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Skills, Tasks, and the Complexity Premium

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The Complexity Premium*

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Abstract

We introduce a task-based framework for modeling production in which certain tasks are too complex for many workers to perform. In such an environment, workers' wages may significantly diverge from their relative productivities: Workers with marginally higher skill levels may obtain a large additional wage premium on top of the skill premium, which we call *complexity premium*. We apply our framework to explain past employment and wage polarization and estimate model parameters for the U.S. labor market between 2001 and 2019. Beyond a rising skill level and skill premium, we find that the complexity of tasks increases and employees performing more complex tasks earn a significant complexity premium, which accounts for up to 43 percent of their wages. Finally, we explore the effects of artificial intelligence and find it may aggravate wage inequality, with an ambiguous effect on complexity premia.

Keywords: Skills, technological change, tasks, complexity

JEL Classification: J24, J30, J31

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1 Introduction

We propose a novel task-based model, the *task-complexity model*, in which workers are substitutes for one another across tasks as long as they satisfy an additional constraint: their skill level must be sufficient to master the task's complexity (for empirical evidence, see Autor et al., 2003, and Frey and Osborne, 2017). Therefore, less-skilled labor may not merely be an imperfect substitute for higher-skilled labor but no substitute at all. This constraint, which we denote as the *sufficient skill condition*, can translate into a scarcity of labor and wage premia that cannot be explained by productivity alone. Specifically, a worker with a skill level that just satisfies the sufficient skill condition for one task may earn significantly more than another worker with a marginally lower skill level. We call the difference in the wages of workers with virtually the same skill level a *complexity premium*, and a labor market exhibiting such jumps in the wage scheme *disintegrated*.

Our model contributes to the existing literature in various dimensions. First, from a theoretical perspective, the task-complexity model provides a formal framework to explain how wage inequality across skills can be caused by the supply and the demand side of the labor market, both of which have been found to be relevant in the empirical literature (Goldin and Katz, 2018). On the supply side, skill may determine productivity, resulting in skill premia. On the demand side, the distribution of task complexities may or may not cause complexity premia of different sizes. This novel demand-side mechanism is the model's key contribution.

Second, our approach helps explain stylized outcomes driven by skill-biased technological change. In our example, we focus on the computerization phase that began around 1980. The task-complexity model can reconcile the following two stylized facts. First, Autor and Dorn (2013) observe a polarizing distribution of wages and employment across the complexity space, i.e., increasing wages and employment shares particularly in low- and high-complexity occupations. Second, (Acemoglu and Autor, 2011; Autor, 2019) find decreasing real wages among less educated individuals, which might appear to be at odds with rising wages in low-complexity occupations as implied by the first fact.

Third, the complexity premium is an important explanatory factor of wage differences in the data. The estimated model can be used as a measurement tool to decompose wages into skill and complexity premia and identify drivers of observed wage inequality. We find that, in the period of 2001–2019, a disintegration of the U.S. labor market has significantly contributed to wage differences among low-, medium-, and high-skilled

employees. The complexity premium explains up to 43 percent of the wages of the high-skilled. Such high complexity premia create strong incentives for firms to reduce the complexity of their tasks and for workers to increase their skill level. In the estimation results, only the latter is observed.

Fourth, we argue that the sufficient skill condition creates a new mechanism by which artificial intelligence (AI) may affect wage inequality. AI exerts downward pressure on wages in tasks where it can substitute for human labor, and affected workers cannot transition to exclusively human tasks that are too complex for them to perform. However, we also study settings in which workers can switch to low-complexity tasks—such as manual labor—in which AI cannot be used, thereby avoiding nominal wage losses. Even in this scenario, our model predicts that AI exacerbates wage inequality, and declining real wages for low- and medium-skilled workers cannot be ruled out. The effect of AI on complexity premia in high-complexity tasks performed by humans is ambiguous and driven by price effects in the goods market.

The task-complexity model builds on the existing task-based model literature. In contrast to the earlier, so-called canonical model, workers in task-based models allocate endogenously into tasks based on their individual skill levels (Acemoglu and Autor, 2011; Acemoglu and Zilibotti, 2001; Autor et al., 2003). While workers may have a comparative advantage in one task, they are in principle able to perform any task. The essential difference between our model and previous task-based models is the sufficient skill condition. This constraint implies that workers are substitutes in some tasks but not in others.

We stress that our model features a continuous distribution of skill levels. In this regard, our model is similar to the early task-based model proposed by Kremer and Maskin (1996) and a model by Jung and Mercenier (2011). In prominent models by Acemoglu and Zilibotti (2001), Acemoglu and Autor (2011), Acemoglu and Restrepo (2022), and Acemoglu (2025), there is a finite and mostly small number of skill levels. We argue that a continuous skill level distribution is more realistic, and it enables us to stay agnostic about the relevance of discrete proxies like educational degrees for skill in production. Instead, we can estimate parameters of the skill level distribution from the data.

The remainder of the paper is organized as follows. We formally describe the model in Section 2 and illustrate major mechanisms in a two-firm example in Section 3. In Section 4, we replicate stylized facts about wages and employment observed in the past (1980–2005). In Section 5, we estimate our model on recent U.S. labor market data from 2001 to 2019. In Section 6, we discuss the effects that the emergence of AI can be

expected to have on wages in the future. Section 7 concludes and suggests directions for future research with the task-complexity model.

2 The Model

2.1 Households

There is a continuum of households of mass one. We refer to households and workers interchangeably, and index them by their skill level r . We assume that the skill level is distributed according to some continuous density function $f(r)$ and cumulative distribution function $F(r)$ with support $\mathcal{R} = [0, 1]$. For notational convenience, we denote the density of households with skill level r by $L^r = f(r)$. Hence, L^r is a continuous function of r .

Households derive utility from consuming final goods $i \in \mathcal{I}$. Household r 's utility is given by a CES function

$$U^r \left(\{c_i^r\}_{i \in \mathcal{I}} \right) = C^r, \quad (1)$$

where

$$C^r := \left(\sum_{i \in \mathcal{I}} (c_i^r)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.$$

C^r represents both the utility and the consumption basket of household r . In the consumption basket, c_i^r is the amount of final good i consumed by household r . The parameter σ is the elasticity of substitution between goods. We assume that $\sigma > 1$.

Household r receives wage w^r and receives firm profits Π^r . We do not make any assumptions about the distribution of firm ownership across households since equilibria can be determined independently. Accordingly, the budget constraint of household r is

$$\sum_{i \in \mathcal{I}} p_i c_i^r \leq L^r w^r + \Pi^r, \quad (2)$$

where p_i denotes the price of final good i . The demand of household r for a good i is

$$c_i^r = \left(\frac{p_i}{P} \right)^{-\sigma} C^r,$$

where

$$P = \left(\sum_{i \in \mathcal{I}} p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \quad (3)$$

is the aggregate price index. The derivation is detailed in Appendix A.1. Then, total demand for a representative product i is

$$c_i = \left(\frac{p_i}{P} \right)^{-\sigma} C, \quad (4)$$

where $C := \int_0^1 C^r L^r dr$ is total consumption. Aggregating budget constraints across households yields

$$PC = \int_0^1 w^r L^r dr + \int_0^1 \Pi^r L^r dr,$$

where PC denotes the total nominal consumption expenditures.

2.2 Firms

Final good i is produced in industry i in quantity x_i and is a CES aggregate of intermediate goods ij :

$$x_i = \left(\int_0^1 x_{ij}^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}} \quad (5)$$

such that p_i is the index

$$p_i = \left(\int_0^1 p_{ij}^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}. \quad (6)$$

Each intermediate good ij is produced by a firm indexed accordingly. Firms act as monopolistic competitors. For tractability, we assume that the elasticity of substitution is the same between final and intermediate goods.

Demand for the intermediate good ij follows as

$$x_{ij}^d = \left(\frac{p_{ij}}{p_i} \right)^{-\sigma} \left(\frac{p_i}{P} \right)^{-\sigma} C. \quad (7)$$

Please refer to Appendix A.2 for the derivation.

Production Technology

All firms in industry i use the same technology that involving a single task. Thus, we index final goods, industries, and tasks by i . We refer to the complexity level of a task as its *task complexity* and denote it by τ_i . The task complexity indicates the degree of difficulty connected with the successful completion of a task, i.e., the higher the task complexity, the more difficult the production process.

Indices in the set $\mathcal{I} = \{1, \dots, \bar{i}\}$ are ordered according to related task complexities such that $\tau_i < \tau_{i+1} \forall i$. Hence, task \bar{i} is the most complex, and task 1 is the least complex

in the economy. Assuming strict inequality in $\tau_i < \tau_{i+1} \forall i$ implies a bijective mapping between industries and task complexities. We denote the set of task complexities as $T = \{\tau_1, \dots, \tau_i, \tau_{i+1}\}$.

The following assumption is essential:

Assumption 1 (Sufficient Skill Condition)

Labor with skill level r can only perform at task complexity τ_i if $r \geq \tau_i$.

We assume $\tau_i \in [0, 1) \forall i \in \mathcal{I}$ such that for each industry i there will be some worker r satisfying the sufficient skill condition given in Assumption 1. Note that task complexity τ_{i+1} does not correspond to an industry. It is set to one and is used in expressions derived later.

As a side note, Assumption 1 implies that a fraction of workers $F(\tau_1)$ remains unemployed whenever $\tau_1 > 0$ and $f(r) > 0$ for some $r < \tau_1$. As discussed in Brynjolfsson and McAfee (2014), Harari (2016), and Keynes (2010), such a class of unemployed workers may indeed emerge as a result of technological progress, reflected in our model as a shift of task complexities. However, unemployment is not the focus of our paper. Instead, we concentrate on studying wage differences across employed workers due to what we call *complexity premia*.

The production function of firm ij relates the amount $l_{ij}(r)$ of labor of skill level r to x_{ij} :

$$x_{ij} = \int_{r \in \mathcal{R}_{ij}} \kappa(r) l_{ij}(r) dr , \quad (8)$$

where \mathcal{R}_{ij} denotes the set of skill levels used in production by firm ij and $\kappa(r) : [0, 1] \rightarrow \mathbb{R}_+$ denotes the productivity of worker r . We assume $\kappa'(\cdot) > 0$, i.e., productivity is strictly increasing in the worker's skill level.

Profit Maximization

The profit maximization problem of firm ij is

$$\begin{aligned} \max_{\mathcal{R}_{ij}, p_{ij}, \{x_{ij}(r)\}_{r \in \mathcal{R}_{ij}}, \{l_{ij}(r)\}_{r \in \mathcal{R}_{ij}}} \quad & \int_{r \in \mathcal{R}_{ij}} \left(p_{ij} x_{ij}(r) - x_{ij}(r) \frac{w^r}{\kappa(r)} \right) dr , \\ \text{s.t.} \quad & x_{ij} = \int_{r \in \mathcal{R}_{ij}} x_{ij}(r) dr = x_{ij}^d = \left(\frac{p_{ij}}{p_i} \right)^{-\sigma} \left(\frac{p_i}{P} \right)^{-\sigma} C , \\ & r \geq \tau_i \quad \forall r \in \mathcal{R}_{ij} , \\ & \mathcal{R}_{ij} \subseteq \mathcal{R} . \end{aligned} \quad (9)$$

Firm ij chooses a set of skill levels for production \mathcal{R}_{ij} , an amount of labor input $l_{ij}(r)$ for each skill level in \mathcal{R}_{ij} , and a price p_{ij} . The total output of firm ij , x_{ij} , must equal the total demand for its variety, x_{ij}^d .

Firm ij 's profit maximization problem is solved by dividing it into two subproblems:

(i) *Cost minimization*

As there is a continuum of firms in each industry and workers do not have preferences for working in certain firms, we assume perfect competition among firms in the labor market, such that firms take wages as given. Firm ij minimizes the cost per unit of output, $\frac{w^r}{\kappa(r)}$, by choosing a subset of skills $\mathcal{R}_{ij} \subseteq \mathcal{R}$ that minimizes this cost and satisfies Assumption 1.

All firms in industry i are symmetric, and the mass of firms in industry i is one, so $\mathcal{R}_{ij} = \mathcal{R}_i \forall i, j$. Further, we expect $\frac{w^r}{\kappa(r)}$ to be homogeneous and equal to a constant ω_i for all $r \in \mathcal{R}_i$. If this were not the case—i.e., if there existed $r, r' \in \mathcal{R}_i$ for which $\frac{w^{r'}}{\kappa(r')} > \frac{w^r}{\kappa(r)} = \omega_i$ —then the firm would prefer not to hire labor of skill level r' , contradicting $r' \in \mathcal{R}_i$. We refer to ω_i as the *scaling factor* of industry i , and define $\omega = \{\omega_1, \dots, \omega_i\}$ as the set of scaling factors for all industries. These definitions will be used in the next section to solve for the equilibrium.

(ii) *Profit maximization*

Given the cost-minimizing set of skill levels in production \mathcal{R}_i , firm ij chooses a price that solves the profit maximization problem given in (9). With $\omega_i = \frac{w^r}{\kappa(r)}$, the problem becomes

$$\begin{aligned} \max_{p_{ij}} \quad & \int_{r \in \mathcal{R}_i} (p_{ij} x_{ij} - \omega_i x_{ij}) dr , \\ \text{s.t.} \quad & x_{ij} = x_{ij}^d = \left(\frac{p_{ij}}{p_i} \right)^{-\sigma} \left(\frac{p_i}{P} \right)^{-\sigma} C . \end{aligned}$$

Note that the integrand is independent of r . The solution to the maximization problem is

$$p_{ij} = p_i = \frac{\sigma}{\sigma - 1} \omega_i \quad \forall i, j. \quad (10)$$

Thus, the price equals the constant mark-up, $\frac{\sigma}{\sigma - 1}$ times the marginal cost $\omega_i = \frac{w^r}{\kappa(r)}$ for all firms in industry i .

Because firms in industry i are symmetric and there is a unit mass of firms in each industry, we may index output and labor demand of a representative firm in industry i by i , such that $x_i = x_{ij}$ and $l_i(r) = l_{ij}(r) \forall i, j$.

2.3 Equilibrium

We begin by defining an equilibrium.

Definition 1 (Equilibrium) *An equilibrium consists of:*

- (i) *a set of skill levels $\mathcal{R}_i \subseteq \mathcal{R}$ for each representative firm i , with $i \in \mathcal{I}$, that this firm is willing to employ,*
- (ii) *output levels, $\{x_i\}_{i \in \mathcal{I}}$, and labor, $\{l_i(r)\}_{(i,r) \in \mathcal{I} \times \mathcal{R}_i}$, chosen by representative firm i ,*
- (iii) *a set of consumption levels, $\{c_i^r\}_{(i,r) \in \mathcal{I} \times \mathcal{R}}$, for each household r 's consumption of each final good i ,*
- (iv) *a set of goods prices, $\{p_i\}_{i \in \mathcal{I}}$,*
- (v) *a set of wages, $\{w^r\}_{r \in \mathcal{R}}$,*

such that

- (A) *$x_i, \{l_i(r)\}_{r \in \mathcal{R}_i}$, and p_i solve the representative firm i 's profit maximization problem (9) $\forall i \in \mathcal{I}$,*
- (B) *$\{c_i^r\}_{i \in \mathcal{I}}$ maximizes the utility of household r in (1) subject to the household's budget constraint (2) $\forall r \in \mathcal{R}$,*
- (C) *goods markets clear for all products,*
- (D) *labor markets clear, and*
- (E) *\mathcal{R}_i fulfills Assumption 1 for all $i \in \mathcal{I}$.*

To characterize the equilibrium, we define *effective labor demand* \tilde{l}_i of a representative firm or industry i as

$$\tilde{l}_i := \int_{\mathcal{R}_i} l_i(r) \frac{\kappa(r)}{\kappa(1)} dr . \quad (11)$$

Intuitively, effective labor demand is labor demand in terms of the number of workers adjusted for their respective productivity differences.

On the supply side, we may define *effective labor supply* as the analog. Specifically,

$$\tilde{L}_i^\Delta := \int_{\tau_i}^{\tau_{i+1}} \frac{\kappa(r)}{\kappa(1)} L^r dr \quad (12)$$

denotes effective labor that can be employed in task i (or less complex tasks), but not in task $i + 1$.

To close our model, we introduce a labor market clearing condition—henceforth LMCC—represented by inequality (13):

LMCC (Labor Market Clearing Condition)

$$\sum_{\tilde{i} \geq i} \tilde{L}_{\tilde{i}}^\Delta \geq \sum_{\tilde{i} \geq i} \tilde{l}_{\tilde{i}}(\mathcal{W}) \quad \forall i \in \mathcal{I}, \quad (13)$$

where $\mathcal{W} = \{w^r\}_{r \in \mathcal{R}}$ denotes the *wage scheme* that describes the wages paid to workers of different skill levels. The left-hand side is the supply of effective labor that is able to perform task i with complexity τ_i or more complex tasks. The right-hand side is the cumulative demand of all industries $\tilde{i} \geq i$ for effective labor given wage scheme \mathcal{W} . In an equilibrium, Condition (13) must hold for every task $i \in \mathcal{I}$ and with equality for the least complex task 1. If the latter was not the case, there would be aggregate excess labor supply or demand. Whenever the LMCC is satisfied as an inequality for some $i > 1$, this reflects a case in which more skilled labor with $r \geq \tau_i$ is available than needed for all tasks $\tilde{i} \geq i$. In equilibrium, this does not translate into excess labor supply for any task as workers $r \geq \tau_i$ can still be hired for some task $\hat{i} < i$.

Recall that we determined the *scaling factor* ω_i of industry i as

$$\omega_i = \frac{w^r}{\kappa(r)} \quad \forall r \in \mathcal{R}_i,$$

implying $w^r = \omega_i \kappa(r) \quad \forall r \in \mathcal{R}_i$. Therefore, we can write the wage scheme \mathcal{W} as

$$\mathcal{W} = \{\omega_i \kappa(r)\}_{(i,r) \in \mathcal{I} \times \mathcal{R}}. \quad (14)$$

When we adopt scaling factor $\omega_i = 1$ for all firms, wage scheme \mathcal{W} implies $\frac{w^r}{\kappa(r)} = \frac{w^{r'}}{\kappa(r')} \quad \forall r, r' \geq \tau_1$. However, Equation (14) also covers cases with heterogeneity in the scaling factor across tasks. Intuitively, ω_i will adjust due to perfect competition on the labor market: If there is excess labor demand for $\omega_i = 1$, atomistic producers ij will increase wages up to the point where demand equals supply.

Whenever $\omega_i = 1 \forall i$, we speak of an *integrated labor market* equilibrium, henceforth ILM. If, on the other hand, the demand for high-skilled workers exceeds the supply with this scheme, we need a scaling factor $\omega_i \neq 1$. In that case, we speak of a *disintegrated labor market* equilibrium, henceforth DLM. We may also observe a *locally integrated labor market* within a disintegrated labor market when the scaling factor is equal across at least two neighboring tasks but different for some other task, i.e., $\omega_i = \omega_{i+1} \neq \omega_{\tilde{i}}$ for some $\tilde{i} \notin [i, i+1]$.

Having introduced the scaling factor and the definition of a disintegrated labor market, we may now formally define a complexity premium:

Definition 2 (Complexity Premium) *In a disintegrated labor market with $\omega_i > 1$ for some task i , the **complexity premium** earned by worker $r \in \mathcal{R}_i$ is the difference between wage and productivity, i.e., $w^r - \kappa(r) = (\omega_i - 1) \kappa(r)$.*

We now proceed to solve for the equilibrium. Given p_i from (10) and the aggregate price index (3), we obtain

$$P = \frac{\sigma}{\sigma - 1} \left(\sum_{i \in \mathcal{I}} \omega_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}}.$$

Substituting p_i from (10) and P into the household demand function (4) yields

$$c_i = \omega_i^{-\sigma} \left(\sum_{\tilde{i}} \omega_{\tilde{i}}^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}} C. \quad (15)$$

Market clearing for good i implies $c_i = x_i$. Combining the definition of effective labor demand (11) and technology (8), we find $c_i = \kappa(1) \tilde{l}_i$. Substituting this into (15) gives

$$\tilde{l}_i = \omega_i^{-\sigma} \kappa(1)^{-1} \left(\sum_{\tilde{i}} \omega_{\tilde{i}}^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}} C \quad \forall i \in \mathcal{I}. \quad (16)$$

Total wages earned by households must equal total wages paid by firms TW , i.e.,

$$TW = \kappa(1) \sum_i \omega_i \tilde{L}_i^\Delta = \kappa(1) \sum_i \omega_i \tilde{l}_i. \quad (17)$$

Multiplying (16) by ω_i and summing over all industries yields total consumption as a function of total wages:

$$C = \left(\sum_{i \in \mathcal{I}} \omega_i^{1-\sigma} \right)^{\frac{1}{\sigma-1}} TW. \quad (18)$$

Thus, given a solution for ω_i , C becomes a linear function of TW , and TW itself depends on exogenous labor supply \tilde{L}_i^Δ and productivity $\kappa(1)$ only. Labor demand from industries (16) and demand for final good i (15) both depend on scaling factors and C . Thus, all that is needed to solve the model is the equilibrium solution for the scaling factors and thereby the wage scheme, ω^* and \mathcal{W}^* .

We impose three restrictions on \mathcal{W}^* . First, we set the wages of unemployable workers with $r < \tau_1$ to zero. Second, we normalize the scaling factor of the industry with the lowest task complexity to one, i.e., $\omega_1^* := 1$. Thus, the lowest-skilled employable worker always earns his productivity such that $w^{\tau_1} = \kappa(\tau_1)$. Third, equilibrium scaling factors ω_i^* have to be non-decreasing in i . Otherwise, wages as a function of skill levels would not be increasing in the skill level everywhere, which cannot be an equilibrium.

To simplify further derivations and language, we define a *group* as a set of tasks with a common scaling factor. A group is indexed by $g \in \mathbb{N}$. Note that a group may encompass multiple task complexities. We order groups by their scaling factors and refer to a collection of such groups as a group structure, denoted \mathcal{G} . The number of groups, $|\mathcal{G}|$, equals the number of labor market separations plus 1, i.e., the number of jumps in the scaling factor plus one. Thus, if $|\mathcal{G}| = 1$, there is no labor market disintegration and the economy is in an ILM equilibrium. If $|\mathcal{G}| > 1$, the economy is in a DLM equilibrium. If $|\mathcal{G}| \in (1, |\mathcal{I}|)$, the economy is in a DLM equilibrium with at least one locally integrated labor market.

The scaling factor within a group is denoted by ω_g . By construction,

$$\omega_{g+1} > \omega_g \quad \forall g \in \mathcal{G} , \quad (19)$$

i.e., the higher a group's index, the higher its scaling factor. The following proposition establishes the existence and uniqueness of the equilibrium.

Proposition 1 (Existence and Uniqueness)

There exists a unique equilibrium with group structure \mathcal{G}^ .*

The proof is given in Appendix B.1. In the proof, we also establish the following properties regarding the equilibrium wages:

Corollary 1 (Wage Scheme)

1. *The uniquely determined wage scheme \mathcal{W}^* can be characterized by*

$$\mathcal{W}^* := \{\omega^*(r)\}_{r \in \mathcal{R}}, \text{ where } \omega^*(r) = \omega_i^* \text{ if } r \in [\tau_i, \tau_{i+1}).$$

2. *If two workers of skill levels r and r' are in the same group, then $\omega^*(r) = \omega^*(r')$.*

In Appendix C, we present the algorithm used to solve for \mathcal{W}^* .

From the previous considerations, we obtain a full characterization of the equilibrium:

Proposition 2 (Equilibrium)

The equilibrium is characterized by a group structure \mathcal{G}^ and by*

- (i) $\mathcal{W}^* = \{\omega^*(r)\kappa(r)\}_{r \in \mathcal{R}}$,
- (ii) $\mathcal{R}_i^* \subseteq \{r \in \mathcal{R} | r \geq \tau_i\} \cap \{r \in \mathcal{R} | \omega^*(r) = \omega_i^*\} \quad \forall i \in \mathcal{I}$,
- (iii) $p_i^* = \frac{\sigma}{\sigma-1} \omega_i^* \quad \forall i \in \mathcal{I}$,

$$P^* = \frac{\sigma}{\sigma-1} \left(\sum_{i \in \mathcal{I}} (\omega_i^*)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \quad ,$$
- (iv) $TW^* = \kappa(1) \sum_{i \in \mathcal{I}} \omega_i^* \tilde{L}_i^\Delta$,
- (v) $\tilde{l}_i^* = \frac{\omega_i^{*\sigma}}{\sum_{i \in \mathcal{I}} (\omega_i^*)^{1-\sigma}} \kappa(1)^{-1} TW^* \quad ,$
- (vi) $x_i^* = \kappa(1) \tilde{l}_i^* \quad ,$
- (vii) $\pi_i^* = \frac{\kappa(1) \omega_i^* \tilde{l}_i^*}{\sigma-1} \quad ,$
- (viii) $C^* = \left(\sum_{i \in \mathcal{I}} (\omega_i^*)^{1-\sigma} \right)^{\frac{1}{\sigma-1}} TW^* \quad ,$
and $P^* C^* = \frac{\sigma}{\sigma-1} TW^*.$

Item (ii) indicates that representative firm i hires only labor that satisfies Assumption 1 and is paid according to representative firm i 's scaling factor.

Note that any equilibrium characterized by Proposition 2 is Pareto-efficient. There are no externalities and all conditions of the First Welfare Theorem are fulfilled. Yet, this Pareto efficiency may be associated with a high degree of wage inequality, especially in DLM equilibria with complexity premia.

3 A Two-Industry Example

In this section, we provide a simple example with only two industries. We begin by analyzing general properties of the equilibrium and its solution. Next, we present both an ILM and a DLM equilibrium.

3.1 Setup

Assume that skill levels are uniformly distributed, i.e., $L^r = 1$ on $\mathcal{R} = [0, 1]$. Further, skill is transformed one-to-one into productivity, so that $\kappa(r) = r$. Finally, the economy consists of only two industries, each with a representative firm and a corresponding task $i \in \{1, 2\}$, with associated task complexities satisfying $0 < \tau_1 < \tau_2 < 1$. Recall that we normalize $\omega_1 = 1$. Hence, the labor market is integrated if $\omega_2 = 1$ and disintegrated if $\omega_2 > 1$.

To study the equilibrium, we begin with the labor market clearing condition (13). Substituting (12) for \tilde{L}_i^Δ and using (16), (18), and (17) to express \tilde{l}_i , we obtain:

$$\int_{\tau_i}^1 \frac{\kappa(r)}{\kappa(1)} L^r dr \geq \frac{\sum_{i \geq \hat{i}} \omega_i^{-\sigma}}{\sum_{\tilde{i}} \omega_{\tilde{i}}^{1-\sigma}} \sum_{\tilde{i}} \omega_{\tilde{i}} \int_{\tau_{\tilde{i}}}^{\tau_{\tilde{i}+1}} \frac{\kappa(r)}{\kappa(1)} L^r dr \quad \forall \hat{i} \in \mathcal{I}. \quad (20)$$

Exploiting the assumptions $L^r = 1$ and $\kappa(r) = r \forall r \in \mathcal{R}$, this simplifies to

$$1 - \tau_1^2 \geq \frac{1 + \omega_2^{-\sigma}}{1 + \omega_2^{1-\sigma}} (\tau_2^2 - \tau_1^2 + \omega_2 (1 - \tau_2^2)) \quad \text{for } \hat{i} = 1, \quad (21)$$

$$1 - \tau_2^2 \geq \frac{\omega_2^{-\sigma}}{1 + \omega_2^{1-\sigma}} (\tau_2^2 - \tau_1^2 + \omega_2 (1 - \tau_2^2)) \quad \text{for } \hat{i} = 2. \quad (22)$$

First, assume an integrated labor market, i.e., $\omega_2 = 1$. This yields

$$\begin{aligned} 1 &\geq 1 && \text{for } \hat{i} = 1, \\ \sqrt{\frac{1}{2}(\tau_1)^2 + \frac{1}{2}} &\geq \tau_2 && \text{for } \hat{i} = 2. \end{aligned}$$

The first line is trivially satisfied, but this is not necessarily the case for the second. If τ_2 is large relative to τ_1 , the condition will be violated, indicating that the ILM is not an equilibrium. The respective threshold value of τ_2 for a given τ_1 is illustrated in Figure 1. Note that for any τ_1 , there is a τ_2 such that the labor market is integrated. This implies that the absolute level of task complexities is irrelevant for disintegration; what matters is their relative position. Similarly, the level of σ does not affect whether the labor market is integrated or disintegrated.

These findings generalize as follows. Labor market disintegration and scaling factors depend on the size of the labor force satisfying the sufficient skill condition for tasks i compared to the total labor supply satisfying it for task 1 (see (B.9) and Appendix C). Thus, it holds in general that the relative position of some τ_i compared to τ_1 is crucial. Moreover, we confirm that altering σ alone cannot cause disintegration although it influences the magnitude of scaling factors in DLM equilibria.

Figure 1: Disintegrated and Integrated Labor Markets, for Levels of τ_2 Given τ_1 , in a Two-Firm Example.

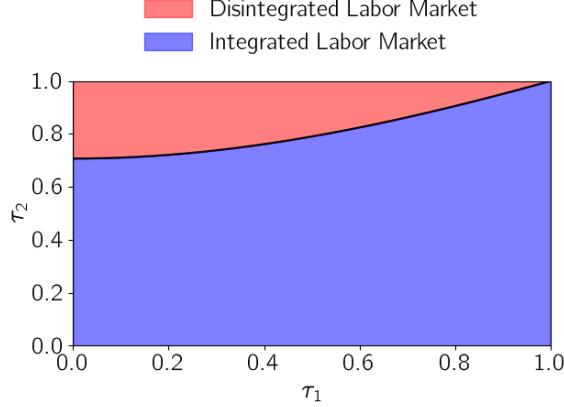
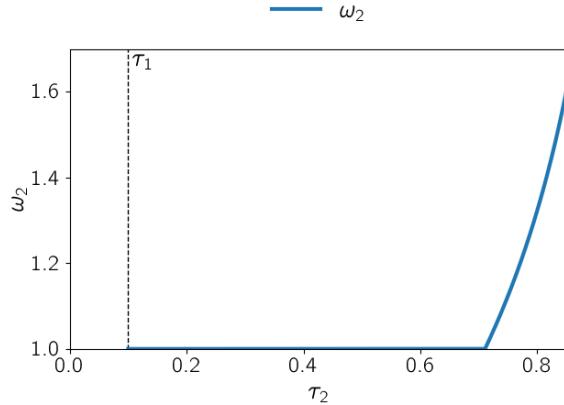


Figure 2: Scaling Factor ω_2 as a Function of τ_2 in a Two-Firm Example, Fixing $\tau_1 = 0.1$ and $\sigma = 2$. Figure Cut off at $\tau_2 = 0.85$.



We now continue to study the two-industry example. In the case where (22) is violated as $\tau_2 > \sqrt{\frac{1}{2}(\tau_1)^2 + \frac{1}{2}}$, the solution for ω_2 can be obtained by solving (22) under equality for ω_2 or using the general solution derived in Appendix C. With both approaches, we obtain $\omega_2 = \left(\frac{1-\tau_2^2}{\tau_2^2-\tau_1^2}\right)^{-\frac{1}{\sigma}}$. As $\omega_2 = 1$ up to $\tau_2 = \sqrt{\frac{1}{2}(\tau_1)^2 + \frac{1}{2}}$ and ω_2 is strictly increasing in τ_2 for $\tau_2 > \sqrt{\frac{1}{2}(\tau_1)^2 + \frac{1}{2}}$, it follows that

$$\omega_2 = \max \left\{ 1, \left(\frac{1-\tau_2^2}{\tau_2^2-\tau_1^2} \right)^{-\frac{1}{\sigma}} \right\}, \quad (23)$$

for any levels of τ_1 , τ_2 , and σ . Given $\tau_1 = 0.1$ and $\sigma = 2$, the relationship between ω_2 and τ_2 is depicted in Figure 2. In a disintegrated labor market, ω_2 grows rapidly as τ_2 increases and goes to infinity as $\tau_2 \rightarrow 1$.

3.2 Comparison of Two Scenarios

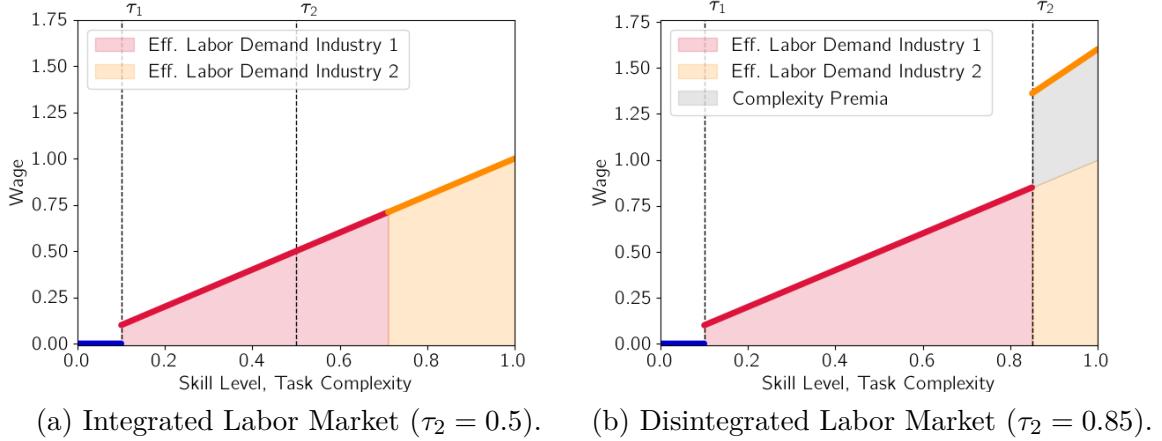
Next, we calibrate the model for two scenarios to study the effects of a shift towards a more heterogeneous task-complexity distribution. Holding all other parameters constant, we first compute wages for task complexities $\tau_1 = 0.1$ and $\tau_2 = 0.5$. In the second case, we set $\tau_2 = 0.85$. Given that the threshold for a disintegrated labor market at $\tau_1 = 0.1$ is at 0.71, the first scenario corresponds to an integrated and the second to a disintegrated labor market (see Figure 2). The resulting wage schemes and effective labor demand for both firms are depicted in Figure 3.

We can summarize our findings as follows. In the integrated labor market (ILM) scenario, illustrated in Figure 3a, effective labor demand—represented by shaded areas—is homogeneous across both firms. The same holds for scaling factors, prices, output, and profits. Consequently, aside from each firm's chosen skill level set \mathcal{R}_i and the wages paid, firms are fully symmetric in the ILM equilibrium.

In the presented disintegrated labor market (DLM) scenario shown in Figure 3b, τ_2 is sufficiently high that the supply of effective labor satisfying $r \geq \tau_2$ is smaller than firm 2's demand in the ILM. Thus, scaling factor ω_2 increases as predicted in Figure 2. Total production and consumption decline by 5 percent, and the wage share in the economy rises by 12 percentage points due to higher wages for high-skilled workers. The Gini coefficient of labor income increases from 0.35 to 0.42, indicating a rise in wage inequality. Since prices are given by $p_i = \frac{\sigma}{\sigma-1} \omega_i$, the aggregate price level P increases by approximately 23 percent. As a result, workers employed in industry 1 experience declining real wages. Firms have become heterogeneous not only in terms of prices but also in terms of output and labor demand—industry 2's share of output and effective labor declines from 50 to less than 28 percent.

Plotting effective labor demand for firms 1 and 2 in Figure 3a requires to pin down labor allocation for the integrated labor market case. One way to do this is imposing Assumption 2.

Figure 3: Wage Distributions and Labor Demand in a Two-Firm Example of an Integrated and a Disintegrated Labor Market.



Assumption 2 (Allocation of Skill Levels to Tasks)

For all tasks $i \in \mathcal{I}$,

1. \mathcal{R}_i is an interval,
2. firm i hires all available labor with skill level $r \in \mathcal{R}_i$, i.e. $l_i(r) = L^r$ and $l_i(r) = 0 \forall \tilde{i} \in \mathcal{I} : \tilde{i} \neq i$.

Without this assumption, it would be indeterminate which workers $r \geq \tau_2$ are allocated to firm 1 and firm 2. Importantly, Assumption 2 has no impact on wages, prices, firms' effective labor demand or any other equilibrium variable included in Proposition 2.

In the next section, we examine how the model can be used to explain stylized patterns observed in labor market data.

4 The Past: Explaining Stylized Facts since 1980 in the Task-Complexity Model

4.1 Stylized Facts

Investigating a period from 1980 to 2005 in the United States, Autor and Dorn (2013) document a U-shaped growth of wages and employment across the skill distribution: both increased most for low- and high-skilled labor and only weakly for medium-skilled employees. In terms of wages, high-skilled individuals benefited more than low-skilled

ones, but wages for both of these groups rose substantially more than those of the medium-skilled.

Autor and Dorn (2013) use the occupational mean wage in 1980 as a proxy for skill level. To measure wage growth, they calculate the difference in mean wages per occupation between 2005 and 1980. Using this approach, real wage growth was positive across all skill groups, although growth was weaker among medium-skilled workers. In contrast, studies relying on educational attainment as a proxy for skill find real wage losses for less-educated individuals over the same period (Acemoglu and Autor, 2011; Autor, 2019).

We argue that this apparent discrepancy arises from differences in skill measurement. While education may be related to skills on the supply side of the labor market, occupational mean wages are an equilibrium outcome shaped by the allocation of skills across occupations. This allocation may change over time, as it is the case in our application of the task-complexity model below. Accordingly, we interpret the findings of Autor and Dorn (2013) as referring wage and employment growth across occupations of varying complexity.

Autor and Dorn (2013) conclude that the observed increase in wages and employment for workers in—using our terminology—high- and low-complexity occupations was driven by demand-side factors. Specifically, they argue that service occupations at the lower end of the task-complexity distribution are comparably costly to automate as complex, non-routine tasks performed by high-skilled labor. In contrast, workers in medium-complexity occupations involving routine tasks were more easily substituted by information technology.

Broad empirical evidence qualitatively confirms the trend of employment polarization in developed countries, for instance Autor et al. (2006) for the United States, Spitz-Oener (2006) and Dustmann et al. (2009) for Germany, and Goos et al. (2009) for 16 European countries. However, the quantitative findings are heterogeneous, suggesting that technological change alone cannot fully account for these patterns. The same holds for wage polarization: Antonczyk et al. (2018) find that rising skill premia played a more prominent role in the United States than in Germany.

4.2 Replicating Trends in the Model

We now illustrate how empirical trends in wages and employment can be explained within the task-complexity model, where computerization is represented by a polarization of the task-complexity distribution. Cross-country differences in wage and

employment distributions are consistent with our model particularly if the skill level distribution varies over countries.

In what follows, we use a simple calibration of the task-complexity model to simulate labor market conditions before and after computerization. For the pre-computerization state, we assume five industries $i \in \mathcal{I} = \{1, 2, 3, 4, 5\}$ with associated task complexities $\tau_1 = 0.1$, $\tau_2 = 0.35$, $\tau_3 = 0.475$, $\tau_4 = 0.6$, and $\tau_5 = 0.85$. Dividing the complexity space into three equally-spaced segments, we classify tasks between $\tau = \frac{1}{3}$ and $\tau = \frac{2}{3}$ as medium-complexity, and those outside this range as low- and high-complexity tasks, respectively. All other assumptions from the previous section remain unchanged.

Given this calibration, the labor market is integrated (see Figure 4a). As more skilled workers allocate to more complex tasks (occupations) and wages are increasing in the skill level, the occupational mean wage is informative about the average skill level in a task, as proposed by Autor and Dorn (2013).

Following Autor and Dorn (2013), we assume that tasks performed by medium-skilled labor are automated in the computerization phase. In our framework, this leads to a polarization of the task-complexity distribution. Specifically, as shown in Figure 4b, we assume that τ_2 shifts closer to τ_1 , whereas τ_4 approaches τ_5 , and τ_3 remains unchanged. As a result, there is only one medium-complexity task left. Importantly, the number of industries remains constant with this setup, which eliminates an additional demand-side effect on the goods market through the consumers' CES preferences. The labor market is now disintegrated. Remarkably, shifting only two task complexities is sufficient to replicate all four stylized facts observed by Autor and Dorn (2013):

First, as shown in Figure 4c, employment in low-complexity tasks increases as relative demand for medium-complexity tasks declines. Since the labor market remains locally integrated across low- and medium-complexity tasks, some workers previously employed in medium-complexity occupations can be reallocated to low-complexity tasks. This reallocation effect has also been discussed by Goos and Manning (2007).

Second, as depicted in Figure 4d, real wages in low-complexity tasks rise substantially more than wages in medium-complexity tasks. This pattern arises from the higher productivity of medium-skilled labor now employed in low-complexity tasks, which raises the average wage in these occupations. Autor and Dorn (2013) assume complementarity between goods and services in consumption to explain this observation.

Third, employment in high-complexity tasks increases. As industries concentrate at the upper end of the task-complexity distribution, high-skilled labor becomes scarce, and industries 4 and 5 employ all workers with sufficiently high skill levels. Prior to

computerization, some individuals with skill levels $r \geq 0.85$ were employed in medium-complexity tasks.

Fourth, wages in high-complexity tasks increase due to the scarcity of high-skilled labor and the resulting complexity premium. As in the data shown by Autor and Dorn (2013), this wage growth exceeds that observed in low-complexity tasks.

It is worth noting why wages also increase in the remaining medium-complexity task (task 3, see Figure 4d). Labor demand from firms 1 and 2 is too large to be satisfied by workers with $r < \tau_3$ alone. Therefore, the least-skilled workers with $r \geq \tau_3$ are still hired for low-complexity tasks (by Assumption 2). As a result, the workers remaining in task 3 exhibit higher average productivity than before computerization, leading to higher wages.

In summary, our model can simultaneously explain four stylized facts through an adjustment in task complexities only. In addition, it can explain why empirical results depend on the variable that is used to measure skill. As discussed, Autor and Dorn (2013) approximate the skill level by the occupational mean wage in the base year 1980. They implicitly assume that the allocation of skills to occupations remains constant to compute the U-shaped growth patterns of wages and employment across skill levels from 1980 to 2005. In contrast, Acemoglu and Autor (2011) and Autor (2019) use educational attainment as a proxy for skill and observe negative—rather than merely lower—wage growth for low- and medium-skilled labor.

In our model, both results can be reconciled if the allocation of skills to occupations is endogenous. As shown in Figures 4d and 4e, wage growth is positive across firms regardless of task complexity, but negative for all workers with skill levels too low to qualify for high-complexity tasks after computerization. This occurs because all low- and medium-skill workers with $r < 0.85$ experience constant nominal wages before and after computerization, while the aggregate price level rises due to higher wages paid in tasks 4 and 5. Importantly, these falling real wages for low- and medium-skilled workers do not contradict rising wages in low- and medium-complexity *tasks*, which result from the reallocation of more productive labor into these tasks.

Finally, our simulations help explain rising wage dispersion within the group of high-skilled labor, another phenomenon documented in the literature (Lemieux, 2006; Antonczyk et al., 2018). In our model, wage inequality among workers with skill levels above $r = 0.85$ increases because the scaling factor for this group rises, steepening the slope of the wage curve and amplifying differences in earnings. Simultaneously, the model predicts greater wage dispersion within low-complexity tasks, as more skilled

Figure 4: Replication of Stylized Facts Before and After Computerization.



workers are reallocated to these occupations. This outcome is also consistent with empirical findings (Antonczyk et al., 2018).

It is important to note that Assumption 2 is required in this section not only to plot effective labor demand but also to compute employment and wage growth across complexity levels (Figures 4c and 4d). In Appendix D, we relax Assumption 2 and show that all results remain qualitatively unaltered.

5 The Present: Estimating the Task-Complexity Model on Recent Data

In the previous section, we demonstrated that shifts in task complexities can qualitatively account for observed labor market trends, specifically the rise in employment and wages in both low- and high-complexity tasks.

In this section, we propose an approach for estimating the model parameters using U.S. labor market data on occupational wages from 2001 to 2019. Since wages are influenced by various factors beyond the scope of our model, our primary objective is not to precisely replicate the observed wage distribution across all occupations. Instead, we apply the model to uncover underlying labor market trends related to the relationship between skill and productivity, the distribution of skill levels in the population, and the task-complexity distribution on the firms' side. The strength of our framework lies in its ability to structurally disentangle these distinct labor demand and supply effects.

In the remainder of this section, we begin by presenting the data underlying our estimation. Next, we investigate the relation of skill levels and wages and derive stylized facts based on our data in a descriptive analysis. Finally, we estimate the model parameters and present the results in detail.

5.1 Data and Measure of Skill

To obtain data on skill levels and related wages, we draw on the O*NET database, version 28.3, (National Center for O*NET Development, 2024) and the Occupational Employment and Wage Statistics (Bureau of Labor Statistics, 2024).

The O*NET database provides a range of indicators suitable as complexity measures for over 700 occupations. We focus on variables from the “Work Activities” sub-database and construct a single task-complexity measure by aggregating two scales for non-routine cognitive activities proposed by Acemoglu and Autor (2011). These scales have

been shown to correlate positively with higher educational attainment and employment in high-wage occupations. Details on the construction of the task-complexity variable are provided in Appendix E. We treat the resulting measure as cardinal and normalize its domain to $[0, 1]$.

The indicators on which the task-complexity variable is based were observed only once per occupation and in different years between 2004 and 2022. We assume them to be stable over the whole period from 2001 to 2019. This is common practice in the literature but may not be accurate if the within-occupation composition of tasks changes over time, as discussed in Autor (2013). While occupational task complexities are time-invariant in our data, we can still estimate movements in the task-complexity distribution by leveraging variation in observed wages, which reflect skill and complexity premia and thereby allow us to infer the scarcity of the corresponding skill levels.

We retrieve occupational wage data from the Occupational Employment and Wage Statistics (OEWS). The occupation-specific version of this database covers the period from 1997 to 2023. A major revision to the occupational classification system occurred in 1999. To reduce sampling errors, the U.S. Bureau of Labor Statistics aggregates data from three consecutive years for each annual dataset. As a result, the first year with a complete sample is 2001 (Bureau of Labor Statistics, 2001), which marks the beginning of our estimation period.

After merging OEWS and O*NET data via occupation codes and retaining only occupations observed in both datasets, we obtain a sample of 639 unique occupations. Of these, we observe an average of 571 occupations per year, with a rising trend over time: the number of observed occupations ranges from 543 in 2003 to 617 in 2019.

We do not observe the distribution of skill levels within occupations, and therefore cannot directly link a specific skill level to the average wage paid in an occupation. However, we do observe occupational task complexities in the O*NET database. We exploit this information to infer a relationship between skill levels and wages. We assume that the least-skilled worker in an occupation possesses a skill level that just matches the occupational task complexity. This worker should be expected to earn the lowest wage among all workers in the occupation. Accordingly, we interpret the lowest wage paid in an occupation as corresponding to a skill level equal to the occupational complexity. Note that we need to impose Assumption 1 for occupations and not only for tasks to make our argument.

Unfortunately, the OEWS does not report the lowest occupational wage but only the 10th-percentile wage. We proceed by treating this wage as described above, linking it

to the skill level equal to the occupational complexity reported in O*NET. Although skipping nine percentiles may introduce some inaccuracy, it can also be interpreted as excluding outliers potentially driven by measurement error or factors outside the scope of our model. Furthermore, robustness checks presented in Appendix F indicate that the estimation results remain qualitatively unchanged even when using the 25th- or 50th-percentile occupational wage instead. To remove business cycle effects from each occupation’s wage series, we apply an HP filter with a smoothing parameter of $\lambda = 6.25$, as recommended by Ravn and Uhlig (2002) for annual data.

As discussed in the previous section, alternative skill measures proposed in the literature include wages in a base year (Autor and Dorn, 2013; Acemoglu and Restrepo, 2022) and educational degrees (Lemieux, 2006; Acemoglu and Autor, 2011; Antonczyk et al., 2018). For our model, these measures are inadequate, as we require a cardinal skill-level measure to locate observations in the skill-level space \mathcal{R} . Educational degrees are inherently ordinal, while base-year wages—though cardinal—translate into an ordinal skill-level measure unless the equilibrium wage scheme \mathcal{W}^* is known. We therefore rely on O*NET/OEWS data to construct our skill-level variable. In the following descriptive analysis, we show that this measure is positively correlated with observed wages.

5.2 Descriptive Analysis

Relation between Skill Level and Wages

Our model predicts that wages will increase in the skill level, with the steepness of the increase depending on labor market integration or disintegration and the effect of skill on productivity. A scatter plot showing skill levels and wages in our data—or, equivalently, occupational 10th-percentile wages and task complexities—in the first year of our estimation period (2001) is presented in Figure 5.

We observe a positive and potentially more-than-linear relationship between wages and skill levels. When each observation is weighted by the number of employees, the correlation coefficient is 0.66, suggesting a strong positive association between our skill-level variable and wages. A linear regression, again weighted by employee shares, yields a slope coefficient of 3.60; the resulting regression line is included in Figure 5. By 2019, both the correlation coefficient and the regression slope increase to 0.68 and 4.49, respectively, following a dip before 2010 that may reflect disturbances caused by the Great Recession (see Figure 6).

Figure 5: Hourly Wages and Skill Levels (or, Equivalently, Hourly 10th-Percentile Occupational Wages and Task Complexities) in 2001 and Fitted Values, Computed by Weighting each Occupation by the Number of Employees. Sources: National Center for O*NET Development (2024) and Bureau of Labor Statistics (2024).

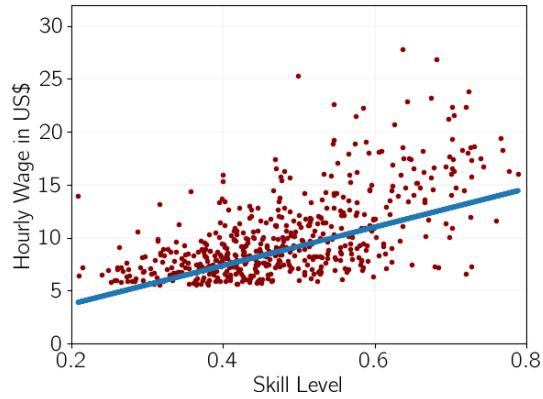
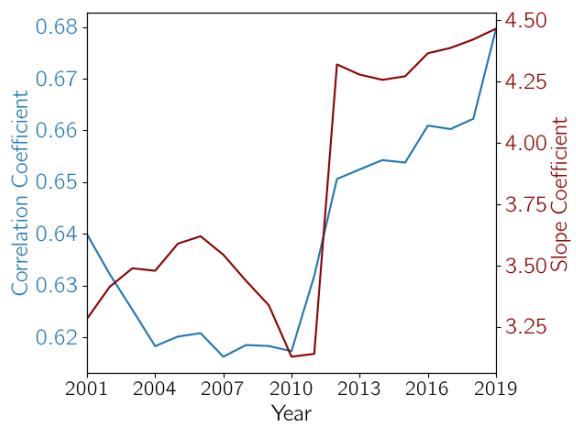


Figure 6: Correlation Coefficients and Linear Regression Slope Coefficients for Skill Levels and Wages (or Occupational Task Complexities and 10th-Percentile Wages). Observations are Weighted by the Number of Employees.



Stylized Facts in the Joint O*NET/OEWS Dataset

Next, we identify trends in wages and employment using the joint O*NET/OEWS dataset. Recall that Autor and Dorn (2013) measured skill as the mean wage per occupation in the base year of their observation period, 1980. Between 1980 and 2005, they documented a U-shaped pattern of wage and employment growth over this skill-level proxy. We have shown that the allocation of skills across occupations may vary over time, implying that the base-year mean wage is better interpreted as a proxy for occupational complexity rather than for skill level.

As described above, we can establish a direct relationship between skill and wages in our data. Changes in this relationship between 2001 and 2019 are depicted in Figure 7a. Observations are weighted by the number of employees per occupation, both for determining the size of each occupation’s marker and for computing a degree-3 spline. Nominal wages are deflated using consumption price data from U.S. Bureau of Economic Analysis (2025), ensuring comparability over time. We observe real wage gains across nearly all skill levels, with the largest increases occurring at skill levels around 0.3 and between 0.6 and 0.7. At the very highest skill levels, the spline indicates lower wage growth, though this appears to be driven by a small number of occupations with relatively few employees.

While we can observe pairs of skill levels and wages in the data, they do not provide information on the mass of workers with a respective skill level. Nevertheless, we can relate changes in employment to the complexity of occupations. This relationship is depicted in Figure 7b. Similar to wages and skill levels, employment shares increased in both low- and high-complexity occupations, with the exception of the most complex occupations at the upper end of the distribution.

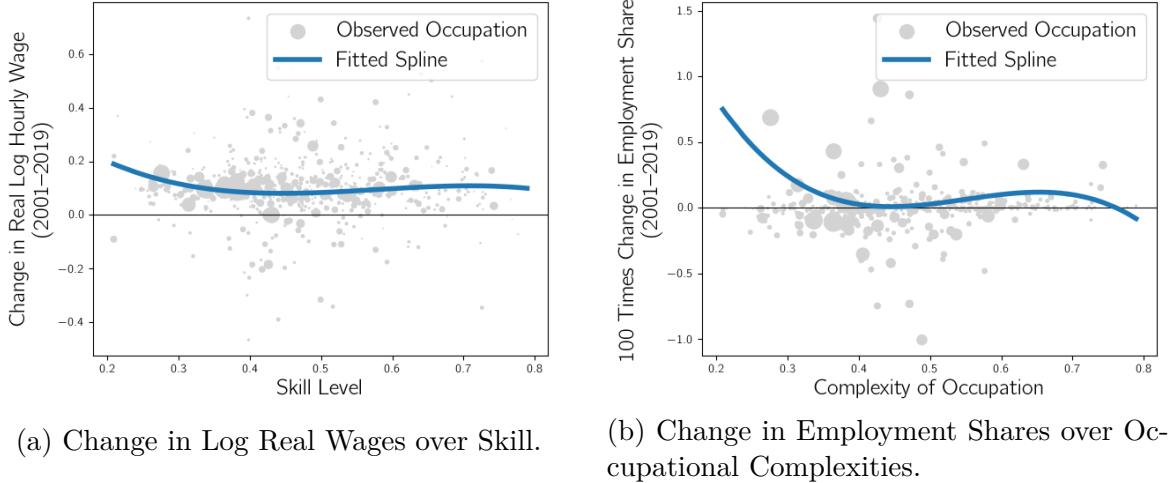
In summary, we still observe a U-shaped growth of wages and employment over the skill level and the occupational complexities. In the next section, we turn to identifying the mechanisms underlying these stylized facts.

5.3 Estimation

Procedure

We estimate the model for a 19-year period between 2001 and 2019. For each year, we estimate the productivity-skill mapping $\kappa(r)$, the skill-level distribution in the population L^r , and the task-complexity distribution T .

Figure 7: Trends in Wages and Employment over Skill and Complexity, 2001–2019.



For productivity, we assume the functional form $\kappa(r) = r^q$ and estimate the parameter q . Note that a linear transformation of productivity, such as $\kappa(r) = br^q$ with $b \neq 1$, does not affect the estimation results, as it leaves relative wages and effective labor unchanged for all r .

For the skill-level distribution L^r , we assume it follows a beta distribution with the probability density function

$$f(r) = r^{\alpha_L-1}(1-r)^{\beta_L-1},$$

and estimate the parameters α_L and β_L . This specification ensures that all workers' skill levels lie within the closed interval $[0, 1]$. Moreover, the beta distribution is flexible: it can approximate uniform or normal distributions, and it can accommodate concentrations of mass both at the center and at the margins of the skill-level space \mathcal{R} . We remain ex-ante agnostic about the shape of the skill-level distribution due to the lack of suitable alternatives. As discussed earlier, educational degrees or base-year wages translate into ordinal skill level variables only.

Finally, we assume that task complexities across industries follow a second beta distribution with the probability density function

$$h(r) = \left(\frac{r - \tau_1}{\bar{\tau} - \tau_1}\right)^{\alpha_{\tau}-1} \left(1 - \frac{r - \tau_1}{\bar{\tau} - \tau_1}\right)^{\beta_{\tau}-1} \quad \text{for } r \in [\tau_1, \bar{\tau}].$$

Here, τ_1 and $\bar{\tau}$ denote the lowest and highest occupational task complexities observed in our dataset. We assume ten tasks and compute $\tau_2, \tau_3, \dots, \tau_{10}$ as the 10th, 20th, \dots , and 90th percentile of the distribution. This approach ensures that all industries in our

model are relatable to some observed occupation in the data and vice versa. With ten industries, we allow for a wide range of locations for possible labor-market breaks in the task-complexity space. In a robustness exercise, we vary the number of tasks from 8 to 15 without significantly affecting the estimation results (see Appendix F). Note that increasing the number of industries arbitrarily is infeasible, as the number of possible labor-market arrangements $2^{|\mathcal{I}|-1}$ and thereby code runtimes grow exponentially with $|\mathcal{I}|$.

As in the case of the skill-level distribution in the population, we lack alternatives to being agnostic about the distribution of industries in the complexity space. One alternative would be to treat occupations or groups of occupations listed in the O*NET database as tasks. As the number of employees per occupation is recorded in the OEWS, this could serve as a proxy for the number of firms hiring workers for each task. However, firm and industry sizes may vary significantly across the complexity space, depending on labor supply and the complexity of further tasks. Thus, the number of employees per occupation is uninformative about the distribution of industries. A second alternative would be to exploit another dataset that directly includes firms or industries. Yet, no dataset exists that links firms or industries to occupation codes and, by extension, to skill levels. For instance, the Statistics of U.S. Businesses (United States Census Bureau, 2021) provides data on industries but not on occupations.

We estimate all of the aforementioned parameters on grids since the potential discontinuity of the wage function in our model poses a challenge for gradient-descent-based optimizers. For q in $\kappa(r) = r^q$, we select 21 evenly spaced grid points between $10e^{-5}$ (chosen to be close to but strictly greater zero, as we assume $\kappa' > 0$) and 2. For all beta distribution parameters, we construct grids with evenly spaced values between 4 and 20 in steps of 2. To improve coverage near unity, where small changes can significantly affect the shape of the beta distribution, we add the values 1, 1.5, 2, and 3 to each grid. This results in 13 grid points per beta parameter. In total, we evaluate 599,781 parameter combinations.

For each year, we further drop the subset of grid points for α_L and β_L that cannot be aligned with the employment data reported in the OEWS. Let τ_k^{occ} denote the complexity of occupation k in the O*NET data. Following Assumption 1, we compare the share of the working population employed in occupations above some complexity τ_k^{occ} in the data to $1 - F(\tau_k^{\text{occ}})$, where $F(r)$ denotes the cumulative distribution function of L^r according to the estimated beta distribution. If the computed share $1 - F(\tau_k^{\text{occ}})$ is smaller than the observed one, we discard the respective grid points as the supply of skilled labor in the model is too low to be aligned with observed employment in the

data. Conversely, $1 - F(\tau_k^{\text{occ}})$ can never be too large to be in line with the data as higher-skilled labor can always work in less complex occupations.

The only parameter we calibrate externally is the elasticity of substitution, σ . For each year, it is chosen such that the firms' markups $\frac{\sigma}{\sigma-1}$ matches the average markup estimated by De Loecker et al. (2020) for the period 2001–2016. To remove business cycle effects, we compute the HP-filtered trend of the markup series, again with the smoothing parameter $\lambda = 6.25$. We extend the time series beyond 2016 by assuming that σ remains constant thereafter.

Finally, for each year and each grid point, we compute the mean squared error (MSE) between observed wages and model-predicted wages at the corresponding skill levels. Each observation is weighted by the number of employees in the respective occupation. We select the parameter combination that minimizes the weighted MSE as the final estimate.

Results

The yearly parameter estimates are reported in Table 1, which also includes the means, modes, and standard deviations of the distributions $f(r)$ and $h(r)$. Three key trends emerge from the data.

First, the mean skill level in the population has increased substantially over time, rising from 0.54 in 2001 to 0.70 in 2019, while the standard deviation of the skill-level distribution has remained relatively stable. This suggests that skill levels have improved across all groups in the population.

Second, the mean of the task-complexity distribution has increased from 0.60 in 2001 to 0.69 in 2019 whereas its standard deviation has almost halved from 0.09 to 0.05. Effectively, this means that tasks have become more complex on average and are also more concentrated in the high-complexity region.

Third, productivity, modeled as $\kappa(r) = r^q$, has become significantly more sensitive to skill over time, with q rising from 0.2 in 2001 to 0.7 in 2019. Accordingly, even in the absence of labor market disintegration, wages have become more dependent on skill in later years.

To illustrate these findings, we present the estimated wage schemes for 2001 and 2019 alongside observed wages and skill levels in Figure 8, and the corresponding estimated skill-level and task-complexity distributions in Figure 9. Wages in Figure 8 are expressed in real US dollars, adjusted to 2001 prices.

Table 1: Estimation Results for the Skill-Level Distribution, Task-Complexity Distribution, and Productivity.

Year	Parameters					Distribution Moments					
	α_L	β_L	α_τ	β_τ	q	μ^L	mode^L	σ^L	μ^τ	mode^τ	σ^τ
2001	14	12.00	6	3	0.20	0.54	0.54	0.10	0.60	0.62	0.09
2002	14	12.00	6	3	0.20	0.54	0.54	0.10	0.60	0.62	0.09
2003	14	12.00	6	3	0.20	0.54	0.54	0.10	0.60	0.62	0.09
2004	14	12.00	6	3	0.20	0.54	0.54	0.10	0.60	0.62	0.09
2005	14	12.00	6	3	0.20	0.54	0.54	0.10	0.60	0.62	0.09
2006	14	12.00	6	3	0.30	0.54	0.54	0.10	0.60	0.62	0.09
2007	8	4.00	14	3	0.60	0.67	0.70	0.13	0.69	0.71	0.05
2008	8	4.00	14	3	0.60	0.67	0.70	0.13	0.69	0.71	0.05
2009	12	6.00	14	3	0.60	0.67	0.69	0.11	0.69	0.71	0.05
2010	8	4.00	14	3	0.50	0.67	0.70	0.13	0.69	0.71	0.05
2011	8	4.00	14	3	0.50	0.67	0.70	0.13	0.69	0.71	0.05
2012	8	4.00	14	3	0.50	0.67	0.70	0.13	0.69	0.71	0.05
2013	8	4.00	14	3	0.50	0.67	0.70	0.13	0.69	0.71	0.05
2014	8	4.00	14	3	0.50	0.67	0.70	0.13	0.69	0.71	0.05
2015	12	6.00	14	3	0.50	0.67	0.69	0.11	0.69	0.71	0.05
2016	12	6.00	14	3	0.50	0.67	0.69	0.11	0.69	0.71	0.05
2017	12	6.00	14	3	0.50	0.67	0.69	0.11	0.69	0.71	0.05
2018	4	1.50	14	3	0.60	0.73	0.86	0.17	0.69	0.71	0.05
2019	14	6.00	14	3	0.70	0.70	0.72	0.10	0.69	0.71	0.05

Figure 8: Estimated and Observed Wages in 2001 and 2019.

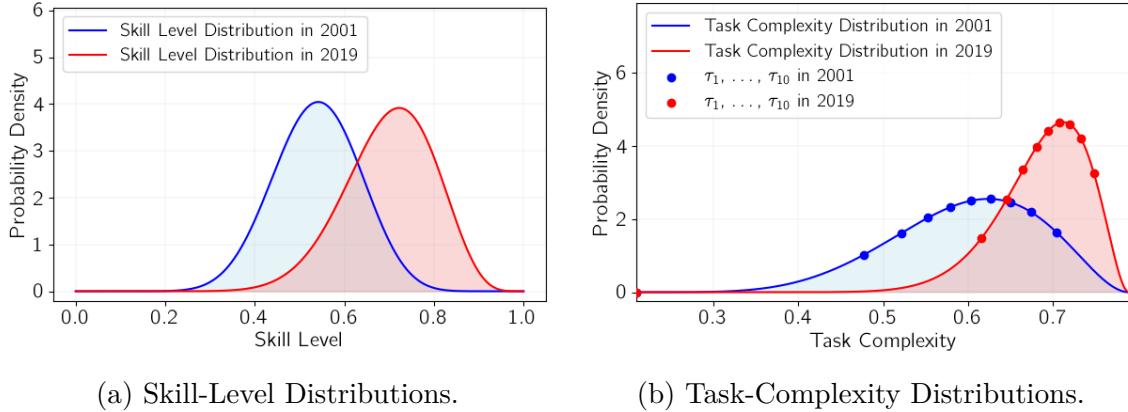


In 2001, labor market breaks were present for all tasks except $i = 10$ (Figure 8a), reflecting the scarcity of medium- and high-skilled labor (Figure 9a). By 2019, all task complexities except τ_1 had increased (Figure 9b), while high-skilled labor supply soared, too (Figure 9a). Although this increase in supply was not sufficient to prevent a large scaling factor jump for ω_2 (Figure 8b), it ensured that this remained the only estimated labor-market break.

Despite the lower number of jumps in the wage function in 2019, high-skilled workers earned more relative to low-skilled workers than in 2001: The ratio of wages estimated for the highest and lowest observed skill levels increases from 2.28 to 3.14. This trend cannot be explained by the complexity premium. In fact, the complexity premium associated with the highest observed skill level ($r = 0.79$) decreased from a share of 43 percent of the wage in 2001 to 19 percent in 2019. Instead, growing wage inequality is primarily driven by the significant increase in the q parameter, i.e., the rising convexity of $\kappa(r) = r^q$. At the same time, our results underscore that complexity premia consistently accounted for a substantial share of wages throughout the sample period and play a crucial role in explaining wage differences across skill levels.

Figures illustrating wage schemes for all years are provided in Appendix E. In Appendix F, we demonstrate that our results are robust to changes in the assumed number of industries and the percentile of the wage distribution used to link wages with task complexities and skills.

Figure 9: Estimated Skill-Level and Task-Complexity Distributions in 2001 and 2019.



Interpretation

Based on our estimates, we identify the key drivers behind the stylized facts discussed in the descriptive analysis section, namely a U-shaped growth of wages across skill levels and a U-shaped growth of employment across occupational complexities. The estimation results suggest that wages at the bottom and top of the skill level distribution rose disproportionately as medium-skill workers suffered from a shift of the first labor market break to higher skill levels, eliminating their complexity premia. Wages for high-skilled workers additionally benefited from the increased convexity of productivity with respect to skill. The general shift of task complexities to higher levels explains the rise in employment in low-complexity occupations: workers previously employed in medium-complexity occupations were reallocated to less complex occupations (this is the same mechanism as described in Section 4). Employment in high-complexity occupations increased because of greater demand for highly complex tasks, accompanied by an increase in skill levels within the labor force.

How can these results be explained intuitively? The estimates suggest that firms have not reduced the complexity of their tasks (see Figure 9b), despite the strong incentive to do so created by the high complexity premia at the beginning of the sample period. This finding speaks against the hypothesis of directed technical change on the firm side—or at least suggests that such change has not been strong enough to offset the trend toward higher complexity. Nevertheless, the increasing convexity of $\kappa(r) = r^q$ could reflect rising productivity among high-skilled workers due to complementarities between high-skilled labor and information technology, as discussed in Acemoglu and Autor (2011) and Autor and Dorn (2013). Recall that increasing labor productivity raises the supply of *effective* labor even if the number of workers stays constant. Equipping high-skilled

workers with information technology could therefore be interpreted as a strategy on the firm side to reduce complexity premia without lowering task complexities. While firms may be incentivized to reduce complexity premia, workers can be expected to increase their skill level in response to high wage premia, whether driven by complexity or skill premia. This hypothesis aligns with our results in Figure 9a.

6 The Future: Artificial Intelligence in the Task-Complexity Model

After applying the task-complexity model to replicate stylized facts observed between 1980 and 2005 and to identify labor market trends from 2001 until 2019, this section explores potential future effects of artificial intelligence (AI) within the framework. While our analysis of computerization since 1980 in Section 4 focused on shifts in the task-complexity distribution on the firm side—that is, the demand side of the labor market—we think of AI as competing with workers on the supply side.

6.1 The AI-Augmented Task-Complexity Model

We assume that AI is able to perform tasks $i \in [\underline{i}^a, \overline{i}^a]$ as a perfect substitute for labor. The lower bound \underline{i}^a reflects the idea that AI is not suitable for replacing workers in low-complexity, often manual tasks, but may be capable of performing medium- or even high-complexity clerical work.

AI's productivity is normalized to one. We assume an infinite supply of AI at a rental rate (or AI "wage") denoted by w^a . The demand of representative firm ij for AI is denoted by A_{ij} . The profit maximization problem of a firm ij in industry $i \in [\underline{i}^a, \overline{i}^a]$ is given by:

$$\begin{aligned}
& \max_{\mathcal{R}_{ij}, p_{ij}, A_{ij}, \{l_{ij}(r)\}_{r \in \mathcal{R}_{ij}}} p_{ij} x_{ij} - \int_{r \in \mathcal{R}_{ij}} w^r l_{ij}(r) dr - w^a A_{ij} \\
& \text{s.t.} \\
& x_{ij} = \int_{r \in \mathcal{R}_{ij}} \kappa(r) l_{ij}(r) dr + A_{ij}, \\
& x_{ij} = x_{ij}^d = \left(\frac{p_{ij}}{p_i} \right)^{-\sigma} \left(\frac{p_i}{P} \right)^{-\sigma} C, \\
& r \geq \tau_{ij} \quad \forall r \in \mathcal{R}_{ij}, \\
& \mathcal{R}_{ij} \subseteq \mathcal{R}.
\end{aligned}$$

As firms in each industry i remain symmetric and each industry comprises a unit mass of firms, we may continue to consider one representative firm per industry i . Since AI's productivity is normalized to one, the cost of producing a marginal unit of output with AI is w^a , compared to $\frac{w^r}{\kappa(r)}$ when human labor is employed, where $\frac{w^r}{\kappa(r)} = \omega_i \quad \forall r \in \mathcal{R}_i$. Based on this comparison, we can distinguish three cases regarding the use of AI in industries $i \in [\underline{i}^a, \bar{i}^a]$:

1. If $w^a > \omega_i$, the representative firm does not rent AI, and we find $A_i = 0$.
2. If $w^a < \omega_i$, the firm does not employ any workers, i.e., $\tilde{l}_i = 0$.
3. If $w^a = \omega_i$, the firm is indifferent between hiring labor and renting AI. We assume that firms hire all available labor in this case and satisfy any additional consumer demand by renting AI.

The profit maximization problem for firms in industries $k \notin [\underline{i}^a, \bar{i}^a]$ remains as defined in (9), and we impose $A_k = 0$. A full formal derivation of the AI-augmented model and its solution is provided in Appendix G.

6.2 Two Scenarios for the Transition to AI

In the following, we present a baseline labor market setting and then introduce AI to examine its effects on wages and employment under two scenarios. In both scenarios, we fix the AI rental rate at $w^a = 0.5$, which is lower than the smallest scaling factor $\omega_1 = 1$ observed in any labor market without AI. This implies that AI will always reduce wages in the tasks it is able to perform.

We assume a task-complexity distribution $T = \{0.1, 0.5, 0.8, 0.95\}$, which enables a detailed analysis of AI's impact on complexity premia in detail. We distinguish between a setting in which AI can versus a setting in which AI cannot perform the least complex task, imposing $\underline{i}^a = 1$ or $\underline{i}^a = 2$, respectively. In both scenarios, the most complex task for which AI can be used is $\bar{i}^a = 2$. All remaining parameters are calibrated as described in Section 3.

Before AI is introduced into the model economy, the labor market is locally integrated across industries 1 and 2 and disintegrated for all further tasks (Figure 10a).

In the scenario in which AI can be rented for the least complex two tasks (Figure 10b), we observe that wages in these tasks decline from $\kappa(r)$ to $w^a \kappa(r)$. Due to the sufficient skill condition, workers $r < \tau_3$ are trapped in their industries and cannot avoid this drop in nominal wages. Notably, wages in industries 3 and 4 also decline, as complexity premia vanish entirely in industry 3 and decrease in industry 4. The Gini coefficient of the wage distribution increases from 0.40 to 0.51, indicating a rise in wage inequality driven by the drop in wages for tasks 1 and 2.

If AI cannot perform task 1 but can perform task 2 only (Figure 10c), wages for workers in task 1 still equal their productivity $\kappa(r)$. Therefore, all workers $r < \tau_3$ are attracted to industry 1, and no human labor is employed in task 2 (consequently, no wages are depicted for this task). AI now has the opposite effect on complexity premia in tasks 3 and 4, as they increase relative to the pre-AI scenario. The Gini coefficient of the wage distribution is 0.46 and thus in between its levels in the pre-AI scenario and the scenario in which AI is used in both task 1 and 2.

Why do complexity premia drop in the first case and rise in the latter relative to the pre-AI scenario? *Ceteris paribus*, labor demand in an exclusively human industry decreases if wages decrease in a task that can be performed by AI (see equation G.1 in the Appendix), leading to lower equilibrium complexity premia. The underlying mechanism is downward price pressure on the goods market from tasks that are (partially) performed by AI, which explains the differing complexity premia observed across the two AI scenarios.

This channel does not, however, explain why complexity premia in the second AI scenario are higher even compared to the pre-AI scenario. This is due to the large supply of final good 1, resulting from the increased number of workers hired by industry 1 relative to the pre-AI scenario. On the goods market, this higher supply can only match demand if the relative price of good 1 decreases (see Equation 4). As the price of good 1 is fixed—industry 1 pays workers according to their productivity $\omega_1 = 1$

and $p_1 = \frac{\sigma}{\sigma-1} \omega_1$ —the prices of other goods must rise to establish equilibrium relative prices. Given constant mark-ups, this requires an increase in ω_3 and ω_4 . Indeed, we observe that the aggregate price level P is 39 percent higher in the second AI scenario than in the first.

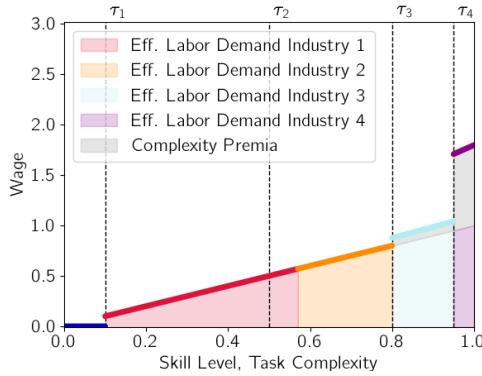
In the context of price levels, we shall discuss the effects of AI on real wages within our model. Note that the price level declines in both scenarios, but more significantly in the setting where AI can perform two tasks, as described in the previous paragraph. When AI can be used in the two least complex tasks, the reduction in the price level is insufficient to prevent declining real wages for workers trapped in these two tasks. The opposite holds for workers in tasks 3 and 4 that cannot be performed by AI—for them, the decreasing price level outweighs the reduction in complexity premia, resulting in rising real wages (Figure 10d). In the scenario where AI is limited to task 2, no worker experiences a decline in nominal wages (Figure 10e). Although the price level in this case remains higher than in the other AI scenario, it is still lower than before AI’s introduction, leading to real wage growth across all skill levels—most pronounced in industries not directly impacted by AI.

Note, however, that even when workers can switch to low-complexity, exclusively human tasks, real wage growth may not be guaranteed for all workers. We can construct an example with real wage losses for workers $r < 0.8$ by setting w^a sufficiently high—still assuming that AI can only perform task 2 and keeping all other parameters constant. In this case, the general price level rises following the introduction of AI, as complexity premia in industries 3 and 4—and the associated goods prices—increase, while the drop in the price of final good 2 is insufficient to outweigh this effect. Strikingly, not only reallocated workers who were previously employed in industry 2, but also those who have worked in industry 1 before, are negatively affected. Real wage growth across skill levels for a calibration with $w^a = 0.9$ is illustrated in Figure 11.

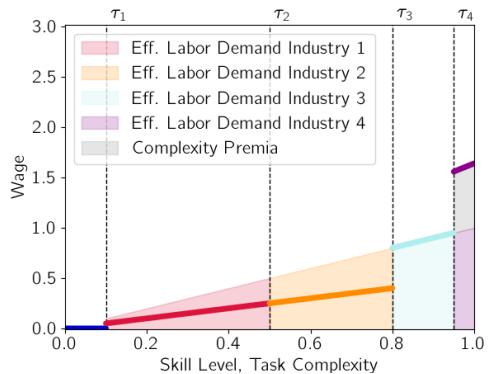
6.3 Discussion

In the task-complexity model, AI lowers wages in tasks for which it is rented. This may have no effect on workers’ wages if they can switch to less complex tasks that cannot be performed by AI. Furthermore, the use of AI might decrease or increase complexity premia in tasks in which it cannot be used. This ambiguous effect arises from opposing price effects on the goods market. Across all scenarios, we observe that AI increases wage inequality relative to the pre-AI baseline scenario, and it may even trigger a drop in real wages for workers in low-complexity industries.

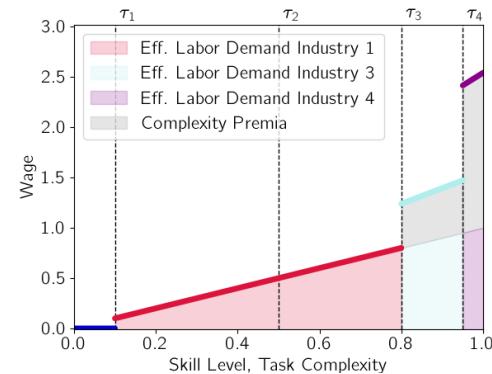
Figure 10: The Effect of AI on Wages and Labor Demand in 2 Different Scenarios.



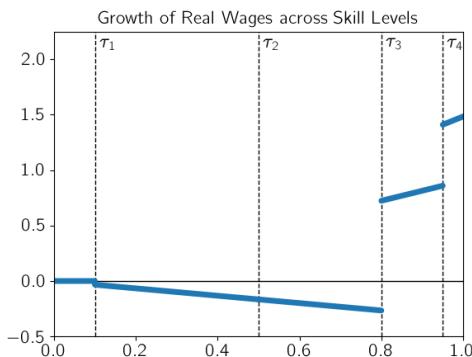
(a) Pre-AI Scenario.



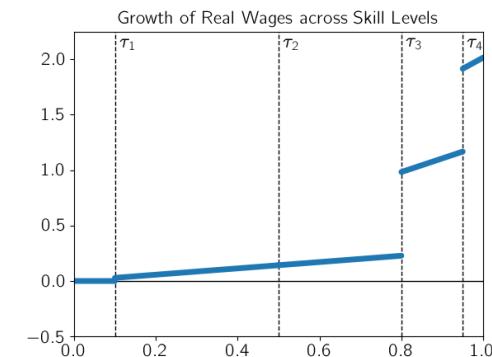
(b) AI Can Perform 2 Least Complex Tasks.



(c) AI Can Only Perform Task 2.

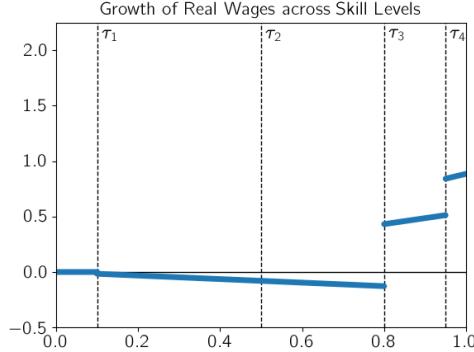


(d) Real Wage Growth across Skill Levels Relative to Pre-AI Scenario if AI Can Perform 2 Least Complex Tasks.



(e) Real Wage Growth across Skill Levels Relative to Pre-AI Scenario if AI Can Only Perform Task 2.

Figure 11: Real Wage Growth across Skill Levels Relative to Pre-AI Scenario if AI Can Only Perform Task 2, $w^a = 0.9$.



How do these predictions compare to findings in the existing literature? Our approach to modeling complexity has been extended by Korinek and Suh (2024) in the version of their model with heterogeneous worker skills, with the key difference that skilled labor is never scarce in their framework. As a result, labor in tasks $i \notin [\underline{i}_a, \bar{i}^a]$ —those not directly affected by AI—is always paid according to its marginal product, implying that AI does not influence wages for these workers. In contrast, the task-complexity model allows for unequal complexity premia across tasks that are not automated by AI, and this inequality may be either dampened or amplified if AI is used in other tasks.

In contrast to our model and that of Korinek and Suh (2024), Acemoglu (2025) assumes that both low- and high-skilled labor can perform any task, with differences arising only in productivity. Thus, as in Korinek and Suh (2024), labor is never scarce, and there are no complexity premia. In such a framework, all labor reallocates to tasks that AI cannot perform, and it is a priori unclear whether wages will grow or shrink due to AI-based automation. Wage inequality is affected by AI only if it modifies the relative productivity of skill groups through complementarities—a mechanism that is absent in our model.

Finally, how realistic are the presented scenarios? Experiments by Shojaee et al. (2025) demonstrate that AI’s productivity drops to zero once the complexity of the task it is asked to solve exceeds a certain threshold. Eloundou et al. (2024) show that many, though not all, tasks included in the O*NET database may be (partially) performed by AI. We interpret both findings as evidence for $\bar{i}^a < \bar{i}$. At the same time, Felten et al. (2021) find that tasks AI cannot perform are especially prevalent in “blue-collar industries that involve manual labor”, supporting $\underline{i}_a > 1$. In summary, we expect a

scenario as depicted in Figure 10c to be the most realistic. According to our model, we therefore expect the reallocation of medium-skill workers to low-complexity tasks and increasing wage inequality. Real wages of reallocated workers may decrease if AI is less expensive than human labor but still relatively costly. Potentially, such a setting could be observed if AI was supplied in an insufficiently competitive environment.

7 Conclusion

We propose a new task-based model, which we call the task-complexity model. In this framework, the labor force exhibits a continuous skill-level distribution. On the demand side, there is a finite set of tasks, each associated with a distinct level of task complexity. Workers whose skill level falls below a task’s complexity cannot be hired for this task. In such a setting, high-skilled workers receive wage premia exceeding their marginal productivity if they are sufficiently scarce. We refer to these premia as *complexity premia*, and we term a labor market in which they arise a *disintegrated labor market*.

We have shown that a calibrated version of the task-complexity model is consistent with the stylized facts documented by Autor and Dorn (2013) for the period from 1980 to 2005, namely rising wages and employment in both low- and high-complexity tasks. In our model, wages at the top of the skill distribution increase because high-skilled labor is scarce and earns a complexity premium. Wages at the bottom rise as relatively productive medium-skill workers are forced to work in low-complexity tasks as a result the automation of medium-complexity tasks.

Next, we derived stylized facts for a more recent period up to 2019 and estimated model parameters to match annual data on occupational wages and skill levels. We found that labor market disintegration persisted, yet complexity premia have become less relevant, accounting for up to 43 percent of wages in 2001 and only 19 percent in 2019. Also, labor market disintegration is observed for higher skill levels only in later years, explaining relatively low wage growth for medium-skilled workers. As during the earlier automation period beginning in 1980, medium-skill workers were reallocated into low-complexity tasks. Combined with rising skill levels in the population, this yields a U-shaped pattern in employment growth across occupations ranked by task complexity.

Finally, we discussed the potential effects of AI within our framework. We assume that, per unit of output, employing AI is less costly than hiring human labor. Therefore, its availability exerts downward pressure on wages in the tasks it can perform. In our

model, the sufficient skill condition prevents workers from switching to high-complexity tasks in such a case. However, if low-complexity tasks exist that AI cannot perform—a scenario supported by recent empirical findings—workers may shift toward these tasks. Nevertheless, wage inequality can be expected to increase, and real wages may decline for low- and medium-skilled workers.

The framework provides several avenues for extension. First, we have imposed the simplifying assumption that each industry’s production involves only a single task. In an extended version of the model, one could allow for production processes—or even individual jobs—that bundle multiple tasks with varying levels of complexity. Suppose that a job’s complexity is determined by the most complex task it includes. If technology automates some tasks but not the most complex one, the job’s overall complexity would remain unchanged, while productivity and skill premia would rise. By contrast, if technology automates or simplifies the most complex task in a job’s task bundle, the job’s complexity and associated complexity premia would decline. Such a setup could generate a non-monotonic impact of technological progress on wages over time, a pattern been observed in empirical data (Acemoglu and Autor, 2011; Carneiro and Lee, 2011).

Second, one could endogenize movements in the skill-level and task-complexity distributions. On the workers’ side, we would expect responses in terms of education and migration if either skill or complexity premia are sufficiently large. On the firms’ side, we may assume investments in complexity-reducing technologies if complexity premia are large, whereas skill premia do not alter firms’ incentives. Furthermore, high complexity premia could even induce market exits of firms in high-complexity industries and thereby relax the scarcity of high-skilled labor if a fixed cost of production is added to the model.

Finally, one might extend the setting to encompass more than one dimension of task complexity along which workers’ skills can be ordered. With multiple dimensions of task complexity, matching skills to task complexities across these dimensions becomes essential for the functioning of the labor market. Work by Lise and Postel-Vinay (2020) demonstrates that both endogenous skill accumulation and multiple skill dimensions can be incorporated into a job-search model. The integration of these features into the task-complexity model is a promising direction for future research.

Appendix

A Derivation of Demand Functions

A.1 Household Demand for Final Good i

To derive the demand of a household r for final good i , we solve the household's optimization problem by setting up the Lagrangian

$$\begin{aligned}\mathcal{L} = & \left(\sum_{i \in \mathcal{I}} (c_i^r)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \\ & - \lambda \left(\sum_{i \in \mathcal{I}} p_i c_i^r - L^r w^r - \Pi^r \right) .\end{aligned}$$

We take the derivative with respect to consumption of one final good i :

$$\frac{\partial \mathcal{L}}{\partial c_i^r} = \left(\frac{c_i^r}{C^r} \right)^{-\frac{1}{\sigma}} - \lambda p_i = 0 .$$

Thus, the ratio of the marginal utilities of consumption of good c_i^r and of good c_k^r must be proportional to their respective prices

$$\frac{p_i}{p_k} = \left(\frac{c_i^r}{c_k^r} \right)^{-\frac{1}{\sigma}} . \quad (\text{A.1})$$

Using Equation (A.1) and the budget constraint, we can derive the following condition:

$$c_i^r = p_i^{-\sigma} \left(\sum_{\tilde{i} \in \mathcal{I}} p_{\tilde{i}}^{1-\sigma} \right)^{-1} (L^r w^r + \Pi^r) . \quad (\text{A.2})$$

Using the price index $P := \left(\sum_{\tilde{i} \in \mathcal{I}} p_{\tilde{i}}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ and the fact that the budget constraint must be binding, i.e., $PC^r = L^r w^r + \Pi^r$, we can rewrite the equation as household demand for good i as a function of P and C^r :

$$c_i^r = \left(\frac{p_i}{P} \right)^{-\sigma} C^r . \quad (\text{A.3})$$

A.2 Demand for Variety ij

We assume that in each industry i a final good producer operates under perfect competition. Its profit maximization problem reads as

$$\max_{x_{ij}} p_i x_i - \int_0^1 p_{ij} x_{ij} dj$$

with the corresponding first-order condition

$$p_i \frac{\partial x_i}{\partial x_{ij}} = p_{ij}.$$

Using (5), we obtain an explicit expression for $\frac{\partial x_i}{\partial x_{ij}}$ and find

$$x_{ij} = \left(\frac{p_{ij}}{p_i} \right)^{-\sigma} x_i. \quad (\text{A.4})$$

Use the market-clearing condition $c_i = x_i$ and combine (A.4) with (4) to arrive at (7).

B Proofs

B.1 Proof of Proposition 1

We prove the uniqueness of the equilibrium. Using an induction argument, we see that there are $2^{|\mathcal{I}|-1}$ possible arrangements of the labor market. For a given set of industries \mathcal{I} , there are $|\mathcal{I}| - 1$ possible locations for jumps of ω_i for $i \in \mathcal{I}$. Since at each potential location of a jump there are two possibilities (jump or no jump), there are $2^{|\mathcal{I}|-1}$ possible arrangements of the labor market. Examples:

- For $|\mathcal{I}| = 2$, there are 2 potential arrangements, $1 \times \text{ILM}$ and $1 \times \text{DLM}$.
- For $|\mathcal{I}| = 3$, there are 4 potential arrangements, $1 \times \text{ILM}$, $2 \times \text{a single separation}$, and $1 \times \text{a double separation}$.
- For $|\mathcal{I}| = 4$, there are 8 potential arrangements, $1 \times \text{ILM}$, $3 \times \text{a single separation}$, $3 \times \text{a double separation}$, and $1 \times \text{three separations}$.

Next, we use the group notation. We can rewrite the problem as a system of nonlinear equations. We denote the demand for a group g 's effective labor by \tilde{L}_g^d . Using (16), (18), and (17), we can derive $\tilde{L}_g^d = \sum_{i \in g} \tilde{l}_i$, which is

$$\begin{aligned}\tilde{L}_g^d &= \omega_g^{-\sigma} \left(\sum_{\hat{g} \in \mathcal{G}} \sum_{i \in \hat{g}} \omega_i^{1-\sigma} \right)^{-1} \sum_{\hat{g} \in \mathcal{G}} \sum_{i \in \hat{g}} \omega_i \tilde{L}_i^\Delta \\ &= \omega_g^{-\sigma} \left(\sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}}^{1-\sigma} \right)^{-1} \sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}} \sum_{i \in \hat{g}} \tilde{L}_i^\Delta.\end{aligned}$$

We denote $\tilde{L}_g^\Delta = \sum_{i \in g} \tilde{L}_i^\Delta$ (this is the effective labor supply in group g). Using this, we rewrite the demand for group g 's effective labor,

$$\tilde{L}_g^d = \omega_g^{-\sigma} \left(\sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}}^{1-\sigma} \right)^{-1} \sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}} \tilde{L}_{\hat{g}}^\Delta. \quad (\text{B.1})$$

In equilibrium, for each group the labor market must clear, i.e., $\tilde{L}_g^d = \tilde{L}_g^\Delta$. Thus,

$$1 = \frac{\omega_g^{1-\sigma}}{\sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}}^{1-\sigma}} \frac{\sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}} \tilde{L}_{\hat{g}}^\Delta}{\omega_g \tilde{L}_g^\Delta}. \quad (\text{B.2})$$

Accordingly, we need to find a solution to system (B.2) with a number of $|\mathcal{G}|$ unknowns under the constraint given in (19), i.e. $\omega_{g+1} > \omega_g \forall g \in \mathcal{G}$. Without loss of generality, we can normalize $\omega_{g_1} \equiv 1$. Furthermore, for each group g it can be shown that the partials for the demand for a group's effective labor, denoted by \tilde{L}_g^d , fulfill the following properties in the auxiliary Lemma 1:

Lemma 1

The partial derivatives with respect to scaling factors for a group's effective labor demand are

$$\frac{\partial \tilde{L}_g^d}{\partial \omega_g} < 0 \text{ for } |\mathcal{G}| \geq 2 \text{ and } \frac{\partial \tilde{L}_g^d}{\partial \omega_g} = 0 \text{ for } |\mathcal{G}| = 1, \quad (\text{B.3})$$

$$\frac{\partial \tilde{L}_g^d}{\partial \omega_{g'}} = \frac{\partial \tilde{L}_g^d}{\partial \omega_g} \frac{\tilde{L}_{g'}^\Delta}{\tilde{L}_g^\Delta} > 0 \forall g, g' \in \mathcal{G} \setminus g, \quad (\text{B.4})$$

A proof is given in Section B.2. Lemma 1 implies that groups are gross substitutes. Furthermore, (B.2) is homogeneous of degree 0 in wages, and Walras' Law holds. Thus, the system is equivalent to a demand system with fixed supply.

The proof of Proposition 1 now proceeds in three steps. First, we show that for each group structure \mathcal{G} the system described in (B.2) has a unique solution. Since there are $2^{|\mathcal{I}|-1}$ group structures, there are $2^{|\mathcal{I}|-1}$ potential solutions. Second, using (19) we show that all potential solutions are mutually exclusive, i.e., only one can qualify as an equilibrium solution for the wage scheme. Third, we show that there exists an equilibrium solution which, given Step 2, must be unique.

Step 1: Unique solution for given \mathcal{G}'

EXISTENCE: An economy with a group structure \mathcal{G}' can be interpreted as a pure exchange economy with $|\mathcal{G}'|$ goods and $|\mathcal{G}'|$ agents, where there is a demand for labor and a given labor supply. Then the aggregate excess demand can be denoted by

$$z(\vec{\omega}) = \sum_{g \in \mathcal{G}'} \left(\tilde{L}_g^d(\vec{\omega}, \omega_g \tilde{L}_g^\Delta) - \tilde{L}_g^\Delta \right) \equiv 0 ,$$

where $\vec{\omega}$ denotes the vector of scaling factors. Since preferences are described by CES utility functions, demand functions are continuous, and we can define continuous functions $s_g(\vec{\omega})$,

$$s_g(\vec{\omega}) = \frac{\omega_g + \max \{0, z_g(\vec{\omega})\}}{1 + \sum_{\hat{g} \in \mathcal{G}'} \max \{0, z_{\hat{g}}(\vec{\omega})\}} \quad \forall g \in \mathcal{G}' ,$$

where $z_g(\vec{\omega})$ is defined as $z_g(\vec{\omega}) = \tilde{L}_g^d(\vec{\omega}, \omega_g \tilde{L}_g^\Delta) - \tilde{L}_g^\Delta$. Then, by using Kakutani's fixed-point theorem (see Proposition 17.C.1 in Mas-Colell et al. (1995), applied to a sufficiently large compact and convex set in $\mathbb{R}^{|\mathcal{G}'|}$ that contains all feasible vectors $\vec{\omega}$), there exists $\vec{\omega}^*$ with $\vec{\omega}^* = s(\vec{\omega}^*)$. This implies

$$z_g(\vec{\omega}^*) = \tilde{L}_g^d(\vec{\omega}^*, \omega_g^* \tilde{L}_g^\Delta) - \tilde{L}_g^\Delta = 0 \quad \forall g \in \mathcal{G}' .$$

Thus, $\vec{\omega}^*$ is a price vector solution for the economy with a group structure \mathcal{G}' that satisfies the equilibrium condition (B.2).¹

UNIQUENESS: Partial derivatives expressed in (B.4) and (B.3) show that group labor demand are gross substitutes. Thus the solution of (B.2) for a given group structure \mathcal{G} is unique.

NUMBER OF SOLUTIONS: There are $2^{|\mathcal{I}|-1}$ ways to build a group structure, and for every particular group structure \mathcal{G}' there is a unique solution. Thus there must be $2^{|\mathcal{I}|-1}$ unique solutions.

Step 2: Mutually exclusive solution

¹If $|\mathcal{G}'| = 3$, for example, market clearing is given by

$$\begin{aligned} \tilde{L}_{g_1, g_1}^d(\vec{\omega}^*, \omega_{g_1}^* \tilde{L}_{g_1}^\Delta) + \tilde{L}_{g_1, g_2}^d(\vec{\omega}^*, \omega_{g_2}^* \tilde{L}_{g_2}^\Delta) + \tilde{L}_{g_1, g_3}^d(\vec{\omega}^*, \omega_{g_3}^* \tilde{L}_{g_3}^\Delta) - \tilde{L}_{g_1}^\Delta &= 0 , \\ \tilde{L}_{g_2, g_1}^d(\vec{\omega}^*, \omega_{g_1}^* \tilde{L}_{g_1}^\Delta) + \tilde{L}_{g_2, g_2}^d(\vec{\omega}^*, \omega_{g_2}^* \tilde{L}_{g_2}^\Delta) + \tilde{L}_{g_2, g_3}^d(\vec{\omega}^*, \omega_{g_3}^* \tilde{L}_{g_3}^\Delta) - \tilde{L}_{g_2}^\Delta &= 0 , \\ \tilde{L}_{g_3, g_1}^d(\vec{\omega}^*, \omega_{g_1}^* \tilde{L}_{g_1}^\Delta) + \tilde{L}_{g_3, g_2}^d(\vec{\omega}^*, \omega_{g_2}^* \tilde{L}_{g_2}^\Delta) + \tilde{L}_{g_3, g_3}^d(\vec{\omega}^*, \omega_{g_3}^* \tilde{L}_{g_3}^\Delta) - \tilde{L}_{g_3}^\Delta &= 0 , \end{aligned}$$

where $\tilde{L}_{g_2, g_1}^d(\vec{\omega}^*, \omega_{g_1}^* \tilde{L}_{g_1}^\Delta) := \omega_{g_2}^{* -\sigma_I} c_{g_2} \left(\sum_{g \in \mathcal{G}} \omega_g^{*1-\sigma_I} \right)^{-1} \omega_{g_1}^* \tilde{L}_{g_1}^\Delta$ denotes the demand of group g_2 from group g_1 .

Given a set of task complexities \mathcal{I} , we show that Inequality (19) implies that potential solutions are mutually exclusive.

From Proposition 2 we know that effective labor demand from a representative firm, price indices, and total wages are

$$\tilde{l}_i = \frac{\omega_i^{*-\sigma}}{\sum_{i \in \mathcal{I}} (\omega_i^*)^{1-\sigma}} \kappa(1)^{-1} TW^* \quad (\text{B.5})$$

$$p_i^* = \frac{\sigma}{\sigma-1} \omega_i^* \quad (\text{B.6})$$

$$P^* = \frac{\sigma}{\sigma-1} \left(\sum_{i \in \mathcal{I}} (\omega_i^*)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

$$P^* C^* = \frac{\sigma}{\sigma-1} TW^* .$$

Rearrange the price indices and plug them into Equation (B.5) to obtain

$$\tilde{l}_i = \frac{\sigma}{\sigma-1} \kappa(1)^{-1} \left(\frac{p_i^*}{P^*} \right)^{-\sigma} \frac{TW^*}{P^*} .$$

Substituting for total wages yields

$$\tilde{l}_i = \kappa(1)^{-1} \left(\frac{p_i^*}{P^*} \right)^{-\sigma} C^* . \quad (\text{B.7})$$

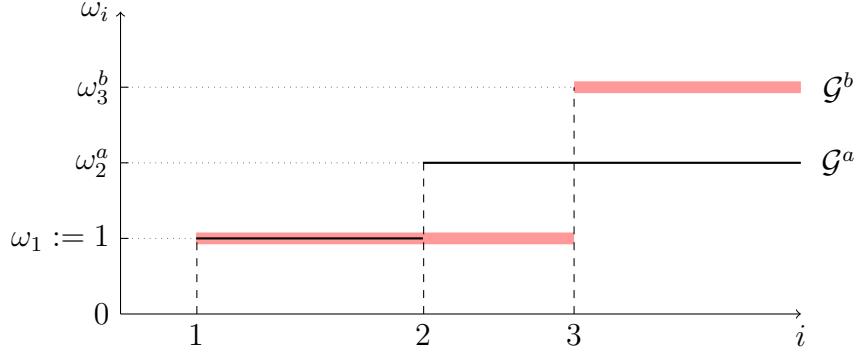
We first note that since production is linear in labor, and the labor market clears, the total amount of effective labor employed is constant, no matter how it is allocated across industries and groups. We define two group structures: $\mathcal{G}^a = \{g_1^a, g_2^a, \dots\}$, $\mathcal{G}^b = \{g_1^b, g_2^b, \dots\}$, where $\mathcal{G}^a \neq \mathcal{G}^b$.

We need to analyze two cases:

- *CROSSING*: Assume that there are two group structures— \mathcal{G}^a and \mathcal{G}^b —as depicted in Figure 12, i.e., their wage schemes cross at $i = 3$. We observe that $\omega_1^a = \omega_1^b = 1$ and that $\omega_2^a > \omega_2^b$. Then, \mathcal{G}^a implies that $\tilde{l}_1^a = \tilde{L}_1^\Delta$, and \mathcal{G}^b implies that $\tilde{l}_1^b \geq \tilde{L}_1^\Delta$, i.e., the demand for effective labor from representative firm 1 is greater than or equal to the supply of low-skilled effective labor in group structure b . Thus, higher skill levels (with $r \geq \tilde{\tau}_1$ and $r < \tau_2$) may also work in firm 1. With \mathcal{G}^b , the labor market is locally integrated and $i = 1, 2$ form the group $g_1^b \in \mathcal{G}^b$. Now, \mathcal{G}^a also implies that $\tilde{l}_2^a \geq \tilde{L}_2^\Delta$ and \mathcal{G}^b implies that $\tilde{l}_2^b \leq \tilde{L}_2^\Delta$.

To see that crossing cannot be a solution, note that:

Figure 12: Crossing



- i. $\tilde{l}_1^a \leq \tilde{l}_1^b$ and Equation (B.7) imply that $\left(\frac{p_1^a}{P^a}\right)^{-\sigma} C^a \leq \left(\frac{p_1^b}{P^b}\right)^{-\sigma} C^b$.
- ii. $\tilde{l}_2^a \geq \tilde{l}_2^b$ and Equation (B.7) imply that $\left(\frac{p_2^a}{P^a}\right)^{-\sigma} C^a \geq \left(\frac{p_2^b}{P^b}\right)^{-\sigma} C^b$.
- iii. Now, $p_1^a = p_1^b$ (as $\omega_1^a = \omega_1^b = 1$) implies $C^a P^{a\sigma} \leq C^b P^{b\sigma}$ through (i).
- iv. Then, as $C^a P^{a\sigma} \leq C^b P^{b\sigma}$, it must be the case that $p_2^{a-\sigma} \geq p_2^{b-\sigma}$ (see (ii)).
- v. As $p_2^{a-\sigma} \geq p_2^{b-\sigma}$, it must be the case that $\omega_2^a \leq \omega_2^b$, given price index p_i in equilibrium (B.6).

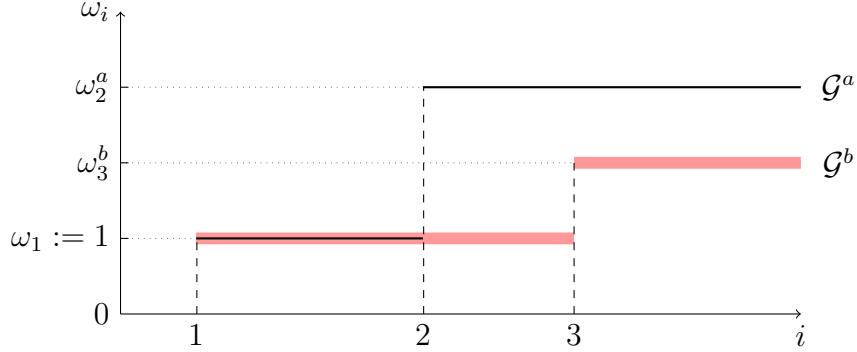
This contradicts $\omega_2^a > \omega_2^b$.

- **DOMINANCE:** Assume now that the two group structures \mathcal{G}^a and \mathcal{G}^b correspond to Figure 13. Again, \mathcal{G}^a implies that $\tilde{l}_1^a = \tilde{L}_1^\Delta$, and \mathcal{G}^b implies that $\tilde{l}_1^b \geq \tilde{L}_1^\Delta$. Thus, with \mathcal{G}^b , the labor market is locally integrated from $i = 1$ to $i = 2$ (forming group $g_1^b \in \mathcal{G}^b$), and we can use the same rationale as above (see *CROSSING*) to conclude that $\omega_2^a \leq \omega_2^b$.

This, again, contradicts $\omega_2^a > \omega_2^b$.

Thus, we have proven that under Condition (19), potential solutions are mutually exclusive, i.e., there is at most one solution fulfilling Condition (19).

Figure 13: Dominance



Step 3: At least one solution

We now show that there must be at least one solution that fulfills Condition (19) and derive this solution. Use the labor-market clearing condition as given in (13) and plug in (16):

$$\sum_{\tilde{i} \geq i} \tilde{L}_i^\Delta \geq \kappa(1)^{-1} \left(\sum_{\tilde{i}} \omega_{\tilde{i}}^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}} C \sum_{\tilde{i} \geq i} \omega_{\tilde{i}}^{-\sigma}. \quad (\text{B.8})$$

Further, define $\tilde{L}_i^s := \sum_{\tilde{i} \geq i} \tilde{L}_i^\Delta$ as the aggregate effective labor satisfying the sufficient skill condition $r \geq \tau_i$. As (B.8) needs to hold with equality for $i = 1$, we find

$$\kappa(1)^{-1} \left(\sum_{\tilde{i}} \omega_{\tilde{i}}^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}} C = \tilde{L}_1^s \left(\sum_{\tilde{i}} \omega_{\tilde{i}}^{-\sigma} \right)^{-1},$$

which can be plugged back into (B.8). Now, we have derived a version of the LMCC that only depends on labor supply satisfying $r \geq \tau_i$ relative to total labor supply and scaling factors:

$$\tilde{L}_i^s \geq \frac{\sum_{\tilde{i} \geq i} \omega_{\tilde{i}}^{-\sigma}}{\sum_{\tilde{i}} \omega_{\tilde{i}}^{-\sigma}} \tilde{L}_1^s, \quad (\text{B.9})$$

Let us assume $\omega_i = 1 \forall i \in \mathcal{I}$. If the labor market is integrated, (B.9) is satisfied and we have found the solution. If (B.9) is not satisfied, denote the first \hat{i} for which it is violated as i_b . Then, we observe

$$\frac{\tilde{L}_{i_b}^s}{\tilde{L}_1^s} < \frac{\sum_{i \in \mathcal{I}: i \geq i_b} \omega_i^{-\sigma}}{\sum_{i \in \mathcal{I}} \omega_i^{-\sigma}}.$$

We now show that ω_{i_b} can always be adjusted such that the labor market clearing condition for i_b is satisfied again. Note that for all ω_i with $i > i_b$ Condition (19)

requiring that ω_i is weakly increasing in i is binding. Thus, the above inequality becomes

$$\frac{\tilde{L}_{i_b}^s}{\tilde{L}_1^s} < \frac{(\bar{i} + 1 - i_b)\omega_{i_b}^{-\sigma}}{\sum_{i \in \mathcal{I}: i < i_b} \omega_i^{-\sigma} + (\bar{i} + 1 - i_b)\omega_{i_b}^{-\sigma}}.$$

Now, we can solve for an ω_{i_b} such that the inequality becomes an equality. This is given as

$$\omega_{i_b} = \left(\frac{\Lambda_{i_b}}{\bar{i} + 1 - i_b} \sum_{i \in \mathcal{I}: i < i_b} \omega_i^{-\sigma} \right)^{-\frac{1}{\sigma}},$$

with $\Lambda_{i_b} = \frac{\tilde{L}_{i_b}^s}{\tilde{L}_1^s - \tilde{L}_{i_b}^s}$.

After plugging in the solution for all ω_i with $i \geq i_b$, we can search for the next \hat{i} that violates (B.9) and apply the same procedure repeatedly until there are no labor market breaks left.

In sum, we have proven that, given $\sigma > 1$, there is one unique solution \mathcal{W}^* for any set of task complexities \mathcal{I} and skill distribution L^r .

□

B.2 Proof of Lemma 1

We use (B.1). We first prove (B.3), then we prove (B.4).

For $|\mathcal{G}| \geq 2$, the derivative of (B.1) with respect to its own scaling factor ω_g is given by

$$\frac{\partial \tilde{L}_g^d}{\partial \omega_g} = \omega_g^{-\sigma} \left(\sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}}^{1-\sigma} \right)^{-1} \left(-\sigma \omega_g^{-1} \sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}} \tilde{L}_{\hat{g}}^\Delta + (\sigma - 1) \omega_g^{-\sigma} \left(\sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}}^{1-\sigma} \right)^{-1} \sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}} \tilde{L}_{\hat{g}}^\Delta + \tilde{L}_g^\Delta \right),$$

We now show that the sum of the terms in brackets is negative. We multiply by scaling factor ω_g and divide by $\sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}} \tilde{L}_{\hat{g}}^\Delta$ to obtain

$$\begin{aligned} & -\sigma + (\sigma - 1) \frac{\omega_g^{1-\sigma}}{\sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}}^{1-\sigma}} + \frac{\omega_g \tilde{L}_g^\Delta}{\sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}} \tilde{L}_{\hat{g}}^\Delta} \\ &= \sigma \left(\frac{\omega_g^{1-\sigma}}{\sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}}^{1-\sigma}} - 1 \right) - \frac{\omega_g^{1-\sigma}}{\sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}}^{1-\sigma}} + \frac{\omega_g \tilde{L}_g^\Delta}{\sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}} \tilde{L}_{\hat{g}}^\Delta}. \end{aligned}$$

From (B.2) we know that $-\frac{\omega_g^{1-\sigma}}{\sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}}^{1-\sigma}} + \frac{\omega_g \tilde{L}_g^\Delta}{\sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}} \tilde{L}_{\hat{g}}^\Delta} = 0$. Observe that $\sigma > 0$ and that, as $\frac{\omega_g \tilde{L}_g^\Delta}{\sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}} \tilde{L}_{\hat{g}}^\Delta} < 1$, $\frac{\omega_g^{1-\sigma}}{\sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}}^{1-\sigma}} - 1 < 0$. Thus, we have proven that $\frac{\partial \tilde{L}_g^d}{\partial \omega_g} < 0$.

The derivative of (B.1) with respect to some scaling factor $\omega_{g'}$ ($g' \neq g$) is given by

$$\frac{\partial \tilde{L}_g^d}{\partial \omega_{g'}} = \omega_g^{-\sigma} \left(\sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}}^{1-\sigma} \right)^{-1} \left((\sigma - 1) \omega_{g'}^{-\sigma} \left(\sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}}^{1-\sigma} \right)^{-1} \sum_{\hat{g} \in \mathcal{G}} \omega_{\hat{g}} \tilde{L}_{\hat{g}}^{\Delta} + \tilde{L}_{g'}^{\Delta} \right) > 0.$$

Thus we have proven that $\frac{\partial \tilde{L}_g^d}{\partial \omega_{g'}} > 0$.

□

C Solution Algorithm

In Appendix B.1, we proved that there exists a unique solution in the task-complexity model. In this section, we present the solution algorithm that we apply to determine this solution.

When proving the existence of the solution, we have already proposed a method for determining scaling factor ω_{i_b} when i_b is the index of a labor market break. This method is based on verifying the satisfaction of the labor market clearing condition (LMCC) for each industry and repeatedly increasing scaling factors if necessary. The algorithm introduced in this section is fast as it omits the sequential checking of the LMCC for each industry. Instead, it computes solutions for all possible labor market arrangements first and then drops solutions that violate the LMCC or Condition 19.

Starting with (B.9) and defining $\Lambda_i \equiv \frac{\tilde{L}_i^s}{\tilde{L}_1^s - \tilde{L}_i^s}$, we find

$$\omega_i \geq \left(\Lambda_i \sum_{\tilde{i} < i} \omega_{\tilde{i}}^{-\sigma} - \sum_{\tilde{i} > i} \omega_{\tilde{i}}^{-\sigma} \right)^{-\frac{1}{\sigma}}.$$

Define the right-hand side as

$$\Omega_i(\omega_{\tilde{i} \in \mathcal{I} \setminus i}) := \left(\Lambda_i \sum_{\tilde{i} < i} \omega_{\tilde{i}}^{-\sigma} - \sum_{\tilde{i} > i} \omega_{\tilde{i}}^{-\sigma} \right)^{-\frac{1}{\sigma}}. \quad (\text{C.1})$$

If we also impose Condition (19) and normalize $\omega_1 = 1$, we obtain

$$\omega_i = \begin{cases} 1 & \text{for } i = 1, \\ \max \{ \omega_{i-1}, \Omega_i \} & \text{for } i > 1. \end{cases} \quad (\text{C.2})$$

This holds with equality, as $\omega_i > \Omega_i > \omega_{i-1}$ would imply a labor market break despite the lack of excess demand.

We can now solve the model by applying the following procedure:

Table 2: All Possible Labor Market Arrangements in a Model with 3 Industries.

Arrangement	1	2	3	4
ω_1	1	1	1	1
ω_2	1	1	$\Omega_2(1)$	$\Omega_2(1)$
ω_3	1	$\Omega_3(1, 1)$	$\Omega_2(1)$	$\Omega_3(1, \Omega_2(1))$

1. List all possible $2^{|\mathcal{I}|-1}$ labor market arrangements.
2. Determine each ω_i for all arrangements by either setting $\omega_i = \omega_{i-1}$ or by computing $\Omega_i(\omega_{\tilde{i} \in \mathcal{I} \setminus i})$.

In Table 2, we present all possible labor market arrangements for $|\mathcal{I}| = 3$ as an example. For arrangement 2, computing Ω_3 is straightforward as ω_1 and ω_2 are known and equal to one. For the two remaining arrangements, we can replace ω_3 in $\Omega_2(\omega_{\tilde{i} \in \mathcal{I} \setminus 2})$ either by Ω_2 (arrangement 3) or Ω_3 (arrangement 4) such that we are left with $\Omega_2(\omega_1)$. In cases with more industries and arrangements, we can repeat this substitution until we have an expression for Ω_i that depends on $\omega_{\tilde{i} < i}$ only. In general, it reads as

$$\Omega_i = \left(\frac{\Lambda_i - \Lambda_{i_{b+}}}{(i_{b+} - i)(1 + \Lambda_{i_{b+}})} \sum_{\tilde{i} < i} \omega_{\tilde{i}}^{-\sigma} \right)^{-\frac{1}{\sigma}}, \quad (\text{C.3})$$

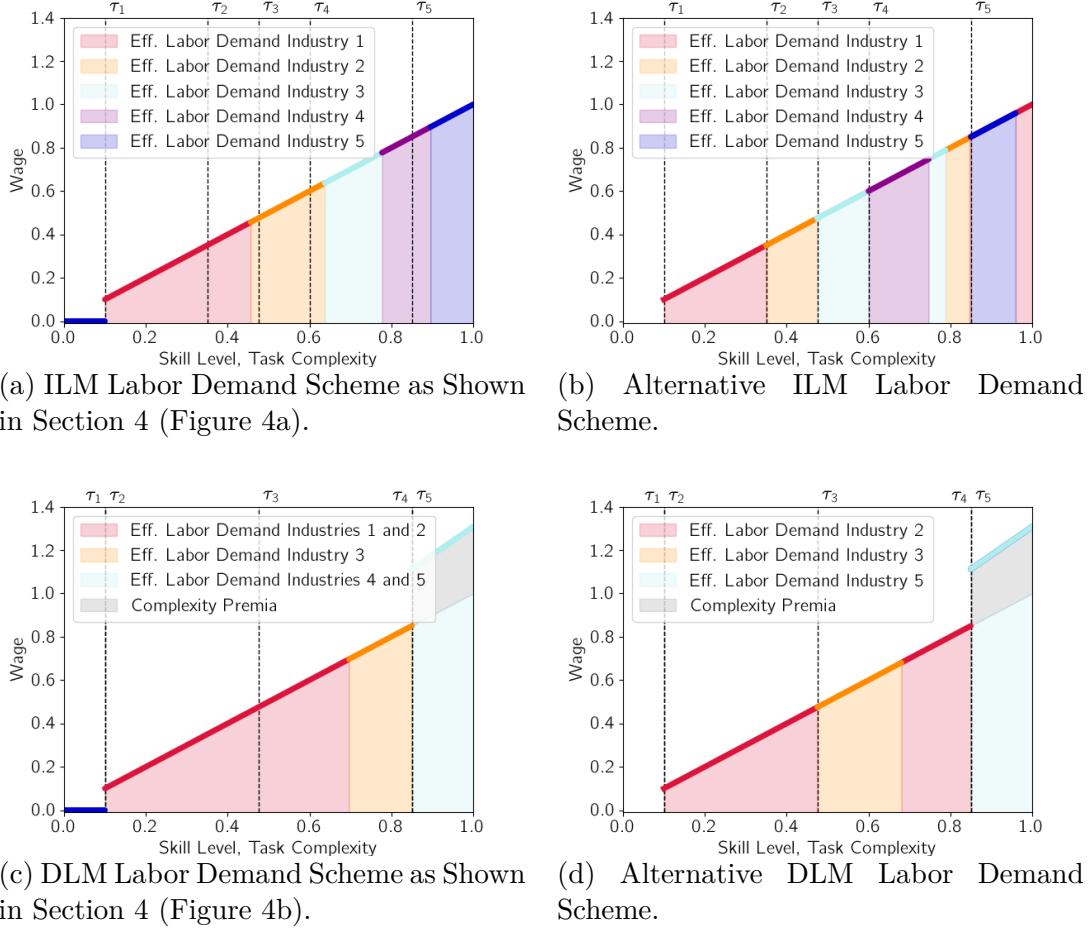
where i_{b+} is the index of the next scaling factor break. If there are no further breaks, we set $i_{b+} = |\mathcal{I}| + 1$ and $\Lambda_{i_{b+}} = 0$. Starting from $i = 2$, we can straightforwardly compute all Ω_i since Ω_i depends on $\omega_{\tilde{i}}$ with $\tilde{i} < i$ only.

3. Delete the labor market arrangements for which at least one ω_i violates (19), i.e., where there is some i such that $\omega_i < \omega_{i-1}$.
4. Check which of the remaining solution candidates satisfies the labor market clearing condition (B.9). The uniqueness of the solution has already been proven.

D Robustness of the Stylized Facts Replication in Section 4

To compute changes in employment and wages across complexities in Figures 4a and 4b, we relied on Assumption 2. It states that, for each representative firm i , \mathcal{R}_i is an interval and the firm hires all available labor with skill level $r \in \mathcal{R}_i$, i.e., $l_i(r) = L^r$ and $\tilde{l}_i(r) = 0 \forall \tilde{i} \neq i$. Effectively, this means that, given \tilde{l}_1 and the sufficient skill condition,

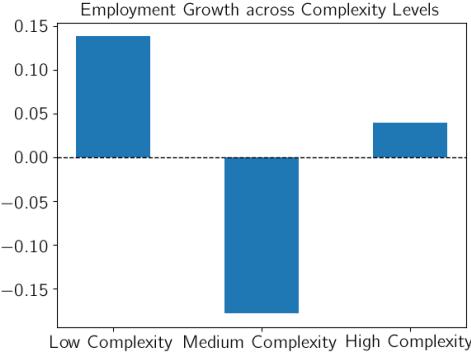
Figure 14: Alternative Labor Demand Schemes.



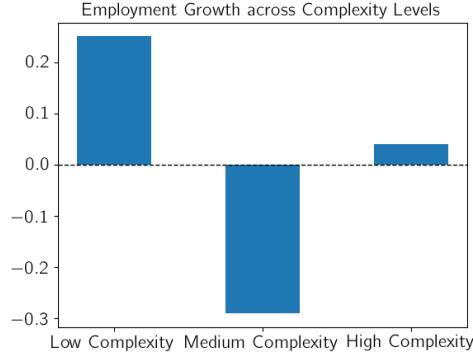
the average skill level employed in representative firm 1 is minimized, followed by a minimization of the average skill level in firm 2 and subsequent firms.

In what follows, we continue to assume that each representative firm hires all available labor with skill level $r \in \mathcal{R}_i$, i.e., $l_i(r) = L^r$. However, we drop the assumption that \mathcal{R}_i is a single interval. Instead of minimizing the average skill level per firm, starting with the least complex task, we maximize it. This means that firm 1 still hires all labor up to τ_2 , as no other firm can employ these workers. But after that, it switches to hiring the most productive available workers. “Available” in this sense means that firm 1 can hire them without causing excess labor demand for another firm. The same procedure applies for firm 2, then firm 3, and so on. As a result, \mathcal{R}_i may be a union of intervals instead of a single one. In Figure 14, we compare the resulting allocation of skills to firms under this alternative rule to the outcome under Assumption 2.

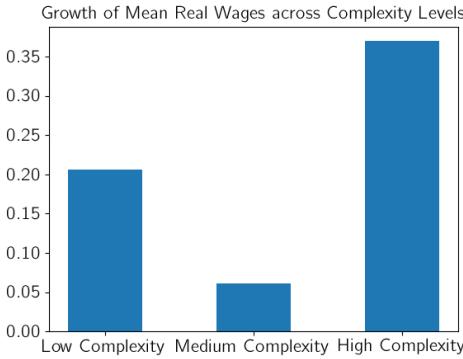
Figure 15: Employment and Wage Growth over Task Complexities Given an Alternative Labor Demand Scheme.



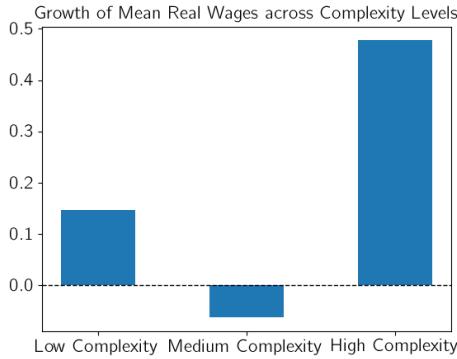
(a) Employment Growth Computed in Section 4 (Figure 4c).



(b) Employment Growth Given the Alternative Labor Demand Scheme.



(c) Wage Growth Computed in Section 4 (Figure 4d).



(d) Wage Growth Given the Alternative Labor Demand Scheme.

Again, we compute employment and wage growth across three complexity groups from the pre- to the post-automation scenario, where medium-complexity tasks are defined as those with task-complexities $\tau_i \in [0.33, 0.67]$. In Figure 15, we compare the results to those shown in Section 4. We find that they are qualitatively equivalent, with a U-shaped pattern of employment and wage growth across complexities. Thus, the robustness analysis confirms that the model's ability to replicate the stylized facts observed by Autor and Dorn (2013) does not hinge on Assumption 2 but sustains even under the opposite allocation rule.

E Data and Estimation

E.1 Construction of the Task-Complexity Variable

The task-complexity variable used for the estimation of our model is constructed as the sum of six indicators from the “Work Activities” table given in the O*NET database, version 28.3. These indicators are:

1. analyzing data or information (4.A.2.a.4),
2. thinking creatively (4.A.2.b.2),
3. interpreting the meaning of information for others (4.A.4.a.1),
4. establishing and maintaining interpersonal relationships (4.A.4.a.4),
5. guiding, directing, and motivating subordinates (4.A.4.b.4),
6. coaching and developing others (4.A.4.b.5).

The first three indicators in this list are used by Acemoglu and Autor (2011) as a composite measure for analytical non-routine cognitive tasks, while the remaining three serve as a measure for interpersonal non-routine cognitive tasks. The measure for analytical non-routine cognitive tasks is also adopted by Braxton et al. (2023). We sum the level scores for all six indicators to construct our task-complexity variable.

As the scale for each original indicator ranges from 0 to 7, the total score goes from 0 to 42 for each occupation. We divide the score by 42 to normalize the variable to the domain $[0, 1]$.

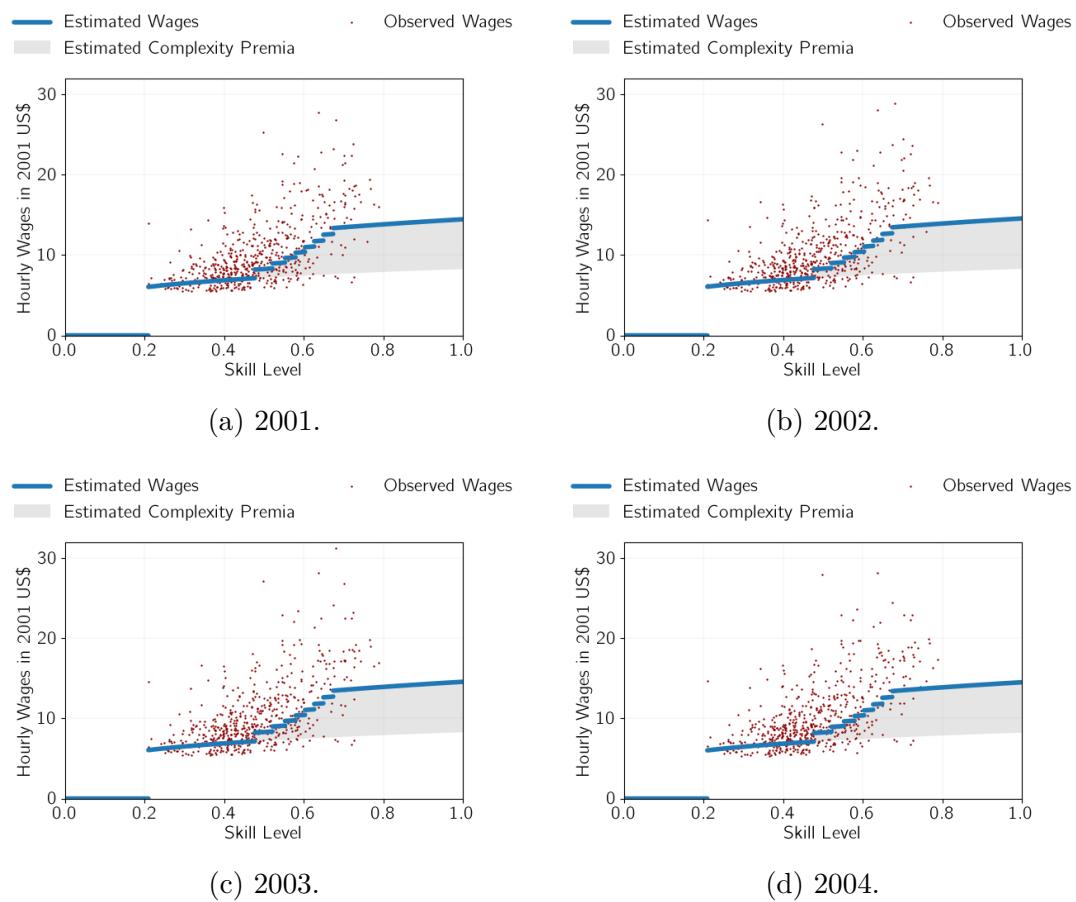
E.2 Plots of Estimated Wage Schemes and Complexity Premia for 2001–2019

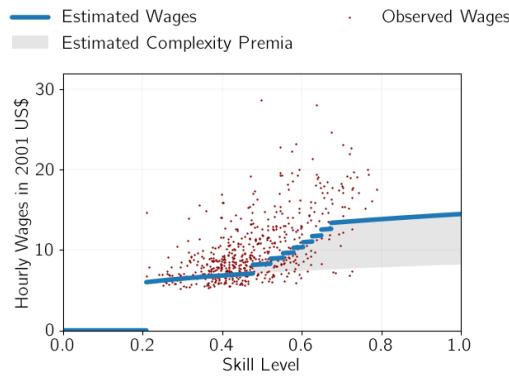
Figure 16 displays estimated wage schemes, estimated complexity premia, and observed occupational wages in 2001 prices. The underlying parameter values for the skill-level distribution, task-complexity distribution, and productivity are given in Table 1.

F Robustness of the Estimation

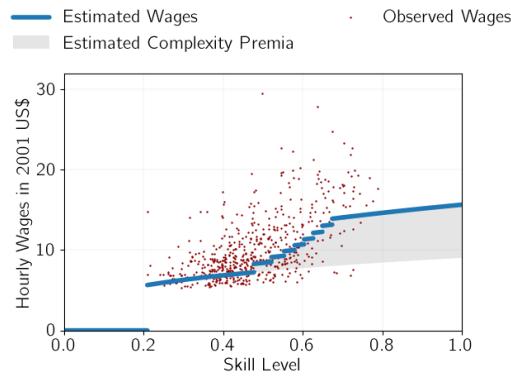
We assess the robustness of the estimation results presented in Table 1 by varying two parameters. First, we vary the number of industries (or, equivalently, tasks) in the

Figure 16: Estimated Wage Schemes and Observed Wages across Skills, 2001–2019.

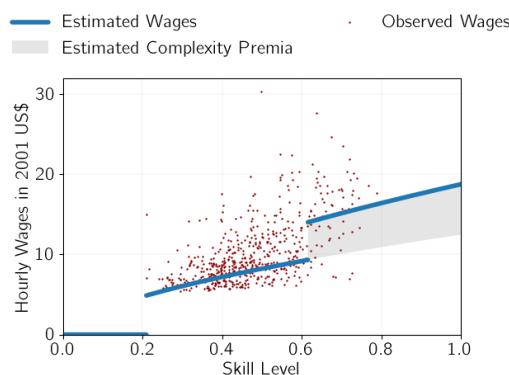




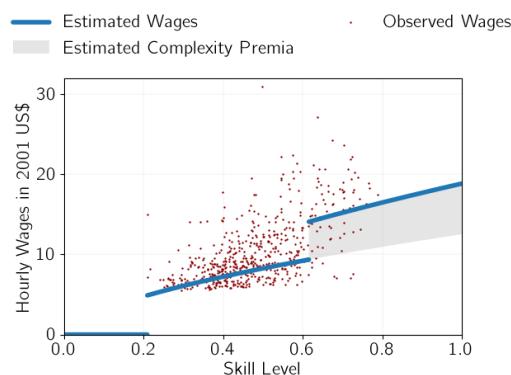
(e) 2005.



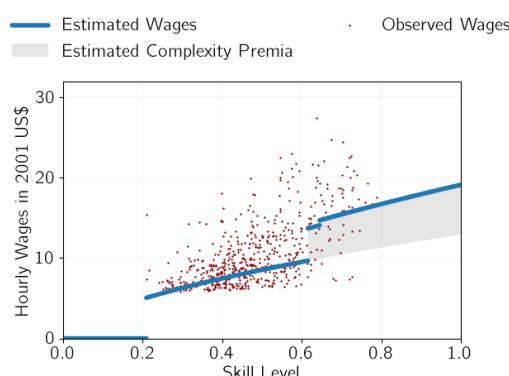
(f) 2006.



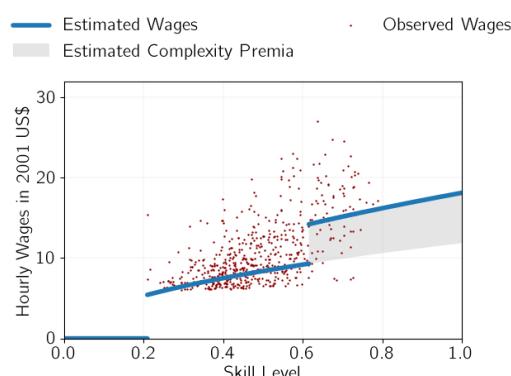
(g) 2007.



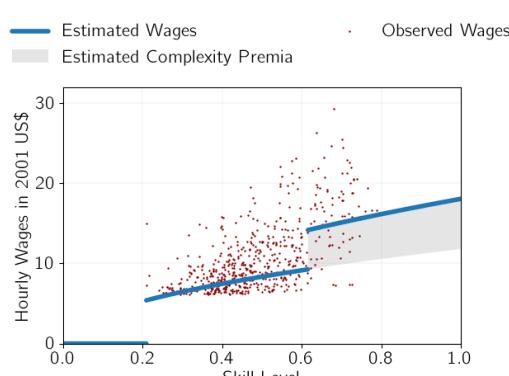
(h) 2008.



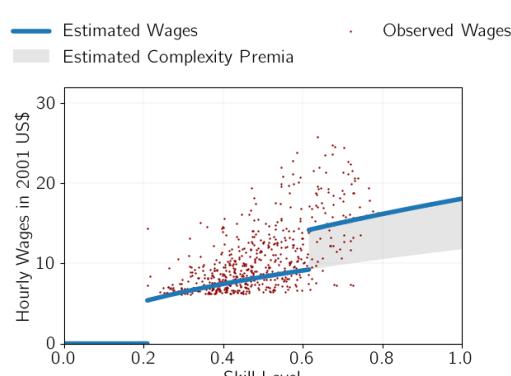
(i) 2009.



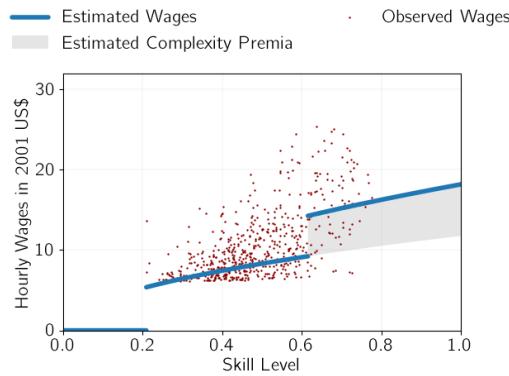
(j) 2010.



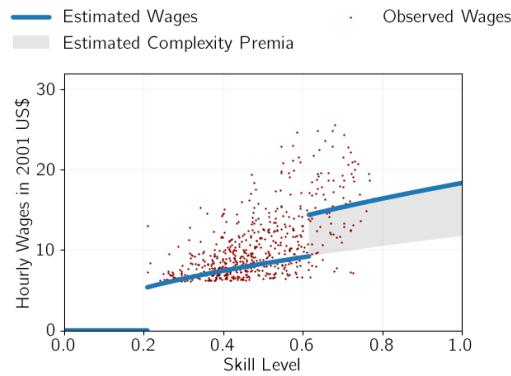
(k) 2011.



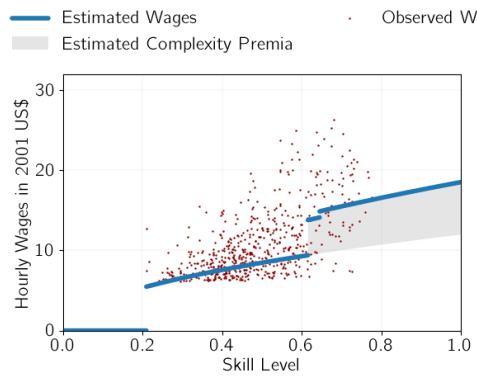
(l) 2012.



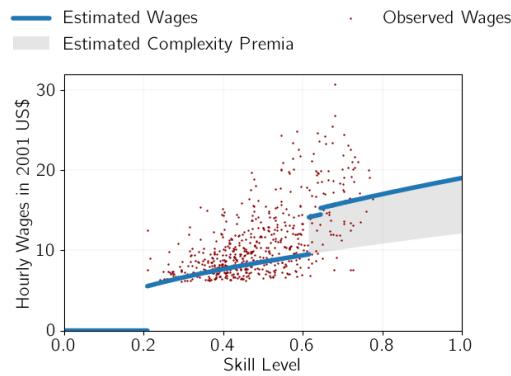
(m) 2013.



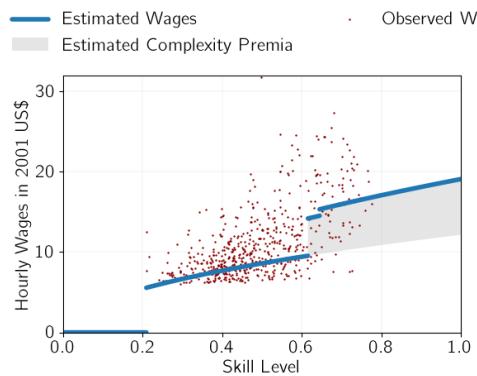
(n) 2014.



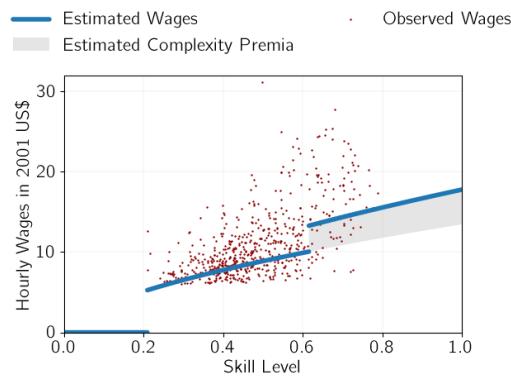
(o) 2015.



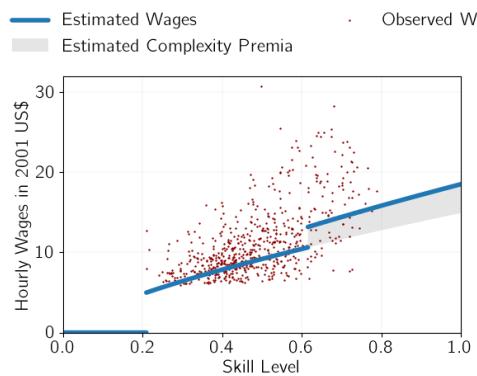
(p) 2016.



(q) 2017.



(r) 2018.



(s) 2019.

economy. Second, we retrieve additional moments of the occupational wage distribution from the OEPS data to match skills to wages.

Estimation results for different numbers of industries in the economy are reported in Tables 3 to 5. Recall that the complexity of each task i is determined as the $100 \cdot \frac{i-1}{|T|}$ th percentile of an estimated beta distribution, and that $|T|$ was set to 10 in the main body. In our robustness exercise, we additionally test $|T| = 8, 12, 15$. Estimation results for these values of $|T|$ confirm three trends identified in the main body.

First, productivity has become more convex in the skill level, as the q parameter has increased over the years. Second, tasks have become more complex on average, measured by the mean of the task-complexity distribution μ^τ . Additionally, tasks are more concentrated in the high-complexity region as σ^τ has declined. Third, the average skill level in the labor force μ^L has increased as well, without a clear trend in the dispersion of skill levels, captured by σ^L . Results are less stable only for the case of twelve tasks in years 2018 and 2019.

In a second analysis, we vary the occupational hourly wage percentile used to map skills to wages. Recall that we assumed that the skill level of the least-paid worker in an occupation to matches the occupational task complexity. As we observe the occupational task complexity in the O*NET database, this assumption allows us to establish a relationship between skills and wages. Finally, matching this relationship was the objective when estimating the skill-level and task-complexity distribution, as well as parameter q , which determines the convexity of productivity in skill.

As the OEPS data do not include the wage of the least-paid worker in each occupation but only the 10th percentile of the hourly wage distribution, we proceeded with this percentile, arguing that it may at least exclude outliers resulting from measurement errors or mechanisms not captured by our model. Still, one may argue that using the 10th percentile specifically is arbitrary. We address this critique by switching from the 10th percentile to the 25th percentile (Table 6) and 50th percentile (Table 7) of the hourly wage distribution, which are the next two moments observed in OEPS. Comparing the robustness results to those of the original estimation presented in Table 1, we observe no significant differences for the trends in productivity, the skill-level distribution, or the task-complexity distribution. Thus, we are confident that our conclusions do not hinge on using a specific percentile of the wage distribution.

Table 3: Robustness Results for 8 Industries.

Year	Parameters					Distribution Moments					
	α_L	β_L	α_τ	β_τ	q	μ^L	mode^L	σ^L	μ^τ	mode^τ	σ^τ
2001	12.00	12.00	4.00	3.00	0.40	0.50	0.50	0.10	0.54	0.56	0.10
2002	12.00	12.00	4.00	3.00	0.40	0.50	0.50	0.10	0.54	0.56	0.10
2003	8.00	4.00	14.00	3.00	0.60	0.67	0.70	0.13	0.69	0.71	0.05
2004	8.00	4.00	14.00	3.00	0.60	0.67	0.70	0.13	0.69	0.71	0.05
2005	8.00	4.00	14.00	3.00	0.60	0.67	0.70	0.13	0.69	0.71	0.05
2006	8.00	4.00	14.00	3.00	0.60	0.67	0.70	0.13	0.69	0.71	0.05
2007	8.00	4.00	14.00	3.00	0.60	0.67	0.70	0.13	0.69	0.71	0.05
2008	8.00	4.00	14.00	3.00	0.60	0.67	0.70	0.13	0.69	0.71	0.05
2009	8.00	4.00	14.00	3.00	0.60	0.67	0.70	0.13	0.69	0.71	0.05
2010	8.00	4.00	14.00	3.00	0.60	0.67	0.70	0.13	0.69	0.71	0.05
2011	6.00	3.00	14.00	3.00	0.60	0.67	0.71	0.15	0.69	0.71	0.05
2012	6.00	3.00	16.00	4.00	0.50	0.67	0.71	0.15	0.67	0.69	0.05
2013	6.00	3.00	16.00	4.00	0.50	0.67	0.71	0.15	0.67	0.69	0.05
2014	6.00	3.00	16.00	4.00	0.50	0.67	0.71	0.15	0.67	0.69	0.05
2015	8.00	4.00	16.00	4.00	0.50	0.67	0.70	0.13	0.67	0.69	0.05
2016	8.00	4.00	16.00	4.00	0.50	0.67	0.70	0.13	0.67	0.69	0.05
2017	8.00	4.00	16.00	4.00	0.50	0.67	0.70	0.13	0.67	0.69	0.05
2018	8.00	4.00	16.00	4.00	0.50	0.67	0.70	0.13	0.67	0.69	0.05
2019	16.00	8.00	16.00	4.00	0.60	0.67	0.68	0.09	0.67	0.69	0.05

Table 4: Robustness Results for 12 Industries.

Year	α_L	β_L	α_τ	β_τ	q	μ^L	mode^L	σ^L	μ^τ	mode^τ	σ^τ
2001	16.00	14.00	6.00	3.00	0.20	0.53	0.54	0.09	0.60	0.62	0.09
2002	14.00	12.00	6.00	3.00	0.20	0.54	0.54	0.10	0.60	0.62	0.09
2003	14.00	12.00	6.00	3.00	0.20	0.54	0.54	0.10	0.60	0.62	0.09
2004	14.00	12.00	6.00	3.00	0.20	0.54	0.54	0.10	0.60	0.62	0.09
2005	14.00	12.00	6.00	3.00	0.20	0.54	0.54	0.10	0.60	0.62	0.09
2006	14.00	12.00	6.00	3.00	0.20	0.54	0.54	0.10	0.60	0.62	0.09
2007	14.00	12.00	6.00	3.00	0.20	0.54	0.54	0.10	0.60	0.62	0.09
2008	14.00	12.00	6.00	3.00	0.20	0.54	0.54	0.10	0.60	0.62	0.09
2009	14.00	12.00	6.00	3.00	0.20	0.54	0.54	0.10	0.60	0.62	0.09
2010	16.00	14.00	6.00	3.00	0.10	0.53	0.54	0.09	0.60	0.62	0.09
2011	4.00	1.50	16.00	3.00	0.60	0.73	0.86	0.17	0.70	0.72	0.05
2012	8.00	4.00	10.00	2.00	0.50	0.67	0.70	0.13	0.69	0.73	0.06
2013	4.00	1.50	18.00	4.00	0.60	0.73	0.86	0.17	0.68	0.70	0.05
2014	4.00	1.50	18.00	4.00	0.50	0.73	0.86	0.17	0.68	0.70	0.05
2015	4.00	1.50	18.00	4.00	0.50	0.73	0.86	0.17	0.68	0.70	0.05
2016	4.00	1.50	18.00	4.00	0.50	0.73	0.86	0.17	0.68	0.70	0.05
2017	4.00	1.50	10.00	2.00	0.50	0.73	0.86	0.17	0.69	0.73	0.06
2018	14.00	12.00	6.00	3.00	0.10	0.54	0.54	0.10	0.60	0.62	0.09
2019	14.00	12.00	6.00	3.00	0.20	0.54	0.54	0.10	0.60	0.62	0.09

Table 5: Robustness Results for 15 Industries.

Year	α_L	β_L	α_τ	β_τ	q	μ^L	mode^L	σ^L	μ^τ	mode^τ	σ^τ
2001	10.00	8.00	4.00	1.50	0.30	0.56	0.56	0.11	0.63	0.71	0.10
2002	10.00	8.00	4.00	1.50	0.30	0.56	0.56	0.11	0.63	0.71	0.10
2003	10.00	8.00	4.00	1.50	0.30	0.56	0.56	0.11	0.63	0.71	0.10
2004	10.00	8.00	4.00	1.50	0.30	0.56	0.56	0.11	0.63	0.71	0.10
2005	10.00	8.00	4.00	1.50	0.30	0.56	0.56	0.11	0.63	0.71	0.10
2006	10.00	8.00	4.00	1.50	0.30	0.56	0.56	0.11	0.63	0.71	0.10
2007	10.00	4.00	16.00	3.00	0.60	0.71	0.75	0.12	0.70	0.72	0.05
2008	10.00	4.00	16.00	3.00	0.60	0.71	0.75	0.12	0.70	0.72	0.05
2009	10.00	4.00	16.00	3.00	0.60	0.71	0.75	0.12	0.70	0.72	0.05
2010	10.00	4.00	16.00	3.00	0.60	0.71	0.75	0.12	0.70	0.72	0.05
2011	4.00	1.50	16.00	3.00	0.50	0.73	0.86	0.17	0.70	0.72	0.05
2012	4.00	1.50	10.00	1.50	0.50	0.73	0.86	0.17	0.71	0.76	0.06
2013	4.00	1.50	12.00	2.00	0.50	0.73	0.86	0.17	0.71	0.74	0.05
2014	4.00	1.50	12.00	2.00	0.50	0.73	0.86	0.17	0.71	0.74	0.05
2015	8.00	3.00	12.00	2.00	0.50	0.73	0.78	0.13	0.71	0.74	0.05
2016	8.00	3.00	12.00	2.00	0.50	0.73	0.78	0.13	0.71	0.74	0.05
2017	8.00	3.00	12.00	2.00	0.50	0.73	0.78	0.13	0.71	0.74	0.05
2018	8.00	3.00	12.00	2.00	0.50	0.73	0.78	0.13	0.71	0.74	0.05
2019	8.00	3.00	12.00	2.00	0.60	0.73	0.78	0.13	0.71	0.74	0.05

Table 6: Robustness Results for the 25th Hourly Wage Percentile.

Year	α_L	β_L	α_τ	β_τ	q	μ^L	mode^L	σ^L	μ^τ	mode^τ	σ^τ
2001	16.00	14.00	6.00	3.00	0.30	0.53	0.54	0.09	0.60	0.62	0.09
2002	16.00	14.00	6.00	3.00	0.30	0.53	0.54	0.09	0.60	0.62	0.09
2003	16.00	14.00	6.00	3.00	0.30	0.53	0.54	0.09	0.60	0.62	0.09
2004	14.00	12.00	6.00	3.00	0.30	0.54	0.54	0.10	0.60	0.62	0.09
2005	14.00	12.00	6.00	3.00	0.40	0.54	0.54	0.10	0.60	0.62	0.09
2006	14.00	12.00	6.00	3.00	0.40	0.54	0.54	0.10	0.60	0.62	0.09
2007	14.00	12.00	6.00	3.00	0.40	0.54	0.54	0.10	0.60	0.62	0.09
2008	6.00	3.00	14.00	3.00	0.70	0.67	0.71	0.15	0.69	0.71	0.05
2009	6.00	3.00	14.00	3.00	0.70	0.67	0.71	0.15	0.69	0.71	0.05
2010	8.00	4.00	14.00	3.00	0.70	0.67	0.70	0.13	0.69	0.71	0.05
2011	6.00	3.00	14.00	3.00	0.70	0.67	0.71	0.15	0.69	0.71	0.05
2012	6.00	3.00	14.00	3.00	0.70	0.67	0.71	0.15	0.69	0.71	0.05
2013	6.00	3.00	14.00	3.00	0.70	0.67	0.71	0.15	0.69	0.71	0.05
2014	8.00	4.00	14.00	3.00	0.70	0.67	0.70	0.13	0.69	0.71	0.05
2015	8.00	4.00	14.00	3.00	0.70	0.67	0.70	0.13	0.69	0.71	0.05
2016	8.00	4.00	14.00	3.00	0.60	0.67	0.70	0.13	0.69	0.71	0.05
2017	8.00	4.00	14.00	3.00	0.60	0.67	0.70	0.13	0.69	0.71	0.05
2018	8.00	4.00	14.00	3.00	0.60	0.67	0.70	0.13	0.69	0.71	0.05
2019	8.00	4.00	14.00	3.00	0.70	0.67	0.70	0.13	0.69	0.71	0.05

Table 7: Robustness Results for the 50th Hourly Wage Percentile.

Year	α_L	β_L	α_τ	β_τ	q	μ^L	mode^L	σ^L	μ^τ	mode^τ	σ^τ
2001	12.00	12.00	4.00	2.00	0.20	0.50	0.50	0.10	0.60	0.64	0.10
2002	12.00	12.00	4.00	2.00	0.20	0.50	0.50	0.10	0.60	0.64	0.10
2003	12.00	12.00	4.00	2.00	0.20	0.50	0.50	0.10	0.60	0.64	0.10
2004	12.00	12.00	4.00	2.00	0.20	0.50	0.50	0.10	0.60	0.64	0.10
2005	12.00	12.00	4.00	2.00	0.30	0.50	0.50	0.10	0.60	0.64	0.10
2006	12.00	12.00	4.00	2.00	0.30	0.50	0.50	0.10	0.60	0.64	0.10
2007	14.00	12.00	6.00	3.00	0.50	0.54	0.54	0.10	0.60	0.62	0.09
2008	16.00	14.00	6.00	3.00	0.50	0.53	0.54	0.09	0.60	0.62	0.09
2009	16.00	14.00	6.00	3.00	0.50	0.53	0.54	0.09	0.60	0.62	0.09
2010	16.00	14.00	6.00	3.00	0.50	0.53	0.54	0.09	0.60	0.62	0.09
2011	6.00	3.00	14.00	3.00	0.90	0.67	0.71	0.15	0.69	0.71	0.05
2012	6.00	3.00	14.00	3.00	0.90	0.67	0.71	0.15	0.69	0.71	0.05
2013	6.00	3.00	14.00	3.00	0.90	0.67	0.71	0.15	0.69	0.71	0.05
2014	6.00	3.00	14.00	3.00	0.90	0.67	0.71	0.15	0.69	0.71	0.05
2015	6.00	3.00	14.00	3.00	0.80	0.67	0.71	0.15	0.69	0.71	0.05
2016	6.00	3.00	14.00	3.00	0.80	0.67	0.71	0.15	0.69	0.71	0.05
2017	6.00	3.00	14.00	3.00	0.80	0.67	0.71	0.15	0.69	0.71	0.05
2018	8.00	4.00	14.00	3.00	0.80	0.67	0.70	0.13	0.69	0.71	0.05
2019	8.00	4.00	14.00	3.00	0.80	0.67	0.70	0.13	0.69	0.71	0.05

G Solving the AI-Augmented Task-Complexity Model

G.1 Formal Derivation of the Model

As outlined in the main body, AI can perform tasks $i \in [\underline{i}^a, \bar{i}^a]$, and its productivity is normalized to one. There is an infinite supply of AI at rental rate w^a . Here, we only provide the solution for the case $w^a < 1$, which is consistent with the calibration in the main body. A representative firm i 's demand for AI is denoted by A_i .

In what follows, we first go through the necessary amendments to the model equations summarized in Section 2. We then introduce the required adaptations to the solution algorithm presented in Appendix C.

The goods market clearing condition now reads:

$$\begin{aligned} c_i &= x_i = \kappa(1)\tilde{l}_i + A_i \\ \Leftrightarrow \tilde{l}_i &= \kappa(1)^{-1}(c_i - A_i) \end{aligned}$$

for any $i \in [\underline{i}^a, \bar{i}^a]$.

To solve the model, we introduce Assumption 3:

Assumption 3 (Allocation of Skills in the Presence of AI)

No labor allocates to tasks $i \in [\underline{i}^a, \bar{i}^a]$ if $\underline{i}^a > 1$.

If $\underline{i}^a = 1$, labor with $r \geq \tau_{\bar{i}^a+1}$ does not allocate to tasks $i \in [\underline{i}^a, \bar{i}^a]$.

Intuitively, this assumption reflects the preference of workers to not be employed in an industry which only pays wage $w^a\kappa(r)$, with $w^a < 1$, if they are skilled enough to work in another industry i paying $\omega_i\kappa(r)$ with $\omega_i \geq 1$. If $\underline{i}^a > 1$, workers with skill level $r \in [\tau_{\underline{i}^a}, \tau_{\bar{i}^a})$ prefer to “flee” to low-complexity tasks $i < \underline{i}^a$. In practice, this could mean that medium-skilled workers move from medium-complexity office jobs to manual work that cannot be carried out by AI. If $\underline{i}^a = 1$, there are no exclusively human low-complexity jobs left, such that any worker with $r < \tau_{\bar{i}^a+1}$ works in tasks with wage $w^a\kappa(r)$.

Further, we assume that in case $\underline{i}^a = 1$ representative firms $i \in [\underline{i}^a, \bar{i}^a]$ hire all skill levels $r \in [\tau_i, \tau_{i+1})$, i.e., \tilde{L}_i^Δ . This assumption is needed to solve for specific \tilde{l}_i and A_i for $i \in [\underline{i}^a, \bar{i}^a]$ but not otherwise.

With these assumptions, effective labor demand (16) becomes

$$\tilde{l}_i = \begin{cases} \tilde{L}_i^\Delta & \text{for } i \in [\underline{i}^a, \bar{i}^a] \text{ if } \underline{i}^a = 1, \\ 0 & \text{for } i \in [\underline{i}^a, \bar{i}^a] \text{ if } \underline{i}^a > 1, \\ \kappa(1)^{-1} \omega_i^{-\sigma} \left(\sum_{\tilde{i} \in \mathcal{I}} \omega_{\tilde{i}}^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}} C & \text{for } i \notin [\underline{i}^a, \bar{i}^a]. \end{cases} \quad (\text{G.1})$$

In all cases, A_i follows as the residual of output produced by labor and consumer demand:

$$A_i = c_i - \kappa(1) \tilde{l}_i. \quad (\text{G.2})$$

Using (15), we can write c_i in (G.2) as a function of C and scaling factors.

We can solve for C in (G.1) and (G.2) using total wages TW . Use (G.1) to obtain

$$\begin{aligned} TW &= \kappa(1) \sum_i \omega_i \tilde{l}_i \\ &= \begin{cases} w^a \left(\tilde{L}_{\underline{i}^a}^s - \tilde{L}_{\bar{i}^a+1}^s \right) + \sum_{\tilde{i}=\underline{i}^a+1}^{\bar{i}^a} \omega_{\tilde{i}}^{1-\sigma} \left(\sum_{\tilde{i}} \omega_{\tilde{i}}^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}} C & \text{if } \underline{i}^a = 1, \\ \sum_{\tilde{i} \notin [\underline{i}^a, \bar{i}^a]} \omega_{\tilde{i}}^{1-\sigma} \left(\sum_{\tilde{i}} \omega_{\tilde{i}}^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}} C & \text{if } \bar{i}^a > 1. \end{cases} \\ \Leftrightarrow C &= \begin{cases} \frac{\left(TW - w^a \left(\tilde{L}_{\underline{i}^a}^s - \tilde{L}_{\bar{i}^a+1}^s \right) \right) \left(\sum_{\tilde{i}} \omega_{\tilde{i}}^{1-\sigma} \right)^{\frac{\sigma}{\sigma-1}}}{\sum_{\substack{\tilde{i}=\underline{i}^a+1 \\ \tilde{i} \in [\underline{i}^a, \bar{i}^a]}} \omega_{\tilde{i}}^{1-\sigma}} & \text{if } \underline{i}^a = 1, \\ \frac{TW \left(\sum_{\tilde{i}} \omega_{\tilde{i}}^{1-\sigma} \right)^{\frac{\sigma}{\sigma-1}}}{\sum_{\tilde{i} \notin [\underline{i}^a, \bar{i}^a]} \omega_{\tilde{i}}^{1-\sigma}} & \text{if } \underline{i}^a > 1. \end{cases} \end{aligned}$$

Here, we used $\tilde{L}_i^s := \sum_{\tilde{i} \geq i} \tilde{L}_{\tilde{i}}^\Delta = \int_{\tau_i}^1 \frac{\kappa(r)}{\kappa(1)} L^r dr$ as defined in Appendix B. Now, we have obtained C as a function of TW and the scaling factors. TW can be computed as

$$TW = \begin{cases} \kappa(1) \sum_{i \in \mathcal{I}} \omega_i \tilde{L}_i^\Delta & \text{if } \underline{i}^a = 1, \\ \kappa(1) \left(\sum_{i \notin [\underline{i}^a, \bar{i}^a]} \omega_i \tilde{L}_i^\Delta + \omega_{\underline{i}^a-1} \sum_{i \in [\underline{i}^a, \bar{i}^a]} \tilde{L}_i^\Delta \right) & \text{if } \underline{i}^a > 1. \end{cases}$$

The expression for case $\underline{i}^a > 1$ follows from Assumption 3—no workers will allocate to tasks $i \in [\underline{i}^a, \bar{i}^a]$ that only pay $w^a \kappa(r)$ if $\underline{i}^a > 1$.

Knowing C and TW , we have fully derived a solution for \tilde{l}_i and A_i given the scaling factor scheme and can characterize the equilibrium.

Proposition 3 (Equilibrium in the AI-Augmented Task-Complexity Model)
The equilibrium in the AI-augmented task-complexity model is characterized by a group structure \mathcal{G}^ and by*

$$(i) \quad \mathcal{W}^* = \{\omega^*(r)\kappa(r)\}_{r \in \mathcal{R}},$$

$$(ii) \quad \mathcal{R}_i^* \subseteq \{r \in \mathcal{R} | r \geq \tau_i\} \cap \{r \in \mathcal{R} | \omega^*(r) = \omega_i^*\} \quad \forall i \in \mathcal{I},$$

$$(iii) \quad p_i^* = \frac{\sigma}{\sigma-1} \omega_i^*,$$

$$(iv) \quad P^* = \frac{\sigma}{\sigma-1} \left(\sum_{i \in \mathcal{I}} (\omega_i^*)^{1-\sigma} \right)^{\frac{1}{1-\sigma}},$$

$$(v) \quad TW^* = \begin{cases} \kappa(1) \sum_{i \in \mathcal{I}} \omega_i^* \tilde{L}_i^\Delta & \text{if } \underline{i}^a = 1, \\ \kappa(1) \left(\sum_{i \notin [\underline{i}^a, \bar{i}^a]} \omega_i^* \tilde{L}_i^\Delta + \omega_{\underline{i}^a-1}^* \sum_{i \in [\underline{i}^a, \bar{i}^a]} \tilde{L}_i^\Delta \right) & \text{if } \underline{i}^a > 1, \end{cases}$$

$$(vi) \quad C^* = \begin{cases} \frac{\left(TW^* - w^a (\tilde{L}_{\underline{i}^a}^s - \tilde{L}_{\bar{i}^a+1}^s) \right) \left(\sum_{\tilde{i}} (\omega_{\tilde{i}}^*)^{1-\sigma} \right)^{\frac{\sigma}{\sigma-1}}}{\sum_{\tilde{i}=\bar{i}^a+1}^{\underline{i}^a} (\omega_{\tilde{i}}^*)^{1-\sigma}} & \text{if } \underline{i}^a = 1, \\ \frac{TW^* \left(\sum_{\tilde{i}} (\omega_{\tilde{i}}^*)^{1-\sigma} \right)^{\frac{\sigma}{\sigma-1}}}{\sum_{\tilde{i} \notin [\underline{i}^a, \bar{i}^a]} (\omega_{\tilde{i}}^*)^{1-\sigma}} & \text{if } \underline{i}^a > 1, \end{cases}$$

$$(vii) \quad \tilde{l}_i^* = \begin{cases} \tilde{L}_i^s - \tilde{L}_{i+1}^s & \text{for } i \in [\underline{i}^a, \bar{i}^a] \text{ if } \underline{i}^a = 1, \\ 0 & \text{for } i \in [\underline{i}^a, \bar{i}^a] \text{ if } \underline{i}^a > 1, \\ \kappa(1)^{-1} (\omega_i^*)^{-\sigma} \left(\sum_{\tilde{i} \in \mathcal{I}} (\omega_{\tilde{i}}^*)^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}} C^* & \text{for } i \notin [\underline{i}^a, \bar{i}^a], \end{cases}$$

$$(viii) \quad x_i^* = \left(\frac{p_i^*}{P^*} \right)^{-\sigma} C^*,$$

$$(ix) \quad A_i^* = x_i^* - \kappa(1) \tilde{l}_i^*,$$

$$(x) \quad \pi_i^* = \frac{\omega_i^*}{\sigma-1} x_i^*$$

G.2 Solution Algorithm in the Presence of AI

Next, we derive the solution to the scaling factor scheme in the AI-augmented task-complexity model. Let us start with the labor market clearing condition in its general form (13), using $\tilde{L}_i^s := \sum_{\tilde{i} \geq i} \tilde{L}_{\tilde{i}}^\Delta$:

$$\tilde{L}_i^s \geq \sum_{\tilde{i} \geq i} \tilde{l}_{\tilde{i}} \quad \forall i \in \mathcal{I}. \quad (\text{G.3})$$

We proceed by distinguishing the cases given in the labor demand scheme (G.1):

1. Case $\underline{i}^a = 1$:

By Assumption 3, the labor market clearing condition (LMCC) needs to hold with equality for $i = \bar{i}^a + 1$, the least complex task AI cannot perform. Using (G.1), we find

$$\begin{aligned}\tilde{L}_{\bar{i}^a+1}^s &= \kappa(1)^{-1} \sum_{\tilde{i} \geq \bar{i}^a+1} \omega_{\tilde{i}}^{-\sigma} \left(\sum_{\tilde{i}} \omega_{\tilde{i}}^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}} C \\ \Leftrightarrow \kappa(1)^{-1} \left(\sum_{\tilde{i}} \omega_{\tilde{i}}^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}} C &= \tilde{L}_{\bar{i}^a+1}^s \left(\sum_{\tilde{i} \geq \bar{i}^a+1} \omega_{\tilde{i}}^{-\sigma} \right)^{-1}.\end{aligned}$$

This can be plugged into the LMCC for any $i \geq \bar{i}^a + 1$:

$$\tilde{L}_i^s \geq \frac{\sum_{\tilde{i} \geq i} \omega_{\tilde{i}}^{-\sigma}}{\sum_{\tilde{i} \geq \bar{i}^a+1} \omega_{\tilde{i}}^{-\sigma}} \tilde{L}_{\bar{i}^a+1}^s.$$

Observe that this is equivalent to (B.9) with index 1 replaced by $\bar{i}^a + 1$. Therefore, we can follow the steps outlined in Appendix C. Find

$$\omega_i = \begin{cases} w^a & \text{for } i \in [\underline{i}^a, \bar{i}^a], \\ 1 & \text{for } i = \bar{i}^a + 1, \\ \max \left\{ \omega_{i-1}, \Omega_i^{\underline{i}^a=1} \right\} & \text{for } i > \bar{i}^a + 1, \end{cases}$$

with

$$\Omega_i^{\underline{i}^a=1} = \left(\frac{\Lambda_i^{\underline{i}^a=1} - \Lambda_{i_b+}^{\underline{i}^a=1}}{(i_{b+} - i) (1 + \Lambda_{i_b+}^{\underline{i}^a=1})} \sum_{\tilde{i} \in (\bar{i}^a, i)} \omega_{\tilde{i}}^{-\sigma} \right)^{-\frac{1}{\sigma}} \quad \forall i > \bar{i}^a,$$

$$\text{where } \Lambda_i^{\underline{i}^a=1} \equiv \frac{\tilde{L}_i^s}{\tilde{L}_{\bar{i}^a+1}^s - \tilde{L}_i^s} \quad \forall i \geq \bar{i}^a + 1.$$

2. Case $\underline{i}^a > 1$:

Combining (G.3) and (G.1), we can rewrite the LMCC as

$$\tilde{L}_i^s \geq \kappa(1)^{-1} \sum_{\tilde{i} \geq i, \tilde{i} \notin [\underline{i}^a, \bar{i}^a]} \omega_{\tilde{i}}^{-\sigma} \left(\sum_{\tilde{i}} \omega_{\tilde{i}}^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}} C. \quad (\text{G.4})$$

Imposing equality for $i = 1$, we find

$$\kappa(1)^{-1} \left(\sum_{\tilde{i}} \omega_{\tilde{i}}^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}} C = \tilde{L}_1^s \left(\sum_{\tilde{i} \geq 1, \tilde{i} \notin [\underline{i}^a, \bar{i}^a]} \omega_{\tilde{i}}^{-\sigma} \right)^{-1},$$

which can be used in (G.4) to arrive at

$$\tilde{L}_i^s \geq \frac{\sum_{\tilde{i} \geq i, \tilde{i} \notin [\underline{i}^a, \bar{i}^a]} \omega_{\tilde{i}}^{-\sigma}}{\sum_{\tilde{i} \geq 1, \tilde{i} \notin [\underline{i}^a, \bar{i}^a]} \omega_{\tilde{i}}^{-\sigma}} \tilde{L}_1^s.$$

Observe that this is equivalent to (B.9) with all indices $i \in [\underline{i}^a, \bar{i}^a]$ excluded in the summation terms. Following the steps outlined in Appendix C, we find

$$\omega_i = \begin{cases} 1 & \text{for } i = 1, \\ \max \left\{ \omega_{i-1}, \Omega_i^{\underline{i}^a > 1} \right\} & \text{for } i \in (1, \underline{i}^a) \text{ and for } i > \bar{i}^a + 1, \\ w^a & \text{for } i \in [\underline{i}^a, \bar{i}^a], \\ \max \left\{ \omega_{\underline{i}^a-1}, \Omega_{\bar{i}^a+1}^{\bar{i}^a > 1} \right\} & \text{for } i = \bar{i}^a + 1. \end{cases}$$

with

$$\Omega_i^{\underline{i}^a > 1} = \left(\frac{\Lambda_i - \Lambda_{i_{b+}}}{(i_{b+} - i) (1 + \Lambda_{i_{b+}})} \sum_{\tilde{i} < i, \tilde{i} \notin [\underline{i}^a, \bar{i}^a]} \omega_{\tilde{i}}^{-\sigma} \right)^{-\frac{1}{\sigma}} \quad \forall i \notin [\underline{i}^a, \bar{i}^a].$$

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