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Doctoral Thesis
Incentive models of the labour market

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Incentive Models of the Labour Market

Inauguraldissertation
zur Erlangung des akademischen Grades eines
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1 Introduction

In the new stage of globalization the broad and substantial reorganization of production and work leads to a geographic decomposition of value chains, a rising importance of personal relationships, and an increasing flexibility, heterogeneity and versatility of work. These developments lead to changing, often unpredictable, patterns of winners and losers from globalization and pose a challenge of enormous proportions to the welfare state - in developed as well as in emerging economies -, which broadly suffers from being inefficient and not adjusting to people’s welfare needs. It must protect losers and spread the benefits from globalization to keep new protectionist tendencies at bay, but it must do it in ways that do least damage to people’s incentives to work, to educate and train themselves, and to save and invest. What is required, is not a passive welfare state, but a policy framework that makes people more adaptable and versatile. The welfare state must provide people with the tools to adjust to changing market conditions to turn themselves from losers into winners through their own efforts.

This dissertation aims at investigating policies that enhance people’s adaptability and seeks to providing suitable analytical frameworks for this analysis which are rich enough to capture the necessary heterogeneities and simple enough to generate straight-forward, intuitively transparent policy recommendations.

In doing so, the main focus of this dissertation lies on economic incentives and it provides various simple analytical frameworks to explicitly model the incentives of labour market agents. This dissertation does not rely on the workhorse model of the labour market, the search and matching model by Mortensen and Pissarides (1994), which is often applied to analyze the effects of labour market policies. This literature rests on the assumption of a stable matching function, which describes the relation between unemployment and vacancies and the number of matches. It is admissible to use the matching function for macroeconomic analysis and to investigate the effectiveness of labour market policies provided that neither macroeconomic variables nor the policies have any significant influence on the matching process itself. It seems implausible that the matching function is invariant with respect to labour market policies that are actually designed to improve the matching process. The resulting instability of the matching function prohibits its use for the analysis of labour market policies or macroeconomic fluctuations, since it could run afoul of the Lucas critique (see Lucas, 1976).

Instead of assuming a policy-invariant matching function, using purely microfounded models
the dynamics of the labour market are described as the outcome of optimizing behaviour by economic agents. These analytical frameworks are behavioural Markov models of the labour market in which the dynamics of employment and unemployment are determined by transition probabilities among various labour market states. These transition rates between employment and unemployment are a function of the employment incentives of economic agents and are derived from optimization principles. Furthermore, these models explicitly incorporate different heterogeneities and frictions, and take a variety of common labor market imperfections into account (insider wage bargaining, hiring and firing costs, and imperfections related to the tax and transfer system, etc.).

This dissertation exploits these attractive features of the models to analyze the economic incentives generated by welfare policies, derive the behavioral implications and quantify the resulting responses.

In line with policymakers’ actual concerns budgetary constraints of the government are explicitly taken into account and the effectiveness of the policies under consideration is often measured not just in terms of employment, but also in terms of inequality or welfare.

In what follows, three policies to respond to the challenges for the welfare state from the new stage of globalization are analyzed with differently focussed incentive-based models; in addition a new modelling framework for policy analysis is proposed.

The following Chapter 2 proposes to replace the current unemployment benefit system with unemployment accounts, which promote adaptability by giving people property rights in welfare services. A simple household model is used to analyze how switching from the existing system to a system of unemployment accounts would affect people’s employment incentives. Then this model is calibrated for the high-unemployment countries of Europe and the unemployment and welfare implications of the switch are then examined.

Chapter 3 investigates the effectiveness of differently targeted employment subsidies in supporting the losers from globalization by considering various groups of workers with heterogeneous and partly endogenous skills. A firm side model is calibrated exemplarily to the German labour market and the implications for the targetting scheme as well as for the magnitude of intervention are derived.

The household side model from Chapter 2 and the firm side model from Chapter 3 are then brought together in Chapter 4 to enable a more holistic analysis of the interactions of a set of policies and their effects. Specifically, this chapter performs the policy experiment of introducing the Danish flexicurity policies in Germany. This flexicurity concept aims at providing employment security by creating flexible and adaptable labour markets.

In Chapter 5 the usefulness of the matching function for policy analysis is examined and a new modelling strategy with heterogenous workers and jobs is proposed for moving beyond the matching function in future policy evaluation. Furthermore, the new framework’s performance in accounting for important empirical regularities of the US economy is examined in a business cycle analysis.


2 Unemployment Accounts and Employment Incentives

2.1 Introduction

This chapter\textsuperscript{4} explores the implications of reforming labour market policy to replace the unemployment benefit (UB) system, in which the unemployed receive benefits that are financed through taxes on the employed, with an unemployment accounts (UAs) system. Under unemployment accounts, people have individual unemployment accounts, to which they make ongoing contributions when they are employed. The balances in these accounts are then drawn upon during periods of unemployment. These withdrawals from UAs substitute for unemployment benefits. UAs are hence compulsory saving to provide security against the income loss of unemployment. An UAs system need not, and in our view should not, remove redistribution and equity as design considerations. To achieve its equity objectives in a UAs system, the government can make balanced-budget interpersonal redistributions among the UAs, taxing the accounts of higher-income people and subsidizing those of lower-income people. At the end of their working lives, people could transfer the remaining balances on their UAs into the pension accounts.

The same principle can equally well be applied to incapacity benefits: Current incapacity benefits could be transformed into incapacity accounts. People could draw on these accounts while they are incapacitated and, when they retire, use the remaining balances to top up their pensions.

This chapter presents a simple model of how switching from the UB to the UAs system would affect people’s incentives to work and search for jobs. We then calibrate this model for the high-unemployment countries of Europe and examine the employment and welfare implications of the switch.

In practice, the UAs system would run along the following general lines.\textsuperscript{5} Each employed worker contributes a fixed mandatory minimum amount to his or her account each month. Voluntary contributions in excess of the minimum amount are allowed. Upon becoming unemployed an individual is entitled to withdraw a predetermined maximum amount per month. Smaller withdrawals are also allowed.

When a person’s account balance is zero, the person is entitled to unemployment assistance, on the same terms and conditions as under the current UB system. In addition, as noted, the government can subsidize the contributions of low-income people. Both these expenditures are financed by taxing contributions of other unemployment account holders. When people’s UAs balances are sufficiently high, they can use the surplus funds for other purposes; and, as noted, when they retire, their remaining UAs balances can be used to top up their pensions.\textsuperscript{6}

The UAs system can be run on a pay-as-you-go (PAYG) or fully funded basis.\textsuperscript{7} If the UAs

\textsuperscript{4}For a different version of this chapter see Brown et al. (2008).

\textsuperscript{5}In the model below, we simplify several aspects of this account for the sake of analytical simplicity and transparency.

\textsuperscript{6}An unemployed person could also be permitted to use a portion of his UAs balance to provide employment vouchers to employers who employ him. See Orszag and Snower (2000) and Brown et al. (2007a).

\textsuperscript{7}This aspect is potentially important, for a standard criticism of personalized accounts in other areas of
system is fully funded, then the contribution rates can be set in an actuarially fair manner so that, for all the UAs of a particular age cohort in the economy, the discounted value of aggregate minimum benefits is equal to the discounted value of aggregate contributions.\(^8\) If the UAs system is run on a PAYG basis, cross-subsidization of accounts would also extend across generations. In particular, a part of the UAs balances of young people then finances the withdrawals of older people.

Since the UAs system is compatible with both PAYG and fully funded schemes, the transition from the former to the latter can proceed at any pace desirable. The closer the system is to being fully funded, the more discretion people can be given in determining who is to manage their UAs, the government or private sector financial institutions. The investment activity of the latter institutions would of course have to be regulated so as to protect individuals.\(^9\)

While UAs are in principle savings accounts, they involve two main advantages over the laissez-faire stance of simply letting people save whatever they want to protect themselves from the income loss of unemployment. First, UAs with mandatory contributions mitigate the moral hazard problem, namely, that individuals - knowing that the government will support them in unemployment regardless of how much they have saved - will have insufficient incentive to save enough.\(^10\) And second, the UAs system also fulfills a redistributive function, whereby people who are unable to support themselves out of their savings receive support from others.\(^11\)

Intuitively, the case for switching from the UB to the UAs system is straightforward. Current UB systems, broadly speaking, provide unemployment benefits under the condition that the recipients are unemployed and benefits are financed through taxes falling primarily on the employed. When unemployed people find jobs, their benefits generally are withdrawn (in whole or in part) and taxes are imposed. In effect, under an UB system, people are rewarded for being unemployed (through unemployment benefits) and penalized for being employed (through taxes). The UB system thereby creates an externality, distorting the incentives to work and save, since the unemployed impose costs on the employed.

The unemployed do not take the full social costs of their unemployment into account when seeking jobs. In this way, the UB system depresses job search and thereby stimulates unem-

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\(^8\)This method could ensure that generational accounts are in balance. But since some of the UAs balances of higher-income individuals would be used to subsidize the contributions of low-income individuals and finance unemployment assistance, the contribution rates would not be actuarially fair for each individual.

\(^9\)Implementing a fully funded system poses many choices on how to invest the funds. Whether these yield additional gains is subject to dispute, e.g. Shiller (2005) criticises the specific life-cycle portfolio plan of the personal accounts proposal for social security in the US. We focus on different implications: our accounts cover unemployment (instead of pensions) and we focus on the resulting employment incentives.

\(^10\)Along the same line, maximum withdrawal rates avoid excessive withdrawals.

\(^11\)Barro (2005) argues that, for pension accounts, there is no good reason to go beyond the minimum standard of living, as voluntary saving should be sufficient above this minimum payout. Our proposal, by contrast, deals with unemployment accounts and provides support up to the level of existing unemployment benefits, which presumably often exceed Barro’s “minimum standard of living.” It is worth noting that in a stochastic world, guaranteeing a minimum standard of living naturally has adverse employment incentives on people whose income lies above the minimum since, ex ante, they are not certain whether their income will exceed or fall short of the minimum level. We show that such disincentives are substantially smaller under UAs than under a UB system.
ployment. Furthermore, the employed do not receive full compensation for the social benefits from their employment and thus, if the relevant substitution effect dominates the income effect, they will work less hard than they otherwise would have. Thereby, the UB system may depress productivity and thereby reduce employment.

Not all of the unemployment benefits and taxes under the UB system are interpersonal redistributions. On the contrary, most of the people who are unemployed at one point in time are employed at other times, and thus part of the taxes they pay when they are employed serve to pay the benefits when they are unemployed, i.e. they are in effect paying themselves. This is an "intrapersonal" redistribution in the form of intertemporal income smoothing, rather than an interpersonal redistribution. These intrapersonal redistributions are handled inefficiently under the UB system, since both the taxes and the benefits create externalities that promote unemployment.\textsuperscript{12}

The UAs system alleviates these externality problems. For when an unemployed person makes withdrawals from his UAs, he is thereby diminishing the amount of funds that are available to him later on. Thus, in comparison to the UB system, the unemployed internalize more of the social costs of their unemployment and thus have greater incentives to search for jobs. When an employed person makes contributions to her UAs, she is thereby increasing the account balance that she can draw on in the future. Hence, employed people internalize more of the social benefit of their employment than under the UB system and thus have greater incentives to work.

Of course the interpersonal redistributions in the UAs system do create externalities that generate disincentives for job search and for work. But these disincentives are lower in the UAs system than in the UB system. The reason is that the UAs system redistributes income more efficiently: Since intrapersonal redistributions are conducted through the UAs rather than through taxes, the costs and benefits of these redistributions are internalized by the account holder, whereas under the UB system an employed person whose taxes pay for her subsequent unemployment benefits does not internalize the costs and benefits. These taxes discourage work effort and these unemployment benefits discourage job search under UBs, whereas the corresponding intrapersonal redistributions under UAs do not. Hence, UAs generates less unemployment and thus there is less need for interpersonal redistributions.

In short, under the UAs system the intrapersonal redistributions lead to lower interpersonal redistributions and thereby to higher employment, lower unemployment and higher productivity without making the unemployed worse off. We will take a first step towards quantifying these effects below.

This chapter provides an analytical framework for assessing the labour market implications of switching from the UB to the UAs system. The chapter is organized as follows. Section

\textsuperscript{12}Several empirical studies have illustrated that intrapersonal redistributions play a dominant role in total social expenditures. Among others, Björklund (1993) reports, that lifetime incomes are distributed more equally than annual incomes. Estimates on which portion of total social expenditure actually is redistributed between individuals' lifetime incomes (i.e. interpersonal redistribution) have been provided for a number of countries: for Sweden 24\%, Fölster (1997); for Denmark 26\%, Sørensen et al.; for Australia 48-62\% and for Great Britain 29-38\%, Falkingham and Harding (1996).
2.2 surveys the related literature. Section 2.3 depicts the UB and the UAs system in the context of a simple overlapping-generations model and derives the incentives for job search and work. Section 2.4 specifies functional forms for this model and calibrates it for European high-unemployment countries. The calibration is used to derive how the switch from the UB to the UAs system affects unemployment and welfare. Section 2.5 concludes.

### 2.2 Related Literature

Individual welfare accounts are widely discussed in the public debate on welfare reform and have also been put into practice in several countries. Practical proposals for unemployment accounts (individual accounts to cover income loss from unemployment) have been made by various authors, but have been only implemented recently in Chile. The UAs system in Chile includes a so-called Solidarity Fund to provide minimum unemployment benefits if the account balances are insufficient to provide the benefit payment. Unemployment accounts in a different sense - essentially severance savings accounts - have been implemented several Latin American countries and in Austria. In contrast to our proposal, these are not a substitute for the unemployment benefit system.

Our theoretical analysis of UAs must be distinguished from the literature on optimal unemployment insurance. While the latter is concerned with the efficient design of the UB system (support levels, duration, eligibility, etc.), we take the design of the existing unemployment benefit systems as given and focus on how employment incentives would be affected by replacing the existing systems by personalized unemployment accounts. As is well known, the existing unemployment benefit systems pursue two potentially conflicting goals: insurance and redistribution. Optimal insurance under adverse selection implies that insurance premia must be positively related to employment risk; redistribution dictates the opposite. The aim of this chapter is not to analyze the roles of insurance and redistribution in current benefit systems,

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15For the mechanics of the system in Chile see Acevedo et al. (forthcoming), Schenbruch (2006), Conerly (2002) and ILO (2001).

16For Latin America see Mazza (2000), Jaramillo and Saavedra (2005), Ferrer and Riddell (forthcoming), specifically for the Colombian reform see Kugler (1999) and (2005) and for the Austrian reform see Koman et al. (forthcoming). For a proposal for Korea see Hur (forthcoming).

17While severance and seniority payments are similar to unemployment benefits in that they provide cash compensation in the case of unemployment, they differ as well in objectives as in creating externalities and in crucial design features. For a general discussion see Parsons (2004), Holzmann (2005), Vodopivec (2006) and Holzman et al. (forthcoming). Furthermore, in these countries severance payment accounts exist parallel to unemployment benefit systems, thereby, our proposal applies also to these countries. Both types of these accounts could well be integrated, but this goes beyond the scope of this paper.

18See for a survey Karni (1999) and for a recent paper e.g. Shimer and Werning (2005).
but rather to ask whether the support provisions of the existing systems (who gets how much under which conditions) can be fulfilled more efficiently through UAs. In particular, we ask whether UAs can create more favourable employment incentives without sacrificing the support provisions of the existing systems.

These effects are largely ignored in the existing applied literature on mandatory savings accounts in general and UAs specifically. Specifically, much of this literature has two undesirable features: (i) the analysis is static and thereby ignores the intertemporal effects that are inherently important in an account system and (ii) only mechanical impact effects (based on existing behaviour patterns) are investigated, rather than changes in employment behaviour as result of the institutional switch to UAs. Various authors examine empirically the feasibility of establishing an accounts system, as well as its distributional and government budget impacts: Feldstein and Altmann (2007) examine UAs for the US, Vodopivec (2008) for Slovenia, Vodopivec and Rejec (forthcoming) for Estonia, Fölster (1999), (2001) and Fölster et al. (2003a) investigate comprehensive welfare accounts for Sweden, Yun (forthcoming) simulates a specific integrated proposal for Korea and Kling (2006) simulates his specific reform proposal for the US. While these studies claim that UAs would improve employment incentives substantially, they do not take people’s response to these incentives into account. While Sørensen et al. (2006), who estimate a specific proposal for reform for the Danish welfare state, take people’s responses into account, their analysis is static and thereby ignores intertemporal effects.

We explicitly derive the resulting incentive effects within an intertemporal behavioural model and quantify the resulting responses. Similar models have been used to assess the welfare and government budget effects of integrated account systems encompassing unemployment insurance and pensions. These analyses, however, do not address the incentive and employment effects resulting from a shift from an unemployment benefit system to UAs, which are of concern here. For example, Hopenhayn and Hatchondo (2002) provide a cost-benefit analysis in terms of welfare and government cost savings for alternative specifications of UAs for Estonia. Bovenberg and Sørensen (2004) and Sørensen (2003) examine a specific proposal for comprehensive welfare accounts for Denmark and show that it could be self-financing and lead to a welfare gain.

Moreover, Stiglitz and Yun (2005) examine the optimal degree of integration of tax-financed unemployment benefits with government provision of pension-funded borrowing. The authors show improved job search incentives and the resulting welfare gains, which though are mainly a result of their assumption of capital market imperfections. In contrast, we examine the incentive effects for search and work effort under the assumptions that agents can freely borrow and lend at the prevailing interest rates and that borrowing contracts are enforced. Thereby, we explicitly disregard any effects resulting from creating a channel for consumption smoothing. Our effects result from a higher efficiency in the redistribution of income. Goerke (2007) investigates in a trade union model the employment and welfare consequences of integrated UAs which operate

\[19\] For example, Feldstein and Altmann (2007) merely present results for the assumption that as a result of possible behavioral responses unemployment duration is reduced by 10% or 30%.

\[20\] This analysis complements the accounting analysis by Vodopivec and Rejec (forthcoming)
solely via the wage, he thereby fully ignores individual work and search effort decisions. We explicitly shed light on the channels whereby the shift from an UB to an UAs system raises the search and work incentives and show that a more efficient redistribution of income via UAs would improve welfare and significantly reduce unemployment.

2.3 The General Model

Figure 2.1: The structure of the model.

Workers in our model live for two periods: in the first period the worker is "young", in the second period she is "old". The worker’s possible labour market states are illustrated in Figure 2.1. Upon entering the workforce, each worker faces a predetermined probability $h_e$ of becoming employed and a probability $(1 - h_e)$ of remaining unemployed. Let $V(y, n)$ and $V(y, u)$ be the discounted lifetime utilities of an employed and unemployed young worker, respectively. Then the discounted lifetime utility of an entrant $(e)$ to the workforce is:

$$V(e) = h_e V(y, n) + (1 - h_e) V(y, u) \quad (2.1)$$

A young, employed worker $(y, n)$ faces a probability $f_y$ of being fired at beginning of the second period, in which case he turns into an old, short-term unemployed $(o, su)$ worker. With probability $(1 - f_y)$ he is retained in the second period, in which case he and turns into an insider $(i)$, i.e. an employed incumbent worker.

Finally, a young, unemployed worker $(y, u)$ faces a probability $h_y$ of being hired at the beginning of the second period, whereupon he becomes an old, short-term employed worker $(o, sn)$. With probability $(1 - h_y)$ the young, unemployed $(y, u)$ does not find a job in the second period and becomes an old, long-term unemployed worker $(o, lu)$.

Thus, the unemployment rate of young workers is

$$u_y = (1 - h_e) \quad (2.2)$$

and the unemployment rate of old workers is

$$u_o = (1 - h_e)(1 - h_y) + h_e f_y. \quad (2.3)$$

For simplicity, we define the categories "young" and "old" so that both generations are of equal size and the overall unemployment rate is $u = \frac{1}{2}(u_y + u_o)$. 
Unemployed workers divide their time between leisure and job search; employed workers divide their time between on-the-job leisure (e.g. shirking) and work. The hiring rates in our model depend on search intensity (i.e. the length of time unemployed workers spent searching), and the firing rates depend on work effort (i.e. the length of time spent working), which determines the worker’s productivity. Workers make their leisure and consumption decisions so as to maximize their discounted lifetime utilities, taking into account the effects of their leisure choices on the hiring and firing rates.\(^{21}\)

For simplicity, entrants are assumed to devote all their time to job search, and thus the entrants’ hiring rate \(h_e\) may be taken as an exogenously given constant. All old workers are assumed to exert a given, fixed level of effort, since they maintain their current (un)employment state and die in the following period. Thus search and work incentives in our model can be assessed simply by examining the leisure and consumption decisions of young workers.

Workers are assumed to have access to capital markets, so that they are able to save their current income or borrow against their future incomes at the market interest rate. This assumption is of particular interest in our context, since it allows us to explore the degree to which savings are a substitute for unemployment accounts. By assuming perfect capital markets, we bias our model against unemployment accounts. With imperfect capital markets, unemployment accounts would increase economic efficiency, at least with voluntary contribution levels, by providing households with a channel for transferring purchasing power through time.

2.3.1 Job Search and Work Effort Decisions

As noted, the hiring rate \(h_y\) for young unemployed workers \((y, u)\) depends inversely on their leisure \(l_{y,u}\): the more leisure they consume, the less time they spend on job search and thus the fewer jobs they are likely to find. The firing rate \(f_y\) for young, employed workers \((y, n)\) depends positively on their leisure \(l_{y,n}\): workers who shirk (indulge in more leisure) when young are less likely to be productive when old because of "learning by doing", and thus, more likely to be fired by the firm. (The microfoundations of the hiring and firing functions are presented in appendix 2.A.1.)

A young, employed worker \((y, n)\) has the period utility \(v(c_{y,n}, l_{y,n})\), where \(c_{y,n}\) is consumption and \(l_{y,n}\) is the worker’s leisure. In the second time period, he receives an old, short-term unemployed worker’s utility \(V(o, su)\) with probability \(f_y(l_{y,n})\), and an insider’s utility \(V(i)\) with probability \((1 - f_y(l_{y,n}))\). Since the leisure of the old worker is fixed and the young worker transfers unconsumed income into the second period, \(V(i)\) and \(V(o, su)\) are determined by the young worker’s consumption decision.

The young, employed worker maximizes the present value of utility over leisure \(l_{y,n}\) and consumption \(c_{y,n}\):

\[
V(y, n) = \max_{l_{y,n}, c_{y,n}} \left[ v(c_{y,n}, l_{y,n}) + \beta (f_y(l_{y,n})V(o, su) + (1 - f_y(l_{y,n}))V(i)) \right] \tag{2.4}
\]

\(^{21}\)The model is a simple two-period variant of the labour market model developed by Phelps (1994a). Our innovations include the incorporation of job search and the analysis of unemployment accounts in this setting.
where $\beta$ is the discount factor. The first-order conditions for this decision problem are:

\[
v_{y,u,n} = \beta f_{y,u,n} [V(i) - V(o, su)] \tag{2.5}
\]

\[
v_{c_{y,u},n} = -\beta \left[ f_{y}(l_{y,u,n}) \frac{dV(o, su)}{dc_{y,u,n}} + (1 - f_{y}(l_{y,u,n})) \frac{dV(i)}{dc_{y,u,n}} \right] \tag{2.6}
\]

In other words, the marginal utility of leisure must be set equal to the discounted marginal firing propensity ($\beta f_{y,u,n}$) times the reward for keeping a job ($[V(i) - V(o, su)]$). Given diminishing marginal utility of leisure, the optimal leisure depends inversely on the reward for keeping a job.\(^{22}\) Furthermore, the marginal utility of consumption when young ($v_{c_{y,u},n}$) must be equal to the discounted marginal disutility of not consuming when old, which is a weighted average of the consumption sacrificed when being an old, short-term unemployed worker ($\frac{dV(o, su)}{dc_{y,u,n}}$) and when being an insider ($\frac{dV(i)}{dc_{y,u,n}}$). The weights are given by the respective probabilities of being fired ($f_{y}(l_{y,u,n})$) and being retained ($1 - f_{y}(l_{y,u,n})$).

Along similar lines, a young, unemployed worker has a period utility $v(c_{y,u}, l_{y,u})$, where $c_{y,u}$ is the worker’s consumption and $l_{y,u}$ his leisure. In the second time period, she receives the utility of an old, short-term employed worker $V(o, sn)$ with probability $h_{y}(l_{y,u})$, and the utility of an old, long-term unemployed worker $V(o, lu)$ with probability $(1 - h_{y}(l_{y,u}))$. As above, $V(o, sn)$ and $V(o, lu)$ are fixed by the young worker’s consumption decision.

Thus, the young, unemployed worker’s decision problem is to find the level of leisure $l_{y,u}$ and consumption $c_{y,u}$ that maximizes the present value of utility:

\[
V(y, u) = \max_{l_{y,u}, c_{y,u}} \left[ v(c_{y,u}, l_{y,u}) + \beta (h_{y}(l_{y,u})V(o, sn) + (1 - h_{y}(l_{y,u}))V(o, lu)) \right] \tag{2.7}
\]

The first-order conditions for this problem are:

\[
v_{y,u} = -\beta h_{y,u} [V(o, sn) - V(o, lu)] \tag{2.8}
\]

\[
v_{c_{y,u}} = -\beta \left[ h_{y}(l_{y,u}) \frac{dV(o, sn)}{dc_{y,u}} + (1 - h_{y}(l_{y,u})) \frac{dV(o, lu)}{dc_{y,u}} \right] \tag{2.9}
\]

Here, the marginal utility of leisure must be set equal to the discounted marginal hiring propensity ($-\beta h_{y,u}$) times the reward for seeking a job ($[V(o, sn) - V(o, lu)]$). As above, diminishing marginal utility of leisure implies that the optimal level of leisure depends inversely on the reward for seeking a job.\(^{23}\) Accordingly, the marginal utility of consumption when young ($v_{c_{y,u}}$) must be equal to the discounted marginal disutility of not consuming when old, which is a weighted average of the consumption sacrificed when being an old, short-term employed worker

\(^{22}\)This is true if (as assumed below) leisure and consumption are complements or weak substitutes. If they were sufficiently strong substitutes, then the decrease in period one consumption might lead to a sufficient increase in the marginal utility of leisure to counterbalance the effect on leisure from the reward for remaining employed.

\(^{23}\)As in the previous footnote, this is true if (as assumed below) leisure and consumption are complements or weak substitutes.
and when being an old, long-term unemployed worker \( \frac{dV(o,lu)}{dc_{y,u}} \). Here the weights are given by the respective probabilities of being hired \( h_y(l_{y,u}) \) and remaining unemployed \( (1 - h_y(l_{y,u})) \).

An attractive feature of this model is that both job search and work effort are determined by the difference between the value of being employed and that of being unemployed (by the first-order conditions for leisure, eqs. 2.5 and 2.8). Below, we will show how the UB and UAs systems have different effects on this difference, which will help explain why the two systems have different unemployment and welfare outcomes.

### 2.3.2 The Unemployment Benefit (UB) System

In an unemployment benefit system each unemployed worker receives an exogenously given real unemployment benefit \( b \), and unemployment benefits are financed through a payroll tax, where \( \tau \) is the payroll tax rate. For simplicity, all employed workers are assumed to receive the same, exogenously given wage \( w \), normalized to unity.\(^{24}\)

The government is in balance in each time period, i.e. the government’s expenditures on unemployment benefits are equal to its tax receipts. Specifically, the value of unemployment benefits received by the unemployed workers (young, unemployed workers \( y,u \), old, long-term unemployed workers \( o,lu \), and old, short-term unemployed workers \( o,su \)) must be equal to the value of taxes paid by the employed workers (young, employed workers \( y,n \), insiders \( i \), and old, short-term employed workers \( o,sn \)) in each period:\(^{25}\)

\[
\begin{align*}
\nu u &= \tau (1 - u) \\
(2.11)
\end{align*}
\]

with the unemployment rate \( u \) determined by eq. 2.2 and 2.3 and with the replacement ratio\(^{26}\)

\[\nu u = \tau (1 - u)\]

---

\(^{24}\)By implication, we do not investigate the general equilibrium effects of the shift from the UB to the UAs system that operate via the wage. A switch from UB to UAs will affect not only the households’ work incentives and thus shift the labour supply curve outwards; it will also affect the firms’ incentives (via their hiring and firing rates) and thus shift the labour demand curve outwards. For this reason alone the effect on the wage is ambiguous. Which way the wage will move will depend on the relative size of the labour demand and supply curve shifts. Since this part of the story is well-understood and we wish to concentrate on our novel contribution, the direct incentive effects resulting from switching to UAs, we let wages be constant for our analysis. The resulting reduction in wage effects illustrated by Goerke (2007) is a result of the specific institutional assumption of collective wage determination.

\(^{25}\)Since our analysis focuses on the stationary steady state, this is of course equivalent to the condition that the present value of government expenditures is equal to the present value of government tax receipts. An equivalent formulation is that the deficit (surplus) generated by the young generation must be equal to the surplus (deficit) generated by the old generation.

\(^{26}\)Naturally, since we have normalized the wage to unity, the replacement ratio \( \nu \) is equal to the unemployment benefit \( b \) in our analysis. We nevertheless distinguish between these parameters to aid the reader’s intuition.
\( \nu = \frac{b}{w} \). Thus, the payroll tax rate under the UB system is:

\[
\tau = \frac{\nu u}{1 - u} = \frac{\nu ((1 - h_e) + (1 - h_e)(1 - h_y(l_{y,u})) + h_e f_y(l_{y,n}))}{(h_e + h_e (1 - f_y(l_{y,n})) + (1 - h_e)h_y(l_{y,u}))} \tag{2.12}
\]

By eq. 2.12, the payroll tax level is increasing in both the leisure of the employed and unemployed.

### 2.3.3 The Unemployment Accounts (UAs) System

Under the UAs system unemployed workers are assumed to receive a payment equal to the unemployment benefit \( b \) out of their UAs.\(^{27} \) This enables us to compare the incentives under both systems when the unemployed receive identical support. Thus, for a real interest rate \( r \), the contribution of a young, employed worker to her UAs must be \( \frac{b}{1 + r} \).

The payroll tax rate \( \kappa \) must be set so that total tax receipts by young and old employed workers are equal total expenditures on unemployment benefits. In contrast to the UB system old, short-term unemployed workers \((o, su)\) finance their own unemployment using their accounts.\(^{28} \) Thus, payments of \( b \) must be financed via taxes only for young, unemployed workers \((y, u)\) and the old, long-term unemployed workers \((o, lu)\), who have no balances on their accounts. The balanced budget constraint is:

\[
b ((1 - h_e) + (1 - h_e)(1 - h_y(l_{y,u}))) = \kappa w (h_e + h_e (1 - f_y(l_{y,n})) + (1 - h_e)h_y(l_{y,u})) \tag{2.13}
\]

which can be rewritten as

\[
\nu \left( u - \frac{1}{2} h_e f_y(l_{y,n}) \right) = \kappa (1 - u) \tag{2.15}
\]

with the unemployment rate \( u \) determined by eq. 2.2 and 2.3 and with the replacement ratio \( \nu = \frac{b}{w} \). Hence the tax rate is

\[
\kappa = \nu \frac{(u - \frac{1}{2} h_e f_y(l_{y,n}))}{(1 - u)} = \nu \frac{(1 - h_e) + (1 - h_e)(1 - h_y(l_{y,u}))}{h_e + h_e (1 - f_y(l_{y,n})) + (1 - h_e)h_y(l_{y,u})} \tag{2.16}
\]

Comparing the tax rates under the UB system (eq. 2.12) and UAs system (eq. 2.16), it is clear that, for any given unemployment rate, the tax rate is lower under the UAs system than under the UB system.

We now proceed to analyze how the job search and work effort decisions are affected by these UB and UAs systems.

\(^{27}\)This payment is financed either (a) by past forced savings or (if the account balances are insufficient to provide the payment (b) by government transfers.

\(^{28}\)To achieve better comparability to the UB system instead of taxing the contributions to the accounts, we simply tax the wage. Levying taxes just in the first period - on the contributions to the accounts - leads to the same qualitative results.
2.3.4 Comparison of Employment Incentives

As we have seen in the first-order conditions for leisure (eqs. 2.5 and 2.8), the leisure decisions depend negatively on the reward for keeping a job, which is the reward for work effort, and the reward for seeking a job, namely the reward for search effort. Thus, to understand why the UB and UAs systems generate different employment incentives, it is useful to consider what workers stand to lose from being unemployed under the two systems.

Table 2.1: Old workers' consumption and the associated rewards for keeping a job and seeking a job as a function of their past and current employment states.

<table>
<thead>
<tr>
<th>The Unemployment Benefit (UB) System</th>
<th>Employed, when old</th>
<th>Unemployed, when old</th>
<th>Reward $\Delta^B_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Employed, when young</td>
<td>$w(1 - \tau) + s_n(1 + r)$</td>
<td>$b + s_n^B(1 + r)$</td>
<td>$\Delta^B_n = w(1 - \tau) - b$</td>
</tr>
<tr>
<td>Unemployed, when young</td>
<td>$w(1 - \tau) + s_n^B(1 + r)$</td>
<td>$b + s_n^B(1 + r)$</td>
<td>$\Delta^B_u = w(1 - \tau) - b$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The Unemployment Accounts (UAs) System</th>
<th>Employed, when old</th>
<th>Unemployed, when old</th>
<th>Reward $\Delta^A_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Employed, when young</td>
<td>$w(1 - \kappa) + b + s_n^A(1 + r)$</td>
<td>$b + s_n^A(1 + r)$</td>
<td>$\Delta^A_n = w(1 - \kappa)$</td>
</tr>
<tr>
<td>Unemployed, when young</td>
<td>$w(1 - \kappa) + s_n^A(1 + r)$</td>
<td>$b + s_n^A(1 + r)$</td>
<td>$\Delta^A_u = w(1 - \kappa) - b$</td>
</tr>
</tbody>
</table>

Table 2.1 compares the two systems by describing old workers’ consumption as a function of the worker’s labour market history and also presents the associated rewards for keeping a job ($\Delta_n$) and seeking a job ($\Delta_u$). In what follows, the superscript "B" stands for the unemployment "benefit" system and the superscript "A" for the unemployment "accounts" system.

As noted above, workers are assumed to have access to capital markets and $s$ is the purchasing power transferred through saving into the second period by young workers, which earns interest at the interest rate $r$.\(^{29}\) This saving may be described as "voluntary", in the sense that it is the outcome of the the workers’ optimization decisions under the prevailing institutional setting (UB or UAs); it may be contrasted with the "forced saving" in the form of contributions to the UAs. As our workers live only two periods, in the second period they withdraw their total savings.

Under the UB system old workers receive $w(1 - \tau)$ plus their savings when employed and $b$ plus their savings when unemployed; thus they stand to lose $\Delta^B = w(1 - \tau) - b$ from being unemployed, regardless of whether they were employed or unemployed when they were young. Thus, in the UB system the reward for keeping a job ($\Delta^B_n$) is equal to the reward for seeking a job ($\Delta^B_u$).

Under the UAs system, by contrast, the respective rewards ($\Delta^A_n$ and $\Delta^A_u$) are not equal, and hence, the incentives are different - especially for young, employed workers. If they continue to be employed, they receive $w(1 - \kappa)$ and the sum of their savings, namely their interest-bearing voluntary savings $s_n^A(1 + r)$ and forced savings $b$ from the UAs. If they become unemployed, they still receive in addition to their voluntary savings $s_n^A(1 + r)$, their forced savings $b$ from the

---

\(^{29}\) As above, the subscript "n" refers to the employment state and "u" to the unemployment state.
accounts. Hence, in contrast to the UB system, the reward for keeping a job is \( \Delta^A_n = w(1 - \kappa) \).

As for old, short-term employed workers, they receive \( w(1 - \kappa) \) plus their interest-bearing savings \( s^A_n(1 + r) \) and old, long-term unemployed workers receive \( b \) plus their interest-bearing savings \( s^A_n(1 + r) \). Now the resulting difference, the reward for seeking a job, is \( \Delta^A_n = w(1 - \kappa) - b \).

In sum, in the UAs system workers stand to lose more from being unemployed: the rewards for keeping and seeking a job in consumption terms are higher, particularly the former, which is the reward for work effort to young, employed workers. Under an UAs system, these workers will not benefit from becoming unemployed (through the payment of unemployment benefits), in contrast to the UB system. The reason is, that under the UAs system these workers are forced to redistribute their income intrapersonally via their UAs. By financing their own possible future unemployment fully themselves via their accounts, these workers completely internalize the cost of their own unemployment and hence stand to lose more from being unemployed than under the UB system.\(^{30}\) We call this the internalization effect.

As the leisure decision depends negatively on the reward for keeping a job, the higher reward for keeping a job with UAs induces young, employed workers to increase their work effort (take less leisure at work). Consequently firing rates will fall and unemployment of old workers will be lower.

Additionally, under the UAs system young, employed workers stand to lose more from being unemployed when old (\( \Delta^A_n = w(1 - \kappa) \)) than workers who were unemployed when young under the UAs system and thus did not contribute to their UAs (\( \Delta^A_n = w(1 - \kappa) - b \)). Due to the internalization effect under the UAs system, the employment incentives depend on workers’ labour market history.

Since young, employed workers under the UAs system save for their own unemployment, taxes are only required to finance unemployment assistance for young, unemployed workers and long-term unemployed workers. The cost of unemployment of old, short-term unemployed workers is not imposed on others. Consequently the tax rate is lower under the UAs system than under the UB system (as indicated by eqs. 2.12 and 2.16). Lower taxes mean that workers receive higher rewards for keeping a job and seeking a job. We call this the tax reduction effect.

Hence, the tax reduction effect not only raises the reward for keeping a job under the UAs system relative to the UB system, it also generates a higher reward for seeking a job. Thus, young, unemployed workers have an incentive to search harder for jobs (take less leisure while unemployed)\(^{31}\) and, since hiring rates depend positively on search intensity, hiring rates will rise.

To be precise, there are in fact two tax reduction effects in our model. In addition to the direct tax reduction effect above (whereby young, employed workers finance their own unem-

\(^{30}\)In utility terms for any concave utility function this result depends on the respective size of the "voluntary" savings in both systems. For any concave utility function this result holds under the reasonable parameter values in our calibration and under the assumption that \( s^A_n(1 + r) + b > s^B_n(1 + r) > s^A_n(1 + r) \). Then the reward for keeping a job in utility terms \( \Lambda_n \) is greater under the UAs than the UB system: \( \Lambda^A_n = v(w(1 - \kappa) + s^A_n(1 + r)) - v(s^A_n(1 + r) + b) > \Lambda^B_n = v(w(1 - \kappa) + s^B_n(1 + r)) - v(b + s^B_n(1 + r)) \).

\(^{31}\)Again, the result in utility terms for any concave utility function depends on the size of the savings in the two systems. Under reasonable parameter values in our calibration this result holds: \( \Lambda^A_n = v(w(1 - \kappa) + s^A_n(1 + r)) - v(b + s^A_n(1 + r)) > \Lambda^B_n = v(w(1 - \kappa) + s^B_n(1 + r)) - v(b + s^B_n(1 + r)) \).
ployment support rather than receiving unemployment benefits financed through taxes), there is also an *indirect* tax reduction effect: The increased employment broadens the tax base and the lower unemployment implies that there are fewer unemployed workers with insufficient UAs balances to support themselves. Accordingly, the tax rate that is required in the UAs system to finance the unemployment support is even lower. (This in turn improves the incentives for job search and work effort even further, leading to another round of unemployment reductions, and so on.)

Summing up, firing rates are lower and hiring rates are higher under the UAs system than under the UB system and thus (by eqs. 2.2 and 2.3) unemployment is lower under the UAs system.

### 2.4 Evaluation of Employment Incentives

In this Section we explore whether the incentive effects identified above could be potentially important in practice. For this purpose, we specify particular functional forms for the behavioural relations above and derive the optimal search and work effort decisions for the UB and UAs systems. We then proceed to calibrate the resulting model, and evaluate the unemployment and welfare implications of moving from the UB to the UA system. Our analysis shows that, for reasonable parameter values, the unemployment reductions can be substantial in Europe’s high-unemployment countries, namely France, Germany, Italy and Spain. Naturally, these results need to be interpreted with extreme caution since the underlying model is extremely simple; but the potential effectiveness of UAs in reducing unemployment is nevertheless highlighted through this exercise.

#### 2.4.1 Specification

We start by specifying the functional forms.: Let the workers’ utility function be Cobb-Douglas:

\[ v(c, l) = \left(\frac{c^\alpha l^{(1-\alpha)}}{\gamma}\right)^\gamma \]  

(2.17)

and let hiring and firing rates be linear (where \( \theta \), \( a \) and \( \phi \) are parameters whose microfoundations are derived in appendix 2.A.1):

\[ h_y(l_{y,u}) = \theta (1 - al_{y,u}) \]  

(2.18)

\[ f_y(l_{y,n}) = \phi l_{y,n} \]  

(2.19)

For these functional forms we now proceed to examine incentives under the UB and UAs systems and derive the optimal search and work effort decisions for the UB and UAs systems.
The Unemployment Benefit System

Under the UB system, the optimization problem of a young, employed worker is:

\[ V^B(y, n) = \max_{l_{y,n}, s_n} \frac{1}{\gamma^\gamma} \left( \left( (w \left(1 - \tau\right) - s_n)^\alpha l_{y,n}^{1-\alpha} \right)^\gamma \right) \]

\[ + \beta \left( f_y(l_{y,n}) \left( b + s_n (1 + r) \right)^{\alpha\gamma} \right) \]

subject to \(0 \leq l_{y,n} \leq \frac{1}{\phi}\) \(2.21\)

The resulting optimal leisure decision is:

\[ l^B_{y,n} = \left( \frac{\beta \phi}{\gamma (1 - \alpha)} \right) \left( \left( w(1 - \tau) + s_n (1 + r) \right)^{\alpha\gamma} - (b + s_n (1 + r))^{\alpha\gamma} \right) \]

\[ \left( w(1 - \tau) - s_n \right)^{\frac{1}{\gamma} \left( \frac{1}{(1-\alpha)\gamma} - 1 \right)} \]

The optimal saving decision is given by the following implicit function:

\[ 0 = (w \left(1 - \tau\right) - s_n)^{\alpha\gamma - 1} f_y(l_{y,n})^{\frac{1}{\gamma}} \]

\[ - (1 + r) \beta \left( f_y(l_{y,n}) (b + s_n (1 + r))^{\alpha\gamma - 1} \right) \]

The optimization problem of a young, unemployed worker is:

\[ V^B(y, u) = \max_{l_{y,u}, s_u} \frac{1}{\gamma^\gamma} \left( (b - s_u)^\alpha l_{y,u}^{1-\alpha} \right)^\gamma \]

\[ + \beta \left( h_y(l_{y,u}) \left( b + s_u (1 + r) \right)^{\alpha\gamma} \right) \]

subject to \(\max \left[ 0, \frac{1}{a} \left( 1 - \frac{1}{\theta} \right) \right] \leq l_{y,u} \leq \frac{1}{a}\) \(2.25\)

---

32 The superscript "B" stands for the Unemployment “Benefit” System.
33 This condition ensures that the firing rate \(f_y\) is non-negative and not greater than 1.
34 We substitute the firing rate, eq. 2.19, into the optimization problem, eq. 2.20. We focus on interior solutions.
35 We express consumption as income minus saving and optimize with respect to saving.
36 This condition ensures that the hiring rate \(h_y\) is non-negative and not greater than 1.
The resulting optimal leisure decision is:\(^{37}\)

\[
l_{y,u}^B = \left( \frac{\beta \theta \alpha}{\gamma (1 - \alpha)} \right) \left( (w(1 - \tau) + s_u (1 + r))^{\alpha \gamma} - (b + s_u (1 + r))^{\alpha \gamma} \right)^{\frac{1}{(1 - \alpha) \gamma - 1}}
\]

and the following implicit function yields the optimal saving decision:

\[
0 = (b - s_u)^{\alpha \gamma - 1} l_{y,u}^{(1 - \alpha) \gamma}
\]

\[
= -(1 + r) \beta \left( l_{y,u}^{(1 - \alpha) \gamma} + (1 - f_y(l_{y,u})) \left( (w(1 - \kappa) + b + s_n (1 + r))^{\alpha \gamma} \right) \right)
\]

The system of equations 2.22, 2.23, 2.26, 2.27 and the government budget constraint, eq. 2.11, describe the equilibrium levels of job search and work effort under the UB system, to be calibrated in Section 2.4.2.

**The Unemployment Accounts (UAs) System**

Under the accounts system, the young, employed worker’s decision problem is to solve the problem\(^{38}\)

\[
V^A(y, n) = \max_{l_{y,n}, s_n} \frac{1}{\gamma (1 - \alpha)} \left( \left( w(1 - \kappa) - \frac{b}{1 + r} - s_n \right)^{\alpha} l_{y,n}^{1 - \alpha} \right)^{\gamma}
\]

\[
+ \beta \left( f_y(l_{y,n}) \left( b + s_n (1 + r) \right)^{\alpha \gamma} + (1 - f_y(l_{y,n})) \left( (w(1 - \kappa) + b + s_n (1 + r))^{\alpha \gamma} \right) \right)
\]

subject to the leisure constraint, eq. 2.21. The resulting optimal leisure decision is:\(^{39}\)

\[
l_{y,n}^A = \left( \frac{\beta \phi}{\gamma (1 - \alpha)} \right) \left( (w(1 - \kappa) + b + s_n (1 + r))^{\alpha \gamma} - (b + s_n (1 + r))^{\alpha \gamma} \right)^{\frac{1}{(1 - \alpha) \gamma - 1}}
\]

\[
\left( w(1 - \kappa) - \frac{1}{1 + r} b - s_n \right)^{\frac{\alpha \gamma}{(1 - \alpha) \gamma - 1}}
\]

and the optimal saving decision is given by the following implicit function:

\[
0 = (w(1 - \kappa) - \frac{1}{1 + r} b - s_n)^{\alpha \gamma - 1} l_{y,n}^{(1 - \alpha) \gamma}
\]

\[
= -(1 + r) \beta \left( f_y(l_{y,n}) \left( b + s_n (1 + r) \right)^{\alpha \gamma - 1} + (1 - f_y(l_{y,n})) \left( (w(1 - \kappa) + b + s_n (1 + r))^{\alpha \gamma - 1} \right) \right)
\]

\(^{37}\)We substitute the hiring rate, eq. 2.18, into the optimization problem, eq. 2.24.

\(^{38}\)The superscript "A" stands for the Unemployment "Accounts" System.

\(^{39}\)We substitute the firing rate, eq. 2.19, into the optimization problem, eq. 2.28.
The young unemployed worker’s optimization problem is:

\[ V^A(y, u) = \max_{l_{y,u}, s_u} \left( \frac{1}{\gamma} \left( (b - s_u)^{\alpha} l_{y,u}^{1-\alpha} \right)^\gamma \right. \]

\[ + \beta \left( h_y(l_{y,u}) l_{y,u}^{\frac{1}{\gamma}} \left( (w (1 - \kappa) + s_u (1 + r))^{\alpha \gamma - 1} \right) \right. \]

subject to the leisure constraint, eq. 2.25. In this case, the resulting optimal leisure decision is:

\[ l_{y,u}^A = \left( \frac{\beta \phi}{\gamma (1 - \alpha)} \left( (w (1 - \kappa) + s_u (1 + r))^{\alpha \gamma - 1} - (b + s_u (1 + r))^{\alpha \gamma - 1} \right) \right) \]

and the first-order condition for saving:

\[ 0 = (b - s_u)^{\alpha \gamma - 1} l_{y,u}^{\frac{1}{\gamma} (1-\alpha) \gamma} \]

Under the UAs system the equilibrium levels of job search and work effort are described by the system of Eq. 2.29, 2.30, 2.32, 2.33 and the government budget constraint, Eq. 2.15, also to be calibrated in the next Section.

### 2.4.2 Calibration

The period of analysis is one year. The interest rate \( r \) is set at 4% per year, which corresponds to the average real interest rate in the OECD over the last four decades, and we set \( \beta = \frac{1}{1+r} \). We let the utility coefficient \( \alpha \) be 0.85. In our 2-period model an unemployment spell lasts for half the workers’ lifetime, thereby it creates an unrealistically high amount of risk for households. In order order to compensate for this, we use of a correspondingly low parameter of risk aversion \((1 - \gamma)\) of 0.25.

The parameters of the hiring function \( a, \phi \) of the firing function \( \phi \) and the hiring rate \( h_e \) for each country are assigned the values necessary for the model to reproduce the net replacement ratio, the average duration of job tenure, and the unemployment rate of the four high-unemployment countries, as shown in Table 2.2. These variables are defined as follows:

(i) The net replacement ratio \((\nu^{net}, \text{for the current UB systems})\) is taken to be the after-tax replacement ratio for 2002 (OECD, 2004a), so that the unemployment benefit in our model is given by \( b = w\nu^{net} (1 - \tau) \), where \( \nu^{net} = \frac{\nu}{(1-\tau)} \).

(ii) The average job tenure (in years) is that

\[ \text{Average job tenure} = \frac{1}{1+\frac{1}{n}} \]

40We substitute the hiring rate, eq. 2.18, into the optimization problem, eq. 2.31.

41The net replacement ratio is the average of net replacement rates for six family types and different earning levels for the initial phase of unemployment (i.e. upon entering unemployment following any benefit waiting period) for somebody who was previously employed on a full-time basis, 2002.
for 2002 in Auer et al. (2005), and it is computed as as the inverse of the rate of outflow from employment $1/f_y$ (see appendix 2.A.2). (iii) The unemployment rate is the standardized unemployment rate for 2002 (OECD, 2005b).

<table>
<thead>
<tr>
<th></th>
<th>$v^{\text{net}}$</th>
<th>$1/f_y$</th>
<th>$u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>78.4</td>
<td>11.2</td>
<td>8.9</td>
</tr>
<tr>
<td>Germany</td>
<td>76.5</td>
<td>10.7</td>
<td>8.2</td>
</tr>
<tr>
<td>Italy</td>
<td>60.2</td>
<td>12.1</td>
<td>8.6</td>
</tr>
<tr>
<td>Spain</td>
<td>72.4</td>
<td>9.9</td>
<td>11.3</td>
</tr>
</tbody>
</table>

Table 2.2: Net replacement ratio in percent, average job tenure in years and standardized unemployment rate in percent for 2002.

2.4.3 Results

As noted (in Section 2.3.4), the incentive improvement from switching from an UB to an UAs system is generated by an internalization effect (the internalization of the cost of unemployment increases the reward for keeping a job) and the resulting tax reduction effects (the resulting reductions in the tax rate increase the reward for keeping and seeking a job). For the calibrated model above, the plots in Figure 2.2 shed light on the relative importance of these two effects, by comparing the rewards for keeping and seeking a job under both systems for varying replacement ratios.

We have seen in Section 2.3.4 that the employment incentives under the UB system are independent of a worker’s employment history, i.e. the reward for seeking a job is equal to the reward for keeping a job ($\Delta_B^A = \Delta_B^B = w (1 - \tau) - b$), as shown in Figure 2.2. When moving from the UB to the UAs system, the reward for keeping a job rises substantially, as indicated in Figure 2.2, where this reward is measured in consumption terms (compare $\Delta_n^A = w(1 - \kappa)$ and $\Delta_n^B = w (1 - \tau) - b$). The increased reward - implying increased incentives for work effort - is due to both the internalization and the tax reduction effects. The reward for seeking a job also rises, as shown, but by substantially less, since this change - implying increased job search incentives - is driven only by the tax reduction effects (compare $\Delta_u^A = w(1 - \kappa) - b$ and $\Delta_u^B = w (1 - \tau) - b$).

Figure 2.2 also shows that the replacement ratio has a weaker influence on the reward for keeping a job under UAs ($\Delta_u^A = w(1 - \kappa)$) than under UBs ($\Delta_n^B = w (1 - \tau) - b$). Naturally, the replacement ratio does have an indirect effect under UAs, since the higher replacement ratio implies more interpersonal redistribution to those unemployed workers who are unable to support themselves and thus a correspondingly higher tax rate ($\kappa$) on the employed workers. But under the UAs system, workers internalize more of the cost of their own unemployment (specifically, unemployed workers who were previously employed pay for themselves), and thus the tax rate rises less with the replacement ratio and work incentives are reduced by correspondingly less as well. This is the reason for the weak reaction of the reward for keeping a job under UAs to a rise in the replacement rate.
Figure 2.2: The relation between the net replacement ratio and the reward for keeping and seeking a job, in consumption terms, under the UB and the UAs system for France, Germany, Italy and Spain.

Table 2.3 summarizes the implications of these incentive effects from switching to the UAs system for unemployment ($u$) and welfare, which is quantified by the consumption equivalent measure\(^{42}\) ($CE$) for our calibrated model. (The effects are given in terms of percentage changes.)\(^{43}\)

It is worth emphasising that these substantial reductions in unemployment are achieved even though unemployed people receive the same amount of unemployment support in both systems.

Furthermore, as Tables 2.4 and 2.5 show, these results appear to be reasonably robust, in the sense that they are not very sensitive to changes in the utility parameters $\alpha$ and $RRA = (1 - \gamma)$.

\(^{42}\) We quantify the welfare change from the switch from an UB to an UAs system for an entrant to the labour market by asking by how much in percent this individual's consumption has to be increased in all periods and labour market states (keeping leisure constant) in the UB system so that his present value of utility equals that under the UAs system. Thus, it can be easily calculated as: $CE = \left( \frac{V^u(e)}{V^u(e)} \right)^{\frac{1}{\alpha \gamma}} - 1$.

\(^{43}\) The cross-country rankings of changes in unemployment and welfare do not coincide with those of the net replacement ratio since these countries differ in terms of variables other than the replacement ratio as well, viz., they also differ in terms of average job tenure and average unemployment rates, implying differences in hiring rates for entrants. Consequently, there are cross-country differences in the hiring and firing functions.
Table 2.3: The percentage change in unemployment rates and welfare (calculated as consumption equivalents) resulting from a shift from the UB to the UAs system.

<table>
<thead>
<tr>
<th></th>
<th>%Δu</th>
<th>%CE</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>−46.3</td>
<td>2.8</td>
</tr>
<tr>
<td>Germany</td>
<td>−50.9</td>
<td>2.6</td>
</tr>
<tr>
<td>Italy</td>
<td>−34.4</td>
<td>1.2</td>
</tr>
<tr>
<td>Spain</td>
<td>−37.7</td>
<td>2.6</td>
</tr>
</tbody>
</table>

Table 2.4: The percentage change in unemployment rates and consumption equivalents resulting from a shift from the UB to the UAs system for alternative values for the parameter α of the utility function for a RRA value of 0.25.

<table>
<thead>
<tr>
<th></th>
<th>α</th>
<th>%Δu</th>
<th>%CE</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>0.90</td>
<td>−43.5</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>0.95</td>
<td>−40.0</td>
<td>2.2</td>
</tr>
<tr>
<td>Germany</td>
<td>0.90</td>
<td>−48.2</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>0.95</td>
<td>−45.4</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Table 2.5: The percentage change in unemployment rates and consumption equivalents resulting from a shift to the UAs system for alternative values for RRA for an α value of 0.98 for France and Germany, 0.96 for Italy and 0.97 for Spain.

<table>
<thead>
<tr>
<th></th>
<th>RRA</th>
<th>%Δu</th>
<th>%CE</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>0.50</td>
<td>−40.7</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>0.75</td>
<td>−42.2</td>
<td>1.4</td>
</tr>
<tr>
<td>Germany</td>
<td>0.50</td>
<td>−46.4</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>0.75</td>
<td>−48.1</td>
<td>1.2</td>
</tr>
</tbody>
</table>

The improved employment incentives depicted in Figure 2.2 imply unemployment reductions that are depicted in Figure 2.3, which specifically shows how the unemployment reductions are related to the net replacement ratio.

Figure 2.4 depicts the tax reductions associated with a switch to the UAs system for varying replacement ratios. (The greater is the replacement ratio, the greater is the tax rate necessary to finance this replacement ratio under the UAs and UB systems.) The greater is the replacement ratio, the greater is the difference between the externalities generated by the UBs and those generated by the UAs. The reason is that a higher replacement ratio under the UAs system means that more support for the unemployed is paid out of the UAs and the greater is the associated internalization effects and the resulting tax reduction effects. It is for this reason that, as the replacement ratio rises, the switch from the UB to the UAs system leads to progressively larger reductions in unemployment and taxes.
Figure 2.3: The relation between the net replacement ratio and the percentage reduction of the unemployment rate.

Figure 2.4: The relation between the net replacement ratio and the percentage reduction in tax rates resulting from a shift to the UAs system.

Figure 2.5 shows the difference in saving, in the UAs and UB systems, associated with a range of replacement ratios. We compare the total saving of an young, employed worker under the two systems, namely, the sum of "voluntary" and "forced" saving under the UAs system.
with saving under the UB system. The vertical axis depicts the percentage difference of young worker’s saving between the UAs and UB systems.

![Graph showing the relation between the net replacement ratio and the percentage difference in saving of young workers between the UAs and the UB system.](image)

Figure 2.5: The relation between the net replacement ratio and the percentage difference in saving of young workers between the UAs and the UB system.

The positive differences show that the mandatory contributions to the UAs are not fully crowded out by less voluntary savings. The reason is that under the UB system an employed worker has less incentive to save. First, his incentive to save to support himself is lower, as the government will provide benefit payments in the case of unemployment, i.e. he does not fully internalize the cost of unemployment. Second, higher interpersonal redistributions reduce his incentive to save. This explains why forcing individuals to save can raise welfare, even though they are already optimizing.

Clearly, the greater is the replacement ratio, the greater are the differences in saving, for the UAs and UB systems, since the higher replacement ratio implies more interpersonal redistribution to those unemployed workers who are unable to support themselves and thereby, more strongly distorted incentives to save.

2.5 Conclusion

This chapter has analyzed the implications of moving from the UB to the UAs system. To promote understanding of major incentive effects, we have focused on some central characteristics of these systems, assuming that unemployment benefits are financed by payroll taxes. Our simple models are meant to clarify important channels whereby the policy change affects job search.

---

44 Topel (1990) and Cortázar (1996) argued that forced savings would be simply offset accordingly by voluntary savings. We show that this is not the case.
and work effort. They also show how the redistribution of income, performed through the UB system, can be accomplished more efficiently through the UAs system, permitting significant declines in unemployment rates and improvements in welfare.

In particular, we have shown how UAs permit people to internalize a portion of a significant policy-induced externality: the support of unemployed workers imposes a tax cost on the employed workers. Under the UAs system people finance more of their own unemployment support than under the UB system and thus the externality is reduced. The reason is that every system of unemployment support involves both interpersonal and intrapersonal redistribution. The switch to the UAs system reduces the need for the latter through taxes and transfers, since employed people can use their UAs to support themselves should they become unemployed in the future. Lower taxes (uncompensated costs on the employed) and lower transfers (uncompensated benefits to the unemployed) means greater incentives for job search and work effort. The resulting rise in hiring rates and reduction in firing rates leads to a fall in unemployment. This in turn broadens the tax base and shrinks the number of people requiring support, leading to further reductions in tax rates and unemployment benefit expenditures, and so on.

Our calibration exercises suggest that these unemployment reductions could be measurable in Europe’s high-unemployment countries. It is important to emphasize that these reductions are achieved without reducing the level of support to the unemployed. Our analysis also shows that switching to the UAs system makes unemployment less sensitive to the replacement ratio and that, the higher is this replacement ratio, the greater is the achievable reduction in unemployment. Naturally, in providing a transparent way of describing how the policy change can affect labour market behaviour, our models of course make strong simplifying assumptions and thus our results must be interpreted with caution, indicating only general orders of magnitude.

2.A Appendix

2.A.1 Hiring and Firing Rates

Having assumed that the only way workers perceive they can influence hiring and firing is through the choice of leisure, we provide some microfoundations for such hire and fire rates with a particularly simple model.

There is a large number $M$ of firms, each of which has workers and maximizes its present value of profits. In a steady state this is equivalent to maximizing one-period profit $\Pi(L, 1)$:

$$
\Pi(L, 1) = \left[ \Gamma(0, l_{y,n}) - w_y - df_y \right] L + \left( \Gamma(\psi, l_{y,n}) - w_i \right) (1 - f_y)L \\
+ \left( \Gamma(0, l_{o,n}) - w_o - \omega \right) \omega N \frac{U_y}{M}
$$

Here, $\psi$ captures learning by doing in production, $\Gamma$ is productivity which depends on experience and effort, $w_y$ is the wage in period 1, $w_o$ is the wage of those unemployed when young and then become hired, $w_i$ is the insider wage, $d$ is the cost of firing a worker, $\omega$ is the cost of hiring a worker, $l_{y,n}$ is the leisure of the young employed and $l_{o,n}$ of the old employed, $N$ is the number of interviews conducted with each of the young, unemployed workers ($U_y$) and
\( \omega \) is the hiring rate at each interview.

Since the purpose of this section is to derive the microfoundations of hire and fire rates, we treat the wages \( w_y \), \( w_o \) and \( w_i \) in the model as predetermined. We introduce these wages as separate parameters here because they separately influence the hire and fire decisions.

The first order condition for hiring is that, if the firm is hiring, the shadow value of a worker exceeds the marginal hiring cost:

\[
\lambda = \Gamma(0, l_{o,n}) - w_o > \omega \tag{2.36}
\]

When a worker comes for interviews, the firm’s hiring decision is based on comparing the estimated shadow values \( \lambda - \varepsilon \) (where \( \varepsilon \) is a random variable) from hiring the additional employee with marginal training costs \( \omega \). At the interview time, the firm does not know how active the worker has been searching so that its estimates of the shadow value are independent of the amount of search of the employee. The hire rate \( \omega \) of the profit-maximising firm then is:

\[
\omega = H(\lambda - \omega), \tag{2.37}
\]

where \( H \) is the cumulative distribution function of \( \varepsilon \).

Workers know the hire rate and have a time endowment of 1 when unemployed and obtaining an interview takes \( g \) units of time. Workers who do one interview are hired with a probability \( \omega \); if they are not hired (with probability \( 1 - \omega \)), they may proceed to a second interview and be hired with a probability \( \omega \). Thus each worker’s hiring rate (the total probability of being hired) is:

\[
h_y = \omega \sum_{j=0}^{N-1} (1 - \omega)^j = 1 - (1 - \omega)^N \tag{2.38}
\]

The hiring rate may now be expressed in terms of the unemployed worker’s leisure. Given that the total time endowment (to be split into search and leisure) is 1, then \( N \) interviews take \( gN \) units of time. Thus, leisure when unemployed is \( 1 - gN \) so that \( N = (1 - l_{y,u})/g \). Hence

\[
h_y(l_{y,u}) = 1 - (1 - \omega)^{(1-l_{y,u})/g} \tag{2.39}
\]

which is decreasing in leisure when unemployed. A linear approximation of eq. 2.39 is

\[
h_y(l_{y,u}) = \log(1 - \omega)((l_{y,u} - 1)/g) \tag{2.40}
\]

which can be rewritten as eq. 2.18, with hiring propensity \( \theta \) and search cost parameter \( a \). The hiring rate function formulation implicitly assumes that workers take the wage as given.

The first order condition of the profit maximisation problem eq. 2.34 for firing is that a worker will be fired if

\[
\Gamma(\psi, l_{y,u}) - w_i < -d \tag{2.41}
\]

so that a worker is fired when her discounted future contribution to profits falls below minus
marginal firing costs \( d \). Because the worker is working on projects which may take more than one period, her first period effort will influence her second period productivity. This effect is captured through the random parameter \( \psi \) which measures "learning by doing".

Since the learning by doing parameter is random, firing is stochastic and the probability of firing a worker is given by the probability that the following equation is negative:

\[
\Gamma (\psi, l_{y,n}) - w_i - d < 0
\] (2.42)

To simplify analysis, we assume that \( \Gamma (\psi, l_{y,n}) \) is linear:

\[
\Gamma (\psi, l_{y,n}) = (\zeta - l_{y,n}) \psi
\] (2.43)

Hence, the probability of firing the worker is:

\[
f = G \left( \frac{\chi}{\zeta - l_{y,n}} \right),
\] (2.44)

where \( G \) is the cumulative density function of \( \psi \) and \( \chi = \frac{\beta w - d}{\beta} \). Here, the firing rate is increasing in the level of leisure on the job as well as in the wage \( w \). The cumulative density function \( G \) can take a variety of forms but we can construct a first order approximation in terms of \( l_{y,n} \) about \( l_{y,n} = \bar{l} \):

\[
f_y(l_{y,n}) = \varphi + \phi l_{y,n},
\] (2.45)

with firm firing propensity \( \phi \).

Someone who exhibits full effort and does not shirk at all should not be fired which can be achieved by setting \( \varphi = 0 \), as in eq. 2.19. The parameters will depend on a number of other parameters including the wage.\(^{45}\)

### 2.A.2 Mean Duration of Job Tenure

Assume duration independent transition and a steady state. The probability of being fired each period is \( f_y \). Then the probability of being fired after \( t \) periods is

\[
f_y(1 - f_y)^{t-1}
\] (2.46)

Thus the mean duration of employment is

\[
f_y \sum_{t=1}^{\infty} t (1 - f_y)^{t-1}
\] (2.47)

which can be rewritten as

\[
f_y \sum_{t=i}^{\infty} \sum_{i=1}^{\infty} (1 - f_y)^{i-1} = \frac{1}{f_y}
\] (2.48)

Hence, if the rate of outflow from employment is \( f_y \), the mean duration of job tenure is \( 1/f_y \).\(^{45}\)

\(^{45}\)Another way of justifying this functional form for the firing function is in terms of a shirking model (see Phelps, 1994a).
3 Comparing the Effectiveness of Employment Subsidies

3.1 Introduction

This chapter\textsuperscript{46} compares the effectiveness of alternative employment subsidy policies. These policies have been popular since they are often meant to reduce both unemployment and earnings inequality together. The quest for such measures has been a prime objective of employment policy throughout the OECD and continues to be central to the policy debate in the large continental European countries.\textsuperscript{47}

We make a number of contributions. First, we examine the policy effects in the context of a model that takes a variety of common labour market imperfections into account: insider wage bargaining, hiring and firing costs, and imperfections related to the tax and transfer system. These imperfections are important because (a) they are responsible for unemployment that is inefficiently high, (b) the institutions underlying these imperfections can be changed only gradually and with considerable delay, so that it is useful to examine the relative effectiveness of different employment policies while these institutions are in place. Since unemployment is inefficiently high in our analysis, this analysis is appropriate to policy design in high-unemployment OECD countries.

Second, our analysis allows us to compare the effects of different targeting schemes for employment subsidies. Alternative employment subsidy policies differ primarily in terms of these targeting schemes. To capture them, our model allows skills to depend both on heterogeneous abilities and heterogeneous durations of employment and unemployment.\textsuperscript{48}

Third, in accord with policy makers’ actual concerns, we measure policy effectiveness in terms of employment and welfare, and also give explicit consideration to earnings inequality and government budgetary outlays.

Fourth, we derive the policy effects of employment subsidies with a purely microfounded model, without assuming a policy-invariant matching function. The reason for this approach is that the matching process can itself be influenced through employment subsidies, so that the use of an exogenously given matching function could run afoul of the Lucas critique.\textsuperscript{49}

Finally, since Pareto welfare-improving policies often insuperably difficult to identify in practice, we introduce a new criterion for evaluating policies: "approximate welfare efficiency" (AWE). A policy is approximately welfare efficient when it

1. improves aggregate employment and welfare (defined in terms of the utility functions of the households),

\textsuperscript{46}For a different version of this chapter see Brown et al. (2007a).
\textsuperscript{47}For a detailed discussion of currently applied employment subsidy programmes in Germany and recent reform proposals see Boss (2006).
\textsuperscript{48}We have sought a model that is rich enough to capture the various groups of workers relevant to the alternative targeting approaches, while at the same time being simple enough to generate straightforward policy guidelines.
\textsuperscript{49}This comes at the cost of focusing on the firm side of the labor market, which we believe is appropriate for high unemployment countries, where labour demand is the short side of the market and thus may be expected to have a dominant influence on employment.
2. does not increase earnings inequality (measured in terms of the Gini coefficient), and

3. is self-financing (i.e. it does not require an additional government budgetary allocation).\(^{50}\)

We argue that approximate welfare efficiency is a useful concept for policy making, since policies that are approximately welfare efficient are not only desirable for Benthamite reasons (the greatest happiness of the greatest number of people), but are unlikely to be blocked through the political process.\(^{51}\)

This chapter addresses two important questions: (i) How should employment policies be targeted? (ii) What should the magnitude of the policy intervention be? There is much disagreement on these issues among policy makers.

In practice, there are two broad policy approaches to targeting employment subsidies: The first favours targeting workers with low incomes and low abilities; and the second focuses on targeting the unemployed. This chapter compares the effectiveness of the following employment subsidy policies: wage subsidies targeted at workers with low abilities and hiring vouchers targeted at long-term unemployed workers or at workers with low abilities or at both.

We evaluate the effectiveness of alternative employment subsidy policies by calibrating the model for the German labour market and deriving the corresponding policy implications. This exercise is meant to be understood as illustrative of our novel approach, which is relevant to other high-unemployment OECD countries. We show that, for the calibrated model, hiring vouchers targeted at the duration of unemployment, specifically at the long-term unemployed, is particularly effective in raising employment and welfare, without increasing earnings inequality or requiring an additional government budget outlay. Moreover, while low wage subsidies can also reduce earnings inequality, they are a relatively expensive and ineffective instrument for reducing unemployment and are not AWE at all.

We also investigate the employment and equity effects of implementing employment subsidies in excess of the magnitudes that are self-financing. Specifically, we examine how much employment could be created by each of the policy measures under consideration if the government’s net budgetary allocation for this measures were increased by a specified amount. Here, too, we find that hiring vouchers targeted at the long-term unemployed have relatively strong employment creating effects, without increasing inequality.

The chapter is organized as follows. Section 2 relates our work to the existing literature. Section 3 presents the theoretical labour market model. Section 4 calibrates the model for Germany, shows the driving forces to make a policy effective and derives the policy implications. In Section 5, we provide some intuition on the robustness of our results. Finally, Section 6 concludes.

\(^{50}\)Clearly, approximate welfare efficiency is not equivalent to Pareto welfare efficiency, because an employment policy can obviously satisfy the three conditions above and still generate uncompensated losers.

\(^{51}\)The reason is that the fear of rising earnings equality is the most common reason for blocking efficiency-improving employment reforms. See also Orszag and Snower (1998), Saint Paul (1995, 1996 and 1998).
3.2 Relation to the Literature

There is a large theoretical and empirical literature on the impact and optimal design of employment subsidies, originating with the work by Pigou (1933) and Kaldor (1936). The search and matching framework of Mortensen and Pissarides (1994) is frequently used to analyze the effect of employment subsidies (see e.g. Boone and van Ours, 2004, Bovenberg et al., 2000, Cardullo and van der Linden, 2006, Mortensen and Pissarides, 2003, Pierrard, 2005, and Vereshchagina, 2002). The matching technology - describing the relation between the inputs and output of the matching process - is assumed to be stable through time. This assumption is admissible provided that the matching technology (described by the functional form of the matching function) can be considered independent of the inputs and output of the matching process. However, very often a negative time trend is found when estimating the search and matching function, thus casting doubt on the stability through time (Blanchard and Diamond, 1989, for the United States, and Fahr and Sunde, 2001, 2004, for Germany). It is admissible to use the matching function to analyze labour market policies, provided that these policies have no significant influence on the matching process itself. However, it seems implausible that active labour market policies should have no effect on the matching process and we know of no rationale why this should be the case.

To avoid running afoul of the Lucas Critique, we do not take the short-cut of assuming a policy-invariant matching function. Instead, we derive the policy effects in a microfounded way from the intertemporal maximization of economic agents and model their incentives explicitly. We give special emphasis to the firm side in our model since labour demand is the short side of the market in economies with high unemployment. The household side comes into play through the wage formation.

Many theoretical analyzes of employment policies are static and thus suffer from the serious drawback that they do not take into account the dynamic feedback effects of employment policies. There are various significant dynamic interconnections. For example, hiring in response to employment policy takes time and may have persistent effects since incumbent employees’ probability of being retained generally exceeds the unemployed people’s probability of being hired. We explicitly capture the dynamic effects of subsidies by specifying the transition rates between employment and unemployment as a function of the employment incentives of the firm.

52For a survey of the empirical literature, see for example Katz (1998). For US evidence, see Woodbury and Spiegelman (1987) and O’Leary et al. (2006). For international evidence, see for example N.E.R.A. (1995), and for British evidence, see Bell et al. (1999). As follows, we will focus on theoretical papers and the calibration thereof.

53Furthermore, many empirical studies reject the hypothesis of constant returns to scale (e.g. Warren, 1996, for the United States, Fahr and Sunde, 2001 for Germany). The number of matches \( M \) is a function of unemployment and vacancies \( M = f(U, V) \), typically specified in Cobb Douglas form \( M = U^\alpha V^\beta \). If \( \alpha + \beta \) do not sum up to 1, the results are input dependent.

54Furthermore, in contrast to a big part of the search and matching literature, we use an endogenous job destruction rate. It can for example be expected that a wage subsidy reduces the firing rate, while a hiring subsidy does not do so. Omitting this feature would bias the results.

55See, e.g., Layard et al. (1991), pp. 490-492, and Snower (1994). Orszag and Snower (2000) have shown that the dynamic, long-run effects of employment subsidies, once the associated lagged adjustment processes have worked themselves out, differ from what may be expected in the short run.
We contribute to the existing literature by considering skills depending unemployment duration as well as on different levels of ability. This detailed grid allows us to analyze and contrast the effects of employment subsidies targeted at different skill classes under the criteria approximative welfare efficiency - explicitly taking the complete budgetary effects into account.56

This is in stark contrast to the existing literature which only considers a small subset of possible targets for employment subsidies. A large part examines the rationale and economic effects of subsidies for the low skilled (e.g., Phelps, 1994b, 1997a, 1997b, Dréze and Snessens, 1997, and Oskamp and Snower, 2006)57, while less attention has been given to subsidies to long-term unemployed workers (Hui and Trivedi, 1986, Snower, 1994, Vereshchagina, 2002).

There is a significant body of literature which proposes the introduction of a low-wage subsidy, either in order to stimulate employment (e.g. SVR, 2006, Sinn et al., 2006) or on grounds of equity considerations (e.g. Phelps, 1994b, 2003). In the search and matching framework with endogenous job destruction, a low wage subsidy would always show positive employment effects, while the effects are ambiguous for hiring vouchers (as they increase job creation and job destruction at the same time, see Pissarides, 2000, p. 217 f.). In this chapter, we develop a new framework which does not run afoul of the Lucas Critique and comes to striking new results which differ from this existing literature.

We now proceed to present how these heterogeneities are modelled in our analytical framework.

### 3.3 The Model

We construct a Markov model of the labour market in which the dynamics of employment and unemployment are determined by transition probabilities among various labour market states. We derive these transition probabilities from optimization principles.

Workers’ productivities are *ability-dependent* and *duration-dependent*. This distinction is important for policy purposes. Whereas the duration of employment and unemployment is readily affected through the standard employment policy instruments, ability can be affected primarily though education and training policy and this latter influence generally takes much longer to manifest itself. Since our focus is on employment policy, we let the duration-dependent productivity differences be endogenous in our model (influenceable by the employment subsidies), whereas the ability-dependent productivity differences are defined as exogenous (not influenceable by the subsidies).58

Our model contains workers in three ability classes, the exogenous component of skill dif-

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56 Orszag and Snower (2003a and 2003b) pointed out the fact that the literature disregarded the complete impact of employment subsidies on the government budget constraint by requiring that aggregate payroll taxes finance aggregate employment subsidies and thereby ignoring the reduction of unemployment benefit payments, which result from reduced employment. In this paper we follow their line of reasoning.

57 Mortensen and Pissarides (2003) analyze low wage and hiring subsidies, but do not take different unemployment durations into account.

58 Our analysis can be extended to education and training policy, then these ability classes would become endogenous; see, for example, Oskamp and Snower (2006).
ferences: low-ability, medium-ability and high-ability workers, denoted by \( \alpha = l, m, h \), respectively. Within each ability class, workers’ productivity depends on whether they are employed or unemployed and for how long. Specifically, there are five labour market states, two for the unemployed:

(i) the *long-term unemployed* \( U^L \), who have been unemployed for more than a year (the period of analysis),

(ii) the *short-term unemployed* \( U^S \), who have been unemployed up to one year, and three for the employed:

(iii) the *primary entrants* \( N^E_1 \), who are short-term employed workers (employed up to one year) that were previously short-term unemployed,

(iv) the *secondary entrants* \( N^E_2 \), who are short-term employed workers that were previously long-term unemployed, and

(v) the *insiders* \( N^I \), who are long-term employed, i.e. employed for more than a year.

The short-term unemployed workers are more productive than the long-term unemployed. Insiders are more productive than primary entrants who, in turn, are more productive than secondary entrants.

We assume constant returns to labour. Let \( a_{\alpha}^{d_u} \) be the labour productivity of an employee in duration class \( d_u \) and ability class \( \alpha \), where \( d_u = I, E1, E2 \) for employed workers \( d_u = S, L \) for unemployed workers. The firm faces a random cost \( \varepsilon_{\alpha,t} \), which is iid across workers and time within the ability class \( \alpha \). This cost may be interpreted as an operating cost or a negative productivity shock. The expected operating cost conditional on being retained or hired is normalized to zero and its cumulative distribution \( \Gamma_\alpha (\varepsilon_\alpha) \) is time-invariant.

Agents in our model pursue the following sequence of decisions. First the government sets the income tax rate to ensure that its tax receipts are equal to its net budgetary allocation on employment subsidies. Second, the random operating costs are revealed. Third, wages are determined through bargaining and then employment decisions are made.

### 3.3.1 The Government Budget Constraint

For simplicity, our model considers only four instruments of government policy: (i) a proportional payroll tax, with a tax rate \( \tau \), (ii) an unemployment benefit \( b_\alpha \), (iii) an employment subsidy, specifically a hiring voucher \( \sigma_{a,t}^{d_u} \) targeted at workers of duration-dependent groups and ability-dependent groups or a wage subsidy \( \sigma_{a,t} \) for employees of certain abilities \( \alpha \), and (iv) the net allocation of government expenditures \( G_t \) to employment subsidies.

The government budget is given by:

\[
G_t + \sum_\alpha n_{\alpha,t} \sum_\alpha l_{\alpha,t} w_{\alpha,t} \tau = \sum_\alpha \sum_{d_u} u_{\alpha,t}^{d_u} \sum_\alpha \sum_{d_u} \sigma_{a,t}^{d_u} \sum_\alpha \sum_{d_u} \sigma_{a,t}^{d_u} \sum_\alpha \sum_{d_u} \sigma_{a,t} \sum_\alpha \sum_{d_u} \sigma_{a,t} \sum_\alpha \sum_{d_u} \sigma_{a,t} + \sum_\alpha \sigma_{a,t} n_{\alpha,t}. \tag{3.1}
\]

---

59 We follow the notational convention that only those variables have time subscripts that actually vary through time in our model.

60 As we calculate the long-run effects of different policies (i.e. the new steady states), the static budget constraint is relevant.
i.e. the net government allocation $G_t$ plus tax receipts is equal to government spending on unemployment benefits and the employment subsidies. For simplicity, we assume that the payroll tax is set so as to finance the unemployment benefits in the absence of subsidies and of a net government allocation (i.e., $\sigma_{\alpha,t} = \sigma_{\alpha,t}^u = 0$ and $G_t = 0$).

### 3.3.2 Wage Bargaining

Let the wage $w_{\alpha,t}$ for each ability class $\alpha$ in period $t$ be the outcome of a Nash bargain between the median insider of that ability class and her firm. Our wage bargaining model is analogous to the median voter model, where the utility of the median voter is maximized. The firm bargains with a union which maximizes the utility of the median insider.\(^{61}\) When the bargaining decision takes place, nobody has been fired yet. The median insider is the worker who is situated exactly in the middle of the distribution and who is assumed to face no risk of dismissal at the negotiated wage. The wage is renegotiated in each period $t$. Thus, the present value in period $t$ is independent of the present value in period $t+1$.

Each worker has the following utility function:

$$v_t(c) = c^{\xi},$$

which depends positively on consumption $c_t$, where $(1-\xi)$ is the relative risk aversion parameter of the workers’ utility function.\(^{62}\)

Under bargaining agreement, the insider receives the wage $w_{\alpha,t}(1-\tau)$, where $\tau$ is the payroll tax rate, and the firm receives the expected profit $(a_{\alpha}^I - \epsilon_{\alpha}^{MI} - w_{\alpha,t} + \sigma_{\alpha,t})$ in each period $t$, where $\epsilon_{\alpha}^{MI}$ is the operating cost of the median insider, $\sigma_{\alpha}$ is a wage subsidy for workers of class $\alpha$ and $a_{\alpha}^I$ is the productivity of an insider of ability class $\alpha$. Thus the expected present value of the insider’s utility $V_{\alpha,t}^I$ under bargaining agreement is

$$V_{\alpha,t}^I = (w_{\alpha,t}(1-\tau))^\xi + \delta \left( (1-\phi_{\alpha,t+1}) V_{\alpha,t+1}^I + \phi_{\alpha,t+1} V_{\alpha,t+1}^S \right),$$

where $\phi_{\alpha,t+1}$ is the firing rate, the time discount factor is $\delta$, and $V_{\alpha,t+1}^S$ the expected present value of a short-term unemployed workers’ returns. The expected present value of firm’s returns under bargaining agreement are

$$\Pi_{\alpha,t}^I = (a_{\alpha}^I - \epsilon_{\alpha}^{MI} - w_{\alpha,t} + \sigma_{\alpha,t}) + \delta \left( (1-\phi_{\alpha,t+1}) \Pi_{\alpha,t+1}^I - \phi_{\alpha,t+1} f_{\alpha,t+1} \right),$$

where $f_{\alpha,t+1}$ are firing costs.

Under disagreement, the insider’s fallback income is $b_{\alpha,t}$, assumed for simplicity to be equal to the unemployment benefit. The firm’s fallback profit is $-f_{\alpha,t}$, i.e. during disagreement the insider imposes the maximal cost on the firm (e.g. through strike, work-to-rule, sabotage) short

\(^{61}\)Collective bargaining coverage in central European countries such as Austria, Belgium, France, Germany or the Netherlands lies within the range of 68% to 97.5 % (see OECD, 2004b). Furthermore collective wage agreements anchor wage settings also for firms that are not covered by collective agreements (for Germany see Schnabel, 2005, and Kohaut and Schnabel, 2004).

\(^{62}\)In our model, for simplicity, workers consume all their income.
of inducing dismissal. Assuming that disagreement in the current period does not affect future returns, the present values of utility under disagreement for the insider are

\[ V^{\prime}_{a,t} = (b_{a,t})^{\xi} + \delta \left( (1 - \phi_{a,t+1}) V^{I}_{a,t+1} + \phi_{a,t+1} V^{S}_{a,t+1} \right) \]  

(3.5)

and for the firm are

\[ \Pi^{I}_{a,t} = -f_{a,t} + \delta \left( (1 - \phi_{a,t+1}) \Pi^{I}_{a,t+1} - \phi_{a,t+1} f_{a,t+1} \right). \]  

(3.6)

The insider’s bargaining surplus is

\[ V^{I}_{a,t} - V^{\prime I}_{a,t} = (w_{a,t}(1 - \tau))^{\xi} + \delta \left( (1 - \phi_{a,t+1}) V^{I}_{a,t+1} + \phi_{a,t+1} V^{S}_{a,t+1} \right) \]
\[ - (b_{a,t})^{\xi} - \delta \left( (1 - \phi_{a,t+1}) V^{I}_{a,t+1} + \phi_{a,t+1} V^{S}_{a,t+1} \right) \]
\[ = (w_{a,t}(1 - \tau))^{\xi} - (b_{a,t})^{\xi}, \]

(3.7)

and the firm’s surplus is

\[ \Pi^{I}_{a,t} - \Pi^{\prime I}_{a,t} = (a^{I}_{\alpha} - \varepsilon^{\alpha}_{M} - w_{a,t} + \sigma_{a,t}) + \delta \left( (1 - \phi_{a,t+1}) \Pi^{I}_{a,t+1} - \phi_{a,t+1} f_{a,t+1} \right) - \]
\[ (-f_{a,t} + \delta \left( (1 - \phi_{a,t+1}) \Pi^{I}_{a,t+1} - \phi_{a,t+1} f_{a,t+1} \right)) \]
\[ = a^{I}_{\alpha} - \varepsilon^{\alpha}_{M} - w_{a,t} + \sigma_{a} + f_{a,t}, \]

(3.8)

The negotiated wage maximizes the Nash product (\( \Lambda \)):

\[ \Lambda = \left( (w_{a,t}(1 - \tau))^{\xi} - (b_{a,t})^{\xi} \right)^{\gamma} \left( a^{I}_{\alpha} - \varepsilon^{\alpha}_{M} - w_{a,t} + \sigma_{a} + f_{a,t} \right)^{1-\gamma}, \]

(3.9)

where \( \gamma \) represents the bargaining strength of the insider relative to the firm. For the bargained wage, the following relationship holds:

\[ (1 - \gamma) \left( [w_{a,t}(1 - \tau)]^{\xi} - b_{a,t}^{\xi} \right) = \gamma \xi \left[ w_{a,t}(1 - \tau) \right]^{\xi-1} \left( a^{I}_{\alpha} - \varepsilon^{\alpha}_{M} - w_{a,t} + \sigma_{a} + f_{a,t} \right) (1 - \tau). \]

(3.10)

In the labour market equilibrium, let firing costs be proportional to the wage, \( f_{a,t} = \rho w_{a,t} \), and the unemployment benefit be proportional to the wage as well, \( b_{a,t} = \beta_{a}(1 - \tau) w_{a,t} \), where \( \beta_{a} \) is the net replacement ratio. Then the negotiated wage is

\[ w_{a,t} = \frac{\gamma \xi}{(1 - \gamma) \left( 1 - \beta_{a}^{\xi} \right) + \gamma \xi \left( 1 - \rho \right)} \left( a^{I}_{\alpha} - \varepsilon^{\alpha}_{M} + \sigma_{a,t} \right). \]

(3.11)

In words, the wage depends positively on the median insider’s productivity, the magnitude of the subsidy the insider receives, the replacement rate and the magnitude of firing costs relative to the wage.\(^{63}\)

\(^{63}\)In the analysis above, the profit functions were interpreted as those pertaining to a fixed number of firms. However, since there are constant returns to labor and since labor is heterogeneous in terms of the random cost \( \varepsilon \), these profit functions can also be interpreted as the outcome under a free firm entry condition. In this latter interpretation, each firm offers a job to a single worker and firms enter the economy until the marginal firm’s
3.3.3 Transitions Among Labour Market States

The transitions among the labour market states are summarized in Figure 3.1. The short-term unemployed $U^S$ are hired with probability $\eta^S$ and then become primary entrants $N^{E1}$; with probability $(1 - \eta^S)$ they remain unemployed and then join the long-term unemployed $U^L$, thereby losing productivity. The long-term unemployed are hired with probability $\eta^L$ and then become secondary entrants $N^{E2}$; with probability $(1 - \eta^L)$ they remain long-term unemployed.

Human capital not only depreciates with the duration of unemployment, but also appreciates with the duration of employment. At the end of a period, the primary entrants turn into insiders $N^I$, and thereby gain productivity. As insiders, they lose their jobs with probability $\phi$ and then become short-term unemployed; with probability $(1 - \phi)$ they are retrained. The same holds for the secondary entrants: they, too, turn into insiders, who have a $\phi$ chance of losing their jobs and a $(1 - \phi)$ chance of retaining them.

Figure 3.1: Transitions among labor market states
The labour market system for each ability group $a$ in period $t$ may be described as follows:

\[
S_{a,t} = T_{a,t} S_{a,t-1},
\]

where $S_t$ is a vector of the labour market states:

\[
S_{a,t} = (N^{I}_{a,t}, N^{E1}_{a,t}, N^{E2}_{a,t}, U^{S}_{a,t}, U^{L}_{a,t})',
\]

and $T_{a,t}$ is a Markov matrix of transition probabilities:

\[
T_{a,t} =
\begin{pmatrix}
(1 - \phi_{a,t}) & (1 - \phi_{a,t}) & (1 - \phi_{a,t}) & 0 & 0 \\
0 & 0 & 0 & \eta^S_{a,t} & 0 \\
0 & 0 & 0 & 0 & \eta^L_{a,t} \\
\phi_{a,t} & \phi_{a,t} & \phi_{a,t} & 0 & 0 \\
0 & 0 & 0 & (1 - \eta^S_{a,t}) & (1 - \eta^L_{a,t})
\end{pmatrix}
\]

(3.14)

We now proceed to derive these transition probabilities between the labour market states from microeconomic foundations.

### 3.3.4 Hiring and Firing

#### The Firing Rate for Insiders

The expected present value of profit generated by an insider, after the random cost $\varepsilon_{a,t}$ at time $t$ is observed, is \(^6\)

\[
\pi^I_{a,t} = (a^I_{a} - w_{a} - \varepsilon_{a,t} + \sigma_{a}) + \sum_{i=1}^{\infty} \delta^i \left[(1 - \phi_{a})^i (a^I_{a} - w_{a} + \sigma_{a}) - \phi_{a} f_{a} (1 - \phi_{a})^{i-1}\right],
\]

(3.15)

so that

\[
\pi^I_{a,t} = \frac{a^I_{a} - w_{a} + \sigma_{a} - \delta \phi_{a} f_{a}}{1 - \delta (1 - \phi_{a})} - \varepsilon_{a,t},
\]

(3.16)

where $\sigma_{a}$ is the wage subsidy.

The expected incentive of retaining the insider ($\nu^I_{a}$) is defined as the difference between the expected gross profit from a retained insider \(^6\)

\[
E(\pi^I_{a,t}) = \frac{a^I_{a} - w_{a} + \sigma_{a} - \delta \phi_{a} f_{a}}{1 - \delta (1 - \phi_{a})} + \frac{\varepsilon_{a,t}}{\delta}
\]

and the expected profit from firing an insider ($-f_{a}$). Thus this insider retention incentive is

\[
\nu^I_{a} = \frac{a^I_{a} - w_{a} + \sigma_{a} - \delta \phi_{a} f_{a}}{1 - \delta (1 - \phi_{a})} + f_{a}.
\]

(3.17)

An insider is fired in period $t$ when the realized value of the random cost $\varepsilon_{a,t}$ is greater than the

\(^6\)In the first period, profit is $(a^I_{a} - w_{a} - \varepsilon_{a,t} + \sigma_{a})$; in the second period, the insider is retained with probability $(1 - \phi_{a})$ and then generates an expected profit of $a^I_{a} - w_{a} + \sigma_{a}$, and the insider is fired with a probability of $\phi_{a}$ and then generates a firing cost of $f_{a}$; and so on. Recall that $\varepsilon^I$ is normalised by $E(\varepsilon \leq \nu^I_{a}) = 0$.

\(^6\)This "gross" profit is the expected profit generated by retaining a worker, without taking the operating cost into account.
insider employment incentive.\footnote{Equivalently, the insider is fired when the profit from retaining the insider is less than the firing cost: $\phi_\alpha - \sigma_\alpha > f_\alpha$.} $\varepsilon_{\alpha,t} > \nu_\alpha^I$. Since the cumulative distribution of the operating cost is $\Gamma_{\alpha} (\varepsilon_{\alpha,t})$, the insider’s firing rate is
\[
\phi_\alpha = 1 - \Gamma_{\alpha} (\nu_\alpha^I).
\] (3.18)

**The Hiring Rate for Short-Term Unemployed Workers**

The expected present value of profit generated by a primary entrant (a worker who has been hired after being short-term unemployed), after the random cost $\varepsilon_{\alpha,t}$ at time $t$ is observed, is\footnote{Equivalently, the primary entrant is hired when the profit from employing this worker is greater than the hiring cost: $a_{\alpha}^{E1} - w_\alpha - \varepsilon_{\alpha,t} + \sigma_\alpha + \sigma_S^a + \frac{\delta (1 - \phi_\alpha) (a_{\alpha}^f - w_\alpha + \sigma_\alpha) - \phi_\alpha f_\alpha (1 - \phi_\alpha)}{1 - \delta (1 - \phi_\alpha)} > h_\alpha$.}

\[
\pi_{\alpha,t}^{E1} = (a_{\alpha}^{E1} - w_\alpha - \varepsilon_{\alpha,t} - h_\alpha + \sigma_\alpha + \sigma_S^a) + \sum_{i=1}^{\infty} \delta^i \left( (1 - \phi_\alpha)^i (a_{\alpha}^f - w_\alpha + \sigma_\alpha) - \phi_\alpha f_\alpha (1 - \phi_\alpha)^{i-1} \right),
\] (3.19)

where $\sigma_S^a$ is a hiring voucher for a short-term unemployed worker with ability $\alpha$.\footnote{For simplicity we assume that $E (\varepsilon_{\alpha,t} \leq \nu_\alpha^S) = 0$.}

The expected incentive to hire a short-term unemployed worker ($\nu_\alpha^S$) is defined as the difference between the expected gross profit from an employed primary entrant ($E (\pi_{\alpha,t}^{E1}) = a_{\alpha}^{E1} - w_\alpha - h_\alpha + \sigma_\alpha + \sigma_S^a + \frac{\delta (1 - \phi_\alpha) (a_{\alpha}^f - w_\alpha + \sigma_\alpha) - \phi_\alpha f_\alpha \delta}{1 - \delta (1 - \phi_\alpha)}$)\footnote{For simplicity we assume that $E (\varepsilon_{\alpha,t} \leq \nu_\alpha^S) = 0$.} and the expected profit from not employing a short-term unemployed (i.e. zero). Thus, the short-term unemployed hiring incentive is

\[
\nu_\alpha^S = a_{\alpha}^{E1} - w_\alpha - h_\alpha + \sigma_\alpha + \sigma_S^a + \frac{\delta (1 - \phi_\alpha) (a_{\alpha}^f - w_\alpha + \sigma_\alpha) - \phi_\alpha f_\alpha \delta}{1 - \delta (1 - \phi_\alpha)}.
\] (3.20)

A primary entrant is hired in period $t$ when the realized value of the random cost $\varepsilon_{\alpha,t}$ is less than the primary entrant hiring incentive: $\varepsilon_{\alpha,t} < \nu_\alpha^S$. Thus the hiring rate for short-term unemployed is

\[
\eta_\alpha^S = \Gamma_{\alpha} (\nu_\alpha^S).
\] (3.21)

**The Hiring Rate for Long-Term Unemployed**

The expected present value of profit generated by a secondary entrant (a worker who has been hired after being long-term unemployed),\footnote{This "gross" profit is the expected profit generated by hiring a worker, without taking the operating cost into account.} after the random cost $\varepsilon_{\alpha,t}$ at time $t$ is observed, is
\[
\pi_{a,t}^{E2} = a_{a}^{E2} - w_a - \varepsilon_{a,t} - h_a + \sigma_a + \sigma_a^L + \frac{\delta (1 - \phi_a) \left( a_{a}^L - w_a + \sigma_a \right) - \phi_a f_a \delta}{1 - \delta (1 - \phi_a)},
\] (3.22)

where \( \sigma_a^L \) is a hiring voucher for a long-term unemployed worker with ability \( a \). The expected incentive to hire a long-term unemployed (\( \nu_{a}^{L} \)) is defined as the difference between the expected gross profit from an employed secondary entrant (\( E(\pi_{a,t}^{E2}) \)) and the expected profit from not employing a long-term unemployed (i.e. zero). The long-term unemployed hiring incentive is

\[
\nu_{a}^{L} = a_{a}^{E2} - w_a - h_a + \sigma_a + \sigma_a^L + \frac{\delta (1 - \phi_a) \left( a_{a}^L - w_a + \sigma_a \right) - \phi_a f_a \delta}{1 - \delta (1 - \phi_a)}.
\] (3.23)

A secondary entrant is hired in period \( t \) when the realized value of the random cost \( \varepsilon_{a,t} \) is less than the secondary entrant hiring incentive:76 \( \varepsilon_{a,t} < \nu_{a}^{L} \). Thus the hiring rate for long-term unemployed workers is

\[
\eta_{a}^{L} = \Gamma_{a} (\nu_{a}^{L}).
\] (3.24)

### 3.3.5 Employment, Unemployment and the Labour Market Equilibrium

The change in employment in each ability group (\( \Delta N_{a,t} \)) is the difference between the outflow from the unemployment pool (\( \eta_{a,t}^{S} U_{a,t-1}^{S} + \eta_{a,t}^{L} U_{a,t-1}^{L} \)) and the outflow from the employment pool (\( \phi_{a,t} N_{a,t-1} \)) of that ability group: \( \Delta N_{a,t} = \eta_{a,t}^{S} U_{a,t-1}^{S} + \eta_{a,t}^{L} U_{a,t-1}^{L} - \phi_{a,t} N_{a,t-1} \). Assuming a constant labour force \( L_a \) in each ability class and defining the employment rate to be \( n_{a,t} = N_{a,t}/L_{a,t} \), we obtain the following employment dynamics equation:78

\[
n_{a,t} = \eta_{a,t}^{S} u_{a,t-1}^{S} + \eta_{a,t}^{L} u_{a,t-1}^{L} + (1 - \phi_{a,t}) n_{a,t-1}.
\] (3.25)

The long-term unemployed comprise those workers who were either short- or long-term unemployed in the previous period and who have not been hired in the current period. Thus the long-run unemployment dynamics equation is

\[
u_{a,t}^{L} = (1 - \eta_{a,t}^{S}) u_{a,t-1}^{S} + (1 - \eta_{a,t}^{L}) u_{a,t-1}^{L}.
\] (3.26)

The short-term unemployment rate is the difference between the aggregate unemployment rate and the long-term unemployment rate:

\[
u_{a,t}^{S} = 1 - n_{a,t} - \nu_{a,t}^{L}.
\] (3.27)

The labour market equilibrium is the solution of the system comprising

- the employment and unemployment dynamics, eqs. 3.25, 3.26 and 3.27,
- the government budget constraint, eq. 3.1,

---

76 Equivalently, the secondary entrant is hired when the profit from employing this worker is greater than the hiring cost: \( a_{a}^{E2} - w_a - \varepsilon_{a,t} + \sigma_a + \sigma_a^L \frac{\delta (1 - \phi_a) \left( a_{a}^L - w_a + \sigma_a \right) - \phi_a f_a \delta}{1 - \delta (1 - \phi_a)} > h_a \)

78 Note that \( \Delta n_{a,t} = \eta_{a,t}^{S} \left(1 - n_{a,t-1}^{S}\right) + \eta_{a,t}^{L} \left(1 - n_{a,t-1}^{L}\right) - \phi_{a,t} n_{a,t-1} \).
• the firing and hiring rates, eqs. 3.18, 3.21, 3.24, and
• the negotiated wage, eq. 3.11.

3.4 Evaluation Of Employment Subsidies

We now proceed to calibrate the model above for German data and compare the effectiveness of alternatively targeted employment subsidies. First we describe the calibration; second we provide an intuitive account of what determines policy effectiveness. Third, we turn to the numerical results when the subsidies are self-financing. Finally, we relax the self-financing constraint.

3.4.1 Calibration

Naturally, in calibrating our model, we take data commonly used in the literature. Although we performed numerous robustness checks which did not affect the qualitative ranking of hiring vouchers vs low-wage subsidies, our quantitative results should nevertheless be interpreted with caution. The model is, after all, very simple.

The period of analysis is one year. The real interest rate \( r \) is set at 4% per year, which corresponds to the average real interest rate in the OECD over the last four decades, and we set \( \delta = \frac{1}{1+r} \). For simplicity, we choose \( \xi = 1 \).

Firing costs and hiring costs are set proportional to 60\% \( (f_\alpha = \rho w_\alpha \text{ with } \rho = 0.6) \) and 10\% \( (h_\alpha = \mu w_\alpha \text{ with } \mu = 0.1) \) of the labour costs respectively (Chen and Funke, 2003).\(^{80}\) The net replacement rates \( \beta_\alpha \) are set to 78.25\% for low-ability, 68.25\% for medium-ability, and 64.67\% for high-ability workers (OECD, 2006), respectively.\(^{81}\) The tax-rate \( \tau \) that balances the government’s budget in the absence of subsidies amounts to 6.9\%.

Keane and Wolpin (1997) estimated rates of skill depreciation during unemployment: white collar workers lose about 30\% of their skills after being unemployed for one year, whereas the number is about 10\% for blue collar workers (see Keane and Wolpin, 1997, p. 500). In Ljungqvist and Sargent (1998, p. 527) the rate of depreciation of skills during unemployment is twice the rate of accumulation. In line with these studies we assume an insider productivity advantage is 10\% and a skill depreciation of 20\% of the respective productivity due to long-term unemployment.

Table 3.1\(^{82}\) shows the percentage values for Germany for the three ability classes of the employment dynamics equations. The percentage share of the labour force \( l_\alpha \) for each ability class for Germany (2002) is taken from the OECD (2005a), the respective

\(^{79}\)In other words, we varied the parameters below within sensible parameter ranges. Results are available on request.

\(^{80}\)See Appendix 3.A.1 for the calculation of the ability specific labour costs and wages.

\(^{81}\)The replacement rates of a 67\% average productivity worker (APW), a 100\% APW and a 150\% APW were chosen to represent the respective replacement rates of low, medium and high-ability group. For simplicity, we took the unweighted average across six family types as well as over the initial period of unemployment and long-term unemployment.

\(^{82}\)Note that variables with subscript "0" denote the value at the "old" steady state, i.e. before any policy exercise.
Table 3.1: Steady state values of the labor share, unemployment, hiring and firing rates for each skill class and in aggregate in percentage for Germany.

<table>
<thead>
<tr>
<th></th>
<th>low-ability</th>
<th>medium-ability</th>
<th>high-ability</th>
<th>aggregate</th>
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<tbody>
<tr>
<td>$l_a$</td>
<td>16.6</td>
<td>59.4</td>
<td>24</td>
<td>100</td>
</tr>
<tr>
<td>$u_{a,0}$</td>
<td>18</td>
<td>10.2</td>
<td>5.2</td>
<td>10.3</td>
</tr>
<tr>
<td>$u^S_{a,0}$</td>
<td>7.7</td>
<td>5.1</td>
<td>2.8</td>
<td>5</td>
</tr>
<tr>
<td>$u^L_{a,0}$</td>
<td>10.3</td>
<td>5</td>
<td>2.4</td>
<td>5.3</td>
</tr>
<tr>
<td>$u^L_{a,0}/u_{a,0}$</td>
<td>57</td>
<td>49</td>
<td>46</td>
<td>50</td>
</tr>
<tr>
<td>$\eta^S_{a,0}$</td>
<td>49</td>
<td>59</td>
<td>55</td>
<td>56.4</td>
</tr>
<tr>
<td>$\eta^L_{a,0}$</td>
<td>38</td>
<td>42</td>
<td>51</td>
<td>43.5</td>
</tr>
<tr>
<td>$\phi_{a,0}$</td>
<td>9.4</td>
<td>5.7</td>
<td>2.9</td>
<td>5.6</td>
</tr>
</tbody>
</table>

aggregate unemployment rates for Germany (2002) $u_{a,0}$ from the OECD (2005b, 2005c). The actual hiring rates for 1996 of each ability and duration group $\eta^S_{a,0}$ and $\eta^L_{a,0}$ are taken from Wilke’s (2005) Kaplan-Meier functions for Germany. According to the OECD (2005b, 2005d) the average share of long-term unemployment ($u^L_{a,0}/u_{a,0}$) is around 50 percent and similar across all ability classes. The firing rates $\phi_{a,0}$ are assigned the values necessary for the model to reproduce the unemployment rates of the respective ability classes. We interpret these numbers as steady state values.

We linearize the model around the old steady state and calculate the long-run effects of the policy exercises (new steady state) as permanent deviations from the old steady state. Thus, we have to choose the first derivative of the cumulative density functions in our model ($\Gamma^\prime_{a,0}$), which determines the hiring and firing elasticities. For this purpose, we use empirical estimates, as summarized in Orszag and Snower (1999, p. 208). The first derivative of the cumulative function for the hiring rate ($\eta^\prime_{a,0}$) (denoted with subscript $\eta$) is set in such a way that the hiring elasticity with respect to a hiring voucher is equal to 0.5. Also in line with the aforementioned empirical literature, we set the first derivative of the cumulative function ($\phi^\prime_{a,0}$) for the firing rate (denoted with subscript $\phi$) in such a way that a one-period reduction of the wage has an elasticity of 0.125.

To double check that we have chosen appropriate hiring and firing elasticities, we compare the endogenous reactions of our model to the empirical labour demand literature. A permanent 10 percent wage cut (ceteris paribus) for low-ability workers generates for example an increase in the employment rate of 8.7 percent in the long-run, which yields an long-term labour demand elasticities.

83 The firing rate of 9% for low-ability employees is close to what can be found in the literature (e.g. Brussig and Erlinghagen, 2005, Fitzenberger et al., 2003, and Wilke, 2005). The firing rate for high-ability is somewhat lower than in reality since many high-ability workers rotate back into work quickly. This phenomenon cannot be captured by our model since it is calibrated on a yearly basis and workers stay unemployment for at least a year. However, this property does not affect the model dynamics for the performed exercises.

84 See Appendix 3.A.2.

85 The hiring elasticity is defined as the reaction of the hiring rate to a hiring voucher for short-term unemployed, which is permanently paid during the first year of the employment spell ($\chi_a = \frac{\partial \eta^S_{a,0}}{\partial \eta_{a,0}/w_a} \cdot \frac{\partial \eta_{a,0}/w_a}{\partial \eta_{a,0}/w_a}$). For simplicity, we choose the same $\eta^\prime_{0,a}$ for short-term and long-term unemployed in each ability group.

43
elasticity of -0.87.\textsuperscript{86}

Empirical labour demand elasticities for Germany are generally estimated in a range from -0.3 to -0.9 (see Riphan et al., 1999). Sinn et al. (2006, p. 10) point out that these estimation results rather reflect short-term than long-term elasticities, refer to studies where considerable higher estimates have been found for the low-wage sector and consider an elasticity of -1 as realistic. Thus, we see ourselves well in line with the empirical labour demand literature for Germany.

3.4.2 Determinants of Policy Effectiveness

In our analysis, the policies under consideration exhibit diminishing returns, in the sense that equal incremental increases in each employment subsidy leads to progressively smaller incremental increases in employment and social welfare. We show that, for each employment subsidy, once a critical level is exceeded, it ceases to be self-financing. Recalling that our notion of approximate welfare efficiency involves the satisfaction of three constraints - an employment and welfare constraint, an earnings inequality constraint, and a self-financing constraint - we find, in our calibration exercises, that as each subsidy is increased, the self-financing constraint is reached first. Thus the self-financing constraint determines the magnitude of each policy intervention that is compatible with approximate welfare efficiency.

We begin by examining the potential of alternative policies to be self-financing.

Wage Subsidies versus Hiring Vouchers

Let us start with comparing the two most general types of subsidies:

- a wage subsidy $\sigma_\alpha$ paid to the firm for each employed worker of a specific ability class,
- 1-period hiring voucher $\sigma_{d,\alpha,t}$ paid to the firm for hiring a worker of a specific target group (duration and ability).

Our quantitative analysis below shows that hiring vouchers are more likely to be self-financing than wage subsidies, for the following reasons:\textsuperscript{87}

**Deadweight Effect**

The deadweight effect - defined as the amount of subsidy payments which are paid to workers who would have been employed in absence of the subsidy - is much larger for wage subsidies than for hiring vouchers. For any particular ability group, the proportion of unemployed workers who would have been hired without the subsidy is clearly smaller than the proportion of all employed workers who would have been retained without the subsidy.\textsuperscript{88}

\textsuperscript{86}Note that the endogenous labour demand elasticity in our model varies with the size of the wage movement. The bigger the change in the wage, the smaller is the labour demand elasticity (in absolute terms).

\textsuperscript{87}Since the following effects are strongly interrelated, we will not try to disentangle them in the numerical exercise below.

\textsuperscript{88}For example, 82\% of the low-skilled workers in Germany are already employed in absence of a low-wage subsidy, while this groups contains 16.6\% of all workers. Thus, 13.6\% of the workforce would receive a low wage subsidy, although these people would be employed without a subsidy. For hiring vouchers deadweight subsidy payments cover only about 5\% of the workforce.
Wage Effect

The wage-effect is defined as the proportion of the subsidy that goes into wage increases and thus does not encourage employment creation. In the context of our model, wage subsidies raise insider wages, since these subsidies are paid to all employed workers of a specific ability class, including the median insiders. By contrast, hiring subsidies to the unemployed do not affect negotiated wages, since these subsidies do not affect the position of the median insiders.

The assumption that wages are negotiated by the median insider is of course extreme, in that it makes this difference in subsidy effectiveness particularly stark. At the opposite extreme, insider wages could be negotiated individualistically, so that both the wage subsidies and the hiring subsidies affect the negotiated wage. But even in the latter case, wage subsidies may be expected to have a larger wage effect than hiring subsidies. The reason is that, in practice, the positions of newly hired workers are less protected by labour turnover costs than are the positions of the established insiders. Thus the established insiders will be able to capture more of the bargaining surplus than the newly hired workers.

In short, both the deadweight and wage effects imply that hiring vouchers are more likely to be self-financing than wage subsidies.

Targeting Hiring Vouchers at Duration versus Ability

We now proceed to analyze whether hiring vouchers should be targeted at specific duration or ability groups. What targeting schemes are most likely to be self-financing? The answer depends on the following effects:

Deadweight Effect

The greater is the hiring rate, the larger will be the deadweight implied by a hiring voucher. Empirically, as can be seen in Table 3.1, hiring rates increase with productivity. Thus hiring vouchers should be targeted at workers with the lowest productivity, namely low-ability workers as well as long-term unemployed workers.

Replacement Rate Effect

As shown in Section 3.4.1, the lower the income and ability, the higher is the replacement rate. Thus, increased employment in the group with the lowest income will generate the largest reduction in government expenditure in terms of the respective wage. Thus the higher is the replacement rate, the more likely is the subsidy to be self-financing. Since unemployed, low-ability workers have the highest replacement rates, hiring vouchers for these groups are more likely to be self-financing than other subsidies, including wage subsidies.

Transition Effect

When hiring vouchers bring workers back to work, their human capital appreciates. In our model, the human capital appreciation implies that the formerly short- and long-term unemployed have the same productivity as insiders after one period of employment. As a consequence, their low hiring probability \( \eta_{d_{a_{i}}} \) is exchanged for a considerably higher retention
probability \((1 - \phi_{\alpha,t})\). This effect is strongest for long-term unemployed since they have been most affected by human capital loss. Thus, the bigger the likelihood that workers’ human capital appreciates, the bigger are the long-run effect on the government budget. Clearly, this effect favours hiring vouchers for long-term unemployed workers.

### 3.4.3 Numerical Results: AWE Subsidies

We simulate our above model for Germany, and compare the effectiveness of the following employment subsidy policies:

(i) A low-wage subsidy \((\sigma_l)\) which is paid (each period) for each low-wage/ability employee. It will reduce the firing rate, by making employees more profitable for the firm. Thus, it raises the insider retention incentive, whereby the firm retains more workers with high operating costs (low productivity).

At the same time the hiring rate will increase since the subsidy provides the incentive to hire more low productivity workers, who would not have been hired otherwise.

(ii) A hiring voucher targeted at low-ability workers \((\sigma_{dlu})\), which is paid for hiring unemployed, low-ability workers. Following the same rationale as above, the firm will hire more workers than without a voucher. In contrast to the first policy, the firing rate will not be affected since the voucher is only paid for new hires and not for the entire employment stock.\(^8\)

(iii) A hiring voucher targeted at long-term unemployed workers \((\sigma_{La})\), which is paid if a long-term unemployed worker is hired.

(iv) A hiring voucher targeted at the low-ability, long-term unemployed workers \((\sigma_{Ll})\).

Our simulation reveals that a low-wage subsidy is not an AWE (approximately welfare efficient) policy for Germany. While a low-wage subsidy creates employment and reduces inequity, it is not self-financing. This result is driven by the deadweight effect and the wage effect above. (Thus low wage subsidies can only be implemented if the government is willing to provide extra resources permanently.) Furthermore, our results show that hiring vouchers for Germany can be self-financing and thereby AWE, depending on the target group.

To determine the most effective employment subsidy, we examine the approximately welfare efficiency of hiring vouchers targeted at the low-productivity groups, namely at long-term unemployed as well as at the low-ability unemployed, and compare their employment, welfare and equity effects.

For both groups there are two possible options for hiring vouchers (HV):

**Option 1:** a same lump sum voucher is paid for hiring a long-term unemployed worker (low-ability worker) irrespectively of his ability class (unemployment duration),

**Option 2:** a different voucher is paid for hiring a long-term unemployed worker (low-ability worker) depending on his ability class (unemployment duration).

While option 1 implies a voucher which is self-financing across ability classes (unemployment duration), option 2 is determined to be self-financing within each ability class (unemployment duration).

---

\(^8\)This holds for constant returns to labour; we will discuss the effect of decreasing returns and displacement effect in section 3.4.4.
duration), thereby, preventing cross-subsidization across ability classes (unemployment duration).

**Targeting Long-Term Unemployed:**

Vouchers targeted at long-term unemployed (LTU) workers are AWE for Germany. Table 3.2 compares the effectiveness of the two design options by describing their unemployment, welfare and equity implications, the latter given by the Gini coefficient.90

If a same lump sum hiring voucher is paid for all long-term unemployed compared to an ability specific payment, the self-financing restriction is hit much earlier. While only 947 € per worker are AWE in the former case, up to 4390 € (2503 €) can be paid for low-ability (medium-ability) workers in the latter. The intuition is straightforward: option 2 fully exploits the larger self-financing areas for long-term unemployed workers in the low-ability and medium-ability class, thereby, it prevents costly cross-subsidization. The self-financing AWE subsidy decreases with productivity due to a smaller deadweight effect and the bigger replacement rate effect.

The more an approximately welfare efficient policy raises employment and welfare, the more "effective" we denote the policy to be. By comparing the results of these two exercises, we can clearly infer that hiring vouchers of different magnitudes for each ability group are most effective. They perform better in terms of unemployment reduction, welfare improvement and earnings inequality reduction. For example, according to our calibration the long-term unemployment among low-ability workers can be reduced by 9% for free (i.e. self-financing).

<table>
<thead>
<tr>
<th></th>
<th>HV for LTU Option 1</th>
<th>HV for LTU Option 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Subsidy</td>
<td>947</td>
<td>4390 / 2503 / 0</td>
</tr>
<tr>
<td>2 Subsidy in % of respective wage</td>
<td>3.7 / 3.1 / 2.2</td>
<td>16.9 / 8.4 / 0</td>
</tr>
<tr>
<td>3 % Change of Low-Ability Long-Term Unemployment</td>
<td>-2.1</td>
<td>-8.9</td>
</tr>
<tr>
<td>4 % Change of Low-Ability Unemployment</td>
<td>-0.8</td>
<td>-4.4</td>
</tr>
<tr>
<td>5 % Change of Long-Term Unemployment</td>
<td>-2.0</td>
<td>-5.9</td>
</tr>
<tr>
<td>6 % Change of Overall Unemployment</td>
<td>-0.9</td>
<td>-2.8</td>
</tr>
<tr>
<td>7 Change of Welfare</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>8 Gini coefficient (old steady state 11.47)</td>
<td>11.45</td>
<td>11.41</td>
</tr>
</tbody>
</table>

Table 3.2: Approximately welfare efficient hiring vouchers (HV) for long-term unemployed (LTU) workers in design options 1 and 2 differentiated in those for low-, medium- and high-ability and the resulting unemployment, welfare and equity implications.

---

90 The welfare of the workforce is calculated as the sum of the utility of the workers over the various labour market states.

\[
\Omega_t = \sum_{\alpha} v(w_{\alpha,t}(1 - \tau))I_{\alpha,n_{\alpha}} + \sum_{\alpha} \sum_{d_{\alpha}} v(b_{\alpha}) u_{\alpha}^{d_{\alpha}} I_{\alpha}.
\]  

A "+" for welfare changes indicates an increase in welfare. The cross-policy ranking of changes in welfare corresponds to the ranking of changes in overall unemployment. The utility parameter \( \xi \) does not affect the cross-policy rankings.

91 Note that the Gini coefficient generated by our model is lower than in reality, as our model does not generate income differentials within ability groups and it does not take non-wage related inequalities into account (e.g., due to the wealth distribution).
Targeting Low-Ability Unemployed:

If a lump sum hiring voucher is targeted at all low-ability unemployed (LAU) (option 1), there is no self-financing area at all. But as shown in Table 3.3 differentiating the vouchers for short-term and long-term unemployed workers reveals an approximately welfare efficient hiring voucher for low-ability workers (4390 €), which though is present only for long-term unemployed workers. The reason is that short-term unemployed workers have a higher productivity than the long-term unemployed, thereby a higher hiring rate, which implies a higher deadweight effect and a smaller transition effect. Both impede a self-financing areas for these workers.

So, for the German calibration above, targeting vouchers at long-term unemployed workers (targeted at the low- and medium-ability workers) is more effective than targeting low-ability workers.92

<table>
<thead>
<tr>
<th></th>
<th>HV for LAU Option 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Subsidy</td>
</tr>
<tr>
<td>2</td>
<td>Subsidy in % of respective wage</td>
</tr>
<tr>
<td>3</td>
<td>% Change of Low-Ability Long-Term Unemployment</td>
</tr>
<tr>
<td>4</td>
<td>% Change of Low-Ability Unemployment</td>
</tr>
<tr>
<td>5</td>
<td>% Change of Long-Term Unemployment</td>
</tr>
<tr>
<td>6</td>
<td>% Change of Overall Unemployment</td>
</tr>
<tr>
<td>7</td>
<td>Change of Welfare</td>
</tr>
<tr>
<td>8</td>
<td>Gini coefficient (old steady state 11.47)</td>
</tr>
</tbody>
</table>

Table 3.3: Approximately welfare efficient hiring vouchers (HV) for low-ability unemployed (LAU) workers in design option 2 differentiated in those for long-term and short-term unemployed and the resulting unemployment, welfare and equity implications.

Employment-Equity Trade-Off:

Interestingly, the self-financing hiring voucher reduces unemployment and inequality at the same time. In other words, a hiring voucher does not move the economy along an employment-equity trade-off, but rather shifts this trade-off in a favourable direction. By contrast, a reduction of the replacement rate for low-ability workers (which are the most unemployment prone) would buy more employment at the cost of a higher Gini coefficient; thus facing an employment-equity trade-off.

3.4.4 Numerical Results: Relaxing the Self-Financing Constraint

We now examine the effectiveness of these policies when we relax the self-financing condition, by allowing a small increase in government spending.93 We say that a policy "outperforms"...
other policies if it delivers the largest reduction in unemployment\footnote{The largest reduction in unemployment can be shown to be associated with the largest increase in welfare.} and in inequality.

Specifically, we assume that in the long-run the government allocates a net expenditure of € 50 in real terms ($G = 50$, per year and per person of the workforce) to active labour market policies. This expenditure is allocated to the targeted groups by increasing the subsidy (in equal € steps for all targeted groups) until the (new steady state’s) budget constraint is reached.

Table 3.4 compares the effects of hiring vouchers for long-term unemployed (LTU) and for low-ability (LAU) workers and low-wage subsidies, with regard to unemployment and inequality. The "marginal unemployment reduction" refers to the changes beyond the self-financing subsidy, whereas the "total reduction" is calculated relative to the pre policy steady state.\footnote{Naturally, as low-wage subsidies are not approximately efficient, the marginal reduction is equal to the overall reduction.}

\begin{table}[h!]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\textbf{1} & \textbf{Total Subsidy (% of respective wage)} & \textbf{HV for LTU (Option 2)} & \textbf{HV for LAU (Option 2)} \textbf{Low-Wage Subsidy} \\
\hline
\textbf{2} & \textbf{Addtional Subsidy} & \textbf{9033} & \textbf{12363} & \textbf{403} \\
\hline
\textbf{3} & \textbf{% Marginal Reduction of Low-Ability Unemployment} & \textbf{-7.8} & \textbf{-18.7} & \textbf{-1.5} \\
\hline
\textbf{4} & \textbf{% Marginal Reduction of Long-Term Unemployment} & \textbf{-15.1} & \textbf{-11.4} & \textbf{-0.8} \\
\hline
\textbf{5} & \textbf{% Marginal Reduction of Overall Unemployment} & \textbf{-7.1} & \textbf{-5.3} & \textbf{-0.4} \\
\hline
\textbf{6} & \textbf{% Total Reduction of Low-Ability Unemployment} & \textbf{-11.8} & \textbf{-22.3} & \textbf{-1.5} \\
\hline
\textbf{7} & \textbf{% Total Reduction of Long-Term Unemployment} & \textbf{-20.1} & \textbf{-14} & \textbf{-0.8} \\
\hline
\textbf{8} & \textbf{% Total Reduction of Overall Unemployment} & \textbf{-9.7} & \textbf{-6.6} & \textbf{-0.4} \\
\hline
\textbf{9} & \textbf{Gini coefficient} & \textbf{11.29} & \textbf{11.38} & \textbf{11.32} \\
\hline
\end{tabular}
\caption{The effects on increased government spending on hiring vouchers (HV) for long-term unemployed (LTU) and low-ability unemployed (LAU) workers.}
\end{table}

**Employment Effects**

Low wage subsidies perform worse in reducing unemployment. The first reason is the dead-weight: while hiring vouchers targeted at the long-term unemployed are paid to roughly 2.2% of the overall workforce, it is almost 13.6% for low-wage subsidies. The deadweight is substantial for wage subsidies as 99.7% of the recipients would also have been employed in the absence of a subsidy. This number is considerably smaller for hiring vouchers targeted at long-term unemployed (68.5%). In addition, there is the wage effect (see Section 4.2 for details). Thus, in contrast to low-wage subsidies, higher hiring vouchers (targeted at long-term unemployed, see column one of Table 3.4, or the low-ability unemployed, see column two) can be allocated for a given net government expenditure and thus, deliver a bigger employment effect.

Hiring vouchers targeted at long-term unemployed, instead of low-ability unemployed workers, deliver a bigger unemployment effect.\footnote{Besides the transition effect, another reason can be found in the employment dynamics equation (4.31). In our model it is much easier to obtain small employment effects for a given ability group compared to a large effect, i.e. labour demand elasticities are higher, the smaller the marginal expenses are. This is most easy to see under homogenous labour where the long-run employment is equal to $n = \frac{\eta}{\varphi}$ (see Snower and Merkl, 2006). The marginal employment effect of an increasing hiring rate obviously is positive, but decreasing ($\frac{\partial n}{\partial \eta} > 0, \frac{\partial^2 n}{\partial \eta^2} < 0$).} The net expenditure for subsidies should be spent for hiring vouchers targeted at long-term unemployed workers in different ability classes and could cut long-term unemployment by 20% and overall unemployment by 10%.
Displacement Effects

Thus far, we have ignored the displacement effect - viz., hiring vouchers induce firms to substitute the subsidized workers for existing employees - due to our assumption of constant returns to labour. We now extend our model to include this displacement effect, which weakens the employment-creating influence of the hiring vouchers. The simplest way to do so is to assume decreasing returns to labour in the short run, under a constant stock of physical capital.\textsuperscript{97} We assume a Cobb-Douglas function $Y = AN^{1-\alpha}K^\alpha$, where capital is a constant $K$.

When we assume that the capital share for Germany is 33\% (i.e. $\alpha = 1/3$, see Statistisches Bundesamt, 2006), we find that the size of AWE hiring vouchers for low-ability, long-term unemployed workers is reduced from 17 to 14\% of the wage. In short, the displacement effect is rather weak. The reason is the transition effect: if the hiring voucher pushes a long-term unemployed worker back to work, her human capital appreciates. Thus when the voucher expires, former long-term unemployed workers face a retention probability which is considerably higher than their initial hiring probability. Furthermore, even if workers are displaced or the worker is fired once the voucher expires, a long-term unemployed worker is exchanged for a short-term unemployed worker with higher human capital and thereby a higher reemployment probability. This transition effect has been shown to be both statistically and economically significant in different German labour market programs. See, for example, the evaluation of the "Hamburger Modell", a lump-sum hiring voucher adopted in a model experiment in Hamburg (Jirjahn et al., 2009), and the evaluation of the so-called "Eingliederungszuschuss", which is a limited hiring subsidy (Bernhard et al., 2008).

Finally, displacement is of course a short-run phenomenon. In the long run, the capital adjusts to the larger workforce with higher human capital, and this adjustment naturally reduces, and possibly eliminates, displacement.\textsuperscript{98} Thus we conclude that displacement effects have no substantial effect on our results.

Equity Effects

Our model shows that hiring vouchers are a more effective in reducing unemployment than low wage subsidies, the relative effectiveness is less straightforward with respect to equity. Both, wage subsidies and hiring vouchers, improve the Gini coefficient, but through different mechanisms. Whereas low-wage subsidies improve equity by raising the wages of low-ability workers, hiring subsidies improve equity by bringing the long-term unemployed back to work. Our calibration shows that the hiring subsidies have a stronger equity-reduction effect.\textsuperscript{99}

\textsuperscript{97}Clearly, the more sharply decreasing the returns to labour are, the more the employment of subsidized workers will reduce the marginal product of the existing workers, and thus, the greater the incentive to dismiss these existing workers.

\textsuperscript{98}Although our quantitative analysis above omits displacement effects, we consider our calibration to be rather conservative (regarding the size of the approximately efficient subsidy). While we used a tax rate (6.9 percent) to finance unemployment related expenses, in reality other tax revenues would also increase with the employment rate. This would raise the self-financing, and thereby, the approximately efficient subsidy and thus, lead to a higher effectiveness (in terms of employment, welfare and inequality).

\textsuperscript{99}To gain some perspective on our analysis in the appendix 3.A.3 we discuss some important effects - lying beyond the scope of our model (namely, substitution effects, asymmetric information, skill acquisition effects, households’ job acceptance incentives) - some of which may be expected to weaken or even reverse our qualitative
3.5 Concluding Thoughts

In the context of a new labour market model, which is illustratively calibrated for Germany, we have shown that low-wage subsidies (targeted at low-income/ability workers) are not approximately welfare efficient (AWE), i.e. no positive low-wage subsidies are self-financing. By contrast, hiring vouchers can be AWE. Our calibrated model suggests that hiring subsidies for the long-term unemployed are more effective than hiring vouchers for low-income/ability workers. The same ranking, in terms of employment, holds for employment subsidies financed through government expenditures extending beyond the AWE limit.

3.6 Appendix

3.6.1 Labour Costs

The different abilities’ labour costs and wages are calculated as follows: the aggregate producer wage and gross value added per worker can be obtained from Statistische Ämter des Bundes und der Länder (2006). The aggregate producer wage is defined as the average real gross wage per employee plus social security payments. We took the 2003 values for gross value added (50334 Euros) and real labour costs (32672 Euros) since the OECD numbers which we used for further calculations were only available until this point in time.

Using the wage equation (3.11), we calculated the average bargaining power in the economy, where the variables denote aggregate values:

\[ w = (1 - \gamma) \beta w + \gamma \left( (a - \epsilon^{MI}) + \rho w \right), \]  
\[ \gamma = \frac{w - \beta w}{(a - \epsilon^{MI}) + \rho w - w\beta}. \]  

We obtain \( \gamma = 0.2134 \).

Ability group specific relative labour costs for Germany are calculated as follows (OECD, 2005c): high-ability workers earn 148\% of their medium-ability counter-parts’ wage and low-ability 87\%, respectively.\(^{101}\) Low-ability workers’ highest education level is lower secondary education, whereas it is upper secondary education or post-secondary non-tertiary education for medium-ability and tertiary education for high-ability. Assuming that the bargaining power is the same in all ability groups and using the respective replacement rates\(^{102}\) we get for each ability group \( \alpha \)

\[ (a^I_{\alpha} - \epsilon^{MI}) = \frac{w_{\alpha} - (1 - \gamma_{\alpha}) \beta_{\alpha} w_{\alpha} - \gamma_{\alpha} \rho w_{\alpha}}{\gamma_{\alpha}}. \]  

\(^{100}\) We interpret this as the productivity of the median insider \((a^I - \epsilon^{MI})\).

\(^{101}\) Similar relations can be found in Wienert (2006).

\(^{102}\) Furthermore, we assumed that the firing costs are 60\% of the labour costs, see Chen and Funke (2003).
Table 3.5: Relevant labour cost values.

<table>
<thead>
<tr>
<th></th>
<th>low-ability</th>
<th>medium-ability</th>
<th>high-ability</th>
<th>aggregate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_\alpha$</td>
<td>16.6</td>
<td>59.4</td>
<td>24</td>
<td>100</td>
</tr>
<tr>
<td>$w_\alpha$</td>
<td>25948</td>
<td>29940</td>
<td>44100</td>
<td>32672</td>
</tr>
<tr>
<td>$(a_\alpha^I - \varepsilon^{M_I})$</td>
<td>31179</td>
<td>47012</td>
<td>75069</td>
<td>51109</td>
</tr>
</tbody>
</table>

Table 3.5 summarizes the relevant values.\(^{103}\) Starting from this steady state we perform policy exercises and compare the resulting new steady states.\(^{104}\)

3.A.2 Linearisation

**Firing Rate**

Non-linear equation:

$$\phi_\alpha = 1 - \Gamma_\alpha \left( \frac{a_\alpha^I - w_\alpha + \sigma_\alpha - \phi_\alpha f_\alpha \delta}{1 - \delta (1 - \phi_\alpha)} + f_\alpha \right), \quad (3.32)$$

where $\sigma_\alpha$ is a wage subsidy for ability class $\alpha$. Linearisation:

$$\phi_{\alpha,\text{new}} = \phi_{\alpha,0} - \phi \Gamma_{\alpha,0} \left[ \frac{1}{1 - \delta (1 - \phi_\alpha)} \right]_0 \frac{1}{1 + V_\alpha} \left[ \left( a_{\alpha,\text{new}}^I - w_{\alpha,\text{new}} + \sigma_{\alpha} \right) - (a_{\alpha,0}^I - w_{\alpha,0}) \right] \quad (3.33)$$

with

$$V_\alpha = \phi \Gamma_{\alpha,0} \left[ \frac{\delta (f_\alpha (\delta - 1) - (a_\alpha^I - w_\alpha))}{(1 - \delta (1 - \phi_\alpha))^2} \right]_0$$

where variables with subscript "0" are at the old steady and variables with subscript "new" are at the new steady state.\(^{105}\)

**Hiring Rates**

Non-linear equation:

$$\eta_\alpha^S = \Gamma_\alpha \left( a_\alpha^E - w_\alpha + \sigma_\alpha^S + \sigma_\alpha + \frac{\delta (1 - \phi_\alpha) (a_\alpha^I - w_\alpha + \sigma_\alpha) - \phi_\alpha f_\alpha \delta}{1 - \delta (1 - \phi_\alpha)} - h_\alpha \right), \quad (3.34)$$

where $\sigma_\alpha^S$ is the hiring voucher for short-term unemployed workers of ability class $\alpha$.

Linearisation:

\(^{103}\)Due to the aggregation the value for the aggregate labour cost is not equal to the original value for real labour costs (50334), which we used to compile the bargaining strength and the ability group specific relative labour cost.

\(^{104}\)See Appendix 3.A.2.

\(^{105}\)In the calibration, we assume for simplicity that $\frac{\delta E(\varepsilon (1 - \phi))}{\sigma_\alpha} = 0$. 

52
\[ \eta_{\alpha,\text{new}} = \eta_{\alpha,0} + \Gamma_{\alpha,0} \left[ \left( a_{\alpha,\text{new}}^E - w_{\alpha,\text{new}} + \sigma_{\alpha} - \left( a_{\alpha,0}^E - w_{\alpha,0} \right) \right) + \delta \left( 1 - \phi_{\alpha} \right) \right] 
+ \n \Gamma_{\alpha,0} \left[ \phi_{2}\delta \left( f_{\alpha,\text{new}} - f_{\alpha,0} \right) \right. 
- \delta \left( 1 - \phi_{2} \right) \left( h_{\alpha,\text{new}} - h_{\alpha,0} \right) 
+ \eta_{\alpha,0} \left[ -\delta \left( a_{\alpha,\text{new}} - w_{\alpha} \right) \right] 
\left[ 1 - \delta \left( 1 - \phi_{\alpha} \right) \right]^2 \left( \phi_{\alpha,\text{new}} - \phi_{\alpha,0} \right). \] (3.35)

And equivalently for the second unemployment duration group.

3.A.3 Extensions

In this Section we discuss various other effects - lying beyond the scope of our model - some of which may be expected to weaken or even reverse our qualitative results.

Substitution Effects

In our setting workers of different ability groups are not substitutable. A strong substitutability would make it profitable for firms to substitute medium and high ability workers with low ability workers. First, this problem would be more severe for low wage subsidies as they are paid permanently and as they are exclusively targeted at low wages. Second, the empirical literature delivers evidence that the substitutability between different ability groups is quite low (see e.g. Buslei and Steiner, 1999 and SVR, 2006, for German evidence as well as Kremer and Maskin, 1996, for cross country evidence). Thus, substitution effects are not strong, but would reinforce our ranking of policies.

Asymmetric Information

Subsidies, which are targeted at low wage workers, may provide an incentive for households to shift from a full-time to a part-time position in order to cash the subsidy. If the subsidy is restricted to full time employees only, there may be an incentive for firms and workers to take advantage of asymmetric information to collude and cheat the government to qualify for the subsidy (i.e. falsely claim a lower hourly wage and more hours; and maybe follow a black market activity at the same time). While this problem may not be entirely absent for hiring vouchers which are targeted at long-term unemployed workers, for low wage subsidies it is a lot more difficult to abuse such a scheme. Low-wage workers are only a subset of the target group and they are only eligible if they were unemployed for longer than one year. Furthermore, the voucher is only paid out for a limited period of time. This problem is though particulary relevant for low-wage subsidies, as these are exclusively targeted at low wages and paid permanently. Thus, we conclude that asymmetric information problems again reinforce the ranking in favour of hiring vouchers for long-term unemployed.
Households’ Job Acceptance Incentives

In our model above, as noted, hiring and firing decisions are made by firms, and the households’ employment incentives enter the model only through wage bargaining. We have argued that this is reasonable in countries with high unemployment (like Germany), for then labour demand is generally the short side of the labour market, so that firms’ labour demands play a leading role in determining employment, while most unemployed households may be expected to accept jobs willingly. The critical reader however may wonder if our ranking of policies (hiring subsidies for long-term unemployed outperform low wage subsidies) would be overturned if we took account of households’ job acceptance incentives. Then hiring vouchers may reduce short-term unemployed households’ job acceptance rate, since they increase the households’ present value of income from unemployment. On the other hand, hiring vouchers raise firms’ job offer rate and this may also raise long-term unemployed households’ job acceptance rate due to strategic complementarities.

To assess the potential importance of these possibilities, consider the following thought experiment. Let us interpret our hiring rate as a matching rate, i.e. as the product of the job offer rate (i.e. the probability that a firm offers a job to an outsider) and the job acceptance rate (i.e. the probability that the outsider accepts the job offer). Assume that the government allocates a net expenditure of €50 (per year and per person of the workforce) for employment subsidies. Then assume, that the new matching rate under low-wage subsidies of 50% (39%) for short-term (long-term) low-ability unemployed consists of a job offer rate of 66% (52%) and a job acceptance rate of 75% (in both cases). To make the low wage subsidy equivalent with the hiring voucher for long-term unemployed (in terms of job creation) an increase of the job acceptance rate for the former case to 100%, while the job acceptance rate stays constant for the latter, would not be sufficient.

Skill-Acquisition Incentives

Low-wage subsidies may create disincentives to acquire human capital. While we assume that the composition of different ability groups is exogenously given, in the long-run employment subsidies may affect workers’ incentives to acquire human capital. Oskamp and Snower (2006) show that the positive short-run employment effects of a low wage subsidy may be undone by negative long-run employment effects, as fewer people choose to invest in skills and thus the more unemployment-prone low ability group becomes larger. This effect is particulary strong for low wage subsidies and much less severe with hiring vouchers for long-term unemployed, as they are non-permanent and not exclusively targeted at the lowest ability group.

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106 These matching rates correspond to the hiring rates which result from the policy exercise in Section 4.3.3.
107 For short-term and long-term unemployed.
108 With different numerical values it would be mathematically feasible to change the ranking. However, such an exercise remains economically highly implausible. With a job acceptance rate of 50%, an increase of this rate by roughly 50% would be necessary (for a low wage subsidy) in order to obtain equivalence between a low wage subsidy and a hiring voucher for long-term unemployed. Note that the job acceptance rate after the introduction of a hiring voucher for long-term unemployed would in addition have to stay unaffected.
4 Incentives and Complementarities of Flexicurity

4.1 Introduction

The Danish miracle of low and stable unemployment has been attributed to the Danish labour market policy of flexicurity, which combines very flexible labour markets, i.e. low job security, with generous unemployment support and active labour market policies. Consequently, the Danish flexicurity has drawn substantial attention and been praised as a role model by the ILO (e.g. Auer, 2000, Auer and Casez, 2003 and Egger et al., 2003) and by the OECD (e.g. OECD, 2004b). Also the European Commission has embraced the broader concept of flexicurity by developing guidelines for national flexicurity reforms.109

This chapter110 analyzes how and to which degree the Danish flexicurity concept and its various elements achieve a low unemployment rate and thus, a higher employment security. Since the various policy components of flexicurity interact with each other, it is essential not to examine these policies in isolation, but to evaluate their relative importance by exploring their complementarities and substitutabilities. This analysis will provide significant insights on the role and composition of flexicurity in achieving low unemployment as well as for the implementation of the Danish model in other countries.

To understand the interactions between these three components of flexicurity we develop a microfounded model of searching workers and employing firms, calibrate it to Germany and perform the policy experiment of implementing the full Danish Flexicurity set of policies in Germany, namely low employment protection, high unemployment benefits and active labour market policies, specifically workfare activation requirements, which are seen as the decisive element (Andersen and Svarer, 2007 and 2008). We analyze the unemployment and inequality effects of the set of policies as well as the single policies and their complementarities.

Flexible labour markets enable firms to adapt flexibly to face the challenge of world competition. In exchange for their job security workers receive generous unemployment support (income security) which is combined with workfare policy to strengthen employment incentives and ensure workers’ employment. This set of policies not only implies strong economic complementarities, as our analysis shows, but also entails political complementarities in the sense that the ability to gain political consent for one policy depends on the acceptance of other policies (see Orszag and Snower, 1998). Political implementation of broad labour market reforms is often opposed to if the burden is placed narrowly on a specific group of workers. The joint implementation of the flexicurity policies directly addresses distributional consequences of more flexible labour markets, which are spread through the whole workforce, by providing more generous income support. These political complementarities might generate stronger support for this reform. At the same time the active labour market policies, esp. the workfare activation requirements put workers’ employment incentives back in place and generate strong economic

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109 Specifically, the European Commission has developed a common set of flexicurity principles, endorsed by the European Council (European Commission, 2007 and Council of the European Union, 2007) to guide national reforms as well as flexicurity pathways (European Expert Group on Flexicurity, 2007) as different avenues member countries can follow to reform their labour markets.

110 For a different version of this chapter see Brown and Snower (2009).
complementarities and, thus, enable the sustainability of the flexicurity policy.

Our results show that implementing the Danish flexicurity concept in Germany would reduce unemployment by 50% and would significantly reduce earnings inequality. Furthermore our analysis illustrates that the Danish flexicurity policies have some apparent complementarities in Germany, in the sense that the reduction of unemployment effect is approx. 40% greater when the policies are implemented in conjunction than in isolation. The strongest economic complementarities in reducing unemployment are generated by the joint implementation of higher unemployment benefits and the introduction of workfare. Flexible firing rules and workfare are not complementary at all, while flexible firing rules and higher unemployment benefits are weak.

The remainder is structured as follows: Section 2 discusses the flexicurity concept, its transferability and relates our work to the existing literature. Section 3 presents the theoretical labour market model. Section 4 calibrates the model for Germany, shows and discusses the single and joint effects of the flexicurity policies and their complementarities. Finally, Section 5 concludes.

4.2 Flexicurity

Danish flexicurity is seen as variant of a wider concept which encompasses various existing combinations. Wilthagen and Tros (2004) summarize the various dimensions of flexibility (external numerical, internal numerical, functional and flexible pay) and security (job security, income security, employment and combination security) in a matrix. While the discussion on flexicurity triggered a vast literature on this topic, it has not resulted in a consensus definition. It is either seen as a result of the evolution of labour market institutions and social dialogue (Bredgaard et al., 2006, Madsen, 2006b) or as a policy strategy (Wilthagen, 1998, Wilthagen and Rogowski, 2002, Wilthagen and Tros, 2004, Wilthagen et al., 2004).

The flexicurity concept originated in the Netherlands in the mid 1990s when labour market regulation was reformed. The reform introduced flexible and atypical employment contracts which were entitled to similar social security and working condition rights as for standard employment contracts. As the Dutch version, Danish flexicurity focuses on external numerical flexibility and stemmed from a social dialogue. Danish flexicurity is a result of the combination of three central components, which form a “golden triangle” (Madsen, 2004):

1. very flexible labour markets, resulting from low employment protection for all employees - high external numerical flexibility,
2. extensive unemployment benefits providing income security to the unemployed - up to a replacement rate of 90% for low-skilled workers and
3. active labour market policies aimed at bringing workers back into employment - by

\[111\] For examples see e.g. Auer and Cazes (2003), Wilthagen and Tros (2004) and European Expert Group on Flexicurity (2007).

\[112\] I.e., the ability to combine work and private life.

strengthening employment incentives via activation and workfare requirements, by facilitating 
reintegration as well as by skills upgrading.\textsuperscript{114}

Generally the idea behind flexicurity is that the two components flexibility and security are 
complementary policies. As Andersen and Svarer (2007) though point out these two elements 
have been part of the existing policy framework in Denmark since the mid 1970s, when unem-
ployment was high and persistent. The low unemployment rate was achieved only when in the 
mid 1990s these two features were augmented by a third, namely active labour market policies, 
especially the introduction of workfare activation requirements.\textsuperscript{115} This policy implied as shift 
from a focus on income security to one on employment security or as Torfing (1999) names it, 
a shift from a safety-net to a trampoline, which ensured the transition back to employment. 
Employment incentives were strengthened by reducing the 9 years of passive unemployment 
benefit entitlement successively to four years, with maximally 1 year of passive entitlement 
which is not renewable through activation as before.\textsuperscript{116}

Andersen and Svarer (2007) underline the point that this reform and the introduction of 
workfare had a decisive effect on unemployment and motivated by the Danish flexicurity An-
dersen and Svarer (2008) look exclusively at the sole role of workfare on unemployment within 
a search and matching framework with exogenous separations. It is admissible to use the search 
and matching framework of Mortensen and Pissarides (1999) to analyze labour market pol-
cies, provided that these policies have no significant influence on the matching process itself. 
However, it seems implausible that active labour market policies should have no effect on the 
matching process. To avoid running afoul of the Lucas Critique, we do not take this short-cut 
but derive the policy effects microfoundedly from the intertemporal maximization of economic 
agents and model their incentives explicitly. Additionally, while examining the interplay be-
tween all elements of the Danish flexicurity, we use an endogenous job destruction rate. Clearly, 
omitting this feature would bias the results.

In international comparisons the Danish employment security was perceived very high (Auer 
and Cazes, 2003) and at the same time the Danish flexicurity generated the lowest European 
inequality (Dennis and Guio, 2004). Consequently the issue of transferability of Danish flexicu-
ritiy receives a lot of attention. Some authors (Bredgaard et al, 2005, and Andersen and Svarer, 
2007) regard the transfer of Danish flexicurity into other countries as a quick and dirty copy-
and-paste approach which neglects complex political, historical and social preconditions.\textsuperscript{117} We 
wish to point out that balancing flexible firing rules and workfare requirements with higher un-
employment benefits provides the ability to gain political consent for the full set of policies. And 
in the lines of Coe and Snower (1997) thereby, the flexicurity concept as a broad labour market

\textsuperscript{114}Andersen and Svarer (2007) and Madsen (2008) point out these three key elements are aided by the other 
arms of the extensive Danish welfare state, e.g. a comprehensive educational service, encompassing adult 
vocational training and education, a well-functioning childcare system and publicly financed health care.

\textsuperscript{115}For the Danish flexicurity and the labour market reforms see among many others Auer and Cazes (2003), 

\textsuperscript{116}In line with this rights and duties approach benefit entitlements are lost if workers do not accept activation 
offers.

\textsuperscript{117}Further transferability issues as optimal sequencing and institutional capacity are pointed out by Wilthagen 
reform policy exploits the political complementarities among individual policy measures. The focus in this chapter though lies on the economic complementarities of the flexicurity policies.

In a political-economic model Boeri et al.(2006) analyze the emergence of labour market institutions, specifically, combinations of employment protection legislation strictness and generosity of unemployment benefits. They report that a flexicurity composition arises when wage differentials are significant or the unemployment benefit system sufficiently generous and progressive to generate consent to such a flexicurity policy strategy. The authors though completely neglect active labour market policies which according to Andersen and Svarer (2007) and many others were the decisive component in achieving Denmark’s low unemployment rate.

Algan and Cahuc (2006) argue that the feasibility of the Danish flexicurity model is strongly dependent on the public-spiritedness of the workers. They theoretically analyze the implication of feelings of guilt due to workers cheating on unemployment benefits for the design of labour market institutions. They illustrate that the higher these feelings of guilt, the lower will be the employment protection, the higher the unemployment benefits and the higher the labour market participation. Furthermore, in their empirical analysis the authors show that civic attitudes depend on cultural values and thus, can not be easily changed by reforming labour market institutions. They thereby conclude that the implementation of flexicurity necessitates a comprehensive policy, thereby affecting civic behaviour of its citizens. The authors did not take into account the third element of the Danish flexicurity model, the active labour market policy, especially the activation requirements we consider in our analysis, which completes a comprehensive flexicurity policy package which could change civic behaviour.

Also Zhou (2007) addresses the question whether it is feasible and desirable to transfer the Danish flexicurity model to other countries. He empirically examines to what extend the components of flexicurity have contributed to the low unemployment rate. Regarding the single effects of the components the author qualitatively achieves similar results as in our analysis, but he completely disregards any complementarities in his analysis. In the theoretical model of Zhou (2008) and (2007) the author adopts a simple theoretical two equation model which he calibrates to the French economy to analyze the feasibility of financing the Danish concept. He concludes that the implementation from a high unemployment level is very costly and implies very limited short-run reductions of unemployment. In contrast to his analysis our approach employs a richer and microfounded model which also incorporates the government budget but in which the transition probabilities between the various labour market states depend on workers’ incentives. This model will enable us to analyze the effect of the single policies and their complementarities on workers’ employment incentives, thereby, their transitions and finally, the feasibility of the Danish flexicurity concept in Germany.

In the following Section we will derive this model.

4.3 The Model

We construct a Markov model of the labour market in which the dynamics of employment and unemployment are determined by transition probabilities among various labour market states.
We derive these transition probabilities from optimization principles.

Our model is meant to be both rich enough to capture endogenous wage bargaining, hiring and firing as well as household search, but it also aims to be simple enough to generate straightforward, intuitively transparent, policy guidelines. Accordingly, our model involves some judicious compromizes between analytical simplicity and the depiction of heterogeneous labour market behaviour of workers and firms. The model structure is closely related to Brown et al. (2007a), specifically, it extends their firm side and wage bargaining with household search, as in Brown et al. (2008).

Specifically, our model contains workers in three labour market states:

1. the employed \( (N) \),
2. the unemployed \( (U) \), who are not activated, yet,
3. the activated \( (A) \), who are unemployed workers in activation, specifically in workfare programmes.

Our model describes labour market activity for workers as a Markov process involving these three states. Apart from the probability of activation which is a policy variable, the transition probabilities among these states are derived from microeconomic foundations.

We assume constant returns to labour. Let \( qe^N \) be the labour productivity of an employee, where \( e^N \) is the work effort of the employed worker. The firm faces a random cost \( \varepsilon_t \), which is iid across workers and time. This cost may be interpreted as, say, an operating cost or a negative productivity shock. Its mean is normalized to zero and its cumulative distribution \( \Gamma(\varepsilon_t) \) is time-invariant.

Workers’ instantaneous utility \( u_t(c_t, l_t) \) depends on consumption \( c_t \) and leisure \( l_t \), where the time endowment is normalized to 1. For simplicity we assume that workers consume all their income. Employed workers receive a wage \( w_t \) and pay payroll taxes with a rate of \( \tau_t \) and unemployed workers receive unemployment benefits \( b_t \). Employed workers provide work effort \( e^N \), which creates disutility as it restricts the available time for leisure. Unemployed workers divide their time between leisure and job search \( e^U_t \). Activated unemployed workers \( (A) \) in addition to searching for a job \( e^A_t \) have to provide the required workfare effort \( e^A \). Furthermore, in line with Andersen and Svarer (2008) we allow the utility function of employed workers \( (N) \) to differ from the unemployeds’ \( (U, A) \) to capture potential stigmatisation effects of unemployment.

Agents in our model pursue the following sequence of decisions. First the government sets its policy variables and balances its budget. Second, workers make their search decision. Third, the operating costs are revealed. Then, wages are determined through bargaining and fourth, firms make their employment decisions.

\(^{118}\)See also Snower and Merkl (2006). Lechthaler et al. (2008) included this model into a DSGE framework.

\(^{119}\)We follow the notational convention that only those variables have time subscripts that actually vary through time in our model.

\(^{120}\)Thus, leisure time is the time which is not spent working, being on workfare or searching for a job: \( l^A_t = 1 - e^A_t - e^A, l^N_t = 1 - e^N_t, l^U_t = 1 - e^U_t \).
4.3.1 Government Budget

Our model considers 4 instruments of government policy:

(i) a proportional payroll tax, with a tax rate $\tau_t$, paid by all employed workers, set to balance the governments’ budget,

(ii) unemployment benefits $b_t$, paid to all unemployed workers,

(iii) legislation on the flexibility of the labour market, i.e. firing costs $f_t$, at no expense,

(iv) active labour market policy in form of workfare activation, specifically, unemployed workers are required to participate in workfare programmes to remain eligible for unemployment benefits, with an activation probability $\alpha_t$ and a work requirement $e_t^A$ which creates some costs $\kappa_t$ per activated worker.\footnote{For simplicity in our model active labour market policy does not affect workers’ human capital, thereby we bias the model against us.}

Assuming a constant labour force $L$ the government budget is given by

$$n_tw_t\tau_t = (a_t + u_t)b_t + a_t\kappa_t$$

where $w_t$ is the wage, $n_t$, $u_t$, $a_t$ the rate of workers employed, passively unemployed and activated, respectively.

4.3.2 Transitions Among Labour Market States

The transitions among the labour market states are summarized in Figure 4.1.\footnote{For simplicity we assume that the effort on workfare does not influence the cost of workfare.} For analytical simplicity, we choose to describe these transitions in terms of a small number of transition variables.

\footnote{We assume that workers who are activated will always accept going on workfare since otherwise they would suffer severe reductions in unemployment benefits.}

\footnote{To simplify notation, we suppress the subscripts referring to time ($t$) here.}
The unemployed \((U)\) are matched with probability \(\mu^U\) and then become employed; alternatively if they are not matched with probability \((1 - \mu^U)\) they are activated with probability \(\alpha\) and enter workfare programmes; with the probability \((1 - \alpha)\) \((1 - \mu^U)\) they remain unemployed on passive benefits. The activated workers are matched with probability \(\mu^A\) and then become employed \((N)\); with probability \((1 - \mu^A)\) they are not hired and remain unemployed and in activation. At the end of a period, new hires turn into insiders. As insiders, they lose their jobs with probability \(\phi\) and then become unemployed; with probability \((1 - \phi)\) they are retrained.\(^{126}\)

Thus the labor market system in period \(t\) may be described as follows:

\[
S_t = T_t S_{t-1} \tag{4.2}
\]

where \(S_t\) is a vector of the labor market states:

\[
S_t = (N_t, N^E_t, U_t, A_t)'
\tag{4.3}
\]

and \(T_t\) is a Markov matrix of transition probabilities:

\[
T_t = \begin{pmatrix}
1 - \phi_t & \mu^U_t & \mu^A_t \\
\phi_t & (1 - \alpha_t)(1 - \mu^U_t) & 0 \\
0 & \alpha_t(1 - \mu^U_t) & (1 - \mu^A_t)
\end{pmatrix}
\tag{4.4}
\]

We now proceed to derive the transition probabilities from microeconomic foundations.

### 4.3.3 Households’ Search Decisions

As noted, workers’ utility \(v_t\) depends on consumption and leisure. As pointed out above workers can be employed \((N)\) or unemployed and when unemployed, they can be on passive benefits \((U)\) or activated in workfare programmes \((A)\). The passive unemployed \((U)\) and the activated unemployed \((A)\) workers determine the amount of effort \(e^U_t, e^A_t\) they expend in searching for a job. The probability of an unemployed or activated unemployed worker being employed depends on his search intensity (i.e. the length of time unemployed workers spent searching). The harder unemployed search for a job relative to the other workers the more likely they are to find a a firm, which is hiring.\(^{127}\) Thus, an unemployed worker’s job expected finding probability \(E_t(\mu^U_t)\) is

\[
E_t(\mu^U_t) = \frac{\zeta e^U_{t-1}}{e^U_{t-1}} E(\eta_t),
\tag{4.5}
\]

and an activated unemployed worker’s expected job finding probability \(\mu^A_t\) is

\(^{125}\)Frederiksson and Holmund (2006) argue that a fixed time duration of activation can be approximated by a stochastic transition into activation.

\(^{126}\)Entrants turn into insiders at the end of a period. In case they are fired at the beginning of the next period these entrants have been insiders just for an instant.

\(^{127}\)There is no on-the-job search.
\[ E_t (\mu^A_t) = \zeta \frac{\bar{e}^A_t}{\bar{e}^{A-1}_t} E (\eta_t), \]  

(4.6)

where \( \eta_t \) is the expected firm’s hiring rate, \( \zeta \) is a scale parameter and \( \bar{e}_t \) is the average amount of job search effort:

\[
\bar{e}_t = \left( \frac{\sum_{i=1}^U e_{t,i}^U + \sum_{j=1}^{A_t} e_{t,i}^A}{U_t + A_t} \right) = \frac{(u_t e_t^U + a_t e_t^A)}{u_t + a_t} \tag{4.7}
\]

and where \( U_t \) is the number of unemployed job searchers and \( A_t \) the number of activated job searchers and \( u_t = \frac{U_t}{L} \) and \( a_t = \frac{A_t}{L} \) are the respective rates given the constant labour force \( L \). Thereby each household takes \( \bar{e}_t \) as exogenously given when it makes its search decision.\(^{128}\)

Workers choose the search effort which maximizes their expected present value of lifetime utility taking into account the effects of their leisure choices on the job finding rates.

A unemployed worker maximizes his expected present value of utility \( E_t (V_t^U) \) over his search effort \( e_t^U \):

\[
E_t (V_t^U) = \max_{e_t^U} v_t^U (b_t, 1 - e_t^U) + \delta E_t \left( \mu_{t+1}^U (e_t^U) V_{t+1}^N + (1 - \mu_{t+1}^U (e_t^U)) \alpha_{t+1} V_{t+1}^A \right), \tag{4.8}
\]

where \( v_t \) is the contemporaneous utility, which depends on consumption and leisure, \( \delta \) is the discount factor, \( E_t (V_{t+1}^N) \) and \( E_t (V_{t+1}^A) \) an employed worker’s and an activated unemployed workers’ present value of utility in \( t + 1 \), respectively, which are:

\[
E_t (V_{t+1}^N) = E_t \left( v_{t+1}^N (w_{t+1} (1 - \tau_{t+1}), 1 - e_{t+1}^N) \right) + \delta E_t \left( (1 - \phi_{t+2}) V_{t+2}^N + \phi_{t+2} V_{t+2}^U \right), \tag{4.9}
\]

\[
E_t (V_{t+1}^U) = E_t \left( v_{t+1}^U (b_{t+1}, 1 - e_{t+1}^U) \right) + \delta E_t \left( \mu_{t+2}^U (e_{t+1}^U) V_{t+2}^N + (1 - \mu_{t+2}^U (e_{t+1}^U)) \alpha_{t+2} V_{t+2}^A \right), \tag{4.10}
\]

and

\[
E_t (V_{t+1}^A) = E_t \left( v_{t+1}^A (b_{t+1}, 1 - e_{t+1}^A - e_{t+1}^A) \right) + \delta E_t \left( \mu_{t+2}^A (e_{t+1}^A) V_{t+2}^N + (1 - \mu_{t+2}^A (e_{t+1}^A)) V_{t+2}^A \right). \tag{4.11}
\]

Along the same line, an activated unemployed worker maximizes his present value of utility \( E_t (V_t^A) \) over search effort \( e_t^A \):

\(^{128}\)Note that we are assuming that the effort decisions of an entrant affects the probability of getting hired (relative to the probability that other job searchers get hired), but it does not affect the productivity of the entrants, once they are at work. In other words, the unemployed’s effort decision affects the probability of getting hired, but not the productivity on the job.
\[ E_t \left( V_t^A \right) = \max_{\epsilon_t^A} u_t^A \left( b_t, 1 - \epsilon_t^A - \epsilon_t^A \right) + \delta E_t \left( \mu_{t+1}^A \left( \epsilon_t^A \right) V_{t+1}^N + (1 - \mu_{t+1}^A \left( \epsilon_t^A \right)) V_{t+1}^A \right). \] (4.12)

The first-order conditions for these decision problems for the passive and activated unemployed, respectively, are:

\[ -v_{t+1}^U = \frac{\delta \zeta E_t \left( \eta_{t+1} \right)}{e_t} E_t \left( V_{t+1}^N - \alpha_{t+1} V_{t+1}^A - (1 - \alpha_{t+1}) V_{t+1}^U \right)^{129} \] (4.13)

\[ -v_{t+1}^A = \frac{\delta \zeta E_t \left( \eta_{t+1} \right)}{e_t} E_t \left( V_{t+1}^N - V_{t+1}^A \right). \] (4.14)

In other words, the marginal utility of leisure must be set equal to the discounted marginal job finding propensity \( \left( \frac{\zeta \delta E_t \left( \eta_{t+1} \right)}{e_t} \right) \) times the reward for seeking a job. This reward is the difference between the expected present value of lifetime utility of being employed \( E_t \left( V_{t+1}^N \right) \) and the present value of lifetime utility of not finding a job. For activated workers the latter value corresponds to the the expected present value of lifetime utility of being an activated unemployed worker \( E_t \left( V_{t+1}^A \right) \), and for not activated unemployed workers it is equal to the weighted average of the expected present value of lifetime utility of a passive and an activated unemployed worker \( E_t \left( V_{t+1}^N - \alpha_{t+1} V_{t+1}^A - (1 - \alpha_{t+1}) V_{t+1}^U \right) \). The weights are given by the respective probabilities of being activated and remaining on passive benefits. Given diminishing marginal utility of leisure, the optimal leisure depends inversely on the reward for seeking a job.

Once workers have decided on their search effort, the idiosyncratic operating cost is revealed and insiders bargain for wages.

### 4.3.4 Wage Determination

For simplicity, let the wage \( w_t \) be the outcome of a Nash bargain between the median insider and her firm. Our wage bargaining model is analogous to the median voter model, where the utility of the median voter is maximized. The firm bargains with a union which maximizes the utility of the median insider. When the bargaining decision takes place, nobody has been fired yet. The median insider is the worker who is situated exactly in the middle of the distribution and who faces no risk of dismissal at the negotiated wage. She has average operating costs normalized to zero. The wage is renegotiated in each period \( t \).

Under bargaining agreement, the insider receives the wage \( w_t(1 - \tau_t) \) where \( \tau_t \) is the payroll tax rate, and has some disutility from regular employment \( e_t^N \), and the firm receives the expected profit \( \left( q e_t^N - w_t \right) \) in each period \( t \). Thus the expected present value of the insider’s utility \( E_t \left( V_t^I \right)^{130} \) under bargaining agreement is

\[ E_t \left( V_t^I \right)^{130} = \max_{\epsilon_t^A} u_t^A \left( b_t, 1 - \epsilon_t^A - \epsilon_t^A \right) + \delta E_t \left( \mu_{t+1}^A \left( \epsilon_t^A \right) V_{t+1}^N + (1 - \mu_{t+1}^A \left( \epsilon_t^A \right)) V_{t+1}^A \right). \] (4.12)

\[ -v_{t+1}^U = \frac{\delta \zeta E_t \left( \eta_{t+1} \right)}{e_t} E_t \left( V_{t+1}^N - \alpha_{t+1} V_{t+1}^A - (1 - \alpha_{t+1}) V_{t+1}^U \right)^{129} \] (4.13)

\[ -v_{t+1}^A = \frac{\delta \zeta E_t \left( \eta_{t+1} \right)}{e_t} E_t \left( V_{t+1}^N - V_{t+1}^A \right). \] (4.14)

129 This can also be written as \( -v_{t+1}^U = \frac{\delta \zeta E_t \left( \eta_{t+1} \right)}{e_t} \left( V_{t+1}^N - V_{t+1}^U - \lambda_t \left( V_{t+1}^A - V_{t+1}^U \right) \right) \). In absence of workfare, for \( \lambda_t = 0 \) this reduces to \( -v_{t+1}^U = \frac{\delta \zeta E_t \left( \eta_{t+1} \right)}{e_t} \left( V_{t+1}^N - V_{t+1}^U \right) \).

130 Please note that \( V_t^I \), the expected present value of utility of an insider, is equal to \( V_t^N \), that of an employed
where $\phi_{t+1}$ is the firing rate and $V_{t+1}^{U}$ the expected present value of a unemployed workers’ returns. The expected present value of firm’s returns under bargaining agreement are

$$E_t \left( \Pi_t \right) = \left( \epsilon^N q - w_t \right) + \delta E_t \left( \left(1 - \phi_{t+1} \right) \Pi_{t+1} - \phi_{t+1} f_{t+1} \right) \quad (4.16)$$

where $f_t$ are firing costs.

Under disagreement insiders workers go on strike and cause a cost $-\theta f_t$ with $0 \leq \theta \leq 1$ to the firm up to short of inducing dismissal, and the insider’s fallback income is assumed for simplicity to be equal to the unemployment benefits $b_t$. Under disagreement the firm (goes to a secondary, temporary market and) tries to hire temporarily activated workers (in partial replacement of the workers on strike). As it has to search for these workers, hiring of temporary workers takes place with the probability $\chi$.

Assuming that disagreement in the current period does not affect future returns, the present values of utility under disagreement for the insider are

$$E_t \left( V_t^{II} \right) = v_t \left( b_t, 1 \right) + \delta E_t \left( \left(1 - \phi_{t+1} \right) V_{t+1}^{I} + \phi_{t+1} V_{t+1}^{U} \right) \quad (4.17)$$

and for the firm are

$$E \left( \Pi_t^{II} \right) = -\theta f_t + \chi \left( \epsilon^A q - b_t \right) + \delta E_t \left( \left(1 - \phi_{t+1} \right) \Pi_{t+1}^{I} - \phi_{t+1} f_{t+1} \right). \quad (4.18)$$

Thus, the insider’s bargaining surplus is

$$E_t \left( V_t^{I} \right) - E_t \left( V_t^{II} \right) = v_t \left( w_t \left(1 - \tau_t \right), 1 - \epsilon^N \right) - v_t \left( b_t, 1 \right) \quad (4.19)$$

and the firm’s surplus is

$$E_t \left( \Pi_t \right) - E_t \left( \Pi_t^{I} \right) = \epsilon^N q - w_t + \theta f_t - \chi \left( \epsilon^A q - b_t \right) \quad (4.20)$$

The negotiated wage maximizes the Nash product ($\Lambda$):

$$\Lambda_t = \left( v_t^N \left( w_t \left(1 - \tau_t \right), 1 - \epsilon^N \right) - v_t^U \left( b_t, 1 \right) \right)^\gamma \left( \epsilon^N q - w_t + \theta f_t - \chi \left( \epsilon^A q - b_t \right) \right)^{1-\gamma}, \quad (4.21)$$

---

131 This cost may be generated through activities such as picket lines, work-to-rule, sabotage, etc. Employed workers have an incentive to impose this cost if these activities per se are costless to them (but of course costly to the firm). The reason is that reducing the firm’s fallback profit raises the bargaining rent, some of which is captured by the workers. However the incumbent workers have no incentive to drive the firm’s fallback profit below $-f_t$, for then the firm would find it worthwhile to fire them.

132 For this to happen, the following must hold $\epsilon^A q - b > 0$.

133 Once an agreement has been reached, insiders go back to work and temporarily hired workers, who are in activation, are fired.
where $\gamma$ represents the bargaining strength of the insider relative to the firm, and satisfies:

$$1 = \frac{v_t u_t \gamma \left( e^N q - w_t + \theta f_t - \chi_t \left( e^A q - b_t \right) \right)}{(1 - \gamma) \left( v_t(u_t(1 - \tau_t), 1 - e^N) - v_t \right) (b_t, 1))} \quad (4.22)$$

### 4.3.5 Firms’ Hiring and Firing Decisions

First consider the firing rate $\phi_t$ for insiders. An insider is associated with the wage $w_t$ and the firing cost $f_t$. Let the time discount factor be $\delta$. Recalling that the insider’s productivity is $e^N q_t$, the expected present value of profit generated by an insider, after the random cost $\varepsilon_t$ at time $t$ is observed, is

$$E_t (\pi_t^I) = (e^N q - w_t - \varepsilon_t) + \sum_{i=t+1}^{\infty} \delta^i \left( (1 - \phi_i)^i \left( e^N q - w_t - E_t (\varepsilon_i | \varepsilon_i < \nu_t^I) \right) - \phi_i f_t (1 - \phi_i)^{i-1} \right), \quad (4.23)$$

so that

$$E_t (\pi_t^I) = \frac{e^N q - w_t - \delta \phi_i f_t - \delta (1 - \phi_i) E_t (\varepsilon_i | \varepsilon_i < \nu_t^I)}{1 - \delta (1 - \phi_i)} - \varepsilon_t. \quad (4.24)$$

The expected incentive to retain the insider $(\nu_t^I)$ is defined as the difference between the expected gross profit from retaining the insider $(E_t (\pi_t^I) = \frac{e^N q - w_t - \delta \phi_i f_t - \delta (1 - \phi_i) E_t (\varepsilon_i | \varepsilon_i < \nu_t^I)}{1 - \delta (1 - \phi_i)})$, the expected profit from firing him $(-f_t)$, i.e. this insider retention incentive is

$$\nu_t^I = \frac{e^N q - w_t + (1 - \delta) f_t - \delta (1 - \phi_i) E_t (\varepsilon_i | \varepsilon_i < \nu_t^I)}{1 - \delta (1 - \phi_i)} \quad (4.25)$$

An insider is fired in period $t$ when the realized value of the random cost $\varepsilon_t$ is greater than the insider employment incentive. Since the cumulative distribution of the operating cost is $\Gamma (\varepsilon_t)$, the insider’s firing rate is

$$\phi_t = 1 - \Gamma (\nu_t^I) \quad (4.26)$$

Next consider the hiring rate $\eta_t$ for unemployed and unemployed, activated workers. The expected present value of profit generated by an entrant (a worker who has been hired after being unemployed), after the random cost $\varepsilon_t$ at time $t$ is observed, is

$$E_t (\pi_t^E) = (e^N q - w_t - \varepsilon_t - h) + \sum_{i=t+1}^{\infty} \delta^i \left( (1 - \phi_i)^i \left( e^N q - w_t - E_t (\varepsilon_i | \varepsilon_i < \nu_t^I) \right) - \phi_i f_t (1 - \phi_i)^{i-1} \right), \quad (4.27)$$

134 In the first period, profit is $(q e^N - w_t - \varepsilon_t)$; in the second period, the insider is retained with probability $(1 - \phi_i)$ and the insider is fired with a probability of $\phi_i$ and so on.

135 $E_t (\varepsilon_i | \varepsilon_i < \nu_t^I)$ is the expected value of the operating cost conditional on being retained.

136 The "gross" profit is expected profit generated by retaining him, without accounting for the operating cost.

137 Equivalently, the insider is hired when the profit from retaining the insider is less than the firing cost $(e^N q - w_t - \delta \phi_i f_t - \delta (1 - \phi_i) E_t (\varepsilon_i | \varepsilon_i < \nu_t^I) / (1 - \delta (1 - \phi_i)) - \varepsilon_t < -f_t).$
where $h$ is the hiring cost, so that

$$E_t \left( \pi^E_t \right) = \frac{e^N q - w_t - \delta \phi_t f_t - \delta (1 - \phi_t) E_t \left( \varepsilon_t | \varepsilon_t < \nu^U_t \right)}{1 - \delta (1 - \phi_t)} - \varepsilon_t - h. \quad (4.28)$$

The expected incentive to hire an entrant ($\nu^E_t$) is defined as the difference between the expected gross profit from employing the entrant\(^{138}\) and the expected profit from not doing so (i.e. zero). Thus the unemployed hiring incentive is

$$\nu^E_t = \frac{e^N q - w_t - \delta \phi_t f_t - \delta (1 - \phi_t) E_t \left( \varepsilon_t | \varepsilon_t < \nu^U_t \right)}{1 - \delta (1 - \phi_t)} - h \quad (4.29)$$

An unemployed worker is hired in period $t$ when the realized value of the random cost $\varepsilon_t$ is less than the entrant hiring incentive:\(^{139}\) $\varepsilon_t < \nu^E_t$. Thus the hiring rate for unemployed (passive and activated) is

$$\eta_t = \Gamma \left( \nu^E_t \right). \quad (4.30)$$

### 4.3.6 Employment, Unemployment and the Labour Market Equilibrium

The change in employment ($\Delta N_t$) is the difference between the outflow from the total unemployment pool ($\mu^U_t U_{t-1} + \mu^A_t A_{t-1}$) and the outflow from the employment pool ($\phi_t N_{t-1}$):

$$\Delta N_t = \mu^U_t U_{t-1} + \mu^A_t A_{t-1} - \phi_t N_{t-1}. \quad (4.31)$$

Assuming a constant labour force $L$ and defining the employment rate to be $n_t = N_t / L$, we obtain the following employment dynamics equation:\(^{140}\)

$$n_t = \mu^U_t u_{t-1} + \mu^A_t a_{t-1} + (1 - \phi_t) n_{t-1} \quad (4.31)$$

The passive unemployed comprise those workers who were unemployed in the previous period and who have neither been hired or activated in the current period and those who have been fired. Thus the passive unemployment dynamics equation is

$$u_t = (1 - \mu^U_t) \left( 1 - a_t \right) u_{t-1} + \phi_t n_{t-1} \quad (4.32)$$

The activated unemployment rate is the difference between 1 and the aggregate employment and unemployment rates:

$$a_t = 1 - n_t - u_t \quad (4.33)$$

$$= (1 - \mu^U_t) a_{t-1} + (1 - \mu^A_t) a_{t-1}. \quad (4.34)$$

---

\(^{138}\) This "gross" profit is the expected profit generated by hiring an unemployed worker, without taking the operating cost into account.

\(^{139}\) Equivalently, the entrant is hired when the profit from employing this worker is greater than the hiring cost: $\frac{e^N q - w_t - \delta \phi_t f_t - \delta (1 - \phi_t) E_t \left( \varepsilon_t | \varepsilon_t < \nu^U_t \right)}{1 - \delta (1 - \phi_t)} - \varepsilon_t > h$.

\(^{140}\) Note that $\Delta n_t = \mu^U_t (1 - n_{t-1} - a_{t-1}) + \mu^A_t (1 - n_{t-1} - u_{t-1}) - \phi_t n_{t-1}$. 

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The labour market equilibrium is the solution of the system comprising

- the employment and unemployment dynamics, eqs. 4.31, 4.32, 4.33,
- the firing and hiring rate as well as the job finding rates, eqs. 4.26, 4.30, 4.5, 4.6,
- the entrant hiring incentive and the insider retention incentive, eqs. 4.29, 4.25,
- the search equations, eqs. 4.13, 4.14, 4.7,
- the negotiated wage, eq. 4.22, and
- the government budget constraint (eq. 4.1).

We now proceed to calibrate the model above for German data and analyze the effectiveness of Danish flexicurity on unemployment and inequality. We will proceed as follows: first, the calibration, then the numerical results and an intuitive discussion of the policy effects.

### 4.4 Quantitative Evaluation

In this Section we evaluate the unemployment and inequality effects of implementing the Danish flexicurity concept in Germany. Our analysis shows that, for reasonable parameter values, the Danish flexicurity policy has huge incentive effects and sizeable complementarities in terms of unemployment. For our purpose, we specify particular functional forms for the behavioural relations above and calibrate the resulting model.

#### 4.4.1 Specification

We start by specifying the functional forms: We assume a logistic distribution for the operating cost \( \varepsilon_t \), such that the time-invariant cumulative density function is

\[
\Gamma(\nu_t^{L,E}) = \frac{1}{1 + e^{-(\nu_t^{L,E} - E(\varepsilon_t))/s}}; \tag{4.34}
\]

recall that due to normalisation \( E(\varepsilon_t) \) is zero, \( s \) is the scale parameter of the distribution. In line with Andersen and Svarer (2008) and Fredriksson and Holmlund (2006) workers’ utility is additively separable as well as logarithmic in consumption and leisure and the function differs between employed and unemployed (non-activated and activated) workers:

\[
u_t^{U,A}(c_t^{U,A}, l_t^{U,A}) = \ln c_t^{U,A} + \ln l_t^{U,A} \tag{4.35}\]

\[
u_t^{N}(c_t^{N}, l_t^{N}) = \ln xc_t^{N} + \ln l_t^{N}, \tag{4.36}\]
where $x > 1$ is a non-monetary payoff from employment. This yields the wage

$$w_t = \frac{\gamma \left( (e^N - \chi \epsilon_A^t) q + \theta f_t + \chi b_t \right)}{(1 - \gamma) \ln \left( \frac{xw_t(1 - \tau_t)(1 - e^N)}{b_t} \right)} + \gamma. \quad (4.37)$$

For these functional forms, we now proceed to calibrate the resulting model.

### 4.4.2 Calibration

The period of analysis is one quarter. The quarterly interest rate $r$ is set to yield a rate of 3% per year, which corresponds to the average money market rate over the last 10 years in Germany, and we set the discount factor to $\delta = \frac{1}{1 + r}$. Hiring costs are set to 60% of quarterly productivity as used by Mortensen and Pissarides (1999).

We apply institutional features of the German labour market by calibrating the unemployment benefit in the steady state to $b_0 = \beta_0 w_0 (1 - \tau_0)$ with a net replacement rate for Germany of $\beta_0 = 0.62$ and quarterly firing costs relative to the wage to $f_0 = \rho_0 w_0$ with $\rho_0 = 2.4$, in line with yearly values provided by Chen and Funke (2003) and Bentolila and Bertola (1990).

The unemployment rate is set to the standardized unemployment rate of Germany 2005, namely 0.095, see OECD (2007). Given this unemployment rate the quarterly job finding rate $\mu u_0 = 3.2\%$, taken from Wilke’s (2005) Kaplan-Meier functions for Germany, yields a steady state firing rate of $\phi_0 = \mu u_0 n = 3.4\%$. The free parameter $s = 1.68$ of the CDF of the operating costs and $\zeta = 0.36$ are set to match the job finding and the firing rate and generate a long-run wage elasticity of labour demand, which is within the range of $-0.1$ and $-1.3$ found for Germany, see Riphahn et al. (1999), specifically of $-0.86$.

Andersen and Svarer (2008) assume that workers spend 60% of their time at work ($e^N$). The relative time use for effort and leisure of employed workers per weekday for Western Europe from Krueger and Mueller (2008) suggests a value for $e^N$ of 69%, while the relative allocation in Freeman and Schettkat (2005) points at a value between 61-64%. Similarly, from the relative time use for effort and leisure of unemployed workers we can determine the effort of unemployed workers: the analysis of Krueger and Mueller (2008) suggests a value of 10%.

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142 Thus, the utility of consumption for unemployed differs from that of employed. This can be interpreted e.g. as the converse of the circumstance that being unemployed negatively affects workers’ subjective well being (stigmatising effects), see Krueger and Mueller (2008) also for previous research on this topic. Separability implies that leisure does not depend on current income.

143 See Deutsche Bundesbank (2008).

144 These costs consist of 30% recruiting and 30% training costs; see Mortensen and Pissarides (1999).

145 For simplicity, we took the unweighted average across six family types as well as over the initial period of unemployment and long-term unemployment for 2005, see OECD (2008).

146 Please note that variables and parameters with subscript "0" denote the value at the calibrated steady state for Germany, i.e. before any flexicurity policy exercise.

147 Employed workers spend 395 minutes per weekday on the effort considered here - 395 for work and 0 for job search - and 179 minutes for leisure and socialising, see Table 3 from Krueger and Mueller (2008). Thus, employed workers divide their time in 69% effort and 31% leisure.

148 Here we compare market work and leisure for women and men, respectively. Western Europe here comprises Netherlands, Italy and Austria. Freeman and Schettkat (2001) provide values for Germany but do not distinguish between employed and unemployed.

149 Unemployed workers from Western Europe spend 33 minutes on effort - 19 for work and 14 for job search - and 313 minutes for leisure and socialising, see Table 3 from Krueger and Mueller (2008). Thus, workers divide...
and Freeman and Schettkat (2005) offer values from 11% to 18%. 150 In line with these studies we set \( e^N = 65\% \) and \( e^U_I = 15\% \). These values then yield the value for the utility parameter of \( x \approx 1.84 \). Furthermore, for simplicity \( q \) is normalized to 1 and we use a bargaining power \( \gamma \) equal to 0.6 which corresponds to the average of the values estimated for union bargaining power in Germany by Dumont et al. (2006). This in turns determines a value for \( \theta \approx 0.03 \). The resulting elasticity of the wage to firing costs with a value of 0.05 is very conservative compared to the results from van der Wiel (2008), which suggest a value of 0.1. 151

With this calibration our model generates reasonable values for various elasticities which can be found in the literature: our calibration yields an elasticity of unemployment duration to unemployment benefits of \(-0.5\), Hornstein et al. (2005) refering to various studies report values between \(-0.1\) and \(-1\). 152 Furthermore, the amplification mechanism of our calibrated model with an elasticity of unemployment to productivity of approximately \(-5\) is nearer to the data than e.g. the standard search and matching model. 153

Before the policy exercises a tax rate of \( \tau_0 \approx 0.06 \) balances the government’s budget. In the following policy exercise we apply Danish values to the poicy variables: the net replacement rate will be increased to the Danish level of \( \beta_1 = 0.69 \). 154 The firing cost parameter \( \rho \) for the policy exercise is set to \( \rho_1 = 0.3 \), which is suggested by the relative value (50%) of the employment protection indice of Denmark relative to Germany by the OECD (2004b) for regular employment as well as by Lazear (1990) in terms of severance pay. 155

Andersen and Svarer (2007) point out that unemployed workers receive an activation offer not later than after 12 months of unemployment, thus we set \( \lambda_1 = 0.25 \). Furthermore, we assume that workfare requirements correspond to the work effort of employed workers, thus, we set \( e^A = e^N = 65\% \) and will assume that the cost of workfare is comparable to the one applied in Denmark, thus, we choose a cost per activated worker \( \kappa_1 \) equal to 2.5% in line with Andersen and Svarer (2008). 156

The only parameter where to the best of our knowledge the literature does not supply any

150 Again we compare market work and leisure for women and men, respectively.
151 Van der Wiel (2008) estimated that the reduction of the period of notice of 3.4 months by one month leads to a wage reduction of 3%. This suggests an elasticity of 0.1
152 Furthermore, our calibrated model implies a wage elasticity to productivity of 0.43 which matches approximately Hagedorn and Manovskii (2008)’s estimate for the cyclicality of wages of 0.45, though for the US.
153 Shimer (2005) shows that while for the U.S. unemployment is 10 times more volatile than productivity, the search and matching model can only generate a value of 0.5.
154 For simplicity, we took the unweighted average across six family types and over the initial period of unemployment and long-term unemployment for 2005, see OECD (2008). Madsen (2007) and Andersen and Svarer (2007) illustrate that unemployment benefits amount to 90% of previous earnings but face an absolute ceiling implying that gross compensation rates decline rapidly with previous income once the limit is reached.
155 This value might well be too high: Emerson (1988), Bertola (1990) and Garibaldi (1998) rank Denmark in terms of employment protection far above Germany near the UK. Bentolila and Bertola (1990) suggest a firing cost value for the UK which is one third of the value for Germany. Various indices on employment protection provide ranges for the relative values of firing costs of Denmark from 0% to 80% of the German value: the World Bank (2008) 0.3 (difficulty of firing) or 0 (firing costs); Belot et al. (2007) 0.6 (for regular jobs) and 0.3 (for overall protection); OECD (2004) 0.6 or 0.7 for overall strictness (version 1 or 2); Botero et al. (2004) 0.8 (employment laws index); Garibaldi (1998) 0.4 (strictness of procedural constraints).
156 They argue that this corresponds to a cost of workfare in Denmark of around 3% of GDP. They point out that if the activation results in output, it could be interpreted as the net cost per activated worker.

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estimates is $\chi$ which determines the wage effect of workfare. We will set this parameter to replicate the unemployment rate of Denmark to evaluate the strength of this channel, but we will provide a robustness analysis. Tables 4.1, 4.2 and 4.3 summarize the calibrated parameters, the values of the variables in the calibrated steady state and the flexicurity policy parameters.

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<tr>
<th>Parameter</th>
<th>Description</th>
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<tr>
<td>$\delta$</td>
<td>discount factor</td>
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<tr>
<td>$\epsilon^w$</td>
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<td>$h$</td>
<td>hiring cost in terms of productivity</td>
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</tr>
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<td>$p$</td>
<td>quarterly productivity</td>
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<td>$s$</td>
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<tr>
<td>$\theta$</td>
<td>parameter determining the wage effect of firing costs</td>
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<td>$\zeta$</td>
<td>job finding rate parameter</td>
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<tr>
<td>$x$</td>
<td>utility parameter</td>
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<td>$\beta_0$</td>
<td>net replacement rate</td>
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<td>$\rho_0$</td>
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<td>$\tau_0$</td>
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Table 4.1: Parameter values.

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<tr>
<td>$\phi_0$</td>
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<td>$\mu u_0$</td>
<td>job finding rate</td>
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<td>$u_0$</td>
<td>unemployment rate</td>
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<td>$e_U^0$</td>
<td>search effort</td>
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Table 4.2: Steady state labour market values.

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<th>Description</th>
<th>Value</th>
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</thead>
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<tr>
<td>$\beta_1$</td>
<td>flexicurity net replacement rate</td>
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</tr>
<tr>
<td>$\epsilon^w_1$</td>
<td>work effort in workfare</td>
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</tr>
<tr>
<td>$\kappa_1$</td>
<td>cost per activated worker</td>
<td>0.025</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>probability of activation</td>
<td>0.25</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>flexicurity firing cost parameter relative to the wage</td>
<td>1.2</td>
</tr>
<tr>
<td>$\chi_1$</td>
<td>wage effect of workfare parameter</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table 4.3: Flexicurity policy parameters.

### 4.4.3 Results and Intuition

As pointed out above we apply policy variables which match the Danish flexicurity approach in line with Andersen and Svarer (2008). Assuming a probability for a firm of finding an activated unemployed worker of $\chi_1 = 40\%$ the flexicurity approach of adopting Denmarks’ level of unemployment benefit, employment protection and introducing workfare reduces unemployment by 50%, yielding an unemployment rate of 4.8% which is equal to Denmark’s 2007 unemployment rate, OECD (2007), and reduces inequality, in terms of the Gini coefficient\footnote{Note that the Gini coefficient generated here is lower than in reality, as our model does not generate income differentials, does not distinguish ability groups and does not take non-wage related inequalities into account.} from 3.4% to
1.6%; see Table 4.4. In the following we will discuss the single effects and complementarities leading to these results.

Table 4.4: Single and joint effects of the flexicurity instruments on unemployment and inequality.

<table>
<thead>
<tr>
<th>Flexicurity Instrument</th>
<th>% Change of Unemployment</th>
<th>Gini Coeff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Workfare Introduction ($\alpha_1, \epsilon_1^L$)</td>
<td>$-85%$</td>
<td>$0.5%$</td>
</tr>
<tr>
<td>Firing Cost Reduction ($\rho_1$)</td>
<td>$-16%$</td>
<td>$2.8%$</td>
</tr>
<tr>
<td>Unemployment Benefit Increase ($\beta_1$)</td>
<td>$65%$</td>
<td>$5.4%$</td>
</tr>
<tr>
<td>Joint effect</td>
<td>$-50%$</td>
<td>$1.6%$</td>
</tr>
</tbody>
</table>

Single Effects

Reducing Firing Costs

Figure 4.2 displays the effects of reducing the firing costs parameter $\rho_1$.158 Reducing firing costs reduces the insider retention incentive and increases the outsider hiring incentive and thereby, increases the firing rate as well as the hiring rate. The resulting direct effect on unemployment is ambiguous, which is commonly pointed out in the literature, e.g. Nickell (1997) and Bentolila and Bertola (1990). At the same time lower firing costs though lower the wage and consequently increase both retention and hiring incentives. This indirect wage effect of reducing firing costs counterbalances the positive direct effect on the firing rate and strengthens the positive direct effect on the hiring and thereby on the job finding rate. In our empirical exercise the indirect effect on the firing rate outweighs the respective direct effect, thereby the firing rate falls, and this in sum leads to a reduction of the unemployment rate.159

Furthermore, our empircial exercise indicates that a reduction of firing costs induces households to search more for jobs. The reason is that lower wages lead to a reduction of the reward for searching for a job, thereby, providing less incentives to search.160 This wage effect outweighs the direct positive effect resulting from strategic complementarities, namely households search more when facing a higher hiring rate and a lower firing rate.

The Gini coefficient is lower with lower firing costs since inequality is reduced directly through a lower wage and indirectly through lower unemployment.

Increasing Unemployment Benefits

Raising unemployment benefits to the Danish level of a replacement rate of $\beta_1 = 0.69$ leads to an increase of unemployment by $65\%$ (see Figure 4.3) and increases inequality to a Gini coefficient of $5.4\%$.161

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158 Recall that $f_1 = \rho_1 w_0$. We reduce $\rho$ for a given calibrated wage of the prepolicy steady state, which is equivalent to reducing $f$.

159 The stronger effect of lower firing costs on firings relative to hireings is in line with empirical results by Messina and Valanti (2007) showing that firing costs have stronger effects on job destruction relative to creation.

160 This effect is counterbalanced by a lower tax on wages resulting from lower unemployment.

161 Recall that $b_1 = \beta_1 w_0 (1 - \tau_0)$. We reduce the net replacement rate $\beta$ for the calibrated wage $w_0$ and tax rate $\tau_0$ of the prepolicy steady state.
Effects of Reducing of Firing Costs

![Graph showing the effects of percentage changes of the firing costs parameter \( \rho \) on unemployment \( u \), firing rate \( \phi \), the job finding rate \( \mu \), the wage \( w \) and search effort \( \epsilon_U \).]

Figure 4.2: The effects of percentage changes of the firing costs parameter \( \rho \) on unemployment \( u \), firing rate \( \phi \), the job finding rate \( \mu \), the wage \( w \) and search effort \( \epsilon_U \).

Effect of Increasing Benefit on Unemployment

![Graph showing the effects of changes in the replacement rate \( \beta \) on unemployment \( u \), firing rate \( \phi \), the job finding rate \( \mu \) and the wage \( w \).]

Figure 4.3: The effects of changes in the replacement rate \( \beta \) on unemployment \( u \), firing rate \( \phi \), the job finding rate \( \mu \) and the wage \( w \).

The effect is straightforwardly due to the resulting higher wage, which reduces firms’ unemployed hiring incentive and insider retention incentive. Thereby employment falls. At the same time higher unemployment benefits reduce the reward for searching for a job. This results since the increase in unemployment benefits and the resulting increase in taxes as well as the lower hiring and higher firing rates raise the present value of utility of an unemployed worker.
relative to that of an employed worker - despite the wage increase. Thus, workers search less. Furthermore, increasing unemployment benefits to the Danish level increases inequality mainly due to higher unemployment which reduces the group receiving the higher income.

**Introducing Workfare**

As pointed out in the previous Section we introduce the Danish workfare requirements in line with Andersen and Svarer (2008). Assuming a probability for a firm, whose workers are on strike, of temporarily employing activated unemployed of $\chi_1 = 40\%$ the introduction of workfare reduces unemployment by 85% and thereby significantly reduces inequality, the Gini coefficient falls to 0.5%. For an increasing value of $\epsilon_1^A$ Figure 4.4 illustrates the three effects of workfare, which are also commonly reported in the literature. The locking-in effect, namely that workers’ employment probability is reduced while they are on workfare, since they have less time to search for a job, is reflected by the decreasing search effort of activated workers with increasing work effort on workfare. The so-called threat effect, which refers to the fact that unemployed workers exit unemployment more quickly before being activated, since remaining in unemployment becomes less attractive relative to employment, is revealed by the increasing search effort of unemployed workers. Furthermore, also employed workers react to the introduction of workfare - they bargain a lower wage.

---

Figure 4.4: The effects of increasing workfare effort $\epsilon^A$ with $\lambda = 0.25$ on wage and search effort of the unemployed and the activated, reflecting wage, threat and locking-in effect, respectively.

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162 See Andersen and Svarer (2007), (2008). The following results are in line with Andersen and Svarer (2008).
164 We assume that the active labour market policy (ALMP) is represented by workfare and this instrument does not have any effect on workers’s productivity. Thus, our model does not take the post-programme effect into account. This effect refers to the better employment probability after having taken part in the ALMP. By omitting this positive effect, we bias our analysis against ALMP. The model adopted here could easily be extended to incorporate other instruments.
The reason for the reduction in the negotiated wage lies in the fact that firms whose workers are on strike draw back on activated workers. Clearly, via the wage reduction, the introduction of workfare raises firms’ unemployed hiring incentive and insider retention incentive. Thereby due to workfare indirectly more workers are hired and less fired and thus, employment falls.

Since to the best of our knowledge the empirical literature does not supply any estimates for $\chi$ (wage effect of workfare), in Table 4.5 we provide the unemployment and inequality results for alternative values of this parameter. The weaker the wage effect the weaker is the unemployment reducing effect of this policy as well as of the full set of flexicurity policies. Since according to Andersen and Svarer (2007) the wage effect is one of the the dominant channels of workfare, the value adopted for $\chi$ seems reasonable.

<table>
<thead>
<tr>
<th>$\chi$-Value</th>
<th>Effect of Workfare</th>
<th>Effect of Flexicurity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>on Unemployment</td>
<td>on Inequality</td>
</tr>
<tr>
<td>0.1</td>
<td>-26%</td>
<td>2.5</td>
</tr>
<tr>
<td>0.2</td>
<td>-50%</td>
<td>1.7</td>
</tr>
<tr>
<td>0.3</td>
<td>-70%</td>
<td>1.0</td>
</tr>
<tr>
<td>0.4</td>
<td>-85%</td>
<td>0.5</td>
</tr>
<tr>
<td>0.5</td>
<td>-94%</td>
<td>0.2</td>
</tr>
<tr>
<td>0.6</td>
<td>-98%</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 4.5: The unemployment and inequality effects of workfare and the full set of flexicurity policies for various values of the wage effect of workfare.

The direct effect of workfare on the household side is the relative increase of the reward for seeking a job for all unemployed, especially for the activated as illustrated in Figure 4.5.

![Figure 4.5: The effect of increasing workfare effort on the reward for seeking a job for passive and activated unemployed workers in comparison to the reward in absence of workfare.](image)
The higher reward amplified by a higher hiring rate as in eqs. 4.13, 4.14 leads to a reduction of leisure time. This increased effort though only translates into higher search effort for the not yet activated unemployed workers (threat effect), since the activated workers need to incur work effort on workfare, in line with our specification of the utility function and calibration, they have less time to search for a job (locking-in effect). This locking-in effect manifests itself via eq. 4.6, whereby the relatively less hard a worker searches for a job the respectively lower will be his job finding rate. Thereby, while the hiring rate increases, only unemployed, non-activated workers’ job finding rate rises, the job finding rate for activated workers falls, in despite of a much higher reward for seeking a job because they search less than the not activated workers.

Despite of the cost of workfare, the tax rate falls since employment increase and unemployment decreases.

Joint Effects and Complementarities

As can be seen from Table 4.4, the flexicurity group of labour market policies we are considering here are complementary for Germany in the sense that the unemployment effect of each policy is greater when implemented in conjunction with the other policies than in isolation (Coe and Snower, 1997), or in other words the effectiveness of one policy depends on the implementation of other policies (Orszag and Snower, 1998).\textsuperscript{165} As pointed out be Coe and Snower, who analyzed policy complementarities in a static world, a wide range of labour market institutions have complementary effects on unemployment and thus, labour market policies targeted at reforming these institutions are also complementary.

The three flexicurity instruments have some apparent complementarities. Implementing the three instruments jointly leads to a percentage reduction of unemployment which is 39\% (size of the complementarity in \%) higher than summing up the respective single effects.\textsuperscript{166}

\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|c|}
\hline
Policy & Sum of Single Effects & Joint Effect & Complementarity \\
\hline
Flexicurity & -36\% & -50\% & 39\% \\
Workfare ($\alpha_i,\epsilon_i^f$) and Higher Benefits ($\beta_1$) & -20\% & -36\% & 83\% \\
Lower Firing Costs ($\rho_1$) and Higher Benefits ($\beta_1$) & 49\% & 38\% & 22\% \\
Workfare ($\alpha_i,\epsilon_i^f$) and Lower Firing Costs ($\rho_1$) & -100\% & -90\% & -10\% \\
\hline
\end{tabular}
\caption{Unemployment effects of various combinations of the flexicurity instruments.}
\end{table}

Table 4.6 illustrates the unemployment effects of the flexicurity policies as well as pairwise combinations of the three policies. The strongest economic complementarities in reducing unemployment are generated by the joint implementation of higher unemployment benefits and the introduction of workfare. Flexible firing rules and workfare are not complementary at

\textsuperscript{165}Formally presented by Coe and Snower (1997) as follows: a set of policy instruments $x_i$, $i = 1,...,n$, has complementary effects on a policy objective $y$ when $\frac{\partial^2 y}{\partial x_i \partial x_j} > 0$ for $i \neq j$.

\textsuperscript{166}We measure the size of the complementarity in percentage terms as Oskamp and Snower (2008), as the relative difference between the joint effect and the sum of the single effects of the set of policies under consideration. In other words the percentage reduction in the total unemployment rate is 14 percentage points higher.
all, while flexible firing rules and higher unemployment benefits are. In the following we will discuss these complementarities generated by the policies.

**Increasing Unemployment Benefits and Reducing Firing Costs**

As pointed out above higher unemployment benefits increase the unemployment rate, but implementing them with lower firing costs jointly increases the unemployment rate by less than when implementing them separately. Thus, for the above calibrated model for Germany these two policies are complementary in terms of reducing unemployment. This implies that a reduction of firing costs has a bigger employment effect if unemployment benefits are high. The reasons is the following: high firing costs and high unemployment benefits give leverage to each other. Higher unemployment benefits reduce firms’ hiring and retention incentives and thereby, leverage the effect of firing costs in reducing these incentives. This interaction also holds vice versa, while firing costs reduce firms’ hiring incentives, they thereby magnify the weakening of the reward for seeking a job. Thereby, the positive impact on unemployment of increasing the replacement rate when firing costs are high - due to the high leverage effect - can not be compensated by the negative impact of reducing firing costs when replacement rates are low. The joint implementation of low firing costs and a high replacement rate avoids this leverage effect.\(^{167}\)

**Increasing Unemployment Benefits and Introducing Workfare**

The strongest complementarity is reached when higher unemployment benefits and workfare requirements are implemented jointly.

The joint introduction of these two policies couldn be justified by the concept of political complementarities, see Orszag and Snower (1998), which arise when the ability to gain voters’ approval for a policy depends on the implementation of another policy. Similarly, Andersen and Svarer (2007) illustrate that the strong egalitarian foundations of the Danish welfare system ruled out general reductions in unemployment benefits to strengthen incentives.

On the economic side one could expect a reduction of unemployment benefits and workfare instruments to be equivalent from an utility perspective and thereby, make a distinction unnecessary. But as pointed out be Andersen and Svarer (2008), the labour market effects of these two policies are distinct and will differ across the three groups of workers, namely employed, unemployed and activated. This is so for several reasons. First, employed workers’ contemporaneous (consumption) utility is affected by both policies - in both cases indirectly via the wage. But while a reduction in benefits has the same effect on unemployed and activated workers, this is not the case for workfare policies. The reason is that workfare has no direct effect on the instantaneous utility of the unemployed, but on the activated workers’. Furthermore, benefit changes and workfare requirements affect search incentives differently. The reason is

\(^{167}\) Alternatively, one can argument via the tax-benefit multiplicator: since all workers behave the same and reduce their search effort due to a rise in unemployment benefits, their behaviour does not affect their job finding probability and the only channel through which the unemployment benefits changes unemployment is via the wage. A benefit increase implies a higher tax rate, which raises the wage even more and leads to higher unemployment, which in turn necessitates a higher tax rate and so on. A reduction of the firing costs weakens this tax-benefit multiplicator, via its employment enhancing effect it reduces the tax rate.
that workfare requirements affect the marginal cost of search directly, whereas benefits have an effect via an income effect.

The strong complementarity is based on these effects of the two instruments. Analogous to the above joint implementation of reduced firing costs and increased unemployment benefits, an introduction of workfare requirements has much stronger effects when the disincentive effects due to unemployment benefits are high. While the disincentive effects are the same, workfare has much stronger effects on unemployment than the flexible firing rules, thereby also the complementarity is stronger.

**Reducing Firing Costs and Introducing Workfare**

While being powerful tools to reduce unemployment, clearly, flexible firing rules and workfare requirements are weak substitutes in this respect, since the reduction of unemployment is reduced by 10% if both instruments are implemented jointly compared to summing up the single effects. As pointed out above, these policies have stronger effects when the disincentive for workers and firms to match are high. Since workfare has strong effects on unemployment and increases firms’ and workers’ incentive effects, a reduction of firing cost does not contribute to the reduction of unemployment as it would do in the absence of workfare.

**Flexicurity**

As illustrated above the unemployment effect of the joint implementation of the three flexicurity instruments for Germany implies a complementarity of these instruments of 39%. To understand how this complementarity of all three instruments can be rationalized it is useful to compare their unemployment effect with the sums of the effects of the pairwise joint implementation and the respective single third instrument, as presented in Table 4.7:

<table>
<thead>
<tr>
<th>Policy</th>
<th>Joint Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexicurity ($\alpha_1, \epsilon^4_1, \beta_1$ and $\rho_1$)</td>
<td>-50%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sum of Pairwise and Single Effects</th>
<th>Joint Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Workfare and Higher Benefits ($\alpha_1, \epsilon^4_1$ and $\beta_1$) + Lower Firing Costs ($\rho_1$)</td>
<td>-52%</td>
</tr>
<tr>
<td>Lower Firing Costs and Higher Benefits ($\rho_1$ and $\beta_1$) + Workfare ($\alpha_1, \epsilon^4_1$)</td>
<td>-47%</td>
</tr>
<tr>
<td>Workfare and Lower Firing Costs ($\alpha_1, \epsilon^4_1$ and $\rho_1$) + Higher Benefits ($\beta_1$)</td>
<td>-25%</td>
</tr>
</tbody>
</table>

Table 4.7: Unemployment effects of various joint implementations of flexicurity instruments.

These results underline our previous argumentation. First, the sum of the effects of the pairwise implementation of workfare and high unemployment benefits ($\alpha_1, \epsilon^4_1$ and $\beta_1$) and the single effects of a reduction of firing costs ($\rho_1$) is greater than the effect of flexicurity, since the former includes the high complementarity between workfare and high unemployment benefits but not the substitutability between workfare and the firing cost reduction. Second, flexicurity has a stronger effect on unemployment than the sum of the combination of low firing costs and high benefits, which implies only a weak complementarity, together with workfare. Third, the combination of the pairwise introduction of workfare and lower firing costs ($\alpha_1, \epsilon^4_1$ and $\rho_1$) and
the single implementation of higher unemployment benefits has a much smaller unemployment mitigation effect since it only takes into account the substitutability between the former two instruments.

Finally, our analysis replicates the Danish experience,\textsuperscript{168} whereby the workfare component is the decisive element in generating the economic effectiveness of the flexicurity policy.

### 4.5 Concluding Remarks

This chapter analyzes the channels and complementarities of the Danish flexicurity concept in reducing unemployment and inequality. We perform the experiment of implementing Danish flexicurity in Germany using a calibrated, microfounded model, which is derived from the agents’ labour market incentives.

This policy experiment replicates the Danish miracle in Germany and illustrates the strong complementarities of nearly 40\% underlying the Danish flexicurity concept when implemented in Germany. Furthermore, our results emphasize the strong role of workfare policies in setting employment incentives right.

Our results underline the need for fundamental labour market reforms with a set of broad and deep policies which imply strong economic complementarities and at the same time encompass political complementarities by taking distributional objectives into account thereby facilitating the consent for implementing the reforms.

The flexicurity policy enables firms to adapt to the global market, supports workers and at the same time enhances their adaptability, which is strongly required in the new wave of globalisation.\textsuperscript{169} Thereby, this reform policy with a focus on employment security is a viable and option for Germany which tends to emphasize income and job security.

\textsuperscript{168} See Andersen and Svarer (2007) and (2008).

\textsuperscript{169} See Snower et al. (2009).
5 An Incentive Theory of Matching

5.1 Introduction

The literature on search and matching in the labour market rests heavily on the assumption of a stable matching function (Mortensen and Pissarides, 1994). According to Pissarides (2000, p. 3-4), the matching function aims to summarize “heterogeneities, frictions and information imperfections” and represent “the implications of the costly trading process without the need to make the heterogeneities and the other features that give rise to it explicit.” Various authors (e.g. Lagos, 2000) have noted that policies which affect labour market heterogeneities (e.g. retraining programs), frictions (e.g. job counselling) and information imperfections (e.g. job exchanges) may naturally be expected to influence the matching function. In short, there is no reason to believe that the matching function is invariant with respect to labour market policies that are designed to improve the matching process.\(^{170}\)

This chapter\(^{171}\) provides a simple analytical framework that makes the above heterogeneities and frictions explicit, and describes the matching process as the outcome of optimizing behaviour by firms and workers. Our analysis thereby obviates the need for a matching function to capture the above heterogeneities and frictions.\(^{172}\) In this context, we show that, in general, the matching process is indeed affected by labour market policies, as well as a variety of labour and macroeconomic shocks. This calls into question the usefulness of matching functions for policy analysis and prediction. In short, the matching function runs afoul of the Lucas critique.

We present two simple models of heterogeneous variations in the characteristics of workers and jobs, combined with adjustment costs in responding to these variations. In this context, we derive the job offer and firing decisions of firms and the job acceptance and quit decisions of workers, and show how these decisions generate labour market matches and separations. Needless to say, we make no attempt to be comprehensive in covering the wide variety of frictions that are prevalent in labour markets, but we believe that our analysis is sufficient to clarify a general modeling strategy for moving beyond the matching function. Our analysis may be called an incentive theory of matching, since it explains the microeconomic decisions relevant to the matching process in terms of the incentives that economic agents face.

We calibrate our incentive model for the U.S. economy and show that it can account for some important empirical regularities that the conventional matching model cannot. First, our model generates labour market volatilities that are close to what can be found in the empirical data, specifically for the unemployment rate, the job finding rate and the separation rate. This is remarkable, as we do not rely on any type of real wage rigidity. Instead, our calibration permits us to replicate the stylized fact that wages are as volatile as productivity (see, for example,

\(^{170}\) Several empirical studies indicate instabilities of the matching function. Very often a negative time trend is found when estimating the search and matching function, thus casting doubt on the stability through time (Blanchard and Diamond, 1989, for the United States, and Fahr and Sunde, 2001, 2004, for Germany).

\(^{171}\) For a different version of this chapter see Brown et al. (2009).

\(^{172}\) As shown in the Appendix 5.A.1, the matching function may remain useful to capture information imperfections regarding workers and jobs that, on the basis of the imperfect information, appear homogeneous to the searchers.
Hornstein et al., 2005). The standard calibration of the conventional matching model\textsuperscript{173} is unable to generate these high volatilities of labour market variables (see Shimer, 2005). Second, it generates a strong negative correlation between the job finding rate and the unemployment rate. And third, it can account for a strong negative correlation between job creation and job destruction. The standard calibrations of the matching model, with endogenous job destruction (see Krause and Lubik, 2007), cannot account for these last two stylized facts.\textsuperscript{174}

The rest of this chapter is organized as follows. Section 2 covers the conceptual issues underlying our approach to matching. Section 3 sets the stage by presenting a particularly simple and transparent incentive model of matching. Section 4 presents an extended incentive model. Section 5 discusses the calibration strategy. Section 6 presents the numerical results and Section 7 concludes.

### 5.2 Conceptual Issues

It is convenient to think about the coexistence of unemployed workers alongside unfilled jobs in terms of heterogeneities and imperfect information. In the presence of heterogeneous workers and jobs, some workers are offered employment because they are sufficiently profitable; others remain unemployed because they are insufficiently profitable. These differences may arise since workers differ in terms of their productivities at various jobs, their costs of being identified as searching and employable, their relocation and training costs, etc. In the same vein, some job offers are accepted because they offer sufficient remuneration; others are rejected because they are not sufficiently remunerative. These differences may exist because jobs differ in terms of job productivities associated with various workers, costs of being identified as vacant, etc.

In the presence of imperfect information, jobs that are in fact heterogeneous may look homogeneous to a searching worker, who then may choose randomly among these jobs. Similarly, workers that are in fact heterogeneous may look homogeneous to a searching firm, leading the firm to make a random choice among these workers. (In addition, of course, there is also random choice among jobs and workers that are in fact homogeneous.)

The matching function is an analytic device that is meant to capture both the heterogeneities and the imperfect information. As aptly noted by Petrongolo and Pissarides (2001, p. 390): “The attraction of the matching function is that it enables the modeling of frictions in otherwise conventional models, with a minimum of added complexity. Frictions derive from information imperfections about potential trading partners, heterogeneities...”

The heterogeneities are particularly important in accounting for unemployed workers along-
side unfilled jobs. In practice, workers are rarely indistinguishable in terms of all the characteristics relevant for job acquisition and thus firms are rarely in a position of choosing randomly among interchangeable workers. Similarly, workers are rarely in a position of choosing randomly among interchangeable jobs. Often firms and workers draw on a store of prior information, so that their objects of choice are heterogeneous. Of course firms and workers also assemble opportunities blindly — as when they place and read advertisements — but these activities are usually far less costly than the subsequent ones of choosing among heterogeneous options. So if the matching function is to capture the costly process of labour market choice "without the need to make the heterogeneities ... explicit" (Pissarides, 2000, p.4), then it is important that it be able to capture matching behaviour among heterogeneous agents.

This chapter focuses on these heterogeneities. We examine whether a stable matching function can reproduce the behaviour of optimizing agents among heterogeneous options. We do not dispute that a matching function may be appropriate in describing the outcome of choices among options that look homogeneous, due to imperfect information. What is at issue is whether it can describe heterogeneous choice. On this account, we present choice-theoretic models of hiring and job acceptance, as well as firing and quits, for heterogeneous workers and jobs.175

Shimer (2007, p. 1074) makes an insightful distinction between search and mismatch: “According to search theory, unemployed workers have left their old jobs and are actively searching for a new employer. In contrast, this dynamic model of mismatch emphasizes that unemployed workers are attached to an occupation and a geographic location in which jobs are currently scarce. Mismatch is a theory of former steel workers remaining near a closed plant in the hope that it reopens. Search ... is a theory of former steel workers moving to a new city to look for positions as nurses.” While this distinction is undoubtedly important, it is worth emphasizing that adjustment costs are responsible for both mismatch and search. The reason that the former steel workers remain near the closed plant is that the cost of adjusting to another job (net of the expected benefits) is greater than the cost of waiting for the plant to open (again net of the expected benefits). Moreover, if the former steel workers can’t find jobs as nurses despite the existence of nursing vacancies, then the reason is that either it was too costly for the workers to find the available vacancies or it was too costly for the employers to find the available workers. So, under both mismatch and search, unemployed workers and vacancies coexist because it is too costly to fill the vacancies with the unemployed workers. Otherwise, obviously, the match would have been made. Under mismatch, the costs are frequently large and persistent, whereas under search they are often small and transient; but the fact remains that adjustment costs are responsible for the friction in both cases.

We investigate the role of these adjustment costs as a source of labour market frictions. The adjustment costs of course come in many guises. Some are costs of geographic and occupational mobility (as in Shimer’s analysis); others are costs of obtaining the relevant information (the

175In the Appendix 5.A.1, we present a model of both heterogeneous choice as well as choice among agents who appear homogeneous, combining our analysis (dealing with heterogeneities) with a matching function analysis (dealing with random choice among homogeneous agents).
standard search costs, e.g. costs of advertising, monitoring, screening).

While various other authors have modeled the matching process without resorting to a matching function (e.g. Hall, 1977, Lagos, 2000, Shimer, 2007, and others), our analysis explicitly focuses on two-sided search (i.e. search by both workers and firms at the same time) and an optimizing framework that replaces the matching function. Due to our focus on heterogeneities and adjustment costs, it is not necessary for our analysis to make a sharp distinction between search and mismatch. Our analysis is meant to cover both phenomena, depending on how we choose to interpret the nature of the heterogeneities and adjustment costs. In this context, as noted above, we show that our analysis can explain various stylized facts that are not accounted for in the conventional search and matching literature.

5.3 A Simple Model

To set the stage, we begin by constructing a particularly simple model of the incentive theory of matching, based on heterogeneous jobs and workers. Our model has the following sequence of labour market decisions. First, the realized values of the shocks are revealed. Second, the firms make their hiring and firing decisions and the households make their job acceptance and refusal decisions, taking the wage as given. Unemployed workers search for jobs; employed workers do not.

For simplicity, we will not consider vacancies in our model. While it is of course possible to include them (as shown in the Appendix 5.A.1), doing so would complicate our analysis without amplifying the basic point of this chapter, namely, to describe the matching process without need for a matching function, which is vulnerable to the Lucas critique. Beyond that, it is worth noting that vacancy data is the Achilles’ heel of conventional empirical matching models. For long time series, we have only a very rough proxy for U.S. vacancies, namely, the Conference Board help-wanted advertising index, measuring the number of help-wanted ads in 51 major newspapers. Over the past decade, this index shows a clear downward trend (adjusting for the business cycle), which may well be due to internet advertising. Although an internet advertising index exists, it is far from clear how this index can be made comparable to the newspaper index. Moreover, while the Conference Board advertising index and the JOLTS survey on vacancies exhibit similar dynamics for the limited sample periods in which comparable data sets are available (Shimer, 2005), it appears, surprisingly, that the number of vacancies (as defined by the JOLTS survey) is consistently and substantially smaller than the number of new hires! There are two obvious reasons why this should be so, both highlighting weaknesses of vacancies data: (i) Only a fraction of the jobs that get filled are preceded by vacancy postings. (The matching function has nothing to say about the many hires that occur without formal advertising.) (ii) The JOLTS survey, like all other surveys, ignores high-frequency vacancy movements. In particular, JOLTS measures end-of-month reported job openings, not job openings that get filled before the month is over. Overall, such considerations indicate that vacancy data is much less reliable than the other data (e.g. unemployment rates, productivity) used in conventional empirical matching models. On account of this as well as analytical simplicity, our incentive
model will not cover vacancies.

To keep our analysis as simple as possible, we assume in this Section that the real wage $w$ is exogenously given. This assumption is relaxed in the next Section, where wages are determined through bargaining. Our basic result concerning the stability of the matching function, however, can be derived straightforwardly under an exogenous wage.

Furthermore, to provide a maximally transparent comparison of our incentive model and the standard matching model, we assume that workers and firms are myopic (i.e. their rates of time discount are 100%). This assumption is also relaxed in the next Section.

### 5.3.1 The Firm’s Behavior

We assume that the profit generated by a particular worker at a particular job is subject to a random shock $\epsilon$, which is meant to capture idiosyncratic variations in workers’ suitability for the available jobs. For example, workers in a particular skill group and sector may exhibit heterogeneous profitabilities due to random variations in their state of health, levels of concentration, and mobility costs, or to random variations in firms’ operating costs, screening, training, and monitoring costs, and so on. In short, the random shock $\epsilon$ is a short-hand for workplace heterogeneities. It is positive and iid across workers, with a stable probability density function $G_\epsilon(\epsilon)$, known to the firm.\footnote{Our analysis can of course be extended straightforwardly to shocks with AR and MA components. For mobility costs, the shocks are often serially correlated in practice.} Let the corresponding cumulative distribution be $C_\epsilon(\epsilon)$. The period of analysis is equal to the period between successive realizations of $\epsilon$.

The average productivity of each worker is $a$, a positive constant. The hiring cost $h$ per worker is a constant. The profit generated by an entrant (a newly hired worker) is

$$\pi^E = a - \epsilon - w - h,$$

where the superscript “$E$” stands for “entrant” and $w$ is the real wage.

The firm’s “job offer incentive” (its payoff from hiring a worker) is the difference between its gross profit\footnote{This "gross" profit is the expected profit generated by hiring an unemployed worker, without taking the operating cost into account.} from hiring an entrant worker $(a - w - h)$ and its profit from not doing (namely, zero):

$$\nu^E = a - w - h.$$

The firm offers this job to a worker whenever that worker generates positive profit: $\epsilon < \nu^E$. Thus the job offer rate is

$$\eta = C_\epsilon(\nu^E).$$

The firm’s “retention incentive” (its payoff from retaining a worker) is the difference between its gross profit from retaining a worker is $(a - w)$ and the (negative) profit from firing that worker:

$$\nu^I = a - w + f.$$
where the superscript “I” stands for the incumbent employee who has been retained, and \( f \) is the firing cost per worker, assumed constant. The firm with a filled job will fire an incumbent worker whenever she generates negative profit: \( \varepsilon > \nu^I \). Thus the firing rate is:

\[
\phi = 1 - C_\varepsilon (\nu^I) .
\] (5.5)

Note that due to the hiring and firing costs, the retention incentive exceeds the job offer incentive \( (\nu^I > \nu^E) \) and thus the retention rate exceeds the job offer rate \( (1 - \phi) > \eta) \).

### 5.3.2 The Worker’s Behavior

The worker faces a discrete choice of whether or not to work. If she works, her disutility of work effort is \( e \), which is a random variable, which is iid, with a stable probability density function \( G_e (e) \), known to the worker. The corresponding cumulative distribution is \( C_e (e) \). The random variable captures heterogeneities in the disagreeability of work, due to such factors as temporary variations in health, moods, idiosyncratic reactions to particular workplaces, and personal circumstances. If the worker does not work, her utility is \( b \) (a constant). Her utility is linear in consumption and work effort.\(^{178}\) She consumes all her income. Thus the utility of an employed worker is \( V^N = w - e \), and the utility of an unemployed worker is \( V^U = b \).

A worker’s “work incentive” (her payoff from choosing to work) is the difference between her gross utility from working \( (w) \) and her utility from not working \( (b) \):

\[
\iota = (w - b) .
\] (5.6)

Assuming that \( w > b \) and letting \( E (e) = 0 \), all unemployed workers have an ex ante incentive to seek work.

An unemployed worker will accept a job offer whenever \( e < \iota \). This means that the job acceptance rate is

\[
\alpha = C_e (\iota) .
\] (5.7)

Along the same lines, an employed worker will decide to quit when \( e > \iota \). This means that the quit rate is

\[
\chi = 1 - C_e (\iota) .
\] (5.8)

Note that, for simplicity, we have assumed that the job acceptance rate is identical to the job retention rate \( (\alpha = 1 - \chi) \). When unemployed workers face costs of adjusting to employment (e.g. buying a car to get to work, or psychic costs of changing one’s daily routine) or when employed workers face costs of adjusting to unemployment (e.g. building networks of friends with potential job contacts, psychic costs of adjusting to joblessness), then the job acceptance rate would fall short of the job retention rate.\(^{179}\)

\(^{178}\) Observe that on the firm’s side, we distinguish between entrants \( (E) \) and incumbent workers \( (I) \); whereas on the workers’ side, we distinguish between employed \( (N) \) and unemployed \( (U) \) workers. The rationale for these two distinctions is that the firm can hire two types of workers (entrants and incumbents), whereas the worker can be in two states (employment and unemployment).

\(^{179}\) Specifically, for example, the unemployed worker’s job acceptance incentive could be expressed as \( \iota^U = \)
5.3.3 Employment

An unemployed worker gets a job when two conditions are fulfilled: (i) she receives a job offer and (ii) she accepts that offer. Thus the match probability ($\mu$) is the product of the job offer rate ($\eta$) and the job acceptance rate ($\alpha$):

$$\mu = \eta \alpha. \quad (5.9)$$

Consequently the number of unemployed workers who get jobs in period $t$ is $\mu U_{-1}$, where $U_{-1}$ is the number of unemployed in the previous period.\(^{180}\)

An employee separates from her job when at least one of two conditions is satisfied: (i) she is fired or (ii) she quits. Thus the separation probability is

$$\sigma = \phi + \chi - \phi \chi. \quad (5.10)$$

This implies that the number of employed who separate from their jobs in period $t$ is $\sigma N_{-1}$, where $N_{-1}$ is the number of employed in the previous period.

The change in employment is $\Delta N = N - N_{-1} = \mu U_{-1} - \sigma N_{-1}$. The labour force $L$ is assumed constant. Thus $U = L - N$ and employment may be described by

$$N = \mu U_{-1} + (1 - \sigma) N_{-1} = \mu L + (1 - \mu - \sigma) N_{-1}. \quad (5.11)$$

Expressing the equation in terms of the employment rate, $n = N/L$, yields the following employment equation:

$$n = \mu + (1 - \mu - \sigma) n_{-1}. \quad (5.12)$$

(Although we have not included vacancies in our model - since doing so would complicate our analysis without contributing substantially to our main message - the Appendix 5.A.1 shows how that can be incorporated straightforwardly into our analysis.)

5.3.4 The Matching-Function Representation

We now juxtapose the model above with its matching-function counterpart in order to investigate the stability of the matching function. For this purpose, let us assume that the model above describes the real world, and then let us ask whether the behaviour of this model can be replicated by a corresponding model containing a matching function. We will show that such replication cannot occur unless the matching function changes whenever the underlying parameters of the model change. These parameter changes include macroeconomic variables (such as productivity, $a$) and policy variables (such as unemployment benefits, underlying the parameter $b$). Thus, in this analytical context, the matching function runs afoul of the Lucas
critique: policy analysis and comparative static prediction on the basis of a stable matching function would yield misleading results.

Naturally, the incentive model above is extremely simple, but it is precisely this simplicity that allows us to bring the Lucas critique of the matching function into sharp relief. Needless to say, the same critique can be formulated with respect to more complicated models (such as the one in the next Section), since the underlying idea is quite general: For any given matching function - specified independently of the optimizing decisions relevant to the matching process - it is always possible to construct a microfounded macro model that systematically fools this matching function. In this sense, the difficulty of the matching functions is analogous to that of expectation-generating mechanisms in traditional macro models that were incompatible with rational expectations.

Let the matching function be

$$x = x(u, v),$$

(5.13)

where $u$ is the unemployment rate and $v$ is the vacancy rate (number of vacant jobs relative to the labour force). This function satisfies the standard conditions: $x_i > 0$, $x_{ii} < 0$, $i = u, v$; $x(u, 0) = x(0, v) = 0$; and there are constant returns to scale: $gx(u, v) = x(gu, gv)$ where $g$ is a positive constant.

Let $\theta = v/u$ denote labour market tightness, so that $q(\theta) = x(u/v, 1)$ is the probability that a job is matched with a worker, and $\theta q(\theta)$ is the probability that a worker is matched by a job. Along the lines of the simple labour market matching models, we assume that jobs are destroyed at an exogenous rate $\lambda$, $0 < \lambda < 1$. Then the change in the employment rate is\(^{181}\)

$$\Delta n = \theta q(\theta) \left( 1 - n_{-1} - \lambda n_{-1} \right),$$

implying the following employment dynamics equation:

$$n = \theta q(\theta) + (1 - \theta q(\theta) - \lambda) n_{-1}. \quad (5.14)$$

Vacancies are posted until the expected profit is reduced to zero: $a - w = \kappa/\theta q(\theta)$, where $\kappa$ is a vacancy posting cost, $\kappa/q(\theta)$ is the expected vacancy posting cost per worker. Expressing this zero-profit condition in terms of labour market tightness:

$$\theta = g \left( \frac{\kappa}{a - w} \right), \quad (5.15)$$

where $g = q^{-1}$.

The equilibrium employment rate $n$ is obtained by substituting the zero-profit condition (5.15) into the employment dynamics equation (5.14).

### 5.3.5 Equivalence Conditions

In order for the two models to be comparable, let the exogenous wage $w$ be identical in both models and suppose that the separation rate $\sigma$ in the incentive model is a constant equal

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\(^{181}\)To keep this model comparable with our the simple incentive model above, we assume (without loss of generality) the same timing in both models. Matches are not destroyed in the match period and they become immediately productive.
to the job destruction rate $\lambda$ in the conventional matching model. Then the two models are observationally equivalent when

$$\theta q(\theta) + (1 - \theta q(\theta) - \sigma) n_{t-1} = \mu + (1 - \mu - \sigma) n_{t-1},$$

so that

$$\theta q(\theta) = \mu,$$  \hspace{1cm} (5.16)

which we call the “equivalence condition.” By implication,

$$\frac{\kappa}{a - w} g\left(\frac{\kappa}{a - w}\right) = C_e(a - w - h) C_e'(w - b).$$  \hspace{1cm} (5.17)

Differentiating with respect to $b$,

$$0 = -C_e(a - w - h) C_e'(w - b).$$  \hspace{1cm} (5.18)

Furthermore, differentiating with respect to productivity $a$ yields:

$$-\frac{\kappa}{(a - w)^2} \left(\frac{\kappa}{a - w} g'(\frac{\kappa}{a - w}) + g\left(\frac{\kappa}{a - w}\right)\right) = C_e'(a - w - h) C_e'(w - b).$$  \hspace{1cm} (5.19)

Condition (5.18) holds only when the slope of the cumulative distribution is $C_e'(w - b) = 0$, which implies that the underlying density is $G_e = 0$. This occurs when there exist no households with a marginal disutility of effort $e$ over the relevant range. (It is on this account that the number of households that accept jobs is not affected by $(w - b)$.) A distribution of $e$ with zero mass is indeed a special case; it amounts to excluding the possibility of heterogeneous workers in our model.

Condition (5.19) can only hold by accident; in general, this condition is not met. Alternatively, it is clear that for any change in one or more of the parameters $(a, w, \kappa, h, b)$, that matching function (and thus the $g$ function) needs to change in order to ensure that condition (5.19) holds.

By assumption, the matching function is a shorthand for heterogeneities and frictions addressed explicitly in the incentive model of matching. It is a useful shorthand if it can be shown that it is observationally equivalent to the explicit model of the underlying heterogeneities and frictions, so that it is stable with respect to the macroeconomic and policy variables whose effects the matching model aims to analyze (to avoid the Lucas critique).

The comparison above indicates that these two conditions are not satisfied. The standard matching model cannot reproduce the labour market dynamics of the incentive model above. This non-equivalence is not a special case to be ascribed to the particular specification of the incentive model. It is easy to see that the reasoning above is applicable to a broad family of models.

The source of the non-equivalence is analogous to the non-equivalence of adaptive-expectations and rational-expectations macro models. Adaptive-expectations models were unable to reproduce the dynamics of rational-expectations models because, for any given function specifying adaptive expectations, it is always possible to find a hypothetically “true” stochastic generating
process which produces predictable errors, that is, errors not reconcilable with rational expectations. Along the same lines, the comparison above makes clear that for any given matching function, it is always possible to find a hypothetically “true” model of the underlying heterogeneities and frictions which produces labour market dynamics that cannot be replicated through the matching function. Just as an expectations generating mechanism that is specified a priori (independently of the underlying macro model) is not a reliable tool for investigating the influence of macro policy, so a matching function that is specified a priori is also not a reliable tool to explore the influence of labour market policy. The same can be said regarding the influence of other macro and labour parameters.

Alternatively, we can say that the matching function is not stable with respect to the parameters whose influence the matching models are meant to analyze. If the incentive model above is assumed to be the “true” model of the labour market, then the standard matching model can reproduce the “true” employment effects of variations in all the relevant parameters - the wage $w$, productivity $a$, the hiring cost $h$, or the leisure utility $b$ - only if we assume that the matching function is modified whenever these parameters are changed. This instability of the matching function makes it an inappropriate tool for investigating the effectiveness of policy changes or macroeconomic fluctuations.

Although the simple model above is useful to examine why the matching function is subject to the Lucas critique, we now need to relax several restrictive assumptions of the incentive model above - that households and firms are myopic, wages are exogenous, and productivity is constant - in order to examine the relative performance of the incentive model and the standard matching model in accounting for well-known stylized facts. In the context of conventional calibrations, we will show that the incentive model fares better than the standard matching model in reproducing the volatilities of major labour market variables.

5.4 An Extended Incentive Model

We now extend the simple model above by

- including aggregate risk: the average aggregate productivity parameter $a$ is now subject to random productivity shocks;
- allowing for rates of time discount that are less than 100%, so that workers and firms become intertemporal optimizers and
- introducing wage determination through bargaining.

The first extension enables us to simulate productivity shocks as done in Hall (2005), Shimer (2005) and numerous other papers and to make our framework quantitatively comparable to the matching theory. The second and third extensions provide a richer depiction of the determinants of employment and wages.

In this context, the new sequence of decisions may be summarized as follows. First, the aggregate productivity shock and the idiosyncratic shocks are revealed. Second, the wage is set
through bargaining. Third, the firms make their hiring and firing decisions and the households make their job acceptance and refusal decisions, taking the wage and the realization of the aggregate and idiosyncratic shocks as given.

5.4.1 The Firm’s Behavior

Since the firm is not myopic in this model, its hiring and firing decisions depend on its expected profits not only in the current time period, but also in future time periods.

The Firing Decision

The expected present value of profit generated by an incumbent employee, after the random profitability term \( \varepsilon_t \) is observed, is

\[
E_t \left( \pi^I_t \right) = (a_t - w_t - \varepsilon_t) + \delta E_t \left( \pi^I_{t+1} \right),
\]

where \( \delta \) is the time discount factor, \( a_t \) is the incumbent employee’s productivity, and

\[
E_t \left( \pi^I_{t+1} \right) = E_t \left[ (1 - \sigma_{t+1}) (a_{t+1} - w_{t+1} - E_t \left( \varepsilon_{t+1} | (\varepsilon_{t+1} < \nu^I_t) \right)) + \delta \pi^I_{t+2} \right] - \phi_{t+1} f. \quad (5.21)
\]

\( E(\varepsilon|\varepsilon < \nu^I_t) \) is the expectation of the random term \( \varepsilon \), conditional on this shock falling short of the incumbent employee’s retention incentive \( \nu^I_t \), which is defined as

\[
\nu^I_t = a_t - w_t + \delta E_t \left( \pi^I_{t+1} \right) + f, \quad (5.22)
\]

i.e. the retention incentive is the difference between the gross expected profit from retaining the employed worker \( (a_t - w_t + \delta E_t \left( \pi^I_{t+1} \right)) \) and the expected profit from firing her \( (-f) \).

An incumbent worker is fired in period \( t \) when the realized value of the random cost \( \varepsilon_t \) is greater than the incumbent worker employment incentive: \( \varepsilon_t > \nu^I_t \). Since the cumulative distribution of \( \varepsilon \) is \( C^\varepsilon(\nu^I_t) \), the employed worker’s firing rate is

\[
\phi_t = 1 - C^\varepsilon(\nu^I_t). \quad (5.23)
\]

The Job Offer Decision

The expected present value of profit generated by an entrant, after the random cost \( \varepsilon_t \) has been observed, is

\[
E_t \left( \pi^E_t \right) = a_t - w_t - \varepsilon_t - h + \delta E_t \left( \pi^I_{t+1} \right). \quad (5.24)
\]

We define the firm’s expected job offer incentive \( \nu^E_t \) as the difference between the gross expected profit from a hired worker \( (a_t - w_t - h + \delta E_t \left( \pi^I_{t+1} \right)) \) and the profit from not hiring

\[\text{In the first period, profit is } (a_t - w_t - \varepsilon_t); \text{ in the second period, the worker is retained with probability } (1 - \phi_t) \text{ and then generates an expected profit of } a_t - w_t, \text{ and the worker is fired with a probability of } \phi_t \text{ and then generates a firing cost of } f_t; \text{ and so on.}\]
him (i.e. zero):

\[ \nu_t^E = a_t - w_t - h + \delta E_t(\pi_{t+1}) \]  

(5.25)

A job is offered when \( \nu_t^E > \varepsilon_t \). Thus the job offer rate is

\[ \eta_t = C_\varepsilon(\nu_t^E) . \]  

(5.26)

5.4.2 The Worker’s Behavior

The incumbent worker’s expected present value of utility ex post (once the realized value of the disutility shock \( e_t \) is known) is

\[ E_t(\nu_t^N) = w_t - e_t + \delta E_t((1 - \sigma_{t+1}) V_{t+1}^N + \sigma_{t+1} V_{t+1}^U) , \]  

(5.27)

where \( E_t(\nu_t^N) \) is the expected present value of utility of the following period (before the realized value of the shock \( e_t \) is known):

\[ E_t(\nu_t^N) = E_t(w_{t+1} - E_t(e_{t+1} | e_{t+1} < u_{t+1}) + \delta ((1 - \sigma_{t+2}) V_{t+2}^N + \sigma_{t+2} V_{t+2}^U)) . \]  

(5.28)

The expected present value utility from unemployment is

\[ E_t(\nu_t^U) = b_t + \delta E_t(\mu_{t+1} V_{t+1}^N + (1 - \mu_{t+1}) V_{t+1}^U) . \]  

(5.29)

An unemployed’s expected “work incentive” \( \iota_t \) (the incentive for an unemployed to accept work) is the expected difference between the gross\(^{183} \) present value from working \( E_t(\nu_t^N) = w_t + \delta E_t((1 - \sigma_{t+1}) V_{t+1}^N + \sigma_{t+1} V_{t+1}^U) \) and the present value from not working \( E_t(\nu_t^U) \) in the current period:

\[ \iota_t = E_t(\tilde{V}_t^N - \tilde{V}_t^U) . \]  

(5.30)

Thus the unemployed accepts a job offer when \( e_t < E_t(\tilde{V}_t^N - \tilde{V}_t^U) \), so that \( e_t < \iota_t \). Consequently, the job acceptance rate is

\[ \alpha_t = C_\varepsilon(\iota_t) . \]  

(5.31)

The incumbent worker decides to quit his job when the present value of becoming unemployed exceeds the present value of remaining employed \( (E_t(\tilde{V}_t^N) - e_t < E_t(\nu_t^U)) \), so that his expected work incentive is lower than the utility cost \( e_t > E_t(\tilde{V}_t^N - \tilde{V}_t^U) = \iota_t \). Thus the quit rate is

\[ \chi_t = 1 - C_\varepsilon(\iota_t) . \]  

(5.32)

5.4.3 Employment

As in the previous model, the match probability is

\(^{183}\)The employed worker’s “gross” expected present value from working is the employed worker’s expected present value of utility without taking the utility shock into account.
\[ \mu_t = \eta_t \alpha_t, \]  
\( (5.33) \)

and the separation probability is

\[ \sigma_t = \phi_t + \chi_t - \phi_t \chi_t, \]  
\( (5.34) \)

and the associated employment dynamics equation is

\[ n_t = \mu_t + (1 - \sigma_t - \mu_t) n_{t-1} \]  
\( (5.35) \)

where the employment persistence parameter \((1 - \sigma_t - \mu_t)\) depends inversely on the match probability \(\mu_t\) and the separation probability \(\sigma_t\). An alternative interpretation of the persistence parameter is given by

\[ 1 - \sigma_t - \mu_t = (1 - \phi_t) (1 - \chi_t) - \eta_t \alpha_t, \]  
\( (5.36) \)

where \((1 - \phi_t) (1 - \chi_t)\), the product of the incumbents’ retention rate and staying rate, is the incumbents’ survival rate. Thus the persistence parameter is the difference between the incumbents’ survival rate and the unemployed workers’ match probability.

### 5.4.4 Wage Determination

We now endogenize the real wage through bargaining. The conventional matching models assume that the real wage is the outcome of Nash bargaining, which takes place after the match has been made. This sequence of decisions is conceptually problematic, particularly when match productivities are heterogeneous. Should not the firms’ and workers’ incentives to match depend on the wage offered? If workers and jobs differ in terms of their productivities, will not a change in the wage lead to a change in the number of matches that are productive? In practice, of course, we don’t find workers and firms agreeing to match before the terms of the employment contract have been set.

This difficulty is easy to overlook in the conventional matching models, where matches are generated mechanically through a matching function, all matches generate a bargaining surplus, and this bargaining surplus is shared by the worker and the firm through the subsequent wage negotiation. But once the matching process is endogenized in terms of the worker’s and firm’s incentive to match - as is done in our incentive model - the difficulty comes into sharp relief. Then we see that the match probability depends on the firm’s job offer rate and the worker’s job acceptance rate, and these rates in turn depend on the wage. Similarly, the separation probability depends on the firm’s firing rate and the worker’s quit rate, and these rates are also wage-dependent. In this context, it is clear that the number of matches made and destroyed per period of time cannot be determined without prior knowledge of the wage.

On this account, we assume here that wage bargaining takes place before the job offer, acceptance, firing and quit decisions are made. Our aim is to formulate a wage determination model that is (i) simple and tractable, (ii) comparable to the wage bargaining process in the
conventional matching models (with the exception of the timing issue above) and (iii) able to reproduce the stylized fact that wages are as volatile as productivity. For this purpose, we let the incumbent workers and entrants receive the same wage $w_t$, determined through Nash bargaining between the firm and its median incumbent worker. The median worker faces no risk of dismissal, as he is at the middle of the $\varepsilon$ distribution. These assumptions satisfy the three aims above, because (i) the simplify the analysis by allowing the employment rate to depend on the wage, but not vice versa, (ii) the Nash bargaining between the firm and the median incumbent is comparable to the wage bargaining in the conventional matching models, and (iii) the negotiated wage turns out to be as volatile as productivity.

Needless to say, other models of wage negotiations could be incorporated into our analysis (e.g. individualistic wage bargaining, monopoly union wage setting, separate wage negotiations for incumbents and entrants, etc.), but we do not do so here since they would substantially complicate the model, without affecting the main points of our analysis, namely, that the matching and separation rates can be determined endogenously through the job offer, job acceptance, firing and quit decisions, and that these decisions are not replicable through a stable matching function.

The wage bargain takes place in each period of analysis. In the current period $t$, under bargaining agreement, the median incumbent worker receives the wage $w_t$ incurs effort cost $\varepsilon^M$ and the firm receives the expected profit $(a_t - w_t - \varepsilon^M)$ in each period $t$. Thus the expected present value of the median incumbent worker’s utility $E(V_t^M)$ under bargaining agreement is

$$E_t(V_t^M) = w_t - \varepsilon^M + \delta E_t \left( (1 - \sigma_{t+1}) V_{t+1}^N + \sigma_{t+1} V_{t+1}^U \right). \quad (5.37)$$

The expected present value of firm’s returns under bargaining agreement are

$$E_t \left( \Pi_t^M \right) = (a_t - w_t - \varepsilon^M) + \delta E_t \left( (1 - \sigma_{t+1}) \Pi_{t+1}^N - \phi_{t+1} \right). \quad (5.38)$$

Under disagreement in bargaining, the incumbent worker’s fallback income is $d$, which can be conceived as financial support from family and friends, strike pay out of a union fund, or other forms of support. The firm’s fallback profit is $-z$, a constant. Assuming that disagreement in the current period does not affect future returns, the present value of utility under disagreement for the incumbent worker is

$$E_t \left( \tilde{V}_t^M \right) = d + \delta E_t \left( (1 - \sigma_{t+1}) V_{t+1}^N + \sigma_{t+1} V_{t+1}^U \right). \quad (5.39)$$

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184 This assumption also implies that an increase in wages leads to a fall in employment. This employment effect can of course also be generated when incumbent workers and entrants have different wages. For example, Lindbeck and Snower (2001) provide a variety of reasons why entrants do not receive their reservation wage and thus a rise in incumbent workers’ wages is not met a countervailing fall in entrant wages, and thus a rise in incumbent workers’ wage lead to a fall in employment. In the context of a Markov model, Diaz-Vazquez and Snower (2003) show that incumbent workers’ wages are inversely related to aggregate employment even when entrants receive their reservation wages.

185 Wages would depend on the time path of employment, while employment depends on the time path of wages.
and the present value of profit under disagreement for the firm is
\[ E \left( \Pi_t^M \right) = -z + \delta E_t \left( (1 - \sigma_{t+1}) \Pi_t^N - \phi_{t+1} \right). \] (5.40)

The incumbent worker’s bargaining surplus is
\[ E_t \left( V_t^M \right) - E_t \left( \tilde{V}_t^M \right) = \left( w_t - e^M + \delta E_t \left( (1 - \sigma_{t+1}) V_{t+1}^N + \sigma_{t+1} V_{t+1}^U \right) \right) \]
\[ - d - \delta E_t \left( (1 - \sigma_{t+1}) V_{t+1}^N + \sigma_{t+1} V_{t+1}^U \right) \]
\[ = w_t - d - e^M, \] (5.41)

and the firm’s surplus is
\[ E_t \left( \Pi_t^M \right) - E_t \left( \tilde{\Pi}_t^M \right) = \left( a - w_t - \epsilon^M \right) + \delta E_t \left( (1 - \sigma_{t+1}) \Pi_t^N - \phi_{t+1} \right) - \]
\[ E_t \left( -z + \delta \left( (1 - \sigma_{t+1}) \Pi_t^N - \phi_{t+1} \right) \right) \]
\[ = a_t - w_t - \epsilon^M + z. \] (5.42)

The negotiated wage maximizes the Nash product (\( \Lambda \)):
\[ \Lambda = \left( w_t - e^M - d \right) \gamma \left( a_t - w_t + z - e^M \right)^{1-\gamma}. \] (5.43)

Thus the negotiated wage is
\[ w_t = \gamma \left( a_t + z - e^M \right) + (1 - \gamma) \left( e^M + d \right), \] (5.44)

where \( \gamma \) represents the bargaining strength of the incumbent worker relative to the firm.

5.4.5 The labour Market Equilibrium

The labour market equilibrium is the solution of the system comprising the following equations:

- **Incentives:** the incumbent worker retention incentive \( \nu_t^I \) (eq. 5.22), the job offer incentive \( \nu_t^E \) (eq. 5.25), the work incentive \( \iota_t \) (eq. 5.30).
- **Employment decisions:** the firing rate \( \phi_t \) (eq. 5.23), and the job offer rate \( \eta_t \) (eq. 5.26).
- **Work decisions:** the job acceptance rate \( \alpha_t \) (eq. 5.31) and the quit rate \( \chi_t \) (eq. 5.32).
- **Match and separation probabilities:** the match probability \( \mu_t \) (eq. 5.33) and the separation probability \( \sigma_t \) (eq. 5.34).
- **Employment and wage determination:** the employment level \( N_t \) (eq. 5.35) and the negotiated wage \( w_t \) (eq. 5.44).
5.5 Calibration

We now calibrate our incentive model for the US economy. The calibration is done on a monthly basis. The simulation results are aggregated to quarterly frequency to make them comparable to the empirical data, as for example in Shimer (2005). For the discount factor $\delta = \frac{1}{1+r}$ we apply the real interest rate $r = 1.04^{1/12} - 1$. We normalize the average productivity ($a$) to 1. As in Hall (2005) and Shimer (2005), we set $b$ by applying a replacement rate of $\beta = 40\%$ of the wage. For simplicity, we set $d = b$. As commonly found in the literature we adopt a bargaining power parameter $\gamma$ of 0.5.

Vacancy posting costs are usually set to around 30 percent of the quarterly productivity in the conventional matching model calibrations. To make our calibration as comparable as possible to conventional ones, we divide this number by the typical quarterly worker job finding rate of 0.7 (see, e.g., Krause and Lubik, 2007 and den Haan et al., 2000) to obtain the hiring costs, $h$. This gives us a value of 43 percent of the quarterly productivity or roughly 130 percent of the monthly productivity.

The literature does not provide reliable direct estimates of the magnitude of US firing costs. Thus we assess these costs indirectly. For this purpose, note that Belot et al. (2007) provide index measures of employment protection for regular jobs in the US and UK, and that Bentolila and Bertola (1990) provide estimates of the average magnitude of UK firing costs on a yearly basis. Assuming that the index measures of employment protection are proportional to the estimates of the magnitude of firing costs, we multiply the magnitude of the UK firing costs by the ratio of the US to the UK employment protection indices to derive a rough estimate of the magnitude of US firing costs. Accordingly, the magnitude of monthly US firing costs, relative to productivity, is 0.08. The same exercise based on other industrialized countries (France, Germany and Italy), however, yields higher estimates of US firing costs. Thus we choose a value of 0.1 for our baseline calibration, but provide a robustness analysis for other values in Appendix 5.A.2. For simplicity, we set the firm’s fallback profit $-z$ equal to $-f$.188

We assume that the random profitability term $\varepsilon$ and the utility shock $e$ have cumulative distributions given by logistic functions with scale factors $s_\varepsilon$ and $s_e$ and expected values $\bar{\varepsilon}$ and $\bar{e}$, respectively. We calibrate our model such that it replicates the stylized fact that wages are as volatile as productivity. This is achieved by setting $\bar{e} = 0.19$. Thereby our calibration excludes the possibility that our results are driven by real wage rigidity. We assign values to the remaining free parameters of the model ($\bar{\varepsilon}$, $s_e$, $s_\varepsilon$) so as to replicate the following steady state values (summarized in Table 5.1): The match probability $\mu$, which is the probability for a
worker to find a new job within one period, is calibrated to 45%\textsuperscript{192}, as in Shimer (2005) and Hagedorn and Manvoskii (2008). The unemployment rate $u$ is calibrated to 12%.\textsuperscript{193} According to our employment dynamics equation (5.12) steady state unemployment is $u = \frac{\mu}{\mu + \sigma}$ which implies a separation rate of 6.14%. Based on Hall (2006), who shows that fires and quits have approximately the same share in separation, we assume firings to account for 50% of the separations, namely $\phi = 3.1\%$. Eq. (5.34) then yields the quit rate of $\chi = 3.2\%$. Since $\alpha$ is equal to $1 - \chi$, the job acceptance rate is set at 96.8%. Recalling that $\mu = \alpha \eta$, we find that the resulting job offer rate $\eta$ is 46.5%.

We normalize the autocorrelation ($\rho_a$) of the aggregate productivity shock and normalize the standard error such that we obtain the empirical values for the autocorrelation and the volatility of productivity in the model simulation below. Table 5.2 summarizes our calibrated parameter values.

<table>
<thead>
<tr>
<th>Variable</th>
<th>In Words</th>
<th>Steady State Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>unemployment rate</td>
<td>0.120</td>
</tr>
<tr>
<td>$\mu$</td>
<td>match probability</td>
<td>0.450</td>
</tr>
<tr>
<td>$\eta$</td>
<td>hiring/job offer rate</td>
<td>0.465</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>separation rate</td>
<td>0.061</td>
</tr>
<tr>
<td>$\phi$</td>
<td>firing rate</td>
<td>0.031</td>
</tr>
<tr>
<td>$\chi = 1 - \alpha$</td>
<td>job quit rate</td>
<td>0.032</td>
</tr>
</tbody>
</table>

Table 5.1: Steady state values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>In Words</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>productivity</td>
<td>1</td>
</tr>
<tr>
<td>$\beta$</td>
<td>replacement rate $\frac{b}{w}$, $\frac{d}{w}$</td>
<td>0.4</td>
</tr>
<tr>
<td>$f$</td>
<td>firing cost</td>
<td>0.1</td>
</tr>
<tr>
<td>$h$</td>
<td>hiring cost</td>
<td>1.3</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>workers’ bargaining strength</td>
<td>0.5</td>
</tr>
<tr>
<td>$r$</td>
<td>discount factor</td>
<td>0.997</td>
</tr>
<tr>
<td>$-z$</td>
<td>firm’s fallback profit</td>
<td>-0.1</td>
</tr>
<tr>
<td>$\bar{e}$</td>
<td>average value of leisure</td>
<td>0.17</td>
</tr>
<tr>
<td>$\bar{\xi}$</td>
<td>average operating costs</td>
<td>0.465</td>
</tr>
<tr>
<td>$s_{\bar{e}}$</td>
<td>scale factor of the cumulative distribution of $\bar{e}_t$</td>
<td>0.390</td>
</tr>
<tr>
<td>$s_e$</td>
<td>scale factor of the cumulative distribution of $e_t$</td>
<td>0.078</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>autocorrelation of the aggregate productivity shock</td>
<td>0.975</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>standard error of the aggregate productivity shock</td>
<td>0.007</td>
</tr>
</tbody>
</table>

Table 5.2: Parameter values (rounded to the third decimal)

\textsuperscript{192}Note: in our model the worker finding rate (i.e., the probability of a firm to find a new worker) and the job finding rate (i.e., the probability of a worker to find a new) are the same.

\textsuperscript{193}This value also considers potential participants in the labor market such as discouraged workers and workers loosely attached to the labor force, see Krause and Lubik (2007) and den Haan et al. (2000).
5.6 Description of Results

5.6.1 Labour Market Volatilities

Costain and Reiter (2007) and Shimer (2005) show that the conventional calibration of the matching model is unable to replicate the volatility of the job finding rate, the unemployment rate, and other labour market variables in response to productivity shocks. Table 5.3 shows that the empirical volatilities for the United States (from 1951-2003, HP filtered data with smoothing parameter 100,000, as calculated by Shimer) are far greater than the corresponding volatilities in response to productivity shocks, as generated by the simulation of the conventional matching model (in its standard calibration, as calculated by Shimer).

<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>U. Rate</td>
</tr>
<tr>
<td>Standard deviation</td>
</tr>
<tr>
<td>Relative to productivity</td>
</tr>
<tr>
<td>Quarterly autocorrelation</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Volatilities by Shimer’s (2005) Search and Matching Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>U. Rate</td>
</tr>
<tr>
<td>Standard deviation</td>
</tr>
<tr>
<td>Relative to productivity</td>
</tr>
<tr>
<td>Quarterly autocorrelation</td>
</tr>
</tbody>
</table>

Table 5.3: Empirical volatilities and volatilities generated by the search and matching model from Shimer (2005).

To compare our model with the conventional matching theory, we use our baseline calibration (with robustness checks in the Appendix 5.A.2) to simulate our model for 200 quarters (i.e. 600 months). We repeat this exercise 10,000 times and report the average of the macroeconomic volatilities (HP filtered simulated data with smoothing parameter 100,000) in Table 5.4.

<table>
<thead>
<tr>
<th>Standard Calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>U. Rate</td>
</tr>
<tr>
<td>Standard deviation</td>
</tr>
<tr>
<td>Relative to productivity</td>
</tr>
<tr>
<td>Quarterly autocorrelation</td>
</tr>
</tbody>
</table>

Table 5.4: Volatilities generated by the incentive model of matching.

The differences between our model and the conventional matching model are striking. Our model can generate the high macroeconomic volatilities found in the data. Our results are all the more remarkable, as we do not neither have to resort to Hall’s (2005) real wage rigidity assumption nor to Hagedorn and Manovskii’s (2008) small surplus calibration.
Specifically, the more rigid the wage in the conventional matching model (Hall, 2005), the greater the share of productivity variations that is captured by the firm and thus the greater the volatility of vacancies. However, there is evidence against the rigid-wage hypothesis both from the microeconometric and the macro perspective. Haefke et al. (2008) infer that wages for newly created jobs (i.e., those modeled in the matching model) are completely flexible on a microeconomic level. Hornstein et al. (2005) show that wages are roughly as volatile as the labour productivity on a macroeconomic level. By contrast, our model generates high labour market volatilities, even though it replicates the stylized fact that wages are as volatile as productivity.

Hagedorn and Manovskii (2008) choose a small-surplus calibration to resolve the volatility puzzle of the matching model. Under this calibration, aggregate profits are only a very small share of the overall production in the steady state, so that a positive productivity shock sharply increases the relative profits. This gives a large incentive to firms to post more vacancies (due to the free entry condition). Consequently all labour market variables become volatile. This type of calibration has several shortcomings. Besides the unrealistically low profit share, the utility value of unemployment is extremely high and workers’ bargaining power is very low in the calibration. Therefore workers are almost indifferent between working and not working. We do not need to rely on any of these mechanisms in our calibration. As noted, we assume that worker’s bargaining power is 50 percent. The labour income divided by overall production is roughly 80 percent in our model. Furthermore, the average worker’s disutility of labour and unemployment benefits make up only 80 percent of the current wage. As a consequence, the average worker is not indifferent between unemployment and employment.

5.6.2 Correlations

Our model features several additional advantages compared to the conventional matching framework. Krause and Lubik (2007) show that the matching framework with endogenous job destruction and flexible wages cannot generate a strong negative correlation between the job finding rate and the unemployment. In all of our model simulations, the correlation between these two variables is very strongly negative, in magnitude between -0.95 and -0.99, i.e., slightly higher than in the US data (-0.95, see Shimer, 2005). Further, Krause and Lubik (2007) show that the matching model with endogenous job destruction and flexible wages cannot account for the negative correlation between job destruction and job creation. In our model, the correlation between these two variables is always negative and close to -1.\(^{194}\)

\(^{194}\)The job finding rate and the job destruction rate are both driven by the same underlying shock, resulting in this strong negative correlation. We could get a lower correlation if we introduced another shock to drive a wedge between the shocks underlying job destruction and those underlying job creation. However, for simplicity we do not choose this option.
5.7 Conclusion

This chapter has presented a choice-theoretic theory of labour market matching in the presence of frictions, heterogeneous jobs and heterogeneous workers. This theory does not rely on a matching function. We have presented simple analytical models that derive labour market matches and separations from the optimizing behaviour of workers and firms. Since the matching function is meant to encapsulate frictions and heterogeneities, we have examined whether it can replicate the optimizing behaviour above. Our analysis indicates that the matching function is vulnerable to the Lucas critique, since it is not stable with respect to changes in policy and macroeconomic variables. Thus its use for policy analysis and prediction becomes problematic.

The general intuition is straightforward. Although it is often claimed that the matching function is analogous to a production function, an important difference stands out. A firm’s production function captures the portfolio of available technologies, and these are often invariant with respect to many government policies and macroeconomic variations. By contrast, a matching function summarizes the upshot of the many individual decisions by firms and workers, responding to their individual incentives to offer, accept, quit and destroy jobs, and these incentives are in general affected by policies and macroeconomic shocks. In this respect, the matching function appears to face difficulty analogous to the adaptive expectations hypothesis, which sought to predict expectations without reference to the actual stochastic processes in the economy. Just as predictable changes in policies and and macroeconomic variables could be expected to influence agents’ expectations, so these changes can also be expected to influence agents’ incentives to generate labour market matches.

Our incentive theory provides a different view of the matching process than that presented by a matching function. Whereas the matching function depicts matches as the output of a “matching technology” that mechanically pairs unemployed workers and vacant firms, the incentive theory explains the matching probability in terms of the firm’s job offer incentive and the worker’s job acceptance incentive. Similarly, the separation probability is explained in terms of the firm’s firing incentive and the worker’s quit incentive. These incentives depend on all the parameters of the model, including policy and macro parameters.

To keep our formal analysis as simple as possible, we have made some radically simplifying assumptions, such as those concerning wage determination, the depiction of heterogeneities in terms of only two additive shocks $\varepsilon$ and $e$, and the depiction of adjustment costs in terms of only two additive costs $h$ and $f$. Whereas these simplifying assumptions naturally affect the quantitative predictions of our model, they are not essential to basic idea that motivates this chapter: namely, that the matching and separation probabilities can be understood in terms of job offer, job acceptance, firing, and quit probabilities, which may be derived from the optimizing decisions of firms and workers. These optimizing decisions - in the presence of heterogeneous workers and jobs, as well as costs of adjustment - explain why some job-seeking workers remain unemployed and some vacant jobs remain unfilled.

Needless to say, the incentive models presented above are merely a first step towards a
choice-theoretic understanding of the matching process. Much research remains to be done. Although relaxing our simplifying assumptions regarding wage determination, heterogeneities and adjustment costs will not affect the basic idea above, it will help us refine the quantitative predictive properties of the incentive model.

Nevertheless, we have shown that even on the basis of our radically simplifying assumptions, our calibrated incentive model can account for various important empirical regularities that have eluded the conventional matching models. In particular, our model comes close to generating the empirically observed volatilities of the unemployment rate, the job finding rate and the separation rate. Furthermore, our model can also account for the observed strong negative correlations between the job finding rate and the unemployment rate, and between job creation and job destruction.

5.A Appendix

5.A.1 Vacancies

The purpose of this appendix is to incorporate vacancies in the incentive model above and derive the associated equivalence condition. The firm has two costs of employing new entrants: a vacancy posting cost $\kappa$ and a hiring cost $h$. The vacancy posting cost is expended regardless of whether a worker is hired. The hiring cost, by contrast, is expended only once a worker is hired. The vacancy posting cost $\kappa$ is assumed constant per vacancy and the hiring cost $h$ is assumed constant per worker hired. Thus the profit generated by a new entrant is $\pi^E = a - \varepsilon - w - h - \varpi \frac{\kappa}{q(\theta)}$ where $\varepsilon$ is positive and iid across workers (as above), $\kappa/q(\theta)$ is the vacancy cost per worker hired and $\varpi$ is the proportion of jobs that require vacancy postings before they are filled, where $0 \leq \varpi \leq 1$. (If the vacancy posting cost is dwarfed by the hiring cost and $\varpi$ is negligible, this general model reduces to the model above.)

Let the number of job applicants be $A = x(V, mU)$, where $m$ is the number of search actions per worker and the function $x$ exhibits constant returns to scale. Then the number of committed applicants (i.e. applicants who do not defect to other jobs) is $A/m = x(V/m, U)$. Then the number of committed applicants per vacancy is $q(\theta) = \frac{A}{mV} = x(\frac{1}{m}, \frac{U}{V})$. Thus the expected profit from an entrant is

$$\pi^E = a - \varepsilon - w - h - \frac{\varpi \kappa}{x(\frac{1}{m}, \frac{U}{V})}$$

where the number of vacancies $V$ and the number of unemployed $U$ are both exogenous to the firm when it makes its hiring decision.

We assume that vacancies are set before the random variable $\varepsilon$ is known. Thus the number of vacancies is determined by the following zero expected profit condition: $E(\pi^E) = a - E(\varepsilon) - w - h - \frac{\varpi \kappa}{x(\frac{1}{m}, \frac{U}{V})} = 0$, where $E(\varepsilon) = 0$. Thus the equilibrium number of vacancies satisfies

$$x\left(\frac{1}{m}, \frac{U}{V^*}\right) = \frac{\varpi \kappa}{a - w - h}$$

(5.45)
where \( u \) is the unemployment rate \((U/L)\) and \( v \) is the vacancy rate \((V/L)\).

Furthermore, we assume that the number of search actions per worker is set before the random variable \( e \) is known. Let the utility of a currently unemployed worker be \( E(V) = (\eta m) w + (1 - \eta m) b - \eta m E(e) - \psi(m) \), where \( \psi(m) \) is the cost of search actions. The utility-maximizing number of search actions \( m^* \) satisfies \( \frac{\partial E(V)}{\partial m} = \eta (w - b - E(e)) - \psi'(m) = 0 \), so that

\[
m^* = k (\eta (w - b - E(e)))
\]

where \( k = (\psi')^{-1} \).

Since a worker is hired iff \( \pi^E > 0 \), the job offer rate is

\[
\eta = C_e \left( a - w - h - \frac{\pi \kappa}{x \left( \frac{1}{m}, \frac{u}{v} \right)} \right).
\]

Thus the match probability is

\[
\mu = \eta \alpha = C_e \left( a - w - h - \frac{\pi \kappa}{x \left( \frac{1}{m}, \frac{u}{v} \right)} \right) C_e (w - b)
\]

The firing rate \( \phi \) is given by equation (5.5), the job acceptance rate \( \alpha \) by (5.7), and the quit rate \( \chi \) by (5.32). Thus the separation probability is given by equation (5.34).

The change in employment is

\[
\Delta N = \mu x (V_{-1}, m U_{-1}) - \sigma N_{-1}
\]

Thus the employment dynamics equation satisfies

\[
n = \mu x (v_{-1}, m (1 - n_{-1})) + (1 - \sigma) n_{-1}
\]

The equilibrium vacancies rate, unemployment rate, search actions are the solutions of the equation system (5.46), (5.45), (5.47), (5.48) and (5.49).

We now examine whether the optimized matching behaviour above can be mimicked through the mainstream matching model, namely, a model based on a stable matching function and the assumption that all jobs require prior posting of vacancies. This stable matching function is of course not identical with that above, since it is required to mimic not only the standard matching of homogeneous workers and jobs, but also the matching of heterogeneous workers and jobs, as described above. In order to provide maximal flexibility in finding such a stable matching function, we have specified it as a general functional form: \( x = x(u, v) \).

The corresponding employment dynamics equation is (5.14) and the zero-profit condition is (5.15).

The employment dynamics equations for the two models, (5.12) and (5.14), are identical when the following equivalence condition holds:
\[
\frac{\kappa}{a-w}g \left( \frac{\kappa}{a-w} \right) = C_e \left( a - w - \frac{\varpi \kappa}{x \left( \frac{1}{m} \right. \left. \frac{u}{v} \right)} \right) C_e (w - b) \quad (5.50)
\]

Differentiating this condition with respect to \( \kappa \), we obtain

\[
\frac{1}{a-w} \left( \frac{\kappa}{a-w} g' + g \right) = -C_e' C_e \left( \frac{\varpi}{x \left( \frac{1}{m} \frac{u}{v} \right)} + \kappa \frac{\partial \varpi}{\partial \kappa} \right) \quad (5.51)
\]

and differentiating the equivalence condition with respect to \( a \), we get

\[
-\frac{\kappa}{(a-w)^2} \left( \frac{\kappa}{a-w} g' + g \right) = C_e' C_e \left( 1 - \frac{\partial \varpi}{\partial a} \right) \left( \frac{1}{m} \frac{u}{v} \right) \quad (5.52)
\]

Rewriting equation (5.19),

\[
\frac{1}{a-w} \left( \frac{\kappa}{a-w} g' + g \right) = -\frac{a-w}{\kappa} C_e' C_e \left( 1 - \frac{\partial \varpi}{\partial a} \right) \left( \frac{1}{m} \frac{u}{v} \right) \quad (5.51)
\]

Substituting this into (5.51),

\[
\left( \frac{\varpi}{x \left( \frac{1}{m} \frac{u}{v} \right)} + \kappa \frac{\partial \varpi}{\partial \kappa} \right) = \frac{a-w}{\kappa} \left( 1 - \frac{\partial \varpi}{\partial a} \right) \left( \frac{1}{m} \frac{u}{v} \right)
\]

It is clear by inspection that this equation does not hold for all values of the parameters \( \varpi, \kappa, a \), and so on.

5.A.2 Robustness

Table 5.5 provides a robustness analysis of the labour market volatilities implied by our model for values of the firing cost \( f = 0.08 \) and \( f = 0.20 \).

<table>
<thead>
<tr>
<th></th>
<th>U. Rate</th>
<th>Match. Rate</th>
<th>Sep. Rate</th>
<th>Product.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Volatilities for ( f = 0.08 )</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.23</td>
<td>0.14</td>
<td>0.09</td>
<td>0.02</td>
</tr>
<tr>
<td>Relative to productivity</td>
<td><strong>10.7</strong></td>
<td><strong>6.6</strong></td>
<td><strong>4.4</strong></td>
<td>1</td>
</tr>
<tr>
<td>Quarterly autocorrelation</td>
<td>0.90</td>
<td>0.88</td>
<td>0.88</td>
<td>0.88</td>
</tr>
<tr>
<td><strong>Volatilities for ( f = 0.2 )</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.11</td>
<td>0.09</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>Relative to productivity</td>
<td><strong>5.2</strong></td>
<td><strong>4.5</strong></td>
<td><strong>1.5</strong></td>
<td>1</td>
</tr>
<tr>
<td>Quarterly autocorrelation</td>
<td>0.91</td>
<td>0.88</td>
<td>0.88</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Table 5.5: Robustness analysis of the labour market volatilities implied by our model for values of the firing cost \( f=0.08 \) and \( f=0.20 \).
6 Conclusion

Globalization has entered a new stage, with new and varying constellations of winners and losers and thus, calls for a new policy paradigm. In contrast to the traditional way of thinking - the welfare state redistributes purchasing power to the disadvantaged to support them in the event of adverse shocks - now workers must be made more adaptable and be encouraged to stabilize themselves if hurt from globalization.

This dissertation has examined several labour market policies with various incentive models of the labour market, which explain the microeconomic decisions relevant to the transitions in the labour market in terms of the incentives that economic agents face. To fulfill its new role the following three reform policies for the welfare state have been proposed.

Unemployment accounts, which could be structured to reproduce all the main regulations and provisions of the existing system,\(^{195}\) introduce a more efficient redistributive scheme and an incentive instrument for rewarding good risks, which gives people some property rights in this welfare service. Thereby unemployment accounts do not only improve employment incentives by reducing negative externalities, but also promote adaptability in the labour market, since the accumulation of positive account balances, in the current stage of globalization, would increasingly require workers to adapt to the rapidly changing environment. The quantitative evaluation in Chapter 2 has shown that the incentive effects of this reform would be more than sufficient for unemployment accounts to be self-financing and the resulting reductions in unemployment could be substantial.\(^{196}\)

Chapter 3 has concluded that employment subsidies should be targeted as hiring vouchers at the duration of unemployment, specifically at the long-term unemployed.\(^{197}\) Redirecting funds from passive unemployment support towards hiring vouchers, which redistribute incentives in favour of the disadvantaged, equalizes workers’ employment opportunities. Hiring vouchers enable previously unemployed workers to become productive by adapting to their changed circumstances and thereby, avoid further depreciation of skills. The quantitative analysis has illustrated that hiring vouchers for long-term unemployed are “approximately welfare efficient,” i.e., they improve aggregate employment and welfare, do not increase earnings inequality and are self-financing.

The Danish flexicurity concept shifts the focus of social security from job and income security towards employment security. By increasing flexibility in the labour market and increasing the employment incentives this policy package makes the labour market more adaptable. In Chapter 4 the experiment of introducing the Danish flexicurity set of policies in Germany has revealed the strong complementarities of this set of policies, which ensure its viability and

\(^{195}\) Such that the unemployed would not be worse off than they are in the existing system.

\(^{196}\) How unemployment accounts could work in the UK is discussed in Snower and Brown (2009). Boss et al. (2008) present a concrete proposal how unemployment accounts could be integrated in Germany.

\(^{197}\) Brown et al. (2007b) and Boss et al. (2007) show how hiring vouchers could be implemented in Germany. The size of the hiring voucher rises with the duration of unemployment and falls with the duration of subsequent employment. Unskilled workers receive higher vouchers than their skilled counterparts, because the present value of the vouchers is tied to the expected present value of unemployment benefits that would have been received and unskilled workers have higher unemployment rates.
generate strong reductions in unemployment and inequality. These findings also emphasize the need for a more holistic approach to labour market reforms, which enables not only to exploit substantial economic complementarities but by addressing distributional concerns generates political complementarities thus, potentially facilitating implementation.

Furthermore, it is straightforward to combine these policy proposals, e.g. workers could use their unemployment account balances to purchase hiring vouchers and additionally, the government could provide subsidies for the long-term unemployed to use their unemployment account to purchase hiring vouchers.

This dissertation has provided simple micro-founded analytical frameworks that derive labour market transitions from the optimizing behaviour of workers and firms without relying on a matching function. By exposing that the matching function is unstable with respect to labour market policies and macroeconomic shocks Chapter 5 has indicated that the matching function runs afoul of the Lucas critique. A new modelling strategy of two-sided search with frictions and heterogeneities, which is able to account for empirical stylized facts, has been proposed for future policy analysis and prediction. This analytical framework is seen as a first step towards a new approach in labour market modelling and policy analysis, obviating the need for a matching function.
References


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Ich erkläre hiermit an Eides Statt, dass ich meine Doktorarbeit „Incentive Models of the Labour Market“ selbstständig angefertigt habe und dass ich alle von anderen Autoren wörtlich übernommenen Stellen, wie auch die sich an die Gedanken anderer Autoren eng anlehnenden Ausführungen meiner Arbeit, besonders gekennzeichnet und die Quellen zitiert habe.

Kiel, Juni 2009