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Friese, Max; Fenge, Robert

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The minimum wage in a state union with unemployment insurance

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The minimum wage in a state union with unemployment insurance

Robert Fenge
University of Rostock, 18057 Rostock, Germany
CESifo Munich
robert.fenge@uni-rostock.de

Max Friese
University of Rostock, 18057 Rostock, Germany
max.friese@outlook.com

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Abstract
In a union of states with partial mobility of households and firms we analyze the setting of minimum wages against the background of unemployment insurance. We find that unemployment insurance is always efficiently organized by decentral decisions. In contrast, for symmetric states only the central setting of minimum wages is efficient. Decentralized minimum wages can be shown to be generically inefficient because the decision makers either externalize the cost of unemployment via the pooled insurance budget or exploit migrational externalities. Only with full mobility, the opposing migration effects outweigh the pooling effect. Our results suggest that contrary to insurance which can be efficiently organized decentrally, a pure redistribution like minimum wages should be centralized in a state union.

Keywords: Minimum wage, unemployment insurance, centralization, international migration, common pool problem

JEL classifications: E24, F22, H77, J38, J65

1 Introduction

The centralization of decision-making is subject of an ongoing debate in the European Union. In recent years, two social policy fields stand out: the harmonization of relative minimum wage levels and the introduction of a union-wide unemployment insurance (see e.g. Beblavý and Lenaerts, 2017).

In this paper we consider the welfare implications and interdependencies of (de)centralized decisions on minimum wages and unemployment insurance parameters. We use a theoretical model of two states joint in a state union.[1] The centralized decision on minimum wages in the state union is compared with the decentralized decision in the two states. We consider those decisions against the background of central versus decentral unemployment insurances. Firms and households relocate between the states taking account of their profit and expected utility, respectively. For symmetric states, we show that a centralized decision on minimum wages is superior to a decentralized setting. The latter suffers from horizontal migration effects (see Zodrow and Mieszkowski, 1986) and/or vertical fiscal effects (see Keen and Kotsogiannis, 2002). These effects do also occur with centralized minimum wage setting, but are there either directly internalized or do not develop a weight in the decision-making.

The paper is organized as follows. Chapter 2 gives a review of the relevant literature. Chapter 3 explains the basic model. The benchmark scenario yielding a second best optimum by a social planner is presented in chapter 4. Chapter 5 discusses the governmental decisions on minimum wages with decentralized unemployment insurance and in chapter 6 with centralized insurance. Chapter 7 concludes.

2 Literature review

Minimum wages have been analysed extensively in the economic literature, empirically as well as theoretically. Major directions of research consider the effects of minimum wages on employment, distributions of wages and income, skills, schooling, prices and profits (see Neumark, 2008, for a comprehensive overview about the literature and a summary of results). The setting of minimum wages itself has been received little attention as several authors note (see e.g. Boeri, 2012, Bosch *et al.*, 2019, Fukumura and Yamagishi, 2020, Kampelmann *et al.*, 2013). Indeed, this observation only applies to governmental minimum wage setting, because minimum wages agreed upon by collective wage bargaining have been elaborated intensively and constitute a research field with many distinct branches (for an overview see OECD, 2019). Furthermore, governmental decision-making about mandatory minimum wages can be encountered

[1] We follow up the model developed in Fenge and Friese (2022). Here we analyze the governmental setting of a minimum wage instead of wage bargaining by trade unions. For that we consider a different sequence of decisions.

in models of optimal income redistribution and optimal taxation.[2]

Theoretical minimum wage setting models that put an explicit focus on governmental decision-making are rare. An early contribution is made by Gabszewicz and van Ypersele (1996) who consider a two-country model characterized by capital mobility, labor immobility and a median-voter decision on the locally optimal minimum wage level. They show that international competition in the capital market always weakens social protection, because it drives down the ly optimal minimum wage.

A similar result is obtained by Fukumura and Yamagishi (2020) for the case of partial labour mobility in a model with two regions. They consider a minimum wage setting government that pays attention to regional GDP as well as the utility of minimum wage workers. In the open economy the effect of a minimum wage rise on the job destruction rate determines whether individuals immigrate or emigrate. If the effect is strong, individuals emigrate due to loss of employment. In the opposite case the higher minimum wage attracts immigrant workers, because negative effects on employment are mild. Thus, depending on whether job destruction is affected mildly or not, a race to the top or a race to the bottom in minimum wage setting results.

Boeri (2012) develops and tests a model, which indicates under which conditions a governmental minimum wage is lower than a collectively agreed upon minimum wage. The theoretical one-sector model predicts that the governmental minimum wage is lower, if the distributional weight of the profit earners in the social welfare function is larger than their bargaining power. A similar reasoning applies for a two-sector model. Econometric results confirm the theoretical findings.

Lee and Saez (2012) consider minimum wage setting in a single jurisdiction with a competitive labor market. They show that a binding minimum wage is justified, if the government values redistribution to minimum wage workers and an additional unemployed has zero surplus from being employed.

Simon and Wilson (2021) build on Lee and Saez (2012) and consider a federal setting with two regional and one federal government. They ask under which conditions the setting of a binding minimum wage is optimal for each type of government. A higher minimum wage creates a horizontal migration externality, where high-skilled workers emigrate to the other jurisdiction the higher the minimum wage is set by the government. This hinders the local government's ability to redistribute through the minimum wage. In turn, the central government is restricted to a uniform policy.[3] Three authority settings are examined: (i) only a central government,

[2] A binding minimum wage has redistributive implications per se but can also be used as an efficiency enhancing instrument (see e.g. Allen, 1987, Gerritsen and Jakobs, 2019, Lee and Saez, 2012, Marceau and Boadway, 1994). However, in our paper we do not consider the question whether a minimum wage should be set at all. We assume a situation where a binding minimum wage is socially desirable for redistributive reasons.

[3] Horizontal inter-jurisdictional externalities are well understood in the literature on fiscal federalism (see Zodrow and Mieszkowski, 1986; Wildasin, 1991; Dahlby, 1996; among others). The interplay of horizontal and vertical fiscal externalities, in turn, were initially and extensively elaborated by Keen and Kotsogiannis (2002).

(ii) only decentral governments, (iii) tiered (both). In scenario (iii), the central uniform policy mitigates the externalities arising form local governments' horizontal competition. Thus, Simon and Wilson (2021) conclude that decentralized and centralized policy setting are strategic complements.
In our paper we also consider the setting of the contribution rate to the unemployment insurance system and the interdependencies with the minimum wage setting, which distinguishes us from other papers in the small field of related literature.

3 The model

We consider a state union with two member states, $i = A, B$ where governmental decision-making concerns the minimum wage and subsequently the extent of unemployment insurance. Thereby, both decisions take the mobility of households and firms between both member states into account.

3.1 Households

In either state i lives the total number of N_i households such that $N = N_A + N_B$ individuals live in the whole state union. An individual is either employed (e) or unemployed (u) but always supplies inelastically one unit of labor in the region of residence. The number of employed individuals is L_i and the number of unemployed is $N_i - L_i$. We assume that the probability to be unemployed is equal to the unemployment rate, which is the number of unemployed individuals divided by the number of households: $\frac{N_i - L_i}{N_i}$ (compare Harris and Todaro, 1970). An income tax t_i is levied on the gross wage w_i such that the net income of an employed individual is given by $\tilde{w}_i = w_i(1 - t_i)$.Unemployed individuals receive an unemployment benefit b_i.[4]

Utility is represented by a monotonically increasing and strictly concave von Neumann-Morgenstern utility function $U(\cdot)$. Expected utility of an individual living in state i is given by

$$EU_i = \frac{L_i}{N_i} U_{i,e}(\tilde{w}_i) + \left[1 - \frac{L_i}{N_i}\right] U_{i,u}(b_i), \tag{1}$$

which sums up the expected utility gains from being employed or unemployed.
An exogenous number of individuals $\hat{N}_i$ in each state is immobile such that the total number in the state union is given by $\hat{N} = \hat{N}_A + \hat{N}_B \leq N$. Then, the total number of mobile individuals $\vec{N} = N - \hat{N}$ in the state union is given exogenously, too, while the country-specific amount of mobile individuals $\vec{N}_i$ is endogenous.

[4] In the case of centralized minimum wage we have $w_A = w_B = w$. For the centralized unemployment insurance applies accordingly $t_A = t_B = t$ and $b_A = b_B = b$.

In either country, the share β_i of mobile households is defined as the ratio of $\vec{N}_i$ to N_i which is assumed to be identical to the ratio of mobile employed persons $\vec{L}_i$ to all employed persons L_i

$$\beta_i = \frac{\vec{N}_i}{N_i} = \frac{\vec{L}_i}{L_i} = 1 - \frac{\hat{N}_i}{N_i}. \tag{2}$$

Mobile individuals relocate as long as their expected utilities differ between the states. The worker's migration equilibrium is given by

$$\frac{\beta_A L_A}{\vec{N}_A} U_A(\tilde{w}_A) + \frac{\vec{N}_A - \beta_A L_A}{\vec{N}_A} U_A(b_A) - \frac{\beta_B L_B}{\vec{N}_B} U_B(\tilde{w}_B) - \frac{\vec{N}_B - \beta_B L_B}{\vec{N}_B} U_B(b_B) = 0. \tag{3}$$

3.2 Firms

The number of firms operating in region i is denoted by M_i such that $M = M_A + M_B$ is the total number of firms in the state union. In contrast to households firms do not face different states of the world. They are always active in one of both regions. We assume regional aggregate labor demand to be determined by a firm association. A single firm does not maximize individual profit but receives a proportional share of total profit P_i in the region of choice.[5]
The total profit in state i is given by

$$P_i = F_i(L_i) - w_i L_i, \tag{4}$$

where $F_i(\cdot)$ denotes the production function, L_i regional labor demand and w_i the gross wage level. Regional profit is maximized if the marginal product of an additional worker in region i equals the marginal cost, that is

$$\frac{\mathrm{d}F_i}{\mathrm{d}L_i}(L_i) \stackrel{!}{=} w_i. \tag{5}$$

implying $\frac{\mathrm{d}L_i}{\mathrm{d}w_i} < 0$, because $\frac{\mathrm{d}^2 F_i}{\mathrm{d}(L_i)^2} < 0$. We assume that the different minimum wage levels are always binding in both regions, e.g. higher than the local competitive wage levels. Note that regional labor demand L_i defined by (5) is fully immobile while labor supply N_i is mobile by share β_i. Therefore, the immobile workplaces in region i are always filled by both mobile and immobile workers. It follows that $\beta_i L_i$ denotes the number of workplaces occupied by mobile employees.

[5] By these key assumptions we avoid corner solutions in the firms' relocation equilibrium. At the same time we ensure the separation of firms' relocation decisions from households' migration decisions. That is, the migration of households does not depend on firm relocation and vice versa. However, our key assumptions come at the cost of the absence of production or workplace relocation. Labor demand in each region is exogenous and only dependent on the wage level but not on the number of firms. This approach allows us to focus on the redistribution mechanics between firms and households via the minimum wage without intermingling their migration and relocation incentives. Then, a firm's relocation into the other region does only follow the incentive to operate in the region with higher profits.

Regional profit is assumed to be distributed equally among firms of residence such that we denote average profit in state i by

$$AP_i = \frac{P_i}{M_i}. \tag{6}$$

A certain exogenous number of firms $\hat{M}_i$ in each state is immobile such that the total number of immobile firms in the state union is given by $\hat{M} = \hat{M}_A + \hat{M}_B \leq M$. In analogy to households, the total number of mobile firms $\vec{M} = M - \hat{M}$ in the state union is given exogenously and the country-specific amount of mobile firms $\vec{M}_i$ is endogenous.
In either country, the share α_i of mobile firms is the ratio of $\vec{M}_i$ to the number of all firms M_i:

$$\alpha_i = \frac{\vec{M}_i}{M_i} = 1 - \frac{\hat{M}_i}{M_i}. \tag{7}$$

Mobile firms relocate as long as their expected profits differ between the states. The firms' relocation equilibrium is given by

$$\frac{\alpha_A P_A}{\vec{M}_A} - \frac{\alpha_B P_B}{\vec{M}_B} = 0. \tag{8}$$

3.3 Unemployment insurance

Unemployment insurance is organized either in each state or centrally for the whole state union. In general, the level of the unemployment benefit is equal to tax revenue per unemployed individual.

Decentralized insurance
The benefit b_i of a decentralized unemployment insurance in each region is either given by

$$b_i = \frac{L_i t_i w_i}{N_i - L_i}, \tag{9}$$

if the minimum wage is set by the state government. The right-hand side shows the tax revenues divided by the number of unemployed. If the minimum wage is chosen centrally for the whole union the benefit ist given by:

$$b_i = \frac{L_i t_i w}{N_i - L_i}. \tag{10}$$

Centralized insurance
In the case of centralized unemployment insurance with a common benefit b and a common tax rate t, revenues are pooled and redistributed to the unemployed in both states. Then, depending on the wage setting regime, the centralized unemployment benefits with decentralized minimum

wages is given by:

$$b_z = \frac{L_A t_z w_A + L_B t_z w_B}{N_A + N_B - L_A - L_B} \tag{11}$$

and with centrally determined minimum wages:

$$b_z = \frac{L_A t_z w + L_B t_z w_z}{N_A + N_B - L_A - L_B}. \tag{12}$$

3.4 Governments

Decentralized governments maximize the sum of expected utility EU_i and average profit AP_i in each state. The objective function is represented by

$$V_i = EU_i + AP_i = \frac{L_i}{N_i} U_{i,e} + \left[1 - \frac{L_i}{N_i}\right] U_{i,u} + \frac{P_i}{M_i}. \tag{13}$$

Note that $M_i = \hat{M}_i + \vec{M}_i(\cdot)$ and $N_i = \hat{N}_i + \vec{N}_i(\cdot)$ are determined endogenously by migration decisions. Centralized governments maximize the sum of the regional governments' objective functions:

$$V_z = V_A + V_B = \sum_{i=A}^{B} \left(\frac{L_i}{N_i} U_{i,e} + \left[1 - \frac{L_i}{N_i}\right] U_{i,u} + \frac{P_i}{M_i} \right). \tag{14}$$

3.5 Sequence of decisions

Decision-making is sequential in our model. The government that decides on the minimum wage takes into account how the minimum wage affects the setting of the contribution rate. In turn, the government that sets the unemployment contribution takes the wage levels as given.[6] The governments take account of optimal regional labor demand and worker migration as well as firm relocation. Table 1 summarizes the sequence of decisions.

Table 1: Sequence of decisions

Stage	Decision variable		Decision maker
1	Wage level:	w	wage setting government
2	Contribution rate:	t	Contribution setting government
3	Employment:	l	Firm association
4	Migration:	n, m	Work force and firms

There are four different combinations of minimum wage and contribution rate setting possible. We consider all four of them. Additionally we specify a social planner scenario as benchmark.

[6] Fenge and Friese (2022) consider the opposite case with collective wage bargaining instead of governmental minimum wage setting.

Table 2 gives an overview about all combinations of (de)centralized minimum wage and contribution rate setting investigated in this paper.

Table 2: Scenarios

Wage/Insurance	Decentralized (budget)	Centralized (budget)
Decentralized	1.1: w_A, w_B, t_A, t_B	2.1: w_A, w_B, t_z
Centralized	1.2: w_z, t_A, t_B	2.2: w_z, t_z

4 Social optimum

Our benchmark for efficiency is a social planner who is restricted by regional production and who is required to maintain the social insurance system in each region.[7] The budget for unemployment insurance is equivalent to a decentralized setting as described in equation (9) while the social planner's objective function sums up the welfare of both regions as described in equation (14). The social planner sets simultaneously the contribution rate t_i, the minimum wage level w_i, the number of individuals N_i and the number of firms M_i in each state.
The resulting first-order-condition with respect to t_i is given by

$$\frac{\mathrm{d}V_i}{\mathrm{d}t_i} = \underbrace{\frac{L_i}{N_i}\frac{\partial U_{i,e}}{\partial \tilde{w}_i}\frac{\partial \tilde{w}_i}{\partial t_i} + \left[1 - \frac{L_i}{N_i}\right]\frac{\partial U_{i,u}}{\partial b_i}\frac{\partial b_i}{\partial t_i}}_{\text{insurance condition}} \stackrel{!}{=} 0. \tag{15}$$

The first term of the insurance condition denotes the marginal effect on the expected utility for an employed individual and the second term for an unemployed person. Thus, by changing the value of t_i, the social planner redistributes income between employment statuses within the labor force (i.e., between employed and unemployed). A higher contribution rate decreases the net income and increases the unemployment benefit. The welfare maximizing level of t_i balances the marginal utilities of being employed and unemployed. This implies the actuarial fair contribution rate which is equal to the unemployment probability: $t_i = \frac{N_i - L_i}{N_i}$. If the contribution rate matches the risk of becoming unemployed, we say that unemployment insurance is *efficient*. Furthermore, we refer to (15) as the *insurance condition*.
Setting the minimum wage w_i implies redistribution between the firms and the workers as a whole. First-order-condition (16) shows that the social planner balances four main effects with respect to w_i:

$$\frac{\mathrm{d}V_i}{\mathrm{d}w_i} = \underbrace{\frac{\frac{\partial L_i}{\partial w_i}}{N_i}\left[U_{i,e} - U_{i,u}\right] + \frac{L_i}{N_i}\frac{\partial U_{i,e}}{\partial \tilde{w}_i}\frac{\partial \tilde{w}_i}{\partial w_i} + \left[1 - \frac{L_i}{N_i}\right]\frac{\partial U_{i,u}}{\partial b_i}\left[\frac{\partial b_i}{\partial L_i}\frac{\partial L_i}{\partial w_i} + \frac{\partial b_i}{\partial w_i}\right] + \frac{\frac{\partial P_i}{\partial w_i}}{M_i}}_{\text{wage setting condition}} \stackrel{!}{=} 0. \tag{16}$$

[7] This implies given regional profit maximizing labor demand, equation (5).

The first term describes the indirect effect on individuals' expected utility due to a change of their employment probability.[8] The second term depicts the direct effect on the employed individuals' income and utility through the change in the gross wage. A higher wage level directly increases the utility of the employed individuals. The third term shows the effect on the benefit level and thus the income of the unemployed individuals. This benefit effect is assumed to be negative. This means that with a higher minimum wage level, the increasing number of beneficiaries reduces the benefit level stronger (rising expenditures) than the higher wage level increases it (increasing tax revenue). The fourth term shows the effect on firms' labor cost, which increase with a higher wage level and directly reduce firms' profits. Condition (16) defines the efficient minimum wage level to which we refer in the following as the *wage setting condition*.

The social planner allocates the number of individuals and firms among regions. With respect to individuals, the planner balances the effects on the employment probability with the direct effects on the number of beneficiaries and social security expenditures:

$$\frac{\mathrm{d}V_A}{\mathrm{d}N_A} + \frac{\mathrm{d}V_B}{\mathrm{d}N_B}\frac{\partial N_B}{\partial N_A} = -\frac{L_A}{N_A^2}\left(U_{A,e} - U_{A,u}\right) + \left[1 - \frac{L_A}{N_A}\right]\frac{\mathrm{d}U_{A,u}}{\mathrm{d}b_A}\frac{\mathrm{d}b_A}{\mathrm{d}N_A} + \frac{L_B}{N_B^2}\left(U_{B,e} - U_{B,u}\right) - \left[1 - \frac{L_B}{N_B}\right]\frac{\mathrm{d}U_{B,u}}{\mathrm{d}b_B}\frac{\mathrm{d}b_B}{\mathrm{d}N_B} \stackrel{!}{=} 0. \tag{17}$$

The first term describes the direct effect of a change in the number of residents on the utility loss or gain from a change in the labor market status. E.g. ceteris paribus a higher number of residents decreases the employment probability such that less individuals are able to achieve the employed individuals' level of utility. The second term describes the negative effect of a higher number of residents on the unemployment benefit. The benefit decreases because for given employment a higher number of residents implies a rise in unemployment such that a given amount of tax revenues has to be shared by a higher number of unemployed individuals. Both effects are of negative sign in country A and of positive sign in country B. An inner solution of condition (17) balances both effects.

Furterhmore, the social planner balances the effect on average profit between both regions:

$$\frac{\mathrm{d}V_A}{\mathrm{d}M_A} + \frac{\mathrm{d}V_B}{\mathrm{d}M_B}\frac{\partial M_B}{\partial M_A} = -\frac{P_A}{M_A^2} + \frac{P_B}{M_B^2} \stackrel{!}{=} 0. \tag{18}$$

A higher number of firms in state A decreases profit per firm while it increases profit in state B.

[8] If unemployment insurance is efficient, this term vanishes.

5 Minimum wage setting with decentralized unemployment insurance

Unemployment insurance

Each local government sets the contribution rate t_i independently from the other region. Both regions are connected by free migration of workers and firms. The maximization of local welfare (13) with respect to t_i yields the following first-order-condition, e.g. for country A:

$$\frac{\mathrm{d}V_A}{\mathrm{d}t_A} = \left(\frac{L_A}{N_A}\frac{\partial U_{A,e}}{\partial \tilde{w}_A}\frac{\partial \tilde{w}_A}{\partial t_A} + \left[1-\frac{L_A}{N_A}\right]\frac{\partial U_{A,u}}{\partial b_A}\frac{\partial b_A}{\partial t_A}\right)\left(1-\underbrace{\frac{\frac{\partial EU_A}{\partial N_A}}{\frac{\partial EU_A}{\partial N_A}\frac{1}{\beta_A}+\frac{\partial EU_B}{\partial N_B}\frac{1}{\beta_B}}}_{\text{workers' mobility factor}}\right) \stackrel{!}{=} 0. \tag{19}$$

First-order-condition (19) shows that the decentralized government balances the effect of the contribution rate on the marginal utility of the unemployment benefit with the marginal loss of utility derived from the net wage. This is the *insurance condition*, describing optimal intra-group redistribution within the labor force. Migration mitigates each effect in condition (19) by the workers' mobility factor, which reflects the relative strength of the change of expected utility due to migration in one state compared to the other.

Thus, migration reduces both the utility increase of the unemployed households (due to higher benefits) and the decrease of the employed households (due to lower net wage). The overall effect on expected utility is neutral, because the migration incentives to immigrate as an unemployed are the same as to emigrate as an employed.

With symmetric regions the first-order-condition can be written as

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \left(\frac{L}{N}\frac{\partial U_e}{\partial \tilde{w}}\frac{\partial \tilde{w}}{\partial t} + \left[1-\frac{L}{N}\right]\frac{\partial U_u}{\partial b}\frac{\partial b}{\partial t}\right)\left(1-\beta\frac{1}{2}\right) \stackrel{!}{=} 0, \tag{20}$$

implying the setting of the efficient contribution rate, $t = \frac{N-L}{N}$.

Proposition 1. *For symmetric and asymmetric states the decentralized decision of a government in state i, i = A,B, yields the efficient contribution rate which provides full insurance against the risk of unemployment, irrespective of households' degree of mobility.*

Proof. See appendix. ■

5.1 Decentralized minimum wage setting

With decentralized minimum wages, each government sets the wage rate independently from the other region. Both regions are linked to each other only via relocation of households and firms. The government maximizes the objective function (13) with respect to w_i and takes into

account the effects on the setting of t_i.[9] The following first-order-condition results, e.g. for country A:

$$\frac{\mathrm{d}V_A}{\mathrm{d}w_A} = \left(\frac{\frac{\partial L_A}{\partial w_A}}{N_A} \left[U_{A,e} - U_{A,u} \right] + \frac{L_A}{N_A} \frac{\partial U_{A,e}}{\partial \tilde{w}_A} \frac{\partial \tilde{w}_A}{\partial w_A} + \left[1 - \frac{L_A}{N_A} \right] \frac{\partial U_{A,u}}{\partial b_A} \left[\frac{\partial b_A}{\partial L_A} \frac{\partial L_A}{\partial w_A} + \frac{\partial b_A}{\partial w_A} \right] \right)$$
$$\times \Bigg(1 - \underbrace{\frac{\frac{\partial EU_A}{\partial N_A}}{\frac{\partial EU_A}{\partial N_A} \frac{1}{\beta_A} + \frac{\partial EU_B}{\partial N_B} \frac{1}{\beta_B}}}_{\text{workers' mobility factor}} \Bigg) + \frac{\frac{\partial P_A}{\partial w_A}}{M_A} \Bigg(1 - \underbrace{\frac{\frac{\partial AP_A}{\partial M_A}}{\frac{\partial AP_A}{\partial M_A} \frac{1}{\alpha_A} + \frac{\partial AP_B}{\partial M_B} \frac{1}{\alpha_B}}}_{\text{firms' mobility factor}} \Bigg) \stackrel{!}{=} 0. \tag{21}$$

First-order-condition (21) is equal to the social planner's *wage setting condition* (16), adjusted only by the mobility factors of households and firms. The firms' mobility factor describes the relative strength of the change of profit per firm due to relocation in one state compared to the other. Generically, the state government and and the social planner do not set the same minimum wage level. This is best seen by assuming symmetric regions in condition (21):

$$\frac{\mathrm{d}V}{\mathrm{d}w} = \left(\frac{\frac{\partial L}{\partial w}}{N} \left[U_e - U_u \right] + \frac{L}{N} \frac{\partial U_e}{\partial \tilde{w}} \frac{\partial \tilde{w}}{\partial w} + \frac{\partial U_u}{\partial b} \left[1 - \frac{L}{N} \right] \left[\frac{\partial b}{\partial L} \frac{\partial L}{\partial w} + \frac{\partial b}{\partial w} \right] \right) \left(1 - \beta \frac{1}{2} \right)$$
$$+ \frac{\frac{\partial P}{\partial w}}{M} \left(1 - \alpha \frac{1}{2} \right) \stackrel{!}{=} 0. \tag{22}$$

Without migration and relocation ($\alpha = \beta = 0$) a higher wage increases directly workers' expected utility and lowers average profits. The welfare maximizing minimum wage balances the marginal benefit of workers with the marginal cost of the firms. If some workers in the state union are mobile ($\beta > 0$), a wage increase in some state incentivizes workers to migrate to this state. The local government takes account of this migration effect, which is reflected by the *workers' mobility factor*. Then, maximizing social welfare yields a lower minimum wage compared to a situation without mobile individuals. The mobility of workers partly crowds out the utility increase from higher wage levels. A similar reasoning applies for the relocation of firms ($\alpha > 0$), where a higher minimum wage decreases profits and encourages firms to leave the state.

Assume households are more immobile than firms ($\alpha > \beta$). Then the share by which the labor force increases due to immigration is relatively lower than the share of firms that leave the state. As a consequence of migration, the expected utility of households decreases by less than the profits per firm increase. Thus, the dampening effect of migration on the increasing expected utility of households is smaller than its impact on the decreasing profit of firms. Hence, the minimum wage will be chosen smaller than without migration. Generally, a higher degree of immobility improves workers or firms which we call the *advantage of relative immobility*.

[9] These indirect effects $\frac{\mathrm{d}t_i}{\mathrm{d}w_i}$ vanish via the envelope theorem.

Proposition 2. *In symmetric states with decentralized unemployment insurance, the minimum wage level set by each state is only efficient, if firms and workers are equally mobile:* $1 \geq \alpha = \beta \geq 0$. *Otherwise, if firms are more mobile than workers, the minimum wage is higher than the efficient level:* $1 \geq \alpha > \beta \geq 0$, *and vice versa.*

Proof. See appendix. ■

5.2 Centralized minimum wage setting

The centralized wage setting government considers the welfare of both regions and maximizes objective function (14) with respect to w. The government takes into account that any adjustment of the wage level affects both regions uniformly. Thus, any incentive for firms or workers to relocate e.g. from region A to region B due to a minimum wage adjustment is crowded out by the mirror-effect in region B. With symmetric regions the crowding out is complete and the following first-order-condition results:

$$\frac{\mathrm{d}V}{\mathrm{d}w} = \frac{\frac{\partial L}{\partial w}}{N}\left[U_e - U_u\right] + \frac{L}{N}\frac{\partial U_e}{\partial \tilde{w}}\frac{\partial \tilde{w}}{\partial w} + \left[1 - \frac{L}{N}\right]\frac{\partial U_u}{\partial b}\left[\frac{\partial b}{\partial L}\frac{\partial L}{\partial w} + \frac{\partial b}{\partial w}\right] + \frac{\frac{\partial P}{\partial w}}{M} \stackrel{!}{=} 0. \tag{23}$$

The first three terms in condition (23) describe the effects of a wage adjustment on workers' welfare. There are three channels by which workers are affected: (1) the employment probability, (2) the gross wage of employed workers and (3) the unemployment benefit of unemployed individuals. Firms are affected via labor cost adjustments which are represented by the fourth term in first-order-condition (23). With symmetric regions the welfare effects with centralized wage setting are equal to the social planner's *wage setting condition* (16). This is the case, because the government internalizes directly the migration effects between both regions. With centralized minumum wages relocation incentives between both regions crowd each other out, respectively for firms and workers. Furthermore, the wage setting government is not distorted by the setting of the contribution rate to the unemployment insurance, because the decentralized organization of unemployment insurance is efficient.[10] Thus, we conclude that centralized minimum wage setting with decentralized unemployment insurance is efficient.

Proposition 3. *In symmetric states with decentralized unemployment insurance, the minimum wage set by a central government of the state union internalizes the advantage of relative immobility and is efficient.*

Proof. See appendix. ■

[10] The wage setting government takes into account the effects of minimum wage level adjustments on the contribution rate, $t_i(w)$.

6 Minimum wage setting with centralized unemployment insurance

Unemployment insurance

The government maximizes total welfare in the state union with respect to the common contribution rate t. It considers objective function (14), which sums up the expected utility and average profit of both regions A and B. The following first-order-condtion with asymmetric regions results:

$$\frac{\mathrm{d}V_z}{\mathrm{d}t_z} = \sum_{\substack{i,j=A \\ i\neq j}}^{B} \left[\frac{L_i}{N_i}\frac{\partial U_{i,e}}{\partial \tilde{w}_i}\frac{\partial \tilde{w}_i}{\partial t_z} + \left[1-\frac{L_i}{N_i}\right]\frac{\partial U_{i,u}}{\partial b_z}\frac{\partial b_z}{\partial t_z}\right]\left(1 - \frac{\frac{\partial EU_i}{\partial N_i}}{\frac{\partial EU_A}{\partial N_A}\frac{1}{\beta_A} + \frac{\partial EU_B}{\partial N_B}\frac{1}{\beta_B}}\right)$$
$$+ \left[\frac{L_i}{N_i}\frac{\partial U_{i,e}}{\partial \tilde{w}_i}\frac{\partial \tilde{w}_i}{\partial t_z} + \left[1-\frac{L_i}{N_i}\right]\frac{\partial U_{i,u}}{\partial b_z}\frac{\partial b_z}{\partial t_z}\right]\frac{\frac{\partial EU_j}{\partial N_j}}{\frac{\partial EU_A}{\partial N_A}\frac{1}{\beta_A} + \frac{\partial EU_B}{\partial N_B}\frac{1}{\beta_B}} \stackrel{!}{=} 0. \tag{24}$$

The terms in square brackets denote the *insurance condition*. Increasing the contribution rate leads to a loss in net income of the employed individuals and a gain in the unemployment benefit of the unemployed. For a given number of individuals this may cause a disbalance between expected utilities of both regions. Migration is incentivized, which changes the unemployment probabilities until expected utility between both regions is equalized again. In general migration attenuates the negative as well as the positive effects of a contribution rate adjustment. Additionally, the setting of a common contribution rate causes reverse migration effects in the other region. With symmetric regions the migration effects in (24) outweigh each other such that only the *insurance condition* remains:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \left[\frac{L}{N}\frac{\partial U_e}{\partial \tilde{w}}\frac{\partial \tilde{w}}{\partial t} + \left[1-\frac{L}{N}\right]\frac{\partial U_u}{\partial b}\frac{\partial b}{\partial t}\right] \stackrel{!}{=} 0. \tag{25}$$

Proposition 4. *The centralized unemployment insurance is (i) generically inefficient for asymmetric states and (ii) efficient for symmetric states.*

Proof. See appendix. ■

6.1 Decentralized minimum wage setting

Each minimum wage setting government in the state union maximizes regional welfare (13) with respect to the local minimum wage w_i. The following first-order-condition with asymmet-

ric regions results:

$$\frac{\mathrm{d}V_A}{\mathrm{d}w_A} = \left(\frac{\frac{\partial L_A}{\partial w_A}}{N_A} \left[U_{A,e} - U_{A,u} \right] + \frac{L_A}{N_A} \frac{\partial U_{A,e}}{\partial \tilde{w}_A} \frac{\partial \tilde{w}_A}{\partial w_A} + \left[1 - \frac{L_A}{N_A} \right] \frac{\partial U_{A,u}}{\partial b} \left[\frac{\partial b}{\partial L_A} \frac{\partial L_A}{\partial w_A} + \frac{\partial b}{\partial w_A} \right] \right.$$
$$\left. + \left[\frac{L_A}{N_A} \frac{\partial U_{A,e}}{\partial \tilde{w}_A} \frac{\partial \tilde{w}_A}{\partial t} + \left[1 - \frac{L_A}{N_A} \right] \frac{\partial U_{A,u}}{\partial b} \frac{\partial b}{\partial t} \right] \frac{\partial t}{\partial w_A} \right) \times \left(1 - \frac{\frac{\partial EU_A}{\partial N_A}}{\frac{\partial EU_A}{\partial N_A} \frac{1}{\beta_A} + \frac{\partial EU_B}{\partial N_B} \frac{1}{\beta_B}} \right)$$
$$+ \left(\left[\frac{L_B}{N_B} \frac{\partial U_{B,e}}{\partial \tilde{w}_B} \frac{\partial \tilde{w}_B}{\partial t} + \left[1 - \frac{L_B}{N_B} \right] \frac{\partial U_{B,u}}{\partial b} \frac{\partial b}{\partial t} \right] \frac{\partial t}{\partial w_A} + \left[1 - \frac{L_B}{N_B} \right] \frac{\partial U_{B,u}}{\partial b} \left[\frac{\partial b}{\partial L_A} \frac{\partial L_A}{\partial w_A} + \frac{\partial b}{\partial w_A} \right] \right)$$
$$\times \frac{\frac{\partial EU_A}{\partial N_A}}{\frac{\partial EU_A}{\partial N_A} \frac{1}{\beta_A} + \frac{\partial EU_B}{\partial N_B} \frac{1}{\beta_B}} + \frac{\frac{\partial P_A}{\partial w_A}}{M_A} \times \left(1 - \frac{\frac{\partial AP_A}{\partial M_A}}{\frac{\partial AP_A}{\partial M_A} \frac{1}{\alpha_A} + \frac{\partial AP_B}{\partial M_B} \frac{1}{\alpha_B}} \right) \stackrel{!}{=} 0. \tag{26}$$

First-order-condition (26) differs from the social planner's decision-making with respect to three effects. First, unlike the social planner the decentralized wage setting governments cannot set the contribution rate directly, but take into account the indirect effects of their decision-making on the level of the common contribution rate: $\frac{\partial t}{\partial w_i}$.[11] Second, it is taken into account that the setting of the local minimum wage triggers migrational responses in both regions due to the common unemployment insurance. Third, the common budget allows the externalisation of the cost of higher wages to the other region. With symmetric regions first-order-condition (26) can be written as:[12]

$$\frac{\mathrm{d}V}{\mathrm{d}w} = \left(\frac{\frac{\partial L}{\partial w}}{N} \left[U_e - U_u \right] + \frac{L}{N} \frac{\partial U_e}{\partial \tilde{w}} \frac{\partial \tilde{w}}{\partial w} + \left[1 - \frac{L}{N} \right] \frac{\partial U_u}{\partial b} \left[\frac{\partial b}{\partial L} \frac{\partial L}{\partial w} + \frac{\partial b}{\partial w} \right] \right) \left(1 - \beta \frac{1}{2} \right)$$
$$+ \frac{\frac{\partial P}{\partial w}}{M} \left(1 - \alpha \frac{1}{2} \right) \underbrace{- \left[1 - \frac{L}{N} \right] \frac{1}{2} \frac{\partial U_u}{\partial b} \left[\frac{\partial b}{\partial L} \frac{\partial L}{\partial w} + \frac{\partial b}{\partial w} \right] (1 - \beta)}_{\text{vertical fiscal externality}} \stackrel{!}{=} 0. \tag{27}$$

The first two terms describe the familiar *wage setting condition* and are directly comparable to their equivalents in the social planner's decision-making (16) and decentralized minimum wage setting with a decentralized insurance budget (22). As with decentralized unemployment insurance, the *advantage of immobility* causes a distortion in minimum wage setting, if the degree of firms' mobility differs from the households' degree of mobility.

The third term describes the *vertical fiscal externality* and its migrational counter effect. The pooling of the unemployment insurance budget allows for the setting of an excessively high

[11] In contrast to decentralized wage setting with decentralized unemployment insurance, the indirect effects of w_i on t do not vanish via the enevelope theorem. The reason is that the contribution setting government considers the effects of its decision-making in both regions while the wage setting governments do not.

[12] For better comparability we use the fact that with symmetric regions, the following equation holds in the comparison of decentralized minimum wage setting with centralized unemployment insurance and the social planner's (SP) decision making: $\left[\frac{\partial b_{SP}}{\partial L_{SP}} \frac{\partial L_{SP}}{\partial w_{SP}} + \frac{\partial b_{SP}}{\partial w_{SP}} \right] \frac{1}{2} = \left[\frac{\partial b}{\partial L_A} \frac{\partial L_A}{\partial w_A} + \frac{\partial b}{\partial w_A} \right]$.

minimum wage, because the related costs in terms of higher unemployment or higher expenditures for unemployment benefits can be externalized to the other region such that local expected utility increases.[13] However, the higher minimum wage resulting from this policy attracts individuals from the other region. Ceteris paribus the probability to be employed and to earn the higher wage level decreases. This means that the intended aim of the minimum wage increase - to improve welfare via cost externalisation - is partly crowded out by the related migrational response and the related decrease of local expected utility. With full workers' mobility both effects completely outweigh each other and no distortion from the vertical fiscal externality arises.

Proposition 5. *In symmetric states with centralized unemployment insurance, the minimum wage level set by state governments is generic inefficient due to distortions by the advantage of immobility and the vertical fiscal externality. A decentral minimum wage is only efficient if both workers and firms are fully mobile:* $\alpha = \beta = 1$.

Proof. See appendix. ■

6.2 Centralized minimum wage setting

Similar to centralized wage setting under decentralized unemployment insurance, the government considers the welfare of both regions simultaneously and maximizes objective function (14) with respect to w. Additionally, the wage setting government under centralized unemployment insurance has to take account of the common insurance budget. However, due to the centralized minimum wage, no common pool effect can be exploited by the government. Similar to the two-sided migration effects any wage adjustment affects contributions and expenditures from both regions simultaneously. Then, with symmetric states the following first-order-condition results:

$$\frac{\mathrm{d}V}{\mathrm{d}w} = \frac{\frac{\partial L}{\partial w}}{N}\left[U_e - U_u\right] + \frac{L}{N}\frac{\partial U_e}{\partial \tilde{w}}\frac{\partial \tilde{w}}{\partial w} + \left[1 - \frac{L}{N}\right]\frac{\partial U_u}{\partial b}\left[\frac{\partial b}{\partial L}\frac{\partial L}{\partial w} + \frac{\partial b}{\partial w}\right] + \frac{\frac{\partial P}{\partial w}}{M} \stackrel{!}{=} 0. \tag{28}$$

Condition (28) is equal to the social planner's *wage setting condition* (16) as well as the centralized setting of minimum wages with decentralized unemployment insurance, condition (23). It shows that neither the mobility of households and firms nor the centralized unemployment insurance budget distort governmental decision-making if regions are symmetric.

Proposition 6. *In symmetric states with centralized unemployment insurance, the minimum wage set by a central government of the state union internalizes both the advantage of immobility and the vertical fiscal externality, and is efficient.*

Proof. See appendix. ■

[13] The gains of a higher minimum wage in terms of higher tax revenues are overcompensated by the related cost due to higher unemployment. The net effect is negative, but can be shared with the other region.

7 Conclusion

We consider a theoretical model where two states are organized in a state union with different levels of governmental decision-making. A federal government may set centrally the minimum wage in both states or the state governments may determine the minimum wages in their states. Then both level of governments may choose also the contribution rate to the social unemployment insurance.[14] As a consequence, minimum wage setting takes into account its effects on the level of the contribution rate.

We investigate, which constellation of centralized and decentralized decision-making is efficient. As a benchmark for efficiency we consider a social planner. This approach results in four different scenarios of governmental decision-making. The results of our analysis are summarized in table 3, where brackets describe the asymmetric cases.

Table 3: Results with symmetric (asymmetric) states

Scenarios	**Decision variables**	**Contribution rate**	**Minimum wage**
1.1	t: dc w: dc	efficient (efficient)	inefficient (inefficient)
1.2	t: dc w: c	efficient (efficient)	efficient (inefficient)
2.1	t: c w: dc	efficient (inefficient)	inefficient (inefficient)
2.2	t: c w: c	efficient (inefficient)	efficient (inefficient)

Contribution rate setting is always efficient with symmetric states, which means that the contribution rate is equal to the probability to become unemployed. With decentralized unemployment insurance (scenarios 1.1 and 1.2), migrational responses to the contribution rate setting do not distort the governments' decision-making. Because migrational responses affect the employed and the unemployed equally, no distortions occur. We show that the contribution rate can serve its single purpose, the insurance against the risk of becoming unemployed. Furthermore, firms are not directly affected by a change of the contribution rate such that the contribution setting government does not consider any welfare effects between firms and workers.

Different to the contribution rate the setting of the minimum wage serves to redistribute income between firms and workers. Both groups are directly affected by an adjustment of the minimum wage level. Accordingly, the wage setting government has to weigh migrational responses from different welfare subjects against each other. Wage setting results to be generically inefficent (scenario 1.1) when decision-making is decentralized. With centralized minimum wage setting the migration responses are internalized from the beginning such that no distortions arise and wage setting results to be efficient (scenario 1.2).

In the two cases of centralized unemployment insurance (scenarios 2.1 and 2.2), contribution rate setting is efficient as well. The contribution setting government does always consider the

[14] In our model, centralized unemployment insurance implies a common budget for the whole state union.

whole state union and directly internalizes all migrational and fiscal responses. The same reasoning applies for a centralized wage setting government (scenario 2.2). However, decentralized wage setting governments set the local minimum wages generically inefficient (scenario 2.1). On one hand, local wage setting governments weigh the different firms' and workers' migrational responses against each other. On the other hand, the wage setting governments have an incentive to set excessively high local minimum wages due to the pooling of the social security budget.

In summary, for symmetric states we find that (1) the centralized setting of minimum wages is superior to the decentralized setting and that (2) decentralized unemployment insurance is always efficient even with asymmetric regions. With decentralized unemployment insurance the contribution rate setting government appears to be immune against migrational welfare effects. This is due to the intra-group character of social insurance. Unemployment insurance redistributes income only between the workers' two states of the world - being employed or unemployed - and not between workers and firms (inter-group). This also affects wage-setting favourably, because no migration effects can translate to the minimum wage setting government's decision-making.

The most favourable scenario is where unemployment insurance is organized decentrally and the minimum wage centrally (scenario 1.2). No fiscal externalities occur and migration cannot distort governmental decision-making on either level. Only with asymmetric states, centrality cannot yield optimal results for two regions because only one instrument is available to redistribute income between firms and workers in both regions. The most unfavourable scenario is where contributions are organized centrally and minimum wages decentrally (scenario 2.1). Here, even with symmetric states, the minimum wage setting governments suffer from fiscal externalities under all circumstances except the special case of full mobility.

As a general result of our paper we find that in a state union minimum wages should be organized centrally (redistribution) while unemployment benefits should be organized decentrally (social insurance). This mixture of organizational structures appears to be superior in the case that no common budget between regions exists. In turn, harmonization, e.g organizing each decision-making centrally, is to be chosen if regions are interconnected by a common budget.

8 Appendix

Proof of Proposition 1

Maximizing the objective function (13) with respect to t_A yields the first-order-condition[15]

$$\frac{\mathrm{d}V_A}{\mathrm{d}t_A} = \frac{\partial EU_A}{\partial t_A} + \frac{\partial EU_A}{\partial \vec{N}_A}\frac{\mathrm{d}\vec{N}_A}{\mathrm{d}t_A} + \frac{\partial AP_A}{\partial \vec{M}_A}\frac{\mathrm{d}\vec{M}_A}{\mathrm{d}t_A} \stackrel{!}{=} 0. \tag{29}$$

The effects $\frac{\mathrm{d}\vec{N}_A}{\mathrm{d}t_A}$ and $\frac{\mathrm{d}\vec{M}_A}{\mathrm{d}t_A}$ can be obtained from the implicit differentiation of workers' and firms' migration and relocation equilibra. These are given by

$$\frac{\vec{L}_A}{\vec{N}_A}U_{A,e}\left(\tilde{w}_A\right) + \left[1-\frac{\vec{L}_A}{\vec{N}_A}\right]U_{A,u}\left(\frac{\vec{L}_A t_A w_A}{\vec{N}_A - \vec{L}_A}\right) = \frac{\vec{L}_B}{\vec{N}_B}U_{B,e}\left(\tilde{w}_B\right) + \left[1-\frac{\vec{L}_B}{\vec{N}_B}\right]U_{B,u}\left(\frac{\vec{L}_B t_B w_B}{\vec{N}_B - \vec{L}_B}\right) \tag{30}$$

with $\vec{N}_i = \beta_i N_i$ and $\vec{L}_i = \bar{\beta}_i L_i$. Assume $\bar{\beta}_i = \beta_i$. Implicit differentiation then yields

$$\frac{\mathrm{d}\vec{N}_A}{\mathrm{d}t_A} = -\frac{\frac{\partial EU_A}{\partial t_A}}{\frac{\partial EU_A}{\partial N_A}\frac{1}{\beta_A} + \frac{\partial EU_B}{\partial N_B}\frac{1}{\beta_B}} \tag{31}$$

The relocation equilibrium of the firms is given by

$$\frac{\vec{P}_A}{\vec{M}_A} = \frac{\vec{P}_B}{\vec{M}_B} \tag{32}$$

with $\vec{M}_i = \alpha_i M_i$ and $\vec{P}_i = \bar{\alpha}_i P_i$. Assume $\bar{\beta}_i = \beta_i$ and $\bar{\alpha}_i = \alpha_i$. Then:

$$\frac{\mathrm{d}\vec{M}_A}{\mathrm{d}t_A} = 0 \tag{33}$$

Substituting (31) and (33) into (29) yields first-order-condition (19) for asymmetric states and first-order-condition (20) for symmetric states. Rewriting (20) yields the fair contribution rate:

$$\left(1-\beta\frac{1}{2}\right)\left[\frac{L}{N}\frac{\partial U_e}{\partial \tilde{w}}\frac{\partial \tilde{w}}{\partial t} + \left[1-\frac{L}{N}\right]\frac{\partial U_u}{\partial b}\frac{\partial b}{\partial t}\right] \stackrel{!}{=} 0 \tag{34}$$

$$\Leftrightarrow \quad \frac{\partial U_u}{\partial b}\left(\frac{Ltw}{N-L}\right) \stackrel{!}{=} \frac{\partial U_e}{\partial \tilde{w}}\left(w(1-t)\right) \tag{35}$$

$$\Leftrightarrow \quad t \stackrel{!}{=} \frac{N-L}{N} \tag{36}$$

which is always true for $1 \geq \beta \geq 0$. ■

15 $\frac{\partial EU_A}{\partial t_A} = \frac{L_A}{N_A}\frac{\partial U_{A,e}}{\partial \tilde{w}_A}\frac{\partial \tilde{w}_A}{\partial t_A} + \left[1-\frac{L_A}{N_A}\right]\frac{\partial U_{A,u}}{\partial b_A}\frac{\partial b_A}{\partial t_A}$, $\frac{\partial EU_A}{\partial N_A} = -\frac{L_A}{N_A^2}\left[U_{A,e} - U_{A,u}\right] + \left[1-\frac{L_A}{N_A}\right]\frac{\partial U_{A,u}}{\partial b_A}\frac{\partial b_A}{\partial N_A}$, $\frac{\partial AP_A}{\partial \vec{M}_A} = -\frac{P_A}{M_A^2}$.

Proof of Proposition 2

Maximizing the objective function (13) with respect to w_A yields the first-order-condition[16]

$$\frac{\mathrm{d}V_A}{\mathrm{d}w_A} = \frac{\partial EU_A}{\partial w_A} + \frac{\partial EU_A}{\partial t_A}\frac{\mathrm{d}t_A}{\mathrm{d}w_A} + \frac{\partial EU_A}{\partial N_A}\frac{\mathrm{d}\vec{N}_A}{\mathrm{d}w_A} + \frac{\partial AP_A}{\partial w_A} + \frac{\partial AP_A}{\partial \vec{M}_A}\frac{\mathrm{d}\vec{M}_A}{\mathrm{d}w_A} \stackrel{!}{=} 0 \tag{37}$$

The effects $\frac{\mathrm{d}\vec{N}_A}{\mathrm{d}w_A}$ and $\frac{\mathrm{d}\vec{M}_A}{\mathrm{d}w_A}$ can be obtained from the migration and relocation equilibra. Workers' migration equilibrium is given by

$$\begin{aligned}
&\frac{\vec{L}_A}{\vec{N}_A}U_{A,e}\left(\tilde{w}_A\right) + \left[1 - \frac{\vec{L}_A}{\vec{N}_A}\right]U_{A,u}\left(\frac{\vec{L}_A(w_A)t_A(w_A)w_A}{\vec{N}_A - \vec{L}_A(w_A)}\right) = \\
&\frac{\vec{L}_B}{\vec{N}_B}U_{B,e}\left(\tilde{w}_B\right) + \left[1 - \frac{\vec{L}_B}{\vec{N}_B}\right]U_{B,u}\left(\frac{\vec{L}_B w_B t_B}{\vec{N}_B - \vec{L}_B}\right)
\end{aligned} \tag{38}$$

with $\vec{N}_i = \beta_i N_i$, $\vec{L}_i = \bar{\beta}_i L_i$. Assume $\bar{\beta}_i = \beta_i$. Implicit differentiation then yields

$$\frac{\mathrm{d}\vec{N}_A}{\mathrm{d}w_A} = -\frac{\frac{\partial EU_A}{\partial w_A} + \frac{\partial EU_A}{\partial t_A}\frac{\mathrm{d}t_A}{\mathrm{d}w_A}}{\frac{\mathrm{d}EU_A}{\mathrm{d}N_A}\frac{1}{\beta_A} + \frac{\mathrm{d}EU_B}{\mathrm{d}N_B}\frac{1}{\beta_B}}. \tag{39}$$

The relocation equilibrium of the firms is given by

$$\frac{\vec{P}_A}{\vec{M}_A} = \frac{\vec{P}_B}{\vec{M}_B} \tag{40}$$

with $\vec{M}_i = \alpha_i M_i$. Assume $\bar{\alpha}_i = \alpha_i$. Then, following implicit derivative is obtained

$$\frac{\mathrm{d}\vec{M}_A}{\mathrm{d}w_A} = -\frac{\frac{\partial AP_A}{\partial w_A}}{\frac{\mathrm{d}AP_A}{\mathrm{d}M_A}\frac{1}{\alpha_A} + \frac{\mathrm{d}AP_B}{\mathrm{d}M_B}\frac{1}{\alpha_B}} \tag{41}$$

Substituting (39) and (41) into (37) yields first-order-condition (21) for asymmetric states and first-order-condition (22) for symmetric states.

16 $\frac{\partial EU_A}{\partial w_A} = \frac{\frac{\partial L_A}{\partial w_A}N_A}{N_A^2}\left[U_{A,e} - U_{A,u}\right] + \frac{L_A}{N_A}\frac{\partial U_{A,e}}{\partial \tilde{w}_A}\frac{\partial \tilde{w}_A}{\partial w_A} + \left[1 - \frac{L_A}{N_A}\right]\frac{\partial U_{A,u}}{\partial b_A}\left[\frac{\partial b_A}{\partial L_A}\frac{\mathrm{d}L_A}{\mathrm{d}w_A} + \frac{\partial b_A}{\partial w_A}\right]$,
$\frac{\partial EU_A}{\partial t_A} = \frac{L_A}{N_A}\frac{\partial U_{A,e}}{\partial \tilde{w}_A}\frac{\partial \tilde{w}_A}{\partial t_A} + \left[1 - \frac{L_A}{N_A}\right]\frac{\partial U_{A,u}}{\partial b_A}\frac{\partial b_A}{\partial t_A}$, $\frac{\partial EU_A}{\partial N_A} = -\frac{L_A}{N_A^2}\left[U_{A,e} - U_{A,u}\right] + \left[1 - \frac{L_A}{N_A}\right]\frac{\partial U_{A,u}}{\partial b_A}\frac{\partial b_A}{\partial N_A}$,
$\frac{\partial AP_A}{\partial w_A} = \frac{\frac{\partial P_A}{\partial w_A}M_A}{M_i A^2}$, $\frac{\partial AP_A}{\partial \vec{M}_A} = -\frac{P_A}{M_A^2}$.

Rewrite (22) as follows:

$$\frac{\frac{\partial L}{\partial w}}{N}\left[U_e - U_u\right] + \frac{L}{N}\frac{\partial U_e}{\partial \tilde{w}}\frac{\partial \tilde{w}}{\partial w} + \frac{\partial U_u}{\partial b}\left[1-\frac{L}{N}\right]\left[\frac{\partial b}{\partial L}\frac{\partial L}{\partial w} + \frac{\partial b}{\partial w}\right] \stackrel{!}{=} -\frac{\frac{\partial P}{\partial w}}{M}\frac{1-\alpha^{\frac{1}{2}}}{1-\beta^{\frac{1}{2}}}. \tag{42}$$

For $\alpha = \beta$, $\frac{1-\alpha^{\frac{1}{2}}}{1-\beta^{\frac{1}{2}}} = 1$ such that equation (42) is equal to the social planners wage setting condition (16). For $\alpha > \beta$, $\frac{1-\alpha^{\frac{1}{2}}}{1-\beta^{\frac{1}{2}}} < 1$ and the firms' losses have a lower weight in the first-order-condition such that the government sets a higher minimum wage than the social planner. Vice versa, if $\alpha < \beta$, $\frac{1-\alpha^{\frac{1}{2}}}{1-\beta^{\frac{1}{2}}} > 1$ and the government sets a lower minimum wage than the social planner. ■

Proof of Proposition 3

Maximizing the objective function (14) with respect to w yields the first-order-condition[17]

$$\begin{aligned}\frac{\mathrm{d}V_z}{\mathrm{d}w_z} = &\frac{\partial EU_A}{\partial w_z} + \frac{\partial EU_A}{\partial t_A}\frac{\partial t_A}{\partial w_z} + \frac{\partial EU_A}{\partial N_A}\frac{\partial \vec{N}_A}{\partial w_z} + \frac{\partial AP_A}{\partial w_z} + \frac{\partial AP_A}{\partial \vec{M}_A}\frac{\partial \vec{M}_A}{\partial w_z} \\ &+ \frac{\partial EU_B}{\partial w_z} + \frac{\partial EU_B}{\partial t_B}\frac{\partial t_B}{\partial w_z} - \frac{\partial EU_B}{\partial N_B}\frac{\partial \vec{N}_A}{\partial w_z} + \frac{\partial AP_B}{\partial w_z} - \frac{\partial AP_B}{\partial \vec{M}_B}\frac{\partial \vec{M}_A}{\partial w_z} \stackrel{!}{=} 0\end{aligned} \tag{43}$$

The effects $\frac{\mathrm{d}\vec{N}_A}{\mathrm{d}w_z}$ and $\frac{\mathrm{d}\vec{M}_A}{\mathrm{d}w_z}$ can be obtained from the implicit differentiation of workers' and firms' migration and relocation equilibra. Workers' migration equilibrium is given by

$$\begin{aligned}&\frac{\vec{L}_A(w_z)}{\vec{N}_A}U_{A,e}\left(\tilde{w}_A(w_z,t_A(w_z))\right) + \left[1-\frac{\vec{L}_A(w_z)}{\vec{N}_A}\right]U_{A,u}(b_A(\vec{N}_A,\vec{L}_A(w_z),t_A(w_z),w_z)) = \\ &\frac{\vec{L}_B(w_z)}{\vec{N}_B}U_{B,e}\left(\tilde{w}_B(w_z,t_B(w_z))\right) + \left[1-\frac{\vec{L}_B(w_z)}{\vec{N}_B}\right]U_{B,u}(b_B(\vec{N}_B,\vec{L}_B(w_z),t_B(w_z),w_z))\end{aligned} \tag{44}$$

with $\vec{N}_i = \beta_i N_i$, $\vec{L}_i = \bar{\beta}_i L_i$. Assume $\bar{\beta}_i = \beta_i$. Implicit differentiation then yields

$$\frac{d\vec{N}_A}{dw_z} = -\frac{\frac{\partial EU_A}{\partial w_z} + \frac{\partial EU_A}{\partial t_A}\frac{\partial t_A}{\partial w_z} - \frac{\partial EU_B}{\partial w_z} - \frac{\partial EU_B}{\partial t_B}\frac{\partial t_B}{\partial w_z}}{\frac{1}{\beta_A}\frac{\partial EU_A}{\partial N_A} + \frac{1}{\beta_B}\frac{\partial EU_B}{\partial N_B}} \tag{45}$$

17 $\frac{\partial EU_i}{\partial w_z} = \frac{\frac{\partial L_i}{\partial w_z}N_i}{N_i^2}\left[U_{i,e}-U_{i,u}\right] + \frac{L_i}{N_i}\frac{\partial U_{i,e}}{\partial \tilde{w}_i}\frac{\partial \tilde{w}_i}{\partial w_z} + \left[1-\frac{L_i}{N_i}\right]\frac{\partial U_{i,u}}{\partial b_i}\left[\frac{\partial b_i}{\partial L_i}\frac{\partial L_i}{\partial w_z} + \frac{\partial b_i}{\partial w_z}\right]$,
$\frac{\partial EU_i}{\partial N_i} = -\frac{L_i}{N_i^2}\left[U_{i,e}-U_{i,u}\right] + \left[1-\frac{L_i}{N_i}\right]\frac{\partial U_{i,u}}{\partial b_i}\frac{\partial b_i}{\partial N_i}$, $\frac{\partial AP_i}{\partial w_z} = \frac{\frac{\partial P_i}{\partial w_z}M_i}{M_i^2}$, $\frac{\partial AP_i}{\partial \vec{M}_i} = -\frac{P_i}{M_i^2}$.

The relocation equilibrium of the firms is given by

$$\frac{\vec{P}_A}{\vec{M}_A} = \frac{\vec{P}_B}{\vec{M}_B} \tag{46}$$

with $\vec{M}_i = \alpha_i M_i$. Assume $\bar{\alpha}_i = \alpha_i$. Then, following implicit derivative is obtained

$$\frac{\mathrm{d}\vec{M}_A}{\mathrm{d}w_z} = -\frac{\frac{\partial AP_A}{\partial w_z} - \frac{\partial AP_B}{\partial w_z}}{\frac{\partial AP_A}{\partial M_A}\frac{1}{\alpha_A} + \frac{\partial AP_B}{\partial M_B}\frac{1}{\alpha_B}} \tag{47}$$

Substituting (45) and (47) into (43) yields first-order-condition

$$\begin{aligned}\frac{\mathrm{d}V_z}{\mathrm{d}w_z} = &\sum_{i=A}^{B}\frac{\partial AP_i}{\partial w_z} - \frac{\frac{\partial AP_A}{\partial M_A} - \frac{\partial AP_B}{\partial M_B}}{\frac{1}{\alpha_A}\frac{\partial AP_A}{\partial M_A} + \frac{1}{\alpha_B}\frac{\partial AP_B}{\partial M_B}}\left(\frac{\partial AP_A}{\partial w_z} - \frac{\partial AP_B}{\partial w_z}\right) + \sum_{i=A}^{B}\left(\frac{\partial EU_i}{\partial w_z} + \frac{\partial EU_i}{\partial t_i}\frac{\partial t_i}{\partial w_z}\right) \\ &- \frac{\frac{\partial EU_A}{\partial N_A} - \frac{\partial EU_B}{\partial N_B}}{\frac{1}{\beta_A}\frac{\partial EU_A}{\partial N_A} + \frac{1}{\beta_B}\frac{\partial EU_B}{\partial N_B}}\left(\frac{\partial EU_A}{\partial w_z} + \frac{\partial EU_A}{\partial t_A}\frac{\partial t_A}{\partial w_z} - \frac{\partial EU_B}{\partial w_z} - \frac{\partial EU_B}{\partial t_B}\frac{\partial t_B}{\partial w_z}\right)\end{aligned} \tag{48}$$

for asymmetric states and first-order-condition (22) for symmetric states.

■

Proof of Proposition 4

Maximizing the objective function (14) with respect to t_z yields the first-order-condition[18]

$$\frac{\mathrm{d}V_z}{\mathrm{d}t_z} = \sum_{i=A}^{B}\frac{\partial EU_i}{\partial t_z} + \left(\frac{\partial EU_A}{\partial N_A} - \frac{\partial EU_B}{\partial N_B}\right)\frac{\partial \vec{N}_A}{\partial t_z} + \left(\frac{\partial AP_A}{\partial M_A} - \frac{\partial AP_B}{\partial M_B}\right)\frac{\partial \vec{M}_A}{\partial t_z} \tag{49}$$

The effects $\frac{\mathrm{d}\vec{N}_A}{\mathrm{d}t_z}$ and $\frac{\mathrm{d}\vec{M}_A}{\mathrm{d}t_z}$ can be obtained from the implicit differentiation of workers' and firms' migration and relocation equilibra. Workers' migration equilibrium is given by

$$\begin{aligned}&\frac{\vec{L}_A}{\vec{N}_A}U_{A,e}\left(\tilde{w}_A(t_z)\right) + \left[1 - \frac{\vec{L}_A}{\vec{N}_A}\right]U_{A,u}(b_z(\vec{N}_A,\vec{N}_B,t_z)) = \\ &\frac{\vec{L}_B}{\vec{N}_B}U_{B,e}\left(\tilde{w}_B(t_z)\right) + \left[1 - \frac{\vec{L}_B}{\vec{N}_B}\right]U_{B,u}(b_z(\vec{N}_A,\vec{N}_B,t_z))\end{aligned} \tag{50}$$

with $\vec{N}_i = \beta_i N_i$, $\vec{L}_i = \bar{\beta}_i L_i$.

18 $\frac{\partial EU_i}{\partial t_z} = \frac{L_A}{N_A}\frac{\partial U_{A,e}}{\partial \tilde{w}_A}\frac{\partial \tilde{w}_A}{\partial t_z} + \left[1 - \frac{L_A}{N_A}\right]\frac{\partial U_{A,u}}{\partial b_z}\frac{\partial b_z}{\partial t_z}$,
$\frac{\partial EU_i}{\partial N_i} = -\frac{L_i}{N_i^2}\left[U_{i,e} - U_{i,u}\right]$, $\frac{\partial AP_i}{\partial t_z} = 0$, $\frac{\partial AP_i}{\partial \vec{M}_i} = -\frac{P_i}{M_i^2}$.

Assume $\bar{\beta}_i = \beta_i$. Implicit differentiation then yields

$$\frac{\mathrm{d}\vec{N}_A}{\mathrm{d}t_z} = -\frac{\frac{\partial EU_A}{\partial t_z} - \frac{\partial EU_B}{\partial t_z}}{\frac{1}{\beta_A}\frac{\partial EU_A}{\partial N_A} + \frac{1}{\beta_B}\frac{\partial EU_B}{\partial N_B}} \tag{51}$$

The relocation equilibrium of the firms is given by

$$\frac{\vec{P}_A}{\vec{M}_A} = \frac{\vec{P}_B}{\vec{M}_B} \tag{52}$$

with $\vec{M}_i = \alpha_i M_i$. Assume $\bar{\alpha}_i = \alpha_i$. Then, following implicit derivative is obtained

$$\frac{\mathrm{d}\vec{M}_A}{\mathrm{d}t_z} = 0 \tag{53}$$

Substituting (51) and (53) into (49) yields first-order-condition (24) for asymmetric states and first-order-condition (25) for symmetric states.

■

Proof of Proposition 5

Maximizing the objective function (13) with respect to w_A yields the first-order-condition[19]

$$\frac{\mathrm{d}V_A}{\mathrm{d}w_A} = \frac{\partial EU_A}{\partial w_A} + \frac{\partial EU_A}{\partial t_z}\frac{\partial t_z}{\partial w_A} + \frac{\partial EU_A}{\partial N_A}\frac{\partial \vec{N}_A}{\partial w_A} + \frac{\partial AP_A}{\partial w_A} + \frac{\partial AP_A}{\partial M_A}\frac{\partial \vec{M}_A}{\partial w_A} \stackrel{!}{=} 0 \tag{54}$$

The effects $\frac{\mathrm{d}\vec{N}_A}{\mathrm{d}w_A}$ and $\frac{\mathrm{d}\vec{M}_A}{\mathrm{d}w_A}$ can be obtained from the implicit differentiation of workers' and firms' migration and relocation equilibra. Workers' migration equilibrium is given by

$$\begin{aligned}
&\frac{\vec{L}_A(w_A)}{\vec{N}_A} U_{A,e}\left(\tilde{w}_A(w_A, t_z(w_A))\right) + \left[1 - \frac{\vec{L}_A(w_A)}{\vec{N}_A}\right] U_{A,u}(b_z(\vec{N}_A, \vec{N}_B, \vec{L}_A(w_A), t_z(w_A), w_A)) = \\
&\frac{\vec{L}_B}{\vec{N}_B} U_{B,e}\left(\tilde{w}_B(t_z(w_A))\right) + \left[1 - \frac{\vec{L}_B}{\vec{N}_B}\right] U_{B,u}(b_z(\vec{N}_A, \vec{N}_B, \vec{L}_A(w_A), t_z(w_A), w_A))
\end{aligned} \tag{55}$$

with $\vec{N}_i = \beta_i N_i$, $\vec{L}_i = \bar{\beta}_i L_i$.

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19 $\frac{\partial EU_A}{\partial w_A} = \frac{\frac{\partial L_A}{\partial w_A} N_A}{N_A^2}\left[U_{A,e} - U_{A,u}\right] + \frac{L_A}{N_A}\frac{\partial U_{A,e}}{\partial \tilde{w}_A}\frac{\partial \tilde{w}_A}{\partial w_A} + \left[1 - \frac{L_A}{N_A}\right]\frac{\partial U_{A,u}}{\partial b_z}\left[\frac{\partial b_z}{\partial L_A}\frac{\partial L_A}{\partial w_A} + \frac{\partial b_z}{\partial w_A}\right]$,
$\frac{\partial EU_A}{\partial t_z} = \frac{L_A}{N_A}\frac{\partial U_{A,e}}{\partial \tilde{w}_A}\frac{\partial \tilde{w}_A}{\partial t_z} + \left[1 - \frac{L_A}{N_A}\right]\frac{\partial U_{A,u}}{\partial b_z}\frac{\partial b_z}{\partial t_z}$, $\frac{\partial EU_i}{\partial N_i} = -\frac{L_i}{N_i^2}\left[U_{i,e} - U_{i,u}\right]$,
$\frac{\partial AP_A}{\partial w_A} = \frac{\frac{\partial P_A}{\partial w_A} M_A}{M_A^2}$, $\frac{\partial AP_i}{\partial t_z} = 0$, $\frac{\partial AP_i}{\partial \vec{M}_i} = -\frac{P_A}{M_A^2}$.

Assume $\bar{\beta}_i = \beta_i$. Implicit differentiation then yields[20]

$$\frac{d\vec{N}_A}{dw_A} = -\frac{\frac{\partial EU_A}{\partial w_A} + \left(\frac{\partial EU_A}{\partial t_z} - \frac{\partial EU_B}{\partial t_z}\right)\frac{\partial t_z}{\partial w_A} - \frac{\partial EU_B}{\partial b_z}\frac{\partial b_z}{\partial w_A}}{\frac{1}{\beta_A}\frac{\partial EU_A}{\partial N_A} + \frac{1}{\beta_B}\frac{\partial EU_B}{\partial N_B}} \tag{56}$$

The relocation equilibrium of the firms is given by

$$\frac{\vec{P}_A(L_A(w_A), w_A)}{\vec{M}_A} = \frac{\vec{P}_B}{\vec{M}_B} \tag{57}$$

with $\vec{M}_i = \alpha_i M_i$. Assume $\bar{\alpha}_i = \alpha_i$. Then, following implicit derivative is obtained

$$\frac{\mathrm{d}\vec{M}_A}{\mathrm{d}w_A} = \frac{\frac{\partial AP_A}{\partial w_A}}{\frac{1}{\alpha_A}\frac{\partial AP_A}{\partial M_A} + \frac{1}{\alpha_B}\frac{\partial AP_B}{\partial M_B}} \tag{58}$$

Substituting (56) and (58) into (54) yields first-order-condition (26) for asymmetric states and first-order-condition (27) for symmetric states.
Assume that $0 \leq \alpha = \beta \leq 1$. Divide first-order-condition (27) by $1 - \beta\frac{1}{2}$ to get

$$\begin{aligned}
&\frac{\frac{\partial L}{\partial w}}{N}\left[U_e - U_u\right] + \frac{L}{N}\frac{\partial U_e}{\partial \tilde{w}}\frac{\partial \tilde{w}}{\partial w} + \left[1 - \frac{L}{N}\right]\frac{\partial U_u}{\partial b}\left[\frac{\partial b}{\partial L}\frac{\partial L}{\partial w} + \frac{\partial b}{\partial w}\right] + \frac{\frac{\partial P}{\partial w}}{M} \\
&\quad \stackrel{!}{=} \left[1 - \frac{L}{N}\right]\frac{\partial U_u}{\partial b}\frac{1}{2}\left[\frac{\partial b}{\partial L}\frac{\partial L}{\partial w} + \frac{\partial b}{\partial w}\right]\frac{1-\beta}{1-\beta\frac{1}{2}}.
\end{aligned} \tag{59}$$

For any value $0 \leq \beta < 1$ the minimum wage set by the decentral government is higher than the social planner's, because the vertical fiscal externality on the right-hand-side of (59) is not fully crowded out by the migrational externality. Only for $\alpha = \beta = 1$ equation (59) is equal to the social planner's decision (16). ■

20 $\frac{\partial EU_B}{\partial b_z}\frac{\partial b_z}{\partial w_A} = \left[1 - \frac{L_B}{N_B}\right]\frac{\partial U_{B,u}}{\partial b_z}\left[\frac{\partial b_z}{\partial L_A}\frac{\partial L_A}{\partial w_A} + \frac{\partial b_z}{\partial w_A}\right]$.

Proof of Proposition 6

Maximizing the objective function (14) with respect to w_z yields the first-order-condition[21]

$$\frac{\mathrm{d}V_z}{\mathrm{d}w_z} = \sum_{i=A}^{B} \left(\frac{\partial EU_i}{\partial w_z} + \frac{\partial EU_i}{\partial t_z}\frac{\partial t_z}{\partial w_z} + \frac{\partial AP_i}{\partial w_z} \right) + \left(\frac{\partial EU_A}{\partial N_A} - \frac{\partial EU_B}{\partial N_B} \right) \frac{\partial \vec{N}_A}{\partial w_z} + \left(\frac{\partial AP_A}{\partial M_A} - \frac{\partial AP_B}{\partial M_B} \right) \frac{\partial \vec{M}_A}{\partial w_z} \stackrel{!}{=} 0 \tag{60}$$

The effects $\frac{\mathrm{d}\vec{N}_A}{\mathrm{d}w_z}$ and $\frac{\mathrm{d}\vec{M}_A}{\mathrm{d}w_z}$ can be obtained from the implicit differentiation of workers' and firms' migration and relocation equilibra. Workers' migration equilibrium is given by

$$\begin{aligned} &\frac{\vec{L}_A(w_z)}{\vec{N}_A} U_{A,e}\left(\tilde{w}_A(w_z, t_z(w_z))\right) + \left[1 - \frac{\vec{L}_A(w_z)}{\vec{N}_A}\right] U_{A,u}(b_z(\vec{N}_A, \vec{N}_B, \vec{L}_A(w_z), \vec{L}_B(w_z), t_z(w_z), w_z)) \\ &= \frac{\vec{L}_B(w_z)}{\vec{N}_B} U_{B,e}\left(\tilde{w}_B(w_z, t_z(w_z))\right) + \left[1 - \frac{\vec{L}_B(w_z)}{\vec{N}_B}\right] U_{B,u}(b_z(\vec{N}_A, \vec{N}_B, \vec{L}_A(w_z), \vec{L}_B(w_z), t_z(w_z), w_z)) \end{aligned} \tag{61}$$

with $\vec{N}_i = \beta_i N_i$, $\vec{L}_i = \bar{\beta}_i L_i$. Assume $\bar{\beta}_i = \beta_i$. Implicit differentiation then yields[22]

$$\frac{d\vec{N}_A}{dw_z} = -\frac{\frac{\partial EU_A}{\partial w_z} + \frac{\partial EU_A}{\partial t_z}\frac{\partial t_z}{\partial w_z} - \frac{\partial EU_B}{\partial w_z} - \frac{\partial EU_B}{\partial t_z}\frac{\partial t_z}{\partial w_z}}{\frac{1}{\beta_A}\frac{\partial EU_A}{\partial N_A} + \frac{1}{\beta_B}\frac{\partial EU_B}{\partial N_B}} \tag{62}$$

The relocation equilibrium of the firms is given by

$$\frac{\vec{P}_A(L_A(w_z), w_z)}{\vec{M}_A} = \frac{\vec{P}_B(L_B(w_z), w_z)}{\vec{M}_B} \tag{63}$$

with $\vec{M}_i = \alpha_i M_i$. Assume $\bar{\alpha}_i = \alpha_i$. Then, following implicit derivative is obtained

$$\frac{\mathrm{d}\vec{M}_A}{\mathrm{d}w_z} = -\frac{\frac{\partial AP_A}{\partial w_z} - \frac{\partial AP_B}{\partial w_z}}{\frac{1}{\alpha_A}\frac{\partial AP_A}{\partial M_A} + \frac{1}{\alpha_B}\frac{\partial AP_B}{\partial M_B}}. \tag{64}$$

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21 $\frac{\partial EU_i}{\partial w_z} = \frac{\frac{\partial L_i}{\partial w_z} N_i}{N_i^2}\left[U_{i,e} - U_{i,u}\right] + \frac{L_i}{N_i}\frac{\partial U_{i,e}}{\partial \tilde{w}_i}\frac{\partial \tilde{w}_i}{\partial w_z} + \left[1 - \frac{L_i}{N_i}\right]\frac{\partial U_{i,u}}{\partial b_z}\left[\frac{\partial b_z}{\partial L_A}\frac{\partial L_A}{\partial w_z} + \frac{\partial b_z}{\partial L_B}\frac{\partial L_B}{\partial w_z} + \frac{\partial b_z}{\partial w_z}\right]$,
$\frac{\partial EU_i}{\partial t_z} = \frac{L_i}{N_i}\frac{\partial U_{i,e}}{\partial \tilde{w}_i}\frac{\partial \tilde{w}_i}{\partial t_z} + \left[1 - \frac{L_i}{N_i}\right]\frac{\partial U_{i,u}}{\partial b_z}\frac{\partial b_z}{\partial t_z}$, $\frac{\partial EU_i}{\partial N_i} = -\frac{L_i}{N_i^2}\left[U_{i,e} - U_{i,u}\right]$,
$\frac{\partial AP_i}{\partial w_z} = \frac{\frac{\partial P_i}{\partial w_i} M_i}{M_i^2}$, $\frac{\partial AP_i}{\partial t_z} = 0$, $\frac{\partial AP_i}{\partial \vec{M}_i} = -\frac{P_i}{M_i^2}$.

22 $\frac{\partial EU_B}{\partial b_z}\frac{\partial b_z}{\partial w_A} = \left[1 - \frac{L_B}{N_B}\right]\frac{\partial U_{B,u}}{\partial b_z}\left[\frac{\partial b_z}{\partial L_A}\frac{\partial L_A}{\partial w_A} + \frac{\partial b_z}{\partial w_A}\right]$.

Substituting (62) and (64) into (60) yields first-order-condition

$$\frac{\mathrm{d}V_z}{\mathrm{d}w_z} = \sum_{i=A}^{B}\left(\frac{\partial EU_i}{\partial w_z} + \frac{\partial EU_i}{\partial t_z}\frac{\partial t_z}{\partial w_z}\right) + \sum_{i=A}^{B}\frac{\partial AP_i}{\partial w_z} - \frac{\frac{\partial AP_A}{\partial M_A} - \frac{\partial AP_B}{\partial M_B}}{\frac{1}{\alpha_A}\frac{\partial AP_A}{\partial M_A} + \frac{1}{\alpha_B}\frac{\partial AP_B}{\partial M_B}}\left(\frac{\partial AP_A}{\partial w_z} - \frac{\partial AP_B}{\partial w_z}\right)$$
$$- \frac{\frac{\partial EU_A}{\partial N_A} - \frac{\partial EU_B}{\partial N_B}}{\frac{1}{\beta_A}\frac{\partial EU_A}{\partial N_A} + \frac{1}{\beta_B}\frac{\partial EU_B}{\partial N_B}}\left(\frac{\partial EU_A}{\partial w_z} + \frac{\partial EU_A}{\partial t_z}\frac{\partial t_z}{\partial w_z} - \frac{\partial EU_B}{\partial w_z} - \frac{\partial EU_B}{\partial t_z}\frac{\partial t_z}{\partial w_z}\right) \tag{65}$$

for asymmetric states and first-order-condition (28) for symmetric states. ■

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