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Article

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Facility location based on Adjusted Present Value

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ABSTRACT

Supply chain network design aims to optimize strategic decisions such as facility location decisions.

These decisions have a major impact on the supply chain, but also on the financial value of the company. However, financial considerations are often omitted from facility location mathematical models.

This paper addresses the challenge of identifying a relevant financial indicator that can be practically implemented in facility location models across different industries.

This paper makes several contributions: the Adjusted Present Value (APV) is identified as such a financial indicator; we propose a mathematical formulation that embeds the APV in a facility location model maximizing firm value; computational experiments demonstrate the tractability of the model. Finally, we compare the mathematical model with a sequential approach that first optimizes logistical decisions and then financial decisions. The proposed model improves the sequential approach up to 5.5%, increases the market coverage and anticipates facility location decisions.

1. Introduction

Optimizing strategic decisions in supply chain network design involves key decisions such as locating facilities, determining their capacity, and deciding when to open them. It also involves determining the product flows in the logistics network under consideration. Supply chain network design has been the subject of extensive literature and many reviews (see, e.g. [1,2]). Most of this literature identifies the facility location problem and its extensions as the core of supply chain network design models. The problem is known to be NP-hard.

The primary goals of facility location in the context of supply chain management are (i) to define the network itself, by locating facilities and defining the allocation of product flows to these facilities and (ii) to determine the optimal product flows in this logistics network. Classically, this network is optimized either by minimizing total logistics costs or by maximizing the profit generated by the distribution of goods.

Ideally, the corresponding mathematical models should take into account the interactions between supply chain management and other company departments (e.g. marketing, human resources, finance) as well as external factors (e.g. competitors, consumer behavior, financial markets).

Despite their importance, it is striking that financial instruments rarely appear as full components of facility location models. However, most companies finance their strategic investments by resorting to debt. It therefore becomes clear that the strategic planning of logistical and financial decisions must go hand in hand. This would enhance cost management by enabling more effective capital allocation and debt financing, and reducing operational expenses. By analyzing financial implications, companies can strategically allocate resources to ensure that facility investments are both cost-effective and in line with their overall financial objectives. This assessment of location decisions further enhances resource allocation, leading to investments that are efficient and aligned with financial goals. Such integration improves investment strategies, secures advantageous financing terms, and facilitates better negotiation opportunities, resulting in lower borrowing costs and greater financial stability.

In addition to internal financial benefits, well-informed location decisions can have other positive effects, like local economic development. Choosing sites that contribute to job creation and leveraging available economic incentives can stimulate local economies, creating additional value beyond the company's immediate financial interests [3]. Although these factors contribute to the company's reputation and long-term success, they are beyond the scope of this paper.

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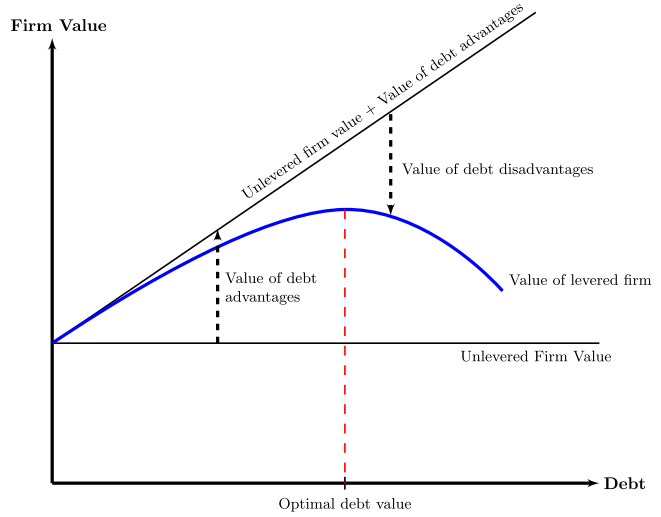

Fig. 1. Trade-off theory of capital structure.

[4] mentioned the strong interaction between the financial factors and the strategic planning. In closed-loop supply chains, [5] noted few studies consider these as decision variables. Traditional supply chain management approaches consider the cash flows generated by operational decisions, but not those resulting from financial decisions. [6] observe that the majority of models only broadly examine the costs and revenues associated with the design of supply chains and ignore other financial elements that are related to its future performance. Therefore, a strong motivation of this paper is to jointly consider the impact of financial and operational decisions on the location strategy.

Financial theory seeks to maximize the value of the company and states that the value of any asset is equal to the present value of the future cash flows discounted at an appropriate discount rate called the *cost of capital*. The discounted cash flows valuation models can be found in any finance text book for readers that would like to explore the model more in detail, for example [7,8].

In corporate finance, key decisions about sources of finance are based on both equity and debt. The theoretical and practical question is what should be the mix of equity and debt that maximizes the company's value? To answer this question, the trade-off theory of capital structure [9] explores two opposite consequences of the debt: *(i)* the tax advantages associated with the fact that paying interest on debt reduces the corporate taxes and *(ii)* the distress disadvantages associated with the fact that, as the firm's leverage increases, so does the probability of default and hence the Expected Bankruptcy Costs (EBC). The trade-off theory states that the advantages of debt (*tax shield benefits*) are balanced by its disadvantages (*bankruptcy costs*).

According to this theory, there is an optimal debt level that maximizes a firm's value (Fig. 1). As debt increases (horizontal axis), both the tax shield benefit and the bankruptcy cost of debt rise. The horizontal solid line represents the unlevered value of the firm, i.e., the value of the firm without taking into account the cash flow consequences of financial decisions. The tax shield, represented by the solid diagonal line, is linearly increasing, while the cost of bankruptcy is convex non-linear. The value of the firm (blue curve) is therefore a concave non-linear function. The peak of this curve indicates the optimal debt value. This paper explicitly introduces the benefits and disadvantages of debt in order to determine the optimal mix of debt and equity a company should settle to fund its facility location strategy.

The starting point for most discussions on the impact of financing on firm valuation is the seminal work of Modigliani and Miller [10]. Since then, several methods have been proposed for incorporating the effects of debt into cash flow discounting valuation models. The

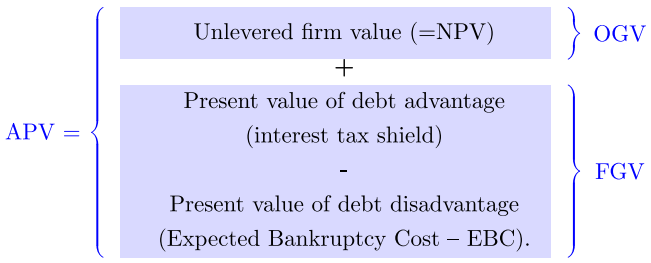

Fig. 2. Calculation of the APV.

two main approaches are either (*i*) to incorporate the consequences of financial decisions into the discounting rate: Free Cash Flow (FCF) discounted at the Weighted Average Cost of Capital (WACC), Capital Cash Flows (CCF) discounted at the pre-tax WACC, Economic Value Added (EVA) discounted at the WACC (also known as the Market Value Added (MVA)) [11]; or (*ii*) to include them in the Adjusted Present Value (APV).

According to [12], all of the methods proposed in the literature lead to the same valuation of the company when properly applied. However, some methods are more appropriate than others in specific cases.

This paper focuses on the APV [13], which represents the value of a leveraged firm (taking into account the cash flow consequences of financial decisions). It is a widely used measure of value creation in the context of discounted cash flow valuation models. The APV has several main advantages: (i) It offers detailed information on the factors contributing to the firm's value [14], distinguishing the unlevered firm value from the value added by financial decisions. (ii) It allows a detailed analysis of the value derived from the choice of a particular financial structure by isolating the contribution of tax benefits to the corporate value creation [14]. (iii) As explained by [15], the change in leverage requires a periodic complex reassessment of the WACC. On the contrary, the APV works under both constant and variable debt ratios over the forecast period. Thus, maximizing APV satisfies our goal of incorporating financial considerations into a facility location model. More specifically, APV allows the advantages and disadvantages of debt to be incorporated into the company's cash flows. If the company does not use debt, APV is reduced to the classic NPV model.

The unlevered firm value, also known as the base case Net Present Value (NPV) [16], is the value generated by the operational decisions alone. This is why we call it *Operationally Generated Value* (OGV), as opposed to the *Financially Generated Value* (FGV). OGV is calculated by discounting future operational cash flows at the unlevered cost of equity (or debt-free cost of capital). FGV is the present value of the advantages and disadvantages of debt.

As shown in Fig. 2, the APV is determined by two types of decisions. First, logistical decisions consist of selecting facilities from a set of candidate locations to deliver goods to a set of customers. This affects only the OGV. Second, financial decisions determine the debt level needed to finance logistics investments, staggered over time.

This only affects the FGV. This led us to compare two approaches, which consist of solving the proposed MILP model sequentially (OGV then FGV) or all at once. These two approaches are described in detail in Section 6.

The main contributions of this paper are: (i) to propose the Adjusted Present Value (APV) as a financial indicator used to optimize the future impact of strategic supply chain decisions on the future value of the firm (instead of the classical cost function), (ii) to propose a mixed integer linear programming (MILP) model that integrates facility location decisions and their financial consequences over a strategic horizon, (iii) to assess the tractability of this MILP by state-of-the-art solvers, and (iv) to evaluate the potential benefits of integrating financial considerations into a facility location model, through a comparison

with a sequential approach that optimizes logistical decisions first and the financial decisions second.

The structure of the paper is as follows. Section 2 positions this work in the related literature. Section 3 describes the assumptions and the global structure of our problem. Section 4 presents the mathematical formulation. Section 5 describes the data generation principles used to create new instances. Section 6 presents the results of our computational experiments. Section 7 presents some managerial insights and concludes with future research directions.

2. State of the art

In their review on facility location and supply chain management, [1] classify financial factors in three categories: (i) *international factors*, including taxes, duties, tariffs, exchange rates, transfer prices, and local content rules, (ii) *financing and taxation incentives* offered by governments and (iii) *investment expenditures*, usually limited by the total available budget. In the first category, [17] pleads for the consideration of different tax regimes and duties, exchange rates, transfer prices and differences in operating costs. [18] propose a facility location model in offshoring context, with both tactical and strategic decision levels. The financial decisions are the transfer pricing and two variables allocating logistics costs to various stakeholders of the supply chain.

Budget constraints can be found in numerous facility location models. For example, [19] addressed a facility location problem considering budget constraints with the aim of minimizing the overall distance traveled. Similarly, [20] proposed a multi-period mathematical model minimizing total business costs. The available budget limitation is the incorporated financial factor. [21] addressed a multi-period stochastic facility location problem to maximize the benefits of the firm, with a budget constraint on investments and the possibility of setting a target for the return on investment. See also, e.g., [22–26]

In many papers, the only financial consideration is to optimize NPV. Although NPV is an appropriate measure of the profitability of an investment, it does not capture the value contributed by financing decisions [16,27].

In the following paragraphs, we review publications according to their approach measuring financial values.

Free cash flow (FCF) discounted at WACC. [27], propose an integrated strategic-tactical model whose objective function, called *Corporate Value*, is the difference between the FCF discounted at WACC and net debt. To cope with the difficulty of a variable WACC rate, they consider it as a fixed parameter over the planning horizon. This approach favorably compares with the traditional NPV-oriented model. [28] extend the work of [27] by introducing the uncertainty of demand, price, and interest rates to the model. The presented stochastic model is then compared with a deterministic model, indicating a significant performance improvement.

Economic value added (EVA). EVA is an absolute key figure based on earnings that focuses on the performance of a single period [29], while all the following publications deal with a multi-period planning horizon. In these cases, the objective function is the sum of forecasted EVAs.

[30] develop a mathematical model for designing a four-echelon supply chain under demand uncertainty, optimizing net created value, measured by EVA. They assume a constant WACC and compare their financial model with a non-financial one that ignores financial analysis. While the financial model creates more shareholder value, the non-financial model shows higher return on equity.

The same authors propose a bi-objective MILP model that captures trade-off values between financial performance, measured with EVA, as the first objective and credit solvency, using Altman Z-score [31], as the second objective function [32] . Our APV model relies on a similar trade-off between the opposite effects of interest tax shield benefit and the present value of the debt. [33] address a three-echelon, multi-period, multi-item closed-loop, strategic and tactical facility location problem. EVA is maximized while the WACC is defined as a constant parameter. [34] propose a multi-product, multi-period, four-echelon model addressing financial decisions like cash and risk management, capital structure, and revenue/cost management. They evaluate financial dimensions (corporate value, Change in Equity, and EVA) against profit maximization.

[35] propose a fuzzy MILP model to design a global supply chain network that considers logistical and financial flows simultaneously. To assess the financial performance, EVA is maximized. [36] propose a MINLP model analyzing the impact of economic uncertainty on supply chain financial health. The model designs a multi-period closed-loop supply chain to maximize EVA under demand uncertainty. Besides EVA, the authors also use NPV and WACC as objective functions, with a fixed WACC rate.

EVA is also used as the objective function in the multi-period model proposed by [37], which considers debt repayments and new capital entries as decision variables. Still, the WACC is assumed to be a fixed parameter. Their model leads to higher EVA in comparison to the model of [30]. [38] address a stochastic supply chain network design model which aims at maximizing EVA. Interest rates (short-term and long-term), expected return on stock market, and risk-free rate of interest are subject to uncertainty.

Market value added (MVA). MVA is a tool to transform EVA to a multi-period basis. It measures the present value of future EVAs by discounting them at the WACC rate [29]. Unfortunately, the MVA loses the EVA's property of being compatible with fluctuating debt ratios. [39] address a multi-period, multi-product sustainable supply chain network design problem maximizing the MVA of the firm. Yet, the authors consider the WACC rate as a fixed parameter.

The reviewed literature provides a range of methodologies for integrating financial considerations into facility location models, each with its strengths and limitations. Free Cash Flow (FCF) and Economic Value Added (EVA) are commonly used but have constraints related to fixed financial parameters. Market Value Added (MVA) addresses multi-period analysis but struggles with fluctuating debt ratios. Notably, Adjusted Present Value (APV) is identified as a promising approach for handling both multi-period models and variable debt ratios. Table 1 summarizes these approaches and highlights the potential for APV to offer a more adaptable and comprehensive financial valuation framework for facility location decisions.

To our knowledge, Capital Cash Flow (CCF) and Adjusted Present Value (APV) have not been employed as valuation methods in facility location models and remain relatively unexplored in this context. This observation is corroborated by the recent review conducted by [6].

3. Problem description and assumptions

In this section, we describe the main settings and assumptions of the multi-period capacitated facility location problem that will be formally presented in Section 4. The notations are summarized in Appendix A.

3.1. General setting

The main goal of this paper is to propose a mathematical model for the joint optimization of facility location and financing decisions. We want to ensure that: (i) the differences between the proposed model and traditional models based on cost minimization must be tangible and interpretable, and (ii) the logistical assumptions must be as "simple" as possible, in order to highlight the financial interpretations.

The first feature motivated us to consider a company starting from a blank page. There is no operating facility at the beginning of the time horizon, nor current loans. Therefore, fulfilling all customer demands is not mandatory. This assumption supports a gradual expansion of

Table 1
Financial valuation approaches in facility locations models.

Valuation approach	Compatibility with		References
	Multiple periods	Fluctuating debt ratio	
Free Cash Flow (FCF)	✓	×	[27] [28] [34]
Economic Value Added (EVA)	×	✓	[30] [32] [34] [33] [35] [36] [37] [38]
Market Value Added (MVA)	✓	×	[39]
Capital Cash Flow (CCF)	✓	×	–
Adjusted Present Value (APV)	✓	✓	*this paper*

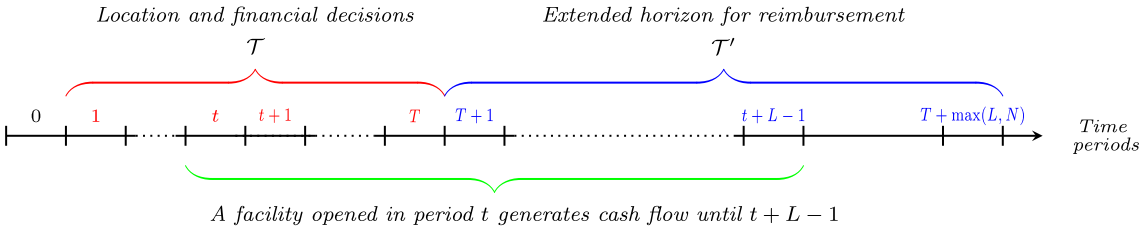

Fig. 3. Time scale of supply chain decisions and their financial impact.

the logistics network, allowing the company to avoid the obligation of opening numerous candidate facilities in period 1 with no additional openings afterward.

Based on the second feature, we consider a logistics network with two layers: a set $\mathcal{J}$ of candidate production facilities and a set $\mathcal{I}$ of customers to be served from the selected facilities.

The model is based on a strategic time horizon (time periods are typically years). All data and parameters are assumed deterministic.

The main goal of the model is to determine which candidate facilities to open, when to open them over the planning horizon, which customers will be served by the chosen facilities and how to finance these decisions with an appropriate mix of debt and equity. The objective function to be maximized is the company's APV.

3.2. Time horizon

Maximizing APV requires computing the present value of the future operational and financial cash flows over multiple periods. The time horizon $\mathcal{T} = \{1, \dots, T\}$ represents the set of periods in which the location decisions are applicable, i.e., the company can borrow and invest money at any period $t \in \mathcal{T}$. Period 0 represents the initial state of the supply chain. The mathematical model presented in Section 4 is assumed to be solved at period 0, and concerns decisions that apply in any period $t \in \mathcal{T}$.

Even so, the logistical and financial decisions taken in this time horizon will have a much longer impact on the company's cash flows. Assume that a new facility starts operating at some period $t \in \mathcal{T}$ and that some money will be borrowed to finance this decision (possibly in addition to the use of internal and/or external equity). Given a payback period of N time periods and a facility lifetime L, the impact of this decision on the debt will be observed until period $t + N$ while the associated cash flow will be observed until period $t + L - 1$.

Time horizon $\mathcal{T}$ needs to be extended. To do so, we define the time horizon extension $\mathcal{T}' = \{T+1, \dots, T+\max(L,N)\}$ as the set of periods during which the selected facilities are still operating and the financial impact of logistical decisions can be observed after the time horizon $\mathcal{T}$. Fig. 3 illustrates the case where a new facility is selected at some period $t \in \mathcal{T}$ and generates cash flow until period $T + L - 1$.

3.3. Logistics and market features

Each candidate facility $j \in J$ has a known capacity C_j. Each facility has a fixed opening cost O_j paid once if the facility $j \in J$ is selected. It also has a fixed yearly running cost F_j paid every year provided the facility is operating. The unitary processing cost μ_j is paid for each unit of product manufactured in the facility.

Since the model is initialized by a blank page, we consider that a new opened facility will not be closed during the horizon $\mathcal{T}$. The list of candidate facilities in facility location models is generally much larger than the total number of facilities actually selected. Thus, we consider an upper bound J_{max} on the total number of selected facilities. The risk associated with the selection and all logistical operations is assumed to be the same in each facility.

Our multi-period model addresses markets with time-variable and deterministic customers' demands. We assume that reliable demand forecast is available for the whole time horizon $\mathcal{T}$. The demand of customer $i \in \mathcal{I}$ in period $t \in \mathcal{T}$ is denoted D_{it}. The selling price to each customer can depend on many factors, including the customer's market, its demand level and its negotiation skills. For these reasons, we assume a selling price P_i for each customer $i \in \mathcal{I}$. We do not force the company to serve all customers at every period. Unsatisfied customers' demands are simply lost and back orders are forbidden. Following the all-or-nothing principle, a customer's demand is either entirely fulfilled or not at all. We apply the idea of incremental service [40]: customers that are being served in a period must be served in all subsequent periods (the allocation of customers to facilities might change in different periods).

3.4. Financial features

We assume all facilities operate in a homogeneous financial environment with a single tax rate η and no exchange rates. Consequently, the equity cost of capital K_E is location-independent and remains constant over time, as all investments are subject to the same business risk within the industry and market. This homogeneity means all candidate facilities and logistical operations share similar systematic risks. Additionally, we assume that investments related to location decisions carry the same risk as the company's "business as usual."

For a new facility $j \in \mathcal{J}$ opened in period $t \in \mathcal{T}$, the firm has to finance its fixed opening cost O_j by an appropriate mix of debt and equity.

3.4.1. Debt financing

Debt financing amounts to borrowing money from a bank. There are $|T|$ discrete periods in which borrowing is possible. In light of this, we define $|T|$ separate loans, each of which is defined at a period $t \in T$

and may or may not be activated. We will refer to the loan that was triggered in period t as l_t throughout this study.

We assume that the loans have the same reimbursement duration N, and a constant annuity repayment method (homogeneous markets). However, the cost of loan l_t, denoted by KD_t, depends on the accumulated debt ratio of period t, such that the higher the debt ratio, the higher the cost of loan. The increase in leverage leads to a higher risk of bankruptcy for the company, prompting debt holders to demand a higher risk premium.

For each loan l_t, the annuity amount A_t associated with a loan amount B_t, a cost of loan KD_t and a number N of equally sized payments is given by:

$$A_t = B_t \frac{KD_t \times (1+KD_t)^N}{(1+KD_t)^N - 1}.$$

Assuming that the company will only use loans with the same maturity N, $N \geq T$, and constant annuity repayment method, the amount borrowed in period t will be repaid from period $t+1$ to period to $t+N$.

3.4.2. Equity financing

In equity financing, money is supported by shareholders in the form of internal financing (company's cash holdings) represented by the variables IE_t and external equity represented by the variables EE_t. The company will first seek internal financing based on available cash and then external financing. The amount of cash available depends on the level of retained earnings and the amortization policy. The parameter δ defines the payout ratio, i.e., the fraction of earnings paid as dividends. The remaining amount will be retained by the company. It will be added to existing cash holdings.

In an accounting approach, depreciation represents a yearly decrease in tangible assets' value over their lifetime. Among different methods to depreciate the assets (see, e.g., [27]) we consider a straight-line depreciation scheme, between the initial value O_j and the salvage value SV_j of the selected assets $j \in \mathcal{J}$. The amount of depreciation, as a non-cash accounting expense, enables the company to build up reserves that can be used for future investments.

As mentioned above, the change from NPV to APV is made by adding two complementary elements: Tax Shield Benefits and Expected Bankruptcy Cost (see Fig. 2). EBC is the present value of direct (legal and accounting costs) and indirect bankruptcy costs, a continuum of costs that increase at an accelerating rate as exposure to bankruptcy increases, e.g., increased interest expenses, lost credit, lost sales, inefficient operations. Direct costs take the form of administrative expenses (trustee's fees, legal fees, referee's fees), and in the time lost by executives in liquidation [41].

[8] calculates EBC as the product of the probability of bankruptcy, denoted p, by the bankruptcy costs $\gamma \times OGV$, where $0 < \gamma < 1$ is a known fixed parameter. We choose to express bankruptcy costs as a fraction of the firm value before bankruptcy as usually done both in theoretical [42] and empirical studies [43,44].

The various approaches only differ on the measures of the firm value (book versus market value, total versus equity value), in the model, we choose the market operational value of the company. A company will go bankrupt in period $t \in \mathcal{T}$ if the market value of its assets falls below the value of debt at this period. The higher the debt threshold relative to the company's assets, the more difficult it will be for the asset value to reach it.

4. Mathematical formulation

This section details the mathematical model maximizing APV subject to logistical and financial constraints. Sections 4.1 and 4.2 enumerate the model constraints related to the Operationally Generated Value (OGV) and the Financially Generated Value (FGV), respectively.

4.1. OGV: Operationally generated value

For each candidate facility $j \in \mathcal{J}$, the opening cost O_j is paid once if the facility is selected, and yearly operating fixed costs F_j paid at every period when the facility is operating. In addition, each operating facility has a processing cost μ_j for each unit of product processed by this facility. We assume a lifetime $L > T$ for all candidate facilities which is consistent in a strategic point of view.

The distance between a customer $i \in \mathcal{I}$ and a facility $j \in \mathcal{J}$ is denoted as $Dist_{ij}$. We assume that the transportation cost between two locations is proportional to the distance traveled and the load carried, with a unit transportation cost ω over the whole network. Finally, we consider a selling price P_i to customer $i \in \mathcal{I}$.

Due to trade rules between geographical areas as well as various logistical constraints, some customers might not be delivered by some facilities. Thus, we introduce an accessibility binary parameter V_{ij} which takes the value 1 if the customer $i \in \mathcal{I}$ is accessible from facility $j \in \mathcal{J}$.

We consider two families of binary decision variables and one family of continuous decision variables. The variable y_{jt}, takes the value 1 if the facility $j \in \mathcal{J}$ is operating in period $t \in \mathcal{T}$, and 0 otherwise. The variable x_{it} takes value 1 if customer $i \in \mathcal{I}$ is served in period $t \in \mathcal{T}$, and 0 otherwise. The variable q_{ijt} denotes the quantity delivered by facility $j \in \mathcal{J}$ to customer $i \in \mathcal{I}$ in period $t \in \mathcal{T}$.

$$y_{j0} = 0 \qquad \forall j \in \mathcal{J} \quad (1)$$

$$y_{j,t-1} \leq y_{jt} \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \quad (2)$$

$$\sum_{j \in \mathcal{J}} y_{j,T} \leq J_{max}. \quad (3)$$

$$x_{i,t-1} \leq x_{it} \qquad \forall i \in \mathcal{I}, t \in \mathcal{T} \quad (4)$$

$$q_{ijt} \leq V_{ij}\ D_{it}\ y_{jt} \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}, t \in \mathcal{T} \quad (5)$$

$$\sum_{j \in \mathcal{J}} q_{ijt} = D_{it}\ x_{it} \qquad \forall i \in \mathcal{I}, t \in \mathcal{T} \quad (6)$$

$$\sum_{i \in \mathcal{I}} q_{ijt} \leq C_j\ y_{jt} \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \quad (7)$$

$$e_{jt} = F_j\ y_{jt} + \mu_j \sum_{i \in \mathcal{I}} q_{ijt} + \omega \sum_{i \in \mathcal{I}} (Dist_{ij}\ q_{ijt}) \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \quad (8)$$

$$r_{jt} = \sum_{i \in \mathcal{I}} (P_i\ q_{ijt}) \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \quad (9)$$

Constraints (1) establish initial conditions: in period 0, none of the candidate facilities is selected. Constraints (2) state that selected facilities cannot be closed during the time horizon $\mathcal{T}$ (neither during the complementary time horizon $\mathcal{T}'$ whereas $y_{j,t}$ is frozen when $t \in \mathcal{T}'$). Constraints (3) define an upper bound J_{max} on the number of selected facilities. Constraints (4) impose to serve in period t a customer who was served in period $t-1$.

Constraints (5) state that the variable q_{ijt} is strictly positive only if facility j is opened and accessible ($y_{jt} = 1$) and $V_{ij} = 1$. This quantity cannot exceed the demand D_{it}. Constraints (6) calculate the total quantity delivered to each customer, which is either 0 when $x_{it} = 0$ or the total demand D_{it} when $x_{it} = 1$ (no partial satisfaction of a particular customer's demand). Some customers may be delivered from several different facilities, at a given period. The capacity constraints (7) enforce the total quantity shipped by a selected facility $j \in \mathcal{J}$ to be at most equal to its capacity C_j.

Constraints (8) calculate the total amount of logistics expenses e_{jt} related to facility $j \in \mathcal{J}$ in period $t \in T$. This amount is the sum of the yearly fixed cost, the processing cost and the transportation cost. Constraints (9) calculate the total revenue r_{jt} generated by facility $j \in \mathcal{J}$ in period $t \in \mathcal{T}$.

4.1.1. Calculation of OGV

We consider a tax rate η and the equity cost of capital of the unlevered company K_E. For each candidate facility $j \in \mathcal{J}$, the initial value and the salvage value and denoted O_j and SV_j, respectively.

A facility $j \in \mathcal{J}$ opened in period $\tau \in \mathcal{T}$ has a lifetime $L > T$. It generates variable cash flows between period τ and T and a constant cash flow CF_{jT} after period T, as long as is it active.

The calculation of OGV requires determining which facilities are operating at each period $t \in \mathcal{T} \cup \mathcal{T}'$. We extend the definition of binary variables y_{jt} to $\mathcal{T}'$ as follows: for any $j \in \mathcal{J}, t \in \mathcal{T}'$, y_{jt} is equal to 1 if facility j is operating in period t, and 0 otherwise. If $y_{jT} = 1$ and the facility was opened in period $\tau \in \mathcal{T}$, then $y_{jt} = 1$ for all $T+1 \le t \le \tau + L - 1$, and 0 otherwise. Then, for all $t \ge \tau + L$, $y_{tj} = 0$. If $y_{jT} = 0$, all variables y_{jt} are equal to 0.

$$Dep_j = \frac{O_j - SV_j}{L} \qquad \forall j \in \mathcal{J} \qquad (10)$$

$$EBIT_{jt} = r_{jt} - e_{jt} - Dep_j\ y_{j,t-1} \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \qquad (11)$$

$$CF_{jt} = (1-\eta)EBIT_{jt} + Dep_j\ y_{j,t-1} \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \qquad (12)$$

$$FCF_{jt} = (1-\eta)EBIT_{jt} - (O_j(y_{jt} - y_{j,t-1}) - Dep_j\ y_{j,t-1}) \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \qquad (13)$$

$$y_{j,t-1} \ge y_{jt} \qquad \forall t \in \mathcal{T}' \backslash \{T+1\} \qquad (14)$$

$$\sum_{t \in \mathcal{T}} y_{jt} + \sum_{t \in \mathcal{T}'} y_{jt} = L\ y_{jT} \qquad \forall j \in \mathcal{J} \qquad (15)$$

Constraints (10) calculate the straight-line depreciation Dep_j of facility $j \in \mathcal{J}$ as a linear function of its initial value (opening cost) O_j, salvage value SV_j, and lifetime L.

Constraints (11) calculate the Earning Before Interest and Taxes $EBIT_t$ associated with facility $j \in \mathcal{J}$ in period $t \in \mathcal{T}$. It is the difference between the revenues and expenses of facility $j \in \mathcal{J}$ (before interest and tax) in period t and the depreciation factor Dep_j occurring if the facility j operates in period $t-1$.

Constraints (12) calculate the operating cash flow associated with facility $j \in \mathcal{J}$ in period $t \in \mathcal{T}$. The first term is the accounting result after tax. The second term reintroduces the depreciation because it is not a cash outflow.

In constraints (13), the free cash flows FCF_{jt} measures the ability of the firm to generate cash. It is the difference between operating and non-operating cash inflows and outflows associated with each facility.

Constraints (14) and (15) model the extension of variables y_{jt} to the periods $t \in \mathcal{T}'$.

The Operationally Generated Value (OGV) is defined by:

$$OGV = \sum_{j \in \mathcal{J}} \left(\sum_{t \in \mathcal{T}} \frac{FCF_{jt}}{(1+K_E)^t} + \sum_{t \in \mathcal{T}'} \frac{CF_{jT}\ y_{jt}}{(1+K_E)^t} \right). \qquad (16)$$

This expression is non-linear due to the product of variables CF_{jT} and y_{jt}. The linearization process is explained in Appendix C.2.

4.2. FGV: Financially generated value

If a facility $j \in \mathcal{J}$ is opened in period $t \in \mathcal{T}$, the firm has to decide how to finance its initial value O_j. The company can mix debt financing, detailed in Section 4.2.1 and equity financing, detailed in Section 4.2.2. We consider three families of continuous decision variables: $borrow_t$ represent the amount of money borrowed in period $t \in \mathcal{T}$; IE_t and EE_t represent the internal equity the external equity in period $t \in \mathcal{T}$, respectively.

Eq. (17) states that facility sunk costs are funded by the debt, internal funding or by external equity.

$$\sum_{j \in \mathcal{J}} O_j(y_{jt} - y_{j,t-1}) = borrow_t + IE_t + EE_t \qquad \forall t \in \mathcal{T}. \qquad (17)$$

4.2.1. Debt financing constraint

For loan $l_{\bar{t}}$, we define $balance_{\bar{t}t}$, $repay_{\bar{t}t}$ and $interest_{\bar{t}t}$ respectively as the total amount still alive, the annuity amount repaid, and the value of the interest associated with that loan in period $t \in \mathcal{T} \cup \mathcal{T}'$.

$$balance_{\bar{t}t} = \begin{cases} 0 & t < \bar{t} \\ borrow_{\bar{t}} & t = \bar{t} \\ balance_{\bar{t},t-1} - repay_{\bar{t}t} + interest_{\bar{t}t} & t > \bar{t} \end{cases} \qquad (18)$$

$$repay_{\bar{t}t} = \begin{cases} 0 & t \le \bar{t} \\ borrow_{\bar{t}} \frac{KD_{\bar{t}}(1+KD_{\bar{t}})^N}{(1+KD_{\bar{t}})^N - 1} & \bar{t} < t \le \bar{t} + N \\ 0 & t > \bar{t} + N \end{cases} \qquad (19)$$

$$interest_{\bar{t}t} = KD_{\bar{t}}\ balance_{\bar{t},t-1} \qquad \forall t \in \mathcal{T} \cup \mathcal{T}' \qquad (20)$$

Constraints (18) calculate the value of the loan balance associated with loan $l_{\bar{t}}$, in period $t \in \mathcal{T} \cup \mathcal{T}'$. Its value is 0 until its activation in period $t = \bar{t}$. As it is activated ($t = \bar{t}$), the balance equals the amount borrowed (= $borrow_{\bar{t}}$), then for $t > \bar{t}$, it is decreased by the debt amortization during the period, which is equal to $repay_{\bar{t}t}$ - $interest_{\bar{t}t}$.

The repayment formula is given by (19). After loan $l_{\bar{t}}$ is activated in period $\bar{t} \in \mathcal{T}$, its repayment starts at the beginning of the following period ($t = \bar{t} + 1$) until it is fully repaid ($t = \bar{t} + N$).

Constraints (20) calculate $interest_{\bar{t}t}$. Both constraints (19) and (20) are non-linear. Their linearization is explained in Appendices C.3, and C.4, respectively.

4.2.2. Equity financing constraint

$$NOPAT_t = (1-\eta)\left(\sum_{j \in \mathcal{J}} EBIT_{jt} - \sum_{\bar{t} \in \mathcal{T}} interest_{\bar{t}t}\right) \qquad \forall t \in \mathcal{T} \qquad (21)$$

$$cash_t = cash_{t-1} + (1-\delta)NOPAT_t + EE_t + \sum_{\bar{t} \in \mathcal{T}} (borrow_{\bar{t}t} - repay_{\bar{t}t} + interest_{\bar{t}t}) - \sum_{j \in \mathcal{J}} (O_j(y_{jt} - y_{j,t-1}) - Dep_j\ y_{j,t-1}) \qquad \forall t \in \mathcal{T} \qquad (22)$$

$$IE_t \le \max(0, cash_{t-1}) \qquad \forall t \in \mathcal{T} \qquad (23)$$

To ease the presentation calculation of internal equity variables IE_t, we introduce two intermediate variables: the Net Operating Profit After Taxes of period $t \in \mathcal{T}$, denoted $NOPAT_t$, is obtained by removing the interest expenses from the EBIT (see constraints (11)) in period $t \in \mathcal{T}$, and then multiplying the result by the term $(1-\eta)$, where η is the firm tax rate. The cash flows $cash_t$ generated in period t and owned by shareholders (constraints (22)) are equal to the residual cash flows once all stakeholders, including lenders and State, have been remunerated.

Finally, constraints (23) state that IE_t cannot exceed the cash available at the end of the preceding period, $cash_{t-1}$. The linearization of this equation is explained in Appendix C.5.

4.2.3. Calculation of FGV and APV

The probability of default is denoted by p. The variable $equity_t$ represents the value of the equity in period $t \in \mathcal{T}$. The parameter $\beta > 1$ models the volatility of the company's assets. The parameter $0 < \zeta < 1$ is the maximum percentage of the total assets that can be used for debts related to new facilities. We recall that the parameter $0 < \gamma < 1$, defined in Section 3.4 is used to calculate the bankruptcy cost. Since cash flows and tax shield benefits share the same systematic risk, they are discounted at the same rate K_E.

$$equity_t = equity_{t-1} + (1-\delta)NOPAT_t + EE_t \qquad \forall t \in \mathcal{T} \qquad (24)$$

$$p = \left(\frac{\sum_{\bar{t} \in \mathcal{T}} balance_{\bar{t}T}}{\sum_{\bar{t} \in \mathcal{T}} balance_{\bar{t}T} + equity_T} \right)^{\beta} \qquad (25)$$

$$\sum_{\bar{t} \in \mathcal{T}} balance_{\bar{t}t} \le \zeta\ (cash_t + \sum_{j \in \mathcal{J}} \sum_{t' \le t} (O_j(y_{jt'} - y_{j,t'-1}) - Dep_j\ y_{j,t'-1})) \qquad \forall t \in \mathcal{T}. \qquad (26)$$

$$FGV = (1-p)\eta \sum_{t\in\mathcal{T}\cup\mathcal{T}'} \frac{\sum_{\tilde{\imath}\in\mathcal{T}} interest_{\tilde{\imath}t}}{(1+K_E)^t} - p\,\gamma\,OGV \tag{27}$$

In constraints (24), the value of equity in period $t \in \mathcal{T}$ is defined as its value at time $t-1$ plus the retained earnings at time t plus the new cash provided by shareholders at time t. This value is used to calculate the probability of default in constraint (25). This probability is an increasing function of the debt ratio $\sum_{\tilde{\imath}\in\mathcal{T}} balance_{\tilde{\imath}T}/(\sum_{\tilde{\imath}\in\mathcal{T}} balance_{\tilde{\imath}T} + equity_T)$ in period T.

The probability of default is influenced by the company's capacity to generate enough cash flow to meet its debt obligations. The debt ratio quantifies the relative significance of the debt in relation to cash flow. Since we do not directly model cash flow risk, we approximate it indirectly using the coefficient β. The linearization of this constraint is explained in Appendix C.8.

To avoid financial distress in earlier periods, we consider an upper bound on the level of debt, set by Constraints (26). The right-hand side represents the net value of the total assets in period $t \in T$, multiplied by parameter ζ. Constraint (27) calculates the value of FGV. The first term corresponds to the tax shield benefit (TSB) and the second one to the expected bankruptcy cost (EBC) as illustrated in Fig. 2. As debt levels increase, so do interest payments and the resulting tax shield benefits. However, as shown in Eq. (25), a higher debt level also raises the probability of default and the corresponding bankruptcy costs.

Finally, the objective function to be maximized is the APV, which is the sum of OGV and FGV, as written in constraint (28):

$$APV = (1-p\gamma)OGV + (1-p)\eta\left(\sum_{t\in\mathcal{T}\cup\mathcal{T}'} \frac{\sum_{\tilde{\imath}\in\mathcal{T}} interest_{\tilde{\imath}t}}{(1+K_E)^t}\right). \tag{28}$$

The present value of tax shields is computed by discounting the annual interest amount $\sum_{\tilde{\imath}\in\mathcal{T}} interest_{\tilde{\imath}t}$ at the rate K_E, multiplied by the firm tax rate η.

The company will receive the interest tax shields with a probability $1-p$ (i.e., if it is not bankrupted) and pay a bankruptcy cost γ OGV with probability p. From this equation, it is clear that the leverage has a mixed impact on APV. On the one hand, it has a negative impact through the bankruptcy risk (p) and, on the other hand, a positive one through the tax shield benefit (last term of the APV function).

5. Data generation

As explained in Section 2, we did not find a related benchmark instance that could be used to validate and experiment with this model. Therefore, new instances were generated following the generation rules used by different authors. As mentioned above, APV is particularly relevant for evaluating strategic decisions in large-scale activities. Thus, the generated instances mimic a supply chain with multiple markets, each market having its own costs, product price, etc.

We generated instances with 60 to 270 customers. Following [45], the number $|\mathcal{J}|$ of candidate facilities is defined as 10% of the number of customers. The maximum number of open facilities (J_{max}) is defined as $\lceil 0.5|\mathcal{J}|\rceil$. In all instances, $|\mathcal{T}| = 5$ and $|\mathcal{T}'| = 10$ periods were considered. The next subsections detail the steps followed by the data generation.

5.1. Logistics data and parameters

All locations were generated in a 1000×1000 grid. Both axes of the grid are decomposed into 5 intervals of size 200, defining 25 squared areas called *regions*.

Markets. The grid is partitioned into two to five *markets*, that is a connected set of regions. Each market receives an *economic index* that is used as a proxy to indicate its economic situation and generate the costs and prices in accordance. The affected costs and prices are, specifically the processing cost (μ), the opening cost parameter (φ), and the selling price (P) as shown in Table 2. The economic indices are generated with a uniform distribution between 50 and 150. The higher the economic index, the higher the market's costs and prices.

Customers and candidate facilities. To generate the location of customers and candidate facilities, we used two approaches: the coordinates of the customers and candidate facilities were either randomly generated with a uniform distribution over the entire grid or we used a clustered pattern. In the latter case, we located around 60% of customers (resp. candidate facilities) in a subset of 4 or 5 regions, then the remaining 40% were randomly located with a uniform distribution over the grid.

Customers' demand. We generated the customers' demand according to two different demand profiles: First, following [46], the customers' demands at each period are generated with a uniform distribution in the interval [100, 300]. Then, each customer's demand still lies in the interval [100, 300], with the additional property that the total demand grows by a factor in the interval [1.05, 1.25] between two successive periods. Note that although the sum of customers' demands grows over time, the individual demand of some customers may decrease between two successive periods due to the uneven distribution of growth."

Capacity of the facilities. Each candidate location has a given capacity. We have generated three sizes of facilities, named *small*, *medium* and *large*, representing 80%, 100% and 130% of the ratio D/J_{max} respectively, where D is the average demand per period. The capacity of each facility is randomly chosen such that about 1/3 of facilities are small, 1/3 are medium and 1/3 are large.

Logistics costs. Several costs in the mathematical model are affected by variations in the cost of living across different market areas. To account for this, we introduced an intermediate parameter, referred to as the *economic index*, to generate these costs. For each market, the economic index is generated with a uniform distribution between values 50 and 150. The range $[50, 150]$ is decomposed into 5 intervals, and for each interval, the processing cost, the opening cost and the selling price are generated according to a uniform distribution, as detailed in Table 2. Note that as we generate the yearly fixed cost of facility as a percentage of the opening cost, this data is also directly affected by the economic index.

- The **Processing costs** (μ_j) of small-sized facilities is defined with a uniform distribution on an interval depending on the economic index of the market where facility j lies (see line 2 of Table 2). The fictitious monetary unit used in the rest of this article is called *relative money unit* (*rmu*). To model economies of scale, the processing costs at average-sized and large-sized facilities are obtained by multiplying these values by 0.98 and 0.96, respectively. These parameters have been adjusted in such a way that the relative part of processing costs roughly represents around 35% – 45% of the total costs in each instance.
- The value of the **fixed opening cost** (O_j), represented in monetary value, is strongly related to the value of the real estate. To model economies of scale as the capacity grows, following [47], we assume that the fixed opening cost of a facility j is roughly proportional to the square root of its capacity. We set $O_j = \varphi_j \sqrt{C_j}$, where C_j represents the capacity of facility j and φ_j is the opening cost parameter at location j (mainly determined by the cost of the local real estate). The value of φ_j is generated randomly with a uniform distribution between 625 and 750 (see line 3 of Table 2). These intervals have been set by successive adjustments in such a way that the relative part of the fixed opening costs roughly represent 25% – 35% of the total logistics costs (see, e.g. [47,48] for similar approaches).
- The **Fixed yearly running cost** (F_j) is set at 5% of the fixed opening cost of each facility, O_j, per year. Thus, it represents around 10% – 15% of the total costs.
- The **transportation costs** are considered proportional to the Euclidean distance traveled. We assume they are similar in all markets. To have the transportation cost representing 10% to 20% of total costs [49], the unit transportation cost, ω, is set at 0.002 *rmu* in all the instances.

Table 2
Generation of logistical data.

Economic index	[50, 70[	[70, 90[	[90, 110[	[110, 130[	[130, 150]
Processing cost μ (rmu/unit)	[1, 1.1]	[1.1, 1.2]	[1.2, 1.3]	[1.3, 1.4]	[1.4,1.5]
Opening cost parameter φ	[625, 650]	[650, 675]	[675, 700]	[700, 725]	[725, 750]
Selling price P (rmu/unit)	[3, 3.4]	[3.4, 3.8]	[3.8, 4.2]	[4.2, 4.6]	[4.6, 5]

Table 3
Calculation of the cost of loan KD_t.

Debt ratio interval (in %)	Cost of loan (in %)
(0,30]	3.2
(30,40]	3.5
(40,50]	4.0
(50,60]	5.0
(60,70]	6.6
(70,80]	9.6

Selling price. The price P_i proposed to customer $i \in \mathcal{I}$ depends on the market in which i lies. Its value lies in the interval $[3, 5]$ (see line 4 of Table 2).

Accessibility parameters. The accessibility parameter V_{ij} is set to value 1 if the distance between facility $j \in \mathcal{J}$ and customer $i \in \mathcal{I}$ is less than or equal to 750 (roughly half of the longest possible distance in the grid), and 0 otherwise.

5.2. Financial parameters

The average cost of equity K_E is set at 9% [50]. In order to estimate the cost of loan KD_t in relation to the probability of bankruptcy, we employ an arbitrage equation used by a risk-neutral bank[1]. This approach allows us to quantify the expected rate of return on loans by taking into account their associated risks.

Using the arbitrage equation, we modeled the cost of loan as a piecewise linear function varying between 3.2% and 9.6%. We considered six debt ratio intervals as defined in Table 3.

In all instances, we consider a lifetime value L and a number of annuities N that are both equal to 10 years. The depreciation Dep_j calculated with this lifetime value, and the salvage value SV_j is assumed to be negligible [27]. The value of the bankruptcy cost parameter γ is set to 0.5 and the upper bound of the debt ratio ζ is set to 0.8 in order to ensure an acceptable financial situation. The value of the bankruptcy probability parameter β in constraint (25) is set to 3. The corporate tax rate η is set at 30%.

5.3. Set of instances

Following the principles described above, 32 instances were generated: 16 instances have random locations and 16 instances have clustered locations, 16 instances have random demands and 16 instances have growing demands.

Instance names are formed by their size followed by one of the letters A, B, C, or D. Note that there are 4 different types of instances due to applying four patterns to generate the customers' and facilities' coordinates as well as the customers' demands.

For each size of instance, Table 4 enumerates the number of binary and continuous variables associated to the logistical and financial part of the model, as well as the number of constraints. Note that the binary variables in the financial part of the model come from the linearization of nonlinear constraints.

6. Numerical experiments

6.1. Integrated and sequential approaches

As represented in Fig. 2, APV can be decomposed into a logistical part (OGV) and a financial part (FGV). The whole mathematical model can be decomposed into two sub-problems presented in Sections 4.1 and 4.2, respectively. The *logistical sub-problem* determines which facilities should be opened, which customers should be served as well as the associated product flows. Once the logistical decisions have been fixed, the *financial sub-problem* optimizes the value of APV by maximizing FGV (see Fig. 4).

The *Sequential Approach* mimics the decision process followed in classical facility location models. The logistical and financial sub-problems are solved sequentially: First, the OGV formula defined by Eq. (16) is maximized, subject to constraints (1)—(15). Then, the OGV is considered constant and the FGV is optimized. This amounts to maximizing the APV formula defined in constraint (28), subject to constraints (17)—(26). The *Integrated Approach* considers the whole mathematical model (1)–(28) at once and maximizes APV by simultaneously determining the value of all logistical and financial variables.

By nature, the objective function values of the optimal solutions in the integrated approach are higher than those in the sequential approach. The relative gap between the optimal solutions of the two approaches measures the benefit of introducing financial considerations into facility location models. This comparison underscores the tangible advantages of integrating financial decisions into classical facility location models, demonstrating the benefits of adopting an integrated financial perspective. In return, the integrated approach is likely to yield computational difficulties due to the size of the mathematical model. An objective of the numerical experiments is to explore the practical limits of such a model.

6.2. Assessment of the sequential and the integrated approaches

All numerical experiments were run on an Intel(R) Xeon(R) Gold 6230 CPU @ 2.10 GHz using ten cores. The mathematical model was solved with the IBM ILOG CPLEX Optimization Studio 20.1.0 solver, with a time limit of 6 h (21 600 seconds).

Table 5 compares the computational time needed to solve the integrated and the sequential approaches, respectively. Columns 2 and 4 indicate the computational time (in seconds) to get an optimal solution. Columns 3 and 5 indicate the optimality gap when no optimal solution could be obtained after 3 h of computation.

This table shows that both approaches are tractable for instances with up to 150 customers. As expected, the sequential approach is easier to solve than the integrated approach: 29 instances could be solved to optimality with the sequential approach within 6 h, only

[1] The arbitrage equation used by a risk-neutral bank can be expressed in the form of the following equation:

$$F(1+r_f)^N = (1+KD)^N(pR+(1-p)F).$$

Where F, r_f, N, KD, p, and R are loan facial value, risk-free rate, loan duration, cost of loan, probability of bankruptcy, and value given default or the amount that lender recovers if the company defaults on its debt, respectively. Note that, we set the risk-free rate to 3% in accordance with the cost of debt reported by the KPMG report [50]. According to this report, the cost of debt for industrial manufacturing companies with a debt ratio of 25%–30% is around 3%. Setting r_f to 3% lets us obtain almost the same cost of debt for the same amount of debt ratios.

Table 4
Number of constraints and variables of the model.

Instance size	Constraints			Binary variables			Continuous variables		
	Total	Logistical	Financial	Total	Logistical	Financial	Total	Logistical	Financial
60	6079	3087	2992	559	462	97	3191	2108	1083
90	8976	5979	2997	791	693	98	5595	4511	1084
120	12 768	9771	2997	1022	924	98	8898	7814	1084
150	17 460	14 463	2997	1253	1155	98	13 101	12 017	1084
180	23 052	20 055	2997	1484	1386	98	18 204	17 120	1084
210	29 549	26 547	3002	1716	1617	99	24 208	23 123	1085
240	36 941	33 939	3002	1947	1848	99	31 111	30 026	1085
270	45 233	42 231	3002	2178	2079	99	38 914	37 829	1085

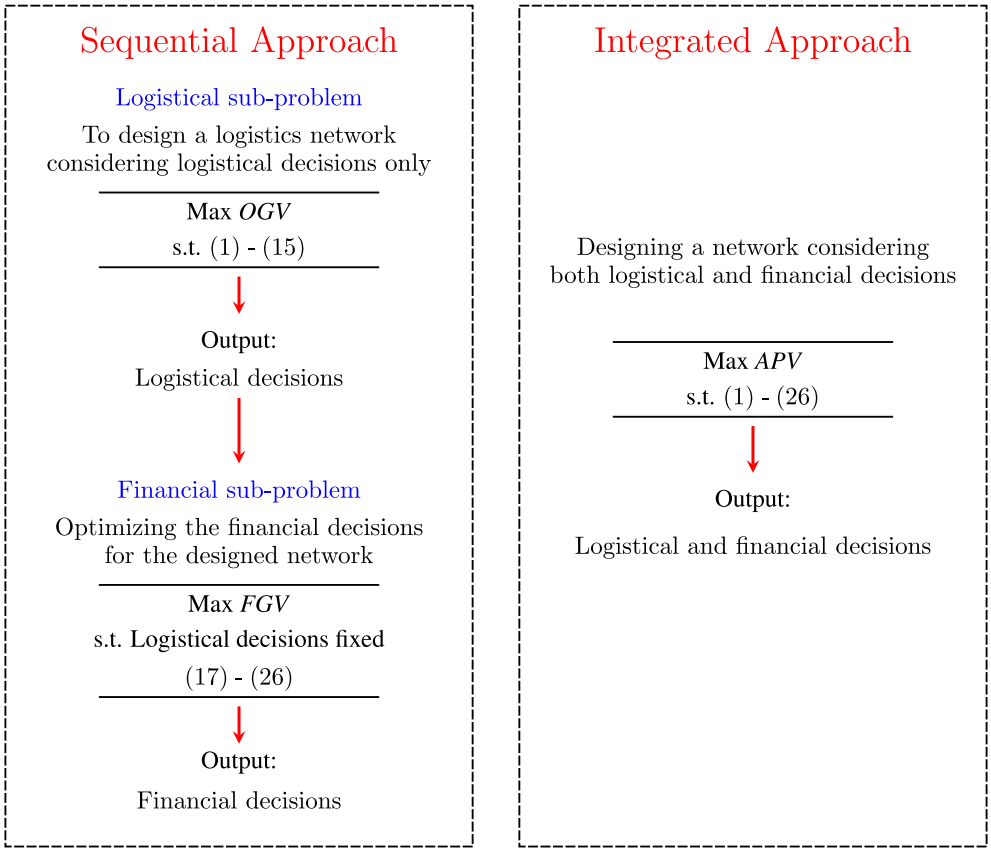

Fig. 4. Sequential versus Integrated approaches.

15 with the integrated approach. The optimality gap of the integrated approach exceeds that of the sequential approach, particularly for the large-sized instances. This table suggests that specific solution methods (either exact or heuristics) should be used for larger instances.

reports the performance of the two approaches using five indicators. The first indicator (column 3) is the APV. Since the APV is the objective function to be maximized in the integrated approach, the results of this approach are generally better than (23 instances) or equivalent to (3 instances) those of the sequential approach. The relative gap between the APV of the two approaches is reported in column 4.

Small gap values confirm that the sequential approach is a good alternative to the integrated approach. Obviously, solutions obtained with the integrated approach may have lower OGV, which is offset by the value contributed by the financial decisions.

The second indicator (column 5) is the fill rate, which describes the ability to satisfy customers' demands. The model is profit-oriented and unsatisfied demands are lost. The fill rate of the integrated approach is generally higher than that of the sequential approach. The integrated approach favors larger investments that are made possible through the efficient use of financial decisions. Since OGV does not benefit from the leverage effect of the tax shield benefit, the sequential approach appears to be more conservative.

The third indicator is the Return On Equity (ROE) which represents the company's profitability from the shareholders' point of view. It is computed as the ratio of the net income ($NOPAT$) and the shareholders' equity ($Equity$). Hence, it expresses the net profit made by each unit (rmu) of the shareholders' equity.

The fourth indicator, Return On Assets (ROA), measures the company's global profitability (paid income to both shareholders and debtholders). It equals the net operating profit after taxes plus the after-tax interest ($NOPAT + interest \times (1 - \eta)$) divided by total assets (i.e., investment made by both shareholders and debt-holders). This indicator does not consider the impact of leverage on profitability.

The fifth indicator is the leverage ratio, which calculates the ratio of the debt and total assets ($Debt/total\ assets$). Following Modigliani and Miller's reasoning, ROA and leverage should positively affect ROE.

In column 7, we report the average value of the ROA over the facilities' economic life. The ROA of the sequential approach is always higher than or equal to that of the integrated approach for two reasons. On the one hand, the additional investments with the integrated approach,

Table 5
Comparison of CPU times and optimality gaps of the sequential and the integrated approaches.

Instance	Integrated		Sequential	
	CPU (in s)	Gap	CPU (in s)	Gap
60-A	857		13	
60-B	1154		15	
60-C	495		16	
60-D	728		54	
90-A	2606		246	
90-B	2044		88	
90-C	2232		78	
90-D	5975		129	
120-A	7974		1023	
120-B	8962		809	
120-C	6351		1570	
120-D	7458		726	
150-A	10 625		1350	
150-B	21 600	*0.02%*	2355	
150-C	3551		1682	
150-D	21 489		3842	
180-A	21 600	*0.99%*	5861	
180-B	21 600	*15.98%*	9177	
180-C	21 600	*0.03%*	8159	
180-D	21 600	*2.49%*	3040	
210-A	21 600	*2.09%*	3295	
210-B	21 600	*34.18%*	3836	
210-C	21 600	*1.41%*	14 749	
210-D	21 600	*2.87%*	4903	
240-A	21 600	*41.22%*	6923	
240-B	21 600	*33.37%*	12 755	
240-C	21 600	*5.02%*	21 600	*0.02%*
240-D	21 600	*49.53%*	6787	
270-A	21 600	*77.69%*	21 600	*0.06%*
270-B	21 600	*70.56%*	21 600	*0.05%*
270-C	21 600	*15.48%*	19 913	
270-D	21 600	*29.54%*	15 910	

in terms of profitability, are not as efficient as those chosen by the sequential approach. On the other hand, these additional investments increase the level of total assets (increasing the ratio's denominator).

Column 8 shows the average leverage ratio over the economic life of the facilities. This ratio is not systematically higher or lower for the integrated approach. Probably since, in some cases, the integrated approach leads not only to more debt but also to higher assets level.

Similarly to ROA and leverage ratio, ROE, reported in column 6, is calculated as the average value over the economic life of facilities. This indicator is significantly related to the two previously mentioned indicators. For 23 out of 32 instances, larger or equal ROE is obtained with the sequential approach. In all other cases, the leverage ratio of the integrated approach is higher than that of the sequential approach. The higher leverage ratio offsets the negative effect of lower ROA on ROE.

6.3. Detailed solutions

To better highlight the differences obtained by both approaches, Table 7 provides a detailed list of candidate facilities selected at each period (columns $t = 1$ to $t = 5$). Additionally, we have reported the accumulated capacity installed at each period and the total investment for each solution (sum of the fixed costs of the selected facilities) in Table B.12 in Appendix B.

For most instances, these tables show slight differences between the networks found by both approaches. These tables confirm that the integrated approach tends to favor larger investments than the sequential approach. For example, in the result of instance 60-D, facility 6 is selected by the integrated approach and not by the sequential approach. When exactly the same facilities are selected, the integrated approach favors earlier investments. For example, in the result of instance 60-C, facility 4 is selected in period 1 by the integrated approach and in period 4 by the sequential approach.

To analyze the effect of different instance types, we focused on instances that could be solved to optimality or exhibited a negligible optimality gap. Consequently, our analysis includes instances up to size 210, while excluding instance 210-B.

Table 8 summarizes our findings regarding the relationship between instance types. For each group of instance (A, B, C or D), we report the differences in APV and fill rate between the results of the integrated and sequential approaches.

We observed no systematic relationship between the coordinate pattern, the demand profile and the value of APV. Only minor differences (between 0% and 5%) are observed between the optimal value of the sequential and integrated approaches. For growing demand, APV ranged from 1.04% to 2.48% for Type B and 00% to 5.13% for Type D, making it difficult to determine any significant influence from a particular demand profile. Similarly, no consistent relationship was found between fill rates and demand profiles.

However, clustered instances seem to sometimes yield much larger differences between the results of the sequential and integrated approaches than random instances. This suggests that clustering can magnify the difference in fill rates. When the sequential and integrated approaches do not select the same facilities, we notice an "all-or-nothing" effect: a well-located facility is likely to service an entire cluster, while an alternative solution may entirely ignore that cluster.

We also conduct a sensitivity analysis to assess the impact of various parameters on the objective function (APV) and fill rate within the integrated approach. Five key parameters were considered: Tax Rate (η), Debt Ratio Upper Bound (ζ), Bankruptcy Cost Parameter (γ), Facility Fixed Yearly Running Cost (F), and Unit Transportation Cost (ω). The results, shown in Appendix D, indicate that the variations in numerical results across the key parameters are consistent for nearly all parameters and instances.

Table 6
Performance comparison of the sequential and integrated approaches.

1st part							
Instance	Approach	APV (rmu)		Fill rate (%)	ROE (%)	ROA (%)	Leverage ratio (%)
60-A	Integrated	40 848	*2.04%*	85	16.77	8.15	31.11
	Sequential	40 031		76	18.07	11.25	41.41
60-B	Integrated	45 055	*1.35%*	89	15.16	8.49	29.50
	Sequential	44 454		76	19.16	9.18	31.75
60-C	Integrated	45 556	*0.57%*	87	18.78	11.49	41.20
	Sequential	45 298		61	25.09	11.84	33.67
60-D	Integrated	48 523	*4.72%*	80	24.34	9.52	31.54
	Sequential	46 337		60	22.36	10.10	25.70
90-A	Integrated	48 698	*5.54%*	95	13.90	8.02	31.98
	Sequential	46 144		73	15.27	8.55	29.95
90-B	Integrated	51 711	*2.48%*	85	14.20	8.50	31.87
	Sequential	50 461		74	16.35	8.64	29.05
90-C	Integrated	58 061	*0.00%*	66	19.96	9.45	28.99
	Sequential	58 061		66	19.96	9.45	28.99
90-D	Integrated	59 764	*0.00%*	60	24.50	10.28	31.09
	Sequential	59 764		60	24.50	10.28	31.09
120-A	Integrated	73 903	*0.67%*	86	16.67	8.58	30.38
	Sequential	73 413		81	17.45	8.86	32.81
120-B	Integrated	74 147	*0.00%*	82	18.41	8.80	28.57
	Sequential	74 147		82	18.41	8.80	28.57
120-C	Integrated	59 225	*5.16%*	80	15.14	8.04	29.02
	Sequential	56 317		58	18.30	8.71	26.54
120-D	Integrated	60 823	*5.13%*	83	17.65	8.31	29.78
	Sequential	57 853		64	23.11	9.43	34.01
150-A	Integrated	191 945	*1.26%*	95	19.42	10.02	26.89
	Sequential	189 557		84	21.30	11.16	26.94
150-B	Integrated	184 077	*1.04%*	91	30.79	11.13	28.96
	Sequential	182 190		86	30.22	11.46	28.72
150-C	Integrated	160 934	*1.36%*	97	15.73	9.31	27.61
	Sequential	158 772		87	18.85	9.90	28.01
150-D	Integrated	152 863	*0.20%*	93	23.02	9.91	29.81
	Sequential	152 561		89	22.04	10.02	28.22

2nd part							
Instance	Approach	APV (rmu)		Fill rate (%)	ROE (%)	ROA (%)	Leverage ratio (%)
180-A	Integrated	131 962	*0.82%*	90	18.00	9.88	37.05
	Sequential	130 885		82	16.93	10.65	34.33
180-B	Integrated	132 613	*0.22%*	94	16.67	8.74	27.82
	Sequential	132 317		85	19.19	9.23	29.69
180-C	Integrated	143 515	*1.54%*	87	14.73	8.81	28.75
	Sequential	141 344		84	14.37	8.96	28.12
180-D	Integrated	140 417	*0.46%*	97	18.10	8.75	29.62
	Sequential	139 780		85	20.85	9.32	30.02
210-A	Integrated	152 812	*1.46%*	84	17.38	8.65	28.39
	Sequential	150 606		77	18.44	9.18	32.51
210-B	Integrated	149 438	*2.01%*	89	17.10	8.86	31.43
	Sequential	146 490		77	18.63	9.24	31.28
210-C	Integrated	161 064	*0.36%*	82	23.72	10.54	40.95
	Sequential	160 487		79	22.71	10.59	34.62
210-D	Integrated	157 895	*0.91%*	83	19.90	9.24	31.05
	Sequential	156 467		78	19.69	9.43	30.76
240-A	Integrated	196 749	*-4.32%*	89	12.95	8.80	23.85
	Sequential	205 257		94	13.34	8.57	29.95
240-B	Integrated	171 933	*-2.61%*	94	12.95	8.27	25.11
	Sequential	176 424		94	16.34	8.68	27.97
240-C	Integrated	203 582	*0.01%*	94	13.32	8.57	29.98
	Sequential	203 566		94	13.32	8.57	29.97
240-D	Integrated	169 404	*-5.91%*	94	20.24	10.51	35.02
	Sequential	179 413		95	21.20	10.69	36.91
270-A	Integrated	142 548	*-6.18%*	68	24.67	10.34	44.45
	Sequential	151 357		69	17.01	10.69	37.69
270-B	Integrated	157 160	*-4.37%*	79	20.22	9.40	38.10
	Sequential	164 025		66	19.46	9.48	29.62
270-C	Integrated	264 244	*0.17%*	87	16.10	9.67	25.69
	Sequential	263 796		75	22.85	10.62	28.53
270-D	Integrated	276 141	*-1.71%*	83	18.27	9.90	21.57
	Sequential	280 868		81	27.35	10.65	28.59

Table 7
Comparison of the facilities selected by the sequential and the integrated approaches.

1st part						
Instance	Approach	Facilities selected at each period				
		t=1	*t=2*	*t=3*	*t=4*	*t=5*
60-A	Integrated	2 3			6	
	Sequential	2 3				
60-B	Integrated	2 3			4	
	Sequential	3	5		4	
60-C	Integrated	2 4				
	Sequential	2			4	
60-D	Integrated	2	6		4	
	Sequential	2			4	
90-A	Integrated	2 6 9			4	
	Sequential	6 9			4	
90-B	Integrated	6 9		2	4	
	Sequential	6	9		2 4	
90-C	Integrated	5 6			4	
	Sequential	5 6			4	
90-D	Integrated	6	5		4	
	Sequential	6	5		4	
120-A	Integrated	9 10 12	6		4	
	Sequential	9 10 12		6	4	
120-B	Integrated	9 12	10	6	4	
	Sequential	9 12	10	6	4	
120-C	Integrated	4 9 12			2 6	
	Sequential	4 12			2 6	
120-D	Integrated	4 12	9	2	6	
	Sequential	12	4	9	6	
150-A	Integrated	6 7 9 12 13 14		3	15	
	Sequential	6 7 9 13 14		12	3	
150-B	Integrated	7 9 13 14	6	12	3 15	
	Sequential	7 9 13 14	6	12	3	
150-C	Integrated	2 6 7 12 13 15		11	8	
	Sequential	2 6 7 13 15		12	8	
150-D	Integrated	6 7 13 15	2	12	8 11	
	Sequential	6 7 13 15	2	12	8	

2nd part						
Instance	Approach	Facilities selected at each period				
		t=1	*t=2*	*t=3*	*t=4*	*t=5*
180-A	Integrated	3 4 8 12 16 17	7		2	
	Sequential	3 4 8 12 16 17			2	
180-B	Integrated	3 4 6 8 12	7	17	2	
	Sequential	3 4 8 12	6	7	2 17	
180-C	Integrated	2 4 7 8 12 17			11 13	
	Sequential	2 4 7 8 12 17			11	
180-D	Integrated	2 4 8 12 17	13	7 14	11	
	Sequential	4 8 12 17	2	7	11 13	
210-A	Integrated	1 2 5 12 15	4	10 13	7 8	
	Sequential	1 2 5 12 15		4 13	8 10	
210-B	Integrated	1 2 5 12 14	4	10	8 13 15	
	Sequential	1 2 12 15	5	13	4 8 10	
210-C	Integrated	1 5 10 14 15		7 8	2 11 18	
	Sequential	1 5 10 14 15		7 8	2 18	
210-D	Integrated	5 8 10 15	1	14	2 7 11 18	
	Sequential	1 5 10 15	18	14	2 8 11	
240-A	Integrated	6 8 11 15 16 22 24	2 10	17 19		
	Sequential	2 6 8 10 11 15 16 19 22 24			17	
240-B	Integrated	2 8 10 12 16 19 24	11	6	13 22	
	Sequential	2 6 10 11 16 19 24	15	8	22	
240-C	Integrated	2 4 6 8 15 17 20 22 23 24			10	
	Sequential	2 4 6 8 15 17 20 22 23 24			10	

(*continued on next page*)

Table 7 (*continued*).

2nd part						
Instance	Approach	Facilities selected at each period				
		t=1	*t=2*	*t=3*	*t=4*	*t=5*
240-D	Integrated	2 8 10 16 17 22	4 23	15	9	6
	Sequential	2 8 10 17 20 22 23	4	15	6	
270-A	Integrated	9 11 21	6 19 22 25	24	20 26 27	
	Sequential	19 20 21 22 24 25 26			16	
270-B	Integrated	8 19 21 22 25 26	20 27	6	16 24	
	Sequential	8 19 20 21 22	25	26	16 24	
270-C	Integrated	1 8 16 20 21 23 24 26		3 11	7	
	Sequential	1 8 16 20 21 23 24			3 7 19	
270-D	Integrated	8 11 20 21 23 24	1 26	16	3 7 19	
	Sequential	8 20 21 23 24 26	16	1	3 7 11 19	

Table 8
Analysis of results by instance type (difference between the sequential and integrated approaches).

Instance	Coordinate pattern	Demand profile	Range of APV difference (%)	Range of fill rate difference (%)
Type A	Random	Random	0.67 – 5.54	15 – 19.16
Type B	Random	Growing	1.04 – 2.48	15 – 19.19
Type C	Clustered	Random	0 – 5.16	0 – 30
Type D	Clustered	Growing	0 – 5.13	0 – 30

Table 9
Instance 120-D: Market parameters.

Market	Economic index	Opening cost parameter (φ) (rmu)	Selling price (P) (rmu/unit)
M1	88	670	3.62
M2	57	630	3.22
M3	140	740	4.67
M4	124	725	4.48

Detailed results for each instance are provided as supplementary material. These results are organized in an Excel spreadsheet containing two tabs (Integrated and Sequential). For each instance, the file indicates the facilities selected at each time period, their capacity, and the total quantity of goods processed by them. Additionally, the file shows the percentage of total demand handled by each facility and the percentage of the facility's capacity that was utilized. It can be observed that this percentage is always at least 75%.

6.4. Detailed results of instance 120-D

To get a detailed picture of the decision-making mechanism, we illustrate the solutions found by the integrated and sequential approaches on instance 120-D. This instance has four markets (denoted M1–M4) and 12 candidate facilities, whose features are detailed in Tables 9 and 10, respectively.

Figs. 5 and 6 present maps with the optimal network configurations obtained under the integrated and sequential approaches, respectively. These maps show the 4 markets with their borders (solid red lines). The facilities selected at each period appear in different shades of green. Non-selected facilities appear in gray color. The served customers are represented by blue circles while the customers with unsatisfied demands are represented in gray color.

As presented in Table 7, the integrated approach first opens facilities 12 and 4, that are located near customer clusters. Then, facilities 9, 2, and 6 are open at subsequent periods. The sequential approach opens only facility 12 in period 1, then facility 4 in period 2, facility 9 in period 3, and facility 6 in period 4. Table 11 details the value of equity, debt, tax shield benefit, expected bankruptcy cost and probability of

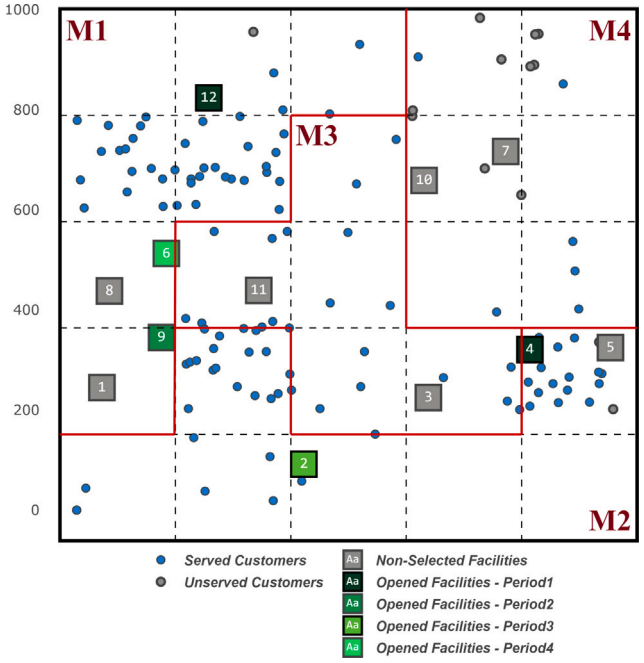

Fig. 5. Instance 120-D — Solution obtained with the integrated approach.

bankruptcy for both solutions (columns 2 to 6). It also compares the value of OGV and APV (columns 7 and 8)

The first difference between both solutions is the opening time of facility 4. Opening this facility in period 1 in the integrated approach slightly reduces the value of OGV but increases the value of total assets in period 1; this increases the debt ratio's denominator and gives more opportunity to borrow ($borrow_1 = 87\,479$ *rmu* with the integrated approach, versus $borrow_1 = 44\,142$ *rmu* with the sequential approach). An immediate consequence is the increase of the tax shield benefit. As the value of the debt ratio at the first period is 0.8 in both approaches, the additional investment by the integrated approach does not increase the probability of bankruptcy. A similar mechanism explains why facility 9 is opened earlier with the integrated approach. The last difference between both solutions concerns facility 2. The total assets at the end of the second period are higher with the integrated approach. It allows the firm to use debt financing for opening facility 2 which results in a higher tax shield benefit. But, this investment slightly decreases the OGV.

In the integrated approach, higher investment results in an increase in both the liabilities and total assets. However, the increase in total assets is greater than the increase in liabilities. Consequently, the integrated approach results in a lower debt ratio, which leads to a lower probability of bankruptcy. This can also be explained by the difference

Table 10
Instance 120-D: Facility parameters.

Facility	Market	Capacity (C) (units/period)	Opening cost (O) (rmu)	Fixed yearly running cost (F) (rmu/year)	Processing cost (μ) (rmu/unit)
1	M1	3500	39 600	1980	1.03
2	M2	4500	42 300	2115	1.31
3	M3	3500	37 300	1865	1.03
4	M2	6000	48 800	2440	1.29
5	M2	4500	42 300	2115	1.00
6	M1	6000	51 900	2595	0.99
7	M4	3500	42 900	2145	1.15
8	M1	4500	44 900	2245	1.00
9	M1	6000	51 900	2595	1.10
10	M4	4500	44 900	2245	1.12
11	M3	3500	42 900	2145	1.34
12	M1	6000	48 800	2440	0.99

Table 11
Value of the financial indicators in the integrated and sequential based solutions.

Solution approach	Equity	Loan balance	Tax shield benefit	EBC	Proba. of bankruptcy	Objective function	
						OGV	APV
Integrated	150 311	100 160	14 492	1507	6.4%	47 838	**60 823**
Sequential	115 844	95 838	12 099	2206	9.3%	**47 960**	57 853

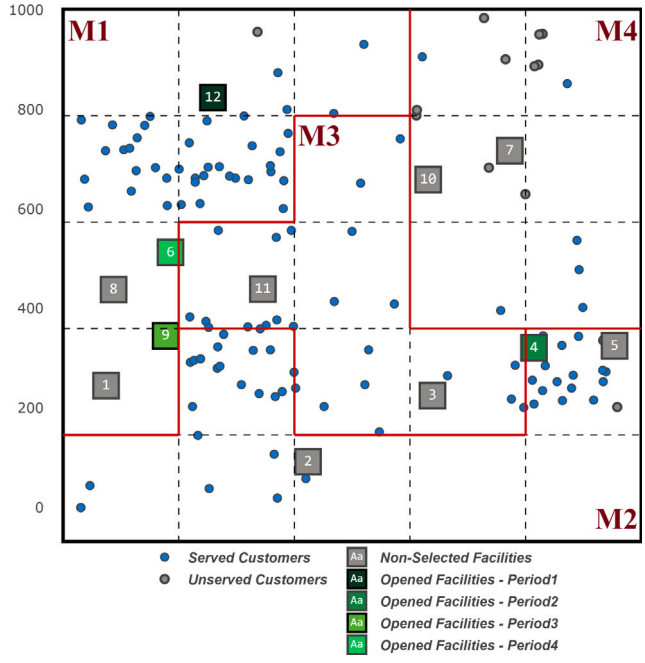

Fig. 6. Instance 120-D — Solution obtained with the sequential approach.

between the leverage ratios of the two approaches. The integrated strategy eventually results in a somewhat greater business value due to larger tax shield benefits and lower expected bankruptcy costs.

Another consequence concerns the market coverage. A higher fill rate is achieved with the integrated approach ($fill\ rate = 83\%$, versus 64%), due to larger and earlier investments.

The results for ROA, ROE, and leverage ratio (shown in) are also consistent. As expected, the ROA of the integrated approach is lower than the sequential approach (8.31% versus 9.43%). Although the average income paid to both shareholders and debt-holders ($NOPAT + interest \times (1 - \eta)$) is higher on average with the integrated approach (16 431 *rmu* versus 14 805 *rmu*), the value of total assets is also higher on average (206 304 *rmu* versus 169 645 *rmu*).

The leverage ratio of the integrated approach is also lower than the sequential approach (29.78% versus 34.01%). These two factors positively affect the ROE. As both are lower with the integrated approach, a lower ROE with the integrated approach is expected (17.65% versus 23.11%). Correspondingly, the lower leverage explains why the gap between ROE is higher than the gap between ROA. The integrated approach generates more money for the investors but at a lower rate of return.

7. Managerial insights and conclusion

In this paper, we proposed using the adjusted present value as an objective function in a facility location problem. This objective function integrates both the classical components of the NPV and additional costs associated with debt or equity financing. The proposed mixed-integer linear programming formulation can be solved to optimality by state-of-the-art MILP solvers, at least for reasonable-size instances. We compared two approaches that consist of either decomposing the model into a logistical part and a financial part, or solving an integrated model.

Valuable managerial insights can be gained from these results, allowing strategic supply chain decisions that are consistent from both a logistical and financial perspective.

First, the integration of financial factors into logistics decisions has a positive impact on a company's financial position. It encourages firms to make larger and earlier investments. This changes both the spatial layout of the logistics network and its implementation on a strategic time horizon. This impact on the supply chain is made possible by an improved use of financial resources, in particular the tax shield benefit. An immediate benefit of these larger and earlier investments is the significant increase in the customer demand coverage rate. Strikingly, the integrated approach does not increase the firm's risk of bankruptcy compared to the sequential approach.

Secondly, as observed in , the classical logistical KPI are improved with an integrated approach. This result lightens the positive role of the financial strategy on the efficiency of the supply chain.

In addition, the financial metrics provide other insights. Companies can generate more cash flow by integrating financial decisions. However, this comes at the cost of a slightly lower rate of return. Exploring this trade-off is an avenue for potential future research.

For practical implementation of the integrated decision model, company management must consider the optimal mix of debt and equity financing for its strategic projects. The benefits from debt-related tax savings are relatively straightforward to calculate, as they depend on the easily observable corporate tax rates in the countries where the locations are established. In contrast, estimating bankruptcy costs is far more challenging. The probability of a company's bankruptcy can be estimated using several factors: (i) the credit rating assigned to the company's debt by rating agencies, (ii) the interest rate set by lenders (with higher rates indicating a greater risk of bankruptcy), (iii) econometric models based on the company's financial ratios (scoring functions). Bankruptcy costs will be assessed by considering the costs of bankruptcy procedures in each country for direct costs and by comparing the bankruptcy costs of previously defaulted companies for indirect costs. Another managerial challenge is bringing the operational and financial teams together to collaboratively develop the implementation strategy.

Since the goal of this paper was to highlight the interactions between logistical and financial decisions rather than describing a particular supply chain, we considered a simple two-echelon supply chain. This model can be extended in many directions. Considering a supply chain with already active facilities, customers and current loans only requires modifying the initial conditions. Relaxing the incremental customer satisfaction, allowing partial customer satisfaction or setting single-sourcing constraints would slightly modify the logistics constraints, but these rules do not affect the financial part of the model. Extending the model to more complex supply chains (e.g., with additional echelons, selection of raw material suppliers, resizing of facilities, selection of subcontractors, selection of transportation modes [51]) would require a set of more elaborated logistical constraints but, once again, these logistical rules do not affect the financial part of the model.

However, introducing the possibility of closing facilities has direct financial consequences. A realistic assumption is that, once opened, facilities should be operating for a minimum number of periods (which is generally larger than T). Assuming that facilities already operating in period 0 can be closed before the end of their lifetime has consequences both on the value of OGV and FGV. Closing a facility and selling it modifies the cash flows. All associated loans must be the object of early reimbursement, with possible financial penalties. Finally, constraints (15), which assumed $y_{jT} = 1$ for any operating facility, should be adapted to the case of closing facilities. Our model can easily apply to a realistic case where the taxes and cost of debt differ from one location to another. In that case, the difference between both approaches would probably be more significant.

A further possible extension is the consideration of stochastic data or parameters. Several factors, such as demand, logistics costs, disruption probabilities, and financial parameters, are subject to uncertainty. A stochastic model incorporating multiple uncertain parameters would provide a more realistic representation. However, managing several stochastic parameters simultaneously remains a significant challenge in stochastic optimization. Robust optimization might be a more tractable approach, though it may yield very conservative results. A possible solution is to utilizing methods like Approximate Dynamic Programming and Stochastic Dynamic Optimization, which handle both uncertainty and time-dependent decision-making.

Other possible extensions of the model would be to explore financial risk-aware facility location models [52], or to extend the model by integrating tactical variables such as the working capital.

The costly linearization mechanism prevents MILP solvers to solve much larger instances than those presented in this paper.

Table 5 illustrates the model's limited scalability. While the solutions obtained from the integrated approach are relatively close to those from the sequential approach, a significant gap between the lower and upper bounds emerges for instances with more than 210 customers. For even larger instances, such as those with 270 customers, the sequential approach also struggles to find optimal solutions. To tackle larger instances or more complex logistics networks not addressed in this study, alternative solution methods are required. Utilizing heuristic methods or a combination of exact and heuristic approaches could provide an effective strategy for solving these larger problems.

CRediT authorship contribution statement

Hamidreza Rezaei: Writing – review & editing, Writing – original draft, Validation, Software, Methodology, Investigation, Formal analysis, Conceptualization. **Nathalie Bostel:** Writing – review & editing, Writing – original draft, Validation, Supervision, Methodology, Investigation, Formal analysis, Conceptualization. **Vincent Hovelaque:** Writing – review & editing, Writing – original draft, Supervision, Project administration, Methodology, Investigation, Funding acquisition, Conceptualization. **Olivier Péton:** Writing – review & editing, Writing – original draft, Validation, Supervision, Methodology, Investigation, Formal analysis, Conceptualization. **Jean-Laurent Viviani:** Writing – review & editing, Writing – original draft, Validation, Methodology.

Declaration of competing interest

None

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Appendix A. Notations

Sets	
$\mathcal{I}$	Set of customers (i)
$\mathcal{J}$	Set of candidate facilities (j)
$\mathcal{T}$	Set of time periods (t)
$\mathcal{T}'$	Extended set of time periods (t)

Decision variables		
y_{jt}	= 1 if $j \in \mathcal{J}$ is operating in period $t \in \mathcal{T} \cup \mathcal{T}'$, 0 otherwise	$\{0,1\}$
$borrow_t$	Amount borrowed in period $t \in \mathcal{T}$	$[0,\infty[$
EE_t	External equity in period $t \in \mathcal{T}$	$[0,\infty[$
IE_t	Invested amount of internal equity in period $t \in \mathcal{T}$	$[0,\infty[$

Intermediate logistical variables		
x_{it}	= 1 if $i \in \mathcal{I}$ is served in period $t \in \mathcal{T}$, 0 otherwise	$\{0,1\}$
e_{jt}	Expenses occurred in period $t \in T$ for $j \in \mathcal{J}$	$[0,\infty[$
q_{ijt}	Quantity shipped from $j \in \mathcal{J}$ to $i \in \mathcal{I}$ in period $t \in T$	$[0,\infty[$
r_{jt}	Revenues obtained in period $t \in T$ related to $j \in \mathcal{J}$	$[0,\infty[$

Intermediate financial variables		
$balance_{\bar{\imath}t}$	Value of loan balance $l_{\bar{\imath}}$ in period $t \in \mathcal{T}$	$[0, \infty[$
$cash_t$	Cash level in period $t \in \mathcal{T}$	$]-\infty, \infty[$
CF_{jt}	Cash flow generated by $j \in J$ in period $t \in \mathcal{T}$	$]-\infty, \infty[$
$EBIT_{jt}$	Earnings before interest and tax by $j \in \mathcal{J}$ in period $t \in \mathcal{T}$	$]-\infty, \infty[$
$equity_t$	Amount of equity in period $t \in \mathcal{T}$	$[0, \infty[$
FCF_{jt}	Free cash flow associated with $j \in \mathcal{J}$ in period $t \in \mathcal{T}$	$]-\infty, \infty[$
FGV	Financially generated value of the firm	$]-\infty, \infty[$
$interest_{\bar{\imath}t}$	Interest of loan $l_{\bar{\imath}}$ in period $t \in \mathcal{T}$	$[0, \infty[$
KD_t	Cost of loan l_t	$[0, 1]$
OGV	Operationally generated value of the firm	$]-\infty, \infty[$
$NOPAT_t$	Net operating profit after tax in period $t \in \mathcal{T}$	$]-\infty, \infty[$
p	Probability of bankruptcy	$[0, 1]$
$repay_{\bar{\imath}t}$	Repayment of loan $l_{\bar{\imath}}$ in period $t \in \mathcal{T}$	$[0, \infty[$

Logistical parameters	
C_j	Yearly production capacity of facility $j \in \mathcal{J}$
D_{it}	Demand of $i \in \mathcal{I}$ in period $t \in \mathcal{T}$
Dep_j	Depreciation of $j \in \mathcal{J}$
F_j	Fixed yearly running cost of $j \in \mathcal{J}$
J_{max}	Max number of facilities to be located
L	Facility lifetime
O_j	Opening cost of $j \in \mathcal{J}$
P_i	Product selling price for $i \in \mathcal{I}$
SV_j	Salvage Value of $j \in \mathcal{J}$
V_{ij}	Equals 1 if $j \in \mathcal{J}$ can serve $i \in \mathcal{I}$, 0 otherwise
μ_j	Processing cost of $j \in \mathcal{J}$
ω	Unit transportation cost

Financial parameters	
K_E	Cost of equity rate
N	Number of annuities
β	Bankruptcy probability parameter
γ	Bankruptcy cost parameter
δ	Dividend payout ratio
η	Firm tax rate
ζ	Upper bound of debt ratio

Appendix B. Detailed results: accumulated capacity and total investment

Table B.12 reports the accumulated capacity installed at each period along with the total amount of investment in each solution (sum of the fixed cost of the selected facilities). For the sake of clarity, the capacity and investment values in Table B.12 represent thousands of units.

Appendix C. Linearization procedures

In the proposed mathematical model, several constraints are non-linear due to the product of two decision variables. We first recall several well-known linearization techniques (Appendix C.1) and then explain how these techniques are applied to our model.

C.1. Classical linearization procedures

- **Linearization 1: product of a real and a binary variable**
Let u and v be two real positive variables and b a binary variable. Given an upper bound U of variable u, the expression $v = bu$ can be linearized by:

$$v \leq u$$
$$v \geq u - U(1 - B)$$
$$v \leq Ub$$
$$v \geq 0$$

- **Linearization 2: product of two continuous variables**
There is no exact way to linearize a product of two continuous variables u and v. In our model, most continuous variables represent large monetary values. The consequence of rounding down these variables to the nearest integer value is then negligible. Assuming that variable u is rounded down, we use a power-of-two decomposition to represent u as a set of binary variables b_i:

$$u = b_0 + 2b_1 + 4b_2 + 8b_3 + \cdots + 2^{\lfloor log_2 U \rfloor} b_{log_2 U},$$

where U is an upper bound of v. For example, the value 100 = 4 + 32 + 64 can be represented by the vector (0,0,1,0,0,1,1).
The product uv can now be rewritten as follows:

$$uv = v \sum_{i=0}^{i=\lfloor log_2 U \rfloor} 2^i b_i.$$

It is a weighted sum of the terms vb_i, where v is a continuous variable and b_i is an integer variable. These terms can be linearized with Linearization 1.

- **Linearization 3: Piece-wise linearization**
Consider a general non-linear function $f(u)$ of a single variable u, where $u \in [u_0, u_h]$.
We consider intermediate values $u_1, \ldots, u_{h-1}$ and intervals of the form $[u_i, u_{i+1}]_{0 \leq i \leq h-1}$. Let λ_i be a vector of binary variables, where $\lambda_i = 1$ if and only if $u \in [u_i, u_{i+1}]$.
We define a vector ξ of continuous variables such that

$$u = \sum_{i=0}^{h-1} \xi_i$$
$$\lambda_i u_i \leq \xi_i \leq \lambda_i u_{i+1}$$
$$\sum_{i=0}^{h-1} \lambda_i = 1.$$

Only one value in vector ξ is strictly positive and it corresponds to the value u. Hence, the function $f(u)$ is approximated by selecting the appropriate interval and considering the piece-wise linear approximation of $f(u)$ in this interval:

$$f(u) = \sum_i \lambda_i f(u_i) + \sum_i \left((\xi_i - \lambda_i u_i) \frac{f(u_{i+1}) - f(u_i)}{u_{i+1} - u_i} \right).$$

Since only one value of vectors λ and ξ is strictly positive, this expression reduces to

$$f(u) = \lambda_i f(u_i) + (\xi_i - u_i) \frac{f(u_{i+1}) - f(u_i)}{u_{i+1} - u_i}$$

for some $0 \leq i \leq h - 1$.

- **Linearization 4: Logical constraints**
Logical constraints allow the expression of logical operators such as logical-or, logical-and, and conditional statements (if ... then ...) in the linear programming context.

  - **Or-condition:** Consider a general mathematical expression $g(x)$ and two parameters b_1 and b_2. The expression

    $$(g(x) \leq b_1) \vee (g(x) \leq b_2)$$

Table B.12
Accumulated capacity installed at each period and overall investment with the sequential and the integrated approaches.

1st part

Instance	Approach	Total capacity installed ($\times 10^3$)					Investment
		t=1	*t=2*	*t=3*	*t=4*	*t=5*	($\times 10^3$ rmu)
60-A	Integrated	10.5	10.5	10.5	14	14	146.1
	Sequential	10.5	10.5	10.5	10.5	10.5	104.7
60-B	Integrated	10.5	10.5	10.5	16.5	16.5	162.4
	Sequential	4.5	9	9	15	15	154.7
60-C	Integrated	12	12	12	12	12	111.9
	Sequential	6	6	6	12	12	111.9
60-D	Integrated	6	9.5	9.5	15.5	15.5	153.3
	Sequential	6	6	6	12	12	111.9
90-A	Integrated	16.5	16.5	16.5	22.5	22.5	211.3
	Sequential	12	12	12	18	18	165.7
90-B	Integrated	12	12	16.5	22.5	22.5	211.3
	Sequential	6	12	12	22.5	22.5	211.3
90-C	Integrated	10.5	10.5	10.5	16.5	16.5	154.8
	Sequential	10.5	10.5	10.5	16.5	16.5	154.8
90-D	Integrated	6	10.5	10.5	16.5	16.5	154.8
	Sequential	6	10.5	10.5	16.5	16.5	154.8
120-A	Integrated	16.5	22.5	22.5	28.5	28.5	250.6
	Sequential	16.5	16.5	22.5	28.5	28.5	250.6
120-B	Integrated	12	16.5	22.5	28.5	28.5	250.6
	Sequential	12	16.5	22.5	28.5	28.5	250.6
120-C	Integrated	18	18	18	28.5	28.5	243.7
	Sequential	12	12	12	22.5	22.5	191.8
120-D	Integrated	12	18	22.5	28.5	28.5	243.7
	Sequential	6	12	18	24	24	201.4
150-A	Integrated	32	32	36.5	41	41	341.2
	Sequential	27.5	27.5	32	36.5	36.5	296.6
150-B	Integrated	21.5	27.5	32	41	41	341.2
	Sequential	21.5	27.5	32	36.5	36.5	296.6
150-C	Integrated	33	33	36.5	41	41	370.5
	Sequential	28.5	28.5	33	37.5	37.5	331.2
150-D	Integrated	22.5	28.5	33	41	41	370.5
	Sequential	22.5	28.5	33	37.5	37.5	331.2

2nd part

Instance	Approach	Total capacity installed ($\times 10^3$)					Investment
		t=1	*t=2*	*t=3*	*t=4*	*t=5*	($\times 10^3$ rmu)
180-A	Integrated	33	37.5	37.5	43.5	43.5	391.5
	Sequential	33	33	33	39	39	346.9
180-B	Integrated	28.5	33	39	45	45	400.7
	Sequential	22.5	28.5	33	45	45	400.7
180-C	Integrated	34.5	34.5	34.5	42.5	42.5	385.4
	Sequential	34.5	34.5	34.5	39	39	346.6
180-D	Integrated	30	33.5	41.5	46	46	424.2
	Sequential	24	30	34.5	42.5	42.5	385.4
210-A	Integrated	28.5	33	43.5	53	53	502.1
	Sequential	28.5	28.5	37.5	49.5	49.5	463.1
210-B	Integrated	28.5	33	39	55.5	55.5	514.2
	Sequential	22.5	28.5	33	49.5	49.5	463.1
210-C	Integrated	30	30	39.5	53.5	53.5	505
	Sequential	30	30	39.5	49	49	460.7
210-D	Integrated	24	30	36	53.5	53.5	505
	Sequential	24	27.5	33.5	50	50	466
240-A	Integrated	40.5	52.5	63	63	63	559.1
	Sequential	57	57	57	63	63	559.1
240-B	Integrated	39	43.5	49.5	59	59	539.4
	Sequential	39	45	51	57	57	505.8
240-C	Integrated	57	57	57	63	63	553.9
	Sequential	57	57	57	63	63	553.9

(*continued on next page*)

Table B.12 (*continued*).

2nd part							
Instance	Approach	Total capacity installed ($\times 10^3$)					Investment
		t=1	*t=2*	*t=3*	*t=4*	*t=5*	($\times 10^3$ rmu)
240-D	Integrated	36	45	51	54.5	60.5	541.9
	Sequential	40.5	45	51	57	57	503.2
270-A	Integrated	15	34.5	40.5	57	57	521.4
	Sequential	39	39	39	45	45	402.1
270-B	Integrated	33	43.5	48	60	60	543
	Sequential	28.5	33	39	51	51	453.8
270-C	Integrated	45	45	54	58.5	58.5	535.9
	Sequential	39	39	39	54	54	497.6
270-D	Integrated	33	43.5	49.5	64.5	64.5	592.2
	Sequential	34.5	40.5	45	64.5	64.5	592.2

can be linearized by:

$$g(x) \le b_1 + M \times \vartheta$$
$$g(x) \le b_2 + M \times (1 - \vartheta)$$

where M is a large enough positive value and ϑ a binary variable.

– **Conditional expression:** Consider the following expression

$$\begin{cases} g(x) \ge 0 \Rightarrow \rho = 1 \\ g(x) < 0 \Rightarrow \rho = 0, \end{cases}$$

where ρ is a binary variable. This conditional expression is linearized by:

$$\begin{cases} g(x) + \epsilon \le \rho \times M \\ -g(x) \le (1 - \rho) \times M \end{cases}$$

where ϵ is a relatively small positive value.

C.2. Linearization of the OGV second term

The second term of the OGV formula (16) is non-linear due to the product of continuous variables CF_{jT} and binary variables y_{jt}. This expression can be linearized with Linearization 1 of Appendix C.1.

Since the cash flow variables CF_{jT} measure the cash flows of facility $j \in \mathcal{J}$ after period T, its maximal value corresponds to the case where all customers' demands are served by facility j. Hence, the upper bound U can be set to $\sum_{i \in \mathcal{I}} P_i \, D_{iT}$.

C.3. Linearization of constraints (19)

According to the constraints (19), the repayment for loan $l_{\bar{t}}$ can be a positive value only for periods $\bar{t} < t \le \bar{t} + N$. This amount depends on the amount borrowed ($borrow_{\bar{t}}$), and its associated interest rate, $KD_{\bar{t}}$. We linearize the product of $borrow_{\bar{t}}$ and $KD_{\bar{t}}$. First, $KD_{\bar{t}}$ is written as follows:

$$KD_{\bar{t}} = \sum_{m=1}^{n} ir_m \times \theta_{\bar{t}m} \qquad \forall \bar{t} \in \mathcal{T},$$

where n represents the number of debt ratio intervals and ir_m is the interest rate associated with the interval $m \in [1, n]$. $\theta_{\bar{t}m}$ is a binary variable that takes the value of 1 if the debt ratio after activating loan $l_{\bar{t}}$ belongs to the interval m, and 0 otherwise.

Thus, if $\theta_{\bar{t}m} = 1$, the interest rate of loan $l_{\bar{t}}$ is ir_m. Now the repayment formula can be rewritten as follows:

$$\begin{aligned} repay_{\bar{t}t} &= borrow_{\bar{t}} \times \frac{KD_{\bar{t}}(1 + KD_{\bar{t}})^N}{(1 + KD_{\bar{t}})^N - 1} \\ &= borrow_{\bar{t}} \times \sum_{m=1}^{n} \frac{ir_m(1 + ir_m)^N}{(1 + ir_m)^N - 1} \times \theta_{\bar{t}m} \\ &= \sum_{m=1}^{n} \frac{ir_m(1 + ir_m)^N}{(1 + ir_m)^N - 1} \times \theta_{\bar{t}m} \times borrow_{\bar{t}}, \end{aligned}$$

where the term $\theta_{\bar{t}m} \times borrow_{\bar{t}}$ is a product of real and binary variables, that can be linearized according to the Linearization 1 of Appendix C.1, where the upper bound U can be set to the maximum investment size which equals $\sum_{j \in J} O_j$.

C.4. Linearization of constraints (20)

The linearization of product $KD_{\bar{t}} \times balance_{\bar{t},t-1}$ follows the same principle as in Appendix C.3.

We rewrite $KD_{\bar{t}} = \sum_{m=1}^{n} ir_m \times \theta_{\bar{t}m}$, and obtain the following equation:

$$interest_{\bar{t}t} = \sum_{m=1}^{n} ir_m \times \theta_{\bar{t}m} \times balance_{\bar{t},t-1}.$$

Using again the procedure explained in Appendix C.1 concludes the linearization. For the product of binary ($\theta_{\bar{t}m}$) and real ($balance_{\bar{t},t-1}$) variables, the upper bound U is set to the maximum possible amount to borrow i.e., $\sum_{j \in J} O_j$.

C.5. Linearization of constraints (23)

In constraints (23), the expression $\max(0, cash_{t-1})$ is non-linear due to the max operator. We introduce the continuous variables $IEMAX_t$ and auxiliary binary variables w_t satisfying:

$$\begin{aligned} &IEMAX_t \ge cash_{t-1} && \forall t \in \mathcal{T} \\ &IEMAX_t \le cash_{t-1} + U\,(1 - w_t) && \forall t \in \mathcal{T} \\ &IEMAX_t \le U\, w_t && \forall t \in \mathcal{T} \\ &IEMAX_t \ge 0 && \forall t \in \mathcal{T}, \end{aligned}$$

where $U = \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} P_i \, D_{it}$.

C.6. Linearization of the tax shield benefit - objective function's second term

The tax shield benefit is calculated as:

$$(1 - p)\, \eta \sum_{t \in \mathcal{T} \cup \mathcal{T}'} \frac{\sum_{\bar{t} \in \mathcal{T}} interest_{\bar{t}t}}{(1 + K_E)^t}$$

where both p and $\sum_{\bar{t} \in \mathcal{T}} interest_{\bar{t}t}$ are continuous variables. We use Linearization 2 presented in Appendix C.1.

U can be set to the maximum value of interest i.e., the interest paid if all candidate facilities are opened using debt financing. Ignoring the impact of the time value of money: $U = N \times KD_{\bar{t}} \times \sum_{j \in J} O_j$, where $KD_{\bar{t}}$ is set to its maximum possible value.

C.7. Linearization of the expected bankruptcy cost (EBC) - objective function's third term

EBC is estimated as $p \times \gamma \times OGV$ where p and OGV are continuous variables. We use Linearization 2 of Appendix C.1.

The maximum value for OGV can be obtained if all customers are served at all periods without any cost. Thus:

Table D.13
Parameter values for sensitivity analysis.

Parameter	Baseline	Alternative value 1	Alternative value 2
Tax rate (η)	30%	20%	40%
Debt ratio upper bound (ζ)	0.8	0.7	0.9
Bankruptcy cost parameter (γ)	0.5	0.4	0.6
Facility fixed yearly running cost (F)	5%	7.5%	10%
Unit transport cost (ω)	0.002 rmu	0.001 rmu	0.003 rmu

$$U = (1-\eta) \times \Big(\sum_{i\in I} \sum_{j\in J} \sum_{t\in\mathcal{T}} P_i\, D_{it} + \sum_{i\in I} \sum_{j\in J} \sum_{t\in\mathcal{T}'} P_i\, D_{iT} \Big).$$

C.8. Linearization of the probability of bankruptcy p - eq. (25)

In Eq. (25), the probability of bankruptcy is given by $p = \left(\frac{\sum_{\bar{t}\in\mathcal{T}} balance_{\bar{t}T}}{\sum_{\bar{t}\in\mathcal{T}} balance_{\bar{t}T} + equity_T} \right)^{\beta}$.

We first consider the debt ratio:

$$\kappa = \frac{\sum_{\bar{t}\in\mathcal{T}} balance_{\bar{t}T}}{\sum_{\bar{t}\in\mathcal{T}} balance_{\bar{t}T} + equity_T}.$$

Then, we introduce the notation $TA = \sum_{\bar{t}\in\mathcal{T}} balance_{\bar{t}T} + equity_T$ representing the total assets in period t.

Using a power-of-two decomposition, TA can be represented by a set of binary variables b_j.

The debt ratio κ can then be rewritten as:

$$\kappa = \frac{\sum_{\bar{t}\in\mathcal{T}} balance_{\bar{t}T}}{\sum_{j=0}^{j=U} 2^j b_j}.$$

Using a cross multiplication, we have $\kappa \times \sum_{j=0}^{j=U} 2^j b_j = \sum_{\bar{t}\in\mathcal{T}} balance_{\bar{t}T}$.

This formulation does not contain any ratio, but still multiplies the continuous variable κ with a set of binary variables b_j. Thus, Linearization 1 of Appendix C.1 can be used, with $U = 1$. Finally, we use a piece-wise linearization (Linearization 3 of Appendix C.1) to linearize the expression $p(\kappa) = \kappa^{\beta}$.

C.9. Linearization of debt ratio intervals

The debt ratio intervals impose different interest rates, i.e., $KD_{\bar{t}}$, for loan $l_{\bar{t}}$.

$$\alpha_{1m} \le \kappa_{\bar{t}} \le \alpha_{2m} \Rightarrow KD_{\bar{t}} = ir_m,$$

where m represents an interval number, $\kappa_{\bar{t}}$ denotes the debt ratio at $\bar{t}$, and α_{1m} and α_{2m} are the debt ratio's lower and upper bounds associated with interval m.

Interval m is a conditional expression that can be linearized using Linearization 4 of Appendix C.1. To this end, the interest rate of loan $l_{\bar{t}}$, is replaced by the term $\sum_{m=1}^{n} ir_m \times \theta_{\bar{t}m}$, where $\theta_{\bar{t}m}$ takes the value of 1 if $\kappa_{\bar{t}}$, the debt ratio after activating loan $l_{\bar{t}}$, stands in the interval m, and 0 otherwise.

Appendix D. Sensitivity analysis

We conduct a sensitivity analysis to assess the impact of various parameters on the objective function (APV) and fill rate within the integrated approach. Five key parameters are examined: Tax Rate (η), Debt Ratio Upper Bound (ζ), Bankruptcy Cost Parameter (γ), Facility Fixed Yearly Running Cost (F), and Unit Transport Cost (ω). For each parameter, three values are tested: the baseline value used in prior numerical experiments and two alternative values as defined in Table D.13. In this table, the Facility Fixed Yearly Running Cost (F) is calculated as a percentage of the facility opening cost (O).

The sensitivity of these parameters is tested on a representative subset of six instances. These selected instances — 60-A, 60-C, 120-B, 120-D, 210-A, 210-B — represent different sizes and patterns with an equal distribution across instance types. All instances were solved under the same settings as those specified in Section 6.2. The key observations regarding the influence of these parameters on APV and fill rate are outlined in the following subsections. Fig. D.7 visualizes the detailed results of this analysis.

D.1. Tax rate

APV:. The impact of the tax rate on APV is almost linear. A reduction in the tax rate will result in an increase in APV, due to the decrease in the tax burden on the firm's net income, which will in turn lead to an increase in OGV. However, the advantage of the tax shield is reduced as the tax rate is lowered. While lower debt levels reduce bankruptcy costs, the overall impact on APV remains positive at lower tax rates. Conversely, higher tax rates increase the benefit from the tax shield, partially offsetting the decline in OGV. This is accompanied by higher debt and a greater risk of bankruptcy, which ultimately reduces APV at higher tax rates.

Fill rate:. Lowering the tax rate generally reduces the fill rate for most instances, except for 210-A. Conversely, raising the tax rate increases the fill rate for half of the instances, while the others remain close to the baseline.

When the tax rate decreases, the firm experiences a lower tax burden, which leads to an increase in OGV. Consequently, the expected bankruptcy cost – directly linked to OGV – tends to rise as a result of this increase, making the firm more cautious about increasing its debt ratio.

This reduces the incentive to raise debt, while the diminished tax shield further lowers FGV. These combined effects reduce the motivation to open new facilities, aligning with earlier findings (obtained through comparison of the integrated and sequential approaches) that financial considerations make investment more appealing. Similar reasoning applies to the cases where the fill rate increases when the tax rate is raised. Thus, the observed decline in the fill rate under lower and higher tax rates can be explained by the interplay between rising bankruptcy costs, reduced debt, and reduced tax shield benefits.

D.2. Debt ratio upper bound

Considering value 0.9 as debt ratio upper bound amount to create a new debt ratio interval $(80\%, 90\%]$ in Table 3. We associate this interval with a 15.9% cost of loan.

APV:. The results of the sensitivity analysis reveal a positive correlation between APV and the debt ratio upper bound. While increasing the firm's debt ratio raises the probability of bankruptcy, it also increases the benefit of the tax shield. In all instances, the increase in the tax shield benefit outweighs the increase in expected bankruptcy costs, resulting in a higher APV when the firm is allowed to borrow more. Conversely, decreasing the upper bound to 0.7 reduces APV across all instances.

The increase in APV when raising the upper bound to 0.9 is larger than the decrease when lowering it to 0.7. This asymmetry is due to the sharp rise in loan costs as the debt ratio increases. A higher debt ratio, like 0.9, raises loan interest rates and boosts the tax shield benefit. In contrast, lowering the upper bound to 0.7 reduces loan costs, but the decrease in the tax shield is less significant than the benefit gained at a 0.9 debt ratio.

Fill rate:. Decreasing the debt ratio upper bound generally results in a decreased or unchanged fill rate, while increasing the upper bound typically leads to a higher fill rate. A higher tax shield makes opening new facilities more appealing, while a lower one has the opposite effect. The greater fill rate increase with a higher debt ratio upper bound is due to the larger tax shield benefit outweighing the reduction from a lower bound.

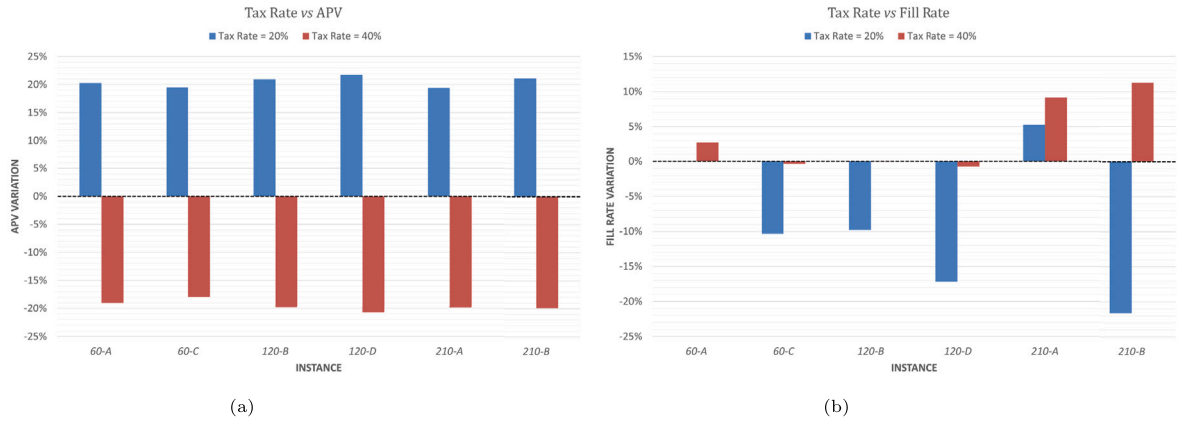

Fig. D.7. Impact of variation of tax rate η on APV (a) and on the fill rate (b)

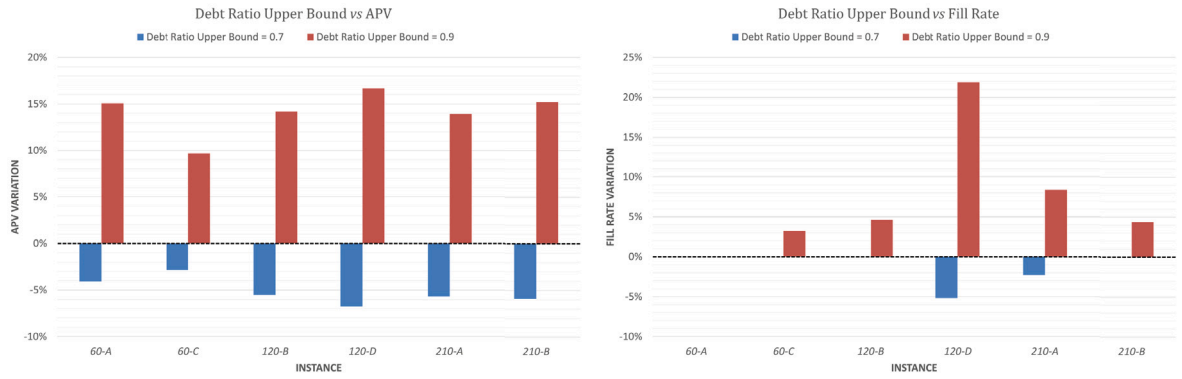

Fig. D.8. Impact of variation of the debt ratio upper bound ζ on APV (a) and on the fill rate (b)

Table D.14
Calculation of the cost of loan KD_i for the alternative γ levels.

Debt ratio interval	Cost of Loan (in %)		
	$\gamma = 0.5$ *(baseline)*	$\gamma = 0.4$	$\gamma = 0.6$
(0,30]	3.2	3.2	3.2
(30,40]	3.5	3.4	3.5
(40,50]	4.0	3.9	4.1
(50,60]	5.0	4.9	5.1
(60,70]	6.6	6.5	6.8
(70,80]	9.6	9.4	9.8

D.3. Bankruptcy cost parameter

Changing the bankruptcy cost parameter impacts the loan cost. A higher (or lower) bankruptcy cost parameter results in a lower (or higher) recovery value in the event of default, affecting how much the company can recover. As a result, the loan cost adjusts accordingly, and we updated it using the arbitrage equation, as shown in Table D.14.

APV:. Reducing the bankruptcy cost parameter increases both OGV and the debt ratio, potentially resulting in the same or higher APV. In all instances, lower bankruptcy costs lead to higher APV. Conversely, increasing bankruptcy costs consistently reduces APV, confirming a negative correlation between the bankruptcy cost parameter and APV.

Fill rate:. Decreasing the bankruptcy cost parameter may have a positive effect on the fill rate. It provides an opportunity to invest more, which in turn can lead to serving more customers. This is reflected in the results, where lowering the bankruptcy cost either increases the fill rate or keeps it unchanged. Conversely, increasing the bankruptcy cost reduces the attractiveness of further investments. As the cost increases, the impact of FGV diminishes, making it less profitable to invest further or serve additional customers. This results in a decreased or unchanged fill rate. The sharp reduction in fill rate for instance 60-C is primarily due to the limited number of candidate facilities in this instance. When the bankruptcy cost increases, the FGV effect decreases and there are no viable alternative facilities to replace the one(s) that are closed.

This finding supports the earlier observation that the integration of financial dimensions can enhance the attractiveness of investment, ultimately leading to improved customer satisfaction.

D.4. Facility fixed yearly running cost

APV:. There is a negative linear relationship between APV and the facility's fixed yearly running cost. This cost, generated as a percentage of the facility's opening cost, directly impacts APV. The higher the percentage, the higher the yearly running cost, leading to a reduction in OGV and, consequently, a lower APV.

Fill rate:. A similar trend can be observed between the increase in fixed yearly running costs and the fill rate. As this cost rises, the fill rate tends to decrease or remain constant across all instances. For most instances, increasing the fixed yearly running cost from 5% to 7.5% and then to 10% consistently reduces the fill rate, indicating that fewer facilities are being opened as the costs rise (see Figs. D.8–D.11).

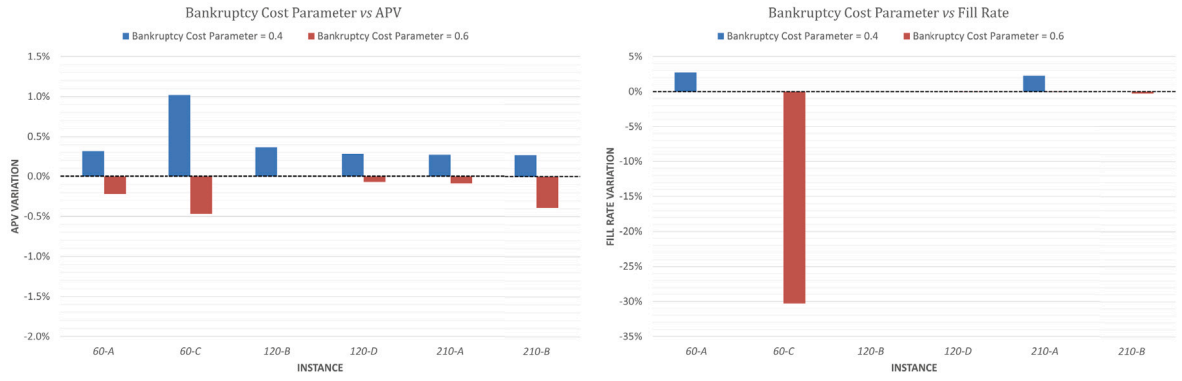

Fig. D.9. Impact of variation of the bankruptcy cost parameter γ on APV (a) and on the fill rate (b)

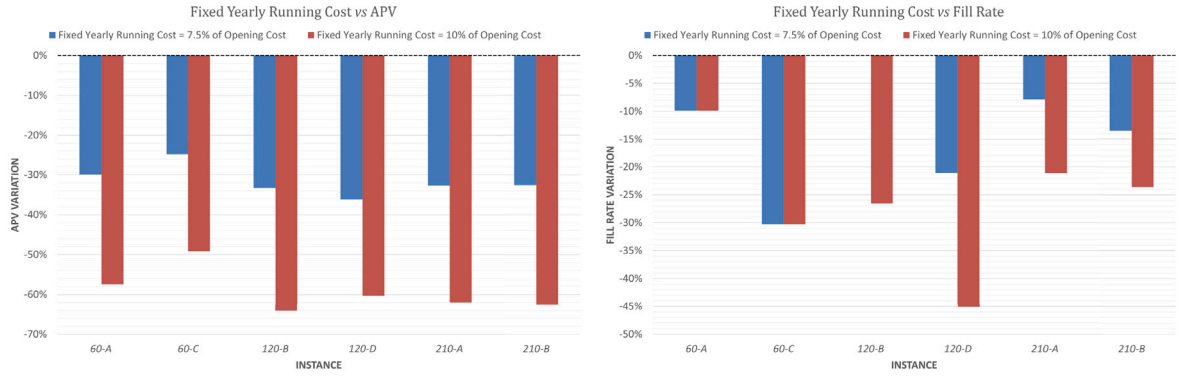

Fig. D.10. Impact of variation of the facility fixed yearly cost F on APV (a) and on the fill rate (b)

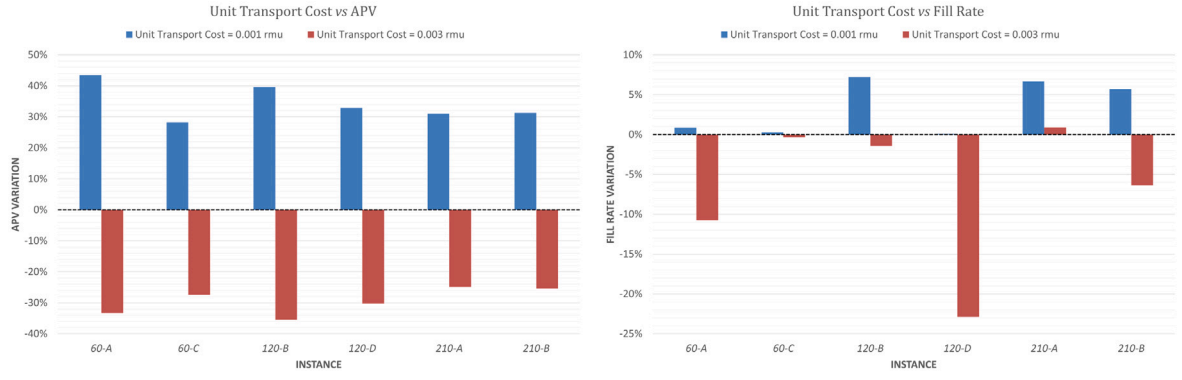

Fig. D.11. Impact of variation of the unit transport cost ω on APV (a) and on the fill rate (b)

D.5. Unit transport cost

APV:. This parameter has a negative effect on APV across all instances. Decreasing the unit transport cost leads to a higher APV, while increasing it reduces APV. This is because increasing transport costs raises the firm's total expenses, leading to a lower OGV, which directly lowers APV.

Fill rate:. The relationship between unit transport cost and fill rate is more complex. Lowering the unit transport cost either increases the fill rate or keeps it constant for all instances. Detailed solutions reveal two reasons for the higher fill rate: for half of the instances, lower transport costs make it more profitable to serve additional customers, while for the other half, more facilities are opened, contributing to the increase in fill rate. Conversely, increasing the unit transport cost reduces the fill rate in most instances, except for 210-A. In the remaining seven instances, the opened capacity is either less than or equal to that of the baseline, indicating insufficient profitability to justify opening additional facilities despite lower transport costs.

Data availability

Data will be made available on request.

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