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The effect of an uncertain commission rate on the decisions of a capital-constrained developer

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ABSTRACT

This study investigates a green supply chain consisting of a capital-constrained developer who sells a product via a platform. The parties interact via an agency contract, in which the platform charges a fixed proportion of the revenue gained from each sold unit and the developer receives the remaining sum. Since the development process is relatively protracted, at the early stages of this process, the commission rate to be charged by the platform is random from the developer's perspective. Upon receiving information about the amount of capital the developer has committed to investing in greenness from his own resources, an external investor offers the developer a loan at a certain interest rate (to further enhance the developer's investment in greenness), based on which the developer sets the product's greenness level and selling price. The study provides a game-theoretic analysis of this model and compares its equilibrium solution with the optimal solution of a fully self-financing developer. The innovative feature of the study lies in its comparison between the case of a developer who might not be able to repay the loan, because his revenue from selling the product might be lower than the amount he is required to repay the investor (the loan plus interest), and the case in which it is certain that the developer will be able to repay any debt to the investor. Our study shows that, in the case where the investor takes on the financing risk, the customers benefit from a higher greenness level (albeit at a higher price), resulting in greater demand for the product.

1. Introduction

In recent years, technology has become a significant contributor to global greenhouse gas emissions, and thus public concerns about saving energy and protecting the environment have dramatically increased. The digital carbon footprint, which originates from sources such as the emissions produced during the manufacture of digital products, the energy required to run devices and equipment, the transfer of data (i.e., product distribution), and the energy consumed by servers and data centers has become a key contributor to global carbon emissions.¹ According to a report published in 2019 by the Shift Project,² digital technologies are responsible for 3.7% of global gas emissions and are causing an annual increase in the carbon footprint of 9%, where video streaming is one of the biggest contributors to these figures. Manufacturers are strongly motivated to invest in carbon emission reduction during the product life-cycle, and in some countries, governments have

started to impose green regulations with respect to digital technology. For example, France is planning to introduce a green software legislation, known as the Digital Environmental Footprint Reduction Legislation, which comes into effect in 2024 and includes mandatory digital emissions disclosures. As a result, Netflix in France will be required to report to the government and to users on the environmental cost of its products.³ In another example, according to the US Securities and Exchange Commission, the USA is expected to strengthen emissions disclosure requirements, which will include digital emissions, as is stated in their proposed Rules to Enhance and Standardize Climate-Related Disclosures for Investors.⁴ In general, industries and companies are expending increased efforts in implementing green technologies during the production process in order to enhance their reputation, satisfy the environmental concerns of consumers, and consequently, increase demand.

To create greener digital products, companies and individuals should minimize environmental impacts from the early stages of product

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¹ <https://www.techtarget.com/searchcio/feature/Understand-the-digital-carbon-footprint-of-enterprise-IT#:~:text=A%20digital%20carbon%20footprint%20refers,of%20sight%2C%20out%20of%20mind>

² https://theshiftproject.org/wp-content/uploads/2019/03/Lean-ICT-Report_The-Shift-Project_2019.pdf

³ <https://greensoftware.foundation/articles/is-there-a-wave-of-green-software-legislation-and-standards-coming-our-way>

⁴ <https://www.sec.gov/news/press-release/2022-46>

development. This can be achieved using strategies such as investing in a green hosting provider, adopting an efficient cache policy, removing unused features, improving battery use, and optimizing the use of location services.⁵ Such strategies require a vast monetary investment, which is usually beyond the financial capability of developers in their early stages. Therefore, many developers choose to obtain a green loan from an external financial institution (e.g., bank, venture capitalist). Unlike a conventional loan, in the case of a green loan, the company or developer is required to commit the money from the loan to the task of improving the greenness level and sustainability of the product.⁶

In recent years, online marketplace platforms have widely adopted the agency selling format [1] according to which sellers are granted direct access to the platform's customers in exchange for a fixed commission from each financial transaction. As evidence for the importance of this distribution channel, we note that in 2019, 47% of digital purchases worldwide were executed via online marketplace platforms.⁷ In this work, we consider a dyadic supply chain comprising a developer and an online marketplace platform. The latter serves as a mediator and publicly publishes its universal commission rate, but does not engage in operational or marketing decisions. The developer, who is capital-constrained, produces a virtual product (such as software, an application or an e-service) and sells it via the platform. In order to increase the greenness level and reduce the energy consumption of his product (thereby increasing demand), the developer can augment his investment in greenness from his own resources by borrowing money from an external investor. As explained in Section 3, the developer's income from selling the product is relatively uncertain at the early stages of the development process. Therefore, we consider two model variants with regard to the developer's ability to repay the loan: (a) no bankruptcy risk; and (b) bankruptcy risk. For the first model variant, we use analytical investigation to determine properties of the equilibrium greenness level and equilibrium selling price of the product, as well as properties of the equilibrium interest rate charged by the investor. For the second model variant, since its mathematical complexity precludes analytical investigation, we conduct a numerical analysis to derive conjectures.

This study addresses two main research questions:

- How does the amount of capital that the developer commits to investing in greenness from his own resources affect the greenness level of the product, its selling price, the interest rate for taking a loan from an external investor, and the profits of the developer, the platform and the investor?
- Which model – with regards to whether or not the developer faces a bankruptcy risk – is preferred by each party and by consumers?

The results of the first model variant, according to which the developer's bankruptcy risk is not considered, show that the product's selling price increases in its greenness level. Furthermore, as the amount of capital that the developer commits to investing in greenness from his own resources (hereafter referred to as "the developer's capital") increases, the investor charges a lower interest rate for the loan, which in turn leads to a higher greenness level of the product. As expected, the higher the developer's capital, the lower the investor's profit, but the higher the developer's and the system's expected profits. Under the second model variant, according to which the developer's bankruptcy risk is taken into account, the results of the numerical analysis demonstrate that when the developer expects the commission rate to fall randomly within the range 3% – 30%, his bankruptcy risk

can be disregarded. However, when the expected commission rate is spread over its entire potential domain, disregarding the developer's bankruptcy risk results in lower profits for both the developer and the investor. Clearly, the interest rate charged by the investor is higher when the latter faces the risk of the developer's bankruptcy. In such a case, because the developer is not concerned about the financial consequences of bankruptcy, he is willing to invest more in greenness compared with the model without bankruptcy risk.

The contribution of this paper is threefold. First, while most studies that consider a capital-constrained party have opted to model the interaction using a wholesale-price contract, we assume an agency contract, which is more commonly used in platform selling. Second, we use analytical tools to provide properties of the equilibrium and, in particular, to examine these properties as a function of the developer's capital. Third, based on numerical examples, we show that randomness of the platform's commission rate within its commonly-used range (3–30%) results in an overlap between the two models — with and without risk of the developer's bankruptcy.

The remainder of the paper is organized as follows. Section 2 reviews related literature on capital-constrained parties, agency contracts and green production. In Section 3, we describe the general model. Section 4 analyzes the optimal solution for the case of a self-financing developer, which serves as a benchmark model. In Section 5, we provide a full theoretical equilibrium analysis under the assumption of no bankruptcy risk, while Section 6 presents a numerical analysis to illustrate the effects of including bankruptcy risk. Section 7 concludes and provides managerial insights.

2. Related work

This paper is related to four streams of the operations management literature: (i) capital-constrained parties; (ii) agency contracts; (iii) supply chains of digital products; and (iv) green production. To better explicate our contribution, in the following paragraphs we review each of these research topics.

Capital-constrained parties. Previous studies dealing with a capital-constrained party in a supply chain have generally been based on the newsvendor model, but have differed in terms of the identity of the capital-constrained party (either downstream or upstream) and the financing source (either internal or external). For example, Dada and Hu [2] examined a capital-constrained newsvendor who borrows capital from a bank for procurement purposes, and analyzed the influence of the interest rate on the newsvendor's equity. Kouvelis and Zhao [3] compared trade credit with bank financing and concluded that trade credit should be preferred by the supply-chain parties. Yang and Birge [4] assumed that the retailer is able to borrow from both a bank and the supplier, where the former has seniority. They found that, in some cases, the borrower may use both bank and supplier financing, and they argued that this happens because of bankruptcy costs. Yan et al. [5] analyzed the equilibrium financing strategies under a partial credit guarantee contract incorporating bank credit financing and the manufacturer's trade credit guarantee. Chen et al. [6] extended the scenario of partial credit guarantee financing by examining buyback guarantee financing for a capital-constrained retailer. Xu and Fang [7] further investigated partial credit guarantee and trade credit financing for a capital-constrained manufacturer. Gupta and Chen [8] considered a supplier who is able to accept a retailer's offer of a loan, but in the case where he rejects the offer, he is able to borrow money from the bank.

Agency contracts. While the above papers assume that the interaction between the parties is based on a wholesale-price contract, in practice, the agency contract is more common in platform selling. In the supply-chain management literature, the latter contract can be considered as a consignment contract with revenue sharing [9]. To the best of our knowledge, only a few papers have studied a capital-constrained

⁵ <https://medium.com/swlh/green-it-a-sustainable-approach-to-app-development-1ef4234faf51>

⁶ <https://www.worldbank.org/en/news/feature/2021/10/04/what-you-need-to-know-about-green-loans>

⁷ <https://www.statista.com/statistics/897678/online-shopper-preference-for-first-time-repeat-purchases-platform-global/>

party operating under an agency contract. In particular, Wang et al. [10] considered a capital-constrained retailer that can choose between platform financing and bank financing. They concluded that platform financing yields a larger order quantity and higher profits for the parties than those yielded through bank financing. Gong et al. [11] considered a powerful e-tailing platform and a budget-constrained small/medium-sized seller. The authors showed that the optimal selling price can decrease when a budget constraint exists and they identified a budget threshold below which the seller should accept a loan. They further demonstrated that it is always beneficial for the platform to provide a loan to a seller who is short on funds. Gupta and Chen [8] formulated a supply chain in which the retailer offers trade credit to the supplier in a consignment selling setting. After setting the loan terms (composed of a debt seniority requirement and an interest rate), the supplier can either accept the retailer's offer, borrow only from the bank, or rely solely on its own capital. The authors concluded that direct financing from the retailer under equilibrium loan terms weakly improves the expected profits of both the retailer and the supplier relative to bank financing. Tao et al. [12] considered a capital-constrained retailer selling products through an e-commerce platform, which also offers financial services to retailers. They designed specific revenue-cost sharing contracts to coordinate the supply chain and demonstrated that these contracts are feasible. Moreover, they found that the retailer consistently prefers platform financing over bank financing. Chang et al. [13] modeled the interaction between an e-commerce platform and a retailer as a Stackelberg game. Specifically, they studied the relationship between the financing strategy of the retailer (trade credit or bank credit) and the channel cooperation contract of the e-commerce platform (reselling or marketplace). They showed that the capital level of the retailer affects the preferences of both parties. In a paper closely related to the present study, Avinadav and Bunker [14] analyzed an app developer who is financed by an external investor in exchange for a share of his revenue, where the financing amount is determined via bargaining. In contrast to the latter paper, in our study, the developer can borrow money to be repaid at a given interest rate. Also, we consider how the risk of the developer's bankruptcy affects the decisions of the parties.

Supply chains of digital products. Digital products, such as software and mobile apps, are characterized by the ability to supply any demanded quantity at zero lead time and without holding costs [15]. In recent years, this topic has been receiving increasing attention in the operations management literature. Chernonog et al. [16] studied decisions related to pricing, quality investment and sales-effort investment in a supply chain of digital products under bi-criteria and various risk attitudes of the parties. Avinadav et al. [17] considered a two-echelon supply chain in which the digital-product developer has information superiority about demand. Hu et al. [18] studied an online-game supply chain, composed of a game developer and a distributor, in which the distributor has superior information about demand. The authors investigated the impact on the developer's level of investment in game quality of the distributor's decision about whether or not to share demand information. In a later study, Hu and Meng [19] examined the effects of sales-effort efficiency and risk attitude on a supply chain of mobile games in which the platform invests in sales effort and the developer invests in product quality. Avinadav and Bunker [14] investigated a supply chain of digital products, comprised of a distribution platform and an app developer, which operates under the freemium business model and the agency selling mode. In a recent study, Avinadav et al. [20] analyzed the equilibrium decisions of a distribution platform and an app developer, each of which holds private information about demand. Similar to the above papers, we investigate equilibrium decisions in a supply chain of digital products. However, the focus of the current work is a capital-constrained developer who is financed by an external investor. Although Avinadav and Bunker [14] also investigated a financing scenario, they examined the case where a non-capital constrained developer acquires financing in return for a share of the revenue, whereas the focus of the current work is a capital-constrained developer who obtains interest-based financing.

Green production. Green production mainly focuses on carbon emission reduction and green supply chains. For example, Swami and Shah [21] studied the problem of coordinating a manufacturer and a retailer in a green supply chain. Ghosh and Shah [22] investigated the effect of using a cost-sharing contract on the supply-chain measures and the product's greenness level. El Ouardighi et al. [23] studied how double marginalization affects pollution in a supply chain composed of a single manufacturer and a single retailer. Bai et al. [24] investigated the effects of carbon emission reduction on a supply chain with a single manufacturer and two competing retailers, where the demand depends on the manufacturer's green technology level and the selling prices of the two competing retailers, and showed that coordination can be achieved. Other papers have taken into account eco-friendly awareness of the consumer. Liu et al. [25] studied the impact of competition and consumer environmental awareness on supply-chain profits. Zhu and He [26] examined how supply-chain decisions under competition are affected by factors such as the supply-chain structure, the green product type and the type of competition. Yang and Chen [27] investigated the impact of revenue-sharing and cost-sharing schemes, offered by a retailer, on a manufacturer's carbon emission abatement efforts when taking consumer environmental awareness into consideration. Wang and Sun [28] considered a green supply chain in which a single manufacturer produces a green product and sells it through both a retail channel and its own direct channel. They found that the manufacturer has a greater incentive to invest in greenness when a higher proportion of consumers buy the green product through the direct channel. Wang and Choi [29] explored firms' production and emission decisions in a demand uncertainty market assuming carbon-emission restriction.

Several papers have considered the role of green investment in capital-constrained supply chains. Wu et al. [30], for example, investigated a green supply chain consisting of a single manufacturer and a single capital-constrained retailer, where both bank financing and trade credit financing are possible. They showed that the retailer orders a larger quantity of products when the manufacturer invests more efforts to reduce carbon emissions, and that this effect is more marked under trade credit financing than under bank financing. Unlike the aforementioned study, which used a wholesale-price contract, Fang and Xu [31] considered an interaction between supply-chain members that is based on a pull contract, where the manufacturer is capital-constrained. Other papers on green supply chains have dealt with financial incentives and have concentrated on two main aspects: (a) government policy incentives to promote the adoption of environmentally friendly products [32–34]; and (b) the economic effect of green credit policies [35,36]. In particular, An et al. [37] studied the impact of a precondition of green credit on the financing supply-chain's equilibrium.

In line with the above studies, we also study the effect of carbon emission reduction in a green supply chain. Our model, however, differs from previous studies in three main dimensions. First, it is unique in considering the following combination of features related to financial constraint: the developer who invests in carbon emission reduction faces capital constraint, can borrow additional capital from an external investor, and might face bankruptcy due to failure to pay off the loan. Second, our model assumes that the interaction between the two parties – the developer and the distribution platform – involves uncertainty in the sense that the commission rate to be charged by the platform for each transaction is random, from the developer's perspective, when the latter chooses the greenness level of the product. The last major difference between prior studies on green supply chains with financing and the current work is that we focus on digital products, which, in contrast to tangible ones, do not bear any leftover or shortage costs [16–20,38,39]. To the best of our knowledge, our paper is the first to connect the four research topics discussed in this literature review, and is positioned at the intersection between the domains of green supply chains and supply-chain finance.

Table 1
Overview of the related literature from recent years, classified according to a variety of characteristics.

Authors (year)	Virtual/Tangible	Demand sensitive to: Price/Quality/Greenness/Random	Wholesale/Revenue Sharing	Financing party	Capital-constrained party: Manufacturer/Retailer/Supplier	Financing model: Revenue Based/Interest Rate	Includes bankruptcy?: Yes/No	Production cost: Fixed/Variable
Avinadav and Bunker [14]	V	Q,P	RS	External investor	–	RB	N	F
Xu and Fang [7]	T	G	W	Bank	M	IR	Y	V
Gupta and Chen [8]	T	R	RS	Bank, Retailer, both	S	IR	N	V
Chang et al. [13]	T	R	W	Bank, Platform	R	IR	Y	V
An et al. [37]	T	G	W	Bank, Supplier	M	IR	N	V
Fang and Xu [31]	T	P,G	W	Bank, Retailer	M	IR	Y	V
Jin et al. [36]	T	G	RS	Government, Bank	M	IR	Y	V
The current paper	V	P,G	RS	Investor	M	IR	Y	F

To summarize, Table 1 provides an overview of related literature from recent years, where the models have been classified according to a variety of characteristics. As can be seen, none of these papers has investigated the revenue-sharing model with a capital-constrained manufacturer who produces a virtual product and receives interest-based financing from an external investor.

3. General model

Glossary of notations.

Symbol	Description
<u>Abbreviations</u>	
<i>D</i> (subscript)	Developer
<i>I</i> (subscript)	Investor
<i>P</i> (subscript)	Platform
<u>Random variables</u>	
η	The developer’s revenue share (a non-negative random variable with mean μ and variance σ^2)
<u>Decision variables</u>	
<i>k</i>	Interest rate demanded by the investor for a loan
<i>p</i>	Selling price of the product (set by the developer)
<i>g</i>	Greenness level of the product (set by the developer)
<u>Functions</u>	
$D(\cdot)$	Demand function of the product
$C(\cdot)$	Cost of carbon emission reduction
π_i	Profit function of party $i \in \{D, I, P\}$
Π_i	Expected profit of party $i \in \{D, I, P\}$
$E[\cdot]$	Expected value of a random variable
$V[\cdot]$	Variance of a random variable
<u>Parameters</u>	
<i>a</i>	The base demand
<i>m</i>	The amount of capital that the developer commits to investing in greenness from his own resources
β	Demand sensitivity to price
θ	Demand sensitivity to greenness
γ	A scale coefficient that reflects the investment efficiency of the developer in carbon emission reduction

Consider a developer who plans to sell a virtual product via a distribution platform, where the parties interact using the agency contract, which is common in platform selling [1,40]. More specifically, the platform charges a fraction $1 - \eta$ of the revenue gained from each transaction, and the developer receives the remaining sum. Since the development process is relatively protracted, the developer’s share of the revenue, η , is random (from his perspective) when he chooses his

total level of investment in carbon emission reduction (i.e., during both the development process and the app’s lifecycle). In particular, this share is a non-negative random variable defined generally over the support $(0, 1)$ with probability density function $f(\cdot)$, cumulative distribution function $F(\cdot)$, and $E[\eta] = \mu$. Note that although platforms have traditionally used a fixed, universal commission rate that applies to their interactions with all app developers, in recent years, this commission rate has become dynamic. For example, until mid-2016 (for Apple) and 2018 (for Google Play), the commission fee for all apps was 30%. Later, both platforms reduced its commission fee to 15% for users who had been subscribed for more than one year. After the legal dispute between Apple and Epic Games, followed by the creation of the App Equity Coalition at the end of 2020, Apple changed its policy such that it now offers a commission discount to small developers under the App Store Small Business Program. In July 2021, Google launched a similar program, with the same conditions as those stipulated by Apple. Yet shortly after (in October 2021), Google reduced the commission fee to 15% for all subscription apps, starting from day one of each subscription and regardless of the total revenue gained by the developer.⁸ Another example of the volatility of the commission rate in the app market is the policy change implemented by Steam, which is one of the biggest online stores for desktop games. Steam had traditionally charged a 30% fee from all game sales on its platform. Recently, however, Steam changed its fee policy to one in which the fee depends on the market performance of the game. Specifically, it takes 30% from the first 10 million in sales; then, for sales above 10 million and below 50 million, the commission fee drops to 25%; finally, beyond 50 million, the fee drops further to 20%.⁹ Therefore, we conclude that during the development process of an app (which can be relatively long), the developer cannot be certain of the exact fee that will be charged upon signing the contract.

Since this study deals with virtual products (e.g., mobile apps, video games and streaming content), the demand is fulfilled precisely, meaning that there are no shortages or leftover units. This is different from the case where a market of tangible products is modeled. In addition, virtual products are characterized by a substantial development cost, followed by a negligible unit production cost. Following the law of diminishing returns, and in order to facilitate mathematical tractability, we adopt the quadratic cost function, which is prevalent in the operations management literature [31,41–45]:

$$C(g) = \frac{\gamma g^2}{2}, \tag{1}$$

where γ is a scale coefficient reflecting the efficiency in reducing carbon emissions (a higher value of γ implies lower efficiency). In line with closely-related papers [17,18,20,46–50], we further assume that demand is jointly linear in the app’s greenness level (increasing) and selling price (decreasing):

$$D(p, g) = a - \beta p + \theta g, \tag{2}$$

⁸ <https://qonversion.io/blog/how-to-determine-apple-google-service-fees/>
⁹ <https://techcult.com/how-much-does-steam-charge-to-sell-a-game/>

where a is the base demand, and β and θ are parameters representing consumer sensitivity to price and to greenness level, respectively. Note that, in our model, $g = 0$ represents a minimal greenness level to which the developer has already committed (prior to the contract) in order to meet industry and market standards, and it can be considered as a sunk cost.

4. Self-financing developer

In practice, the developer might not have sufficient capital to develop his product to the desired greenness level, and thus might need to borrow money from an external investor to accomplish this process. In order to evaluate the effect of external financing of a capital-constrained developer on the supply-chain measures, we start by analyzing the simple case where the developer is self-financing. To represent the case where the developer does not need any external financing, we assume that the capital available to the developer for investment in carbon emission reduction is unlimited. According to this model, the profit function of the developer is

$$\pi_D(p, g) = \eta p D(p, g) - C(g). \quad (3)$$

The sequence of events in this model is as follows: (1) The developer sets the greenness level of the product g ; (2) The commission rate charged by the platform, $1 - \eta$, is realized; and (3) The developer sets the selling price of the product p . Note that, in this case, the greenness level is set under commission-rate uncertainty, whereas the selling price is determined once this uncertainty has disappeared. Due to the uncertainty at the first stage of the process, we use stochastic programming to obtain the optimal solution, which comprises the following two stages:

Second stage of decision making: setting the selling price. At this stage of the stochastic-programming problem, the decision about the greenness level of the product has already been made, which subsequently prompted the platform to declare its commission rate. Note that due to the structure of the developer's profit function, the optimal selling price for a given g , denoted by $p(g)$, is obtained by maximizing the revenue, $pD(p, g)$, regardless of the actual value of η . Consequently:

$$p(g) = \frac{a + \theta g}{2\beta}. \quad (4)$$

This relation implies that a greener product leads to a higher selling price.

First stage of decision making: setting the product's greenness level. At the first stage of decision making, the commission rate is not yet known. However, the developer knows that at the second stage, he will set the selling price according to $p = p(g)$. Thus, at the first stage, the developer's profit is formulated as $\pi_D(g) = \frac{(a+\theta g)^2 \eta - 2\beta \gamma g^2}{4\beta}$, and a risk-neutral developer solves

$$\max_g \{ E[\pi_D(g)] = \frac{(a + \theta g)^2 \mu - 2\beta \gamma g^2}{4\beta} \}.$$

To satisfy the second-order condition for the concavity of the developer's expected profit in g , we assume the following:

Assumption 1. $\gamma > \frac{\theta^2 \mu}{2\beta}$

Recall that a higher value of γ implies a lower efficiency in reducing carbon emissions. Thus, by Assumption 1, in order to avoid unrealistic cases where investment in green technologies results in infinitely large profits, the developer's efficiency in reducing carbon emissions should be limited. Under this assumption, the developer's optimal greenness level under self financing (denoted by the superscript SF) is:

$$g^{SF} = \frac{a\theta\mu}{2\beta\gamma - \theta^2\mu} \quad (5)$$

from which we calculate the expected selling price:

$$p^{SF} \equiv p(g^{SF}) = \frac{a\gamma}{2\beta\gamma - \theta^2\mu}. \quad (6)$$

Consequently, at equilibrium, the developer's investment in reducing carbon emissions is

$$m^{SF} \equiv \frac{\gamma(g^{SF})^2}{2} = \frac{\gamma a^2 \theta^2 \mu^2}{2(2\beta\gamma - \theta^2\mu)^2}, \quad (7)$$

the demand is

$$D^{SF} \equiv D(p(g^{SF}), g^{SF}) = \frac{a\beta\gamma}{2\beta\gamma - \theta^2\mu}, \quad (8)$$

and the expected profit of the developer is

$$\Pi_D^{SF} \equiv E[\pi_D(g^{SF})] = \frac{\gamma a^2 \mu}{2(2\beta\gamma - \theta^2\mu)}. \quad (9)$$

5. Investor financing: no bankruptcy risk

When the developer is capital constrained, we model the interaction between the developer and the investor according to the six sequential stages shown in Fig. 1, which are the stages that would arise in practice. The events proceed as follows: (1) The developer announces the amount of capital m that he commits to investing in carbon emission reduction from his own resources (hereafter "the developer's capital"); (2) The investor determines the interest rate k for financing the developer's green activities; (3) The developer sets the greenness level of the product g ; (4) The commission rate charged by the platform, $1 - \eta$, is revealed; (5) The developer sets the selling price of the product p ; and (6) The selling period starts and demand is realized.

In this section, we assume that the revenue of the developer from selling the product is greater than the interest to be paid for the loan, such that the investor is fully refunded (although the developer can lose when the realization of η is sufficiently low). According to this model, the profit function of the developer (a random variable) and that of the investor, are, respectively:

$$\pi_D(p, g) = \eta p D(p, g) - m - \left(\frac{\gamma g^2}{2} - m\right)(1 + k) \quad (10)$$

$$\pi_I(k) = \left(\frac{\gamma g^2}{2} - m\right)k$$

where the cost of investment in greenness is divided into two components: self financing according to the amount m and a loan for the remaining sum (at interest rate k). Note that an equilibrium exists under the condition $m < \frac{\gamma g^2}{2}$, which ensures that the developer's capital is insufficient to accomplish the desired greenness level. Below, in Lemma 2(ii), we show that this condition is satisfied for all $m < m^{SF}$, that is, up to the optimal amount of capital a self-financing developer with no capital constraint would invest (see (7)). Although the platform is a non-strategic player in our model, i.e., the commission rate $1 - \eta$ is universal and is not determined by the developer's properties, we are interested in exploring the effect of the developer's capital on the platform's profit, which is expressed by

$$\pi_P(p, g) = (1 - \eta)pD(p, g). \quad (11)$$

Due to the attributes of the agency model, the platform's profit is proportional to the revenue from selling the product, regardless of the level of investment in greenness and the financial capability of the developer.

5.1. Solution

Final stage of decision making: setting the selling price. Similar to the self-financing model, at this stage of the stochastic-programming problem, the commission rate $1 - \eta$ has been revealed. This realization occurs after the decisions about the greenness level of the product, g , and the interest rate, k , have been made. Therefore, the optimal selling price for

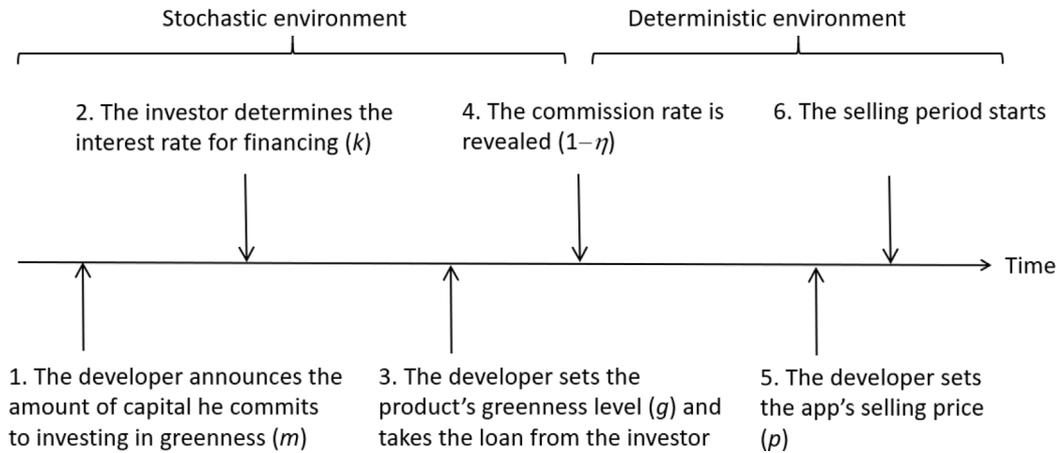


Fig. 1. Sequence of events.

a given g is obtained by maximizing the revenue, $pD(p, g)$, with respect to p , resulting in:

$$p(g) = \frac{a + \theta g}{2\beta}. \tag{12}$$

Note that due to the profit structure of the developer under the agency model, the optimal selling price is independent of the value of the commission rate.

Second stage of decision making: setting the product's greenness level. At this stage, the decision about the investor's interest rate k has already been made, whereas the commission rate $1 - \eta$ has not yet been determined. Thus, the developer's profit is a random variable expressed by (after substituting $p = p(g)$):

$$\pi_D(g) = \frac{(a + \theta g)^2 \eta - 2\beta(\gamma g^2(1 + k) - 2mk)}{4\beta}. \tag{13}$$

The risk-neutral developer now solves $\max_g E[\pi_D(g)]$ where:

$$E[\pi_D(g)] = \frac{(a + \theta g)^2 \mu - 2\beta(\gamma g^2(1 + k) - 2mk)}{4\beta}. \tag{14}$$

To avoid triviality and ensure that $E[\pi_D(g)]$ is a concave function of g , for a given value of k (which has already been set by the investor at the previous stage), we restrict the analysis to $k > \frac{\theta^2 \mu}{2\beta\gamma} - 1$ (to satisfy the second-order condition $\frac{d^2 E[\pi_D(g)]}{dg^2} = -\frac{2\beta\gamma(1+k) - \theta^2 \mu}{2\beta} < 0$). Under this restriction (which has to be satisfied for equilibrium k), the developer's best response to the interest rate k set by the investor is:

$$g(k) = \frac{a\theta\mu}{2\beta\gamma(1+k) - \theta^2\mu}. \tag{15}$$

As expected, a higher interest rate causes the developer to reduce the app's greenness level in order to decrease the financing amount.

The platform's profit is also a random variable expressed by $\pi_P(g) = \frac{(1-\eta)(a+\theta g)^2}{4\beta}$ (after substituting $p = p(g)$); thus the reward of a risk-neutral platform is

$$E[\pi_P(g)] = \frac{(1-\mu)(a+\theta g)^2}{4\beta} \tag{16}$$

First stage of decision making: determining the investor's interest rate. At this stage, the investor maximizes his profit by controlling k , given the developer's best response $(p(g(k)), g(k))$. Therefore, the investor's objective to be maximized is:

$$\pi_I(k) = \frac{\gamma a^2 \theta^2 \mu^2 k}{2(2\beta\gamma(1+k) - \theta^2\mu)^2} - mk. \tag{17}$$

To derive the investor's strategy, we differentiate $\pi_I(k)$ with respect to k , and obtain $\pi'_I(k) = Y(k) - m$, where:

$$Y(k) = \frac{\gamma a^2 \theta^2 \mu^2 (2\beta\gamma(1-k) - \theta^2\mu)}{2(2\beta\gamma(1+k) - \theta^2\mu)^3}. \tag{18}$$

The first-order condition for a maximum is $Y(k) = m$, which results in a cubic equation in k . Although closed-form solutions for this equation can be obtained, they do not provide any insight (due to their complexity) and thus are not presented. The following lemma, which presents the properties of $Y(k)$, is used to ensure an interior-point solution, such that there exists a unique value of k that satisfies the first-order condition, and is crucial for studying the strategy of the investor. In its proof, we justify the rationale behind [Assumption 1](#), and show that under this assumption, the restriction stated beneath Eq. (16) holds in equilibrium. For simplicity of analysis, let $m_0 \equiv Y(0) = \frac{\gamma a^2 \theta^2 \mu^2}{2(2\beta\gamma - \theta^2\mu)^2}$,

$k_0 \equiv 1 - \frac{\theta^2\mu}{2\beta\gamma}$ (such that $Y(k_0) = 0$) and $k_{va} \equiv -k_0$. By [Assumption 1](#), $-1 < k_{va} < 0 < k_0 < 1$.

Lemma 1. $Y(k)$ is a quasi-convex function with the following properties: a negative minimum value (in fact, $Y(k)$ is a convex function until its minimum), a vertical negative asymptote $k = k_{va}$ and a horizontal asymptote $Y = 0$.

Fig. 2(a) illustrates the properties of $Y(k)$ given in [Lemma 1](#) under the setting $a = 100$, $\beta = 0.9$, $\gamma = 1.28$ and $\theta = 1.1$, (similar to the parameter values considered by Avinadav et al. [17] and Avinadav and Levy [40], which also satisfy the condition in [Assumption 1](#)). As for the platform's commission rate $1 - \eta$, the two most prominent distribution platforms, Apple App Store and Google Play, charge a commission of 15–30% from each transaction,¹⁰ leaving 70–85% of the revenue for the developer. Furthermore, according to [51], the platform's commission rate in practice rarely exceeds 25% (5% for Tmall.com, 15% to 25% for Baidu.com and Meituan.com, and 15% for Amazon). Therefore, for practicality and simplicity, we first consider cases where $\eta \sim U(0.7, 0.97)$, implying $\mu = 0.835$.

Based on the properties of $Y(k)$ presented in [Lemma 1](#), we can state the following property of the investor's expected profit:

Proposition 1. $\pi_I(k)$ is a unimodal quasi-concave function.

¹⁰ <https://support.google.com/googleplay/android-developer/answer/10632485?hl=en>
<https://developer.apple.com/support/downloads/terms/schedules/Schedule-2-and-3-20220225-English.pdf>

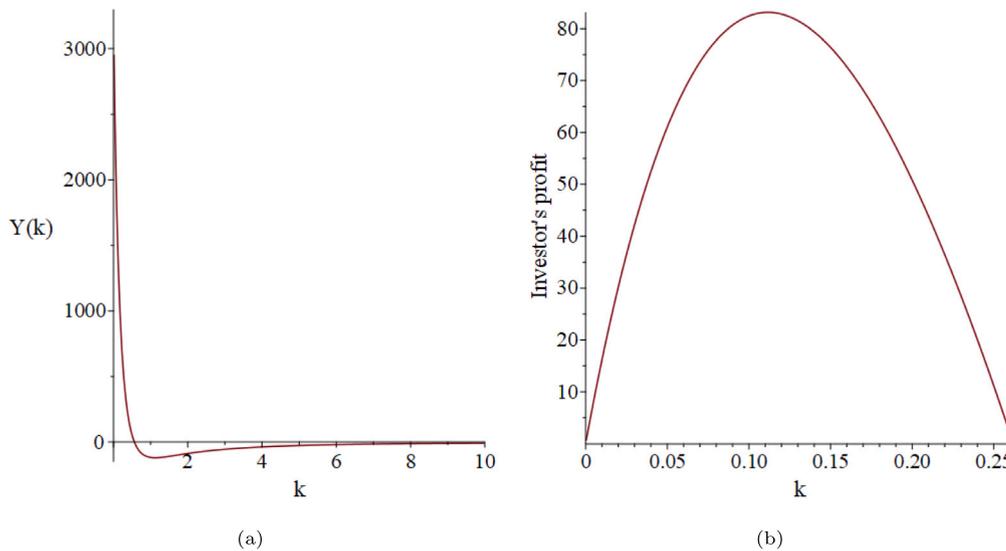


Fig. 2. (a) $Y(k)$ for $a = 100, \beta = 0.9, \gamma = 1.28, \theta = 1.1$ and $\mu = 0.835$; and (b) $\pi_I(k)$ for the same setting with $m = 1500$.

We conclude from Proposition 1 that the investor's expected profit first increases in the interest rate and then decreases, implying that there is a unique value for which it is maximized. To illustrate the unimodality of $\pi_I(k)$, Fig. 2(b) plots the investor's profit as a function of the interest rate it charges for the loan, when the developer's capital is $m = 1500$ (which is within the range $(0, m^{SF})$, where $m^{SF} = 3226.3$) and the setting used is the same as in Fig. 2(a).

Several properties of the equilibrium interest rate are revealed in the next theorem, which arise from the properties of $Y(k)$ presented in Lemma 1. We use superscript IF to denote equilibrium outcomes under investor financing.

Theorem 1. The equilibrium interest rate $k^{IF} \equiv k(m)$ is the unique solution of $Y(k) = m$ (i.e., $k^{IF} = Y^{-1}(m)$). The equilibrium greenness level and selling price are calculated, respectively, by $g^{IF} = g(k^{IF})$ and $p^{IF} = p(g^{IF})$. The corresponding profit of the investor at equilibrium is $\pi_I^{IF} \equiv \pi_I(k^{IF})$ and the expected profit of the developer at equilibrium is $E[\pi_D^{IF}] \equiv E[\pi_D(g^{IF})]$. In addition, the expected profit of the system at equilibrium is calculated by $E[\pi_S^{IF}] \equiv \pi_I^{IF} + E[\pi_D(g^{IF})]$.

5.2. The effect of the developer's capital on equilibrium

Until now, we have assumed that m is fixed. In what follows, we relax this assumption and investigate the effect of m on the investor's profit and on the developer's and the system's expected profits. We study this issue numerically in the next section, and the following propositions provide essential properties (analytically proven) to support the numerical analysis. First, we investigate the effect of m on the equilibrium investment in greenness, and we determine the maximal amount of capital that the investor agrees to lend the developer by solving the condition $m < C(g(k(m)))$; i.e., the capital that the developer commits to investing in carbon emission reduction from his own resources must be lower than the cost of achieving the desired greenness level.

Lemma 2.

- (i) $C(g(k(m)))$ is an increasing concave function of m .
- (ii) $m < C(g(k(m)))$ if and only if $m < m^{SF}$.

By Lemma 2, at equilibrium: (i) the investment in greenness is an increasing function of m , but it increases at a decelerating rate; and (ii) the highest greenness level is bounded by the level obtained in a supply chain with a self-financing developer.

Proposition 2. At equilibrium:

- (i) k^{IF} is a convex decreasing function of the developer's capital m , where $k^{IF} = k_0$ for $m = 0$ and $k^{IF} = 0$ for $m = m^{SF}$.
- (ii) p^{IF}, g^{IF} and $D(p^{IF}, g^{IF})$ are concave increasing functions of the developer's capital m .

According to Proposition 2, the greater the amount the developer commits to investing in greenness, the lower the interest rate charged by the investor for the loan.

Theorem 2. Properties of the developer's and platform's expected profits, and properties of the investor's profit, with respect to the developer's capital $m \in [0, m^{SF}]$:

- (i) $E[\pi_D(g(k(m)))]$ is a concave increasing function of m .
- (ii) $E[\pi_P(g(k(m)))]$ is a concave increasing function of m .
- (iii) $\pi_I(k(m))$ is a convex decreasing function of m .

Theorem 2 shows that it is in the developer's best interests to utilize his entire capital to invest in greenness (up to the amount a self-funding developer would invest), whereas the investor prefers that the developer invests as little capital as possible in greenness. We now investigate the effect of the developer's capital on the profit of the system, which includes the developer and the investor. Accordingly, the system's profit at equilibrium is captured by $\pi_S(m) \equiv \pi_D(g(k(m))) + \pi_I(k(m)) = \eta p(g(k(m)))D(p(g(k(m))), g(k(m))) - C(m)$.

Corollary 1. $E[\pi_S(m)]$ is a concave increasing function of m .

By Theorem 2, as his capital increases, the developer enjoys a higher expected profit, whereas the investor gains a lower profit. However, by Corollary 1, in total, the positive effect on the developer's profit outweighs the negative effect on the investor's profit. Fig. 3 depicts the developer's expected profit (Panel (a)), the investor's profit (Panel (b)) and the system's expected profit (Panel (c)) as a function of the developer's capital m . The setting used is the same as in Fig. 2. The results are consistent with Theorem 2 and Corollary 1.

6. Investor financing: bankruptcy risk

In this section, we assume that the revenue of the developer from selling the product may be lower than the amount he is obliged to return to the investor (i.e., $(\frac{\gamma g^2}{2} - m)(1 + k)$), so the developer faces a bankruptcy risk. In such a case, the developer transfers his entire sales

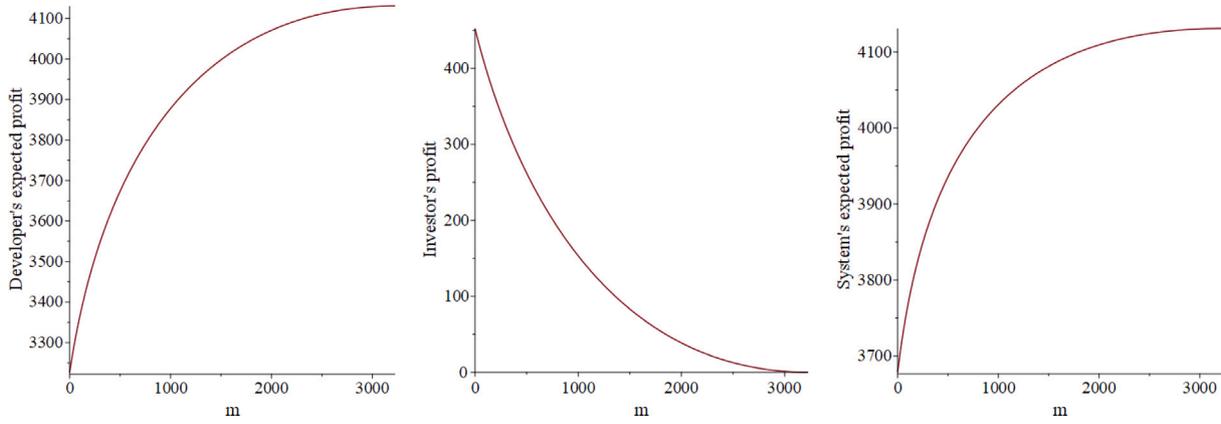


Fig. 3. (a) Developer's expected profit; (b) Investor's profit; and (c) System's expected profit, depicted as a function of m , for $a = 100$, $\beta = 0.9$, $\gamma = 1.28$, $\theta = 1.1$ and $\mu = 0.835$.

revenue to the investor at the end of the selling season, not necessarily covering the loan, and thus potentially causing losses to the investor. According to this model, the profit functions of the developer and the investor (both random variables) are, respectively:

$$\begin{aligned} \pi_D(p, g) &= \max\{\eta p D(p, g) - (\frac{\gamma g^2}{2} - m)(1 + k), 0\} - m \\ \pi_I(k) &= \min\{(\frac{\gamma g^2}{2} - m)k, \eta p D(p, g) - (\frac{\gamma g^2}{2} - m)\}. \end{aligned} \tag{19}$$

We distinguish between two cases. In the case where the developer's revenue exceeds the loan repayment (i.e., $\eta p D(p, g) > (\frac{\gamma g^2}{2} - m)(1 + k)$), the parties' profits are as given in (10). In the case where the developer cannot cover the loan repayment, his profit is $-m$ (loss of capital invested in greenness) and the investor's profit is $\eta p D(p, g) - (\frac{\gamma g^2}{2} - m)$ (i.e., the developer's revenue minus the funding amount). In contrast to the model in the previous section, here, the investor is exposed to the risk that the commission rate charged by the platform will be so high that the developer will not be able to return the loan. Our goal is to investigate how this exposure to financial risk affects the decisions of the parties, as well as their expected profits.

6.1. Solution

Final stage of decision making: setting the selling price. Similar to the no-bankruptcy case, at this stage of the stochastic-programming problem, the commission rate $1 - \eta$ has been realized. This realization occurs after the decisions about the greenness level of the product, g , and the interest rate, k , have been made. We assume that even if the developer is not profitable, he sets a price that maximizes the revenue from selling the product in order to return the highest possible amount to the investor. Therefore, the optimal selling price for a given g , denoted by $p(g)$, is obtained by maximizing the revenue $pD(p, g)$ with respect to p , resulting in (12).

Second stage of decision making: setting the product's greenness level. At this stage, the decision about the investor's interest rate k has already been made, whereas the commission rate $1 - \eta$ has not yet been determined. Thus, the developer's profit, after substituting $p = p(g)$ and using some algebraic manipulations, is a random variable expressed by:

$$\pi_D(g) = \max\left\{\frac{(a + \theta g)^2 \eta - 2\beta(\gamma g^2(1 + k) - 2mk)}{4\beta}, -m\right\}. \tag{20}$$

Let $\eta_1 \equiv \frac{2\beta(\gamma g^2(1+k)-2mk)-4\beta m}{(a+\theta g)^2}$ be the threshold value of η for which the developer faces bankruptcy (i.e., loses his entire investment in greenness). Therefore, the optimal greenness level for a given k , denoted by

$g(k)$, is set by solving $\max_g E[\pi_D(g)]$ where:

$$\begin{aligned} E[\pi_D(g)] &= \int_0^{\eta_1} -mf(\eta)d\eta + \int_{\eta_1}^1 \frac{(a + \theta g)^2 \eta - 2\beta(\gamma g^2(1 + k) - 2mk)}{4\beta} f(\eta)d\eta \\ &= -mF(\eta_1) + \frac{(a + \theta g)^2}{4\beta} \int_{\eta_1}^1 \eta f(\eta)d\eta - 0.5(\gamma g^2(1 + k) - 2mk)(1 - F(\eta_1)) \\ &= \frac{(a + \theta g)^2}{4\beta} \int_{\eta_1}^1 \eta f(\eta)d\eta - (0.5\gamma g^2 - m)(1 + k)(1 - F(\eta_1)) - m \\ &= \frac{(a + \theta g)^2}{4\beta} \int_{\frac{2\beta(\gamma g^2(1+k)-2mk)-4\beta m}{(a+\theta g)^2}}^1 \eta f(\eta)d\eta \\ &\quad - (0.5\gamma g^2 - m)(1 + k) \left(1 - F\left(\frac{2\beta(\gamma g^2(1 + k) - 2mk) - 4\beta m}{(a + \theta g)^2}\right)\right) - m. \end{aligned} \tag{21}$$

First stage of decision making: determining the investor's interest rate. At this stage, the investor maximizes its profit by controlling k given the developer's best response $(p(g(k)), g(k))$. Therefore, the equilibrium interest rate, denoted by k^B (where B stands for bankruptcy), is obtained by solving $\max E[\pi_I(k)]$ where:

$$\begin{aligned} E[\pi_I(k)] &= \int_0^{\eta_1} \left(\eta p(g(k))D(p(g(k)), g(k)) - (\frac{\gamma g^2(k)}{2} - m)\right) f(\eta)d\eta \\ &\quad + \int_{\eta_1}^1 (\frac{\gamma g^2(k)}{2} - m)k f(\eta)d\eta \\ &= \frac{(a + \theta g(k))^2}{4\beta} \int_0^{\frac{2\beta(\gamma(g(k))^2(1+k)-2mk)-4\beta m}{(a+\theta g(k))^2}} \eta f(\eta)d\eta \\ &\quad + (\frac{\gamma g^2(k)}{2} - m) \\ &\quad \times \left(k - (1 + k)F\left(\frac{2\beta(\gamma(g(k))^2(1 + k) - 2mk) - 4\beta m}{(a + \theta g(k))^2}\right)\right). \end{aligned} \tag{22}$$

6.2. Numerical illustration

In what follows, we use numerical analysis to investigate the case in which the developer might face bankruptcy. In particular, we explore the effect of the developer's capital on the equilibrium interest rate, greenness level, selling price, and probability of bankruptcy ($P(\eta < \eta_1)$), as well as on the parties' expected profits. We use the same normalized values as in Fig. 2 ($\beta = 0.9$, $\gamma = 1.28$, $\theta = 1.1$, and $a = 100$), which satisfy the conditions for an interior-point solution (Assumption 1).

To evaluate the effect of bankruptcy on financing, we compare the results with those obtained for the case of no bankruptcy. Fig. 4 illustrates the investor's interest rate (Panel (a)), the product's greenness level (Panel (b)), the product's selling price (Panel (c)), the demand

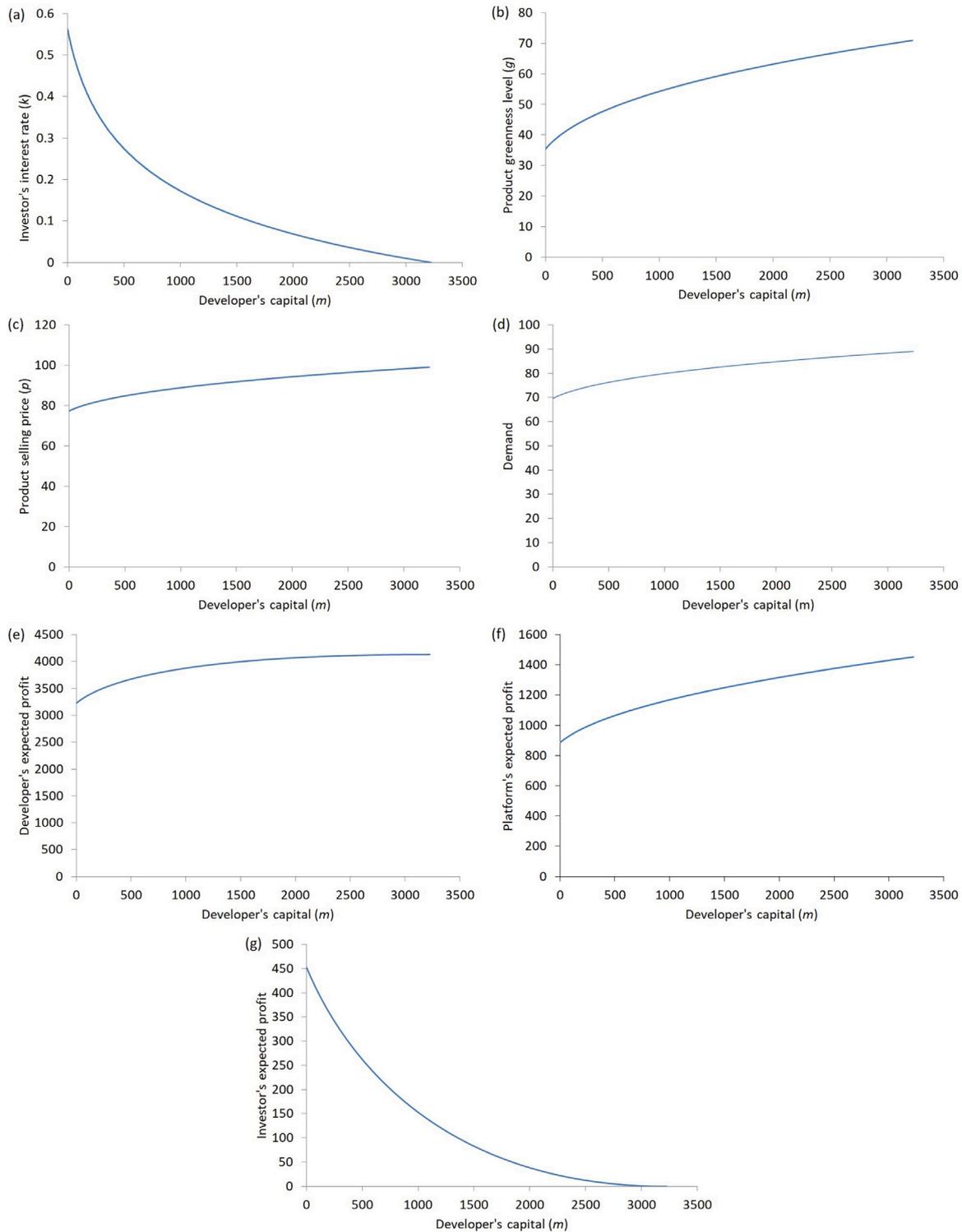


Fig. 4. (a) Investor's interest rate; (b) Product's greenness level; (c) Product's selling price; (d) Demand; (e) Developer's expected profit; (f) Platform's expected profit; and (g) Investor's expected profit, for $a = 100$, $\beta = 0.9$, $\gamma = 1.28$, $\theta = 1.1$ and $\eta \sim U(0.7, 0.97)$.

(Panel (d)), the developer's expected profit (Panel (e)), the platform's expected profit (Panel (f)), and the investor's expected profit (Panel (g)), as a function of the developer's capital m , for both settings (with and without bankruptcy risk). As can be seen, the interest rate charged by the investor, as well as the investor's expected profit, decrease with the developer's capital, whereas the product's greenness level, as well as the developer's expected profit, increase with m . In particular, we observe that an additional unit of capital has a diminishing positive

effect on the developer's and platform's expected profits due to the fact that the decrease in the interest rate charged by the investor decreases with the developer's capital. It is worth noting that for all the examples considered in the illustration, the results are identical for both models. To further investigate this result, we conduct a factorial experimental design with regard to the model parameters. In particular, and without loss of generality, we normalize the consumers' sensitivity coefficients to price and greenness level to one (i.e., $\beta = \theta = 1$), and in line with

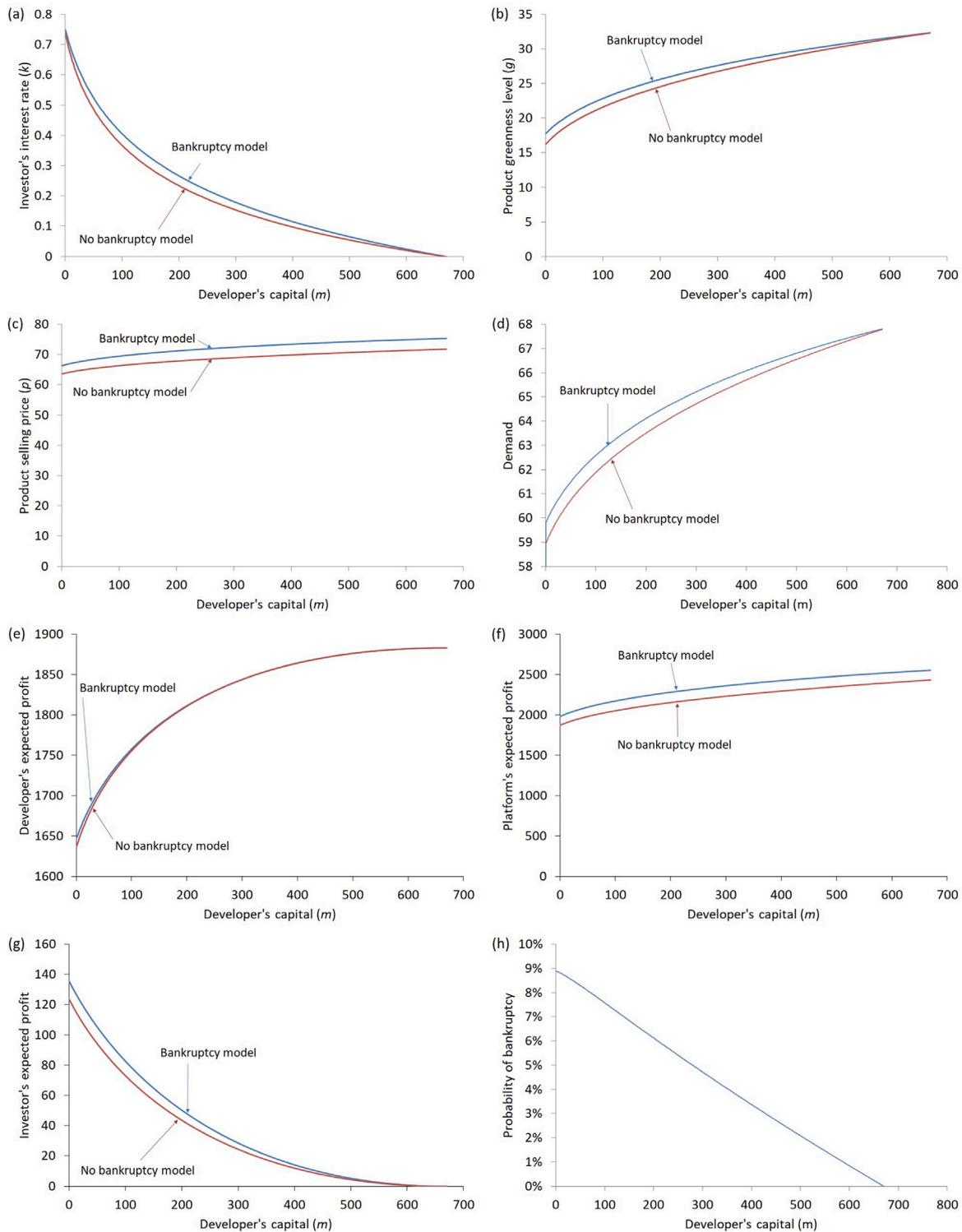


Fig. 5. (a) Investor's interest rate; (b) Product's greenness level; (c) Product's selling price; (d) Demand; (e) Developer's expected profit; (f) Platform's expected profit; (g) Investor's expected profit; and (h) Probability of bankruptcy, for $a = 100$, $\beta = 0.9$, $\gamma = 1.28$, $\theta = 1.1$ and $\eta \sim U(0, 1)$.

Assumption 1, we consider three possible levels of a and γ : low ($a = 100$, $\gamma = 1$), medium ($a = 500$, $\gamma = 5$) and high ($a = 1000$, $\gamma = 10$), resulting in nine combinations. All trends are similar to those obtained under the original setting, and most importantly, the results under both cases – no bankruptcy and bankruptcy – coincide. From this factorial design, we conclude that under the common range of commission rates used in platform selling, the investor does not need to take into account the

risk of not being fully refunded by the developer due to changes in the commission rate.

In order to investigate whether the results under the two models (with and without bankruptcy) always coincide, we now analyze an extreme case in which the developer's share of the revenue, η , distributes uniformly over $(0, 1)$. Such an extreme case might occur when the platform is a monopoly or when η takes into account not only the share

of the revenue that the developer receives, but also a potential reduction in demand¹¹ due to the arrival of competitors selling substitutable products (which might render the developer's product useless such that demand ceases). Fig. 5 uses the same setting as Fig. 4 with the exception of the developer's revenue share, which now follows $\eta \sim U(0, 1)$. As can be seen, for all values of the developer's capital m , the investor charges a higher interest rate under the bankruptcy model than under the no-bankruptcy model, and the gap between the two models first increases and then decreases. Despite the higher interest rate, however, the bankruptcy model dominates the no-bankruptcy model in terms of the product's greenness level as well as the developer's, platform's and investor's expected profits. This counter-intuitive result can be explained by the fact that under the bankruptcy model, the risk shifts from the developer (the only party that invests in greenness) to the investor, where the latter is not involved in the operational decisions.

7. Discussion and conclusions

In this paper, we consider a capital-constrained developer who, in addition to investing capital from his own resources, can borrow money from an external investor in order to produce a greener product, where the loan should be returned to the investor with interest. We consider two model variants: (a) the developer does not face a bankruptcy risk, meaning that it is guaranteed that the investor will be fully refunded; and (b) the developer faces a bankruptcy risk, and thus the investor might only recover part of the loan (if any). As is common in platform selling, in this work, we assume that the interaction follows the agency model, where the proportional commission from each transaction is determined by the platform. The sequence of events is as follows. The investor determines the loan's interest rate, after which the developer sets the greenness level of the product. Next, the platform reveals the commission rate, and finally, the developer sets the selling price.

Under the first model variant (no bankruptcy risk), we prove that at equilibrium, the price increases in the product's greenness level, which is uniquely determined by the investor's interest rate. We also find that the latter decreases with the amount of capital that the developer commits to investing in greenness from his own resources. With regard to the effect of the developer's capital on the parties' and system's profits, we show that the greater the amount of capital the developer commits to investing in greenness, the lower the investor's profit, and the higher the developer's and system's expected profits. Therefore, we conclude that it is in the best interests of the developer to invest his entire capital in greenness, up to the amount that a non capital-constrained developer would invest. An important managerial insight for platforms with strong green awareness is that interacting with developers who have more capital would lead to a larger investment in greenness.

Under the second model variant, the price is dictated by greenness level in exactly the same manner as under the first model variant; however, the problem of analytically determining the equilibrium greenness level and interest rate is intractable due to the complexity of the objective functions. Therefore, we use numerical analysis to explore the effects of the developer's capital on the equilibrium, the probability of bankruptcy and the parties' expected profits. A managerial insight that emerges from this numerical study is that under a commission rate that varies between 3% and 30% (as is common in practice), the two model variants overlap. However, when the randomness of η is spread over its entire potential domain, i.e., 0%–100% (for example, due to the need to take into account demand uncertainty), it is possible that the developer will face bankruptcy, and thus, the second model variant needs to be applied in order to achieve optimality.

¹¹ Avinadav and Levy [52] investigate a case in which demand uncertainty has a multiplicative effect on the demand function.

Comparing the two model variants, the numerical analysis reveals that, from the developer's and investor's perspectives, shifting the risk to the investor (by adopting the second model variant) is preferable in terms of the expected profits of both parties. In addition, consumers enjoy a higher greenness level (albeit at a higher price) when the developer's bankruptcy risk is taken into account; this results in higher demand for the product, indicating greater social welfare. A further important insight that emerges from the numerical study is that when the revenue is subject to a high level of uncertainty (either due to large uncertainty in the commission rate or due to demand interruption, where the latter is beyond the scope of this work), all parties are better off when the investor assumes the risk of suffering losses, because the latter is not directly involved in setting the greenness level of the product. Although the investor charges a higher interest rate for the loan under this setting, the developer has the incentive to increase the greenness towards the optimal level (from the system's perspective) because he is not concerned about bankruptcy.

In this work, we provide analytical results for the model variant in which there is no bankruptcy risk, while the other model variant is studied numerically due to its mathematical complexity. As a direction for future work, we suggest expanding the numerical analysis, using more powerful computing resources, to fully cover the domain of the model parameters. Furthermore, we suggest validating our theoretical results using an empirical study that evaluates the performance of the parties as a function of the identity of the party that faces the bankruptcy risk.

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CRedit authorship contribution statement

Tal Avinadav: Conception and design of study, Analysis and/or interpretation of data, Writing – original draft, Writing – review & editing. **Priel Levy:** Conception and design of study, Analysis and/or interpretation of data, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no conflicts of interests.

Data availability

No data was used for the research described in the article.

Declaration of Generative AI and AI-assisted technologies in the writing process

None

Appendix. Proofs

Proof of Lemma 1. By (18), $Y(k)$ has a horizontal asymptote $\lim_{k \rightarrow \infty} Y(k) = 0$ and a vertical asymptote [at] $k = k_{va}$, where $\lim_{k \rightarrow (k_{va})^+} Y(k) = +\infty$ (since the numerator of $Y(k)$ at $k = k_{va}$ equals $2\gamma a^2 \theta^2 (2\beta\gamma - \theta^2 \mu) \mu^2$, which is positive under Assumption 1). By differentiating $Y(k)$ with respect to k , we obtain

$$Y'(k) = \frac{4\beta\gamma a^2 \theta^2 \mu^2 (\beta\gamma(k-2) + \theta^2 \mu)}{(2\beta\gamma(1+k) - \theta^2 \mu)^4}.$$

Since the denominator of $Y'(k)$ is always positive and its numerator is a linear increasing function of k , $Y(k)$ has a unique minimum at

$k_{min} = 2 - \frac{\theta^2 \mu}{2\beta\gamma} > 0$ (under Assumption 1) and $Y(k_{min}) = -\frac{\gamma a^2 \theta^2 \mu^2}{54(2\beta\gamma - \theta^2 \mu)^2} < 0$. Since $k_{va} < 0 < k_{min}$, $Y(k) = m$ has a solution for any $m \geq 0$. The second derivative of $Y(k)$ is

$$Y''(k) = \frac{12\beta^2 \gamma^3 a^2 \theta^2 \mu^2 (2\beta\gamma(3-k) - 3\theta^2 \mu)}{(2\beta\gamma(1+k) - \theta^2 \mu)^5}.$$

By Assumption 1, the denominator of $Y''(k)$ is always positive, so the sign of $Y''(k)$ is dictated by its numerator, which is a scalar multiplied by a linear decreasing function of k . Since this linear decreasing function equals $2\beta\gamma - \theta^2 \mu$ for $k = k_{min}$, by Assumption 1, it is positive for all $k \leq k_{min}$. This implies that $Y''(k) > 0$ whenever $Y(k) > 0$, because $Y(k_{min}) < 0$. \square

Proof of Proposition 1. By Lemma 1, for any $m \geq 0$, the first-order condition $\pi'_I(k) = 0$ (which can be written as $Y(k) = m$) has a unique solution, denoted by k^* , which decreases in m because $Y(k)$ decreases in k whenever $Y(k) > 0$. Based on the latter property, for a given value of m , $\pi'_I(k) > 0$ for $k_{va} < k < k^*$ and $\pi'_I(k) < 0$ for $k > k^*$, implying that $\pi_I(k)$ first increases and then decreases. Therefore, $\pi_I(k)$ is a unimodal quasi-concave function. \square

Proof of Theorem 1. Straightforward from the properties of $Y(k)$ indicated in Lemma 1. \square

Proof of Lemma 2. Let $C(m) = C(g(k(m)))$ be the investment in greenness at equilibrium for a given m . In order to prove the lemma, we show that the following properties hold: (i) the first and second derivatives of $C(m)$ satisfy $C'(m) > 0$ and $C''(m) < 0$ respectively; and (ii) $C(m) > m$ iff $0 \leq m < m^{SF}$, where $C(m^{SF}) = m^{SF}$.

(i) By its definition, $k(m)$ is obtained by solving $Y(k) = m$, where $Y(k)$ is given in (18). Therefore, $k'(m) = \frac{1}{Y'(k(m))}$ and $\frac{d}{dm} Y'(k(m)) = Y''(k(m))k'(m) = \frac{Y''(k(m))}{Y'(k(m))}$. By substituting $k(m)$ into (15), we obtain $g(k(m)) = \frac{a\theta\mu}{2\beta\gamma(1+k(m)) - \theta^2 \mu}$. Thus,

$$C'(m) = \gamma g(k(m)) \frac{d}{dm} g(k(m)) = \frac{-2\beta\gamma^2 a^2 \theta^2 \mu^2}{(2\beta\gamma(1+k(m)) - \theta^2 \mu)^3 Y'(k(m))} \tag{A.1}$$

and

$$C''(m) = 2\beta\gamma^2 a^2 \theta^2 \mu^2 \frac{6\beta\gamma Y'(k(m)) + (2\beta\gamma(1+k(m)) - \theta^2 \mu) Y''(k(m))}{(2\beta\gamma(1+k(m)) - \theta^2 \mu)^4 (Y'(k(m)))^3}. \tag{A.2}$$

By substituting the expressions for $Y'(k(m))$ and $Y''(k(m))$ given in the proof of Lemma 1 into the numerator of $C''(m)$ and using algebraic manipulations, we obtain

$$C''(m) = \frac{24\beta^3 \gamma^5 a^4 \theta^4 \mu^4 (2\beta\gamma - \theta^2 \mu)}{(2\beta\gamma(1+k(m)) - \theta^2 \mu)^8 (Y'(k(m)))^3}. \tag{A.3}$$

By Lemma 1, $Y'(k(m)) < 0$ for any $m \geq 0$ (because $Y(k)$ is a convex decreasing function over the interval (k_{va}, k_{min}) with $Y(k_{min}) < 0$), implying that $C'(m) > 0$ (since, by Theorem 1(iii), $k^{IF} > k_{va}$) and also implying that $C''(m) < 0$ (by Assumption 1).

(ii) Equilibrium k is obtained by solving $Y(k) = m$. Suppose $m = 0$; then, $C(0) = C(g(k(0))) = C(g(k_0)) > 0$ since $g(k_0) = \frac{a\theta\mu}{2(2\beta\gamma - \theta^2 \mu)} > 0$ (by Assumption 1). Now suppose that $m = m^{SF}$ (where m^{SF} is given in (7)); then, by solving the cubic equation in k , $Y(k) = m^{SF}$, where $Y(k)$ is given in (18), we obtain the unique solution $k(m^{SF}) = 0$ (because after dividing the cubic equation by k , we obtain a quadratic equation in k with discriminant $\Delta = -1.75\beta^4 \gamma^6 a^4 \theta^4 \mu^4 (2\beta\gamma - \theta^2 \mu)^2 < 0$, implying that there are no additional real roots). Thus, $g(k(m^{SF})) = \frac{a\theta\mu}{2\beta\gamma - \theta^2 \mu}$, implying that $C(m^{SF}) = \frac{\gamma(g(m^{SF}))^2}{2} = \frac{\gamma a^2 \theta^2 \mu^2}{2(2\beta\gamma - \theta^2 \mu)^2} = m^{SF}$ and that $C'(m^{SF}) = \frac{1}{2} < 1$. The claim that $C(m) > m$ if and only if $0 \leq m < m^{SF}$ is proved by

the concavity of $C(m)$ combined with $C(0) > 0$, $C(m^{SF}) = m^{SF}$ and $C'(m^{SF}) < 1$. \square

Proof of Proposition 2.

- (i) By Lemma 1(i), $Y(k)$ is a convex decreasing function of k whenever $Y(k) \geq 0$. Therefore, $k(m)$, as defined in Theorem 1, can be written as $k(m) = Y^{-1}(m)$ for $m \in [0, m^{SF}]$. Since $Y'(k) < 0$ and $Y''(k) > 0$ whenever $Y(k) > 0$ (see proof of Lemma 1), then, for $m \in [0, m^{SF}]$: $\frac{d}{dm} Y^{-1}(m) = \frac{1}{Y'(Y^{-1}(m))} < 0$ and $\frac{d^2}{dm^2} Y^{-1}(m) = \frac{-Y''(Y^{-1}(m))}{(Y'(Y^{-1}(m)))^3} > 0$. The proof is completed since $k(0) = k_0$ and $k(m^{SF}) = 0$ (see the proof of Lemma 2(ii)).
- (ii) According to (15) and similar to (i), by substituting the expressions for $Y'(k)$ and $Y''(k)$, and using algebraic manipulations, $\frac{d}{dm} g^{IF} = -\frac{a(1-k_0)k'(m)}{\theta(k(m)+k_0)^2} = \frac{2\beta(k(m)+k_0)^2}{\theta\mu a(2k_0-k(m))} > 0$ and $\frac{d^2}{dm^2} g^{IF} = -\frac{a(1-k_0)\left((k(m)+k_0)k''(m) - 2(k'(m))^2\right)}{\theta(k(m)+k_0)^3} = -\frac{4\beta^2(k(m)+k_0)^5(5k_0-k(m))}{a^3(1-k_0)\theta\mu^2(2k_0-k(m))^3} < 0$. According to (12), p is linear in g and so p^{IF} is also a concave increasing function of m . By substituting p^{IF} into the demand function, we obtain $D(p^{IF}, g^{IF}) = \frac{a+\theta g^{IF}}{2}$, which is a linear function of g^{IF} ; thus, $D(p^{IF}, g^{IF})$ is also a concave increasing function of m . \square

Proof of Theorem 2.

- (i) By substituting $g(k)$ (given in (15)) into $E[\pi_D(g)]$ (given in (16)), and using the relations $k_0 = 1 - \frac{\theta^2 \mu}{2\beta\gamma}$ and $k = k(m)$, the expected profit of the developer is $E[\pi_D(g(k(m)))] = q(m) + h(m)$, where $q(m) = \frac{\mu}{4\beta} \left(\frac{a^2(1-k_0)}{k(m)+k_0} + a^2 \right)$ and $h(m) = mk(m)$. In order to complete the proof, it is sufficient to show that: (a) $q'(m) < 0$ and $h'(m) < 0$ (so that $\frac{d^2}{dm^2} E[\pi_D(g(k(m)))] < 0$); and (b) $q'(m^{SF}) + h'(m^{SF}) = 0$ (so that $\frac{d}{dm} q'(m) + h'(m) > 0$ for $m \in [0, m^{SF}]$). To do so, we use the following relations: (i) $m = Y(k(m))$ (obtained from the definition of $k(m)$); and (ii) $k'(m) = \frac{1}{Y'(k(m))} < 0$ and $k''(m) = -\frac{Y''(k(m))}{(Y'(k(m)))^2} k'(m) = -\frac{Y''(k(m))}{(Y'(k(m)))^3} > 0$ for all $m \geq 0$ (obtained by implicit differentiation of $m = Y(k(m))$ with respect to m and using the properties of $Y(k)$ presented in Lemma 1). By differentiation, $q'(m) = -\frac{\mu a^2(1-k_0)}{4\beta(k(m)+k_0)^2} k'(m)$ and $h'(m) = k(m) + mk'(m)$, and thus:

$$q'(m) + h'(m) = k(m) + k'(m) \left(m - \frac{\mu a^2(1-k_0)}{4\beta(k(m)+k_0)^2} \right). \tag{A.4}$$

By Proposition 2(i), $k(m^{SF}) = 0$; thus,

$$q'(m^{SF}) + h'(m^{SF}) = k'(m^{SF}) \left(m^{SF} - \frac{\mu a^2(1-k_0)}{4\beta k_0^2} \right). \tag{A.5}$$

By (7) and since $k_0 \equiv 1 - \frac{\theta^2 \mu}{2\beta\gamma}$, (A.5) equals zero. The concavity of $E[\pi_D(g(k(m)))]$ is proved as follows:

$$q''(m) = -\frac{\mu a^2(1-k_0)}{4\beta(k(m)+k_0)^3} \left((k(m)+k_0)k''(m) - 2(k'(m))^2 \right) = \frac{\mu a^2(1-k_0)}{4\beta(k(m)+k_0)^3 (Y'(k(m)))^2} \left(\frac{(k(m)+k_0)Y''(k(m))}{Y'(k(m))} + 2 \right),$$

so $q''(m) < 0$ for $m \geq 0$ if $\frac{(k+k_0)Y''(k)}{Y'(k)} + 2 < 0$ for $k \in (0, k_0)$. By substituting the expressions for $Y'(k)$ and $Y''(k)$, and using algebraic manipulations, the latter inequality is equivalent to the following quadratic inequality in k :

$$\left(\beta\gamma(k-2) + \theta^2 \mu \right) \left(2\beta\gamma(k-5) - 5\theta^2 \mu \right) > 0,$$

the solution of which is $k < k_1$ or $k > k_2$, where $k_1 = 2 - \frac{\theta^2 \mu}{\beta\gamma}$ and $k_2 = 5 + \frac{5\theta^2 \mu}{2\beta\gamma}$. By Assumption 1, $k_1 > 1$. Since $k(m) \leq k_0 < 1$, then $q''(m) < 0$ for $m \geq 0$.

In addition,

$$h''(m) = 2k'(m) + mk''(m) = \frac{2}{Y'(k(m))} - \frac{Y(k(m))Y''(k(m))}{(Y'(k(m)))^3}$$

$$= \frac{2(Y'(k(m)))^2 - Y(k(m))Y''(k(m))}{(Y'(k(m)))^3},$$

so $h''(m) < 0$ for $m \geq 0$ if $2(Y'(k))^2 - Y(k)Y''(k) > 0$ for $k \in (0, k_0)$. By substituting the expressions for $Y(k)$, $Y'(k)$ and $Y''(k)$, and using algebraic manipulations, the latter inequality is equivalent to the following quadratic inequality in k , which always holds:

$$4(\theta^2\mu - \beta\gamma(2 - k))^2 + 3(\theta^2\mu - 2\beta\gamma)^2 > 0.$$

Thus, $h''(m) < 0$ for $m \geq 0$.

- (ii) By Proposition 2(ii), $g(k(m))$ is a concave increasing function of m . Since $g^2(k(m)) = \frac{2C(g(k(m)))}{\gamma}$, according to Lemma 2, $g^2(k(m))$ is also a concave increasing function of m . Hence, $(a + \theta g)^2$ is also a concave increasing function of m , and thus, by (16), $E[\pi_P(g)] = \frac{(1-\mu)(a+\theta g)^2}{4\beta}$ is a concave increasing function of m .
- (iii) By (17), and after substituting $m = Y(k(m))$ as given in (18), we obtain:

$$\pi_I(k(m)) = \frac{\mu a^2(1 - k_0)k^2(m)}{2\beta(k(m) + k_0)^3}. \tag{A.6}$$

By differentiation, $\frac{d\pi_I(k(m))}{dm} = \frac{d\pi_I(k)}{dk(m)} k'(m) = \frac{\mu a^2(1-k_0)(2k_0-k(m))k(m)}{2\beta(k(m)+k_0)^4} k'(m)$. By Proposition 2(i), it is clear that $\frac{d\pi_I(k(m))}{dm} < 0$. By differentiating further,

$$\frac{d^2\pi_I(k(m))}{dm^2} = \frac{\mu a^2(1 - k_0)}{2\beta} \left(2 \frac{k^2(m) - 4k(m)k_0 + k_0^2}{(k(m) + k_0)^5} (k'(m))^2 + \frac{(2k_0 - k(m))k(m)}{(k(m) + k_0)^4} k''(m) \right).$$

By substituting $k'(m)$ and $k''(m)$ from the proof of Proposition 2(i), as well as the expression $\frac{Y''(k(m))}{Y'(m)}$ (which can be evaluated from the expressions given in the proof of Lemma 1), and using algebraic manipulations, we obtain:

$$\frac{d^2\pi_I(k(m))}{dm^2} = \frac{\mu a^2(1 - k_0)(k_0 + k(m))(2k_0 - k(m))}{2\beta(Y'(k(m)))^2(k(m) + k_0)^5}. \tag{A.7}$$

By Proposition 2(i), it is clear that $\frac{d^2\pi_I(k(m))}{dm^2} > 0$. \square

Proof of Corollary 1. By summing (16) and (17) and substituting (15), $E[\pi_S(m)] = \frac{\mu a^2(k^2(m)+2k(m)+k_0)}{4\beta(k(m)+k_0)^2}$ and $\frac{d}{dm} E[\pi_S(m)] = -\frac{\mu a^2(1-k_0)k(m)}{2\beta(k(m)+k_0)^3} k'(m)$. By Proposition 2(i), it is clear that $\frac{d}{dm} E[\pi_S(m)] > 0$.

$$\frac{d^2}{dm^2} E[\pi_S(m)] = \frac{\mu a^2(1 - k_0) \left((k_0 + k(m))k(m)k''(m) + (k_0 - 2k(m))(k'(m))^2 \right)}{2\beta(k(m) + k_0)^4}.$$

By substituting $k'(m)$ and $k''(m)$ from the proof of Proposition 2(i), as well as the expression $\frac{Y''(k(m))}{Y'(m)}$ (which can be evaluated from the expressions given in the proof of Lemma 1), and using algebraic manipulations, we obtain:

$$\frac{d^2}{dm^2} E[\pi_S(m)] = \frac{2\beta(k(m) + k_0)^4(k^2(m) - 4k(m)k_0 - 2k_0^2)}{\mu a^2(1 - k_0)(2k_0 - k(m))^3}.$$

The sign of $\frac{d^2}{dm^2} E[\pi_S(m)]$ is dictated by the sign of $k^2(m) - 4k(m)k_0 - 2k_0^2$, which is a convex parabolic function of $k(m)$. The function is negative for $(2 - \sqrt{6})k_0 < k(m) < (2 + \sqrt{6})k_0$, and thus for any $0 < k(m) < k_0$. Therefore, the claim is proved. \square

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