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Article

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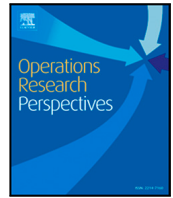
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The impact of information disclosure and smart technology integration on e-retailing performance: A production delivery policy framework

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ABSTRACT

The electronic retailers face distinct challenges in information sharing compared to their purely offline counterparts, particularly in transparently communicating their environmental practices to increasingly eco-conscious consumers. This complexity increases in e-retailing due to the absence of direct interaction and makes it difficult for consumers to evaluate the sustainability efforts of retailing channel's stakeholders. In response to it, manufacturers and e-retailers are leveraging social media and blockchain technology for personalized advertising to bridge this information transparency gap. This research presents a sustainable multi-item integrated model for manufacturer–retailer collaboration in e-retailing by incorporating multiple delivery policies and investments in technology aimed at information disclosure and environmental footprint reduction. The manufacturer adopts a smart production system and reuse returned goods in the manufacturing process while investing in Green Emissions Reduction Technology. Meanwhile, the e-retailer enhances product demand through Information Disclosure Technology on social media and blockchain by showcasing their environmental protection efforts. By employing a hybrid analytic-metaheuristic approach, we determine optimal production and delivery policies to improve green consumer service under varying budgetary and spatial constraints. The results demonstrate a 4.39% increase in online consumer demand through information sharing and a 3.86% improvement in profitability of the collaborative retailing system under single-setup multi-delivery policy that confirms robustness of the proposed model. Scenario analysis further provides decision-makers with actionable insights by showcasing 8.44% increment in the system profit by converting traditional production into smart production system. Moreover, the sensitivity of the proposed model to balancing the technology investments among emission control and information disclosure efforts suggests keeping track of the efficiency parameters of these investment options before making technology budget allocations.

1. Introduction

In the recent years, consumers have shown growing concern over the global energy crisis and resultant global warming. Consequently, they have been compelled to prioritize environmental conservation and the use of low-carbon commodities [1]. The 2030 Agenda regarding Sustainable Development (SD) from the UN makes sustainable consumption a strategic goal, saying that the individuals should contribute to the altering unsustainable consumption and production patterns. Therefore, the production and retail chains of daily consumables are making efforts to mitigate greenhouse gas emissions and communicating their efforts to the potential customers through various channels. However, determining the sustainability of a product is challenging for customers due to the subjective nature of environmental sustainability as a belief-based characteristic.

E-retailers face exceptional difficulties in communicating their environmental practices to eco-conscious consumers in a transparent manner. This issue originates from the lack of direct physical interaction inherent in online sales settings. This opacity in the information disclosure makes it difficult for the product consumers to evaluate the sustainability efforts of e-tailers and their respective supply chain partners. For instance,

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while a brick-and-mortar store can simply display certifications like Fair Trade labels directly on the products, online retailers can rely on digital representations only that can be easily manipulated or overlooked. Furthermore, explaining complicated environmental initiatives including carbon offsetting or sustainable packaging choices requires detailed descriptions that may not effectively translate to the online environment. Consider a fast fashion industry where the online retailers like ASOS and Boohoo have faced scrutiny for “greenwashing” to promote environmentally friendly collections without sufficient evidence of genuine sustainable practices. Another obvious case is of Amazon, which faces inquiry about its carbon footprint and has responded to it by commencing initiatives including “Climate Pledge Friendly” with the objective of highlighting sustainable products [2]. Similarly, a globally famous retail chain named Patagonia has faced communication challenges in leveraging its e-commerce platform to transmit its environmental efforts information by showcasing detailed lifecycle analyses of its products. However, the lack of direct consumer interaction in e-retailing adds to the difficulty of verifying these claims which leads to skepticism about the greenwashing. This lack of transparency in information disclosure can directly wear away consumer trust and negatively impact the brand reputation. This highlights the practical implications of the green information-sharing challenges for the e-tailing industry.

The continuous growth of information technology such as social media and blockchain has transformed individuals’ methods of communication and their decision-making processes about purchases, while also reshaping the digital marketing environment [3]. The emergence of personalized advertisements has prompted advertising personnel to investigate its efficacy, including its impact on customer attitudes, ad engagement (such as clicks or purchases), and the factors that influence it [4]. Therefore, blockchain and social media advertising offer powerful platforms for the retail sector to inform and educate their consumers about their initiatives to reduce carbon emissions by making investments in advanced low carbon technology. By virtue of this, production and retail industries have become capable of effectively drawing in a larger customer base and attract more demand.

Production-delivery reliability in addition to the sustainability is of utmost importance in today’s globalized world. Failing to satisfy customer expectations may lead to a permanent loss of customers and a decrease in competitiveness. As a result, the supply chains must optimize their production and inventory systems to succeed in the long run despite the complicated global environment. Achieving success in manufacturing depends on effectively integrating production and inventories of the whole system. The major objective in the supply chains of integrated systems includes value creation through well-coordinated production quantities and delivery lot size decisions. The goal is to minimize total cost of the supply chain while making a wise trade-off among setup cost and inventory carrying costs of the system. However, basic inventory management models such as EOQ and EPQ presented by Harris and Taft in 1913 and 1918, respectively have certain limitations in terms of practical applicability [5]. Therefore, several studies have extended these models by incorporating many real world conditions. One of the recent addition is Vendor Managed Inventory (VMI) in the production inventory systems [6]. Several supply chains including NIKE, Procter & Gamble, and BOSCH implement Vendor Managed Inventory (VMI) to make decisions around delivery lot sizes and number of deliveries by taking full control of the retailer inventories. On the other hand, some production industries implement Just In Time (JIT) inventory control systems to develop a long-term economic viability through mutual cooperation with the downstream supply chain partners where the optimal lot size is broken down into smaller multi-shippments. In this way, the coordinated supply chains ensure higher service levels while maximizing economic sustainability. Recent applications of JIT have established that Single-Setup-Multi-Delivery-Policy (SSMD) has economic superiority over the Single-Setup-Single-Delivery (SSSD) policy [7]. However, there exists a high likelihood of shortages in the system, especially under the cases of rush deliveries and emergency deadlines faced in automotive industry such as Toyota and technology companies like DELL [8]. Therefore, the consideration of possible shortages in model formulation through deployment of a robust backordering policy is critical in determining optimal production delivery policies.

Given the above key considerations, we propose a sustainable multi-item integrated production delivery model for a single-manufacturer single-retailer e-retailing management under SSMD policy and technology investments for advertisement and greenhouse gas emissions control. The manufacturer operates a smart production system and utilizes predetermined proportion of returned goods in producing the new products. He invests in green emissions reduction technology (GERT) in his production system, and the retailer invests in social media advertisement in the form of information disclosure technology (IDT) to communicate the emissions control efforts of the system to the customers in order to enhance product demand. Information Disclosure Technology (IDT) includes technological and digital platforms utilized to transparently communicate product and process information regarding sustainability efforts to the customers. This includes social media campaigns, interactive e-pages and websites, and product labeling initiatives that elaborates the supply chain’s environmental protection efforts. For instance, Patagonia uses its website and social media to share information about its sustainable raw-material sourcing and fair labor practices, whereas Unilever’s Sustainable Living Plan is communicated through various digital channels and product labels. In this research, IDT represents the retailer’s investment in social media advertising to inform consumers about the manufacturer’s greenhouse gas emissions reduction efforts. Further, Green Emissions Reduction Technology (GERT) refers to the investments in manufacturing and conversion processes, tools and equipment, and the technologies designed to minimize greenhouse gas emissions [7]. Some of the examples of GERT include carbon capture and storage systems, renewable energy integration in manufacturing plants such as solar panels or wind turbines and the utilization of energy-efficient equipment. In this study, GERT represents the manufacturer’s investment in technologies to reduce emissions during the production process which is then communicated to the customers through retailer’s IDT initiatives. Green technology investment plays a key role in supply chain dynamics, impacting costs, consumer behavior, and product demand. By implementing sustainable practices, the companies can not only lower operational costs through efficient use of resources and waste management but also increase profit of the system [9]. Consumers are increasingly willing to pay more for environmentally friendly products, encouraging manufacturers to adopt green technologies [10]. This shift in consumer behavior, particularly among those sensitive to carbon emissions, drives the need for firms to invest in sustainable practices. For example, Walmart’s investment in renewable energy and green supply chains has not only reduced costs but attracted eco-conscious consumers, and Tesla’s focus on electric vehicles and sustainable manufacturing has enhanced its market appeal [11]. Overall, green technology investments help businesses align with consumer preferences, driving demand and fostering sustainability [12].

To the best of author’s knowledge, no such sufficing model has been presented yet that considers this optimal e-advertisement and emissions control strategy while maximizing the system profit under shortage-prone SSMD product delivery mode. We develop two major cases in the proposed advertisement and emissions control efforts based sustainable integrated model. Case I develops a non-capacitated integrated model before advertisement and emissions control efforts which is solved through a combination of analytical optimization technique and a proposed heuristic approach. Further, Case II presents a capacitated integrated model with technology investments for IDT and emissions control which is solved through a metaheuristic approach to identify the optimal advertisement and emission control policy under the smart production rate. The proposed model cases are validated through numerical experiment and analysis to infer significant managerial insights.

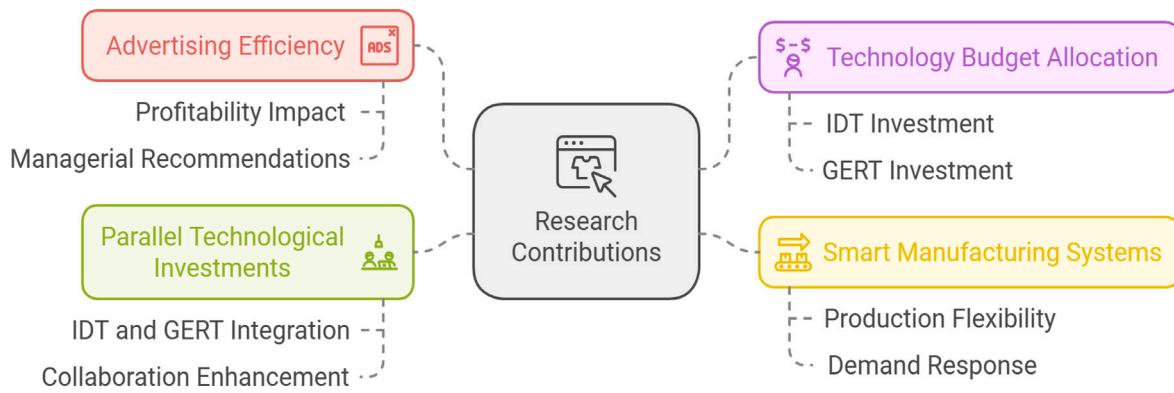


Fig. 1. Contribution of the proposed study.

1.1. Research motivation

Although the importance of flexibility, technology, and advertising [13] for enhancing profitability in offline retailing is studied, their integration within an SSMD policy adoption has not been well researched in e-retailing mechanisms. Further, as the SSMD policy adoption has shown economic benefits in JIT systems [14], it increase the likelihood of shortages in the integrated systems. Hence, consideration of backordering policy in such manufacturer–retailer coordinated arrangement is crucial. This study addresses the aforementioned research gap by presenting a comprehensive inventory model that incorporates technology investments for environmental protection and advertisement efforts within a multi-item supply chain comprising of a single manufacturer and a single retailer. Further, several global companies including Amazon faces inquiry about its carbon footprint and has responded to it by commencing initiatives including “Climate Pledge Friendly” with the objective of highlighting sustainable products [2]. Similarly, a globally famous retail chain named Patagonia has faced communication challenges in leveraging its e-commerce platform to transmit its environmental efforts information by showcasing detailed lifecycle analyses of its products. However, the lack of direct consumer interaction in e-retailing adds to the difficulty of verifying these claims which leads to skepticism about the greenwashing. This lack of transparency in information disclosure can directly wear away consumer trust and negatively impact the brand reputation. This highlights the practical implications of the green information-sharing challenges for the e-tailing industry. Therefore, this study examines the most effective combination of smart production, technological investments, and dynamic advertising methods to optimize overall profit in the presence of SSMD and planned backordering strategies under collaborative e-retailing. We investigate the strategic choices of technology budget distribution among IDT and GERT under various realizations of the budgetary and space constraints. Further, we identify the critical parameters through sensitivity analysis and devise recommendations to improve economic viability of the system under the variations in these parameters. The obtained managerial insights provide decision-makers with the necessary recommendations to deploy efficient strategies and attain environmental and economic sustainability. Therefore, our study aims to answer the below research questions:

1. How can the integration of smart production systems, technological investments, and dynamic advertising methods enhance the profitability of a manufacturer–retailer collaborative e-retailing system?
2. What is the optimal allocation of the technology investment budget between IDT for the retailer and GERT for the manufacturer to maximize overall profitability?
3. How does the adjustment of production rates within predefined ranges in response to fluctuating e-product demand contribute to the profitability and flexibility of the smart production system?
4. What is the economic role of advertising efficiency in enhancing the overall profitability of a sustainable integrated production-delivery strategy under e-retailing?

1.2. Contribution of the research

On the basis of aforementioned discussion, this study contributes to the e-retailing literature in the following aspects.

- This study highlights the significance of advertising efficiency by establishing a clear association between advertisement efficiency and profitability of the sustainable integrated production-delivery strategy under e-retailing. This understanding offers managerial recommendations on improving advertisement efforts in order to generate encouraging effects on overall profitability.
- This study offers valuable insights into the strategic distribution of the technology investment budget among Information Disclosure Technology (IDT) for the e-retailer and Green Emissions Reduction Technology (GERT) for the manufacturer. This contribution enables managers and practitioners in improving economic viability through intelligent technology budget allocation under varied circumstances.
- This study demonstrates a constructive impact of converting traditional fixed manufacturing systems into smart ones by allowing adjustment of production rates within the predefined ranges in response to varied e-product demand. Through an experimental study, we answer the question; What is the strategic role of flexibility in enhancing overall profitability of the system?
- This study contributes to the literature by verifying the significant impact of incorporating parallel technological investments in Information Disclosure Technology (IDT) and Green Emissions Reduction Technology (GERT) to improve system profitability. This insight enables managers and practitioners in making strategic technology integration decisions for collaboration among the manufacturer and e-retailer.

Fig. 1 provides graphical representation of the major contribution of this research.

1.3. Orientation of the research

Rest of this research is structured as below. Section 2 provides a thorough review of the research to identify the key milestones and need of the study in the field of production and retail management. Next section explains the problem and emphasizes the nomenclature and assumptions used to formulate the proposed mathematical model. Both cases of the proposed sustainable integrated production-delivery model are developed in the Section 4, and Section 5 illustrates hybrid solution methodology for the proposed sustainable integrated inventory models. An experimental study is setup in Section 6 to validate real world application of the model and the experimental outcomes are analyzed for various instances of the model to devise significant managerial insights. Eventually, Section 7 presents conclusions, limitations, and future research avenues of the study.

2. Literature review

This section presents an overview of the current literature in the field of e-retailing management with a focus on technology integration for green products manufacturing and delivery under a smart production management system.

2.1. E-retail management

E-retail management is a process of managing online retail operations including sales, inventory, and customer engagement. For instance, Amazon uses AI-driven recommendations to personalize shopping experience and shopify platforms enable small businesses to set up and manage online stores. The businesses can establish their online presence with minimal effort and it has significantly lowered the barriers to entry into the e-retailing industry. The online marketplace including platform selling is primarily open to all types of businesses. However, this convenience to develop e-retailing channels has created a high degree of homogenization in this industry. The business models developed around e-retailing can be easily replicated due to their wide availability and affordability. This makes it even more complex to differentiate one online retailer from another in the same sector. This constructs a highly competitive landscape encompassing a high threat of new entrants, quite readily available distribution channels, reduced economies of scales, and eventually low customer switching costs. Further, the online retailers face persistent pressure to enhance their delivery process performance. This is due to the fact that online consumers have high expectations regarding product quality, delivery speed and quality, and the overall purchasing experience [15]. As a result, building a strong strategic position that offers a sustainable competitive advantage is important for the e-retailers. Current research on e-retailing has explored various aspects of improvement and has identified respective key factors for success in e-retailing [11,16–19]. Ani et al. [16] emphasize the importance of managing security concerns as a critical success factor in e-retailing. Further, Chen et al. [18] focused on the operational aspects of e-retailing and suggested that a successful e-retailing model should integrate simple information search, numerous communication channels, and safe trading and transaction processes. Cataldo et al. [17] has highlighted the importance of a highly integrated e-retailing application and a robust organizational operating system. Edghiem et al. [11] point out the crucial role of business networks in creating value. They have observed that e-retailers operate within highly interconnected business ecosystems that extend beyond their own organizational boundaries. Recently, Zhang et al. [19] emphasize the impact of key factors including responsiveness, personalization, ease of use, aesthetics, and perceived risk on the success of e-retailing businesses.

2.2. Information sharing in e-retailing

Information disclosure through technology integration is a practice of sharing relevant business information with stakeholders to ensure transparency and build trust. Apple Inc. shares its annual environmental report to expose customers to its carbon footprint and sustainability efforts. Similarly, Tesla shares its quarterly earnings reports that provide insights into its financial performance and future plans. Green information disclosure efforts significantly impact consumer behavior by improving environmental awareness and promoting informed purchasing choices. According to the theory of planned behavior (TPB), consumer intentions to buy sustainable products are shaped by their attitudes, subjective norms, and the perceived behavioral control. Here, the environmental disclosure acts as a key factor in rationalizing behavioral decisions. Further, the Stakeholder theory highlights the mediating role of sustainability reporting in fostering stakeholder participation and improving sustainable information transfer [20]. In addition, the legitimacy and resource dependency theories emphasize that environmental reporting enhances corporate image of the firm, its legitimacy, and the consumer trust on the firm [21]. Now a days, increasing environmental consciousness is altering the preferences of consumers, placing more emphasis on sustainability and ethical manufacturing and delivery through e-retail channels. This transition requires clear and open communication about the green production and delivery practices. Customized advertising on technology platforms such as social media and Blockchain enables firms to directly convey their environmentally friendly activities, informing and involving customers in their efforts to reduce carbon emissions by using green technologies [22]. This not only cultivates customer loyalty but also broadens consumer demographics and stimulates demand for eco-friendly goods. However, the effectiveness of this communication relies on an efficient supply chain that guarantees product availability and fulfills consumer expectations. Therefore, environmentally responsive manufacturers and retailers including P&G, IKEA, and Walmart have expanded their operations from physical to digital platforms in response to the growth of e-commerce and are using AI-powered tools and IoT-based tracking systems. The online platform provides enhanced flexibility in response to consumer demands with regards to pricing, quality, selection, and convenience. The study conducted by Li et al. [23] explains that retailers operating in online settings have significantly improved demand related information capabilities due to considerable investments in information disclosure technology. Wei et al. [24] suggests that the retailers should be motivated to communicate sensitive demand information to the product suppliers to increase the green supply chain efficiency. The research conducted by Shang et al. [25] verify the sound impact that information sharing puts on supply chain decisions. In particular, Wang and Zhuo [26] states that retailers who possess advanced market intelligence can effectively negotiate more advantageous supply contracts and transaction terms with their suppliers by exchanging demand information. Cai et al. [27] have observed that the retailer's choice of approach for disclosing information under voluntary information disclosure depends on the offered product price and the degree of greenness. Sarkar et al. [12] has focused on the impact of upstream asymmetric information and the bullwhip effect on profit and customer satisfaction under sustainable delivery practices like advanced transportation and automated inspection for defect management, but they ignored downstream information sharing. They demonstrated that information sharing among supply chain players enhances profits and reduces losses from information asymmetry with numerical examples validating these findings. Recently, Liu et al. [28] have examined the role of advertisement on supply and retail performance of the commodity products. Taking into account the positive effects of social media marketing in disseminating information regarding the environmentally conscious production of products to consumers and the subsequent rise in product demand, we consider IDT investment in advertisement of multi-product's green manufacturing initiatives to the customers.

Table 1
Major contribution of the research.

Author(s)	Model type	Advertisement investment	Product type	Product demand	Production system	Production delivery policy	Product shortages	Technology investment for emissions control	Solution methodology
Kang et al. [33]	Non-integrated production	No	Single-item	Constant	Smart	SSSD	Not allowed	No	Analytical optimization
Sarkar et al. [34]	Supply chain management	No	Single-item	Constant	Traditional	SSMD	Not allowed	No	Analytical optimization
Taleizadeh et al. [35]	Vendor managed inventory	No	Single-item	Uncertain	Traditional	SSSD	Partially backordered	No	Analytical optimization – heuristic
Sana [36]	News vendor	No	Single-item	Price and emissions control efforts dependent	Traditional	SSSD	Not allowed	Yes	Analytical optimization
Sadeghi et al. [8]	Integrate vendor–buyer	No	Single-item	Constant	Traditional	SSMD	Fully backordered	No	Analytical optimization – heuristic
Dey et al. [37]	Integrate vendor–buyer	Yes	Single-item	Advertisement dependent	Smart	SSSD	Fully backordered	No	Analytical optimization
Tayyab et al. [38]	Non-integrated production	No	Single-item	Constant	Traditional	SSSD	Fully backordered	No	Analytical optimization
Kar et al. [39]	Non-integrated production	Yes	Multi-item	Price and advertisement dependent	Smart	SSSD	Not allowed	Yes	Analytical optimization
Umar et al. [7]	Supply chain management	No	Single-item	Constant	Traditional	SSMD	Not allowed	No	Metaheuristic
Saxena et al. [10]	Integrate vendor–buyer	No	Single-item	Random	Smart	SSSD	Not allowed	No	Analytical optimization
Datta et al. [40]	Dynamic retailing model	Yes	Single-item	Advertisement dependent	Traditional	NA	Not allowed	No	Analytical optimization
Morshedin et al. [41]	Coordinated supply chain	Yes	Single-item	Constant Demand	Traditional	SSSD	Not allowed	No	Metaheuristic
Sebatjane and Adetunji [42]	Integrated production model	No	Single-item	Price and freshness dependent	Traditional	SSSD	Not allowed	No	Iterative procedure
Sasanuma et al. [43]	Retailer cost reduction	No	Multi-item	Random demand	Traditional	SSSD	Allowed	No	Analytical optimization
Nobil et al. [44]	Vendor economic decision	No	Single-item	Constant demand	Traditional	SSSD	Allowed	No	Analytical optimization
This study	Integrated manufacturer–retailer	Yes	Multi-item	Price and advertisement dependent	Smart	SSMD	Fully backordered	Yes	Analytical optimization – Metaheuristic

2.3. Multi-shipment policy

Multi-shipment policy is a production and delivery strategy that allows manufacturers to produce the goods in a single-setup but deliver it in multiple shipments to improve logistics efficiency. Zara's multi-shipment approach for international orders to optimize shipping costs and delivery times is one of the successful implementation of this strategy. The inventory management solutions such as Vendor Managed Inventory (VMI) and Single-Setup-Multi-Delivery (SSMD) policy play a key role in optimizing production and delivery strategies by ensuring timely deliveries and reducing the possibility of shortages. Ben-Daya et al. [29] presented a three-layer integrated supply chain model considering SSMD policy. Their research outcomes verified economic benefits of SSMD policy over the SSSD policy. Sarkar and Chung [30] presented a mathematical model to minimize total cost of a flexible manufacturing system. The research objective was to reduce system cost in the situations where demand during the lead time is considered as following a normal distribution. The experimental results verified the importance of making technology investments in developing a flexible manufacturing system by understanding its impact on total cost of the complete system. They implemented a classical optimization method along with a heuristic approach to derive exact and approximate solutions for the model variables. The sensitivity analysis indicated that their study has achieved lowest cost by optimizing the decision variables under the presence of technology investments and SSMD policy. A recent study conducted by Mridha et al. [31] seeks to enhance the efficiency of a multi-layer sustainable supply chain model by specifically addressing control of carbon emissions and maximizing profitability of the system under SSMD policy. They presented a smart manufacturing system that incorporates a model consisting of a single manufacturer, a single supplier, and multiple retailers. Their model effectively reduces imperfect production and carbon emissions by implementing a two-stage inspection policy, flexible production rate strategy, and selling prices based on demand. Their experimental results show positive environmental and economic impacts. Jauhari et al. [32] has developed an integrated inventory model for a supply chain involving a vendor and a buyer by accounting for stochastic demand, imperfect production, and a hybrid system combining regular and green production.

By optimizing factors including shipment quantity, production allocation, and defect rate, their model minimizes supply chain costs while balancing environmental impact and production efficiency. However, they did not consider multi-shipment policy to minimize transportation costs

of the system. Chen et al. [45], Sarkar et al. [46], and Kang et al. [33] are among several other researchers who implement SSMD policy under manufacturer–retailer integrated supply chain management. Given the economic benefits of SSMD production delivery policy within a coordinated e-retailing, we incorporate this policy in the proposed sustainable integrated model to improve profitability of the system.

2.4. Green technology integration

Green technology integration is the process of incorporating eco-friendly technologies into business processes to reduce its environmental impact. For instance, IKEA uses solar panels technology and energy-efficient lighting in its stores to reduce energy consumption, and Patagonia has adopted recycled materials in the clothing production. Technology investments mark outstanding contribution to improving global economy through effectively reducing carbon emissions in the global supply chains. As customer environmental awareness grows, manufacturers' competition no longer just dependent on profits. It also includes their environmental performance. Thus, global production and retailing industries need carbon emissions reduction strategies and low-carbon product pricing. Song et al. [47] provided a multi stage stochastic optimization model to examine logistics capacity increase under various carbon emissions legislations. They showed that capacity investment cost volatility affects optimal capacity expansion decisions more than production cost for low and high carbon tax rates. Ganda [48] examined how innovation and technology investments affected Organization for Economic Co-operation and Development member nations' carbon emissions from the year 2000 to 2014. Their analysis verified that renewable energy integration and R&D spending have a statistically significant negative correlation with the greenhouse gas emissions. They suggested that incorporating environmental responsibility in patents and training researchers in green skills may help them meet zero-emission ambitions. Sana [36] investigated a price competition between environmentally friendly and non-environmentally friendly manufacturers, taking into account product demand influenced by sales price, investments in technology for reducing carbon emissions, and corporate social responsibility index. Their results suggested continuous investments in greenhouse gas emissions reduction and consumer awareness may increase product demand in the longer run. Bose et al. [9] and Sarkar et al. [49] developed online-to-offline retailing models to optimize pricing, advertising strategy, customer care efforts and investment decisions to increase profit under the consideration of budget and space constraints, respectively, but they ignored green technology investments. Their research findings highlight that offering free home delivery and managing defective rates effectively can significantly enhance profitability of the system. Recently, Kang and Tan [50] have utilized an evolutionary-based model to investigate the technology investment choices made by suppliers and product manufacturers in a supply chain management under carbon cap-and-trade emissions control policy. They studied the fact that how revenue and cap-and-trade-related factors affect green technology investment choices. Their research provides insights for policy makers on how to handle the interplay between these investments and resulting revenues. Jauhari et al. [51] has developed a mathematical model for a closed-loop supply chain involving a manufacturer and retailer in different countries by considering stochastic conditions, imperfect production, and carbon emissions. By incorporating a carbon tax policy and green technology investments, their model aims to minimize total costs while addressing key factors like exchange rate uncertainty, production defectives and delivery decisions. Jauhari et al. [52] focused on minimizing emissions in a supply chain involving a vendor and a buyer by addressing carbon regulations through green technology investments, supported by government incentives and carbon tax policies. Their proposed model optimizes operational decisions and green investments to reduce costs and emissions, demonstrating improvements in both economic and environmental performance. However, they did not consider the impact of green investments on consumer demand through green information sharing. Given the technology investment benefits for environmental protection, our research incorporates technology investments for green emission reduction technology (GERT) as a strategic choice.

2.5. Smart production system

Smart Production System is an advanced manufacturing system that uses automation, IoT, and data analytics to adjust production rates in order to enhance efficiency and productivity. For instance, Siemens uses digital twins to simulate and optimize production processes, and Tesla has fully automated its Gigafactories for electric vehicle manufacturing. Being able to adjust the production rates in a smart production system in accordance with the changes in demand and delivery time requirements is an essential factor that impacts manufacturing costs. Such smart production system can address issues associated to the random, uncertain and sometimes unpredictable characteristics of the market demand, and the resulting shortages. In this direction, Khouja and Mehrez [53] provided a method for determining the optimal lot size using Economic Production Quantity (EPQ) by considering both variable production rates and the dependent unit production costs. The production cost was taken into account as a function of the changes in production rate using that approach. This approach enables even more rigorous investigation of the cost components related to the production systems by taking in both flexibility of production rates and its impact on unit production cost. Further, several other researchers including AlDurgam et al. [54], and Sarkar and Chung [30] have implemented a similar type of approach by considering variability in the production rates to derive optimal inventory management policies to improve profitability of the whole production delivery system. Dey et al. [37] have considered the impact of flexible production rate on economic performance of the supply chain by incorporating the advantages of reducing lead time across various situations. The equation presented by them shows a precise relationship between total cost and lead time, indicating that the cost savings from reducing lead time drop as lead time increases. Their research outcomes suggest making continuous investments to improve the reliability of the production process. Sadeghi et al. [8] provided an integrated inventory model with shortages to minimize system cost without considering flexible production rate. Recently, Kar et al. [39] presented a dual-channel advertisement and pricing model for multiple manufacturers and retailers while ignoring product shortages. They incorporated flexible production rate in their model and proved that such policy provides better economic benefits in comparison to the traditional fixed production rate systems. To acquire an in-depth understanding of how variable production rates impact the performance of inventory management systems, it is advised that interested readers refer to the research conducted by Glock and Grosse [55] as their study offers a comprehensive appreciation of this impact. Considering aforementioned discussion and analysis of the literature, we consider flexible production rate and planned backorders in our proposed sustainable integrated inventory model for a manufacturer and retailer coordinated system. Fig. 2 presents the graphical illustration of the proposed study and Table 1 presents contribution of this study.



Fig. 2. Graphical representation of the proposed collaborative e-retailing model.

3. Problem definition, notation, and assumptions

This research proposes a sustainable multi-item integrated model to maximize total profit for the manufacturer–retailer coordinated decision-making under online sales mechanism. Within this collaborative system, a controllable production rate ($P_i \in \{P_{i,min} - P_{i,max}\}$) for the manufacturer [38] is considered to take advantage of a smart production system in making production-delivery decisions under price and advertisement dependent product demand. The manufacturer invests in green emissions reduction technology (GERT) in his production system and the retailer invests in advertisement in the form of information disclosure technology (IDT) to communicate the emissions control efforts of the system to the customers in order to enhance product demand. Product demand is taken as a function of product price and advertisement efforts of the retailer. We use the dependent demand function as $D_i = \Delta_i - \gamma_i p_i + \psi_i \eta_i$, where Δ_i is the initial market size of product type- i , γ_i is the price sensitivity, and ψ_i is IDT sensitivity. Similar type of demand function has been utilized by Umar et al. [7] and Kar et al. [39] in their manufacturer–retailer optimization models. The e-retailer receives annual product- i demand of D_i units and the manufacturer delivers products to the retailer in multiple equal shipments using SSMD policy, where the ordered lot size is $Q_i = n_i q_i$. In this way, inventory holding cost of the retailer is reduced while compromising on the product transportation cost. We consider discrete IDT investment of $\eta_i = \alpha_i W_{i,Tech}$, where α_i is the proportion of discrete technology investment budget $W_{i,Tech}$ allocated for product type- i . The manufacturer invests in GERT technology to reduce the carbon emissions of the system during production. We consider a similar discrete GERT investment function of type $E_i = \beta_i W_{i,Tech}$, where E_i is the GERT investment for product type- i , and $E_i = \beta_i W_{i,Tech}$. It is worth noting that $\alpha_i + \beta_i = 1$. The impact of discrete GERT investment on greenhouse gas emissions mitigation is incorporated as $G_i = \lambda_i (1 - e^{-\rho_i E_i}) = \xi_i e^{-\rho_i E_i}$, where E_i is GERT investment, and ρ_i is its efficiency as a scaling parameter. Further, λ_i is the savings in emissions due to GERT. It is to be noted that when $E_i \rightarrow \infty$, then $G_i = \lambda_i$, and when $E_i = 0$, then $G_i = 0$. Similar type of investment function has been implemented by Kar et al. [39] for greenhouse gas emissions reduction. A planned backorder strategy is introduced in the system to compensate the shortages originated in the smart integrated system. Fig. 3 presents inventory behavior of the sustainable integrated system at manufacturer and retailer. We develop two major case in the proposed model. Case I develops a non-capacitated integrated model before advertisement and emissions control efforts which is solved through a combination of analytical optimization technique and a proposed heuristic approach. Further, Case II presents a capacitated sustainable integrated model with technology investments for advertisement and emissions control which is solved through a metaheuristic approach to identify the optimal advertisement and emission control policy under the flexible production rate. The proposed model cases are validated through numerical experiment and analysis to infer significant managerial insights.

3.1. Notation

Below notation are utilized for the proposed model formulation and analysis.

Indices	
i	index for product type ($i = 1, 2, 3 \dots y$)
\hat{m}	index for the vendor/manufacturer
\hat{r}	index for the retailer
Decision Variables	
p_i^*	Optimal online selling price of product type- i (\$ per item)
Q_i^*	Optimal batch quantity (items)
q_i^*	Optimal shipment quantity per shipment (items)
B_i^*	Optimal backorder quantity (items)
P_i^*	Production rate for product type- i (items per year)
$W_{i,Tech}^*$	Optimal technology investments (\$)
α_i^*	Percentage of $W_{i,Tech}^*$ allocated to IDT (percentage)

β_i^*	Percentage of $W_{i,Tech}^*$ allocated to GERT (percentage)
X_i^j	Decision vector of this paper
Parameters	
D_i	Retailer's estimated demand for product- i (items per year)
T_i	The cycle time for product type- i (years)
T_{1i}	Maximum inventory consumption time (years)
T_{2i}	Time for which system faces shortages (years)
T_{3i}	Time to compensate the product shortage (years)
T_{4i}	Time for inventory fulfillment (years)
t_i	Time interval among successive deliveries (years)
K_i	Setup cost per setup (\$ per setup)
h_i	Inventory holding cost (\$ per item per unit time)
b_i	Fixed backorder cost (\$)
g_i	Variable backorder cost (\$ per item)
δ_i	Batch transportation cost (\$ per item)
C_{iv}	Cost of purchasing and processing virgin raw material (\$ per item of raw-material)
C_{ir}	Cost of incentive and processing returned material (\$ per item of raw-material)
θ_i	Percentage of D_i fulfilled through remanufacturing (percentage)
$I_{i,max}$	Maximum attainable inventory level for product type- i (items)
n_i	Number of deliveries per cycle for product type- i (number)
d_i	Initial market size of product type- i (items per year)
u_i	Scaling parameter for the order processing cost with respect to flexible production rate (number)
v_i	Shape parameter for the order processing cost with respect to flexible production rate (number)
γ_i	Sale price sensitivity of market demand (number)
ψ_i	IDT investment sensitivity (number)
λ_i	Savings in CO_2 emissions through GERT investment (units per item)
ξ_i	CO_2 emissions per item during processing (units per item)
$\xi_{i,CO2}$	Emissions cost per unit of emissions (\$ per unit)
z_i	Storage space required for one item type- i (cubic units)
Expressions	
$TP_{i,wot}$	Profit function for product type- i in Case-I (\$ per unit time)
TP_{wt}	Profit function for sustainable multi-item in Case-II (\$ per unit time)

3.2. Assumptions

We use below set of assumptions for the proposed sustainable integrated production delivery model formulation.

1. The proposed study considers a single-manufacturer single-retailer integrated system for production and delivery of multi-items under e-retailing.
2. A dependent product demand that is sensitive to the product price and IDT advertisement investments is considered in this research as $D_i = \Delta_i - \gamma_i p_i + \psi_i \eta_i$, where Δ_i is the initial market size of product type- i , γ_i is the price sensitivity, and ψ_i is IDT sensitivity [4].
3. A smart production system is considered where the production rate for each product type- i can be varied within the interval $\{P_{i,min} - P_{i,max}\}$ in response to the fluctuations in online product demand for the retailer [38].
4. In order to determine an optimal trade-off among inventory holding cost and batch transportation cost, an SSMD policy is implemented in this research where the manufacturer ships the product in smaller batches to the e-retailer's warehouse [30].
5. Shortages are allowed in this study and are fully backordered in the proposed sustainable integrated system, where both of the fixed and variable portions of backorder cost are considered.
6. Products are made with virgin and returned material, where pre-defined θ_i percentage of the online demand for product type- i is fulfilled using return products as a raw material.
7. Initial delivery of each product type- i ($i = (1, 2, 3, \dots, n)$) takes place at time $t = 0$.
8. The successive production cycles are identical in nature.

4. Mathematical model

Analytical derivation of the proposed sustainable integrated model is initiated by determining the size of individual shipment to the e-retailer's warehouse. The manufacturer produces a quantity $q_i = P_i t_i$ over the time interval t_i and the retailer receives $D_i T_i$ units of demand from the market during this time period. If the warehouse inventory of the retailer is enough, this demand will be met and otherwise the shortage will occur. Hence, given the n_i^{th} delivery, the consumed inventory over the last $n_i - 1$ deliveries can be determined as $(n_i - 1)D_i t_i$.

As the batch size is $Q_i = n_i q_i$, the maximum quantity for the retailer can be obtained from Fig. 3 as

$$\begin{aligned}
 I_{i,max} &= n_i q_i - (n_i - 1)D_i t_i - B_i, \\
 &= \frac{1}{P_i} [D_i q_i - B_i P_i - D_i Q_i + P_i Q_i].
 \end{aligned} \tag{1}$$

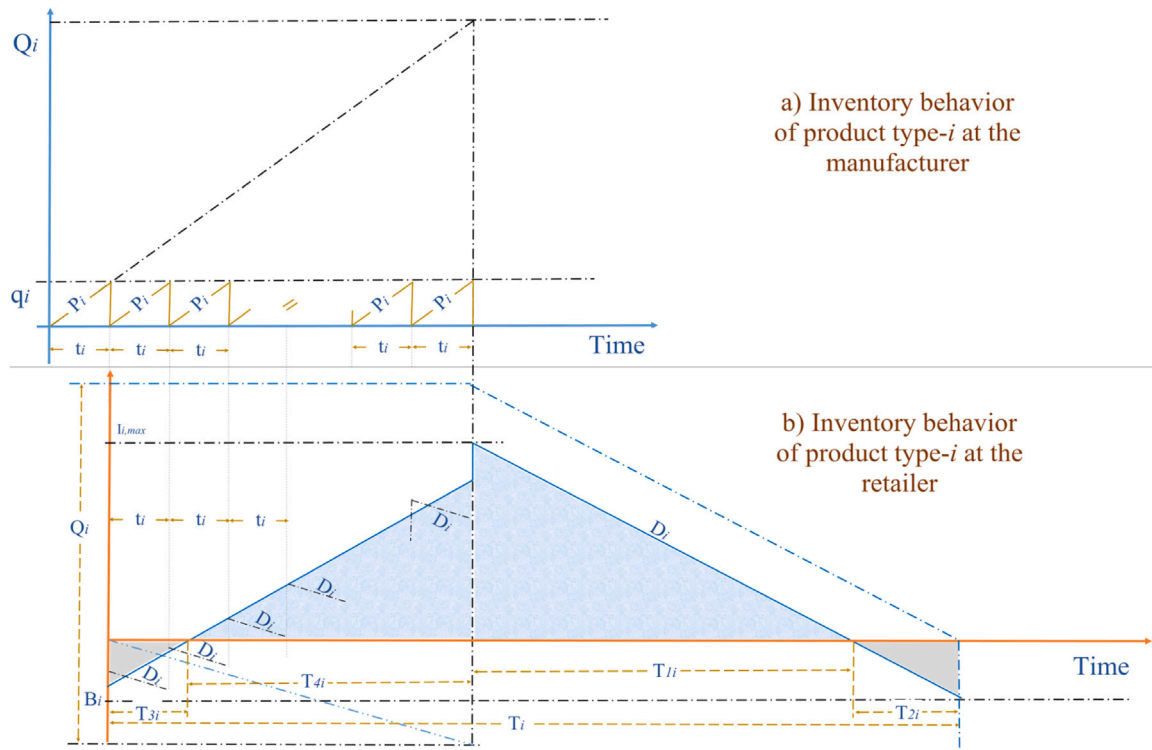


Fig. 3. Inventory behavior of product type- i at (a) manufacturer and (b) e-retailer [ref to Sadeghi et al. [8]].

The total time (T_{1i}) required for consumption of this inventory in result of $I_{i,max}$ obtained in (1) can be computed as

$$T_{1i} = \frac{I_{i,max}}{D_i} = \frac{1}{P_i D_i} [D_i q_i - B_i P_i - D_i Q_i + P_i Q_i]. \quad (2)$$

As the proposed model allows planned backorders against shortages to improve profitability of the system, certain portion of the cycle time should be set aside to compensate these shortages. Now the time (T_{2i}) for which the integrated system faces the product shortages is obtained as

$$T_{2i} = \frac{B_i}{D_i}, \quad (3)$$

and the time (T_{3i}) to compensate for these shortages can be determined as

$$T_{3i} = \frac{\left(B_i - \frac{q_i}{2}\right) t_i}{q_i - D_i t_i} = \frac{2B_i - q_i}{2(P_i - D_i)}. \quad (\text{since } q_i = P_i t_i) \quad (4)$$

During the above time period, the inventory is being restocked at a consistent rate, which is represented by q_i units. The value of q_i can be simply obtained by multiplying the demand rate (D_i) with the time interval (t_i). Thus, the total inventory refilled over the time period T_{3i} can be calculated as the difference between q_i and product of the demand rate (D_i) with the replenishment lead time (t_i), and then multiplied by the time interval T_{3i} . At time $t_i = 0$, the first delivery begins and immediately compensates for a shortage of B_i units by providing q_i units from the inventory. During the time interval T_{3i} , which is marked as having null inventory, the inventory starts increasing by a constant amount $(q_i - D_i \times t_i)$ at each interval, reducing the total deficiency of inventory. Fig. 3 demonstrates that the entire backorder (B_i) is equal to half of the cumulative replenishment ($q_i/2$). Hence, in order to fully compensate for the whole shortage during this interval, replenishment should be established as the quantity $(q_i - D_i \times t_i) \times T_{3i}/t_i$.

It is important to acknowledge that the gradual process of restocking inventory in the proposed model makes it difficult to calculate the costs associated with inventory holding and backorders. Therefore, we utilize the inventory calculation approach suggested by Sadeghi et al. [8] where the area of stepwise section is calculated to indicate the area of shaded region. This method initially supports in calculating the time (T_{4i}) required to produce and achieve the maximum inventory as below.

$$\begin{aligned} T_{4i} &= \frac{\left(I_{i,max} - \frac{q_i}{2}\right) t_i}{q_i - D_i t_i}, \\ &= \frac{P_i (2Q_i - 2B_i - q_i) - 2D_i (Q_i - q_i)}{2P_i (P_i - D_i)}. \end{aligned} \quad (5)$$

Now the inventory of the whole system for product type- i is calculated as follows.

$$I_{i,inv} = \frac{1}{2} \left[I_{i,max} T_{1i} + T_{4i} \left(I_{i,max} - \frac{q_i}{2} \right) \right]. \quad (6)$$

Similarly, total inventory of the backorders for product type- i is computed from Fig. 3 as below.

$$I_{i,backorder} = \frac{1}{2} \left[T_{2i} B_i + T_{3i} \left(B_i - \frac{q_i}{2} \right) \right]. \quad (7)$$

Now we calculate different cost components of the system including (i) ordering and setup cost, (ii) production cost, (iii) carbon emissions cost, (iv) inventory holding cost, (v) backorder cost, (vi) transportation cost, and (vii) technology investment cost for IDT and GERT.

1. Setup and ordering cost

Ordering cost for a retailer corresponds to the costs incurred in preparing for placing orders, and arranging inbound order delivery. Setup cost for a manufacturing system corresponds to the costs incurred in preparing a machine or manufacturing line for another batch of items. The setup cost plays a crucial role in industrial operations as significantly affects overall production efficiency and cost-effectiveness of the system. The relevance of minimizing setup costs lies in its direct impact on inventory management, as frequent setups can lead to increased wastages. Additionally, the significant upfront costs might adversely affect the profitability of the system by reducing the economic order quantity and increasing the cost per unit produced. Therefore, production systems tend to optimize their setup operations in order to enhance operational efficiency and economic results. On the similar grounds, an ordering cost corresponds to the retailer's costs incurred in placing the subsequent order. The setup and ordering cost of the proposed model are given by

$$TC_{i,setup} = K_{i,setup} + K_{i,order} = K_i. \quad (8)$$

For simplicity purpose, we use the term setup cost to denote K_i in rest of the paper.

2. Order processing cost

Production system efficiency and sustainability depend on order processing costs. Optimizing resource utilization, operational effectiveness, and market competitiveness requires focus in reducing order processing costs. Raw materials, labor force, operating equipment, and overhead costs affect order processing costs. Cutting down this cost without sacrificing quality can improve profits. For a smart production system proposed in this research, changes in production rate (P_i) affect resource usage and efficiency, which affects the system's cost structure. Further, utilizing virgin raw material and returned items as a raw material reduces the overall production cost of the system while contributing in sustainability of the system by improving environmental protection. Order processing cost of the proposed integrated production inventory model including impact of the flexible production rate, and the consideration of virgin and used raw material is provided as below.

$$TC_{i,man} = Q_i \left[u_i P_i + \frac{v_i}{P_i} + C_{iv} - \theta_i (C_{iv} - C_{ir}) \right]. \quad (9)$$

3. Carbon emissions cost

Due to growing worldwide environmental concerns, costs related to carbon emission in production systems are becoming crucial in modern industries. The production systems need to keep track of their carbon footprint to comply with strict environmental regulations and corporate social responsibility. Carbon emissions are especially important in production delivery decisions as production processes emitting huge amounts of greenhouse gases may affect required resources, suppliers, and manufacturing techniques. Carbon emission costs affect system profitability by increasing operating costs and related penalties. Recognizing this fact, the production systems can reduce their carbon emissions related costs by investing in technology upgrades and low-emission equipment for promoting environmental sustainability and economic viability. The proposed research considers discrete technology investments for carbon emissions cost reduction. Hence, carbon emissions cost after technology investments is given by

$$TC_{i,CO2} = \xi_i e^{-\rho_i E_i} Q_i. \quad (10)$$

4. Inventory holding cost

The inventory holding cost in a production system is the cost of keeping and maintaining goods over time and it plays a crucial role in devising production delivery policies. Continuous monitoring and control of this cost is critical as it affects supply chain inventory stocking choices. Higher levels of inventory holding costs increase total cost of the system. For this, through effective inventory management, organizations may improve output, minimize surplus stock, and eventually increase profitability. Therefore, a focused inventory holding cost analysis is essential for making educated choices that improve production system profitability. This holding cost is incurred during the intervals T_{1i} and T_{4i} in the proposed model. We can simply derive the holding cost of the system as below.

$$TC_{i,hold} = \frac{h_i}{2} \left[I_{i,max} T_{1i} + T_{4i} \left(I_{i,max} - \frac{q_i}{2} \right) \right]. \quad (11)$$

5. Backorder cost

In a production system, planned backorder cost is incurred for purposely delaying customer orders. Effective production planning is required to optimize this cost. Backorder costs help companies plan inventory, production, and customer fulfillment. It impacts production delivery choices, affecting the balance between serving customer wants quickly and saving expenses. The appropriate balance is needed to fulfill customer expectations and reduce total cost of the coordinated system. The proposed model considers fixed and variable backorder costs. Total cost of backorders is determined as below.

$$TC_{i,border} = \frac{g_{iv}}{2} \left[T_{2i} B_i + T_{3i} \left(B_i - \frac{q_i}{2} \right) \right] + b_i B_i. \quad (12)$$

6. Transportation cost

The cost of batch transportation is a significant factor in the process of moving items from the producer to the consumer. The objective of the supply chain is not only to transport items from one location to another, but to transport it in a most economic manner. The cost of transporting product batches strongly impacts decisions about production quantities and delivery timing. The production systems must strike a wise balance between timely delivery and minimizing transportation costs. Batch transportation cost for the proposed production model is provided as below.

$$TC_{i,trans} = n_i \delta_i = \frac{Q_i}{q_i} \delta_i. \quad (13)$$

7. Technology investment cost for advertisement and emissions control

Amidst the evolving industrial landscape, making the strategic choice to integrate investments in Information Disclosure Technology (IDT) and Green Emissions Reduction Technology (GERT) is becoming critical. Embracing GERT involves making a deliberate effort to support environmental sustainability and adopting IDT to communicate it to the customers improve production system's credibility. These strategic investments not only improve corporate responsibility but also establish firms as forward-thinking enterprises committed to both transparency and environmental care. Technology investment costs including GERT and IDT for the proposed model are given as below.

$$TC_{i,tech} = (\alpha_i + \beta_i) W_{i,Tech}. \quad (14)$$

Now we present two major cases of the proposed model. Model Case I provides a non-capacitated integrated model before advertisement and emissions control efforts, and Case II provides a capacitated sustainable integrated model with technology investments for advertisement (IDT) and emissions control (GERT).

Case-I: Non-capacitated integrated model before advertisement and emissions control efforts

We develop a base-case scenario before capacity limitations and technology intervention for advertisement and carbon emissions control. First term in the below objective Eq. (15) depicts total revenue of the integrated supply chain network. The second term presents setup cost, and the third term of the function shows manufacturing cost of the system. Further, fourth to seventh terms depict carbon emissions cost, inventory carrying cost, backordering cost, and the transportation cost, respectively.

$$\begin{aligned} TP_{i,wot} &= \sum_{i=1}^y [Revenue_i - (TC_{i,setup} + TC_{i,man} + TC_{i,CO2} + TC_{i,holding} + TC_{i,backorder} + TC_{i,trans})], \\ &= \sum_{i=1}^y \left[p_i (\Delta_i - \gamma_i p_i) - \frac{\Delta_i - \gamma_i p_i}{n_i q_i} \left[\left(b_i B_i + \frac{1}{8} g_i \left(\frac{4B_i^2}{\Delta_i - \gamma_i p_i} + \frac{(q_i - 2B_i)^2}{P_i - (\Delta_i - \gamma_i p_i)} \right) \right) \right. \right. \\ &\quad + \frac{1}{8} h_i \left(\frac{4(B_i - n_i q_i)^2}{\Delta_i - \gamma_i p_i} + \frac{(q_i - 2B_i)^2}{P_i - (\Delta_i - \gamma_i p_i)} - \frac{4(n_i - 1)n_i q_i^2}{P_i} \right) \\ &\quad \left. \left. + n_i q_i \left(-\theta_i (c_{vi} - c_{ri}) + c_{vi} + P_i u_i + \frac{w_i}{P_i} \right) + k_i + \frac{\delta_i (n_i q_i)}{q_i} + n_i \xi_i q_i \right] \right]. \end{aligned} \quad (15)$$

Case-II: Capacitated sustainable integrated model with technology investments for advertisement and emissions control

The objective of the proposed sustainable integrated retailing model is to maximize the profitability of the system under technology investments and shortages. Now we develop the objective function to express the expected profit of the system by incorporating aforementioned cost components including setup cost, manufacturing cost, carbon emissions cost, inventory holding cost, backordering cost, transportation cost, and technology investment cost.

$$\begin{aligned} TP_{wt} &= \sum_{i=1}^y \left[Revenue_i - (TC_{i,setup} + TC_{i,man} + TC_{i,CO2} + TC_{i,holding} + TC_{i,backorder} + TC_{i,trans} + TC_{i,tech}) \right], \\ &= \sum_{i=1}^y \left[p_i (\Delta_i - \gamma_i p_i + \psi_i \alpha_i W_{i,Tech}) - \frac{\Delta_i - \gamma_i p_i + \psi_i \alpha_i W_{i,Tech}}{n_i q_i} \left(\left(b_i B_i + \frac{1}{8} g_i \left(\frac{4B_i^2}{\Delta_i - \gamma_i p_i + \psi_i \alpha_i W_{i,Tech}} \right. \right. \right. \right. \\ &\quad \left. \left. \left. + \frac{(q_i - 2B_i)^2}{P_i - (\Delta_i - \gamma_i p_i + \psi_i \alpha_i W_{i,Tech})} \right) \right) + \frac{1}{8} h_i \left(\frac{4(B_i - n_i q_i)^2}{\Delta_i - \gamma_i p_i + \psi_i \alpha_i W_{i,Tech}} + \frac{(q_i - 2B_i)^2}{P_i - (\Delta_i - \gamma_i p_i + \psi_i \alpha_i W_{i,Tech})} \right. \right. \\ &\quad \left. \left. - \frac{4(n_i - 1)n_i q_i^2}{P_i} \right) + n_i q_i \left(-\theta_i (c_{vi} - c_{ri}) + c_{vi} + P_i u_i + \frac{w_i}{P_i} \right) + k_i + \frac{\delta_i (n_i q_i)}{q_i} + n_i \xi_i q_i e^{-\rho_i \beta_i W_{i,Tech}} \right. \\ &\quad \left. - (\alpha_i + \beta_i) W_{i,tech} \right]. \end{aligned} \quad (16)$$

4.1. Constraint formulation

Building on the real-world intricacies of the vendor–buyer coordinated-decision making under sustainable retailing mechanism, the following set of functional constraints are considered.

$$(i) \quad \sum_{i=1}^y [z_i (Q_i - B_i)] \leq Z_r \in [\min Z_{r1}], \quad (17)$$

$$(ii) \quad p_i \leq \frac{A_i}{\gamma_i} \quad \forall i, (i = 1, 2, \dots, y), \quad (18)$$

$$(iii) \quad P_i \geq P_{i,min} \quad \forall i, (i = 1, 2, \dots, y), \quad (19)$$

$$(iv) \quad P_i \leq P_{i,max} \quad \forall i, (i = 1, 2, \dots, y), \quad (20)$$

$$(v) \quad q_i > 0 \quad \forall i, (i = 1, 2, \dots, y), \quad (21)$$

$$(vi) \quad T_i > 0 \quad \forall i, (i = 1, 2, \dots, y), \quad (22)$$

$$(vii) \quad W_{i,tech} \leq \epsilon W_{TECH}, \quad (23)$$

$$(viii) \quad \sum_{i=1}^y [(\alpha_i + \beta_i) W_{i,tech}] \leq W_{TECH}. \quad (24)$$

Constraint (17) ensures that the sum the batch sizes for all product types does not exceed the maximum storage capacity. Constraint (18) limits the product price for each item type- i , ($i = 1, 2, \dots, y$). Constraints (19) and (20) keeps the production rate P_i within the interval $\{P_{i,min} - P_{i,max}\}$, whereas the constraints (21) and (22) ensure that the shipment size and production cycle length for each item type- i , ($i = 1, 2, \dots, y$) is positive, respectively. Further, the constraint (23) ensures that the proportion of technology investment allocated to product type- i does not exceed ϵ %age of the total budget for technology investments in the system. Finally, the constraint (24) puts budgetary limit on the technology investments including GERT and IDT, where $(\alpha_i + \beta_i)W_{i,tech} = W_{i,tech}$ and $\alpha_i + \beta_i = 1$ for each item type- i , ($i = 1, 2, \dots, y$).

5. Solution methodology

In this section, we initially introduce an efficient hybrid solution approach to determine optimal solution set for Case I where we implement analytical optimization approach along with the heuristic method to determine the global maximum profit of the system. As the proposed objective function in this case is a complex mixed-integer non-linear (MINLP) problem, we present a solution algorithm for profit maximization of the system for product type- i .

We first utilize the below theorem to examine the concavity of the profit function for Case-I formulated in Eq. (15).

Theorem 1. *The profit function $T P_{i,wot}$ attains a maximum value if $X^t A X < 0$ in the interval $[0, t]$.*

Proof. There are four decision variables $[q_i, n_i, B_i, p_i] \in X_i[\cdot]_i^t$ in the profit function of Case I. Initially, to obtain the critical points, first order partial derivatives of the objective function with respect to each decision variable ($\frac{\partial T P_{i,wot}}{\partial X_i}$) should be individually set to zero. Eventually, to maximize the total profit, all principle minors must be negative, i.e. it must be presented that the four order Hessian Matrix is a negative definite. Below gradient equalities must hold to satisfy the necessary conditions of optimality.

$$\frac{\partial T P_{i,wot}}{\partial q_i} = 0, \quad \frac{\partial T P_{i,wot}}{\partial n_i} = 0, \quad \frac{\partial T P_{i,wot}}{\partial B_i} = 0, \quad \text{and} \quad \frac{\partial T P_{i,wot}}{\partial p_i} = 0.$$

Hence, from the necessary conditions for Case-I, we have

$$\begin{aligned} \frac{\partial T P_{i,wot}}{\partial q_i} &= \frac{1}{n_i q_i^2} \left[(\Delta_i - \gamma_i p_i) \left(b_i B_i + \frac{1}{8} g_i \left(\frac{4 B_i^2}{\Delta_i - \gamma_i p_i} + \frac{(q_i - 2 B_i)^2}{-\Delta_i + \gamma_i p_i + P_i} \right) + \frac{1}{8} h_i \left(\frac{4 (B_i - n_i q_i)^2}{\Delta_i - \gamma_i p_i} + \frac{(q_i - 2 B_i)^2}{-\Delta_i + \gamma_i p_i + P_i} \right) \right. \right. \\ &\quad \left. \left. - \frac{4(n_i - 1)n_i q_i^2}{P_i} \right) + n_i q_i \left(-\theta_i (c_{vi} - c_{ri}) + c_{vi} + P_i u_i + \frac{w_i}{P_i} \right) + k_i + \delta_i n_i + n_i \xi_i q_i \right] - \frac{1}{n_i q_i} \left[(\Delta_i - \gamma_i p_i) \right. \\ &\quad \times \left(\frac{g_i (q_i - 2 B_i)}{4(-\Delta_i + \gamma_i p_i + P_i)} + \frac{1}{8} h_i \left(-\frac{8 n_i (B_i - n_i q_i)}{\Delta_i - \gamma_i p_i} + \frac{2(q_i - 2 B_i)}{-\Delta_i + \gamma_i p_i + P_i} - \frac{8(n_i - 1)n_i q_i}{P_i} \right) \right. \\ &\quad \left. \left. + n_i \left(-\theta_i (c_{vi} - c_{ri}) + c_{vi} + P_i u_i + \frac{w_i}{P_i} \right) + n_i \xi_i \right) \right] = 0. \end{aligned} \quad (25)$$

$$\begin{aligned} \frac{\partial T P_{i,wot}}{\partial n_i} &= \frac{1}{n_i^2 q_i} \left[(\Delta_i - \gamma_i p_i) \left(b_i B_i + \frac{1}{8} g_i \left(\frac{4 B_i^2}{\Delta_i - \gamma_i p_i} + \frac{(q_i - 2 B_i)^2}{-\Delta_i + \gamma_i p_i + P_i} \right) + \frac{1}{8} h_i \left(\frac{4 (B_i - n_i q_i)^2}{\Delta_i - \gamma_i p_i} + \frac{(q_i - 2 B_i)^2}{-\Delta_i + \gamma_i p_i + P_i} \right) \right. \right. \\ &\quad \left. \left. - \frac{4(n_i - 1)n_i q_i^2}{P_i} \right) + n_i q_i \left(-\theta_i (c_{vi} - c_{ri}) + c_{vi} + P_i u_i + \frac{w_i}{P_i} \right) + k_i + \delta_i n_i + n_i \xi_i q_i \right] - \frac{1}{n_i q_i} \left[(\Delta_i - \gamma_i p_i) \right. \\ &\quad \times \left(\frac{1}{8} h_i \left(-\frac{8 q_i (B_i - n_i q_i)}{\Delta_i - \gamma_i p_i} - \frac{4(n_i - 1)q_i^2}{P_i} - \frac{4 n_i q_i^2}{P_i} \right) + q_i \left(-\theta_i (c_{vi} - c_{ri}) + c_{vi} + P_i u_i + \frac{w_i}{P_i} \right) \right. \\ &\quad \left. \left. + \delta_i + \xi_i q_i \right) \right] = 0. \end{aligned} \quad (26)$$

$$\frac{\partial T P_{i,wot}}{\partial B_i} = -\frac{1}{n_i q_i} \left[(\Delta_i - \gamma_i p_i) \left(b_i + \frac{1}{8} g_i \left(\frac{8 B_i}{\Delta_i - \gamma_i p_i} - \frac{4(q_i - 2 B_i)}{-\Delta_i + \gamma_i p_i + P_i} \right) + \frac{1}{8} h_i \left(\frac{8(B_i - n_i q_i)}{\Delta_i - \gamma_i p_i} - \frac{4(q_i - 2 B_i)}{-\Delta_i + \gamma_i p_i + P_i} \right) \right) \right] = 0. \quad (27)$$

$$\begin{aligned} \frac{\partial T P_{i,wot}}{\partial p_i} &= \frac{1}{n_i q_i} \left[\gamma_i \left(b_i B_i + \frac{1}{8} g_i \left(\frac{4 B_i^2}{\Delta_i - \gamma_i p_i} + \frac{(q_i - 2 B_i)^2}{-\Delta_i + \gamma_i p_i + P_i} \right) + \frac{1}{8} h_i \left(\frac{4(B_i - n_i q_i)^2}{\Delta_i - \gamma_i p_i} + \frac{(q_i - 2 B_i)^2}{-\Delta_i + \gamma_i p_i + P_i} \right. \right. \right. \\ &\quad \left. \left. - \frac{4(n_i - 1)n_i q_i^2}{P_i} \right) + n_i q_i \left(-\theta_i(c_{vi} - c_{ri}) + c_{vi} + P_i u_i + \frac{w_i}{P_i} \right) + k_i + \delta_i n_i + n_i \xi_i q_i \right] - \frac{1}{n_i q_i} \\ &\quad \times \left[(\Delta_i - \gamma_i p_i) \left(\frac{1}{8} g_i \left(\frac{4 B_i^2 \gamma_i}{(\Delta_i - \gamma_i p_i)^2} - \frac{\gamma_i (q_i - 2 B_i)^2}{(-\Delta_i + \gamma_i p_i + P_i)^2} \right) + \frac{1}{8} h_i \left(\frac{4 \gamma_i (B_i - n_i q_i)^2}{(\Delta_i - \gamma_i p_i)^2} - \frac{\gamma_i (q_i - 2 B_i)^2}{(-\Delta_i + \gamma_i p_i + P_i)^2} \right) \right] + \Delta_i - 2 \gamma_i p_i = 0. \end{aligned} \quad (28)$$

The partial derivatives presented in Eqs. (25)–(28) are highly complex non-linear equations. Therefore, for the simplification purpose, we use few abbreviations provided in Appendix A. From the simultaneous solution of Eqs. (25)–(28) and Appendix A, we determine below closed-form solutions for the decision variables.

$$n_i^* = \sqrt{\frac{P_i(\mu_i(8b_i B_i P_i + q_i(q_i - 4B_i)(g_i + h_i) + 8k_i P_i) - 8\mu_i^2(b_i B_i + k_i) + 4B_i^2 P_i(g_i + h_i))}{4h_i q_i^2 (P_i - \mu_i)^2}}, \quad (29)$$

$$q_i^* = \sqrt{\frac{4P_i(2b_i B_i \mu_i(P_i - \mu_i) + B_i^2 P_i(g_i + h_i) + 2\mu_i(P_i - \mu_i)(k_i + \delta_i n_i))}{g_i \mu_i P_i + h_i(4n_i^2(P_i - \mu_i)^2 + 4\mu_i n_i(P_i - \mu_i) + \mu_i P_i)}}, \quad (30)$$

$$B_i^* = \frac{\mu_i(2b_i(\mu_i - P_i) + g_i q_i) + h_i q_i(\mu_i + 2n_i(P_i - \mu_i))}{2P_i(g_i + h_i)}, \quad (31)$$

where $\mu_i = \Delta_i - \gamma_i p_i$, and

$$p_i^* = \frac{\gamma_i(\gamma_i \mathcal{L} - \mathcal{H})}{\left(6 \left(\sqrt{6\gamma_i^{15} (54\gamma_i \chi^2 \mathcal{H}^2 + (\gamma_i \mathcal{L} - \mathcal{H})^3)} + 18\gamma_i^8 \chi \mathcal{H} \right) \right)^{\frac{1}{3}}} - \frac{\left(\sqrt{6\gamma_i^{15} (54\gamma_i \chi^2 \mathcal{H}^2 + (\gamma_i \mathcal{L} - \mathcal{H})^3)} + 18\gamma_i^8 \chi \mathcal{H} \right)^{\frac{1}{3}}}{6^{2/3} \gamma_i^4} - \frac{\chi}{\gamma_i}. \quad (32)$$

For profit maximization of the base case scenario, we use Hessian Matrix approach to show that all the principle minors are negative definite. From the sufficient conditions of optimality, the Hessian matrix $\mathbf{H}_{T P_{i,wot}}$ is given by:

$$\mathbf{H}_{T P_{i,wot}} = \begin{vmatrix} \frac{\partial^2 T P_{i,wot}}{\partial q_i^2} & \frac{\partial^2 T P_{i,wot}}{\partial q_i \partial n_i} & \frac{\partial^2 T P_{i,wot}}{\partial q_i \partial B_i} & \frac{\partial^2 T P_{i,wot}}{\partial q_i \partial p_i} \\ \frac{\partial^2 T P_{i,wot}}{\partial n_i \partial q_i} & \frac{\partial^2 T P_{i,wot}}{\partial n_i^2} & \frac{\partial^2 T P_{i,wot}}{\partial n_i \partial B_i} & \frac{\partial^2 T P_{i,wot}}{\partial n_i \partial p_i} \\ \frac{\partial^2 T P_{i,wot}}{\partial B_i \partial q_i} & \frac{\partial^2 T P_{i,wot}}{\partial B_i \partial n_i} & \frac{\partial^2 T P_{i,wot}}{\partial B_i^2} & \frac{\partial^2 T P_{i,wot}}{\partial B_i \partial p_i} \\ \frac{\partial^2 T P_{i,wot}}{\partial p_i \partial q_i} & \frac{\partial^2 T P_{i,wot}}{\partial p_i \partial n_i} & \frac{\partial^2 T P_{i,wot}}{\partial p_i \partial B_i} & \frac{\partial^2 T P_{i,wot}}{\partial p_i^2} \end{vmatrix} < 0.$$

Appendix B presents the analysis of sufficient conditions for optimality, where we show that all four principle minors of the Hessian matrix $\mathbf{H}_{T P_{i,wot}}$ are negative definite.

Eqs. (29)–(32) present optimal values of the number of shipments (n_i), shipment size (q_i), backorder quantity (B_i), and market price (p_i). However, these do not guarantee integer solutions for (q_i) and (n_i). Further, an algorithm-supported approach is required to determine $P_i^* \in \{P_{i,min} - P_{i,max}\}$. Therefore, we provide below heuristic to obtain integer solutions for q_i and n_i , and optimal production rate P_i^* while maintaining concavity of the objective function (15) for product type- i .

Step I. Incorporate parametric information in objective function (15) and the optimal decision variable equations obtained through satisfying the necessary conditions of optimality in (29)–(32), and set the counter loop.

Step II. Start with the base production rate $P_o \in \{P_{i,min} - P_{i,max}\}$.

Step III. Put all the values of $P_i \in \{P_{i,min} - P_{i,max}\}$ with appropriate step size to determine $n_i^j \in J$, $q_i^j \in J$, $B_i^j \in J$, and $p_i^j \in J$ (where $\text{COUNT}[J] = \text{COUNT}[P_i]$), and objective function value as $T P_{i,wot}^1[P_i] \forall P_i \in \{P_{i,min} - P_{i,max}\}$.

Step IV. Ascertain optimal production rate P_i^* from the interval $\{P_{i,min} - P_{i,max}\}$ by determining maximum value of $T P_{i,wot}^1[P_i]$ as $T P_{i,wot} = \text{MAX}[T P_{i,wot}^1[P_i] \forall P_i \in \{P_{i,min} - P_{i,max}\}]$, and identify corresponding $n_i^{j*} \in J$, $q_i^{j*} \in J$, $B_i^{j*} \in J$, and $p_i^{j*} \in J$; $\forall j$.

Step V. Set $P_i^* = P_i$, $n_i^{j*} = n_i^*$, $q_i^{j*} = q_i^*$, $B_i^{j*} = B_i^*$, and $p_i^{j*} = p_i^*$ for further proceeding.

Step VI. Using optimal number of shipments n_i^* from Step V, determine $n_{1i} = \lfloor n_i^* \rfloor$ and $n_{2i} = \lfloor n_i^* \rfloor + 1$.

Step VII. Using optimal shipment size q_i^* from Step V, determine $q_{1i} = \lfloor q_i^* \rfloor$ and $q_{2i} = \lfloor q_i^* \rfloor + 1$.

Step VIII. Given the values of n_i^* and q_i^* from Step V and $\{n_{1i}, n_{2i}\}$ and $\{q_{1i}, q_{2i}\}$ from Step VI and Step VII, respectively; determine $Q_{1i}^1 = ([n_i^*] * [q_i^*])$, $Q_{2i}^1 = ([n_i^*] + 1) * ([q_i^*] + 1)$, $Q_{3i}^1 = ([n_i^*] + 1) * [q_i^*]$, and $Q_{4i}^1 = ([n_i^*] * [q_i^*] + 1)$.

Step IX. For all Q_{zi}^1 , $\forall z, z = (1, 2, 3, 4)$ obtained in Step VIII, determine optimal backorder quantity B_{zi}^1 and product selling price p_{zi}^1 , $\forall z, z = (1, 2, 3, 4)$ by replacing Q_i with Q_{zi}^1 in Eqs. (31) and (32), respectively.

Step X. Put $\{n_{i1}^*$ and $q_{i1}^*\} \in Q_{zi}^1, B_{zi}^1$, and $p_{zi}^1 \forall z, z = (1, 2, 3, 4)$ from Step IX in profit objective function (15) to determine estimated profit of the system for each instance $TP_{i,wot,z} \forall z, z = (1, 2, 3, 4)$ as $TP_{i,wot,z} [Q_{zi}^1, B_{zi}^1, p_{zi}^1] \forall z, z = (1, 2, 3, 4)$.

Step XI. Considering $TP_{i,wot,z} [Q_{zi}^1, B_{zi}^1, p_{zi}^1] \forall z, z = (1, 2, 3, 4)$ from Step X and Eq. (15), determine optimal profit of the system as $TP_{i,wot}^* = \text{MAX}[TP_{i,wot,z} [Q_{zi}^1, B_{zi}^1, p_{zi}^1] \forall z, z = (1, 2, 3, 4)]$. Finally, state $\{[Q_i^1]^*, B_i^{1*}, p_i^{1*}\} \in TP_{i,wot}^*$ as optimal solution of the decision variables, where shipment size (q_i) and number of shipments (n_i) are presented as positive integers.

The proposed model formulation in Case II presents a multi-item capacitated integrated production model with flexible production rate and technology investment opportunity for IDT and GERT. The profit maximization objective presented in is a complex non-linear constrained function that lies beyond the scope of analytical optimization methodologies. Hence considering the NP-Hard nature of the proposed model, we implement a metaheuristic approach to obtain optimal solution for the proposed model. Recent studies in this domain including Cárdenas-Barrón et al. [56], Sarkar et al. [57], Tayyab et al. [14], and Umar et al. [7] have showed the superiority of Genetic Algorithm (GA) in effectively handling such NP-Hard problems. Therefore, we implement a GA-based solution approach for the proposed integrated production management model.

The initial set of information necessary to begin GA for solving the proposed model is:

- (i) Population size denoted as N ,
- (ii) Crossover rate denoted as P_c , and
- (iii) Mutation rate denoted as P_m .

We follow below steps to solve the problem using GA approach in MATLAB optimization software.

Step I. Initialization.

Step I.I Parameter setting: Set initial values of the parameters for population size, crossover rate, and mutation rate to $N = 100$, $P_c = 0.80$, and $P_m = 0.80$, respectively. Specify *stochastic uniform* selection strategy, and *uniform* crossover and mutation operations. Set stopping criteria at 1800 generations as population size \times number of variable).

Step I.II Set *random initialization* of the population.

Step II. Fitness function evaluation. Repeat.

Step III. Selection of the individual solutions for the mating pool (N =mating pool size).

Step IV. Implying crossover for each consecutive pair with probability P_c .

Step V. Implying mutation for each subsequent generation with probability P_m .

Step VI. Substitution of the current population with the elicited pool.

Step VII. Continuous fitness function evaluation until the stopping criteria is met.

Step VIII. Record the fitness function and variable values at the stopping iteration.

6. Model implementation

In this section, we present an experimental illustration to exhibit the practical implementation of the proposed collaborative e-retailing model for a manufacturer and a retailer, analyze behavior of the model under several real world instances, and draw significant managerial insights.

6.1. Experimental analysis

Parametric information for the numerical experiment is adopted from Sadeghi et al. [8] and Kar et al. [39] with appropriate units and modifications to ensure the degree of precision and model convergence and implement the solution for three product types, $i \in y = \{1, 2, 3\}$. Table 2 presents input parameter values for numerical experiment of the proposed study. The aforementioned GA-supported solution algorithm has been applied with feasible lower and upper bounds on the variables using MATLAB R2020a optimization software package on a laptop computer with an 11th Gen Intel(R) Core(TM) i7-1165G7 @ 2.80 GHz 2.70 GHz Processor and 16.00 GB of RAM.

Table 2
Input data for numerical experiments.

Parameter	Value	Parameter	Value
y	3	c_{vi}	[50 60 40] \\$/item
Δ_i	[1000 1400 800] units/year	c_{ri}	[40 50 30] \\$/item
γ_i	[5 3 4] units	v_i	[90 90 90]
$P_{i,min}$	[1500 1800 1200] units/year	Z_r	1950 units ³
$P_{i,max}$	[2200 2500 1800] units/year	u_i	[0.02 0.01 0.02]
ψ_i	[0.03 0.015 0.02] units	k_i	[100 150 80] \\$/setup
η	0.40 percent	ρ_i	[0.012 0.021 0.009] units
g_i	[45 50 40] \\$/item/year	δ_i	[10 8 13] \\$/batch
h_i	[20 20 20] \\$/item/year	z_i	[4 4 4] units ³
θ_i	[0.25 0.30 0.20] percent	W_r	3000 (\$)

Table 3
Optimal solution for Case I.

Model type	Optimal solution		Model type	Optimal solution	
$n, q \notin \text{Int.}$	q^*	33.5	$n, q \in \text{Int.}$	q^*	27
	B^*	10.49		B^*	10.60
	p^*	134.46		p^*	134.60
	n^*	2.31		n^*	3
	TP_{wot}^*	20,749.75		TP_{wot}^*	20,742.60

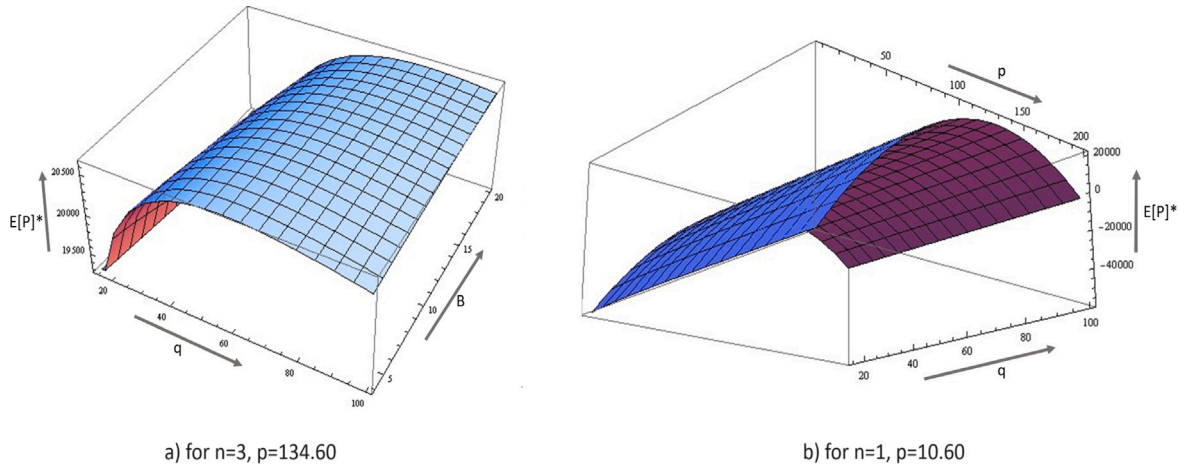


Fig. 4. Behavior of the objective function under variations in decision variables for Case I.

6.2. Computational outcomes and discussion

The proposed model Case I without technology investments for IDT and GERT is solved through hybrid methodology for product type-1 presented in Section 5.

Remark 1. A traditional production system with fixed production rate of $P_0 = 2000$ units per year generates a profit of \$19,133.8 per unit time with optimal values of the decision variables as $q^* = 36.68$ units, $n^* = 1.96$ shipments, $p^* = \$137.225$ per unit, and $B^* = 9.28$ units. However, for a smart production system, varying the production rate with appropriate step size within the interval $P \in \{P_{min} - P_{max}\}$ generates a superior solution at $P = 1500$ units per year. The optimal solution for this smart production system in this case is $q^* = 33.50$ units, $n^* = 2.31$ shipments, $p^* = \$134.46$ per unit, $B^* = 10.49$ units, and the total profit of \$ 20,749.75 per unit time. One can notice a 8.44% improvement in the system profit in this case by transforming a fixed system into a smart production system. Hence, we consider smart production systems with $P^* = 1500$ for making further analysis.

As indicated earlier, the analytical solution approach for the proposed smart integrated production management model does not guarantee integer solutions for the optimal shipment size (q^*) and number of shipments (n^*). Hence we apply further steps (Step VI – Step XI) of the proposed heuristic to determine integer solution of these variables in order to satisfy $q^* \in \text{Integers}$ and $n^* \in \text{Integers}$. Table 3 presents a comparative analysis of the integer and non-integer solutions. A reduction of \$7.15 per unit time in optimal profit of the system is observed to compensate the integer solution requirements in devising optimal shipment size and number of shipments.

Fig. 4 illustrates the concavity of profit maximization objective presented in Case 1 for different values of the decision variables as proved in Section 5 through analytical optimization technique.

The proposed multi-item constrained integrated model in Case II is solved through a GA-supported metaheuristic approach presented in Section 5. Table 4 presents optimal solution for the proposed model under capacity limitation and technology investments for advertisement (IDT) and emissions control (GERT), and Fig. 5 shows an instance of a GA-supported solution algorithm.

One can observe that the optimal solution for Case II presented in Table 4 develops a wise trade-off among technology investments for information disclosure technology and green emissions reduction technology while satisfying the capacity limitations.

Table 4
Optimal solution for Case II under capacity limitation and technology investments.

Item type	q_i^* (units)	B_i^* (units)	P_i^* (units per year)	$W_{i,tech}^*$ (\$)	α_i^* (%)	β_i^* (%)	p_i^* (\$ per unit)	n_i^* (number)	TP_{wt}^* (\$ per unit time)
1	55.59	63.50	1500	1200.00	0.84	0.16	136.47	5.28	\$ 144,757.29
2	31.04	0.00	1800	156.46	0.00	1.00	280.78	5.13	
3	37.07	14.66	1200	156.37	0.00	1.00	126.75	3.06	

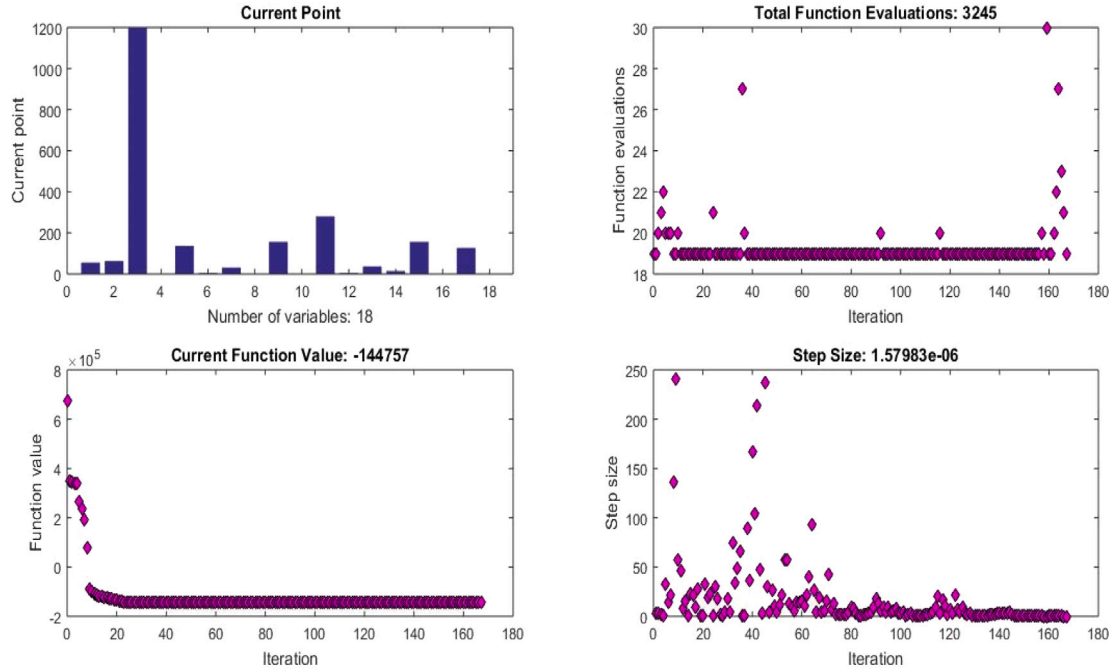


Fig. 5. Solution algorithm implementation for Case II on MATLAB.

Table 5
Optimal solution for Case II under capacity constraint without technology investments.

Item type	q_i^* (units)	B_i^* (units)	P_i^* (units per year)	$W_{i,tech}^*$ (\$)	α_i^* (%)	β_i^* (%)	p_i^* (\$ per unit)	n_i^* (number)	TP_{wt}^* (\$ per unit time)
1	20.19	8.33	1500	0	0	0	134.68	3.69	\$ 139,375.39
2	21.59	0	1800	0	0	0	284.55	4.44	
3	35.44	0	1200	0	0	0	127.89	1.54	

Remark 2. Technology investments IDT and GERT are considered in Case II of the proposed model under capacity constraints. Analysis of the model outcomes under capacity constraints and no technology investments scenario indicates an inferior solution as shown in Table 5. One can observe a decrease of \$5,381.90 in case of no investments for technology integration, as the model without technology investments produce a maximum profit of \$139,375.39 per unit time. Whereas, the model case with technology investments generates a profit of \$144,757.29 per unit of time. This substantiates a constructive account of technology investments for improvements in system profitability.

Remark 3. For a non-capacitated scenario under Case II with technology investments for IDT and GERT, the model generates a superior solution in comparison to the capacitated model with and without technology investments. Table 6 shows optimal solution for this case, where the model attains maximum profit of \$145,507.33. It is evident from the model solution of this case that a trade-off among capacity expansion cost and technology investment efficiency is crucial for economic advancement of the production system. The managers can identify a level of capacity expansion for which the following inequality holds.

$$\frac{\text{New revenue after capacity expansion} - \text{Capacity expansion cost} - \text{Additional technology investment}}{\text{Current revenue with no capacity expansion}} \geq 1, \quad (33)$$

where capacity expansion cost is calculated as units of increment in capacity times per unit increment cost.

We utilize Eq. (33) to identify optimal capacity level for Case II the proposed model under technology investment. Fig. 6 presents maximum capacity level of the system as 2805 units for which system profit attains maximum value of \$145,507.33. Any further incremental capacity under the current parametric information and technology investment budget is surplus for the system.

Table 6
Optimal solution for Case II under technology investments and no capacity limitation.

Item type	q_i^* (units)	B_i^* (units)	P_i^* (units per year)	$W_{i,tech}^*$ (\$)	α_i^* (%)	β_i^* (%)	p_i^* (\$ per unit)	n_i^* (number)	TP_{wt}^* (\$ per unit time)
1	55.59	63.5	1500	1200	0.84	0.16	136.47	5.28	\$ 145,507.33
2	44.71	43.34	1800	1200	0.86	0.14	283.89	8.56	
3	37.06	14.66	1200	156.37	0	1	126.75	3.06	

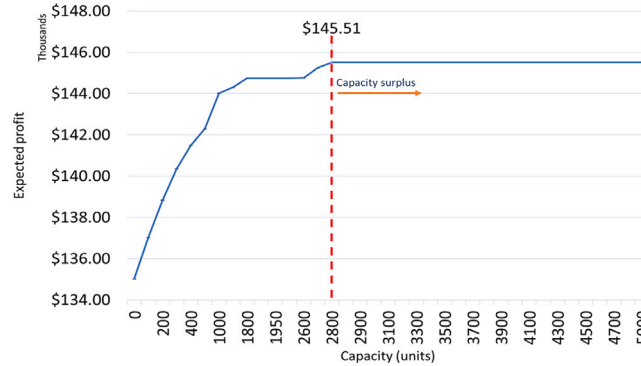


Fig. 6. Maximum capacity level for Case II under technology investment.

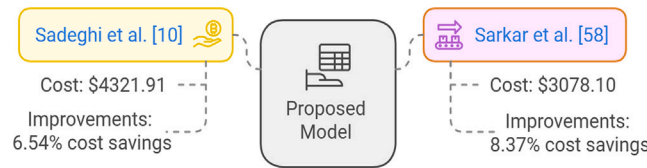


Fig. 7. Comparison of the proposed study with existing research in the literature.

6.3. Comparison with existing studies in the literature

In this section, we compare the numerical results of proposed model with the existing studies in the literature. If IDT and GERT investments, smart production system, multi-item production delivery mechanism, and green emissions reduction efforts-dependent demand are ignored, then our study reduces to Sadeghi et al. [8]. Our proposed model further reduces to Sarkar et al. [58] if multi-shipment policy and manufacturer-retailer collaborative decision making are also ignored.

Sadeghi et al. [8] proposed a mathematical model to optimize total cost of a single-manufacturer single-retailer supply chain by considering multi-shipment policy. However, they ignored technology investments to reduce environmental impact of the supply chain and improve product demand. Further, they considered traditional production system with constant production rate to produce a single-type of product with constant market demand. Their numerical results provided by their proposed model generated a minimum cost of \$4321.91. Our model provides a minimum cost of \$4039.26 with the similar data set and considering IDT and GERT investments for emissions reduction to fulfill the environmental awareness-based consumer demand. This provides a 6.54% cost savings, which illustrates the superiority of the proposed model. Moreover, Sarkar et al. [58] presented a single-stage production model with random defective rate by considering fully backordered shortages. However, they ignored the environmental impact and multi-shipment policy in their model. The numerical experiment of their model presented a minimum cost of \$3078.10 where the random defective rate follows a beta distribution. Our model provides a minimum cost of \$2820.16 for the similar parametric information and the consideration of green technology integration for emissions reduction. This demonstrates a cost saving of 8.37% in comparison to the production model provided by Sarkar et al. [58]. Fig. 7 illustrates the superiority of the proposed model in comparison to the existing studies in the literature.

6.4. Impact of technology investment for advertisement and emissions control

Case II of the proposed integrated production model considers technology investment for IDT and GERT. Fig. 8 shows the impact of allocated technology investment budget on overall profitability of the system following diminishing return on investment. We observe the variation in distribution of the technology investment budget among IDT and GERT by varying α_i , and among different product types by varying η_i within the interval $[0, 1]$ for Case II. Fig. 9 presents an interplay between α , β , and expected profit of the system and Fig. 10 illustrates the impact of variation in η_i on total profit of the system. One can observe that the variation in maximum allowed proportion (η_i) of technology budget to the product type- i has a direct impact on profitability of the system. As we reduce the limit of distribution of maximum available technology investment budget from $\eta_i = 80\%$ ($\forall i, i = (1, 2, 3)$), total profit of the system reduces. However, any such limitation above this value ($\eta_i \geq 80\%$) shows no improvement in the profit above \$146,369.13 per unit time. Therefore, the managers should pay close attention to the interplay between allocated investment budget amount and its distribution strategy among different aspects of the production system while focusing on system profitability.

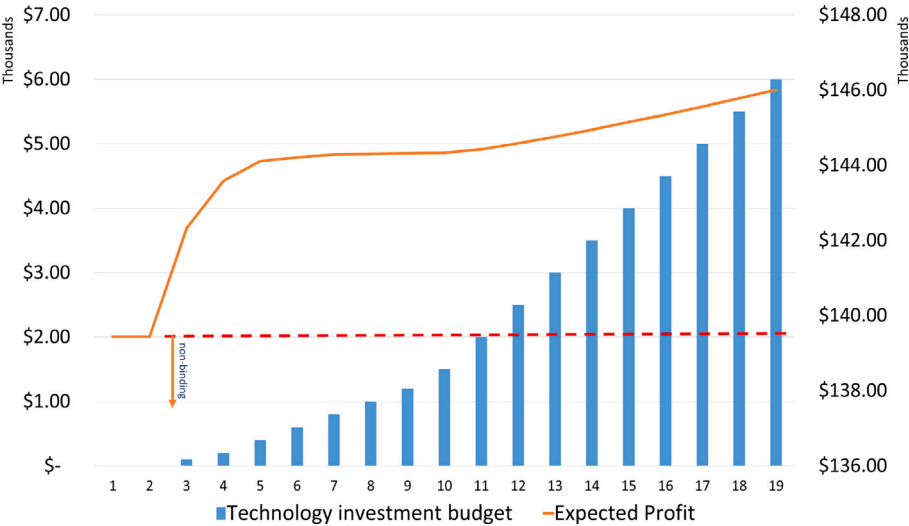


Fig. 8. Impact of technology investment budget on system profitability for Case II.

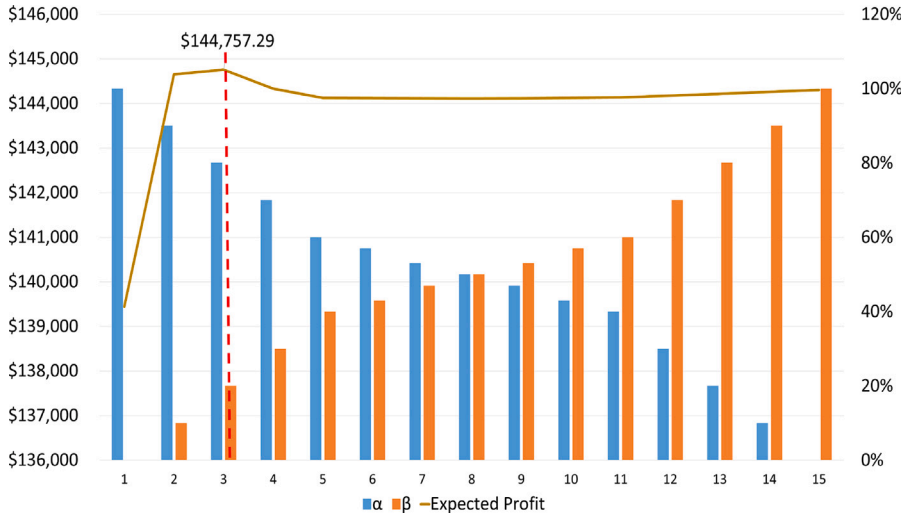


Fig. 9. Impact of technology investment budget distribution (α, β) on system profitability for Case II.

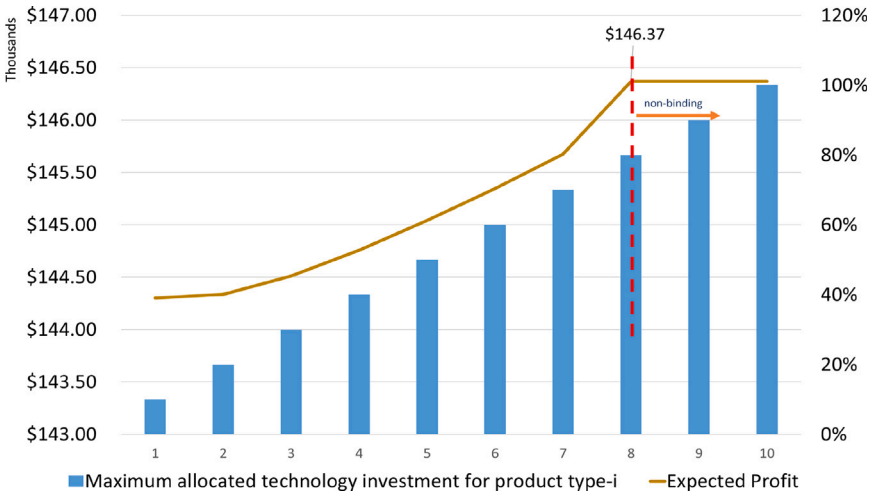


Fig. 10. Impact of technology investment budget distribution (η) on system profitability for Case II.

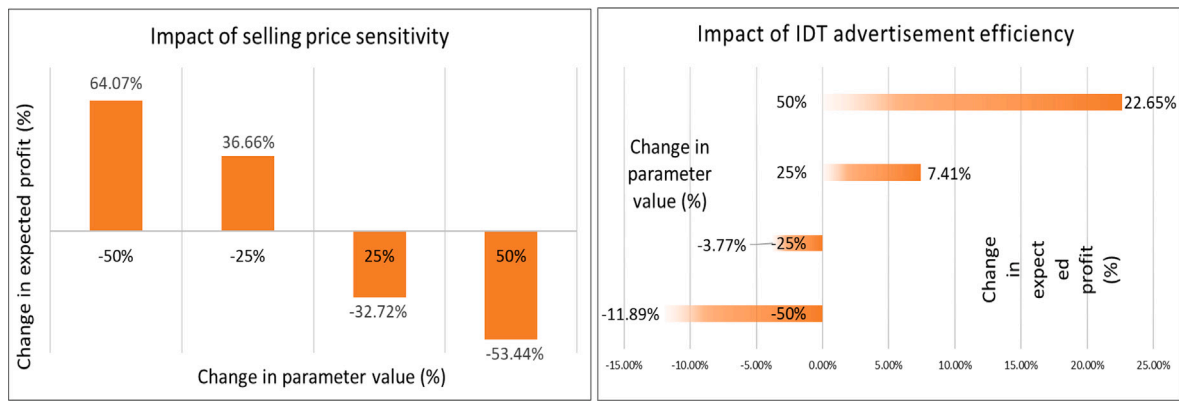


Fig. 11. Impact of price sensitivity and technology investment efficiency for advertisement on system profit for Case II.

Table 7

Comparison of model outcomes for SSSD and SSMD policy for Case I.

Production delivery policy	Optimal solution		Production delivery policy	Optimal solution	
SSSD policy	q^*	64.64	SSMD policy	q^*	27
	B^*	10.83		B^*	10.60
	p^*	135.00		p^*	134.60
	n^*	1		n^*	3
	TP_{tot}^*	20,648.58		TP_{tot}^*	20,742.60

6.5. Impact of advertisement efficiency

The level of advertising expenditure relies primarily on how effective a company's advertising efforts are in terms of reaching and influencing the target audience through the chosen advertising channels. Advertising firms define their target audience and then suggest spending plans for advertising Sarkar and Dey [4]. Therefore, managers of production systems should carefully examine the interaction between the incremental gains obtained from advertising activities across various channels. Managers may strategically choose the most beneficial channel by carefully evaluating the incremental benefits associated with each option and selecting the one with the most incremental benefit.

Remark 4. In the proposed model, we consider a pre-determined advertisement channel with advertisement efforts efficiencies of $\psi_i = 0.03$, $\psi_2 = 0.015$, and $\psi_3 = 0.02$ for product type 1, 2, and 3, respectively. Adoption of this advertisement policy as IDT generates a profit of \$144,757.29 per unit time, which is higher than the no-advertisement case presented in Remark 2 (with a profit of \$139,375.39 per unit time). Now we examine the impact of -50% to $+50\%$ variation in the advertisement efficiencies of all product types under this channel on expected profit of the proposed multi-item integrated system (see Fig. 11). It is observed that 25%, and 50% reduction in advertisement efficiency tends to reduce the system profit by 3.77% and 11.89%, respectively. However, 25%, and 50% increase in advertisement efficiency improves the profit of the system by 7.41% and 22.65%, respectively. This verifies the importance of putting efforts in improving the advertisement efficiency constants for the system.

6.6. Impact of product selling price sensitivity of consumer market

Selling price sensitivity parameter (γ_i) plays a crucial role in limiting the maximum selling price asked for the product type- i . The interaction between initial market size and the product selling price sensitivity of the consumer market (Δ_i/γ_i) puts an upper bound on maximum asking price of the product. Product selling price sensitivities for each product type in the proposed model are initially set at $\gamma_1 = 5$, $\gamma_2 = 3$, and $\gamma_3 = 4$; resulting in the per unit time profit of \$144,757.29. Making a 25%, and 50% simultaneous reduction in product price sensitivity of all the product types tend to improve the system profit by 36.66% and 64.07%, respectively. Whereas, 25%, and 50% increase in these reduce the profit by 32.72% and 53.44%, respectively (see Fig. 11).

6.7. Comparison with SSSD policy

The proposed integrated model suggests SSMD policy to make a trade-off between setup cost, inventory holding cost, and transportation cost. We implement SSSD policy for Case I and Case II by strictly limiting $n_i = 1$ and observe the variation in system profit (see Fig. 12). Tables 7 and 8 present a comparative analysis between SSSD and SSMD policy implementation for Case I and Case II, respectively. For a model with SSSD policy implementation, we can observe that the system profit reduces by \$94.02 for Case I and \$1535.83 per unit time for Case II. This analysis verifies the superiority of SSMD policy adoption over the prior one as supported by the recent research including Sarkar et al. [57], Sadeghi et al. [8], and Umar et al. [7].

Table 8
Comparison of model outcomes for SSSD and SSMD policy for Case II.

Production delivery policy	Item type	q_i^* (units)	B_i^* (units)	P_i^* (units per year)	$W_{i,tech}^*$ (\$)	α_i^* (%)	β_i^* (%)	p_i^* (\$ per unit)	n_i^* (number)	TP_{wt}^* (\$ per unit time)
SSSD policy	1	106.95	25.52	1500	154.92	0.00	1.00	133.39	1.00	\$ 143,221.46
	2	108.72	0.00	1800	138.33	0.00	1.00	281.99	1.00	
	3	85.07	13.47	1200	124.36	0.00	1.00	127.58	1.00	
SSMD policy	1	55.59	63.50	1500	1200.00	0.84	0.16	136.47	5.28	\$ 144,757.29
	2	31.04	0.00	1800	156.46	0.00	1.00	280.78	5.13	
	3	37.07	14.66	1200	156.37	0.00	1.00	126.75	3.06	

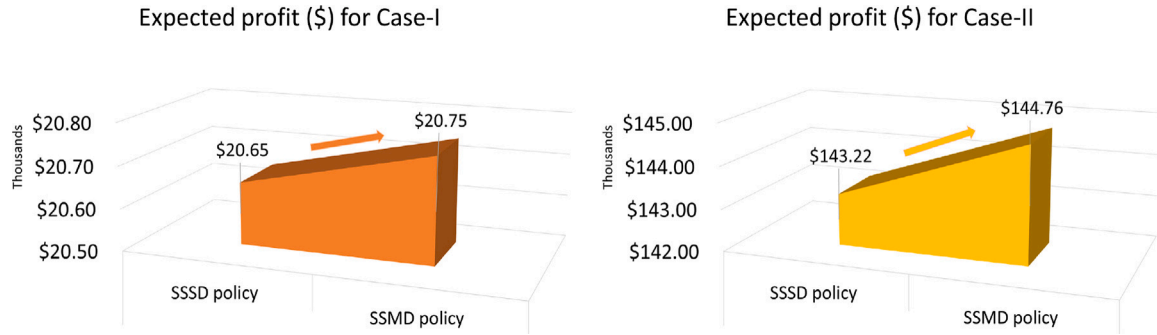


Fig. 12. Comparison of model outcomes for SSSD and SSMD policy adoption.

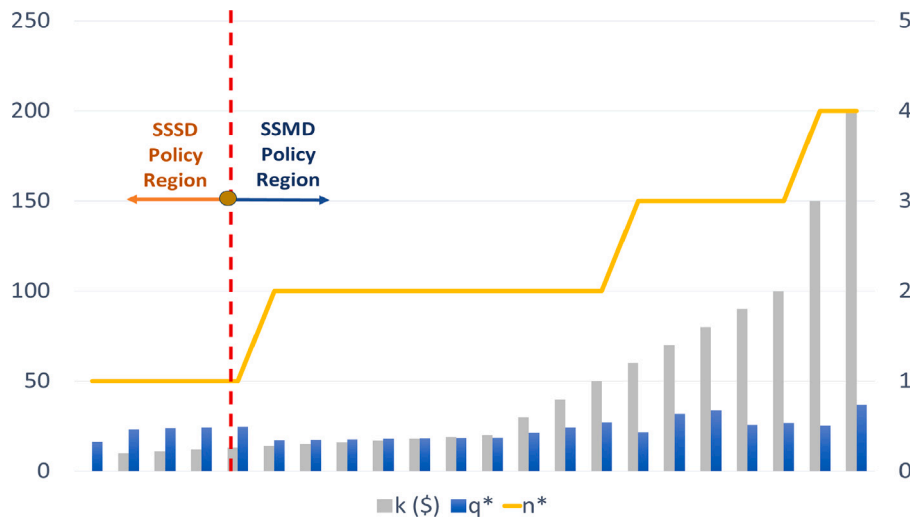


Fig. 13. Optimal production delivery policy suggestion for Case I.

6.7.1. Optimal production delivery policy suggestion

In this section, we analyze the impact of setup cost variation on the optimal production delivery policy suggestion for Case I. Fig. 13 illustrates the impact of setup cost on optimal policy. It is observed from this analysis that lower setup costs suggest SSSD as an optimal policy for the system. Whereas, a setup cost of \$13.08 is a critical point, above which SSMD policy becomes feasible. The step-wise incremental behavior of n^* -curve in Fig. 13 indicates optimal breakdown of the batch size into specific number of shipments that maximizes the system profit. This analysis is completely in line with the observations of Ben-Daya et al. [29], Sarkar et al. [57], and Sadeghi et al. [8] that lower setup costs tend to reduce the average batch size and associated inventory holding cost of the production system which eradicates the need of SSMD policy. However, as the setup cost of the system increase to the critical point, the batch size and related inventory holding cost increase and the model suggests shifting from SSSD production delivery policy to SSMD policy. Similar type of policy suggestions can be devised for other parameters of the system.

6.8. Sensitivity analysis

In this section, we examine the impact of variation in other key model parameters on total profit of the system for Case II. The parameter values are varied from -50% to $+50\%$ within equal intervals for each product type and the corresponding changes in total profit per unit time of the system are recorded. Table 9 shows percentage change in total profit of the system for corresponding percentage changes in the model parameters.

It is observed from the sensitivity analysis that any decrease in setup cost (k_i), variable backorder cost (b_i), inventory holding cost (h_i), carbon emissions cost (ξ_i), and batch shipment cost (δ_i) improves total profit of the system. Whereas, any decrease in percentage of returned products

Table 9
Sensitivity analysis of key model parameters.

Parameter(s)	Changes in parameter value	Changes in system profit	Parameter(s)	Changes in parameter value	Changes in system profit
k_i	-50%	+0.31%	ξ_i	-50%	+0.33%
	-25%	+0.15%		-25%	+0.13%
	+25%	-0.17%		+25%	-0.35%
	+50%	-0.35%		+50%	-0.76%
	-50%	+0.40%		-50%	+0.15%
b_i	-25%	+0.095%	δ_i	-25%	+0.087%
	+25%	-0.06%		+25%	-0.18%
	+50%	-0.11%		+50%	-0.26%
	-50%	+1.139%		-50%	-1.08%
	-25%	+0.56%		-25%	-0.54%
h_i	+25%	-0.49%	θ_i	+25%	+0.54%
	+50%	-0.91%		+50%	+1.08%

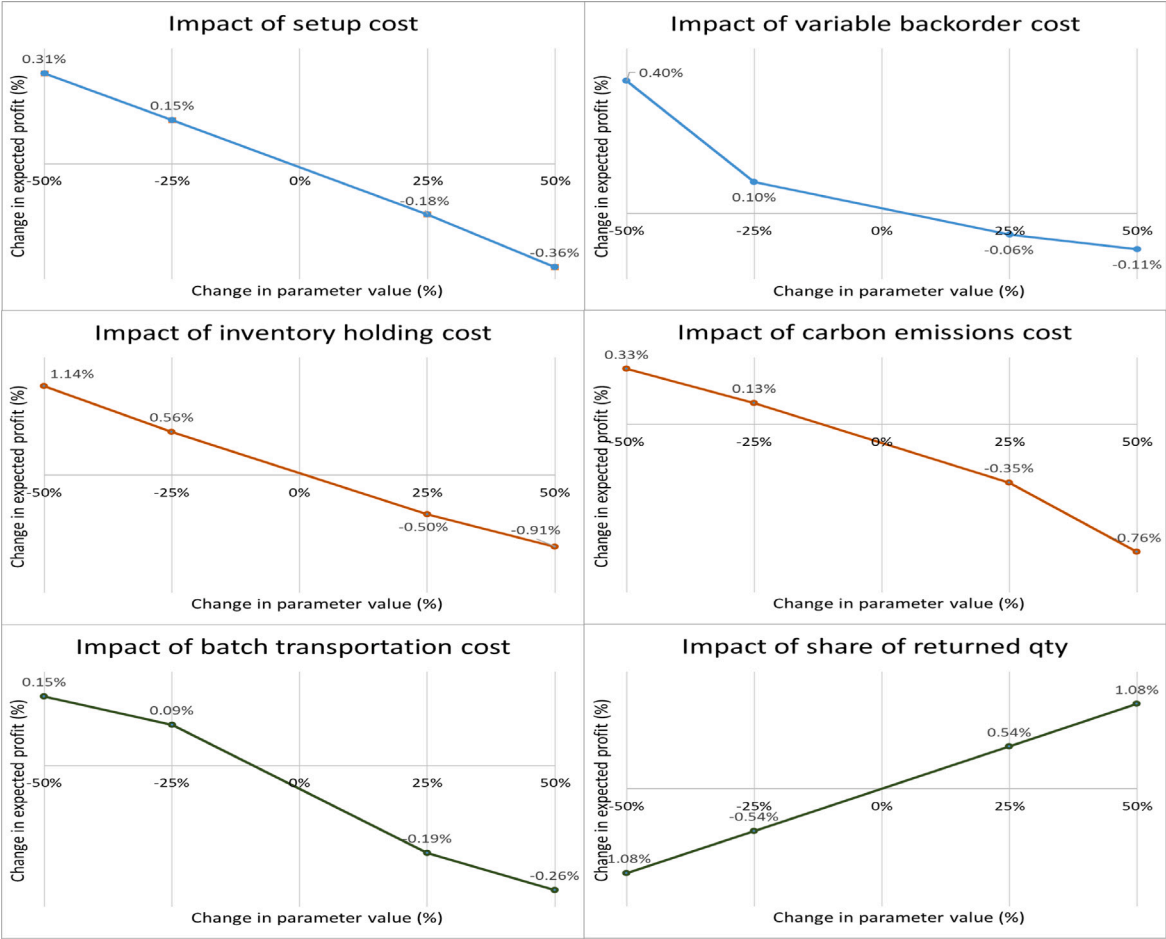


Fig. 14. Sensitivity analysis of key model parameters for Case II.

(θ_i) used in manufacturing reduces the total profit. One can see that the system profit is highly sensitive to h_i and θ_i , where 50% decrease in h_i improves the profit by 1.139% and the similar decrease in θ_i reduces the profit by 1.08%. On the other hand, 50% increase in h_i reduces the profit by 0.91% and the similar level of increment in θ_i improves the profit by 1.08%. Any reduction in k_i and ξ_i induces similar level of increment in the profit, where 25% decrease in these variables increases the profit by 0.15% and 0.13%, and 50% decrease in these variables increases the profit by 0.31% and 0.33%, respectively. However, any increase in values of these variables show different effects on the profit, where 25% increase in these variables reduce the profit by 0.17% and 0.35%, and 50% increase in these variables reduces the profit by 0.35% and 0.76%, respectively. Fig. 14 presents a graphical representation of the changes in profit for these variations in key model parameters.

6.9. Managerial insights

In this section, we devise significant managerial insights from the experimental outcomes and extensive analysis of the proposed integrated model that can support managers and decision makers in determining optimal production delivery policies for profit maximization under collaborative

e-retailing to improve consumer service.

Research outcomes of the proposed model illustrate that converting a rigid manufacturing system into a smart one, capable of modifying output rates within a defined range, results in a significant 8.44% augmentation in the system's profitability. Hence, the managers should contemplate using smart production systems, as seen in Case I, where modifying the production rate within the range $P \in \{P_{min} - P_{max}\}$ led to a more optimum result. Upon analyzing Case II, it becomes evident that information sharing and technological investments (IDT and GERT) have a significant and favorable effect on the product demand and profitability of the system. The model that incorporates technological investments generates a profit of \$144,757.29 per unit of time, whereas the profit without technology integration is \$139,375.39. It is advisable for managers to allocate resources towards incorporating technology into their operations, namely for the sake of information disclosure and emissions reduction. This investment will ultimately improve the overall profitability of the system.

This study verifies the significance of advertising effectiveness in impacting the profitability of the e-retailing system. A fluctuation in advertisement efficiency ranging from -50% to +50% directly correlates with fluctuations in system profit. Hence Managers should proactively focus on improving the efficiency constants of the advertising system in order to generate favorable effects on profitability. It is crucial to strike a balance between capacity and investment in technology for information disclosure. Analysis of the Case II in the proposed research emphasizes the significance of achieving a suitable equilibrium between capacity growth and technological investment in order to foster economic advancement. The most efficient approach, taking into account both limitations on capacity and expenditures in technology results in a maximum profit of \$145,507.33. Managers should thoroughly assess this equilibrium, taking into account the expenses associated with expanding capacity and the effectiveness of investing in technology, in order to attain economic advancement in the production system.

The research emphasizes the need of managers strategically distributing the monetary budget for technology investment between Information Disclosure Technology (IDT) and Green Emissions Reduction Technology (GERT). The correlation between the designated investment budget and its distribution across different components of the production system has a substantial impact on the overall profitability. Managers should prioritize increasing profitability by strategically allocating the technology budget among various product categories and technologies.

7. Conclusions

A growing trend towards online shopping and environmental awareness is transforming consumer choices as they are now placing a strong emphasis on sustainability and environmentally friendly products they purchase. In order to deal with these environmentally conscious commitments, it is essential for the e-retailing industry to have effective communication with the consumers regarding the green practices of their production and retailing channels. Information technology platforms such as blockchain and social media offer efficient platforms for individualized advertising that can be used to educate the customers about the emissions control efforts put by manufacturers of the said products. However, the effectiveness of this policy heavily relies on a collaboration among the supply chain players. The inventory management strategies like Vendor Managed Inventory (VMI) and single-setup-multi-delivery (SSMD) policy play a crucial part in improving production and delivery efficiency by guaranteeing timely and most economic availability of the product.

In the aforementioned context, this research introduces a comprehensive e-retailing model for a collaborative supply chain management between a manufacturer and a retailer. The model takes into account SSMD production delivery policy and technological investments for advertising and controlling greenhouse gas emissions. The manufacturer employs a smart manufacturing system and incorporates a predefined ratio of returned items into the production of new products. He makes technological investments in Green Emissions Reduction Technology (GERT) for the production system, while the retailer invests in Information Disclosure Technology (IDT) on social media to promote the system's efforts in controlling emissions through information sharing and increase customer demand for the product. Multiple model instances are developed and a hybrid analytical-heuristic method is utilized to derive optimal production delivery policy, investment distribution, and product price. The study examines the strategic allocation of technology budget between IDT and GERT, considering several scenarios of budget and stocking capacity limitations. The sensitivity analysis identifies critical parameters, and key recommendations are provided to enhance the economic viability of the system considering the fluctuations in these parameters. The acquired management insights offer decision-makers the essential focus to implement effective investment and production delivery policies in order to achieve environmental and economic sustainability of the coordinated system.

The research outcomes provide significant managerial insights for the decision-makers seeking to maximize profits through joint optimization of sustainable production delivery policies for a manufacturer and e-retailer. Model results verify that the adoption of a smart manufacturing system over a fixed one yields a substantial 8.44% rise in profitability of the system. Furthermore, significant increases in profitability are positively correlated with investments in Green Emissions Reduction Technology (GERT) and Information Disclosure Technology (IDT). This underscores the critical need for managers to allocate resources towards the smooth integration of smart manufacturing technology and information sharing into their production and retailing processes, respectively. Further, the results indicate that advertising efficiency plays a pivotal role in improving economic and environmental sustainability of the system. This insight suggests to take proactive measures in improving the efficiency constants of the advertising channel. Model analysis recognized that accomplishing a balanced approach between expanding production capacity and investing in technology is vital for fostering economic success in the manufacturer-retailer collaborative scheme. The policy suggestion analysis indicates identification and monitoring of critical points of the model parameters should be focused to determine the suitable production delivery policy.

While this study makes significant contributions in the literature, a few limitations may affect its practical applicability in certain scenarios. We assume constant cost components for model formulation and evaluation while ignoring inherent uncertainty in the costs. Additionally, we have considered the perfect production process by neglecting the possibility of defective products production due to the production process shifting to an *out-of-control* state [34] in several real-world situations. Furthermore, the proposed integrated model is limited to a single manufacturer and a single e-retailer, whereas several collaborations may involve multiple suppliers [10] or retailers [9] for enhanced economic and environmental sustainability, and robust optimization [59]. Therefore, to enhance the effectiveness of this study, potential research directions include developing an inspection schema [31], incorporating imperfect product production process [49], exploring integrated decisions with multi-manufacturers and multi-retailers [3], and addressing uncertainty issues in the model cost parameters [60] by implementing deep reinforcement learning [61], and fuzzy optimization [62] approaches. These extensions may contribute to a more comprehensive and adaptive manufacturer-retailer integrated model for online and offline sales.

CRediT authorship contribution statement

Muhammad Tayyab: Writing – review & editing, Writing – original draft, Software, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Hira Tahir:** Writing – review & editing, Visualization, Software, Data curation. **Muhammad Salman Habib:** Visualization, Supervision, Resources, Project administration, Methodology, Investigation, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

$$\begin{aligned}\Omega &= B_i h_i - \frac{B_i^2 g_i}{2n_i q_i} - \frac{B_i^2 h_i}{2n_i q_i} - \frac{h_i n_i q_i}{2}, \\ \tau_i &= -\theta_i (c_{vi} - c_{ri}) + c_{vi} + P_i u_i + \frac{w_i}{P_i} + \xi_i, \\ \sigma &= \frac{-b_i B_i}{n_i q_i} - \frac{k_i}{n_i q_i} - \frac{h_i q_i}{2P_i} + \frac{h_i n_i q_i}{2P_i} - \frac{\Delta_i}{q_i} - \tau_i, \\ \Gamma &= \frac{B_i g_i}{2n_i} + \frac{B_i h_i}{2n_i} - \frac{B_i^2 g_i}{2n_i q_i} - \frac{B_i^2 h_i}{2n_i q_i} - \frac{g_i q_i}{8n_i} - \frac{h_i q_i}{8n_i}, \\ F &= \gamma_i \Gamma, \\ \chi &= P_i - \Delta_i \\ \mathcal{G} &= \Gamma \Delta_i, \\ \mathcal{H} &= \mathcal{G} \gamma_i + F \gamma_i, \quad \text{and} \\ \mathcal{L} &= \mathcal{G} - F.\end{aligned}$$

Appendix B

First principle minor $|H_{11}|$ at the optimal values is

$$|H_{11}| = \frac{-2b_i B_i \mu_i}{n_i q_i^3} - \frac{B_i^2 P_i (g_i + h_i)}{n_i q_i^3 (P_i - \mu_i)} - \frac{2\mu_i (k_i + \delta_i n_i)}{n_i q_i^3} < 0.$$

Second principle minor $|H_{22}|$ at the optimal values is

$$|H_{22}| = \frac{1}{64n^4 P^2 q^4 (P - \mu)^2} \left[\begin{aligned} &8B^2 P^2 \left(24b^2 \mu^2 (P - \mu)^2 + 2g \left(4\mu(P - \mu)(-2b\mu q + 3kP + 2\delta nP) \right. \right. \\ &\quad \left. \left. + hq^2(3\mu P - 2n^2(P - \mu)^2) \right) + 8h\mu(P - \mu)(-2b\mu q + 3kP + 2\delta nP) \right. \\ &\quad \left. + 3g^2 \mu P q^2 + h^2 q^2(3\mu P - 4n^2(P - \mu)^2) \right) - 64B^3 \mu P^3 (g + h)(3b(\mu - P) \\ &\quad + q(g + h)) - 16B\mu P(P - \mu) \left(b \left(\mu P(-3gq^2 - 8(P - \mu)(3k + 2\delta n)) \right. \right. \\ &\quad \left. \left. + hq^2(4n^2(P - \mu)^2 - 3\mu P) \right) + 8\mu P q(g + h)(k + \delta n) \right) + 48B^4 P^4 (g + h)^2 \\ &\quad - 2h\mu P q^2 \left(gq^2(\mu P - 4n^2(P - \mu)^2) + 8k(P - \mu)(4n^2(P - \mu)^2 - 3\mu P) \right. \\ &\quad \left. + 16\delta \mu n P(\mu - P) \right) + \mu^2 P^2 \left(16k(P - \mu)(3gq^2 + 16\delta n(P - \mu)) - gq^2 \right. \\ &\quad \left. \left(gq^2 + 32\delta n(\mu - P) \right) + 192k^2(P - \mu)^2 \right) - h^2 q^4 (\mu P - 4n^2(P - \mu)^2)^2 \end{aligned} \right] < 0.$$

Third principle minor $|H_{33}|$ at the optimal values is

$$|(H_{33})| = -\frac{1}{64n^5 P q^5 \Omega^3} \left[\begin{aligned} & (g+h) \left(16P^2(4B\Gamma(2b\Omega - q(g+h)) - 4B^2P(g+h) + \Gamma(q^2(g+h) + 8k\Omega)) (2bB\Gamma\Omega - B^2P(g+h) + 2(k+\delta n)\Gamma\Omega) \right. \\ & - \left. (-8bBP\Gamma\Omega + 4B^2P^2(g+h) - P\Gamma(8k\Omega - gq^2) + hq^2(4n^2\Omega^2 + P\Gamma))^2 \right) + 4\left(\Gamma(q(g+h) - 2b\Omega) + 2BP(g+h) \right) \left(4P \right. \\ & \left. \left(\Gamma(q(g+h) - 2b\Omega) + 2BP(g+h) \right) (2bB\Gamma\Omega - B^2P(g+h) + 2(k+\delta n)\Gamma\Omega) - (BP(g+h) - b\Gamma\Omega) \left(8bBP\Gamma\Omega - 4B^2P^2 \right. \right. \\ & \left. \left. (g+h) + P\Gamma(8k\Omega - gq^2) - hq^2(4n^2\Omega^2 + P\Gamma) \right) \right) - 4\left(BP(g+h) - b\Gamma\Omega \right) \left(\left(\Gamma(q(g+h) - 2b\Omega) + 2BP(g+h) \right) \left(8bBP \right. \right. \\ & \left. \left. \Gamma\Omega - 4B^2P^2(g+h) + P\Gamma(8k\Omega - gq^2) - hq^2(4n^2\Omega^2 + P\Gamma) \right) - 4P(BP(g+h) - b\Gamma\Omega) \left(4B\Gamma(2b\Omega - q(g+h)) - 4B^2 \right. \right. \\ & \left. \left. P(g+h) + \Gamma(q^2(g+h) + 8k\Omega) \right) \right) \end{aligned} \right] < 0.$$

Fourth principle minor $|(H_{44})|$ at the optimal values is

$$|(H_{44})| = \frac{\gamma}{512n^6 P^2 q^6 \Omega^6} \left[\begin{aligned} & -\gamma \left(\frac{(g+h)(q-2B)(2B+q)P^2}{\Omega^2} - 8(bB+k+n\delta)P - 4h(n-1)nq^2 \right) \left(2 \left(4\Gamma\Omega b^2 + 4hq(-\Gamma)b + g^2q^2 + h^2q^2 + 8gkP \right. \right. \\ & \left. \left. + 8hkP + 2gq(2(-\Gamma)b + hq) \right) \Gamma\Omega \left((g+h)(4B^2 - q^2)P^2 + 4(h(n-1)nq^2 + 2P(bB+k+n\delta))\Omega^2 \right) - \left(4 \left(2B(g+h)P \right. \right. \right. \\ & \left. \left. + \Gamma((g+h)q - 2b\Omega) \right) (B(g+h)P - b\Gamma\Omega) - (g+h) \left(4B^2(g+h)P^2 - (8k\Omega - gq^2)\Gamma P - 8bB\Gamma\Omega P + hq^2(4n^2\Omega^2 + P \right. \right. \\ & \left. \left. \Gamma) \right) \right) \left((g+h)P^2(q-2B)^2 + 4hn^2q^2\Omega^2 + 8(bB+k)P\Omega^2 \right) + 2(2b\Omega^2 + 2B(g+h)P - (g+h)Pq) \left(4P \left(4(g+h)PB^2 \right. \right. \\ & \left. \left. + 4\Gamma((g+h)q - 2b\Omega)B - ((g+h)q^2 + 8k\Omega)\Gamma \right) (B(g+h)P - b\Gamma\Omega) - (2B(g+h)P + \Gamma((g+h)q - 2b\Omega)) \left(4B^2(g+h) \right. \right. \\ & \left. \left. P^2 - (8k\Omega - gq^2)\Gamma P - 8bB\Gamma\Omega P + hq^2(4n^2\Omega^2 + P\Gamma) \right) \right) \Omega^2 - \left(2P(8nq\Omega^3 + (g+h)P(q-2B)^2\gamma) \left(4 \left(2B(g+h)P \right. \right. \right. \\ & \left. \left. + \Gamma((g+h)q - 2b\Omega) \right) (B(g+h)P - b\Gamma\Omega) - (g+h) \left(4B^2(g+h)P^2 - (8k\Omega - gq^2)\Gamma P - 8bB\Gamma\Omega P + hq^2(4n^2\Omega^2 + \right. \right. \\ & \left. \left. P\Gamma) \right) \right) - \gamma \left(2(2B(g+h)P + \Gamma((g+h)q - 2b\Omega)) (2b\Omega^2 + 2B(g+h)P - (g+h)Pq) - (g+h) \left((g+h)P^2(q-2B)^2 \right. \right. \\ & \left. \left. + 4hn^2q^2\Omega^2 + 8(bB+k)P\Omega^2 \right) \right) \left((g+h)(4B^2 - q^2)P^2 + 4(h(n-1)nq^2 + 2P(bB+k+n\delta))\Omega^2 \right) + 2\gamma \left(2b\Omega^2 + 2B \right. \\ & \left. (g+h)P - (g+h)Pq \right) \left(4B^2(g+h)P^2 - (8k\Omega - gq^2)\Gamma P - 8bB\Gamma\Omega P + hq^2(4n^2\Omega^2 + P\Gamma) \right) \left(2b\Omega^2 + 2B(g+h)P - \right. \\ & \left. (g+h)Pq \right) - 2(B(g+h)P - b\Gamma\Omega) \left((g+h)P^2(q-2B)^2 + 4hn^2q^2\Omega^2 + 8(bB+k)P\Omega^2 \right) \left(-4B^2(g+h)P^2 + 8bB\Gamma \right. \\ & \left. \Omega P + \Gamma(8k\Omega - gq^2)P - hq^2(4n^2\Omega^2 + P\Gamma) \right) + 8P \left(4P \left(4\Gamma\Omega b^2 + 4hq(-\Gamma)b + g^2q^2 + h^2q^2 + 8gkP + 8hkP + 2gq \left(2(-\Gamma) \right. \right. \right. \\ & \left. \left. b + hq \right) \Gamma\Omega (8nq\Omega^3 + (g+h)P(q-2B)^2\gamma) - \gamma \left(2(2B(g+h)P + \Gamma((g+h)q - 2b\Omega)) \left(2b\Omega^2 + 2B(g+h)P - (g+h) \right. \right. \right. \\ & \left. \left. Pq \right) - (g+h) \left((g+h)P^2(q-2B)^2 + 4hn^2q^2\Omega^2 + 8(bB+k)P\Omega^2 \right) \right) \left((g+h)P^2(q-2B)^2 + 4hn^2q^2\Omega^2 + 8(bB+k) \right. \\ & \left. P\Omega^2 \right) + 2\gamma(2b\Omega^2 + 2B(g+h)P - (g+h)Pq) \left(2P(4(g+h)PB^2 + 4\Gamma((g+h)q - 2b\Omega)B - ((g+h)q^2 + 8k\Omega)\Gamma) \left(2b\Omega^2 \right. \right. \\ & \left. \left. + 2B(g+h)P - (g+h)Pq \right) - (2B(g+h)P + \Gamma((g+h)q - 2b\Omega)) \left((g+h)P^2(q-2B)^2 + 4hn^2q^2\Omega^2 + 8(bB+k)P \right. \right. \\ & \left. \left. \Omega^2 \right) \right) \left(2b\Gamma\Omega B - B^2(g+h)P + 2\Gamma(k+n\delta)\Omega \right) + 4(B(g+h)P - b\Gamma\Omega) \left(-2P(8nq\Omega^3 + (g+h)P(q-2B)^2\gamma) \left(4P \left(4 \left(g+h \right. \right. \right. \right. \\ & \left. \left. P B^2 + 4\Gamma((g+h)q - 2b\Omega)B - ((g+h)q^2 + 8k\Omega)\Gamma \right) (B(g+h)P - b\Gamma\Omega) - (2B(g+h)P + \Gamma((g+h)q - 2b\Omega)) \left(4 \right. \right. \\ & \left. \left. B^2(g+h)P^2 - (8k\Omega - gq^2)\Gamma P - 8bB\Gamma\Omega P + hq^2(4n^2\Omega^2 + P\Gamma) \right) \right) + \gamma \left((g+h)(4B^2 - q^2)P^2 + 4(h(n-1)nq^2 + 2P \right. \\ & \left. (bB+k+n\delta))\Omega^2 \right) \left(2P(4(g+h)PB^2 + 4\Gamma((g+h)q - 2b\Omega)B - ((g+h)q^2 + 8k\Omega)\Gamma) \left(2b\Omega^2 + 2B(g+h)P - (g+h) \right. \right. \\ & \left. \left. Pq \right) - (2B(g+h)P + \Gamma((g+h)q - 2b\Omega)) \left((g+h)P^2(q-2B)^2 + 4hn^2q^2\Omega^2 + 8(bB+k)P\Omega^2 \right) \right) - \gamma \left((g+h)P^2(q- \right. \\ & \left. 2B)^2 + 4hn^2q^2\Omega^2 + 8(bB+k)P\Omega^2 \right) \left(4B^2(g+h)P^2 - (8k\Omega - gq^2)\Gamma P - 8bB\Gamma\Omega P + hq^2(4n^2\Omega^2 + P\Gamma) \right) \left(2b\Omega^2 + 2B \right. \\ & \left. (g+h)P - (g+h)Pq \right) - 2(B(g+h)P - b\Gamma\Omega) \left((g+h)P^2(q-2B)^2 + 4hn^2q^2\Omega^2 + 8(bB+k)P\Omega^2 \right) \end{aligned} \right] < 0.$$

where

$$\Omega = (P + p\gamma - \Delta),$$

$$\Gamma = (p\gamma - \Delta).$$

Data availability

No data was used for the research described in the article.

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