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# Is there PPP in the data? New evidence from a structural DSGE model

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## Abstract

There has been a long-running debate in international macroeconomics about whether there is purchasing power parity (PPP) in the data. But while the literature has been dominated by reduced-form evidence focusing on the real exchange rate dynamics including its long-run behaviour, little attention has been paid to establishing whether a full structural model with PPP can empirically explain the data. In this paper we test a DSGE model in which the long-run forces of trade force the exchange rate back to PPP, while shocks to preferences, technology and policy drive it away from PPP in the short run. We do so by using the method of Indirect Inference where we test whether the model's simulated behaviour can match the reduced-form VARX behaviour found in the data. We find, using a sample of the UK and US data observed between 1997 and 2020, that this PPP-constrained model fits the data well. Thus, even over quite a short recent sample the real exchange rate is being consistently driven back to a PPP equilibrium, in spite of all the shocks and policy pressures creating divergence in this turbulent period, which includes the global financial crisis, Brexit and Covid. As the model suggests, such convergence is driven by forces of trade generated by cross-country competition which drives price equivalence as productivity converges globally, as implied by classical trade theory.

*Keywords:* PPP; real exchange rate behaviour; two-country DSGE model; indirect inference test

*JEL Classification:* C12, E12, F41

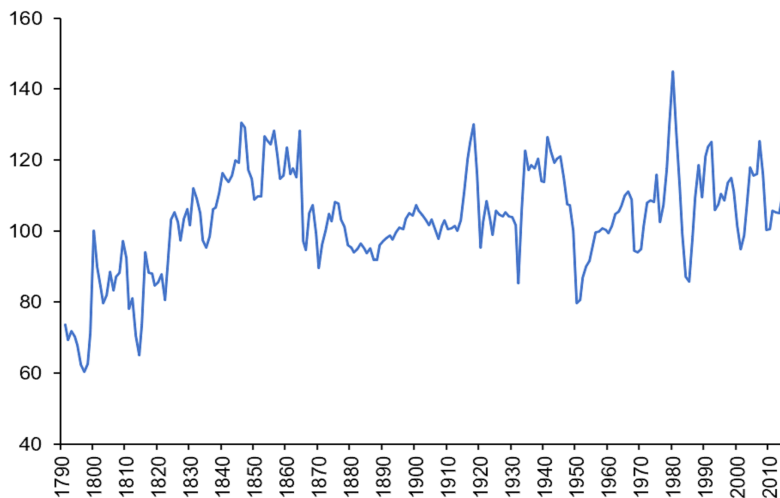
## 1 Introduction

There has been a long-running debate in international macroeconomics about whether purchasing power parity (PPP) holds in the long run – the ‘PPP hypothesis’ – and if so the dynamics of its convergence. In 2004 Alan Taylor and Mark Taylor argued in a comprehensive survey – Taylor and Taylor (2004) – that the reduced-form data on real exchange rates was widely seen as favouring the PPP hypothesis, with convergence modelled by a Smooth Transition Autoregressive (STAR) model. The data on the UK-US real exchange rate over the past two centuries, as plotted in Figure 1, seems by its considerable stability to back up the PPP hypothesis over the long run:

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Figure 1: Real \$/£ exchange rate (CPI based)



Authors' chart based on figures from a millennium of macroeconomic data, Bank of England

But while this reveals that reduced-form behaviour of exchange rates gives much support for the hypothesis, there has been little attention paid to whether structural DSGE models of the open economy obeying the PPP hypothesis can predict this behaviour. This is what has motivated our investigation here where we ask whether a structural DSGE model with PPP can predict the actual data behaviour of the economy and the real exchange rate in particular; we assume in line with the earlier literature that the reduced form data behaviour shows PPP convergence when quite large samples are used. Here, we investigate whether a structural DSGE model with PPP matches such reduced form behaviour in our small recent sample although such behaviour may not show actual convergence to PPP because of the actual shocks hitting the sample.

We set out a model of two open economies with the usual short term properties of product and productivity heterogeneity across countries but which in the long run are driven by competition and free entry to homogeneity in both so that traded prices are equal for all commodities across countries as is their productivity, so that we also get factor price equality; with productivity in the non-tradable sector also converging, this in turn causes non-tradable price equality, so that all home prices equate with foreign prices – PPP. Our proposal rests on two component models: the first one is a macro model of short-run behaviour, which we take to the data as our sole explanatory model of behaviour; the second is a classical trade model of long-run behaviour and comparative advantage, from which we extract the long-run properties that we impose as terminal conditions on our explanatory model. We propose that the second model governs long-run equilibrium behaviour which must in turn constrain the responses in our macro model. Our structural model embeds the long-run model into the short-run macro model via these terminal conditions. Agents in the short run respond to expectations of the future which ultimately are set as implied by these; these expectations move the short-run economy gradually towards the long-run equilibrium, so supplying the dynamics by which the real exchange rate and other macro variables evolve.

The central question we ask in this work is whether our model – the structural macro model constrained in the long run by the PPP hypothesis implied by the classical model – can pass as a whole a formal statistical test on the data from the UK and the US, which has been the focus of much of the literature. The

model is tested by the method of Indirect Inference, which evaluates the distance between the model and the data based on a Wald test, which allows one to formally reject or ‘accept’ a model based on its statistical significance. We know from previous work that open economy models like our structural macro model but without the long-run constraints of PPP fit a wide variety of countries’ general dynamic behaviour according to this test; thus we have estimated models of this type, paired with the US as major trading partner, for ten countries – including the UK, Australia and the eurozone – in Minford, Ou and Zhu (2022, 2024). In these tests the ‘basic dynamic behaviour’ we use as our descriptive summary of the data behaviour is the lag VAR coefficients of a VARX(1) in the two outputs of a country pair, where the ‘X’ are the exogenous non-stationary variables such as productivity and trade policies – as we detail these below. Hence our test matches the model predictions of the rate of convergence found in the data. In this paper we strengthen our test to include the lag VAR coefficients in the real exchange rate as well as the real exchange rate coefficients on these X variables, tightening the test to check how closely the model predicts the effects of these variables on the real exchange rate data. The US-UK model we have estimated here fits the basic dynamic behaviour in our short data sample for the UK and US, as we would expect, when unconstrained by long-run PPP conditions. But this model is rejected by the test that includes the X variable effects on the exchange rate – revealing that it cannot fully match real exchange rate behaviour. By contrast, the PPP-constrained model passes the full test comfortably, so confirming that a PPP-based DSGE model can fully predict real exchange rate behaviour. We interpret this to mean that a structural model with PPP is fully consistent with the data behaviour, whereas the unconstrained model is not – it can only match measures of the rate of convergence not those of how the exchange rate reacts to exogenous shocks.

We have looked at a 23-year sample of recent data, mostly for the current century, from the beginning of 1997 to the end of 2020, for which we have the full data on non-tradable sector variables we need for our complete model; this is the sample period on which we focus in this paper. We find for this recent post-war data period that our model passes the indirect inference Wald test comfortably. This is quite striking, since it shows that even over quite a short recent sample the real exchange rate is being consistently driven back to a PPP equilibrium, in spite of all the shocks and policy pressures creating divergence in this turbulent period, which includes the global financial crisis, Brexit and Covid. Our contribution in this paper is therefore to show that the data support a structural model in which the equilibrium exchange rate is PPP even over quite a short and turbulent sample period.

The remainder of the paper is organised as follows: we begin with an account of the long-run, ‘classical’ model of trade in Section 2, which implies the long-run conditions imposed on our short-run model; in Section 3 we describe our structural model of behaviour, which is a New Keynesian open economy model of short-run behaviour constrained by terminal conditions implied by this long-run classical model; in Section 4 we explain the method of indirect inference and how it is used for testing our model, and report the results; in Section 5 we analyse the behaviour of the model with its impulse response functions; in Section 6 we analyse the impact of the model shocks during the sample period; we discuss the implications of our findings in Section 7; Section 8 concludes.

## 2 The classical model of trade

We begin with the ‘classical’ model of world trade, whose intellectual origins lie in the work of Ricardo (1817), Heckscher (1919), Ohlin (1933), Stolper and Samuelson (1941), and Rybcynski (1955). In this model output is determined by factor supplies and sectoral productivity. Outputs here are divided into primary (agriculture and raw materials), manufactures, traded services and non-traded output. All supplies of each product are assumed, due to the long run effects of competition, to be perfectly substitutable, as if defined commodities. Capital is freely available and mobile from the rest of the world at the world’s exogenous cost of capital. The model of trade is, as in Minford et al. (1997, 2015), a computable general equilibrium model of trade, output, factor supply and demand; with four products, four factors and here two ‘countries’, the UK and the US. The model adopts the key assumptions of the Heckscher-Ohlin-Samuelson set-up as follows. Production functions are assumed to be Cobb-Douglas and identical across countries, up to a differing productivity multiplier factor; thus factor shares are constant, so that the model can be calibrated parsimoniously from the available detailed UK data. Three immobile factors of production are identified: unskilled and skilled labour and land. All sectors are perfectly competitive and prices of traded goods of each sector are equalised across borders. This is assumed to occur via competition and free entry in supply and switching in demand; thus if a good in one country differed from the same good in another country in quality or price, then consumers would switch between them and suppliers would switch supply to where the margins were higher, until price, quality and margins were equalised. Minford et al. (1997, 2015) developed this model for assessing the effects of globalisation on the world economy. It performed well empirically in accounting for the trade trends of the 1970-1990 period; it identified a group of major causal ‘shocks’ during this period which between them gave a good fit to the salient features of the period – including terms of trade, production shares, sectoral trade balances, relative wage movements and employment/unemployment trends.

This set-up gives rise to a well-known set of equations:

1. *given world prices of traded goods, price=average costs determines the prices of immobile factors of production;*
2. *domestic supplies of these factors are largely exogenous but may respond partially to these factor prices;*
3. *outputs of each sector are determined by these immobile factor supplies; the non-traded sector output is fixed by demand, the traded sector outputs by the supplies of immobile factors not used in the non-traded sector;*
4. *demands for traded goods are set by the resulting level of total GDP;*
5. *world prices are set by world demand=world supply.*

Given this set-up, if productivity in each traded industry is equal at home and abroad, the price-cost equations will imply factor price equalisation. If furthermore productivity in the non-traded sector is also equal at home and abroad, then these equal factor prices will imply equal non-traded costs, and so equal non-traded prices. Given that capital and technology are mobile across borders, it is natural to assume that both will flow across until returns on both are equalised by identical productivity.

### 3 A New Keynesian short-run model constrained by a classical model long-run equilibrium

In recent years this classical trade model has been challenged among trade theorists by what has come to be called the ‘gravity’ model, developed to rationalise the ‘gravity equation’ regressions first discovered by Tinbergen (1962). In this model the assumption of goods homogeneity made in the classical model is discarded in favour of the assumption of country heterogeneity made by Armington (1969) and adopted in many open economy macro models, just as we will adopt it below for our short-run macro model. Were we to adopt it also for our long-run trade model, it would destroy the long-run traded price equalisation the classical model provides us with, which is a necessary condition for long-run PPP. However, we justify not using this gravity trade model from recent empirical tests reported in Minford and Xu (2018), Chen et al. (2021) and Minford et al. (2021); in those tests this gravity model was widely rejected by the data whereas the classical model was almost invariably not rejected. Essentially what these tests show is that open economies’ long-run behaviour matches this classical model of long-run trade in the economy, even though short-run economic behaviour is well matched by New Keynesian macro models that assume product heterogeneity as well as other typical features we will set out below.

Thus for our long-run model we will use the classical model; in addition we will impose equal productivity in the long run across countries’ industries, both traded and non-traded. Our use of this model is restricted to imposing its equilibrium behaviour on the short-run model via terminal conditions. We will then test the resulting model by the powerful method of Indirect Inference, in a way to be described below. We will first describe the full macro model for which the trade model provides the terminal conditions. For this model we follow a New Keynesian setting, with heterogeneous country outputs. In effect we assume that in the short run the competitive pressures that force homogeneity in the long run simply have no strength beyond the short-run substitution possibilities embodied in the Armington set-up. These pressures only affect expectations for the future of the economy, its prices and output.

In the short-run macro model, we assume that households maximise utility in choosing consumption and labour supply, the latter being a combination of skilled and unskilled labour in a fixed weighting that reflects the household’s skills endowment. The production functions of firms in the country’s aggregate traded and in its non-traded output is a labour-only one, with productivity contributed by an endowment of saved capital, technology and land. This is a shock process in the model, and in the long run it is equalised across countries as described above. Firms set prices in a Calvo manner as a mark-up on labour costs, where a fixed proportion are able to change prices freely, with the rest holding them constant; real wages are flexible, clearing the labour market. There is uncovered interest parity connecting the real interest differential to the expected real exchange rate change. The current account balance depends on demands at home and abroad and on the real exchange rate. In the long run, when the terminal conditions hold, this balance (= the change in net foreign assets) must be zero. Thus our short-run macro model is a typical New Keynesian open economy two-country model; this model has been found to fit the data behaviour of a typical small open economy like the UK (Lyu et al., 2023) as well as a large open economy like the eurozone (Minford et al., 2022).

We go on to outline the structure of the combined model below. To save space, we only present the ‘home’ (UK) part of the two-country model; the foreign (US) economy shares the same structure but is

characterised with different parameter values. The full, linearised model is detailed in the appendix.

### 3.1 Micro-foundations

#### 3.1.1 Households

There is a continuum of measure one of representative households who work, consume and save. They have life-time utility:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_{j,t} \left[ \ln c_t + \phi \ln c_{NT,t} - \frac{\psi}{1+\eta} \left( n_{T,t}^{1+\xi} + n_{NT,t}^{1+\xi} \right)^{\frac{1+\eta}{1+\xi}} \right] \quad (1)$$

where  $c_t$  is a composite consumption on tradable goods defined as:

$$c_t = [(1-\alpha)^{\frac{1}{v}} c_{T,t}^{\frac{v-1}{v}} + \alpha^{\frac{1}{v}} im_t^{\frac{v-1}{v}}]^{\frac{v}{v-1}} \quad (2)$$

where  $c_{T,t}$  is domestic products,  $im_t$  is imports,  $v$  ( $> 0$ ) is the substitutability between  $c_{T,t}$  and  $im_t$ , and  $\alpha$  is the degree of openness.  $c_{NT,t}$  is consumption of non-tradable goods.  $n_{T,t}$  and  $n_{NT,t}$  are labour hours spent on tradable and non-tradable goods production, and  $\xi$  is their substitutability.  $\phi$  and  $\psi$  are the relative preference for non-tradable goods consumption and leisure, respectively.  $\eta$  is the inverse of the wage elasticity.  $\beta$  is the time discount factor.  $\varepsilon_{j,t}$  is the time preference shock.

The household budget constraint is:

$$\begin{aligned} & c_{T,t} + p_{NT,t} c_{NT,t} + q_t im_t + b_t + q_t bf_t + t_t \\ = & w_{T,t} n_{T,t} + w_{NT,t} n_{NT,t} + (1 + r_{t-1}) b_{t-1} + (1 + r_{t-1}^*) q_t bf_{t-1} + \Pi_{T,t} + \Pi_{NT,t} \end{aligned} \quad (3)$$

where  $p_{NT,t}$  is the relative price of non-tradable goods,  $q_t$  is the real exchange rate (defined as the units of domestic products per unit of foreign products),  $b_t$  and  $bf_t$  are holdings of home and foreign bonds,  $r_{t-1}$  and  $r_{t-1}^*$  are the home and foreign real interest rates,  $w_{T,t}$  and  $w_{NT,t}$  are the real wage rates in the tradable and non-tradable sectors,  $t_t$  is the lump-sum tax payment,  $\Pi_{T,t}$  and  $\Pi_{NT,t}$  are lump-sum profits received from the tradable and non-tradable sectors, respectively.

The household problem is to maximise (1) by choosing  $c_{T,t}$ ,  $c_{NT,t}$ ,  $im_t$ ,  $n_{T,t}$ ,  $n_{NT,t}$ ,  $b_t$  and  $bf_t$ , subject to (3). The first order conditions imply the demand for tradable and non-tradable goods consumption, imports, labour supply in the two sectors, and the demand for home and foreign bonds which implies the UIP condition:

$$E_t \hat{q}_{t+1} - \hat{q}_t = r_t - r_t^* \quad (4)$$

which suggests that home currency must depreciate/appreciate to eliminate any arbitrage opportunity should there be a positive/negative margin between the home and foreign interest rates<sup>1</sup>.

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<sup>1</sup>‘ $\Lambda$ ’, here in (4) and hereafter, denotes the percentage deviation of a variable from its steady-state value.



### 3.1.2 Firms

There are two production sectors, one producing tradable goods while the other producing non-tradable goods. In each of these there is a continuum of measure one of representative firms which produce via a labour-only production function:

$$y_{i,t} = z_{i,t} n_{i,t} \quad (5)$$

where  $i = T$  or  $NT$ ,  $y_{i,t}$  is the sectoral aggregate output,  $z_{i,t}$  is the sectoral productivity.

We assume Calvo (1983) pricing, where in each period only a fraction  $(1 - \omega)$  of the firms are able to reset prices so that the standard profit maximisation problem under the assumptions of a zero-inflation steady state and no past-inflation indexation implies the following Phillips curve for domestic price inflation:

$$\hat{\pi}_{i,t}^D = \beta E_t \hat{\pi}_{i,t+1}^D + \kappa \widehat{mc}_{i,t}$$

where  $i = T$  or  $NT$ ,  $\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega}$ , and  $mc_{i,t} (= w_{i,t}/z_{i,t})$  is the real marginal cost of production. The all-sector domestic inflation, defined as the weighted average of sectoral inflation with the weight being each sector's economic size, is given by:

$$\hat{\pi}_t^D = \frac{y_{T,t}}{y_{T,t} + y_{NT,t}} \hat{\pi}_{T,t}^D + \frac{y_{NT,t}}{y_{T,t} + y_{NT,t}} \hat{\pi}_{NT,t}^D \quad (6)$$

Given the definition of CPI that:

$$P_t = [(1 - \alpha)P_{h,t}^{1-v} + \alpha(Q_t P_{T,t}^*)^{1-v}]^{\frac{1}{1-v}} \quad (7)$$

where  $P_{h,t}$  is the price level of home products,  $P_{T,t}^*$  is that of tradable goods of the foreign country and  $Q_t$  is the nominal exchange rate (defined as the units of home currency per unit of foreign currency), CPI inflation can be shown to be:

$$\hat{\pi}_t = (1 - \alpha)\hat{\pi}_t^D + \alpha\hat{\pi}_{T,t}^{D*} + \alpha\Delta\hat{Q}_t + \hat{\varepsilon}_{\pi,t} \quad (8)$$

which is the weighted average of the home-product price inflation and foreign-tradable-goods price inflation, adjusted by the nominal exchange rate movement<sup>2</sup>.  $\hat{\varepsilon}_{\pi,t}$  is an exogenous 'inflation shock' which may be interpreted as the price mark-up shock.

Firms are assumed to be owned by households to whom they transfer their profits in each period in a lump sum. The profit from the tradable sector is given by  $\Pi_{T,t} = y_{T,t} - w_{T,t}n_{T,t}$ ; that from the non-tradable sector is given by  $\Pi_{NT,t} = p_{NT}y_{NT,t} - w_{NT,t}n_{NT,t}$ <sup>3</sup>.

### 3.1.3 Monetary and fiscal policy

Monetary policy is set by the central bank following a Taylor rule:

<sup>2</sup>In deriving this, it is assumed that full PPP holds in the long run, such that  $\frac{P_h}{P} = \frac{Q P_T^*}{P} = 1$ .

<sup>3</sup>Thus, in a zero-profit equilibrium  $p_{NT} = w_{NT,t}n_{NT,t}/y_{NT,t}$ .

$$1 + R_t = (1 + R_{t-1})^{\rho_R} (1 + \pi_t)^{(1-\rho_R)\phi_\pi} \left( \frac{gdp_t}{gdp_{t-1}} \right)^{(1-\rho_R)\phi_y} (1 + r)^{(1-\rho_R)} \varepsilon_{R,t} \quad (9)$$

where nominal interest rate responds to both inflation ( $\phi_\pi$ ) and GDP growth ( $\phi_y$ ), subject to policy inertia ( $\rho_R$ ) and a monetary policy shock ( $\varepsilon_{R,t}$ ). For simplicity, we include the GDP growth rate in place of GDP deviation from its steady-state value; but the model's behaviour is hardly affected.

Fiscal policy is set by the government, which adjusts its spending following an exogenous, stationary process around the steady-state level:

$$g_t = \varepsilon_{g,t} \bar{g}^{1-\rho_g} g_{t-1}^{\rho_g} \quad (10)$$

where  $\rho_g$  is the policy's inertia,  $\varepsilon_{g,t}$  is the government spending shock.

### 3.1.4 Identities and shock processes

Goods market clearing in the tradable and non-tradable sectors requires:

$$y_{T,t} = c_{T,t} + im_t^* \quad (11)$$

and

$$y_{NT,t} = c_{NT,t} + g_t \quad (12)$$

where  $im_t^*$  in (11) is imports by the foreign economy, hence home exports.

GDP is defined as:

$$gdp_t = y_{T,t} + p_{NT,t} y_{NT,t} \quad (13)$$

Balance of international payments requires:

$$q_t [bft + im_t - (1 + r_{t-1}^*) bft_{-1}] = im_t^* \quad (14)$$

The real exchange rate is defined as  $q_t = Q_t P_{T,t}^* / P_{T,t}$  (noting that  $P_{T,t}$  and  $P_{T,t}^*$  are the home and foreign tradable price, respectively), which implies:

$$\Delta \hat{q}_t = \Delta \hat{Q}_t + \hat{\pi}_{T,t}^{D*} - \hat{\pi}_{T,t}^D \quad (15)$$

The real interest rate is calculated by the Fisher equation:

$$\hat{r}_t = \hat{R}_t - E_t \pi_{t+1} \quad (16)$$

All shocks of the model, except for the productivity shock which we describe in the next section, are assumed to follow an AR(1) process in natural logarithm; hence:

$$\hat{\varepsilon}_{i,t} = \rho_i \hat{\varepsilon}_{i,t-1} + u_{i,t} \quad (17)$$

where  $i = j, \pi, R, g, u_{i,t} \sim iid$ .

### 3.2 Long-run convergence

Productivity in different sectors across the two countries is assumed to converge in the long run to enforce factor price equality for PPP to hold. In particular, we assume that the effect of productivity shocks is permanent, and productivity growth in any sector and country will adjust for itself to keep up with the growth rate in other sectors and countries. We let this dynamics take the form of a VECM, such that:

$$\begin{aligned} \begin{bmatrix} \Delta \hat{z}_{T,t} \\ \Delta \hat{z}_{NT,t} \\ \Delta \hat{z}_{T,t}^* \\ \Delta \hat{z}_{NT,t}^* \end{bmatrix} &= \begin{bmatrix} \Gamma_T \\ \Gamma_{NT} \\ \Gamma_T^* \\ \Gamma_{NT}^* \end{bmatrix} + \begin{bmatrix} -f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & -f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & -f_{33} & f_{34} \\ f_{41} & f_{42} & f_{43} & -f_{44} \end{bmatrix} \begin{bmatrix} \Delta \hat{z}_{T,t-1} \\ \Delta \hat{z}_{NT,t-1} \\ \Delta \hat{z}_{T,t-1}^* \\ \Delta \hat{z}_{NT,t-1}^* \end{bmatrix} + \begin{bmatrix} u_{ZT,t} \\ u_{ZNT,t} \\ u_{ZT,t}^* \\ u_{ZNT,t}^* \end{bmatrix} \\ &+ \begin{bmatrix} -d_{11} & -d_{12} & -d_{23} & 0 & 0 & 0 \\ d_{21} & 0 & 0 & -d_{22} & -d_{23} & 0 \\ 0 & d_{31} & 0 & d_{32} & 0 & -d_{33} \\ 0 & 0 & d_{41} & 0 & d_{41} & d_{43} \end{bmatrix} \begin{bmatrix} \hat{z}_{T,t-1} - \hat{z}_{NT,t-1} \\ \hat{z}_{T,t-1} - \hat{z}_{T,t-1}^* \\ \hat{z}_{T,t-1} - \hat{z}_{NT,t-1}^* \\ \hat{z}_{NT,t-1} - \hat{z}_{T,t-1}^* \\ \hat{z}_{NT,t-1} - \hat{z}_{NT,t-1}^* \\ \hat{z}_{T,t-1}^* - \hat{z}_{NT,t-1}^* \end{bmatrix} \end{aligned} \quad (18)$$

where  $\Gamma$ 's are the constants,  $f$ 's and  $d$ 's are the error-correction parameters, and  $u$ 's are the innovations which in this context may include the effect of trade policies on productivity such as the impact of tariffs on tradable prices; these productivities are assumed to converge to their steady-state value,  $\hat{z}_{SS}$ , where  $\hat{z}_{SS} = \frac{1}{4} (\hat{z}_{T,SS} + \hat{z}_{NT,SS} + \hat{z}_{T,SS}^* + \hat{z}_{NT,SS}^*)$ .

Since when PPP holds the real exchange rate equals one, we also impose  $q_{SS} = 1$  as a terminal condition. We also impose that  $\Delta b f_{SS} = 0$ , i.e., the stock of foreign bonds reflects the history of current account deficits up to the terminal date and so is non-stationary.

## 4 Confronting the model with the data: can it fit the facts?

### 4.1 The method of Indirect Inference

In this section we assess the empirical validity of the combined model formally. We do so by first estimating the model with the sample, and then testing the estimated version (which would have come closest to the data) by comparing its implied data features to those of the sample with a Wald test – the whole procedure known as the method of Indirect Inference (II).

We choose the II method, rather than the popular Bayesian method, because it is our aim here to test the model rather than just estimating it alone. The Bayesian method, which is a method of estimating models, does not test the models themselves; it only finds parameter values (based on set priors of them) that maximise the likelihood of a model which, nevertheless, may still be statistically rejected by the data. The DSGE-VAR method suggested by Del Negro and Schorfheide (2006) does evaluate model fit; however it

only does so informally by reporting a hyper parameter (between 0 and 1) that measures a model’s ‘goodness-of-fit’, but is itself not a test and hence, provides no indication as to when the model should/should not be rejected. The Maximum Likelihood method does provide a likelihood test of data fit, which could be used for our task. But as Le et al. (2016) have shown, in small samples typical in macro modelling (and here in particular) likelihood tests on macro models generally suffer from low power compared with indirect inference tests, and ML estimates are also highly biased, as is well-known.

Indirect inference was originally designed for estimating a model when the model’s likelihood function was too complex for it to be estimated directly. Early applications of the method can be dated back to Smith (1993), Gregory and Smith (1991, 1993), Gouriéroux, Monfort and Renault (1993) and Gouriéroux and Monfort (1996). The basic idea is to first use an auxiliary model whose likelihood function is relatively simple for inferential, indirect inferences to be found; the algorithm then searches for the parameter values of the structural model that enable it to best replicate the inferences implied by the auxiliary model.

The method has been substantially developed by Minford, Theodoridis and Meenagh (2008), Meenagh, Minford and Wickens (2009), Le et al. (2011, 2016) and Minford, Wickens and Xu (2019) in recent years for it to be used as a formal statistical test on an already estimated or calibrated model. The idea in this testing task is to first describe the data’s behaviour using the auxiliary model; and then simulate the main model to create parallel simulations from each of which the counterpart auxiliary model estimates are found, generating a distribution of them under the null hypothesis that ‘the main model is the true model’; it then asks whether the auxiliary model estimates found with the sample data came from this distribution with a high-enough probability according to the Wald test – see Liu, Minford and Ou (2024) for elaboration of a three-step procedure.

In our application of the method here, where we are testing an open-economy model, we are interested in the model’s capacity to explain the international business cycle and real exchange rate dynamics; so we focus on both the UK and US outputs, and the pound/dollar real exchange rate. According to Meenagh et al. (2012), for non-stationary data, the general solution of a log-linearized DSGE model can be written as a VARMA model, which, in turn, can be further approximated as a VECM model. In this context, non-stationary exogenous variables are introduced as conditioning variables to achieve cointegration, ensuring that the stochastic trends within the endogenous vector are appropriately accounted for. This leads to stationary errors within the VAR framework, which can be reexpressed as a VARX(1) model. Thus in practical terms our auxiliary model can be represented as:

$$\begin{bmatrix} \widehat{gdp}_t \\ \widehat{gdp}^*_t \\ \hat{q}_t \end{bmatrix} = A \begin{bmatrix} \widehat{gdp}_{t-1} \\ \widehat{gdp}^*_{t-1} \\ \hat{q}_{t-1} \end{bmatrix} + B \begin{bmatrix} \hat{z}_{T,t-1} \\ \hat{z}_{NT,t-1} \\ \hat{z}^*_{T,t-1} \\ \hat{z}^*_{NT,t-1} \\ tr \end{bmatrix} + \begin{bmatrix} e_{1,t} \\ e_{2,t} \\ e_{3,t} \end{bmatrix} \quad (19)$$

where productivity shocks ( $\hat{z}_{T,t}$ ,  $\hat{z}_{NT,t}$ ,  $\hat{z}^*_{T,t}$ ,  $\hat{z}^*_{NT,t}$ ) account for the stochastic trend and the time trend  $tr$  captures the deterministic trend in the data. This framework allows us to effectively capture and analyze the reduced-form data dynamics while accommodating the necessary adjustments for non-stationarity and cointegration.

In testing ordinary short-term open economy models (without PPP) we typically focus just on the  $A$

matrix plus the variances of the VARX residuals, since previous work using Monte Carlo experiments on macro models have shown that this test has good, but not excessive, power in testing these models (Le et al., 2016; Meenagh et al., 2019). Nevertheless, as in this work we are also interested in knowing whether our PPP-constrained model can match the data relationships between the real exchange rate and productivity, we include also the  $B$  matrix coefficients for the real exchange rate into our auxiliary model. This raises the power of our II test so that it will reject models that fail also to capture the real exchange rate behaviour. Notice that we do not test the PPP hypothesis on the reduced form itself; that has already been done in the literature surveyed by Taylor and Taylor (2004) in which it is concluded that the real exchange rate does converge to PPP in the long run, typically using long samples; furthermore, we may note that in our short sample there may not be actual convergence in the reduced form because of the particular shocks drawn. Rather, we ask whether our PPP-constrained DSGE model could have produced the sample behaviour with fair probability, using the output and real exchange behaviour as estimated in our VARX(1) as the description that we ask our DSGE model to match. Hence we are asking whether a DSGE model with PPP convergence is likely to be the model producing the actual sample behaviour.

The distance between our DSGE model and the sample data is measured by the Wald statistic:

$$Wald = (\Phi^{Act} - \bar{\Phi})' \sum_{(\Phi\Phi)}^{-1} (\Phi^{Act} - \bar{\Phi}) \quad (20)$$

where  $\Phi^{Act}$  is a vector of the chosen VARX estimates found with the sample data,  $\bar{\Phi}$  and  $\sum_{(\Phi\Phi)}$  are, respectively, a vector of the mean values, and the variance-covariance matrix, of the same set of VARX estimates found with parallel simulations generated by the DSGE model.

The DSGE model is first estimated by a grid-search for its parameter values which minimise (20). The significance of the estimated model is then evaluated by its p-value, calculated by:

$$p = (100 - WP)/100 \quad (21)$$

where  $WP$  is the percentile of (20) under the distribution of it generated by the parallel simulations. Under the null hypothesis ( $H_0$ ) that ‘the DSGE model is true’, the model is rejected/not rejected should the calculated p-value be below/above the usual 1%, 5% or 10% threshold.

## 4.2 Data

Our data sample spans the period from 1997Q1 to 2020Q4. The observed variables are those which are employed by the auxiliary model and the other variables involved in calculating the sample DSGE shocks with the linearised model, viz. the real exchange rate, and GDP, non-tradable consumption, imports, government spending, labour hours, the real wage rate, domestic-price inflation (in both the tradable- and non-tradable sectors), CPI inflation and nominal interest rate of both countries. Both nominal interest rate (which we measure with the lending rate which never touched the zero lower bound), inflation and the real wage rate are measured as quarterly rates. The other variables, which are defined in real and per-capita term, are measured in natural logarithm. Unfiltered data are used. The data sources, the time series collected, and the adjustments to the raw data are detailed in the appendix.

### 4.3 Model parameters and fit

We fix some model parameters that are hard to identify or depend on the data, at their calibrated values; thus we set the discount factor for both economies to 0.99, and the model steady-state ratios to the mean value of them as found with the sample. The other model parameters, for preferences and other behavioural functions, are estimated by grid-searching (using the simulated annealing algorithm) as described above. The parameter values are reported in Table 1.

Table 1: II estimates and p-value of the DSGE model

Parameter	Definition	PPP-constrained model		Unconstrained model	
		UK	US	UK	US
$\beta$	Time discount factor	0.99	0.99	0.99	0.99
$\frac{p_{NT}c_{NT}}{gdp}$	Steady-state non-tradable goods consumption ratio	0.29	0.55	0.29	0.55
$C_T/Y$	Steady-state tradable goods consumption ratio	0.34	0.11	0.34	0.11
$IM/Y$	Steady-state imports ratio	0.20	0.18	0.20	0.18
$Y_{NT}/Y_T$	Steady-state non-tradable to tradable output ratio	0.80	5.56	0.80	5.56
$BF/Y$	Steady-state ratio of foreign assets	1.18	0.81	1.18	0.81
$G/Y$	Steady-state government expenditure ratio	0.21	0.18	0.21	0.18
$\phi$	Relative preference on non-tradable consumption	3.27	1.41	2.43	1.27
$\alpha$	Degree of openness	0.33	0.44	0.36	0.49
$\psi$	Preference to leisure	2.93	2.10	2.88	2.32
$\eta$	Inverse of wage elasticity of labour	0.78	0.77	0.54	0.86
$\xi$	Substitutability between labour skills	0.46	0.59	0.48	0.43
$\nu$	Substitutability between home and foreign products	1.74	1.63	1.47	1.91
$\omega$	Calvo non-adjusting probability	0.33	0.36	0.31	0.32
$\kappa_\alpha$	Slope of the Phillips curve	1.37	1.13	1.53	1.48
$\rho$	Monetary policy inertia	0.49	0.29	0.56	0.23
$\phi_\pi$	Monetary policy response to inflation	2.58	2.14	2.26	2.78
$\phi_y$	Monetary policy response to output gap	0.17	0.30	0.15	0.26
$\rho_b$	Persistence of the preference shock	0.83	0.92	0.82	0.92
$\rho_{PP}$	Persistence of the mark-up shock	0.91	0.88	0.91	0.89
$\rho_R$	Persistence of the monetary policy shock	0.74	0.91	0.73	0.91
$\rho_g$	Persistence of the fiscal shock	0.94	0.92	0.94	0.92
$\delta_T$	tradable productivity shock	0.21	0.28	0.21	0.28
$\delta_{NT}$	nontradable productivity shock	0.18	0.09	0.18	0.09
Model p-value		10.1%		0.01%	

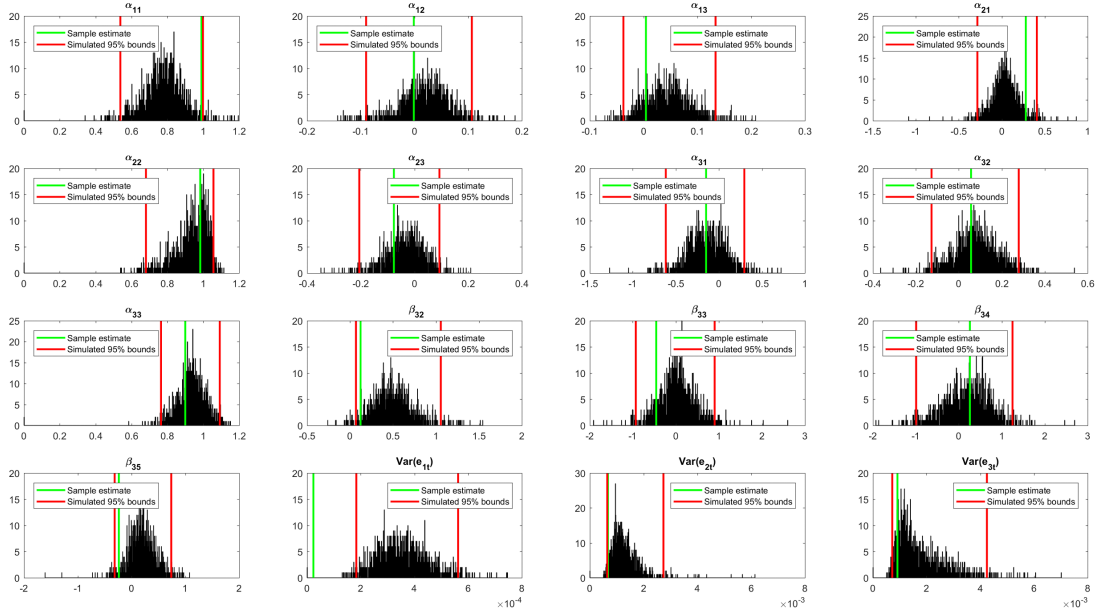
What we find here is that the UK and US models have generally quite different parameter values. For example, the non-tradable consumption ratio in the UK ( $p_{NT}c_{NT}/gdp$ ) is only about half that of the US, while the tradable consumption ratio ( $c_T/gdp$ ) in the UK triples the US counterpart; the relative size of the non-tradable sector ( $Y_{NT}/Y_T$ ) is just under unity in the UK, but is close to six in the US; the relative preference on non-tradable consumption over tradable goods ( $\phi$ ), and monetary policy inertia ( $\rho$ ), in the UK are about double those in the US. The other model parameters are less different between the two economies. In particular, the data suggest a very active interest rate response to inflation ( $\phi_\pi$ ), and generally high

persistence of shocks ( $\rho'_i s$ ); the productivity shocks are non-stationary and hence permanent but have little autocorrelation.

Can this model fit the facts? According to the reported p-value, which is 10.1%, this PPP-constrained model passes the Wald test comfortably. However, when the model is reestimated and retested the PPP restrictions removed, the p-value drops significantly to literally zero, which indicates a strong rejection of the model. Thus we find evidence that the data behaviour of the economy and the real exchange rate supports the empirical validity of PPP; while the short-run model may still fit the data to some extent, on its own it would fit only up to the short-run data dynamics but will be rejected by the long-run real exchange rate behaviour as the rate converges to the steady-state set by productivity which itself will converge across different sectors and countries.

Figure 2 shows the estimated value together with the histogram of the VARX coefficients employed by our II Wald test. It can be seen that the 95% confidence interval of these auxiliary model coefficients implied by the PPP model encloses their sample estimate almost in all cases; the high joint p-value of the PPP model just reported is therefore a parsimonious reflection of such fit.

Figure 2: Histogram of VARX estimates implied by parallel simulations



## 5 The behaviour of the structural PPP model in response to shocks

In this section we describe the PPP model's behaviour in response to shocks. As Figure 3 shows, a rise in preference towards current utility (against future utility) raises non-tradable consumption, total output and inflation. The expansion triggers a rise in nominal interest rate due to the Taylor rule and, as the rate rises more than inflation so that the real interest rate is raised, home currency appreciates (shown as a fall in the real exchange rate as defined here) and imports rise while the demand for home tradable goods falls.

The relative price of non-tradable goods compared to tradable goods rises as a result. The shock spills over to the foreign economy via the home currency appreciation first triggering a fall in foreign imports. The ‘relative-CPI-based real exchange rate’, calculated here by  $\hat{P}_t - \hat{P}_t^* - \hat{Q}_t$  which measures the ‘full-price’ deviation from PPP, shows that the shock disrupts PPP as pound over-depreciates in its nominal value as it causes a rise in the UK relative price.

Figure 3: Effect of the preference shock

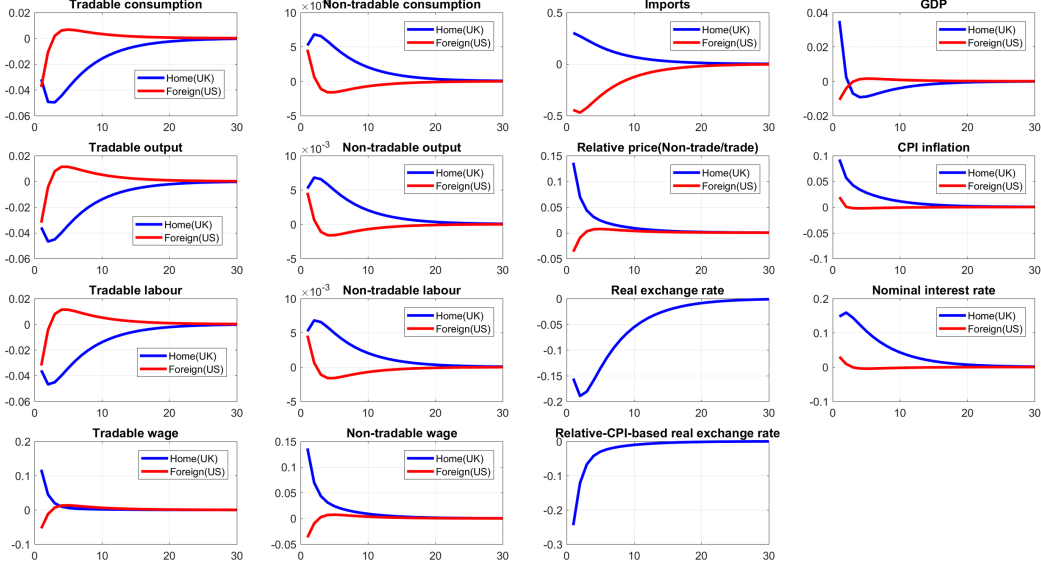


Figure 4 shows the effect of a price mark-up shock. The shock raises inflation and lowers total output, following which nominal interest rate rises to respond and home currency appreciates. Imports rise, but only moderately, due to an offsetting substitution effect caused by a fall in non-tradable goods prices which raises non-tradable consumption while reducing tradable consumption (The fall in non-tradable goods prices reflects the fall in real wages in that sector as labour flows in from the tradable sector due to lower demand for tradable goods). The shock has a similar spillover effect on the foreign economy via the exchange rate channel as described earlier. However, in this case PPP is disrupted as while the shock opens a gap in the UK-US relative price, the nominal value of pound fails to depreciate sufficiently, thus impairing the competitiveness of the UK.



Figure 4: Effect of the price mark-up shock

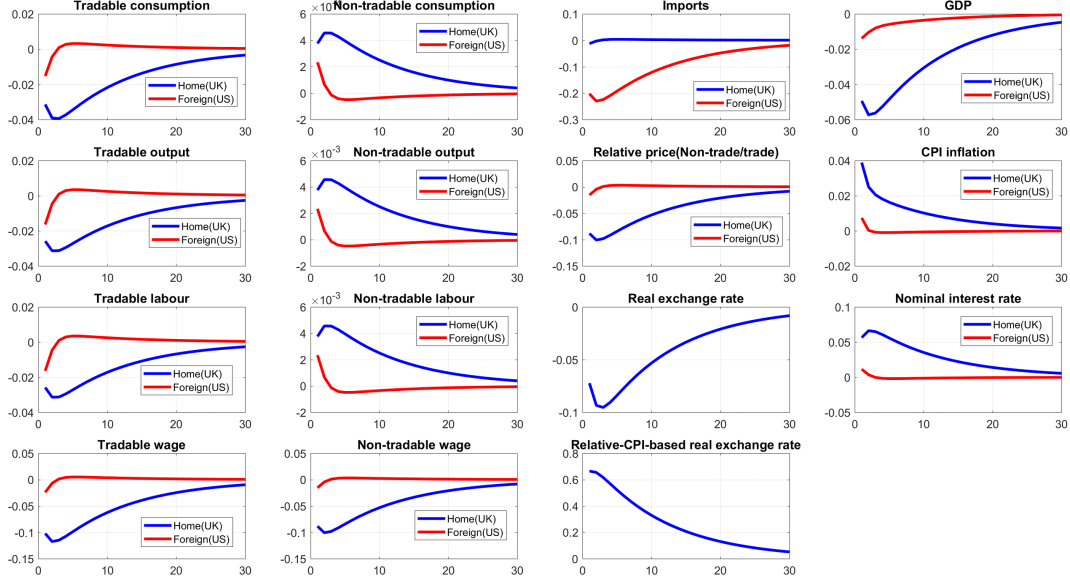
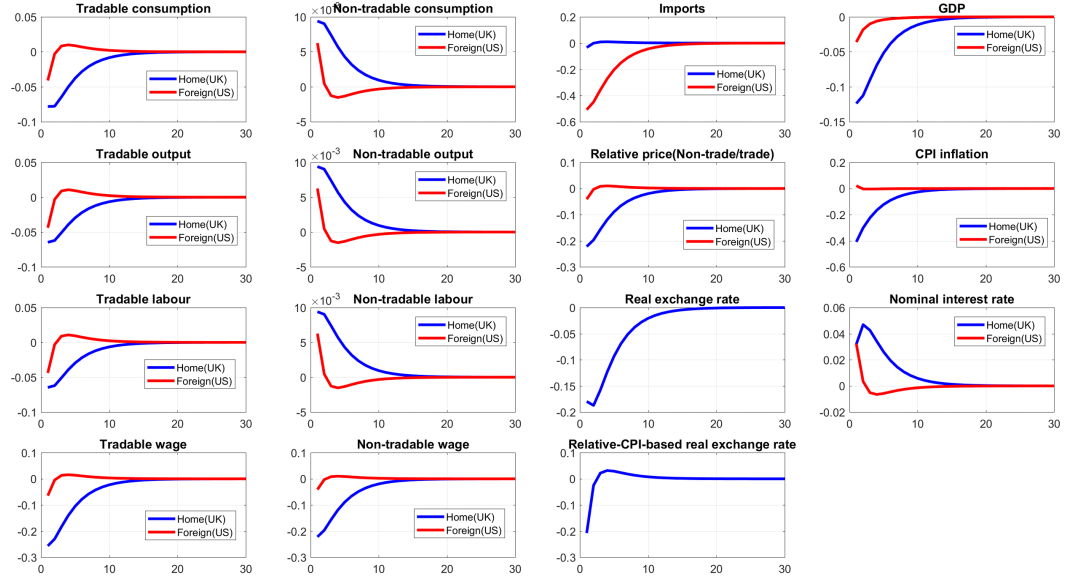


Figure 5: Effect of the interest rate shock



The effect of an interest rate shock, as shown by Figure 5, is similar to that of the price mark-up shock, except that in this case inflation falls (instead of rising) due to the tightened monetary condition and there is nominal exchange rate overshooting (instead of undershooting). By contrast, a government spending shock (Figure 6) raises both total output and inflation due to the familiar multiplier effect (which comes from the non-tradable sector as it shows). The rise in nominal interest rate, as enforced by the Taylor rule, crowds out both tradable and non-tradable consumption, and raises the home currency value and therefore imports. The boost in the demand for non-tradable goods (due to the spending itself) raises the demand for labour,

and wages, in that sector while the tradable sector contracts. Again, the shock spills over to the foreign economy through trade; but because it only has a limited impact on the foreign interest rate, the spillover effect, as well as its impact on PPP, is trivial.

Figure 6: Effect of the government spending shock

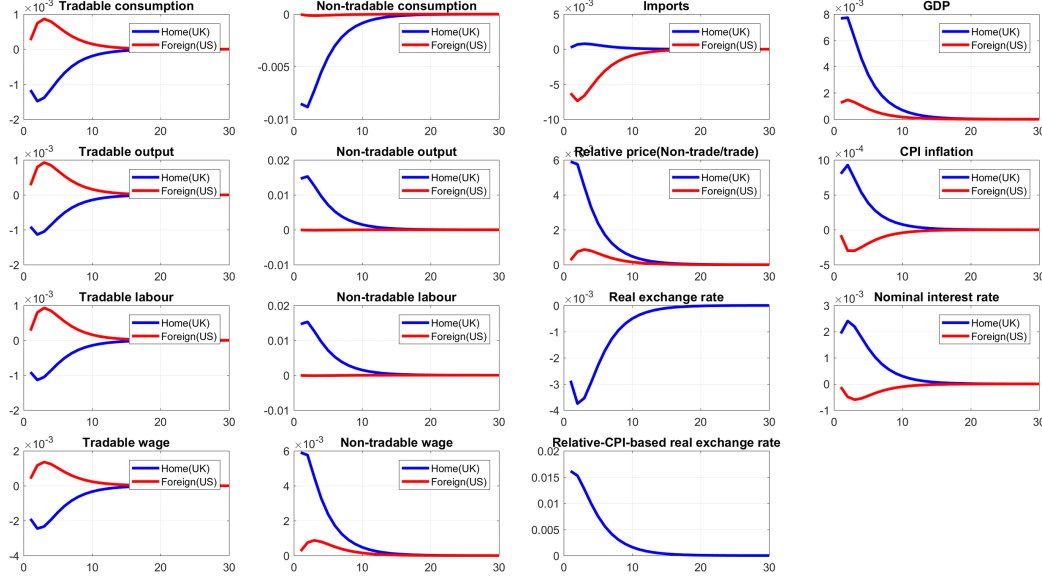
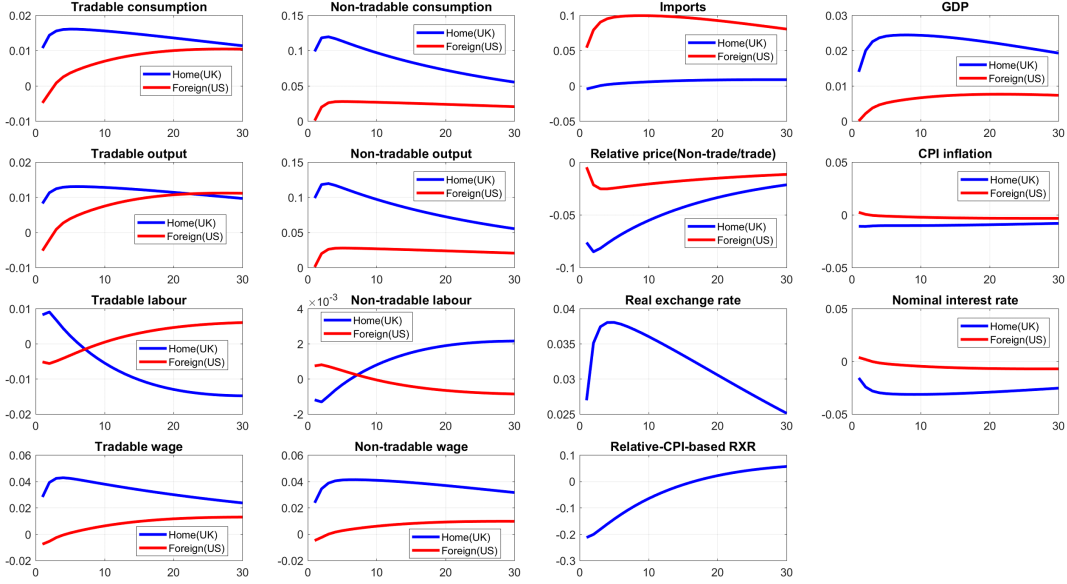


Figure 7: Effect of the productivity shock (Tradable-goods sector)

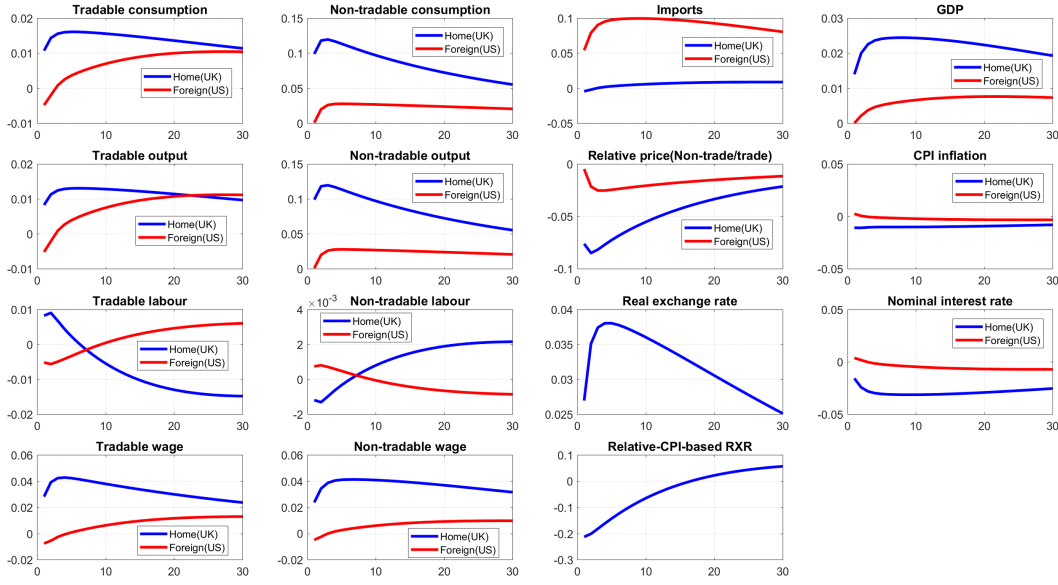


Finally, the effect of the productivity shock is shown by Figures 7 and 8. A shock to traded productivity raises tradable output, and lowers inflation (as tradable prices fall). Nominal interest rate falls to respond to these changes, which depreciates the home currency and raises consumption in both the tradable and non-tradable sectors. The higher demand for goods then causes a general rise in wages, although wages rise more

in the non-tradable sector with workers being shed in the tradable sector with higher productivity. Because the relative price of non-tradable goods rises, which generates a substitution effect dominating the home currency depreciation, imports rise slightly in this case. The shock spills over to the foreign economy via trades as usual; but on this occasion it also does so via cross-country and cross-sector productivity spillovers as assumed by the long-run model. Nevertheless these productivity spillovers are weak in the short run, as the IRFs suggest; this also results in the slow convergence to PPP, as the IRF of the relative-CPI-based real exchange rate shows.

A shock to non-tradable productivity has similar effects to the ones for tradable productivity, except that the sectoral effects are reversed; it also has more lasting impact compared to the shock to tradable productivity.

Figure 8: Effect of the productivity shock (Non-tradable-goods sector)



## 6 The relative importance of the shocks affecting the UK and US economies

In this section we go on to evaluate the importance of the PPP model's shocks in determining the dynamics of the key variables. We start with a variance decomposition analysis to identify the main drivers of these variables in the short and long runs. We then decompose the data of these variables against the historical shocks computed with the model to reveal how these shocks have driven the sample.

### 6.1 Variance decomposition

Table 2 decomposes the variance of total output, inflation, nominal interest rate and the real exchange rate against all the twelve model shocks from a short-run perspective (Eight quarters ahead of the rise of the shocks). As can be seen, UK output is dominated by the domestic interest rate shock, which accounts

for 56% of its variation; the traded productivity shock and the domestic price mark-up shock account for 19% and 15%, respectively, while the other shocks affect little. Inflation is almost solely influenced by the domestic interest rate shock, which accounts for 95%. Nominal interest rate is dominated by shocks to its own (50%), but is also disturbed by the domestic price mark-up shock (20%), time preference shock (16%) and traded productivity shock (11%). The US shocks generally do not have a material impact on the UK variables, as the real exchange rate – driven mainly by interest rate shocks in both the economies (which sum up to near 47%) – adjusts to minimise cross-boarder spillovers via trades. The US variables are affected in a similar manner.

Table 2: Variance decomposition (Eight quarters ahead)

	UK variables			US variables			RXR
	GDP	CPI Infl	NIR	GDP	CPI Infl	NIR	
UK shocks							
Preference	1.3	1.4	16.2	0.3	0.1	0.1	6.5
Mark-up	14.9	3.2	19.6	0.1	0	0.0	21.1
Interest	55.7	94.6	50.4	1.1	0.2	0.9	23.9
Gov Spn	0.2	0	0	0	0	0	0
Prod (T)	19.4	0.5	10.5	2.6	0.1	0.3	5.3
Prod (NT)	2.9	0.1	1.4	0.3	0.1	0.0	1.2
US shocks							
Preference	1.6	0.1	0.1	6.6	2.9	1.4	4.8
Mark-up	0.3	0.0	0.0	14.7	4.4	19.1	11.8
Interest	1.3	0.0	0.0	57.6	91.2	76.1	23.1
Gov Spn	0	0	0	0.1	0	0	0
Prod (T)	2.0	0.1	1.6	10.5	0.6	1.3	0.1
Prod (NT)	0.4	0	0.1	6.1	0.3	0.8	2.0

The long-run impact of the shocks (40 quarters ahead of their occurrence), as shown by Table 3, is by and large similar. But as the productivity shocks – which follow a random walk by assumption – accumulate, they become dominant in the UK output determination; the domestic shocks account for 40%, of which 30% is due to the traded productivity shock. The US output is also much more affected by its own productivity shocks – 29% in total, of which 24% is due to the traded sector; however, it remains governed by the home interest rate shock which contributes near 49%.

In sum, for both the economies, domestic monetary policy is always decisive in determining output, inflation and the real exchange rate. Productivity – in the tradable-goods sector in particular – plays a moderate role in the short run, but affects output substantially in the long run. The two economies, while connected by trade and UIP, are, however, quite mutually impervious as cross-boarder shock spillovers are trivial.

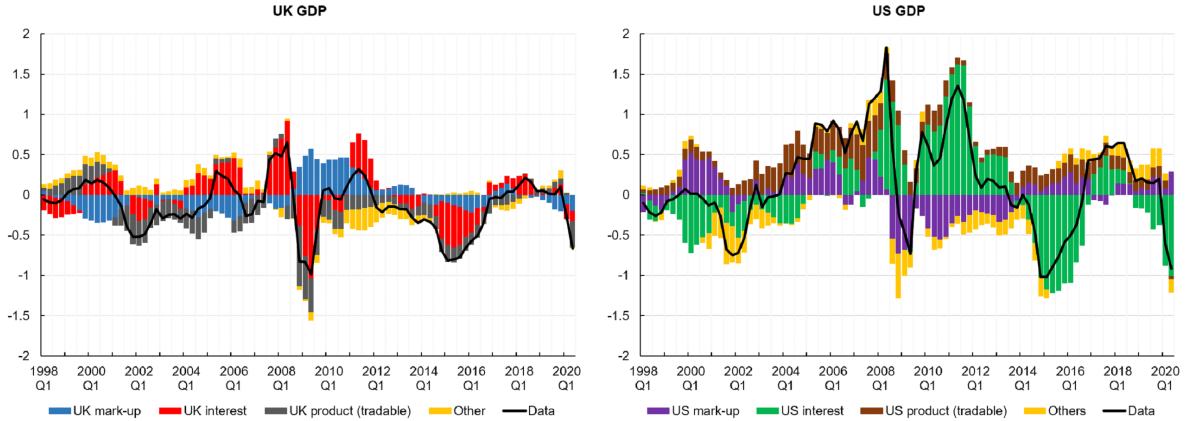
Table 3: Variance decomposition (40 quarters ahead)

	UK variables			US variables			RXR
	GDP	CPI Infl	NIR	GDP	CPI Infl	NIR	
UK shocks							
Preference	0.7	1.2	15.1	0.2	0.1	0.2	6.9
Mark-up	12.4	3.3	19.1	0.1	0	0.1	16.9
Interest	35.1	89.7	36.8	0.4	0.2	0.4	22.0
Gov Spn	0.1	0	0	0	0	0	0
Prod (T)	30.3	3.8	17.3	3.9	0.6	3.3	8.4
Prod (NT)	10.2	1.0	4.6	1.5	0.1	1.2	3.4
US shocks							
Preference	1.0	0	0.7	3.9	2.5	4.8	4.0
Mark-up	0.1	0.1	0.1	12.2	3.4	11.7	11.9
Interest	1.3	0.1	0.1	48.8	92.1	53.3	24.3
Gov Spn	0	0	0	0.1	0	0	0
Prod (T)	8.3	0.8	5.9	23.5	0.6	23.3	2.1
Prod (NT)	0.5	0	0.3	5.4	0.4	1.7	0.1

## 6.2 Historical decomposition – how did the shocks shape major US and UK variables?

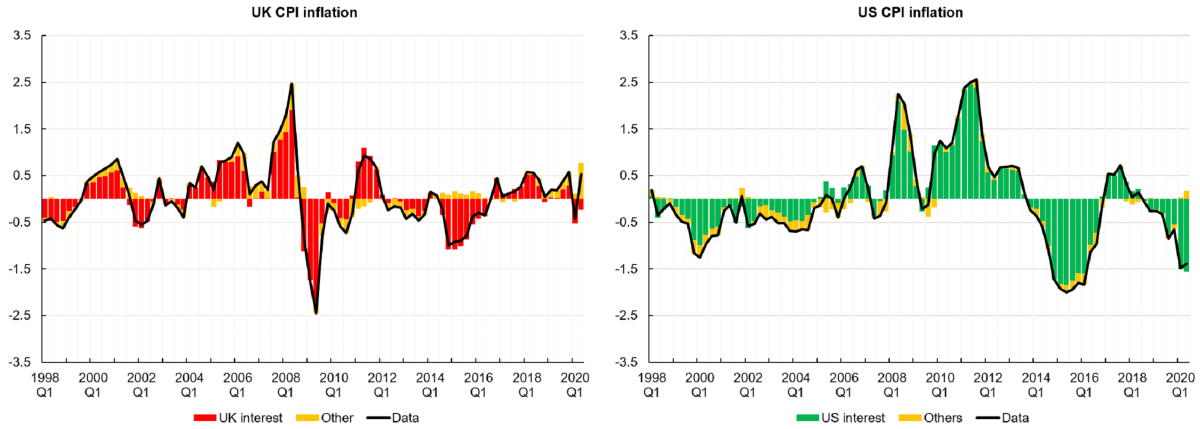
While outputs (Figure 9) in the two economies evolved in a similar pattern, they generally reflected quite different forces driving them. The UK output, which was more stable than the US's, was mainly led by monetary policy, moderated by the price mark-up of production; the interest rate shock had mainly been a force supporting growth before the global Financial crisis, but a key impeding factor in the aftermath of it. The US output was boosted by enhanced traded productivity and low price mark-up cost before the Crisis, but swung drastically thereafter following the interest rate shock. The other shocks (combined) only had a minor impact on the time paths.

Figure 9: Historical decomposition – output



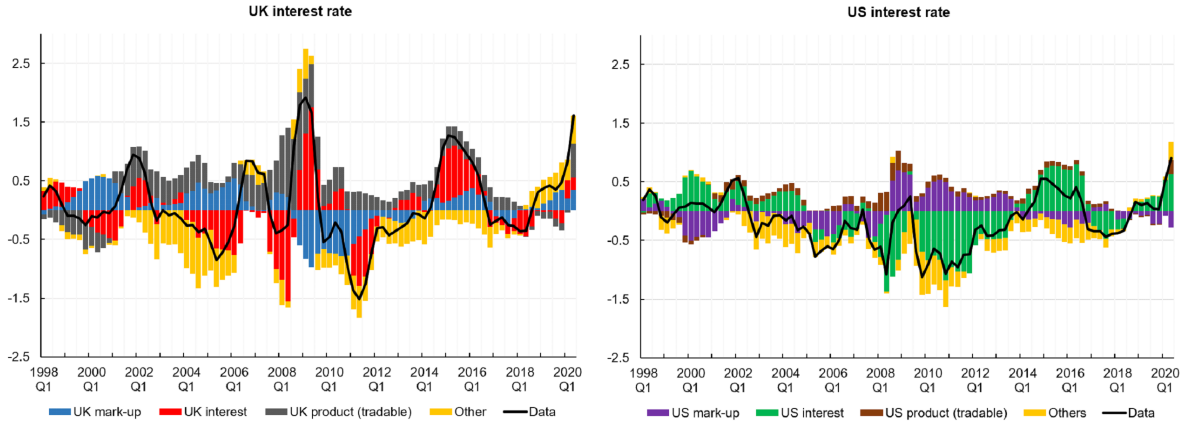
Inflation (Figure 10) in both the economies were governed by monetary policy; however, the price cycle in the two economies varied. The UK policy shocks generally had a similar size, whereas those of the US were modest before the Crisis, but much more sizable thereafter.

Figure 10: Historical decomposition – CPI inflation



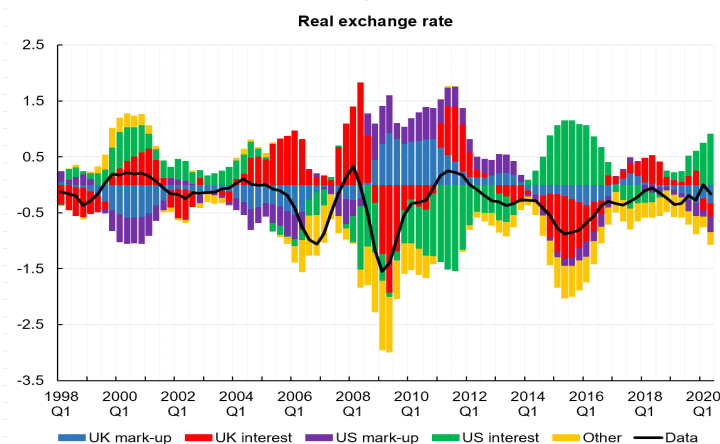
Nominal interest rate (Figure 11) in both the economies were driven by the policy shock itself, the price mark-up cost, and traded productivity. These shocks much offset one another in the US but less so in the UK; hence the much more volatile US rate.

Figure 11: Historical decomposition – nominal interest rate



The real exchange rate (Figure 12) was driven from PPP by monetary policy and mark-up shocks. It is striking how it fluctuated around the PPP value, returning towards it robustly, throughout the period. This would also apply to the relative-CPI-based real exchange rate, even though it would have experienced larger shocks.

Figure 12: Historical decomposition – real exchange rate



Overall, we find that, while the UK and US variables shared broadly similar time paths, the UK had enjoyed more output stability while the US had enjoyed more financial stability; price stability in both countries is similar, nevertheless. For both these economies, the uncertainty had come from both the demand side – due to interest rate variations, and the supply side – due to shocks to traded productivity and production costs, with little from the other sources including international shock spillovers. The central conclusion from this model is that the real exchange rate responds sharply to the full menu of model shocks but tracks robustly towards its PPP value, which is set constant at unity (thus, zero in log) throughout the period.

## 7 Conclusion

In this paper we have been concerned to test a micro-founded structural (DSGE) model in which the long-run forces of trade force the exchange rate back to PPP, while shocks to preferences, technology and policy drive it away from PPP in the short run. We tested our structural model by indirect inference to see whether its simulated behaviour could match the reduced form VARX behaviour found in the data; we found that it could do so rather well. Our conclusion is that there are good theoretical reasons why the real exchange rate converges to PPP in the long run. They stem from the forces studied in classical trade theory in which in the long run the cross-border mobility of capital and technology forces productivity, and competition forces prices, to converge across countries.

We have known for a long time now that open economy models with a short-run macro focus can match the basic dynamic behaviour of the data. However, what we have discovered in this paper is that for the UK and the US such a model cannot adequately mirror real exchange rate behaviour and is rejected by our indirect inference test when this aspect is taken into account. However the US-UK model with a long-run focus, where long-run classical forces drive the economy to PPP, is not rejected by this test but instead passes it comfortably. This validation of classical trade theory and PPP is of much importance for policy towards trade, capital movements and exchange rate intervention. It implies that we need to evaluate this policy using a model embodying PPP and taking account of the full long-run policy effects this implies.

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# Appendix

## A Full listing of the linearised model

### A.1 Home economy (UK)

Demand for tradable goods:

$$\begin{aligned} \hat{c}_{T,t} = & \frac{p_{NT}c_{NT}}{gdp} \frac{(\hat{p}_{NT,t} + \hat{c}_{NT,t})}{\phi \left[ \frac{c_T}{gdp} + \left( \frac{\alpha}{1-\alpha} \right)^{\frac{1}{v}} \left( \frac{im}{gdp} \right)^{\frac{v-1}{v}} \left( \frac{c_T}{gdp} \right)^{\frac{1}{v}} \left( \frac{1}{v} \right) \right]} \\ & - \frac{\left( \frac{\alpha}{1-\alpha} \right)^{\frac{1}{v}} \left( \frac{im}{gdp} \right)^{\frac{v-1}{v}} \left( \frac{c_T}{gdp} \right)^{\frac{1}{v}} \left( \frac{v-1}{v} \right)}{\frac{c_T}{gdp} + \left( \frac{\alpha}{1-\alpha} \right)^{\frac{1}{v}} \left( \frac{im}{gdp} \right)^{\frac{v-1}{v}} \left( \frac{c_T}{gdp} \right)^{\frac{1}{v}} \left( \frac{1}{v} \right)} \widehat{im}_t \end{aligned} \quad (A.1)$$

Demand for non-tradable goods:

$$\hat{c}_{NT,t} = E_t \hat{c}_{NT,t+1} + (E_t \hat{p}_{NT,t+1} - \hat{p}_{NT,t}) - \frac{\hat{r}_t}{1+r} - (E_t \hat{\varepsilon}_{j,t+1} - \hat{\varepsilon}_{j,t}) \quad (A.2)$$

Imports:

$$\begin{aligned} \widehat{im}_t = & \frac{p_{NT}c_{NT}}{gdp} \frac{\hat{p}_{NT,t} + \hat{c}_{NT,t} - \hat{q}_t}{\phi q \left[ \frac{im}{gdp} + \left( \frac{1-\alpha}{\alpha} \right)^{\frac{1}{v}} \left( \frac{c_T}{gdp} \right)^{\frac{v-1}{v}} \left( \frac{im}{gdp} \right)^{\frac{1}{v}} \left( \frac{1}{v} \right) \right]} \\ & - \frac{\left( \frac{1-\alpha}{\alpha} \right)^{\frac{1}{v}} \left( \frac{c_T}{gdp} \right)^{\frac{v-1}{v}} \left( \frac{im}{gdp} \right)^{\frac{1}{v}} \left( \frac{v-1}{v} \right)}{\frac{im}{gdp} + \left( \frac{1-\alpha}{\alpha} \right)^{\frac{1}{v}} \left( \frac{c_T}{gdp} \right)^{\frac{v-1}{v}} \left( \frac{im}{gdp} \right)^{\frac{1}{v}} \left( \frac{1}{v} \right)} \hat{c}_{T,t} \end{aligned} \quad (A.3)$$

Supply of labour (Tradable sector):

$$\hat{n}_{T,t} = \frac{w_{T,t} - p_{NT,t} - c_{NT,t} - (\eta - \xi) \left( 1 + \frac{y_T}{y_{NT}} \right)^{-1} \hat{n}_{NT,t}}{(\eta - \xi) \left( 1 + \frac{y_{NT}}{y_T} \right)^{-1} + \xi} \quad (A.4)$$

Supply of labour (Non-tradable sector):

$$\hat{n}_{NT,t} = \frac{w_{NT,t} - p_{NT,t} - c_{NT,t} - (\eta - \xi) \left( 1 + \frac{y_{NT}}{y_T} \right)^{-1} \hat{n}_{T,t}}{(\eta - \xi) \left( 1 + \frac{y_T}{y_{NT}} \right)^{-1} + \xi} \quad (A.5)$$

Production function (Tradable sector):

$$\hat{y}_{T,t} = \hat{z}_{T,t} + \hat{n}_{T,t} \quad (A.6)$$

Production function (Non-tradable sector):

$$\hat{y}_{NT,t} = \hat{z}_{NT,t} + \hat{n}_{NT,t} \quad (\text{A.7})$$

Price of non-tradable goods (derived from the demand for labour equation):

$$\hat{p}_{NT,t} = \hat{w}_{NT,t} - \hat{y}_{NT,t} + \hat{n}_{NT,t} \quad (\text{A.8})$$

Real marginal cost (Tradable sector):

$$\widehat{mc}_{T,t} = \hat{w}_{T,t} - \hat{z}_{T,t} \quad (\text{A.9})$$

Real marginal cost (Non-tradable sector):

$$\widehat{mc}_{NT,t} = \hat{w}_{NT,t} - \hat{z}_{NT,t} \quad (\text{A.10})$$

Domestic inflation (Tradable goods):

$$\hat{\pi}_{DT,t} = \beta E_t \hat{\pi}_{DT,t+1} + \kappa \widehat{mc}_{T,t} \quad (\text{A.11})$$

where  $\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega}$ .

Domestic inflation (Non-tradable goods):

$$\hat{\pi}_{DNT,t} = \beta E_t \hat{\pi}_{DNT,t+1} + \kappa \widehat{mc}_{NT,t} \quad (\text{A.12})$$

Domestic inflation (All-sector):

$$\hat{\pi}_{D,t} = \frac{y_{T,t}}{y_{T,t} + y_{NT,t}} \hat{\pi}_{DT,t} + \frac{y_{NT,t}}{y_{T,t} + y_{NT,t}} \hat{\pi}_{DNT,t} \quad (\text{A.13})$$

CPI inflation:

$$\hat{\pi}_t = (1 - \alpha) \hat{\pi}_{D,t} + \alpha \hat{\pi}_{DT,t}^* + \alpha \Delta \hat{Q}_t + \hat{\varepsilon}_{\pi,t} \quad (\text{A.14})$$

Taylor rule:

$$\hat{R}_t = \rho \hat{R}_{t-1} + (1 - \rho) [\phi_{\pi} \hat{\pi}_t + \phi_y (\widehat{gdp}_t - \widehat{gdp}_{t-1})] + \hat{\varepsilon}_{R,t} \quad (\text{A.15})$$

Government spending:

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \hat{\varepsilon}_{g,t} \quad (\text{A.16})$$

Balance of payment:

$$\frac{bf}{gdp} \widehat{bf}_t = \frac{bf}{gdp} \hat{r}_{t-1}^* + (1 + r^*) \frac{bf}{gdp} \widehat{bf}_{t-1} + \frac{1}{q} \frac{im^*}{gdp} (\widehat{im}_t^* - \hat{q}_t) - \frac{im}{gdp} \widehat{im}_t \quad (\text{A.17})$$

Uncovered interest parity:

$$\hat{q}_t - E_t \hat{q}_{t+1} = \hat{R}_t^* - \hat{R}_t - (E_t \hat{\pi}_{t+1}^* - E_t \hat{\pi}_{t+1}) \quad (\text{A.18})$$

Nominal exchange rate:

$$\Delta \hat{Q}_t = \Delta \hat{q}_t - \hat{\pi}_{DT,t}^* + \hat{\pi}_{DT,t} \quad (\text{A.19})$$

Fisher equation:

$$\hat{r}_t = \hat{R}_t - E_t \hat{\pi}_{t+1} \quad (\text{A.20})$$

Market clearing (Tradable sector):

$$\frac{y_T}{gdp} \hat{y}_{T,t} = \frac{c_T}{gdp} \hat{c}_{T,t} + \frac{im^*}{gdp} \widehat{im_t^*} \quad (\text{A.21})$$

Market clearing (Non-tradable sector):

$$\frac{p_{NT} y_{NT}}{gdp} \hat{y}_{NT,t} = \frac{p_{NT} c_{NT}}{gdp} \hat{c}_{NT,t} + \frac{p_{NT} g}{gdp} \hat{g}_t \quad (\text{A.22})$$

GDP:

$$\widehat{gdp}_t = \frac{y_T}{gdp} \hat{y}_{T,t} + \frac{p_{NT} y_{NT}}{gdp} (\hat{p}_{NT,t} + \hat{y}_{NT,t}) \quad (\text{A.23})$$

## A.2 Foreign economy (US)

Demand for tradable goods:

$$\begin{aligned} \hat{c}_{T,t}^* &= \frac{p_{NT}^* c_{NT}^*}{gdp^*} \frac{\hat{p}_{NT,t}^* + \hat{c}_{NT,t}^*}{\phi^* \left[ \frac{c_T^*}{gdp^*} + \left( \frac{\alpha^*}{1-\alpha^*} \right)^{\frac{1}{\nu^*}} \left( \frac{im^*}{gdp^*} \right)^{\frac{\nu^*-1}{\nu^*}} \left( \frac{c_T^*}{gdp^*} \right)^{\frac{1}{\nu^*}} \left( \frac{1}{\nu^*} \right) \right]} \\ &\quad - \frac{\left( \frac{\alpha^*}{1-\alpha^*} \right)^{\frac{1}{\nu^*}} \left( \frac{im^*}{gdp^*} \right)^{\frac{\nu^*-1}{\nu^*}} \left( \frac{c_T^*}{gdp^*} \right)^{\frac{1}{\nu^*}} \left( \frac{\nu^*-1}{\nu^*} \right)}{\frac{c_T^*}{gdp^*} + \left( \frac{\alpha^*}{1-\alpha^*} \right)^{\frac{1}{\nu^*}} \left( \frac{im^*}{gdp^*} \right)^{\frac{\nu^*-1}{\nu^*}} \left( \frac{c_T^*}{gdp^*} \right)^{\frac{1}{\nu^*}} \left( \frac{1}{\nu^*} \right)} \widehat{im_t^*} \end{aligned} \quad (\text{A.24})$$

Demand for non-tradable goods:

$$\hat{c}_{NT,t}^* = E_t \hat{c}_{NT,t+1}^* + (E_t \hat{p}_{NT,t+1}^* - \hat{p}_{NT,t}^*) - \frac{1}{1+r^*} \hat{r}_t^* - (E_t \hat{\varepsilon}_{j,t+1}^* - \hat{\varepsilon}_{j,t}^*) \quad (\text{A.25})$$

Imports:

$$\begin{aligned} \widehat{im_t^*} &= \frac{p_{NT}^* c_{NT}^*}{gdp^*} \frac{q (\hat{p}_{NT,t}^* + \hat{c}_{NT,t}^* + \hat{q}_t)}{\phi^* \left[ \frac{im^*}{gdp^*} + \left( \frac{1-\alpha^*}{\alpha^*} \right)^{\frac{1}{\nu^*}} \left( \frac{c_T^*}{gdp^*} \right)^{\frac{\nu^*-1}{\nu^*}} \left( \frac{im^*}{gdp^*} \right)^{\frac{1}{\nu^*}} \left( \frac{1}{\nu^*} \right) \right]} \\ &\quad - \frac{\left( \frac{1-\alpha^*}{\alpha^*} \right)^{\frac{1}{\nu^*}} \left( \frac{c_T^*}{gdp^*} \right)^{\frac{\nu^*-1}{\nu^*}} \left( \frac{im^*}{gdp^*} \right)^{\frac{1}{\nu^*}} \left( \frac{\nu^*-1}{\nu^*} \right)}{\frac{im^*}{gdp^*} + \left( \frac{1-\alpha^*}{\alpha^*} \right)^{\frac{1}{\nu^*}} \left( \frac{c_T^*}{gdp^*} \right)^{\frac{\nu^*-1}{\nu^*}} \left( \frac{im^*}{gdp^*} \right)^{\frac{1}{\nu^*}} \left( \frac{1}{\nu^*} \right)} \hat{c}_{T,t}^* \end{aligned} \quad (\text{A.26})$$

Supply of labour (Tradable sector):

$$\hat{n}_{T,t}^* = \frac{w_{T,t}^* - p_{NT,t}^* - c_{NT,t}^* - (\eta^* - \xi^*) \left(1 + \frac{y_T^*}{y_{NT}^*}\right)^{-1} \hat{n}_{NT,t}^*}{(\eta^* - \xi^*) \left(1 + \frac{y_T^*}{y_{NT}^*}\right)^{-1} + \xi^*} \quad (\text{A.27})$$

Supply of labour (Non-tradable sector):

$$\hat{n}_{NT,t}^* = \frac{w_{NT,t}^* - p_{NT,t}^* - c_{NT,t}^* - (\eta^* - \xi^*) \left(1 + \frac{y_{NT}^*}{y_T^*}\right)^{-1} \hat{n}_{T,t}^*}{(\eta^* - \xi^*) \left(1 + \frac{y_{NT}^*}{y_T^*}\right)^{-1} + \xi^*} \quad (\text{A.28})$$

Production function (Tradable sector):

$$\hat{y}_{T,t}^* = \hat{z}_{T,t}^* + \hat{n}_{T,t}^* \quad (\text{A.29})$$

Production function (Non-tradable sector):

$$\hat{y}_{NT,t}^* = \hat{z}_{NT,t}^* + \hat{n}_{NT,t}^* \quad (\text{A.30})$$

Price of non-tradable goods (derived from the demand for labour equation):

$$\hat{p}_{NT,t}^* = \hat{w}_{NT,t}^* - \hat{y}_{NT,t}^* + \hat{n}_{NT,t}^* \quad (\text{A.31})$$

Real marginal cost (Tradable sector):

$$\widehat{mc}_{T,t}^* = \hat{w}_{T,t}^* - \hat{z}_{T,t}^* \quad (\text{A.32})$$

Real marginal cost (Non-tradable sector):

$$\widehat{mc}_{NT,t}^* = \hat{w}_{NT,t}^* - \hat{z}_{NT,t}^* \quad (\text{A.33})$$

Domestic inflation (Tradable goods):

$$\hat{\pi}_{DT,t}^* = \beta^* E_t \hat{\pi}_{DT,t+1}^* + \kappa^* \widehat{mc}_{T,t}^* \quad (\text{A.34})$$

where  $\kappa^* = \frac{(1-\omega^*)(1-\beta^*\omega^*)}{\omega^*}$ .

Domestic inflation (Non-tradable goods):

$$\hat{\pi}_{DNT,t}^* = \beta^* E_t \hat{\pi}_{DNT,t+1}^* + \kappa^* \widehat{mc}_{NT,t}^* \quad (\text{A.35})$$

Domestic inflation (All-sector):

$$\hat{\pi}_{D,t}^* = \frac{y_{T,t}^*}{y_{T,t}^* + y_{NT,t}^*} \hat{\pi}_{DT,t}^* + \frac{y_{NT,t}^*}{y_{T,t}^* + y_{NT,t}^*} \hat{\pi}_{DNT,t}^* \quad (\text{A.36})$$

CPI inflation:

$$\hat{\pi}_t^* = (1 - \alpha^*)\hat{\pi}_{D,t}^* + \alpha^*\hat{\pi}_{DT,t} - \alpha^*\Delta\hat{Q}_t + \hat{\varepsilon}_{\pi,t}^* \quad (\text{A.37})$$

Taylor rule:

$$\hat{R}_t^* = \rho^*\hat{R}_{t-1}^* + (1 - \rho^*)[\phi_\pi^*\hat{\pi}_t^* + \phi_y^*(\widehat{gdp}_{t-1}^* - \widehat{gdp}_{t-1}^*)] + \hat{\varepsilon}_{R,t}^* \quad (\text{A.38})$$

Government spending:

$$\hat{g}_t^* = \rho_g^*\hat{g}_{t-1}^* + \hat{\varepsilon}_{g,t}^* \quad (\text{A.39})$$

Balance of payment:

$$\frac{bf^*}{gdp^*}\widehat{bf}_t^* = \frac{bf^*}{gdp^*}\hat{r}_{t-1} + (1 + r)\frac{bf^*}{gdp^*}\widehat{bf}_{t-1}^* + q\frac{im}{gdp^*}(\widehat{im}_t + \hat{q}_t) - \frac{im^*}{gdp^*}\widehat{im}_t^* \quad (\text{A.40})$$

Fisher equation:

$$\hat{r}_t^* = \hat{R}_t^* - E_t\hat{\pi}_{t+1}^* \quad (\text{A.41})$$

Market clearing (Tradable sector):

$$\frac{y_T^*}{gdp^*}\hat{y}_{T,t}^* = \frac{c_T^*}{gdp^*}\hat{c}_{T,t}^* + \frac{im}{gdp^*}\widehat{im}_t \quad (\text{A.42})$$

Market clearing (Non-tradable sector):

$$\frac{p_{NT}^*y_{NT}^*}{gdp^*}\hat{y}_{NT,t}^* = \frac{p_{NT}^*c_{NT}^*}{gdp^*}\hat{c}_{NT,t}^* + \frac{p_{NT}^*g^*}{gdp^*}\hat{g}_t^* \quad (\text{A.43})$$

GDP:

$$\widehat{gdp}_t^* = \frac{y_T^*}{gdp^*}\hat{y}_{T,t}^* + \frac{p_{NT}^*y_{NT}^*}{gdp^*}(\hat{p}_{NT,t}^* + \hat{y}_{NT,t}^*) \quad (\text{A.44})$$

### A.3 Shock processes

UK preference, price mark-up, monetary policy and government spending shocks:

$$\hat{\varepsilon}_{i,t} = \rho_i\hat{\varepsilon}_{i,t-1} + u_{i,t} \quad (\text{A.45})$$

where  $i = j, \pi, R, g$ ,  $u_{i,t} \sim iid$ .

US preference, price mark-up, monetary policy and government spending shocks:

$$\hat{\varepsilon}_{i,t}^* = \rho_i^*\hat{\varepsilon}_{i,t-1}^* + u_{i,t}^* \quad (\text{A.46})$$

where  $i = j, \pi, R, g$ ,  $u_{i,t} \sim iid$ .

Evolution of productivity shocks:

$$\begin{aligned}
\begin{bmatrix} \Delta \hat{z}_{T,t} \\ \Delta \hat{z}_{NT,t} \\ \Delta \hat{z}_{T,t}^* \\ \Delta \hat{z}_{NT,t}^* \end{bmatrix} &= \begin{bmatrix} \Gamma_T \\ \Gamma_{NT} \\ \Gamma_T^* \\ \Gamma_{NT}^* \end{bmatrix} + \begin{bmatrix} -f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & -f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & -f_{33} & f_{34} \\ f_{41} & f_{42} & f_{43} & -f_{44} \end{bmatrix} \begin{bmatrix} \Delta \hat{z}_{T,t-1} \\ \Delta \hat{z}_{NT,t-1} \\ \Delta \hat{z}_{T,t-1}^* \\ \Delta \hat{z}_{NT,t-1}^* \end{bmatrix} + \begin{bmatrix} u_{ZT,t} \\ u_{ZNT,t} \\ u_{ZT,t}^* \\ u_{ZNT,t}^* \end{bmatrix} \\
&+ \begin{bmatrix} -d_{11} & -d_{12} & -d_{23} & 0 & 0 & 0 \\ d_{21} & 0 & 0 & -d_{22} & -d_{23} & 0 \\ 0 & d_{31} & 0 & d_{32} & 0 & -d_{33} \\ 0 & 0 & d_{41} & 0 & d_{41} & d_{43} \end{bmatrix} \begin{bmatrix} \hat{z}_{T,t-1} - \hat{z}_{NT,t-1} \\ \hat{z}_{T,t-1} - \hat{z}_{T,t-1}^* \\ \hat{z}_{T,t-1} - \hat{z}_{NT,t-1}^* \\ \hat{z}_{NT,t-1} - \hat{z}_{T,t-1}^* \\ \hat{z}_{NT,t-1} - \hat{z}_{NT,t-1}^* \\ \hat{z}_{T,t-1}^* - \hat{z}_{NT,t-1}^* \end{bmatrix}
\end{aligned} \tag{A.47}$$

where  $u_{ZT,t}, u_{ZNT,t}, u_{ZT,t}^*, u_{ZNT,t}^* \tilde{iid}$ .

#### A.4 Terminal conditions

There is no change in the UK's holding of US bonds ( $\Delta bf_{SS} = 0$ ), which implies:

$$\widehat{bf}_{SS} = \left( r^* \frac{bf}{gdp} \right)^{-1} \left[ \frac{im}{gdp} \widehat{im}_{SS} - \frac{1}{q} \frac{im^*}{gdp} \left( \widehat{im}_{SS}^* - \hat{q}_{SS} \right) \right] \tag{A.48}$$

There is no change in the US's holding of UK bonds ( $\Delta bf_{SS}^* = 0$ ), which implies:

$$\widehat{bf}_{SS}^* = \left( r \frac{bf^*}{gdp^*} \right)^{-1} \left[ \frac{im^*}{gdp^*} \widehat{im}_{SS}^* - q \frac{im}{gdp^*} \left( \widehat{im}_{SS} + \hat{q}_{SS} \right) \right] \tag{A.49}$$

The real exchange rate is equal to unity, which implies:

$$\hat{q}_{SS} = 0 \tag{A.50}$$

Productivity in different sectors converges to the long-run level ( $\hat{z}_{SS}$ ), such that:

$$\hat{z}_{T,SS} = \hat{z}_{NT,SS} = \hat{z}_{T,SS}^* = \hat{z}_{NT,SS}^* = \hat{z}_{SS} \tag{A.51}$$

where  $\hat{z}_{SS} = \frac{1}{4} (\hat{z}_{T,SS} + \hat{z}_{NT,SS} + \hat{z}_{T,SS}^* + \hat{z}_{NT,SS}^*)$ .

## B Data sources, time series collected, and adjustments to the raw data

The data sample spans from 1997Q1 to 2020Q4. The observed variables are the real exchange rate, and GDP, non-tradable consumption, imports, government spending, labour hours, the real wage rate, domestic-price inflation (in both the tradable and non-tradable sectors), CPI inflation and nominal interest rate of both countries. Nominal interest rate, inflation and the real wage rate are measured as quarterly rates. The other variables, which are defined in real and per-capita term, are measured in natural logarithm. Unfiltered data are used. The data sources, the time series collected, and the adjustments to the raw data are detailed Table B.1.

Table B.1: Measurement, sources & adjustments of the raw data

Observable variables	Time series collected	Source <sup>a</sup>	Divided by CPI?	Divided by pop.?	Seasonally adjusted ?
GDP	Real GDP	ONS, FRED	✓	✓	✓
Cons. (Non-tradable) <sup>b</sup>	Real consumption	ONS, FRED	✓	✓	✓
Imports	Real imports	ONS, FRED	✓	✓	✓
Gov. spending	Real gov. expenditure	ONS, FRED	N.A.	N.A.	✓
Lab. hours (Tradable)	Working hours	ONS, FRED	N.A.	N.A.	✓
Lab. hours (Non-tradable)	Working hours	ONS, FRED	N.A.	N.A.	✓
Real wage rate (Tradable)	Real wage	ONS, FRED	N.A.	N.A.	✓
Real wage rate (Non-tradable)	Real wage	ONS, FRED	N.A.	N.A.	✓
Domestic infl. (Tradable)	GDP deflator	ONS, FRED	N.A.	N.A.	✓
Domestic infl. (Non-tradable)	GDP deflator	ONS, FRED	N.A.	N.A.	✓
CPI inflation	Consumer price index	ONS, FRED	N.A.	N.A.	✓
Nom. interest rate	Bank lending rate	IMF	N.A.	N.A.	N.A.
Real exchange rate	$SP_t^*/P_t$	FRED	N.A.	N.A.	N.A.

*a*: ONS (Office for National Statistics); FRED (Federal Reserve Economic Data); IMF (International Monetary Fund).

*b*: Tradable goods are defined as those whose export-to-total-production ratio is  $> 10\%$  according to the

OECD Input-output Supply table; non-tradable goods are those whose export-to-total-production ratio is  $\leq 10\%$ .