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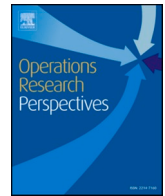
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Modeling Bayesian inspection game for non-performing loan problems

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ABSTRACT

This study compiled a Bayesian inspection game as a branch in game theory to deal with non-performing loans (NPLs). Three types of games are analyzed, which are false alarm (FA), non-detection (ND), and bull's eye (BE). A Bayesian Nash equilibrium calculation process took place to formulate the player's strategy proportion. The equilibrium solution indicates the causative factors and develops the strategies to anticipate NPLs. The identified factors causing NPLs include customers' utility and disutility, inspection error in the form of false alarm and non-detection, operational costs to conduct an inspection, and bank utility related to inspection. The results showed that some examinations of type I and II errors to the game model could provide more comprehensive and interesting insights in managing NPL problems.

1. Introduction

One of the problems encountered by banks is non-performing loan (NPL), which refers to loans that do not provide calculated profits. Such risk is the one that banks are trying to avoid [1–2] mentioned that risk allocation calculation is a fundamental problem of portfolio management, and coherent risk measures also provide an axiomatic approach to the problems of portfolio risk. The incoming portfolios will have different characteristics, one of which is due to the behavior pattern of the prospective debtors. Behavioral patterns that indicate a problematic track record will end up with a risky portfolio [3]. In NPL cases, a limitless optimization model with an objective function of a credit score could help managers to select the best loan portfolio. One of the studies on credit score calculation conducted by [4] explained how to calculate credit scores by assessing variable positions at the quartile level. Other methods such as inferential statistics have also contributed to evaluating the feasibility of debtors using the classification and clustering principles. Several studies that applied this method include feature selection algorithms and ensemble learning classifiers [5], advanced support vector machine [6], and multi-objective soft subspace clustering algorithm [7]. The research, as mentioned earlier, focused on evaluating the feasibility of the debtor's portfolio. The proposed methods were only able to answer the problem of the cause of NPL in the category of the ability of the debtor to repay the loans. The optimization model and inferential statistics cannot accommodate the factors that cause a debtor to commit NPL. Since the NPL are transaction problems between

corporate as loan inspector and customer as loan inspectee, game-theoretical approaches are needed to find a win-win solution [8]. Other studies employed a leader-follower game theory approach for lending strategies [9], a game theory approach for credit behavior analysis in small and medium enterprises [10], and stochastic replicator dynamics game theory on the relationship between banks and companies [11]. The findings in game theory research displayed game theory's capability to uncover the hidden factors that other methods cannot reveal. This statement encourages this research to explore more game theory models that could overcome NPL problems.

Another game theory model developed to solve NPL problems in this study is the inspection game. An inspection game is a mathematical model of a condition in which one player performs an inspection, commonly called an inspector, to supervise other players; in this case, it is an inspectee to comply with the applicable regulations [12]. The reasons for selecting inspection games to be developed in this study include the similarity of conditions in the decision-making process where creditors can act as inspectors and debtors as inspectees. Furthermore, some findings in several studies on inspection games in other sectors reveal some factors that drive one of the players, the inspectee, to commit an offense in most cases. Several studies on inspection games include the ones by [13] and [14] that changes in the payoff value of a player do not affect the player's strategy but affect the opponent's strategy at the equilibrium point. Andreozzi stated that the magnitude of the sentence is irrelevant to the number of violations. However, law enforcement incentives as studied by Handoyo and

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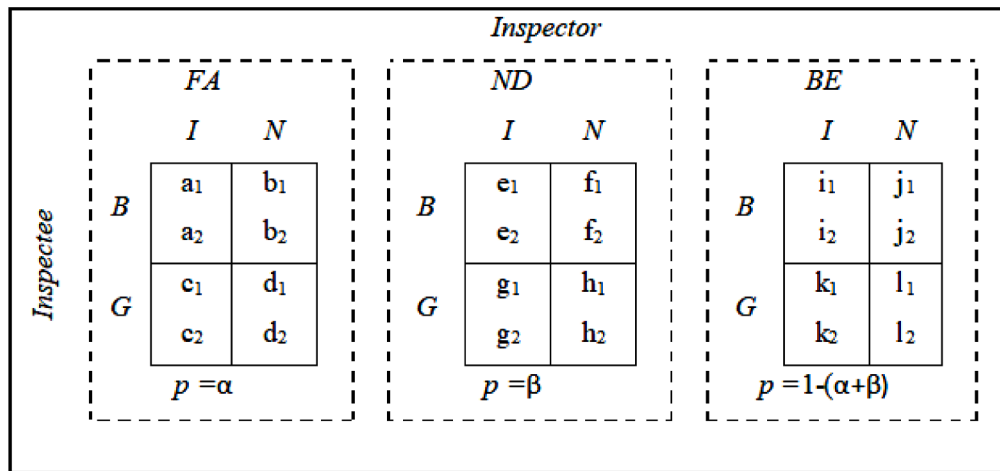


Fig. 1. Conceptual model of Bayesian inspection game for non-performing loans.

Kautsar [15] by developing an inspection game model for corruption cases in Indonesia. The results of the study showed several findings. The first finding is that citizens tend to commit criminal acts of corruption with the number of funds corrupted greater than one billion. The second finding suggests that citizens who have the opportunity to commit corruption are "obliged" to do so because the payoff received will be very large. Then, the third finding reveals that such sanctions do not reduce the frequency of corruption but increase the probability of law enforcement and corruption crimes so that the players play in the scenario of (Corruption, Acting). Another work by [16] revealed that doping tendency in sports could be effectively managed by utilizing inspection games. There were three scenarios, namely tyrannic, draconian and lenient. Surprisingly, the game analysis showed that a lenient scenario provided the highest probability of avoiding doping practices amongst athletes.

In NPL problems, some of the studies mentioned mainly focused on predicting the feasibility of the debtor's portfolio and only on the game theory method that raises the causes of NPL and must take steps to suppress NPL. [17] claimed that an inspection occurs in a mechanism similar to a statistical test process in which there are two types of errors that may arise, i.e., false alarm and non-detection. The game inspection model that involves inspection errors can provide a holistic picture of the events or cases to model. A game-theoretical of credits and loan for several players are conducted by Welbrun and Hausken [18]. However, their model does not consist and asymmetry information between the players. The existence of stochastic parameters in decision-making certainly requires to be studied systematically so that the possibility of bias can be minimized [19]. The possibility of an inspection error can explain one of the reasons why debtors still tend to commit arrears despite knowing that there will be sanctions for the delay. The debtor may have the perception that he can avoid arrears if a non-detection type error occurs. Furthermore, the opportunity for other conditions to occur is one of Bayes' characteristics in game theory. Therefore, this research will also apply Bayes' characteristics to compile a Bayesian inspection game model from the bank's point of view to identify, analyze and anticipate the factors that cause NPLs.

2. Problem description

2.1. Conceptual model formulation

The development of a conceptual model of the game model is carried out by identifying the components that make up the game. The first component of a game is the player. This study assumed that the player

Table 1
Scenarios in the game model.

No	Type	Description
1	Scenario 1	Scenario <i>B-I</i> in false alarm game type
2	Scenario 2	Scenario <i>B-N</i> in false alarm game type
3	Scenario 3	Scenario <i>G-I</i> in false alarm game type
4	Scenario 4	Scenario <i>G-N</i> in false alarm game type
5	Scenario 5	Scenario <i>B-I</i> in non-detection game type
6	Scenario 6	Scenario <i>B-N</i> in non-detection game type
7	Scenario 7	Scenario <i>G-I</i> in non-detection game type
8	Scenario 8	Scenario <i>G-N</i> in non-detection game type
9	Scenario 9	Scenario <i>B-I</i> in bull's eye game type
10	Scenario 10	Scenario <i>B-N</i> in bull's eye game type
11	Scenario 11	Scenario <i>G-I</i> in bull's eye game type
12	Scenario 12	Scenario <i>G-N</i> in bull's eye game type

who acted as an inspector was the credit risk analyst of Bank X, while the player who acted as an inspectee was the debtor. The second component is the strategy that each player has. The strategy of each player in this study is defined as follows. The inspector has two strategies: conducting an inspection (*I*) or not conducting an inspection (*N*). The inspectee has two strategies: paying arrears (*B*) or not paying arrears (*G*). The third component is the payoff for each player, which results from the scenario by meeting the strategy *i* of the first player with the strategy *j* of the second player. In Fig. 1, the notations are from $a_1, a_2; b_1, b_2; \dots$ to i_1, i_2 respectively. The fourth component is the type that each player has. In this type of Bayesian game, there is uncertainty that occurs in a game. In this case, uncertainty occurs in the type of condition experienced by the inspector. A condition that the inspector may experience is a false alarm (*FA*), which is a condition that describes the situation when the inspector conducts an inspection (*I*) on the inspectee who makes arrears payments (*B*). However, the inspectee is not considered making payment by the inspector. So, the inspector and inspectee experience some form of the disutility of the error that occurs in this condition. The probability of a false alarm condition is denoted by α . The second condition that the inspector may undergo is non-detection (*ND*), which is a condition that describes the situation when the inspector performs an inspection (*I*) on the inspectee but fails to meet the inspectee so that the inspectee gets a form of utility by not paying installments in arrears and the inspector experiences a form of disutility due to an error that happens. The probability of occurrence of a non-detection condition is denoted by β . The third condition that an inspector may encounter is a bull's eye (*BE*), which is a condition that describes a situation when the inspector does not make a false alarm and a non-detection error. The chance of a bull's eye condition is denoted by $1 - (\alpha + \beta)$.

Table 2
Scenarios in the game model.

Notation	Description
R_D	debtor credential value, which has been harmed because of the offense committed, in contrast to the actual condition of being innocent
R_B	bank reputation value that has inspected error in a false alarm situation
C_B	operational costs in conducting an inspection
U_D	profit value or utility obtained by a debtor from the number of installments that are not paid during the arrears period
D_D	disutility value resulting from the penalty by the bank upon successful inspection
U_B	profit value or utility obtained by the bank when giving a penalty on the inspection to the debtor
D_B	the disutility value experienced due to the unpaid installments

2.2. Identification of payoff composing variables

Based on the conceptual model and influence diagram, the payoff for each player can be composed following the constituent variables and the scenario of the occurrence. The game model in this study refers to the conceptual models consisting of twelve scenarios, as presented in Table 1.

Among the twelve scenarios, there are several with the same payoff for each player. Scenario 2, scenario 6, and scenario 10 have the same payoff. Scenario 3 and Scenario 11 also have the same payoff. Then, Scenario 4, scenario 8, and scenario 12 have the same payoff. Scenario 5 and scenario 9 have the same payoff as well. Finally, scenario 1 and scenario 7 are the scenarios with unique conditions that have a different payoff from other scenarios because scenario 1 is a false alarm game type and scenario 7 is a non-detection game type. In addition, the following Table 2 gives the summary of some important parameters affecting our payoff matrix.

2.2.1. Scenario 1

Scenario 1 illustrates the condition of the debtor performing strategy B and the bank performing strategy I in the false alarm game type. The false alarm game type is a condition where the bank makes an error by assuming that the debtor does not pay the arrears in installments, while the actual situation is that the debtor has paid it. In the conceptual model, the payoff for debtors is denoted by a_1 and the payoff for the bank is denoted by a_2 . The a_1 value is affected by the debtor (R_D) credential value, which has been harmed because of the offense committed, in contrast to the actual condition of being innocent. The a_2 value is influenced by the bank reputation value that has inspected error in the form of a false alarm (R_B) and the operational costs for conducting an inspection (C_B). Consequently, a_1 and a_2 can be formulated as follows:

$$a_1 = -R_D \quad (1)$$

$$a_2 = -R_B - C_B \quad (2)$$

2.2.2. Scenario 2, scenario 6, and scenario 10

Scenario 2, Scenario 6, and Scenario 10 are scenarios that describe the condition when the debtor performs strategy B , and the bank performs strategy N in the types of false alarm, non-detection, and bull's eye games, respectively. The condition with the $B-N$ scenario is a condition that is considered "ideal" for the bank because, without inspection, the debtor completes his obligations by paying installments in arrears. In the conceptual model, the payoff for debtors and the bank in the types of false alarm, non-detection, and bull's eye games are denoted by (b_1, b_2) , (f_1, f_2) , and (j_1, j_2) respectively. The values of (b_1, b_2) , (f_1, f_2) , and (j_1, j_2) are zero (0) because both players are doing what they should do, so they do not get anything in this scenario.

2.2.3. Scenario 3 and scenario 11

Scenarios 3 and 11 describe a situation of when the debtor carries out strategy G and the bank carries out the strategy I in the types of false alarm and bull's eye games, respectively. The $G-I$ scenario is a condition when the debtor chooses not to pay the installments in arrears until the maximum allowed period. Let us assume this due date is by the end of the 90th day or three months. Such a situation strongly indicates the potential of a non-performing loan. In this scenario, the bank expects that when conducting an inspection and taking certain actions on debtors who choose not to pay, they will experience a deterrent effect so that they do not make more delinquent payments. In the conceptual model, the payoff for the debtor and the bank in the false alarm and bull's eye game types is denoted by (c_1, c_2) and (k_1, k_2) respectively. The values of c_1 and k_1 are influenced by the value of the profit or utility obtained from the number of installments that are not paid during the arrears period (U_D) and the disutility value resulting from the penalty of the bank upon successful inspection (D_D). The values of c_2 and k_2 are influenced by the value of the profit or utility obtained when giving a penalty on the inspection to the debtor (U_B), the disutility value experienced due to the unpaid installments (D_B), and the operational costs for conducting inspections (C_B). Therefore, (c_1, c_2) and (k_1, k_2) can be formulated as follows:

$$c_1 = k_1 = U_D - D_D \quad (3)$$

$$c_2 = k_2 = U_B - D_B - C_B \quad (4)$$

2.2.4. Scenario 4, scenario 8, and scenario 12

Scenario 4, Scenario 8, and Scenario 12 are scenarios that illustrate the condition when the debtor implements strategy G and the bank implements strategy N in the false alarm, non-detection, and bull's eye types, respectively. The condition under the $G-N$ scenario is a condition that the bank tries to avoid because this condition is considered detrimental to the bank if it continues. In the conceptual model, the debtor's and the bank's payoff on the types of false alarms, the non-detection, and the bull's eye game is denoted respectively by (d_1, d_2) , (h_1, h_2) , and (l_1, l_2) . In this condition, the values of d_1 , h_1 , and l_1 will have the same amount or equal to the value of U_D . Meanwhile, the values of d_2 , h_2 , and l_2 will have the same amount or equal to the value of $-D_B$. Hence, the formulations for (d_1, d_2) , (h_1, h_2) , and (l_1, l_2) are as follows:

$$d_1 = h_1 = l_1 = U_D \quad (5)$$

$$d_2 = h_2 = l_2 = -D_B \quad (6)$$

2.2.5. Scenario 5 and scenario 9

Scenario 5 and Scenario 9 are scenarios that describe the condition when the debtor applies strategy B , and the bank applies strategy I in the types of non-detection and bull's eye games, respectively. This condition is a normal condition expected by the bank when conducting an inspection, i.e., there is no error in the inspection, and the debtor pays the installments in arrears. In the conceptual model, the payoff for the debtor and bank in the non-detection and bull's eye types is denoted by (e_1, e_2) and (i_1, i_2) respectively. In this condition, the values of e_1 and i_1 will be 0 (zero). Meanwhile, the values of e_2 and i_2 will have the same amount or equal to the value of $-C_B$.

2.2.6. Scenario 7

Scenario 7 is a scenario that illustrates the condition when the debtor undertakes strategy G , and the bank launches strategy I in the type of non-detection game. This scenario is included in a situation that the bank does not want to occur due to the bank's failure to inspect debtors in arrears. Such a situation makes the debtor remains in a non-paying condition. In the conceptual model, the debtor's payoff is denoted by g_1 and the bank payoff is denoted by g_2 . The value of g_1 is influenced by the utility value of the debtor (U_D). Meanwhile, the value of g_2 is influenced by the disutility value experienced due to the unpaid

		Inspector					
		FA		ND		BE	
		I	N	I	N	I	N
Inspector	B	$-R_D$	0	0	0	0	0
	G	$-R_B-C_B$	0	$-C_B$	0	$-C_B$	0
	B	U_D-D_D	U_D	U_D	U_D	U_D-D_D	U_D
	G	$U_B-D_B-C_B$	$-D_B$	$-D_B-C_B$	$-D_B$	$U_B-D_B-C_B$	$-D_B$
		$p = \alpha$		$p = \beta$		$p = 1-(\alpha+\beta)$	

Fig. 2. Bayesian inspection game model.

installments (D_B) and the operational costs for conducting inspections (C_B). Therefore, g_1 and g_2 can be formulated as follows:

$$g_1 = U_D \quad (7)$$

$$g_2 = -D_B - C_B \quad (8)$$

2.3. Bayesian-Nash equilibrium

Based on the conceptual model formulation process in Section 2.1 and the process of variable identification in Section 2.2, a Bayesian inspection game model is obtained for NPL problems. Fig. 2 presents the model.

In order to obtain the equilibrium point from the game model, a Bayesian Nash equilibrium calculation approach by Leyton-brown and Shoham [20] could be utilized.

2.3.1. Compilation of support arguments

The strategy that the inspectee can employ is to purely pay the arrears $\{B\}$ or purely not to pay the arrears $\{G\}$; thus, the strategy set for the inspectee is $\{B, G\}$. The strategy that the inspector can utilize is to purely perform the inspection $\{I\}$ or purely not to perform the inspection $\{N\}$ on each type of game; hence, the strategy set for the inspector is $\{III, IIN, INI, NII, INN, NIN, NNI, NNN\}$. Based on the existing combination of each player, the support arguments for this game are: $\{B\} \times \{III\}$, $\{B\} \times \{IIN\}$, $\{B\} \times \{INI\}$, $\{B\} \times \{NII\}$, $\{B\} \times \{INN\}$, $\{B\} \times \{NIN\}$, $\{B\} \times \{NNI\}$, $\{B\} \times \{NNN\}$, $\{G\} \times \{III\}$, $\{G\} \times \{IIN\}$, $\{G\} \times \{INI\}$, $\{G\} \times \{NII\}$, $\{G\} \times \{INN\}$, $\{G\} \times \{NIN\}$, $\{G\} \times \{NNI\}$, and $\{G\} \times \{NNN\}$.

2.3.2. Evaluation of dominant strategy

Before searching for the dominant strategy, the interpretation of the variables in the game payoff into a new notation to show the relativity of the payoff value in one cell to another payoff is carried out.

PROPOSITION. In this inspection game to analyze NPL, a payoff sign can be utilized to determine domination. The rules are:

- 1 A strategy with $(- -)$ payoff sign will be dominated by that with $(-)$ payoff sign,
- 2 A strategy with $(-)$ payoff sign will be dominated by that with (0) payoff sign,
- 3 A strategy with (0) payoff sign will be dominated by that with $(+)$ payoff sign,
- 4 And finally, a strategy with $(+)$ payoff sign will be dominated by that with $(+ +)$ notation.

subject to some constrains, namely:

- 1 $U_D > D_D$
- 2 $U_D = D_D$
- 3 $U_D < D_D$

For example :

$e_1 = -D_D - R_D$	$(- -)$
$g_1 = -D_D$	$(-)$

$g_1 > e_1$, so strategy G dominates strategy B
Therefore, $(-)$ dominates $(- -)$

$e_1 = -R_D$	$(-)$
$g_1 = 0$	(0)

$g_1 > e_1$, so strategy G dominates strategy B
Therefore, (0) dominates $(-)$

$e_1 = 0$	(0)
$g_1 = U_D$	$(+)$

$g_1 > e_1$, so strategy G dominates strategy B

		Inspector					
		FA		ND		BE	
		I	N	I	N	I	N
Inspector	B	-	0	0	0	0	0
	G	+	++	++	++	+	++
		$p = \alpha$		$p = \beta$		$p = 1-(\alpha+\beta)$	

Fig. 3. Visualization of the inspectee's dominant strategies when $U_D > D_D$.

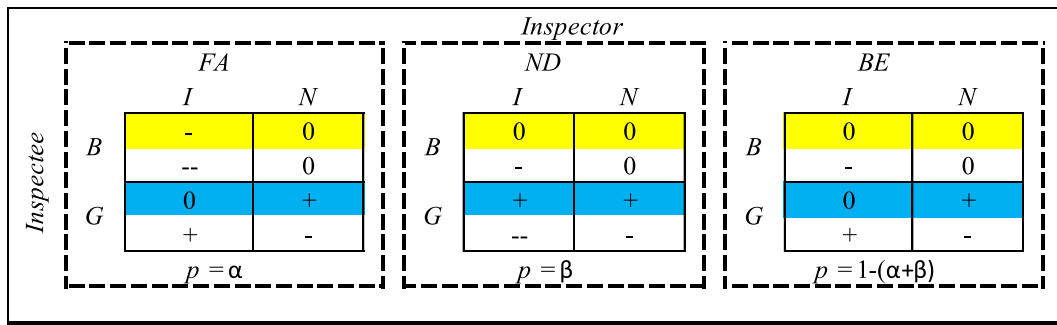
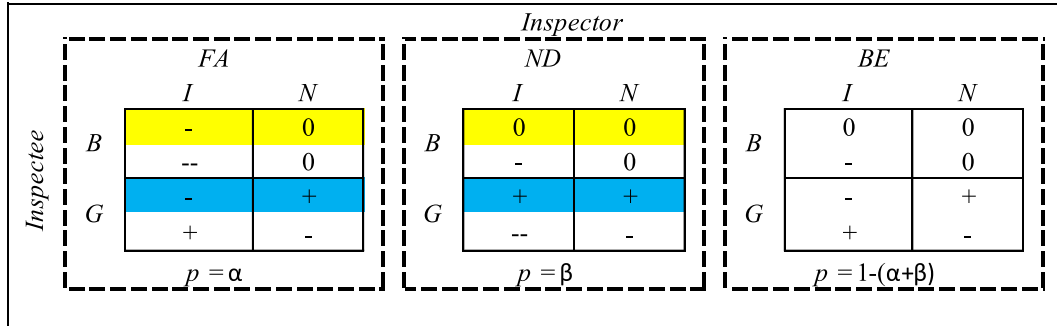
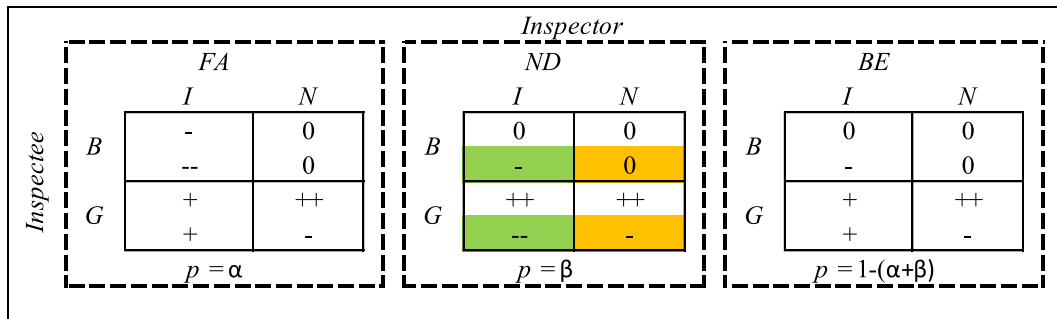
Fig. 4. Visualization of the inspectee's dominant strategies when $U_D = D_D$.Fig. 5. Visualization of the inspectee's dominant strategy when $U_D < D_D$.

Fig. 6. Visualization of the inspector's dominant strategy in all conditions.

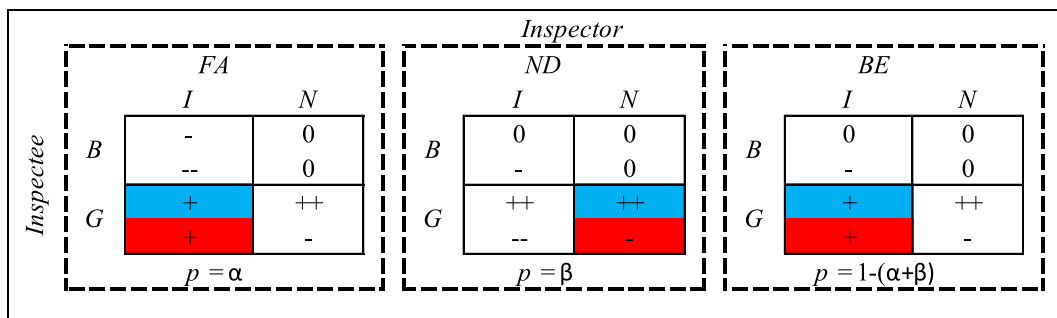
Therefore, (+) dominates (0)

$e_1 = U_D$	(+)
$g_1 = U_D + R_D$	(++)

$g_1 > e_1$, so strategy G dominates strategy B

Therefore, (++) dominates (+)

According to these rules, the evaluation process of the dominant strategy will be easier to carry out. The evaluation begins by looking at the perspective of the inspectee. Under the conditions of $U_D > D_D$ and $U_D = D_D$, strategy G dominates strategy B in all types of games no matter what strategy the inspector employs. In Figs. 3 and 4, the whole cells in

Fig. 7. Visualization of the best response of each player when $U_D \geq D_D$.

		Inspector					
		FA		ND		BE	
Inspectee		I	N	I	N	I	N
	B	-	0	0	0	0	0
G	B	--	0	-	0	-	0
	G	-	+	+	+	-	+
		+	-	--	-	+	-
		$p = \alpha$		$p = \beta$		$p = 1-(\alpha+\beta)$	

Fig. 8. Visualization of the best response of each player when $U_D < D_D$.

strategy G (blue highlight) completely dominate the entire cells in strategy B (yellow highlight). Meanwhile, in the condition of $U_D < D_D$, strategy G dominates strategy B in FA and ND types, while there is no dominant strategy for BE type. Fig. 5 displays that strategy G only dominates strategy B in FA and ND types only.

For the inspector, from the entire conditions, there is only one strategy in one type that dominates, which is strategy N in the ND type. In other types, there is no dominating strategy. Fig. 6 shows that strategy N (orange highlight) dominates strategy I (green highlight).

2.3.3. Best response strategy

Figs. 7 and 8 exhibit the best response strategy for each player in each condition, with the blue highlight for the inspectee and the red highlight for the inspector. By examining Figs. 3 and 4, we get a remain-the-same domination result between ($U_D > D_D$) and ($U_D = D_D$) situation. Since \geq sign means "> or =" at the same time, then as long as inspector and inspectee are both rational, they will opt to the maximum value of payoffs. When we combine payoffs from Figs. 3 and 4 (inspectee viewpoint) using this maximization criterium and payoffs from Fig. 6 (inspector viewpoint), the results are presented in the following figures.

2.3.4. Bayesian-Nash equilibrium calculation

In this study, two calculations in finding equilibria were carried out due to the two conditions that may occur namely $U_D \geq D_D$ and $U_D < D_D$.

2.3.4.1. Bayesian-Nash equilibrium under $U_D \geq D_D$ condition. The evaluation results of the best response strategy from each player viewpoint indicated that the inspectee continues to use the original dominant strategy, i.e., the pure strategy {G} and the inspector employs the initial best response, namely the mixed strategy {INI}. When the best response strategies from all players are determined and matched, an equilibrium point is identified [20]. Consequently, under $U_D \geq D_D$ condition, the equilibrium point that occurs is that the inspectee uses a pure strategy {G} and the inspector uses a mixed strategy {INI}. The calculation of the proportion of strategy I and N in the mixed strategy {INI} of the inspector could be carried out in the following way:

Strategy I is carried out in the mixed strategy for the FA and BE types.

$$p(I) = p(I, FA) + p(I, BE)$$

$$p(I) = p(FA) + p(BE)$$

$$p(I) = \alpha + [1 - (\alpha + \beta)]$$

$$p(I) = 1 - \beta \quad (9)$$

Strategy N is carried out in the mixed strategy for the ND type.

$$p(N) = p(N, ND)$$

$$p(N) = p(ND)$$

$$p(N) = \beta \quad (10)$$

		Inspector	
		INI	INN
Inspectee	B	$-\alpha R_D$ $-\alpha R_B - (1 - \beta) C_B$	$-\alpha R_D$ $-\alpha R_B - \alpha C_B$
	G	$U_D - (1 - \beta) D_D$ $(1 - \beta)(U_B - C_B) - D_B$	$U_D - \alpha D_D$ $\alpha(U_B - C_B) - D_B$

Fig. 9. Game model with ex-ante expected payoff.

with,

- $p(I)$: inspector's proportion in using strategy I
- $p(I, FA)$: inspector's proportion in using strategy I in FA type
- $p(I, BE)$: inspector's proportion in using strategy I in BE type
- $p(N)$: inspector's proportion in using strategy N
- $p(N, ND)$: inspector's proportion in using strategy N in ND type
- $p(FA)$: probability of FA type occurrence
- $p(BE)$: probability of BE type occurrence
- $p(ND)$: probability of ND type occurrence

2.3.4.2. Bayesian-Nash equilibrium under $U_D < D_D$ condition. In this condition, the calculation of the equilibrium point will be carried out using two approaches. The Approach I will refer to Leyton-Brown and Shoham [20]. These authors stated that determining an equilibrium point in a Bayesian game is necessary to calculate the expected payoff for each player based on the actions taken by each player. This statement follows theorem I and theorem II in [21].

• Approach I

The calculation of expected payoff according to [19] has three points of view, which include ex-ante, ex-interim, and ex-post expected payoffs.

- *Ex-ante expected payoff*: expected values in a game in which calculated on the basis of all possible outcomes. Such calculation is performed before the reality happens. Hence, these payoffs represent the sum of all multiplication between all payoffs and their corresponding probabilities to occur. In this paper, the payoffs are all payoff values as provided in both inspectee's strategies/rows {B, G}, and, however, only those in inspector's strategies/columns {INI, INN} as the result of inspector's possible mixed strategies. The result of this calculation is given in the following Fig. 9.

Fig. 9 is a form of the 2×2 game and its strategy that will be exercised by each player to find their mixed strategies. Hence, the calculation of such equilibria may apply Tsebelis theorem. This theorem is effectively

		Player 2	
		S ₂₁	S ₂₂
Player 1	S ₁₁	u ₁	v ₁
	S ₁₂	w ₁	z ₁
		S ₂₁	S ₂₂
		u ₂	v ₂
		w ₂	z ₂

Fig. 10. Game model under Tsebelis Theorem.

used to provide a solution even in a bi-matrix game by decomposing this matrix into two separated games. The equilibrium probability for mixed strategies may be appropriately calculated. Suppose there is a game matrix in Fig. 10 as follows then the equilibrium probability for Player 1 to select S₁₁ is given by

$$p(S_{11}) = \frac{z_1 - w_1}{u_1 - v_1 - w_1 + z_1}$$

For sure,

$$p(S_{12}) = 1 - p(S_{11})$$

In contrast, the equilibrium probability for Player 2 to select S₂₁ is given by

$$p(S_{21}) = \frac{z_2 - v_2}{u_2 - v_2 - w_2 + z_2}$$

Thus,

$$p(S_{22}) = 1 - p(S_{21})$$

Equilibrium of the inspectee

$$p(B) = \frac{z_2 - w_2}{u_2 - v_2 - w_2 + z_2}$$

$$p(B) = \frac{\alpha(U_B - C_B) - D_B - (1 - \beta)(U_B - C_B) - D_B}{[-\alpha R_B - (1 - \beta)C_B] - (-\alpha R_B - \alpha C_B) - [(1 - \beta)(U_B - C_B) - D_B] + [\alpha(U_B - C_B) - D_B]}$$

$$p(B) = \frac{\alpha(U_B - C_B) - (1 - \beta)(U_B - C_B)}{-(1 - \beta)C_B + \alpha C_B - (1 - \beta)(U_B - C_B) + \alpha(U_B - C_B)}$$

$$p(B) = \frac{[\alpha - (1 - \beta)](U_B - C_B)}{[\alpha - (1 - \beta)](C_B + U_B - C_B)}$$

$$p(B) = \frac{U_B - C_B}{U_B}$$

$$p(B) = 1 - \frac{C_B}{U_B} \quad (11)$$

$$p(B) + p(G) = 1$$

$$1 - \frac{C_B}{U_B} + p(G) = 1$$

$$p(G) = \frac{C_B}{U_B} \quad (12)$$

		Inspector	
		INI	INN
Inspectee	B	$-\alpha R_D$	0
	G	$\alpha U_D - \alpha D_D$	αU_D
		INI	INN
		$-\alpha R_B - \alpha C_B$	0
		$\alpha U_B - \alpha D_B - \alpha C_B$	$-\alpha D_B$

Fig. 11. Game model with an ex-interim expected payoff in FA type.

		Inspector	
		INI	INN
Inspectee	B	0	0
	G	βU_D	βU_D
		INI	INN
		$-\beta D_B - \beta C_B$	$-\beta D_B$

Fig. 12. Game model with an ex-interim expected payoff in ND type.

with,

$p(B)$: inspectee's proportion in using strategy B

$p(G)$: inspectee's proportion in using strategy G

Equilibrium of the inspector

$$p(INI) = \frac{z_1 - v_1}{u_1 - v_1 - w_1 + z_1}$$

$$p(INI) = \frac{(U_D - \alpha D_D) - (-\alpha R_D)}{-\alpha R_D - (-\alpha R_D) - [U_D - (1 - \beta)D_D] + (U_D - \alpha D_D)}$$

$$p(INI) = \frac{U_D - \alpha D_D + \alpha R_D}{-U_D + D_D - \beta D_D + U_D - \alpha D_D}$$

$$p(INI) = \frac{U_D - \alpha D_D + \alpha R_D}{(1 - \alpha - \beta)D_D}$$

$$p(INI) = \frac{U_D - \alpha(D_D - R_D)}{(1 - \alpha - \beta)D_D} \quad (13)$$

		Inspector	
		INI	INN
Inspectee	B	0	0
	G	$(1 - \alpha - \beta)C_B$	$(1 - \alpha - \beta)U_D$
		INI	INN
		$(1 - \alpha - \beta)(U_D - D_D)$	$(1 - \alpha - \beta)U_D$
		$(1 - \alpha - \beta)(U_B - D_B - C_B)$	$-(1 - \alpha - \beta)D_B$

Fig. 13. Game model with ex-interim expected payoff in BE type.

		Inspector	
		I	N
Inspectee	B	$-R_D$	0
	G	$-R_B - C_B$	0
		$U_D - D_D$	U_D
		$U_B - D_B - C_B$	$-D_B$

Fig. 14. Game model with an ex-post expected payoff in FA type.

		Inspector	
		I	N
Inspectee	B	0	0
	G	$-C_B$	0
		U_D	U_D
		$-D_B - C_B$	$-D_B$

Fig. 15. Game model with an ex-post expected payoff in ND type.

$$p(INI) + p(INN) = 1$$

$$\frac{U_D - \alpha(D_D - R_D)}{(1 - \alpha - \beta)D_D} + p(INN) = 1$$

$$p(INN) = 1 - \frac{U_D - \alpha(D_D - R_D)}{(1 - \alpha - \beta)D_D} \quad (14)$$

with,

$p(INI)$: inspector's proportion in using mixed strategy INI
 $p(INN)$: inspector's proportion in using mixed strategy INN

- *Ex interim expected payoff*: partial expected values in a game which are calculated on the basis of each Bayesian condition. Such calculation should take place when the reality is about to happen. Hence, these payoffs represent the sum of all multiplication between all payoffs and their corresponding probabilities in each Bayesian type. In this paper, the payoffs are all payoff values as provided in both inspectee's strategies/rows {B,G}, and, however, only those in inspector's strategies/columns {INI, INN} as the result of inspector's possible mixed strategies in each Bayesian type, namely FA, ND, and BE. Thus, there are three payoff matrices in providing this representation. The result of this calculation is given in the following Figs. 11, 12, and 13, respectively.

The game model with a payoff almost identical to the initial value of the conceptual model in the FA type indicates a dominant strategy in the inspectee. This occurs due to the multiplication of the α value on the entire payoffs of each player. Therefore, it does not change the rules for the evaluation of dominant strategy in Section 2.3.2. As a result, the values of $p(B) = 0$, $p(G) = 1$, $p(I) = 1$, and $p(N) = 0$ were obtained, with:

$p(I)$: inspector's proportion in using a pure strategy I
 $p(N)$: inspectee's proportion in using a pure strategy N

The game model with a payoff almost identical to the initial value of the conceptual model in the ND type indicates a dominant strategy in the

		Inspector	
		I	N
Inspectee	B	0	0
	G	$-C_B$	0
		$U_D - D_D$	U_D
		$U_B - D_B - C_B$	$-D_B$

Fig. 16. Game model with an ex-post expected payoff in BE type.

		Inspector	
		INI	INN
Inspectee	GGB	$(\alpha + \beta)U_D - \alpha D_D$ $\alpha U_B - (\alpha + \beta)D_B - (1 - \beta)C_B$	$(\alpha + \beta)U_D - \alpha D_D$ $\alpha U_B - (\alpha + \beta)D_B - \alpha C_B$
	GGG	$U_D - (1 - \beta)D_D$ $(1 - \beta)(U_B - C_B) - D_B$	$U_D - \alpha D_D$ $\alpha(U_B - C_B) - D_B$

Fig. 17. Game model with approach II.

inspectee. This happens because of the multiplication of the β value on the entire payoffs of each player. Therefore, it does not change the rules for the evaluation of the dominant strategy in Section 2.3.2. As a result, the values of $p(B) = 0$, $p(G) = 1$, $p(I) = 0$, and $p(N) = 1$ was obtained.

The game model with a payoff that is almost identical to the initial value of the conceptual model in the BE type indicates a dominant strategy in the inspectee. The cause is the multiplication of the $(1 - \alpha - \beta)$ value on the entire payoffs of each player. Therefore, each player's proportion of each strategy will be the same as the calculation for BE type. Thus, the values of $p(B) = p(B, BE) = 1 - \frac{C_B}{U_B}$, $p(G) = p(G, BE) = \frac{C_B}{U_B}$, $p(I) = p(I, BE) = \frac{U_D}{D_D}$, and $p(N) = p(N, BE) = 1 - \frac{U_D}{D_D}$ were obtained, with:

$p(B, BE)$: inspectee's proportion in using strategy B in BE type
 $p(G, BE)$: inspectee's proportion in using strategy G in BE type
 $p(I, BE)$: inspector's proportion in using strategy I in BE type
 $p(N, BE)$: inspector's proportion in using strategy N in BE type

- *Ex post expected payoff*: actual values in a game determined on the basis of final outcomes. Such values might be known after the reality happens. Hence, these payoffs are basically the original payoff values illustrated in each Bayesian type. In this paper, these payoffs are all payoff values as provided in both inspectee's strategies/rows {B,G}, and both inspector's strategies/columns {I, N} in each FA, ND, and BE Bayesian type. These payoffs are provided in the following Figs. 14, 15, and 16.

The game model with a payoff identical to the initial value of the conceptual model in the FA type displays a dominant strategy in the inspectee and the inspector. As a result, the values of $p(B) = 0$, $p(G) = 1$, $p(I) = 1$, and $p(N) = 0$ were obtained.

The game model with a payoff identical to the initial value of the conceptual model in the ND type menu exhibits a dominant strategy in the inspectee and the inspector. Therefore, the values of $p(B) = 0$, $p(G) = 1$, $p(I) = 0$, and $p(N) = 1$ was obtained.

The game model with a payoff identical to the initial value of the conceptual model in the BE type indicates a dominant strategy in the inspectee. Therefore, each player's proportion of each strategy will be the same as the calculation for BE type. Hence, the values of $p(B) = p(B, BE) = 1 - \frac{C_B}{U_B}$, $p(G) = p(G, BE) = \frac{C_B}{U_B}$, $p(I) = \frac{U_D}{D_D}$, and $p(N) = p(N, BE) =$

$1 - \frac{U_D}{D_D}$ were obtained.

• Approach II

Meanwhile, this second approach in finding an equilibrium point will refer to the following corollary:

COROLLARY. There are some equilibria under several game types that a player encounter. These equilibria are as many as the number of dominant interim strategies. Hence the calculation of these equilibria could be performed by involving the strategy set in the existing game types.

When each player's evaluation of the dominant strategy and the best response strategy was performed, a combination of strategy pairs for each type of game was obtained. In the *FA* type, the best response combination of $\{G\} \times \{I\}$ was obtained. Then, in the *ND* type, the best response combination of $\{G\} \times \{N\}$ was obtained. Meanwhile, in the *BE* type, the best response combination of $\{B, G\} \times \{I, N\}$ was obtained (Fig. 17).

The form of the 2×2 game and the strategy that each player will utilize is the mixed strategy; thus, the equilibrium point calculation can use the *Tsebelis* theorem.

Equilibrium of the inspectee

$$p(GGB) = \frac{z_2 - w_2}{u_2 - v_2 - w_2 + z_2}$$

$$p(GGB) = \frac{[\alpha(U_B - C_B) - D_B] - [\alpha(1 - \beta) - (U_B - C_B) - D_B]}{[\alpha U_B - (\alpha + \beta)D_B - (1 - \beta)C_B] - [\alpha U_B - (\alpha + \beta)D_B - \alpha C_B] - [\alpha(1 - \beta) - (U_B - C_B) - D_B] + [\alpha(U_B - C_B) - D_B]}$$

$$p(GGB) = \frac{\alpha(U_B - C_B) - (1 - \beta)(U_B - C_B)}{-(1 - \beta)C_B + \alpha C_B - (1 - \beta)(U_B - C_B) + \alpha(U_B - C_B)}$$

$$p(GGB) = \frac{\alpha(U_B - C_B) - (1 - \beta)(U_B - C_B)}{-(1 - \beta)C_B + \alpha C_B - (1 - \beta)(U_B - C_B) + \alpha(U_B - C_B)}$$

$$p(INI) = \frac{(U_D - \alpha D_D) - [(\alpha + \beta)U_D - \alpha D_D]}{[(\alpha + \beta)U_D - \alpha D_D] - [(\alpha + \beta)U_D - \alpha D_D] - [U_D - (1 - \beta)D_D] + (U_D - \alpha D_D)}$$

$$p(GGB) = \frac{[\alpha - (1 - \beta)](U_B - C_B)}{[\alpha - (1 - \beta)](C_B + U_B - C_B)}$$

$$p(GGB) = \frac{U_B - C_B}{U_B}$$

$$p(GGB) = 1 - \frac{C_B}{U_B}$$

$$p(GGB) = p(B, BE) = 1 - \frac{C_B}{U_B}$$

Proportion calculation of strategy G.

$$p(GGB) + p(GGG) = 1$$

$$1 - \frac{C_B}{U_B} + p(GGG) = 1$$

$$p(GGG) = \frac{C_B}{U_B}$$

$$p(GGG) = p(G, BE) = \frac{C_B}{U_B}$$

Aggregate calculation of strategy B.

$$p(B) = p(GGB) \times p(BE)$$

$$p(B) = \left(1 - \frac{C_B}{U_B}\right) \times (1 - \alpha - \beta)$$

$$p(B) = 1 - \alpha - \beta - \frac{C_B}{U_B} + \alpha \frac{C_B}{U_B} + \beta \frac{C_B}{U_B} \quad (15)$$

Aggregate calculation of strategy G.

$$p(G) = p(GGB) \times p(FA) + p(GGB) \times p(ND) + p(GGG) \times p(FA) + p(GGG) \times p(ND) + p(GGG) \times p(BE)$$

$$p(G) = \left(1 - \frac{C_B}{U_B}\right) \times \alpha + \left(1 - \frac{C_B}{U_B}\right) \times \beta + \frac{C_B}{U_B} \times \alpha + \frac{C_B}{U_B} \times \beta + \frac{C_B}{U_B} \times (1 - \alpha - \beta)$$

$$p(G) = \alpha + \beta + \frac{C_B}{U_B} - \alpha \frac{C_B}{U_B} - \beta \frac{C_B}{U_B} \quad (16)$$

Equilibrium of the inspector

$$p(INI) = \frac{z_1 - v_1}{u_1 - v_1 - w_1 + z_1}$$

$$p(INI) = \frac{U_D - \alpha U_D - \beta U_D}{-U_D + D_D - \beta D_D + U_D - \alpha D_D}$$

$$p(INI) = \frac{(1 - \alpha - \beta)U_D}{(1 - \alpha - \beta)D_D}$$

$$p(INI) = \frac{U_D}{D_D}$$

$$p(INI) = p(I, BE) = \frac{U_D}{D_D}$$

Proportion calculation of strategy INN.

$$p(INI) + p(INN) = 1$$

$$\frac{U_D}{D_D} + p(INN) = 1$$

$$p(INN) = 1 - \frac{U_D}{D_D}$$

$$p(INN) = p(N, BE) = 1 - \frac{U_D}{D_D}$$

Aggregate calculation of strategy I.

$$p(I) = p(INI) \times p(FA) + p(INI) \times p(BE) + p(INN) \times p(FA)$$

$$p(I) = \frac{U_D}{D_D} \times \alpha + \frac{U_D}{D_D} \times (1 - \alpha - \beta) + \left(1 - \frac{U_D}{D_D}\right) \times \alpha$$

$$p(I) = \alpha + \frac{U_D}{D_D} - \alpha \frac{U_D}{D_D} - \beta \frac{U_D}{D_D} \quad (17)$$

Aggregate calculation of strategy N.

$$p(N) = p(INI) \times p(ND) + p(INN) \times p(ND) + p(INN) \times p(BE)$$

$$p(N) = \frac{U_D}{D_D} \times \beta + \left(1 - \frac{U_D}{D_D}\right) \times \beta + \left(1 - \frac{U_D}{D_D}\right) \times (1 - \alpha - \beta)$$

$$p(N) = 1 - \alpha - \frac{U_D}{D_D} + \alpha \frac{U_D}{D_D} + \beta \frac{U_D}{D_D} \quad (18)$$

3. Numerical experiments

The experiment carried out in this research came in the form of numerical experiments by testing the equilibrium point equation, which was formulated according to the approach method utilized in four classes of the debtor types.

By observing the characteristics of the data obtained with the value of $p(G) \neq 0$, the equilibrium point equation used was the result of the calculation of Approach I with the ex-interim expected payoff or ex-ante expected payoff in the *BE* type or ex-post expected payoff in the *BE* type since the value of $p(G)$ in the three conditions was the same. The experiment was also carried out with the calculation results of Approach II.

4. Result and discussion

4.1. Bayesian-Nash equilibrium analysis

The difference in the results of the equilibrium point obtained occurred because of two factors. The first factor is the payoff composition of the game model. It is distinct that two main payoff compositions happened in the game model. The first composition was the condition when $U_D \geq D_D$, which caused strategy *G* to dominate strategy *B* because the inspectee felt that the profit or utility of arrears was greater than the loss or disutility. Therefore, the proportion of strategy *G* for the inspectee was valued one, and strategy *B* was valued 0. The strategy carried out by the inspector was the best response strategy from strategy *G* employed by the inspectee. The inspector's best response strategy was to carry out the pure strategy *I* on *FA* and *BE* types. The proportion of strategy *I* has the value of $1 - \beta$ and the inspector performed pure strategy *N* on type *ND* so that the proportion of strategy *N* has the value of β . Consequently, in the conditions when $U_D \geq D_D$, the first set of equilibrium points of $p(B) = 0$, $p(G) = 1$, $p(I) = 1 - \beta$, and $p(N) = \beta$ was obtained.

The second factor is the calculation point of view on the conditions of the game type. The second set was the result of the calculation by method 1 of ex-ante expected payoff. This method considers the possibility of both players participating in all types of games that may occur. Likewise, the sixth set of money was obtained by method 2. In this case,

the two players will join every game that will occur without knowing what type of game they will play. In the process of the optimal strategic proportion calculation, both players must include the chances of the type of game in the calculation process. This is the process that leads to the equilibrium point value in the second set to have an element of the chance value of the type of game in the equilibrium point set. In the calculation condition in which there is no dominant strategy, the mixed strategy is used.

4.2. Analysis of numerical experiment results

The numerical experiment carried out only calculated one indicator, i.e., the value of $p(G)$, since the initial objective of the study was to identify the factors that cause non-performing loans and formulate a strategy that can anticipate this condition.

In the calculations with Approach I, a relatively large difference was obtained in all existing debtor classes. In grade one, grade two, grade three, and grade four, the differences were 43.59, 38.97, 20.60, and 12.90, respectively. The differences were in the deviation value in the existing data by 96.08%, 97.06%, 88.60%, and 89.71%, respectively. These deviation values are relatively higher than the standard commonly used, namely the significant value of 0.05 or 5% of the existing data. This fact occurred due to the point of view of the calculations used in the fifth set. The second set only focused on the *BE* game type and ignored the *FA* and *ND* types even though the *FA* and *ND* types are very possible to happen in the field and were proven to occur, resulting in a relatively significant difference with the existing data. However, the calculations with this second set illustrate that the steps taken by the bank are proven to be correct to reduce the value of *non-performing loans* if the bank does not make an inspection error. This fact is indicated by the value of $p(G)$ which was relatively small compared to the commonly used standard, which is the significant value of 0.05 or 5%.

In the calculation with Approach II, there was a slight difference in class three and class four, i.e., 0.16 and 0.09, only deviating by 0.69% and 0.63% from the existing data. These values are smaller than the standard commonly used, which is the significant value of 0.05 or 5% of the current data. This fact indicates that calculations using the sixth set can accurately describe the conditions in the third and fourth-grade debtors. In class one and class two, the differences were 18.05 and 18.22, deviating by 39.78% and 45.38% from the existing data. These values are greater than the standard commonly used, namely the significant value of 0.05 or 5% of the current data. This considerable difference shows that the calculations using the sixth set fail to describe the conditions for the first and second-class debtors. This situation may occur due to the loan's duration or tenor that exists in the two classes. Inaccurate calculation of $p(G)$ value can occur because the model compiled is only played at one point in time, while the $p(G)$ value in the data was obtained from several points of time when the debtor was in arrears of payments, resulting in a significant difference in the values.

4.3. Analysis of NPL cause factors and NPL Anticipation strategies

LEMMA 1. Suppose the disutility value of the delinquent payments experienced by the debtor is reduced until the value is equal to or even less than the utility. In that case, the chance of a non-performing loan occurring will increase.

PROOF. When $U_D < D_D$, $p(G) = \alpha + \beta + \frac{C_B}{U_B} - \alpha \frac{C_B}{U_B} - \beta \frac{C_B}{U_B}$ or $p(G) = \frac{C_B}{U_B}$ which means that $p(G) < 1$. When $U_D \geq D_D$, $p(G) = 1$, then $p(G, U_D \geq D_D) - p(G, U_D < D_D) > 0$.

LEMMA 2. If the error probability of false alarm type is increased by δ , the chance of a non-performing loan to occur will also increase by $\delta \alpha \left(1 - \frac{C_B}{U_B}\right)$.

PROOF. . If $0 < \delta < 1$; $G_0 = G$ without error increase; and $G_1 = G$ with error increase

$$p(G_0) = \alpha + \beta + \frac{C_B}{U_B} - \alpha \frac{C_B}{U_B} - \beta \frac{C_B}{U_B}$$

$$p(G_1) = (1 + \delta)\alpha + \beta + \frac{C_B}{U_B} - (1 + \delta)\alpha \frac{C_B}{U_B} - \beta \frac{C_B}{U_B}$$

then

$$p(G_1) - p(G_0) = \delta\alpha - \delta\alpha \frac{C_B}{U_B} = \delta\alpha \left(1 - \frac{C_B}{U_B}\right)$$

LEMMA 3. If the error probability of a non-detection type is increased by δ , the chance of a non-performing loan to occur will also increase by $\delta\beta \left(1 - \frac{C_B}{U_B}\right)$.

PROOF. If $0 < \delta < 1$; $G_0 = G$ without error increase; and $G_1 = G$ with error increase

$$p(G_0) = \alpha + \beta + \frac{C_B}{U_B} - \alpha \frac{C_B}{U_B} - \beta \frac{C_B}{U_B}$$

$$p(G_1) = \alpha + (1 + \delta)\beta + \frac{C_B}{U_B} - \alpha \frac{C_B}{U_B} - (1 + \delta)\beta \frac{C_B}{U_B}$$

then

$$p(G_1) - p(G_0) = \delta\beta - \delta\beta \frac{C_B}{U_B} = \delta\beta \left(1 - \frac{C_B}{U_B}\right)$$

LEMMA 4. If the inspection operational costs clue is increased by δ , the chance of a non-performing loan to occur will also increase by $\delta(1 - \alpha - \beta) \frac{C_B}{U_B}$.

PROOF. If $0 < \delta < 1$; $G_0 = G$ without cost increase; and $G_1 = G$ with cost increase

$$p(G_0) = \alpha + \beta + (1 - \alpha - \beta) \frac{C_B}{U_B}$$

$$p(G_1) = \alpha + \beta + (1 - \alpha - \beta) \frac{(1 + \delta)C_B}{U_B}$$

then

$$p(G_1) - p(G_0) = \delta(1 - \alpha - \beta) \frac{C_B}{U_B}$$

LEMMA 5. If there is an increase of δ in the utility obtained by the bank, the chance of a non-performing loan to occur will decrease by $\frac{\delta}{1 + \delta} (1 - \alpha - \beta) \frac{C_B}{U_B}$.

PROOF. If $0 < \delta < 1$; $G_0 = G$ without utility increase; and $G_1 = G$ with utility increase

$$p(G_0) = \alpha + \beta + (1 - \alpha - \beta) \frac{C_B}{U_B}$$

$$p(G_1) = \alpha + \beta + (1 - \alpha - \beta) \frac{C_B}{(1 + \delta)U_B}$$

then

$$p(G_1) - p(G_0) = -\frac{\delta}{1 + \delta} (1 - \alpha - \beta) \frac{C_B}{U_B}$$

Table 3

Debtor Class Data.

Class	n	p (FA)	p (ND)	p (BE)	p (G)	Tenor
1	42	10,00%	16,00%	74,00%	45,37%	≥ 300
2	37	6,00%	15,00%	79,00%	40,15%	≥ 300
3	44	9,00%	12,00%	79,00%	23,25%	< 300
4	22	4,00%	9,00%	87,00%	14,38%	< 300

Table 4

Results of Numerical Experiments with Approach I.

Class	Existing	Approach I	Δ
1	45.37%	1.78%	43.59%
2	40.15%	1.18%	38.97%
3	23.25%	2.65%	20.60%
4	14.38%	1.48%	12.90%

Table 5

Results of Numerical Experiments with Approach II.

Class	Existing	Approach II	Δ
1	45.37%	27.32%	18.05%
2	40.15%	21.93%	18.22%
3	23.25%	23.09%	0.16%
4	14.38%	14.29%	0.09%

5. Conclusion

The Bayesian inspection game model that has been compiled consists of three types of games due to three types of player inspectors. These three types of games represent the conditions that occur on the field. The FA game type represents the possibility of an inspection error in the form of an error in assuming that the debtor has not made payment although he has paid the arrears. Then, the ND game type represents the possibility of an inspection error in the form of a failure to meet a debtor during an inspection. This type also accommodates the debtor's perception of the opportunity to escape a penalty when making a payment arrears. Meanwhile, the BE game type represents the possibility of the inspection process occurring without making inspection errors.

The results of identifying the factors causing the occurrence of non-performing loans include the proportion of utility and disutility of the customer, the possibility of an inspection error in the form of a false alarm that considers the debtor has not made a payment. However, he has paid the arrears, the chance of an inspection error in the form of non-detection, which refers to the failure to meet debtors when conducting inspections, the high operational costs for conducting inspections, and the low utility that the bank gets as a result of the inspection. The proportion of the customer's utility will cause a shift in the equilibrium point and result in a higher NPL level if the utility value is greater than or equal to the debtor's disutility. The chance of an inspection error, both false alarm and non-detection types will cause an increase in the NPL value if the opportunity value increases. The increase in NPL value is also caused by an increase in the operational cost of inspections. Increasing bank utility, however, will lower the NPL rate.

The strategies to take in anticipating the occurrence of non-performing loans are: increasing the level of late payment fines and also the interest from late fees following the maximum limit allowed by laws and regulations, improving the banking information system that can provide the latest updates on arrears payments to inspectors so that they may cancel the inspection when the debtor has made a payment, looking for information on the whereabouts of the debtor from the debtor's colleagues, relatives, or family on D-1 of conducting the inspection to confirm the whereabouts of the debtor and optimizing the inspection schedule on the inspector to achieve cost efficiency.

The numerical experiment results showed that in the calculation

with Approach II, the differences in deviation were 0.69% and 0.63% from the existing data. These values are smaller than the standard commonly used, i.e., the significant value of 0.05 or 5% of the current data. This result shows that calculations using Approach II can accurately describe the conditions in the third and fourth-grade debtors. Meanwhile, in classes one and two, the differences in deviation were 39.78% and 45.38% from the existing data. These values are greater than the standard commonly used, namely the significant value of 0.05 or 5% of the current data. In addition, the results of numerical experiments showed that if the bank can minimize the α and β values to reach zero, the collectability level in class 1 will increase by 96.08%, that in class 2 will increase by 97.06%, that in class 3 will increase by 88, 60%, and that in class 4 will increase by 89.71%.

The experimental results also showed the limitations of the model being developed. Therefore, further research could consider making a game model that performs inspections not only once but also repeated n times to obtain complete model results in evaluating class one and two debtors (Tables 3–5).

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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