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Green retailer: A stochastic bi-level approach to support investment decisions in sustainable energy systems

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ABSTRACT

This paper presents a bi-level approach to support retailers in making investment decisions in renewable-based systems to provide clean electricity. The proposed model captures the strategic nature of the problem and combines capacity sizing decisions for installed technologies with pricing decisions regarding the electricity tariffs to offer to a reference end-user, representative of a class of residential prosumers. The interaction between retailer and end-user is modeled using the Stackelberg game framework, with the former acting as a leader and the latter as follower. The reaction of the follower to the electricity tariff affects the retailer's profit, which is calculated as the difference between the revenue generated from selling electricity and the total investment, operation and management costs. To account for uncertainty in wholesale electricity prices, renewable resource availability and electricity request, the upper-level problem is formulated as a two-stage stochastic programming model. First-stage decisions refer to the sizing of installed technologies and electricity tariffs, whereas second-stage decisions refer to the operation and management of the designed system. The model also incorporates a safety measure to control the average profit that can be achieved in a given percentage of worst-case situations, thus providing a contingency against unforeseen changes. At the lower level, the follower reacts to the offered tariffs by defining the procurement plan in terms of energy to purchase from the retailer or potential competitors, with the final aim of minimizing the expected value of the electricity bill. A tailored approach that exploits the specific problem structure is designed to solve the proposed formulation and extensively tested on a realistic case study. The numerical results demonstrate the efficiency of the proposed approach and validate the significance of explicitly dealing with the uncertainty and the importance of incorporating a safety measure.

1. Introduction

Nowadays, there is a broadened awareness of the potentially severe consequences of global warming and climate change on our planet's preservation. This growing concern has been pushing all the countries worldwide to set greenhouse gas reduction targets, with European Union envisioning climate neutrality by 2050. To achieve this ambitious goal, a substantial rise in production from Renewable Energy Sources (RESs) becomes mandatory, necessitating a shift away from carbon emitting generation mix in so far as possible. The energy transition brings new complexities to the management of the electricity grid, with an evident impact on the energy markets. Within this evolving context, a large instability of the electricity prices could be observed, further compounded by emerging geopolitical factors such as the ongoing Ukrainian conflict.

In this paper, we focus on a timely and challenging problem faced by retailers operating in the liberalized electricity market. Acting as

intermediaries between the wholesale electricity providers and final consumers, they face the problem of defining appropriate tariffs that can ensure profit margin, while remaining sufficiently attractive in relation to competitive offers. With the growing emphasis on sustainability, retailers may play an increasingly active role integrating RESs into their portfolios rather than only limiting to a resale function. Renewable energy portfolios would also allow to mitigate the impact of market price fluctuations and to eventually attract customers who prioritize environmentally friendly options.

In this paper, we focus on the pricing problem from a strategic perspective by proposing an integrated approach to support retailers in their investment decisions in RES-based systems. To mitigate the effect of uncertain production from RES, the energy system would integrate electricity storage devices. Investment decisions are clearly influenced by the potential revenues from electricity sales and, therefore, depend on the prices at which electricity is sold to end-users, whose reaction

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should be anticipated. We point out that we study the problem from the perspective of the retailer, but the same approach can be used when considering other entities in the electricity market, such as, for example, aggregators coordinating energy communities.

While many scientific contributions focus on the pure electricity pricing problem, to the best of our knowledge, there is a gap in the literature regarding integrated approaches combining sizing decisions with pricing strategies. To fill this gap, we propose a mathematical model to support retailers in profitable and environmentally sustainable projects.

We formally model the interaction between retailer and end-users by using the Stackelberg game framework and we formulate the problem by the Bi-Level (BL) paradigm [1]. Here, the retailer acts as leader and decides first regarding to the optimal size of the technologies to install and the electricity tariffs to offer. Then, the follower reacts to the communicated prices by determining the best procurement plan in terms of amount of electricity to purchase from the retailer and from competitors. The follower's reaction is fed back to the retailer and affects the actual profit, which is computed as the difference between the revenue from electricity sale and the total costs defined, as the sum of investment and operation and management costs. In our formulation, the follower embodies a class of prosumers, i.e., consumers also producing electricity, primarily through RESs.

Data uncertainty contributes to make the BL problem even more challenging. Market electricity prices are clearly not known when investment decisions have to be made. In addition, weather-related variables affect the retailer's procurement plan, which in turn depends on the uncertain prosumer's demand also influenced by the tariffs offered by potential competitors. To incorporate uncertainty features into the BL approach, we use the stochastic programming paradigm [2]. In particular, the strategic nature of the leader problem suggests the adoption of the two-stage framework. First-stage decisions refer to actions that should be taken "here and now" in the face of uncertainty, i.e., capacity sizing and setting electricity prices. Recourse decisions involve corrective actions that the retailer should take in order to guarantee the follower's energy supply under any situation that may arise. On the basis of the tariffs offered, the follower defines the procurement plan, which is influenced by the uncertain RES production.

Both the leader's and follower's objective functions are affected by uncertainty. While for the follower we consider minimizing the expected electricity bill, we assume that the retailer is interested in controlling the random profit. To this end, we introduce a safety measure aimed at controlling the expected profit that can be achieved in a given percentage of worst-case realizations. This measure is known by various names in the literature, of which the conditional value-at-risk (CVaR) is by far the most widely used [3], when referring to the loss function. The terms safety and risk are used in text, interchangeably. Depending on the choice of the safety level, a different risk attitude can be modeled, thus providing the retailer with a wide range of alternative solutions.

The paper unfolds as follows. Section 2 reviews the related scientific literature and provides motivation for this work. Section 3 introduces the problem and Section 4 presents the proposed stochastic BL formulation. Section 5 is devoted to the presentation of the designed solution approach, whereas extensive computational experimentation, carried out on a test case based on data of the Italian retail electricity market, is discussed in Section 6. Finally, conclusions and future research directions are drawn in Section 7.

2. Relevant studies and research gaps

This section provides an overview and discussion of recent relevant scientific contributions related to the addressed problem. While the literature on the electricity pricing problem is rich and steadily growing, to the best of our knowledge, no contribution proposes a stochastic BL approach, where sizing and pricing decisions are jointly considered.

The problem is analyzed at the retail level and involve small-scale installation, differently from capacity expansion problems that refer to broader energy infrastructure requiring much larger investment [4].

The BL programming represents the elective mathematical framework used to model electricity pricing problems, as it naturally fits the interaction between retailers and end-users. Most of the contributions focus on the design of pricing policies that encourage customers to modify their consumption patterns in response to changes in the electricity prices. The aim is to spur electricity usage at off-peak hours through low prices, while penalizing peak hours with higher prices. In [5] the authors compare various flexible pricing schemes, i.e., time-of-use pricing (ToU), real-time-pricing (RTP), and critical-peak-pricing (CPP) tariffs, coming to the conclusion that the RTP scheme yields the highest additional profits for the retailer (see also [6]). We also mention [7], where the authors propose personalized RTP schemes and [8] where the prices vary depending on the time and the level of consumption. More recently, Oggioni et al. propose in [9] a BL formulation to determine RTP tariffs that accounts for competition and strategic interactions between retailers.

Unlike the above mentioned contributions that focus on dynamic pricing schemes, where tariffs are communicated with short notice, our problem considers a static scheme, where electricity tariffs remain fixed for a given time horizon. This choice is suggested by the strategic nature of the problem addressed. We point out that in the proposed formulation, tariffs play an instrumental role, as they affect the profitability of the investment project and could serve as suitable limit for electricity prices, that could be redefined in a dynamic setting. In the model, we consider a ToU scheme with three time slots, peak, mid-peak and off-peak, to reflect the structure commonly adopted in retail markets, such as the Italian market analyzed in this study.

Uncertainty affecting data and/or decisions of the players has been seldom dealt in the electricity pricing problems. We refer interested readers to the recent contribution [10] which provides an updated survey on the main approaches proposed in the literature to address uncertainty in BL optimization. Recently, Jacquet et al. propose in [11] a BL formulation where the probabilistic nature of the customers' choice is modeled by using a Logit function. To circumvent the resolution difficulties of the proposed formulation, a quadratically regularized model of customer's response is considered. Data uncertainty in BL formulations for electricity pricing problem analyzed from the retailer's perspective has been dealt in few contributions. Here, we mention [12, 13], where the authors propose stochastic formulations to account for the uncertain electricity market prices and weather-related variables by adopting a stochastic programming approach.

The other stream of literature related to our problem refers to the evaluation of investments in local energy systems, including RES and storage devices. Recognizing the importance of explicitly dealing with the uncertain nature of main input data, several contributions propose stochastic formulations aimed at determining the optimal capacity of the different devices that provide the minimum total cost, both in terms of investment and management of the system. For example, in [14] the authors propose a two-stage stochastic programming formulation for jointly determining the optimal sizes of various distributed renewable energy resources, considering both economic benefits and resilience performance. In [15] the authors present a stochastic programming formulation for the optimal sizing and management of an energy system integrating renewable sources, namely wind turbines and PV panels with electricity storage devices and electric vehicles. The results presented confirm that the optimized system can enhance efficiency, self-sufficiency, while being cost-effective by reducing both investment and operating costs.

While the contributions mentioned above are modeled as single-level problems, we observe that the use of the BL framework is often adopted when considering investment decisions in generation and transmission since the involved decision makers have different objectives. For example, in [16] Grimm et al. propose a formulation where

a Transmission System Operator (TSO) decides on network investment at the first level anticipating private generation expansion and spot market trade at a second level. We also refer the reader to [17] where the authors propose a BL formulation where the upper level refer to the TSO that has to decide on optimal line investments as well as on a corresponding network fee anticipating the market outcomes of the lower level where competitive firms trade energy and invest in storage facilities given the realized network extensions of the TSO. We finally mention the contribution [18] where the authors present a stochastic BL formulation to deal with investment problem in power generation. Here, at the upper level, an investor decides on investments while anticipating the market results. At the lower level, a market operator maximizes welfare given consumer demand and installed generation assets as well as producer price bids. The problem is solved in a rolling-horizon fashion splitting the investment process into multiple stages, allowing the modification of wait-and-see decisions.

Despite the rich literature dealing with the pricing and the investment problems separately, to the best of our knowledge, no BL approach jointly considers both the decisions and analyze the investment problem from a retailer's perspective. Consequently, this paper makes the following contributions to existing literature.

- (1) From a modeling perspective, we propose a BL model to support retailers in their strategic decision to invest in a sustainable energy system to provide clean energy to end-users, identified by a representative follower. Decisions concerning the electricity request in response to offered tariffs are under the follower's control and they affect the evaluation of the entire project investment. Data uncertainty related to future market prices and production from renewables is explicitly considered by formulating the leader's problem by the two-stage stochastic programming paradigm, whereas the follower takes wait and see decisions also in response to data uncertainty. The leader's model incorporates a safety measure to control the variability of the profit, whereas the follower aims at minimizing the expected electricity bill.
- (2) From a methodological standpoint, we contribute to the scientific literature by proposing a solution approach that exploits the specific problem structure. This approach relies on the partial enumeration of the BL feasible solutions. Additionally, we provide a comparison with the traditional approach, which involves deriving and solving a single-level reformulation of the original problem using a general-purpose solver.
- (3) Finally, the paper provides extensive computational experiments carried out on a case study calibrated on data of the Italian electricity market. The results demonstrate the effectiveness of the proposed solution approach when considering an increasing number of scenarios. We validate the significance of explicitly dealing with data uncertainty in the BL formulation and the importance of incorporating a safety measure. The findings of this study provide valuable insights for retailers prioritizing sustainable energy solutions.

3. Problem description

We consider the problem faced by an electricity retailer who wants to assess the benefits of investing in a local RES-based system for the production of clean electricity. In particular, the sustainable solution involves the integration of solar production, from PV panels, with battery energy storage (BES) technologies. PV-BES is currently a popular solution, widely promoted by national policies. By integrating the BES technology, the unpredictable and intermittent nature of the RES can be mitigated by decoupling production from consumption. Excess energy can be stored for later use. In addition, electricity can be purchased from the market during off-peak hours and used later to meet the demand when it is needed. The problem at hand involves defining the optimal configuration in terms of technology sizing. While investment

decisions have a distinct strategic dimension, the effective operation and management of the devised system directly influence the optimal sizing. As the retailer primary aim is to sell energy, tariff optimization becomes crucial and, given the competitive nature of the market, it becomes imperative to anticipate the response of end-users as this reaction influences the overall investment profitability. To represent the interplay between the strategic retailer decisions and end-users responsive behaviors, we adopt the BL paradigm. In the following, we specifically focus on the class of residential prosumers that we identify by a reference follower.

Data uncertainty clearly affects the entire decision process. Market electricity prices are not known in advance and the operation of the designed system is affected by weather-related variables and by the reaction of the follower which, in turns, also depends on potentially competitive offers. To account for uncertainty, we adopt the stochastic programming framework and we model the uncertain parameters as random variables defined on a given probability space (Ω, F, \mathcal{P}) , assumed to be discrete. Thus, we consider a given set of possible realizations (scenarios), each occurring with a given probability value.

Fig. 1 depicts the structure of the proposed stochastic BL formulation. The retailer acts as leader and solves the upper-level (UL) problem that is modeled as a two-stage problem with recourse. Here, first-stage decisions represent here and now actions that should be taken in face of uncertainty. They refer to the design of the energy system in terms of number of PV panels to install, and capacity of the storage device. In the proposed formulation, pricing are also considered as first-stage decisions as they influence the profitability of the investment and subsequent operational decisions related to the system utilization. Second-stage decisions, besides being affected by the realization of the uncertain parameters, as in traditional stochastic programming paradigm where only a single decision maker is present, also depend on the decisions that are under the follower's control. More specifically, at the lower level (LL), the follower reacts to the tariffs by the defining the best procurement plan. Being a prosumer, the follower energy request is partially supplied by his local RES-based generation system. Thus, the follower takes wait and see decisions, referring to the energy amount (EA) purchased from the retailer and/or competitive provider. The scenario-based follower's reaction is fed back in the UL problem where the leader takes recourse decisions referring to the operation and management of the energy system and to the involvement in the wholesale electricity market (EM) to meet the follower's request under every circumstance that may occur. The follower's reaction affects the retailer's profit which is also influenced by the future electricity market prices. In the proposed formulation, we assume that the retailer is risk-averse and we model this attitude by introducing a safety measure as the UL objective function. More specifically, we use the CVaR as function of a safety level α to control the expected profit that can be earned in a given percentage $(1-\alpha)$ of worst-case realizations. As for the LL, the minimization of the expected electricity bill is considered as follower's objective.

4. Problem formulation

This section is devoted to the presentation of the leader's and the follower's problems. The complete list of decision variables, parameters and sets used in the formulations can be found in Table 1.

4.1. The retailer problem

We model the retailer problem by considering a planning horizon of one year, and since the operational decisions refer to time periods of one hour (or even less), to get rid of the computational effort we consider a set of representative days $d \in \mathcal{D}$, one for each season, and a daily horizon with hourly time steps $t \in \mathcal{T}$. We do not distinguish between working and non-working days.

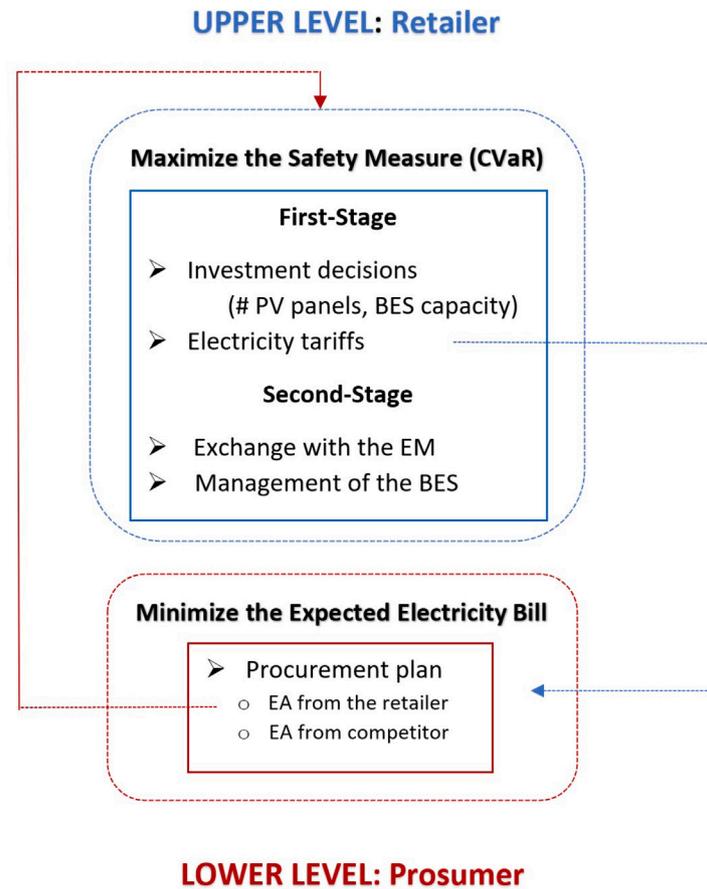


Fig. 1. Overview of the proposed BL approach.

In the first stage, the retailer takes pricing and size decisions. In particular, we assume that the retailer wants to offer a static pricing scheme defined according to a ToU structure, i.e., tariffs are differentiated according to the hours of the day. Following the organization of the Italian electricity market, which is used as a reference, we consider three main blocks, corresponding to peak, intermediate and off-peak hours, respectively.

Considering the components entering the definition of the electricity rates, we express the prices as the sum of a base value \underline{p}_f that accounts for the forecast cost of energy in the different time blocks and for the different charges and levies, and a spread corresponding to the surcharge. This last component represents a variable value on which the retailer is called to decide. Thus, for each block f , we can express p_f as

$$p_f = \underline{p}_f + \Delta_f \quad \forall f \in \mathcal{F}. \quad (1)$$

Upper bounds are introduced to account for the competitiveness of the retail market, as expressed by the constraints:

$$p_f \leq \bar{p}_f \quad \forall f \in \mathcal{F}. \quad (2)$$

In addition to deciding on electricity tariffs, the retailer also makes strategic decisions regarding the capacity of the technologies to be installed. For the PV panels, assuming to denote by A_{max} the available space, the decision concerns the number n of PV panels to install, given the required individual space A :

$$A n \leq A_{max}. \quad (3)$$

As for the BES, since predefined sizes are available on the market, we formulate the size selection by the following set of constraints:

$$BS = \sum_{q \in \mathcal{Q}} \lambda_q v_q, \quad (4)$$

$$\sum_{q \in \mathcal{Q}} v_q \leq 1, \quad (5)$$

where λ_q represents the size of model q and v_q denotes a binary variable taking the value 1 if the size q is selected and 0 otherwise.

The management of the BES is modeled by a set of constraints that are imposed for each scenario s . Storage devices serve as a buffer to mitigate the unpredictable nature of the PV production and/or can be used to properly redefine the retailer's procurement plan. The following constraints are introduced:

$$soc_{dt}^s = \tau_m BS + \eta_c in_{dt}^s - \frac{1}{\eta_d} out_{dt}^s \quad d = 1, t = 1 \quad \forall s \quad (6)$$

$$soc_{dt}^s = soc_{d-1|T}^s + \eta_c in_{dt}^s - \frac{1}{\eta_d} out_{dt}^s \quad \forall d > 2, t = 1 \quad \forall s \quad (7)$$

$$soc_{dt}^s = soc_{d,t-1}^s + \eta_c in_{dt}^s - \frac{1}{\eta_d} out_{dt}^s \quad \forall d, \forall t > 1 \quad \forall s \quad (8)$$

$$soc_{|D||T}^s = \tau_m BS \quad \forall s \quad (9)$$

$$\tau_m BS \leq soc_{dt}^s \leq \tau_M BS \quad \forall d, t, s \quad (10)$$

$$in_{dt}^s \leq \phi_c BS \quad \forall d, t, s \quad (11)$$

$$out_{dt}^s \leq \phi_d BS \quad \forall d, t, s \quad (12)$$

In particular, constraints (6)–(8) relate the state of charge over two consecutive periods to the amount of energy charged and discharged, where the parameters η_c and η_d account for the energy loss in the storage operations. We point out that for the first hour of the first day (6), the initial state of charge is a known value that we impose to be equal to the selected size BS by τ_m . For the successive days, the state of charge of the first hour is equal to the stage of charge of the last hour of the previous day (7). Moreover, we impose by (9) that at the end of the planning horizon (last hour of the last day) the state of charge is equal to the initial value. Constraints (10) ensure that the amount of energy

Table 1
Nomenclature.

Sets	
D	Set of day types, indexed by d
F	Set of time blocks, indexed by f
\mathcal{T}	Set of elementary time periods (hours) indexed by t
$\mathcal{T}(f)$	Set of time periods in the time block f
Q	Set of the BES sizes indexed by q
S	Set of scenarios, indexed by s
Parameters	
π_s	Probability associated with scenario s
θ_d	Number of days of type d in one year
rf	Capital recovery factor
i	Discount rate
\bar{p}_f	Upper bound for the tariff offered in the time block f
\underline{p}_f	Lower bound for the tariff offered in the time block f
A_{max}	Available space to install the PV
A	Space required by a single PV module
C_{PV}	Cost of a single PV module
C_{BES}	Specific investment cost of the BES
λ_q	Specific capacity of the BES of size q
c_{sm}	Cost related to the storage management
η_c, η_d	Efficiency rates for energy injection and withdrawal from the BES
ϕ_c, ϕ_d	Charging and discharging rates from the BES
τ_m, τ_M	Operation range of the BES as percentage of the nominal capacity
ξ_{dt}^s	Electricity production from a single PV module at t on the day d under scenario s
EP_{dt}^{s+}	Electricity purchasing cost at t on the day d under scenario s
EP_{dt}^{s-}	Electricity selling price at t on the day d under scenario s
b_{dt}^s	Electricity demand at t on the day d under scenario s
cp_{dt}^s	Electricity price of the competitor at t on the day d under scenario s
Decision variables	
Upper level decision variables	
n	Number of installed PV panels
p_f	Electricity rate for the time block f
x_{dt}^{s+}	Energy purchased from the market at time t on the day d under scenario s
x_{dt}^{s-}	Energy fed back to the grid at time t on the day d under scenario s
in_{dt}^s	Energy charged into the BES at t on the day d under scenario s
out_{dt}^s	Energy discharged from the BES at t on the day d under scenario s
soe_{dt}^s	State of charge of the BES at t on the day d under scenario s
v_q	Binary variable taking value 1 if a BES of size q is selected and 0 otherwise
z_s	Auxiliary variable under each scenario
v	Auxiliary variable
Lower level decision variables	
y_{dt}^s	Amount of electricity purchased from the retailer at time t on the day d under scenario s
g_{dt}^s	Amount of electricity purchased from competitors at time t on day d under scenario s

in the BES is always within a minimum and a maximum level, defined as a function of the selected capacity. Finally, the set of constraints in (11)–(12) impose an upper bound on the total amount of power that can be charged into and/or discharged from the BES at each time t of each day d , under each scenario s .

For each day d , the retailer defines the procurement and operational plan in order to satisfy the follower’s demand pattern, represented by the variables y_{dt}^s . These latter decisions are under the follower’s control who can react to the offered prices by modifying the consumption profile or opting to purchase energy from potential competitors, if this is more advantageous. In addition to using self-produced energy, which depends on the number of installed panels and it is clearly uncertain, the retailer can also purchase energy from the wholesale market and/or use the energy stored in the BES. The corresponding constraints are mathematically represented as:

$$x_{dt}^{s+} + n \xi_{dt}^s + out_{dt}^s - in_{dt}^s - x_{dt}^{s-} = y_{dt}^s \quad \forall d, t, s, \tag{13}$$

where x_{dt}^{s+} and x_{dt}^{s-} denotes the amount of electricity exchanged with the market.

4.1.1. The objective function

The aim of the retailer is to maximize the profit, which is defined as the difference between revenue and costs. The first term depends on the amount of electricity sold to the follower and on the offered tariffs. We remind that the revenue is increased by the quota resulting from the electricity fed back to the grid. More specifically, the annual revenue,

under each scenario s , is defined as:

$$RV_s = \sum_{d \in D} \theta_d \sum_{f \in F} p_f \sum_{t \in \mathcal{T}(f)} y_{dt}^s + \sum_{d \in D} \theta_d \sum_{t \in \mathcal{T}} EP_{dt}^{s-} x_{dt}^{s-}, \tag{14}$$

where θ_d represents the number of days of type d in the year. As for the costs, they include two main components related to the investment and to the operational management. Since the investment spans a time horizon greater than one year, in the model we consider the annualized cost, computed by using a recovery factor rf defined as:

$$rf = \frac{i(1+i)^L}{(1+i)^L - 1},$$

where i represents a proper discount rate and L is the lifetime assumed to be the same for all the considered technologies. The annualized investment cost denoted by INV can be defined as:

$$INV = rf(C_{BES} n + C_{PV} n),$$

where C_{BES} and C_{PV} denote the BES and PV panels specific costs, respectively.

The annual operational and management cost (OMC) is related to the retailer’s procurement plan and to the management of the selected technologies. More specifically, for each scenario s , the OMC_s can be defined as:

$$OMC_s = \sum_{d \in D} \theta_d \sum_{t \in \mathcal{T}} (EP_{dt}^{s+} x_{dt}^{s+} + c_{sm} (out_{dt}^s + in_{dt}^s)),$$

where the first term accounts for the market operation and the last two are related to the charging and discharging operation of the BES.

The maintenance costs are not included since they are assumed to be negligible.

Thus, the annual profit PR_s is defined as the difference between revenue and cost:

$$PR_s = RV_s - (INV + OMC_s).$$

We assume that the retailer is risk averse and is interested in maximizing the CVaR as safety measure rather than the expected profit. Following the results presented in [19], in the case of discrete distributions, the CVaR can be written as:

$$CVaR = v - \frac{1}{(1-\alpha)} \sum_{s \in S} \pi_s z_s.$$

Here v is a free decision variable, whereas the supporting variables z_s are defined as

$$z_s \geq v - PR_s \quad \forall s \quad (15)$$

$$z_s \geq 0 \quad \forall s \quad (16)$$

with π_s denoting the probability of occurrence of scenario s . By varying the safety level α , different solutions could be obtained, reflecting the decision maker's attitude towards risk, from risk neutrality (α approaching 0) to more conservative positions (α approaching 1).

4.2. The lower level problem

At the lower level, the follower reacts to the offered tariff by deciding on the amount of electricity to purchase from the retailer. To account for price competition, we consider the possibility that the prosumer's demand could be eventually supplied by competing retailers. For each day d , the follower's problem aims to define the procurement plan that minimizes the expected value of the daily electricity bill. Formally, the problem is defined as:

$$\min z_L = \sum_{s \in S} \pi_s \left(\sum_{f \in \mathcal{F}} p_f \sum_{t \in \mathcal{T}(f)} y_{dt}^s + \sum_{t \in \mathcal{T}} pc_{dt}^s g_{dt}^s \right) \quad (17)$$

$$y_{dt}^s + g_{dt}^s \geq b_{dt}^s \quad \forall t, s \quad (18)$$

$$y_{dt}^s, g_{dt}^s \geq 0 \quad \forall t, s \quad (19)$$

Here constraints (18) include the amount of energy g_{dt}^s that is purchased from the competitor and ensure the satisfaction of the demand b_{dt}^s no matter which the supplier is selected.

5. The solution approach

The proposed model encompasses two sources of difficulty related to the BL structure and to the stochastic nature of the problem. If finding a global solution in the deterministic setting is already challenging, the problem becomes even more complex in the stochastic case. We refer the interested readers to the recent survey on the BL optimization under uncertainty [10].

We deal with the problem by applying a traditional approach usually employed when the LL problem is convex and satisfies suitable constraint qualification conditions (i.e., Slater's constraint qualification, in the convex case). The approach relies on the derivation of a single-level reformulation of the original problem. To this end, either the Karush–Kuhn–Tucker (KKT) conditions of the LL problem or the strong duality theorem can be applied. We point out that both the approaches are only equivalent to the original BL problem if globally optimal solutions are considered and if Slater's constraint qualification holds (see, e.g., [20,21]). Regardless the approach used, the single-level reformulations optimize over an extended space of variables since we additionally have to include the LL dual variables. In our case, such variables also depend on the scenario number, drastically increasing the size of the reformulated problem and preventing the application of general-purpose solvers when considering instances that are meaningful from an application standpoint.

In the following, we first derive the single-level reformulation used as basis of comparison and then we introduce the proposed approach which exploits the specific structure our BL formulation.

5.1. The single level reformulation

Looking at the problem structure, we may note that for a given upper level first-stage solution p_f , the LL is a linear problem always feasible and bounded. We can, thus, derive the dual problem reported below:

$$\max w_L = \sum_{s \in S} \sum_{t \in \mathcal{T}} b_{dt}^s \gamma_{dt}^s \quad (20)$$

$$\gamma_{dt}^s \leq \pi_s p_f \quad \forall f \quad \forall t \in \mathcal{T}(f) \quad \forall s \quad (21)$$

$$\gamma_{dt}^s \leq \pi_s pc_{kdt}^s \quad \forall t, s \quad (22)$$

$$\gamma_{dt}^s \geq 0 \quad \forall t, s \quad (23)$$

where γ_{dt}^s denote the dual variables associated with the constraints (18). It is worth noting that, the dual problem can be decomposed with respect to scenarios and solved by inspection. We derive the single-level reformulation by applying the strong duality theorem. More specifically, we replace the LL problem by the primal and dual feasibility conditions, together with the strong duality equation, which enforces the equality between the primal and dual objective functions:

$$\sum_{s \in S} \pi_s \left(\sum_{f \in \mathcal{F}} p_f \sum_{t \in \mathcal{T}(f)} y_{dt}^s + \sum_{t \in \mathcal{T}} pc_{dt}^s g_{dt}^s \right) = \sum_{s \in S} \sum_{t \in \mathcal{T}} b_{dt}^s \gamma_{dt}^s. \quad (24)$$

In (24), it is easy to recognize the presence of bilinear terms defined by the product of two continuous variables p_f and y_{dt}^s . Working out on p_f , we may express, for each f and s , the product as:

$$\sum_{f \in \mathcal{F}} \left(\underline{p}_f + \Delta_f \right) \sum_{t \in \mathcal{T}(f)} y_{dt}^s = \sum_f \sum_{t \in \mathcal{T}(f)} \left(\underline{p}_f y_{dt}^s + \Delta_f y_{dt}^s \right).$$

We note that price changes are typically considered discrete and expressed in cents. Therefore, in the following, we express the spread Δ_f in terms of binary expansion. More specifically, we denote by μ the size of the price increase, and we introduce the following conditions:

$$\Delta_f = \mu \sum_{l=1}^{L_f} l \delta_{lf},$$

$$\sum_{l=1}^{L_f} \delta_{lf} \leq 1.$$

Here, for each time block f , δ_{lf} denotes a binary variable taking value 1 if l -increment is considered and 0 otherwise, and L_f is computed as $L_f = \lceil \frac{\bar{p}_f - \underline{p}_f}{\mu} \rceil$.

Following this approach, the bilinear terms are expressed by the product of binary and continuous decision variables and a classical reformulation can be used. In particular, we replace the product $\delta_{lf} y_{dt}^s$ by the variables w_{dltf}^s and we add the constraints:

$$w_{dltf}^s \leq y_{dt}^s \quad \forall f \quad \forall t \in \mathcal{T}(f) \quad \forall d \quad \forall s \quad (25)$$

$$w_{dltf}^s \geq y_{dt}^s + u (\delta_{lf} - 1) \quad \forall f \quad \forall t \in \mathcal{T}(f) \quad \forall d \quad \forall s \quad (26)$$

$$w_{dltf}^s \leq u \delta_{lf} \quad \forall f \quad \forall t \in \mathcal{T}(f) \quad \forall d \quad \forall s, \quad (27)$$

where u represents a known upper bound on the amount of electricity that can be purchased from the retailer.

5.2. The proposed approach

The large size of the single-level reformulation limits its application to solving test instances of moderate size. Our approach exploits the specific structure of the proposed model and, in particular, the binary expansion reformulation of the electricity tariffs. Looking at the problem, we can observe that all linking leader variables are binary, and that for each leader decision, the parametric follower problems are always feasible, as leader variables only appear in the LL objective function.

Before presenting the solution approach, we rewrite the formulation in more general terms. Let \mathbf{x} and $\mathbf{y}(\omega)$ denote the first and second stage UL decision variables, with ω representing the source of uncertainty. We assume that the probability space is discrete and we consider a set of realizations, each occurring with a given probability value. We observe that in the proposed formulation, the set of the first-stage UL variables \mathbf{x} can be partitioned into two subsets, denoted by \mathbf{x}_1 and \mathbf{x}_2 . The first subset contains the leader variables entering into the LL problem, i.e., the binary variables related to the tariff δ_{lf} , whereas the second one is associated with the sizing decisions, i.e., the decision variables n and λ_q . The decision variables $\mathbf{y}(\omega)$ represent the leader's recourse actions, i.e., variables related to the storage management and to the involvement in the electricity market, together with the variables used to linearize the bilinear terms. These variables enter into the coupling constraints relating to the follower's response, represented by variables $\mathbf{z}(\omega)$. We note that the follower decides in a wait-and-see manner, taking the leader's decision \mathbf{x}_1 as well the realization of the uncertainty into account. In general terms, the proposed formulation can be rewritten as:

$$\begin{aligned} \max_{\mathbf{x} \in \mathbf{X}, \mathbf{y}(\omega) \in \mathbf{Y}(\omega)} & F(\mathbf{x}, \mathbf{y}(\omega), \mathbf{z}(\omega)) \\ G(\mathbf{x}, \mathbf{y}(\omega), \mathbf{z}(\omega)) & \leq 0 \quad \forall \omega \in \Omega \\ \mathbf{z}(\omega) & \in Z(\mathbf{x}_1, \omega) \quad \forall \omega \in \Omega \end{aligned} \quad (28)$$

where $Z(\mathbf{x}_1, \omega)$ denotes the set of optimal solutions of the LL problem parameterized in \mathbf{x}_1 :

$$\begin{aligned} \min & f(\mathbf{x}_1, \mathbf{z}(\omega)) \\ g(\mathbf{z}(\omega)) & \geq 0 \quad \forall \omega \in \Omega \end{aligned} \quad (29)$$

Here F and f represent the objective function of the leader and follower, i.e., the CVaR of the profit for a fixed value of the safety level and expected value of the electricity bill, respectively. The function G is used to represent the leader constraints relating leader and follower decisions, i.e., the constraints on the demand satisfaction (13) and those introduced to linearize the bilinear terms. The other constraints in the UL problem are represented by the constraint sets \mathbf{X} and $\mathbf{Y}(\omega)$, i.e., the design constraints and those related to the management of the storage device. Finally, the function g is used to represent the LL constraints (18)–(19).

The proposed approach relies on an enumeration scheme of the BL feasible solutions generated by exploiting the structure of our formulation. Rather than working on the single-level reformulation, we apply a decomposition approach, where lower and upper bounds on the objective function of the leader's problem are iteratively determined. More specifically, upper bounds are obtained by solving the High Point Relaxation (HPR) problem, defined as:

$$\begin{aligned} \max_{\mathbf{x} \in \mathbf{X}, \mathbf{y}(\omega) \in \mathbf{Y}(\omega)} & F(\mathbf{x}, \mathbf{y}(\omega), \mathbf{z}(\omega)) \\ G(\mathbf{x}, \mathbf{y}(\omega), \mathbf{z}(\omega)) & \leq 0 \quad \forall \omega \in \Omega \\ g(\mathbf{z}(\omega)) & \geq 0 \quad \forall \omega \in \Omega \end{aligned} \quad (30)$$

The HPR problem does not consider the optimality of the follower and, thus, abstracting his reaction, provides an upper bounds on the BL problem solution. In the algorithm, the HPR problem is incrementally enriched of specific cuts. We call this new problem Enriched HPR (EHPR, for short). Let k denote the iteration counter, initially set to 0. At each iteration, we start solving the EHPR problem, defined by adding to the HPR (30) a set of inequalities denoted by C^k . We note that, for $k = 0$, the set C^k is empty and the EHPR coincides with the HPR. Moreover, because of the structure of the constraints that define the EHPR in our specific case, it is always feasible and bounded. Let $(\mathbf{x}^k, \mathbf{y}^k(\omega), \mathbf{z}^k(\omega))$ denote the solution of the EHPR at a given iteration k . We solve the parametric follower's problem that corresponds to problem (29) with fixed $\mathbf{x}_1 = \mathbf{x}_1^k$. Again the structure of the problem guarantees that a feasible solution can always be determined. Let us

denote such a solution by $\bar{\mathbf{z}}^k(\omega)$. In order to get a BL feasible solution, since the reaction of the follower (i.e., the amount of electricity purchased from the retailer) enters into the leader constraints, we solve a parametric leader's problem defined as:

$$\begin{aligned} \max_{\mathbf{x} \in \mathbf{X}, \mathbf{y}(\omega) \in \mathbf{Y}(\omega)} & F(\mathbf{x}, \mathbf{y}(\omega), \bar{\mathbf{z}}^k(\omega)) \\ G(\mathbf{x}, \mathbf{y}(\omega), \bar{\mathbf{z}}^k(\omega)) & \leq 0 \quad \forall \omega \in \Omega \\ \mathbf{x} & = \mathbf{x}^k \end{aligned} \quad (31)$$

The optimal solution of problem (31) clearly returns a BL feasible solution and, thus, a lower bound.

We now introduce the cuts which are progressively added into the set C^k . The structure of these cuts is similar to the form of cuts known in the literature as no good cuts (see, e.g., [22]). While these cuts are typically used to eliminate feasible or unwanted solutions, in our case they are used with the aim of generating a different configuration of the electricity tariffs at each iteration. Let δ_{lf}^k the solution at a given iteration k . The cut is defined as:

$$\sum_{f \in F} \sum_{l \in L_f: \delta_{lf}^k=0} \delta_{lf} + \sum_{f \in F} \sum_{l \in L_f: \delta_{lf}^k=1} (1 - \delta_{lf}) \geq 1. \quad (32)$$

We note that (32) represents a valid cut. Since the best solution for a fixed configuration of the upper and lower level variables is obtained, then no optimal solution can be missed. In such a way at every iteration a different leader's solution is obtained leading to the derivation of a new BL feasible solution. The sketch of the algorithm is reported below.

Algorithm 1 Pseudo code of the proposed approach

```

1: Initialization:  $k \leftarrow 0$ ,  $C^0 \leftarrow \emptyset$ ,  $LB \leftarrow -\infty$ ,  $UB \leftarrow \infty$ 
2: while ( $LB < UB$ ) do
3:   Solve the EHPR and obtain an optimal solution  $(\mathbf{x}^k, \mathbf{y}^k(\omega), \mathbf{z}^k(\omega))$ 
4:   Set  $UB = F(\mathbf{x}^k, \mathbf{y}^k(\omega), \mathbf{z}^k(\omega))$ .
5:   Solve the follower problem (29) with  $\mathbf{x}_1 = \mathbf{x}_1^k$  and get the optimal solution  $\bar{\mathbf{z}}^k(\omega)$ 
6:   Solve the leader problem (31) with  $\mathbf{z}(\omega) = \bar{\mathbf{z}}^k(\omega)$  and get the optimal solution.
7:   Set  $LB = \max\{LB, F(\mathbf{x}^k, \mathbf{y}^k(\omega), \bar{\mathbf{z}}^k(\omega))\}$ 
8:   Generate the cut (32) and add it to  $C^{k+1}$ 
9:   Set  $k \leftarrow k + 1$ 
10: end while
11: Return  $LB$ 

```

Theorem 1. *Algorithm 1 terminates in a finite number of iterations and provide the optimal solution of the single-level reformulation.*

Proof. Since the number of linking leader variables is finite, the space of feasible solutions is finite and the algorithm converges in a finite number of iterations to the optimal solution.

We point out that the specific structure of the problem is also used for the solution of the follower's problem (Line 5 in Algorithm 1). Indeed, for a fixed value of the upper level linking variables, the LL solution is trivial and can be carried out by inspection. Moreover, we consider the optimistic version of the BL formulation, i.e., when the solution of the LL problem is not unique, the follower is expected to choose the solution that the leader prefers.

6. Computational experiments

This section is devoted to the presentation and discussion of the computational experiments carried out to assess the effectiveness of the proposed approach on a realistic case study. The model and the solution algorithms have been coded in GAMS 38 and solved using ILOG CPLEX [23]. All the experiments have been performed on an Intel®Core i7 2.6 GHz, with 16.0 GB of RAM memory. After introducing the case study, we first comment on the performance of the solution approach and then we present and analyze the results.

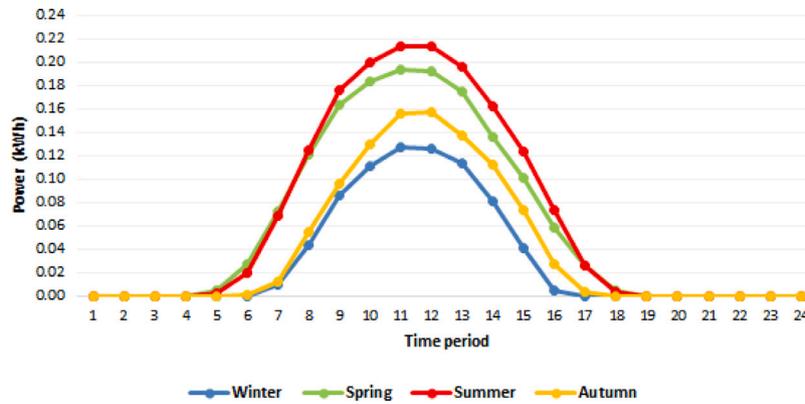


Fig. 2. Hourly electricity production from PV for the different typical days.

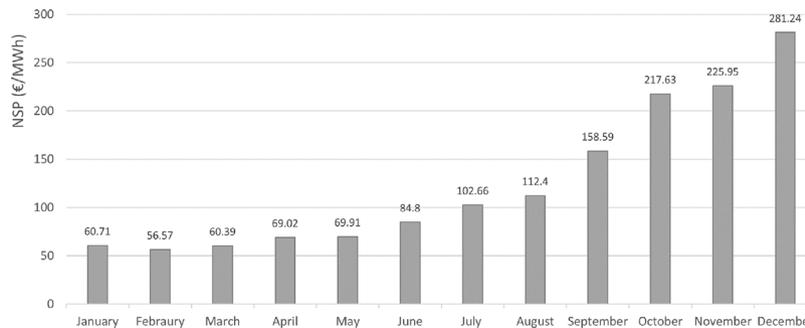


Fig. 3. Average monthly NSP in 2021.

6.1. Experimental setting and data

We have considered a realistic test case where the leader is represented by a retailer operating in the Italian electricity market and the follower is representative of a group of residential prosumers assumed to be identical in terms of required load. Under this assumption, the leader-follower game unfolds on an individual level. The extension to the more general multi-follower case where different tariffs can be proposed to each class of prosumer, is quite straightforward. However, the addition of more variables and constraints introduces an even greater level of complexity.

We have assumed that the leader has an area available for PV panel installation. Preliminary, a technical analysis of the storage devices available in the market has been carried out with the aim of identifying suitable models and sizes taking into account the potential electricity requirements and space availability. All relevant economic and technical details regarding the considered technologies are reported in Appendix. In the experiments, we have assumed a lifespan time of 20 years for both technologies and a yearly interest rate of 2%.

Sizing decisions are affected by the management strategy which, in turn, relies on the time of year (represented by the day-type and hour of the day), and on the uncertainty affecting weather-related variables, energy demand, and market prices. Figs. 2 reports the average electricity production from a single PV panel¹ for each day type, whereas Fig. 3 shows the average Italian National Single Price (NSP) for all months of 2021.²

Looking at the Figure, we may notice the significant rise in electricity prices experienced throughout the considered year. Fig. 4 displays the average hourly electricity price on a typical summer day.

The scenarios used in the experiments have been generated by considering random variations from the average values. This simplified approach is suggested by the strategic nature of the decision problem. To evaluate the impact of the scenario number on the model's recommendations, different experiments have been carried out by progressively increasing the number of scenarios in the set.

In the experiments, we have considered three time-of-use blocks. According to the Italian organization, the first one covers the hours from 8:00 a.m. to 7:00 p.m., the second one includes the hours in the two sub-intervals 7:00–8:00 a.m. and 7:00–11:00 p.m. Finally, the last block refers to the hours between 11:00 p.m. and 7:00 a.m. For the sake of simplicity, we have not distinguished between working and non-working days. The extension to this case, as well as the consideration of a different number of blocks, is quite straightforward.

The follower is representative of a group of prosumers, each with a daily electricity demand of around 24 kWh. The consumption pattern is deemed uncertain as the prosumer is assumed to have a system PV panels.

6.2. Computational performance

Different experiments have been carried out with the aim of assessing the performance of the proposed solution approach. Compared to the deterministic BL formulation, the problem is clearly more challenging as variables and constraints, related to the operation of the system, are duplicated across multiple scenarios. Furthermore, the solution time is influenced by the number of binary variables employed to obtain the binary expansion, which depends on the value of μ . To gain a more precise understanding of this impact, we have performed some preliminary experiments by keeping the number of scenarios constant and selecting two different values for μ , specifically 0.04 and 0.05. As a result, when considering the upper and lower limits of the prices, the values of L_f for the three time-of-use blocks were set to 6, 5, and 4 for the first option, and 5, 4, and 3 for the second option. The subsequent

¹ <https://re.jrc.ec.europa.eu>

² <https://www.mercatoelettrico.org/it>

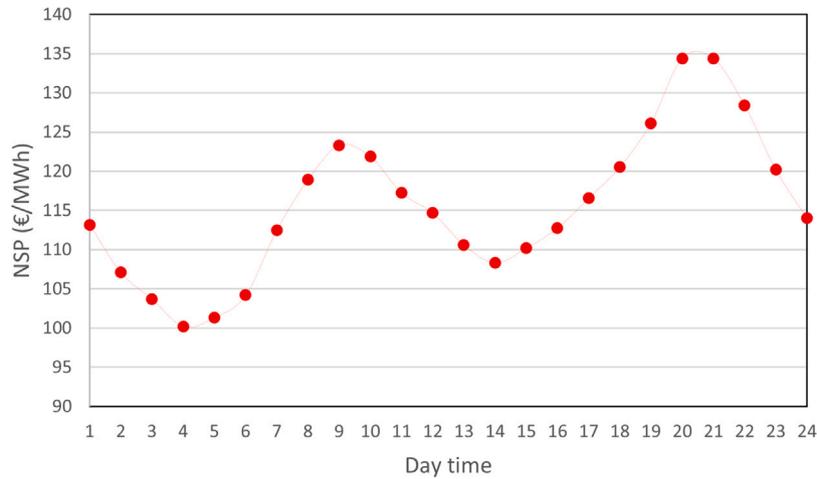


Fig. 4. Average NSP for a typical summer day.

Table 2
Solution times (in seconds).

# Scenarios	α	SL	PA
10	0	26.2	32.75
10	0.5	53.98	62.08
10	1	62.45	72.08
20	0	87.11	86.90
20	0.5	176.23	172.861
20	1	214.97	210.23
30	0	409.12	396.73
30	0.5	876.27	832.21
30	1	1192.74	1132.81
40	0	950.22	899.54
40	0.5	1354.45	1218.62
40	1	1696.13	1001.81
50	0	1223.26	988.4
50	0.5	2424.51	1722.82
50	1	2760.33	1877.46
60	0	3001.11	2255.56
60	0.5	-	2775.44
60	1	-	3075.81

results refer to the first case, where it is worth noting that the solution time is more than twice higher than in the second case.

Table 2 reports the solution times (in seconds) of the proposed approach referred with the acronym PA and the time required to solve (by CPLEX) the single level reformulation, referred to as SL. Different instances have been generated by varying the number of scenarios between 10 and 60 and the safety level between 0 and 1. In particular, we have considered three different values of α equal to 0, 0.5 and 0.9999, which are associated with a neutral, medium and extreme attitude towards risk, respectively. For the sake of simplicity, in the Table we report the value 1 instead of 0.9999. The reported results are the average solution times computed by running 10 different instances of the problem with the same number of scenarios. Moreover, we have set a time limit of 3600 s. In case the solution time reaches this limit a “-” is reported.

Looking at the results, we may observe that the choice of the safety level impacts on the computational performance. When the value of α is set to 0, for all instances and regardless the approach, the solution time is the lowest. We remind that in this case, the retailer aims at maximizing the expected profit and is not concerned about risk. When the value of α is increased the problem is more computationally challenging and the solution time is on average more than twice longer than that required to solve the BL problem under risk neutrality.

When comparing the two approaches, it becomes evident that the single level reformulation is competitive with the proposed approach

as long as the number of scenarios remains limited. In effect, for the instances with only 10 scenarios the solution time of the SL is even lower than that of the PA. However, the savings in computational time become more evident when considering a higher number of realizations. More importantly, starting from 60, the SL approach fails to provide the optimal solution within the fixed time limit, while the PA does. When we consider a number of scenarios greater than 60 also the PA fails to deliver the optimal solution, but the returned feasible solution is always significantly better than the solution provided by the SL approach.

Analyzing the results in more details, we observe that starting from 50 scenarios, at least for the considered test instances, the solution exhibits good in sample stability properties, in that the same values of first-stage solutions are obtained when considering different sets of scenarios with the same cardinality. The results reported in the next subsection does refer to the case of 50 scenarios.

6.3. Results and discussion

A first set of experiments have been carried out to assess the impact of risk aversion on design decisions. To this aim, we have solved different instances of the problem by varying the value of α from 0 to 1, with a step increment of 0.1. We recall that when the safety margin is set to 0, the retailer’s objective is to maximize the expected value with no consideration for possible profit losses in adverse situations. As expected, this approach yields the highest objective function value. As the value of α increases, a decrease in the objective function value is observed. In such instances, the retailer would prioritize “safer” design solutions, even if they may be less profitable. As an extreme case, the retailer may aim to maximize the profit in the worst-case scenario. When this criterion is selected, the minimum value for the objective function is achieved.

The results are shown in Fig. 5 where we report the values of the retailer’s objective function for different levels of α .

Looking at the Figure, we can appreciate the difference in the objective function value associated with the two extreme cases, i.e., α equal to 0 and 1. Specifically, in the case of risk neutrality, the value of the objective function is 17% higher than the corresponding value obtained for the risk aversion case. To enhance the analysis of the results, we also report in Fig. 6 the expected profit and the standard deviation for different values of the safety level. The results clearly indicate that smaller α values correspond to higher expected profits, but they also entail greater risk. The retailer shall select the optimal configuration from the various options offered by the problem solutions according to her attitude towards risk. In this regard, the proposed

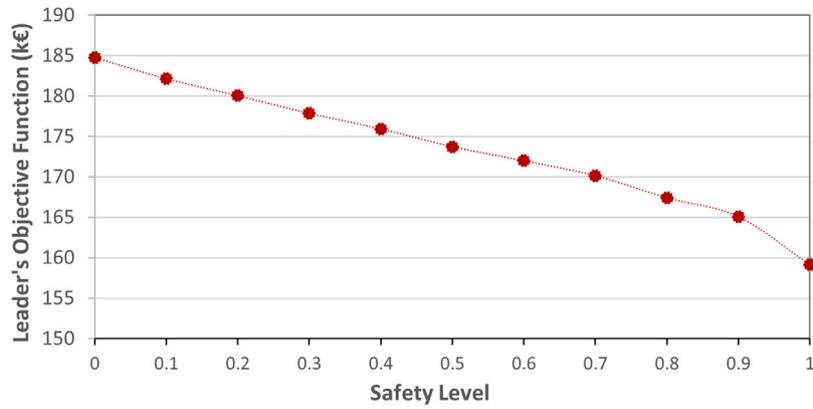


Fig. 5. Retailer's objective value as function of the safety level.

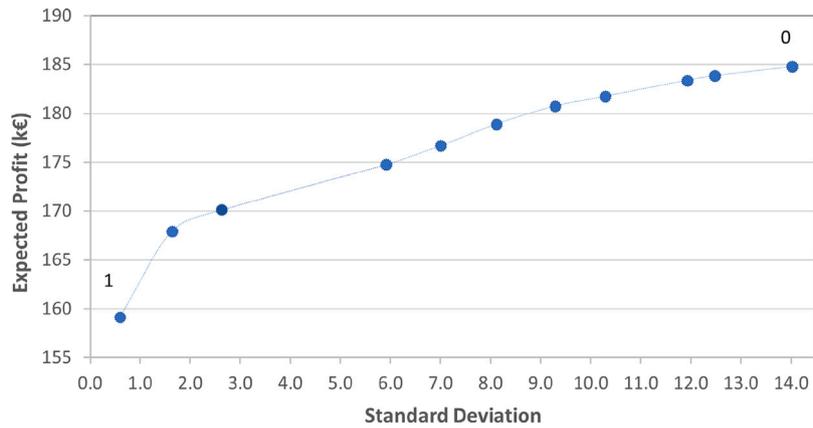


Fig. 6. Expected profit vs standard deviation as function of the safety level.

approach provides decision makers with a valuable tool to evaluate and identify the best solution, while also considering other relevant factors.

To conduct a more comprehensive analysis of the results, we now analyze the solutions associated with three different safety levels, 0, 0.5 and 1. Specifically, we examine the first-stage decisions, which are crucial strategic actions taken before the operational management phase.

The analysis of the results reveals that the same number of PV panels is installed regardless of the selected safety margin. This solution confirms that the PV represents a low-cost energy source for fulfilling the energy demand. If the constraint on the available space were relaxed, even more panels could be installed, at least to some extent. This solution would enable the sale of any surplus energy generated and/or its release when required if stored.

Regarding the storage device, the higher the leader risk aversion, the more investment is made in the storage solution. For example, in the analyzed instances, the sizes selected were 250, 300 and 700 for the three risk aversion levels, respectively. This trend has been consistently observed across all the instances tested. This solution is in line with what ones expects, as batteries can be used to balance supply and energy demand, especially during periods of high market prices and/or limited renewable energy generation. By investing in larger storage devices, the leader could effectively manage unfavorable situations related to instability in electricity market prices.

6.3.1. The value of dealing with uncertainty and risk

It is widely recognized that stochastic programming poses greater computational challenges compared to using deterministic models even in the conventional single-level case. To highlight the significance

of managing uncertainty and risk, we have carried out a comparison between our proposed model and a deterministic BL formulation wherein the uncertain parameters are replaced with their expected values. In particular, we have computed the Value of Stochastic Solution (VSS) [2], a traditional measure which was originally proposed for the risk-neutral setting and modified in [24] to account for formulations with a mean-risk objective function. More specifically, the *VSS* (in percentage) is computed as:

$$VSS_{\alpha} = \frac{z_{\alpha}(SP) - z_{\alpha}(EEV)}{z_{\alpha}(EEV)}$$

Here $z_{\alpha}(SP)$ denotes the leader's objective function for a fixed value α , whereas $z_{\alpha}(EEV)$ indicates the value of leader's objective function computed by solving the proposed model with the upper level first-stage variables fixed to the optimal values obtained by solving the deterministic (with expected values) BL counterpart.

To assess how the safety level affects this measure, we report the VSS values in Fig. 7 for the neutral, medium, and extreme level of risk aversion.

Looking at the results, we may observe that in the risk-neutral case, the benefit gained from solving the stochastic model over its deterministic counterpart is relatively small, approximately round 4%. However, the advantage increases as the safety level rises, and as expected, the highest value is achieved in the case of extreme risk aversion, with a value approximately around 14%. We may notice that the gap between the computed values is higher when moving from an extreme risk-averse position to a moderate one compared to what happens when passing from a moderate to a neutral position. This trend highlights the importance of dealing with uncertainty and risk in order to provide the decision maker with more reliable solutions.

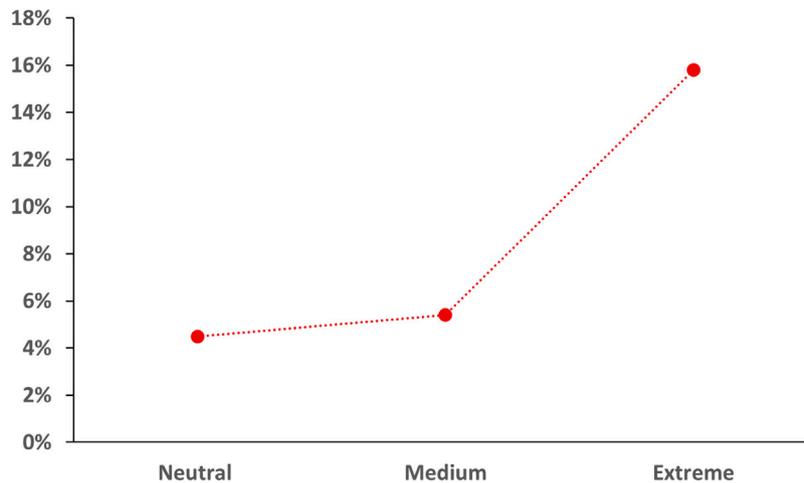


Fig. 7. The VSS as function of the safety value.

6.4. Discussion

The comprehensive computational analysis carried out on the case study provides several valuable insights that could support the retailer in the choice of adopting sustainable solutions.

The first takeaways refers to the selection of the energy technologies. The results confirm the benefit of investing in RESs and of integrating the BES to mitigate the negative effects deriving from their intermittent and uncertain nature. Moreover, the stored energy can be used during periods of high prices when the retailer's energy supply is insufficient to fulfill the demand.

To assess the impact of investment decisions on profitability, we have solved a problem formulation that disregards investment and we have compared the corresponding leader's objective functions. The analysis has been performed for the three different values of safety levels. On average, incorporating in the problem the investment decisions results in an 18% increase in profit, with the highest rise in the case of extreme risk aversion (around 24%). Moreover, upon closer examination of the results, we may observe that investing in renewable energy enables decision makers to offer more affordable tariffs to the follower. For the considered case study, the reduction amounts to approximately 6% with higher values when a high risk aversion level is considered. The results referring to this case are shown in Fig. 8, where we report the electricity tariffs for the three time of use block in both the renewable-based energy system and the conventional solution. These additional findings add to the existing evidence that investing in renewable-based energy systems can be a profitable opportunity, while contributing to energy transition.

A second important takeaways refers to the importance of explicitly dealing with uncertainty. The problem has indeed a strategic nature and ignoring uncertainty that naturally affects the decision problem may lead to inferior solutions. For example, changes in market conditions, such as energy prices and government regulations, can significantly impact the financial viability of investment project. This consideration is confirmed by the values of the VSS that measures the importance of explicitly accounting for uncertainty to take decisions which could be resilient to future changes.

Another important insight refers to the incorporation of a safety measure in the stochastic BL formulation. The results indicate that the size of the storage device is influenced by the safety level, with larger sizes when higher risk aversion level is considered. More prudent decision makers may prefer to support higher investment costs selecting a system configuration that could be less vulnerable to fluctuations in the long-term. This is confirmed by the values of the VSS which are higher when the retailer is more risk-averse.

A final consideration refers to the importance of integrating sizing and pricing decisions in the BL formulation. Price is the key factor in determining the profitability of the investment, since it influences the revenue. The determined tariffs can be used as bounds in operational models defined with the aim of determining dynamic tariffs that may be offered, for example on a daily basis.

7. Conclusions and future research directions

This paper presents a BL approach to support electricity retailers in making investment decisions in sustainable energy systems to produce clean electricity. The interaction between the retailer acting as leader and the follower representative a set of residential prosumers is modeled as a Stackelberg game. Both players solve an optimization problem subject to uncertainty related to varying market prices, weather-related variables and prosumer's load. In particular, the leader solves a strategic problem referring to the optimal sizing of the energy system conjecturing the possible reaction of the follower to the offered tariffs considered as upper level decision variables. The follower reacts to the communicated prices establishing the best procurement plan in terms of amount of electricity to purchase from the retailer and from competitors so to minimize the average electricity bill. The follower's reaction affects the actual profit computed as difference between the revenue from electricity selling and the total cost defined as the sum of investment and operation and management cost. To account for the retailer's attitude towards risk, the upper level problem includes a safety measure to control the average profit that can be gained in a given percentage of unfavorable realizations, whereas at the lower level, the follower aims at minimizing the expected electricity bill.

To solve the stochastic BL formulation we have proposed a solution approach that exploits the specific structure of the problem. The basic idea is to perform a partial enumeration of the BL feasible solutions. The approach is compared with the traditional strategy relying on the derivation of a single-level reformulation of the original problem solved by a general-purpose solver. The numerical results shows that the proposed method becomes competitive when the number of scenarios is increased.

Comprehensive computational experiments have been carried out on a case study calibrated considering real data referring to the Italian market. Our results confirm the importance of explicitly addressing uncertainty and risk in the BL formulation.

Different issues are left for future investigation. The most natural extension refers to the possibility of considering different types of followers (e.g. residential, public, and private consumers) for whom

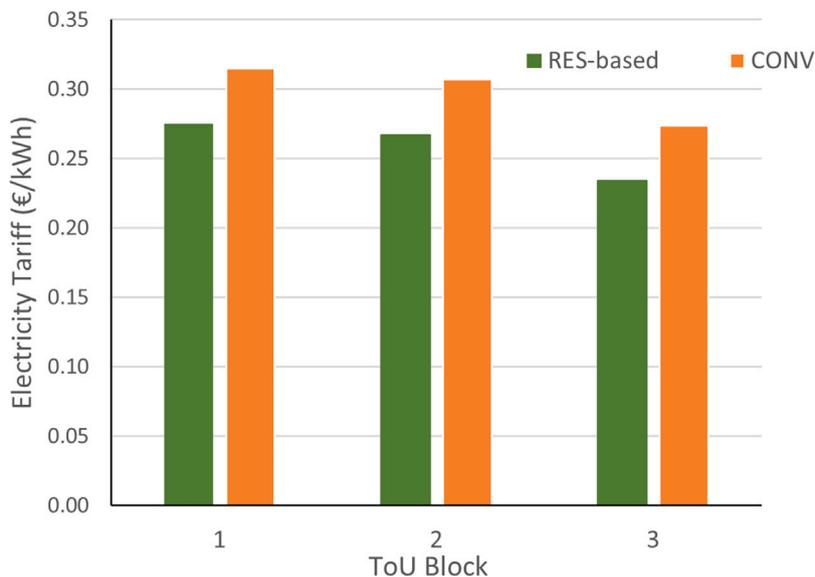


Fig. 8. Electricity tariffs: comparison with the conventional solution.

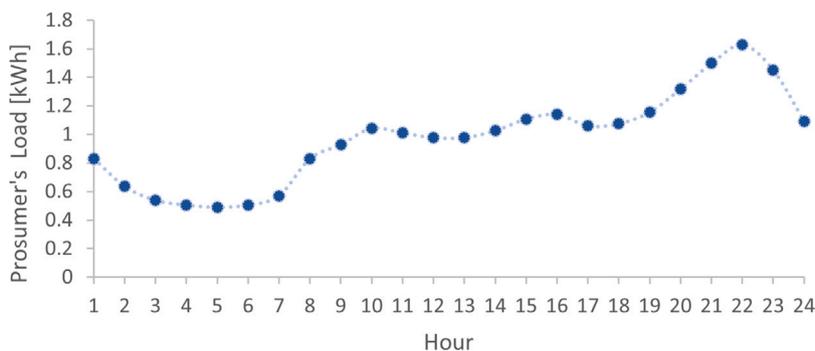


Fig. 9. Prosumer's load.

different tariffs may be proposed. This multi-follower setting could require additional effort pertaining in designing more effective solution methods.

The approach has the potential to serve as the core of a decision support system that, by monitoring the performance of the energy system, would eventually suggest adjusting investment decisions over time with the aim of aligning with changing market conditions, evolving technology options, and emerging sustainability goals.

CRedit authorship contribution statement

Patrizia Beraldi: Writing – review & editing, Writing – original draft, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Appendix

In the numerical experiments we have considered as electric storage devices the lithium batteries. We have assumed a specific cost of 420 €/kW and a lifetime of 20 years. As for the technical parameters, we have assumed an efficiency rate for energy injection and withdrawal, namely η_c and η_d equal to 0.98, whereas the operation range of the BES has been set between the 5% and the 99% of the nominal capacity. As for the sizes, we have considered 15 different alternatives ranging from 100 to 800 kW.

Regarding the PV, we have considered modules of 1.67 m² with a specific cost of 1200 €/m². The total available area is of 1000 m².

Finally, we report in Fig. 9 the electricity load of a single prosumer. We have assumed the same request for all the day types.

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