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# **Article**

Optimising a closed-loop supply chain inventory system with product, material, and energy recoveries under different coordination structures

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# Optimising a closed-loop supply chain inventory system with product, material, and energy recoveries under different coordination structures

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#### ABSTRACT

The increasing recognition of environmental concerns and the adoption of Extended Producer Responsibility (EPR) have contributed significantly to the development of sustainable industries. Reverse logistics (RL) and closed-loop supply chain (CLSC) are two concepts that involve effective management of product returns to minimise consumer waste. In this paper, the authors develop a mathematical model for inventory management in CLSC systems with multiple recovery options, including product, material and energy recoveries. The model was developed based on a supply chain structure that includes a supplier, a manufacturer, a retailer, and a material recovery facility (MRF). The proposed model helps to maximise the profit of the supply chain. A hybrid method of analytical and numerical approaches is used to determine the optimal inventory decisions, including order cycle time and number of shipments between parties. Solution procedures are proposed for decentralised (DDMS) and centralised decision-making structures (CDMS). Furthermore, a profit-sharing mechanism is also analysed in the model. A sensitivity analysis is carried out to investigate the model's behaviour concerning variations in crucial parameters, including demand, product returns, recycling cost, post-consumer recycled content, and energy recoverable item rate. The results of this study show that the CDMS, without profit-sharing, generates the highest profits for the system. On the other hand, implementing a profit-sharing mechanism provides a fairer profit enhancement to the parties involved. Applying the energy recovery at the supplier results in financial benefits for the system. Additional discussion is carried out to understand the impact of energy recovery on the model's optimal solution.

### 1. Introduction

The rise of sustainability has led to significant transformations in supply chain management, compelling companies to re-evaluate their strategies and operations [1]. At its core, sustainability refers to the ability to uphold or preserve a process continuously over time without harming or diminishing resources [2]. It consists of three major areas: economic, social and environmental. Sustainability in the supply chains cannot be achieved effectively unless all the processes in the supply chain are integrated and synchronised at all stages of the supply chain [3,4]

One of the key sustainability strategies is using reverse logistics/RL [5]. RL is concerned with the management of used items, which are also called end-of-use (EoU) or end-of-life (EoL) products and is aimed at the recovery and redistribution of these products within the supply chain [6]. This process helps to decrease the reliance on the earth's resources

and the energy used in the process, supporting the formation of efficient and environmentally friendly supply chain systems [7]. RL is one of the key processes of a closed-loop supply chain (CLSC). CLSC involves RL activities, and these include the collection of used products or materials for recovery, such as through recycling, refurnishing, remanufacturing, or even reuse. Manufacturing companies, for example, DuPont Inc., have adopted RL, which added recycled polyethene into its Tyvek® products [8]. Also, Lenovo Group Ltd. has decided to use recycled plastics to manufacture tablets, all-in-one PCs, monitors, Laptops and accessories. Other industries have adopted RL practices like textile, automotive, printers, copiers, refrigerators, air conditioners, washing machines, tyres, and PCBs [9].

RL and CLSC are being recognised in many countries due to the recent policies on carbon emissions, extended producer responsibility, and waste management. In Indonesia, carbon emission is regulated by Law No 7 of 2021 concerning the Harmonisation of Tax Regulations

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(HPP). The tax rate is Rp 30 per kilogram of CO₂eq and above. The regulation aligns with the Paris Agreement, which states that global emissions will be cut by 45 % by 2030 and net zero by 2050. China's government offers subsidies for remanufactured goods and encourages eco-design standards. In addition, carbon pricing in Japan has included a tax for climate change mitigation, which was introduced in 2012. It is based on the carbon content applied to fossil fuels, which promotes energy efficiency and renewable energy sources.

RL and CLSC are gaining significance in energy-extensive industries, including the textiles and clothing (T&C) industry, which is the second most significant contributor to waste and emissions [10]. The T&C industry is one of the most energy-consuming sectors, contributing about  $10\,\%$  of global carbon emissions [11]. This has made manufacturers and fashion retailers like Zara, H&M, and UNIQLO integrate RL in their waste management systems for post-consumer products and product recovery, and thus reduce their environmental impacts [12–15].

Advancements in product recovery technologies create additional opportunities for RL and CLSC. Regarding the product recovery options, some products, such as garments, electronics, batteries, and cooler products, have unique characteristics that make waste recovery possible in any of the following ways [16]. Besides product and material recovery, used clothing products can be reused as sustainable energy through energy recovery [17]. This enables firms to regain value from the products that cannot be recycled. Energy recovery is the process of converting waste into energy in the form of heat, electricity or fuel through methods such as incineration, gasification or pyrolysis, hence decreasing the dependence on fossil fuel and minimising emissions [18-21]. All these have helped reduce the environmental impacts and the dependence on fossil fuels. For instance, the T&C industry in Japan manages 7.35 million tons of textile waste for RPF, which helps to avoid 1.63 million tons of CO2eq and has better combustion properties than coal [22]. This technology is used mainly in highly populated areas where there is not enough land to create landfills, for instance, in Japan, Europe, and several developing Southeast Asia countries, including Indonesia and Thailand [23,24].

Energy recovery can be incorporated into supply chains to produce thermal energy in a sustainable way. It enables companies to find substitutes for conventional fuels [25]. For instance, a study conducted by [25] indicated that the energy potential of cotton briquettes from textile waste can cut down energy costs by 80 % compared to fuel oil. Moreover, energy recovery makes it possible to implement closed-loop recycling, which enhances the economic viability of manufacturers and recyclers [26,27].

Uncertainty concerning the quantity of product returns, technological accessibility, and operational efficiency in the CLSC system are just a few of the challenges that energy recovery faces upon implementation. There is an urge to study how integrating energy recovery in the CLSC system can improve the economic and ecological performance of the overall supply chain. In supply chain management, inventory models and optimisation are among the key areas that many researchers have focused on in order to study the role of RL and CLSC. The majority of inventory models in the literature primarily focus on product recovery without addressing the integration of multiple recovery alternatives, including energy recovery, on CLSC performance [28]. To fill this gap, this study develops an inventory model for a four-echelon CLSC system, incorporating product refurbishing, material recycling, and energy recovery. The proposed model also examines the effectiveness of coordistructures—decentralisation, nation centralisation, profit-sharing-in enhancing financial performance. Specifically, this study addresses the following research questions:

(1) How can optimal inventory decisions, such as replenishment cycles and shipment decisions, in a CLSC incorporating energy recovery maximise overall system profits?

- (2) How do different coordination structures—centralised, decentralised, and profit-sharing—impact the financial performance of a CLSC under various operational conditions?
- (3) How does energy recovery—along with recycling and refurbishing— impact optimal inventory decisions and benefit the overall performance of the CLSC?

The remainder of this paper is structured as follows: Section 2 reviews relevant literature, Section 3 presents the problem description, and Section 4 develops the mathematical model. Section 5 outlines the solution procedure, Section 6 discusses numerical results, and Section 7 provides sensitivity analysis. Finally, Section 8 concludes the study and discusses some insights and implications for practice.

#### 2. Literature review

This section briefly reviews several CLSC inventory models as a fundamental support for the paper. The authors construct the review based on three relevant streams of literature, which are (1) inventory models in RL and CLSC systems with multi-recovery processes, (2) inventory models with CLSC coordination, and (3) environmental investigation to control emissions in RL and CLSC. Each stream is discussed briefly in the following sections.

#### 2.1. Inventory models with multiple recovery processes

Reverse logistics plays a crucial role in the management of CLSC [29]. It not only focuses on the collection of product returns but also on finding ways to recover value from these items rather than simply disposing of them [30,31]. In addition to the available technologies, recovery options typically depend on the characteristics of returned products and materials. For example, waste from textiles and clothing products can be recycled, refurbished, or subjected to energy recovery [17]. Another example is waste from used tyres that can be reprocessed by material recycling and energy recovery [32]. Furthermore, electronic waste (e-waste) can be recovered by several options, including reuse, refurbishing, remanufacturing, material recycling, and energy recovery [331].

In addition to product characteristics, the heterogeneous quality of product returns is another reason the recovery system should be designed to manage multi-recovery processes. Several studies have incorporated multiple recovery processes in inventory and supply chain modelling. Konstantaras [34] developed an inventory model that considers inspection and sorting for two recovery processes: remanufacturing and refurbishing. In the model, remanufactured products are assumed to have the same quality level as new products used to meet customer demand. Meanwhile, refurbished products will be of lower quality and sold on the secondary market. Benkherouf et al. [35] developed the model of Konstantaras [34] by considering time-varying demand and finite planning horizons. The model evaluated two types of policies regarding the number of batches of inspection and sorting, recovery, and new items.

Another relevant study was done by Tahirov et al. [36] who developed a model with disassembly, remanufacturing, and recycling. The objective is to evaluate which strategy—pure remanufacturing, pure production, or mixed strategy—is more applicable. Masoudipour et al. [37] Masoudipour et al. [37] developed a CLSC model comprising single-manufacturer and distribution centres. The model incorporated a segmentation policy based on conditional quality of product return. Their idea came from an Iranian textile company that integrated three quality-based recovery options: repair, remanufacturing, and recycling into a secondary supply chain. Similarly, Moshtagh and Taleizadeh [38] developed a multi-period-multi-echelon sustainable closed-loop supply chains (SSCs) model with two recovery alternatives, e.g. remanufacturing and recycling, under quality-dependent discount for returned products. They build the multi-objective optimisation model based on

Pareto principles, which includes cost, environmental, and social objective functions. The model is applied to a case of CFL light bulb supply chain. Later on, Devoto et al. [39] proposed an inventory model for managing heterogeneous returns, focusing on the inspection, sorting, and remanufacturing processes. Their model helps minimise costs by determining when and how much to inspect, remanufacture, or dispose of, ensuring that resources are used as efficiently as possible. The incoming and inspected-and-graded returns, as well as serviceable products, are held in separate inventories.

The need for an SC model considering multiple recovery processes urges along with the development of recycling technologies. Converting waste from consumer products into sustainable energy, also referred to as energy/thermal recovery, has become a potential alternative to address the ongoing energy crisis and reduce environmental waste [40]. Several industries have started to take action on the application of waste-to-energy (WTE), including cement [41], electronics [42], textiles [43], and plastics [44]. In Japan, energy recovery has been adopted by industries like textiles, plastics, paper, and batteries [22]. Semba et al. [22] specifically highlighted how thermal recycling of  $7.35 \times 10^7$  kg of consumer clothing waste into refuse paper and plastic fuel (RPF) resulted in a significant reduction of GHG emissions by  $1.63 \times 10^8$  kg CO<sub>2</sub>e. This approach has become more common in industries such as pulp and paper, textiles, and cement, where waste is converted into RPF and RDF (refuse-derived fuel) to produce electricity and other energy-intensive products [45-48].

The above literature reviews underscore the growing need for supply chain models that not only consider multiple recovery processes but also incorporate energy recovery into the CLSC system. As recycling technologies advance, integrating energy recovery into supply chains is a potential way for industries to reduce environmental impact while creating economic and operational value.

### 2.2. Inventory models with CLSC coordination

Chung et al. [49] developed a CLSC inventory model to maximise joint total profits between the retailer, the manufacturer, the supplier, and the third-party collector. Yuan and Gao [50] extended this work by incorporating multiple production and recovery cycles into their model. Meanwhile, Mitra [51] introduced a CLSC model with two main parties—the depot and the distributor—focusing on minimising setup, holding, and shortage costs under deterministic and stochastic demand and return conditions. Mitra further extended this model by considering situations where demand and returns are correlated [52].

In a more recent development, Parsa et al. [53] proposed a Joint Economic Lot Size (JELS) model for a five-echelon CLSC, including a retailer, manufacturer, supplier, material recovery facility (MRF), and recycling facility. The model's goal was to maximise the total profit of the CLSC system, considering two profit-sharing mechanisms: disproportionate profit sharing (DPS) and proportionate profit sharing (PPS). Their model also addressed price-sensitive demand and quantity discounts while determining the optimal fraction of recycled material in new products (i.e., post-consumer recycled content). Several other vital contributions to CLSC inventory modelling have been made by Jaber et al. [54], Gu and Tagaras [55], Giri and Sharma [56], Cobb [57], Bazan et al. [58], and Dwicahyani et al. [59], all focusing on various facets of inventory optimisation in closed-loop systems.

Another development was done by Jauhari [60] who developed a sustainable inventory model for a CLSC with energy usage, imperfect production, and green investment. He proposed an inventory model for a manufacturer-multi-retailer system with an adjustable production rate, which accounted for energy use, rework, and carbon emissions. The manufacturer invests in green production technologies to reduce emissions and meet carbon tax regulations. They found that an effective green investment can reduce emissions and increase profits on used products. Giri and Dash [61] developed a single-manufacturer-single-retailer inventory model for an imperfect

production system under demand sensitive to the product's price, advertisement, and greening level. They evaluated a cost-sharing contract between the manufacturer and the retailer and found that the contract significantly enhances the environmental and financial performance of the supply chain.

Mondal and Giri [62] developed a model in which consumer demand is influenced by price, quality, and product marketing factors, employing various payment methods and cost-sharing agreements. Dash et al. [63] explored how demand responds to price and sales efforts in a two-level supply chain. The study focused on an imperfect production system that implemented a trade credit policy and wholesale price discount contract. Herbon and David [64] examined the integration of inventory supply chains and analysed the manufacturer's optimal cost-sharing ratio, finding that the shipment cost share played a key role in controlling total costs. Jauhari et al. [65] developed a single-manufacturer-single-retailer CLSC model and compared three scenarios: centralised, decentralised, coordination manufacturer-led Stackelberg game. Their study suggested that green investments, promotional efforts, and technology licensing could significantly enhance both financial and environmental performance in the CLSC. Moreover, they concluded that centralised coordination is the most profitable approach compared to decentralised manufacturer-led models.

#### 2.3. Inventory models with environmental investigation

The further development of the RL and CLSC inventory models is the investigation of environmental impacts and carbon emissions, which are analysed through various policies [66]. This is driven by the widespread implementation of greenhouse gas (GHG) emission regulations in various countries, such as carbon tax policies in Finland, Sweden, Poland, Switzerland, Canada, Singapore, Japan, and Indonesia, and carbon cap-and-trade policies in South Korea, Australia, New Zealand, Kazakhstan, Switzerland, and several EU countries. Bonney and Jaber [67] argue that environmental costs, including GHG emissions, energy consumption, and landfill disposal, cannot be ignored when developing RL and CLSC models. Consequently, environmental impact analysis has become a key consideration in these models.

Bazan et al. [68] studied the impacts of environmental investigation on manufacturing-remanufacturing inventory models under a limited number of remanufacturing generations. The energy used from manufacturing and remanufacturing operations and the greenhouse gas emissions (GHG) from manufacturing, remanufacturing, and transportation activities were analysed. They considered emission penalty tax as a carbon regulatory mechanism. Furthermore, Bazan et al. [69] studied how the energy used in production, GHG emission under penalty tax policy, and a limited number of remanufacturing generations affect the optimal decisions of the CLSC model under two different coordination mechanisms, i.e. classical and VMI-CS coordination.

Dwicahyani et al. [59] developed a model of a CLSC system comprising a depot and a distributor and analysed the effect of carbon tax regulation and energy usage on inventory decisions. Konstantaras et al. [70] studied the impact of carbon tax regulation in a supply chain system integrating manufacturing, remanufacturing, and repair activities. Jauhari et al. [71] developed a CLSC model with stochastic demand, hybrid production, carbon emissions, and take-back incentives. Carbon taxes were applied for the green and regular production operations, which performed manufacturing and remanufacturing processes.

Golpîra and Javanmardan [72] developed a sustainable CLSC model that considers various carbon emission schemes (CESs), including carbon caps, carbon tax, and carbon cap-and-trade. Their study involved a supply chain structure with a supplier, manufacturer, distribution centre, collection centre, disposal centre, and recycling centre. Using Conditional Value at Risk (CVaR) as a risk measure under demand uncertainty, they found that the carbon cap-and-trade scheme provided the best cost performance while maintaining appropriate levels of

robustness. Integrating carbon emission analysis is essential in both the RL and CLSC models as it strongly corresponds to green, environmental friendliness, and sustainability principles. Therefore, it becomes relevant to acknowledge and integrate the aspect into the model.

#### 2.4. Research contributions

The proposed model develops a four-echelon closed-loop supply chain (CLSC) inventory system. The CLSC structure consists of a retailer, a manufacturer, a supplier, and a material recovery facility (MRF). The authors incorporate multiple recovery options into the model, including product refurbishing, material recycling, and energy recovery. Instead of just disposing of the non-recoverable used items as waste, the model incorporates the waste-to-energy option. The energy generated from non-recoverable used items is utilised by the supplier to produce raw materials. To better illustrate the distinctions between our model and other relevant models, the authors provide a comparative table that highlights the differences in terms of the SC structure, recovery alternatives, objective function, decision variables, and environmental investigation, which is presented in Table 1.

This study extends the CLSC inventory model developed by Parsa et al. [53] by incorporating product refurbishing, material recycling, and energy recovery. The authors consider different markets for the newly produced and refurbished products. The proposed model illustrates how quality-based recovery alternatives benefit the entire CLSC system. Energy recovery is incorporated into the model as an alternative to waste management instead of landfill waste disposal. As part of the energy recovery, the supplier utilises energy from waste as an alternative energy source in the production process of virgin/pure raw materials. An environmental investigation under carbon tax policy is applied to control the emissions generated from production and transportation activities. This study proposes two decision-making (DM) approaches, i. e., decentralised DM and centralised DM. The decentralised DM structure is based on a retailer-led supply chain, where the optimisation is conducted sequentially, starting from the retailer, manufacturer, supplier, and MRF. Meanwhile, the centralised structure optimises the CLSC problem simultaneously under joint optimisation. A profit-sharing mechanism is evaluated for the centralised problem to illustrate how each party in the CLSC system can benefit from the joint optimisation. The following section will discuss the problem descriptions, notations, and assumptions used to develop the model.

# 3. Problem descriptions, notations, and assumptions

### 3.1. Problem descriptions

The authors consider an inventory model of a retailer-led CLSC system consisting of a supplier, a manufacturer, a retailer, and an MRF. Fig. 1 depicts the material flow in the investigated CLSC system.

The flow of materials and products, as depicted in Fig. 1, begins with the supplier delivering pure/virgin raw materials to the manufacturer in l shipments. The manufacturer processes these materials into finished products and then delivers them to the retailer, with shipments occurring n times during the retailer's order cycle of  $T_r$ . The retailer collectes returned items from primary market customers and delivers these returned items to the Material Recovery Facility (MRF) in m shipments. The MRF then classifies the items into three quality categories, including:

- (1) recoverable product, comprising a portion of f,
- (2) recoverable material, comprising a portion of  $\alpha$ , and
- (3) recoverable energy, comprising the remaining portion  $(1 \alpha f)$ .

Returned items with the highest quality level, with a portion of f, will be classified as recoverable and will go through a refurbishing process.

The MRF will then sell the refurbished products to the secondary market at a lower price. The medium-quality items will be classified into the recoverable material category and reprocessed as recycled materials. The MRF delivers the recycled materials to the manufacturer as a substitute for virgin/pure/virgin raw materials. Delivery is done in k shipments. The proposed model introduces an energy recovery mechanism for items of the lowest quality. The MRF sends these energy-recoverable items to the supplier in i deliveries, which are used as an alternative energy source in the supplier's production process. The material flow in the investigated system forms a closed-loop recycling starting from the supplier, the manufacturer, the retailer, and the MRF.

Fig. 2 depicts the inventory profile of the CLSC system. The manufacturer holds the pure/virgin and recycled materials, supplied by the supplier and the MRF, in different inventories with a unit holding cost of  $h_{mp}$  and  $h_{mr}$  per year, respectively. Both are used in the manufacturer production process with a production rate of P, setup cost of  $A_m$  per run, and unit production cost of  $C_p$ . The manufacturer incorporates a mix of pure/virgin and recycled materials in producing new products, with recycled materials comprising X% and virgin materials making up (1-X)%. The X percentage denotes the post-consumer recycled content level, which is calculated from the mass proportion of the recycled materials to the mass of the product [53]. The finished products are sold to the retailer at a unit price of  $P_r$  and will be hold at the retailer's serviceable inventory with a unit holding cost of  $h_r$  per year to meet customer demands with a rate of D units per year.

The reverse flow begins by the retailer who collects used items from customers at a rate of R units per year, with customers receiving incentives of  $P_R$  per unit. The collected used items are hold at the retailer's collected used item inventory with a unit holding cost of  $h_{ru}$  per year. It will then be delivered to the MRF in m shipments and stored at the MRF's used items inventory with a unit holding cost of  $h_U$  per year. The MRF conducts refurbishing process to the f portion of returned items, with a refurbishment cost of  $C_f$  per unit and a unit holding cost of  $h_f$  per year. In addition, the MRF also performs recycling process to the  $\alpha$  portion of returned items, with a recycling cost of  $C_R$  per unit and a unit holding cost of  $h_M$  per year. The remaining returned items  $(1 - \alpha - f)$ , will be delivered to the supplier for energy recovery process. The supplier conducts energy recovery at a rate of  $E_n$ , incurring a unit cost of  $C_E$  and unit holding cost of  $C_E$  per year.

# 3.2. Notations

The authors employ the following notations to formulate the model. Parameters to the retailer:

Symbol	Description
D	Annual customer demands
$R_u$	Annual product returns
$A_r$	Ordering cost to the manufacturer
$h_r$	Holding cost for the serviceable inventory
$P_c$	Retail price of new products
$P_r$	Wholesale price of new products
$A_{ru}$	Collection setup cost for the return items
$h_{ru}$	Holding cost for the return items inventory
$P_{ru}$	Return item purchasing price

# Parameters to the manufacturer:

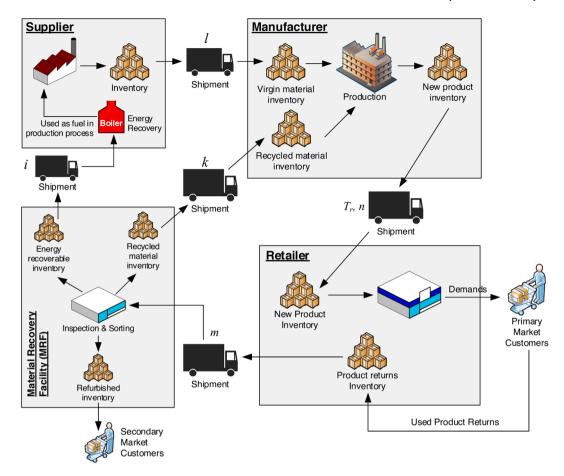
Symbol	Description
X	Postconsumer recycled content of new products
$T_P$	Production cycle time
P	Production rate $(P > D)$
$A_m$	Production setup cost
$A_{mp}$	Cost to order virgin/pure material to the supplier
$A_{mr}$	Cost to order recycled material to the MRF
$h_m$	Holding cost of serviceable inventory
$h_{mp}$	Holding cost of virgin/pure material inventory
$h_{mr}$	Holding cost of recycled material inventory

(continued on next page)

 Table 1

 Comparison of the proposed model with several relevant existing models.

Study	CLSC Structure	Decision-making Process	Demand	Return	Types of recovery	Disposal	Environmental Investigation	Objective	Decision Variable	Consideration
Giri and Sharma [56]	- Single supplier - Single manufacturer - Single retailer - Single collector	Centralised	Deterministic	Deterministic	Remanufacturing	Yes	No	Profit maximisation	-Number of shipments -Cycle length -Returns quality -Number of production/ remanufacturing cycle	-
Taleizadeh et al. [73]	- Single producer - Single remanufacturer - Single retailer - Single recycling centre - Single collection centre - Single disposal centre - Single collector	Centralised	Deterministic	Quality dependent	-Remanufacturing -Recycling	Yes	-GHG emisssions -Waste water -Energy usage	-Profit maximisation -Environmental effect minimisation -Social objective minimisation	-Cycle length -Number of production/ remanufacturing cycle -Number of shipments -Returns quality	-Newly product and remanufactured product have different markets and demands -Discount offer for used product
Mawandiya et al. [74]	- Single manufacturer - Single remanufacturer - Single retailer	Centralised	Normally distributed	Normally distributed	Remanufacturing	Yes	No	Cost Minimisation	-Number of shipments -Cycle length	Collection and remanufacturing are performed by the remanufacturer
Parsa et al. [53]	- Single supplier - Single manufacturer - Single retailer - Single MRF - Single recycling facility	-Decentralised -Centralised	Deterministic	Deterministic	Recycling	Yes	No	Profit maximisation	-Ordering cycle time -Number shipments	-Profit sharing mechanisms
Gilotra and Pareek [75]	- Single vendor - Single buyer	Centralised	Deterministic	Deterministic	Reuse	Yes	No	Cost Minimisation	Order quantity	-Screening cost and defective items after delivery
Giri and Masanta [76]	-Single manufacturer -Single retailer	Centralised	Price dependent demand	Random returns	Remanufacturing	Yes	No	Profit maximisation	-Number of shipments -Order/batch quantity -Retail price	-Consignment stock -Learning and forgetting in manufacturing -Single and batch shipments
Giri and Masanta [77]	-Two suppliers -Single manufacturer -Single retailer	Centralised	Price and quality dependent	Random (uniform distribution)	Remanufacturing	No	No	Profit maximisation	-Shipments lot size -Retail quality -Retail price -Number of shipments	-Stochastic lead time -Supplier disruption
Devoto et al. [39]	Single manufacturer	Decentralised	Deterministic	Random quality	Remanufacturing with several categories	Yes	No	Cost minimisation	Time and quantity to: inspect -remanufacture -discard -and produce	Separate inventories for each grade
This study	-Single supplier -Single manufacturer -Single retailer -Single Material Recovery Facility	-Decentralised -Centralised	Deterministic	Deterministic	-Refurbishing -Material recycling -Energy recovery	No (100 % recovery)	Carbon tax	Profit maximisation	-Ordering cycle time -Number of shipments	Waste-to-energy conversion is employed by the supplier



 $\textbf{Fig. 1.} \ \ \textbf{The material flow of the investigated four-echelon CLSC} \ \ \textbf{system}.$ 

# (continued)

Symbol	Description
$C_p$ $P_{mp}$	Unit production cost Unit purchase price of the virgin/pure materials from the supplier
P <sub>mr</sub> O <sub>m</sub>	Unit purchase price of the recycled materials from the MRF Cost to process each order from the retailer

## Parameters to the MRF:

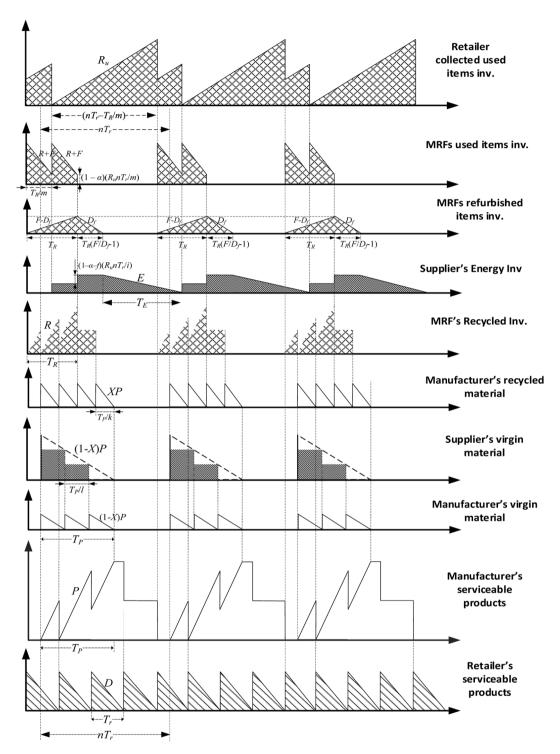
Symbol	Description
$T_R$	Recycling cycle time
R	Recycling rate
F	Refurbishing rate
$\alpha$	The proportion of recyclable items
f	The proportion of refurbishable items
$A_U$	Returned items ordering cost to the retailer
$h_U$	Holding cost of returned items inventory
$A_R$	Recycling setup cost
$h_R$	Holding cost of recycled items inventory
$A_f$	Refurbishing setup cost
$h_f$	Holding cost of refurbished items inventory
$C_R$	Unit recycling cost
$C_F$	Unit refurbishing cost
$P_R$	Wholesale price of used items from the retailer
$P_f$	Selling price of refurbished products to the secondary market
$O_M$	Cost to process each order from the manufacturer
$O_E$	Cost to process each order from the supplier

# Parameters to the supplier:

Symbol	Description
En	Rate of energy recovery process
$h_s$	Holding cost of virgin/pure material inventory
$O_s$	Cost to process each order from the manufacturer
$A_s$	Virgin/pure material ordering cost
$P_{s}$	Unit purchase price of virgin/pure material
$P_E$	Wholesale price of energy-recoverable items from the MRF
$A_E$	Ordering cost of energy recoverable items to the MRF
$h_E$	Holding cost of energy recoverable items inventory
$C_E$	Unit processing cost of energy recovery

### Parameters related to carbon emission:

Symbol	Description
$c_{ec}$	Carbon tax per kg CO <sub>2e</sub> emissions (\$/kg)
$t_c$	Truck capacity (unit)
$g_t$	Truck fuel consumption per mileage (gallons/truck)
$e_t$	The amount of GHG emissions per gallon of fuel from diesel trucks (ton/gallon)
$a_r$	The mass of one unit of recycled material (kg/unit)
$e_r$	Carbon emissions resulting from recycling 1 kg of material (kg CO <sub>2e</sub> /kg)
$a_p$	The mass of one unit of finished product (kg/unit)
$e_p$	Carbon emissions resulting from producing 1 kg of product (kg CO <sub>2e</sub> /kg)
$a_s$	The mass of a unit of virgin/pure material (kg/unit)
$e_s$	Carbon emissions resulting from processing 1 kg of virgin material (kg
	CO <sub>2e</sub> /kg)
$a_E$	Conversion of one unit of energy recoverable items into energy (kg/unit)
$e_E$	Carbon emissions resulting from processing 1 kg of energy from waste (kg
	$CO_{2e}/kg)$



 $\textbf{Fig. 2.} \ \ \textbf{Inventory profile of the four-echelon CLSC system.}$ 

#### Decision variables:

Symbol	Description
$T_r$	Retailer order cycle time
k	Number of shipments of recycled material from the MRF to the manufacturer
1	Number of shipments of virgin material from the supplier to the manufacturer
m	Number of shipments of returned items from the retailer to the MRF
n	Number of shipments of new products from the manufacturer to the retailer
i	Number of shipments of energy-recoverable items from the MRF to the supplier

### 3.3. Assumptions

Several assumptions are made to develop the model, which include:

- (1) The model considers a deterministic situation with a known and constant demand rate over time. Deterministic inventory models are particularly relevant in industries with stable demand and established supply chains. Some examples are standard garment products, such as uniforms and basic garments, which experience consistent and predictable demand. Likewise, demand for automotive services, parts, and maintenance in the automotive and construction materials industries experiences steady demand influenced by maintenance schedules and project timelines ([78]; Mordor [79]). In addition, as noted by Parsa et al. [53], deterministic inventory models are well-suited for long-term coordination in scenarios where demand can be forecasted with high accuracy [80,81].
- (2) Some literature indicates that lead time does not influence the optimal inventory replenishment policy [53,82]. Incorporating a non-zero lead time alters the timing of orders by L units [53,83]. This aligns with Parsa et al. [53], who assume zero lead time to simplify the mathematical model and demonstrate the simultaneous changes in inventory levels across echelons over time [53,81].
- (3) The manufacturer's production rate is always greater than the demand rate (P>D), and shortages are not permitted. This assumption is fundamental in inventory modelling to ensure continuous inventory replenishment and avoid stockouts. Industries like automotive manufacturing, where high production rates are required to meet steady demand and prevent supply disruptions, are relevant to this setting. This assumption is critical for ensuring smooth operations and customer satisfaction in mass-production environments.
- (4) All parties use diesel trucks with the same characteristics to deliver the products. This simplifies the modelling process while reflecting practical scenarios in industries that prioritise standardisation for efficiency. This assumption is particularly relevant for industries with centralised coordination or collaborative logistics systems, where standardisation reduces operational uncertainties. In practice, uniformity in fleet characteristics aids in optimising logistics and is common in retail and e-commerce industries.
- (5) The recoverable items delivered from the retailer to the Materials Recovery Facility (MRF) are stored in the MRF's inventory of

used items, where they are promptly categorised based on their quality for use in the refurbishment and recycling processes. Therefore, the rates for refurbishment and recycling have included sorting, inspection, and the actual processing of items. This assumption simplifies the model and ensures that the system operates based on the principle of inventory minimisation.

#### 4. Model formulation

This section outlines the development of the investigated four-level CLSC inventory model. The proposed model considers a carbon tax policy to reduce emissions released from production and transportation activities. Here, the CLSC inventory model is developed in order, starting from the retailer, MRF, manufacturer, and finally, the supplier.

#### 4.1. Profit formulation for the retailer

The retailer gains revenue for selling D units of products at the selling price  $P_c$ . The revenue function of the retailer is known as

$$TR_r = P_c \times D \tag{1}$$

The retailer carries several costs associated with the inventory of serviceable products, such as placing orders, purchasing the products, and holding the inventory. In this case, the retailer plays the role of a collector for the used items. Consequently, the retailer incurs several costs associated with the inventory of used items, such as collecting, purchasing, and holding costs. The retailer's holding cost is calculated based on the average inventory level of collected used items and serviceable products, as depicted in Fig. 2. Furthermore, the retailer incurs taxes on emissions generated while transporting collected used items to the MRF. Consequently, the retailer's total inventory cost  $(TC_r)$ , encompassing both serviceable and used item inventories, is expressed in Eq. (2).

$$\begin{split} TC_{r} &= \frac{A_{r}}{T_{r}} + P_{r} \times D + \frac{h_{r}DT_{r}}{2} + \frac{A_{Ru}}{n T_{r}} + P_{ru} \times R_{u} \\ &+ h_{ru} \left( \frac{R_{u}T_{r}(2R + m(n-1)(R - R_{u}\alpha))}{2mR} \right) + \left( \frac{R_{u}}{t_{c}} \right) g_{t}e_{t}c_{ec} \end{split} \tag{2}$$

The retailer's profit can be calculated as

$$TP_r = TR_r - TC_r \tag{3}$$

#### 4.2. Profit formulation for the MRF

Here, the authors adopt a system with three recovery options, which are refurbishing, recycling, and energy recovery. The MRF's revenue comes from selling refurbished items to the secondary market, selling recycled materials to the manufacturer, and selling energy-recoverable items to the supplier. The revenue function of the MRF is known as

$$TR_{\rm M} = P_f f R_{\rm u} + P_{mr} XD + P_E (1 - \alpha - f) R_{\rm u}$$

$$\tag{4}$$

The MRF incurs costs related to the inventory of used items, including ordering costs to the retailer, purchasing, and holding costs. As for the recycled materials inventory, the MRF is charged with setup, processing, and holding costs. Furthermore, the inventory cost of the refurbished items of the MRF consists of processing and holding costs. The MRF's holding cost is determined by the average inventory of MRF's recoverable, refurbished, and recycled items, as illustrated in Fig. 2. Here, the MRF also pays costs to process orders from the manufacturer

and the supplier. The MRF pays carbon emissions taxes released from transporting shipments to the manufacturer and supplier and processing the recycling activity. Carbon emissions from refurbishing activities are ignored. Therefore, inventory cost related to the MRF is presented by Eq. (5)

$$TR_{s} = (1 - X)DP_{mp} \tag{10}$$

Supplier inventory cost comprises the cost to order and purchases energy-recoverable items from the MRF, the cost of processing used items into energy, the cost to set up and produce virgin/pure/virgin materials, the cost to process orders from the manufacturer, the holding

$$\begin{split} &TC_{M} \; = \; \frac{A_{U}m}{nT_{r}} + P_{R}R_{u} + \frac{h_{U}\alpha R_{u}^{2}nT_{r}(2-\alpha)}{2mR} + \frac{A_{R}}{nT_{r}} + XDC_{R} \\ & + h_{R}\left(\frac{\alpha R_{u}((2-k)\alpha R_{u}nT_{r} - (k-1)kR)}{2kR}\right) + fR_{u}C_{f} \\ & + h_{F}\left(\frac{f^{2}FnR_{u}^{2}T_{r}}{2D_{f}(F-D_{f})}\right) + \frac{O_{M}k}{nT_{r}} + \frac{O_{E}i}{nT_{r}} + \left(\frac{XD}{t_{c}} + \frac{(1-\alpha-f)R_{u}}{t_{c}}\right)g_{t}e_{t}c_{ec} + (\alpha R_{u})a_{r}e_{r}c_{ec} \end{split}$$

Then, the authors calculate the MRF profit function as

$$TP_{\rm M} = TR_{\rm M} - TC_{\rm M} \tag{6}$$

### 4.3. Profit formulation for the manufacturer

The manufacturer gains revenue from selling serviceable products to the retailer, as given by

$$TR_{\rm m} = P_r \times D \tag{7}$$

In this model, the manufacturer has two raw materials sources for production: virgin/pure raw materials from the supplier and recycled raw materials from the MRF. As shown in Fig. 2, the manufacturer inventory consists of recycled material, virgin/pure material, and serviceable inventory. Here, the manufacturer incurs several costs related to inventory, including ordering cost to the MRF, ordering cost to the supplier, production setup cost, purchasing cost of virgin/pure and recycled materials, cost to process orders from the retailer, holding costs for the serviceable products, recycled materials, and virgin/pure materials, and carbon tax for the transportation and production activities. The manufacturer's holding cost is determined by the average level of the manufacturer's serviceable products, virgin materials, and recycled materials inventories, as depicted in Fig. 2. Therefore, the inventory cost related to the manufacturer is given below.

cost from the virgin/pure/virgin materials, holding cost of the energy recoverable items, and carbon tax from the transportation, production, and thermal recovery. The supplier's holding cost is calculated based on the average level of the supplier's virgin materials and energy inventories, as depicted in Fig. 2. Therefore, the inventory cost of the supplier can be formulated as

$$\begin{split} &TC_{s} = \frac{A_{E}i}{nT_{r}} + \frac{P_{E}(1-\alpha-f)R_{u}nT_{r}}{nT_{r}} + \frac{C_{E}(1-\alpha-f)R_{u}nT_{r}}{nT_{r}} + \frac{A_{s}}{nT_{r}} \\ &+ P_{s}(1-X)D + \frac{O_{s}l}{nT_{r}} + \frac{h_{S}D^{2}nT_{r}(l-1)(1-X)}{2lP} \\ &+ \frac{h_{E}nR_{u}{}^{2}T_{r}(1-f-\alpha)\left(\frac{i(1-f-\alpha)}{E} + \frac{\alpha(1+i)}{R}\right)}{2i} + \\ &+ \frac{c_{ec}\left(\left(\frac{(1-X)D}{t_{c}}\right)g_{t}e_{t} + (1-X)Da_{s}e_{s} + (1-\alpha-f)R_{u}a_{E}e_{E}\right) \end{split} \tag{11} \end{split}$$

Then, the supplier profit function can be calculated as

$$TP_{\rm s} = TR_{\rm s} - TC_{\rm s} \tag{12}$$

#### 4.5. The optimisation problem

The objective function of the proposed mixed-integer non-linear programming (MINLP) problem is to maximise the profit of all parties

$$\begin{split} TC_{m} &= \frac{O_{m}}{T_{r}} + \frac{A_{mp}l}{nT_{r}} + \frac{A_{mr}k}{nT_{r}} + \frac{A_{m}}{nT_{r}} + C_{p}D + (1-X)DP_{mp} + XDP_{mr} \\ &+ h_{m} \left( \frac{D(2DT_{r} - nDT_{r} + P(n-1)(2T_{r}-1))}{2P} \right) + h_{mp} \left( \frac{D^{2}(1-X)nT_{r}}{2lP} \right) + h_{mr} \left( \frac{D^{2}XnT_{r}}{2kP} \right) \\ &+ \left( \frac{XD}{t} \right) g_{t}e_{t}c_{ec} + Da_{p}e_{p}c_{ec} \end{split}$$

Then, the authors calculate the manufacturer profit function as

$$TP_{\rm m} = TR_{\rm m} - TC_{\rm m} \tag{9}$$

# 4.4. Profit formulation for the supplier

The supplier obtains revenue from selling virgin/pure raw materials to the manufacturer as given by

$$Max TP = TP_r + TP_m + TP_M + TP_s$$
 (13)

subject to

$$k < k_0, l < l_0, m < m_0, n < n_0, i < i_0$$
 (14)

$$k, l, m, n, i \in \{1, 2, 3, ...\}$$
 (15)

$$T_r > 0 \tag{16}$$

#### 5. Solution procedure

In this study, the optimisation was done under decentralised and centralised decision-making structures. The proposed solution procedure will be explained for each decision-making structure.

### 5.1. Decentralised decision-making structure (DDMS)

The DDMS procedure begins at the retailer, who chooses the optimal values for  $T_n$ , n, and m. Consequently, the manufacturer will determine the optimal solution for k and l, and finally, the supplier will determine the optimal solution for i. The DDMS strategy applies to an SC structure where the retailer is the leader. A typical supply chain, referred to as a retailer-led supply chain, is common in the T&C industry and is known as the Fashion Retail Supply Chain (FRSC). The procedure begins as follows.

**Theorem 1.** By assuming m to be constant, the  $TP_r$  function is concave with respect to  $T_r$  and n.

**Proof of Theorem 1.** The first and second derivatives of the  $TP_r$  function with respect to  $T_r$  and n are as follows.

$$\frac{\partial TP_r}{\partial T_r} = \frac{h_r D}{2} + \frac{A_r}{T_r^2} + \frac{A_{ru}}{n T_r^2} - \frac{h_{ru} R_u ((2 + m(n-1))R - m(n-1)R_u \alpha)}{2mR}$$
 (17)

$$\frac{\partial TP_r}{\partial n} = \frac{A_{ru}}{n^2 T_r} - \frac{h_{ru} R_u T_r (R - R_u \alpha)}{2R}$$
(18)

$$\frac{\partial^2 TP_r}{\partial^2 T_r} = -\frac{2A_r}{T_r^3} - \frac{2A_{ru}}{nT_r^3} < 0 \tag{19}$$

$$\frac{\partial^2 TP_r}{\partial^2 n} = -\frac{2A_{ru}}{n^3 T_r} < 0 \tag{20}$$

$$\frac{\partial^2 TP_r}{\partial T_r \partial n} = \frac{\partial^2 TP_r}{\partial n \partial T_r} = -\frac{A_{ru}}{n^2 T_r^2} - \frac{h_{ru} R_u (R - R_u \alpha)}{2R}$$
 (21)

So, the Hessian matrix of  $TP_r$  is  $H_1 = \begin{bmatrix} \frac{\partial^2 TP_r}{\partial^2 T_r} & \frac{\partial^2 TP_r}{\partial T_r \partial n} \\ \frac{\partial^2 TP_r}{\partial n \partial T_r} & \frac{\partial^2 TP_r}{\partial^2 n} \end{bmatrix}$ . The determinant

minant of matrix  $H_1$  becomes

 $TP_r(n, m)$  as given by

$$\begin{split} TP_r & \left( n, \, \mathbf{m} \right) = D \left( P_c - P_r \right) - P_{ru} R_{\mathbf{u}} - \frac{c_{ee} e_t g_t R_{\mathbf{u}}}{t_c} \\ & - \frac{\left( A_{ru} + n A_r \right) \sqrt{2nR(D\mathbf{h}_r \mathbf{m} + \mathbf{h}_{ru}(2 + \mathbf{m}(n-1))R_{\mathbf{u}}) - 2\mathbf{h}_{ru} m n(n-1)R_{\mathbf{u}}^{2} \alpha}}{n\sqrt{mR(A_{ru} + A_r n)}} \end{split}$$

By relaxing the integrality constraint of n and assuming m is constant, the authors get the optimal solution of n denoted by  $n_{rx}$  which is obtained by setting  $\frac{\partial TP_r(n)}{\partial n} = 0$ . The value of  $n_{rx}$  that fulfil the equality  $\frac{\partial TP_r(n)}{\partial n} = 0$ . is

$$n_{rx} = \frac{\sqrt{A_{ru}(Dh_{r}mR + h_{ru}R_{u}(mR_{u}\alpha + (2-m)R))}}{\sqrt{mA_{r}h_{ru}R_{u}(R - R_{u}\alpha)}}$$
(25)

Since m and n are positive integers,  $1 \le m \le m_0$  and  $1 \le n \le n_0$ , their optimal value in the DDMS approach, denoted by  $m^{*DDMS}$  and  $n^{*DDMS}$ , are obtained by using Theorem 1 and its proposition through the following B&B (branch and bound) algorithm:

Step 1 For m from 1 to  $m_0$ , do step 2

Step 2 Calculate  $n_{rx}$  in Eq. (25)

Step 2.1 For each given value of m, calculate the value of  $T_r^*$  in Eq. (23) in  $n_1$  and  $n_2$ , with

$$n_1 = \max(1, \min(\text{floor}(n_{rx}), n_0))$$
(26)

$$n_2 = \min(\operatorname{ceil}(n_{rx}), \ n_0) \tag{27}$$

Step 2.2 For each given value of m, find the maximum value of  $TP_r$  using Eq. (3) in  $(T_r^{*DDMS}, n_1)$  or  $(T_r^{*DDMS}, n_2)$ 

Step 3 Find the global maximum of  $TP_r(T_p, n, m)$  among the recorded maximums of Steps 1 to 2. The optimal value of  $T_p$ , n, and m for the decentralised structure, denoted by  $T_r^{*DDMS}$ ,  $n^{*DDMS}$ , and  $m^{*DDMS}$ , are obtained.

**Theorem 2.** Using the value of  $T_r^{*DDMS}$ ,  $n^{*DDMS}$ , and  $m^{*DDMS}$ ,  $TP_m$  is concave to k and l.

**Proof of Theorem 2.** The first and second derivatives of the  $TP_m$  function with respect to k and l are known as

$$\det(H_1) = \frac{12A_{ru}^2R^2 - h_{ru}^2n^4R_u^2T_r^4(R - R_u\alpha)^2 + 4A_{ru}nR(4A_rR + h_{ru}nR_uT_r^2(R_u\alpha - R))}{4n^4R^2T_r^4}$$
(22)

Since  $\frac{\partial^2 TP_r}{\partial^2 T_r} < 0$  and  $\det(H_1) > 0$ , therefore the authors prove that  $TP_r$  is negative definite and concave to  $T_r$  and n.

**Proposition 1.** Based on Theorem I, by assuming m is constant, the optimal solution for  $T_r$  in the DDMS procedure, denoted by  $T_r^{*DDMS}$ , is obtained by setting  $\frac{\partial TP_r}{\partial T_r} = 0$ . The value of  $T_r^{*DDMS}$  that fulfil the equality  $\frac{\partial TP_r}{\partial T_r} = 0$  is

$$T_r^{*DDMS} = \frac{\sqrt{2m(A_{ru} + A_r n)R}}{\sqrt{n(Dh_r mR + h_{ru}R_u((2 + m(-1 + n))R - m(-1 + n)R_u\alpha))}}$$
 (23)

By substituting  $T_r^{*DDMS}$  into  $TP_r$  the authors obtain the function of

$$\frac{\partial TP_m}{\partial k} = -\frac{A_{mr}}{n^{*\text{DDMS}}T_r^{*\text{DDMS}}} + \frac{D^2 h_{mr} n^{*\text{DDMS}} T_r^{*\text{DDMS}} X}{2k^2 P}$$
(28)

$$\frac{\partial TP_m}{\partial l} = -\frac{A_{mp}}{n^{*\text{DDMS}}T_r^{*\text{DDMS}}} + \frac{D^2 h_{mp} n^{*\text{DDMS}} T_r^{*\text{DDMS}} (1 - X)}{2l^2 P}$$
(29)

$$\frac{\partial^2 TP_m}{\partial^2 k} = -\frac{D^2 h_{mr} n^{*\text{DDMS}} T_r^{*\text{DDMS}} X}{k^3 P} < 0$$
(30)

$$\frac{\partial^{2} TP_{m}}{\partial^{2} I} = -\frac{D^{2} h_{mp} n^{*\text{DDMS}} T_{r}^{*\text{DDMS}} (1 - X)}{l^{3} P} < 0$$
(31)

$$\frac{\partial^2 TP_m}{\partial k \partial l} = \frac{\partial^2 TP_m}{\partial l \partial k} = 0 \tag{32}$$

The Hessian matrix of 
$$TP_m$$
 is  $H_2 = \begin{bmatrix} \frac{\partial^2 TP_m}{\partial^2 k} & \frac{\partial^2 TP_m}{\partial k \partial l} \\ \frac{\partial^2 TP_m}{\partial l \partial k} & \frac{\partial^2 TP_m}{\partial^2 l} \end{bmatrix}$ . The determinant

minant of matrix  $H_2$  becomes

$$|H_2| = \frac{D^4 h_{mp} h_{mr} n^{*\text{DDMS}^2} T_r^{*\text{DDMS}^2} (1 - X) X}{k^3 l^3 P^2} > 0$$
(33)

Since the first-order and second-order principal minors of matrix  $H_2$  are always negative  $\left(\frac{\partial^2 TP_m}{\partial^2 k} < 0\right)$  and positive  $(|H_2| > 0)$ , therefore the authors prove that  $TP_m$  is negative definite and concave to k and l.

**Proposition 2.** Based on Theorem 2, by using the value of  $n^{*DDMS}$  and  $T_r^*$  and  $T_r^*$  and relaxing the integrality constraint of k, the solution of k, denoted by  $k_{rx}$ , is obtained by setting  $\frac{\partial TP_m}{\partial k} = 0$ . The value of  $k_{rx}$  that fulfil the equality  $\frac{\partial TP_m}{\partial k}$ 

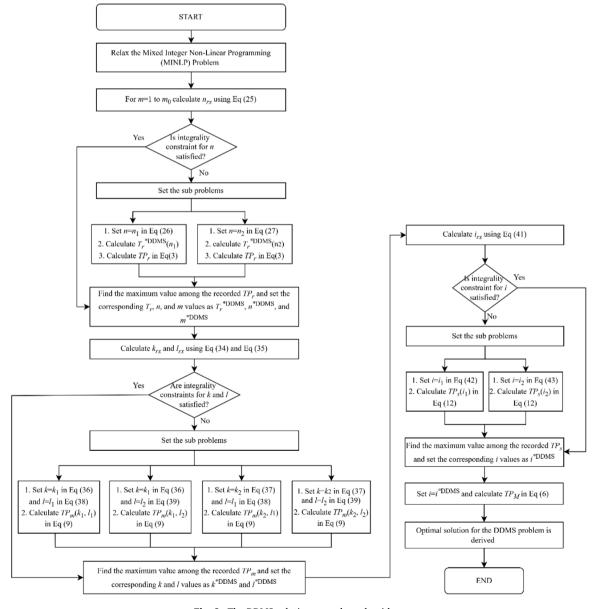
$$k_{rx} = \frac{Dn^{*DDMS}T_r^{*DDMS}\sqrt{Xh_{mr}}}{\sqrt{2PA_{mr}}}$$
(34)

With the same procedure, the optimal value of l (by relaxing the integrality constraint of l), denoted by  $l_{rx}$ , is obtained by setting  $\frac{\partial TP_m}{\partial l}=0$ . Then, the authors obtain the following function of  $l_{rx}$ 

$$l_{rx} = \frac{\sqrt{D^2 h_{mp} n^{*DDMS^2} T_r^{*DDMS^2} (1 - X)}}{\sqrt{2PA_{mr}}}$$
(35)

Similar to n, since k and l are positive integers,  $1 \le k \le k_0$  and  $1 \le l \le l_0$ , their optimal values denoted by  $k^{^*\mathrm{DDMS}}$  and  $l^{^*\mathrm{DDMS}}$  are obtained by using Theorem 2 and its proposition through the following B&B algorithm:

Step 1 Calculate  $k_{rx}$  and  $l_{rx}$  from Eqs. (34) and (35) respectively. Step 2 Find the maximum value of  $TP_m$  using Eq. (9) in  $(k_1, l_1)$ , or  $(k_1, l_2)$ , or  $(k_2, l_1)$ , or  $(k_2, l_2)$ , with



 $\textbf{Fig. 3.} \ \ \textbf{The DDMS solution procedure algorithm.}$ 

$$k_1 = \max(1, \min(\text{floor}(k_{rx}), k_0)) \tag{36}$$

$$k_2 = \min(\operatorname{ceil}(k_{rx}), \ k_0) \tag{37}$$

$$l_1 = \max(1, \min(\text{floor}(l_{rx}), l_0)) \tag{38}$$

$$l_2 = \min(\operatorname{ceil}(l_{rx}), \ l_0) \tag{39}$$

Step 3 Find the global maximum of  $TP_m(k, l)$  using Eq. (9), denoted by  $TP_m^{*DDMS}$ , among the recorded maximums of Step 2. Finally, the optimal values of k and l for the decentralised structure, denoted by  $k^{*DDMS}$  and  $l^{*DDMS}$ , are obtained.

**Theorem 3.** Using the value of  $T_r^{*DDMS}$  and  $n^{*DDMS}$ ,  $TP_s$  is concave to i.

**Proof of Theorem 3.** The second derivative of  $TP_s$  w.r.t i is known as

$$\frac{\partial^{2} TP_{s}}{\partial^{2} i} = -\frac{h_{E} n^{*DDMS^{2}} R_{u}^{2} T_{r}^{*DDMS^{2}} \alpha (1 - f - \alpha)}{i^{3} R} < 0$$
 (40)

Since  $(f+\alpha)$  is always less than 1, then it is proven that  $\frac{\partial^2 TP_s}{\partial^2 i} < 0$ , therefore, the authors know that  $TP_s$  is negative definite and concave to i.

**Proposition 3.** By relaxing the integrality constraint of i, the authors obtain the optimal value of i, denoted by  $i_{rx}$ , which fulfil the equality of  $\frac{\partial TP_s(i)}{\partial i}$  = 0. The value of  $i^*$  is known as

$$i_{rx} = \frac{\sqrt{h_E n^{*DDMS^2} R_u^2 T_r^{*DDMS^2} \alpha (1 - f - \alpha)}}{\sqrt{2RA_E}}$$
(41)

Since i is a positive integer,  $1 \le i \le i_0$ , and  $TP_s$  is strictly concave in i, the optimal value of i in the decentralised structure, denoted by  $i^{*\mathrm{DDMS}}$  is derived at

$$i_1 = \max(1, \min(\text{floor}(i_{rx}), i_0)) \tag{42}$$

$$i_2 = \min(\operatorname{ceil}(i_{rx}), \ i_0) \tag{43}$$

Therefore, the authors obtain the total profit of the system as follows.

$$TP^{*DDMS} = TP_r^{*DDMS} + TP_m^{*DDMS} + TP_M^{*DDMS} + TP_s^{*DDMS}$$
 (44)

Fig. 3 outlines the solution procedure for the decentralised structure and provides a comprehensive overview of the key steps involved in the

#### 5.2. Centralised decision-making structure (CDMS)

The CDMS approach is conducted under the joint economic lot size (JELS) policy. In the CDMS, all parties decide to optimise the inventory simultaneously. Although it can contribute to an overall increase in the joint total profit of the system, some studies discussed that centralised decision-making under JELS policy may not benefit all players in the SC [53,84]. In this study, the authors evaluate two CDMS scenarios, i.e. without a profit-sharing (Scenario 1) and under a disproportionate profit-sharing (DPS) mechanism (Scenario 2). Hence, the CDMS-MINLP problem becomes

$$Max JTP = TP_r + TP_m + TP_M + TP_s$$
(45)

subject to

$$k \le k_0, \ l \le l_0, \ m \le m_0, \ n \le n_0, \ i \le i_0$$
 (46)

$$k, l, m, n, i \in \{1, 2, 3, ...\}$$
 (47)

$$T_r > 0 (48)$$

As for Scenario 2, the following constraint is imposed

$$\begin{split} JTP^{*\text{CDMS}} &\geq TP^{*\text{DDMS}}, \ TP_r^{*\text{CDMS}} \geq TP_r^{*\text{DDMS}}, \ TP_m^{*\text{CDMS}} \geq TP_m^{*\text{DDMS}}, \ TP_M^{*\text{CDMS}} \\ &\geq TP_M^{*\text{DDMS}}, \ TP_s^{*\text{CDMS}} \geq TP_s^{*\text{DDMS}} \end{split} \tag{49}$$

The CDMS solution procedure begins as follows.

**Corollary 1.** Ignoring the integrality constraints of m, k, l, i, and by ensuring that the conditions (50)–(55) are satisfied, the optimal solutions for m, k, l and i in the centralised structure can be realised.

$$y_8 > \frac{\left(2A_U m^2 R^2 + nR_u T_r^2 y_5\right)^2}{8mnR^2 R_u T_r^2 (y_5)} > 0$$
(50)

$$R^2 < \frac{nR_u T_r^2 y_5}{m^2}; (51)$$

$$R(4my_8 - h_{ru}nR_uT_r^2) > h_Un^2R_u^2T_r^2\alpha((1-f) + (F-R))$$
(52)

$$\alpha < 2 - 2f \tag{53}$$

$$y_{5} < \frac{knPy_{6} \left( \left( 2A_{U}m^{2}R^{2} + nR_{u}T_{r}^{2}y_{5} \right)^{2} - 8mny_{8}R^{2}R_{u}T_{r}^{2}y_{5} \right)}{mRR_{u} \left( 2k^{2}PR(A_{mr} + O_{M}) + n^{2}T_{r}^{2}y_{6} \right)^{2}}$$
(54)

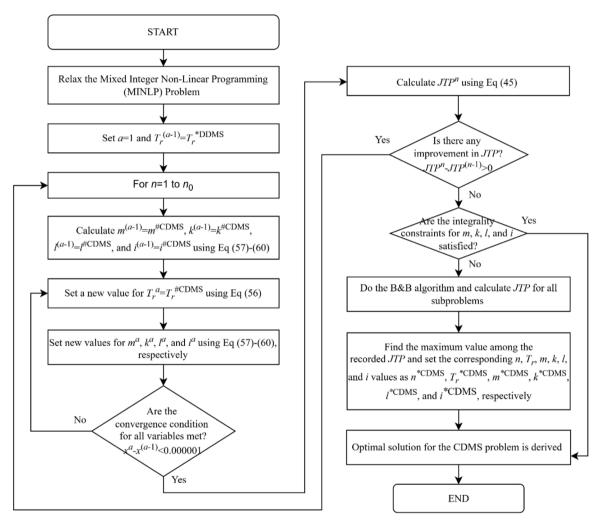
$$D^{2}klmPRy_{5}y_{6}y_{7}\left(2i^{2}R(A_{E}+O_{E})+h_{E}n^{2}R_{u}^{2}T_{r}^{2}\alpha(1-f-\alpha)\right)^{2} < \left(h_{E}iR_{u}\alpha(1-f-\alpha)\left(lD^{2}y_{7}\left(mRR_{u}\left(2k^{2}PR(A_{mr}+O_{M})+n^{2}T_{r}^{2}y_{6}\right)^{2}y_{5}\right)\right) - knPy_{6}\left(8mR^{2}nR_{u}T_{r}^{2}y_{5}y_{8}-\left(2A_{U}m^{2}R^{2}+nR_{u}T_{r}^{2}y_{5}\right)^{2}\right)\right) - kmR^{2}R_{u}y_{5}\left(2l^{2}P(A_{mp}+O_{s})+D^{2}y_{6}y_{7}n^{2}T_{r}^{2}\right)^{2}\right)\right)$$

$$(55)$$

solution process under the DDMS scenario.

The next section will discuss the proposed solution procedure for the centralised structure.

**Proof of Corollary 1.** The Hessian matrix of  $JTP(T_r, m, k, l, and i)$ , denoted as  $H_3$ , is given as follows



 $\textbf{Fig. 4.} \ \ \textbf{The CDMS solution procedure algorithm.}$ 

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$$H_{3} = \begin{bmatrix} \frac{\partial^{2}JTP}{\partial^{2}T_{r}} & \frac{\partial^{2}JTP}{\partial m\partial T_{r}} & \frac{\partial^{2}JTP}{\partial k\partial T_{r}} & \frac{\partial^{2}JTP}{\partial l\partial T_{r}} & \frac{\partial^{2}JTP}{\partial i\partial T_{r}} \\ \\ \frac{\partial^{2}JTP}{\partial T_{r}\partial m} & \frac{\partial^{2}JTP}{\partial^{2}m} & \frac{\partial^{2}JTP}{\partial k\partial m} & \frac{\partial^{2}JTP}{\partial l\partial m} & \frac{\partial^{2}JTP}{\partial i\partial m} \\ \\ \frac{\partial^{2}JTP}{\partial T_{r}\partial k} & \frac{\partial^{2}JTP}{\partial m\partial k} & \frac{\partial^{2}JTP}{\partial^{2}k} & \frac{\partial^{2}JTP}{\partial l\partial k} & \frac{\partial^{2}JTP}{\partial i\partial k} \\ \\ \frac{\partial^{2}JTP}{\partial T_{r}\partial l} & \frac{\partial^{2}JTP}{\partial m\partial l} & \frac{\partial^{2}JTP}{\partial k\partial l} & \frac{\partial^{2}JTP}{\partial l} & \frac{\partial^{2}JTP}{\partial i\partial l} \\ \\ \frac{\partial^{2}JTP}{\partial T_{r}\partial l} & \frac{\partial^{2}JTP}{\partial m\partial l} & \frac{\partial^{2}JTP}{\partial k\partial l} & \frac{\partial^{2}JTP}{\partial l\partial l} & \frac{\partial^{2}JTP}{\partial l\partial l} \\ \\ \frac{\partial^{2}JTP}{\partial T_{r}\partial l} & \frac{\partial^{2}JTP}{\partial m\partial l} & \frac{\partial^{2}JTP}{\partial k\partial l} & \frac{\partial^{2}JTP}{\partial l\partial l} & \frac{\partial^{2}JTP}{\partial l\partial l} \\ \\ \frac{\partial^{2}JTP}{\partial T_{r}\partial l} & \frac{\partial^{2}JTP}{\partial m\partial l} & \frac{\partial^{2}JTP}{\partial k\partial l} & \frac{\partial^{2}JTP}{\partial l\partial l\partial l} & \frac{\partial^{2}JTP}{\partial l\partial l} \\ \\ \end{bmatrix}$$

$$\begin{split} \mathbf{y}_2 &= D^2 E_n \textit{im} R^2 \big( \mathbf{h}_m \textit{kl}(n-2) + \textit{n} \big( \mathbf{h}_\textit{mp} \textit{k}(\textit{X}-1) + \mathbf{h}_\textit{s} \textit{k}(l-1)(\textit{X}-1) - \mathbf{h}_\textit{mr} \textit{lX} \big) \big) \\ &- \textit{DE}_n \textit{iklm} (\mathbf{h}_r + 2\mathbf{h}_m \ (n-1)) \textit{PR}^2 \end{split}$$

$$\begin{split} y_3 &= lP_{\textit{Ru}} \big( h_{\textit{E}} \textit{ikmn} R^2 R_u (1-f-\alpha)^2 + E_n h_{\textit{ru}} \textit{ik} R ((m-mn-2)R \\ &+ m(n-1)R_u \alpha) - E_n n R_u \alpha (mR(h_{\textit{E}}(1+i)k(1-f-\alpha) + h_{\textit{M}} \textit{i}(2+k)\alpha) \\ &+ h_{\textit{U}} \textit{ik} (F\alpha - R(2-2f-\alpha))) \big) \end{split}$$

$$m^{\text{\#CDMS}} = \frac{T_r^{\text{*CDMS}}}{R} \sqrt{\frac{nR_u y_5}{2A_U}}$$

$$(57)$$

$$\begin{bmatrix} -\frac{2y_8}{nT_r^3} & \left(\frac{h_{ru}R_u}{m^2} + \frac{A_U}{nT_r^2} + \frac{y_5 - 2h_{Ru}R^2}{2m^2R^2}\right) & \left(\frac{A_{mr}}{nT_r^2} + \frac{ny_6}{2k^2PR}\right) & \left(\frac{A_{mp} + O_s}{nT_r^2} + \frac{D^2ny_7}{2l^2P}\right) & \left(\frac{A_E}{nT_r^2} + \frac{h_EnR_u^2\alpha(1 - f - \alpha)}{2i^2R}\right) \\ \frac{h_{ru}R_u}{m^2} + \frac{A_U}{nT_r^2} + \frac{y_5 - 2h_{Ru}R^2}{2m^2R^2} & -\frac{R_uT_ry_5}{m^3R^2} & 0 & 0 & 0 \\ = \begin{bmatrix} \frac{A_{mr}}{m^2} + \frac{O_M}{nT_r^2} + \frac{ny_6}{2k^2PR} & 0 & -\frac{nT_ry_6}{k^3PR} & 0 & 0 \\ \frac{A_{mp} + O_s}{nT_r^2} + \frac{D^2y_7}{2l^2P} & 0 & 0 & -\frac{D^2nT_ry_7}{l^3P} & 0 \\ \frac{A_E}{nT_r^2} + \frac{O_E}{nT_r^2} + \frac{h_EnR_u^2\alpha(1 - f - \alpha)}{2i^2R} & 0 & 0 & -\frac{h_EnR_u^2T_r\alpha(1 - f - \alpha)}{i^3R} \end{bmatrix}$$

with,

$$y_5 = \left(2h_{Ru}R^2 + h_UnR_u\alpha(F\alpha + R(2-2f-\alpha))\right)$$

$$y_6 = \left(D^2 h_{mr} RX + 2h_M P R_u^2 \alpha^2\right)$$

$$y_7 = (h_{mp} - h_s)(1 - X)$$

$$y_{8} = (A_{F} + A_{m} + A_{R} + A_{ru} + A_{s} + A_{E}i + A_{mr}k + A_{mp}l + A_{U}m + A_{r}n + iO_{E} + nO_{m} + kO_{M} + lO_{s})$$

Matrix  $H_3$  is negative definite only if  $(-1)^k |H_k| \rangle 0$  for all leading principal minors, with  $(1 \le k \le 5)$ . The leading principal minors of the matrix  $H_3$  are given in Appendix 2. If the necessary conditions in Appendix 2 are satisfied, the leading principal minors of the matrix  $H_3$  are fulfilled. Therefore, the Hessian matrix of  $JTP(T_r, m, k, l, i)$  is negative definite and concave in  $T_r$ , m, k, l, and i.

**Proposition 5.** By ignoring the integrality constraints of m, k, l and i and setting the first partial derivative of JTP, w.r.t to  $T_r$ , m, k, l, and i, respectively, to zero and solving for  $T_r$ , m, k, l, and i, respectively, the optimal values for  $T_r^{*CDMS}$ ,  $\mathbf{m}^{\#CDMS}$ ,  $\mathbf{k}^{\#CDMS}$ ,  $\mathbf{l}^{\#CDMS}$ , and  $\mathbf{i}^{\#CDMS}$  are given below.

$$T_{r}^{*}CDMS = \sqrt{\frac{2D_{f}E_{n}P_{R}(D_{f} - F)iklmy_{1}}{n\left(E_{n}f^{2}Fh_{F}iklmnPR^{2}R_{u}^{2} + y_{2} + D_{f}^{2}(y_{2} + y_{3})\right)}}$$
(56)

with,

$$y_1 = (A_F + A_m + A_R + A_{R_u} + A_s + A_{Um} + i(A_E + O_E) + n(A_R + O_m) + k(A_{mr} + O_M) + l(A_{mn} + O_s))$$

$$k^{\text{\#CDMS}} = inT_r^{\text{*CDMS}} \sqrt{\frac{y_6}{P_R(\mathbf{h_M} n R_u T_r \alpha - 2(A_{mr} + O_{\mathrm{M}}))}}$$
(58)

$$l^{\text{\#CDMS}} = DnT_r^{\text{*CDMS}} \sqrt{\frac{y_7}{2P(A_{mp} + O_s)}}$$
 (59)

$$i^{\text{\#CDMS}} = R_{\text{u}} n T_r^{\text{*CDMS}} \sqrt{\frac{h_E \alpha (1 - f - \alpha)}{2R(A_E + O_E)}}$$
(60)

Finally, since n, m, k, l, and i are positive integers, with  $1 \le n \le n_0$ , their optimal values in the CDMS approach, denoted by  $n^*_{CDMS}$ ,  $m^*_{CDMS}$ ,  $k^*_{CDMS}$ ,  $l^*_{CDMS}$ , and  $i^*_{CDMS}$  are obtained through the following procedure.

- 1 For  $1 \le n \le n0$ , do Step 2.
- 2 Set Tr=Tr\*DDMS from the decentralised solution as an initial value of Tr\*CDMS.
- 3 Calculate m=m#CDMS, k=k#CDMS, l=l#CDMS, and i=i#CDMS using Eq. (57) (60).
- 4 Calculate Tr=Tr#CDMS using Eq. (56).
- 5 Repeat Step 3 to 4 until there is no change in the values of Tr, m, k, l, and i.
- 6 As the value of m, k, l, and i should be integers, do the B&B algorithm to find m\*CDMS, k\*CDMS, l\*CDMS and i\*CDMS which maximise the JTP in Eq. (45).
- 7 For each given value of n, record the maximum value of JTP in Eq. (45) and their corresponding values for Tr, n, m, k, l, and i.
- 8 Finally, the optimal values for Tr, n, m, k, l, and i for the centralised structure, denoted by Tr\*CDMS, n\*DDMS, m\*CDMS, k\*CDMS, l\*CDMS, and i\*CDMS are obtained.

**Table 2** Input parameters of the model.

Parameters to	Parameters to the supplier			Parameters to the retailer		
Input parameter	Value	Units	Input parameter	Value	Units	
$E_n$	250	units/ year	D	500	unit/year	
$A_s$	450	\$/order	$R_u$	400	unit/year	
$A_E$	200	\$/order	$P_c$	175	\$/unit	
$h_s$	0.5	\$/unit/ year	$P_{ru}$	10	\$/unit	
$h_E$	0.5	\$/unit/ year	$A_{ru}$	1000	\$/collection	
$P_s$	45	\$/unit	$h_r$	4.5	\$/unit/year	
$P_E$	12	\$/unit	$h_U$	0.3	\$/unit/year	
$C_E$	5.0	\$/unit	$t_c$	50	unit/truck	
$O_s$	150	\$/order	$g_t$	375	gallon/truck	
$a_s$	1.0	kg/unit	$e_t$	10.08414	kgCO <sub>2</sub> eq/ gallon	
$e_s$	45.26	kg CO <sub>2</sub> eq/kg	$c_{ec}$	0.049	\$/kgCO <sub>2</sub> eq	
$a_E$	0.8	kg/unit				
$e_E$	2.218	kg CO <sub>2</sub> eq/kg				

Parameters to	the manufac	cturer	Parameters to the MRF		
Input parameter	Value	Units	Input parameter	Value	Units
X	40.0	%	α	50.0	%
P	1000	unit/year	f	20.0	%
$C_p$	60	\$/unit	R	800	Unit/year
$A_m$	10,000	\$/cycle	F	300	Unit/year
$A_{mp}$	250	\$/order	$P_R$	12	\$/unit
$A_{mr}$	240	\$/order	$P_f$	85	\$/unit
$h_m$	1.5	\$/unit/ year	$A_R$	5000	\$/cycle
$h_{mp}$	0.7	\$/unit/ year	$A_F$	1000	\$/cycle
$h_{mr}$	0.8	\$/unit/ year	$A_U$	100	\$/order
$P_r$	150	\$/unit	$C_F$	35	\$/unit
$P_{mr}$	60	\$/unit	$C_R$	25	\$/unit
$P_{mp}$	65	\$/unit	$h_U$	0.3	\$/unit/ year
$A_r$	750	\$/order	$h_M$	0.4	\$/unit/ year
$O_m$	250	\$/order	$h_F$	2.5	\$/unit/ year
$a_p$	1.00	kg/unit	$O_E$	100	\$/order
$e_p$	37.03	kg CO <sub>2</sub> eq/ kg	$O_M$	100	\$/order
			$a_r$	0.8	kg/unit
			$e_r$	1.14212	kg CO <sub>2</sub> eq/ kg

Fig. 4 illustrates the flow of the algorithm used to solve the MINLP problem within the CDMS solution procedure, offering a comprehensive outline of the key steps involved in the solution process under the centralised scenario. A numerical illustration and discussion will follow to demonstrate the application of the proposed CLSC inventory model.

### 6. Numerical example

In this section, we present a numerical example to illustrate the application of the proposed model. Suppose a CLSC system consists of a supplier, a manufacturer, a retailer, and a material recovery facility (MRF). Table 2 gives a set of input parameters used to illustrate the proposed model.

The system deals with a CLSC inventory problem defined by a set of hypothetical input parameters, according to Parsa et al. [53]. The input value for the emission parameters are actual figures from textile and clothing products, as given in Appendix 3. We assume that all companies use diesel trucks to deliver the products, with the following

**Table 3** The optimisation result of m, n, and  $T_r$  at the retailer.

m	$n_{rx}$	N	$T_{r}$	$\mathrm{TP}_{\mathrm{r}}$
1	7.2265	7	0.8029	\$9593.57
		8	0.7864	\$9592.21
2	7.1024	7	0.8148	\$9625.93
		8	0.7977	\$9623.89
3	7.0606	7	0.8188	\$9636.82
		8	0.8016	\$9634.55
4	7.0396	7	0.8209	\$9642.28
		8	0.8036	\$9639.90
5	7.0269	7	0.8221	\$9645.57
		8	0.8048	\$9643.12
6	7.0185	7	0.823	\$9647.76
		8	0.8056	\$9645.27
7	7.0125	7	0.8236	\$9649.33
		8	0.8061	\$9646.80
8	7.0079	7	0.824	\$9650.51
		8	0.8066	\$9647.95
9	7.0044	7	0.8244	\$9651.42
		8	0.8069	\$9648.85
10	7.0016	7	0.8246	\$9652.16
		8	0.8072	\$9649.57

**Table 4** The optimisation results of k and l at the manufacturer.

$k_{rx}$	$l_{rx}$	K	1	$TP_m$
2.3566	2.6452	2	2	\$7579.77
		2	3	\$7586.97
		3	2	\$7576.67
		3	3	\$7583.87

characteristics:  $t_c$ =50 units;  $g_t$ =375 gallons/truck;  $e_t$ =10.08414 kgCO<sub>2</sub>eq/gallon [68]. The following section explains the optimisation result for each decision-making scenario.

### 6.1. Decentralised DM structure (DDMS)

The DDMS procedure begins by the retailer who determine their optimal solution, following by the manufacturer, the MRF, and the supplier. By setting  $m_0$ =10 and applying the proposed DDMS procedure, we first obtain the optimal solutions at the retailer side, including  $m^{*}$ ,  $n^*$ , and  $T_r^*$ , which is summarised in Table 3.

As shown in Table 3, the maximum retailer profit of \$9652.16 is obtained at m\*=10, n\*=7, and  $T_r*=0.8246$  year. Then, we continue to the next step which is to determine the optimal solutions for k and l at the manufacturer side as shown in Table 4.

From Table 4, we find that the maximum profit for the manufacturer for DDMS procedure is \$7586.97, and the optimal solutions for  $k^*$  and  $l^*$  are 2 and 3, respectively. Finally, we can determine the optimal solution for i at the supplier side. For  $n^*=7$  and  $T_r^*=0.8246$ , we obtained  $i_{rx}=1.1178$ . Hence, the optimal value of  $i^*$  is derived at 1 or 2. Since  $TP_s(i=1)=\$1667.89$  is greater than  $TP_s(i=2)=\$1654.89$ , we derived the optimal value of  $i^*=1$ . Finally, the optimal DDMS solutions of the given example are summarised in Table 5.

The maximum total profit that the DDMS procedure can obtain is \$23,213.78, with the retailer's ordering cycle time of 0.8246 years and number of shipments between parties are m\*=10, n\*=7, k\*=2, l\*=3, and i\*=1. By employing the DDMS scenario, the retailer, who acts as the SC leader, will get the highest profit ( $TP_r$ =\$9652.16), followed by the manufacturer ( $TP_m$ =\$7586.97), the recycling facility ( $TP_M$ =\$4306.77), and the supplier ( $TP_s$ =\$1667.89).

#### 6.2. Centralised DM structure (CDMS)

CDMS procedure involves integrated optimisation, where the objective function aims to maximise the joint total profit (*JTP*) of all

**Table 5**The optimal DDMS solution for the given case.

m*	n*	T <sub>r</sub> *	k*	1*	i*	TP <sub>r</sub> *	TP <sub>m</sub> *	$TP_M^*$	TP <sub>s</sub> *	TP*
10	7	0.8246	2	3	1	\$9652.16	\$7586.97	\$4306.77	\$1667.89	\$23,213.78

**Table 6**The experimental GA results.

$N_{pop}$	Max Gen	$P_{c}$	$P_{\rm m}$	T <sub>r</sub> * (year)	Number of shipments				JTP	Iteration	
					n*	m*	k*	1*	i*		
10	100	0.5	0.05	0.341920	20	3	7	1	1	\$ 26,598.71337	77
10	100	0.9	0.3	0.341888	20	3	7	1	1	\$ 26,598.71000	100
10	20	0.5	0.05	0.323485	20	3	5	1	1	\$ 26,583.28857	20
10	20	0.5	0.15	0.398549	16	5	10	2	2	\$ 25,565.00807	20
10	20	0.5	0.3	0.354187	20	2	10	1	1	\$ 26,572.11356	20
10	20	0.75	0.05	0.348703	20	4	8	1	1	\$ 26,594.33996	20
10	60	0.5	0.05	0.339839	20	3	6	1	1	\$ 26,598.80454	60
10	60	0.75	0.15	0.340120	20	3	6	1	1	\$ 26,598.81141	60
50	100	0.5	0.05	0.340216	20	3	6	1	1	\$ 26,598.81190	68
50	100	0.9	0.3	0.340217	20	3	6	1	1	\$ 26,598.81190	72
50	20	0.5	0.05	0.348659	20	3	8	1	1	\$ 26,595.15268	20
50	20	0.5	0.15	0.341001	20	3	7	1	1	\$ 26,598.66995	20
50	20	0.5	0.3	0.339185	20	3	6	1	1	\$ 26,598.75707	20
50	20	0.75	0.05	0.342728	20	3	7	1	1	\$ 26,598.68022	20
50	60	0.5	0.05	0.340220	20	3	6	1	1	\$ 26,598.81190	60
50	60	0.75	0.15	0.340217	20	3	6	1	1	\$ 26,598.81190	60←
100	100	0.5	0.05	0.340217	20	3	6	1	1	\$ 26,598.81190	66
100	100	0.9	0.3	0.340217	20	3	6	1	1	\$ 26,598.81190	69
100	20	0.5	0.05	0.343097	20	3	7	1	1	\$ 26,598.64298	20
100	20	0.5	0.15	0.340914	20	3	6	1	1	\$ 26,598.78696	20
100	20	0.5	0.3	0.341451	20	3	7	1	1	\$ 26,598.70206	20
100	20	0.75	0.05	0.345783	20	3	7	1	1	\$ 26,597.96032	20
100	60	0.5	0.05	0.340228	20	3	6	1	1	\$ 26,598.81190	60
100	60	0.75	0.15	0.340217	20	3	6	1	1	\$ 26,598.81190	60
← The b	est value										



**Fig. 5.** The convergence path of GA with N = 50, max gen=60,  $P_c$ =0.75 and  $P_m$ =0.15.

parties. In the proposed CDMS procedure, the decentralised solution is taken as an initial solution to find the centralised solution. We use MATLAB R2018b to compute the optimisation program according to the proposed CDMS procedure. By setting  $n_0$ =20, the CDMS optimisation result yielded an annual *JTP* of \$26,598.81. The retailer's ordering cycle time was determined to be 0.3402 years, with the number of shipments between parties being  $n^*$ =20 shipments,  $n^*$ =3 shipments,  $n^*$ =6 shipments,  $n^*$ =1 shipment, and  $n^*$ =1 shipment.

The optimisation problem in Eq. (13) is a complex MINLP problem, and ensuring a global optimum solution is quite challenging. As discussed by Gantovnik et al. [85], Wangsa and Wee [86], Xin et al. [87], and Jauhari et al. [88], genetic algorithm (GA), as one of the global optimisations (GO) methods, is considered efficient for solving NLP and MINLP problems. To evaluate the effectiveness of the proposed procedure, we will compare it to the solution obtained by the metaheuristic genetic algorithm (GA). The developed GA is run using the global

**Table 7**Comparison of the DDMS, CDMS, and DPS strategies.

Optimal Solution	DM Policy								
	Decentralised Scenario (DDMS)	Centralised Scenario (CDMS)	%improvement	Dispropornionate Profit-sharing (DPS)	%improvement				
$T_r^*$	0.8246	0.3402		0.8141					
n*	7	20		7					
m*	10	3		10					
k*	2	6		3					
1*	3	1		2					
i*	1	1		1					
$TP_r$	\$9652.16	\$8880.57	− <b>7.99</b> %	\$9652.16	+0.00 %				
$TP_m$	\$7586.97	\$11,309.40	+49.06 %	\$7587.90	+0.01 %				
$TP_M$	\$4306.77	\$4554.00	+5.74 %	\$4343.51	+0.85 %				
$TP_s$	\$1667.89	\$1854.84	+11.21 %	\$1731.38	+3.81 %				
JTP	\$23,213.78	\$26,598.81	+14.58 %	\$23,314.94	+0.44 %				

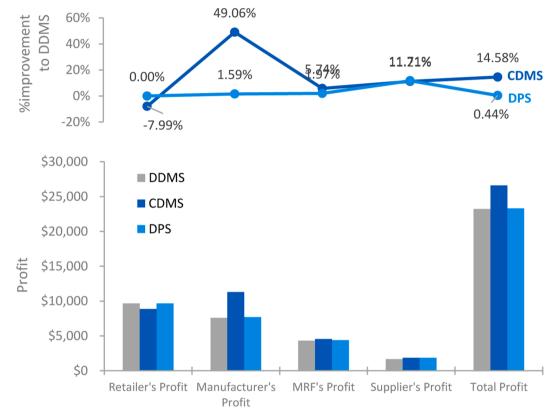


Fig. 6. Profit improvements of the CDMS and DPS compared to the DDMS solution.

optimisation toolbox in MATLAB R2018b. We apply the same numerical inputs to those employed in illustrating the proposed model.

The GA is conducted using the following parameters:

- (1) population size  $(N_{pop}) = [10, 50, 100]$
- (2) maximum number of generations (Max gen) = [20, 60, 100]
- (3) crossover probability ( $P_c$ ) = [0.5, 0.75, 0.9]
- (4) mutation probability  $(P_m) = [0.05, 0.15, 0.3]$

The experimental GA results are summarised in Table 6.

The results from Table 6 show that the best value for *JTP* with GA ( $JTP_{GA}$ ) can be obtained on 50 populations, a maximum generation of 60, a crossover fraction of 0.75, and a mutation rate of 0.15. We obtain the maximum  $JTP_{GA}$  at \$26,598.81/year, with  $T_r$ =0.340217, n = 20, m = 3, k = 6, l = 1, and i = 1. The convergence path graph of the GA solution is presented in Fig. 5. Furthermore, the deviation of the  $JTP_{CDMS}$  and  $JTP_{GA}$  can be calculated as follows

$$= \left| \frac{JTP_{CDMS} - JTP_{GA}}{JTP_{CDMS}} \right| = \left| \frac{26,598.81187 - 26,598.81190}{26,598.81187} \right| \times 100\%$$
$$= 0.0000000752\%$$

This indicates that the proposed CDMS procedure can produce a relatively effective solution with less than 0.0000001 % deviation from the GA result.

The CDMS solution provides higher profits for the system ( $JTP_{CDMS}$ =\$26,598.81) rather than the DDMS solution ( $JTP_{DDMS}$ =\$23,213.78). Nevertheless, further investigation shows that applying the CDMS scenario will drop the retailer's profit by 7.99 %, in contrast to the solution provided by DDMS. Conversely, the other parties involved will experience increases in their profits, particularly the manufacturer who benefits from a substantial profit enhancement of 49.06 %. The next section will discuss a profit-sharing mechanism to offer a fairer solution to all parties involved in the SC.

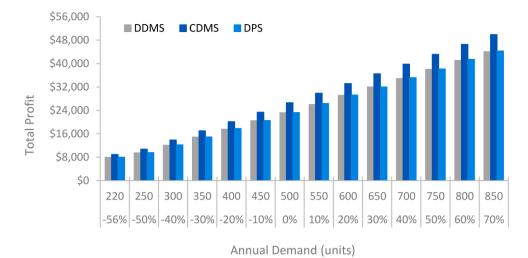


Fig. 7. Profit improvement over changes in demand.

#### 6.3. Profit-sharing mechanism

Centralised optimisation leads to a significant improvement (14,58%) in the overall profit of the system, but it subsequently decreases the profit of the retailer (-7.99%). To mitigate any potential conflict of interest, a profit-sharing mechanism is incorporated into the model. A disproportionate profit-sharing (DPS) mechanism, which is referred to Parsa et al. [53], is evaluated based on Eq. (48). Optimisation of the DPS problem is done using GA global optimisation toolbox in MATLAB R2018b, with  $N_{pop}=50$ , max\_gen=60,  $P_c$ =0.75, and  $P_m$ =0.15. The results are comprised in Table 7 and Fig. 6.

Several findings are obtained:

1. The CDMS strategy with no profit-sharing provides the highest total profits to the system. It increased the JTP by 14.58 % compared to the DDMS solution (see Table 7 and Fig. 6). However, the CDMS solution drops the retailer profit by 7.99 %. On the other hand, the manufacturer will experience a substantial increase in profits (49.06) %), followed by the supplier (11.21 %) and the MRF (5.74 %). This outcome is ineffective in providing fair improvements in profit for all parties involved. Certain parties may experience a financial loss due to the CDMS scenario. As discussed by Ben-Daya et al. [89], the CDMS solution provides significant insights and is particularly suitable for vertically integrated supply chains or chains that are predominantly controlled by the same company. However, if each of the parts of a supply chain are owned by various companies, it may not result in equal benefits for all of them. Profit-sharing coordination becomes a significant concern in such situations. A vertical integration is commonly applied in the fashion retail strategy of SPA (Specialty Store Retailer of Private Label Apparel). The SPA was established with the purpose of consolidating the whole value chain, encompassing the processes of design, manufacture, delivery, and sales. The objective is to optimise the performance of each component within a value chain [90,91]). UNIQLO, Gap, Zara, and H&M are examples of SPA companies that utilise vertical integration throughout their supply chain [92]. In this case, vertically integrated companies, such as SPA fashion companies, might employ the CDMS strategy to enhance their total profit.

2. The DPS strategy increased *JTP* by 0.44 % compared to the DDMS solution. Applying the DPS strategy keeps the retailer's profit at \$9652.16, while the manufacturer, supplier, and MRF improve profits by 0.01 %, 0.85 %, and 3.81 %, respectively (see Table 6 and Fig. 6). DPS offers a relatively small profit enhancement compared to CDMS. However, it ensures that no one is disadvantaged by the centralised optimisation. At this point, the profit increase will not be

as significant as it could be. The other parties must compensate to ensure no financial losses due to the centralised solution. As discussed by Parsa et al. [53], the DPS strategy might be preferable for SC with dominant player(s).

#### 7. Sensitivity analysis

This section discusses a sensitivity analysis to examine the impact of critical parameters on the models' optimal solution. The outcomes of the sensitivity analysis are summarised as follows:

1. Figs. 7 and 8 depict the effect of demand on the model's optimal solution. Fig. 7 shows that the total profit of the CLSC system is directly proportional to the demand. When the demand goes from 500 to 850, which is a 70 % increase, the total profit for the DDMS, CDMS, and DPS scenarios increases by 90 %, 88 %, and 90 %, respectively. The CDMS scenario shows a relatively small gain in total profit compared to the other scenarios.

2. Moreover, it is observed that the retailer's order cycle time in the DDMS scenario is more sensitive to changes in demand than in other scenarios (see Fig. 8). The results suggest an inverse relationship between demand and the retailer's order cycle time. The retailer's order cycle time for DDMS, CDMS, and DPS scenarios will decrease by 23 %, 16 %, and 24 %, respectively, in response to a 70 % increase in demand. Additionally, it has been discovered that the number of shipments from the manufacturer to the retailer is sensitive to the demand in both the DDMS and DPS scenarios. In contrast, the number of returned product shipments from the retailer to the MRF is more sensitive to demand in the CDMS scenario. The number of shipments of materials, k and l, are both influenced by changes in demand, where the effect is much more significant for the DDMS and PPS scenarios. Furthermore, demand does not significantly affect the number of shipments of energy-recoverable material from the MRF to the supplier.

3. Figs. 9 and 10 present the findings of the sensitivity analysis conducted on the product returns (Ru). In this case, any changes in Ru will be accompanied by an adjustment in the parameter X, which refers to the percentage of recycled material in a product. If there is an increase in the number of returned products, with a fixed value of  $\alpha$ , X should be adjusted to a higher level. From Fig. 9, the impact made by Ru on the total profit of the three scenarios is relatively low. In the DPS scenario, a decline in Ru can consequently lead to an increase in TP. However, with a+25% increase in Ru, the DPS scenario can increase its total profit by +1%, whereas as Ru continues to increase, TP would gradually decrease. In the DDMS

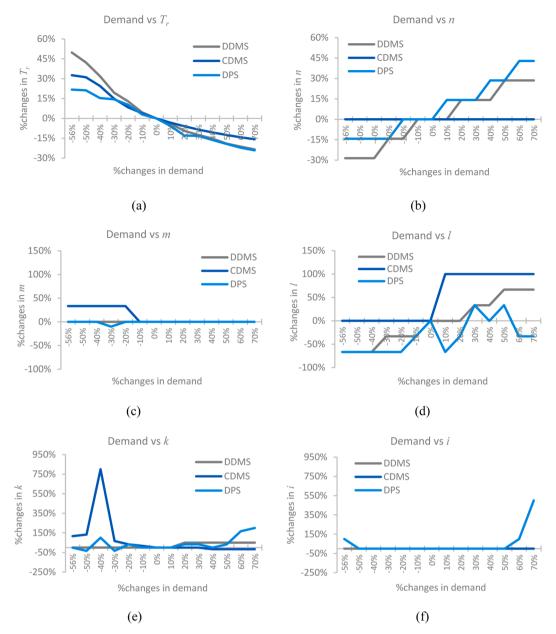


Fig. 8. Effect of demand on decision variables.

scenario, a reduced Ru by -25% increases TP by +8%. However, as the value of Ru continues to decline, the profit will eventually decrease. Compared to the DDMS and DPS scenarios, the impact of Ru on the CDMS scenario is insignificant. It creates a slight concave downward slope, with the highest total profit (+0.3%) attained at a Ru of +25%.

4. Furthermore,  $R_u$  impacts the optimal solutions of n,  $T_n$ , k, l, and i (see Fig. 10). The impact of a variation in  $R_u$  on the retailer's ordering cycle time,  $T_r$ , is most significant for the DPS and CDMS scenarios. In the case of the CDMS scenario, an increase in the number of returned items should lead to a decrease in the ordering cycle time. Conversely, a decline in the number of returned products should increase in the ordering cycle time. An increase in the return parameter,  $R_u$ , is accompanied by a decrease in the number of shipments made by the manufacturer to the retailer, n, and the number of shipments made by the supplier to the manufacturer, l. The influence of a change in  $R_u$  on n is most significant in the DPS scenario, whereas in the CDMS scenario, n remains unaffected by alterations in  $R_u$ . In contrast, the impact of  $R_u$  on the number of

shipments of product returns, m, is highest in the CDMS scenario. When the quantity of  $R_u$  is decreased by 25 %, it is found that the optimal solution for delivering energy-recoverable items from the MRF to the supplier is  $i^*=0$ , which means no shipment is necessary. When the amount of collected used products is very low, it becomes costly for the CLSC system to use them as an energy source due to ordering, holding, and emission costs.

5. As studied by Parsa et al. [53], the postconsumer recycled content parameter, X, has an important relationship with TP. For a given range of recycling costs,  $C_R$  suggested that the maximum profits will be obtained at a certain level of X. In this study, we investigate how different decision-making scenarios react to changes in  $C_R$ . Fig. 11 summarises the effect of changes in  $C_R$  on total profits and the optimum level of X. An increase in recycling costs significantly influences the selection of the optimal value for parameter X. The results also suggest that the total profits in DDMS and DPS scenarios are more sensitive to changes in the  $C_R$  parameter, compared to the CDMS scenario. However, the optimal value for post-consumer recycled content in the CDMS scenario is relatively more sensitive

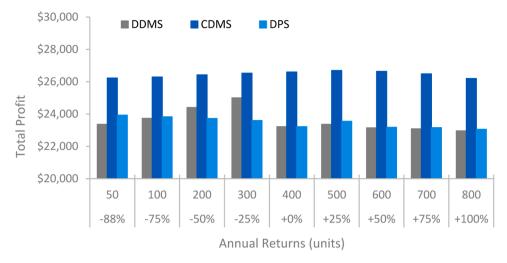


Fig. 9. Profit improvement over changes in product return.

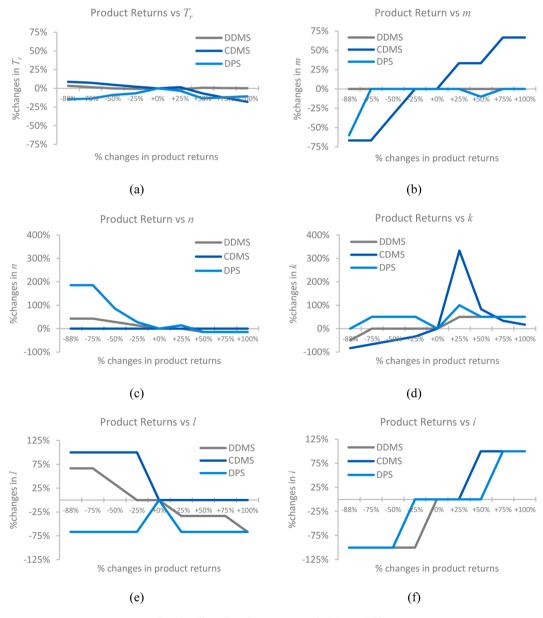
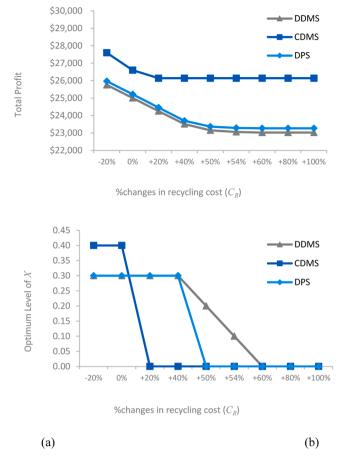


Fig. 10. Effect of product returns on decision variables.



**Fig. 11.** Effect of  $C_R$  on total profits (a) and optimum level of X (b).

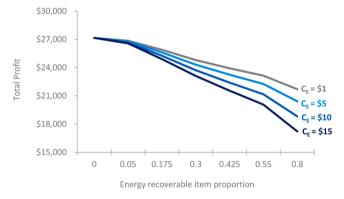


Fig. 12. The effect of different settings of e and  $C_E$  on total profit.

to increased recycling costs. In the CDMS scenario, when the cost of recycling increases by 20 %, the maximum profit is achieved at X=0, suggesting that the system shouldn't contain any recycled material in the product. Meanwhile, in the DDMS and DPS situations, the optimum value for X will gradually decrease from its initial X=0.3 to X=0 as the  $C_R$  increases by +60%. It suggests that when recycling costs are high, the system should respond by decreasing the level of recycled materials in the product.

6. To understand how the energy recovery affects the optimal solution of the model, the authors analyse the impact of different amounts of energy recoverable items on various settings of energy recovery cost,  $C_E$ , while assuming a fixed value for f. The analysis is done for the DDMS scenario. The results of the variations in the

energy recoverable item proportion (e), ranging from 0 to 0.8, at  $C_E = \{\$1, \$5, \$10, \$15\}$ , are depicted in Fig. 12.

Fig. 12 shows that TP will monotonically decrease at a higher proportion of energy-recoverable items. The total profit of the system is maximum at e = 0 or no energy recoverable items at all, suggesting that all product returns may be fully recovered, either as recycled material or refurbished products. This result is evident since the system will benefit more from selling recycled materials and refurbished products, as they have higher levels of added value. Nevertheless, in a practical sense, the quality of product returns is unpredictable, posing a challenge to ensure that all returned products meet high-quality standards. Companies can efficiently handle the range of product returns by employing strategies such as conducting thorough quality inspections throughout the collection phase of product returns and intentionally creating and producing high-quality items with recyclable materials. However, in comparison to waste disposal, energy recovery has the potential to provide higher revenues for the system by extracting value from items that cannot be recovered. Energy recovery can provide economic advantages for the MRF and supplier by converting non-recoverable used items into energy.

#### 8. Conclusions

This study developed an inventory model of a closed-loop supply chain (CLSC) with three recovery alternatives: product refurbishing, material recycling, and energy recovery. The CLSC system consists of a retailer, a manufacturer, a Material Recovery Facility (MRF), and a supplier. The retailer serves as a collector of product returns, and the MRF is responsible for recovering these returns. Accordingly, energy recovery is utilised as an alternative energy source in the upstream supply chain, i.e., the supplier, and it enables the creation of a closedloop recycling system with 100 % value recreation and no disposal. The model's objective is to maximise the total profit of the system and determine the optimal retailer's cycle time and the optimal number of shipments between parties. Three scenarios were analysed, i.e. decentralised decision-making structure (DDMS), centralised decision-making structure (CDMS), and disproportionate profit-sharing (DPS). The MINLP problem is optimised using a hybrid analytical-numerical procedure and subsequently compared with a metaheuristic approach, i.e., the genetic algorithm (GA). Based on the results, the proposed CDMS procedure produced a relatively effective solution with less than 0.0000001 % deviation from the GA result.

This study provides valuable insights into how different supply chain coordination scenarios can impact a CLSC, particularly when energy recovery is involved. The centralised CDMS structure provided the highest total profit for the system, making it an attractive option for vertically integrated supply chains, such as those in SPA fashion companies, where maximising overall profit is a priority. On the other hand, the profit-sharing mechanism under the DPS scenario generated a fairer distribution of profits among all the parties involved. This makes DPS a great option where fairness and incentive alignments across partners are crucial in a more complex supply chain.

The sensitivity analysis revealed that demand significantly impacts the total profit and decision variables, especially in the Decentralised Decision-Making System (DDMS) scenario. For example, the retailer's order cycle time and the number of product return shipments were much more sensitive to changes in demand compared to other scenarios. Interestingly, the number of shipments for energy-recoverable materials remained largely unaffected by demand, suggesting that energy recovery could offer more stability in fluctuating markets. From this findings, an improvement for demand forecasting and plan for demand variability might be crucial to enhance resilience, optimise operations, and maximise profitability. On the other hand, product returns were observed to significantly influence almost all decision variables in the model. However, the impact of product returns on total profit is rather

insignificant. In addition, an increase in recycling costs significantly influenced the selection of the optimal value for the post-consumer recycled content. The results also suggested that the total profits in the DDMS and DPS scenarios are more sensitive to recycling costs than those in the CDMS scenario. The findings implied that when the costs of recycling are high, the system should adjust by reducing the amount of recycled materials used in the product. Therefore, the remaining non-recycled items can be recovered by an alternative process known as energy recovery, which can be utilised at the supplier side or at the secondary supply chain.

The integration of energy recovery from waste into the CLSC system provides economic benefits for the entire supply chain. However, a comprehensive analysis of energy recovery may be required by concurrently evaluating the primary and secondary energy sources utilised by the supplier. In the proposed model, the supplier employs alternative energy sourced from non-recoverable consumer waste. Nonetheless, the volume of waste collected from consumers remains highly uncertain and limited. Consequently, alternative energy from waste will merely serve as a secondary energy source, necessitating that the supplier procures a primary energy source from an outside supplier. The supplier will possess a hybrid production facility that employs two energy sources: regular and green production facilities. The regular production facility depends on a primary energy source procured from an outside supplier. The green production facility employs a secondary energy source derived from consumer waste. Integrating both energy sources into a single model enables a thorough analysis of the procurement allocation of each energy source, the economic advantages of energy recovery, and the carbon emissions generated by the hybrid production facility.

In addition, this model assumes that the supplier did the process of converting waste into energy. Therefore, the non-recoverable consumer waste was delivered from the MRF to the supplier in the form of used items. This needs to be compared with the scenario where energy recovery is carried out by a Material Recovery Facility (MRF). Is it going to be more cost-effective or the opposite? The involvement of a fourth party as a WTE producer may also be considered for further analysis regarding the proposed energy recovery option.

The earlier model developed by Parsa et al. [53] focuses on maximising profits through profit-sharing mechanisms while considering only material recycling for product recovery. In contrast, our model expands this framework by incorporating energy recovery, delivering both financial and environmental benefits. Quantitative results highlight the advantages of our approach. For instance, under a scenario with moderate energy costs, our model achieved a 12 % higher total profit due to the additional value derived from energy recovery. Additionally, while Parsa et al. [53] model showed significant profit declines (up to 10 %) when recycling costs exceeded a critical threshold, our model maintained stability by shifting to energy recovery, demonstrating its resilience and adaptability.

These findings underline the superiority of our model in addressing complex cost structures and highlight the potential for energy recovery to enhance supply chain profitability in dynamic and resource-intensive environments. In conclusion, integrating energy recovery into CLSCs not only enhances profitability but also promotes sustainability. While both models provide valuable insights, our extension to include energy recovery offers a more robust and flexible solution, especially for industries where energy costs and waste management are significant considerations.

This study may be developed by involving supply chain contracts and coordination schemes, such as revenue-sharing, cost-sharing, buy-back contracts, or Vendor-Managed Inventory (VMI). In addition, this study presents a deterministic scenario in which the quantities of consumer demand and returns are predetermined and known. Further studies are required to handle the uncertain environment. This model assumes an instantaneous inspection; hence, it may be possible to alleviate this assumption by considering the inspection rate of product returns. Another possible development is to include imperfect production processes, inspection errors, adjustable production rates, and the analysis of the application of green and production facilities.

#### **Author statements**

The paper is an original contribution, based on the doctoral research of the first author who is supervised by the 2nd and 3rd authors. It is not under consideration in any other publication.

### CRediT authorship contribution statement

Anindya Rachma Dwicahyani: Writing – original draft, Software, Funding acquisition, Formal analysis, Data curation. I Nyoman Pujawan: Writing – review & editing, Supervision, Methodology, Conceptualization. Erwin Widodo: Writing – review & editing, Supervision, Methodology, Formal analysis, Conceptualization.

### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this article.

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### Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.orp.2025.100326.

### Appendix 1

The first partial derivative of JTP w.r.t  $T_r$ , m, k, l, and i, respectively, for the Centralised DM Structure are given as follows.

$$\begin{split} \frac{\partial JTP}{\partial T_r} &= \frac{1}{2} \left( -Dh_r - \frac{D^2h_mn}{P} + \frac{2Dh_m(n-1)(D-P)}{P} + \frac{h_{ru}(m-mn-2)R_u}{m} + \frac{f^2Fh_pnR_u^2}{D_f^2 - D_f F} + \frac{2A_r}{T_r^2} + \frac{2A_r}{nT_r^2} + \frac{2A_m}{nT_r^2} + \frac{2A_{mr}}{nT_r^2} + \frac{2A_{mr}}{nT_r^2} + \frac{2A_{mp}l}{nT_r^2} + \frac{2A_{dm}l}{nT_r^2} + \frac{2IO_g}{nT_r^2} + \frac{2O_m}{T_r^2} + \frac{2IO_s}{nT_r^2} - \frac{D^2h_{mp}n(1-X)}{nT_r^2} \\ &- \frac{D^2h_s(l-1)n(1-X)}{lP} - \frac{D^2h_{mr}nX}{kP} + \frac{h_{ru}(n-1)R_u^2\alpha}{R} - \frac{Fh_unR_u^2\alpha^2}{mR^2} - \frac{h_M(2+k)nR_u^2\alpha^2}{kR} \\ &- \frac{h_E(1+i)nR_u^2\alpha(1-f-\alpha)}{iR} + \frac{h_EnR_u^2(1-f-\alpha)^2}{E_n} - \frac{h_UnR_u^2\alpha(2-2f-\alpha)}{mR} ) \\ &\frac{\partial JTP}{\partial m} = \frac{\left(R_uT_r(2h_{ru}R^2 + h_UnR_u\alpha(F\alpha + R(2-2f-\alpha)))\right)}{(2m^2R^2)} - \frac{A_U}{nT_r} \\ &\frac{\partial JTP}{\partial l} = \frac{1}{2} \left(h_MR_u\alpha + \frac{-2A_{mr} - 2_{OM} + \frac{n^2T_r^2(D^2h_{mr}RX + 2h_MPR_u^2\alpha^2)}{k^2PR}}{nT_r}\right) \\ &\frac{\partial JTP}{\partial l} = \left(\frac{\frac{D^2(h_{mp} - h_s)n^2T_r^2(1-X)}{2nT_r} - 2A_{mp} - 2O_s}{2nT_r}\right) \\ &\frac{\partial JTP}{\partial l} = \left(\frac{\left(\frac{h_En^2R_u^2T_r^2\alpha(1-f-\alpha)}{(l^2R)}\right) - 2A_E - 2O_E}{2nT_r}\right) \end{aligned}$$

#### Appendix 2

The leading principal minors of the matrix  $H_3$  are given as follows

$$\begin{split} |H_1| &= -\frac{2y_8}{nT_r^3} \langle 0, \text{ if } y_8 \rangle 0 \\ |H_2| &= \frac{1}{4m^4n^2R^4T_r^4} \left( 4A_Um^2nR^2R_uT_r^2y_5 + nR_uT_r^2 \left( 2R^2 \left( 4my_8 - h_{ru}nR_uT_r^2 \right) - 2(1-f)h_Un^2RR_u^2T_r^2\alpha - h_Un^2 (F-R)R_u^2T_r^2\alpha^2 \right) y_5 \right) - 4A_U^2m^4R^4x^4 \\ &> 0, \text{ if } R^2 \left\langle \frac{nR_uT_r^2y_5}{m^2}; \ R \left( 4my_8 - h_{ru}nR_uT_r^2 \right) \right\rangle h_Un^2R_u^2T_r^2\alpha ((1-f) + (F-R)); \text{ and } \alpha < 2 - 2f \\ \\ |H_3| &= \frac{1}{4k^4m^4n^2P^2R^5T_r^3} \left( mRR_u \left( 2k^2PR(A_{mr} + O_M) + n^2T_r^2y_6 \right)^2y_5 + knPy_6 \left( \left( 2A_Um^2R^2 + nR_uT_r^2y_5 \right)^2 - 8mny_8R^2R_uT_r^2y_5 \right) \right) < 0, \text{ if } y_5 \\ &< \frac{knPy_6 \left( \left( 2A_Um^2R^2 + nR_uT_r^2y_5 \right)^2 - 8mny_8R^2R_uT_r^2y_5 \right)}{mRR_u \left( 2k^2PR(A_{mr} + O_M) + n^2T_r^2y_6 \right)^2} \\ |H_4| &= \frac{1}{4k^4l^4m^4nP^3R^5T_r^2} \left( kmR^2R_u \left( 2l^2(A_{mp} + O_S)P + D^2y_7n^2T_r^2 \right)^2y_6y_5 - D^2ly_7 \left( mRR_u \left( 2A_{mr}k^2PR + 2k^2O_MPR + n^2T_r^2y_6 \right)^2(y_5) \right. \\ &+ knPy_6 \left( \left( 2A_Um^2R^2 + nR_uT_r^2y_5 \right)^2 - 8mny_8R^2R_uT_r^2(y_5) \right) \right) > 0, \text{ if } y_8 > \frac{\left( 2A_Um^2R^2 + nR_uT_r^2y_5 \right)^2}{8mnR^2R_uT_r^2(y_5)} \end{split}$$

$$\begin{split} |H_{5}| &= \frac{1}{4i^4k^4l^4m^4P^3R^6T_r}R_u \\ &\left(D^2klmPRy_5y_6y_7 \left(2i^2R(A_E+O_E) + h_En^2R_u^2T_r^2\alpha(1-f-\alpha)\right)^2 + h_EiR_u\alpha(1-f-\alpha) \right. \\ &\left(kmR^2R_uy_5 \left(2l^2P(A_{mp}+O_s) + D^2y_6y_7n^2T_r^2\right)^2 - lD^2y_7 \right. \\ &\left(mRR_u \left(2k^2PR(A_{mr}+O_M) + n^2T_r^2y_6\right)^2y_5 - knPy_6 \right. \\ &\left. \left(8mR^2nR_uT_r^2y_5y_8 - \left(2A_Um^2R^2 + nR_uT_r^2y_5\right)^2\right)\right)\right)\right) \\ &< 0, \text{ if } D^2klmPRy_5y_6y_7 \left(2i^2R(A_E+O_E) + h_En^2R_u^2T_r^2\alpha(1-f-\alpha)\right)^2 \\ &< \left(h_EiR_u\alpha(1-f-\alpha) \left(lD^2y_7 \left(mRR_u \left(2k^2PR(A_{mr}+O_M) + n^2T_r^2y_6\right)^2y_5 - knPy_6 \right) \right) \\ &\left. \left(8mR^2nR_uT_r^2y_5y_8 - \left(2A_Um^2R^2 + nR_uT_r^2y_5\right)^2\right)\right) - kmR^2R_uy_5 \\ &\left. \left(2l^2P(A_{mp}+O_s) + D^2y_6y_7n^2T_r^2\right)^2\right)\right) \\ &\text{with,} \\ &y_5 = \left(2h_{Ru}R^2 + h_UnR_u\alpha(F\alpha + R(2-2f-\alpha))\right) \\ &y_6 = \left(D^2h_{mr}RX + 2h_MPR_u^2\alpha^2\right) \\ &y_7 = \left(h_{mp}-h_s\right)(1-X) \\ &y_8 = \left(A_F + A_m + A_R + A_{ru} + A_s + A_{E}i + A_{mr}k + A_{mp}l + A_Um + A_rn + iO_E + nO_m + kO_M + lO_s\right) \end{split}$$

Appendix 3. Emission parameters related to textile and clothing items

Input parameter	Value	Units	Ref
Emission parameters relate	d ro recycling process		
$e_r$	1.14212	kgCO <sub>2</sub> eq/kg	Espinoza Pérez et al. [93]
$a_r$	0.8	kg/unit	
Emission parameters relate	d to supplier production proc	ess	
- Spinning	0.840	kgCO <sub>2</sub> eq/kg	Luo et al. [94]
- Weaving	17.75	kgCO <sub>2</sub> eq/kg	
- Finishing	26.67	kgCO <sub>2</sub> eq/kg	
$e_{\scriptscriptstyle S}$	45.26	kgCO <sub>2</sub> eq/kg	
$a_s$	1.0	kg/unit	
Emission parameters relate	d to manufacturer	C	
- Making-up	16.20	kgCO2eq/kg	Luo et al. [94]
- Denim washing	20.83	kgCO <sub>2</sub> eq/kg	
$e_p$	37.03	kgCO <sub>2</sub> eq/kg	
$a_p$	1.0	kg/unit	
Emission parameters relate	d to energy recovery	-	
$e_E$	2.218	kgCO <sub>2</sub> eq/kg	Semba et al. [22]
$a_e$	0.8	kg/unit	

# Data availability

Data will be made available on request.

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