

Melo, Isotilia Costa et al.

Article

Measuring the performance of retailers during the COVID-19 pandemic: Embedding optimal control theory principles in a dynamic data envelopment analysis approach

Operations Research Perspectives

Provided in Cooperation with:

Elsevier

Suggested Citation: Melo, Isotilia Costa et al. (2023) : Measuring the performance of retailers during the COVID-19 pandemic: Embedding optimal control theory principles in a dynamic data envelopment analysis approach, Operations Research Perspectives, ISSN 2214-7160, Elsevier, Amsterdam, Vol. 10, pp. 1-13,
<https://doi.org/10.1016/j.orp.2023.100282>

This Version is available at:

<https://hdl.handle.net/10419/325767>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

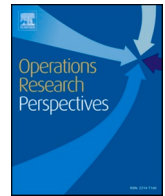
Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



<https://creativecommons.org/licenses/by-nc-nd/4.0/>



Measuring the performance of retailers during the COVID-19 pandemic: Embedding optimal control theory principles in a dynamic data envelopment analysis approach

Isotilia Costa Melo^a, Paulo Nocera Alves Junior^{b,*}, Jéssica Syrio Calfei^c,
Karoline Arguelho da Silva^d, Marcelo Seido Nagano^d, Daisy Aparecida do Nascimento
Rebelatto^d, Athanasios Rentizelas^e

^a ICN Business School, CEREFIGE - Université de Lorraine, France, Paris

^b Escuela de Ingeniería de Coquimbo (EIC), Universidad Católica del Norte (UCN), Chile, Coquimbo

^c State University of Maringá (UEM), Industrial Engineering Department, Brazil, Maringá

^d São Carlos School of Engineering (EESC), University of São Paulo (USP), Brazil, São Carlos

^e School of Mechanical Engineering, National Technical University of Athens, Greece, Zografou

ARTICLE INFO

Keywords:

Data envelopment analysis (DEA)
Optimal control theory (OCT)
Retailing
Dynamic cost efficiency
E-commerce
COVID-19 pandemic

ABSTRACT

Traditional retailers (bricks-and-mortar) have been continuously increasing online sales. However, not all retail companies were able to respond to the increasing sales with the same efficiency level as their competitors. This paper aims to propose a dynamic model – incorporating principles of Optimal Control Theory (OCT) into a Data Envelopment Analysis (DEA) model – for measuring the performance of retailing companies' cost efficiency. It also aims to contribute through the application by investigating the impact of the pandemic on companies from the most prominent developing market in Latin America, Brazil. Twenty-one companies publicly traded in the São Paulo Stock Exchanges (B3) between the third quarter of 2018 (3Q2018) and the third quarter of 2020 (3Q2020) were investigated. Also, six measures – initial inventory cost (IIC), final inventory cost (FIC), net operating income (NOI), cost of goods sold (COGS), cost of the purchased product (CPP), and plant, property, and equipment (PPE) – were considered. In this way, the findings have implications for researchers and practitioners. Practitioners can discover which competitor(s) is (are) adopting the best practices at each operational aspect (e. g., inventory cost). Additionally, the proposed method can be replicated in other markets (developing or not) and for other categories of retailing companies (e.g., small- and middle-sized). Further research directions are presented, and their implications are discussed.

1. Introduction

The global COVID-19 pandemic catalyzed retail transformation through digitalization and channel innovation [1,2]. From the perspective of traditional retailers (bricks-and-mortar), the business model changed drastically. Previously, “marketplace” was understood as a synonym for “website”, a selling channel where the retailer was responsible for buying and reselling all products. Now marketplaces (customarily linked to mobile apps) represent a novel business ecosystem with different taxonomies among retailers, suppliers, and stakeholders [3]. In this competitive market with scarce resources, measuring performance is fundamental.

Data Envelopment Analysis (DEA) was proposed as a tool for measuring the performance of “websites” early in the 2000s [4–6]. However, DEA models have kept continually evolving, while the newest DEA models were not investigated and proposed for this novel retailing context. Thus, there is still a gap regarding DEA approaches for measuring the integrated performance of physical and digital operations of retailers. Once digital activities impact demand and customer satisfaction, more than simply measuring the performance of operations in physical retailing is required in this context [7].

Remarkably, efficient inventory management is a retailing core activity, especially integrating all selling channels (the so-called “omni-channel”) [8]. Inventory is characterized by temporal interdependence

* Corresponding author.

E-mail address: paulo.alves@ucn.cl (P.N. Alves Junior).

<https://doi.org/10.1016/j.orp.2023.100282>

Received 3 November 2022; Received in revised form 27 April 2023; Accepted 21 May 2023

Available online 26 May 2023

2214-7160/© 2023 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

because the availability of a particular product in a certain period depends on activities in previous periods [9]. Also, different from other industries, retailing has the power to plan demand generation, for example, through price reductions and promotions [10]. Given these characteristics, there is a gap regarding dynamic (time-dependent) DEA models for measuring integrated performance. We aim to propose a novel dynamic DEA approach that jointly reflects the performance of physical and digital operations through a unique indicator and enables benchmarking.

This approach represents an advancement due to two reasons. First, it incorporates the inflationary, a non-neglectable factor in retailing [11–14], not considered in previous efforts [15–21]. Second, the proposed approach incorporates fundamental variables in retailing performance, such as inventory and capacity slacks [10]. Also, it embeds principles of Optimal Control Theory (OCT) for considering inventory a temporal-dependent (intermediate) measure [15–21]. Once it is hard to find competitors' data in a real-life context, this approach focuses on available accounting data from public companies. It calculates customized performance targets for each retailer and variable (i.e., for physical and digital operations).

Our research is also relevant due to its application. The growth of e-commerce in developing and emerging markets is notorious. Apart from the US and China (representing more than 60% of the global market jointly), Brazil is the largest e-commerce market on the American continent and has the highest growth [22]. Also, marketplace strategies were consolidated among retailers in 2018 [22]. The analyzed period encompasses the fourth quarter of 2018 (4Q2018) up to the third period of 2020 (3Q2020), i.e., the result of this consolidation. Also, it encompasses the beginning of the pandemic period, and its mid- and long-term impacts are still under investigation [1,2]. The findings show which retailers were: (i) quarterly benchmarks, (ii) consistent benchmarks (during several consecutive periods), and (iii) outliers during the pandemic.

For example, our results pointed out that Lojas Marisa (AMAR3) was a benchmark for Arezzo (ARZZ3) in the second and third quarters of 2020 (during the pandemic). AMAR3 operated as the controller of a unique reselling marketplace. In other words, it purchased products from suppliers and resold them to customers, controlling prices and offers. On the other hand, ARZZ3 operated as the controller of multiple reselling marketplaces (one for each brand) and acted as a seller in multiple third-party marketplaces. The results suggest that ARZZ3 could consider changing its strategy to improve performance, i.e., reducing the number of digital selling channels. The proposed approach can be replicated with different samples, periods, and markets.

Finally, this paper is structured as follows: Section 2 exposes fundamental concepts about DEA; Section 3 details the proposed approach; Section 4 presents and discusses the results; and the paper is concluded in Section 5.

2. Literature review

2.1. Measuring retailing performance

The DEA was proposed for measuring retail performance in 1995 [23,24], i.e., much before the advent of e-commerce. In 2002, Stochastic Frontier Analysis (SFA) was also proposed [25]. The differences between both approaches have been widely explored in the literature [26–29]. Although no approach fits all situations [26], DEA enables easy dealing with multiple metric units and multiple outputs [27,28].

In 2003, a pioneering paper proposed DEA for jointly measuring the website, operational, and financial performance [4]. The authors concluded that a single input or output measure could not measure e-commerce efficiency. Unlike other methods, DEA is suitable because it does not require assigned numeric weights or modeling preferences. However, at the time, the marketplace business model was not relevant enough to be considered, and the authors used the most basic (static)

DEA models, CCR and BCC. In the same decade, DEA was proposed to measure customer satisfaction with retailing websites [30] and the impact of technology on retailing efficiency [31].

The DEA approaches kept been widely used for measuring physical and operational aspects of retailing performance until nowadays, for example, integration between fuzzy Delphi and DEA for measuring the performance of retail electronic chains [32], basic DEA models for measuring the performance of small supermarkets [33], and a basic DEA model for measuring the performance of apparel retailers [34]. Similarly, a paper used lean theory to choose the variables to measure a supermarket's performance with a DEA model [35]. It is also relevant to mention that, aligned with the sustainable development goals, some researchers incorporated sustainable measures in their DEA models, such as [36,37].

On one side, researchers are exclusively applying DEA to measure physical retailing performance. On the other side, researchers are also applying DEA to measure virtual and digital aspects of retailing solely. For example, a paper used basic DEA models integrated with bootstrap regressions to investigate the retail service efficiency in 25 European countries between 2006 and 2015 [38]. More recently, the CCR model combined with a machine learning technique has been proposed for measuring the social media marketing performance of a furniture retailer [39]. It is worth noting that none of the above-mentioned papers considered integrating digital and physical aspects, considered inventory, or measured efficiency dynamically.

Nowadays, retailers search for a seamless customer experience [8,40,41]. This means an ideal integration among all (physical and online) channels, which may be named multi-, cross-, and omnichannel [40]. It implies a synergy among channels, and the management of this synergy affects performance differently than the weighted sum of each channel's performance [41]. For example, the profitability of the products' return policies depends on the nature of the integration between the marketplace and other selling channels [42]. Thus, retailing performance cannot be measured by channel separately.

Two recent papers [43][44] returned to the concept initiated by [4] of integrating digital and physical aspects of retailing performance through DEA. Both are based on the concept of digitalization [45,46]. Also, in both cases, it is only possible to execute the benchmarking having the collaboration of all participants (i.e., competitors). [43] wanted to measure the negative impacts of digitalization on retail logistics. For this, the authors proposed a network DEA model because it enables deconstructing production processes while measuring efficiency. [44] developed the dimensions and variables that could represent digitalization. The authors considered five dimensions (governance and leadership, organizational culture, resources and capabilities, innovation, and the use of technologies) and applied the basic static DEA models, BCC and CCR.

We propose a dynamic DEA model. The dynamicity is based on a temporal-dependent critical variable for the retailing activity (i.e., inventory under inflationary conditions). The model simultaneously considers variables that represent digital and physical aspects of performance. And once it uses publicly available data, it facilitates benchmarking among competitors. Therefore, the current paper represents an advancement because it focuses on the retailers' perspective.

2.2. OCT for inventory control systems

The OCT consists of models based on control variables and state variables whose dynamics depend on the control. The discrete-time OCT model proposed by Hernández-Lerma et al. (2023) [47] to deal with production-inventory systems is expressed in (1).

According to Hernández-Lerma et al. (2023) [47], an Optimal Control Problem (OCP) arises from a controllable dynamic system, and it can be a discrete-time system. For example, one of the most common objective functions of an OCP is a total cost (1):

$$J(\pi, z_0) = \sum_{t=0}^{T-1} C_t(z_t, x_t) + C_T(z_T) \quad (1)$$

where $\pi = \{x_0, \dots, x_{T-1}\}$ denotes the control strategy used, z_0 is the initial state of the system, z_T is the terminal state, t is the period of time, T is the planning horizon, z_t is the state variable, x_t is the control variable, $C_t(z_t, x_t)$ is the stage cost (cost at time t), and $C_T(z_T)$ is the terminal cost function.

And it is possible to have a discrete-time dynamical system of the form (constraint in (2)):

$$z_{t+1} = F_t(z_t, x_t, y_t), \quad (2)$$

where z_t is the state variable, x_t is the control variable, and y_t is the disturbance.

With some given initial state (constraint in (3)):

$$z_0 = z \quad (3)$$

where z_0 is the state variable at $t = 0$ (initial state), and z is the known value of the initial state.

One of the simplest OCPs in Bertsekas (1995) [48] is a problem of ordering quantity at each period t to meet the demand, and one of the simplest OCPs in Hernández-Lerma et al. (2023) [47] is a production-inventory system (it is a dynamical system and, at the same time, a discrete-time control system). Hernández-Lerma et al. (2023) [47] state that, usually, the control variable is the amount to be produced, but the model can also be applied when the control variable is the amount to be ordered. In this problem, we have the constraint in (4):

$$z_t = z_{t-1} + x_t - y_t, \quad (t = 1, 2, \dots, T) \quad (4)$$

where z_t is the inventory level (state variable), x_t is the amount to be ordered or produced (control variable), and y_t is the demand (disturbance).

For minimizing the total costs (1), considering linear total costs of the state and control variables, subject to the initial conditions (3) and dynamics of the system (4), we have the following model in (5):

$$\text{Minimize } J = \sum_{t=1}^T [c_{z_{t-1}} \cdot z_{t-1} + c_{x_t} \cdot x_t] + c_{z_T} \cdot z_T$$

subject to

$$z_t = z_{t-1} + x_t - y_t \quad (t = 1, 2, \dots, T),$$

$$z_0 = z,$$

$$z_t \geq 0 \quad (\forall t), \quad x_t \geq 0 \quad (\forall t) \quad (5)$$

where J is the objective function, T is the number of periods of the analyzed time period (planning horizon), c_{z_t} is the unitary cost of storage at period t , c_{x_t} is the unitary cost of production or order at period t , z_t is inventory at period t , z_0 is the inventory at period $t = 0$, z is the initial inventory of the first period (equal to the final inventory of the period $t = 0$), z_T is the inventory in the last period $t = T$ of the planning horizon.

This is the simplest model and does not contemplate other constraints, besides existing other types of models, for example, Sethi and Thompson [49] develops a continuous-time OCT models with constraints forcing the minimum inventory level to be higher than a value x_{\min} , or the amounts to be produced or ordered to be in an interval $[x_{\min}, x_{\max}]$, among other situations.

The constraint $z_t = z_{t-1} + x_t - y_t$ in (4) describes the relationship among variables. It is related to the variation of inventory from period $t-1$ to period t [50,51]. And it can be incorporated into DEA models.

2.3. State-of-Art DEA cost models

Efficiency can be divided into economic, technical, and allocative. Technical efficiency reflects obtaining more outputs by using fewer inputs. In other words, it results in the optimized efficiency (best weight sets) of each Decision-Making Unit (DMU), i.e., an analyzed unit in a DEA sample. The allocative efficiency measures the efficiency of allocating an optimal proportion of inputs to meet the outputs, given their costs [52].

On the other hand, "cost efficiency" is calculated with a focus on minimizing costs. The definitions and the use of the terms "cost" and "allocative" efficiency changed over time. This may lead to confusion when comparing recent with older papers. According to Cooper, Seiford and Tone [53], "allocative efficiency" is defined by Farrell [52] as income, price, or cost efficiency. Also, "cost efficiency" can also be called "economic efficiency", whereas the term "allocative efficiency" is used to describe the relationship between technical and economic efficiencies. Here, as we exclusively considered costs in the model, we use the term "cost efficiency", meaning an economic efficiency solely based on costs (i.e., not on incomes or prices).

The relationship between these three types of efficiency is given by (6):

$$\theta_{ae} = \frac{\theta_{ce}}{\theta_{te}}, \quad (6)$$

where θ_{ae} is the allocative efficiency, θ_{ce} is the cost efficiency, and θ_{te} is the technical efficiency.

When the ratio between input costs or output prices is known, cost, revenue, or profit efficiency can be calculated. The cost efficiency is the result of a model where the objective function aims at cost minimization, the revenue efficiency at revenue maximization, and the profit efficiency at profit maximization (i.e., simultaneous revenue maximization and cost minimization) [53]. The simplest cost minimization model is based on the CCR model in the form of an envelope oriented to the input [54]. It is formulated from the profit efficiency model of Färe et al. [55].

Alves Junior [18] developed cost-efficiency DEA models integrating principles of the OCT. The dynamic models considered inventory as an intermediate variable, linking the periods. The most common feature of capturing intertemporal behavior in dynamic models is using outputs from one period as inputs from the next. Due to its productive characteristics, inventory is one of the five most used variables to represent this temporal interdependence [9]. At the same time, it is a particularly critical variable for retailing [10]. Also, Goncalves et al. [56] investigated the literature on safety inventory, and their results indicate a gap in studies applying DEA and optimization in inventory theory.

Total cost models are customarily applied to situations where it is not possible to know the unit cost of inventory, e.g., when working with public accounting data. In other words, total-cost models were developed to avoid the problem that arises when the unitary costs q_{jk} of the j -th input (i.e., the lower index j indicates the count of inputs) of the k -th DMU are constant and equal ($q_{jk} = q_k$). In this case, the cost efficiency

becomes only a comparison of quantities $\left(\theta_{ce} = \frac{\sum_{j=1}^n x_{j0}^{p_{j0}}}{\sum_{j=1}^n x_{j0}^{p_{j0}}} \right)$. So, two DMUs could be efficient if the same quantity was reduced. However, in real cases, these quantities may be composed of distinct products with different costs (i.e., stock-keeping units, SKUs), which represent a different economic result for the retailers.

Based on Tone [17], Alves Junior et al. [18–21] proposed a cost-efficiency based on a total cost measure. Instead of using the unit cost information q_j and quantities x_{j0} , the total cost $C(q_j, x_{j0})_{j0} = Cx_{j0}^p$ is used as a proxy of the quantities of products in the inventory during the period under analysis (once they are proportional) and considers the unitary cost as equal to 1 ($q_j = 1$), for the DMUs. Hence, cost efficiency will always be a total cost comparison under all circumstances. Fig. 1 summarizes the conceptual development of these models.

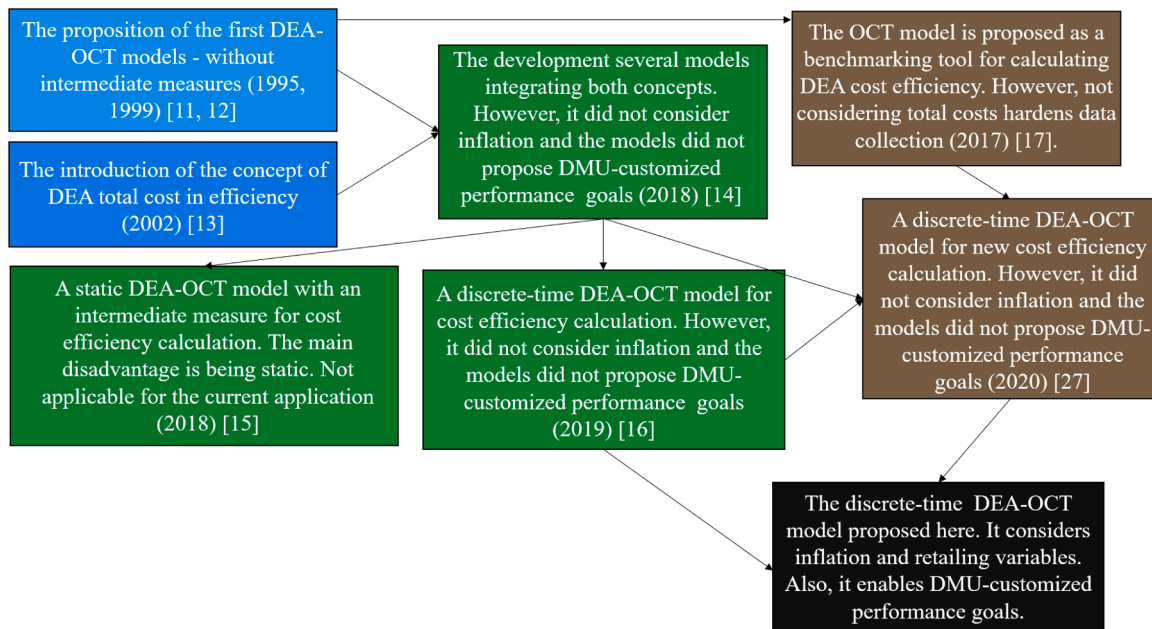


Fig. 1. Historical development of the proposed model.

Another characteristic that describes dynamic behavior is the rate of change of a variable over time, that is, an equation of state or a constraint in which one of the terms is a derivative (or a change in time). This type of model is called variational dynamic due to variations in time. It is used to describe, for example, a depreciation, a capital expansion, or a change in inventory level.

In the dynamic (variational) context, the adopted DEA-OCT dynamic models [18], in addition to being dynamic and considering links between periods, are also a variational model (incorporating OCT restriction) and use inventory as a factor to attribute such interdependence. Alves Junior et al. [18,20,21] suggested that an essential problem to be analyzed is that a very low or zero inventory level is not always desirable. For example, a high variation of an uncertain demand may lead to the occurrence of a lack of inventory, which reduces the demand served. In turn, this can generate a loss greater than the costs of maintaining a small excess of inventory.

The current paper used the discrete DEA-OCT model [18] as a basis due to its ease of processing and because it is characterized by minimizing operational and inventory costs to calculate cost efficiency in the period of interest. The output represents demand, input, and production, and the intermediate variable is inventory. Costs are considered an objective function. The OCT variational constraint is added to describe the relationship between demand, production, and inventory.

Alves Junior et al. [18–21] applied the models to the balance sheet data of publicly traded companies from various industries in Brazil and Chile. However, inventory treatment in the retailing industry is very particular and different from other industries [10,27,57]. Also, the original models did not consider essential variables for retailing, such as Gross Margin (*GM*), Plant, Property, and Equipment (*PPE*), Inventory Slack (*InvSlack*), and Capacity Slack (*CapSlack*) [10]. Finally, inflation is non-neglectable when dealing with financial accounting, especially in emerging markets [11,27,57].

As introduced in subSection 2.1, the proposed approach is a novelty and an advancement [18–21] because it fulfills the gaps in Fig. 1. In other words, it considers the inventory under inflationary conditions [11,27,57] and incorporates a critical variable for retailing activities, *GM* [8,10]. Also, it deals with variables representing digital and physical performance aspects, respectively, *CapSlack* and *InvSlack* [8,10]. Besides, as the approach uses publicly accounting data, it facilitates benchmarking among competitors.

3. Materials and methods

3.1. Retailing variables

Kroes et al. [10] analyzed the relationship between leanness and retailing performance for the US market. Ailawadi and Farris [8] investigated the relationship between retailers and suppliers for e-commerce, proposing performance metrics. Based on both papers, the following concepts are introduced and proposed as variables in the model. It is worth noting that these variables refer to a quarter and do not consider the inflationary effect inside the quarter. A treatment for quarterly inflation is proposed in the model but not calculating variables.

3.1.1. *InvSlack*

Inventory can cushion the impact of other operational issues. In addition, inventory increases cycle time and makes companies less responsive. However, the causal link between inventory and operating performance is likely to be strong in retail environments. When inventory turnover is low, products spend more time on shelves. This increases the window of exposure to damage, theft, and deterioration/expiration [10].

Also, excessive inventories increase the likelihood of lost or misplaced items. When this results in out-of-inventory, a current sale is lost, and more importantly, customers are less likely to return [58]. Then, higher inventory supply chains tend to have longer order cycles making them less responsive to changes in tastes and preferences, which is particularly harmful in segments such as apparel and appliances [59].

Also, customers buy more in stores with fewer shortages [60]. Product variety (i.e., the number of substitute products available to meet a given consumer need) increases demand from individual stores [61]. In addition, enterprise-level retail inventory predicts (positively) future sales [62].

The *InvSlack* is expressed by (7):

$$InvSlack_{it} = \frac{INV_{it} - N}{COGS_{it}} \quad (7)$$

where INV_{it} (continuous) is the value of the quarterly inventory at the period t (discrete, integer, and non-negative) for the firm i (discrete, integer, and non-negative), $COGS_{it}$ (continuous) is the value of the

quarterly cost of goods sold, and N (discrete, integer, and non-negative) represents the number of days in a quarter according to accounting standards in the country. In the case of Brazil, $N = 90$ days [63].

3.1.2. CapSlack

The efficient use of *CapSlack* (the company's physical structure) is another crucial aspect. Retailers focusing on e-commerce, such as Mercado Libre and eBay, had the lowest *CapSlack* and the highest financial value, and there is a causal relationship between the two variables [10]. Therefore, given a specific retailer, *CapSlack* computes the level of its use in e-commerce. The highest level of *CapSlack*, the highest use of e-commerce compared to physical channels.

This paper assumes that *CapSlack* is a measure directly associated with e-commerce activities. However, it is necessary to clarify what "electronic commerce" means. In the case of Kroes et al. [10], only companies in which the marketplace represented the core of their business were considered. For example, the authors did not encompass in this definition large traditional retailers whose e-commerce (through their own or third-party marketplaces) came to represent a significant part of sales.

In this regard, operating through e-commerce simultaneously with other channels (multi-, cross-, or omnichannel) can be understood as an issue of marketing operations and organizational alignment encompassing many levels (operational, tactical, and strategic). Sombultawe and Boon-itt [64] investigated the literature on the theme and concluded that operations alignment is a wide theme spread across several fields. Consequently, no dominant theory can be applied, and there is substantial overlap with concepts of operational integration and coordination.

This work adopts the taxonomy of Tian et al. [3]. It encompasses three models: marketplace, resellers, and hybrid. Retailers, which adopted "pure marketplaces", operate two-sided platforms that allow suppliers to directly sell all categories of products to customers by paying a revenue-sharing fee. This was the case with retailers such as Mercado Libre and eBay [3]. Here is assumed that this model necessarily means no *InvSlack* and minimal *CapSlack*.

On the other hand, retailers that adopt the "reselling marketplace" model purchase all categories of products from suppliers and then resell them to customers. This was the case with Walmart and BestBuy in the US and Zappos in Europe [3]. Here is argued that this model implies *InvSlack* and *CapSlack* are higher than zero. However, slack behaviors may be different from those of retailers which cannot sell products through catalogs and online channels; the integration of physical and online channels requires specific management practices [65].

Between both mentioned marketplace models, retailers can adopt the hybrid model, which consists of reselling certain categories of products and allowing suppliers to directly sell other categories of products to customers by paying a revenue-sharing fee [3]. This was the case with Amazon in the US, JD in China [3], and Magazine Luiza in Brazil [22,27,57,66]. Here is argued that this model also implies *InvSlack* and *CapSlack* are higher than zero. However, the slack behaviors depend on the percentage and category of products allocated to each business model (besides the physical sales).

The *CapSlack* is expressed by (8):

$$CapSlack_{it} = \ln \left(\frac{PPE_{it}}{SALES_{it}} \right), \quad (8)$$

where PPE_{it} (continuous) is the plant, property, and equipment at the end of quarter t (discrete, integer, and non-negative), and $SALES_{it}$ (continuous) is the quarterly sales. PPE is a variable internationally accepted and defined by International Financial Reporting Standards (IFRS) [63]. PPE are tangible items that are held for use in the production or supply of goods or services, for rental to others, or for administrative purposes and are expected to be used during more than one period. Due to the non-linear nature of the relationship between

CapSlack and performance, the natural log was adopted [10].

3.1.3. GM

Comparing Ailawadi and Farris [8] with Kroes et al. [10,10] considered inventory in the calculation of two measures: the *InvSlack* and the accounts payable extraordinary days. In contrast, [8] considered the inventory while calculating GM Return on Inventory Investments (*GMROI*) and inventory turnover. However, inventory turnover was expressed by the ratio of the cost of goods sold to the cost of inventory. That is, it is the inverse of the inventory [10].

The *GM* is calculated by (9):

$$GM_{it} = \frac{(NOI_{it} - COGS_{it})}{NOI_{it}} \quad (9)$$

The concept of the quarterly GM_{it} (continuous) relationship is indirectly added to the current model. It happens through the adoption of the quarterly net operating income (NOI_{it}) (continuous) - as a proxy for $Sales_{it}$ (continuous) in one of the model's constraints - and the adoption of quarterly $COGS_{it}$ (continuous) to calculate the cost of purchased products.

3.2. Proposed model

The proposed model with its sets, indices, parameters, variables, formulation, and explanations of the constraints are described here:

Indices: t (discrete, integer, and non-negative) is the index related to the period t in the planning horizon; k (discrete, integer, and non-negative) is the index related to the DMU; $k = 0$ is the index related to the DMU under analysis; j (discrete, integer, and non-negative) is the index related to the inputs; $j = n$ is the index related to the control variable (input); l (discrete, integer, and non-negative) is the index related to the state variable (intermediate measure); i (discrete, integer, and non-negative) is the index related to the outputs; $i = m$ is the index related to the exogenous variable (output).

Parameters: r_{kt} (continuous) is the inflation of the k -th DMU at the period t ; Cz_{lkt}^- (continuous) is the observed input cost link of the state variable (e.g., initial inventory cost, *IIC* in inventory control systems) of the k -th DMU in period t ; Cz_{lkt}^+ (continuous) is the observed output cost link of the state variable (e.g., final inventory cost, *FIC* in inventory control systems) of the k -th DMU in period t ; Cx_{nkt} (continuous) is the observed cost of the control variable (e.g., cost of the purchased product, *CPP* in inventory control systems) of the k -th DMU in period t ; Cx_{jkt} (continuous) is the observed j -th input cost (e.g., cost of plant, property, and equipment, *PPE*) of the k -th DMU in period t ; Cy_{mkt} (continuous) is the observed cost of the exogenous variable (e.g., cost of goods sold, *COGS* as disturbance in inventory control systems) of the k -th DMU in period t ; Cy_{ikt} (continuous) is the observed i -th output financial value (e.g., net operating income, *NOI*) of the k -th DMU in period t .

Variables: Cz_{l0t}^{-p} (continuous) is the projection of the input cost link of the state variable (e.g., initial inventory cost, *IIC* in inventory control systems) of the DMU under analysis ($k = 0$) in period t ; Cz_{l0t}^{+p} (continuous) is the projection of the output cost link of the state variable (e.g., final inventory cost, *FIC* in inventory control systems) of the DMU under analysis ($k = 0$) in period t ; Cx_{n0t}^p (continuous) is the projection of the cost of the control variable (e.g., cost of the purchased product, *CPP* in inventory control systems) of the DMU under analysis ($k = 0$) in period t ; Cx_{j0t}^p (continuous) is the projection of the j -th input cost (e.g., cost of plant, property, and equipment, *PPE*) of the DMU under analysis ($k = 0$) in period t ; λ_{kt} (continuous) is the contribution of the k -th DMU to a DMU under analysis in period t ; S_{jkt}^- (continuous) is the j -th input slack of the k -th DMU in period t ; S_{ikt}^+

(continuous) is the i th output slack of the k -th DMU in period t ; S_{nkt}^- (continuous) is the control variable slack of the k -th DMU in period t ; S_{mkt}^+ (continuous) is the exogenous variable slack of the k -th DMU in period t ; S_{lkt}^+ (continuous) is the output intermediary-link slack of the k -th DMU in period t ; S_{lkt}^- (continuous) is the input intermediary-link slack of the k -th DMU in period t .

Optimizing the objective function of the proposed cost model is expressed in (10).

$$\text{Minimize } \frac{Cz_{j01}^{-p} + \sum_{t=1}^T Cz_{j0t}^{-p} r_{0t} + \sum_{t=1}^T Cx_{n0t}^p r_{0t} + \sum_{j=1}^{n-1} \sum_{t=1}^T Cx_{j0t}^p r_{0t}}{Cz_{j01} + \sum_{t=1}^T Cz_{j0t}^+ r_{0t} + \sum_{t=1}^T Cx_{n0t} r_{0t} + \sum_{j=1}^{n-1} \sum_{t=1}^T Cx_{j0t} r_{0t}} \quad (10)$$

where, in the numerator, there are the following projections for the DMU under analysis ($k = 0$): Cz_{j01}^{-p} (continuous) is the projection of the input cost link of the state variable of the DMU under analysis ($k = 0$) in period $t = 1$, Cz_{j0t}^{-p} (continuous) is the projection of the output cost link of the state variable in period t , Cx_{n0t}^p (continuous) is the input cost of the control variable in period t , Cx_{j0t}^p (continuous) is the j -th input cost, and r_{0t} (continuous) is the inflation rate.

And in the denominator, there are the following observed values for the DMU under analysis ($k = 0$): Cz_{j01} (continuous) is the observed input cost link of the state variable at the beginning of the period $t = 1$ (equal to the observed output cost link of the state variable in the previous period, $t = 0$), Cz_{j0t}^+ (continuous) is the observed output cost link of the state variable at the end of the period under analysis, t , Cx_{n0t} (continuous) is the observed cost of the control variable in period t , Cx_{j0t} (continuous) is the observed j -th input cost in period t , and r_{0t} (continuous) is the inflation rate.

So, as can be seen in expression (10), the objective function is divided into two parts, one of the projections and the other of observed values. Therefore, the objective function is the sum of all costs related to the inputs and link (input and output links) variables projected in the efficient frontier and compared to the observed costs to estimate the total cost efficiency. Only the input cost link of the first period is used in the objective function because, in other periods, it is equal to the output cost link.

All the constraints are from expression (11) until (20). The constraints originating from the DEA are input constraints (11 and 12), output constraints (13 and 14), intermediary-link constraints (15 and 16), and intermediary-equality constrained (17). Constraints expressed in (15) to (17) promote dynamism. This provides a richer viewpoint for understanding economic relationships [10] than DEA static models applied for transversal studies or DEA window analysis. The constraint expressed in (18) is the variable returns to scale (VRS) assumption. And the constraints that originated from the OCT are the discrete version of the variational constraint of control (19) that also promote dynamism, and the initial value constraint (20), while the constraint expressed in (12) also could be viewed as a control constraint but related to the best-practices.

$$\sum_{k=1}^z Cx_{jkt} r_{kt} \lambda_{kt} + S_{jkt}^- = Cx_{j0t}^p r_{0t}, \quad \{t = 1, 2, \dots, T, j = 1, 2, \dots, n-1, \quad (11)$$

$$\sum_{k=1}^z Cx_{nkt} r_{kt} \lambda_{kt} + S_{nkt}^- = Cx_{n0t}^p r_{0t}, \quad t = 1, 2, \dots, T, \quad (12)$$

$$\sum_{k=1}^z Cy_{ikt} r_{kt} \lambda_{kt} - S_{ikt}^+ = Cy_{i0t} r_{0t}, \quad \{t = 1, 2, \dots, T, i = 1, 2, \dots, m-1, \quad (13)$$

$$\sum_{k=1}^z Cy_{mkt} r_{kt} \lambda_{kt} - S_{mkt}^+ = Cy_{m0t} r_{0t}, \quad t = 1, 2, \dots, T, \quad (14)$$

$$\sum_{k=1}^z Cz_{lkt}^+ \lambda_{kt} r_{kt} - S_{lkt}^+ = Cz_{l0t}^+ r_{0t}, \quad t = 1, 2, \dots, T, \quad (15)$$

$$\sum_{k=1}^z Cz_{lkt}^- \lambda_{kt} r_{kt} + S_{lkt}^- = Cz_{l0t}^- r_{0t}, \quad t = 1, 2, \dots, T, \quad (16)$$

$$\sum_{k=1}^z Cz_{lkt}^+ \lambda_{kt} r_{kt} - Cz_{l(k+1)t} \lambda_{k(k+1)t} r_{k(k+1)t} - S_{lkt}^+ - S_{l(k+1)t}^- = 0, \quad t = 1, 2, \dots, T-1, \quad (17)$$

$$\sum_{k=1}^z \lambda_{kt} = 1, \quad t = 1, 2, \dots, T, \quad (18)$$

$$Cx_{n0t}^p - Cz_{j0t}^{-p} + Cz_{j0t}^{-p} = Cy_{m0t} \quad t = 1, 2, \dots, T, \quad (19)$$

$$Cz_{j01}^{-p} = Cz_{j01}^-. \quad (20)$$

$$Cz_{j0t}^{-p}, Cz_{j0t}^+, Cx_{n0t}^p, Cx_{j0t}^p, \lambda_{kt}, S_{jkt}^-, S_{ikt}^+, S_{nkt}^-, S_{mkt}^+, S_{lkt}^+, S_{lkt}^- \geq 0 (\forall i, j, k, t), \quad (21)$$

Where the indices, parameters, and variables for the k -th DMUs (or DMU under analysis when $k = 0$) are: λ_{kt} (continuous) is the contribution of the k -th DMU to a DMU under analysis ($k = 0$), S_{jkt}^- (continuous) is input slacks, Cy_{ikt} (continuous) is i th output financial value, S_{lkt}^+ (continuous) is output slacks, S_{lkt}^+ and S_{lkt}^- (continuous) are intermediary-link slacks.

A previous paper investigated both DEA assumptions (i.e., the constant returns to scale (CRS) and VRS) [67]. Another paper applied the CRS assumption without justifying it [68]. For verifying the VRS assumption, the multivariate and univariate normality of data and the multivariate and the univariate normality of the data logarithm were tested. The normality of the data logarithm resulted in the best fit. Hence the VRS was assumed and expressed in (18).

3.2.1. Discussions of the optimal solutions

Once the optimal solution ($Cz_{j0t}^{+p*}, Cx_{n0t}^{p*}, Cx_{j0t}^{p*}, \forall(j, t)$) is obtained, the control policy based on the projection of the cost of the control variable $\pi = \{Cx_{n01}^{p*}, \dots, Cx_{n0T}^{p*}\}$ is obtained, as well as the projection of the output cost link of the state variable $Cz_{j0t}^{+p*} \forall(t)$ and input costs $Cx_{j0t}^{p*} \forall(j, t)$, then it

is possible to calculate the cost efficiency of the system as $E_{\text{system}} =$

$$\frac{C_{\text{system}}^*}{C_{\text{system}}} = \frac{Cz_{j01}^{-p*} + \sum_{t=1}^T Cz_{j0t}^{-p*} r_{0t} + \sum_{t=1}^T Cx_{n0t}^{p*} r_{0t} + \sum_{j=1}^{n-1} \sum_{t=1}^T Cx_{j0t}^{p*} r_{0t}}{Cz_{j01} + \sum_{t=1}^T Cz_{j0t}^+ r_{0t} + \sum_{t=1}^T Cx_{n0t} r_{0t} + \sum_{j=1}^{n-1} \sum_{t=1}^T Cx_{j0t} r_{0t}} \quad (\text{continuous}), \quad \text{and}$$

similarly, it is possible to calculate the cost efficiency of each time period

$$t \text{ as } E_t = \frac{C_t^*}{C_t} = \frac{Cz_{j0t}^{-p*} + Cz_{j0t}^{-p*} + Cx_{n0t}^{p*} + \sum_{j=1}^{n-1} Cx_{j0t}^{p*}}{Cz_{j0t}^- + Cz_{j0t}^+ + Cx_{n0t} + \sum_{j=1}^{n-1} Cx_{j0t}} \quad (\text{continuous}); \quad \text{where } C_{\text{system}}^* \quad (\text{continuous})$$

is the optimal cost of the system, C_{system} (continuous) is the actual cost of the system, C_t^* (continuous) is the optimal cost at period t (discrete, integer, and non-negative), C_t (continuous) is the actual cost at period t (discrete, integer, and non-negative).

These optimal costs of the system and per period can be expressed as a projection of the actual costs to the optimal frontier, considering the efficiencies expressed in (22).

$$C_{\text{system}}^* = E_{\text{system}} \cdot C_{\text{system}}$$

$$C_t^* = E_t \cdot C_t \quad (22)$$

Since the optimal cost C_{system}^* (continuous) of the system can be calculated as the sum of the costs, considering the final inventories plus the initial inventory at period $t = 1$ (only as an input for the system in the first period), though the sum of the optimal costs C_t^* (continuous) at each period $t = 1, \dots, T$ considers the initial inventory in each period t (as input for each period); therefore it considers twice the costs of the initial inventories of the middle periods $t = 2, \dots, T-1$.

So, the sum of the optimal costs at each period t can be represented in relation to the optimal cost of the system is expressed in (23).

$$\sum_{t=1}^T C_t^* r_{0t} = C_{system}^* + \sum_{t=2}^T C_{z_{0t}}^{*-ps} r_{0t} \quad (23)$$

Consequently, the efficiency of the system (E_{system}) (continuous) can be represented in relation to the efficiencies of each period t is expressed in (24).

$$E_{system} = \frac{\sum_{t=1}^T E_t \cdot C_t \cdot r_{0t} - \sum_{t=2}^T C_{z_{0t}}^{*-ps} r_{0t}}{C_{system}} \quad (24)$$

3.3. Data sample and empirical application

Here, the input cost link is the initial inventory cost, the output cost link is the final inventory cost, and the input costs refer to the costs of the purchased product and PPE , while the r_{kt} is also considered to deal with the costs in time.

The estimation of the CPP is expressed in (25).

$$C_{x_{nkt}} = C_{z_{ikt}}^+ - C_{z_{ikt}}^- + C_{y_{mkt}} \quad (25)$$

where $C_{y_{mkt}}$ (continuous) is the observed cost of goods sold ($COGS$).

Inventory costs are divided into two temporal factors, initial (IIC) and final (FIC). IIC is between $t = 0, \dots, T-1$, i.e., the inventory from the period before the first quarter (equal to the beginning inventory before the first quarter) to the penultimate period (equal to the beginning of the last quarter).

Due to the DEA constraints, the Eff System (efficiency values for the whole analyzed period) is limited to values between 0 and 1. These constraints create a time frame restriction. In other words, each period's x , y , z is a different variable of the same DMU, and the weighted sum of the y and z minus the x and z - (of all periods) must be less than 0 (zero). This is the same thing as saying the weighted sum of y and z divided by x and z (efficiency) is less than or equal to 1 (one), considering the system. The variables are continuous and non-negative (so the efficiency is greater than or equal to 0). The quarterly efficiencies can be above 1 (one) because there is the dual (in the multiplier form).

- (i) The year 2018 was a year of digital strategic consolidation among the retailers in the Brazilian market. It is assumed that all strategies were operationally consolidated up to Christmas (the fourth quarter of 2018), an important date for retailing. This research was executed in the fourth quarter of 2020. So, we investigated from the fourth quarter of 2018 up to the third quarter of 2020 (last date available). Thus, the period of investigation encompassed eight quarters. During this period, the global COVID-19 pandemic started, changing retailing activities globally. Therefore, we decided to divide it into three periods of investigation: From the fourth quarter of 2018 until the third quarter of 2020 (4Q2018 – 3Q2020), encompassing before and during the pandemic (Period F – full-time).
- (ii) From the fourth quarter of 2018 until the first quarter of 2020 (4Q2018 – 1Q2020), before the pandemic (Period BC – before Covid).
- (iii) From the second quarter of 2020 until the third quarter of 2020 (2Q2020 – 3Q2020), during the pandemic (Period DC – during Covid).

Firm-level quarterly financial data related to $C_{z_{ikt}}^+$, $C_{z_{ikt}}^-$, and $C_{y_{mkt}}$ from (11) were collected in the Economática database. These companies were publicly traded in the São Paulo Stock Exchanges between the third quarter of 2018 (3Q2018) and the third quarter of 2020 (3Q2020). This period encompasses before and during the pandemic. All previous efforts (Fig. 1) worked with quarterly data. According to Kroes et al. [10], when a study focuses on the firm performance implications of

operational slacks, quarterly data is justified because it will be more accurate to reflect the reactions. The reported quarterly firm data is associated with the calendar date (the first quarter ends in March, the second in June, the third in September, and the fourth in December).

Retail firms were identified as those classified as “Commerce”, “Commerce & Distribution”, and “Fabric, Clothing, and Footwear”. Initially, 31 companies were identified. Subsequently, their reference forms¹ were consulted, and those companies that were not within scope were excluded. Only companies, which incorporated the direct management of e-commerce before the pandemic, were considered.

This resulted in a sample of 21 companies and 189 financial observations of each of the six model's measures - initial inventory cost (IIC), final inventory cost (FIC), net operating income (NOI), cost of goods sold ($COGS$), cost of the purchased product (CPP) and plant, property, and equipment (PPE). The data is in the local currency, the Brazilian Real (BRL). The adopted inflation rate was the IPCA (Broad Nacional Consumer Price Index), and the historical series were retrieved from official sources [11,27,57].

As per previous authors [10], the current dataset presents a subtle difference from prior ones, which is the examination of firms at the quarterly versus annual levels. In this way, it is believed that the proposed approach can also contribute to an index that better reflects the reactions of an efficient market and can provide an adequate tool for future investigations regarding operational efficiency and market response.

A summary description of the analyzed companies is presented in Table 1.

Once a novel model based on linear programming is proposed and the data is collected, we develop and run a new algorithm in Matlab software to solve it using a linear programming solver because it is one of the most common solving software for linear programming and operations research [67]. Readers who are interested in using already

Table 1
Description of the analyzed companies.

DMU	Company's Name	B3 Subsector
ALPA3	Alpargatas	Fabric, Clothing, and Footwear
ARZZ3	Arezzo	Fabric, Clothing, and Footwear
BTOW3	B2W Digital	Commerce
CAMB3	Cambuci	Fabric, Clothing, and Footwear
CRFB3	Carrefour BR	Commerce & Distribution
HGTX3	Cia Hering	Fabric, Clothing, and Footwear
GRND3	Grendene	Fabric, Clothing, and Footwear
GUAR3	Guararapes	Commerce
CATA3	Ind Cataguas	Fabric, Clothing, and Footwear
CTKA3	Karsten	Fabric, Clothing, and Footwear
LLIS3	Le Lis Blanc	Commerce
LAME3	Lojas Americanas	Commerce
AMAR3	Lojas Marisa	Commerce
LREN3	Lojas Renner	Commerce
MGLU3	Magazine Luiza	Commerce
PCAR3	Pão de Açúcar	Commerce & Distribution
SLED3	Livraria Saraiva	Commerce
TECN3	Technos	Fabric, Clothing, and Footwear
TEKA3	Teka	Fabric, Clothing, and Footwear
VVAR3	Via Varejo	Commerce
VULC3	Vulcabras	Fabric, Clothing, and Footwear

¹ The Reference Form is an electronic document, periodically and occasionally forwarded, provided for in article 21, item II, of the Brazilian Securities and Exchange Commission (CVM) Instruction No. 480/09, which must be forwarded to the CVM through the Empresas.NET System. The Publicly Held Companies Reference Form gathers all information related to the issuer, such as activities, risk factors, management, capital structure, financial data, management comments on these data, securities issued, and transactions with related parties.

established models are recommended to the interactive online app deaR-Shiny, based on the R software [68].

3.4. Homogeneity and online retailing

Here, the DMUs are publicly traded retailing companies that sell through physical stores as well as virtual channels. In this way, they are assumed to be homogeneous. Data from six measures (of the 21 DMUs) were used, respecting the recommended practice of keeping the number of measures smaller than three times the number of DMUs [69].

For identifying the strategies of marketplaces, the reference forms were consulted. And it was checked through internet research engines whether it was possible to buy products online as described in the companies' official reports. The name of the companies and the name of their brands were searched on Scopus. In this way, it was analyzed the existence of applied research.

4. Results and discussions

4.1. Online retailing

Table 2 shows the identified nine marketplace strategies of the analyzed companies. Companies with similar strategies are separated from others with a horizontal line. Readers interested in the companies can also consult [27,57].

4.2. Efficiency results

Tables 3, 4, and 5 show the results of Periods F (full-period), BC (before Covid), and DC (during Covid), respectively. In each table, the column "Ef System" indicates the global efficiency considering the whole analyzed period, and the other columns show the quarterly efficiencies.

In the broader period (Table 3), three efficient ties exist among the 21 analyzed companies (i.e., 14.3% of the sample). The number of ties equals the period before the pandemic (Table 4). These ties are B2W Digital (BTOW3), Cambuci (CAMB3), and Carrefour Brazil (CRFB3). They belonged to different retailing sectors, i.e., respectively, home appliance and electronics, apparel, and supermarket, and adopted different digital marketplace strategies (Table 2).

During the pandemic (Table 5), seven efficient ties (33.3% of the sample). They were the previous three ties and more Lojas Marisa (AMAR3), Magazine Luiza (MGLU3), Pão de Açúcar (PCAR3), and Via Varejo (VVAR3). This means that these four companies reacted efficiently to the pandemic context. They were an apparel retailer

Table 2
Marketplace strategies of the analyzed companies.

DMUs	Marketplace Strategies
ALPA3, ARZZ3, HGTX3, GRND3, CTKA3, LLIS3, TEKA3, VULC3	Controller of MULTIPLE reselling marketplaces (one for each brand) and seller in MULTIPLE third-party pure marketplaces.
CAMB3, TECN3	Controller of ONE reselling marketplace and seller in MULTIPLE third-party pure marketplaces.
CATA3, SLED3, AMAR3 LREN3	Controller of ONE reselling marketplace. Controller of MULTIPLE reselling marketplaces and ONE hybrid marketplace.
GUAR3, CRFB3 PCAR3	Controller of ONE hybrid marketplace. Controller of TWO hybrid marketplaces.
BTOW3, LAME3	Controller of MULTIPLE pure and hybrid marketplaces (including the brand Americanas, associated to the LAME3).
MGLU3	Controller of ONE reselling marketplace (books), ONE pure marketplace (books), and MULTIPLE hybrid marketplaces (home appliance).
VVAR3	Controller of MULTIPLE hybrid marketplaces.

(AMAR3), two home appliance and electronic retailers (MGLU3 and VVAR3), and a supermarket (PCAR3). Each of them adopted a different marketplace strategy.

The first conclusion that can be drawn from these results is that the pandemic forced retailers to reach efficiency. This observation supports the results of Setiawan et al. [70]. The authors surveyed 99 multi-channel retailers in an emerging market. They concluded that businesses had to become more dynamic during the pandemic. Specifically, supply chain management during the pandemic probably affected business performance through multi-channel retailing with three main indicators: inventory investment, inventory efficiency, and forecasting accuracy.

The second conclusion that can be drawn from these results is that each retailer that reached efficiency had a different marketplace strategy. Based on the definitions presented in Table 2, the only marketplace strategies that did not reach efficiency in any period were controllers of multiple reselling marketplaces (one for each brand) and sellers in multiple third-party pure marketplaces. This strategy was followed by eight retailers, i.e., 38.1% of the sample. And one retailer (LREN3) that followed the strategy of being a controller of multiple reselling marketplaces and one hybrid marketplace.

As presented by the author of the adopted marketplace taxonomy [3], there is no strategy that fits all retailers, and the reasons why retailers choose and change their marketplace strategies keep unknown. The current result indicates that the digital strategy of simultaneously managing multiple reselling marketplaces while also selling products through multiple third-party marketplaces may be too complex. No observed retailer reached efficiency through this strategy.

Table 6 summarizes the comparison of the Eff System among the three periods of analysis (Tables 3, 4, and 5). Lojas Marisa (AMAR3), Lojas Americanas (LAME3), Magazine Luiza (MGLU3), Pão de Açúcar (PCAR3), Tecnos (TECN3), and Teka (TEKA3) presented a better efficiency value during the pandemic compared to before the pandemic. They are in green in Table 6. On the other hand, Arezzo (ARZZ3), Grendene (GRND3), Guararapes (GUAR3), Cia Hering (HGTX3), Le Lis Blanc (LLIS3), Lojas Renner (LREN3), and Livraria Saraiva (SLED3) presented a worse efficiency value during the pandemic (compared to before the pandemic). They are in red in Table 6.

4.2.1. Apparel sector

Interpreting these results based on marketplace strategies, among the retailers that adopted the strategy of being controllers of multiple reselling marketplaces (one for each brand) and sellers in multiple third-party pure marketplaces, one increased performance during the pandemic (TEKA3), but four decreased (ARZZ3, GRND3, HGTX3, and LLIS3). And none reached efficiency. However, it is important to also interpret the results through the sectorial perspective. All mentioned retailers belong to the apparel sector. Also, LREN3, the unique retailer to be a controller of multiple reselling marketplaces and one hybrid marketplace, belongs to the apparel sector and decreased efficiency. Besides, GUAR3 is another representative of the apparel sector that decreased efficiency during the pandemic, even though it adopted a different strategy from the previous ones (i.e., "Controller of one hybrid marketplace."). So, six retailers from the apparel industry decreased efficiency during the pandemic.

However, CAMB3 is representative of the same sector, and it kept efficient during all the investigated periods. CAMB3 and TECN3 followed the strategy of being controllers of one reselling marketplace and sellers in multiple third-party pure marketplaces. And TECN3 also increased efficiency during the pandemic. Besides, AMAR3 reached efficiency during the pandemic, following the strategy of being the controller of one reselling marketplace. Therefore, three retailers from the apparel industry increased efficiency during the pandemic.

These results indicate that apparel retailers suffered the most with the pandemic. However, some of them achieved efficiency even under unfavorable circumstances. Remarkably, those that adopted the

Table 3

DEA-OCT results considering Period F (4Q2018 – 3Q2020).

Comp.	Rank	Ef_System	4Q2018	1Q2019	2Q2019	3Q2019	4Q2019	1Q2020	2Q2020	3Q2020
BTOW3	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
CAMB3	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
CRFB3	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MGLU3	4	0.99	1.00	1.00	1.02	0.97	0.92	0.92	0.91	1.01
VVAR3	5	0.98	1.00	1.00	1.00	0.93	0.96	0.91	0.90	1.00
PCAR3	6	0.97	0.95	0.97	0.90	0.92	1.00	1.00	1.00	1.00
ARZZ3	7	0.94	1.00	1.04	1.05	1.04	0.90	0.96	0.94	0.99
GUAR3	8	0.92	1.00	1.00	1.00	1.00	1.00	0.85	0.87	0.98
AMAR3	9	0.88	1.00	0.78	0.83	0.80	1.00	1.00	1.00	1.00
GRND3	10	0.86	0.94	0.95	0.91	0.98	0.85	0.90	0.81	1.06
LREN3	11	0.85	1.00	0.87	0.95	0.94	1.08	0.80	0.76	0.91
TECN3	11	0.85	1.00	0.96	0.98	0.77	0.77	0.66	0.66	0.71
ALPA3	13	0.81	0.91	0.89	0.95	0.83	0.80	0.69	0.75	0.89
LAME3	14	0.79	0.87	0.71	0.92	0.78	0.91	0.68	0.80	0.78
VULC3	15	0.78	0.80	0.85	0.83	0.77	0.74	0.75	0.61	0.77
HGTX3	16	0.77	0.93	0.90	0.89	0.80	0.71	0.71	0.61	0.55
LLIS3	17	0.76	1.00	0.82	0.90	0.75	0.95	0.71	0.69	0.69
CATA3	18	0.75	0.85	0.82	0.92	0.77	0.78	0.75	0.76	0.73
SLED3	19	0.70	1.00	0.91	0.72	0.67	0.61	0.71	0.75	0.90
CTKA3	20	0.63	0.76	0.71	0.70	0.58	0.63	0.59	0.61	0.70
TEKA3	21	0.53	1.12	0.60	0.62	0.52	0.57	0.55	0.55	0.51

Table 4

DEA-OCT results considering Period BC (4Q2018 – 1Q2020).

Comp.	Rank	Ef_System	4Q2018	1Q2019	2Q2019	3Q2019	4Q2019	1Q2020
BTOW3	1	1.00	0.96	1.10	0.97	1.14	0.77	0.85
CAMB3	1	1.00	0.82	0.83	0.66	1.10	1.29	1.27
CRFB3	1	1.00	1.05	1.06	0.73	0.82	0.72	0.78
MGLU3	4	0.98	1.28	1.20	0.83	0.91	0.57	0.71
VVAR3	4	0.98	1.18	0.97	0.75	1.07	0.73	0.90
ARZZ3	6	0.96	1.04	0.94	0.74	0.96	0.78	0.96
GUAR3	6	0.96	1.20	0.79	0.66	0.94	1.09	1.13
PCAR3	6	0.96	1.39	1.31	0.79	0.92	0.53	0.62
LREN3	9	0.90	1.36	0.73	0.60	0.86	0.97	1.01
GRND3	10	0.88	0.74	0.88	0.78	0.92	0.94	1.06
TECN3	11	0.87	1.09	0.87	0.76	1.06	0.97	0.99
AMAR3	12	0.86	0.94	0.67	0.62	0.79	0.97	1.17
HGTX3	13	0.83	0.86	0.75	0.67	0.89	0.88	0.95
ALPA3	14	0.82	0.73	0.75	0.58	0.83	1.03	1.03
LLIS3	15	0.81	1.14	0.71	0.66	0.94	1.04	1.03
VULC3	15	0.81	0.88	0.79	0.64	0.75	0.63	0.84
LAME3	17	0.80	1.07	0.78	0.68	0.80	0.66	0.69
CATA3	18	0.77	0.79	0.73	0.59	0.90	1.00	1.00
SLED3	19	0.70	1.12	0.64	0.57	0.96	0.75	1.05
CTKA3	20	0.64	0.76	0.66	0.58	0.73	0.64	0.70
TEKA3	21	0.58	0.39	0.50	0.44	0.48	1.25	0.70

strategies that require controlling fewer reselling marketplaces. The adoption of an adequate digital strategy may be one of the reasons for the achieved success.

A previous study [71] demonstrated that retailers from apparel, groceries, and home furniture had a better omnichannel performance during the pandemic in Spain. The degree of online adoption among customers varies according to the sector, with customers from the apparel sector being the most digitalized [71]. Another paper [72] investigated large clothing and apparel retailers in Spain during the pandemic. Their results suggest that retailers were moderately prepared to provide multichannel and omnichannel services and focused on integrating quick and easy-to-implement services.

Given this, it would be expected that apparel retailers had a better performance during the pandemic. However, this is not observed in the investigated sample. The reasons for it may be explained by the difference between a developed and developing market (Spain and Brazil) or an unsuccessful digital strategy choice in the Brazilian case.

4.2.2. Book sector

Retailers in the book sector face the particularity that their core

product (books) can be sold and consumed wholly digitally. Our sample had a book retailer that reduced efficiency during the pandemic (SLED3). This retailer followed the marketplace strategy of being the controller of one reselling marketplace, like AMAR3, one of the three apparel retailers that reached efficiency during the pandemic.

On the other side, the retailer MGLU3 is focused on home appliances and electronics, operated through multiple hybrid marketplaces. However, the retailer also controls one reselling marketplace focused on new books (Livraria Cultura) and one reselling marketplace focused on used books (Estante Virtual), where small bookstores can sell their physical products. MGLU3 reached efficiency during the pandemic.

Our findings indicate that traditional large bookstore chains did not perform well during the pandemic. However, a retailer controlling a marketplace specialized in used books increased performance in the period. In Colombia and Indonesia [73,74], empirical evidence showed that small booksellers intensified their book sales through social media platforms.

4.2.3. Home appliance and electronic sector

Four retailers' main products were related to home appliances and

Table 5

DEA-OCT results considering Period DC (2Q2020 – 3Q2020).

Comp.	Rank	Eff_System	2Q2020	3Q2020
BTOW3	1	1.00	1.00	1.00
CAMB3	1	1.00	1.00	1.00
CRFB3	1	1.00	1.00	1.00
AMAR3	1	1.00	1.00	1.00
MGLU3	1	1.00	1.00	1.00
PCAR3	1	1.00	1.00	1.00
VVAR3	1	1.00	1.00	1.00
TECN3	8	0.99	1.00	0.98
ARZZ3	9	0.87	0.81	0.96
LAME3	10	0.85	0.93	0.84
ALPA3	11	0.80	0.77	0.92
GRND3	12	0.79	0.59	1.00
TEKA3	13	0.77	1.17	0.50
HGTX3	14	0.74	0.82	0.74
VULC3	14	0.74	0.75	0.78
GUAR3	16	0.71	0.71	0.84
CATA3	16	0.71	0.77	0.72
CTKA3	18	0.70	0.76	0.73
LLIS3	19	0.67	0.72	0.73
SLED3	20	0.62	0.62	0.76
LREN3	21	0.59	0.55	0.72

Table 6

Comparison of the Eff_System in Periods F, BF, and DF.

Company	Eff_System Period DC (2Q2020 - 3Q2020)	Period BC (4Q2018 - 1Q2020)	Period F (4Q2018 - 3Q2020)
ALPA3	0.80	0.81	0.81
ARZZ3	0.87	0.94	0.94
BTOW3	1.00	1.00	1.00
CAMB3	1.00	1.00	1.00
CRFB3	1.00	1.00	1.00
HGTX3	0.74	0.77	0.77
GRND3	0.79	0.86	0.86
GUAR3	0.71	0.92	0.92
CATA3	0.71	0.75	0.75
CTKA3	0.70	0.63	0.63
LLIS3	0.67	0.76	0.76
LAME3	0.85	0.79	0.79
AMAR3	1.00	0.88	0.88
LREN3	0.59	0.85	0.85
MGLU3	1.00	0.99	0.99
PCAR3	1.00	0.97	0.97
SLED3	0.62	0.70	0.70
TECN3	0.99	0.85	0.85
TEKA3	0.77	0.53	0.53
VVAR3	1.00	0.98	0.98
VULC3	0.74	0.78	0.78

electronics in our sample (BTOW3, LAME4, MGLU3, and VVAR3). Each of them follows a different marketplace strategy. Two of these companies (BTOW3 and LAME4) have a particular situation. BTOW3 is a controller of several marketplaces, including those from LAME4. On the other hand, in a symbiotic relationship, LAME4 controls the logistics infrastructure of BTOW3. Our results showed BTOW3 is efficient in all periods. This is expected because its InvSlack and CapSlack are very low due to the lack of physical infrastructure, such as warehouses. During the pandemic, LAME4 was among the retailers that improved performance. MGLU3 and VVAR3 reached efficiency during the pandemic.

Furthermore, our results indicate that the pandemic positively affected retailers in this sector. Some cost DEA approaches have already been proposed for measuring the performance of stores from an electronic retail chain in India [32,37,75]. However, these studies did not consider aspects of online performance and the pandemic [32,37,75]. So, the results are not comparable. Our findings are aligned with the empirical evidence that the e-commerce of electronics and household appliances, household goods for home and repairs, cosmetics, and

products for children increased during the pandemic [76].

4.2.4. Supermarket sector

There were only two supermarkets in the sample. They followed different marketplace strategies. One was efficient during the whole period (CRFB3), and another reached efficiency during the pandemic (PCAR3). Both supermarket chains have the same controller. But unlike BTOW3 and LAME4, CRFB3 and PCAR3 have distinct physical and digital operations.

CRFB3 and PCAR3 follow different marketplace strategies. CRFB3 controls one hybrid marketplace, and PCAR3 controls two. CRFB3 was efficient during all analyzed periods, and PCAR3 reached efficiency during the pandemic. Like what happened with the retailers in the apparel sector, the supermarket chain with fewer marketplaces (one) had a better performance.

A DEA approach has already been proposed for measuring the performance of small and medium-sized supermarkets in a specific region of Brazil (Santa Catarina) [33]. Once the authors did not consider the pandemic period and e-commerce aspects, the results are not comparable. There is a gap in applied investigations for this context.

4.3. Defining performance benchmarks

A previous paper proposed a DEA approach for establishing retail performance benchmarks [27,57]. However, this is a pioneering effort with a dynamic DEA model. With the current results, it is also possible to build additional analysis (Tables 7 and 8).

Table 7 shows the ratio between the optimal calculated and observed values for the companies of interest for this discussion (GUAR3 and ARZZ3) of the model's measures (IC, CPP, and PPE). Table 8 shows the lambdas that differed from zero for GUAR3 and ARZZ3. The companies that present lambdas differently from zero are the ones the model points out as benchmarks in the period.

Table 7 shows that GUAR3 had optimal inventory costs 40% higher than the observed, while the ARZZ3 had 2%. The GUAR3 had an optimal CPP 220% higher than the observed value in 2Q2020 (the pandemic's beginning), while ARZZ3 had 56% higher. Both companies reduced CPP in the next quarter, though the ARZZ3 almost reached the optimal value, while the GUAR3 kept above it.

Analyzing the PPE related to digitalization, once digital operations are assumed to require less physical infrastructure, both companies presented optimal value below the observed. However, GUAR3 is more distant from the optimal value than ARZZ3, especially one quarter after the pandemic. This means the ARZZ3 reacted better (reducing infrastructure) during the pandemic than the GUAR3.

Table 8 shows that, in 2Q2020, AMAR3, CAMB3, and TECN3 were benchmarks for the ARZZ3; in 3Q2020, the AMAR3 and the CAMB3 kept being benchmarks, followed by GRND3. In 2Q2020, the AMAR3 and the BTOW3 were benchmarks for the GUAR3; in 3Q2020, AMAR3 and the VVAR3 were the benchmarks.

In this way, although inefficient in the pre-pandemic period, the AMAR3 and the VVAR3 are benchmarks in the pandemic. On the other hand, the TECN3 slightly decreased efficiency from 2Q2020 to 3Q2020 (from 100% to 98%). It became a benchmark just for 1Q2020.

5. Conclusion

Here we demonstrated that the proposed dynamic model is applicable and useful for the context benchmarking of retailing operations, considering digital and physical operations. Besides, through the model's lambda analysis, it is possible to determine which company (or companies) is (are) the benchmark(s) for each inefficient company and to determine for which measure, in which proportion, and in which quarter (i.e., what, how, and when?), the highlighted company (companies) is (are) benchmark(s). Also, it is possible to build the ratio between the optimal calculated and the observed values for directing the

Table 7

Ratio between the optimal calculated and observed values for companies of interest.

	IC - 1Q2020	IC - 2Q2020	IC - 3Q2020	CPP - 2Q2020	CPP - 3Q2020	PPE - 2Q2020	PPE - 3Q2020
GUAR3	1	1.40	1.45	3.20	1.07	0.11	0.27
ARZZ3	1	1.02	1.02	1.56	0.97	0.29	0.83

Table 8

Benchmarks for the companies of interest (in the pandemic period).

	Lambdas - 2Q2020			
	BTOW3	CAMB3	AMAR3	TECN3
ARZZ3	0.00	0.44	0.54	0.02
GUAR3	0.28	0.00	0.72	0.00
	Lambdas - 3Q2020			
	CAMB3	GRND3	AMAR3	VVAR3
ARZZ3	0.27	0.17	0.57	0.00
GUAR3	0.00	0.00	0.86	0.14

companies' improvement efforts, pointing out which measure should focus on improvement.

In terms of lessons learned from the pandemic, our findings showed that the pandemic forced retailers to reach efficiency. However, there was no “always winner” marketplace strategy for all. Each efficient retailer followed a different strategy.

Though it was observed that inefficient retailers controlled several different marketplaces while also selling products through third-party marketplaces. The most efficient retailers adopted more limited strategies in terms of the number of controlled marketplaces. Remarkably, “controlling of one reselling marketplace” and “controlling of one reselling marketplace and seller in multiple third-party pure marketplaces” were the most successful strategies.

Also, beyond their adopted strategies, retailers from different sectors had different responses to the pandemic. In general terms, supermarkets, home appliances, and electronics retailer improved their performance during the pandemic. While traditional bookstore chains decreased performance, new business models (such as pure marketplaces focused on used books) seem to have improved.

Besides, apparel retailers decreased performance, what is contradicting the existing literature [72]. This may be explained by the difference between emerging and developed markets or by the unfocused digital strategy observed in the apparel retailers in this sample.

Finally, the proposed model included the inflation rate and other variables not included in this kind of model. However, one important limitation of the model is that it does not deal with uncertainty. So, it is suggested to extend the development of it to Stochastic DEA-OCT models or Chance-Constrained DEA-OCT models. Also, it should be interesting to explore the possibility of using DEA jointly with reinforcement learning to estimate the efficiency of control systems. Besides, further developments can focus on creating models that concentrate on profit maximization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability

Data will be made available on request.

Acknowledgement

The author Dr Paulo Nocera Alves Junior is supported by Agencia Nacional de Investigación y Desarrollo (ANID), Fondo Nacional de Desarrollo Científico y Tecnológico (FONDECYT) Iniciación en Investigación 2023, Project 11230508. The author Dr Marcelo Seido Nagano is supported by grant number 312585/2021-7 from the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) - Brazil e Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) - Brazil.

References

- [1] Jiang Y, Stylos N. Triggers of consumers' enhanced digital engagement and the role of digital technologies in transforming the retail ecosystem during COVID-19 pandemic. *Technol Forecast Soc Change* 2021;172:121029. <https://doi.org/10.1016/j.techfore.2021.121029>.
- [2] Beckers J, Weekx S, Beutels P, Verhelsel A. COVID-19 and retail: The catalyst for e-commerce in Belgium? *J Retail Consum Serv* 2021;62:102645. <https://doi.org/10.1016/j.jretconser.2021.102645>.
- [3] Tian L, Vakharia AJ, Tan YR, Xu Y. Marketplace, Reseller, or Hybrid: Strategic Analysis of an Emerging E-Commerce Model. *Prod Oper Manag* 2018;27:1595–610. <https://doi.org/10.1111/poms.12885>.
- [4] Wen HJ, Lim B, Huang HL. Measuring e-commerce efficiency: a data envelopment analysis (DEA) approach. *Indust Manage Data Syst* 2003;103:703–10. <https://doi.org/10.1108/02635570310506124>.
- [5] Alpar P, Porembsk M, Pickerodt S. Measuring the Efficiency of Web Site Traffic Generation. *Int J Electron Commer* 2001;6:53–74.
- [6] Yang Z, Shi Y, Yan H. Scale, congestion, efficiency and effectiveness in e-commerce firms. *Electron Commer Res Appl* 2016;20:171–82. <https://doi.org/10.1016/j.elerap.2016.07.003>.
- [7] Ramanathan R, George J, Ramanathan U. The Role of Logistics in E-commerce Transactions: An Exploratory Study of Customer Feedback and Risk. *Supply chain strategies, issues and models*. London: Springer London; 2014. p. 221–33. https://doi.org/10.1007/978-1-4471-5352-8_10.
- [8] Ailawadi KL, Farris PW. Managing Multi- and Omni-Channel Distribution: Metrics and Research Directions. *J Retail* 2017;93:120–35. <https://doi.org/10.1016/j.jretai.2016.12.003>.
- [9] Fallah-Fini S, Triantis K, Johnson AL. Reviewing the literature on non-parametric dynamic efficiency measurement: State-of-the-art. *J Product Anal* 2014;41:51–67. <https://doi.org/10.1007/s11123-013-0349-8>.
- [10] Kroes JR, Manikas AS, Gattiker TF. Operational leanness and retail firm performance since 1980. *Int J Prod Econ* 2018;197:262–74. <https://doi.org/10.1016/j.jipe.2018.01.006>.
- [11] Pinto A da S. A relevância da correção monetária nas demonstrações contábeis das empresas do setor de mineração brasileiro. *Fundação Escola de Comércio Álvares Penteado – FECAP*; 2012.
- [12] Chakraborty D, Garai T, Jana DK, Roy TK. A three-layer supply chain inventory model for non-instantaneous deteriorating item with inflation and delay in payments in random fuzzy environment. *J Indust Prod Eng* 2017;34:407–24. <https://doi.org/10.1080/21681015.2017.1361229>.
- [13] Utama DM, Santoso I, Hendrawan Y, Dania WAP. Integrated procurement-production inventory model in supply chain: A systematic review. *Oper Res Perspect* 2022;9:100221. <https://doi.org/10.1016/j.orp.2022.100221>.
- [14] Saha S, Chakrabarti T. Cost Minimization Policy for Manufacturer in a Supply Chain Management System with Two Rates of Production under Inflationary Condition. *Jurnal Teknik Industri* 2020;21:200–12. <https://doi.org/10.22219/JTIUMM.Vol21.No2.200-212>.
- [15] Sengupta JK. A dynamic efficiency model using data envelopment analysis. *Int J Prod Econ* 1999;62:209–18. [https://doi.org/10.1016/S0925-5273\(98\)00244-8](https://doi.org/10.1016/S0925-5273(98)00244-8).
- [16] Sengupta J.K. Dynamics of Data Envelopment Analysis: Theory of Systems Efficiency. 1995.
- [17] Tone K. A slacks-based measure of super-efficiency in data envelopment analysis. *Eur J Oper Res* 2002;143:32–41. [https://doi.org/10.1016/S0377-2217\(01\)00324-1](https://doi.org/10.1016/S0377-2217(01)00324-1).
- [18] Alves Junior PN. Inventory management and dynamic efficiency: an integrated approach between data envelopment analysis (DEA) and optimal control theory (OCT) [Gestão de estoque e eficiência dinâmica: uma abordagem integrada entre análise envoltória de dados (DEA) e teoria do controle Ótimo (OCT)]. University of São Paulo; 2018.
- [19] Alves Junior PN, Melo IC, Celestini G, Rebelatto DA do N. Allocative control Data Envelopment Analysis (DEA) model to evaluate inventory control systems when

- there is a relationship among variables. *GEPROSGestão Da Produção* 2018;11: 143–60.
- [20] Alves Junior PN, Yushimito WF, Gude JP, Melo IC, Rebelatto DA do N. Insider understanding of country development: a novel application of Optimal Control Theory and Data Envelopment Analysis for benchmarking performance of Chilean and Brazilian companies. *Centr Eur Rev Econ Manage* 2019;3:47. <https://doi.org/10.29015/cerem.578>.
- [21] Alves Júnior PN, Cari EPT. The Use of Optimal Control Theory as a Benchmarking Tool in Production-Inventory Systems. In: Yılmaz ÖF, Tüfekçi S, editors. *Handbook of research on applied optimization methodologies in manufacturing systems*. 1st ed. IGI Global; 2017. p. 400. <https://doi.org/10.4018/978-1-5225-2944-6>.
- [22] Costa Melo I. E-Commerce: compared efficiency of major retailers in Brazil and Canada [E-commerce: comparação da eficiência dos grandes varejistas no Brasil e Canadá]. Universidade de São Paulo (USP); 2021.
- [23] Athanassopoulos AD. Performance improvement decision aid systems (PIDAS) in retailing organizations using data envelopment analysis. *J Product Anal* 1995;6: 153–70. <https://doi.org/10.1007/BF01073409>.
- [24] Athanassopoulos AD, Thanassoulis E. Separating Market Efficiency from Profitability and its Implications for Planning. *J Oper Res Soc* 1995;46:20–34. <https://doi.org/10.1057/jors.1995.3>.
- [25] Parsons L.J. Using stochastic frontier analysis for performance measurement and benchmarking. n.d., p. 317–50. [https://doi.org/10.1016/S0731-9053\(02\)16013-0](https://doi.org/10.1016/S0731-9053(02)16013-0).
- [26] Sellers-Rubio R, Mas-Ruiz F. Different Approaches to the Evaluation of Performance in Retailing. *The International Review of Retail, Distrib Consum Res* 2007;17:503–22. <https://doi.org/10.1080/09593960701631953>.
- [27] Melo IC, Alves Junior PN, Kodama TK, da Silva KA, Chimirri MW, Rebelatto DA do N. B2C E-commerce for Home Appliance in The Brazilian Market: A Cost Efficiency Investigation through a DEA-OCT Model. *Intermet* 2020;15:72–89. <https://doi.org/10.18568/intermet.v15i2.557>.
- [28] Lampe HW, Hilgers D. Trajectories of efficiency measurement: A bibliometric analysis of DEA and SFA. *Eur J Oper Res* 2015;240:1–21. <https://doi.org/10.1016/j.ejor.2014.04.041>.
- [29] Olesen OB, Petersen NC. Stochastic data envelopment analysis - A review. *Eur J Oper Res* 2016;251:2–21. <https://doi.org/10.1016/j.ejor.2015.07.058>.
- [30] Jiang P, Talaga J. Building a customer base in the electronic marketplace: an empirical exploration of the e-tailing industry. *J Serv Market* 2006;20:429–38. <https://doi.org/10.1108/08876040610704865>.
- [31] Park TA, King RP. Evaluating food retailing efficiency: the role of information technology. *J Product Anal* 2007;27:101–13. <https://doi.org/10.1007/s11123-006-0030-6>.
- [32] Gupta A, Pachar N, Jha PC, Darbari JD. Efficiency assessment of Indian electronics retail stores using DEA. *Int J Busi Perform Supply Chain Model* 2019;10:386. <https://doi.org/10.1504/IJBPSM.2019.105719>.
- [33] Silva Junior JCA, Nodari D, Cavalheiro M de O, Victor FG. Performance of supermarkets in Santa Catarina: an analysis of small and medium-sized units. *Gestão & Produção* 2020;27. <https://doi.org/10.1590/0104-530x4685-20>.
- [34] Okur N, Ercan T. Evaluating retail efficiency using DEA and AHP: a case in the Turkish apparel retail industry. *J Fash Market Manage Int J* 2022. <https://doi.org/10.1108/JFMM-06-2021-0154>.
- [35] Shakkor M, Jaber N, Jadayil WA, Qureshi M, Jaber S. A novel model for benchmarking the performance of retail stores for retail operations using lean manufacturing approach. *Int J Appl Eng Res* 2017;12:6686–92.
- [36] Álvarez-Rodríguez C, Martín-Gamboa M, Iribarren D. Sustainability-oriented efficiency of retail supply chains: A combination of Life Cycle Assessment and dynamic network Data Envelopment Analysis. *Sci Total Environ* 2020;705:135977. <https://doi.org/10.1016/j.scitotenv.2019.135977>.
- [37] Pachar N, Darbari JD, Govindan K, Jha PC. Sustainable performance measurement of Indian retail chain using two-stage network DEA. *Ann Oper Res* 2022;315: 1477–515. <https://doi.org/10.1007/s10479-021-04088-y>.
- [38] Cruz Roche I, Romero J, Sellers-Rubio R. Retail services efficiency: impact of country-specific factors. *Int J Ret Distrib Manage* 2019;47:774–92. <https://doi.org/10.1108/IJRDM-12-2018-0275>.
- [39] Kongar E, Adebayo O. Impact of Social Media Marketing on Business Performance: A Hybrid Performance Measurement Approach Using Data Analytics and Machine Learning. *IEEE Eng Manage Rev* 2021;49:133–47. <https://doi.org/10.1109/EMR.2021.3055036>.
- [40] Beck N, Rygl D. Categorization of multiple channel retailing in Multi-, Cross-, and Omni-Channel Retailing for retailers and retailing. *J Retail Consum Serv* 2015;27: 170–8. <https://doi.org/10.1016/j.jretconser.2015.08.001>.
- [41] Cocco H, Demoulin NTM. Designing a seamless shopping journey through omnichannel retailer integration. *J Bus Res* 2022;150:461–75. <https://doi.org/10.1016/j.jbusres.2022.06.031>.
- [42] Shen Y, Zhang Q, Zhang Z, Ma X. Omnichannel retailing return operations with consumer disappointment aversion. *Oper Res Perspect* 2022;9:100253. <https://doi.org/10.1016/j.orp.2022.100253>.
- [43] Klumpp M, Loske D. Sustainability and Resilience Revisited: Impact of Information Technology Disruptions on Empirical Retail Logistics Efficiency. *Sustainability* 2021;13:5650. <https://doi.org/10.3390/su13105650>.
- [44] Kao L-J, Chiu C-C, Lin H-T, Hung Y-W, Lu C-C. Evaluating the Digital Transformation Performance of Retail by the DEA Approach. *Axioms* 2022;11:284. <https://doi.org/10.3390/axioms11060284>.
- [45] Marinho BFD, Costa Melo I. Fostering Innovative SMEs in a Developing Country: The ALI Program Experience. *Sustainability* 2022;14:13344. <https://doi.org/10.3390/su142013344>.
- [46] Queiroz GA, Alves Junior PN, Costa Melo I. Digitalization as an Enabler to SMEs Implementing Lean-Green? A Systematic Review through the Topic Modelling Approach. *Sustainability* 2022;14:14089. <https://doi.org/10.3390/su142114089>.
- [47] Hernández-Lerma O, Laura-Guarachi LR, Mendoza-Palacios S, González-Sánchez D. An introduction to optimal control theory, 76. Cham: Springer International Publishing; 2023. <https://doi.org/10.1007/978-3-031-21139-3>.
- [48] Bertsekas DP. *Dynamic programming and optimal control*. 2nd ed., I. Athena Scientific; 2005.
- [49] Sethi SP, Thompson GL. *Optimal control theory : applications to management science and economics*. Springer; 2006.
- [50] Thompson GL, Sethi SP. Turnpike Horizons for Production Planning. *Manage Sci* 1980;26:229–41. <https://doi.org/10.1287/mnsc.26.3.229>.
- [51] Holt CC, Modigliani F, Muth JF, Simon HA. *Planning production, inventories, and work force*. New Jersey: Prentice-Hall; 1960.
- [52] Farrell MJ. The measurement of productive efficiency. *J Roy Statist Soc Ser C* 1957; 120:253–90.
- [53] Cooper WW, Seiford LM, Tone K. *Data envelopment analysis a comprehensive text with models, applications, references and DEA-Solver software*. 1st ed. Springer; 2007. <https://doi.org/10.1007/978-0-387-45283-8>.
- [54] Charnes A, Cooper WW, Rhodes E. Measuring the efficiency of decision making units. *Eur J Oper Res* 1978;2:429–44. [https://doi.org/10.1016/0377-2217\(78\)90138-8](https://doi.org/10.1016/0377-2217(78)90138-8).
- [55] Fare R, Grosskopf S, Norris M, Zhang Z. *Productivity Growth, Technical Progress, and Efficiency Change in Industrialized Countries*. *Am Econ Rev* 1994;84:66–83.
- [56] Gonçalves JNC, Sameiro Carvalho M, Cortez P. Operations research models and methods for safety stock determination: A review. *Oper Res Perspect* 2020;7: 100164. <https://doi.org/10.1016/j.orp.2020.100164>.
- [57] Costa Melo I, Alves Junior PN, Calfei JS, Kodama TK, Nagano MS, Rebelatto DAN. A performance index for traditional retailers incorporating digital marketplace: benchmarking through data envelopment analysis (DEA). *J Org Comput Electron Commer* 2023;1–21. <https://doi.org/10.1080/10919392.2023.2177070>.
- [58] Ton Z, Raman A. The Effect of Product Variety and Inventory Levels on Retail Store Sales: A Longitudinal Study. *Prod Oper Manag* 2010;19:546–60. <https://doi.org/10.1111/j.1937-5956.2010.01120.x>.
- [59] Martínez S, Errasti A, Rudberg M. Adapting Zara's 'Pronto Moda' to a value brand retailer. *Prod Plan Contr* 2015;26:723–37. <https://doi.org/10.1080/09537287.2014.971526>.
- [60] Dana JD, Petrucci NC. Note: The Newsvendor Model with Endogenous Demand. *Manage Sci* 2001;47:1488–97. <https://doi.org/10.1287/mnsc.47.11.1488.10252>.
- [61] Borle S, Boatwright P, Kadane JB, Nunes JC, Galit S. The Effect of Product Assortment Changes on Customer Retention. *Market Sci* 2005;24:616–22. <https://doi.org/10.1287/mksc.1050.0121>.
- [62] Kesavan S, Mani V. The relationship between abnormal inventory growth and future earnings for U.S. public retailers. *Manufact Serv Oper Manage* 2013;15: 6–23. <https://doi.org/10.1287/msom.1120.0389>.
- [63] IFRS - IAS 16 Property, Plant and Equipment n.d. <https://www.ifs.org/issued-standards/list-of-standards/ias-16-property-plant-and-equipment/> (accessed January 23, 2023).
- [64] Sombultawe K, Boon-itt S. Marketing-operations alignment: A review of the literature and theoretical background. *Oper Res Perspect* 2018;5:1–12. <https://doi.org/10.1016/j.orp.2017.11.001>.
- [65] Melacini M, Perotti S, Rasini M, Tappia E. E-fulfilment and distribution in omnichannel retailing: a systematic literature review. *Int J Phys Distrib Logist Manage* 2018;48:391–414. <https://doi.org/10.1108/IJPDLM-02-2017-0101>.
- [66] *Financial Times*. Magazine luiza's e-commerce strategy. *Financial Times*; 2013.
- [67] Zervopoulos PD, Brisimi TS, Emrouznejad A, Cheng G. Performance measurement with multiple interrelated variables and threshold target levels: Evidence from retail firms in the US. *Eur J Oper Res* 2016;250:262–72. <https://doi.org/10.1016/j.ejor.2015.08.055>.
- [68] Liu J, Gong Y (Yale), Zhu J, Zhang J. A DEA-based approach for competitive environment analysis in global operations strategies. *Int J Prod Econ* 2018;203: 110–23. <https://doi.org/10.1016/j.ijpe.2018.05.029>.
- [69] Cook WD, Tone K, Zhu J. Data envelopment analysis: Prior to choosing a model. *Omega (United Kingdom)* 2014;44:1–4. <https://doi.org/10.1016/j.omega.2013.09.004>.
- [70] Setiawan S, Arif D, Mahmudah S, Agustina H, Martah V. The effect of supply chain management on multi-channel retaining and business performance. *Uncert Supp Chain Manage* 2021;9:823–30. <https://doi.org/10.5267/j.uscm.2021.8.007>.
- [71] Iglesias-Pradas S, Acquila-Natale E, Del-Río-Carazo L. Omnichannel retailing: a tale of three sectors. *Econ Res-Ekonomska Istraživanja* 2022;35:3305–36. <https://doi.org/10.1080/1331677X.2021.1991825>.
- [72] Acquila-Natale E, Chaparro-Peláez J, Del-Río-Carazo L, Cuenca-Enrique C. Do or Die? The Effects of COVID-19 on Channel Integration and Digital Transformation of Large Clothing and Apparel Retailers in Spain. *J Theor Appl Electron Commer Res* 2022;17:439–57. <https://doi.org/10.3390/jtaer17020023>.
- [73] Agudelo Ochoa AM, Marín Colorado PA. Librerías y librerías independientes y de libros leídos en Bogotá y Medellín. Tipologías y funciones. *Revista Interamericana de Bibliotecología* 2022;45. <https://doi.org/10.17533/udea.rib.v45n2e344809>.
- [74] Ayuningtyas P, Saraswati LA. The Transition of Independent Bookselling to Digital Space: A Case Study of Transit Bookstore's Instagram. In: 2022 International Seminar on Application for Technology of Information and Communication

- (iSemantic). IEEE; 2022. p. 238–43. <https://doi.org/10.1109/iSemantic55962.2022.9920368>.
- [75] Pachar N, Gupta A, Jha PC. Bi-level programming DEA approach for efficiency evaluation: A case study of Indian electronics retail stores. *Yugoslav J Oper Res* 2020;30:461–81. <https://doi.org/10.2298/YJOR191124026P>.
- [76] Steklova T.N., Leshcheva M.G., Uryadova T.N., Steklov A.N., Batishcheva E.A. Impact of the COVID-19 Pandemic on the Development of Online Retailing in Russia, 2022, p. 477–81. https://doi.org/10.1007/978-3-031-04289-8_81.