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# DSGE Model Meets Data Gently: the Importance of Trend Modelling <sup>\*</sup>

Petteri Juvonen<sup>†</sup> & Mikko Sariola<sup>‡</sup>

August 27, 2025

## Abstract

DSGE models are often specified so that the long-run variation of variables is driven by one or two common trends, which rarely holds in the data. We find that when this discrepancy exists, high-frequency components (measurement errors) capture variable-specific time variation in trends. When high-frequency components are restricted to be small or ignored, the discrepancy is captured by the model component, which distorts shock decompositions. We show that incorporating variable-specific trend components directly into the measurement equations yields a decomposition in which the high-frequency, model, and trend components each capture what they are intended to. We also find trend modelling useful in forecasting.

**Keywords:** DSGE, trends, business cycles

**JEL Codes:** E17, E32, E37, C52, C53

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<sup>\*</sup>The views expressed are those of the authors and do not necessarily reflect those of the Bank of Finland. This paper has benefited from comments and discussions with Frank Schorfheide, Fabio Canova, Michal Andrle, Juha Kilponen, Antti Ripatti, Markku Lehmus, Adam Gulan, Markus Haavio and Ingvar Strid.

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# 1 Introduction

Dynamic Stochastic General Equilibrium (DSGE) models are usually tailored to explain business cycle fluctuations. However, macroeconomic time series consist of high-frequency, business cycle and long-run variation. When fitting a DSGE model to the data, high-frequency variation is dealt with by adding measurement errors that have little persistence. Long-run variation, i.e. trend in time series, is dealt with by pre-filtering or differencing the time series.<sup>1</sup> A typical approach is to specify the model on differenced data and to include to the model a shock process that produces growth - a process or processes that produce permanent increases to the productivity and this results a trend component which is common to multiple variables. This approach is convenient because it avoids the two steps that pre-filtering requires and, in addition, when using the model for forecasting, this setup produces also for the projection horizon a trend component which pre-filtering does not produce.

In this paper, we highlight issues that can arise when the data include variable-specific variation in mean growth rates of the time series, that is, when time series diverge or converge in a non-constant manner. In this case, the typical assumption that a permanent productivity process produces variation to the trend components of several variables does not work. Our proposed solution is to apply the approach in Canova (2014). We specify the model in (log) levels data and incorporate variable-specific trend components. Our model includes a permanent productivity process that drives a common trend component for multiple variables and variable-specific trend components capture long-run variation that the permanent productivity process does not capture. The variable-specific trend processes are not in the model

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<sup>1</sup>In addition to differencing, either pre-demeaning the time series or allowing for intercepts in the measurement equations.

but are incorporated to the measurement equations.

We compare three different specifications. Two of these specifications are for data in differences. In one of them, the high-frequency component is restricted to be small, while in the other, standard deviations are parameterized to be large enough so that the high-frequency component captures variable-specific changes in the trends. The third specification is specified on (log) levels data. In all specifications, productivity process produces a common trend for real variables but in the third specification we include a component that captures variable-specific deviations from the common trend. We compare the models in terms of historical decompositions and forecast accuracy. We use simulated data and Finnish data. The former is useful because we know the true paths of trend, high frequency and cyclical model components. The latter provides a good example of variable-specific time variation in the trends, though similar developments exist in the data of various countries.

Our experiments suggest that if careful modelling of trend components is overlooked, it may lead to unintended outcomes. That is, the low-frequency developments that should be attributed to trend components might be captured falsely either by the high-frequency or by the model component. We find that when the high-frequency components are allowed to be large and the model is specified on data in difference, high-frequency components capture variable-specific time-variation in trends by accumulating into large and persistent deviations from zero despite being specified to be zero-mean processes with no autocorrelation. When high-frequency components are restricted to be small, the variable-specific time-variation in trends is captured by the model variables. This leads to persistent and large deviations from the steady state for the model variables, although these variables should be cyclical.

When using a model specified on levels data with variable-specific trend components, the trend component captures low-frequency developments. As a result, the trend component, cyclical component and high-frequency component capture what they should capture. This enables correct cyclical analysis with the model. Correct

decomposition into high-frequency, model and trend components is a prerequisite for shock decompositions. If model component explains also long-run variation in the data this requires certain structural shocks to drive this. This can lead to false conclusions when the shock decomposition is thought to give information on cyclical drivers.

In addition to providing more accuracy for structural analysis, we find that modelling trend components is also beneficial for forecasting. When forecasting several years ahead, a significant portion of the forecast is based on the trend component. If the growth rate of a series changes significantly over-time, allowing time-variation for the growth rate of the trend component produces better forecasts than assuming sample mean growth rate for the trend component. Also, the starting point matters. Because there are no new shocks in the projection period, model variables revert toward the steady state during the projection. If, at the beginning of the projection, a model variable is far away from the steady state, a large reversion can distort the projection. It can be that a model variable is far away from the steady state only because the trend of the observed variable is not properly modelled.

Combined with trend modelling, a DSGE model provides a complete statistical model for macroeconomic data. Then the whole data are modelled, all frequencies, even though trend components (and high-frequency components) capture that part of the data that is not explained by the model. This is useful for forecasting and also helps to understand how much the model actually explains of the data and in certain time periods. We highlight this by providing in this paper several figures of decompositions and not just report statistics, which is the more usual approach in the empirical DSGE literature.

Studies including Cogley (2001), Gorodnichenko and Ng (2010), Fukač and Pagan (2010), Canova and Ferroni (2011), Ferroni (2011), Canova (2014) analyse trend treatment in the estimation. These studies focus on biases in the parameter estimates whereas our contribution is to highlight the biases and difficulties that variable-specific time variation in trends can create for forecasting and decompo-

sitions. The importance of modelling trends and fitting the model on data in levels is also emphasised in Andrieu et al. (2009). Fernandez-Villaverde et al. (2016) bring up that the restriction on common growth rate for multiple variables does not hold in the data and suggests alternative approaches from variable-specific detrending to building multiple trends to the model. We follow closely the approach in Canova (2014) that Fernandez-Villaverde et al. (2016) refer to as hybrid approach because it uses non-model trend components. We show that the approach is suitable for a much larger model, commonly used in applied work, compared to the models Canova (2014) considers. Hence, we are able to show that issues arising from variable-specific time-variation in trends can be mitigated by using data in log-levels and modelling variable-specific trend components.

We argue that this hybrid approach is the only viable solution to deal with variable-specific trends. Despite being flexible, pre-filtering has its problems as discussed in Canova (2014) and Ferroni (2011), and in addition, when the DSGE model is used for forecasting, this approach is not suitable because the trend component are also needed for the projection horizon. The most elegant approach would be to build the variable-specific trends to the model itself. However, in the context of larger model, this is not possible because it would require including tens of processes to the model and stationarising the model would become difficult or even impossible.

Much of the recent research on the estimation of DSGE model has focused on more efficient estimation algorithms and filtering techniques, see e.g., Fernández-Villaverde and Guerrón-Quintana (2021). Our analysis suggests that how one links the model to the data and how one handles the unexplained portions of the data—those that the model cannot explain—are crucial considerations that deserve more attention. Typically, the specifications used in applied work and research omit modelling variable-specific trends. Our analysis suggests that the usability of models could be enhanced through more careful trend modelling. This is particularly important if one aims to judge the importance of different shocks on cyclical fluctuations, as in studies such as Justiano, Primiceri and Tambalotti (2010) or Christiano,



Motto and Rostagno (2014). When variable-specific trends are not modeled—as is the case in the aforementioned studies—there is a risk that structural shocks are also needed to explain long-run developments. Typically, a specific shock drives a particular variable, for example a demand shock affects consumption, an investment shock impacts investments, and a markup shock influences prices. If a data variable has long-run development that is neither captured by a trend process nor explained by measurement errors, larger structural shocks are required to drive the corresponding model variable. This can significantly influence the analysis regarding the cyclical drivers.

The DSGE model that we use for the analysis in this paper is a New Keynesian small open economy DSGE model which features a banking sector, and the economy belongs to a monetary union.<sup>2</sup> The external economy is modeled using univariate autoregressive processes for world prices, the export market, the exchange rate, and the euro area interest rate. The model is stationarized around the permanent labor productivity path.

The rest of the paper is organised as follows. In the next section we illustrate problems that variable-specific variation in trend growth rates can cause. In Section 3, we show how to meet these problems by modelling variable-specific trends. In Section 4, we analyse how trend modelling matters for structural analysis and forecasting using simulated data. In Section 5, similar analysis is carried with real data. In Section 6, we offer practical suggestions for implementing trend modelling. Section 7 concludes.

## 2 Problems with the standard approach

Typically, DSGE models are constructed in such a way that the model itself produces only common time-variation for variables in the trend growth rates. Standard solution methods require that a DSGE model produces stationary fluctuations around a steady state. However, a model can include a shock process that produces perma-

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<sup>2</sup>Kilponen et al. (2016), The Aino model of the Bank of Finland.

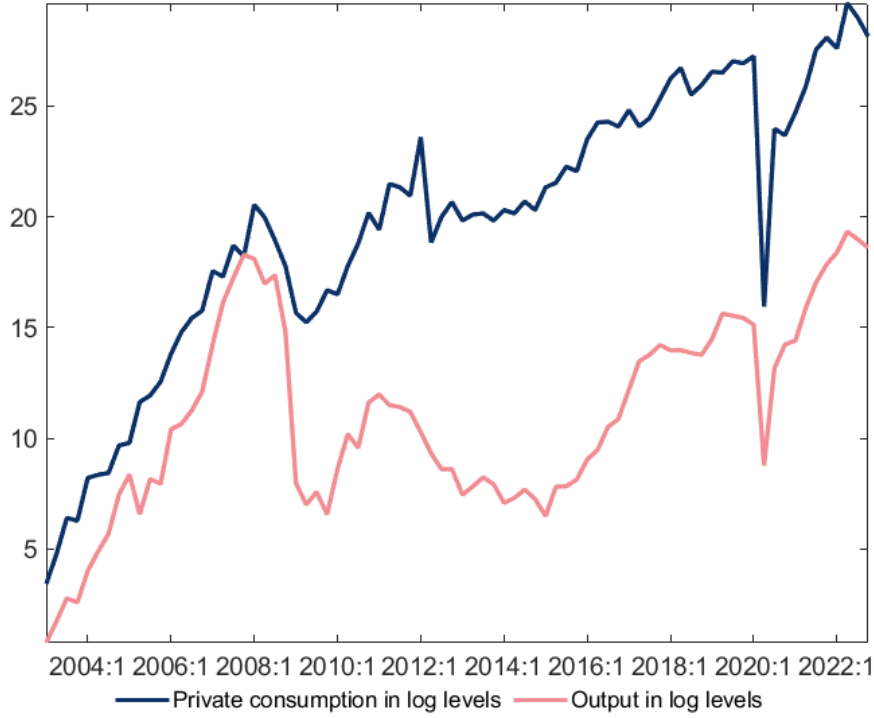


Figure 1. The evolution of the Finnish private consumption and output in log levels. Data indexed to  $\log(1)$  at the beginning of the sample period.

ment level shifts and usually this is a productivity shock process. Then the model needs to be stationarized around the productivity path. The DSGE-model, Aino model, that we use for the illustrations in this paper, and several other models used in policy institutions have this type of feature (e.g. the New Area Wide Model II of the European Central Bank by Coenen et al. (2018) and the FRBNY of the Federal Reserve Bank of New York by Del Negro et al. (2013)<sup>3</sup>). With trending productivity process, a DSGE model can produce both long-run growth and cyclical fluctuations around the growth path. However, this produces a restriction that the variation in trend growth rates is equal across variables, which might not hold in the data.

The evolution of Finnish GDP and consumption in Figure 1 serves as an illustration when the common variation in trend growths does not hold in the data. At the beginning of the sample, the variables share a common trend. Then there is a

<sup>3</sup>See also FRBNY DSGE Model Documentation (2022).

level shift in GDP in 2009 and a period of no growth, whereas consumption grows, albeit with slower pace than in the beginning.

With a trending productivity process the model can be linked to data through the following measurement equations when observed data are in log-differences

$$\Delta \log(Y_t) = \hat{y}_t - \hat{y}_{t-1} + \hat{\mu}_t + \mu^{ss} + hf_t^y \quad (1)$$

$$\Delta \log(C_t) = \hat{c}_t - \hat{c}_{t-1} + \hat{\mu}_t + \mu^{ss} + hf_t^c + (\mu^c) \quad (2)$$

where  $Y_t$  is data series of output and  $C_t$  is data series of private consumption.  $\hat{y}_t$  and  $\hat{c}_t$  are the DSGE model variables of output and consumption and are measured as percentage deviations from the steady state growth path, which is driven by the productivity process.  $\mu^{ss}$  is the steady state growth rate of productivity and  $\hat{\mu}_t$  is the temporary deviation of growth rate from the steady state rate. Jointly they produce the productivity path which is the common trend component for output and consumption. In addition, the system can include variable-specific intercepts like  $\mu^c$  which gives for consumption growth rate a constant deviation from the productivity growth rate. In log-levels,  $\mu^c$  produces stable divergence between output and consumption. Different trend growth rates of time series could also be dealt with by pre-detrending, e.g. by demeaning the series.  $hf_t$  is a high frequency component which is typically referred to as a measurement error and is a white noise process but can be an auto regressive or a moving average process. This process is supposed to capture short run variation in the data that the model does not explain.

For the illustrations of this section, we use a system of observation equations as in (1) and (2) that is specified on data in differences and accumulate the implied trend and high frequency components to log-levels. Figure 2 shows the trend components ( $\hat{\mu}_t$  and constant growth rates,  $\mu^{ss}$  and  $\mu^c$ ), model variables and high-frequency components. There is a discrepancy between the trend components that the system in (1) and (2) implies and the trend components in the data. The system in (1) and (2) allows only for constant divergence between the trends of output and consumption and for equal time variation. In the data, variables evolve differently.

Consumption and output first grow at the same pace. Then there is a significant level drop of output in 2009 and almost a decade of no growth. Consumption does not have a similar level shift in 2009 and also grows steadily after 2009, albeit with a lower rate than in the beginning of the sample period.

The trend of consumption is close to the observed series in the beginning and in the end of the data period. Note that all observed data have been indexed to  $\log(1)$  at the beginning of the sample for illustration purposes. However, most of the time there is a large gap between the data and the trend. This gap is filled by high frequency components which are shown in Figure 2 as an accumulated sum. The outcome is that GDP gets a trend path which is centered on the observed data. In this specification, the time-variation in trend components is given by  $\mu_t$ . This time-variation is different from the time-variation in the trend of growth of consumption, and hence the trend component of consumption is not centered on the observed path of consumption.

It is noteworthy that although high frequency components are specified to be zero mean processes, the accumulated sums can differ largely from what should be under this assumption. Although high-frequency components are specified to capture short-run variation, often referred to as measurement errors, they end up capturing miss-specification in long-run components in our illustration. Just by inspecting Figure 2, one would name the high-frequency components as variable-specific trend deviation components.<sup>4</sup>

The lower panel shows the components when high frequency components innovations are restricted to have 0.1 standard deviations of those in the upper panel. Then the accumulated high frequency components can no longer catch the discrepancy in growth rates, and the common trend component  $\hat{\mu}_t + \mu^{ss}$  is a compromise between several time series because it drives the trends of several variables. In the lower panel, the model component is no longer cyclical fluctuations around zero, but instead showing persistent deviations from it. The model component of con-

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<sup>4</sup>Andrle et al. (2009) also favor specification in levels to prevent high frequency components/measurement errors to accumulate to trends.

sumption is - with an exemption of few observation - above zero for almost twenty years. The structural component of output is negative for over ten years. Under this specification, the structural components capture part of low frequency developments of observed variables and hence, classifying developments to cyclical and long-run developments becomes impossible.

In the lower panel of Figure 2, there is an inconsistency in the sense that while the output gap is negative for a long period of time, the structural component of consumption is most of the time positive.

Besides for the history, trends matter also for the analysis of the current state of the economy. In the upper panel, model components for output and private consumption are close to zero, whereas in the lower panel, both consumption and output are almost five percent below the steady state which suggest a significant negative gap. If the model was used to analyse the current cyclical state of the economy, this would have a large impact on the conclusion.

Finnish data provide a good example of long-run developments that can cause difficulties when fitting a DSGE model to the data. However, data from several other countries in Figure A1 show similar developments as Finnish data show. The euro area consumption and output grew on similar trend paths in the beginning of sample but then output grew on a higher rate which has caused a persistent level shift. The same phenomenon shows up in the data of Germany, the Netherlands, and Spain. Also in Austria, Chile and Iceland output and consumption share a same growth path in the beginning of the sample but diverge afterwards.<sup>5</sup>

Taking the problems illustrated in this chapter as a motivation for alternative approach, the next section introduces how variable-specific time variation to trends can be introduced.

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<sup>5</sup>Illustrations in Lafourcade and de Wind (2012) with Dutch data in the form of nominal and real ratios also show the diverging trends of several variables. Canova (2014) illustrates with series starting from 1950 non-stationarity of some US nominal and real ratios.

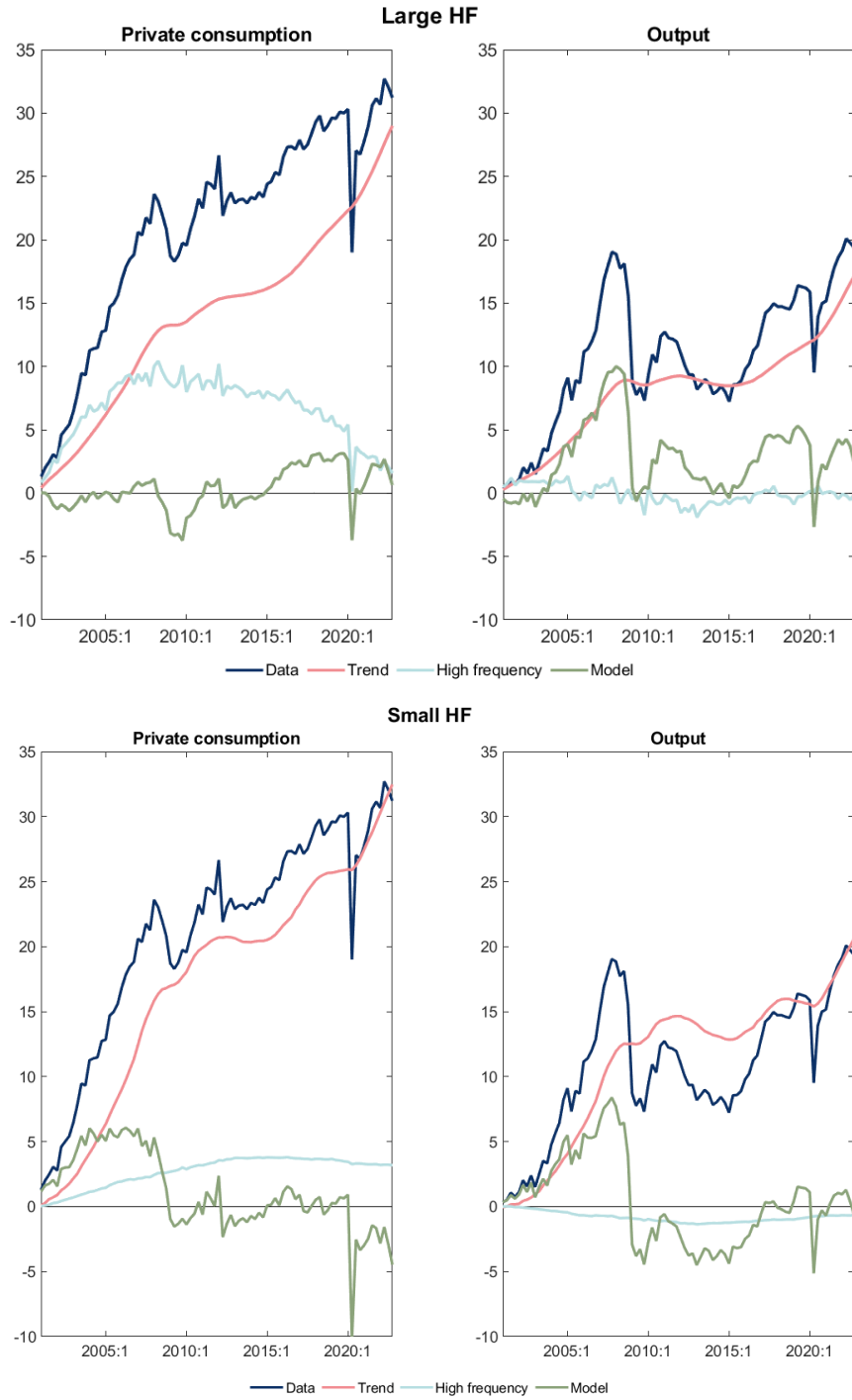


Figure 2. Decompositions when the model is specified on data in log-differences and then accumulated to log-levels. Observed data indexed to  $\log(1)$  at the beginning of the sample period. Finnish private consumption and GDP. Upper figure: specification with large high-frequency components (Large HF). Lower figure: specification with small high-frequency component (Small HF). Model refers to  $\hat{c}, \hat{y}$ .

### 3 Data in log-levels and modelling trends

In this section, we use the same DSGE model as in previous section but link it to data with a different approach. We apply the approach of Canova (2014) which basic idea is to include a trend component, model component and high frequency component:

$$\log(data_t) = trend_t + model_t + hf_t \quad (3)$$

We allow for a permanent productivity process in the model and this gives a common trend component across several variables. Following equations show how aggregate output is linked observed data

$$\begin{aligned} y_t^d &= \mu_t^T + \hat{y}_t + hf_t^y \\ \mu_t^T &= \mu^{ss} + \hat{\mu}_t + \mu_{t-1}^T \\ \hat{\mu}_t &= \rho_\mu \hat{\mu}_{t-1} + e_t^\mu \\ hf_t^y &= \rho_{hf,y} hf_{t-1}^y + \epsilon_t^y \end{aligned} \quad (4)$$

$y_t^d$  is data and  $\hat{y}_t$  is the corresponding variable in the DSGE model, and it is measured as percentage deviations from the productivity path,  $\mu_t^T$ . The trend of output is solely determined by the productivity process. The productivity process  $\mu_t^T$  follows a unit root process with time-varying drift  $\mu^{ss} + \hat{\mu}_t$ , where  $\hat{\mu}_t$  is an AR(1) process with the AR(1) coefficient  $\rho_\mu$  and  $e_t^\mu$  is the respective innovation. We allow for some persistence in the high frequency component,  $hf_t^y$ , and it follows AR(1) process and  $\epsilon_t^y$  is the innovation.

Private consumption is linked to data with the following equations:

$$\begin{aligned}
c_t^d &= c_t^T + \hat{c}_t + hf_t^c \\
c_t^T &= \mu^{ss} + \hat{\mu}_t + c_t^{TD} + c_{t-1}^T \\
c_t^{TD} &= \rho_{td,c} c_{t-1}^{TD} + e_t^c \\
hf_t^c &= \rho_{hf,c} hf_{t-1}^c + \epsilon_t^c
\end{aligned} \tag{5}$$

The permanent productivity process drives also the trend of consumption ( $c_t^T$ ) but we allow also for trend deviations,  $c_t^{TD}$ . Variable-specific trend deviations follow AR(1) processes. This specification produces permanent level deviations from the common trend but the deviations in terms of growth rates are temporary and over the long run the trend converges to common growth rate because the AR(1) coefficient  $\rho_c$  is restricted to be smaller than 1.  $\hat{c}_t$  is the corresponding variable in the model and  $hf_t^c$  is the high frequency component, which follows AR(1) process.

Appendix B provides measurement equations for some additional variables. Similar structure as for consumption is assumed for all the rest demand components of GDP and for some other variables too. All variables that in the model are stationary around the permanent productivity path, have productivity process in their measurement equations. For the prices it is assumed that several price variables, e.g. price of consumption, have the trend of output price deflator in their measurement equations. In our setup, we include common trend components to variables that in the model share common stochastic trends, meaning that we allow for co-integration. However, we also allow trend deviations from the common trends and hence when the common trend assumption does not hold in the data, trend deviation components capture the long-run variation that the common trend component does not capture. For several foreign variables, e.g. price of oil and exchange rate, the trend modelling is completely univariate meaning that their trends do not share common components with other variables.

Figure 3 shows decomposition of output and consumption when the DSGE



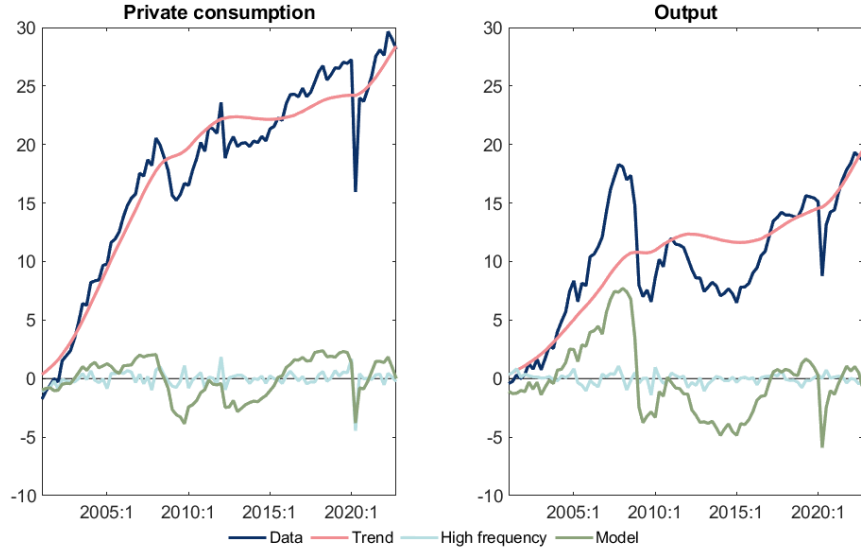


Figure 3. Decompositions when the model is fitted on log-level data and variable-specific trends are modelled. Finnish private consumption and GDP. Model refers to  $\hat{c}$ ,  $\hat{y}$ .

model, Aino model as in Section 2, is linked to data in log-levels. The cyclical model variables,  $\hat{c}$ ,  $\hat{y}$ , are now cyclical and symmetric variation around the zero line. The behavior of model variables is close to that of specification on differenced data with large high frequency components but not exactly the same (Figure 2, upper panel). For example, the cyclical component of consumption under specification in differences with large high frequency components remains below zero after the Covid shock whereas the cyclical component of specification in levels returns to above zero and hence suggesting different cyclical phase of the economy.

In the context of specification in levels, it is the variable-specific component of trends that capture differences in the trends across variables and not the high frequency component. As can be seen from Figure 3, the high frequency component now captures only high frequency variation. In a sense, under the specification in levels with variable-specific trends, the realisations of different components are what they should be. Under the specifications in differences in the previous subsection the trends were not centered on the observed series and accumulated high frequency components produced persistent series with non-zero mean.

To gain insight into how a high-frequency component captures trend shifts when

using specification in differences and how specifications in differences and levels differ, consider the following steps. First, let us rewrite the measurement equation for consumption in periods 1 and 2 in levels by moving the lagged consumption term to the other side.

$$\begin{aligned} \log(C_1) &= \hat{c}_1 - \hat{c}_0 + \hat{\mu}_1 + \mu^{ss} + hf_1^c + \mu^c + \log(C_0) \\ \log(C_2) &= \hat{c}_2 - \hat{c}_1 + \hat{\mu}_2 + \mu^{ss} + hf_2^c + \mu^c + \log(C_1) \end{aligned}$$

Then we can substitute measurement equation of  $C_1$  to measurement equation of  $C_2$

$$\begin{aligned} \log(C_2) &= \hat{c}_2 - \hat{c}_1 + \hat{\mu}_2 + \mu^{ss} + hf_2^c + \mu^c \\ &+ \hat{c}_1 - \hat{c}_0 + \hat{\mu}_1 + \mu^{ss} + hf_1^c + \mu^c + \log(C_0) \end{aligned}$$

which can be simplified to

$$\begin{aligned} \log(C_2) &= \hat{c}_2 + \hat{\mu}_2 + \mu^{ss} + hf_2^c + \mu^c \\ &+ \hat{\mu}_1 + \mu^{ss} + hf_1^c + \mu^c + \log(C_0) - \hat{c}_0 \end{aligned} \tag{6}$$

We can express the measurement equation as "data=model+trend+initial conditions"

$$\begin{aligned} \log(C_2) &= \hat{c}_2 + C_2^T + \log(C_0) - \hat{c}_0 \\ C_2^T &= \hat{\mu}_2 + \mu^{ss} + hf_2^c + \mu^c + C_1^T \end{aligned} \tag{7}$$

Equations in (7) show similarities to those of specification in levels, equations in (5). However, there are striking differences too. In equation (7), there is no high-frequency component in the measurement equation. Consequently, there is no component that could have temporary effects on the level of consumption, which

is what a measurement error should account for. In this specification, the high-frequency component causes permanent level changes. Alternatively, it can be interpreted as temporary deviations in the growth rate of consumption. The growth rate of consumption is determined by the common productivity trend and a constant consumption-specific deviation from it. A noteworthy difference affecting forecasting performance is that the high-frequency component probably exhibits less persistence than the true trend deviation component. Because of the low persistence, it cannot produce deviations in the trend growth rate for a projection period.

When specification in levels is transformed to specification in differences, we get

$$\log(C_t) - \log(C_{t-1}) = \hat{c}_t - \hat{c}_{t-1} + \mu^{ss} + \hat{\mu}_t + C_t^{TD} + hf_t^c - hf_{t-1}^c \quad (8)$$

In (8), lagged high-frequency component ensures that high-frequency components do not produce permanent level changes. <sup>6</sup>

## 4 Evidence from a controlled experiment

In this section, we analyse differences between specifications using simulated data. We have a specification on differenced data with small high-frequency components, a specification on differenced data with large high-frequency components and a specification on log-levels data with variable-specific trends. The data are simulated using the DSGE model with specification on data in levels with variable-specific trends. We analyse the performance of specifications in terms of historical decompositions and forecasting. An advantage of using simulated data is that we know the true paths of each component (trend, cyclical, high frequency) and can compare how well different specifications capture those. Each specification uses the same parameter values as specification that is used to simulate the data. Hence, there is no inaccuracy stemming from parameter estimates for any specification.

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<sup>6</sup>Pagan (2017) also analyses from another perspective problems of measurement errors when the data are in first-differences and also suggest specifying measurement errors in first-differences.

Variable-specific trend components are not explicitly modelled in the specifications on data in differences. As shown in equation (7), in the specification in differences, the high frequency component captures both the true high frequency component and variable-specific trend component. Hence, standard deviations of high frequency component innovations in the specifications in differences are set as a sum of true standard deviations of high frequency innovation and trend deviation innovation. We accumulate variable-specific trends from the common productivity trend and from the high-frequency component when using a specification in differences. The trend of consumption,  $c^T$ , in the specifications in differences consists of  $\mu^T$  and  $hf^C$ . In the small high-frequency component specification, the standard deviations of the shocks of the high-frequency components are set to be 0.1 times those in the large high-frequency component specification. Specifications in differences produce forecasts of variables in differences, and these are accumulated to log levels.

The simulation period is 40 years. The first 20 years are used to filter the data to different components, and from then on out-of-sample forecasts are produced.

To gain insight into how specific features matter, we first produce simulations in which only structural shocks (including the permanent productivity shock process) and the trend deviation shock of private consumption are drawn. Other trend deviation shocks and all high-frequency component shocks are left out. Figure 4 shows outcomes of one simulation and there consumption grows faster than output after the first years and then stays on higher levels than output. When in the specification on data in differences high-frequency components are restricted to be small, high-frequency component does not capture the variable-specific trend changes. For this reason, the trend of consumption,  $c_t^T$ , stays below the true trend path of consumption and equals the common productivity trend,  $\mu_t^T$ . And because the trend of consumption is below the true path, the cyclical model component  $\hat{c}_t$  stays above the true path for most of the time. In this way the sum of components equals the path of observed consumption, but the components themselves are far away from

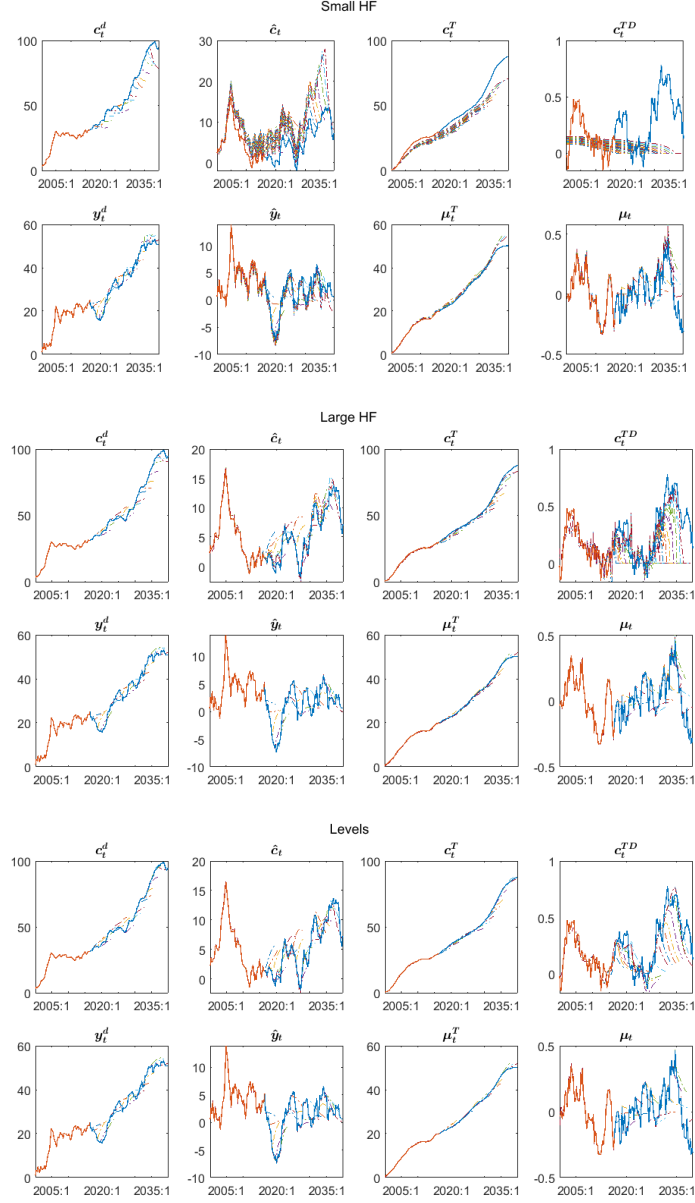


Figure 4. Simulated data from a restricted model. Upper: Specification on data in differences with small high-frequency components. Middle: Specification on data in differences with large high-frequency components. Lower: (log) levels specification and variable-specific trends. Solid lines are true simulated data and dashed lines are decomposition and forecasts of a model. Forecast period is on blue. When the model is specified in differences, projections for observed variables are transformed to log levels.

Table 1. RMSEs, simulated data from a restricted model

	$e_t^\mu$	<u>RMSE</u> consumption demand shock
Diffs-Small HF	0.092	2.53
Diffs-Large HF	0.091	1.08
Levels-trends	0.091	0.78

Note: Root-mean-square errors are calculated for the differences between the simulated data and the decompositions of the model.  $e_t^\mu$  is shock to the growth rate of productivity. Diffs-Small HF: data in differences and small high-frequency components. Diffs-Large HF: data in differences and large high-frequency components. Levels-Trends: data in levels and variable-specific trends.

the true paths.

The middle panel shows the outcomes when the standard deviations of high frequency components equal the true standard deviations of high frequency component and trend deviation innovations. Now the decomposition yields path for the high-frequency component that maps relatively well to the true path of consumption trend deviations. Because there is no gap between the trend series, the cyclical model component of consumption is close to the true one and persistent gaps to the true one do not exist.

Structural analysis of consumption developments would be misleading when using specification in differences with small high-frequency components. It would suggest that for a long time consumption had been above its steady state value which would indicate a consumption boom. However, the true path of the cyclical component of consumption stays much of the time close to zero and goes negative for some years. In addition, because structural shocks are required to drive a certain path for the cyclical component, shock decomposition based on the specification with small high-frequency components would in this case be misleading. Table 1 shows based on 100 simulations that the specification with small high-frequency components performs worse in capturing the true consumption demand shocks than the other two specifications. When high frequency components are restricted to be small, demand shocks are not used only to capture cyclical fluctuations but in addition to make the cyclical model component to capture longer run developments that the trend components does not capture.

The cyclical component of consumption, that the specification with small high-frequency components produces, is well above the steady most of the time. In the projection period, the cyclical component starts to return to steady state because also the structural shocks start to return to the steady state according to their AR(1) processes. This creates the large drops in the forecast of consumption. To the smaller extent the same shows up in the forecasts of output because in some points of time  $\hat{\mu}_t$  is further away from the steady state than the true process.

The specification in differences with unrestricted high frequency components and levels specification produce relatively similar decompositions. In the forecasting period, there is a difference in the behavior of trend deviations. While in the data period high frequency component captures the trend deviations over all relatively well, this does not hold in the projections. The reason is that trend deviations are assumed to be persistent process while high frequency component is a white noise process. For this reason, the trend deviations that high frequency component captures die out right when the projection begins. The results in Table 1 suggest that specification in levels captures the shocks more accurately than specification in differences with large hf's although much larger difference is to specification with small high frequency components.

Next we simulate data from the levels specification so that all shocks (all structural, trend and high frequency shocks) are included. The results based on 100 simulations are presented in Table 2. We provide the forecast RMSEs and the RMSEs for historical decomposition, which allows us to analyse how accurately models pin the cycle and trend in the history. The level model outperforms the diff model in all horizons for observed variables, cycles and trends. The relative performance of the level model is especially good regarding the cyclical components and the trends. For historical decomposition of output's cyclical component, the RMSE of the specification in levels is only 0.78 of the RMSE of the specification in differences with unrestricted high frequency components. Regarding private consumption's cyclical component, the respective ratio is 0.74.

Table 2. RMSEs, simulated data from the full model

			$\underline{\underline{y^d}}$		
	historical	1	4	8	12
Diffs-Small HF	0.00	1.65	3.07	5.25	6.65
Diffs-Large HF	0.00	1.50	2.41	3.72	4.60
Levels-Trends	0.00	1.47	2.36	3.62	4.45
			$\underline{\underline{\hat{y}_t}}$		
	historical	1	4	8	12
Diffs-Small HF	2.47	3.89	3.74	3.78	3.82
Diffs-Large HF	1.43	2.33	2.68	3.23	3.53
Levels-Trends	1.11	2.04	2.51	3.18	3.51
			$\underline{\underline{\mu^T}}$		
	historical	1	4	8	12
Diffs-Small HF	2.71	4.84	5.27	6.13	6.92
Diffs-Large HF	2.14	2.84	2.96	3.32	3.76
Levels-trends	1.07	1.60	1.74	2.16	2.72
			$\underline{\underline{c^d}}$		
	historical	1	4	8	12
Diffs-Small HF	0.00	1.97	3.91	6.73	8.25
Diffs-Large HF	0.00	1.82	2.93	4.49	5.59
Levels-Trends	0.00	1.68	2.75	4.27	5.36
			$\underline{\underline{\hat{c}}}$		
	historical	1	4	8	12
Diffs-Small HF	5.28	5.90	5.22	5.13	5.55
Diffs-Large HF	2.54	3.39	3.61	4.07	4.50
Levels-Trends	1.89	2.63	3.01	3.71	4.19
			$\underline{\underline{c^T}}$		
	historical	1	4	8	12
Diffs-Small HF	5.19	6.29	6.47	7.05	7.75
Diffs-Large HF	2.40	3.35	3.54	4.07	4.71
Levels-Trends	1.82	2.54	2.73	3.29	3.95

Note: Table presents the root mean square errors (RMSE) for historical decompositions and for forecasts of observed variables, model and trend components for horizons of 1, 4, 8 and 12 quarters. Diffs-Small HF: data on differences and small high-frequency components. Diffs-Large HF: data on differences and large high-frequency components. Levels-Trends: data in levels and variable-specific trend components.



Our results suggest that the specification in levels which features variable specific trends produces better forecasting accuracy than the both specifications on differenced data. Moreover, the specification in levels provides not only the best forecasts for the cycle component and the trend but also it pins down their history most accurately. In addition, our results also point out that the specification in differences with small high frequency components performs very poorly. This finding is important more generally as well, because when we constrain the standard deviation of the high frequency component to be very small, it resembles a situation where the measurement equation completely omits measurement error. Consequently, this raises concerns regarding a modelling choice, which is quite common in practice. Results suggest that specification in differences with large high-frequency components performs well. However, in this comparison standard deviations are known and set optimally. In real application it is not guaranteed that when working with a specification in differences, one ends to good specification by ignoring variable-specific trends and allowing high frequency components for each variable.

## 5 An empirical application

In this section, we explore how modelling choices of the trends and data transformations influence the practical use of DSGE model. Because these choices matter for the decomposition to trend and model components, it has implications for the shock contributions which we explore in the first subsection. When trend components are not properly modelled, model variables capture also long-run deviations. These deviations due to miss-specification are attributed to some structural shocks, which importance for the fluctuations can then be over-emphasised. The second subsection analyses how data transformation and trend modelling matters for forecasts.

We use the same parameter values for the three specifications. The parameter estimates are the posterior mode of the specification in levels when using the data of 2001-2022 in the estimation. Therefore, the forecast evaluation in the second sub-

section does not give a true assessment of out-of-sample forecast accuracy. However, it nevertheless allows to compare relative accuracy across specifications. We use the same parameter values across the three specifications because we want to emphasise the effect of data transformations and trend modelling itself on the outcomes that the models produce. The effects of trend treatment on the parameter estimates are discussed in Canova (2014).

## 5.1 Historical decompositions

In this subsection we demonstrate how the specification choices affect the historical shock contributions with examples of output growth, private consumption growth and inflation (measured as a change in the price of output). Figure 5 shows the historical decomposition for the output growth for all three specifications. In the upper panel results for the specification in differences with small high frequency components are presented and in the middle panel for specification in differences with large high frequency components. The lower panel displays results for specification in levels.

There are differences that show up in the Figure 5. When high frequency components are restricted to be small the contribution of high frequency component becomes small, as one would expect, and is digested by the other available components. In turn the supply shocks are notably larger than in the other models. For example, in the midst of the global financial crisis in 2009 there is a disagreement between the specifications whether the slump was driven by a demand or a supply crunch. The specification with small high frequency components would suggest that it was driven by supply, and the specification with large high frequency components would argue in favor of demand crunch. The specification in levels would concur that it was mainly demand. Despite the latter two specifications highlight in this particular case the dominant role of demand shock, the relative importance of the demand shock measured by the magnitude is not the same under specifications in differences with large high-frequency components and specification in levels. Moreover, there

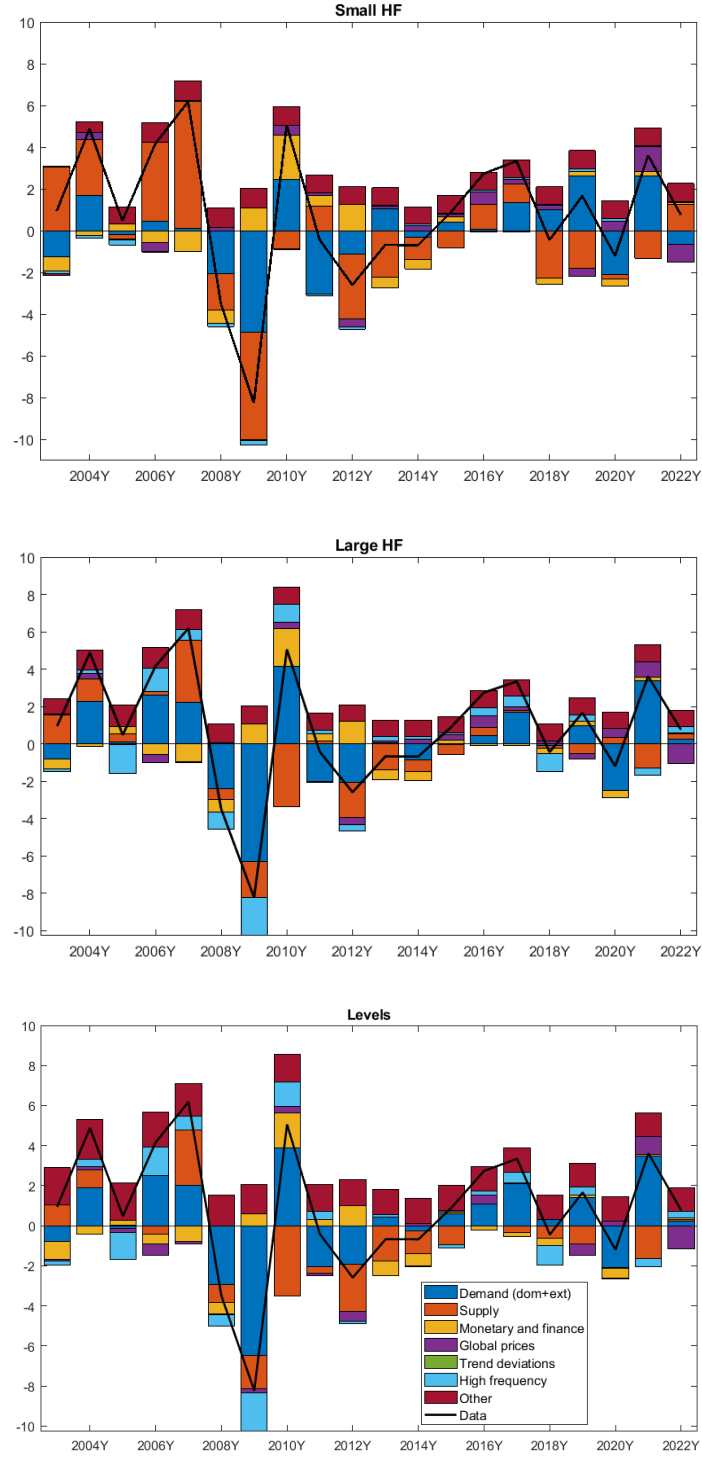


Figure 5. Shock decomposition of Finnish output growth per capita. Upper: Specification on data in differences with small high-frequency components. Middle: Specification on data in differences with large high-frequency components. Lower: Specification on data in levels with variable-specific trends. "Other" is the contribution of the steady state growth rate and initial conditions. "Supply" includes the contribution of the innovations of permanent productivity process,  $\mu_t$ .

are other differences as well between these two specifications. For example, there are variations in terms of how important the global prices or the monetary factors have been for the business cycle.

Figure 6 shows the historical decomposition for private consumption growth in volume per capita. In this example, we observe that the trend deviation component plays an important role when you carefully model the evolution of trends and allow variable specific time variation in trends. This is not surprising while we already earlier made qualitative conclusion by observing Figure 1 that the level of output and consumption has diverged substantially but not with a constant rate. Now we can express also quantitatively trend deviation's impact on growth.

More specifically, at the beginning of the data period, the trend deviation component is important in the specification in levels (Figure 6, lower panel), whereas the same is captured by high frequency component when the data is in differences and large high frequency components are allowed (middle panel). When high frequency components are restricted to be small, demand and supply shocks are more important. This confirms that the underlying cause for the evolution of private consumption varies a lot between models (whether it is trend, cycle or high frequency) and changes the relative importance of structural shocks.

Our last example shows the historical decomposition of inflation measured as a change in the price of output (Figure 7). Earlier we noted that the specification in differences with small high-frequency component needed large supply shocks to match historical output growth (Figure 5). In turn, in the case of output price, this results that larger demand shocks are needed to balance the supply shocks. This shows up in Figure 7 that the specification in differences with small high frequency components has larger contributions both from supply and demand shocks than the specification in differences with large high frequency components and specification in levels. In addition, we note that in decomposition of the specification with small high-frequency components, the sum of absolute contributions is larger compared to the specification with large high-frequency components. This suggests that there

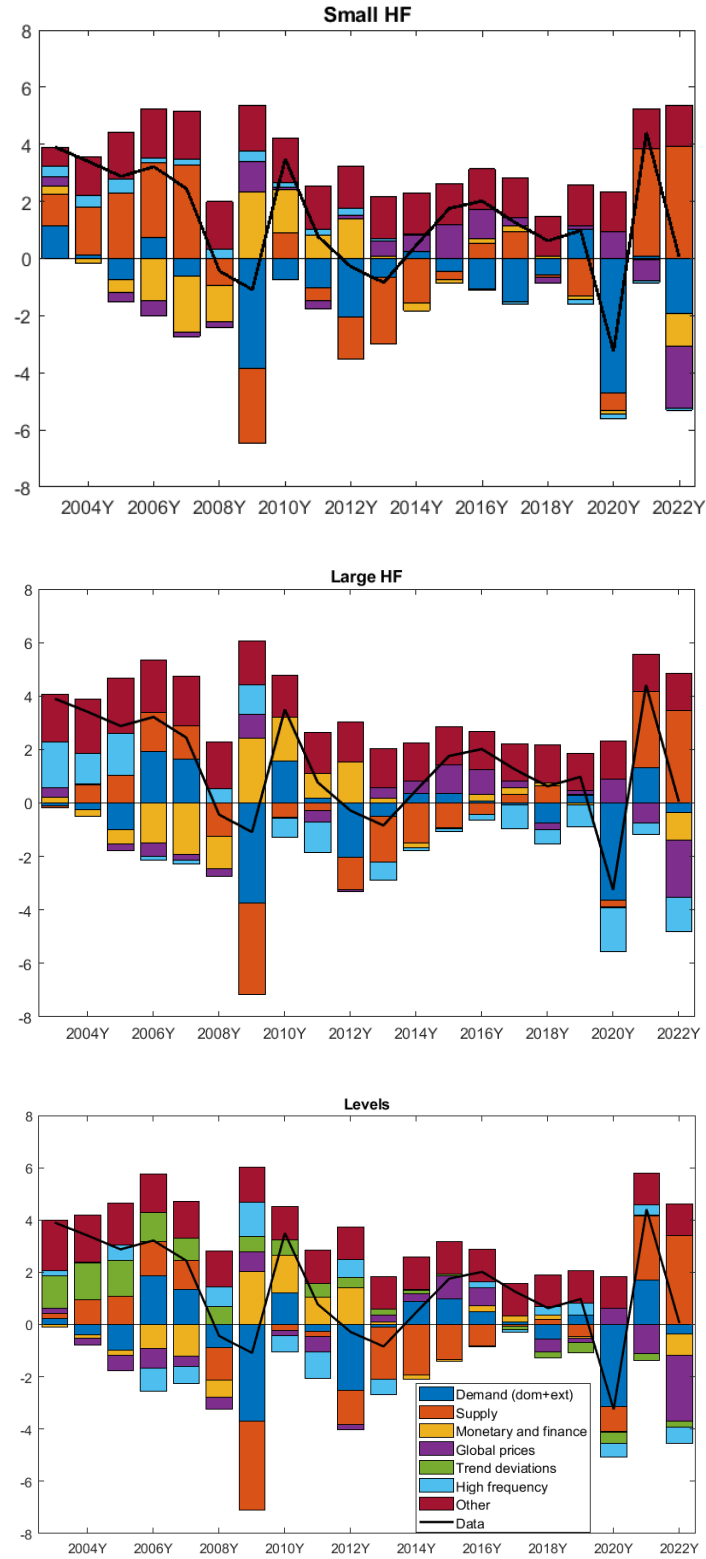


Figure 6. Shock decomposition of Finnish private consumption growth per capita. Upper: Specification on data in differences with small high-frequency components. Middle: Specification on data in differences with large high-frequency components. Lower: Specification on data in levels with variable-specific trends "Other" is the contribution of the steady state growth rate and initial conditions. "Supply" includes the contribution of the innovations of permanent productivity process,  $\mu_t$ .

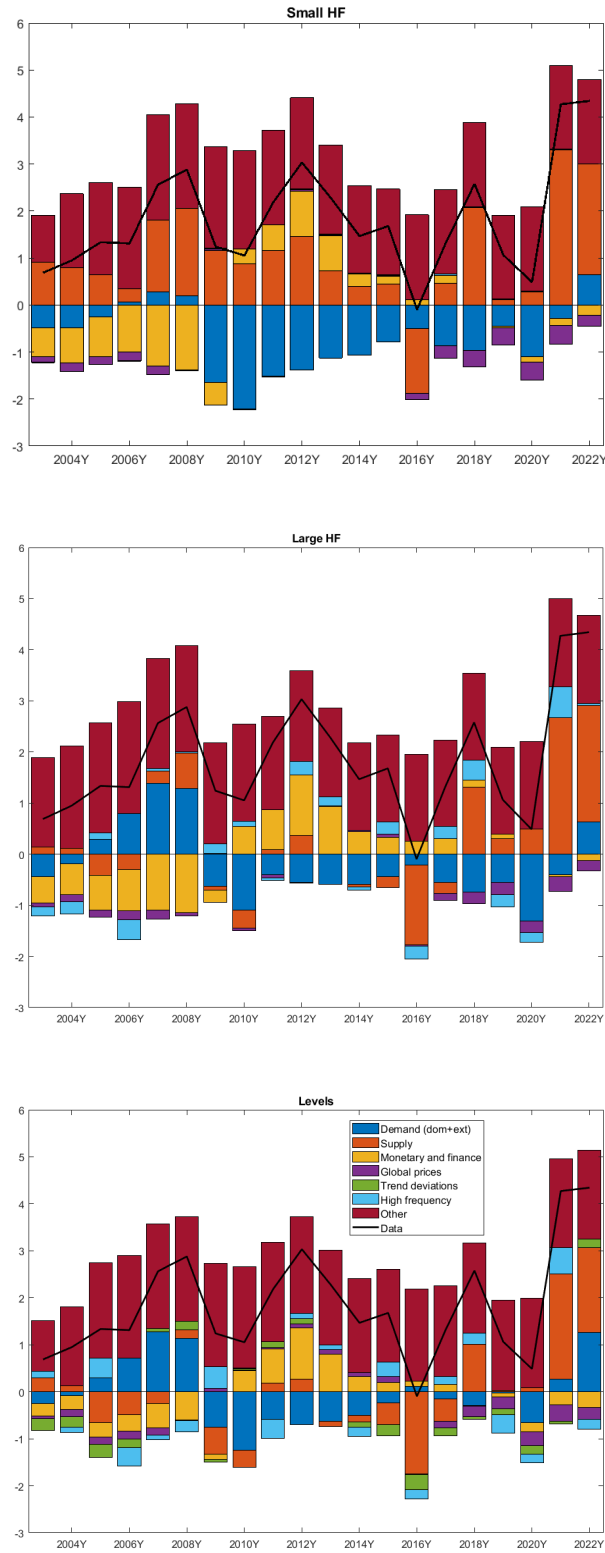


Figure 7. Shock decomposition of Finnish output deflator. Upper: Specification on data in differences with small high-frequency components. Middle: Specification on data in differences with large high-frequency components. Lower: Specification on data in levels with variable-specific trends. "Other" is the contribution of the steady state growth rate and initial conditions. "Supply" includes the contribution of the innovations of permanent productivity process,  $\mu_t$ .

could be a risk of some unintended excessive structural volatility in this specification.

Under the specification in levels, the contributions of monetary and financial shocks are different than in both specifications on data in differences. Modelling of the trends matters also how interest rates and exchange rate series are decomposed to model components and to trend and high-frequency components. The cyclical model component of these series is the only component that matters for the model dynamics. Also, when comparing more closely the results of the specification in differences with large high-frequency components and the specification in levels, we see that the importance of demand shock driving the inflation is more pronounced under the specification in levels.

This illustrates the importance of careful modelling in order to understand what are the factors that have caused acceleration or deceleration of inflation. Has the inflation accelerated because of a demand burst or because of a negative supply shock? For example, this would have important implications for the monetary authority if she would prefer to look through supply shocks and react only to demand shocks.

To sum up this section, we first observe that the trend deviation component plays an important role when you carefully model the evolution of trends and allow variable-specific time variation in trends. This also allows us to quantify how much of the observed economic growth or inflation is attributed to trend deviations. Second, we have demonstrated that the relative importance of shocks varies between specifications and so does the interpretation whether economic developments are due to changes in trend, cycle or high frequency events. Third, from the point of policy perspective, our examples highlight the importance of careful modelling of trends in order to avoid flawed cyclical analysis leading to misguided policy advice at worst. It is, of course, true that when a trend component, a non-model component, explains data, there is no story for the developments it explains. However, the situation is in no way better if the trend component is excluded and long-run developments are explained by the model component, because this can lead to wrong conclusions about the driving shocks.

## 5.2 Forecasting

In this section, we show the results of our forecasting exercise. The forecasts are produced starting from 2016 and new forecasts are made every second quarter. Forecasting horizon is up to 12 quarters. We provide forecast for GDP (Y), private consumption (CH), private investments (I), exports (X), imports (M) and for the corresponding prices with prefix P.

The forecasting exercise is conducted by using the parameters of the posterior mode of the specification in levels with variable specific trends. For the estimation we have used the whole data period 2001-2022. Ideally, we would re-estimate the model on each forecasting round but given that estimation of large models can require tuning from the econometrician during the estimation, we instead run the forecasting exercise with constant parameters. Our forecasting exercise cannot be used to assess the actual forecasting ability of each specification but nevertheless allows to judge the relative forecasting accuracy across specifications.

Figure 8 shows the forecasts. In the upper panel results for the specification in differences with small high frequency are presented and in the middle panel for the specification in differences with large high frequency. The lower panel displays results for the specification in levels. All results in Figure 8 are presented in log levels. Again, largest differences appear between the specification in differences and small high frequency components and the other two specifications.

Close examination of Figure 8 reveals that after 2017 consumption growth slows down and this is best captured by the specification in levels. In case of investments, we observe that the specification in differences seems to project the past to future which leads to over-shooting in the forecasts. An undesirable feature that does not occur with the specification in levels.

Our data period covers the coronavirus pandemic which provides us valuable insights how the different specifications behave in a case of very large, unexpected event. In the beginning of the pandemic the economic activity suddenly collapses in Q2 2020. Consumption and output in both specification in differences continue



to fall and thus cause a significant forecast error. In that point of time, the specification in differences with large high frequency components outperforms the small high frequency components specification. This happens because a large part of the sudden drop in consumption is captured by a high frequency component which is less persistent than the structural shocks and hence affects forecasts less on the longer horizon. All in all, out of the three specifications the level specification with variable-specific trends performs best while it can accommodate part of the sudden drop as high frequency event and also for not allowing the trend to change abruptly.

To provide more formal results of the forecast horse race, we present in Table 3 the forecast root mean square errors (RMSE) for horizons of 1, 4, 8 and 12 quarters. Most of the time the level specification is the most accurate in forecasting.

To be more precise, in terms of GDP, there are not much differences in the forecasting accuracy across specifications. The diff specification with large high frequency components is slightly better than the level specification in horizons 4 and 8 whereas the level model is best at longer horizon. The diff specification with small high frequency component is always the worst.

In terms of demand components, there are more differences, and the level specification with variable-specific trends is always the best one. The differences increase as the horizon increases. For investments, the RMSE of level model is only 67 % of the RMSE of specification in differences with small measurement errors and 76 % of the RMSE of specification in differences with large measurement errors. This suggest that when variable specific trends are more accurately modelled, this also helps in the forecasting.

To sum up, we have provided evidence that specifying the model on data in levels and including variable-specific trends increases forecast accuracy. The careful modelling of low-frequency trends has improved the forecast especially in the longer horizon. We have also illustrated that in the case of large sudden extreme events, such as the coronavirus pandemic, the combination of high frequency and trend

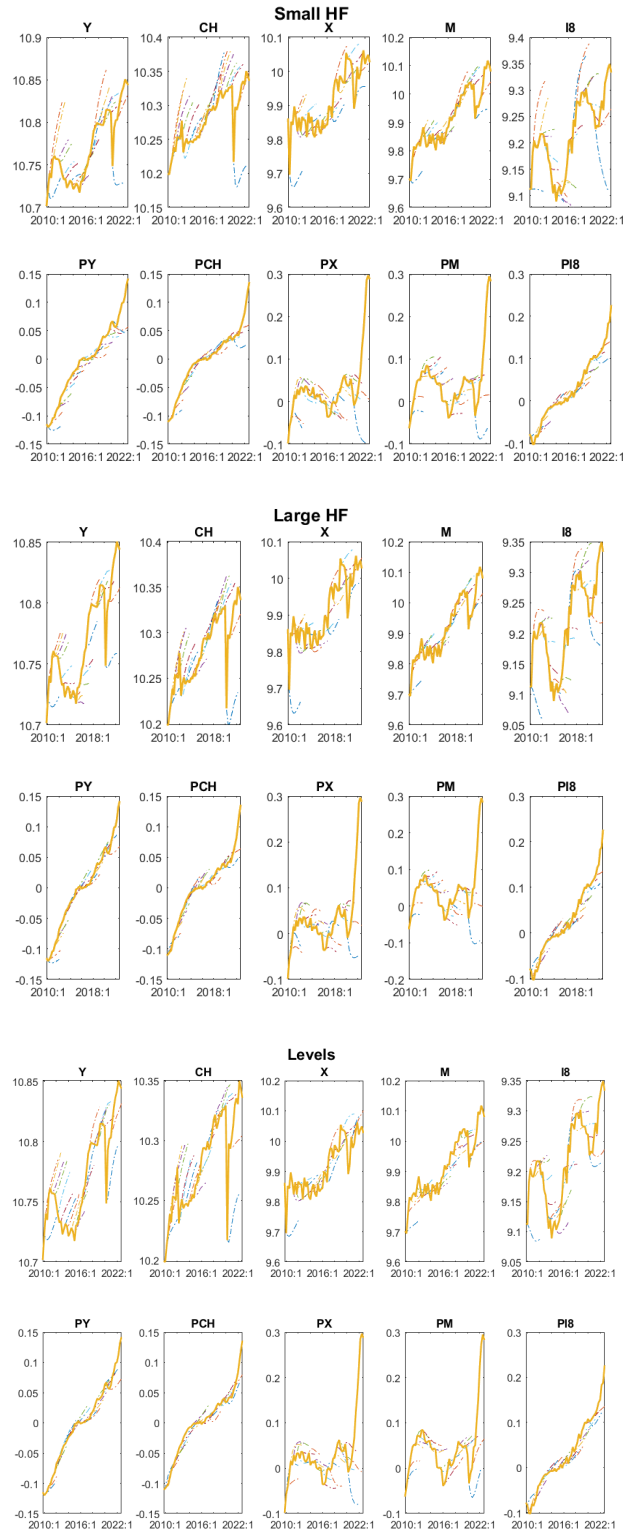


Figure 8. Forecasts for Finnish output (Y), private consumption (CH), exports (X), imports (M), private investments (I8), output deflator (PY), private consumption deflator (PCH), export deflator (PX), import deflator (PM) and private investment deflator (PI8). Upper: Specification on data in differences with small high-frequency components. Middle: Specification on data in differences with large high-frequency components. Lower: Specification on data in (log) levels with variable-specific trends. All results in log levels.

Table 3. RMSEs, forecasts

	<u>GDP</u>			
	1	4	8	12
Diffs-Small HF	0.0087	0.0269	0.0408	0.0542
Diffs-Large HF	0.0083	0.0240	0.0336	0.0450
Levels-Trends	0.0083	0.0254	0.0361	0.0447
	<u>CH</u>			
	1	4	8	12
Diffs-Small HF	0.0141	0.0365	0.0545	0.0671
Diffs-Large HF	0.0122	0.0300	0.0408	0.0455
Levels-Trends	0.0109	0.0272	0.0360	0.0383
	<u>X</u>			
	1	4	8	12
Diffs-Small HF	0.0539	0.0636	0.0662	0.0908
Diffs-Large HF	0.0554	0.0660	0.0705	0.0880
Levels-Trends	0.0514	0.0607	0.0593	0.0626
	<u>I</u>			
	1	4	8	12
Diffs-Small HF	0.0278	0.0543	0.0951	0.1287
Diffs-Large HF	0.0268	0.0547	0.0922	0.1140
Levels-Trends	0.0256	0.0481	0.0746	0.0861

Note: Table presents the forecast root mean square errors (RMSE) for horizons of 1, 4, 8 and 12 quarters for Finnish GDP, private consumption (CH), exports (X) and private investment (I) transformed to log levels. Specification on data in differences with small high frequency components, Small HF. Specification on data in differences with large high frequency components, Large HF. Specification on data in (log) level with variable specific trends, Levels.

components can help avoid large forecast errors by allowing a more rapid pickup in activity and a return to previous trends.

## 6 Practical considerations

Modelling trends carefully and including high frequency components brings several standard deviations that need to be determined. These standard deviations can not be estimated using only data information because free estimation does not necessarily lead to standard deviations that produce components that correspond to trend, cyclical and high frequency components. Therefore, we outline four practical steps how to proceed successfully.

1. Calculate standard deviations for each variable (in log-diffs). Set that the standard deviations of trend deviation shock and high frequency component shock are e.g. 0.1 and 0.3 of the standard deviation of the observed series in log-diffs. Or use statistical filter or unobserved component model to obtain standard deviations for trend and high frequency components.

2. Run Kalman smoother with starting values from step 1. Calculate standard deviations for filtered shock series and adjust calibrated standard deviations accordingly. Visually inspect that trend series look like trend components, model variables look like cyclical components and high frequency component captures mainly noise. Adjust shock standard deviations.
3. After finding satisfactory standard deviations, choose a structural shock for each observed variable that seems to be the most important for the corresponding model variable. E.g. mark-up shock for output price. Alternatively, choose e.g. permanent productivity shock for a variable if it's difficult to find the most important shock for that variable. Calculate the relative standard deviations between trend shock and structural shock, and between high-frequency component shock and structural shock. Write trend and high-frequency shocks as a function of structural shock using the calculated relative standard deviations.
4. Estimate the model, including the standard deviations of structural shocks.
5. Adjust relative standard deviations if needed and estimate the model again.

## 7 Conclusions

We analysed how data transformations and trend modelling impact the use of DSGE models for structural analysis and forecasting. The motivation for this analysis stems from the fact that data often includes series with variable-specific time variation in trend growth rates, whereas DSGE models are constructed so that models themselves produce only time variation in trend growth rates that is common to variables in the model.

Our results show that if variable-specific time variation in trends is not modelled, high-frequency components capture it when the model is specified on differenced data. When high-frequency components are restricted to be small, the cyclical model component captures changes in trends. This, in turn, distorts the shock decompositions and the decomposition into trend and cyclical components. In addition, a

specification with high-frequency components performs poorly in forecasting in our experiments.

The specification with small high-frequency components corresponds to ignoring them in the model. In practice, it is not uncommon to overlook high-frequency components or allow them for only some variables. This raises a concern: there could be a common source of miss-specification in DSGE studies and applications if neither high-frequency components nor properly modelled trends capture time variation in trends. Our results suggest that when conducting empirical analysis with a DSGE model, an important specification check should be whether there is variable-specific time variation in trends and which component in the specification captures this.

We show that the problems arising from time variation in trends can be remedied by fitting the model to (log) level data and incorporating variable-specific trend components. This hybrid approach yields a decomposition where measurement errors, the cyclical model component, and the trend component capture what they should capture. We also find trend modelling to increase forecast accuracy.

Modelling variable-specific trends requires some extra effort but is nevertheless practical and can be used with a larger model than in Canova (2014). We propose some hints how to handle the extra difficulty of setting trend standard deviations that trend modelling requires. In future research, it could be explored how to improve this process.

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# Appendix A

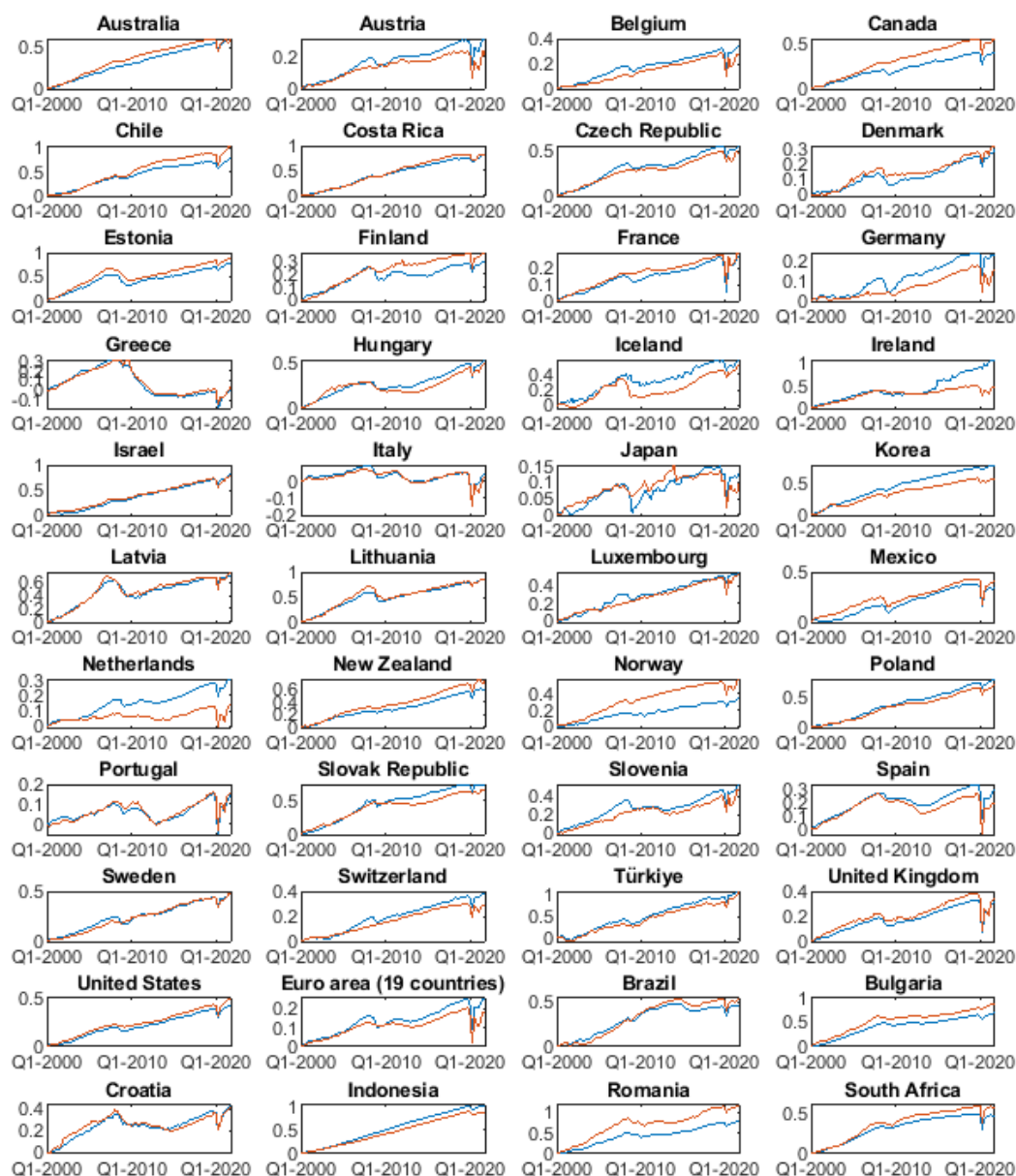


Figure A1. Log level series of real GDP (blue line) and private consumption (red line) by countries. Data period is 2000-2022 and series are indexed to Q1-2000.



## Appendix B

This section shows measurement equations and definitions of trend and high frequency component process.

For the price of output ( $py_t^d$ ), we assume that its trend mean growth is  $\pi^{ss}$  which is the steady state inflation rate in the model. We allow for temporary deviations from this growth rate which are given by  $py_t^T$ .  $py_t^T$  is trend price of output and  $py_t^{TD}$  is the trend deviation which follows an AR(1) process.  $hf_t^{py}$  is the high frequency component.

$$\begin{aligned}
 py_t^d &= py_t^T + \widehat{py}_t + hf_t^{py} \\
 py_t^T &= \pi^{ss} + py_t^{TD} + py_{t-1}^T \\
 py_t^{TD} &= \rho_{py} py_{t-1}^{TD} + e_t^{py} \\
 hf_t^{py} &= \epsilon_t^{py}
 \end{aligned} \tag{9}$$

The price of consumption is assumed to follow the same trend path as price of output, and also deviations are allowed.

$$\begin{aligned}
 pc_t^d &= pc_t^T + \widehat{pc}_t + hf_t^{pc} \\
 pc_t^T &= py_t^T - py_{t-1}^T + pc_t^{TD} + pc_{t-1}^T \\
 pc_t^{TD} &= \rho_{pc} pc_{t-1}^{TD} + e_t^{pc} \\
 hf_t^{pc} &= \epsilon_t^{pc}
 \end{aligned} \tag{10}$$

The trend of nominal wage is linked to trend of output price and to the produc-

tivity process

$$\begin{aligned}
w_t^d &= w_t^T + \hat{w}_t + hf_t^w \\
w_t^T &= py_t^T - py_{t-1}^T + \mu_t^T - \mu_{t-1}^T + w_t^{TD} + w_{t-1}^T \\
w_t^{TD} &= \rho_w w_{t-1}^{TD} + e_t^w \\
hf_t^w &= \epsilon_t^w
\end{aligned} \tag{11}$$

When trend deviations of nominal wages,  $w_t^{TD}$  are zero, the trend of nominal wages is linked to producer price trend inflation and to productivity growth. Hence, the trend of producer price deflated real wages is assumed to follow productivity growth.

The trends of foreign variables are modelled without relation to the trends of domestic variables. For example, the price of oil:

$$\begin{aligned}
poil_t^d &= poil_t^T + \widehat{poil}_t \\
poil_t^T &= poil_t^{TD} + poil_{t-1}^T \\
poil_t^{TD} &= \rho_{poilTD} poil_{t-1}^{TD} + e_t^{poil} \\
\widehat{poil}_t &= \rho_{poil} \widehat{poil}_{t-1} + u_t^{poil}
\end{aligned} \tag{12}$$

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