

Bouveret, Géraldine; Dumitrescu, Roxana; Tankov, Peter

Article

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Operations Research Perspectives

Provided in Cooperation with:

Elsevier

Suggested Citation: Bouveret, Géraldine; Dumitrescu, Roxana; Tankov, Peter (2022) : Technological change in water use: A mean-field game approach to optimal investment timing, Operations Research Perspectives, ISSN 2214-7160, Elsevier, Amsterdam, Vol. 9, pp. 1-13, <https://doi.org/10.1016/j.orp.2022.100225>

This Version is available at:

<https://hdl.handle.net/10419/325722>

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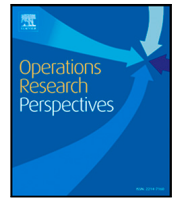
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Technological change in water use: A mean-field game approach to optimal investment timing[☆]

Géraldine Bouveret^{a,*}, Roxana Dumitrescu^b, Peter Tankov^c

^a Nanyang Technological University, School of Physical and Mathematical Sciences, 21 Nanyang Link, 637371, Singapore

^b King's College London, Department of Mathematics, Strand, London, WC2R 2LS, United Kingdom

^c ENSAE, Institut Polytechnique de Paris, 5 avenue Henry Le Chatelier, 91120 Palaiseau, France

ARTICLE INFO

Keywords:

OR in natural resources
Mean-field game
Technology investment
Strategic timing

ABSTRACT

The need for clean water is expected to substantially increase while further reductions of water availability in sufficient quantity and quality are projected owing to climate change and anthropogenic activities. Accordingly, the debate on water security has recently intensified and reached the intergovernmental arena. Industry is, in particular, one of the largest (non-consumptive) water users, accountable for massive toxic wastewater discharges and facing stringent and costly environmental oversight. However, the management of reservoirs is intricate and operational research must be further expanded to design tools that enhance water security while improving operators' profitability.

We therefore consider a game-theoretic framework to study the strategies adopted by a large group of similar producers sharing a water reservoir for their manufacturing activities. Each operator faces random demand for its outputs and chooses the optimal time to invest in a technology that ends its reliance on the reservoir. This technology introduces cost saving opportunities for the operator and benefits for the environment. Each producer therefore solves a so-called optimal stopping problem, and all problems are coupled through the reservoir level. We formulate the problem of finding a Nash equilibrium as a mean-field game (MFG) of optimal stopping. We then apply the model to the paper milling industry, an extensive water user facing a tightening of environmental regulations. This paper provides fresh insights into how to rethink the problem of technological change and water management, by offering an innovative application of operational research that builds on recent mathematical developments made in MFG theory.

1. Introduction

“When the well is dry, we know the worth of water”
Franklin, Benjamin [1]

Water ensures life on Earth and has a critical role in human development. However, the speed with which human activities are exhausting the planet's water resources is “alarming” [2, p. 5]. The WEF [3,4] has warned that water crises will be the most important risk that the world will experience over the next decade in terms of potential consequences and called for an improved water governance. Compared to developed countries, most developing nations are exposed to water insecurity. This is mainly explained by a lack of investments in water security. However, developed nations must continue to adapt and invest in order to sustain water security in the context of “climate change, deteriorating infrastructure, economic development, demographic change, and

rising environmental quality expectations” [5, p. 14]. It is now widely acknowledged that water is an economic good [6–8]. The asymmetry between demand and supply of freshwater is a real challenge. In some countries policymakers may be forced to take tough decisions on the “balance of water availability” for agriculture, industry, municipalities, and the environment [2, p. 5]. Reaching and maintaining water security is therefore a growing and complex global challenge. It is thus critical to design the best ways to sustainably manage water resources to avoid major conflicts that could severely affect societies.

Accordingly, this paper considers a water resource sharing problem to determine some potential water management strategies. In particular, the study conducted here is concerned with the raising

[☆] A preliminary version of the paper has been presented at the 2nd Annual Conference of Global Research Alliance for Sustainable Finance and Investment (GRASFI) and shortlisted for the GRASFI Best Novel Methods Paper Prize.

* Corresponding author at: Nanyang Technological University, School of Physical and Mathematical Sciences, 21 Nanyang Link, 637371, Singapore.

E-mail addresses: geraldine.bouveret@ntu.edu.sg (G. Bouveret), roxana.dumitrescu@kcl.ac.uk (R. Dumitrescu), peter.tankov@polytechnique.org (P. Tankov).

<https://doi.org/10.1016/j.orp.2022.100225>

Received 3 September 2021; Received in revised form 7 January 2022; Accepted 13 January 2022

Available online 22 January 2022

2214-7160/© 2022 The Author(s).

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issues on both water quantity and quality and focuses on industry, which together with municipalities is the largest non-consumptive user of water [9]. Industry is responsible for harmful wastewater discharges and faces a tightening of environmental oversight, impeding its profitability. This study therefore intends to build a model that both helps water management specialists in local authorities fine-tune economic incentives for operators to invest in new technologies that maintain water security, and assists operators in accurately identifying those incentives and timely investing in those assets to improve their profitability. More precisely, this study aims to model the behavior of a large number of agents which belong to the same industry but have specific water needs, and which must share limited (fresh) water for their manufacturing activities (according to some pre-defined rules and under a specific economic and regulatory framework). Each agent has the opportunity to invest at a chosen time in an alternative technology in order to end its reliance on the collective reservoir (e.g. for generating steam and electricity, cooling and cleaning equipment, washing materials, transporting energy or material, or processing chemicals) and build a water conservation/independence program (including e.g. closed-cycle systems, treatment units, and/or rainwater recovery systems). Such a program offers the agent cost saving opportunities by reducing the risk of production disruption due to water scarcity¹ as well as the costs related to water use from pumping to discharging. But more than just inducing economic benefits, water conservation aims to preserve both water levels, by reducing water withdrawals, and water quality, by limiting wastewater discharges and their related pollution issues. The analysis of water management, unlike the management of non-renewable resources, involves attention to the sustainable use of the resource over time. In this study, we neither specify the type of industry we are dealing with (except in the application) nor the type of technologies and financing tools employed. The results are, however, applied to the paper milling industry whose activity relies heavily on water, which has a serious history of water pollution through wastewater discharges, and which is subject to a stringent environmental supervision.

This analysis relies on mean-field game (MFG) theory, allowing for the study of *differential games* involving a large number of competing players, and addresses the question of the agents' optimal time to invest into a technology that serves the purpose of water conservation (see Section 2.3). As far as we know, this question has been overlooked in the water literature (see Section 2.1). The model we use is the so-called *MFG of optimal stopping*, offering an alternative to real option (RO) theory, which accounts for the interactions between the parties involved. However, because of the technicalities of this class of models involving random times and the lack of mathematical tools to handle them, this formalism has been circumvented by economists or operational researchers. Nevertheless, the authors recently developed in [10] new mathematical tools to manage the modeling issues arising in the treatment of a similar class of problems. This study thus builds on these developments to bridge the gap between MFG theory and relevant applications in water security and provides new insights into technological change in water use.

The paper is organized as follows. In Section 2 we provide the reader with some background on game and MFG theory and its use in the water-related literature, while in Section 3 we detail the mathematical modeling of the water resource sharing problem of interest. Then, in Section 4, we apply the model to the paper milling industry. Section 5 offers concluding remarks and suggestions for further research. A supplementary material with a mathematical appendix and the details of the data used in Section 4 is also provided.

¹ In this paper, water scarcity must be understood as a lack of water of sufficient quantity and quality.

2. Related literature

Before defining the water resource sharing problem of interest here, we first provide the reader with some background on game and MFG theory as the framing of our problem falls within this concept.

2.1. Water management & game theory

Game theory, first established by Morgenstern and Von Neumann [11], and then expanded after World War II, is “the mathematical study of competition and cooperation” [12, p. 226]. It integrates the decisions taken by all the agents involved in a system to predict its evolution.

In game theory each agent is willing to achieve its own objectives by anticipating others' decisions, rather than the “best system-wide outcome”, and is aware of the impacts of its and others' decisions on the system [12, p. 225]. A game-theoretic framework offers a “realistic simulation of stakeholders' interest-based behavior” and a dynamic structure of the problem which evolves with agents' reactions [12, p. 226]. The “self-optimizing attitude” of agents may lead to a non-cooperative behavior even when the benefits brought by cooperative behavior are higher [12, p. 226]. Therefore, the overall outcome of a game may not have been planned by any agent [12], depends on the combination of all the behaviors adopted by players, and may not be *Pareto-optimal*. The literature separates *zero- (or constant-) sum games* in which players have conflicting interests from *non-zero-sum games* in which players may cooperate.

Game theory therefore provides valuable insights for resource sharing problems as it offers a framework for the analysis of the impacts of regulatory policies on stakeholders' behaviors. As a result, many researchers have used this framework to study the allocation of environmental resources, along with the costs and benefits of doing so among users (e.g. [13–16]), groundwater management (e.g. [17–19]), water allocation among transboundary users (e.g. [20–22]), water quality management (e.g. [23–25]), and other types of water resource management problems (e.g. [26,27]). We refer the reader to Madani [12] for a review of game theory applications for water. However, despite the growing desire to apply game theory to water-related problems, research remains scant as water scholars may still prefer to circumvent this formalism because of its technicalities and the need for new mathematical tools to handle game complexities. *Cooperative game theory* has actually been more widely explored by water engineers as it is better understood [12]. Indeed, solutions may sometimes be close to those obtained with the optimization, under a set of constraints, of a single-objective function that integrates the conflicting goals within the whole system but assumes that all players are willing to follow the system's optimal solutions [12].

However, regardless of the existence of cooperative Pareto-optimal solutions, non-cooperative actions from stakeholders have been witnessed in many water resource sharing problems worldwide and generated the so-called “tragedy of the commons” outcomes [28, p. 1243]. It is therefore critical to develop models that provide a deep understanding of the structure of games and an accurate prediction of players' reactions to better analyze the consequences of the regulatory measures enforced. We are thus interested in computing *Nash equilibria*, which are the solutions in which no agent can increase its gain by only modifying its own strategy [29], in a *non-cooperative game* in which, in particular, investment decisions are at play.

2.2. Investment games

RO theory is often used to assess an agent's decision to invest in new technologies that reduce water use (see e.g. [30,31]) but it fails to provide a game-theoretic framework. A promising avenue for the analysis of complex interdependent investment problems consists in combining game theory concepts and RO valuation techniques. A

strand of the literature has therefore focused on modeling the so-called *investment games*, also denominated *RO games*. The two prevalent types are *pre-emption games* and *war-of-attrition games*. They are usually formulated as non-zero-sum games and respectively characterized by a first- and second-mover advantage. In particular, the game players are the firms holding the option to invest, the game strategies are the decisions to invest or defer, while the game payoffs are the firms' value functions, usually depending on underlying stochastic variable(s). Smets [32] initiated this literature with his study of the option value of delaying foreign direct investments versus the value of such investments in a symmetric duopolistic market. His preliminary study on symmetric duopoly market was followed by Dixit et al. [33, Chapter 9] and Grenadier [34]. Moreover, Smit and Ankum [35] studied the impact of competition on investment timing and project value. Later, Huisman [36] developed technological change leader–follower game models. These games have mainly been applied to research and development investments, investment (timing) in new technologies, as well as production capacity choices. We refer the reader to Azevedo and Paxson [37] for a review of two decades of RO game models.

Surprisingly, investment games remain scant in the water literature. We can, however, quote the work of Suttinon et al. [38] for an application of option games to water infrastructure investment. We thus investigate the game of investment timing that underlies technological change in water use. However, we focus on the case where the number of parties of the same nature involved in the game is significant, raising an additional difficulty since games with a large number of players are in general intractable. Indeed, a game with n non-cooperative players will involve solving coupled systems of n nonlinear partial differential equations (PDEs) that is one per player. We thus rely on MFG theory which, as detailed in Sections 2.3 and 3.1, allows to handle the limiting game, that is the game for which the number of small players tends to infinity, by solving only a coupled system of two PDEs.

2.3. MFG theory

The MFG approach was first considered by Lasry and Lions [39, 40, 41] and Huang et al. [42, 43] to investigate Nash equilibria when the number of agents tends to infinity for differential games that are “invariant by permutation of similar agents” [44, p. 159]. This wording was borrowed from mean-field theory in physics, which examines a significant number of interacting particles whose individual impact upon the system is negligible. The invariance by permutation is critical as it allows, “in the move to the limit”, the use of the distribution of agents in the state space in order to represent “other agents” for “each atomized agent” [44, p. 159]. Given this distribution, each agent usually solves an optimization problem (the so-called optimal control problem) to maximize its profits over a specific time horizon by playing with admissible controls that influence the evolution of the system. However, the payoff (and possibly the diffusion) depends on the distribution of agents, which is in turn governed by the decisions of all players. Each agent therefore builds a strategy by considering the “anticipated distribution of strategies” and the “contingent decisions of other agents” [44, p. 159]. The MFG approach therefore consists in constructing a “mutually consistent pair of (i) the mass effect and (ii) the individual strategies such that the latter not only each constitute an optimal response to the mass effect but also collectively produce that mass effect” [42, p. 222–223].

In mathematical theory, the analytic approach (e.g. [39–41] or [42]) consists in characterizing equilibria by means of a coupled system of nonlinear partial differential equations: a Hamilton–Jacobi–Bellman (HJB) equation and a Fokker–Planck equation. The HJB equation is backward in time and describes the optimization problem solved by each agent for a given distribution, while the Fokker–Planck equation is forward in time and describes the evolution of the distribution which is affected by the optimal controls. Alternatively, the probabilistic approach (e.g. [45–48]; or [49]) relies on a coupled system of

forward–backward stochastic differential equations of McKean–Vlasov type. MFG theory is a very appealing approach as it builds on the existing “paradigm in economics” (e.g. utility maximization, rational expectations, invariance by permutation of similar agents, and atomization of agents in a continuum) while integrating the impacts of “social interaction between agents”, in contrast to classical economic models [44, p. 158]. More precisely, this theory provides the “mathematical toolbox” allowing to model phenomena usually overlooked because of the absence of appropriate “analytic and numerical tools” [44, p. 158] and can be very useful in the economics of sustainability. In particular, Guéant et al. [44] have applied MFG theory to the production of oil resource and we adapt this approach to tackle the problem of technological change in water use. We also refer the reader to Chan and Sircar [50] and Wang and Huang [51] for additional applications of MFG theory and a survey of the literature on MFG.

3. The model

3.1. Problem definition

We study a problem of water resource sharing in industry and consider a large group of producers sharing, under a specific economic and regulatory framework, a water reservoir for their manufacturing activities (such as freshwater from a river). Producers have the same characteristics and therefore belong to the same *class*. Individually they have a negligible impact on the system, thereby reflecting the perfect competitive framework used in economics. Each producer faces a random demand for its product (and therefore for (fresh)water) which evolves over time according to a stochastic differential equation. Producers may withdraw the required quantity of resource if the reservoir level is sufficient. If the reservoir does not meet the total demand of all producers, resource will be shared among producers proportionally to their demand levels, and the producers will suffer from a production disruption. Each producer has the opportunity to invest in or switch to, at a random time and at a cost, a new technology that stops its reliance on the reservoir. After the switch, the producer is assumed to always be able to meet demand.² This framing defines the rules of a game falling within the category of (non-cooperative) differential games, in which each agent interacts and tries to find its best strategy. We aim to compute Nash equilibria.

In this case, agents must solve an optimal stopping problem in the sense that they must all maximize their reward by looking for the optimal time of switch to the alternative technology in order to increase their profits. In other words, we must consider a MFG of optimal stopping. Such games of timing have only recently been studied in the literature with, in particular, Carmona et al. [52] and Nutz [53] who considered a probabilistic approach with an application to bank runs. However, in our model, the interaction occurs through the proportion of total water demand which may be satisfied given the level of the reservoir. As a consequence, and in contrast to Carmona et al. [52] and Nutz [53], we are facing a war-of-attrition game in which the

² As intimated in Section 1, “new technology” is a generic term that stands for a water independence program that may involve: manufacturing technologies with lower water rates, technologies that collect and treat water from other catchments (e.g. rainwater collection systems, desalination plants), as well as technologies that re-cycle and re-use water (closed-loop systems involving treatment units (e.g. reverse osmosis water system)). Such a program gives the opportunity to an agent to independently and sustainably manage the water resources at its disposal. The model thus suggests that, when a switch occurs, such a program is developed. It is therefore natural to assume that, in this case, the agent has fully hedged its risk of production disruption due to a lack of water. This paper is not concerned with the ways an agent is building this program nor the ways the demand is met after a switch. This paper rather intends to only capture and investigate the mechanisms underlining the decision to switch to that “new technology”.

departure of agents creates an incentive for the agents still in the game (i.e. using the old technology and therefore relying on the reservoir) to stay (i.e. not to switch); see Section 4.2.4. Our analysis therefore builds on the recent theoretical developments made by Bouveret et al. [10] which provides the mathematical resolution of problems sharing similar characteristics in great generality. Their method is based on a relaxed solution approach and their results extend the work of Bertucci [54] relying on an analytic approach.

3.2. Mathematical modeling

We fix a finite time horizon $0 \leq T < \infty$. We consider n producers which are using at time 0, for manufacturing purposes, a technology (hereinafter *technology 1*) involving a dependence on a collective (fresh)water reservoir of finite capacity (flow). They have the possibility to switch at some future (random) date to an innovative technology (hereinafter *technology 2*) that stops their reservoir reliance. The number of producers is fixed: there is no free entry. The reservoir size scales linearly with n and equals nZ_t , where Z_t is a positive water flow. At every time $t \in [0, T]$, producer i faces positive demand for water X_t^i that cannot exceed a given threshold $0 < L < \infty$. Demand already incorporates the conversion factor between the number of outputs ordered by customers and the quantity of water needed to produce the outputs. We assume that output demand never exceeds the production capacity of the operator. Demand for water follows the geometric dynamics

$$\frac{dX_t^i}{X_t^i} = \mu dt + \sigma dW_t^i, \quad X_0^i = x^i \in \mathcal{O} := (0, L), \quad (3.1)$$

where W^1, \dots, W^n are independent Brownian motions, and $\mu \in \mathbb{R}$ and $\sigma > 0$ are constant. We assume that an agent automatically stops using technology 1 once its demand process exits the domain \mathcal{O} and starts using technology 2. We thus consider $\tau_{\mathcal{O}}^i$, the exit time from the domain \mathcal{O} of the state variable X^i . We also consider the time at which producer i decides to switch to technology 2, denoted by τ^i . In particular, τ^i is valued in $[0, T]$ and is a stopping time with respect to the (completed) filtration generated by the Brownian motions of all agents.

If the reservoir flow is insufficient to meet the demand of all producers, the resource is shared among them in proportion to the demand they face. This means that, with technology 1, the output of the i th producer is given by

$$Q_t^i := \begin{cases} X_t^i, & \text{if } Z_t \geq \frac{1}{n} \sum_{j=1}^n X_t^j \mathbf{1}_{t < \tau^j \wedge \tau_{\mathcal{O}}^j} \\ \frac{Z_t}{\frac{1}{n} \sum_{j=1}^n X_t^j \mathbf{1}_{t < \tau^j \wedge \tau_{\mathcal{O}}^j}} X_t^i & \text{otherwise} \end{cases},$$

where for a random time τ , $\mathbf{1}_{\tau > t}$ is the indicator function being equal to 1 if $\tau > t$ and 0 otherwise, and where, for a given a, b , $a \wedge b := \min(a, b)$. In other words, $Q_t^i = \omega_t X_t^i$, where

$$\omega_t := \frac{Z_t}{\frac{1}{n} \sum_{j=1}^n X_t^j \mathbf{1}_{t < \tau^j \wedge \tau_{\mathcal{O}}^j}} \wedge 1, \quad (3.2)$$

is the proportion of total demand which may be satisfied at time t . Recall that in practice industrial water consumers need to obtain a water use permit which usually specifies a fixed maximum water intake rate that may be revised by authorities on an annual basis, depending on the state of the reservoir and the needs of the consumer.

Under this framework, each producer i aims to find the optimal stopping time τ^i that maximizes its reward functional and thus to solve for $\rho > 0$, and p, \hat{p}, K , and \bar{p} real-valued constant parameters

$$\sup_{\tau} \mathbb{E} \left[\int_0^{\tau \wedge \tau_{\mathcal{O}}^i} e^{-\rho t} p \omega_t X_t^i dt - \int_0^{\tau \wedge \tau_{\mathcal{O}}^i} e^{-\rho t} \hat{p} (1 - \omega_t) X_t^i dt - e^{-\rho \tau \wedge \tau_{\mathcal{O}}^i} K + \int_{\tau \wedge \tau_{\mathcal{O}}^i}^T e^{-\rho t} \bar{p} X_t^i dt \right]. \quad (3.3)$$

Here ρ stands for the discount rate, usually the weighted average cost of capital (WACC). The first term represents the discounted net income of the producer before switching, where p is the net income before switching per unit of water withdrawn. The second term is the discounted cost of production disruption due to a lack of water, where \hat{p} is the net cost of disruption per unit of water withdrawn. The third term is the discounted fixed cost of switch. Finally, the fourth term is the discounted net income after switching, where \bar{p} is the net income after switching per unit of water withdrawn. The constants p, \hat{p} , and \bar{p} therefore account for any costs (e.g. production and regulatory costs, taxes, and penalties), revenues (sales) and subsidies (if any) per unit of water withdrawn. In particular, \hat{p} only accounts for costs as manufacturing disruption does not create revenues. The constant K also integrates any regulatory subsidies. Typically, we expect p, \bar{p}, \hat{p} and K to be non-negative, but this is not necessary for the problem to make sense, in particular, a negative K corresponds to the case where the producer is paid a subsidy to switch to the new technology. All the parameters therefore indirectly constitute a specific economic and regulatory framework. We highlight that, by considering the WACC and net income, we account for the effects of financing and accounting decisions in the parameters. The producers interact with one another through ω_t , the proportion of total demand which may be satisfied given the level of the reservoir. The program (3.3) describes a non-cooperative game of investment timing or optimal stopping. We are therefore interested in computing Nash equilibria, whose definition for the game with n players is reminded below, and the MFG approach is used in this regard.

Definition 3.1 (Nash Equilibrium). A set of admissible strategies (τ^1, \dots, τ^n) is said to be a Nash equilibrium of the game of investment timing if

$\forall 1 \leq i \leq n$, for any admissible strategy η_i ,

$$J^i(\tau^1, \dots, \tau^n) \geq J^i(\tau^1, \dots, \tau^{i-1}, \eta_i, \tau^{i+1}, \dots, \tau^n),$$

where for any admissible set of strategies $(\theta^1, \dots, \theta^n)$,

$$J^i(\theta^1, \dots, \theta^n) := \mathbb{E} \left[\int_0^{\theta^i \wedge \tau_{\mathcal{O}}^i} e^{-\rho t} (p \omega_t X_t^i - \hat{p} (1 - \omega_t) X_t^i) dt - e^{-\rho \theta^i \wedge \tau_{\mathcal{O}}^i} K + \int_{\theta^i \wedge \tau_{\mathcal{O}}^i}^T e^{-\rho t} \bar{p} X_t^i dt \right],$$

with ω_t given by (3.2) but with θ^i in place of τ^i for every i .

Before moving forward into the description of the MFG approach adopted here, we end this section with a specific example for which there is no disruption of water supply and the Nash equilibrium thus corresponds to $\omega_t \equiv 1$.

Example of explicit Nash equilibrium. We assume that $T = \infty$. Moreover, we let $\mathcal{O} = (0, \infty)$ (so that $\tau_{\mathcal{O}}^i = +\infty$ for all i), $p + \hat{p} \geq 0$, and we suppose that $\rho > \mu$ for integrability purposes. Moreover, to simplify formulas, we assume that the initial demand level of every agent equals 1. Finally, we assume that (i) $\bar{p} > p$ and (ii) $Z_t \geq C$ for all $t \geq 0$, where $C := \frac{\rho - \mu}{\bar{p} - p} (K + b)$ with b the constant defined in Proposition 1.1 of the supplementary material. In other words, we assume that (i) the net income per unit of withdrawn water obtained from producing with technology 2 is higher than the one obtained from producing with technology 1, and (ii) each agent's share of the reservoir is higher than a certain constant. However, despite (ii), there may not be enough water to satisfy the demand of all agents in all scenarios. We are going to show that, in this setting, a Nash equilibrium is given by

$$\tau^i = \inf \left\{ t \geq 0 : X_t^i \geq C \right\}. \quad (3.4)$$

To this aim, we assume that agents $j = 1, \dots, i-1, i+1, \dots, n$ use candidate optimal strategies τ^j , while the i th agent uses the strategy θ . As $p + \hat{p} \geq 0$, the optimization program of agent i satisfies

$$\sup_{\theta} J^i(\tau^1, \dots, \tau^{i-1}, \theta, \tau^{i+1}, \dots, \tau^n) \leq \sup_{\theta} \mathbb{E} \left[-e^{-\rho \theta} K + \int_{\theta}^{\infty} e^{-\rho t} (\bar{p} - p) X_t^i dt \right]$$

$$\begin{aligned}
& + \mathbb{E} \left[\int_0^\infty e^{-\rho t} p X_t^i dt \right] \\
& = \mathbb{E} \left[-e^{-\rho \tau^i} K + \int_{\tau^i}^\infty e^{-\rho t} (\bar{p} - p) X_t^i dt \right] \\
& + \mathbb{E} \left[\int_0^\infty e^{-\rho t} p X_t^i dt \right], \quad (3.6)
\end{aligned}$$

where (3.6) follows from Proposition 1.1 of the supplementary material. On the other hand, as $Z_t \geq C$, the definition (3.4) implies that, by construction, $\omega_t = 1$ for $t \leq \tau^i$. Hence, (3.5) above actually holds with an equality and the strategies (τ^1, \dots, τ^n) form a Nash equilibrium for the n -player game. Interestingly, this example shows that the optimal switching times can be finite even when there is no disruption of water supply.

3.3. A MFG approach of the game of investment timing/optimal stopping

Generally, water supply may be disrupted at equilibrium, and the n -player game can no longer be solved explicitly. As we are interested in studying the case where the resource is shared among a large number of competing agents, we will simplify the original problem and directly consider the limiting situation where the number of players n goes to infinity. This is the MFG approach.

In order to derive the limiting problem, we will now explicit the dependency in n of the variables involved whenever applicable. In particular, we denote by m_t^n the empirical distribution at time t of the demand levels of the agents that are still using technology 1,

$$m_t^n(dx) := \frac{1}{n} \sum_{j=1}^n \delta_{X_t^j}(dx) \mathbf{1}_{t < \tau^j \wedge \tau_{\mathcal{O}}^j},$$

where $\delta_x(\cdot)$ stands for the probability measure putting mass 1 on the singleton $\{x\}$, and observe that m_t^n is not necessarily a probability measure as the total mass may be less than 1. In the limiting regime, the empirical measure m_t^n is expected, from the law of large numbers, to converge towards a deterministic limiting distribution $m_t \in \mathcal{M}(\mathcal{O})$, the space of bounded positive measures on \mathcal{O} . Similarly, the proportion $\omega_t^n \equiv \omega_t$ of the total demand that may be satisfied given the reservoir level and defined in (3.2) should converge to the deterministic proportion $\bar{\omega}_t$. These quantities are related as follows $\bar{\omega}_t = \frac{Z_t}{\int_{\mathcal{O}} x m_t(dx)} \wedge 1$.

In the MFG paradigm the main question is to understand the behavior of the entire population of agents at equilibrium, rather than individual strategies. The main object of interest is thus the distribution of the states of agents and the technologies they use. As there are only two technologies, it is enough to keep track of the distribution m_t of the states of agents still using technology 1. Besides, since all agents are assumed to have identical properties, there is, in the limit, a decoupling effect in the sense that the statistical properties of the state variable of a single agent that still uses technology 1 can effectively approximate the empirical distribution m_t^n . Therefore, m_t can also be seen as the distribution of the possible states of such a single representative agent. The indexation in the state variable is thus dropped and the latter is now identified by its initial value X_0 as follows

$$\frac{dX_t^{X_0}}{X_t^{X_0}} = \mu dt + \sigma dW_t, \quad (3.7)$$

with W a Brownian motion and where X_0 is valued in \mathcal{O} . For the sake of clarity, we will simply write $X \equiv X^{X_0}$ except when we must explicit the integration of X over its initial values. We then denote by $\tau_{\mathcal{O}} \equiv \tau_{\mathcal{O}}(X_0)$ the first exit time of the process X^{X_0} from \mathcal{O} . The MFG approach of the game problem (3.3) thus consists in:

Step 1: Resolving, for a fixed distribution $m \equiv (m_t)_{0 \leq t \leq T}$, the problem of a representative agent which aims to find the optimal stopping time τ that maximizes its reward functional. The program writes

$$\sup_{\tau \in \mathcal{T}([0, T])} \mathbb{E} \left[\int_0^{\tau \wedge \tau_{\mathcal{O}}} e^{-\rho t} (p \bar{\omega}_t X_t - \hat{p}(1 - \bar{\omega}_t) X_t) dt - e^{-\rho \tau \wedge \tau_{\mathcal{O}}} K \right]$$

$$+ \int_{\tau \wedge \tau_{\mathcal{O}}}^T e^{-\rho t} \bar{p} X_t dt \Big], \quad (3.8)$$

where $\mathcal{T}([0, T])$ denotes the set of stopping times with respect to the (completed) filtration generated by the Brownian motion W and valued in $[0, T]$.

Step 2: Computing, given the optimal stopping time (solution to the problem (3.8)) $\hat{\tau}(m, X_0)$ for the agent with initial demand level X_0 and initial measure m_0^* (initial distribution of demand levels), the limiting distribution m_t satisfying

$$m_t(A) = \int m_0^*(dy) \mathbb{P}[X_t^y \in A; t < \hat{\tau}(m, y) \wedge \tau_{\mathcal{O}}(y)], \quad A \in \mathcal{B}(\mathcal{O}), 0 \leq t \leq T, \quad (3.9)$$

where $\mathcal{B}(\mathcal{O})$ is the Borel σ -field generated by all the open sets in \mathcal{O} , or, equivalently, the limiting proportion $\bar{\omega}_t$ satisfying

$$\bar{\omega}_t = \frac{Z_t}{\int m_0^*(dy) \mathbb{E}[X_t^y \mathbf{1}_{t < \hat{\tau}(m, y) \wedge \tau_{\mathcal{O}}(y)}]} \wedge 1, \quad 0 \leq t \leq T.$$

The flow of measures $m \equiv (m_t)_{0 \leq t \leq T}$, which is the fixed point of the mapping defined by the right-hand side of (3.9) is the solution (Nash equilibrium) to the problem of MFG of investment timing/optimal stopping.

Before moving to the computation of a Nash equilibrium, we end this section with a proposition that reduces the program solved in Step 1 and that will be called in the next section.

Proposition 3.2. *The individual optimization program (3.8) is equivalent (up to a constant) to*

$$\sup_{\tau \in \mathcal{T}([0, T])} \mathbb{E} \left[\int_0^{\tau \wedge \tau_{\mathcal{O}}} e^{-\rho t} f(t, X_t, m_t) dt \right], \quad (3.10)$$

where, on $[0, T] \times \mathcal{O} \times \mathcal{M}(\mathcal{O})$,

$$f(t, x, m_t) := x \left[(p + \hat{p}) \left(\frac{Z_t}{\int_{\mathcal{O}} x m_t(dx)} \wedge 1 \right) - \hat{p} - \bar{p} \right] + \rho K. \quad (3.11)$$

Proof. See supplementary material. \square

3.4. Computation of a Nash equilibrium: the relaxed solution approach

The relaxed solution approach, introduced in [10], simplifies the MFG program (Steps 1 and 2) to solely involve terms in m_t , without making any reference to the stopping time of the representative agent. As intimated above, this is natural in the context of stochastic differential games with a large number of players, where the main object of interest is the dynamics of the state variables of the population involved. The relaxed solution approach builds on two observations. First, the objective function of a single-agent optimal stopping problem defined in (3.10) can be written in terms of the following distribution m_t of the possible states of that agent

$$m_t(A) := \int m_0^*(dy) \mathbb{P}[X_t^y \in A; t < \tau(y) \wedge \tau_{\mathcal{O}}(y)], \quad A \in \mathcal{B}(\mathcal{O}), 0 \leq t \leq T,$$

where m_0^* is a given initial measure. More precisely, assuming that the distribution of the states of the population still using technology 1 is \bar{m}_t , the latter objective function writes

$$\mathbb{E} \left[\int_0^{\tau \wedge \tau_{\mathcal{O}}} e^{-\rho t} f(t, X_t, \bar{m}_t) dt \right] = \int_{[0, T] \times \mathcal{O}} e^{-\rho t} f(t, y, \bar{m}_t) m_t(dy) dt. \quad (3.12)$$

In particular, Step 1 implies that, for a given initial measure m_0^* , the latter quantity must be maximized over a set of admissible flows of measures m corresponding to all the possible distributions for the agents involved. The question on how to properly characterize the admissible m in a computational handy way then arises, which leads us to our second observation. Using Itô's formula, we indeed observe that for a

given stopping time τ and initial measure m_0^* , and for any regular and positive test function $u : [0, T] \times \mathcal{O} \rightarrow \mathbb{R}$, one has

$$\begin{aligned} \mathbb{E}[e^{-\rho\tau} u(\tau, X_\tau)] &= \mathbb{E}[u(0, X_0)] + \mathbb{E}\left[\int_0^\tau (\partial_t u + \mathcal{L}u - \rho u)(t, X_t) dt\right] \\ &= \int_{\mathcal{O}} u(0, X_0) m_0^*(dx) \\ &\quad + \int_0^T \int_{\mathcal{O}} (\partial_t u + \mathcal{L}u - \rho u)(t, x) m_t(dx) dt, \end{aligned}$$

where \mathcal{L} is the infinitesimal generator of $(X_t)_{0 \leq t \leq T}$, defined by $\mathcal{L}u := \mu x \partial_x u + \frac{1}{2} \sigma^2 x^2 \partial_x^2 u$, and where, for a regular function h , the operators $\partial_t h$, $\partial_x h$, and $\partial_{xx} h$ stand for the first and second derivative with respect to the first and second variable of h .

Therefore, for a given measure m_0^* , m_t satisfies, for any positive and regular test function u , the constraint

$$\int_{\mathcal{O}} u(0, x) m_0^*(dx) + \int_0^T \int_{\mathcal{O}} (\partial_t u + \mathcal{L}u - \rho u)(t, x) m_t(dx) dt \geq 0. \quad (3.13)$$

We denote by $\mathcal{A}(m_0^*)$ the set of flows of measures in $\mathcal{M}(\mathcal{O})$ that satisfy (3.13). It turns out that Bouveret et al. [10] actually proved (in a more general context) that the value function of a single-agent optimal stopping problem (3.10) is, under adequate conditions, exactly equal to the maximum of the right-hand side of (3.12) over the set $\mathcal{A}(m_0^*)$. This new formulation of the original problem may appear complicated at first sight. However, when the distribution m_t has a sufficiently regular density $m_t(x)$, this infinite-dimensional constraint is equivalent to the Fokker–Planck inequality

$$-\partial_t m_t(x) + \mathcal{L}^* m_t(x) - \rho m_t(x) \geq 0, \quad (t, x) \in [0, T] \times \mathcal{O}, \quad (3.14)$$

that can be easily computed, where \mathcal{L}^* is the adjoint operator of \mathcal{L} , $\mathcal{L}^* u := -\mu \partial_x(xu) + \frac{\sigma^2}{2} \partial_x^2(x^2 u)$, with the notations introduced above. This inequality constraint expresses the fact that m_t is the distribution of the state variable (following the dynamics described by the stochastic differential Eq. (3.7)) of agents (reduced to particles in the limiting regime) that can exit the game at any time. This result is therefore extremely important as it allows to reduce both Step 1 and Step 2 to the following simpler ones:

Step 1 For a fixed flow of measures $\bar{m} \in \mathcal{A}(m_0^*)$, maximize

$$\int_{[0, T] \times \mathcal{O}} e^{-\rho t} f(t, y, \bar{m}_t) m_t(dy) dt,$$

over $m \in \mathcal{A}(m_0^*)$. Let $\Theta(\bar{m})$ be the set of maximizers (which may contain more than one measure).

Step 2 Find a fixed point of Θ , that is a flow of measures m^* such that $m^* \in \Theta(m^*)$. In other words, if the distribution for all agents is equal to m^* , no individual agent has an incentive to choose a distribution different from m^* .

In [10], it is shown that the set of fixed points is non-empty. In general, there is no uniqueness of the equilibrium, but one can show that the value function of the representative agent at the equilibrium is the same for all solutions. In the following paragraphs, we describe the numerical algorithm that we use for computing the fixed point flow of measures m . It is similar to the one proposed in [55].

Computing the best response. To compute the best response for solving Step 1, we discretize the Fokker–Planck inequality in the definition of the set $\mathcal{A}(m_0^*)$, in time and in space.

The process X (water demand of the representative agent) is discretized over the interval $(0, X_{\max})$, using a uniform grid with $N_X + 1$ points. The time interval is also discretized using a uniform grid with $N_T + 1$ points. We define $\Delta X := \frac{X_{\max}}{N_X}$, $X_j := j \Delta X$ for $j = 0, \dots, N_X$, and similarly for the time discretization. As explained above, every flow of measures $m \in \mathcal{A}(m_0^*)$ satisfies, in the sense of distributions, the Fokker–Planck inequality (3.14). This inequality is discretized using the implicit scheme, leading to the following system of inequalities

$$\frac{m_i^j - m_{i+1}^j}{\Delta T} - \mu \frac{X_{j+1} m_{i+1}^{j+1} - X_{j-1} m_{i+1}^{j-1}}{2 \Delta X}$$

$$+ \sigma^2 \frac{X_{j+1}^2 m_{i+1}^{j+1} + X_{j-1}^2 m_{i+1}^{j-1} - 2 X_j^2 m_{i+1}^j}{2 \Delta X^2} - \rho m_{i+1}^j \geq 0,$$

for $0 \leq j \leq N_X$ and $i = 0, \dots, N_T - 1$, and where, for $j = 0$ and $j = N_X$, these formulas are interpreted by assuming that $m_i^{-1} = m_i^{N_X+1} = 0$ for $i = 0, \dots, N_T$. In the above formula, m_i^j denotes the discretized density at the point (T_i, X_j) . The gain functional is similarly approximated by the discrete sum

$$(m_i^j)_{0 \leq i \leq N_T, 0 \leq j \leq N_X} \mapsto \sum_{i=0}^{N_T} \sum_{j=0}^{N_X} e^{-\rho T_i} f(T_i, X_j, \tilde{m}_i^j) m_i^j \Delta X \Delta T.$$

At a given iteration, the best response is then computed by maximizing this functional under the inequality constraints given above as well as the positivity constraints, using an interior point method for linear programming.

Approximating the MFG equilibrium. To approximate the equilibrium flow of measures m^* for solving Step 2, we successively take steps of decreasing size towards the best response, until a desired convergence criterion is met. The following algorithm details the procedure.

- Choose initial value $m^{(0)} \in \mathcal{A}(m_0^*)$.
- For $i = 1 \dots N_{\text{iter}}$,
 - Compute the best response $\tilde{m}^{(i)} = \arg \max_{m \in \mathcal{A}(m_0^*)} \int_{[0, T] \times \mathcal{O}} e^{-\rho t} f(t, y, m_t^{(i-1)}) m_t(dy) dt$.
 - Choose the step size parameter: $s^{(i)} = \frac{1}{i}$.
 - Update the measure $m^{(i)} = (1 - s^{(i)}) m^{(i-1)} + s^{(i)} \tilde{m}^{(i)}$.

The number of steps of the algorithm may be fixed or chosen based on a convergence criterion. In our implementation, we monitor the relative improvement of the best response, defined by

$$\mathcal{E}(m^{(k)}) = \max_{m \in \mathcal{A}(m_0^*)} \int_{[0, T] \times \mathcal{O}} e^{-\rho t} f(t, y, m_t^{(k)}) (m_t - m_t^{(k)})(dy) dt.$$

Clearly, $\mathcal{E}(m) \geq 0$ for all $m \in \mathcal{A}(m_0^*)$ and the situation when $\mathcal{E}(m) = 0$ corresponds to the Nash equilibrium. In general, $\mathcal{E}(m)$ corresponds to the increase of gain of all agents if they move from the current value m to their best response. Numerically, we observe that $\mathcal{E}(m^{(k)})$ converges to zero at the rate $\frac{1}{k}$. The algorithm thus produces an ε -Nash equilibrium in $O(\varepsilon^{-1})$ steps.

4. Case study: the paper milling industry in Wisconsin

4.1. Rationale for the model application

A key industry at the national and state level. In 2016, over half of the world's production (411 million metric tons) was produced by China, the United States (US), and Japan [56]. Wisconsin, which benefits from the presence of major waterways and the proximity to large Canadian timber stocks, is the first US state producer and hosted 11% of the US paper mills in 2017 [57]. It employs about 31,047 people in this industry [58].

An industry with high levels of water use. In 2016, paper manufacturing accounted for 5% of the total water withdrawal in Wisconsin [59]. Water is used for the washing and transport of pulp, the dilution and preparation of process chemicals, the generation of steam and electricity, the conveyance of energy and raw materials within the mill, and the cleaning and cooling of equipment. Water footprint depends on the mill age, equipment, and products produced. After production, most of the water used is discharged back into waterways as waste.

An environmental rationale for water saving. Paper mills are major sources of pollution in rivers through the discharge of hazardous material by-products [60]. However, despite large groundwater supplies, water in this state must be carefully managed in view of issues raised by pollution, uneven water distribution, the expected increase

of industrial and domestic demand, and environmental stewardship. Water saving aligns with the objectives of the Wisconsin Administrative Code Chapter NR 852 and is part of the Wisconsin Department of Natural Resources (DNR)'s and local authorities' role [61]. In particular, the DNR's Water Use Program has been created. These local actions provide evidence that the subsidiarity principle prevails [62]. Currently wastewater permits are delivered by the state [63], but water allocation caps could be imposed to decrease water use and wastewater discharges, and minimize the social cost of (fresh)water scarcity.

A financial rationale for water saving. The industry must therefore deal with an increasing environmental, political, and economic pressure to curtail the "volume and toxicity of its industrial wastewater" [64, p. 329]. Its operations are regulated by the Clean Water Act, Resource Conservation and Recovery Act, and Safe Drinking Water Act for disposal of liquid and solid waste, as well as the Clean Air Act [57]. The industry also falls within the oversight of the Comprehensive Environmental Response Compensation and Liability Act, allowing the US Environmental Protection Agency (EPA) to take actions against companies harming public health, welfare, or the environment, and the Emergency Planning and Community Right-to-Know Act [57]. However, despite stringent environmental regulations driving the costs of freshwater and wastewater discharge up to 10% of production costs [65], water use remains high in the industry. Reducing energy consumption and environmental compliance costs is therefore a major objective for the industry to move forward [57]. The US EPA works with the American Forest & Paper Association to encourage solutions to reuse and recycle water to decrease energy consumption and compliance costs, and to reduce water use to lower pumping, filtering, demineralizing, heating, treating, and other processing costs, as well as the risk of water shortages.

A need to invest in retrofit/refurbishment measures. The US paper milling industry is subject to stringent regulatory oversight, increasing the costs passed along to downstream consumers, and from aging infrastructure [57]. The industry is thus in need of equipment and machinery with lower water rates, as well as upgraded treatment systems and treatment technologies. However, the issues raised by the rising concentration of dissolved organic matter generated by the repeated recycling of the wastewater cannot be ignored. It is therefore critical to invest not only in additional treatment units (e.g. clarifier, filtration, or evaporation facilities) to closed-cycle systems, but also in the reduction of (non-biodegradable) chemicals use.

An industry with high barriers to entry. The cost of opening and operating a mill is huge. New entrants also face competition for access to distribution channels from larger players [57]. Therefore, large companies operating in a similar industry, such as pulp manufacturing, with the ability to appeal to pre-existing economies of scale are more likely to enter the industry [57]. These barriers sustain our assumption of no free entry made in Section 3.2.

A large concentration of players along a specific waterway. In Wisconsin, the Fox River has the highest concentration of integrated and non-integrated paper mills, with 17³ mills located along the river. According to Huck et al. [66], markets with as few as four firms are never collusive and quite competitive. The group is thus large enough.

A set of players with similar characteristics. Over the five years to 2017, the industry had been facing an annual fall of 2.8% in revenues due to foreign competition, the strength of the US dollar, the cost of regulatory compliance, and a decrease in demand. A further annual drop of 2.5% is expected over the five years to 2022 [57]. Therefore, we can assume that only the actors with the same strongest attributes have survived, as witnessed with mergers and acquisitions to either reach a critical size or diversify activities, and thus that they all belong to the same class.

A diverse portfolio of product buyers. Products are diverse (e.g. printing/writing paper, clay-coated/industrial paper, newsprint, packaging material, and household/sanitary paper) and buyers include individual consumers, businesses, and downstream manufacturers [57]. Each mill can thus be assumed to face independent demand (recall (3.1)).

4.2. Empirical results & discussion

We want to understand the dynamics of technology switching and control of water use through regulatory actions in the paper milling industry. We have illustrated in Section 4.1 the reasons why paper mills in Wisconsin, which share water resources from the Fox River, make a perfect example of MFG of resource sharing and fit the model exposed in Section 3.2. We underline that the model integrates water withdrawn at a given time t to manufacture the final product rather than water footprint. Water that is reused or recycled is thus not integrated in this number. Additionally, the parameters already account for the prevailing regulatory framework, and we do not specify the type of technology we are dealing with. We only know that it ends reliance on the reservoir. Finally, in the model, the authority imposes limits on water rights, according to the sharing rules described in Section 3.2, rather than pollution discharge permits. This section aims to highlight, under different scenarios, the trends in collective water demand and technology change, and the costs and benefits for both the representative agent and authority.

4.2.1. Scenarios

We study four scenarios in Section 4.2.3 to investigate the impacts, in terms of water shortages and costs, of three economic instruments that the authority can employ to control water use. The first one is the "Baseline" scenario describing the prevailing regulatory framework. The other three are:

- (S₁) A subsidy, S_e , in the investment cost of technology 2, decreasing the variable K by 30%. We denote the new value of K by $K - \Delta K$.
- (S₂) A penalty, P_e , for using technology 1 paid through a taxation or an additional lump sum, decreasing the variable p by 30%. We denote the new value of p by $p - \Delta p$.
- (S₃) A subsidy, S_e , for using technology 2 funded through a tax alleviation or an additional lump sum, increasing the variable \bar{p} by 30%. We denote the new value of \bar{p} by $\bar{p} + \Delta \bar{p}$.

Remark 4.1.

- (i) We consider that the regulator cannot modify the cost of production disruption \hat{p} faced by operators in case of water shortages by imposing any additional penalty for not being able to face demand, as this would be economically counter-intuitive in this context.
- (ii) The model does not intend to provide a comparison between the different financial tools available to the regulator (e.g. subsidy, taxation rate).

4.2.2. Outputs

Output 1 For each scenario, we compute the value of variables that are related to agents' water demand and technological change. Those variables are the density of collective water demand (Nash equilibrium) at time t : $\hat{m}_t(x)$, the average collective water demand at time t : $\int_{\mathcal{O}} x \hat{m}_t(dx)$, the proportion of collective water demand satisfied by the reservoir at time t : $\hat{\omega}_t \equiv \hat{\omega}_t$, and the new technology penetration at time t : $\hat{M}_t := 1 - \int_{\mathcal{O}} \hat{m}_t(dx)$. Those computations involve the initial computation of $\hat{m}_t(x)$ that is done following the algorithm defined in Section 3.4 and after updating the values of the parameters involved according to each scenario.

Output 2 For each $t \in [0, T]$, we output the producer's and regulator's average gains seen from $t = 0$. We distinguish the instantaneous gains

³ Data provided upon request by the Wisconsin DNR on 18 January 2019.

$(v_0^{\text{prod}}(t) \& v_0^{\text{reg}}(t))$ from those cumulated over $[0, t]$ ($V_0^{\text{prod}}(t) \& V_0^{\text{reg}}(t)$). Results are stated in dollars and per unit of cubic meter of water withdrawn.

The regulator's gains must integrate the revenues perceived from the taxation of the producer's profits as well as the potential subsidies (resp. penalties) paid to (resp. received from) the producer according to the three scenarios above. We assume that the variation of parameters in the different scenarios only reflects the amount of those subsidies or penalties. A higher level of granularity in the model would allow for the separation of the different sources responsible for that variation following the regulator's actions. For example, it could reflect that regulatory actions favoring switches may further decrease the environmental costs supported by the regulator and passed along to stakeholders, may further lower production costs by enhancing economies of scale and democratizing the new technology, and may further increase demand from buyers with strong environmental considerations as well as productivity by speeding up innovation. The model could also reflect some of the externalities linked to these instruments such as the impacts on other sectors and the society. This is left for further research (see Section 5).

Therefore, in view of the assumption above, recalling that p and \bar{p} are net incomes and denoting by T_c the corporate tax rate, the amount of the penalty P_e per unit of time and per unit of water withdrawn to be received (by the regulator) under scenario (S_2) is $P_e = \frac{\Delta p}{1-T_c}$, while the amount of the subsidy S_e per unit of time and per unit of water withdrawn to be paid (by the regulator) under scenario (S_3) is $S_e = \frac{\Delta \bar{p}}{1-T_c}$. Thus, the total amount perceived by the regulator is

$$\begin{cases} \bar{T}_c(p - \Delta p) + P_e = \bar{T}_c p + \Delta p, & \text{under scenario } (S_2) \\ \bar{T}_c(\bar{p} + \Delta \bar{p}) - S_e = \bar{T}_c \bar{p} - \Delta \bar{p}, & \text{under scenario } (S_3) \end{cases}$$

where $\bar{T}_c := \frac{T_c}{1-T_c}$. Consider, for $x \in \mathcal{O}$, an agent with demand process $(X_t^x)_{0 \leq t \leq T}$ which switches technology at date \hat{t} . According to each scenario (S_i), $1 \leq i \leq 3$, the regulator receives the flow of payments $(\bar{T}_c p + \Delta p) \hat{\omega}_t X_t^x$ between 0 and \hat{t} , makes a lump-sum payment ΔK at time \hat{t} , and receives the flow of payments $(\bar{T}_c \bar{p} - \Delta \bar{p}) X_t^x$ between \hat{t} and T . Averaging over the mean field of producers described by the flow of measures $(\hat{m}_t)_{0 \leq t \leq T}$, the regulator's average gain cumulated over $[0, t]$, $0 \leq t \leq T$, and seen from $t = 0$ is given by

$$V_0^{\text{reg}}(t) := \int_0^t e^{-rs} \left[(\bar{T}_c p + \Delta p) \hat{\omega}_s \int_{\mathcal{O}} x \hat{m}_s(dx) + (\bar{T}_c \bar{p} - \Delta \bar{p}) \int_{\mathcal{O}} x (\hat{m}_s(dx) - \hat{m}_s(dx)) \right] ds - \int_0^t e^{-rs} \Delta K d\hat{M}_s,$$

where, on $[0, T]$, $\hat{m}_t(A) := \int m_t^*(dy) \mathbb{P}[X_t^y \in A]$ for $A \in \mathcal{B}(\mathcal{O})$, and with r being the risk-free interest rate used by the regulator (different from the industry's WACC). It is proved in [10] that the process \hat{M} has finite variation on $[0, T]$ so that the integral above is well defined. If, in addition, the process \hat{M} is absolutely continuous, we can write $V_0^{\text{reg}}(t)$ as a function of the regulator's instantaneous gain, that is $V_0^{\text{reg}}(t) = \int_0^t v_0^{\text{reg}}(s) ds$ where, on $[0, T]$,

$$v_0^{\text{reg}}(t) := e^{-rt} v^{\text{reg}}(t) \text{ with } v^{\text{reg}}(t) := (\bar{T}_c p + \Delta p) \hat{\omega}_t \int_{\mathcal{O}} x \hat{m}_t(dx) + (\bar{T}_c \bar{p} - \Delta \bar{p}) \int_{\mathcal{O}} x (\hat{m}_t(dx) - \hat{m}_t(dx)) - \Delta K \frac{d\hat{M}_t}{dt}.$$

For comparison purposes, we can also define the producer's average gain cumulated over $[0, t]$, $0 \leq t \leq T$, and seen from $t = 0$ as $V_0^{\text{prod}}(t) := \int_0^t v_0^{\text{prod}}(s) ds$ where, on $[0, T]$,

$$v_0^{\text{prod}}(t) := e^{-\rho t} v^{\text{prod}}(t) \text{ with } v^{\text{prod}}(t) := (p - \Delta p + \hat{p}) \hat{\omega}_t \int_{\mathcal{O}} x \hat{m}_t(dx) - \hat{p} \int_{\mathcal{O}} x \hat{m}_t(dx)$$

Table 1
Model parameters.

Parameter	Value (unit)	Source
Net income before switching (p)	1.31 (\$/m ³)	Estimated ^a
Net income after switching (\bar{p})	1.66 (\$/m ³)	Estimated ^a
Cost of production disruption (\hat{p})	5.89 (\$/m ³)	Estimated ^a
Fixed cost of switching (K)	85,000 (\$)	Arbitrary ^b
WACC (ρ)	12 (%)	Provided ^c
Annual water demand growth rate (μ)	-1 (%)	Estimated ^a
Annual water demand volatility (σ)	10 (%)	Arbitrary ^d
Initial water demand distribution (m_0^*)	$\mathcal{N}(\bar{M}_0, 10\%)$	Arbitrary ^{a,e}
Reservoir flow (Z_t)	$f(t)$ (m ³)	Arbitrary ^f
Time horizon (T)	10 (years)	Arbitrary
Risk-free interest rate (r)	2.12 (%)	Estimated ^a
Corporate tax rate (T_c)	39.09 (%)	Estimated ^a

^aSee the supplementary material Section 2.

^bInvestment ranges from a few thousand dollars to several millions.

^cThis is the WACC in (%) for the paper industry provided by RISI.

^dWe use a high volatility to highlight the sensitivity of parameters in Section 4.2.3.

^e $\mathcal{N}(\bar{M}_0, 10\%)$ is the Normal distribution of parameters: $\bar{M}_0 = 13,700$ and 10%.

^fThe reservoir flow is modeled so as to reflect the uncertainty that may arise in future water availability due to the variability of regional stream flows and ground water recharge [68]. We chose a model that exhibits a decreasing trend with two shocks of different amplitude: a moderate and an acute one:

$$f(t) := 10,000 \times \begin{cases} 1.39 - (0.084 \times t)^2 & \text{on } [0, 3.33] \cup [4.66, 6] \cup [6.66, 10] \\ 1.3118 - 0.6762 \times (t - 3.33) & \text{on } [3.33, 3.66] \\ 1.39 - (0.15 \times t)^2 & \text{on } [3.66, 4.33] \\ 0.9681 + 0.8140 \times (t - 4.33) & \text{on } [4.33, 4.66] \\ 1.1360 - 1.9622 \times (t - 6) & \text{on } [6, 6.33] \\ 0.4884 + 1.7836 \times (t - 6.33) & \text{on } [6.33, 6.66] \end{cases} + (\bar{p} + \Delta \bar{p}) \int_{\mathcal{O}} x (\hat{m}_t(dx) - \hat{m}_t(dx)) - (K - \Delta K) \frac{d\hat{M}_t}{dt}.$$

Remark 4.2. We observe that (i) the financial flows of the representative agent are in line with those appearing in (3.8), and (ii) averaging flows over the optimal mean field of producers is natural since the discussion led in Section 3.4. However, the reader may observe that the investment flows are here expressed as continuous flows and not lump-sum payments. This is to render the analysis of instantaneous flows in the next section more comprehensive, but both formulations are equivalent.

Remark 4.3. Consider \tilde{T}_c and \hat{T}_c such that $(\tilde{T}_c p + \Delta p) = \frac{\tilde{T}_c}{1-\tilde{T}_c} p$ and $(\tilde{T}_c \bar{p} - \Delta \bar{p}) = \frac{\tilde{T}_c}{1-\tilde{T}_c} \bar{p}$. If the WACC remains invariant to a change in the corporate tax rate, then \tilde{T}_c (resp. \hat{T}_c) can be thought as a new corporate tax rate applied under scenario (S_2) (resp. (S_3)).

Output 3 A summary table with statistics is provided in Table 2.

Results are computed using the model parameters in Table 1.

4.2.3. Results

Agents' collective water demand We observe from Figs. 1(a) to 1(d)⁴ an improvement in water demand and shortage reduction. Under the Baseline scenario, shortages first occur at year 3.33 and reach a maximum at year 6.33 with a level of 3375.66 m³. The cumulated shortage level is of 7345.86 m³, while the total water demand over the period is of 333,794.98 m³. In (S_1), shortages, which first arise

⁴ Recall that the curve Z_t represents the evolution of the reservoir flow available to the representative agent over t . It is thus independent of the scenario under consideration and of the actions of the population, but rather depends on external geographical conditions.

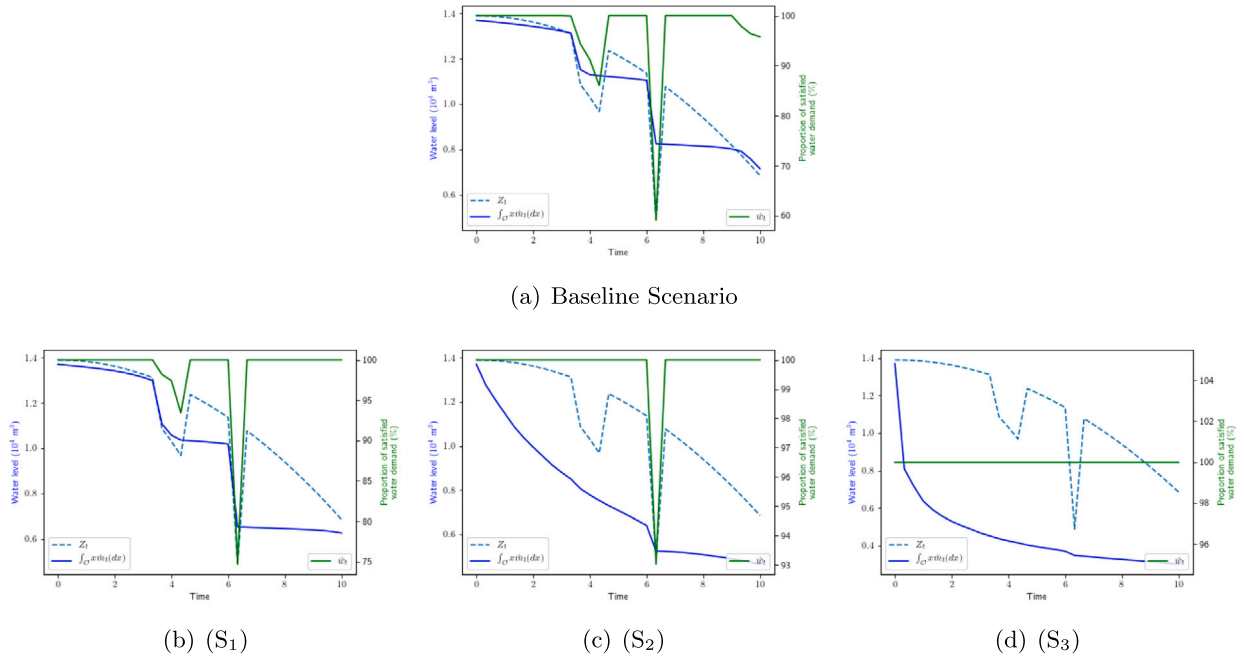


Fig. 1. Plot of $\int_{\mathcal{O}} x \hat{m}_t(dx)$ (10^4 m³) & $\hat{\omega}_t$ (%).

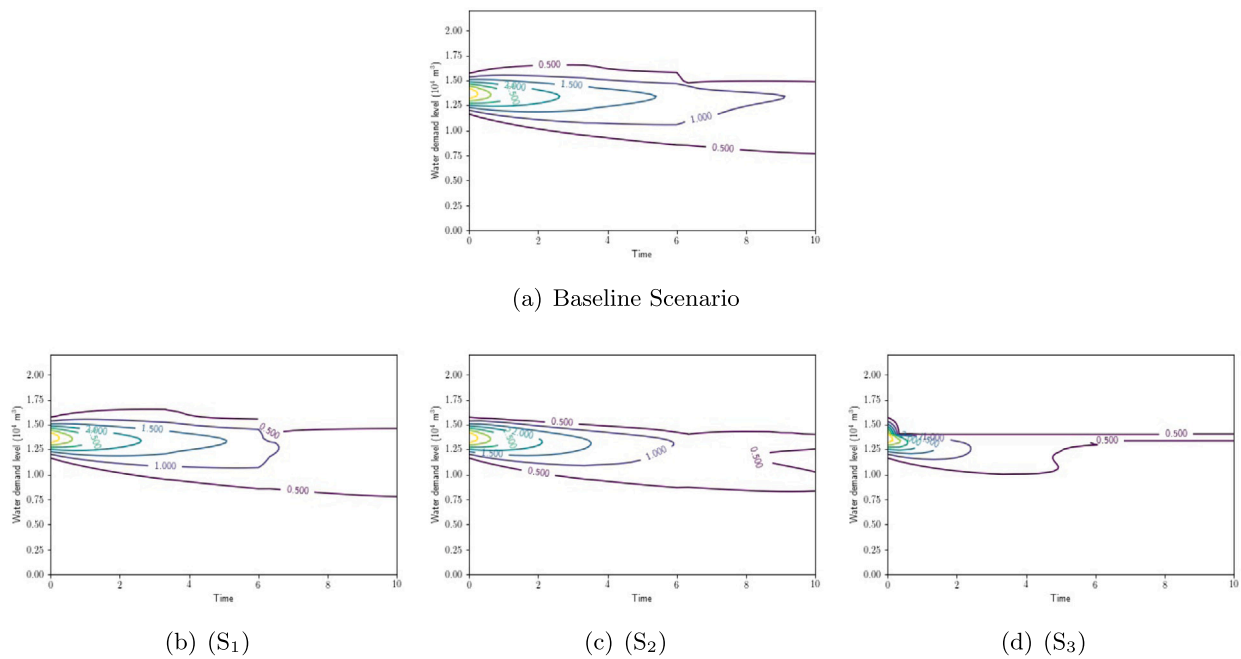
in year 3.67 and peak at 1655.41 m³ in year 6.33, are reduced by 61.76% over the period. Similarly, the total water demand decreases by 7.70%. In (S₂), we observe an important improvement as there is only a shortage in year 6.33, reducing the total shortage amount by 95.02% with respect to the Baseline scenario, while water demand decreases by 29.64% over the period. Finally, in (S₃), no shortage is being observed and the total water demand over the period decreases by 57.43% with respect to the Baseline scenario. As a consequence, a subsidy resulting in a 30%-increase in the variable \bar{p} with respect to the baseline value is the instrument with the most beneficial outcome overall both in terms of water demand and shortage reduction. On the other hand, Figs. 2(a)–2(d) display the contour plots of $\hat{m}_t(x)$ which connect the (t, x) coordinates to a specific $\hat{m}_t(x)$ value. We observe that the density is naturally drifted downwards as the drift term $\mu \leq 0$ and is a decreasing function of time. The contour plots exhibit a decrease in the exercise frontier (i.e. the water demand level at which a switch occurs) after the occurrence of shortages. Finally, as we move gradually from the Baseline scenario to scenarios with a higher level of water demand and shortage reduction, we observe a tightening of the upper bound of the water demand level.

Technological penetration Not surprisingly, in Fig. 3(a), the ordering of the technological penetration curves follows the same pattern as the one observed for water demand and shortage reduction above. In particular, the Baseline and (S₁) scenarios exhibit two main increments in switches that are characterized by the time of the shocks in the reservoir level. In the Baseline (resp. (S₁)) scenario, the first sharp increase occurs from year 3.33 to 3.67 where the penetration goes from 0.01 to 0.10 (resp. 0.02 to 0.13), while the second one happens from year 6 to 6.33 where the penetration jumps from 0.11 to 0.29 (resp. 0.17 to 0.42). The curve ends up at a level of 0.35 (resp. 0.43). On the contrary, switches happen from the beginning in both (S₂) and (S₃) with the penetration jumping from 0 to respectively 0.06 and 0.38. This is the sharpest increment in (S₃) and the second one in (S₂), the first one occurring from year 6 to 6.33 with a jump from 0.44 to 0.53. The curves end up at a penetration level of 0.57 in (S₂) and 0.73 in (S₃).

Producer's gains We observe in Figs. 3(b)–3(c) that the more efficient the instrument is in terms of water demand and shortage reduction, the less erratic the instantaneous gains are at the time of the shocks in the

reservoir level. Under the Baseline and (S₁) scenarios, the instantaneous gains of the producer sharply decrease after the appearance of shocks in the reservoir level. They register their lowest level in year 6.33, with a respective loss of 4537.57 \$ (0.55 \$/withdrawn m³) and 4393.71 \$ (0.67 \$/withdrawn m³). In (S₂), the lowest level is also recorded that year with a loss of 1079.07 \$ (0.21 \$/withdrawn m³). On the contrary, the lowest level recorded in (S₃) is at the beginning with a loss of 26,133.37 \$ (3.23 \$/withdrawn m³), when the Baseline and (S₁) scenarios record their highest level of gains at respectively 5981.12 \$ (0.44 \$/withdrawn m³) and 5981.42 \$ (0.44 \$/withdrawn m³). This is explained by the early high penetration of the new technology under this scenario leading to high investment costs. In Fig. 3(d), we observe the increasing trend in the producer's cumulated gains. Not surprisingly, in (S₃), the curve records its highest loss at the beginning with a level of −26,133.37 \$ and is then positive from year 2.67 onwards. Moreover, the Baseline (resp. (S₁)) scenario exhibits a decrease in cumulated gains of 2.67% and 6.87% (resp. 1.14% and 6.35%), respectively during the shocks on water availability. Similarly, in (S₂), cumulated gains drop by 3.03% during the second shock. The ordering of the curves reflects the cost of the instrument for the producer. A penalty is more expensive than a subsidy, the subsidy for investing in technology 2 being the instrument with the highest total gains over the entire period for the producer, shortly followed by the subsidy for using technology 2. Fig. 3(e) translates the preceding results in terms of value per withdrawn cubic meter of water. The producer's cumulated gains end at 0.24 \$/withdrawn m³ in the Baseline scenario, 0.29 \$/withdrawn m³ in (S₁), 0.24 \$/withdrawn m³ in (S₂), and 0.62 \$/withdrawn m³ in (S₃). In particular, the Baseline (resp. (S₁)) scenario exhibits a decrease around the shocks, i.e. from year 2.67 to 4.67, and year 5.67 (resp. 6) to 6.67.

Regulator's gains Figs. 3(f)–3(g) exhibit the large variations of the instantaneous gains occurring under (S₁) at the time of the shocks in the reservoir level. This is due to the sharp increase in the level of subsidies being paid as switches intensify. A drop of 76% is witnessed in year 3.67 while a fall of 168% occurs in year 6.33, year at which the highest loss is recorded with a level of 2289.16 \$ (0.35 \$/withdrawn m³). Under the Baseline scenario, the instantaneous gains register the most important loss after the appearance of shocks in the reservoir level with a decrease of 21% in year 6.67. Under (S₂), the instantaneous

Fig. 2. Contour Plot of $\hat{m}_t(x)$.

gains drop by 5% after the second shock while the instantaneous gains per cubic meter rise by 20% at the time of the shock, underlying the decrease in water withdrawn over that period. On the contrary, under (S_3) , the highest decrease in instantaneous gains occurs at the beginning, when the amount of subsidies being paid for using technology 2 sharply increases as the number of switches escalates. In Fig. 3(h), we notice the increasing trend in the regulator's cumulated gains (though a decrease of 3.80% at the second shock under (S_1)), with the highest slope recorded under (S_2) . Fig. 3(h) highlights that subsidies are more expensive than penalties. In Fig. 3(i), we observe that once the cumulated gains are expressed in terms of value per withdrawn cubic meter of water, then the ordering changes. A penalty for using technology 1 and subsidy for using technology 2 bring the highest total gains. This is explained by the sharp decrease in water demand over the period under these scenarios. On the contrary, the total gains offered by a subsidy towards the cost of investment tend to be lower than those obtained in the Baseline scenario. The regulator's cumulated gains end at 0.31 \$/withdrawn m^3 in the Baseline scenario, 0.31 \$/withdrawn m^3 in (S_1) , 0.59 \$/withdrawn m^3 in (S_2) , and 0.55 \$/withdrawn m^3 in (S_3) . In particular, (S_1) exhibits a drop of 4.87% and 6.44% at both shocks.

Results summary Table 2 provides a summary of some of the results highlighted above. It is clear that, in terms of water demand and shortage reduction, $(S_3) > (S_2) > (S_1) > \text{Baseline}$. In terms of total gains per withdrawn water over the period, (S_3) leads to the highest and second highest result for respectively the producer and regulator. For the regulator, the amount is however close to the highest total gains reached under (S_2) . In terms of total gains per saved water over the period, (S_1) followed by (S_3) (resp. (S_2)) lead(s) to a positive gain with respect to the Baseline scenario for the producer (resp. regulator). In particular, (S_1) exhibits the highest total loss per unit of water saved for the regulator. Should we combine the producer's and regulator's latter gains under each scenario, we get $(S_2) > (S_1) > (S_3)$. It would be interesting to study the evolution of these results under the integration of social, environmental, and other economic externalities in the model.

4.2.4. The war-of-attrition game

In Figs. 4(a)–4(b), we observe that when the proportion of satisfied water demand sharply increases under the effect of the switch of players, the contour plot of the density of the state variable of players

Table 2

Summary table.

Scenario	WSR ^{a,b}	WUR ^{a,b}	TPG/SW ^{a,b,c}	TRG/SW ^{a,b,c}	TPG/WW ^b	TRG/WW ^b
Baseline	N/A	N/A	N/A	N/A	0.24	0.31
(S_1)	61.76	7.70	0.31	−0.28	0.29	0.31
(S_2)	95.02	29.64	−0.24	0.36	0.24	0.59
(S_3)	100	57.43	0.03	−0.12	0.62	0.55

^aData must be read with respect to the baseline case.

^bAcronyms & units: WSR: Water Shortage Reduction (%), WUR: Water Use Reduction (%), TPG/SW: Total Producer's Gains per Saved m^3 of Water (\$/m³), TRG/SW: Total Regulator's Gains per Saved m^3 of Water (\$/m³), TPG/WW: Total Producer's Gains per Withdrawn m^3 of Water (\$/m³), TRG/WW: Total Regulator's Gains per Withdrawn m^3 of Water (\$/m³).

^cRatio of the difference between the total cumulated gains in the new and baseline scenario, and the total number of saved cubic meters (in terms of demand).

still in the game sharply spreads out towards higher values of water demand. The distribution therefore becomes more asymmetric. This is because, in a war-of-attrition game, the switch of players creates an incentive for the players still relying on the reservoir to stick to the old technology, therefore increasing the level of demand for which agents are willing to switch to the new technology, i.e. the switching frontier.

4.2.5. Key takeaways

By playing on the parameters K, p and \bar{p} , we give evidence that regulation can favor the adoption of clean technology which, in turn, allows a given producer to get a competitive advantage by increasing productivity and reducing production and/or regulatory costs. The increase (resp. decrease) in productivity (resp. costs) is reflected here by the absence of production disruption after the switch and by an after-switch net income higher than the before-switch one. This acts in favor of the ability of environmental policies to counter climate or anthropogenic externalities. This is in line with the *induced innovation hypothesis* [69] according to which a regulated firm is more willing to develop new environmental-friendly technologies when expenditures on regulatory compliance are high compared to other production costs.

The model provides a powerful analytical tool to understand the sensitivity of the game to the different parameters involved and highlights, in particular, a strong sensitivity of the model to the variable \bar{p} in terms of water demand and shortage reduction. The aim of this case

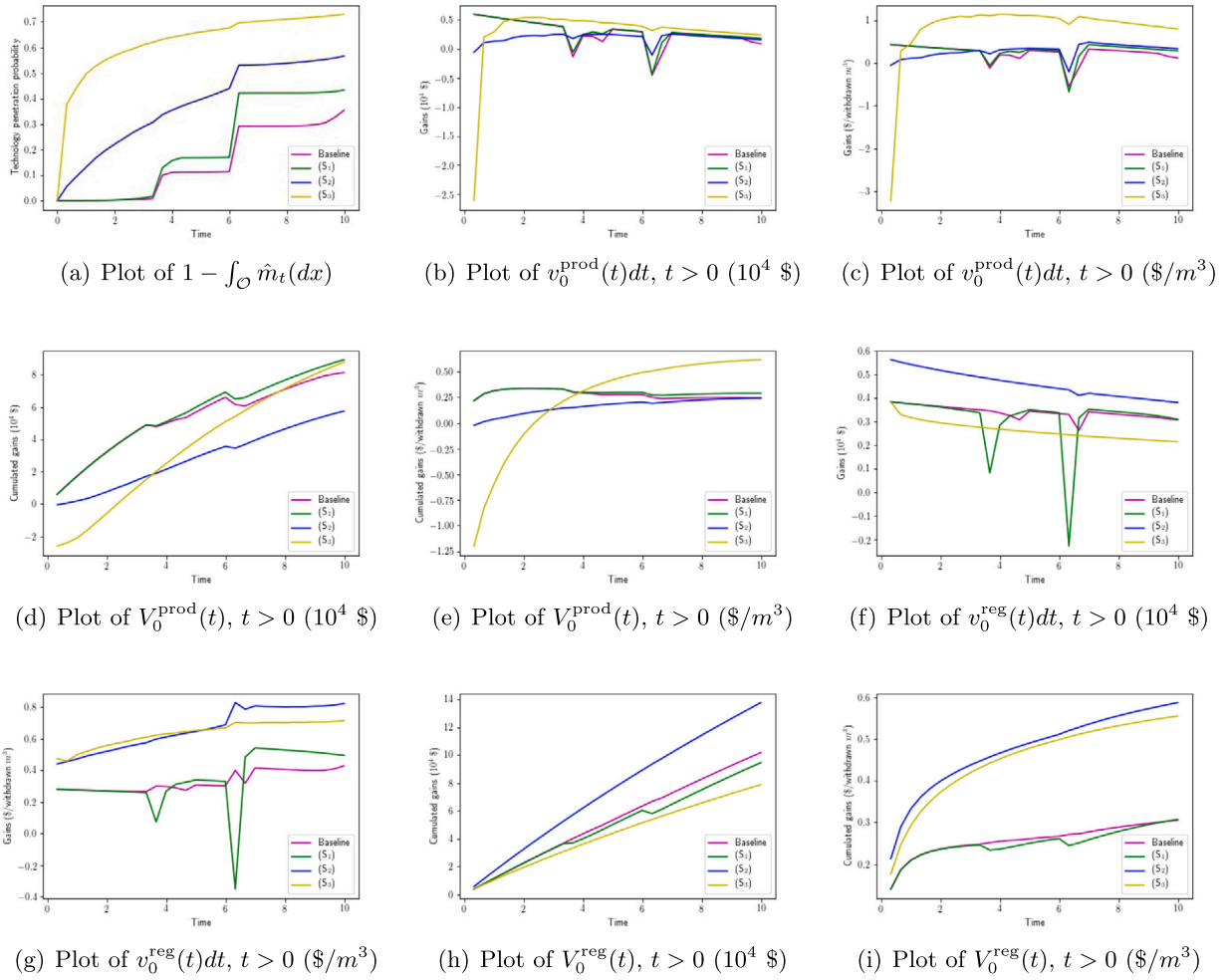
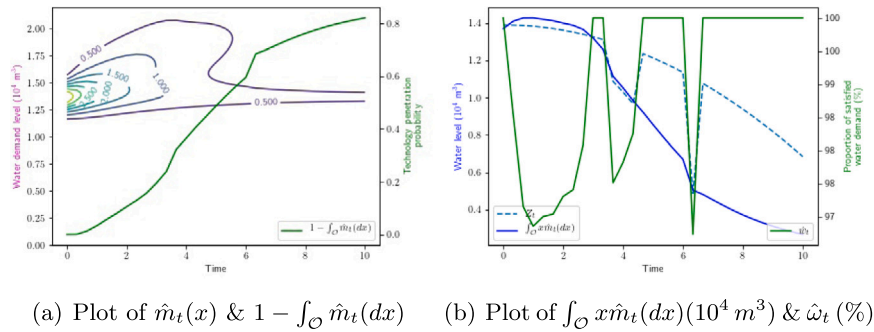


Fig. 3. Technology penetration and producer's & regulator's gains.

Fig. 4. Baseline Scenario but with $\mu = 0.1$.

study was indeed to shed the light on behaviors that could be witnessed in a given manufacturing sector and provide insight on the potential impact of policies.

5. Conclusion & further research

In a context of intensification of the debate on water security in the intergovernmental arena, this paper describes a novel way to tackle technological change in water use. It relies on existing mathematical theory to offer authorities and operators sharing a limited collective reservoir a tool that provides fresh insights on the ways to successfully implement a water program to locally maintain water security and improve operators' profitability.

In this preliminary study, only one class of agents is considered; they all belong to the same industrial sector; and are responsible for substantial (non-consumptive) use of water and toxic wastewater discharges. This study is conducted by means of a mean-field game of optimal stopping involving a large number of players, in which each agent maximizes its reward (depending on the regulator's actions) by looking at the optimal time of switch to/investment in a new technology that ends its reliance on the reservoir. This is the so-called optimal stopping time problem, and the problems of all agents are coupled through the reservoir level. This study invites the reader to reassess the effects of regulatory tools on agents' behaviors. To the best of our knowledge, this is the first time that such a model has been applied to water investment games, essentially because of the lack of mathematical tools that allow

for the handling of the technicalities of the model. However, thanks to the recent developments made by the authors in a companion paper, we are able to provide a complete study of a water resource sharing problem with a case study of the paper milling industry in Wisconsin.

This preliminary study therefore offers a new game-theoretical framework to tackle water-related problems and potential developments are numerous.

From a mathematical perspective, the analysis of our algorithm to compute the MFG equilibrium will be extended in an upcoming paper. Additionally, from an economic perspective, we can spot three main extensions. Firstly, this paper assumes that agents' demand processes are independent. However, they can have a *common noise* in their dynamics, making it necessary to consider a stochastic measure flow $(m_t)_{0 \leq t \leq T}$. Secondly, this paper assumes that there is only one class of agents. However, in many resource-sharing problems, a small number of players has a larger information set and a stronger market power than a large and less influential homogeneous population of players. Bundling the influential players into a single large agent, this leads one to consider *mean-field games with a major player*, which has not yet been addressed in the context of optimal stopping MFG. Thirdly, one could explicitly model the regulator. The mathematical framing would certainly appeal to a model close to the major/minor model introduced above with the regulator standing for the major player. However, rather than competing with other players, the regulator would be expected to maximize an objective function reflecting a social objective.

Further developments would also integrate: higher granularity in the model variables and their dynamics, water markets and the possibility to transfer rights, priority-based allocation systems, the diversity of financial/regulatory tools and their costs, the possibility of pollution leakages, institutional arrangements to coordinate water management, the opportunistic behaviors of organizational actors (e.g. water trusts, basin organizations, and state agencies), as well as the political boundaries of a reservoir, which may prevent water-related investments from achieving their full economic, social, and environmental potential.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The authors are grateful to the Wisconsin Department of Natural Resources for having provided some of the data for the case study.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.orp.2022.100225>.

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