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Two-stage stochastic programming model to locate capacitated EV-charging stations in urban areas under demand uncertainty

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ABSTRACT

Due to the dangerous effects of fossil fuels, policymakers tend to substitute fossil-fuel-based vehicles with electric ones. Thus, the optimal design of a charging station network providing convenient access for the users is of great importance. This paper presents a two-stage stochastic programming model for the problem of locating charging stations in urban areas. Parking lots around the buildings which may be visited by people during the day are considered as potential locations for charger installation. The model determines the parking lots that should be equipped with chargers and the number as well as the type of chargers that must be placed in each parking lot considering the demand as an uncertain parameter. The proposed model is examined on the dataset of a midtown area, taken from the literature, and an efficient heuristic algorithm based on Benders decomposition is utilized to solve the model. The results indicate that the heuristic method can find a near-optimal solution (with the optimality gap of at most 0.05%) in a short time.

Introduction

Motivation

Dangerous effects of greenhouse gases (GHGs) emitted by fossil fuels have caused concerns at the economic, environmental, and industrial levels. According to the GHG Inventory Report in 2011, 30% of GHG emissions in the United States come from the transportation sector, and according to studies provided by the International Energy Outlook in 2011, demand in the transportation sector will increase by 54% until 2035. Furthermore, as pointed out by Veneri (2017), electricity is becoming the preferred energy vector for the next new generation of road vehicles, and policymakers seek strategies to substitute fossil-fuel-based vehicles with electric vehicles (EVs). An EV is a car that instead of an internal combustion engine uses a rechargeable battery for propulsion, and hence, it has the potential to improve air quality. The driving range of an EV is relatively short, and its battery should be charged before it is exhausted; therefore, users may need to charge their vehicles away from home. One of the obstacles to the adoption of EVs is the lack of sufficient publicly accessible charging stations. Hence, the optimal design of a charging station network providing easy and convenient access for the users is of great importance.

Survey

The problem of locating charging stations has received increasing attention in the literature. As pointed out by Faridimehr et al. (2019), many studies have focused on EV charging in large state-wide networks in which EV drivers travel between cities and metropolitan areas. Flow capturing location problems (FCLPs) belong to this category in which a set of traffic flows with specific routes is given and the aim is to locate charging stations (or equivalently, alternative refueling stations) along the routes so that the percentage of traffic flows performing their round trip without running out of charge (or equivalently, alternative fuel) is maximized. For some related work, see Kuby and Lim (2005), Capar and Kuby (2012), and MirHassani and Ebrazi (2013), Hosseini, et al. (2017), Miralinaghi, et al. (2017) and Wang et al. (2018).

Other studies concentrate on the design of charging networks for urban areas or communities. This category is affected by some factors. The first one is the power grid infrastructure. Depending on available technologies for EV charging, two standard charging modes (slow and fast) are defined. If an EV is plugged into a slow charger, the battery would be fully charged within about 8 h. However, if an EV is plugged into a fast charger, the battery would be fully charged in less than 1 h. By simultaneously charging just a few vehicles in a given station of an urban area by fast charging technology, a huge extra demand may be imposed

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on the electric grid and can cause power loss. This important factor has been noted by some researchers. Amongst others, [Schey, et al. \(2012\)](#) and [Mu et al. \(2014\)](#) proposed approaches to evaluate the impact of EV charging on the electric grid. [Waraich et al. \(2013\)](#) proposed a method to assess the bottlenecks in the electric grid caused by EV energy demand. [Sadeghi-Barzani et al. \(2014\)](#) and [Battapothula et al. \(2018\)](#) presented mixed integer nonlinear programming (MINLP) models to place fast-charging stations concerning electric grid loss. The second factor is the EV users' habits (e.g., willingness to walk, preference for charging away, state of charge). [Xu et al. \(2017\)](#) analyzed what factors influence the charging mode and location (home/company or public stations) chosen by EV users. [Andrenacci et al. \(2017\)](#) presented an approach integrating big-data analysis, fuzzy modeling, and simulation techniques to investigate which charging infrastructures provide the best level of service to customers. [Alhazmi et al. \(2017\)](#) introduced the concept of trip-success-ratio to estimate charging station service range considering different driving behaviors and trip types.

Some studies have focused on locating slow charging stations. For example, [Frade, et al. \(2011\)](#) presented a maximum covering location model determining the location and capacity of slow charging stations. [Chen et al. \(2013\)](#) used parking demands in different zones as a base to predict the charging demands and presented a mixed-integer linear programming (MILP) model and an algorithm to assign charging stations to zones. The charging demand of drivers whose destination is a zone not equipped with any charging station may be met by charging stations placed in other zones, and the aim is to minimize the cost of drivers' walking distance and unmet demands. As pointed out by [Cavadas et al. \(2015\)](#), in many European cities, since drivers live in apartment buildings, they have to park on the street, and hence, they cannot charge their EV privately. Thus, they presented an MILP model to locate public slow-charging stations in a city. Other studies addressed the installation of a mix of charging stations (slow and fast). For example, [Wang and Lin \(2013\)](#) presented an MILP model based on the concepts of set coverage and maximum coverage to locate mixed stations taking into account the drivers' behavior. [Shahraki et al. \(2015\)](#) developed an MILP model considering the vehicle travel patterns to determine the location of public charging stations to maximize the EV traveled distance. [He et al. \(2015\)](#) and [He et al. \(2018\)](#) proposed bi-level programming models in which policymakers, as the leader, decide on the location of charging stations, and EV users, as the follower, determine their tour paths and recharging plans. [Guo et al. \(2018\)](#) studied the charging station location problem concerning users' range anxiety and distance deviations, formulated the problem as a bi-level programming model, and solved it via a hybrid heuristic algorithm.

[Yang et al. \(2017\)](#) addressed the allocation of chargers for EV taxis throughout a city to minimize the infrastructure investment. They presented a queuing model to incorporate charging congestion. [Brandstätter et al. \(2017\)](#) combined the electric car-sharing problem with the problem of locating recharging stations. They considered the problem under uncertainty and presented a time-dependent two-stage stochastic programming model. [Hof et al. \(2017\)](#) proposed a variable neighborhood search algorithm with capacitated EVs to solve a variant of vehicle routing problem with intermediate stops at battery swap stations in which in addition to the routing decisions, the location of battery swap stations is also determined. [Verma \(2018\)](#) considered the electric vehicle routing problem by allowing the available stations to serve as recharging and battery swapping stations.

In most of the existing papers on the charging station location problem, demands for charging service are assumed to be deterministic, while in practice, traffic flows are affected by uncertainty (depending on the hour of the day, weekday/weekend, the aim of traveling, destination, etc.). In such cases, the optimal solution to a deterministic model might be sub-optimal or even infeasible in the presence of uncertainties. Thus, uncertainty should be incorporated into the problem. In this regard, [Hosseini and MirHassani \(2015\)](#) proposed a two-stage stochastic programming model to locate permanent and portable charging stations to

maximize served traffic flows. [Wu and Sioshansi \(2017\)](#) combined the EV charging station location and the power distribution problems under the assumption of uncertainty in EV charging loads as a multi-objective multistage model to minimize investment and operational costs while maximizing the annually captured traffic flows. [Kim et al. \(2017\)](#) proposed a stochastic model considering parking times and electricity demand as random parameters. [Mak et al. \(2013\)](#) provided a robust optimization model to locate battery-swapping stations under demand uncertainty. [Yildiz et al. \(2019\)](#) addressed urban recharging station infrastructure design considering stochastic demands, capacity limitations, and route deviation. [Kadri et al. \(2020\)](#) studied the problem of locating EV fast-charging stations in the multi-period case under demand uncertainty.

In some studies, public parking lots are considered as potential locations for installing charging stations in urban areas. [Huang and Zhou \(2015\)](#) addressed the workplace charging strategy and presented an MILP to determine the number and levels of chargers to install while satisfying the charging demand of employees of a given company. [Giménez-Gaydou et al. \(2016\)](#) presented a gradual maximal covering model to locate charging stations for EVs in urban areas. An important feature of their model is that coverage is sought for the groups of places (home, work, etc.) where users park their EVs during the day and not for each one of these places separately. As pointed out by [Morrissey, Weldon, and O'Mahony \(2016\)](#), some EV users of urban areas may prefer to do most of their charging in the evening when they come back home. However, this time is the period of peak demand on the power grid and may cause power loss. If the parking lots are equipped with charging systems, such users may be encouraged to charge their EV in a time other than the peak demand period (e.g. when their vehicle is parked in the parking lot of the workplace, home, shopping center, etc.). [Faridimehr et al. \(2019\)](#) provided a two-stage stochastic programming model to select some parking lots for charger installation under demand uncertainty and presented a heuristic algorithm to solve the model.

Contributions and the general framework

In this paper, public parking lots around the buildings (e.g., urban university, workplace, hospital, shopping center, etc.) that may be visited by users during the day in urban areas are considered as potential locations for installing charging stations. Thus, the vehicles may be charged in the parking lots during the time their users visit the buildings. The EV users' habits (i.e., willingness to walk, state of charge) are taken into account, and it is assumed that parking lots can be equipped with both slow and fast chargers. Although most studies focus on the deterministic case of the problem, we study the problem under demand uncertainty.

Our work can be considered as an extension of the study of [Faridimehr et al. \(2019\)](#) to a more general case. They assumed that each user visits just one building during the day, and all stations are equipped with semi-rapid chargers. However, we address the problem in a more general case assuming that every user may visit one or two buildings during the day, and the installation of both fast and slow chargers is decided.

The main contributions of this paper are as follows: from the modeling aspect, we present a novel model that not only contains the model of [Faridimehr et al. \(2019\)](#) as a special case but also it applies to a more general practical case. From the solution aspect, an efficient heuristic method based on the Benders decomposition (BD) algorithm is presented. The model and solution method is examined in the midtown area of Detroit, Michigan, U.S. as a dataset taken from the literature.

The rest of this paper is organized as follows: Section 2 provides a detailed description of the problem and formulates it as a two-stage stochastic programming model. In Section 3, the BD algorithm is addressed to solve the problem. Computational results are presented in Section 4. Section 5 concludes and suggests directions for future research.

Table 1

Classification of the drivers in terms of the state of charge.

Class	The state of charge when arriving at the first building	Characteristics	Ratio
1	Low	The drivers of this class require to charge their EV during the time they are in the first building $\bar{b}_{1,g}$	β_1
2	Middle	The drivers of this class require to charge their EV during the time they are either in the first building $\bar{b}_{1,g}$ or in the second building $\bar{b}_{2,g}$	β_2
3	High	The drivers of this class require to charge their EV during the time they are in the second building $\bar{b}_{2,g}$	β_3

Mathematical model

Problem description

Consider an urban area with a set of buildings (including workplaces, universities, shopping centers, restaurants, etc.) that may be visited by

people during the day. Public parking lots around the buildings are potential locations for charger installation. The charging station location problem aims at determining the set of parking lots that should be equipped with chargers and the number of slow and fast chargers that should be installed in parking lots. A trade-off between two objectives is sought: minimizing the cost of equipping parking lots with chargers and maximizing the expected coverage of demands. The following assumptions are made:

A1. Drivers visit two buildings during the day.

A2. When drivers park their vehicles in a parking lot near their destination building, they should walk the distance between the parking lot and the building. Whether or not a parking lot is suitable for a given driver, depends on his/her willingness to walk.

A3. EV users are divided into some groups depending on the buildings they visit during the day, the arrival time to the first building, the dwell times in each building, the state of charge of EV battery when arriving at the first building, and their willingness to walk.

A4. The first and the second buildings, the dwell time at each one, the travel time between them, and the arrival time to the first building are known with certainty for each group of drivers.

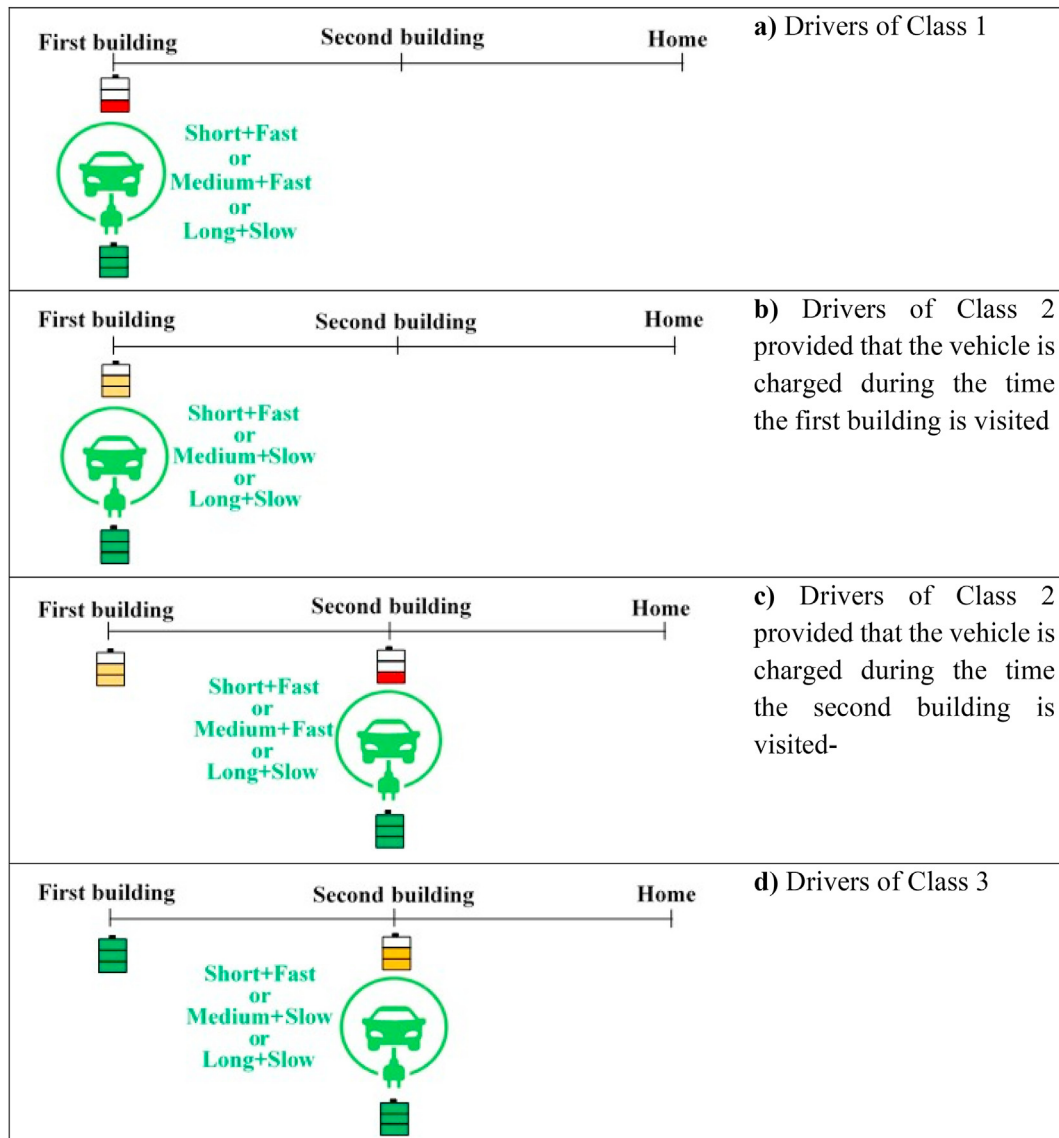


Fig. 1. Charging mode selected by drivers of different classes depending on their dwell time.

Table 2

Charging modes selected by drivers of different classes concerning their dwell time.

#	Class	Building	Dwell time	Charging mode
1	1	First	Short, Medium	Fast
2	1	First	Long	Slow
3	2	First	Short	Fast
4	2	First	Medium, Long	Slow
5	2	Second	Short, Medium	Fast
6	2	Second	Long	Slow
7	3	Second	Short	Fast
8	3	Second	Medium, Long	Slow

A5. For the EVs of each group, single recharging is sufficient to complete the travel from the home to the first and the second buildings and then return to the home.

A6. The charging behavior of each group is dependent on the state of charge when arriving at the first building. In this regard, drivers of each group can be divided into three classes as described in Table 1 where β_i indicates what fraction of users of each group belongs to class i .

A7. For the dwell time in each building, three cases θ_1 , θ_2 , and θ_3 are possible which are corresponding to short, medium, and long durations, respectively.

A8. Drivers of different classes charge their vehicles during the time they visit either the first or the second building as demonstrated in Table 1. Drivers prefer the battery to be fully or nearly fully charged after recharging. If we assume that the charge consumed when traveling from the first building to the second one is about one-third of the capacity of a full battery, Fig. 1 demonstrates how drivers select the charging mode concerning their dwell time and the state of battery charge when visiting their first building. For example, this figure states that for drivers of class 1, if the dwell time in the first building is short or medium, fast charging mode is chosen; otherwise, slow charging mode is selected. Table 2 summarizes the charging modes, shown in Fig. 1.

A9. The fixed cost of equipping a parking lot with fast and slow chargers and the cost of purchasing a single charger (fast or slow) are known with certainty.

A10. A limited number of chargers can be installed. This assumption can be justified due to the physical capacity restriction or the limited budget available for the installation fee.

A11. There is an upper bound on the number of parking lots that can be equipped with chargers.

A12. Depending on the physical capacity and budget limitation, the number of chargers that can be installed in each parking lot is limited.

A13. The urban area is divided into some zones, and the number of equipped parking lots belonging to each zone cannot exceed a given threshold. This assumption ensures that all equipped parking lots are not concentrated in a specific zone, and they are spread throughout the area.

A14. The number of drivers of each group is uncertain, and in this regard, a finite set of scenarios is possible.

A14. The number of drivers of each group is uncertain, and in this regard, a finite set of scenarios is possible.

Concerning the above assumptions, we deal with a two-stage stochastic programming model. In the first stage, the parking lots that should be equipped with chargers and the number of fast and slow chargers installed in each one are determined. Then, the number of drivers of each group is realized, and accordingly, in the second stage, EVs are assigned to parking lots concerning the drivers' willingness to walk and the parking lots capacity. We deal with a trade-off between two objectives: minimizing the cost of equipping parking lots with chargers and

maximizing the expected coverage of demands. We aggregate the two objectives into one single objective by assigning weights to demands covered by fast and slow chargers. We interpret these weights as the income achieved by charging an EV with slow and fast chargers per unit of time.

Concerning the above description, we define the required notations. Let OT and CT be the opening and closing times of parking lots, and assume that the time interval $[OT, CT]$ is divided into time slots with equal lengths. Let \mathbb{T} (indexed by t) be the set of time-slots, \mathbb{B} (indexed by b) be the set of buildings, \mathbb{P} (indexed by p) be the set of parking lots, \mathbb{G} (indexed by g) denote the set of groups, and \mathbb{K} (indexed by k) be the set of zones. Assume that $\bar{b}_{1,g}, \bar{b}_{2,g} \in \mathbb{B}$ are the first and the second buildings visited by EV users of group g , and \bar{t}_g is the time-slot at which the EV users of group g arrive at the first building $\bar{b}_{1,g}$. The dwell time (expressed in the number of time slots) of group g at buildings $\bar{b}_{1,g}$ and $\bar{b}_{2,g}$ are denoted by $d_{1,g}$ and $d_{2,g}$, respectively. Additionally, the travel time from $\bar{b}_{1,g}$ to $\bar{b}_{2,g}$ is denoted by d'_g . The parameters $\alpha_{p,1,g}$ and $\alpha_{p,2,g}$ are defined with respect to the walking willingness of the drivers of group g ; indeed, $\alpha_{p,1,g}$ is a binary parameter that is 1 if drivers of group g prefer to park their vehicles in the parking lot p during the time they visit the building $\bar{b}_{1,g}$, otherwise 0. Similarly, $\alpha_{p,2,g}$ is a binary parameter that is 1 if the drivers of group g prefer to park their vehicles in the parking lot p during the time they visit the building $\bar{b}_{2,g}$, otherwise 0 (see Remark 2 for more details).

The fixed costs of equipping a parking lot with fast and slow chargers are represented by F' and F'' , respectively, and the cost of purchasing one charger of the types fast and slow is denoted by C' and C'' , correspondingly. At most, M parking lots can be equipped with chargers, and totally M' chargers are installable. Additionally, the number of chargers that can be installed in the parking lot p , is limited to M_p'' . The value of M_p'' depends on two factors: the physical capacity of the parking lot p (which is denoted by $Q_{1,p}$) and the maximum number of chargers that can be installed with respect to budget limitation and economic aspects (which is denoted by $Q_{2,p}$), and we have $M_p'' = \min(Q_{1,p}, Q_{2,p})$. The number of equipped parking lots belonging to zone k cannot exceed the fraction λ_k of M . The number of drivers of each group is uncertain, and in this regard, \mathbb{S} (indexed by s) represents the set of possible scenarios. The number of drivers of group g under scenario s and the occurrence probability of s are denoted by dem_g^s and π^s , respectively.

Optimization model

Concerning the description provided in Section 2.1, the sets, indices, and parameters of the problem are summarised as follows:

\mathbb{T}	:	Set of time slots (indexed by t)
\mathbb{B}	:	Set of buildings (indexed by b)
\mathbb{G}	:	Set of groups of EV users (indexed by g)
\mathbb{P}	:	Set of parking lots (indexed by p)
\mathbb{K}	:	Set of zones in the urban area (indexed by k)
\mathbb{S}	:	Set of scenarios (indexed by s)
\mathbb{P}_k	:	A subset of \mathbb{P} containing parking lots placed in the zone k
π^s	:	The occurrence probability of scenario s
\bar{t}_g	:	The time-slot at which the EV users of group g arrive at the first building $\bar{b}_{1,g}$
$\bar{b}_{i,g}$:	The i^{th} building visited by EV users of group g ($i \in \{1, 2\}$)
$d_{i,g}$:	The dwell time (expressed in the number of time slots) of group g at building $\bar{b}_{i,g}$
d'_g	:	The time spent by group g to travel from $\bar{b}_{1,g}$ to $\bar{b}_{2,g}$ (expressed in the number of time slots)
dem_g^s	:	The number of drivers of group g under scenario s
$\alpha_{p,i,g}$:	A binary parameter that is 1 if the drivers of group g prefer to park their vehicles in the parking lot p during the time they are in the building $\bar{b}_{i,g}$ (where $i \in \{1, 2\}$), and 0 otherwise.
$\beta_1, \beta_2, \beta_3$:	Fraction of drivers belonging to classes 1, 2, 3, respectively. See Table 1.
$\theta_1, \theta_2, \theta_3$:	Indicating the duration of short, medium, and long dwell times, respectively (expressed in the number of time slots)

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λ_k	:	The maximum fraction of the total number of equipped parking lots that can belong to zone k
E'	:	The income achieved by charging an EV by a fast charger (expressed in monetary unit per time slot)
E''	:	The income achieved by charging an EV by a slow charger (expressed in monetary unit per time slot)
F'	:	Fixed cost of equipping a parking lot with fast chargers
F''	:	Fixed cost of equipping a parking lot with slow chargers
C'	:	The cost of purchasing a fast charger
C''	:	The cost of purchasing a slow charger
M	:	Maximum number of parking lots that can be equipped with chargers
M'	:	Total number of available chargers
$Q_{1,p}$:	The physical capacity of parking lot p
$Q_{2,p}$:	Maximum number of chargers that can be installed in the parking lot p concerning budget limitation and economic aspects
M_p''	:	The maximum number of chargers that can be installed in the parking lot p . It is defined as $M_p'' = \min(Q_{1,p}, Q_{2,p})$.

We need to define the following subsets of \mathbb{G} for every $i \in \{1, 2\}$, $j \in \{1, 2, 3\}$, and $p \in \mathbb{P}$ to formulate the objective function of the problem:

$$\mathbb{G}_{i,j,p}' = \{g \in \mathbb{G} : \alpha_{p,i,g} = 1 \text{ and } d_{i,g} = \theta_j\}$$

The above set contains those groups whose dwell time in their i^{th} building equals θ_j , and the parking lot p is amongst their preferred parking lots when they want to visit that building. Additionally, the following subsets of \mathbb{G} are defined for every $t \in \mathbb{T}$, and $p \in \mathbb{P}$:

Moreover, we define $\mathbb{G}_{t,p,r}''$ as the set of groups that may use the charging mode mentioned in row $\#r$ of Table 2 and hence, may occupy the parking lot p in time slot t . In other words,

$$\mathbb{G}_{t,p,1}'' = \{g \in \mathbb{G} : (d_{1,g} = \theta_1 \text{ or } \theta_2) \text{ and } \alpha_{p,1,g} = 1 \text{ and } t - d_{1,g} + 1 \leq \bar{t}_g \leq t\}$$

$$\mathbb{G}_{t,p,2}'' = \{g \in \mathbb{G} : d_{1,g} = \theta_3 \text{ and } \alpha_{p,1,g} = 1 \text{ and } t - d_{1,g} + 1 \leq \bar{t}_g \leq t\}$$

$$\mathbb{G}_{t,p,3}'' = \{g \in \mathbb{G} : d_{1,g} = \theta_1 \text{ and } \alpha_{p,1,g} = 1 \text{ and } t - d_{1,g} + 1 \leq \bar{t}_g \leq t\}$$

$$\mathbb{G}_{t,p,4}'' = \{g \in \mathbb{G} : (d_{1,g} = \theta_2 \text{ or } \theta_3) \text{ and } \alpha_{p,1,g} = 1 \text{ and } t - d_{1,g} + 1 \leq \bar{t}_g \leq t\}$$

$$\mathbb{G}_{t,p,5}'' = \{g \in \mathbb{G} : (d_{2,g} = \theta_1 \text{ or } \theta_2) \text{ and } \alpha_{p,2,g} = 1 \text{ and } t - d_{1,g} - d_g' - d_{2,g} + 1 \leq \bar{t}_g \leq t - d_{1,g} - d_g'\}$$

$$\mathbb{G}_{t,p,6}'' = \{g \in \mathbb{G} : d_{2,g} = \theta_3 \text{ and } \alpha_{p,2,g} = 1 \text{ and } t - d_{1,g} - d_g' - d_{2,g} + 1 \leq \bar{t}_g \leq t - d_{1,g} - d_g'\}$$

$$\mathbb{G}_{t,p,7}'' = \{g \in \mathbb{G} : d_{2,g} = \theta_1 \text{ and } \alpha_{p,2,g} = 1 \text{ and } t - d_{1,g} - d_g' - d_{2,g} + 1 \leq \bar{t}_g \leq t - d_{1,g} - d_g'\}$$

$$\mathbb{G}_{t,p,8}'' = \{g \in \mathbb{G} : (d_{2,g} = \theta_2 \text{ or } \theta_3) \text{ and } \alpha_{p,2,g} = 1 \text{ and } t - d_{1,g} - d_g' - d_{2,g} + 1 \leq \bar{t}_g \leq t - d_{1,g} - d_g'\}$$

We need the sets $\mathbb{G}_{t,p,1}'$, $\mathbb{G}_{t,p,3}'$, $\mathbb{G}_{t,p,5}'$, and $\mathbb{G}_{t,p,7}'$ to calculate the number of EVs using fast chargers of parking lot p at the time t , and to satisfy the capacity constraint of parking lots. Similarly, we require the sets $\mathbb{G}_{t,p,2}'$, $\mathbb{G}_{t,p,4}'$, $\mathbb{G}_{t,p,6}'$, and $\mathbb{G}_{t,p,8}'$ to calculate the number of EVs using slow chargers of parking lot p at the time t , and to satisfy the capacity constraint.

Decision variables of the problem are defined as follows:

δ_p	:	A binary variable that is 1 if the parking lot p is equipped with chargers, 0 otherwise.
δ_p'	:	A binary variable that is 1 if the parking lot p is equipped with at least one fast charger, 0 otherwise.
δ_p''	:	A binary variable that is 1 if the parking lot p is equipped with at least one slow charger, 0 otherwise.
γ_p'	:	Number of fast chargers installed in parking lot p
γ_p''	:	Number of slow chargers installed in parking lot p
$y_{p,g}^s$:	Number of drivers of group g which belong to class 1 and charge their EV in parking lot p during the time they are in the first building $\bar{b}_{1,g}$ under scenario s
$x_{p,g}^s$:	Number of drivers of group g which belong to class 2 and charge their EV in parking lot p during the time they are in the first building $\bar{b}_{1,g}$ under scenario s

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$x_{p,g}^{s'}$:	Number of drivers of group g which belong to class 2 and charge their EV in parking lot p during the time they are in the second building $\bar{b}_{2,g}$ under scenario s
$w_{p,g}^s$:	Number of drivers of group g which belong to class 3 and charge their EV in parking lot p during the time they are in the second building $\bar{b}_{2,g}$ under scenario s

Note that for the sake of simplicity, the variables $y_{p,g}^s$, $x_{p,g}^{s'}$, and $w_{p,g}^s$ are considered as nonnegative continuous variables.

The stochastic charging station location problem (SCSLP) is formulated as the following two-stage stochastic programming model: (SCSLP)

$$\begin{aligned} \max \quad & - \sum_{p \in \mathbb{P}} (F' \delta_p' + F'' \delta_p'' + C' \gamma_p' + C'' \gamma_p'') \\ & + \sum_{s \in \mathbb{S}} \pi^s \sum_{p \in \mathbb{P}} \left(E' \left[\theta_1 \left(\sum_{g \in \mathbb{G}_{1,1,p}'} (y_{p,g}^s + x_{p,g}^s) + \sum_{g \in \mathbb{G}_{2,1,p}'} (w_{p,g}^s + x_{p,g}^{s'}) \right) \right. \right. \\ & + \theta_2 \left(\sum_{g \in \mathbb{G}_{1,2,p}'} (y_{p,g}^s) + \sum_{g \in \mathbb{G}_{2,2,p}'} (x_{p,g}^{s'}) \right) \left. \right] + E'' \left[\theta_2 \left(\sum_{g \in \mathbb{G}_{1,2,p}'} (x_{p,g}^s + \sum_{g \in \mathbb{G}_{2,2,p}'} (w_{p,g}^s) \right) \right. \right. \\ & \left. \left. + \theta_3 \left(\sum_{g \in \mathbb{G}_{1,3,p}'} (y_{p,g}^s + x_{p,g}^s) + \sum_{g \in \mathbb{G}_{2,3,p}'} (w_{p,g}^s + x_{p,g}^{s'}) \right) \right] \right) \end{aligned} \quad (1)$$

s.t.

$$\sum_{p \in \mathbb{P}} \delta_p \leq M \quad (2)$$

$$\sum_{p \in \mathbb{P}_k} \delta_p \leq \lambda_k M \quad \forall k \in \mathbb{K} \quad (3)$$

$$\delta_p' + \delta_p'' \leq 2\delta_p \quad \forall p \in \mathbb{P} \quad (4)$$

$$\gamma_p' \leq M_p'' \delta_p' \quad \forall p \in \mathbb{P} \quad (5)$$

$$\gamma_p'' \leq M_p'' \delta_p'' \quad \forall p \in \mathbb{P} \quad (6)$$

$$\gamma_p' + \gamma_p'' \leq M_p'' \quad \forall p \in \mathbb{P} \quad (7)$$

$$\sum_{p \in \mathbb{P}} (\gamma_p' + \gamma_p'') \leq M' \quad (8)$$

$$\sum_{p \in \mathbb{P} : \alpha_{p,1,g}=1} y_{p,g}^s \leq \beta_1 \text{dem}_g^s \quad \forall g \in \mathbb{G}, \forall s \in \mathbb{S} \quad (9)$$

$$\sum_{p \in \mathbb{P} : \alpha_{p,1,g}=1} x_{p,g}^s + \sum_{p \in \mathbb{P} : \alpha_{p,2,g}=1} x_{p,g}^{s'} \leq \beta_2 \text{dem}_g^s \quad \forall g \in \mathbb{G}, \forall s \in \mathbb{S} \quad (10)$$

$$\sum_{p \in \mathbb{P} : \alpha_{p,2,g}=1} w_{p,g}^s \leq \beta_3 \text{dem}_g^s \quad \forall g \in \mathbb{G}, \forall s \in \mathbb{S} \quad (11)$$

$$\sum_{g \in \mathbb{G}_{t,p,1}''} y_{p,g}^s + \sum_{g \in \mathbb{G}_{t,p,3}''} x_{p,g}^s + \sum_{g \in \mathbb{G}_{t,p,5}''} x_{p,g}^{s'} + \sum_{g \in \mathbb{G}_{t,p,7}''} w_{p,g}^s \leq \gamma_p' \quad (12)$$

$$\forall p \in \mathbb{P}, \forall t \in \mathbb{T}, \forall s \in \mathbb{S}$$

$$\sum_{g \in \mathbb{G}_{t,p,2}''} y_{p,g}^s + \sum_{g \in \mathbb{G}_{t,p,4}''} x_{p,g}^s + \sum_{g \in \mathbb{G}_{t,p,6}''} x_{p,g}^{s'} + \sum_{g \in \mathbb{G}_{t,p,8}''} w_{p,g}^s \leq \gamma_p''$$

$$\forall p \in \mathbb{P}, \forall t \in \mathbb{T}, \forall s \in \mathbb{S} \quad (13)$$

$$\delta_p, \delta'_p, \delta''_p \in \{0, 1\}, \gamma'_p, \gamma''_p \geq 0, \text{Integer} \quad \forall p \in \mathbb{P} \quad (14)$$

$$y_{p,g}^s, x_{p,g}^s \geq 0 \quad \forall g \in \mathbb{G}, \forall p \in \mathbb{P} : \alpha_{p,1,g} = 1, \forall s \in \mathbb{S} \quad (15)$$

$$w_{p,g}^s, x_{p,g}^{ts} \geq 0 \quad \forall g \in \mathbb{G}, \forall p \in \mathbb{P} : \alpha_{p,2,g} = 1, \forall s \in \mathbb{S} \quad (16)$$

The objective function (1) makes a trade-off between two objectives: minimizing the cost of equipping parking lots with chargers and maximizing the expected coverage of demands. Constraint set (2) ensures that at most M parking lots are equipped with chargers. Constraint set (3) implies that the number of equipped parking lots belonging to zone k cannot exceed $\lambda_k M$. Constraint set (4) ensures that if $\delta_p = 0$, then $\delta'_p = 0$, and $\delta''_p = 0$. Constraint set (5) ensures that if $\delta'_p = 0$, then $\gamma'_p = 0$. Similarly, constraint set (6) indicates that if $\delta''_p = 0$, then $\gamma''_p = 0$. Constraint set (7) restricts the total number of chargers installed in parking lot p . Constraint set (8) restricts the total number of chargers installed in the network. Constraint sets (9)-(11) specify the classification assumed on the drivers of each group (see Table 1). Constraint set (12) states that the number of EVs using fast chargers of parking lot p at the time t cannot exceed the number of installed fast chargers. Similarly, constraint set (13) guarantees that the number of EVs using slow chargers of parking lot p at the time t cannot exceed the number of installed slow chargers. Constraint sets (14)-(16) define the type of variables.

Remark 1. Although the proposed model is based on the assumption A8, it is fully flexible to adopt this assumption depending on the problem conditions. It is just required to adopt Table 2 with respect to the new circumstances and accordingly, redefine the sets $\mathbb{G}_{i,j,p}$ and $\mathbb{G}_{t,p,r}$, appropriately.

Remark 2. Let $r_{p,b}$ be the distance between the parking lot p and building b . For each group g , and every $b \in \{b_{1,g}, b_{2,g}\}$, the distance between b and its nearest parking lot is calculated as $m_{g,b} = \min_{p \in \mathbb{P}} r_{p,b}$. Considering ψ_g as the admissible percentage of deviation from $m_{g,b}$, the threshold $\bar{r}_{g,b}$ is defined $\bar{r}_{g,b} = m_{g,b} \times (1 + \psi_g)$. Then, the parameter $\alpha_{p,i,g}$ is determined as follows:

$$\alpha_{p,i,g} = \begin{cases} 1 & r_{p,b_{i,g}} \leq \bar{r}_{g,b_{i,g}} \\ 0 & \text{Otherwise} \end{cases} \quad \forall p \in \mathbb{P}, \forall i \in \{1, 2\}, \forall g \in \mathbb{G}$$

It is worth mentioning that due to the excessive data requirement, practically, SCSLP is often applicable on small and moderate urban scale.

Solution method

Benders decomposition (BD) algorithm, first introduced by Benders (1962), is a well-known method, effectively used to solve different practical optimization problems including vehicle routing (Bruck and Lori, 2017), critical node detection (Hooshmand et al., 2019), traffic control (Mohebifard and Hajbabaie, 2019), contamination detection in water networks (Hooshmand et al., 2020), and particularly, charging station location problem (Arslan and Karaşan, 2016; Zheng and Peeta, 2017; Kadri et al., 2020). For a comprehensive overview, see Rahmaniani et al. (2017).

Direct resolution of SCSLP via commercial solvers would be time-consuming in real-world instances. The decision variables of SCSLP can be classified into two groups. The first group includes binary and integer variables $\delta_p, \delta'_p, \delta''_p, \gamma'_p, \gamma''_p$ which we refer to as complicating variables, and the second group contains nonnegative continuous variables $y_{p,g}^s, x_{p,g}^s, x_{p,g}^{ts}$, and $w_{p,g}^s$. By fixing the complicating variables to a given feasible value, the model SCSLP reduces to an easy to solve linear programming (LP) model. This feature motivates us to utilize the BD algorithm to solve

SCSLP. For this purpose, first, we describe how to implement BD on SCSLP. Then, due to the slow convergence of BD, we utilize it as a heuristic by eliminating some non-promising solutions via some cuts. These cuts are not valid for the true model, but it is computationally shown that their addition not only does not result in big deviance from the optimal solutions but also has a great impact on reducing the running time.

Benders decomposition algorithm

BD splits SCSLP into a master problem (MP), involving complicating variables, and a sub-problem (SP). MP is initially constructed from constraints (2)-(8), (14), and (18), where the constraints (2)-(8), and (14) are taken from SCSLP, while (18) is new and prevents the unboundedness of MP in the early iterations of the algorithm by expressing the valid upper bound $\bar{\phi}$ for ϕ . Additionally, constraint set (19) indicates the optimality cut the detail of which is described later. This constraint is not included in the MP at the first iteration but it is added gradually throughout the algorithm.

Master problem (MP)

$$\max - \sum_{p \in \mathbb{P}} (F' \delta'_p + F'' \delta''_p + C' \gamma'_p + C'' \gamma''_p) + \phi \quad (17)$$

$$\text{s.t. (2)-(8), (14)}$$

$$0 \leq \phi \leq \bar{\phi} \quad (18)$$

$$\begin{aligned} \phi \leq & z_{sp}^{(n)} + \sum_p \eta_{1,p}^{(n)} (\delta_p - \delta_p^{(n)}) + \eta_{2,p}^{(n)} (\delta'_p - \delta_p^{(n)}) + \eta_{3,p}^{(n)} (\delta''_p - \delta_p^{(n)}) \\ & + \eta_{4,p}^{(n)} (\gamma'_p - \gamma_p^{(n)}) + \eta_{5,p}^{(n)} (\gamma''_p - \gamma_p^{(n)}) \quad \forall n \\ & \in \mathbb{N} \end{aligned} \quad (19)$$

Considering $(\bar{\delta}, \bar{\delta}', \bar{\delta}'', \bar{\gamma}', \bar{\gamma}'')$ as the optimal solution to MP, the sub-problem $\text{SP}(\bar{\delta}, \bar{\delta}', \bar{\delta}'', \bar{\gamma}', \bar{\gamma}'')$ is formulated as follows:

Sub-problem ($\text{SP}(\bar{\delta}, \bar{\delta}', \bar{\delta}'', \bar{\gamma}', \bar{\gamma}'')$)

$$\begin{aligned} \max & \sum_{s \in \mathbb{S}} \pi^s \sum_{p \in \mathbb{P}} \left(E' \left[\theta_1 \left(\sum_{g \in \mathbb{G}_{1,1,p}} (y_{p,g}^s + x_{p,g}^s) + \sum_{g \in \mathbb{G}_{2,1,p}} (w_{p,g}^s + x_{p,g}^{ts}) \right) \right. \right. \\ & + \theta_2 \left(\sum_{g \in \mathbb{G}_{1,2,p}} (y_{p,g}^s) + \sum_{g \in \mathbb{G}_{2,2,p}} (x_{p,g}^{ts}) \right) \left. \right] + E'' \left[\theta_2 \left(\sum_{g \in \mathbb{G}_{1,2,p}} x_{p,g}^s + \sum_{g \in \mathbb{G}_{2,2,p}} (w_{p,g}^s) \right) \right. \right. \\ & \left. \left. + \theta_3 \left(\sum_{g \in \mathbb{G}_{1,3,p}} (y_{p,g}^s + x_{p,g}^s) + \sum_{g \in \mathbb{G}_{2,3,p}} (w_{p,g}^s + x_{p,g}^{ts}) \right) \right] \right) \end{aligned}$$

$$\text{s.t. (9)-(13), (15), (16)}$$

$$\delta_p, \delta'_p, \delta''_p, \gamma'_p, \gamma''_p \geq 0 \quad \forall p \in \mathbb{P}$$

$$\delta_p = \bar{\delta}_p \quad \forall p \in \mathbb{P} \rightarrow \eta_{1,p} \quad (20)$$

$$\delta'_p = \bar{\delta}'_p \quad \forall p \in \mathbb{P} \rightarrow \eta_{2,p} \quad (21)$$

$$\delta''_p = \bar{\delta}''_p \quad \forall p \in \mathbb{P} \rightarrow \eta_{3,p} \quad (22)$$

$$\gamma'_p = \bar{\gamma}'_p \quad \forall p \in \mathbb{P} \rightarrow \eta_{4,p} \quad (23)$$

$$\gamma''_p = \bar{\gamma}''_p \quad \forall p \in \mathbb{P} \rightarrow \eta_{5,p} \quad (24)$$

Let BUB and BLB represent the best upper and lower bounds found on the optimal objective value of the original model. At the beginning of the algorithm, these bounds are initialized at $BUB := +\infty$ and $BLB := 0$ and

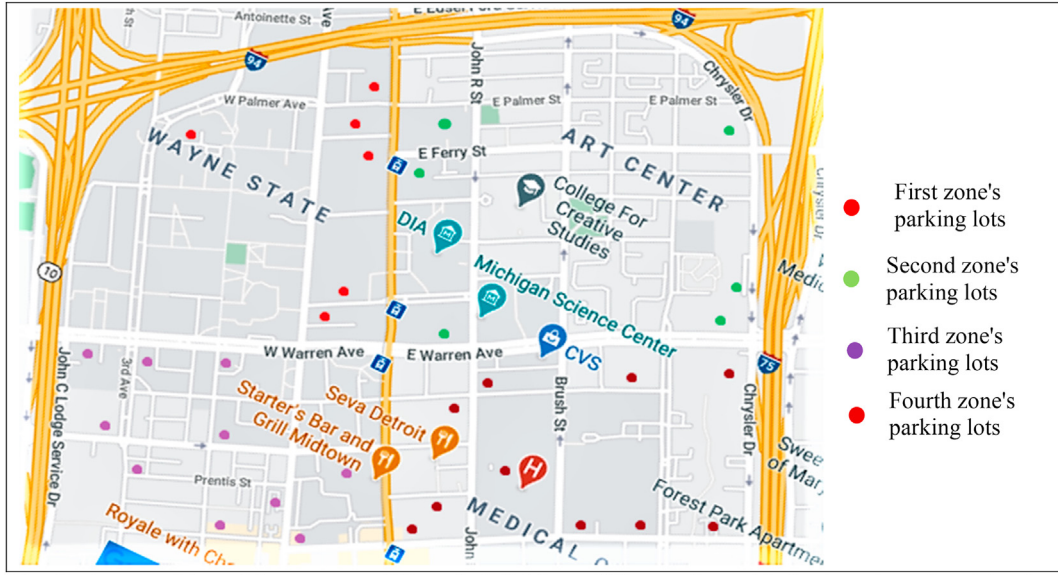


Fig. 2. A part of the midtown area of Detroit, Michigan, US, together with the location of parking lots.

updated throughout the algorithm. In every iteration, namely n , of the BD algorithm, MP is solved and gets an upper bound on the optimal objective value of the original problem. Then, considering $(\delta^{(n)}, \delta'^{(n)}, \delta''^{(n)}, \gamma'^{(n)}, \gamma''^{(n)})$ as the optimal solution to MP, the problem SP is solved with $(\tilde{\delta}, \tilde{\delta}', \tilde{\delta}'', \tilde{\gamma}', \tilde{\gamma}'') = (\delta^{(n)}, \delta'^{(n)}, \delta''^{(n)}, \gamma'^{(n)}, \gamma''^{(n)})$, and accordingly, a lower bound is obtained on the optimal objective value of the original problem. The problem SP is always feasible, and hence, no feasibility cut is needed. Given $\eta_{1,p}^{(n)}$ through $\eta_{5,p}^{(n)}$ as the value of dual variables associated with constraints (20)–(24), and considering $z_{SP}^{(n)}$ as the optimal objective function value of SP at iteration n , the optimality cut is formulated as (19), and added to MP. For more details of how to obtain optimality cut (19), see Birge and Louveaux (2011). The bounds BLB and BUP are updated and the same process is repeated until the bounds get close enough. Algorithm 1

summarizes the main steps of BD.

Algorithm 1. BD algorithm

Implementing BD as a heuristic

Due to the slow convergence of BD, in this section, we develop a heuristic version of BD by adding some inequalities to the MP. These cuts are not valid for the true model, but it is computationally shown that their addition not only does not result in big deviance from the optimal solutions but also has a great impact on reducing the number of iterations of the algorithm and causes the algorithm to be converged to the optimal or a near-optimal solution in a short amount of time.

The first inequality is as follows:

Algorithm 1: BD algorithm

- Consider BLB and BUB , respectively, as the best lower and upper bounds obtained on the optimal objective value of SCSLP, and assume that $(\delta^B, \delta'^B, \delta''^B, \gamma'^B, \gamma''^B)$ is the best solution generated so far, ε is a given accuracy, \hat{n} is the iteration counter, and \mathbb{N} is the index set of optimality cuts generated so far. Initialize $(\delta^B, \delta'^B, \delta''^B, \gamma'^B, \gamma''^B) = 0$, $BLB := 0$, $BUB := +\infty$, $\hat{n} := 1$ and $\mathbb{N} := \emptyset$.
 - While $|BUB - BLB| > \varepsilon$ do
 - Solve MP, represent its optimal solution and the objective value by $(\delta^{(\hat{n})}, \delta'^{(\hat{n})}, \delta''^{(\hat{n})}, \gamma'^{(\hat{n})}, \gamma''^{(\hat{n})}, \tilde{\varphi})$, and \tilde{z}_{MP} , respectively, and set $BUB := \tilde{z}_{MP}$.
 - Solve SP for $(\tilde{\delta}, \tilde{\delta}', \tilde{\delta}'', \tilde{\gamma}', \tilde{\gamma}'') = (\delta^{(\hat{n})}, \delta'^{(\hat{n})}, \delta''^{(\hat{n})}, \gamma'^{(\hat{n})}, \gamma''^{(\hat{n})})$, and represent its optimal objective value by $z_{SP}^{(\hat{n})}$. Set $\mathbb{N} := \mathbb{N} \cup \{\hat{n}\}$, denote the value of dual variables associated with constraints **Error! Reference source not found.-Error! Reference source not found.** by $\eta_{1,p}^{(\hat{n})}$ through $\eta_{5,p}^{(\hat{n})}$, respectively, and add the corresponding optimality cut to MP.
 - If $z_{SP}^{(\hat{n})} + \tilde{z}_{MP} - \tilde{\varphi} > BLB$, update BLB and $(\delta^B, \delta'^B, \delta''^B, \gamma'^B, \gamma''^B)$ to $z_{SP}^{(\hat{n})} + \tilde{z}_{MP} - \tilde{\varphi}$ and $(\delta^{(\hat{n})}, \delta'^{(\hat{n})}, \delta''^{(\hat{n})}, \gamma'^{(\hat{n})}, \gamma''^{(\hat{n})})$, respectively.
 - Set $\hat{n} := \hat{n} + 1$
 - End.
 - Return the $(\delta^B, \delta'^B, \delta''^B, \gamma'^B, \gamma''^B)$ as an optimal solution to SCSLP.
-

Table 3

Characteristics of each type of chargers.

Charger type	E', E'' (\$ per hour)	C', C'' (\$)	F', F'' (\$)
Fast	10	25000	27500
Slow	1	3450	6650

$$\sum_{p \in \mathbb{P}} \delta_p \leq |\mathbb{P}| - \tilde{M} \quad (25)$$

Where, \tilde{M} is a parameter indicating the number of variables δ_p whose value in the LP relaxation of SCSLP is less than the given precision ϵ'_p . In other words, considering $\tilde{\delta}_p$ as the value of the variable δ_p in the LP relaxation of SCSLP, \tilde{M} is defined as follows:

$$\tilde{M} = \left| \left\{ p \in \mathbb{P} : \tilde{\delta}_p < \epsilon'_p \right\} \right|$$

Where, $|\cdot|$ indicates the cardinality of a set. Indeed, inequality (25) indicates that if some parking lots serve a small number of buildings, their conversion to a charging station will not be profitable. The value of ϵ'_p can be determined arbitrarily. However, our preliminary experiments indicate that an appropriate value can be determined with respect to the value of the parameter $\alpha_{p,i,g}$, as follows:

$$\epsilon'_p = \frac{\sum_{g \in G: (a_{p,1,g}=1 \text{ or } a_{p,2,g}=1)} \sum_{s \in S} p^s \text{dem}_g^s}{\sum_{g \in G} \sum_{s \in S} p^s \text{dem}_g^s}$$

If $|\mathbb{P}| - \tilde{M} < M$, the cut (25) may be useful in decreasing the running time of the Benders algorithm.

By a preprocessing manner, we can find a good approximation of the minimum number of required fast chargers, and the maximum number of required slow chargers. This approximation can be obtained with respect to the daily visits of buildings, the type of chargers to which vehicles are plugged in, and the type of destination buildings. Therefore, let μ indicate that at most what percentage of chargers is better to be fast, and suppose that η denotes that at least what percentage of chargers is better to be slow. Thus, the following inequalities are added to MP to accelerate the BD algorithm:

$$\sum_{p \in \mathbb{P}} \gamma'_p \geq \mu \times M' \quad (26)$$

$$\sum_{p \in \mathbb{P}} \gamma''_p \leq \eta \times M' \quad (27)$$

Concerning the above statements, to find a near-optimal solution to SCSLP in a short amount of time, we add the inequalities (25)-(27) to the MP and hence, run the BD algorithm as a heuristic framework which we refer to as BD-based heuristic (BDH) method.

Computational results

In this section, we investigate the performance of the proposed model and algorithm on a dataset taken from Faridimehr et al. (2019). BDH algorithm is coded in the AIMMS mathematical modeling-language (Bisschop, 2012), and all optimization models are solved using the solver CPLEX 12.4, included in the AIMMS software.

Our dataset, taken from Faridimehr et al. (2019), is the community data for the midtown area of Detroit, Michigan, US. A part of this area together with the location of parking lots is shown in Fig. 2 (Anon., 2018), and it is estimated that the average annual daily traffic change from 10,000 to 20,000 (SEMCOG, 2018).

Based on the National Household Travel Survey (NHTS) data, four types of buildings are considered: work, meal, shopping, and school. As pointed out by Faridimehr et al. (2019), the demand pattern for public EV charging services may differ from one day to another. For example, for weekdays, most of the demand is due to people traveling to work or school, and for weekends, most of the demand is due to people traveling to restaurants or shopping centers. Moreover, as pointed out by Krumm (2012), the arrival and dwell time in each destination is dependent on the building type. For example, for schools, the highest rate of arrivals occurs during the morning. Following Zhong et al. (2008), the dwell times are estimated by the Weibull distribution.

We assume that the parking lots are open from 6:00 a.m. to 6:00 p.m. (i.e., $OT = 6, CT = 18$) and the interval $[OT, CT]$ is divided into 12 time-slots of 1 h. We consider 118 buildings (including 67 office buildings, 44 schools, 4 restaurants, and 3 shopping places) and 32 parking lots. It is assumed that in each parking lot, and the total network, at most 400 and 4000 chargers can be installed, respectively (i.e., $Q_{2,p} = 400$, and $M' = 4000$), and at most, 10 parking lots can be equipped with chargers (i.e., $M = 10$). The ratios $\beta_1, \beta_2, \beta_3$, introduced in Table 1, are set at 1/3. The value of admissible percentage of deviation (ψ_g) for each group g , is set on 0 or 0.5 (for 50% of groups, it is set at 0, and for others, at 0.5). Four zones are considered (i.e., $\mathbb{K} = \{1, 2, 3, 4\}$). The parking lots of each zone

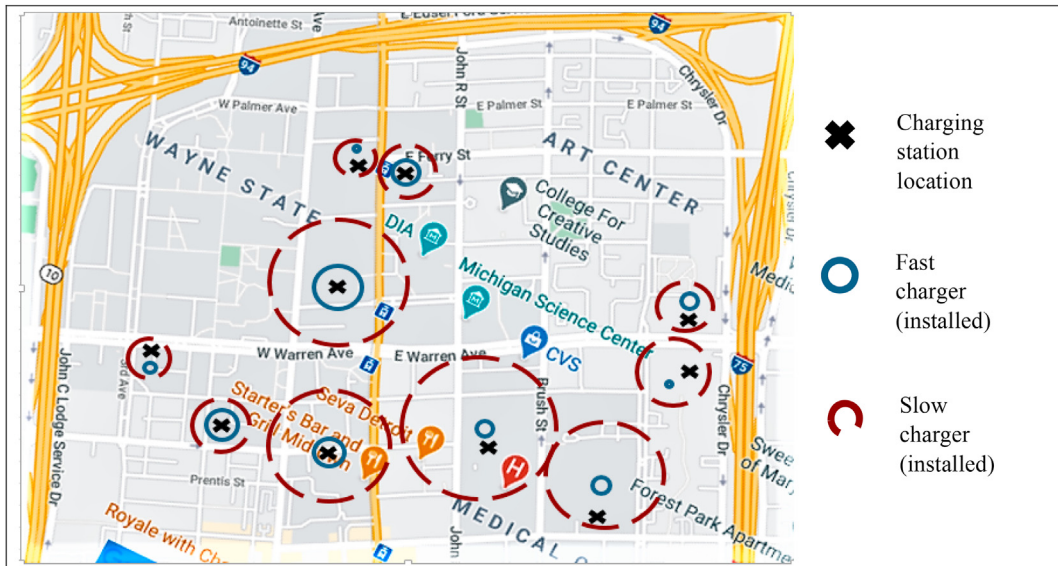


Fig. 3. Location of charging stations in the solution obtained by BDH to the problem and the number of slow and fast chargers installed.

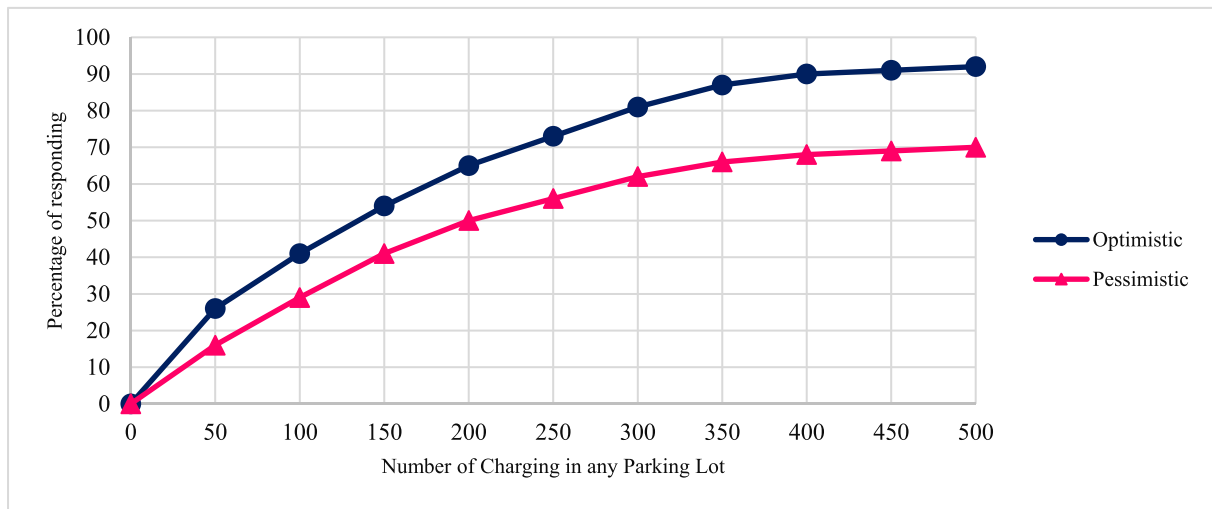


Fig. 4. Percentage of responding to the demand by increasing the value of $Q_{2,p}$ in two cases of driver's willingness.

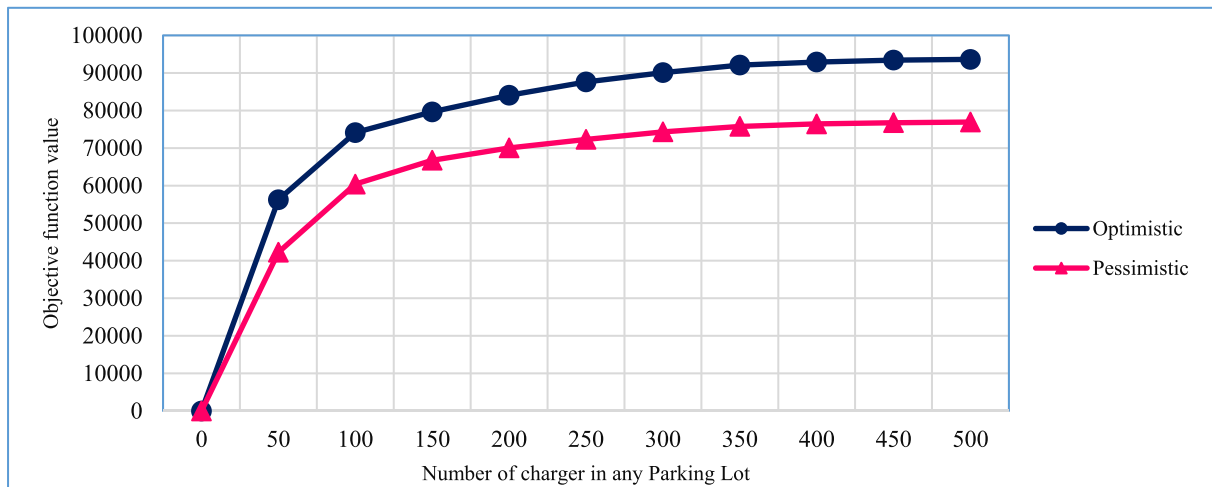


Fig. 5. Variation of the objective function value by increasing the value of $Q_{2,p}$ in two cases of driver's willingness.

Table 4
Characteristics of different instances to compare BD and BDH.

Problem	No. scenarios	Total daily demand under each scenario	No. constraints	No. binary and integer variables	No. continuous variables
P1	50	4087	249664	50	422520
P2	100	4042	497680	50	842170
P3	150	4043	746509	50	1262496
P4	200	4046	994795	50	1682188

are demonstrated in Fig. 2. The value of the parameter λ_k for each zone k is determined with respect to the ratio of the number of parking lots in zone k to the total number of parking lots in the area. Thus, we set $\lambda_1 =$

$\lambda_2 = 0.2$, and $\lambda_3 = \lambda_4 = 0.3$. The value of parameters E' , E'' , F' , F'' , C' , and C'' are presented in Table 3.

The number of scenarios is set at 50, and the occurrence probabilities of scenarios are assumed to be the same. For the number of drivers of each group, first, a nominal value \bar{dem}_g is estimated with respect to the arrival time and the type of buildings, visited by that group. Then, the parameters dem_g^s under different scenarios are generated by making random changes in the value of \bar{dem}_g so that no scenario prevails another one and the total daily demand in each scenario s remains close to $\sum_{g \in G} \bar{dem}_g$.

It is worth mentioning that the sample average approximation method (Aleyweg et al., 2001) can be used to determine how many

Table 5
Evaluation of the BDH compared to BD and the direct method on instances of Table 4.

Instance	Objective function value				Coverage percentage			Running time (s)		
	Direct Method	BD	BDH	Gap (%)	Direct Method	BD	BDH	Direct Method	BD	BDH
P1	21457.55	21457.55	21449.89	0.04	0.73	0.73	0.73	472.3	671.08	102.30
P2	21304.92	21304.92	21291.09	0.06	0.72	0.72	0.72	1955.7	2092.05	316.84
P3	21266.03	21266.03	21257.01	0.04	0.72	0.72	0.72	6633.78	5401.27	469.47
P4	21272.52	21272.52	21265.78	0.03	0.72	0.72	0.72	14056.89	12186.19	586.11

Table 6

Characteristics of a variety of instances to evaluate the efficiency of BDH.

Instance	S	B	P	M	M'	No. constraints	No. binary and integer variables	No. continuous variables
1	10	50	12	4	200	34273	60	63585
2	20	55	13	4	200	81674	65	151376
3	30	60	14	4	200	144864	70	275779
4	40	65	15	6	200	225238	75	433834
5	50	70	16	6	200	324065	80	623041
6	60	75	17	6	200	443329	85	849686
7	70	80	18	6	200	590401	90	1136709
8	80	85	19	8	200	753474	95	1443944
9	90	90	20	8	200	947687	100	1844619
10	100	95	22	8	200	1186790	110	2260749
11	110	100	24	8	200	1439479	120	2755365
12	120	105	26	10	200	1729140	130	3314619
13	130	110	28	10	200	2066135	140	3969291
14	140	115	30	10	200	2421170	150	4546740
15	150	118	32	10	200	2736183	160	5239837
RI	50	118	32	10	4000	1068265	160	2083389

Table 7

Evaluation of BDH in terms of solution quality and running time.

Instance	Objective function value			Coverage percentage		Running time (s)	
	Direct Method	BDH	Gap (%)	Direct Method	BDH	Direct Method	BDH
1	22071.40	22069.63	0.0001	0.63	0.63	374.4	120.2
2	22634.68	22544.01	0.004	0.62	0.62	1481.5	408.3
3	26860.71	26780.04	0.003	0.63	0.63	3742.6	829.0
4	28513.99	28228.53	0.01	0.64	0.63	4210.8	927.1
5	30225.99	29621.91	0.02	0.62	0.62	5563.5	1412.5
6	36081.18	34277.60	0.05	0.68	0.67	7314.9	2089.6
7	37601.03	36473.28	0.03	0.62	0.62	12734.6	3835.2
8	NA	41588.26	NA	NA	0.63	>18000	4036.3
9	NA	45945.81	NA	NA	0.63	>18000	5033.7
10	NA	49148.70	NA	NA	0.64	>18000	6646.9
11	NA	54910.23	NA	NA	0.65	>18000	7862.8
12	NA	61171.83	NA	NA	0.66	>18000	9086.0
13	NA	66492.71	NA	NA	0.65	>18000	11647.8
14	NA	70790.03	NA	NA	0.63	>18000	14382.1
15	NA	73081.23	NA	NA	0.62	>18000	>18000
RI	–	79320.03	–	–	0.66	–	14036.14

scenarios are sufficient to get a robust solution.

For the reference instance, described above, the model SCSLP contains 160 discrete variables, 2083389 continuous variables, and 1068265 constraints. In the solution obtained by BDH for this model, 426 fast chargers and 1913 slow chargers are installed and coverage of 0.66 with the objective function value 79320.3 is obtained. The parking lots equipped with chargers are shown by the sign “×” in Fig. 3. Near each sign “×”, two circles corresponding to two types of chargers (fast and slow) have been plotted, and the diameter of each circle gives an intuition about the number of chargers installed in the corresponding parking lot. The larger the diameter of the circles around each sign “×”, the greater the number of chargers installed in the corresponding parking lot.

To investigate the impact of parameter ψ_g on the demand coverage, for the reference instance, described earlier, we consider two categories, namely pessimistic and optimistic. In the pessimistic case, we have $\psi_g = 0$ meaning that all drivers have the unwillingness to walk and they choose the nearest available parking lot; however, in the optimistic case, we have $\psi_g = 0.5$ meaning that drivers have the willingness to walk and they may choose farther parking lots. Fig. 4 represents how the percentage of responding to the demand increases by increasing the number of installable chargers in each parking lot (i.e., the value of $Q_{2,p}$) from 0 to 500. Similarly, Fig. 5 shows how the objective function value increases by increasing the value of $Q_{2,p}$ from 0 to 500.

As can be seen in Figs. 4 and 5, by increasing the maximum number of chargers that can be installed in each parking lot, the percentage of responding to demand as well as the objective function value increase. The amount of increase is more sensible when the capacity changes from

0 to 200; but it is gradually reduced, and from some point onwards, due to the installation cost of chargers, increasing the capacity has not much effect on improving the system performance. Furthermore, in the pessimistic case, since each driver parks her/his EV only in the nearest parking lot, some parts of the demand cannot be covered. Therefore, in the pessimistic case, it is not economically affordable to install additional chargers since the percentages of responding to the demand do not significantly increase. In contrast, in the optimistic case, by increasing the chargers, about 90 percent of the demand can be met; however, in this case, the percentage remains at 90% after a certain value of the capacity and does not get closer to 100%. Is it because the demand is dispersed and so to reach the total coverage, we need a large number of chargers which is not worth to invest.

To assess the performance of BDH over BD, we investigate the results of both methods on some small sized instances in which the number of buildings and parking lots is 40 and 10, respectively. At most, four parking lots can be equipped with chargers, and in each parking lot and the total network, at most 200 and 800 chargers can be installed, respectively. For the number of scenarios and the total daily demand in each scenario, four cases, as reported in Table 4, are considered. We set $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0.25$, and the willingness to walk is mostly pessimistic. The characteristics of instances and the results of methods are provided in Tables 4 and 5, respectively. In Table 5, the direct method refers to the direct resolution of the model via CPLEX, and the column labeled by Gap is calculated as $\frac{z_{DM} - z_{BDH}}{z_{BDH}} \times 100$ where z_{DM} and z_{BDH} represent the optimal objective function value obtained by the direct method and the best objective function value obtained by BDH,

respectively. It is worth mentioning that the parameter ε in both methods BD and BDH was set at 0.0001. The results, provided in Table 5, indicate that the convergence speed of BD is very slow; while BDH is very effective and can find the optimal or a near-optimal solution (with the optimality gap of at most 0.06%) in a short amount of time.

Furthermore, to investigate the impact of problem size on the runtime and the solution quality of the BDH algorithm, we concentrate on a variety of instances the characteristics of which have been presented in Table 6. It is worth mentioning that the instance RI in the last row of Table 6 refers to the reference instance addressed earlier. All instances of this table are solved by BDH and the direct method in a time limit of 18000 s. The results are reported in Table 7 in which, the column, labeled by Gap, has the same definition as before; the entries, filled by NA, indicate that no feasible solution has been returned by the corresponding method in the given time limit, and the entries, filled by > 18000, indicate that the corresponding method has been stopped due to the time limitation. It is worth mentioning that in the method BDH, the parameter ε has been set at 0.0001, and the total number of iterations is at most 16 in all instances. With respect to the results provided in Table 7, it can be concluded that in those instances for which the optimal solution is available, the BDH has been reached a near-optimal solution with an optimality gap of at most 0.03% in a short amount of time. Additionally, in those instances for which CPLEX cannot find any solution in the given time limit, BDH has obtained a feasible solution with a good percentage of coverage within the given time limit. Such results confirm the efficiency of the proposed BDH. However, as can be seen, by increasing the problem size, the resolution time of BDH increases, and hence, presenting an efficient meta-heuristic method would be a valuable direction for future research.

Conclusions

In this paper, a two-stage stochastic programming model and a heuristic method based on benders decomposition were proposed for the problem of locating charging stations in urban areas. The computational results over a variety of instances indicate that compared to the direct resolution via MIP solvers, the proposed method is able to find a near-optimal solution in a short amount of time. The development of an efficient meta-heuristic method to deal with larger instances of the problem is suggested as future work. Furthermore, the results show that the use of both types of chargers in the network has a significant role in responding to the demand. The extension of the proposed model to incorporate multi-user chargers in the stations would be a valuable direction for future work.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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