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## Sequential information selling: Perfect price discrimination and the role of encryption

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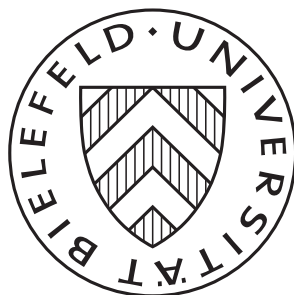


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# Sequential Information Selling: Perfect Price Discrimination and the Role of Encryption

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Manuel Foerster and Fynn Louis Närmann



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# Sequential Information Selling: Perfect Price Discrimination and the Role of Encryption

Manuel Foerster\*      Fynn Louis Närmann<sup>†</sup>

June 24, 2025

## Abstract

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We study a dynamic game in which a monopolistic seller sequentially discloses information about a binary state to a consumer through priced experiments. The consumer privately observes a binary signal which influences her willingness to pay for information. We show that if buyer types favor different actions but their willingness to pay for a state-revealing test is sufficiently close, then the seller can commit to a sequence of priced experiments that extracts the entire surplus of both consumer types simultaneously. The optimal sequence of experiments is such that the high-valuation type assigns a higher probability to outcomes that trigger further information acquisition, thus creating a difference in expected costs. As a key element of the construction, we introduce an ‘encryption protocol’ under which the consumer faces a stopping problem. We then characterize situations in which the seller strictly benefits from a dynamic selling strategy when perfect price discrimination is not feasible. Finally, we illustrate our framework in the context of medical diagnostic testing, showing that a free test followed by a state-revealing test is often sufficient to improve revenue in comparison with a static approach.

**Keywords:** Information design, dynamic mechanism, selling information, encryption, price discrimination.

**JEL:** D42, D81, D82, D83, C73.

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## 1. Introduction

Information goods are an increasingly vital component of modern economies. However, their unique characteristics pose significant challenges to traditional market mechanisms. For instance, they usually entail substantial fixed costs for initial production, yet they can be reproduced and distributed at nearly zero marginal cost. Developing tailored mechanisms is essential for incentivizing the creation and dissemination of valuable information. The digital era has not only amplified the economic importance of information but has also enhanced the technological feasibility of implementing dynamic selling protocols. Online platforms, for instance, allow for interactions to be meticulously staged and tracked, making the sequential disclosure of information more practically relevant than ever.

This paper investigates the strategic interaction between a monopolistic seller who possesses access to informative experiments and consumers with heterogeneous valuations. The seller offers information about an unknown binary state of the world through a finite sequence of priced experiments, which can partially or fully reveal the true state. The buyer's optimal action depends on the state. A critical element of the model is the consumer's private information: she observes a binary signal that shapes her initial belief about the state of the world. This private signal leads to heterogeneity in consumer valuations, distinguishing between a 'high-valuation type' and a 'low-valuation type'.

The seller's objective is to maximize revenue. The ideal scenario is to achieve perfect price discrimination, meaning extracting the entire consumer surplus from each buyer type. The central research question is: What is the optimal sequential information disclosure strategy for a revenue-maximizing monopolist facing privately informed consumers? The sequential nature of the interaction is pivotal because it enables the seller to utilize the buyer's evolving beliefs.

As an illustration, suppose consumers would like to distinguish between two diseases with similar symptoms, such as COVID and the flu. Effective treatment requires knowing the correct illness. Consider two individuals who exhibit general cold symptoms: one's symptoms suggest that COVID is more likely (high type), while the other's tend towards the flu (low type). The seller, say some governmental agency, is interested in raising revenue to keep costs for taxpayers down.<sup>1</sup> Suppose that the high type is 'more uncertain' and thus willing to pay more for a revelation of the state. The seller offers a sequence of two priced tests. The first test may reveal the illness to be the flu or provide partial information indicating towards a case of COVID, resembling a rapid 'antigen test'. If the first test is inconclusive, a second test is offered which determines the illness *conditional*

<sup>1</sup>One may object that the government's primary aim should be to inform consumers. As we will see, consumers obtaining perfect information is a by-product of revenue maximization.

on knowing the result of the first test. As we will see, *encrypting* the second test in this way allows the seller to extract more revenue, as it prevents the buyer from purchasing solely the second test.<sup>2</sup> The seller will price the tests such that the total expected cost for the low type coincides with her valuation. Importantly, the high-type buyer, who perceives a higher likelihood of getting an inconclusive result from the first test, ends up paying more in expectation. Remarkably, we can find parameters such that the seller also extracts the entire surplus of the high type with these two tests although their valuations differ, which is not feasible in a static setting à la Bergemann, Bonatti, and Smolin (2018, henceforth BBS).

Our main result establishes that this insight holds more generally: If buyer types are ‘non-congruent’, meaning their initial private signals predispose them to favor different actions, but their valuations for full state revelation are close enough, then the seller can design a sequence of priced experiments to extract the entire surplus from both types simultaneously. As a key element of the construction of the optimal mechanism, we introduce an ‘encryption protocol’. This protocol includes a coin toss with each experiment, which allows to design the experiments in the sequence such that they are perceived as uninformative unless all preceding outcomes of experiments have been observed. This turns the consumer’s optimal purchase strategy into a stopping problem, which simplifies the analysis. In particular, both types exhibit an identical purchasing behavior under the optimal mechanism: They purchase until the state is revealed (or maturity is reached). The sequence of experiments is structured such that, due to the initial beliefs, the high-valuation buyer assigns a higher ex-ante probability to outcomes that necessitate further information acquisition (i.e., outcomes that do not immediately reveal the state). Therefore, the number of experiments they expect to purchase differs although they follow the same purchasing behavior. This enables the seller to price the experiments so that the total expected costs for each type aligns exactly with their respective valuation for a state-revealing test.

Furthermore, the analysis reveals that the largest ‘valuation gap’ between the two types for which perfect price discrimination remains feasible increases in the length of the sequence of experiments. More periods allow for finer manipulation of beliefs and expected costs. It is worth to note that the constructed sequence of priced experiments that leads to perfect price discrimination involves uniform costs from the third period onward. In the special case that the valuation gap coincides with the highest achievable difference in expected costs in  $T$  periods, our construction leads to a sequence of  $T$  experiments with uniform costs, including also the first two experiments.

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<sup>2</sup>Of course, if it were possible to offer the second test conditional on the purchase of the first test, then such encryption were not necessary. All results would go through under such an assumption.

The paper then explores more generally when a dynamic approach can generate higher revenue than an optimal static menu of experiments à la BBS. They show that the optimal static approach is to sell a single state-revealing test at the valuation of the low type if this type is common enough. In particular, perfect price discrimination is possible only if valuations coincide, i.e., if the valuation gap is zero. We then show that the seller can generate a higher revenue compared to the static benchmark under a large range of parameters, including any non-congruent valuations. Interestingly, it often suffices to employ a dynamic strategy in which a free experiment designed to align valuations if the result is inconclusive is followed by a state-revealing experiment at a price equal to both types' updated valuation. Coming back to the example, the initial free experiment can be understood as a rapid antigen test for COVID, and the subsequent priced experiment resembles a 'PCR test', which precisely determines the illness. Since the high-type buyer perceives a higher likelihood of getting an inconclusive result from the first test, the seller can extract the entire surplus from the low-valuation type, while extracting strictly more than that from the high-valuation type.

### 1.1. Related Literature

There are many authors who have discussed the need for tailored designs of mechanisms for selling information (e.g., Babaioff, Kleinberg, and Paes Leme, 2012), because many of the results that hold for traditional goods fail to apply when dealing with information goods. Varian (1999) provides an elementary discussion of the challenges as well as the intriguing possibilities of information as an economic good.

Our analysis considers a dynamic extension of the work of BBS, who analyzed a static scenario in which a monopolistic seller offers a menu of statistical experiments to a buyer whose willingness to pay for additional information depends on her initial private information. They characterize the optimal menu and show that it generally includes a fully informative experiment and, in some cases, also a partially informative experiment in order to receive different payments from each type of buyer. However, a key limitation of the static framework is that achieving perfect price discrimination when buyer valuations differ is impossible. Our work contrasts with this limitation by demonstrating that a dynamic approach can overcome it, provided that types are non-congruent and their valuations are sufficiently close. The buyer's ability to update beliefs based on observed outcomes and make sequential purchasing decisions is fundamental to this result. In a static model, the seller cannot leverage this belief-updating process: If a menu extracts the low type's full valuation, the high type can mimic the low type's purchasing behavior, thus effectively capping her payment at the low type's valuation.

Hörner and Skrzypacz (2016) explore a dynamic model in which an informed agent

possesses divisible hard evidence that is gradually revealed to an uninformed firm. They show that splitting information over time can help the agent to extract more surplus from the interaction. However, it does not suffice to extract the entire surplus in contrast to our main result. Similarly, Honryo (2018) studies a dynamic communication game, where a sender is endowed with verifiable arguments. He shows that the sender gains from committing to a stochastic mechanism that reports those arguments over multiple periods. While these works highlight the strategic advantages of dynamic disclosure, our paper has a different focus. Here, the seller offers experiments that partially reveal the state of the world instead of releasing verifiable hard evidence.

Babaioff, Kleinberg, and Paes Leme (2012) also investigated optimal mechanisms for a monopolistic data provider. The authors compare different types of mechanisms to sell information. They demonstrate that multiple rounds of partial information disclosure can achieve higher revenue than selling information in a ‘sealed envelope’.<sup>3</sup> In contrast to their analysis, we present a specific construction of a sequential mechanism. In their multi-round protocol, they assume that the seller monitors the purchasing decisions and makes the next round available only if the consumer purchases the previous round. The encryption protocol introduced here provides a microfoundation for implementing such multi-round protocols. Interestingly, the authors claim that it is an open problem to successfully integrate cryptography in information theory.

Urbano and Vila (2002) develop cryptographic techniques that can be applied to the design of strategic communication mechanisms. They state that the main assumption in cryptography is that agents are limited in their computational power. The encryption protocol applied in this paper does not require such an assumption of boundedly rational consumers. Specifically, in our approach, the seller encrypts each test result and embeds a key in the combination of all earlier test results that is required to decrypt it. The gain of implementing such an encryption is that it induces a unique rational purchasing order of tests. In particular, it prevents consumers to buy later experiments, which may be more informative, without having purchased all previous ones.

Our research is also related to the literature on Bayesian persuasion, which was pioneered by Kamenica and Gentzkow (2011). In the Bayesian persuasion model, a sender who does not charge for information commits to an information structure in order to influence the actions of a consumer whose preferences are well-known. However, the model in this paper diverges in several crucial aspects. First, the seller’s objective is to maximize revenue through priced experiments. In classic Bayesian persuasion, the seller does not monetize the information directly via prices, but the induced action has a direct effect on the seller’s utility. Second, the buyer in our model possesses private information,

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<sup>3</sup>This refers to write the information on a paper, put it inside an envelope and treat it as a regular good.

which introduces a problem absent from the basic Kamenica-Gentzkow setup. Third, in our model, the seller controls the belief formation process in a dynamic environment. However, the ‘persuasion’ aspect is embedded in the design of each experiment, which shapes beliefs in a way that encourages the buyer to continue along the seller’s revenue-maximizing path.

Recent research has extended Bayesian persuasion to dynamic settings. For example, Che, Kim, and Mierendorff (2023) analyze a dynamic model where generating and processing information is costly for both the seller and receiver over time, and neither party can commit to future actions. The objective of the seller is to keep the consumer engaged, which differs from the approach in this paper, where each experiment comes with an individual price tag and thus has a different relevance for the seller. Ball (2023) studies the optimal provision of information in a dynamic interaction where the state is evolving, and the seller uses the threat of concealing future information to maintain influence. Specifically, the seller’s optimal dynamic information policy is to report the value of the state with a delay that shrinks over time and to commit to stop reporting once a report is ignored by the consumer.

The seller commitment assumed in this paper (to the sequence of experiments and their prices upfront) is a standard assumption in much of the mechanism design literature. This contrasts with the model of Mora and Rodriguez (2024) that explores a dynamic extension of BBS without such a commitment. Furthermore, they deviate from the latter paper by introducing prior disagreement instead of a private information observed by the consumer. In their model, the seller and receiver agree to disagree about their beliefs. However, their results share some similarities. Using a dynamic programming approach, they prove that the seller benefits from the dynamic framework. However, full surplus extraction (from the viewpoint of the seller) necessitates the commitment ability of the seller and is thus not present in their analysis. Furthermore, they have shown that in a two period environment, it is optimal for the seller to first offer a free sample test and a subsequent fully-revealing test. Due to the additional commitment power in our model, this selling strategy is in general not optimal. However, we show that the seller can benefit, in comparison to a static environment, from a free sample test that aligns the valuations of the types, and characterize situations in which he strictly gains.

To our knowledge, we are the first to study a dynamic extension of BBS in which the seller can commit to a sequence of experiments and show that perfect price discrimination is feasible despite buyers’ valuations being different. Furthermore, we introduce an encryption protocol as a key element of the construction of the optimal mechanism.

The remainder of this paper is organized as follows. We start by providing the illustrative example on medical diagnostic testing in Section 2. Section 3 provides the formal



model, detailing the construction of experiments, the timing of the game as well as the strategies and payoff functions. Section 4 begins by formally introducing the encryption protocol that underpins the sequential disclosure protocol. Afterwards, the main theoretical results are presented: Section 4.3 details the construction of the dynamic mechanism and provides the proof for achieving perfect price discrimination with non-congruent types. Section 4.5 compares the outcomes of this sequential approach with those achievable under a static framework and establishes conditions under which the seller strictly benefits from a dynamic mechanism. Section 5 concludes and discusses relevant modeling assumptions.

## 2. Example: Medical diagnostic Testing

Consider two diseases with similar symptoms, such as COVID (C) and the flu (F). Effective management of each disease requires specific actions – such as appropriate medication and measures to prevent further spread. Formally, let the set of possible states and aligned actions be

$$\Omega = \{C, F\} = A.$$

We assume that taking the correct action yields an ex-post payoff of 1, while taking the incorrect action yields 0, i.e.,  $u(a, \omega) = \mathbb{I}_{[a=\omega]}$  for all  $a \in A$  and  $\omega \in \Omega$ , where  $\mathbb{I}_{[\cdot]}$  denotes the indicator function.

Suppose we have two individuals who both have cold symptoms and can narrow down their illness to be either COVID or the flu. However, one person's set of symptoms suggests that COVID is more likely, while the other person's symptoms tend towards the flu. For concreteness, we treat each set of symptoms as a private signal that classifies the individual into one of two types: a 'high type', with  $\theta_h = \mathbb{P}[\omega = C \mid h] = 0.64$  and a 'low type', with  $\theta_l = \mathbb{P}[\omega = C \mid l] = 0.325$ .

We measure each type's willingness to pay for the revealing information by the expected reduction in misaligned actions. For instance, if the high type does not purchase any additional information, she will take action  $a = C$  and err with probability  $1 - \theta_h$ . Thus, her willingness to pay for the revealing information is given by

$$V(\theta_h) = 1 - \theta_h = 0.36.$$

Similarly, the low type's probability of erring is  $\theta_l$  (since she would choose  $a = F$  absent further information), leading to

$$V(\theta_l) = \theta_l = 0.325.$$

Now consider a monopolistic seller who offers tests that can fully or partially reveal the state of the world. Formally, a *test* (or experiment) is defined by a stochastic mapping:

$$\sigma_t : \Omega \times M^{t-1} \rightarrow \Delta(M),$$

where  $t = 1, 2, \dots, T$  indexes the period within a finite time horizon  $T \in \mathbb{N}$ . This mapping specifies, for each possible state and all previous realizations, a probability distribution over the finite signal space  $M$ . An intuitive interpretation is that the tests are performed one at a time, with the seller deciding which test to perform next based on previous results. However, we will show in Section 4.1 that, for our purposes, it is sufficient to select each subsequent test solely on the basis of events with objective – that is, state-independent and thus agreed upon by both types – probabilities. As a result, there is no need to perform a test that is not purchased; the seller only requires a randomization device to determine the sequence of tests.

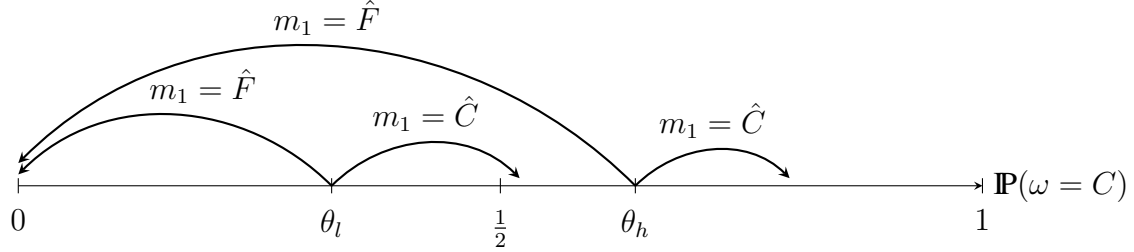
At the beginning of the game, the seller commits to implement a sequence of priced tests  $((\sigma_t, c_t))_{t=1}^T$ . If the buyer decides to purchase the test  $\sigma_t$  in period  $t$ , she pays the costs  $c_t \geq 0$  to the seller, and observes the realized outcome  $m_t \in M$ . The buyer then updates her belief according to Bayes' rule.

BBS analyze a static scenario in which all tests are offered simultaneously and both buyer types choose their preferred tests. In such a setting, the seller can only extract the full surplus from both buyer types if their valuations are equal, i.e.,  $V(\theta_h) = V(\theta_l)$ . The reasoning is straightforward: for the low-type buyer to pay her full valuation, there must be a subset of tests that reveal the state with certainty at a total cost of  $V(\theta_l)$ . However, this implies that the high-type buyer could choose the same subset and also pay  $V(\theta_l) \leq V(\theta_h)$ , preventing the seller from capturing any additional surplus from the high type.

In contrast, our approach adopts a sequential framework: the buyer purchases a test, observes its outcome, and then decides whether to stop or buy another test based on the result. We will show that, for the given valuations, a sequence of two priced experiments is sufficient to perform perfect price discrimination, thus extracting the heterogeneous valuations of both individuals. Moreover, it is sufficient to consider experiments that generate only two possible signals, i.e.,  $M = \{\hat{C}, \hat{F}\}$ . The initial test  $\sigma_1$  is summarized in the table below, which gives the probability distribution over the signal space  $M$  for each state in  $\Omega$ :

$\sigma_1(m_1 \mid \omega)$	$\hat{C}$	$\hat{F}$
$C$	1	0
$F$	$\frac{4}{9}$	$\frac{5}{9}$

Under the first test, observing the signal  $m_1 = \hat{F}$  reveals that  $\omega = F$ . Conversely, receiving  $m_1 = \hat{C}$  shifts the buyer's belief toward  $\omega = C$  due to Bayesian consistency. The resulting posterior beliefs are depicted in Figure 1. Notably, the low-type buyer



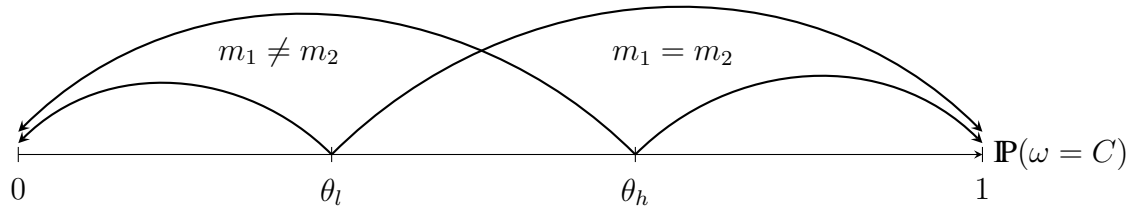
**Figure 1.** Posteriors induced by  $\sigma_1$ .

assigns a higher probability to observing  $m_1 = \hat{F}$  than the high-type buyer does, because she considers  $\omega = F$  to be more likely. Consequently, the low-type buyer expects to identify the state from the first test result with a higher likelihood.

The second test confirms,  $m_2 = \hat{C}$ , or falsifies,  $m_2 = \hat{F}$ , the initial assessment. In other words, a perfect test is performed, but the choice of the test depends on the first outcome: a ‘COVID test’ follows if  $m_1 = \hat{C}$ , while a ‘flu test’ follows if  $m_1 = \hat{F}$ . Formally, the second test is described by the following signal mappings:

$\sigma_2(m_2 \mid \omega, m_1 = \hat{C})$	$\hat{C}$	$\hat{F}$	$\sigma_2(m_2 \mid \omega, m_1 = \hat{F})$	$\hat{C}$	$\hat{F}$
$C$	1	0	$C$	0	1
$F$	0	1	$F$	1	0

This testing procedure ensures that the state is revealed with certainty in the second period if and only if the consumer is aware of the signal realization in the first period. The resulting posterior beliefs are illustrated in Figure 2. Thus, the initial test not only



**Figure 2.** Posteriors induced by  $\sigma_1$  and  $\sigma_2$ .

provides an imperfect signal, but also contains the information about which test will be

performed in the second period. In other words, the first test contains an *encryption key* that is necessary to decipher the full information that is contained in the second test result.

If the buyer chooses to purchase only the second test, she is unaware of the specific stochastic mapping that generates the signal. However, the second test still provides some information. In particular, if  $\omega = C$ , the initial signal realization is always truthful and will therefore be confirmed in the second period. If  $\omega = F$ , the signal probabilities are effectively reversed: a first period test result of  $m_1 = \hat{C}$  will be falsified,  $m_2 = \hat{F}$ , in the second period, while a signal realization of  $m_1 = \hat{F}$  will be confirmed,  $m_2 = \hat{C}$ . As can be seen from the table below, the second test, taken in isolation, is less informative (in the Blackwell sense) than the first test:

$\mathbb{P}(m_2 \mid \omega)$	$\hat{C}$	$\hat{F}$
$C$	1	0
$F$	$\frac{5}{9}$	$\frac{4}{9}$

We now turn to the pricing for each test. To extract the full surplus from both types, each of them must learn the true state with certainty. Otherwise, an immediate action would dominate any purchasing strategy with an expected cost equal to the buyer's valuation. Therefore, it is not sufficient to implement a pricing scheme where only the second test is purchased. Furthermore, if the first test yields  $m_1 = \hat{F}$ , the buyer will identify the state as  $\omega = F$  and will not buy the second test. Therefore, the price of the second test must be set so that both types have an incentive to buy it after observing  $m_1 = \hat{C}$ . In particular, when observing  $m_1 = \hat{C}$  the high type has a lower probability to err (see Figure 1) and thus a lower willingness to pay in comparison to the low type, i.e.:

$$\mathbb{P} \left[ F \mid h, m_1 = \hat{C} \right] = \frac{\frac{4}{9} \cdot 0.36}{0.64 + \frac{4}{9} \cdot 0.36} = 0.2 < 0.48 = \mathbb{P} \left[ F \mid l, m_1 = \hat{C} \right].$$

Since the second test, conditional on  $m_1 = \hat{C}$ , reveals the state with certainty, the seller can ask them to pay their complete residual valuation. Thus, the highest price the seller can charge and still ensure that both types will buy the second test is given by

$$c_2 = \mathbb{P} \left[ F \mid h, m_1 = \hat{C} \right] = 0.2.$$

One might ask how the seller can still extract all of the low type's surplus, given that she is paying strictly less than her willingness to pay for the second test. The key insight is that, in anticipation of receiving this discount at the second test, the low type is willing

to pay more for the first test than its immediate informational value. Consequently, in order to capture all of her surplus, the seller sets the price of the first test so that the total expected cost of purchasing the first test – and, conditional on  $m_1 = \hat{C}$ , the second test – is equal to her valuation. Formally, we choose:

$$c_1 = V(\theta_l) - \mathbb{P} \left[ m_1 = \hat{C} \mid l \right] \cdot c_2 = 0.325 - \left( 0.325 + \frac{4}{9} \cdot 0.675 \right) \cdot c_2 = 0.2.$$

Notably, we arrive at a uniform pricing scheme. More insights on this follow in the analysis.

Neither type of consumer benefits from buying only the second test, as the first test is more informative for the same price. However, we have chosen the price of the second test so that it will be purchased if a consumer has bought the first test and it has realized as  $m_1 = \hat{C}$ . Therefore, both types compare only two purchasing strategies: (i) not buying any test, or (ii) buying the first test and, conditional on  $m_1 = \hat{C}$ , the second test as well. Their expected utility from not purchasing either test is  $1 - V(\theta_s)$ . Under strategy (ii), their action always matches the true state, at an expected cost of:

$$\begin{aligned} c_1 + \mathbb{P} \left[ m_1 = \hat{C} \mid l \right] \cdot c_2 &= 0.325 = V(\theta_l) && \text{for type } \theta_l \text{ and} \\ c_1 + \mathbb{P} \left[ m_1 = \hat{C} \mid h \right] \cdot c_2 &= 0.36 = V(\theta_h) && \text{for type } \theta_h. \end{aligned}$$

Thus, given risk neutrality, their expected utility is also given by  $1 - V(\theta_s)$ . We assume that both types choose the seller-preferred purchasing strategy (ii). The expected costs differ because the high type assigns a higher probability to receiving the test result  $m_1 = \hat{C}$ , which triggers further information acquisition. This discrepancy ensures that both types pay their respective valuations in expectation. Consequently, the seller's sequence of priced experiments is optimal.

Our main result will imply that, given  $\theta_h = 0.64$ ,  $\theta_l = 0.325$  is the lowest value that allows for full surplus extraction in two periods. Achieving perfect price discrimination with a smaller  $\theta_l$  would necessitate additional periods. Furthermore, the uniform pricing across experiments is not coincidental; we will demonstrate that this property generalizes to the  $T$ -period case when the sequence of priced experiments is designed to maximize the discrepancy in expected costs.

### 3. Model

We study a dynamic game featuring a monopolistic data *seller* (he) and an information-seeking *consumer* (or *buyer*, she). The consumer's task is to take an action  $a \in A := \{\underline{a}, \bar{a}\}$ , while facing uncertainty concerning the payoff-relevant state of the world  $\omega \in$

$\Omega := \{\underline{\omega}, \bar{\omega}\}$ . We say that the chosen action matches the realized state ( $a = \omega$ ) if either  $a = \underline{a}$  and  $\omega = \underline{\omega}$  or  $a = \bar{a}$  and  $\omega = \bar{\omega}$ .

At the beginning, both agents share the same prior belief  $\mu := \mathbb{P}[\underline{\omega}] \in (0, 1)$ . However, the consumer privately observes an imperfect signal  $s \in S := \{l, h\}$ . This signal  $s$  is generated by a commonly known experiment  $\pi : \Omega \rightarrow \Delta S$ . The consumer then forms her interim belief via Bayes' rule:

$$\theta_s := \mathbb{P}[\underline{\omega} \mid s] = \frac{\pi(s \mid \underline{\omega})\mu}{\pi(s \mid \underline{\omega})\mu + \pi(s \mid \bar{\omega})(1 - \mu)} \in (0, 1).$$

Consequently, the consumer can fall into one of two ‘types’,  $\theta_s \in \Theta = \{\theta_l, \theta_h\}$ . Without loss of generality (relabeling the signals), we assume that a consumer experiences ‘more uncertainty’ about the state of the world when receiving the signal  $s = h$  and we refer to this consumer as the high-valuation type. Formally:

$$\left| \theta_h - \frac{1}{2} \right| \leq \left| \theta_l - \frac{1}{2} \right|. \quad (1)$$

While the realization of the signal remains private information of the consumer, the seller is aware of its distribution:

$$\mathbb{P}[s] = \mu \cdot \pi(s \mid \underline{\omega}) + (1 - \mu) \cdot \pi(s \mid \bar{\omega}).$$

We consider a dynamic interaction over a finite time horizon  $T \in \mathbb{N}$ . In period  $t = 0$ , the state of the world  $\omega \in \Omega$  is chosen by nature, the consumer privately observes the signal  $s$  generated by  $\pi$  (and thus learns her type  $\theta_s$ ), and the seller publicly commits to a sequence of (Blackwell) experiments  $\sigma = (\sigma_t)_{t=1, \dots, T}$  and a pricing scheme  $c = (c_t)_{t=1, \dots, T} \in \mathbb{R}_+^T$ . In each period  $t$ , the seller will conduct the experiment  $\sigma_t$ . Deciding to observe the experiment outcome  $m_t$  necessitates a payment of  $c_t \geq 0$  to the seller. The seller commits to report the experiment outcome  $m_t \in M$  truthfully, where  $M$  is a finite set of possible signals. The realization  $m_t$  is drawn according to a probability distribution that may depend on the underlying state of the world  $\omega$  as well as on the history of previously realized outcomes  $(m_1, \dots, m_{t-1})$ . Formally, each *experiment* is a stochastic mapping:

$$\sigma_t : \Omega \times M^{t-1} \mapsto \Delta M.$$

We denote  $\Sigma_t$  as the set of all possible experiments in period  $t$ . We prevent the seller to adjust the pricing of the experiments after the initial assessment. Therefore, the seller publicly commits to a pure strategy  $(\sigma, c) \in \prod_{t=1}^T \Sigma_t \times \mathbb{R}_+^T$ .

At this point, it is worth noting that our model differs from Bergemann, Bonatti, and

Smolin (2018) in that the consumer buys the experiments sequentially instead of simultaneously. In particular, the consumer's belief will be updated based on the outcome of each bought experiment, which can influence her decision about subsequent purchases.

In each period  $t$ , the consumer decides whether to observe ( $\hat{\rho}_t = 1$ ) or ignore ( $\hat{\rho}_t = 0$ ) the experiment outcome  $m_t$ . This decision  $\hat{\rho}_t$  may depend on her type  $\theta_s$  as well as the set of outcomes of previously bought experiments  $\{m_k \mid \hat{\rho}_k = 1, k < t\}$ . Furthermore, the decision depends on the strategy of the seller in order to interpret the experiment outcomes and anticipate future gains or losses. The strategy of the consumer requires a purchase decision for any subset of the history of experiment outcomes including non-observed outcomes denoted as  $\emptyset$ . Formally, the behavioral purchase strategy is given by:

$$\rho_t : \Theta \times (M \cup \{\emptyset\})^{t-1} \times \prod_{k=1}^T \Sigma_k \times \mathbb{R}_+^T \mapsto \{0, 1\} \quad \text{for } t = 1, \dots, T.$$

After the final period  $T$ , the consumer chooses an action  $a \in A$ . This action choice  $\rho_a$  depends on her type as well as on the set of all purchased reports  $\{m_k \mid \hat{\rho}_k = 1\}$ . Again, the experiment sequence is required to update the belief correctly. However, we can exclude the pricing scheme as all payments made are already sunk costs and do not influence the terminal action. Analogously, the behavioral action strategy is given by:

$$\rho_a : \Theta \times (M \cup \{\emptyset\})^T \times \prod_{k=1}^T \Sigma_k \mapsto A.$$

The consumer's complete behavioral strategy is  $\rho = ((\rho_t)_{t=1}^T, \rho_a)$ . We make some simplifying assumptions:

**Assumption 1.** (i) *Both agents are risk-neutral and have no time preference.*

(ii) *The seller can design and execute any sequence of experiments  $\sigma \in \prod_{t=1}^T \Sigma_t$  at no cost.*

The ex-post utilities of the seller and consumer depend on the realized purchasing decisions, denoted by  $\hat{\rho}_t$ . The consumer aims to match her action to the state while minimizing cumulative payments. Her preferences are quasi-linear with respect to the payments to the seller, yielding an ex-post utility of:

$$U_R(\omega, a, (\hat{\rho}_t)_{t=1}^T, c) := \mathbb{I}_{[a=\omega]} - \sum_{t=1}^T \hat{\rho}_t \cdot c_t,$$

where  $\mathbb{I}_{[\cdot]}$  denotes the indicator function.

The seller is exclusively interested in maximizing his total revenue. Accordingly, his ex-post utility is given by the sum of received payments:

$$U_S((\hat{\rho}_t)_{t=1}^T, c) := \sum_{t=1}^T \hat{\rho}_t \cdot c_t.$$

Consequently, the seller's utility is independent of the true state and the consumer's action.

Notice that the purchasing behavior can depend on the experiment outcomes that are chosen by nature. Let  $\mathcal{M} = (m_1, \dots, m_T) \in (M \cup \emptyset)^T$  be a sequence of realized signals including non-observed outcomes. Importantly, the probability assigned to observing the sequence of experiment outcomes  $\mathcal{M}$  is type-dependent:

$$\mathbb{P}[\mathcal{M} \mid s, \sigma] = \prod_{t=1}^T \mathbb{P}[m_t \mid s, \sigma, \mathcal{M}|_{t-1}].$$

As the seller commits to the sequence of experiments at the beginning of the game and their outcomes naturally depend on the choice of experiments, we will in the following simplify notation and omit the dependency of the probabilities of experiment outcomes  $\mathcal{M}$  on  $\sigma$ .

These probabilities further depend on the private information of the consumer, thus the same purchasing behavior can lead to a different evaluation of expected costs. The consumer of type  $\theta_s$  evaluates ex-ante her expected payoff from choosing strategy  $\rho$  given the sequence of priced experiments  $(\sigma, c)$  as:

$$\sum_{\mathcal{M} \in (M \cup \emptyset)^T} \mathbb{P}[\mathcal{M} \mid s] \left( \mathbb{I}_{[\rho_a(\theta_s, \mathcal{M}, \sigma) = \omega]} - \sum_{t=1}^T \rho_t(\theta_s, \mathcal{M}|_{t-1}, \sigma, c) \cdot c_t \right).$$

The seller also assigns his subjective probabilities to each sequence of experiment outcomes including non-observed outcomes. However, we can apply the law of total probability and rewrite his expected revenue as the expected payment of each type times the likelihood of the respective type:

$$\sum_{s \in \{l, h\}} \mathbb{P}[s] \cdot \sum_{\mathcal{M} \in (M \cup \emptyset)^T} \mathbb{P}[\mathcal{M} \mid s] \cdot \sum_{t=1}^T \rho_t(\theta_s, \mathcal{M}|_{t-1}, \sigma, c) \cdot c_t.$$

We employ the solution concept of a seller-preferred subgame perfect Nash equilibrium. This implies that the consumer behaves sequentially rational and that she behaves such that her behavior maximizes the seller's expected utility if she is indifferent. Thus, the seller will implement the revenue-maximizing information structure that can be induced by a sequence of priced experiments.



## 4. Analysis

The subsequent analysis is structured as follows: First, Section 4.1 introduces an ‘encryption protocol’. This method simplifies the analysis by ensuring that experiments appear informative only if all preceding experiment outcomes have been observed. Section 4.2 lays further groundwork by defining consumer valuations for a revelation of the state depending on their current information. The core of the analysis is presented in Section 4.3, which details the construction of a sequence of priced experiments, which allows the seller to extract the entire surplus simultaneously from both buyer types. Finally, Section 4.4 investigates conditions under which there exists a sequential selling strategy that yields strictly higher revenue for the seller compared to the static framework.

### 4.1. Encryption Protocol

In our model, the seller cannot observe the buyer’s purchasing decisions. If such observation were possible, the seller could enforce a protocol in which each test becomes available only after all previous tests have been purchased. The following analysis can also be read by allowing for such protocols. However, we show that such a modeling assumption is unnecessary. Rather than preventing the buyer from purchasing a test, we encrypt each test result and embed a key in the combination of all earlier test results that is required to decrypt it. This approach allows the seller to offer the tests regardless of previous purchase decisions, since a test does not provide any information unless all previous test results, and hence the keys need to decrypt it, have been observed. Note in particular that, after the seller has committed to the sequence of priced experiments, the dynamics of the model are driven exclusively by the buyer. Specifically, if the consumer decides to observe an outcome, then she will update her belief and decide whether to purchase an additional experiment or to stop further information acquisition.

We will illustrate the idea of encrypting subsequent test results by revisiting the initial example. We demonstrate how the analysis in Section 2 can be simplified by encrypting the second experiment in such a way that it is only informative if the consumer knows the first test result. In particular, the component of the first-period outcome required to decrypt the second outcome will be chosen independently of the underlying state and any state-depending signal. As a result, it can be generated without performing a state-dependent test, thus the seller only requires a randomization device to select the subsequent test. We then generalize this procedure to  $T$  periods.

Before revisiting the initial example, we consider an extreme but instructive case: two tests that, taken separately, generate no belief update, but together reveal the state with certainty. Intuitively, the first test can be interpreted as randomly ‘claiming’ that

a particular state is true, while the second test indicates whether that claim was correct. Observing the random claim does not in itself reveal any information. Similarly, knowing whether the claim was true or false without knowing the claim is not informative at all. Consequently, each test in isolation is uninformative, but together they reveal the true state with certainty. In particular, the first test effectively serves as a key to decrypt the second test.

To formalize this, we consider a signal space  $M = \{\underline{m}, \overline{m}\}$ . Recall that an experiment is defined by a stochastic mapping,  $\sigma_t : \Omega \times M^{t-1} \rightarrow \Delta(M)$ , which assigns, for each state and every history of past signals, a probability distribution over the signal space  $M$ . In the first period, we use a state-independent coin toss to generate the signal. As a stochastic mapping, the first experiment can be represented as follows:

$\sigma_1(m_1 \mid \omega)$	$\underline{m}$	$\overline{m}$
$\underline{\omega}$	$\frac{1}{2}$	$\frac{1}{2}$
$\overline{\omega}$	$\frac{1}{2}$	$\frac{1}{2}$

In the second period, we introduce an experiment designed to reveal the true state given the outcome of the first experiment. Specifically, if the first experiment yields  $\underline{m}$ , then the second experiment maps  $\underline{\omega}$  to  $\underline{m}$  (and  $\overline{\omega}$  to  $\overline{m}$ ); if it yields  $\overline{m}$ , the mapping is reversed. Hence, although the coin toss itself does not reveal any information about the state, its outcome affects how we interpret the signal in the second period. Formally:

$\sigma_2(m_2 \mid \omega, m_1 = \underline{m})$	$\underline{m}$	$\overline{m}$	$\sigma_2(m_2 \mid \omega, m_1 = \overline{m})$	$\underline{m}$	$\overline{m}$
$\underline{\omega}$	1	0	$\underline{\omega}$	0	1
$\overline{\omega}$	0	1	$\overline{\omega}$	1	0

Viewed in isolation, the second experiment does not update the belief either. This is because in state  $\underline{\omega}$ , the second experiment signals  $\underline{m}$  if and only if the first experiment has realized as  $m_1 = \underline{m}$ , while in state  $\overline{\omega}$  it is the other way around:

$$\mathbb{P}[m_2 = \underline{m} \mid \underline{\omega}] = \mathbb{P}[m_1 = \underline{m} \mid \underline{\omega}] = \frac{1}{2} = \mathbb{P}[m_1 = \underline{m} \mid \overline{\omega}] = \mathbb{P}[m_2 = \underline{m} \mid \overline{\omega}].$$

This means that neither  $m_1$  nor  $m_2$  shifts the belief about the state on its own:

$$\mathbb{P}[\underline{\omega}] = \mathbb{P}[\underline{\omega} \mid m_1] = \mathbb{P}[\underline{\omega} \mid m_2].$$

We can conclude that, while the first and second experiments appear like fair coin tosses in isolation, their combined outcome reveals the underlying state:

$$\mathbb{P}[\omega \mid m_1, m_2] = \begin{cases} 1, & \text{if } m_1 = m_2 = \underline{m} \text{ or } m_1 = m_2 = \overline{m}, \\ 0, & \text{otherwise.} \end{cases}$$

This insight shows that selling information differs from selling traditional economic goods. A binary information can be divided into several components. A consumer will buy each of the components as long as the total cost does not exceed her valuation of the complete information. In other words, it is the total price that matters, not the price of each component. The following analysis demonstrates how the seller can benefit from induced belief updates by providing imperfect information in each component when facing multiple consumers.

#### 4.1.1. Example Revisited: Medical diagnostic Testing

A potential objection to the previous approach used in Section 2 is that the seller must either design correlated tests or perform each test in order to determine subsequent tests. While correlated tests lead to an additional complexity, which makes real-world application difficult, the option to perform each test is heavily dependent on the assumptions that tests can be performed without facing a cost and that agents have no time preference (Assumption 1). In particular, even if the buyer decides against purchasing a test, each test has to be performed.

We now demonstrate how this strict assumption can be relaxed to make real-world application more attainable. Specifically, we will incorporate an independent coin toss, which is the only part of the test that needs to be performed if a test is not purchased. We thereby introduce an *encryption protocol* in which the coin toss serves as a *key* necessary to decrypt the second test result.

We want to address the same situation as in the initial example in Section 2. Recall that the state space is given by  $\Omega = \{C, F\}$ . However, in contrast to the original example, we extend the signal space to incorporate a coin toss, i.e.:

$$M = \{\hat{C}, \hat{F}\} \times \{0 \text{ (Head)}, 1 \text{ (Tail)}\}.$$

In the initial example, the first test always signals  $\hat{C}$  when the state is  $C$ , while in state  $F$ , it signals  $\hat{C}$  with a probability of  $\frac{4}{9}$  and  $\hat{F}$  with a probability of  $\frac{5}{9}$ . We modify this first test to incorporate the outcome of an independent coin toss. For instance, if the state is  $F$ , the signal still includes  $\hat{C}$  with a probability of  $\frac{4}{9}$ , but it is either  $(\hat{C}, 0)$  or  $(\hat{C}, 1)$  with equal

probability. The adjusted test is summarized in the table below.

$\sigma_1(m_1   \omega)$	$(\hat{C}, 0)$	$(\hat{C}, 1)$	$(\hat{F}, 0)$	$(\hat{F}, 1)$
$C$	$\frac{1}{2}$	$\frac{1}{2}$	0	0
$F$	$\frac{4}{18}$	$\frac{4}{18}$	$\frac{5}{18}$	$\frac{5}{18}$

Since the coin toss is independent of the signal containing  $\hat{C}$  or  $\hat{F}$ , the posterior belief induced by observing  $m_1 = (\hat{C}, 0)$  is the same as that induced by  $m_1 = (\hat{C}, 1)$  and analogously for a signal that contains  $\hat{F}$ . Consequently, the resulting posterior beliefs, for both types  $s \in \{l, h\}$ , coincide with those in the original example.

We now turn to the second test. In contrast to the original example, we assume that the choice of the second test depends exclusively on the outcome of the coin toss, rather than on whether the observed signal contains  $\hat{C}$  or  $\hat{F}$ . Consider the following stochastic mapping for each  $\hat{\omega} \in \{\hat{C}, \hat{F}\}$ :

$\sigma_2(m_2   \omega, m_1 = (\hat{\omega}, 0))$	$\hat{C}$	$\hat{F}$	$\sigma_2(m_2   \omega, m_1 = (\hat{\omega}, 1))$	$\hat{C}$	$\hat{F}$
$C$	1	0	$C$	0	1
$F$	0	1	$F$	1	0

Note that we omit the coin toss in the table above, as it is not needed in the second period.<sup>4</sup> Indeed, this procedure simplifies the analysis since the second test becomes uninformative without holding the result of the coin toss, i.e.,  $\mathbb{P}[C | s, m_2] = \theta_s = \mathbb{P}[C | s]$ . Consequently, the consumer will not buy the second test in isolation. However, if the result of the coin toss is known, the second test still reveals the state with certainty. So this approach has the same strategic consequences as in the initial example in Section 2.

#### 4.1.2. Encryption Protocol with $T$ Periods

We next extend the encryption protocol to the general model and  $T$  periods. The idea is to construct ‘encrypted’ experiments that can only be understood if the outcome of each experiment has been observed beforehand. This approach simplifies the analysis: if a consumer decides to ignore an experiment, then her optimal decision is to abstain from buying all subsequent experiments. As a result, analyzing the consumer’s optimal purchase strategy effectively becomes a stopping problem.

<sup>4</sup>Formally, we could include it by assigning probability 1 to heads independently of  $\omega$  and  $m_1$ .

For the construction, consider an arbitrary sequence of independent binary experiments, i.e., experiments which use the signals  $\underline{m}$  and  $\overline{m}$  and do not depend on previous signals. Specifically, in each period  $t$ , signal  $\underline{m}$  is realized with probability  $q_t$  (and signal  $\overline{m}$  with probability  $1 - q_t$ ) if the state is  $\underline{\omega}$  and with probability  $p_t$  if the state is  $\overline{\omega}$ , i.e.:

$\sigma_t(m_t \mid \omega)$	$\underline{m}$	$\overline{m}$
$\underline{\omega}$	$q_t$	$1 - q_t$
$\overline{\omega}$	$p_t$	$1 - p_t$

Suppose the buyer observes a set of experiment outcomes, which is denoted by  $\mathcal{M} \subseteq \{m_1, \dots, m_T\}$ . We define  $l^*(\mathcal{M})$  as the longest consecutive sequence of tests starting with the first test. In other words, the test in period  $l^*(\mathcal{M}) + 1$  is the first test that the consumer does not purchase. Formally, we define:

$$l^*(\mathcal{M}) := \max \{l \mid \{m_1, \dots, m_l\} \subseteq \mathcal{M}\}.$$

We will demonstrate that the seller can design a modified experiment sequence that provides the same information for any consecutive sequence of tests beginning with the first one, but any set of experiment outcomes that follow a non-purchased experiment appear to be independent of the state. Following the example in Section 4.1.1, we do so by including a coin toss with each experiment that serves as an encryption key for subsequent experiments:

$$M = \{\underline{m}, \overline{m}\} \times \{0 \text{ (Head)}, 1 \text{ (Tail)}\}.$$

Specifically, for each period  $t$ , the coin flip determines whether the probability of an experiment outcome  $m$  in state  $\underline{\omega}$  alternates with the probability of  $m$  in state  $\overline{\omega}$  in all subsequent periods. This design ensures that the buyer interprets the  $t$ -th experiment correctly if she has bought all experiments up to period  $t$ , but retains her prior belief about the state if she has skipped at least one test.

**Lemma 1.** *Let  $(\sigma_t)_{t=1}^T$  be a sequence of independent binary experiments, i.e., with outcomes  $(m_t)_{t=1}^T \in \{\underline{m}, \overline{m}\}^T$ . Then the seller can design a sequence of experiments  $(\sigma'_t)_{t=1}^T$  with outcomes  $(m'_t)_{t=1}^T \in M$  such that*

$$\mathbb{P}[\omega \mid \mathcal{M}] = \mathbb{P}[\omega \mid m'_1, \dots, m'_{l^*(\mathcal{M})}] = \mathbb{P}[\omega \mid m_1, \dots, m_{l^*(\mathcal{M})}],$$

where  $\mathcal{M}$  is an arbitrary subset of outcomes of  $(\sigma'_t)_{t=1}^T$ .

The proofs of all Lemmata are relegated to Appendix A. By construction, the modified sequence of experiments in Lemma 1 is such that the information that the buyer gains about the state of the world by acquiring the outcomes in  $\mathcal{M}$  is equivalent to the information that she gains from the longest consecutive sequence of experiments, starting with the first experiment. Consequently, for any positive price assigned to subsequent experiments, it is sequentially rational for the consumer to stop after this sequence.

## 4.2. Consumer Valuations

Consider a consumer of type  $\theta_s \in \Theta$ . If the consumer chooses to ignore all experiment outcomes (i.e.,  $\rho_t = 0$  for all  $t = 1, \dots, T$ ), her optimal action depends exclusively on her private information  $s$ . Since she receives a payoff of 1 for matching the state and 0 otherwise, we obtain:

$$\rho_a^*(\theta_s, \emptyset, \sigma) \in \arg \max_{a \in A} \{\mathbb{P}[a = \omega \mid s]\},$$

with an expected payoff of  $\max\{\theta_s, 1 - \theta_s\}$ . If the consumer learns the state  $\omega$  perfectly, her optimal action would yield a payoff of 1. Thus, the value a consumer assigns to a revelation of the state is the difference between the perfectly informed payoff and the payoff based exclusively on her prior information  $s$ :

$$V(\theta_s) := 1 - \max\{\theta_s, 1 - \theta_s\} = \min\{\theta_s, 1 - \theta_s\}.$$

This valuation represents the highest amount the consumer is willing to pay for a state-revealing experiment. Due to the labeling of the signals (1), it holds that  $V(\theta_h) \geq V(\theta_l)$ .

**Definition 1.** Types  $\theta_l$  and  $\theta_h$  are ‘congruent’ if they share an optimal action based exclusively on their private information  $s$  (without holding any experiment outcomes), i.e.,

$$\bigcap_{s \in S} \arg \max_{a \in A} \{\mathbb{P}[a = \omega \mid s]\} \neq \emptyset.$$

Otherwise, the types are ‘non-congruent’.

We henceforth adopt the convention that the high type  $\theta_h$  prefers the action  $\underline{a}$  based on her private information, which implies that she considers state  $\underline{\omega}$  more likely, i.e.,  $\theta_h \geq 1/2$ . This convention also implies that if  $\theta_h \neq \frac{1}{2}$ , then the types are congruent if and only if  $\theta_h \leq \theta_l$ . Additionally, it allows to simplify the value of information. Specifically, the valuation of the high type is given by  $V(\theta_h) = 1 - \theta_h$  and the valuation of the low

type is given by

$$V(\theta_l) = \begin{cases} 1 - \theta_l & \text{if the types are congruent,} \\ \theta_l & \text{otherwise.} \end{cases}$$

Note that ignoring all experiment outcomes cannot be part of a seller-preferred subgame perfect equilibrium. It follows directly from the static case of BBS that the seller can always extract a revenue equal to the valuation of the low type:

**Corollary 1.** *In any seller-preferred subgame perfect equilibrium  $((\sigma^*, c^*), \rho^*)$ , the expected revenue of the seller is at least  $V(\theta_l)$ , i.e.,*

$$\sum_{s \in \{l, h\}} \mathbb{P}[s] \cdot \sum_{\mathcal{M} \in (M \cup \emptyset)^T} \mathbb{P}[\mathcal{M} \mid s] \cdot \sum_{t=1}^T \rho_t^*(\theta_s, \mathcal{M}|_{t-1}, \sigma^*, c^*) \cdot c_t^* \geq V(\theta_l).$$

Therefore, in equilibrium, the seller offers informative experiments and we need to analyze the optimal terminal action for each set of outcomes. To ease the exposition, we denote the experiment outcome in period  $t$  by  $m_t = \emptyset$  if it remains unobserved by the consumer. Given any set of outcomes  $\mathcal{M} \in (M \cup \{\emptyset\})^T$ , the terminal action in a seller-preferred subgame perfect equilibrium must satisfy

$$\rho_a^*(\theta_s, \mathcal{M}, \sigma) \in \arg \max_{a \in A} \{\mathbb{P}[a = \omega \mid s, \mathcal{M}]\}.$$

Now, suppose that after period  $t$ , the consumer so far has observed outcomes  $\mathcal{M}_t \in (M \cup \{\emptyset\})^t$ . If she decides against acquiring any further experiment, she will choose an optimal terminal action  $\rho_a^*(\theta_s, \mathcal{M}_t, \sigma)$  that yields a payoff of  $\max_{\omega} \{\mathbb{P}[\omega \mid s, \mathcal{M}_t]\}$ . Consequently, her updated valuation of receiving a revelation of the state, given  $\mathcal{M}_t$ , is

$$V(\theta_s, \mathcal{M}_t) := 1 - \max_{\omega} \{\mathbb{P}[\omega \mid s, \mathcal{M}_t]\} = \min_{\omega} \{\mathbb{P}[\omega \mid s, \mathcal{M}_t]\}.$$

Notably, this updated valuation does not necessarily satisfy  $V(\theta_h, \mathcal{M}_t) \geq V(\theta_l, \mathcal{M}_t)$ .

### 4.3. Perfect Price Discrimination

We are now in the position to establish the main result. We will show that by sequentially offering experiments, the seller can perfectly price discriminate between buyer types which are non-congruent, and thus differ in their beliefs about which state is more likely, yet have sufficiently similar valuations. Recall that by convention  $\theta_h \geq 1/2 \geq 1 - \theta_h \geq \theta_l$ . To extract the entire consumer surplus, both types of consumers must learn the state of the world with certainty, and their subjective expected costs must match their respective valuation.

**Theorem 1.** Assume that the consumer types  $\theta_l$  and  $\theta_h$  are non-congruent. If

$$\theta_l \geq \theta_l(\theta_h, T) := \frac{1 - \theta_h - T\theta_h \left(1 - \theta_h^{1/T}\right)}{2(1 - \theta_h) - T \left(1 - \theta_h^{1/T}\right)}, \quad (2)$$

then the seller extracts the entire consumer surplus in a seller-preferred subgame perfect equilibrium; thus, both buyer types  $\theta_s$  pay their respective valuation  $V(\theta_s)$  in expectation to the seller, i.e.,

$$\sum_{\mathcal{M} \in (M \cup \emptyset)^T} \mathbb{P}[\mathcal{M} \mid s] \cdot \sum_{t=1}^T \rho_t^*(\theta_s, \mathcal{M}|_{t-1}, \sigma^*, c^*) \cdot c_t^* = V(\theta_s).$$

In particular, there exists some maturity  $T$  which satisfies the inequality (2) if

$$\theta_l > \frac{1 - \theta_h + \theta_h \ln(\theta_h)}{2(1 - \theta_h) + \ln(\theta_h)}. \quad (3)$$

Note first that  $\theta_l(\theta_h, T) \leq 1 - \theta_h$ , such that extracting the entire consumer surplus is possible at least for equal valuations. Second, call  $V(\theta_h) - V(\theta_l(\theta_h, T))$  the *admissible valuation gap* between the types for perfect price discrimination to be possible. Since  $\theta_l(\theta_h, T)$  is strictly decreasing in the maturity  $T$ , we obtain:

**Remark.** The admissible valuation gap  $V(\theta_h) - V(\theta_l(\theta_h, T))$  is strictly increasing in the maturity  $T$ . In particular,  $T = 1$  means no admissible valuation gap as in BBS, since  $V(\theta_l(\theta_h, 1)) = \theta_l(\theta_h, 1) = 1 - \theta_h = V(\theta_h)$ .

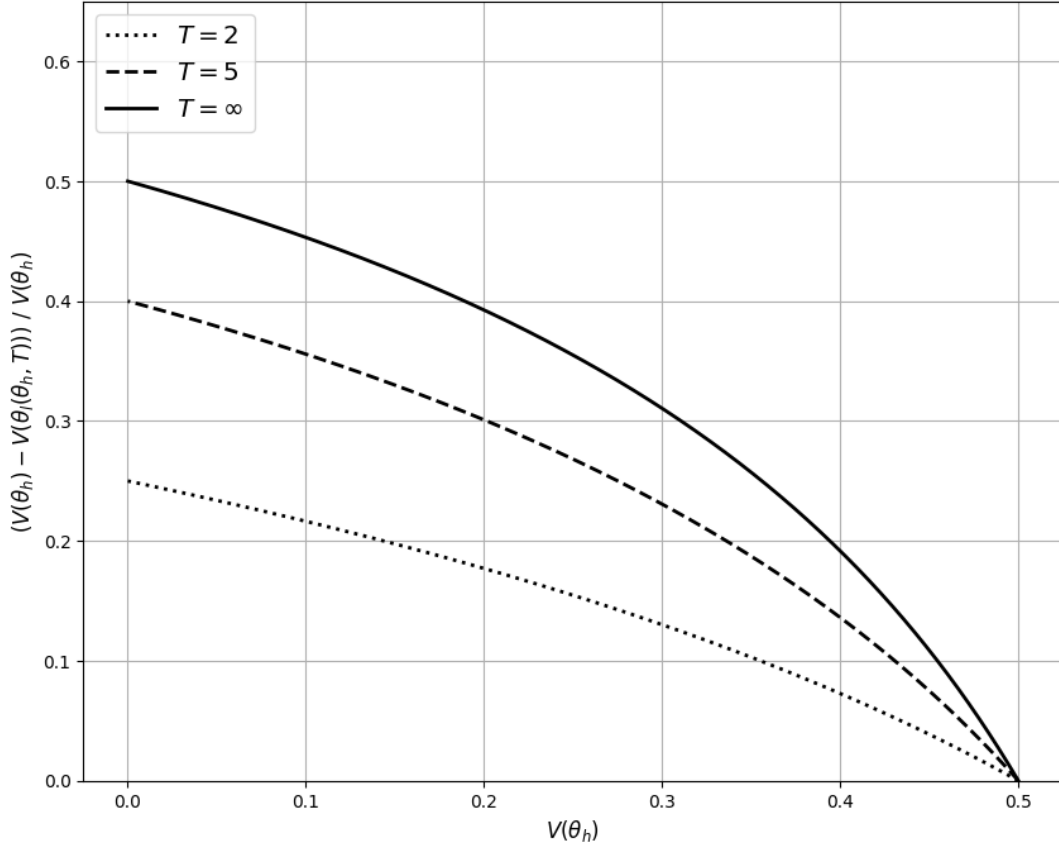
Figure 3 considers the *relative* admissible valuation gap, illustrating how it increases with respect to the maturity for each valuation  $V(\theta_h)$ . Note that the relative admissible valuation gap is larger when the beliefs are more extreme (i.e., when  $V(\theta_h)$  is low), as the types then perceive the likelihood of experiment outcomes as very different, facilitating perfect price discrimination.

The proof of Theorem 1 constructs a sequence of experiments such that the state may only be revealed with certainty when it is equal to  $\bar{\omega}$ :

$\sigma_t(m_t \mid \omega)$	$\underline{m}$	$\bar{m}$
$\underline{\omega}$	1	0
$\bar{\omega}$	$p_t$	$1 - p_t$

We then encrypt the experiments to simplify the analysis as described in Lemma 1, effectively turning the consumer's optimal purchase strategy into a stopping problem. Suppose





**Figure 3.** The relative admissible valuation gap  $(V(\theta_h) - V(\theta_l(\theta_h, T)))/V(\theta_h)$  for perfect price discrimination depending on  $V(\theta_h)$  for  $T = 2$  (dotted),  $T = 5$  (dashed), and  $T = \infty$  (solid) periods.

further that both types follow the same purchasing behavior, namely that they continue buying additional experiments until the state is revealed (or maturity is reached). Since the high-valuation buyer assigns a higher ex-ante probability to the state being  $\underline{\omega}$  (recall that  $\theta_s = \mathbb{P}[\underline{\omega} \mid s]$  and  $\theta_h \geq 1/2 \geq \theta_l$ ), and thus to outcomes that do not immediately reveal the state, she expects to purchase more experiments compared with the low type. This leads to higher expected costs for the high-valuation buyer. The proof then first determines a pricing scheme under which the low type purchases until the state is revealed and pays her valuation in expectation. Second, it proposes probabilities  $p_t$  under which the difference in expected costs between the types becomes large,<sup>5</sup> which yields the admissible valuation gap, and hence  $\theta_l(\theta_h, T)$ . At  $\theta_l = \theta_l(\theta_h, T)$ , the experiments, interestingly, are priced at a uniform price across periods. For smaller valuation gaps, the proof shows that we can simply adapt the initial two experiments and their costs accordingly.

<sup>5</sup>We have verified that these probabilities for many maturities  $T$  maximize this difference.

#### 4.4. Proof of Theorem 1

We proceed in several steps as follows: We first provide a class of experiments that will be used by the seller for perfect price discrimination in Section 4.4.1. Subsequently, Section 4.4.2 introduces a pricing scheme that leads both consumers to buy those experiments. In Section 4.4.3, we search for the parametrization of those experiments that induces the highest achievable difference in expected costs. Since this parametrization can lead to a difference that is more than the valuation gap  $V(\theta_h) - V(\theta_l)$  between consumers, we adjust the parametrization in Section 4.4.4 such that the difference in expected costs coincides with the valuation gap. Finally, Section 4.4.5 summarizes our findings.

##### 4.4.1. Construction of the Experiments

Recall that we assumed that the high-valuation type considers the state  $\underline{\omega}$  to be more likely than  $\bar{\omega}$ , i.e.,  $\theta_h \geq \frac{1}{2}$ . Given non-congruent beliefs, it follows that the low-valuation type considers state  $\bar{\omega}$  more likely. We will construct a sequence of experiments such that a positive probability of state revelation occurs only when the true state is  $\bar{\omega}$ . Specifically, the experiments are constructed as follows:

$\sigma_t(m_t   \omega)$	$\underline{m}$	$\bar{m}$
$\underline{\omega}$	1	0
$\bar{\omega}$	$p_t$	$1 - p_t$

In other words, these experiments are designed to guarantee a non-revealing signal when the state is  $\underline{\omega}$ , but with a positive probability a state-revealing signal when the state is  $\bar{\omega}$ . Consequently, the low type considers it more likely to get a revelation of the state for each experiment  $\sigma_t$  with  $p_t \in (0, 1)$ .

As discussed in Section 4.1, the seller can encrypt each experiment so that its outcome can only be understood if all previous experiment outcomes have been observed beforehand. Specifically, we apply Lemma 1 to the experiments denoted above and from now on consider the encrypted sequence of experiments  $(\sigma'_t)_{t=1}^T$ . Consequently, it is sequentially rational for the buyer to stop purchasing experiments after an unobserved experiment outcome. Alternatively, this could be achieved by monitoring the buyer's purchasing decisions. Exemplary, the seller could punish the consumer by only conducting uninformative experiments after the buyer ignores an experiment.

Furthermore, it is sequentially rational for the buyer to stop purchasing experiments as soon as an observed experiment realizes as  $\bar{m}$ , because in such a case, she is already

aware of the state and assigns no value to holding additional outcomes. Therefore, a purchase of the experiment in period  $t$  is only considered if the consumer holds all previous experiment outcomes and all of them have realized as  $\underline{m}$ . From now on, we will denote

$$\underline{\mathcal{M}}_t := \{m_1 = \underline{m}, \dots, m_t = \underline{m}\}.$$

Let  $\mathcal{M}_{t-1} \in (M \cup \{\emptyset\})^{t-1}$ , by the reasoning above, we can conclude that the following behavior is sequentially rational:

$$\rho_t^*(\theta_s, \mathcal{M}_{t-1}, \sigma', c) = 0 \quad \text{if } \mathcal{M}_{t-1} \neq \underline{\mathcal{M}}_{t-1}.$$

The key insight that enables price discrimination is that the ex-ante probability of observing  $\underline{\mathcal{M}}_t$  depends on the consumer's private information:

$$\mathbb{P}[\underline{\mathcal{M}}_t \mid s] = \theta_s + (1 - \theta_s) \cdot \prod_{r=1}^t p_r.$$

In particular, it holds that the high type assigns a higher probability to the event that all experiments up to period  $t$  result in the non-revealing signal  $\underline{m}$  than the low type:

$$\mathbb{P}[\underline{\mathcal{M}}_t \mid h] \geq \mathbb{P}[\underline{\mathcal{M}}_t \mid l]$$

with strict inequality if  $\theta_h \neq \theta_l$ .

Perfect price discrimination requires that consumers learn the true state with certainty in order to pay their full valuation in expectation. Therefore, the seller must set  $p_t = 0$  for some  $t$ . Without loss of generality, we assume this occurs only in the final period  $T$ . If it occurred earlier, for instance in  $t' < T$ , we have argued that no rational consumer would buy any positively priced experiments thereafter. Thus, such a sequence of experiments is equivalent to a sequence where we cut out all experiments after period  $t'$  and prefix a sequence with  $T - t'$  periods of free, uninformative experiments. This shifts the original sequence so that the experiment in  $t'$  of the original sequence becomes the final experiment of the adjusted sequence and the unique experiment with a certain revelation of the state.

#### 4.4.2. Pricing

In each period  $t$ , conditional on  $\underline{\mathcal{M}}_{t-1}$ , the consumer has to consider two options: either stop acquiring further information and choose the optimal action given her current information, or to buy the next experiment. If she chooses to act (ignore all upcoming

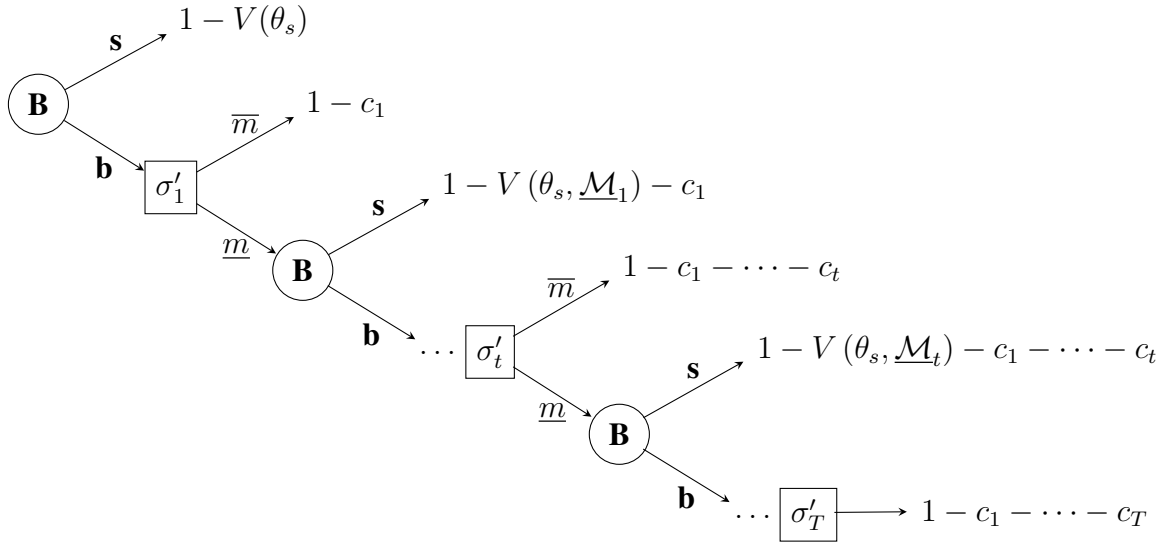
experiments), then she receives a payoff of

$$1 - V(\theta_s, \underline{\mathcal{M}}_{t-1}) - c_1 - \dots - c_{t-1},$$

where we recall that

$$V(\theta_s, \underline{\mathcal{M}}_{t-1}) = 1 - \max_{\omega} \{ \mathbb{P}[\omega \mid s, \underline{\mathcal{M}}_{t-1}] \} = \min_{\omega} \{ \mathbb{P}[\omega \mid s, \underline{\mathcal{M}}_{t-1}] \}.$$

If she purchases the experiment and the signal realizes as  $\bar{m}$ , she identifies the state as  $\bar{\omega}$  and receives a payoff of 1 while incurring a cumulative cost of  $c_1 + \dots + c_t$ . If the signal is  $\underline{m}$ , she faces the same decision problem in period  $t + 1$ . Figure 4 illustrates the stopping problem faced by the buyer.



**Figure 4.** The Buyer decides in each period upon observing signal  $\underline{m}$  whether to buy the next experiment or to stop buying and choose the optimal action given her current information.

To enable perfect price discrimination, the pricing scheme must ensure that each experiment is purchased until the state is identified. In other words, the optimal pricing scheme must incentivize the buyer to purchase the next experiment whenever all previous experiments have been realized as a non-revealing signal. Suppose that the pricing policy from period  $t + 1$  onward is such that the consumer chooses to purchase these experiments. If the buyer purchases the  $t$ -th experiment, she knows that she will choose the state-optimal action – anticipating that she will continue to purchase subsequent experiments until the true state is revealed. Therefore, she will decide to buy the  $t$ -th experiment only if

$$1 - \sum_{u=t}^{T-1} \mathbb{P}[m_t = \underline{m}, \dots, m_u = \underline{m} \mid s, \underline{\mathcal{M}}_{t-1}] c_{u+1} \geq 1 - V(\theta_s, \underline{\mathcal{M}}_{t-1})$$

$$\Leftrightarrow c_t \leq V(\theta_s, \underline{\mathcal{M}}_{t-1}) - \sum_{u=t}^{T-1} \mathbb{P}[m_t = \underline{m}, \dots, m_u = \underline{m} \mid s, \underline{\mathcal{M}}_{t-1}] \cdot c_{u+1}. \quad (4)$$

The following Lemma provides a pricing scheme that leads to the desired purchase behavior. Specifically,  $c_2, \dots, c_T$  are chosen such that equality in (4) holds for  $s = h$ . Under this pricing scheme, the high-valuation type is indifferent in each period  $t \geq 2$  and we can assume that she takes the seller-preferred decision to buy each experiment. Remarkably, the experiments will be designed so that at some point the updated valuation of the low type exceeds the updated valuation of the high type. One might think that the low type would benefit from this effect. However, we will ensure that the low type pays exactly her valuation in expectation, anticipating any future discounts and effectively paying for them in the initial period. We do this by setting the cost in the first period so that equality in (4) holds for  $s = l$ .

**Lemma 2.** *Consider the experiments  $(\sigma'_t)_{t=1}^T$  constructed in Section 4.4.1. If the parameters satisfy non-negativity of the cost in period one, then*

$$c_t = \frac{(1 - \theta_h) \prod_{s=1}^{t-1} p_s (1 - p_t)}{\theta_h + (1 - \theta_h) \prod_{s=1}^{t-1} p_s} \quad (5)$$

and

$$c_1 = \theta_l - \sum_{t=1}^{T-1} \left( \theta_l + (1 - \theta_l) \prod_{s=1}^t p_s \right) \cdot c_{t+1} \quad (6)$$

lead to a sequentially rational purchasing behavior for all  $t \geq 2$ :

$$\rho_t^*(\theta_s, \mathcal{M}, \sigma', c) = 1 \text{ if and only if } \mathcal{M}_{t-1} = \underline{\mathcal{M}}_{t-1}.$$

Furthermore, it is sequentially rational for the low-valuation type to purchase the first experiment, i.e.,  $\rho_1^*(\theta_l, \emptyset, \sigma', c) = 1$ .

Due to the choice of the costs in the first period in Lemma 2, the low type's valuation  $V(\theta_l)$  covers exactly the expected cost of the experiment sequence until a state-revelation:

$$V(\theta_l) = c_1 + \sum_{t=1}^{T-1} \mathbb{P}[\underline{\mathcal{M}}_t \mid l] \cdot c_{t+1}.$$

Furthermore, it is sequentially rational for the high type to buy the first experiment only if

$$V(\theta_h) \geq c_1 + \sum_{t=1}^{T-1} \mathbb{P}[\underline{\mathcal{M}}_t \mid h] \cdot c_{t+1}. \quad (7)$$

Otherwise, the high type will decide against purchasing the first experiment, thereby ig-

noring all experiments. In this case, extending the horizon from  $T$  to  $T + 1$  would allow for perfect price discrimination, as it would create space for an additional state-revealing experiment. Specifically, one could shift all priced experiments one period towards the maturity ( $t \rightarrow t + 1$ ) and offer an additional state-revealing experiment in the first period priced at  $c_1 = V(\theta_h)$  without encrypting the subsequent experiments. Such an experiment would be purchased exclusively by the high type, while the low type behaves sequentially rational by purchasing all following experiments and paying her valuation  $V(\theta_l)$  in expectation. In the following, we will show that this additional period is not required, because the seller can adjust the experiments such that equality in (7) holds.

#### 4.4.3. Cost Discrepancy

Recall that consumers of different types assign different probabilities to the outcomes of the experiments, which in turn leads to different evaluations of the expected costs. Specifically, the experiments are designed to capture the valuation  $V(\theta_l)$  of the low-type while inducing a greater expected cost for the high type. By ‘cost discrepancy’ we refer to the difference in expected costs, given that both consumers will continue to purchase experiments until a state revelation occurs.

In the initial example, we have seen that the seller can make use of the cost discrepancy in order to bridge the gap of the heterogeneous valuations while selling the same sequence of experiments. In particular, with maturity  $T = 2$  and the pricing of Lemma 2, the cost discrepancy between the high and low type is given by

$$(c_1 + \mathbb{P}[\underline{\mathcal{M}}_1 \mid h] \cdot c_2) - (c_1 + \mathbb{P}[\underline{\mathcal{M}}_1 \mid l] \cdot c_2) = (\theta_h - \theta_l) \frac{(1 - p_1)(1 - \theta_h)p_1}{\theta_h + (1 - \theta_h)p_1}. \quad (8)$$

We want to find the probabilities that maximize the expression (8). We first consider the boundaries of  $p_1 \in [0, 1]$ . Choosing  $p_1 = 1$  implies that the first message does not reveal the state, i.e.,  $\mathbb{P}[\underline{\mathcal{M}}_1 \mid s] = 1$ . Thus, both consumer types face the same cost,  $c_1 + c_2$ , and thus there is no difference in expected costs. If the seller were to design the first experiment such that  $p_1 = 0$ , then both types would stop acquiring experiments afterwards, again leading to no cost discrepancy. In particular, inserting  $p_1 = 0$  into the pricing of Lemma 2 yields  $c_1 = V(\theta_l)$  and  $c_2 = 0$ .

We can simplify the expression (8), before searching for the critical points. Specifically, the constant factor  $(\theta_h - \theta_l)(1 - \theta_h)$  can be factored out. Thus, a critical point of (8) is also a critical point of  $\frac{(1-p_1)p_1}{\theta_h + (1-\theta_h)p_1}$ . Using the quotient rule, we can calculate the derivative:

$$\frac{\delta \frac{(1-p_1)p_1}{\theta_h + (1-\theta_h)p_1}}{\delta p_1} = \frac{\theta_h - (1 - \theta_h)p_1^2 - 2\theta_h p_1}{[\theta_h + (1 - \theta_h)p_1]^2}.$$

Setting the numerator of the derivative equal to zero gives a quadratic equation for  $p_1$ . The solutions are  $p_1 = \frac{-\theta_h \pm \sqrt{\theta_h}}{1 - \theta_h}$ . So, there is a unique candidate solution  $p_1^* \in (0, 1)$ :

$$p_1^* = \frac{\sqrt{\theta_h} - \theta_h}{1 - \theta_h}. \quad (9)$$

If one inserts  $p_1^*$  in the cost discrepancy term (8), one gets

$$(\theta_h - \theta_l) \cdot (1 - p_1^*) \cdot \frac{(1 - \theta_h) \cdot p_1^*}{\theta_h + (1 - \theta_h) \cdot p_1^*} = (\theta_h - \theta_l) \left( 2 \cdot \left( \frac{1 - \sqrt{\theta_h}}{1 - \theta_h} \right) - 1 \right) \geq 0.$$

This inequality is strict for  $\theta_l \neq \theta_h$ . Thus, the global maximum of (8) is at  $p_1^*$ . This provides further insight into the initial example.

**Example 1.** Consider the parameter values  $\theta_h = 0.64$ ,  $\theta_l = 0.325$ , and  $p_1 = \frac{4}{9}$  as in our initial example. This choice of  $p_1$  indeed maximizes the cost discrepancy between the two types, as it is the optimal value  $p_1^*$ :

$$p_1^* = \frac{\sqrt{0.64} - 0.64}{1 - 0.64} = \frac{0.8 - 0.64}{0.36} = \frac{0.16}{0.36} = \frac{4}{9}.$$

Inserting  $p_1^*$  into (8) yields the highest attainable cost discrepancy:

$$\begin{aligned} & (\theta_h - \theta_l) \cdot (1 - p_1^*) \cdot \frac{(1 - \theta_h) \cdot p_1^*}{\theta_h + (1 - \theta_h) \cdot p_1^*} \\ &= (0.64 - 0.325) \cdot \left( 1 - \frac{4}{9} \right) \cdot \frac{(1 - 0.64) \cdot \frac{4}{9}}{0.64 + (1 - 0.64) \cdot \frac{4}{9}} \\ &= 0.035. \end{aligned}$$

Here, the cost discrepancy is precisely what is needed to bridge the valuation gap:

$$V(\theta_l) + 0.035 = V(\theta_h).$$

As in the two-period case, the seller can utilize different probability assignments to the experiment outcomes in  $T$  periods. Specifically, the discrepancy in expected costs is given by:

$$\begin{aligned} & \left( c_1 + \sum_{t=1}^{T-1} \mathbb{P}[\underline{\mathcal{M}}_t \mid h] \cdot c_{t+1} \right) - \left( c_1 + \sum_{t=1}^{T-1} \mathbb{P}[\underline{\mathcal{M}}_t \mid l] \cdot c_{t+1} \right) \\ &= \sum_{t=1}^{T-1} (\mathbb{P}[\underline{\mathcal{M}}_t \mid h] - \mathbb{P}[\underline{\mathcal{M}}_t \mid l]) \cdot c_{t+1} \end{aligned}$$

$$\begin{aligned}
&= \sum_{t=1}^{T-1} \left( \theta_h - \theta_l + (1 - \theta_h) \prod_{s=1}^t p_s - (1 - \theta_l) \prod_{s=1}^t p_s \right) \cdot c_{t+1} \\
&= (\theta_h - \theta_l) \cdot \sum_{t=1}^{T-1} \left( 1 - \prod_{s=1}^t p_s \right) \cdot c_{t+1}.
\end{aligned} \tag{10}$$

While the seller's ideal scenario is to find probabilities where the cost discrepancy coincides with the valuation gap, it is also interesting to find the probabilities that maximize the admissible valuation gap and thereby finding the boundary for perfect price discrimination. The following lemma states probabilities that form a critical point of (10) and generalize the two-period maximizing probability (9) to  $T$  periods. These probabilities lead to uniform costs and significant simplifications of (10).

**Lemma 3.** *Consider the experiments  $(\sigma'_t)_{t=1}^T$  constructed in Section 4.4.1. The probabilities*

$$p_t^* = \frac{\theta_h^{t/T} - \theta_h}{\theta_h^{(t-1)/T} - \theta_h} \quad \text{for } t = 1, \dots, T \tag{11}$$

*and the pricing scheme proposed in Lemma 2 lead to uniform costs for  $t \geq 2$ ,*

$$c_t^* = 1 - \theta_h^{1/T} \quad \text{for } t = 2, \dots, T, \tag{12}$$

*and a cost discrepancy of*

$$(\theta_h - \theta_l) \left( T \left( \frac{1 - \theta_h^{1/T}}{1 - \theta_h} \right) - 1 \right) \tag{13}$$

*in  $T$  periods. The limit with respect to  $T \rightarrow \infty$  is given by:*

$$(\theta_h - \theta_l) \frac{1 - \theta_h - \ln(\theta_h)}{1 - \theta_h}. \tag{14}$$

We will show later that  $c_1 = 1 - \theta_h^{1/T}$  holds as well whenever the valuation gap coincides with the highest attainable cost discrepancy.

#### 4.4.4. Lower Bound on the Low Type's Valuation and Adjustment of Experiments

By using Statement (13) of Lemma 3, we can calculate the smallest valuation of the low type such that the cost discrepancy suffices to bridge the valuation gap in  $T$  periods:

$$\theta_l(\theta_h, T) + (\theta_h - \theta_l(\theta_h, T)) \left( T \left( \frac{1 - \theta_h^{1/T}}{1 - \theta_h} \right) - 1 \right) = 1 - \theta_h$$



$$\begin{aligned}
\Leftrightarrow \quad \theta_l(\theta_h, T) &= \frac{1 - 2\theta_h}{1 - \left(T \left(\frac{1 - \theta_h^{1/T}}{1 - \theta_h}\right) - 1\right)} + \theta_h \\
\Leftrightarrow \quad \theta_l(\theta_h, T) &= \frac{(1 - \theta_h) - T \theta_h \left(1 - \theta_h^{1/T}\right)}{2(1 - \theta_h) - T \left(1 - \theta_h^{1/T}\right)}.
\end{aligned} \tag{15}$$

Analogously, we use Statement (14) of Lemma 3 to calculate the smallest valuation of the low type such that the cost discrepancy suffices to bridge the valuation gap when infinitely many periods are available:

$$\begin{aligned}
\theta_l(\theta_h, \infty) + (\theta_h - \theta_l(\theta_h, \infty)) \frac{1 - \theta_h - \ln(\theta_h)}{1 - \theta_h} &= 1 - \theta_h \\
\Leftrightarrow \theta_l(\theta_h, \infty) &= \frac{(1 - \theta_h) + \theta_h \ln(\theta_h)}{2(1 - \theta_h) + \ln(\theta_h)}.
\end{aligned}$$

We can conclude that for every  $\theta_l \in (\theta_l(\theta_h, \infty), \theta_l(\theta_h, 1)] = \left(\frac{(1 - \theta_h) + \theta_h \ln(\theta_h)}{2(1 - \theta_h) + \ln(\theta_h)}, 1 - \theta_h\right]$ , we can find a maturity  $T'$  such that  $\theta_l \in [\theta_l(\theta_h, T'), \theta_l(\theta_h, T' - 1)] \Leftrightarrow$

$$\theta_l \in \left[ \frac{(1 - \theta_h) - T' \theta_h \left(1 - \theta_h^{1/T'}\right)}{2(1 - \theta_h) - T' \left(1 - \theta_h^{1/T'}\right)}, \frac{(1 - \theta_h) - (T' - 1) \theta_h \left(1 - \theta_h^{1/(T'-1)}\right)}{2(1 - \theta_h) - (T' - 1) \left(1 - \theta_h^{1/(T'-1)}\right)} \right]. \tag{16}$$

We will show that the seller can perform perfect price discrimination in  $T' \leq T$  periods. Hence, given (16), we have to find probabilities  $(p_1, \dots, p_{T'})$  such that the valuation gap can be bridged:

$$\theta_l + (\theta_h - \theta_l) \cdot \left( \sum_{t=1}^{T'-1} \left( 1 - \prod_{s=1}^t p_s \right) \cdot c_{t+1} \right) = 1 - \theta_h. \tag{17}$$

The probabilities (11) proposed in Lemma 3 lead to equality in (17) with respect to  $\theta_l = \theta_l(\theta_h, T')$ . We will adjust those probabilities slightly. Notice that they lead to a telescoping product:

$$\prod_{s=1}^t p_s^* = \frac{\theta_h^{1/T'} - \theta_h}{\theta_h^{0/T'} - \theta_h} \cdot \frac{\theta_h^{2/T'} - \theta_h}{\theta_h^{1/T'} - \theta_h} \cdot \dots \cdot \frac{\theta_h^{t/T'} - \theta_h}{\theta_h^{(t-1)/T'} - \theta_h} = \frac{\theta_h^{t/T'} - \theta_h}{1 - \theta_h} \quad \text{for } t = 1, \dots, T'.$$

To allow for equality in (17) across the entire interval (16), we relax this condition to:

$$\prod_{s=1}^t p_s = \frac{\theta_h^{t/T'} - \theta_h}{1 - \theta_h} \quad \text{for } t = 2, \dots, T',$$

while leaving  $p_1 \in \left[ \frac{\theta_h^{2/T'} - \theta_h}{1 - \theta_h}, 1 \right]$  as a variable. Notice that  $p_1 = \frac{\theta_h^{1/T'} - \theta_h}{1 - \theta_h}$  recovers the proposed discrepancy-maximizing case.

The following lemma shows that the range of values of  $\theta_l$  with respect to varying  $p_1$  cover the entire interval (16).

**Lemma 4.** *There exists a probability  $p_1(\theta_l) \in (p_1^*, 1)$  that satisfies Equation (17) for each choice of  $\theta_l$  satisfying (16).*

Finally, we check for non-negativity of the costs. We first determine the individual probabilities  $p_t$ :

$$p_t = \begin{cases} p_1(\theta_l) & \text{for } t = 1, \\ \frac{\prod_{s=1}^2 p_s}{p_1(\theta_l)} = \frac{\theta_h^{2/T'} - \theta_h}{(1 - \theta_h)p_1(\theta_l)} & \text{for } t = 2, \\ \frac{\prod_{s=1}^t p_s}{\prod_{s=1}^{t-1} p_s} = \frac{\theta_h^{t/T'} - \theta_h}{\theta_h^{(t-1)/T'} - \theta_h} & \text{for } t = 3, \dots, T'. \end{cases}$$

As a result, only the second-period probability depends on the variable  $p_1(\theta_l)$  in order to satisfy the relaxed condition. In all subsequent periods, the condition ensures that this variable cancels out.

Let's recall the cost terms  $c_t$  for  $t \geq 2$  (5) from Lemma 2:

$$c_t = \frac{(1 - \theta_h) \prod_{s=1}^{t-1} p_s (1 - p_t)}{\theta_h + (1 - \theta_h) \prod_{s=1}^{t-1} p_s} \geq 0.$$

Substituting the chosen probabilities for  $t \geq 3$  leads to the familiar costs:

$$c_t = 1 - \theta_h^{1/T'} \quad \text{for } t = 3, \dots, T'.$$

However, for  $t = 2$ , the cost depends on  $p_1$ :

$$c_2 = \frac{(1 - \theta_h)p_1(1 - p_2)}{\theta_h + (1 - \theta_h)p_1} = \frac{(1 - \theta_h)p_1 \left( 1 - \frac{\theta_h^{2/T'} - \theta_h}{(1 - \theta_h)p_1} \right)}{\theta_h + (1 - \theta_h)p_1} = \frac{(1 - \theta_h)p_1 - (\theta_h^{2/T'} - \theta_h)}{\theta_h + (1 - \theta_h)p_1}.$$

In order to calculate  $c_1$ , it will prove useful to rearrange (17):

$$\begin{aligned} & \sum_{t=1}^{T'-1} \left( \theta_h + (1 - \theta_h) \prod_{s=1}^t p_s \right) \cdot c_{t+1} - \sum_{t=1}^{T'-1} \left( \theta_l + (1 - \theta_l) \prod_{s=1}^t p_s \right) \cdot c_{t+1} = 1 - \theta_h - \theta_l \\ \Leftrightarrow & - \sum_{t=1}^{T'-1} \left( \theta_l + (1 - \theta_l) \prod_{s=1}^t p_s \right) c_{t+1} = 1 - \theta_h - \theta_l - \sum_{t=1}^{T'-1} \left( \theta_h + (1 - \theta_h) \prod_{s=1}^t p_s \right) c_{t+1}. \end{aligned}$$

Now, we can rewrite the cost in period one (6) from Lemma 2 as:

$$\begin{aligned}
c_1 &= \theta_l - \sum_{t=1}^{T'-1} \left( \theta_l + (1 - \theta_l) \prod_{s=1}^t p_s \right) \cdot c_{t+1} \\
&= \theta_l + 1 - \theta_h - \theta_l - \sum_{t=1}^{T'-1} \left( \theta_h + (1 - \theta_h) \prod_{s=1}^t p_s \right) \cdot c_{t+1} \\
&= 1 - \theta_h - (\theta_h + (1 - \theta_h)p_1) \cdot c_2 - \sum_{t=2}^{T'-1} \theta_h^{t/T'} \cdot (1 - \theta_h^{1/T'}) \\
&= 1 - \theta_h - (1 - \theta_h)p_1(1 - p_2) - \theta_h^{2/T'} + \theta_h \\
&= 1 - \theta_h - (1 - \theta_h)p_1 + \theta_h^{2/T'} - \theta_h + \theta_h^{2/T'} - \theta_h \\
&= 1 - \theta_h - (1 - \theta_h)p_1 \geq 0.
\end{aligned}$$

Thus, the pricing is well defined. In particular, if  $p_1 = p_1^* = \frac{\theta_h^{1/T'} - \theta_h}{1 - \theta_h}$ , then the optimal pricing scheme results in uniform costs, i.e.,  $c_t = 1 - \theta_h^{1/T'}$  for all  $1 \leq t \leq T'$ . Consequently, if the difference in consumer valuations coincides with the highest achievable cost discrepancy in  $T'$  periods, then the seller can implement a sequence of  $T'$  experiments with uniform costs.

**Remark.** In the initial example, we had  $c_1 = c_2 = 1 - \sqrt{0.64} = 0.2$ . As shown, this was no coincidence but holds more generally. However, it requires that the highest admissible valuation gap has to be bridged. As we have seen in Example 1, this holds for the parameters  $\theta_h = 0.64$  and  $\theta_l = 0.325$  in two periods.

#### 4.4.5. Summary

We can conclude that for each pair of  $\theta_l$  and  $\theta_h$  that satisfy inequality (2) from Theorem 1, the seller can restrict himself to a maturity  $T' \leq T$  that satisfies (16) and design priced experiments of the following form:

$\sigma_t(m_t \mid \omega)$	$\underline{m}$	$\overline{m}$
$\underline{\omega}$	1	0
$\overline{\omega}$	$p_t$	$1 - p_t$

with probabilities

$$p_t = \begin{cases} p_1(\theta_l) \in \left[ \frac{\theta_h^{2/T'} - \theta_h}{(1-\theta_h)}, 1 \right] & \text{for } t = 1, \\ \frac{\theta_h^{2/T'} - \theta_h}{(1-\theta_h)p_1(\theta_l)} & \text{for } t = 2, \\ \frac{\theta_h^{t/T'} - \theta_h}{\theta_h^{(t-1)/T'} - \theta_h} & \text{for } t = 3, \dots, T' \end{cases}$$

and costs

$$c_t = \begin{cases} 1 - \theta_h - (1 - \theta_h)p_1(\theta_l) & \text{for } t = 1, \\ \frac{(1-\theta_h)p_1(\theta_l) - (\theta_h^{2/T'} - \theta_h)}{\theta_h + (1-\theta_h)p_1(\theta_l)} & \text{for } t = 2, \\ 1 - \theta_h^{1/T'} & \text{for } t = 3, \dots, T'. \end{cases}$$

Considering the encrypted experiments  $(\sigma'_t)_{t=1}^{T'}$  (in the sense of Lemma 1), we have shown in Lemma 2 that for these costs and  $t \geq 2$ , it is sequentially rational for both consumer types to buy all experiments conditional on  $\mathcal{M}_{t-1} = \underline{\mathcal{M}}_{t-1}$  until a revelation of the state occurs:

$$\rho_t^*(\theta_s, \mathcal{M}_{t-1}, \sigma', c) = \begin{cases} 1 & \text{if } \mathcal{M}_{t-1} = \underline{\mathcal{M}}_{t-1} \\ 0 & \text{otherwise.} \end{cases}$$

If they buy also the first experiment, then we have shown that this behavior leads to a difference in expected costs that coincides with the valuation gap:

$$\left( c_1 + \sum_{t=1}^{T'-1} \mathbb{P}[\underline{\mathcal{M}}_t \mid h] \cdot c_{t+1} \right) - \left( c_1 + \sum_{t=1}^{T'-1} \mathbb{P}[\underline{\mathcal{M}}_t \mid l] \cdot c_{t+1} \right) = 1 - \theta_h - \theta_l.$$

Due to (6) from Lemma 2, it holds that the low type pays her valuation in expectation:

$$c_1 + \sum_{t=1}^{T'-1} \mathbb{P}[\underline{\mathcal{M}}_t \mid l] \cdot c_{t+1} = \theta_l = V(\theta_l).$$

Thus, the same holds for the high type:

$$\begin{aligned} & \left( c_1 + \sum_{t=1}^{T'-1} \mathbb{P}[\underline{\mathcal{M}}_t \mid h] \cdot c_{t+1} \right) - \theta_l = 1 - \theta_h - \theta_l \\ \Leftrightarrow & \quad c_1 + \sum_{t=1}^{T'-1} \mathbb{P}[\underline{\mathcal{M}}_t \mid h] \cdot c_{t+1} = 1 - \theta_h = V(\theta_h). \end{aligned}$$

Hence, we have equality in (7), which implies that it is sequentially rational to take the

seller-preferred decision and buy also the first experiment:

$$\rho_t^*(\theta_s, \emptyset, \sigma, c) = 1.$$

Therefore, the seller extracts the entire consumer surplus:

$$\begin{aligned} & \mathbb{P}[s = h] \cdot \left( c_1 + \sum_{t=1}^{T'-1} \mathbb{P}[\underline{\mathcal{M}}_t \mid h] \cdot c_{t+1} \right) + \mathbb{P}[s = l] \cdot \left( c_1 + \sum_{t=1}^{T'-1} \mathbb{P}[\underline{\mathcal{M}}_t \mid l] \cdot c_{t+1} \right) \\ &= \mathbb{P}[s = h] \cdot V(\theta_h) + \mathbb{P}[s = l] \cdot V(\theta_l). \end{aligned}$$

Since a consumer of type  $\theta_s$  is unwilling to pay more than her valuation  $V(\theta_s)$  in expectation, this sequence of priced experiments is optimal from the viewpoint of the seller. Therefore, we have found a seller-preferred subgame perfect equilibrium and all other equilibria have to extract the entire consumer surplus as well, which concludes the proof of Theorem 1.

#### 4.5. Comparison with the static framework of BBS

Suppose for the moment that the consumer cannot observe the outcomes of experiments until the communication phase concludes. In this scenario, the buyer views a menu of experiments and selects which ones to purchase. This restriction transforms the game into a *static game à la BBS*.

In such a static game, the seller can only extract the entire surplus out of both consumers if their valuations coincide. Therefore, according to Theorem 1, the seller receives a strictly higher revenue compared to the optimal static menu whenever the valuations of the types are sufficiently close. The following theorem lists further conditions under which the seller strictly benefits from a sequential selling strategy.

**Theorem 2.** *Let  $T \geq 2$  and the types satisfy  $0 < V(\theta_l) < V(\theta_h) < \frac{1}{2}$ . If types are either*

(i) *congruent and*

$$\mathbb{P}[s = h] \cdot (1 - \theta_h) < 1 - \theta_l + \mathbb{P}[s = h] \cdot \left( \sqrt{\frac{\theta_h(1 - \theta_h)(1 - \theta_l)}{\theta_l}} - (1 - \theta_l) \right),$$

(ii) *non-congruent and*

$$\frac{\theta_l(2\theta_h - 1)}{\theta_h - \theta_l} + \mathbb{P}[s = h] \cdot \frac{1 - \theta_h - \theta_h\theta_l}{\theta_h - \theta_l} < \theta_l + \mathbb{P}[s = h] \cdot \left( \sqrt{\frac{\theta_l\theta_h(1 - \theta_h)}{1 - \theta_l}} - \theta_l \right),$$

(iii) *non-congruent and  $T \geq 3$ ,*

*then every seller-preferred subgame perfect equilibrium yields a strictly higher revenue than the optimal revenue achievable in the static game.*

To establish the first claim for congruent beliefs, we compare the optimal static menu with a two-period selling strategy of offering a free test that aligns the valuations of each type and offer a subsequent state-revealing test priced at their updated valuations. This comparison leads to the first statement of Theorem 2. Implementing the same idea with non-congruent beliefs yields the second statement of the theorem. Subsequently, a more involved dynamic mechanism, that requires state-dependent encryption, is introduced to prove the third statement.

#### 4.6. Proof of Theorem 2

We prove the theorem by addressing each claim individually because different methodologies are required.

- (i) Consider congruent types. BBS demonstrated that for congruent beliefs, the seller optimally implements a single state-revealing experiment:

$\sigma_1(m_1 \mid \omega)$	$\underline{m}$	$\overline{m}$
$\underline{\omega}$	1	0
$\overline{\omega}$	0	1

The price  $c_1$  of this experiment is determined by comparing the expected revenue from selling exclusively to the high-valuation type consumer versus selling to both consumer types. In the static framework, the seller's optimal menu is as follows:

- (a) If  $\mathbb{P}[s = h] \cdot (1 - \theta_h) \leq 1 - \theta_l$ , then offering the experiment  $\sigma_1$ , priced at  $c_1 = V(\theta_l)$ , is an optimal strategy for the seller in the static framework.
- (b) If  $\mathbb{P}[s = h] \cdot (1 - \theta_h) \geq 1 - \theta_l$ , then offering the experiment  $\sigma_1$ , priced at  $c_1 = V(\theta_h)$ , is an optimal strategy for the seller in the static framework.

In the first case<sup>6</sup>, the following example illustrates how the seller can benefit from providing a free experiment in the first period and selling the state-revealing experiment in the second period. In particular, the seller can design the sequence of priced

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<sup>6</sup>and slightly beyond, i.e., if  $\mathbb{P}[s = h] \cdot (1 - \theta_h) < 1 - \theta_l + \mathbb{P}[s = h] \cdot \left( \sqrt{\frac{\theta_h(1-\theta_h)(1-\theta_l)}{\theta_l}} - (1 - \theta_l) \right)$ .

experiments such that he receives an expected payment of  $V(\theta_l)$  from the low type while receiving strictly more from the high type.

**Example 2.** Let  $T = 2$ . Consider the prior beliefs  $\theta_h = 0.8$  for the high type and  $\theta_l = 0.9$  for the low type. We assume that the low type is more likely, i.e.,  $\mathbb{P}[s = h] \leq \frac{1}{2}$ .

Since the types are congruent, and the probability of the type being high is sufficiently small (specifically,  $\mathbb{P}[s = h] \cdot (1 - \theta_h) \leq (1 - \theta_l)$ ), the optimal static menu consists of a single state-revealing experiment priced at  $c_1 = V(\theta_l) = \frac{1}{10}$ . At this price, it is sequentially rational for both types to buy the experiment, yielding a revenue of  $\frac{1}{10}$ .

We show that a sequential selling strategy over two periods with two signals can yield a strictly higher revenue than in the optimal static menu. Consider a free first-period experiment ( $c_1 = 0$ ) with the following stochastic mapping:

$\sigma_1(m_1 \mid \omega)$	$\underline{m}$	$\overline{m}$
$\underline{\omega}$	$\frac{5}{6}$	$\frac{1}{6}$
$\overline{\omega}$	0	1

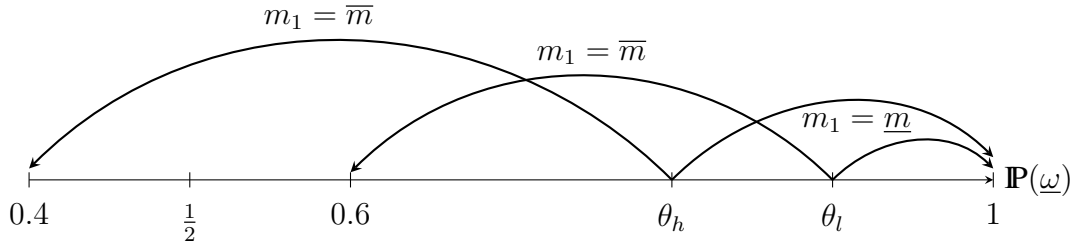
If the consumer receives the signal  $m_1 = \underline{m}$ , they will learn that the state is  $\underline{\omega}$ , and they will rationally ignore further experiments. If they receive  $m_1 = \overline{m}$ , their valuation updates. The posterior beliefs for state  $\underline{\omega}$  given signal  $m_1 = \overline{m}$  are:

$$\mathbb{P}[\underline{\omega} \mid h, m_1 = \overline{m}] = \frac{\theta_h \cdot \frac{1}{6}}{\theta_h \cdot \frac{1}{6} + 1 - \theta_h} = 0.4 < 0.6 = \mathbb{P}[\underline{\omega} \mid l, m_1 = \overline{m}].$$

Figure 5 illustrates the induced posteriors. Importantly, the low type would not pay a positive price for this first experiment in isolation because each possible outcome does not change her optimal action. For this reason, the seller can still extract the full valuation from the low type.

Notice that the stochastic mapping is chosen such that the updated valuations coincide upon observing the non-revealing signal,  $m_2 = \overline{m}$ . Thus, the seller can extract the full updated valuations by offering a state-revealing experiment in the second period at a price of

$$c_2 = V(\theta_l, m_1 = \overline{m}) = V(\theta_h, m_1 = \overline{m}) = 0.4.$$



**Figure 5.** Posteriors induced by  $\sigma_1$ .

After observing  $m_1 = \bar{m}$  from the first experiment, both types find it sequentially rational to purchase the second experiment. The key observation is that the high type considers it more likely than the low type to observe the signal realization  $m_1 = \bar{m}$ :

$$\mathbb{P}[m_1 = \bar{m} \mid h] = \theta_h \cdot \frac{1}{6} + (1 - \theta_h) = \frac{1}{3} > \frac{1}{4} = \theta_l \cdot \frac{1}{6} + (1 - \theta_l) = \mathbb{P}[m_1 = \bar{m} \mid l].$$

Thus, if both types decide to observe the first experiment and only purchase the second experiment upon observing  $m_1 = \bar{m}$ , then the seller's expected revenue is strictly higher than the revenue of the optimal static menu:

$$\begin{aligned} & \mathbb{P}[s = h] \cdot \mathbb{P}[m_1 = \bar{m} \mid h] \cdot c_2 + (1 - \mathbb{P}[s = h]) \cdot \mathbb{P}[m_1 = \bar{m} \mid l] \cdot c_2 \\ &= \mathbb{P}[s = h] \cdot \frac{1}{3} \cdot 0.4 + (1 - \mathbb{P}[s = h]) \cdot \frac{1}{4} \cdot 0.4 > \frac{1}{10}. \end{aligned}$$

We will now generalize the idea from the previous example. We assume that the consumer types are congruent and their valuations satisfy  $0 < V(\theta_l) < V(\theta_h) < \frac{1}{2}$ . This implies that the prior beliefs satisfy  $\theta_l > \theta_h > \frac{1}{2}$ .

As in the previous example, the seller can restrict himself to use only two signals,  $M = \{\underline{m}, \bar{m}\}$ , and two periods. Consider a sequence of two experiments. The first experiment is designed such that each consumer sometimes learns the state, which is already favored by their beliefs. The stochastic mapping of the experiment is as follows:

$\sigma_1(m_1 \mid \omega)$	$\underline{m}$	$\bar{m}$
$\underline{\omega}$	$1 - \sqrt{\frac{(1-\theta_h)(1-\theta_l)}{\theta_h\theta_l}}$	$\sqrt{\frac{(1-\theta_h)(1-\theta_l)}{\theta_h\theta_l}}$
$\bar{\omega}$	0	1

Since an experiment outcome of  $m_1 = \underline{m}$  reveals the state as  $\underline{\omega}$ , a consumer observ-



ing  $m_1 = \underline{m}$  has no need to buy a second experiment. In contrast, an outcome of  $m_1 = \overline{m}$  leads to interim beliefs of:

$$\begin{aligned}\mathbb{P}[\underline{\omega} \mid h, m_1 = \overline{m}] &= \frac{\theta_h \cdot \sqrt{\frac{(1-\theta_h)(1-\theta_l)}{\theta_h \theta_l}}}{\theta_h \cdot \sqrt{\frac{(1-\theta_h)(1-\theta_l)}{\theta_h \theta_l}} + 1 - \theta_h} \\ &= \frac{1}{1 + \sqrt{\frac{(1-\theta_h)\theta_l}{\theta_h(1-\theta_l)}}} \\ &= \frac{1 - \theta_l}{\theta_l \cdot \sqrt{\frac{(1-\theta_h)(1-\theta_l)}{\theta_h \theta_l}} + 1 - \theta_l} = \mathbb{P}[\overline{\omega} \mid l, m_1 = \underline{m}].\end{aligned}$$

The low type's optimal action does not change regardless of the signal realization  $m_1$ . We assume that the seller offers  $\sigma_1$  for free,  $c_1 = 0$ . The seller benefits from the adjustment of the consumers' valuations for further information. Specifically, the seller can offer a second, state-revealing experiment that extracts the updated valuation of both consumer upon observing  $m_1 = \overline{m}$ . The price of it needs to be chosen as:

$$c_2 = V(\theta_h, m_1 = \overline{m}) = V(\theta_l, m_1 = \overline{m}) = \frac{1}{1 + \sqrt{\frac{(1-\theta_h)\theta_l}{\theta_h(1-\theta_l)}}}.$$

At this price, both types, upon observing  $m_1 = \overline{m}$ , consider it sequentially rational to purchase the second experiment,  $\sigma_2$ .

The low type's expected payoff from ignoring all experiments is  $1 - V(\theta_l) = \theta_l$ . Since any of the outcomes of the first experiment do not change her optimal action, her payoff after  $\sigma_1$  alone remains  $\theta_l$ . She will not purchase the second experiment in isolation because its price  $c_2$  exceeds her valuation of a state-revelation,  $V(\theta_l) < c_2$ . She always matches her action to the true state if she observes the outcome of  $\sigma_1$  and then purchases  $\sigma_2$  upon observing  $m_1 = \overline{m}$ . The expected costs are  $\mathbb{P}[m_1 = \overline{m} \mid l] \cdot c_2$ . Crucially,

$$\mathbb{P}[m_1 = \overline{m} \mid l] \cdot c_2 = \mathbb{P}[m_1 = \overline{m} \mid l] \cdot \mathbb{P}[\overline{\omega} \mid l, m_1 = \overline{m}] = \mathbb{P}[\overline{\omega}, m_1 = \overline{m} \mid l] = 1 - \theta_l.$$

Therefore, her expected utility from this purchasing behavior is also  $1 - V(\theta_l) = \theta_l$ . Thus, she is indifferent between this purchasing behavior and ignoring all experiments. We assume that the low type makes the seller-preferred decision and purchases the experiments.

Similarly, the high type's expected payoff from ignoring all experiments is  $1 -$

$V(\theta_h) = \theta_h$ . If she observes the outcome of  $\sigma_1$  and then purchases  $\sigma_2$  only if  $m_1 = \bar{m}$  occurs, her action always matches the state. Her expected costs are

$$\mathbb{P}[m_1 = \bar{m} \mid h] \cdot c_2 = \mathbb{P}[m_1 = \bar{m} \mid h] \cdot \mathbb{P}[\bar{\omega} \mid l, m_1 = \bar{m}] > V(\theta_l)$$

since  $\mathbb{P}[m_1 = \bar{m} \mid h] > \mathbb{P}[m_1 = \bar{m} \mid l]$ . This purchasing behavior is preferred by the high type over buying  $\sigma_2$  exclusively, because there is a positive probability that there is no need to purchase  $\sigma_2$ . Furthermore, it is sequentially rational for the high type to buy  $\sigma_2$  upon observing  $m_1 = \bar{m}$  because her updated valuation exceeds the costs of the second experiment, i.e.,  $V(\theta_h, m_1 = \bar{m}) > c_2$ .

The seller's expected revenue from this sequence of experiments and the implied purchasing behavior is as follows:

$$\begin{aligned} & \mathbb{P}[s = h] \cdot \mathbb{P}[m_1 = \bar{m} \mid h] \cdot c_2 + (1 - \mathbb{P}[s = h]) \cdot V(\theta_l) \\ &= 1 - \theta_l + \mathbb{P}[s = h] \cdot \left( \theta_h \cdot \sqrt{\frac{(1 - \theta_h)(1 - \theta_l)}{\theta_h \theta_l}} - (1 - \theta_l) \right) \\ &= 1 - \theta_l + \mathbb{P}[s = h] \cdot \left( \sqrt{\frac{\theta_h(1 - \theta_h)(1 - \theta_l)}{\theta_l}} - (1 - \theta_l) \right). \end{aligned}$$

To conclude, we compare this dynamic revenue with that of the optimal static menu. Specifically, the sequence of experiments yields a strictly higher revenue than the optimal static menu if

$$\max\{1 - \theta_l, \mathbb{P}[s = h] \cdot (1 - \theta_h)\} < 1 - \theta_l + \mathbb{P}[s = h] \cdot \left( \sqrt{\frac{\theta_h(1 - \theta_h)(1 - \theta_l)}{\theta_l}} - (1 - \theta_l) \right).$$

Since the right-hand side is always greater than  $1 - \theta_l$ <sup>7</sup>, it suffices to check whether the dynamic revenue surpasses  $\mathbb{P}[s = h] \cdot (1 - \theta_h)$ . This verifies the first claim.

- (ii) Consider non-congruent beliefs and  $T = 2$ . The static scenario with non-congruent beliefs has also been analyzed by BBS. Analogously to the congruent case, a state-revealing experiment,  $\sigma_1$ , is part of the optimal menu:

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<sup>7</sup>i.e.,  $\sqrt{\frac{\theta_h(1 - \theta_h)(1 - \theta_l)}{\theta_l}} > 1 - \theta_l \Leftrightarrow \theta_h(1 - \theta_h) > \theta_l(1 - \theta_l)$ , which holds since  $1 - \theta_l < 1 - \theta_h < \theta_h < \theta_l$ .

$\sigma_1(m_1   \omega)$	$\underline{m}$	$\overline{m}$
$\underline{\omega}$	1	0
$\overline{\omega}$	0	1

In contrast to the congruent beliefs scenario, the seller can simultaneously offer an imperfect experiment and sell it to the low type while selling  $\sigma_1$  to the high type. This is possible because the low type benefits from an experiment that reveals the state  $\underline{\omega}$  even with a small probability  $p$ . Such an experiment holds no value for the high type if the probability  $p$  is chosen sufficiently small such that her default action, choosing  $a = \underline{a}$ , remains optimal regardless of the realized signal.

Such an experiment can be designed with a revealing probability  $p^* = \frac{2\theta_h - 1}{\theta_h - \theta_l}$  such that both types get the same additional precision out of it (see BBS). If the seller prices the second imperfect experiment at the low type's additional precision,  $c_2 = p^* \cdot \theta_l$ , then the high type is indifferent between buying or ignoring this experiment and is assumed to ignore it. Instead, we assume that the high type opts for the state-revealing experiment  $\sigma_1$  priced at her valuation  $V(\theta_h)$ .

Hence, the seller compares receiving  $V(\theta_l) = \theta_l$  from both types with receiving  $V(\theta_h) = 1 - \theta_h$  from the high type and  $p^* \cdot \theta_l = \frac{2\theta_h - 1}{\theta_h - \theta_l} \cdot \theta_l$  from the low type. The break-even point, where these revenues are equal, is given by:

$$\theta_l = \mathbb{P}[s = h] \cdot (1 - \theta_h) + (1 - \mathbb{P}[s = h]) \cdot \frac{2\theta_h - 1}{\theta_h - \theta_l} \theta_l \quad \Leftrightarrow \quad \theta_l = \mathbb{P}[s = h] \cdot \theta_h.$$

The optimal static menu is then as follows:

- (a) If types are non-congruent and  $\mathbb{P}[s = h] \cdot \theta_h \leq \theta_l$ , then offering the experiment  $\sigma_1$  priced at  $c_1 = V(\theta_l)$  is an optimal strategy of the seller in the static framework.
- (b) If types are non-congruent and  $\mathbb{P}[s = h] \cdot \theta_h \geq \theta_l$ , then offering the experiment  $\sigma_1$  priced at  $c_1 = V(\theta_h)$  as well as a second imperfect experiment  $\sigma_2$  given by

$\sigma_2(m_2   \omega)$	$\underline{m}$	$\overline{m}$
$\underline{\omega}$	$\frac{2\theta_h - 1}{\theta_h - \theta_l}$	$\frac{1 - \theta_h - \theta_l}{\theta_h - \theta_l}$
$\overline{\omega}$	0	1

priced at  $c_2 = \theta_l \cdot \frac{2\theta_h - 1}{\theta_h - \theta_l}$  is an optimal strategy of the seller in the static framework.

In the first case<sup>8</sup>, similarly to the situation with congruent beliefs, the seller can benefit from providing a free experiment in the first period and offer a state-revealing experiment afterwards. The following example illustrates how the seller can analogously receive an expected payment of  $V(\theta_l)$  from the low type and strictly more than that from the high type.

**Example 3.** *We consider a scenario with prior beliefs of  $\theta_h = 0.8$  and  $\theta_l = 0.1$ . Notice that these beliefs lead to the same valuations as in the previous example. It is assumed that the high type is relatively unlikely, specifically  $\mathbb{P}[s = h] \leq \frac{1}{8}$ .*

*Since these types are non-congruent and the probability of the high type is sufficiently small, i.e.,  $\mathbb{P}[s = h] \cdot \theta_h \leq \theta_l$ , the optimal static menu consists of a single state-revealing experiment. This experiment is priced at  $c_1 = \theta_l = \frac{1}{10}$ . At this price, it is sequentially rational for both types to purchase the experiment, yielding the seller a revenue of  $\frac{1}{10}$ .*

*We now show that a sequential selling strategy over two periods can yield a strictly higher revenue. Consider a first-period experiment,  $\sigma_1$ , offered for free ( $c_1 = 0$ ). The stochastic mapping is given by:*

$\sigma_1(m_1 \mid \omega)$	$\underline{m}$	$\overline{m}$
$\underline{\omega}$	1	0
$\overline{\omega}$	$\frac{2}{3}$	$\frac{1}{3}$

*If a consumer observes the signal  $m_1 = \overline{m}$ , she learns that the state is  $\overline{\omega}$  and will rationally ignore further experiments. If she observes  $m_1 = \underline{m}$ , her valuations updates. The posterior beliefs for state  $\underline{\omega}$  given signal  $m_1 = \underline{m}$  are calculated below.*

$$\mathbb{P}[\underline{\omega} \mid h, m_1 = \underline{m}] = \frac{\theta_h}{\theta_h + (1 - \theta_h) \cdot \frac{2}{3}} = \frac{0.8}{0.8 + 0.2 \cdot \frac{2}{3}} = \frac{6}{7}$$

*and*

$$\mathbb{P}[\underline{\omega} \mid l, m_1 = \underline{m}] = \frac{\theta_l}{\theta_l + (1 - \theta_l) \cdot \frac{2}{3}} = \frac{0.1}{0.1 + 0.9 \cdot \frac{2}{3}} = \frac{1}{7}.$$

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<sup>8</sup>and slightly beyond, i.e., if  $\frac{\theta_l(2\theta_h - 1)}{\theta_h - \theta_l} + \mathbb{P}[s = h] \cdot \frac{1 - \theta_h - \theta_h \theta_l}{\theta_h - \theta_l} < \theta_l + \mathbb{P}[s = h] \cdot \left( \sqrt{\frac{\theta_l \theta_h (1 - \theta_h)}{1 - \theta_l}} - \theta_l \right)$ .

In the second period, a state-revealing experiment is offered at a price of  $c_2 = \frac{1}{7}$ . Upon observing  $m_1 = \underline{m}$  from the first experiment, both types find it sequentially rational to purchase  $\sigma_2$ . A key observation is that the high type considers it more likely than the low type to observe the signal realization  $m_1 = \underline{m}$ :

$$\mathbb{P}[m_1 = \underline{m} \mid h] = \theta_h + (1 - \theta_h) \cdot \frac{2}{3} = \frac{14}{15} > \frac{7}{10} = \theta_l + (1 - \theta_l) \cdot \frac{2}{3} = \mathbb{P}[m_1 = \underline{m} \mid l].$$

Therefore, if both types decide to observe the first experiment and only purchase the second experiment upon observing  $m_1 = \underline{m}$ , the seller's expected revenue is strictly greater than  $\frac{1}{10}$  (the revenue of the optimal static menu):

$$\begin{aligned} & \mathbb{P}[s = h] \cdot \mathbb{P}[m_1 = \underline{m} \mid h] \cdot c_2 + (1 - \mathbb{P}[s = h]) \cdot \mathbb{P}[m_1 = \underline{m} \mid l] \cdot c_2 \\ &= \mathbb{P}[s = h] \cdot \frac{14}{15} \cdot \frac{1}{7} + (1 - \mathbb{P}[s = h]) \cdot \frac{7}{10} \cdot \frac{1}{7} > \frac{1}{10}. \end{aligned}$$

We now generalize this idea. Assume consumer types are congruent and their valuations for a state-revealing experiment satisfy  $0 < V(\theta_l) < V(\theta_h) < \frac{1}{2}$ . This implies the following structure:  $\theta_h > \frac{1}{2} > 1 - \theta_h > \theta_l$ .

Consider a sequence of two experiments with signals  $M = \{\underline{m}, \overline{m}\}$ . The first experiment is offered for free ( $c_1 = 0$ ). It is designed such that the consumers sometimes learn the state  $\overline{\omega}$  (which is already favored by the low type's belief). Furthermore, the probabilities are chosen such that the updated valuations of both types coincide after observing  $m_1 = \underline{m}$ . The stochastic mapping is:

$\sigma_1(m_1 \mid \omega)$	$\underline{m}$	$\overline{m}$
$\underline{\omega}$	1	0
$\overline{\omega}$	$\sqrt{\frac{\theta_l \theta_h}{(1 - \theta_h)(1 - \theta_l)}}$	$1 - \sqrt{\frac{\theta_l \theta_h}{(1 - \theta_h)(1 - \theta_l)}}$

If the outcome is  $m_1 = \overline{m}$ , the state is revealed as  $\overline{\omega}$ , so a consumer observing  $m_1 = \overline{m}$  has no need to buy a second experiment. In contrast, if  $m_1 = \underline{m}$  occurs, the interim beliefs satisfy:

$$\begin{aligned} \mathbb{P}[\underline{\omega} \mid l, m_1 = \underline{m}] &= \frac{\theta_l}{\theta_l + (1 - \theta_l) \sqrt{\frac{\theta_l \theta_h}{(1 - \theta_h)(1 - \theta_l)}}} \\ &= \frac{1}{1 + \sqrt{\frac{(1 - \theta_l) \theta_h}{\theta_l (1 - \theta_h)}}} \end{aligned}$$

$$= \frac{(1 - \theta_h) \sqrt{\frac{\theta_l \theta_h}{(1 - \theta_h)(1 - \theta_l)}}}{\theta_h + (1 - \theta_h) \sqrt{\frac{\theta_l \theta_h}{(1 - \theta_h)(1 - \theta_l)}}} = \mathbb{P}[\bar{\omega} \mid h, m_1 = \underline{m}].$$

In isolation, only the high type strictly benefits from  $\sigma_1$ ; the low type's optimal action does not change regardless of the signal realization  $m_1$ .

Although, the experiment is offered for free, the seller benefits from its implications. Specifically, the seller can offer a second, state-revealing experiment afterwards and price it as the updated valuation after observing  $m_1 = \underline{m}$ :

$$c_2 = V(\theta_h, m_1 = \underline{m}) = V(\theta_l, m_1 = \underline{m}) = \frac{1}{1 + \sqrt{\frac{(1 - \theta_l)\theta_h}{\theta_l(1 - \theta_h)}}}.$$

At this price, both types, holding the signal realization  $m_1 = \underline{m}$ , find it sequentially rational to purchase  $\sigma_2$ .

The low type's expected payoff from ignoring all experiments is  $1 - V(\theta_l) = 1 - \theta_l$ . The free experiment  $\sigma_1$  alone does not alter this payoff, as her optimal action remains unchanged. She would not purchase  $\sigma_2$  in isolation because its price  $c_2$  exceeds her valuation of a state revelation,  $V(\theta_l) < c_2$ . However, if she observes the outcome of  $\sigma_1$  and then purchases  $\sigma_2$  only upon observing  $m_1 = \underline{m}$ , her expected costs match her valuation,  $\mathbb{P}[m_1 = \underline{m} \mid l] \cdot c_2 = \mathbb{P}[\bar{\omega}, m_1 = \underline{m} \mid l] = \theta_l = V(\theta_l)$ . Therefore, her expected utility from this purchasing behavior is also  $1 - \theta_l$ . This makes her indifferent between this sequential purchasing strategy and ignoring all experiments. We assume that the low type makes the seller-preferred decision.

Similarly, the high type's expected payoff from ignoring all experiments is  $1 - V(\theta_h) = \theta_h$ . If she observes the outcome of  $\sigma_1$  and purchases  $\sigma_2$  only if  $m_1 = \underline{m}$  occurs, her action always matches the state. This sequential strategy is preferred by the high type over buying  $\sigma_2$  exclusively, because there is a positive probability that there is no need to pay the cost  $c_2$ . Furthermore, it is sequentially rational for the high type to buy  $\sigma_2$  upon observing  $m_1 = \underline{m}$  because the cost coincides with her updated valuation,  $c_2 = V(\theta_h, m_1 = \underline{m}) = \mathbb{P}[\bar{\omega} \mid h, m_1 = \underline{m}]$ . Since the high type considers the signal realization  $m_1 = \underline{m}$  more likely than the low type,  $\mathbb{P}[m_1 = \underline{m} \mid h] > \mathbb{P}[m_1 = \underline{m} \mid l]$ , her expected costs exceed the valuation of the low type, i.e.,  $\mathbb{P}[m_1 = \underline{m} \mid h] \cdot c_2 > V(\theta_l)$ .

The seller's expected revenue from this sequence of experiments and the implied purchasing behavior is:

$$\mathbb{P}[s = h] \cdot \mathbb{P}[m_1 = \underline{m} \mid h] \cdot c_2 + (1 - \mathbb{P}[s = h]) \cdot V(\theta_l)$$

$$\begin{aligned}
&= \theta_l + \mathbb{P}[s = h] \cdot \left( (1 - \theta_h) \sqrt{\frac{\theta_l \theta_h}{(1 - \theta_h)(1 - \theta_l)}} - \theta_l \right) \\
&= \theta_l + \mathbb{P}[s = h] \cdot \left( \sqrt{\frac{\theta_l \theta_h (1 - \theta_h)}{1 - \theta_l}} - \theta_l \right).
\end{aligned}$$

To conclude, we compare this dynamic revenue with that of the optimal static menu. The sequence of experiments yields a strictly higher revenue than the optimal static menu if:

$$\begin{aligned}
&\max \left\{ \theta_l, \mathbb{P}[s = h] \cdot (1 - \theta_h) + (1 - \mathbb{P}[s = h]) \cdot \frac{2\theta_h - 1}{\theta_h - \theta_l} \theta_l \right\} \\
&< \theta_l + \mathbb{P}[s = h] \cdot \left( \sqrt{\frac{\theta_l \theta_h (1 - \theta_h)}{1 - \theta_l}} - \theta_l \right).
\end{aligned}$$

Since the right-hand side is always higher than  $1 - \theta_l$ <sup>9</sup>, the condition can be simplified to checking whether the dynamic revenue also surpasses the second term inside of the maximum, which can be simplified to

$$\frac{\theta_l(2\theta_h - 1)}{\theta_h - \theta_l} + \mathbb{P}[s = h] \cdot \frac{1 - \theta_h - \theta_h \theta_l}{\theta_h - \theta_l} < \theta_l + \mathbb{P}[s = h] \cdot \left( \sqrt{\frac{\theta_l \theta_h (1 - \theta_h)}{1 - \theta_l}} - \theta_l \right).$$

This verifies the second claim.

- (iii) Consider non-congruent beliefs and  $T \geq 3$ . We demonstrate that a sequence of three experiments can generate a strictly higher revenue for the seller than that of the optimal static menu. We assume  $\mathbb{P}[s = h] \cdot \theta_h > \theta_l$ ; otherwise, the optimal static menu consists of a single state-revealing experiment priced at  $V(\theta_l)$ , and the previous analysis has shown that in this case already two periods are sufficient to achieve a strictly increased revenue.

Let  $T = 3$ . The seller can restrict himself to use the signal set  $M = \{\underline{m}, \underline{m}', \overline{m}\}$ . Consider the following sequence of priced experiments. The first experiment is state-revealing and priced at  $c_1 = 1 - \theta_h$ .

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<sup>9</sup>i.e.,  $\sqrt{\frac{\theta_l \theta_h (1 - \theta_h)}{1 - \theta_l}} > \theta_l \Leftrightarrow \theta_h(1 - \theta_h) > \theta_l(1 - \theta_l)$ , which holds since  $\theta_l < 1 - \theta_h < \theta_h < 1 - \theta_l$ .

$\sigma_1(m_1 \mid \omega)$	$\underline{m}$	$\underline{m}'$	$\overline{m}$
$\underline{\omega}$	1	0	0
$\overline{\omega}$	0	0	1

The subsequent experiments will be designed to ensure that the high-type consumer purchases  $\sigma_1$ , mirroring her choice in the optimal static menu.

The second experiment,  $\sigma_2$ , is a modified version of the imperfect experiment sold to the low type in the optimal static menu. It is defined by the probabilities  $p$  and  $\epsilon$ :

$\sigma_2(m_2 \mid \omega)$	$\underline{m}$	$\underline{m}'$	$\overline{m}$
$\underline{\omega}$	$\frac{p}{2}$	$\frac{p}{2}$	$1 - p$
$\overline{\omega}$	$\frac{\epsilon}{2}$	$\frac{\epsilon}{2}$	$1 - \epsilon$

It is priced at  $c_2 = \theta_l \cdot p - \mathbb{P}[m_2 \neq \overline{m} \mid l] \cdot c_3$ , where  $c_3$  is the price of the third experiment. We choose  $\epsilon > 0$  sufficiently small and  $p$  sufficiently high, specifically

$$\epsilon \leq \frac{\theta_l(2\theta_h - 1)}{(1 - \theta_h)(\theta_h - \theta_l)} \leq \frac{2\theta_h - 1}{\theta_h - \theta_l} \leq p.$$

If we would choose  $\epsilon = 0$ , then  $\sigma_2$  would be as informative as the imperfect experiment in the static menu.

The third experiment will depend on the outcome of the second experiment,  $m_2$ :

- If  $m_2 = \overline{m}$ ,  $\sigma_3$  is uninformative:

$\sigma_3(m_3 \mid \omega, m_2 = \overline{m})$	$\underline{m}$	$\underline{m}'$	$\overline{m}$
$\underline{\omega}$	$\frac{1}{2}$	0	$\frac{1}{2}$
$\overline{\omega}$	$\frac{1}{2}$	0	$\frac{1}{2}$

- If  $m_2 = \underline{m}$  or  $m_2 = \underline{m}'$ ,  $\sigma_3$  is state-revealing.

$\sigma_3(m_3 \mid \omega, m_2 = \underline{m})$	$\underline{m}$	$\underline{m}'$	$\overline{m}$
$\underline{\omega}$	1	0	0
$\overline{\omega}$	0	0	1



$\sigma_3(m_3 \mid \omega, m_2 = \underline{m}')$	$\underline{m}$	$\underline{m}'$	$\overline{m}$
$\underline{\omega}$	0	0	1
$\overline{\omega}$	1	0	0

The price is  $c_3 = \mathbb{P}[\overline{\omega} \mid h, m_2 = \underline{m}] = \mathbb{P}[\overline{\omega} \mid h, m_2 = \underline{m}'] = \frac{\epsilon(1-\theta_h)}{\epsilon(1-\theta_h)+p\theta_h}$ . Let us check that  $c_2$  is well defined for these cost  $c_3$ :

$$c_2 = \theta_l \cdot p - (\theta_l \cdot p + (1 - \theta_l) \cdot \epsilon) \frac{\epsilon(1 - \theta_h)}{\epsilon(1 - \theta_h) + \theta_h \cdot p} \geq 0 \Leftrightarrow \epsilon \leq p \cdot \sqrt{\frac{\theta_l \theta_h}{(1 - \theta_h)(1 - \theta_l)}}.$$

Our initial constraint for  $\epsilon$  satisfies this since:

$$\epsilon < \frac{\theta_l(2\theta_h - 1)}{(1 - \theta_h)(\theta_h - \theta_l)} = \frac{2\theta_h - 1}{\theta_h - \theta_l} \cdot \sqrt{\left(\frac{\theta_l}{1 - \theta_h}\right)^2} < p \cdot \sqrt{\frac{\theta_l \theta_h}{(1 - \theta_h)(1 - \theta_l)}},$$

where we used that  $p > \frac{2\theta_h - 1}{\theta_h - \theta_l}$  and  $\frac{\theta_l}{1 - \theta_h} < \frac{\theta_h}{1 - \theta_l} \Leftrightarrow \theta_l(1 - \theta_l) < \theta_h(1 - \theta_h)$ .

As in Section 4.1, encryption ensures that the third experiment is uninformative if the consumer did not observe the outcome of the second experiment,  $m_2$ . However, the approach that is used here differs from the encryption used in the proof of Lemma 1 as it relies on the state-dependent signal realization of the experiment. Specifically,  $\sigma_3$  is state-revealing only if a signal  $m_2 \neq \overline{m}$  has realized in the second period. This event ( $m_2 \neq \overline{m}$ ) is more likely to occur if the state is  $\underline{\omega}$ , given that  $\epsilon < p$ .

The design ensures that  $\sigma_3$  provides no information if purchased in isolation (i.e., without holding the experiment outcome  $m_2$ ). For any type  $\theta_s$  and any state  $\omega \in \Omega$ :

$$\mathbb{P}[\omega \mid s, m_3 = \underline{m}] = \mathbb{P}[\omega \mid s] = \mathbb{P}[\omega \mid s, m_3 = \overline{m}].$$

If a consumer has observed  $m_2 \neq \overline{m}$  from  $\sigma_2$  (but did not purchase  $\sigma_1$ ), it is sequentially rational to also purchase  $\sigma_3$ . This is because the updated valuation conditional on  $m_2 \neq \overline{m}$  for the low type exceeds that for the high type and the high type is indifferent at price  $c_3$ :

$$V(\theta_l, m_2 = \underline{m}) = V(\theta_l, m_2 = \underline{m}') > V(\theta_h, m_2 = \underline{m}) = V(\theta_h, m_2 = \underline{m}') = c_3.$$

On the other hand, if  $m_2 = \overline{m}$ , the consumer knows that each outcome of  $\sigma_3$  will be uninformative and thus rationally ignores it.

The low type's expected payoff from her default action is  $1 - \theta_l$ . The first experiment is too expensive for her since  $c_1 = 1 - \theta_h > \theta_l$ . The costs of buying the second experiment and the third whenever  $m_2 \neq \bar{m}$  are designed such that her expected costs coincide with her additional precision to match the state. Namely, her expected payoff from this purchasing behavior is  $(1 - \theta_l) + \theta_l \cdot p$ . The additional precision, compared to ignoring all experiments precisely matches the expected costs by construction of  $c_2$ :

$$\underbrace{(1 - \theta_l) + \theta_l \cdot p}_{\text{new expected payoff}} - \underbrace{(1 - \theta_l)}_{\text{initial expected payoff}} = \theta_l \cdot p = \underbrace{c_2 + \mathbb{P}[m_2 \neq \bar{m} \mid l] \cdot c_3}_{\text{expected costs}}.$$

Thus, it is sequentially rational for the low type to purchase the second experiment and the third whenever  $m_2 \neq \bar{m}$ . We will demonstrate that the seller can choose  $p > \frac{2\theta_h - 1}{\theta_h - \theta_l}$  to increase the revenue from the low type, while ensuring the high type still prefers  $\sigma_1$ .

Now, consider the high type. First, we ensure that if the high type observes the signal realization  $m_2 = \bar{m}$ , her belief is updated such that she prefers action  $\bar{a}$  (i.e.,  $\mathbb{P}[\bar{\omega} \mid h, m_2 = \bar{m}] > 1/2$ ). This requires:

$$\begin{aligned} \frac{(1 - \theta_h)(1 - \epsilon)}{\theta_h(1 - p) + (1 - \theta_h)(1 - \epsilon)} &> \frac{1}{2} \quad \Leftrightarrow \quad (1 - \theta_h)(1 - \epsilon) > \theta_h(1 - p) \\ &\Leftrightarrow \quad \epsilon < \frac{1 - \theta_h(2 - p)}{1 - \theta_h}. \end{aligned}$$

This upper bound for  $\epsilon$  is compatible with the initial constraint because the bound  $\frac{1 - \theta_h(2 - p)}{1 - \theta_h}$  is increasing in  $p$ . By choosing  $p > \frac{2\theta_h - 1}{\theta_h - \theta_l}$ , we have that:

$$\epsilon < \frac{\theta_l(2\theta_h - 1)}{(1 - \theta_h)(\theta_h - \theta_l)} = \frac{1 - \theta_h(2 - \frac{2\theta_h - 1}{\theta_h - \theta_l})}{1 - \theta_h} < \frac{1 - \theta_h(2 - p)}{1 - \theta_h}.$$

If the high type purchases the second experiment and the third experiment upon observing  $m_2 \neq \bar{m}$ , then she matches the state with certainty if the state is  $\bar{\omega}$  and with probability  $\mathbb{P}[m_2 \neq \bar{m} \mid \underline{\omega}] = p$  if the state is  $\underline{\omega}$ . Therefore, the additional precision that the consumer has is given by:

$$\underbrace{(1 - \theta_h) + \theta_h \cdot p}_{\text{new expected payoff}} - \underbrace{\theta_h}_{\text{old expected payoff}}$$

and the expected costs are as follows:

$$c_2 + \mathbb{P}[m_2 \neq \bar{m} \mid h] \cdot c_3 = \theta_l \cdot p + (\mathbb{P}[m_2 \neq \bar{m} \mid h] - \mathbb{P}[m_2 \neq \bar{m} \mid l]) \cdot c_3 > \theta_l \cdot p.$$

The inequality holds since, as  $p > \epsilon$ , we have that  $\mathbb{P}[m_2 \neq \bar{m} \mid h] > \mathbb{P}[m_2 \neq \bar{m} \mid l]$ . Let  $p^*$  be the value at which the high-valuation type is indifferent. Then:

$$(1 - \theta_h) + \theta_h \cdot p^* - \theta_h > \theta_l \cdot p^* \quad \Leftrightarrow \quad p^* > \frac{2\theta_h - 1}{\theta_h - \theta_l},$$

which satisfies the requirement for the probability  $p$  and to get a higher payment from the low type compared to that of the optimal static menu.

We can conclude that with this constructed sequence of priced experiments, there exists a sequentially purchasing rational behavior such that the high type purchases  $\sigma_1$  (paying  $c_1 = 1 - \theta_h$ ), while the low type pays  $p^* \cdot \theta_l$  in expectation, where  $p^* > \frac{2\theta_h - 1}{\theta_h - \theta_l}$ . The expected revenue of the seller is:

$$\mathbb{P}[s = h](1 - \theta_h) + (1 - \mathbb{P}[s = l])p^*\theta_l > \mathbb{P}[s = h](1 - \theta_h) + (1 - \mathbb{P}[s = l])\frac{2\theta_h - 1}{\theta_h - \theta_l}\theta_l.$$

This verifies the third statement and concludes the proof of Theorem 2.  $\square$

## 5. Conclusion

This work studies a dynamic game of information selling and demonstrates the construction of a sequence of priced experiments through which a seller can extract the entire surplus from heterogeneous buyer types simultaneously. We have shown that perfect price discrimination is attainable when consumer types are non-congruent, meaning their initial beliefs favor different actions, and their valuations for complete information are close enough. This surplus extraction mechanism relies on a sophisticated design of experiments and pricing: First, it is crucial to design the experiments so that the high-valuation consumer assigns a higher probability to outcomes requiring further information acquisition. Second, we introduce an ‘encryption’ protocol to ensure that information from subsequent experiments is valuable only if the preceding experiments in the sequence have been purchased. This effectively simplifies the buyer’s purchasing behavior to an optimal stopping problem.

The dynamic nature of the model sets this work apart from simpler ‘take-it-or-leave-it’ offers à la BBS. This approach provides a more realistic framework for situations in which information is acquired incrementally and decisions about acquiring more information depend on preliminary findings. Examples of such situations include medical diagnostic

testing as well as research and development processes and social media platforms, whose advertisement revenue depends on how long consumers stay on the platform. This study shows how sequential revelation and strategic belief management allows a monopolistic seller to overcome the limitations of static models.

### 5.1. Discussion of Modeling Assumptions

The results of this study are based on several modeling assumptions that will be discussed below.

First, the model assumes a monopolistic seller. This simplification is crucial because it enables the seller to focus exclusively on optimizing surplus extraction without considering strategic responses from competitive firms. The monopoly assumption is pivotal for achieving perfect price discrimination. Thus, the model establishes an upper bound on what a seller can achieve in terms of revenue.

Second, extending the model to include more than two states or buyer types would introduce substantial complexity. While the general principles of sequential revelation and belief management might still apply, the specific design of experiments and pricing rules would require significant modifications. The binary framework isolates the core logic of how differing initial beliefs and their evolution can be exploited strategically.

Third, both the seller and the buyer are assumed to be risk-neutral with no time preferences. These simplifications allow the analysis to focus on informational and strategic aspects of the interaction, rather than on the added complexities arising from risk attitudes or inter-temporal trade-offs. Including real-world risk aversion and time discounting would lead to offering fewer, but more informative experiments.

Finally, the model assumes that the seller does not incur any costs when designing or conducting the experiments. In practice, however, information generation and dissemination can be costly. Introducing positive costs for experiments would shift the seller's objective from pure revenue maximization to profit maximization. If each experiment would incur a fixed cost, this would lead to fewer experiments being offered. If the cost depends on the complexity of the experiment, it would strengthen the effect of offering imperfect experiments in the beginning in order to increase the updated valuations and thus the willingness to pay of the consumer for more complex tests.

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## A. Appendix: Proofs of Lemmata

### A.1. Proof of Lemma 1

We construct the sequence of experiments  $(\sigma'_t)_{t=1}^T$  as follows. The signal  $m_t$  in period  $t$  contains two components: a state-dependent component  $m \in \{\underline{m}, \bar{m}\}$  and an independent coin flip  $f_t$ . For each period  $t$ , the coin flip  $f_t$  determines whether the probability of an experiment outcome  $m$  in state  $\underline{\omega}$  alternates with the probability of  $m$  in state  $\bar{\omega}$  in all subsequent periods. Thus, the result of the coin flip in  $m_1$  can alternate the probabilities for all periods  $t \geq 2$ , the coin flip in  $m_2$  can alternate the probabilities for  $t \geq 3$ , and so on. Specifically, we make the choice that a ‘tail’ triggers a switch in these probabilities. Since two switches neutralize each other, the net effect on the probabilities in period  $t$  depends only on the parity of the number of ‘tails’ up to period  $t$ . Accordingly, we define the key for interpreting the outcome of the experiment in period  $t$  as follows:

$$K_t := \begin{cases} 1, & \text{if } \sum_{s=1}^{t-1} f_s \text{ is odd,} \\ 0, & \text{otherwise.} \end{cases}$$

In general, the experiment in period  $t$  could depend on all previous test results. However, a simpler approach is sufficient for our analysis: the design of the  $t$ -th experiment

depends only on the key  $K_t$ , which is determined by a sequence of independent coin flips. Specifically, we construct the coin flips such that they are performed independently of each other, the state  $\omega$ , and the state-dependent component  $m$  of the signal. Thus, the following stochastic mapping results from the described modification of the original experimental sequence:

$\sigma'_t(m_t \mid \omega, K_t = 0)$	$(\underline{m}, 0)$	$(\underline{m}, 1)$	$(\overline{m}, 0)$	$(\overline{m}, 1)$
$\underline{\omega}$	$\frac{q_t}{2}$	$\frac{q_t}{2}$	$\frac{1-q_t}{2}$	$\frac{1-q_t}{2}$
$\overline{\omega}$	$\frac{p_t}{2}$	$\frac{p_t}{2}$	$\frac{1-p_t}{2}$	$\frac{1-p_t}{2}$

$\sigma'_t(m_t \mid \omega, K_t = 1)$	$(\underline{m}, 0)$	$(\underline{m}, 1)$	$(\overline{m}, 0)$	$(\overline{m}, 1)$
$\underline{\omega}$	$\frac{p_t}{2}$	$\frac{p_t}{2}$	$\frac{1-p_t}{2}$	$\frac{1-p_t}{2}$
$\overline{\omega}$	$\frac{q_t}{2}$	$\frac{q_t}{2}$	$\frac{1-q_t}{2}$	$\frac{1-q_t}{2}$

By definition,  $K_1 = 0$ . For every other period  $t$ , the key  $K_t$  takes either the value 0 or 1, each with a prior probability of  $\frac{1}{2}$ . The design ensures that the buyer learns the key  $K_t$  with certainty if she buys all tests up to period  $t$ , but retains her prior belief about  $K_t$  if she skips at least one test. Specifically, consider a period  $t$  in which at least one coin flip affecting  $K_t$  is unobserved. Let  $n$  be the number of missed coin flips up to period  $t$ . Note that each outcome (realized sequence of coin flip results) occurs with the same probability. Define  $S_{\text{even}} := \sum_{k: \text{even}}^n \binom{n}{k}$  as the number of outcomes among these  $n$  missed coin flips with an even number of ‘tails’, and analogously,  $S_{\text{odd}} := \sum_{k: \text{odd}}^n \binom{n}{k}$  as the number of outcomes with an odd number of ‘tails’. By the binomial theorem,  $S_{\text{even}}$  and  $S_{\text{odd}}$  coincide, regardless of  $n$ . Formally:

$$S_{\text{even}} - S_{\text{odd}} = \sum_{k=0}^n \binom{n}{k} (-1)^k = (1 - 1)^n = 0 \quad \text{for } n \geq 1.$$

Therefore, if at least one coin flip is missing up to period  $t$ , the probability that  $K_t$  is 0 coincides with the probability that  $K_t$  is 1.

As a next step, we will show that for each period  $t$  following a non-purchased experiment, an experiment outcome  $m_t$ , in isolation, provides no information that updates the belief about the state  $\omega$ . Applying the reasoning from above, we can deduce that the realization of  $K_t \in \{0, 1\}$  remains uncertain since there is at least one coin flip, namely  $f_{l^*(\mathcal{M})+1}$ , that misses. Furthermore, the realization of  $K_t$  is state independent, since it is determined by coin flips, which are independent of the state  $\omega$ . Let  $t > l^*(\mathcal{M}) + 1$  and

$\omega \in \Omega$ , then

$$\mathbb{P}[K_t \mid \omega, m_1, \dots, m_{l^*(\mathcal{M})}] = \begin{cases} \mathbb{P}\left[\sum_{s=l^*(\mathcal{M})+1}^{t-1} f_s \text{ is even} \mid \omega\right] = \frac{1}{2} & \text{if } K_t = K_{l^*(\mathcal{M})+1}; \\ \mathbb{P}\left[\sum_{s=l^*(\mathcal{M})+1}^{t-1} f_s \text{ is odd} \mid \omega\right] = \frac{1}{2} & \text{otherwise.} \end{cases}$$

In the following, we will abbreviate the notation by defining  $\mathbb{P}_{l^*}[\cdot] := \mathbb{P}[\cdot \mid m_1, \dots, m_{l^*(\mathcal{M})}]$ .

To show that the observation  $m_t$  does not update the belief about  $\omega$ , we demonstrate that the probability of observing  $m_t$  given  $\omega$ , is independent of the state  $\omega$ . We compute this probability for each state using the law of total probability, conditional on the key realization  $K_t$ .

For state  $\underline{\omega}$ :

$$\begin{aligned} \mathbb{P}_{l^*}[m_t \mid \underline{\omega}] &= \mathbb{P}_{l^*}[m_t \mid \underline{\omega}, K_t = 0] \mathbb{P}_{l^*}[K_t = 0 \mid \underline{\omega}] + \mathbb{P}_{l^*}[m_t \mid \underline{\omega}, K_t = 1] \mathbb{P}_{l^*}[K_t = 1 \mid \underline{\omega}] \\ &= \mathbb{P}_{l^*}[m_t \mid \underline{\omega}, K_t = 0] \cdot \frac{1}{2} + \mathbb{P}_{l^*}[m_t \mid \underline{\omega}, K_t = 1] \cdot \frac{1}{2}. \end{aligned} \quad (18)$$

For state  $\bar{\omega}$ :

$$\begin{aligned} \mathbb{P}_{l^*}[m_t \mid \bar{\omega}] &= \mathbb{P}_{l^*}[m_t \mid \bar{\omega}, K_t = 0] \mathbb{P}_{l^*}[K_t = 0 \mid \bar{\omega}] + \mathbb{P}_{l^*}[m_t \mid \bar{\omega}, K_t = 1] \mathbb{P}_{l^*}[K_t = 1 \mid \bar{\omega}] \\ &= \mathbb{P}_{l^*}[m_t \mid \bar{\omega}, K_t = 0] \cdot \frac{1}{2} + \mathbb{P}_{l^*}[m_t \mid \bar{\omega}, K_t = 1] \cdot \frac{1}{2}. \end{aligned} \quad (19)$$

We have designed the experiments so that the following symmetry properties hold:

$$\mathbb{P}_{l^*}[m_t \mid \underline{\omega}, K_t = 0] = \mathbb{P}_{l^*}[m_t \mid \bar{\omega}, K_t = 1]$$

and

$$\mathbb{P}_{l^*}[m_t \mid \bar{\omega}, K_t = 0] = \mathbb{P}_{l^*}[m_t \mid \underline{\omega}, K_t = 1].$$

Using these properties, we see that the expression for  $\mathbb{P}_{l^*}[m_t \mid \underline{\omega}]$  in (18) is equal to the expression for  $\mathbb{P}_{l^*}[m_t \mid \bar{\omega}]$  in (19). Since the probability of observing  $m_t$  is the same regardless of the state  $\omega$ , observing  $m_t$  provides no information to distinguish between the states. Consequently, the posterior belief  $\mathbb{P}_{l^*}[\omega \mid m_t]$  is equal to the prior belief  $\mathbb{P}_{l^*}[\omega]$ .

Since all keys to decipher the results in  $\mathcal{M} \setminus \{m_1, \dots, m_{l^*(\mathcal{M})}\}$  are unknown to the buyer, a similar reasoning implies that these test results do not depend on the state from the buyer's perspective:

**Claim:**

$$\mathbb{P}_{l^*}[\mathcal{M} \setminus \{m_1, \dots, m_{l^*(\mathcal{M})}\}] = \mathbb{P}_{l^*}[\mathcal{M} \setminus \{m_1, \dots, m_{l^*(\mathcal{M})}\} \mid \omega].$$

*Proof.* Let  $\mathcal{M} \subseteq \{m_1, \dots, m_T\}$ .

**Probability of Key Sequences.** We first show that for any  $(i_1, \dots, i_n) \in \{0, 1\}^n$ :

$$\mathbb{P}_{l^*}[K_{t_1} = i_1, \dots, K_{t_n} = i_n] = \mathbb{P}_{l^*}[K_{t_1} = i_1, \dots, K_{t_n} = i_n \mid \omega] = \left(\frac{1}{2}\right)^n. \quad (20)$$

Consider the probability that the  $t_j$ -th key  $K_{t_j}$  is equal to  $i_j$  conditioned on the previous key realizations  $K_{t_1}, \dots, K_{t_{j-1}}$ . This event coincides with the parity of the sum of coin flips that happened between the last known key and the current period:

$$\mathbb{P}_{l^*}[K_{t_j} = i_j \mid K_{t_1} = i_1, \dots, K_{t_{j-1}} = i_{j-1}] = \begin{cases} \mathbb{P}\left[\sum_{s=t_{j-1}}^{t_j-1} f_s \text{ is even}\right] & \text{if } i_j = i_{j-1}, \\ \mathbb{P}\left[\sum_{s=t_{j-1}}^{t_j-1} f_s \text{ is odd}\right] & \text{otherwise.} \end{cases}$$

Note that at least one coin flip is missing, namely  $f_{t_{j-1}}$ . By previous reasoning based on the binomial theorem,  $\mathbb{P}\left[\sum_{s=t_{j-1}}^{t_j-1} f_s \text{ is even}\right] = \frac{1}{2} = \mathbb{P}\left[\sum_{s=t_{j-1}}^{t_j-1} f_s \text{ is odd}\right]$ . Furthermore, the coin flip results  $f_s$  are constructed to be independent of the state  $\omega$ . Therefore, conditioning on  $\omega$  does not change these probabilities:

$$\mathbb{P}_{l^*}[K_{t_j} = i_j \mid K_{t_1} = i_1, \dots, K_{t_{j-1}} = i_{j-1}] = \frac{1}{2}$$

and

$$\mathbb{P}_{l^*}[K_{t_j} = i_j \mid \omega, K_{t_1} = i_1, \dots, K_{t_{j-1}} = i_{j-1}] = \frac{1}{2}.$$

Using the chain rule for probability yields:

$$\begin{aligned} & \mathbb{P}_{l^*}[K_{t_1} = i_1, \dots, K_{t_n} = i_n] \\ &= \prod_{j=1}^n \mathbb{P}_{l^*}[K_{t_j} = i_j \mid K_{t_1} = i_1, \dots, K_{t_{j-1}} = i_{j-1}] = \prod_{j=1}^n \frac{1}{2} = \left(\frac{1}{2}\right)^n \end{aligned}$$

and

$$\begin{aligned} & \mathbb{P}_{l^*}[K_{t_1} = i_1, \dots, K_{t_n} = i_n \mid \omega] \\ &= \prod_{j=1}^n \mathbb{P}_{l^*}[K_{t_j} = i_j \mid \omega, K_{t_1} = i_1, \dots, K_{t_{j-1}} = i_{j-1}] = \prod_{j=1}^n \frac{1}{2} = \left(\frac{1}{2}\right)^n, \end{aligned}$$

which confirms (20).



**Independency of the state.** Assume that  $|\mathcal{M} \setminus \{m_1, \dots, m_{l^*(\mathcal{M})}\}| = n$ . Let  $t_1 < \dots < t_n$  be the indices such that  $\mathcal{M} \setminus \{m_1, \dots, m_{l^*(\mathcal{M})}\} = \{m_{t_1}, \dots, m_{t_n}\}$ .

So we have to show that  $\mathbb{P}_{l^*}[m_{t_1}, \dots, m_{t_n}] = \mathbb{P}_{l^*}[m_{t_1}, \dots, m_{t_n} \mid \omega]$ . First, we use the law of total probability, conditioning on the possible sequences  $(i_1, \dots, i_n)$  of key realizations.

$$\begin{aligned} & \mathbb{P}_{l^*}[m_{t_1}, \dots, m_{t_n}] \\ &= \sum_{(i_1, \dots, i_n) \in \{0,1\}^n} \mathbb{P}_{l^*}[m_{t_1}, \dots, m_{t_n} \mid K_{t_1} = i_1, \dots, K_{t_n} = i_n] \mathbb{P}_{l^*}[K_{t_1} = i_1, \dots, K_{t_n} = i_n] \\ &= \sum_{(i_1, \dots, i_n) \in \{0,1\}^n} \mathbb{P}_{l^*}[m_{t_1}, \dots, m_{t_n} \mid K_{t_1} = i_1, \dots, K_{t_n} = i_n] \left(\frac{1}{2}\right)^n. \end{aligned} \quad (*)$$

Note that for every  $(i_1, \dots, i_n)$  there is a unique  $(i'_1, \dots, i'_n)$  with  $i'_j = 1 - i_j$  such that the design of the experiments induces the following symmetry property:

$$\mathbb{P}_{l^*}[m_{t_1}, \dots, m_{t_n} \mid \underline{\omega}, K_{t_1} = i_1, \dots, K_{t_n} = i_n] = \mathbb{P}_{l^*}[m_{t_1}, \dots, m_{t_n} \mid \bar{\omega}, K_{t_1} = i'_1, \dots, K_{t_n} = i'_n].$$

We apply the law of total probability again on (\*), conditioning on  $\omega$ :

$$\begin{aligned} & \sum_{(i_1, \dots, i_n) \in \{0,1\}^n} \mathbb{P}_{l^*}[m_{t_1}, \dots, m_{t_n} \mid K_{t_1} = i_1, \dots, K_{t_n} = i_n] \left(\frac{1}{2}\right)^n \\ &= \sum_{(i_1, \dots, i_n) \in \{0,1\}^n} \sum_{\omega \in \Omega} \mathbb{P}_{l^*}[\omega \mid K_{t_1} = i_1, \dots, K_{t_n} = i_n] \mathbb{P}_{l^*}[m_{t_1}, \dots, m_{t_n} \mid \omega, K_{t_1} = i_1, \dots, K_{t_n} = i_n] \left(\frac{1}{2}\right)^n \\ &= \sum_{(i_1, \dots, i_n) \in \{0,1\}^n} \mathbb{P}_{l^*}[\underline{\omega}] \mathbb{P}_{l^*}[m_{t_1}, \dots, m_{t_n} \mid \underline{\omega}, K_{t_1} = i_1, \dots, K_{t_n} = i_n] \left(\frac{1}{2}\right)^n \\ &+ \sum_{(i'_1, \dots, i'_n) \in \{0,1\}^n} \mathbb{P}_{l^*}[\bar{\omega}] \mathbb{P}_{l^*}[m_{t_1}, \dots, m_{t_n} \mid \bar{\omega}, K_{t_1} = i'_1, \dots, K_{t_n} = i'_n] \left(\frac{1}{2}\right)^n \\ &= \sum_{(i_1, \dots, i_n) \in \{0,1\}^n} \mathbb{P}_{l^*}[m_{t_1}, \dots, m_{t_n} \mid \omega, K_{t_1} = i_1, \dots, K_{t_n} = i_n] \left(\frac{1}{2}\right)^n \\ &= \sum_{(i_1, \dots, i_n) \in \{0,1\}^n} \mathbb{P}_{l^*}[m_{t_1}, \dots, m_{t_n} \mid \omega, K_{t_1} = i_1, \dots, K_{t_n} = i_n] \mathbb{P}_{l^*}[K_{t_1} = i_1, \dots, K_{t_n} = i_n \mid \omega] \\ &= \mathbb{P}_{l^*}[m_{t_1}, \dots, m_{t_n} \mid \omega], \end{aligned}$$

which holds for both states  $\omega \in \Omega$ . Furthermore, we used that

$$\mathbb{P}_{l^*}[\omega \mid K_{t_1} = i_1, \dots, K_{t_n} = i_n] = \frac{\mathbb{P}_{l^*}[K_{t_1} = i_1, \dots, K_{t_n} = i_n \mid \omega]}{\mathbb{P}_{l^*}[K_{t_1} = i_1, \dots, K_{t_n} = i_n]} \cdot \mathbb{P}_{l^*}[\omega] = \mathbb{P}_{l^*}[\omega].$$

We can conclude that  $\mathbb{P}_{l^*}[m_{t_1}, \dots, m_{t_n}] = \mathbb{P}_{l^*}[m_{t_1}, \dots, m_{t_n} \mid \omega]$ .  $\square$

Finally, it holds that:

$$\begin{aligned}
 \mathbb{P}[\omega \mid \mathcal{M}] &= \mathbb{P}[\omega \mid \mathcal{M} \setminus \{m_1, \dots, m_{l^*(\mathcal{M})}\}, \{m_1, \dots, m_{l^*(\mathcal{M})}\}] \\
 &= \frac{\mathbb{P}[\mathcal{M} \setminus \{m_1, \dots, m_{l^*(\mathcal{M})}\} \mid \omega, \{m_1, \dots, m_{l^*(\mathcal{M})}\}]}{\mathbb{P}[\mathcal{M} \setminus \{m_1, \dots, m_{l^*(\mathcal{M})}\} \mid \{m_1, \dots, m_{l^*(\mathcal{M})}\}]} \cdot \mathbb{P}[\omega \mid m_1, \dots, m_{l^*(\mathcal{M})}] \\
 &= \mathbb{P}[\omega \mid m_1, \dots, m_{l^*(\mathcal{M})}],
 \end{aligned}$$

which concludes the proof.  $\square$

## A.2. Proof of Lemma 2

Consider the experiment in the last period  $T$ . This experiment will be purchased only if the buyer has bought all previous experiments, and only if each previous outcome  $m_t$  (for  $1 \leq t \leq T-1$ ) was the non-revealing signal  $\underline{m}$ . If the consumer buys the final experiment, she will learn the true state and select the state-optimal action, yielding a payoff of  $1 - c_1 - \dots - c_T$ . On the other hand, if she refuses to purchase the final experiment, her payoff is  $1 - V(\theta_s, \underline{\mathcal{M}}_{T-1}) - c_1 - \dots - c_{T-1}$ . Thus, buying the last experiment is sequentially rational only if

$$\begin{aligned}
 1 - c_1 - \dots - c_T &\geq 1 - V(\theta_s, \underline{\mathcal{M}}_{T-1}) - c_1 - \dots - c_{T-1} \\
 \Leftrightarrow c_T &\leq V(\theta_s, \underline{\mathcal{M}}_{T-1}).
 \end{aligned}$$

Therefore, the seller must choose  $c_T$  to satisfy this constraint for both buyer types.

Now assume that the pricing policy from period  $t+1$  onward is such that the consumer chooses to purchase these experiments. The experiment in period  $t$  will be purchased only if the buyer has bought all previous experiments and all of them have realized as  $\underline{m}$ . If the consumer decides against purchasing the  $t$ -th experiment outcome, she receives a payoff of  $1 - V(\theta_s, \underline{\mathcal{M}}_{t-1}) - c_1 - \dots - c_{t-1}$ . If the buyer purchases the  $t$ -th experiment, she knows that she will choose the state-optimal action – anticipating that she will continue to purchase subsequent experiments until the true state is revealed – and thus receives a payoff of  $1 - c_1 - \dots - c_t - \sum_{u=t}^{T-1} \mathbb{P}[m_t = \underline{m}, \dots, m_u = \underline{m} \mid s, \underline{\mathcal{M}}_{t-1}] \cdot c_{u+1}$ . Therefore, she will decide to buy the  $t$ -th experiment only if

$$\begin{aligned}
 1 - \sum_{r=1}^t c_r - \sum_{u=t}^{T-1} \mathbb{P}[m_t = \underline{m}, \dots, m_u = \underline{m} \mid s, \underline{\mathcal{M}}_{t-1}] c_{u+1} &\geq 1 - V(\theta_s, \underline{\mathcal{M}}_{t-1}) - \sum_{r=1}^{t-1} c_r \\
 \Leftrightarrow c_t &\leq V(\theta_s, \underline{\mathcal{M}}_{t-1}) - \sum_{u=t}^{T-1} \mathbb{P}[m_t = \underline{m}, \dots, m_u = \underline{m} \mid s, \underline{\mathcal{M}}_{t-1}] \cdot c_{u+1}. \quad (21)
 \end{aligned}$$

Finally, the consumer will purchase the first experiment if and only if

$$\begin{aligned}
1 - V(\theta_s) &\leq 1 - c_1 - \sum_{t=1}^{T-1} \mathbb{P}[\underline{\mathcal{M}}_t \mid s] \cdot c_{t+1} \\
\Leftrightarrow V(\theta_s) &\geq c_1 + \sum_{t=1}^{T-1} \mathbb{P}[\underline{\mathcal{M}}_t \mid s] \cdot c_{t+1} \\
\Leftrightarrow c_1 &\leq V(\theta_s) - \sum_{t=1}^{T-1} \mathbb{P}[\underline{\mathcal{M}}_t \mid s] \cdot c_{t+1}.
\end{aligned} \tag{22}$$

The inequality (22) ensures that the expected cost of acquiring experiments until the state is revealed is less than the consumer's valuation of learning the state.

We choose  $c_2, \dots, c_T$  so that equality holds for  $s = h$  in (21). Note that observing  $\underline{\mathcal{M}}_t$  shifts the belief towards the state  $\underline{\omega}$ . Since we have assumed that the high-type's prior belief already favors the state  $\underline{\omega}$ , we can conclude that  $V(\theta_s, \underline{\mathcal{M}}_t) = \mathbb{P}[\bar{\omega} \mid h, \underline{\mathcal{M}}_t]$ . Thus, the costs of the experiments in period  $2 \leq t \leq T$  are given by

$$c_t = \mathbb{P}[\bar{\omega} \mid h, \underline{\mathcal{M}}_{t-1}] - \sum_{u=t}^{T-1} \mathbb{P}[m_t = \underline{m}, \dots, m_u = \underline{m} \mid h, \underline{\mathcal{M}}_{t-1}] \cdot c_{u+1}. \tag{23}$$

We next derive a closed-form solution of (23) via backward induction.

**Claim:** For  $2 \leq t \leq T$ , it holds that

$$c_t = \frac{(1 - \theta_h) \prod_{s=1}^{t-1} p_s (1 - p_t)}{\theta_h + (1 - \theta_h) \prod_{s=1}^{t-1} p_s} \geq 0.$$

*Proof.* The conditional probabilities involved in the recursive formula for the cost  $c_t$  (23) are given by:

$$\mathbb{P}[\bar{\omega} \mid h, \underline{\mathcal{M}}_{t-1}] = \frac{(1 - \theta_h) \prod_{s=1}^{t-1} p_s}{\theta_h + (1 - \theta_h) \prod_{s=1}^{t-1} p_s}$$

and

$$\mathbb{P}[m_t = \underline{m}, \dots, m_u = \underline{m} \mid h, \underline{\mathcal{M}}_{t-1}] = \frac{\theta_h + (1 - \theta_h) \prod_{s=1}^u p_s}{\theta_h + (1 - \theta_h) \prod_{s=1}^{t-1} p_s}.$$

Therefore, the expression for  $c_t$  simplifies to:

$$c_t = \frac{(1 - \theta_h) \prod_{s=1}^{t-1} p_s}{\theta_h + (1 - \theta_h) \prod_{s=1}^{t-1} p_s} - \sum_{u=t}^{T-1} \frac{\theta_h + (1 - \theta_h) \prod_{s=1}^u p_s}{\theta_h + (1 - \theta_h) \prod_{s=1}^{t-1} p_s} \cdot c_{u+1}. \tag{24}$$

We proceed by backward induction.

**Final Period ( $t=T$ ):** For the final period  $t = T$ , the summation term in (24) is empty (sum from  $T$  to  $T - 1$ ) and thus zero. Therefore:

$$c_T = \frac{(1 - \theta_h) \prod_{s=1}^{T-1} p_s}{\theta_h + (1 - \theta_h) \prod_{s=1}^{T-1} p_s}.$$

This matches the general formula for the defined value  $p_T = 0$ .

**Inductive Step:** Assume the desired formula holds for all  $t \geq t^* + 1$ :

$$c_t = \frac{(1 - \theta_h) \prod_{s=1}^{t-1} p_s (1 - p_t)}{\theta_h + (1 - \theta_h) \prod_{s=1}^{t-1} p_s}. \quad (25)$$

We want to show that it holds as well for  $t = t^*$ . We consider equation (24) and calculate the summation term separately using the induction hypothesis (25):

$$\begin{aligned} & \sum_{u=t^*}^{T-1} \frac{\theta_h + (1 - \theta_h) \prod_{s=1}^u p_s}{\theta_h + (1 - \theta_h) \prod_{s=1}^{t^*-1} p_s} \cdot c_{u+1} \\ &= \sum_{u=t^*}^{T-1} \frac{\theta_h + (1 - \theta_h) \prod_{s=1}^u p_s}{\theta_h + (1 - \theta_h) \prod_{s=1}^{t^*-1} p_s} \cdot \frac{(1 - \theta_h) \prod_{s=1}^u p_s (1 - p_{u+1})}{\theta_h + (1 - \theta_h) \prod_{s=1}^u p_s} \\ &= \sum_{u=t^*}^{T-1} \frac{(1 - \theta_h) \prod_{s=1}^u p_s (1 - p_{u+1})}{\theta_h + (1 - \theta_h) \prod_{s=1}^{t^*-1} p_s} \\ &= \sum_{u=t^*}^{T-1} \frac{(1 - \theta_h) (\prod_{s=1}^u p_s - \prod_{s=1}^{u+1} p_s)}{\theta_h + (1 - \theta_h) \prod_{s=1}^{t^*-1} p_s} \\ &= \frac{1 - \theta_h}{\theta_h + (1 - \theta_h) \prod_{s=1}^{t^*-1} p_s} \sum_{u=t^*}^{T-1} \left( \prod_{s=1}^u p_s - \prod_{s=1}^{u+1} p_s \right) \\ &= \frac{(1 - \theta_h) \prod_{s=1}^{t^*} p_s}{\theta_h + (1 - \theta_h) \prod_{s=1}^{t^*-1} p_s}. \end{aligned}$$

Inserting this result in equation (24) yields  $c_{t^*}$ :

$$\begin{aligned} c_{t^*} &= \frac{(1 - \theta_h) \prod_{s=1}^{t^*-1} p_s}{\theta_h + (1 - \theta_h) \prod_{s=1}^{t^*-1} p_s} - \frac{(1 - \theta_h) \prod_{s=1}^{t^*} p_s}{\theta_h + (1 - \theta_h) \prod_{s=1}^{t^*-1} p_s} \\ &= \frac{(1 - \theta_h) \left( \prod_{s=1}^{t^*-1} p_s - \prod_{s=1}^{t^*} p_s \right)}{\theta_h + (1 - \theta_h) \prod_{s=1}^{t^*-1} p_s} \\ &= \frac{(1 - \theta_h) \prod_{s=1}^{t^*-1} p_s (1 - p_{t^*})}{\theta_h + (1 - \theta_h) \prod_{s=1}^{t^*-1} p_s}. \end{aligned}$$

This is the required form (25) for  $t = t^*$ . By induction, the formula holds for all  $t$ .  $\square$

Under this pricing scheme, the high-valuation type is indifferent in each period  $t \geq 2$  and we can assume that she takes the seller-preferred decision to buy each experiment. This verifies the claim for  $s = h$ .

Remarkably, a non-revealing signal  $\underline{m}$  can update beliefs such that the conditional valuation of the high type decreases while the conditional valuation of the low type increases. In particular, the experiments will be designed so that at some point the updated valuation of the low type exceeds the updated valuation of the high type. One might think that the low type would benefit from this effect. However, we will ensure that the low type pays exactly her valuation in expectation, anticipating any future discounts and effectively paying for them in the initial period. We do this by setting the cost in the first period so that equality holds for  $s = l$  in (22):

$$\begin{aligned} c_1 &= \theta_l - \sum_{t=1}^{T-1} \mathbb{P}[\underline{\mathcal{M}}_t \mid l] \cdot c_{t+1} \\ &= \theta_l - \sum_{t=1}^{T-1} \left( \theta_l + (1 - \theta_l) \prod_{s=1}^t p_s \right) \cdot \frac{(1 - \theta_h) \prod_{s=1}^t p_s (1 - p_{t+1})}{\theta_h + (1 - \theta_h) \prod_{s=1}^t p_s}. \end{aligned}$$

If the parameters are such that the price of the first experiment (6) is non-negative, then the costs are well defined. Finally, to verify the claim for  $s = l$ , it is left to show the following:

**Claim:** The pricing scheme induces a dynamically consistent behavior, i.e., the low type purchases all subsequent experiments until the state is revealed.

*Proof.* By the choice of  $c_1$  (6), the agent's ex-ante valuation  $V(\theta_l)$  covers exactly the expected cost of the experiment sequence until a state-revelation:

$$V(\theta_l) = c_1 + \sum_{t=1}^{T-1} \mathbb{P}[\underline{\mathcal{M}}_t \mid l] \cdot c_{t+1}. \quad (26)$$

We have to show that the consumer will continue to acquire experiments until a state revelation occurs. Consider the agent's decision in period  $t \geq 2$ , given  $\underline{\mathcal{M}}_{t-1}$ . The low-type's updated valuation is

$$V(\theta_l, \underline{\mathcal{M}}_{t-1}) = \min_{\omega \in \{\underline{\omega}, \bar{\omega}\}} \{ \mathbb{P}[\omega \mid l, \underline{\mathcal{M}}_{t-1}] \}.$$

The expected costs, conditional on buying experiment  $t$  and continuing as long as  $\underline{m}$  is

observed, are

$$c_t + \sum_{u=t}^{T-1} \mathbb{P} [m_t = \underline{m}, \dots, m_u = \underline{m} \mid l, \underline{\mathcal{M}}_{t-1}] \cdot c_{u+1}.$$

We want to show that the agent benefits from buying experiment  $t$  and all subsequent experiments until the state is revealed, i.e.:

$$V(\theta_l, \underline{\mathcal{M}}_{t-1}) \geq c_t + \sum_{u=t}^{T-1} \mathbb{P} [m_t = \underline{m}, \dots, m_u = \underline{m} \mid l, \underline{\mathcal{M}}_{t-1}] \cdot c_{u+1}. \quad (27)$$

We consider two cases based on which state yields the minimum posterior probability:

$$V(\theta_l, \underline{\mathcal{M}}_{t-1}) = \mathbb{P} [\underline{\omega} \mid l, \underline{\mathcal{M}}_{t-1}], \quad (i)$$

$$V(\theta_l, \underline{\mathcal{M}}_{t-1}) = \mathbb{P} [\bar{\omega} \mid l, \underline{\mathcal{M}}_{t-1}]. \quad (ii)$$

**Case (i):** We can show inequality (27) directly:

$$\begin{aligned} \mathbb{P} [\underline{\omega} \mid l, \underline{\mathcal{M}}_{t-1}] &\geq c_t + \sum_{u=t}^{T-1} \mathbb{P} [m_t = \underline{m}, \dots, m_u = \underline{m} \mid l, \underline{\mathcal{M}}_{t-1}] \cdot c_{u+1} \\ \Leftrightarrow \mathbb{P} [\underline{\omega} \mid l] &\geq c_t \cdot \mathbb{P} [\underline{\mathcal{M}}_{t-1} \mid l] + \sum_{u=t}^{T-1} \mathbb{P} [\underline{\mathcal{M}}_u \mid l] \cdot c_{u+1} \\ \Leftrightarrow \theta_l &\geq \sum_{u=t-1}^{T-1} \mathbb{P} [\underline{\mathcal{M}}_u \mid l] \cdot c_{u+1}, \end{aligned}$$

This final inequality holds by the initial condition (26) since  $V(\theta_l) = \theta_l$  and the non-negativity of costs  $c_t \geq 0$ .

**Case (ii):** Given a realization of experiment outcomes  $\underline{\mathcal{M}}_{t-1}$ , the following holds. The low type considers state  $\underline{\omega}$  less likely than the high type:

$$\begin{aligned} \mathbb{P} [\underline{\omega} \mid l, \underline{\mathcal{M}}_{t-1}] &= \frac{\theta_l}{\theta_l + (1 - \theta_l) \prod_{s=1}^{t-1} p_s} \\ &\leq \frac{\theta_h}{\theta_h + (1 - \theta_h) \prod_{s=1}^{t-1} p_s} = \mathbb{P} [\underline{\omega} \mid h, \underline{\mathcal{M}}_{t-1}]. \end{aligned}$$

Furthermore, the low type considers subsequent non-revealing experiment outcomes less likely than the high type:

$$\mathbb{P} [m_t = \underline{m}, \dots, m_u = \underline{m} \mid l, \underline{\mathcal{M}}_{t-1}] = \mathbb{P} [\underline{\omega} \mid l, \underline{\mathcal{M}}_{t-1}] + \mathbb{P} [\bar{\omega} \mid l, \underline{\mathcal{M}}_{t-1}] \prod_{s=t}^u p_s$$

$$\begin{aligned}
&= \mathbb{P}[\underline{\omega} \mid l, \underline{\mathcal{M}}_{t-1}] + (1 - \mathbb{P}[\underline{\omega} \mid l, \underline{\mathcal{M}}_{t-1}]) \cdot \prod_{s=t}^u p_s \\
&\leq \mathbb{P}[\underline{\omega} \mid h, \underline{\mathcal{M}}_{t-1}] + (1 - \mathbb{P}[\underline{\omega} \mid h, \underline{\mathcal{M}}_{t-1}]) \cdot \prod_{s=t}^u p_s \\
&= \mathbb{P}[\underline{\omega} \mid h, \underline{\mathcal{M}}_{t-1}] + \mathbb{P}[\bar{\omega} \mid h, \underline{\mathcal{M}}_{t-1}] \prod_{s=t}^u p_s \\
&= \mathbb{P}[m_t = \underline{m}, \dots, m_u = \underline{m} \mid h, \underline{\mathcal{M}}_{t-1}].
\end{aligned}$$

We conclude, by definition of  $c_t$  (23), that:

$$\begin{aligned}
c_t &= \mathbb{P}[\bar{\omega} \mid h, \underline{\mathcal{M}}_{t-1}] - \sum_{u=t}^{T-1} \mathbb{P}[m_t = \underline{m}, \dots, m_u = \underline{m} \mid h, \underline{\mathcal{M}}_{t-1}] \cdot c_{u+1} \\
&\leq \mathbb{P}[\bar{\omega} \mid l, \underline{\mathcal{M}}_{t-1}] - \sum_{u=t}^{T-1} \mathbb{P}[m_t = \underline{m}, \dots, m_u = \underline{m} \mid l, \underline{\mathcal{M}}_{t-1}] \cdot c_{u+1}.
\end{aligned}$$

Since the inequality (27) holds in both cases, the low type finds it optimal to buy experiment  $t$  after any history consisting only of  $\underline{m}$  signals. This holds for all  $t \geq 2$ . Combined with the assumption for  $t = 1$ , the low type buys all experiments until a  $\bar{m}$  signal is received.  $\square$

### A.3. Proof of Lemma 3

In a first step, we aim to find the probabilities  $p_1, \dots, p_T$  with  $p_t \in [0, 1]$  that form a critical point of the cost discrepancy (10). Equivalently, factoring out  $(\theta_h - \theta_l)(1 - \theta_h)$ , we search for a critical point of  $\Delta$ , defined as:

$$\Delta(p_1, \dots, p_T) := \sum_{t=1}^{T-1} \left(1 - \prod_{s=1}^t p_s\right) \frac{c_{t+1}}{1 - \theta_h} = \sum_{t=1}^{T-1} \underbrace{\left(1 - \prod_{s=1}^t p_s\right) \frac{\prod_{s=1}^t p_s (1 - p_{t+1})}{\theta_h + (1 - \theta_h) \prod_{s=1}^t p_s}}_{=: f_t(p_1, \dots, p_{t+1})}.$$

Further define  $P_t := P_t(p_1, \dots, p_t) = \prod_{s=1}^t p_s$  as the cumulative probability of receiving no state-revealing test result up to the  $t$ -th test.

We first observe that only the last term  $f_{T-1}$  depends on  $p_T$ :

$$f_{T-1} = (1 - P_{T-1}) \frac{P_{T-1}(1 - p_T)}{\theta_h + (1 - \theta_h)P_{T-1}}.$$

Since  $P_{T-1}$  and  $\theta_h$  are positive,  $f_{T-1}$  decreases linearly with  $p_T$ . To maximize  $\Delta$ , we must choose  $p_T = 0$ . For  $p_1, \dots, p_{T-1}$ , we find a critical point by setting the partial derivatives

to zero:  $\frac{\partial \Delta}{\partial p_u} = 0$  for  $u = 1, \dots, T-1$ .

**Claim:** The probabilities  $p_1^*, \dots, p_T^*$  defined by

$$p_t^* = \frac{\theta_h^{t/T} - \theta_h}{\theta_h^{(t-1)/T} - \theta_h} \text{ for } t = 1, \dots, T.$$

form a critical point of  $\Delta$ .

*Proof.* We need to show that  $\frac{\partial \Delta}{\partial p_u} \Big|_{p=p^*} = 0$  for  $u = 1, \dots, T-1$ . The partial derivative of  $\Delta$  with respect to  $p_u$  (for  $1 \leq u \leq T-1$ ) is:

$$\frac{\partial \Delta}{\partial p_u} = \sum_{t=1}^{T-1} \frac{\partial f_t}{\partial p_u}.$$

Note that  $f_t$  depends on  $p_u$  only if  $t \geq u-1$ . Hence, the sum reduces to:

$$\frac{\partial \Delta}{\partial p_u} = \frac{\partial f_{u-1}}{\partial p_u} + \sum_{t=u}^{T-1} \frac{\partial f_t}{\partial p_u}. \quad (28)$$

Let's calculate the partial derivatives.

1. For the  $t = u-1$  term:

$$\begin{aligned} \frac{\partial f_{u-1}}{\partial p_u} &= \frac{\partial}{\partial p_u} \left[ (1 - P_{u-1}) \frac{(P_{u-1} - P_u)}{\theta_h + (1 - \theta_h)P_{u-1}} \right] \\ &= -(1 - P_{u-1}) \frac{P_{u-1}}{\theta_h + (1 - \theta_h)P_{u-1}}. \end{aligned}$$

2. For terms  $t \geq u$ :

$$\begin{aligned} \frac{\partial f_t}{\partial p_u} &= \frac{\partial}{\partial p_u} \left[ (1 - P_t) \frac{(P_t - P_{t+1})}{\theta_h + (1 - \theta_h)P_t} \right] \\ &= \frac{(P_t - P_{t+1}) [\theta_h(1 - 2P_t) - (1 - \theta_h)(P_t)^2]}{p_u (\theta_h + (1 - \theta_h)P_t)^2}. \end{aligned}$$

Now, we evaluate these derivatives at the proposed probabilities  $p^*$ . Notice that the product  $P_t$  telescopes, i.e.

$$\begin{aligned} P_t^* &:= P_t(p_1^*, \dots, p_t^*) = \prod_{s=1}^t p_s^* \\ &= \frac{\theta_h^{1/T} - \theta_h}{\theta_h^{0/T} - \theta_h} \cdot \frac{\theta_h^{2/T} - \theta_h}{\theta_h^{1/T} - \theta_h} \cdot \dots \cdot \frac{\theta_h^{t/T} - \theta_h}{\theta_h^{(t-1)/T} - \theta_h} \end{aligned}$$



$$= \frac{\theta_h^{t/T} - \theta_h}{1 - \theta_h}.$$

Using this, we evaluate the derivatives at  $p^*$ :

1. Evaluating  $\frac{\partial f_{u-1}}{\partial p_u}$  at  $p^*$ :

$$\begin{aligned} \left. \frac{\partial f_{u-1}}{\partial p_u} \right|_{p=p^*} &= -(1 - P_{u-1}^*) \frac{P_{u-1}^*}{\theta_h + (1 - \theta_h) P_{u-1}^*} \\ &= - \left( 1 - \frac{\theta_h^{(u-1)/T} - \theta_h}{1 - \theta_h} \right) \frac{\left( \frac{\theta_h^{(u-1)/T} - \theta_h}{1 - \theta_h} \right)}{\theta_h + (1 - \theta_h) \left( \frac{\theta_h^{(u-1)/T} - \theta_h}{1 - \theta_h} \right)} \\ &= - \frac{(1 - \theta_h^{(u-1)/T})(\theta_h^{(u-1)/T} - \theta_h)}{(1 - \theta_h)^2 \cdot \theta_h^{(u-1)/T}}. \end{aligned} \quad (*)$$

2. Evaluating  $\frac{\partial f_t}{\partial p_u}$  for  $t \geq u$  at  $p^*$ :

$$\begin{aligned} \left. \frac{\partial f_t}{\partial p_u} \right|_{p=p^*} &= \frac{(P_t^* - P_{t+1}^*) [\theta_h(1 - 2P_t^*) - (1 - \theta_h)(P_t^*)^2]}{p_u^* (\theta_h + (1 - \theta_h) P_t^*)^2} \\ &= \frac{\left( \left( \frac{\theta_h^{t/T} - \theta_h}{1 - \theta_h} \right) - \left( \frac{\theta_h^{(t+1)/T} - \theta_h}{1 - \theta_h} \right) \right) \left[ \theta_h \left( 1 - 2 \left( \frac{\theta_h^{t/T} - \theta_h}{1 - \theta_h} \right) \right) - (1 - \theta_h) \left( \frac{\theta_h^{t/T} - \theta_h}{1 - \theta_h} \right)^2 \right]}{\frac{\theta_h^{u/T} - \theta_h}{\theta_h^{(u-1)/T} - \theta_h} \left( \theta_h + (1 - \theta_h) \left( \frac{\theta_h^{t/T} - \theta_h}{1 - \theta_h} \right) \right)^2} \\ &= \frac{(1 - \theta_h^{1/T}) (\theta_h^{(T-t)/T} - \theta_h^{t/T})}{(1 - \theta_h)^2} \cdot \frac{\theta_h^{(u-1)/T} - \theta_h}{\theta_h^{u/T} - \theta_h}. \end{aligned}$$

Now, it remains to calculate:

$$\begin{aligned} \sum_{t=u}^{T-1} \left. \frac{\partial f_t}{\partial p_u} \right|_{p=p^*} &= \sum_{t=u}^{T-1} \frac{(1 - \theta_h^{1/T}) (\theta_h^{(T-t)/T} - \theta_h^{t/T})}{(1 - \theta_h)^2} \cdot \frac{\theta_h^{(u-1)/T} - \theta_h}{\theta_h^{u/T} - \theta_h} \\ &= \frac{(1 - \theta_h^{1/T}) (\theta_h^{(u-1)/T} - \theta_h)}{(1 - \theta_h)^2 (\theta_h^{u/T} - \theta_h)} \cdot \left[ \sum_{t=u}^{T-1} \theta_h^{(T-t)/T} - \theta_h^{t/T} \right] \\ &= \frac{(1 - \theta_h^{1/T}) (\theta_h^{(u-1)/T} - \theta_h)}{(1 - \theta_h)^2 (\theta_h^{u/T} - \theta_h)} \cdot \left[ \sum_{t=1}^{T-u} (\theta_h^{1/T})^t - \theta_h^{(u-1)/T} \sum_{k=1}^{T-u} (\theta_h^{1/T})^k \right] \end{aligned}$$

$$\begin{aligned}
&= \frac{(1 - \theta_h^{1/T}) (\theta_h^{(u-1)/T} - \theta_h)}{(1 - \theta_h)^2 (\theta_h^{u/T} - \theta_h)} \cdot (\theta_h^{1/T} - \theta_h^{u/T}) \frac{1 - \theta_h^{(T-u)/T}}{1 - \theta_h^{1/T}} \\
&= \frac{(\theta_h^{(u-1)/T} - \theta_h)}{(1 - \theta_h)^2 \cdot \theta_h^{u/T} (1 - \theta_h^{(T-u)/T})} \cdot \theta_h^{1/T} (1 - \theta_h^{(u-1)/T}) (1 - \theta_h^{(T-u)/T}) \\
&= \frac{(\theta_h^{(u-1)/T} - \theta_h) (1 - \theta_h^{(u-1)/T})}{(1 - \theta_h)^2 \cdot \theta_h^{(u-1)/T}}. \tag{**}
\end{aligned}$$

Comparing (\*) and (\*\*), we see that

$$\sum_{t=u}^{T-1} \frac{\partial f_t}{\partial p_u} \Big|_{p=p^*} = - \frac{\partial f_{u-1}}{\partial p_u} \Big|_{p=p^*}.$$

Substituting this into Equation (28):

$$\frac{\partial \Delta}{\partial p_u} \Big|_{p=p^*} = \frac{\partial f_{u-1}}{\partial p_u} \Big|_{p=p^*} - \frac{\partial f_{u-1}}{\partial p_u} \Big|_{p=p^*} = 0.$$

This holds for all  $u = 1, \dots, T-1$ . Thus, the proposed probabilities  $p^*$  satisfy the first-order conditions for an extrema<sup>10</sup>.  $\square$

Next, by substituting the probability assignments  $p_t^*$  into our expression for the cost in period  $t$  from Lemma 2 (5), we get a uniform cost for all  $t \geq 2$ :

$$\begin{aligned}
c_t^* &= \frac{(1 - \theta_h) \prod_{s=1}^{t-1} p_s^* (1 - p_t)}{\theta_h + (1 - \theta_h) \prod_{s=1}^{t-1} p_s} \\
&= \frac{(1 - \theta_h) \left( \frac{\theta_h^{(t-1)/T} - \theta_h}{1 - \theta_h} \right) \left( \frac{\theta_h^{(t-1)/T} - \theta_h^{t/T}}{\theta_h^{(t-1)/T} - \theta_h} \right)}{\theta_h + (1 - \theta_h) \left( \frac{\theta_h^{(t-1)/T} - \theta_h}{1 - \theta_h} \right)} \\
&= \frac{\theta_h^{(t-1)/T} - \theta_h^{t/T}}{\theta_h^{(t-1)/T}} \\
&= 1 - \theta_h^{1/T}.
\end{aligned}$$

We will show later that  $c_1 = 1 - \theta_h^{1/T}$  holds as well whenever the valuation gap coincides with the highest attainable cost discrepancy.

Now, we can substitute these costs and probabilities in the cost discrepancy term (10)

<sup>10</sup>Numerical maximization suggests that the proposed probabilities maximize the expression  $\Delta$  globally.

and thereby verifying the Statement (13) for any maturity  $T$ :

$$\begin{aligned}
(\theta_h - \theta_l) \sum_{t=1}^{T-1} \left( 1 - \prod_{s=1}^t p_s^* \right) \cdot c_{t+1}^* &= (\theta_h - \theta_l) \sum_{t=1}^{T-1} \left( 1 - \frac{\theta_h^{t/T} - \theta_h}{1 - \theta_h} \right) \cdot (1 - \theta_h^{1/T}) \\
&= (\theta_h - \theta_l) \left( \frac{1 - \theta_h^{1/T}}{1 - \theta_h} \right) \sum_{t=1}^{T-1} (1 - \theta_h^{t/T}) \\
&= (\theta_h - \theta_l) \left( \frac{1 - \theta_h^{1/T}}{1 - \theta_h} \right) \left( T - \sum_{t=0}^{T-1} (\theta_h^{1/T})^t \right) \\
&= (\theta_h - \theta_l) \left( \frac{1 - \theta_h^{1/T}}{1 - \theta_h} \right) \left( T - \frac{1 - \theta_h}{1 - \theta_h^{1/T}} \right) \\
&= (\theta_h - \theta_l) \left( T \left( \frac{1 - \theta_h^{1/T}}{1 - \theta_h} \right) - 1 \right).
\end{aligned}$$

As  $T$  increases, the sequential experiments allow for an increasing gap in the expected costs incurred by the high and low type. The limit of this gap as  $T \rightarrow \infty$  represents the highest cost discrepancy that the seller can exploit to bridge the valuation gap. Applying L'Hôpital's rule yields the desired limit (14):

$$\begin{aligned}
\lim_{T \rightarrow \infty} (\theta_h - \theta_l) \left( T \cdot \left( \frac{1 - \theta_h^{1/T}}{1 - \theta_h} \right) - 1 \right) &= (\theta_h - \theta_l) \left( -1 + \lim_{1/T \rightarrow 0} \frac{1 - \theta_h^{1/T}}{\frac{1}{T}(1 - \theta_h)} \right) \\
&= (\theta_h - \theta_l) \left( -1 + \lim_{1/T \rightarrow 0} \left[ \frac{\theta_h^{1/T} \ln(\theta_h)}{\theta_h - 1} \right] \right) \\
&= (\theta_h - \theta_l) \left( -1 + \frac{-\ln(\theta_h)}{1 - \theta_h} \right) \\
&= (\theta_h - \theta_l) \frac{1 - \theta_h - \ln(\theta_h)}{1 - \theta_h}.
\end{aligned}$$

□

#### A.4. Proof of Lemma 4

Consider an arbitrary  $\theta_l \in \left( \frac{(1-\theta_h) + \theta_h \ln(\theta_h)}{2(1-\theta_h) + \ln(\theta_h)}, 1 - \theta_h \right]$ . From (16), we know there exists a maturity  $T'$  such that  $\theta_l$  lies within the interval:

$$I_{T'} := \left[ \frac{(1 - \theta_h) - T' \theta_h \left( 1 - \theta_h^{1/T'} \right)}{2(1 - \theta_h) - T' \left( 1 - \theta_h^{1/T'} \right)}, \frac{(1 - \theta_h) - (T' - 1) \theta_h \left( 1 - \theta_h^{1/(T'-1)} \right)}{2(1 - \theta_h) - (T' - 1) \left( 1 - \theta_h^{1/(T'-1)} \right)} \right]. \quad (29)$$

Our goal is to demonstrate the existence of probabilities  $(p_1, \dots, p_{T'})$  such that the

discrepancy in expected costs coincides with the valuation discrepancy:

$$\theta_l + (\theta_h - \theta_l) \left( \sum_{t=1}^{T'-1} \left( 1 - \prod_{s=1}^t p_s \right) c_{t+1} \right) = 1 - \theta_h.$$

Rearranging this equation to solve for  $\theta_l$  yields:

$$\theta_l = \frac{1 - 2\theta_h}{1 - \sum_{t=1}^{T'-1} \left( 1 - \prod_{s=1}^t p_s \right) c_{t+1}} + \theta_h. \quad (30)$$

We recall that the probabilities that correspond to the lower bound of  $\theta_l$  in the interval  $I_{T'}$  satisfy  $\prod_{s=1}^t p_s = \frac{\theta_h^{t/T'} - \theta_h}{1 - \theta_h}$  for all  $t = 1, \dots, T'$ . To allow for perfect price discrimination across the entire interval  $I_{T'}$ , we relax this condition by setting:

$$\prod_{s=1}^t p_s = \frac{\theta_h^{t/T'} - \theta_h}{1 - \theta_h} \quad \text{for } t = 2, \dots, T',$$

while leaving  $p_1 \in \left[ \frac{\theta_h^{2/T'} - \theta_h}{1 - \theta_h}, 1 \right]$  as a variable. The lower bound of the interval guarantees that  $p_2$  can be chosen such that  $p_1 \cdot p_2 = \frac{\theta_h^{2/T'} - \theta_h}{1 - \theta_h}$ . Now, we can determine the individual probabilities  $p_t$ :

$$p_t = \begin{cases} p_1 & \text{for } t = 1, \\ \frac{\prod_{s=1}^2 p_s}{p_1} = \frac{\theta_h^{2/T'} - \theta_h}{(1 - \theta_h)p_1} & \text{for } t = 2, \\ \frac{\prod_{s=1}^t p_s}{\prod_{s=1}^{t-1} p_s} = \frac{\theta_h^{t/T'} - \theta_h}{\theta_h^{(t-1)/T'} - \theta_h} & \text{for } t = 3, \dots, T'. \end{cases}$$

The choice of probabilities thus reduces to selecting  $p_1$ . Note that  $p_1 = \frac{\theta_h^{1/T'} - \theta_h}{1 - \theta_h}$  recovers the proposed discrepancy-maximizing case.

Let's recall the cost terms  $c_t$  from (5):

$$c_t = \frac{(1 - \theta_h) \prod_{s=1}^{t-1} p_s (1 - p_t)}{\theta_h + (1 - \theta_h) \prod_{s=1}^{t-1} p_s}.$$

Substituting the chosen probabilities for  $t \geq 3$  leads to the familiar costs:

$$c_t = 1 - \theta_h^{1/T'} \quad \text{for } t = 3, \dots, T'.$$

However, for  $t = 2$ , the cost  $c_2$  depends on  $p_1$ :

$$c_2 = \frac{(1 - \theta_h)p_1(1 - p_2)}{\theta_h + (1 - \theta_h)p_1} = \frac{(1 - \theta_h)p_1 \left(1 - \frac{\theta_h^{2/T'} - \theta_h}{(1 - \theta_h)p_1}\right)}{\theta_h + (1 - \theta_h)p_1} = \frac{(1 - \theta_h)p_1 - (\theta_h^{2/T'} - \theta_h)}{\theta_h + (1 - \theta_h)p_1}.$$

Now, let's evaluate the sum in the denominator of (30), denoted by  $S(p_1)$ :

$$\begin{aligned} S(p_1) &= \sum_{t=1}^{T'-1} \left(1 - \prod_{s=1}^t p_s\right) c_{t+1} \\ &= (1 - p_1)c_2 + \sum_{t=2}^{T'-1} \left(1 - \prod_{s=1}^t p_s\right) c_{t+1} \\ &= (1 - p_1) \frac{(1 - \theta_h)p_1 - \theta_h^{2/T'} + \theta_h}{\theta_h + (1 - \theta_h)p_1} + \frac{(T' - 2) \left(1 - \theta_h^{1/T'}\right) - \theta_h^{2/T'} + \theta_h}{1 - \theta_h}. \end{aligned}$$

The expression  $\theta_l = \frac{1 - 2\theta_h}{1 - S(p_1)} + \theta_h$  depends continuously on  $p_1$  since  $S(p_1) < 1$ . We will show this directly. Note that  $c_1 + S(p_1)$  are the expected costs given that the state is  $\bar{\omega}$ . If  $c_1 = 0$ , the pricing of  $c_t$  for  $t \geq 2$  guarantees that it is sequentially rational for the high valuation type to buy all experiments until the state reveals independent of the choice of probabilities  $(p_1, \dots, p_{T'})$ . In this case, the expected costs of the high valuation type are thus at most equal to her valuation  $V(\theta_h) = 1 - \theta_h$ :

$$\theta_h \cdot \sum_{t=2}^{T'} c_t + (1 - \theta_h) \cdot S(p_1) \leq 1 - \theta_h.$$

Since the costs  $c_t$  for  $t \geq 2$  are strictly positive, it holds that:

$$(1 - \theta_h) \cdot S(p_1) < 1 - \theta_h \Leftrightarrow S(p_1) < 1.$$

We consider the following two values for  $p_1$ :

- (i)  $p_1 = \frac{\theta_h^{1/T'} - \theta_h}{1 - \theta_h}$ . This corresponds to the discrepancy-maximizing probabilities. Equation (30) yields the lower bound of the target interval  $I_{T'}$ :

$$\theta_l = \frac{(1 - \theta_h) - T' \theta_h \left(1 - \theta_h^{1/T'}\right)}{2(1 - \theta_h) - T' \left(1 - \theta_h^{1/T'}\right)}.$$

(ii)  $p_1 = 1$ . Then the first term  $(1 - p_1)c_2$  in  $S(p_1)$  vanishes. The sum becomes:

$$S(1) = \frac{(T' - 2) \left(1 - \theta_h^{1/T'}\right) - \left(\theta_h^{2/T'} - \theta_h\right)}{1 - \theta_h}.$$

Substituting this into (30):

$$\theta_l = \frac{1 - 2\theta_h}{1 - S(1)} + \theta_h = \frac{1 - 2\theta_h}{1 - \frac{(T'-2)(1-\theta_h^{1/T'}) - (\theta_h^{2/T'} - \theta_h)}{1 - \theta_h}} + \theta_h.$$

In particular, if  $T' = 2$ , then  $\theta_l = 1 - \theta_h$  attains the highest possible value for  $\theta_l$ .

By continuity of  $\theta_l$  with respect to  $p_1$ , it holds that for each

$$\theta_l \in \left[ \frac{(1 - \theta_h) - T' \theta_h \left(1 - \theta_h^{1/T'}\right)}{2(1 - \theta_h) - T' \left(1 - \theta_h^{1/T'}\right)}, \frac{1 - 2\theta_h}{1 - S(1)} + \theta_h \right],$$

there exists a  $p_1 \in \left[ \frac{\theta_h^{1/T'} - \theta_h}{1 - \theta_h}, 1 \right]$  such that (30) holds.

It remains to show that this interval covers the target interval  $I_{T'}$  given in (29). This requires verifying that  $\frac{1 - 2\theta_h}{1 - S(1)} + \theta_h$  is greater than or equal to the upper bound of  $I_{T'}$ .

By (15), we can rewrite the upper bound of  $I_{T'}$ :

$$\frac{(1 - \theta_h) - (T' - 1) \theta_h \left(1 - \theta_h^{1/(T'-1)}\right)}{2(1 - \theta_h) - (T' - 1) \left(1 - \theta_h^{1/(T'-1)}\right)} = \frac{1 - 2\theta_h}{1 - \left((T' - 1) \left(\frac{1 - \theta_h^{1/(T'-1)}}{1 - \theta_h}\right) - 1\right)} + \theta_h.$$

We conclude the proof by showing that

$$\begin{aligned} & \frac{1 - 2\theta_h}{1 - \left((T' - 1) \left(\frac{1 - \theta_h^{1/(T'-1)}}{1 - \theta_h}\right) - 1\right)} + \theta_h \leq \frac{1 - 2\theta_h}{1 - S(1)} + \theta_h \\ \Leftrightarrow & (T' - 1) \left(\frac{1 - \theta_h^{1/(T'-1)}}{1 - \theta_h}\right) - 1 \geq \frac{(T' - 2)(1 - \theta_h^{1/T'}) - \theta_h^{2/T'} + \theta_h}{1 - \theta_h} \\ \Leftrightarrow & (T' - 1) \left(1 - \theta_h^{1/(T'-1)}\right) - (1 - \theta_h) \geq (T' - 2) \left(1 - \theta_h^{1/T'}\right) - \theta_h^{2/T'} + \theta_h \\ \Leftrightarrow & T' - 2 - (T' - 1)\theta_h^{1/(T'-1)} \geq T' - 2 - (T' - 2)\theta_h^{1/T'} - \theta_h^{2/T'} \\ \Leftrightarrow & 0 \geq (T' - 1)\theta_h^{1/(T'-1)} - (T' - 2)\theta_h^{1/T'} - \theta_h^{2/T'} \\ \Leftrightarrow & T' - 2 \geq (T' - 1)\theta_h^{1/(T'-1)-1/T} - \theta_h^{1/T'}, \end{aligned}$$

which holds since  $\theta_h^x \leq 1$  for all  $x \geq 0$ . □