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Indirect Inference for the Identification of Star Variables in Macroeconomic Models

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March 11, 2025

Abstract

Star variables, such as potential output and the neutral real interest rate, are fundamental to economic policymaking but challenging to identify due to their latent nature. Buncic, Pagan, and Robinson (2023) highlight the difficulty of identifying star variables within short macroeconomic models, which typically contain more shocks than observable variables. To address this challenge, we propose an indirect inference method that assesses identification by examining how changes in these latent variables impact the behavior of observable economic data. Specifically, we simulate data from structural economic models, summarize their behavior using simplified statistical descriptions (VAR models), and evaluate the consistency between simulated and actual data. If the star variables are identifiable, even small deviations in their specifications will result in significant rejections in our indirect inference test. Applying our method to a standard three-equation New Keynesian model and the widely used Laubach-Williams model, we demonstrate that modest inaccuracies in specifying star variables clearly increase rejection rates. These results support the identification of star variables and indicate that indirect inference provides a reliable method to assess their identification in structural macroeconomic models.

Keywords: Star variables, identification, indirect inference, structural models, neutral real rate, potential output.

JEL Classification Codes: C32, C51, C52, E58.

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1 Introduction

Key macroeconomic models frequently incorporate so-called "star" variables—latent equilibrium values of essential indicators such as potential output, the neutral real interest rate, and equilibrium inflation. These star variables are central to macroeconomic policymaking, helping central banks and economic analysts determine appropriate policy stances. Despite their significance, these latent variables pose substantial estimation challenges because they are inherently unobservable. Influential frameworks, such as that by Laubach and Williams (2003), illustrate how the neutral real interest rate depends critically on unobserved structural shocks, highlighting the inherent difficulty of accurately identifying these key macroeconomic variables.

The complexity of identifying star variables has been explored from various perspectives in recent literature. For instance, Schmitt-Grohe and Uribe (2022) provide alternative structural formulations for linking star variables with model shocks, suggesting the sensitivity of star variables to the underlying assumptions. Furthermore, Buncic, Pagan, and Robinson (2023) emphasize that when macroeconomic models are "short"—meaning that there are more structural shocks than observed economic variables—accurate identification becomes particularly challenging. In such settings, common estimation techniques like the Kalman filter may lead to estimates of star variables that differ significantly from their true underlying processes, compromising the reliability of policy implications drawn from these estimates.

Motivated by these critical issues, this paper proposes a novel approach using indirect inference to address the identification challenges of star variables within structural macroeconomic models. Rather than relying solely on direct methods that attempt to recover star variables from structural shocks, we evaluate identification by observing how even minor inaccuracies in specifying these latent processes alter the simulated behavior of observable economic indicators. Specifically, we simulate data from structural models and summarize their behaviors through reduced-form VAR models, subsequently comparing these simulated outcomes to actual economic data.

Our indirect inference approach employs a robust Wald test to measure the discrepancies between the simulated and empirical reduced-form dynamics. If star variables are genuinely identifiable within the structural model, slight deviations from the true specification should result in statistically significant differences in observed behavior, leading to higher rejection

rates in our indirect inference tests.

We apply this procedure to two widely-used macroeconomic models: the standard three-equation New Keynesian framework and the influential Laubach-Williams (2003) model of the neutral real interest rate. Our empirical results strongly indicate that even minor inaccuracies in specifying star variables significantly increase rejection rates, confirming robust identification. Thus, indirect inference emerges as a powerful diagnostic tool that reliably tests and validates the identification of star variables.

The remainder of this paper proceeds as follows. Section 2 details our indirect inference identification methodology, emphasizing its effectiveness in addressing both narrow and weak identification issues. Section 3 illustrates the application of our procedure through empirical examinations of the three-equation New Keynesian model and the Laubach-Williams model. Section 4 provides an explanation for why star variables can be identified using indirect inference. Finally, Section 5 concludes, highlighting key findings and policy implications.

2 Identification via Indirect Inference

Indirect inference is a powerful framework for both model evaluation and the identification of macroeconomic models. The approach compares features of an auxiliary model—usually a reduced-form VAR—estimated from both simulated and actual data. Let a_T denote the estimates obtained from the actual observable data and $a_S(\theta_0)$ the estimates from N bootstrapped samples generated by the structural model with parameter vector θ_0. The average simulated estimate is $\overline{a_S(\theta_0)}$. The difference between the data-based and simulation-based descriptors is summarized using the Wald statistic:

$$WS = \big(a_T - \overline{a_S(\theta_0)}\big)' W^{-1}(\theta_0) \big(a_T - \overline{a_S(\theta_0)}\big),$$

where, following Guerron-Quintana et al. (2017) and Le et al. (2016, 2019), the weighting matrix is given by

$$W(\theta_0) = \frac{1}{N} \sum_{s=1}^{N} (a_s - \bar{a}_s)(a_s - \bar{a}_s)', \quad \text{with} \quad \bar{a}_s = \frac{1}{N} \sum_{s=1}^{N} a_s.$$

Under standard regularity conditions, WS is asymptotically distributed as $\chi^2(r)$, where r

is the number of elements in a_T. Detailed steps involved in computing the Wald statistic can be found in Le et al. (2016) and Minford et al. (2016, 2018).

If a structural macroeconomic model is accurately specified, the characteristics of the auxiliary model generated from simulated data will closely match those derived from actual economic observations. As a result, the true model will only be rejected at the nominal significance level—for instance, a rejection rate around 5

This indirect inference framework extends naturally to assessing model identification, including the identification of latent star variables. Following the approach outlined by Le et al. (2017), we define identification as the ability of a structural macroeconomic model to produce a unique reduced-form representation not shared by any other specification of the star variables. Thus, failure of identification implies the existence of at least one alternative specification capable of generating the identical reduced form.

Since star variables are latent and unobservable, our identification approach evaluates the impact of slight inaccuracies in specifying these variables on observable economic outcomes. Specifically, we employ Monte Carlo simulations to generate large numbers of data samples from the structural model and summarize their reduced-form dynamics using VAR models. We then compare these simulated VAR characteristics with those estimated from actual economic data.

In large samples, VAR coefficients estimated from the simulated data converge closely to their true underlying values. Consequently, if the structural model accurately specifies the star variables, the rejection rate of the model under indirect inference testing should align closely with the nominal significance level (e.g., 5%). However, any small deviation from the correct specification of star variables should produce significant differences in the VAR-based behavior, resulting in higher rejection rates.

The specific steps of our numerical implementation are as follows: first, we generate numerous samples from the correctly specified structural model using Monte Carlo methods. Next, we estimate VAR models from these simulated samples to characterize their reduced-form behavior based on observed economic variables. We then construct alternative structural models by systematically altering the specification of the star variables. Finally, we apply a Wald test to evaluate whether the discrepancies between these alternative models and the actual data sig-

nificantly exceed the nominal rejection rate. If such deviations produce notably higher rejection rates, we conclude that the star variables are robustly identified.

Our procedure follows these steps:

1. *Monte Carlo Sampling:* Generate a large number of samples from the true macroeconomic model via Monte Carlo simulation.

2. *VAR Coefficient Distribution:* Compute the distribution of VAR coefficients implied by these samples using a VAR estimated on the observed variables.

3. *Test Size Verification:* Validate the testing procedure by comparing the simulated VAR descriptions with historical data at the 95% confidence level, ensuring that the test's nominal size is correct.

4. *Alternative DSGE Models:* Construct alternative DSGE models by systematically falsifying the star variables.

5. *Wald Test Application:* Apply the Wald test to each alternative structural model using the full set of simulated samples, and record the rejection rate at the nominal 5% significance level.

6. *Identification Assessment:* If no alternative model can replicate the nominal rejection frequency (i.e., if any deviation from the true structural model leads to significantly higher rejection rates), we conclude that the star variables are identified. In line with Canova and Sala (2009), a well-identified model exhibits a clear increase in the rejection rate as one moves away from the true specification.

In the Monte Carlo simulation presented below, we perform 5,000 simulations. For each simulation, 500 bootstrap resamples are generated, and the sample size is set to $T = 200$, which is typical in macroeconomic studies.

3 Application to the Identification of the Neutral Real Rate

We now apply our procedure to two popular models that have been used to estimate the neutral real rate.

3.1 Three-Equation New Keynesian Model

The first model is a three-equation New Keynesian framework featuring a labor-only production function, an IS curve derived from the utility and consumption equations, a Calvo Phillips curve, and a Taylor rule, with no government spending or taxation. The model is specified as follows:

$$\pi_t = \omega\, E_t\pi_{t+1} + \lambda\,(y_t - y_t^*) + e_{\pi t}, \quad \omega < 1, \tag{1}$$

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}\Big(r_t - E_t\pi_{t+1}\Big) + e_{yt}, \tag{2}$$

$$r_t = (r_t^* + \bar{\pi}) + \gamma\,(\pi_t - \bar{\pi}) + \eta\,(y_t - y_t^*) + e_{rt}, \tag{3}$$

$$e_{it} = \rho_i\, e_{i,t-1} + \varepsilon_{it}, \quad (i = \pi, y, r).$$

In this model, there are two star variables: potential output, y_t^*, and the neutral real rate, r_t^* , with target inflation $\bar{\pi}$ treated as constant. The observed output is given by

$$y_t = AL_t = A\big(L_t^* + e_{a,t}\big) = y_t^* + A\, e_{a,t},$$

where $e_{a,t}$ is a productivity shock following a unit root process, $e_{a,t} = e_{a,t-1} + \varepsilon_{at}$, and L_t^* represents potential labor input (so that y_t^* corresponds to potential output). The neutral rate r_t^* is defined as the value of r_t for which $y_t = y_t^*$.

Because the model contains four shocks but only three observed variables, it is underidentified (i.e., a "short" system). Buncic, Pagan, and Robinson (2023) argue that the star variables cannot be uniquely identified using only the three observed variables. To assess whether the neutral real rate r_t^* and potential output y_t^* can be inferred, we employ indirect inference.

This model is a simple DSGE specification. In general, DSGE models, including the widely used Smets and Wouters (2007) US model, have the form (possibly after linearization):

$$A_0 E_t x_{t+1} = A_1 x_t + B z_t, \tag{4}$$

$$z_t = R z_{t-1} + \varepsilon_t,$$

where x_t contains the endogenous variables and z_t the exogenous variables. The exogenous variables may be observable or unobservable and are represented by an autoregressive process

with disturbances ε_t. Under the conditions of Fernandez-Villaverde et al. (2007), the solution to this model can be represented by a VAR of the form

$$\begin{bmatrix} x_t \\ \\ z_t \end{bmatrix} = F \begin{bmatrix} x_{t-1} \\ \\ z_{t-1} \end{bmatrix} + G \begin{bmatrix} \xi_t \\ \\ \varepsilon_t \end{bmatrix}. \tag{5}$$

Here, ξ_t represents innovations. (For further details, see the Dynare program of DJuillard (2001) which effectively solves the DSGE model in (4) into the VAR form of (5).) Indirect inference is based on observable endogenous variables x_t and an auxiliary model.

Identification via Indirect Inference

To test whether the star variables are identified, we perturb the parameters π^* and A by $x\%$ and the variance of $e_{a,t}$ (denoted σ_a) by $10x\%$, thereby generating falsified data. If the star variables are identifiable, the indirect inference test should reject these falsified specifications, with the rejection rate increasing as $x\%$ increases.

For the auxiliary VAR model used in the indirect inference test, we estimate a three-variable VAR of order 1 using the observed variables (y_t, r_t, π_t) and include the variance of the VAR residuals. This yields a total of 12 parameters in the Wald statistic (denoted α_T), which serves as the basis for model comparison. Le et al. (2016) demonstrated that using 12 parameters strikes an optimal balance, providing sufficient power for the test without making it excessively sensitive to small specification errors.

The results are reported in Table 1. First, for data simulated from the true model (0% falseness), the rejection rate of the indirect inference test is approximately 5% across all scenarios, confirming that the test has the correct size. When the productivity parameter A and the variance of the productivity shock σ_a are perturbed, the rejection rate increases substantially. For example, with a 5% perturbation, the rejection rate reaches 100%, strongly indicating that the star variables are well identified through the labor and output generation processes. Notably, when both parameters are simultaneously falsified by 3%, the rejection rate approaches 100%, further reinforcing this identification.

In practical scenarios, parameters must be estimated, potentially leading to lower rejection rates and weaker identification. To address this concern, we re-estimate the two parameters,

A and σ_a, for each simulated sample using indirect inference (while holding other structural parameters fixed at their true values), and then conduct the identification test. The results, reported in the final column of Table 1, indicate that estimation only marginally affects rejection rates compared to scenarios without re-estimation. The primary reason is that indirect inference provides unbiased parameter estimates, as demonstrated by Meenagh et al. (2019, 2024). Consequently, the estimated values of A and σ_a remain close to their true values, resulting in rejection rates similar to those obtained without re-estimation.

Table 1 – Rejection Rates for NK 3eq Model

Degree of Falseness	A	σ_a	All	All*
0	0.055	0.055	0.055	0.055
1	0.074	0.088	0.123	0.113
3	0.399	0.663	0.982	0.969
5	0.954	0.999	1.000	1.000
7	0.999	1.000	1.000	1.000
10	1.000	1.000	1.000	1.000
15	1.000	1.000	1.000	1.000
20	1.000	1.000	1.000	1.000

Note: "All" denotes falsifying all parameters. "All*" denotes falsifying all parameters, with re-estimation of these parameters.

Alternatively, we test for identification by modifying the equation for potential labor input. We introduce a random component into the potential labor equation:

$$y_t^* = A\big(L_t^* + \bar{l}_t\big),$$

where $\bar{l}_t$ is drawn from $N(0, \sigma_l)$, with σ_l corresponding to the variance of the observed labor variable l. We then increase the variance σ_l by $10x\%$ to generate falsified data.

Table 2 presents the results. Even with a small perturbation (1 % false), the test rejects the model 8% of the time, confirming that the star variables remain identifiable. As the variance of the random component increases, the rejection rate rises sharply. When the falseness reaches 7%, the rejection rate hits 100%, providing strong evidence of star variable identification.

Why Can the Star Variables Be Identified Using Indirect Inference?

Identification of the star variables r^* and y^* via indirect inference arises from their critical role in shaping the observed variables' reduced-form dynamics. Any alteration in the specifica-

Table 2 – Rejection Rates for NK 3eq Model — False Star Variable

Degree of Falseness	Rejection Rate
0	0.054
1	0.080
3	0.146
5	0.812
7	1.000
10	1.000
15	1.000
20	1.000

tion of these latent processes directly influences the behavior of the model's observable variables, which are summarized by the auxiliary VAR. Thus, even minor perturbations in the equations governing the star variables lead to discrepancies between the simulated and observed auxiliary statistics. These discrepancies, detected as elevated rejection rates by the indirect inference test, reveal that the reduced-form behavior uniquely encodes the underlying structural dynamics of the star variables.

3.2 LM Neutral Real Rate Model

One of the most influential models of the neutral real rate r_t^* over the past two decades is that of Laubach and Williams (2003, hereafter LW). Various extensions of the LW model exist in the literature and are widely used by central banks and policy institutions. Holston, Laubach, and Williams (2017, HLW) provide an updated version using a somewhat different formulation of the Phillips curve equation, estimated over a longer sample period.

The HLW model is specified as follows:

$$y_t = y_t^* + \sum_{i=1}^{2} a_{y,i}\Big(y_{t-i} - y_{t-i}^*\Big) + \frac{1}{2}a_r \sum_{i=1}^{2}\Big(r_{t-i} - r_{t-i}^*\Big) + \sigma_1\varepsilon_{1t}, \tag{6}$$

$$\pi_t = b_y\Big(y_{t-1} - y_{t-1}^*\Big) + b_\pi\pi_{t-1} + \big(1 - b_\pi\big)\pi_{t-2,4} + \sigma_2\varepsilon_{2t}, \tag{7}$$

$$\Delta z_t = \sigma_3\varepsilon_{3t}, \tag{8}$$

$$\Delta y_t^* = g_{t-1} + \sigma_4\varepsilon_{4t}, \tag{9}$$

$$\Delta g_t = \sigma_5\varepsilon_{5t}, \tag{10}$$

$$r_t^* = 4\,g_t + z_t, \quad \text{or} \quad \Delta r_t^* = 4\sigma_5\varepsilon_{5t} + \sigma_3\varepsilon_{3t}. \tag{11}$$

Here, y_t (100 times the log of real GDP) represents real GDP, y_t^* denotes potential GDP, r_t

is the real interest rate, and r_t^* is the neutral real rate. The model also incorporates evolving processes for trend growth g_t and other determinants z_t, which jointly affect r_t^*.

The HLW model can be represented in a standard state-space form:

$$\text{Measurement}: \quad \mathbf{y}_t = \mathbf{A}\mathbf{x}_t + \mathbf{H}\boldsymbol{\xi}_t + \boldsymbol{R}^{1/2}\boldsymbol{\varepsilon}_t^{y}, \tag{12}$$

$$\text{State}: \quad \boldsymbol{\xi}_t = \mathbf{F}\boldsymbol{\xi}_{t-1} + \boldsymbol{Q}^{1/2}\boldsymbol{\varepsilon}_t^{\xi}, \tag{13}$$

where $\mathbf{y}_t = [y_t,\, \pi_t]'$, $\mathbf{x}_t = [y_{t-1},\, y_{t-2},\, r_{t-1},\, r_{t-2},\, \pi_{t-1},\, \pi_{t-2,4}]'$, and

$$\boldsymbol{\xi}_t = [y_t^*,\, y_{t-1}^*,\, y_{t-2}^*,\, g_{t-1},\, g_{t-2},\, z_{t-1},\, z_{t-2}]'.$$

Moreover, the matrix $\mathbf{A}$ is defined as

$$\mathbf{A} = \begin{bmatrix} a_{y,1} & a_{y,2} & \frac{a_r}{2} & \frac{a_r}{2} & 0 & 0 \\ b_y & 0 & 0 & 0 & b_\pi & 1-b_\pi \end{bmatrix},$$

and $\mathbf{H}$ is given by

$$\mathbf{H} = \begin{bmatrix} 1 & -a_{y,1} & -a_{y,2} & -2a_r & -2a_r & -\frac{a_r}{2} & -\frac{a_r}{2} \\ 0 & -b_y & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The model features five shocks, $\eta_{it} = \sigma_i \varepsilon_{it}$ for $i = 1, \ldots, 5$, with standard deviations $\{\sigma_i\}_{i=1}^5$; the error terms $\{\varepsilon_{it}\}$ have unit variances. All relevant model parameters are taken from HLW and are documented in the associated GitHub repository of Buncic, Pagan, and Robinson (2023)[1]. Although HLW primarily focuses on estimating r_t^* (as defined in the equation above), trend growth g_t is also estimated.

Since the model contains five shocks but only two observed variables, it is underidentified. According to the star recovery procedure proposed by Buncic, Pagan, and Robinson (2023), the neutral real rate cannot be uniquely identified from the model and the observed data.

[1]We thank Buncic, Pagan, and Robinson (2023) for providing their codes to replicate the results

Identification via Indirect Inference

Using indirect inference, we test whether the neutral real rate is identifiable. The neutral real rate equation is expressed in differenced form:

$$\Delta r_t^* = 4\sigma_5 \varepsilon_{5t} + \sigma_3 \varepsilon_{3t}.$$

To falsify the neutral real rate, we perturb either the variance of g_t (i.e., σ_5) or the variance of z_t (σ_3). In our Monte Carlo simulation, we randomly perturb three parameters by $10x\%$ to generate falsified data. If the star variable is identifiable, the indirect inference test should reject the falsified specification, with the rejection rate increasing as $x\%$ increases.

For the auxiliary VAR model used in the indirect inference test, we estimate a two-variable VAR of order 2 using the two observed variables (y_t and π_t), including the variance of the VAR residuals. This results in a total of 10 parameters in the Wald statistic used for model comparison.

Table 3 – Rejection Rates for HLW Model

Degree of Falseness	σ_5	σ_3	All	All*
0	0.050	0.050	0.050	0.050
1	0.056	0.166	0.108	0.091
3	0.122	0.468	0.482	0.456
5	0.246	0.522	0.994	0.971
7	0.580	0.654	1.000	1.000
10	0.938	0.866	1.000	1.000
15	0.996	0.944	1.000	1.000
20	1.000	1.000	1.000	1.000

Note: "All" denotes falsifying all parameters. "All*" denotes falsifying all parameters, with re-estimation of these parameters.

The results presented in Table 3 indicate that for the data simulated from the true model (0% falseness), the indirect inference test rejection rate remains consistently around the nominal level of 5%, confirming correct test size. When parameters σ_3 and σ_5 are individually perturbed by $x\%$, the rejection frequency rises rapidly—reaching about 60% at a falseness level of $x = 7\%$ and surpassing 90% at $x = 15\%$. Furthermore, when both parameters are simultaneously falsified, the rejection rate approaches nearly 100% for falseness levels of $x = 5\%$ or higher, clearly indicating strong identification of the star variables.

We also consider a scenario where the parameters σ_3 and σ_5 are re-estimated via indirect in-

ference prior to conducting the identification test. The results, shown in the last column of Table 3, indicate that re-estimation has a minimal impact on the rejection rates. This outcome is consistent with indirect inference being an unbiased estimator, ensuring that parameter estimates remain close to their true data-generating values, and thus leaving the rejection frequencies largely unchanged.

An alternative approach to testing identification is to introduce a random component into the neutral real rate equation:

$$r_t^* = g_t + z_t + \bar{r}_t,$$

where $\bar{r}_t$ is drawn from $N(0, \sigma_r)$ with σ_r being the variance of the observed variable r. By perturbing σ_r by $x\%$, we generate falsified data for further testing.

Table 4 – Rejection Rates for HLW Model — False Star Variable

Degree of Falseness	**Rejection Rate**
0	0.050
1	0.098
3	0.140
5	0.160
7	0.282
10	0.552
15	0.904
20	0.994

The results in Table 4 confirm that the star variables remain identifiable. Even with a small random perturbation, the test rejects the model 9.8% of the time. As the variance of the random component increases, the rejection rate rises sharply, reaching 100% at a falseness level of 20%.

Why Can the Star Variables Be Identified Using Indirect Inference?

Expanding the relations in (12) and (13) and rearranging the terms yields

$$\begin{bmatrix} y_t \\ \\ \\ \\ \\ \pi_t \end{bmatrix} = \begin{bmatrix} y_t^* + a_{y,1}\big(y_{t-1} - y_{t-1}^*\big) + a_{y,2}\big(y_{t-2} - y_{t-2}^*\big) \\ +\frac{1}{2}a_r\Big(\big[r_{t-1} - 4c\, g_{t-1} - z_{t-1}\big] + \big[r_{t-2} - 4c\, g_{t-2} - z_{t-2}\big]\Big) + \sigma_1\varepsilon_{1t} \\ \\ \\ b_y\big(y_{t-1} - y_{t-1}^*\big) + b_\pi\pi_{t-1} + (1-b_\pi)\pi_{t-2,4} + \sigma_2\varepsilon_{2t} \end{bmatrix}, \tag{14}$$

$$\begin{bmatrix} \Delta y_t^* \\ \Delta g_{t-1} \\ \Delta z_{t-1} \end{bmatrix} = \begin{bmatrix} g_{t-1} + \sigma_4 \varepsilon_{4t} \\ \sigma_5 \varepsilon_{5t-1} \\ \sigma_3 \varepsilon_{3t-1} \end{bmatrix}. \tag{15}$$

With $r_t^* = 4g_t + z_t$, we obtain:

$$\Delta r_t^* = 4\sigma_5 \varepsilon_{5t} + \sigma_3 \varepsilon_{3t}. \tag{16}$$

These equations show that y_t^* directly affects y_t and indirectly influences π_t. Moreover, the state equations indicate that changes in r_t^*, g_t, or z_t modify the latent states, which in turn affect the observed data. Thus, the indirect inference test is capable of capturing these structural changes and confirming identification.

3.3 Star Variables Identification Using Indirect Inference

It may seem surprising that, although direct estimation of star variables from the underlying shock processes does not yield unique identification, these variables can nonetheless be identified indirectly through their impact on the model's VAR reduced form. Our Monte Carlo experiments illustrate this point. The key insight is that the reduced-form behavior of the model - reflecting its entire causal structure - embeds information about the star variables. In other words, the response of the observed variables to shocks is influenced not only by the shocks themselves but also by how the model is specified, including the equations that determine the star processes.

A direct estimate of the star variables' behavior in response to shocks would capture both the shock effects and the overall model specification, making it impractical to invert the reduced-form coefficients analytically, as these are complex nonlinear functions of the underlying structural parameters. Indirect inference circumvents this difficulty by simulating the structural model to generate a reduced form that matches the one observed in the data. In an identified model, only the true structural specification will reproduce the data's reduced form. Therefore, the specification of the star variables is confirmed by its unique effect on the reduced-form coefficients.

4 Star Identification

How can the star variables be recovered in practice? Implicit in our indirect inference approach is the idea that one systematically explores alternative model specifications until the simulated auxiliary model closely aligns with the observed data. This iterative process resembles standard model specification searches—provided the underlying structural model is identified, the correct specification of the star variables can, in principle, be recovered.

Several practical methods exist to estimate star variables, each suitable under different circumstances. One prominent method is the use of a Time-Varying Parameter (TVP) VAR, as exemplified by Lubik and Matthes (2015). They estimate a TVP-VAR involving the growth rate of real GDP, the PCE inflation rate, and the real interest rate (as in Laubach and Williams, 2003) to capture evolving dynamics in these macroeconomic indicators. Another approach involves incorporating Stochastic Volatility (SV) into the estimation procedure. Incorporating SV is common in contemporary macroeconomic models, as it better captures evolving uncertainty but may complicate shock recovery and thus potentially obscure the identification of star variables. Beyer and Milivojevic (2023) illustrate this by explicitly modeling stochastic volatility when estimating the neutral real rate for 50 countries. Additionally, smooth-transition models provide another avenue for recovering star variables. For instance, Okimoto (2019) employs a smooth-transition approach to model star variables, which allows for finite but multiple regime changes, thus addressing challenges typically associated with short systems and enhancing the recovery of latent processes.

In essence, our indirect inference procedure serves as an initial diagnostic step, testing the identification of star variables by evaluating if small perturbations from the true specification significantly increase rejection rates. Once identification is confirmed, indicated by high test power even for minor deviations, practitioners can confidently employ these estimation techniques to recover the star variables.

5 Conclusion

In this paper, we propose an indirect inference framework to evaluate the identification and recoverability of star variables in structural macroeconomic models. Using Monte Carlo simula-

tions, we demonstrate that even small perturbations in the specifications of critical star variable processes, such as potential output and the neutral real rate, result in rejection frequencies significantly higher than nominal levels. These findings strongly indicate that the reduced-form behavior of a correctly specified DSGE model uniquely captures its underlying star variable dynamics.

Our contribution is twofold. First, we establish a robust numerical procedure combining Monte Carlo simulations, VAR-based auxiliary model estimation, and Wald tests. This approach allows us to differentiate between narrowly identified models and those that exhibit robust identification. Second, we apply this methodology to two prominent frameworks: a standard three-equation New Keynesian model and the widely cited Laubach and Williams (2003) model. In both cases, our results confirm that even minor deviations from the true specification produce significantly higher rejection rates, thus affirming the unique recoverability of star variables.

The implications of our findings are relevant for researchers and policymakers alike. Given the importance of star variables like potential output and the neutral real rate in macroeconomic policy decisions, reliable identification methods are crucial. Our indirect inference approach serves as a rigorous diagnostic tool, complementing traditional identification checks and aiding in robust model selection.

References

[1] Beyer, R. C. M. and Milivojevic, L. (2023), "Dynamics and Synchronization of Global Equilibrium Interest Rates," *Applied Economics*, 55(28), 3195–3214.

[2] Buncic, D., Pagan, A., and Robinson, T. (2023), "Recovering Stars in Macroeconomics," CAMA Working Paper 43/2023, September 2023, Centre for Applied Macroeconomic Analysis, ANU.

[3] Canova, F. and Sala, L. (2009), "Back to Square One: Identification Issues in DSGE Models," *Journal of Monetary Economics*, 56(4), 431–449.

[4] Dridi, R., Guay, A., and Renault, E. (2007), "Indirect Inference and Calibration of Dynamic Stochastic General Equilibrium Models," *Journal of Econometrics*, 136(2), 397–430.

[5] Fernandez-Villaverde, J., Rubio-Ramirez, F., Sargent, T., and Watson, M. (2007), "ABCs (and Ds) of Understanding VARs," *American Economic Review*, 97, 1021–1026.

[6] Holston, K., Laubach, T., and Williams, J. C. (2017), "Measuring the Natural Rate of Interest: International Trends and Determinants," *Journal of International Economics*, 108(Supplement 1), S59–S75.

[7] Juillard, M. (2001), *Dynare: A Program for the Simulation of Rational Expectation Models* (No. 213), Society for Computational Economics.

[8] Laubach, T. and Williams, J. C. (2003), "Measuring the Natural Rate of Interest," *Review of Economics and Statistics*, 85, 1063–1070.

[9] Le, V. P. M., Meenagh, D., Minford, P., Wickens, M., and Xu, Y. (2016), "Testing Macro Models by Indirect Inference: A Survey for Users," *Open Economies Review*, 27, 1–38.

[10] Le, V. P. M., Meenagh, D., Minford, P., and Wickens, M. (2017), "A Monte Carlo Procedure for Checking Identification in DSGE Models," *Journal of Economic Dynamics and Control*, 76, 202–210.

[11] Meenagh, D., Minford, A. and Xu, Y. (2024). "Indirect inference and small sample bias — Some recent results." Open Economies Review 35, pp. 245-259.

[12] Meenagh, D., Minford, P., Wickens, M., and Xu, Y. (2019), "Testing Macro Models by Indirect Inference: A Survey of Recent Findings," *Open Economies Review*, 30(3), 593–620.

[13] Minford, P., Wickens, M., and Xu, Y. (2016), "Comparing Different Data Descriptors in Indirect Inference Tests on DSGE Models," *Economics Letters*, 145, 157–161.

[14] Minford, P., Wickens, M., and Xu, Y. (2018), "Testing Part of a DSGE Model by Indirect Inference," *Oxford Bulletin of Economics and Statistics*, 81(1), 178–194.

[15] Okimoto, T. (2019), "Trend Inflation and Monetary Policy Regimes in Japan," *Journal of International Money and Finance*, 92, 137–152.

[16] Schmitt-Grohe, S. and Uribe, M. (2022), "The Macroeconomic Consequences of Natural Rate Shocks: An Empirical Investigation," NBER Working Paper No. 30337.

[17] Smets, F. and Wouters, R. (2007), "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97(3), 586–606.

Appendix A: Parameters in Monte Carlo Simulations

Table 5 – Parameters of the True Model Used in the New Keynesian 3-Equation Model

Parameter	Value	Parameter	Value
ω	0.7640	ρ_y	0.8867
σ	3.4550	ρ_π	0.7999
λ	0.0997	ρ_r	0.8383
ρ	0.4029	σ_y	0.52
γ	1.1624	σ_π	0.14
η	0.8830	σ_r	0.24
A	1.61	σ_a	0.45

Table 6 – Parameters of the True Model Used in the LW03 Model

Parameter	Value	Parameter	Value
a_1	1.517	σ_1	0.387
a_2	-0.572	σ_2	0.731
a_r	-0.098	σ_3	0.323
b_y	0.043	σ_4	0.605
c	1.068	σ_5	0.0255