	909.033	5	
C-50-4	.	.	966.377	.	966.377	1015.600	5	
CR-50-2	.	.	1128.833	.	1128.833	1262.480	12	
CR-50-3	.	.	1055.669	.	1055.669	1139.470	8	
CR-50-4	.	.	1211.815	.	1211.815	1248.380	3	
R-50-2	.	.	1166.211	.	1166.211	1355.370	16	
R-50-3	.	.	1430.339	.	1430.339	1543.740	8	
R-50-4	.	.	1169.333	.	1169.333	1274.020	9	8.1

that transport the swap bodies and switch points where they change vehicles. The column generation problems are prize-collecting routing problems, which constructs one to three customer tours. In the second column generation approach, CG_{tour}, the allocation of customer tours for swap bodies to vehicles and switch point reloading is instead made in the master problem. The column generation problems here become prize-collecting routing problems which each constructs a single customer tour.

The clear weakness of both column generation approaches is the computational difficulty of the column generation problems, which are capacitated prize-collecting traveling salesperson problems or generalizations thereof, when they are solved as general mixed-integer problems. This was somewhat unexpected, considering that the numbers of customers are not large and that the vehicle capacities allow each vehicle to visit only relatively few customers. The column generation problems therefore demand for more research; the column generation problem CG 2 appears most promising in this respect, since prize-collecting routing problems are quite well-studied.

A particular weakness of using compact mixed-integer formulations of vehicle routing problems is that they typically include much symmetry, because vehicles are interchangeable. This can severely deteriorate the performance of a solver that is based on branch-and-bound, even though symmetry-breaking constraints can sometimes improve the performance. Similarly to column-oriented for vehicle routing problems in general, the proposed models for HME-VRP do not suffer from any symmetry issues.

Computational tests show that both column generation approaches have great lower bounding qualities. Even for moderately sized instances they can produce significantly stronger lower bounds than the solver Gurobi, and in less time.

In an attempt to accelerate the convergence of CG_{tour}, we introduced re-routing of customer tours to alternative starting points. This re-routing can also be used in CG_{conf} to find improved upper bounds. These upper bounds are however still not as strong as those obtained by the metaheuristic.

A primary goal with this work was to assess the performance of the fast metaheuristic proposed in the earlier work by Tadaros and Kyriakakis [3], by benchmarking the feasible solutions found against strong lower bounds for the optimal values. A comparison of the column UBD_h in Table 4, which contains the upper bounds obtained by the metaheuristic in Tadaros and Kyriakakis [3], and the column LBD in Table 5, which contains the best known lower bounds from the column generation approaches, shows that the objective values found by the metaheuristics are close to, or even coincide with, exact optimality. Hence, the solutions found by the metaheuristic are indeed of high quality. This conclusion warrants that a decision-maker can in a real-life planning situation rely on this metaheuristic, or close relatives, for finding high-quality solutions to the HME-VRP within moderate computation times.

6. Future work

The column generation based on configurations produces strong lower bounds, but the method scales very poorly with respect to problem size. The column generation based on customer tours yields somewhat weaker lower bounds, but the method scales much better. Further, the customer tour problem CG 2 should be amenable to efficient tailored algorithms, while this is far from obvious for CG 2. Finally, the very special and favorable structure of MP 2 makes CG_{tour}

the most promising topic for continued research, and there are several interesting ideas to explore for improving the lower and upper bounding quality of $\mathsf{CG}_{\text{tour}}$.

The integer variables in MP 2, that is, w^1, w^2_p, w^3_p, and w^4_{pq}, are quite few, only $1+2|P|+|P|(|P|-1)$, and they typically take only small values. For improving the lower bound it is therefore reasonable to optimize exactly with respect to these variables. This can be made through a standard branch-and-bound scheme. An interesting alternative is to use Benders decomposition, with the integer variables considered to be complicating. These approaches would both include continued column generation within the optimizing method, in the branch-and-bound nodes and in the Benders subproblem, respectively. This would produce a diversity of customer tours, which should be favorable when searching for an upper bound.

A simpler alternative is to use a metaheuristic search or a diving heuristic with respect to the integer variables; this would also involve continued column generation within the overall method. These alternatives could also yield improved upper bounds, but they do not affect the lower bound.

A bottleneck in the current implementation of $\mathsf{CG}_{\text{tour}}$ is the column generation problem CG 2, and implementing a tailored solution method seems necessary in order to take on larger problem instances. One possibility is to construct customer tours by a metaheuristic method, which can be used as long as negative reduced costs are obtained, such that the column generation method progresses. A more advanced and challenging possibility is to design an optimizing tour-generation based on multi-labeling path search.

CRediT authorship contribution statement

Marduch Tadaros: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Formal analysis, Conceptualization. **Athanasios Migdalas:** Writing – review & editing, Validation, Supervision, Methodology, Formal analysis, Conceptualization. **Nils-Hassan Quttineh:** Writing – review & editing, Writing – original draft, Validation, Software, Methodology, Formal analysis, Conceptualization. **Torbjörn Larsson:** Writing – review & editing, Writing – original draft, Validation, Software, Methodology, Formal analysis, Conceptualization.

Declaration of competing interest

The authors have no competing interests to declare that could have appeared to influence the work reported in this paper.

Appendix. Compact formulation of the HME-VRP

Sets

N:	Set of nodes
A:	Set of arcs
P:	Set of switch points
I:	Set of customers
W:	$\{0\} \cup P$
L:	$P \cup I$
V_o:	Set of depot vehicles
V_s	Set of local vehicles
V:	$V_o \cup V_s$
T:	Set of swap bodies

Variables

$$x^t_{ij} = \begin{cases} 1, & \text{if swap body } t \in T \text{ traverses arc } (i,j) \in A \\ 0, & \text{otherwise} \end{cases}$$

$$y^{vt}_{ij} = \begin{cases} 1, & \text{if depot vehicle } v \in V_o \text{ with swap body } t \in T \\ & \text{traverses arc } (i,j) \in A \\ 0, & \text{otherwise} \end{cases}$$

$$z^{vt}_{ij} = \begin{cases} 1, & \text{if local vehicle } v \in V_s \text{ with swap body } t \in T \\ & \text{traverses arc } (i,j) \in L \\ 0, & \text{otherwise} \end{cases}$$

$$r^v_{ij} = \begin{cases} 1 & \text{if vehicle } v \in V \text{ traverses arc } (i,j) \in A \\ 0 & \text{otherwise} \end{cases}$$

$$f_v = \begin{cases} 1 & \text{if vehicle } v \in V \text{ is used} \\ 0 & \text{otherwise} \end{cases}$$

$$b_t = \begin{cases} 1 & \text{if swap body } t \in T \text{ is used} \\ 0 & \text{otherwise} \end{cases}$$

n_i = auxiliary variable representing the rank-order in which customer $i \in I$ is visited

ds_p = auxiliary variable representing the number of customer tours originating from switch point $p \in P$

Parameters

C	Swap body capacity
c_{ij}	Cost of traversing arc $(i,j) \in A$
T_{ij}	Travel time of arc $(i,j) \in A$
d_i	Demand of customer $i \in I$
s_i	Service time at node $i \in N$
T^{MAX}	Maximum route time
F_{DV}	Fixed cost of using a depot vehicle
F_{LV}	Fixed cost of using a local vehicle
F_{SB}	Fixed cost of using a swap body

Mathematical model

$$[\,\text{FULL}\,] \quad \min \text{Cost} = \sum_{i\in N}\sum_{j\in N}\sum_{v\in V_o} c_{ji} r^v_{ij} + \sum_{i\in L}\sum_{j\in L}\sum_{v'\in V_s}\sum_{t\in T} c_{ji} z^{v't}_{ij} + \sum_{v\in V_o} f_v F_{\text{DV}} + \sum_{v\in V_s} f_v F_{\text{LV}} + \sum_{t\in T} b_t F_{\text{SB}} \tag{A.1}$$

s.t.

$$\sum_{i\in N\setminus\{j\}}\sum_{t\in T} x^t_{ij} = 1 \quad \forall j \in I \tag{A.2}$$

$$\sum_{j\in N\setminus\{i\}}\sum_{t\in T} x^t_{ij} = 1 \quad \forall i \in I \tag{A.3}$$

$$\sum_{i\in N\setminus\{p\}} x^t_{ip} - \sum_{j\in N\setminus\{p\}} x^t_{pj} = 0 \quad \forall t \in T, \quad \forall p \in N \tag{A.4}$$

$$\sum_{i\in I}\sum_{j\in N} d_i \cdot x^t_{ij} \le C \cdot b_t \quad \forall t \in T \tag{A.5}$$

$$\sum_{i\in W}\sum_{j\in I} x^t_{ij} \le 1 \quad \forall t \in T \tag{A.6}$$

$$\sum_{i\in I} x^t_{ji} - \sum_{i\in I} x^t_{ji} = 0 \quad \forall j \in W, \quad \forall t \in T \tag{A.7}$$

$$\sum_{i\in L} x^t_o \le 1 \quad \forall t \in T \tag{A.8}$$

$$\sum_{i\in P}\sum_{j\in I} x^t_{ij} \le \sum_{i\in P} x^t_{oi} \quad \forall t \in T \tag{A.9}$$

$$\sum_{i\in I} x^t_{pi} - \sum_{j\in W} x^t_{jp} = 0 \quad \forall p \in P, \quad \forall t \in T \tag{A.10}$$

$$\sum_{i\in W\setminus\{j\}}\sum_{t\in T} x^t_{ij} \ge ds_j \quad \forall j \in P \tag{A.11}$$

$$x^t_{pp'} \le x^t_{op} + \sum_{i \in I} x^t_{ip} \quad \forall p, p' \in P : \ p \ne p', \ \forall t \in T \tag{A.12}$$

$$2 - (r^v_{op} + r^v_{pp'}) \ge \sum_{i \in I} r^v_{pi} \quad p, p' \in P, \ p \ne p' \ \forall v \in V_o \tag{A.13}$$

$$x^t_{ij} - \sum_{v \in V_o} y^{vt}_{ij} = 0 \quad \forall i, j \in W, \quad i \ne j, \quad \forall t \in T \tag{A.14}$$

$$x^t_{oi} - \sum_{v \in V_o} y^{vt}_{oi} = 0 \quad \forall i \in I, \forall t \in T \tag{A.15}$$

$$x^t_{ij} - \sum_{v \in V_o} y^{vt}_{ij} = \sum_{v' \in V_s} z^{v't}_{ij} \quad \forall i, j \in L, i \ne j, \forall t \in T \tag{A.16}$$

$$\sum_{t \in T} y^{vt}_{ij} \le 3 \cdot r^v_{ij} \quad \forall i, j \in W, \quad i \ne j, \quad \forall v \in V_o \tag{A.17}$$

$$\sum_{t \in T} y^{vt}_{ij} \le r^v_{ij} \quad \forall i \in N, \forall j \in I, i \ne j, \forall v \in V_o \tag{A.18}$$

$$\sum_{t \in T} y^{vt}_{ij} \le r^v_{ij} \quad \forall i \in I, \forall j \in N, i \ne j, \forall v \in V_o \tag{A.19}$$

$$\sum_{i \in L \setminus \{c\}} y^{vt}_{ic} - \sum_{j \in L \setminus \{c\}} y^{vt}_{cj} = 0 \quad \forall v \in V_o, \forall c \in I, \forall t \in T \tag{A.20}$$

$$\sum_{t \in T} y^{vt}_{op} - \sum_{i \in I} \sum_{v' \in V_s} \sum_{t \in T} z^{v't}_{pi} \le \sum_{j \in L \setminus \{p\}} \sum_{t \in T} y^{vt}_{pj} \quad \forall p \in P \quad \forall v \in V_o \tag{A.21}$$

$$\sum_{i \in L \setminus \{c\}} z^{vt}_{ic} - \sum_{j \in l \setminus \{c\}} z^{vt}_{cj} = 0 \quad \forall v \in V_s, \forall c \in I, \forall t \in T \tag{A.22}$$

$$\sum_{i \in W} \sum_{j \in P} \sum_{t \in T} y^{vt}_{ij} \le 1 \cdot f_v \quad \forall v \in V_o \tag{A.23}$$

$$\sum_{i \in P} \sum_{j \in I} \sum_{t \in T} z^{vt}_{ij} \le 1 \cdot f_v \quad \forall v \in V_s \tag{A.24}$$

$$\sum_{i \in L} r^v_{oi} \le 1 \cdot f_v \quad \forall v \in V_o \tag{A.25}$$

$$r^v_{ij} = r^v_{ji} \quad \forall i, j \in W, \quad \forall v \in V_o \tag{A.26}$$

$$\sum_{i \in I} \sum_{t \in T} x^t_{pi} = ds_p \quad \forall p \in P \tag{A.27}$$

$$\sum_{i \in N} \sum_{j \in N} x^t_{ij} \cdot T_{ij} + \sum_{p \in N} \sum_{q \in L} x^t_{pq} \cdot s_q \le T^{\mathrm{MAX}} \quad \forall t \in T \tag{A.28}$$

$$n_j - n_i \ge d_j - u \cdot (1 - x^t_{ij}) \quad \forall (ij) \in A \quad \forall t \in T \tag{A.29}$$

$$d_i \le n_i \le u \quad \forall i \in I \tag{A.30}$$

$$x^t_{ij} \in \{0, 1\}, \quad \forall (i, j) \in A, \quad \forall t \in T; \tag{A.31}$$

$$y^{vt}_{ij} \in \{0, 1\}, \quad \forall (i, j) \in A, \quad \forall v \in V_o, \quad \forall t \in T; \tag{A.32}$$

$$z^{vt}_{ij} \in \{0, 1\}, \quad \forall (i, j) \in L, \quad \forall v \in V_s, \quad \forall t \in T; \tag{A.33}$$

$$r^v_{ij} \in \{0, 1\}, \quad \forall (i, j) \in A, \quad \forall v \in V_o; \tag{A.34}$$

$$f_v \in \{0, 1\}, \quad \forall v \in V; \tag{A.35}$$

$$b_t \in \{0, 1\}, \quad \forall t \in T. \tag{A.36}$$

$$n_i \ge 0 \quad \forall i \in I \tag{A.37}$$

$$ds_p \ge 0 \quad \forall p \in P \tag{A.38}$$

The objective function (A.1) minimizes the total distances traveled and the two vehicle fleets. Constraints (A.2) and (A.3) makes sure that a customer is only visited by one swap body. Constraint (A.4) is route continuity constraint, while constraint (A.5) is the capacity constraint for the swap bodies. Constraint (A.6) states that a swab body can only perform one customer tour, and constraint (A.7) states that the swap body has to return to the origin of the customer tour. Constraints (A.8) and (A.9) state that a swap body can only leave the depot once and that if a swap body performs a customer tour, the same swap body must have left the depot, respectively. Constraint (A.10) states that if a swap body leaves a switch toward a customer, the same swap body must have arrived at the same switch point, directly from the depot or from another switch point. Constraint (A.11) states that the number of incoming swap bodies to a switch point must be larger or equal to the number of customer tours originating from that switch point, whereas constraint (A.12) prohibits swap bodies to go from a switch point to another if it has not arrived to the first one directly from the depot or any customer. Constraint (A.13) prohibits original vehicles from performing customer tours from a switch point if it continues toward another. Constraint (A.14) states that an original vehicle has to accompany a swap body between the depot and switch points, while (A.15) considers the same for swap bodies going directly to customers from the depot. Constraint (A.16) states that there has to be either an original vehicle or local vehicle following each swap body on arcs between switch points and customers, i.e., $(i, j) \in K$. Constraints (A.17)–(A.19) are capacity constraints for original vehicles, constraint (A.17) considers arcs between the depot and switch points while constraints (A.18) and (A.19) considers from the depot toward customers and between customers, respectively. Constraints (A.20)–(A.22) are continuity constraints for the different vehicles. While (A.20) and (A.21) considers original vehicles for arcs between customers and switch points respectively, constraint (A.22) considers local vehicles. The fact that any vehicle, regardless of type, can only perform one customer tour is considered by (A.23) for original vehicles and by (A.24) for local vehicles. An original vehicle can only leave the depot once and is considered by constraint (A.25). Constraint (A.26) states that an original vehicle has to revisit any switch points it had visited before its customer tour. The number of individual customer tours originating from a specific switch point is given by (A.27), while constraint (A.28) limits the total route time for a single swap body. Constraints (A.29) and (A.30) are subtour elimination constraints. Lastly, constraints (A.31)–(A.38) specify the domains of the variables.

Data availability

Data will be made available on request.

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