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Open-Minded

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Abstract

The profitable exploitation of asset portfolios in the European electricity markets has become more challenging in recent years. This is particularly true for combined heat and power (CHP) generation units that are often facing must-run conditions due to heat demands that need to be satisfied. Including the use of flexibility from storage technologies is key to optimize power plant operation margins and therefore it is crucial to adequately account for price uncertainties in the European market design. Stochastic optimization is thus frequently suggested for an optimal bidding and dispatch of said portfolios.

In our contribution, we develop a novel chain of one weekly and five daily two-stage stochastic optimizations with recourse to identify the optimal bidding strategies for CHP portfolios to all relevant markets, including the key European electricity market segments, i.e., hourly day-ahead and quarter-hourly intraday opening auctions, and control reserve markets, i.e., primary (FCR), secondary (aFRR) and tertiary (mFRR) reserve auctions. We test our model by means of a rolling-horizon approach on historical data and contrast our model's performance with regards to objective function improvement and computation time for various numbers of scenarios. We furthermore benchmark the model against its deterministic representation with and without perfect information.

We find that stochastic optimization may substantially increase portfolio returns, without impairing the usability of stochastic optimization frameworks in real-world contexts, a result that is stable with and without the consideration of heat provision and with different market designs regarding FCR provision periods.

Keywords: OR in energy, Stochastic programming, Auctions/bidding, Combined Heat and Power

JEL-Classification : C32, C61, L94

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Content

1 Introduction

The transformation of the energy system towards carbon neutrality drives the need for technologies that enable a coupling of different energy sectors. Decarbonizing heat supply of households and industrial processes is a significant challenge for overall energy system-wide decarbonization. In the EU, the heating sector was responsible for 42 % of the final energy consumption in 2015, and the avoidance of usage of fossil fuels by enabling sector coupling is thus perceived as an attractive strategy to achieve this goal (Thomaßen et al., 2021).

District heating networks and their flexible mix of generation technologies such as combined heat and power generation plants are thereby an established method of simultaneous electricity and heat provision to large-scale and small-scale customers. These portfolios may provide various energy types and services at the same time, such as electricity, heat and various qualities of reserve power. As additionally timestep-overarching restrictions (such as the operation of storage systems) need to be considered, identifying the optimal marketing strategies for such portfolios is challenging. In sequential markets, coordinated decisions thereby need to be made under multiple price uncertainties, which makes portfolio managers often struggle to operate their assets in an optimal manner.

To our knowledge, no comprehensive approach for decision support in a sequence of interdependent markets has been proposed so far. Even though the formulation of optimal marketing problems for CHP portfolios to short-term electricity markets by the help of stochastic optimization has been subject to a number of recent publications, such as Kumbartzky et al. (2017), Ackermann et al. (2019), and Dietrich et al. (2020). However, the approach proposed in this paper extends the existing literature by the consideration of three different reserve power products (FCR, aFRR, mFRR) and corresponding optimized bidding approaches. Also we develop an enhanced spot market bidding approach which robustifies the derived bids. A third novelty is an ex-post evaluation method, which enables to assess consistently the benefits of stochastic optimization in a backtesting setting.

This paper hence addresses the following research questions: 1) How can the sequence of all short-term markets relevant for CHP portfolio management be handled in a stochastic optimization framework? 2) How can uncertainty regarding electricity prices be modelled and corresponding discrete scenarios be generated for use in stochastic optimization? 3) How can optimal reserve power bids be derived based on the generated scenarios? and 4) What are the benefits of stochastic optimization for portfolio managers and what number of scenarios provides the best tradeoff relative to increase in computational burden?

The paper is structured as follows: After reviewing relevant literature on stochastic optimization of CHP in auction-based electricity markets in Section 2, Section 3 focuses on the methodology implemented for the stochastic programming conducted in this paper. Therefore, the stochastic program structure is first designed reflecting the real-world settings for assets participating in short-term electricity and

reserve markets (Section 3.1). Subsequently, the methodology of modelling short-term electricity price uncertainties, including scenario reduction (Section 3.2), and the chosen approach for determination of optimal reserve bidding (Section 3.3) are outlined. In the last step, the portfolio optimization model is described in Section 3.4. Section 4 then describes the investigated cases used for evaluation of the implemented approach, including the characteristics of the assumed portfolio and the investigated sensitivities. In Section 5, the obtained results are summarized and discussed. Finally, Section 6 concludes this paper and answers the research questions above.

2 Literature Review: Stochastic Optimization of CHP portfolios and coordinated bidding in European Electricity Markets

The use of stochastic optimization along with corresponding probability scenarios can provide additional value to generation companies when commercializing a power plant portfolio compared to single deterministic runs (Dietrich et al., 2020). The key issue with such approaches is the required reduction of complexity to keep the model solvable in an acceptable amount of time while maintaining close to optimal results. Earlier studies on the stochastic optimization of CHP units have been reviewed in Kumbartzky et al. (2017) and Dietrich et al. (2020). Subsequently, the focus is therefore on the state of the art regarding the coverage of all relevant short-term markets for CHP plants in stochastic optimization approaches. Key aspects of stochastic optimization models for CHP portfolios in the European markets include the bidding into spot markets based on the bidding curve approach and the possibility to submit optimized bids for the multiple reserve power products (FCR, aFRR, mFRR).

In the European spot market design, a piecewise-linear curve of price-quantity combinations is submitted (EPEXSPOT, 2021). The construction of a bidding curve requires multiple price-quantity combinations. The optimization problem can be formulated as a mixed-integer linear program (Fleten & Kristoffersen, 2007), either using a discrete set of fixed prices or a set of discrete quantities. Two examples of the optimization of CHP units that include the construction of a piecewise linear function are described by Dietrich et al. (2020) and Ackermann et al. (2019). Dietrich et al. (2020) use a multi-stage stochastic program to optimize the bidding of a portfolio including a CHP unit in the German Day-Ahead spot market. Ackermann et al. (2019) optimize the bidding of a single CHP unit with a heat storage. Additionally, Ackermann et al. (2019) compare the simplistic “flat-bidding” approach, where only one price-quantity combination is submitted, with the bidding curve approach. With “flat-bidding” the quantity is not adjusted according to the realized market price, which may lead to costly trades for unexpected market realizations. When using bidding curves without the possibility to make small corrections in the Intraday market, price outliers can still result in expensive start-ups and shutdowns or even infeasibilities (Klaboe & Fosso, 2013). This is because only a subset of all possible price paths is considered and intertemporal constraints apply that cannot be adequately accounted for when submitting simple bidding curves without the ability to submit complex bids. Ackermann et al. (2019) found a

negative value for the bidding curve construction compared to the flat-bidding approach. We suspect that the reason for this counterintuitive result is the missing possibility to adjust the market position after the Day-Ahead market clearing. Hence, we conclude that a stochastic optimization model for the bidding on the electricity spot market should use the bidding curve approach but should as well include the possibility to adjust the market position after the auction.

The inclusion of a reserve market (or balancing market) is notably considered in Kumbartzky et al. (2017). Like the implementation of the Day-Ahead market, Kumbartzky et al. (2017) use a discrete set of prices to include the reserve market in a mixed-integer program. This approach is computationally expensive as many binary variables are added, especially if multiple reserve products are to be considered. Hence, Kumbartzky et al. (2017) consider a simple portfolio without intertemporal constraints and only one reserve product. This is quite common in the literature, e.g. Böhringer et al. (2019) use a similar model. In contrast, Toubeau et al. (2018) pursue a different approach. They formulate a non-linear model with continuous variables for prices and quantities using a genetic algorithm for the optimization. They test the model with a virtual power plant portfolio with data from the Belgian Day-Ahead, reserve, and Intraday market. A typical day approach is used to reduce the computational burden. They justify this approach by stating that they are optimizing reserve markets with a mid-term horizon of a month or a week. While this approach might be appropriate for the medium-term traded markets, the bidding for reserve products in core European markets often happens day-ahead and we can assume that the forecasts are reliable and should be accounted for in the optimization. In short, the inclusion of the reserve markets with a discrete set of prices is often used but computationally expensive and will be suboptimal when multiple reserve products are considered. The approach from Toubeau et al. (2018) can be used for multiple products by reducing the complexity using the typical day approach while neglecting important information from more recent observations.

In a more recent publication, Kraft et al. (2023) develop a three-stage stochastic program including day-ahead and intraday spot markets along with one reserve market. While not considering bidding curves, they further reformulate the model to include the risk exposure while adding the conditional value at risk in the objective. They show that the model can be a valuable tool for portfolio management based on a case study with a simple portfolio consisting of biomass power plants a s set of PV plants.

Nolzen et al. (2025) explore a similar setting including balancing, day-ahead, intraday markets together with a portfolio of industrial multi-energy plants including CHP, (gas and electrode) boilers and chillers. Given that they focus on the marketing of flexibilities, they model the continuous intraday market trading using financial option theory.

Siddique et al. (2024) also consider portfolios related to district heating, yet their application focus is on large-scale heat-pumps rather than CHP units. In terms of balancing markets, they consider a future regulation market with energy bids very shortly before delivery. This market design differs substantially from balancing market designs with early capacity auctions as considered in the other papers. A key

finding is that such a novel design incentivizes day-ahead trading as these quantities may be used for regulation energy markets.

Löhndorf and Wozabal (2022) as well as Miskiw et al. (2025) explore coordinated bidding strategies in the case of price impacts on the markets. While Löhndorf and Wozabal (2022) focus on battery storage, Miskiw et al. (2025) consider combined portfolios based on PV and biomass plants. Also the latter include a (single) reserve market whereas Löhndorf and Wozabal (2022) exclusively consider day-ahead and intraday markets.

Besides this limitation to at most one reserve market, the papers generally also do not evaluate the impact of the number of scenarios on the results of stochastic optimization. To our knowledge, the tradeoff between the improvement and the additional computational burden has not been evaluated in the literature. Dietrich et al. (2020) provide an overview of how many price scenarios have been used in various publications for bidding in the Day-Ahead market. Most authors use ten or fewer price scenarios, while one publication (Ackermann et al. (2019)) stands out by using 100 scenarios.

In conclusion, Ackermann et al. (2019) and Dietrich et al. (2020) provide good examples of the construction of bidding curves in CHP portfolios optimization. In both models, trading in reserve markets was not considered. Kumbartzky et al. (2017) combined the bidding curve approach with trading in reserve markets. However, this approach might be too computationally intensive for a complex portfolio and multiple reserve products. With the typical day approach of Toubeau et al. (2018), the bidding on multiple reserve markets is feasible, but with the drawback that more recent information is neglected. Kraft et al. (2023), as well as Löhndorf and Wozabal (2022) and Miskiw et al. (2025) provide sophisticated approaches but do not include bidding curves and the portfolios do not include CHP units or thermal storages. Especially storage technologies tend to have a huge impact on the performance and are crucial for the flexibility of CHP portfolios. Therefore, the focus of this paper is on a novel model that considers all sequential short-term markets and can be applied to a complex CHP portfolio without taking too much time to solve the model.

3 Methodology

In Figure 1, the general approach for applying stochastic optimization to a given decision problem is sketched. At least the four core aspects listed here – decision structure, uncertainty representation,

scenario generation and stochastic program – need to be considered to investigate an optimization problem under uncertainty by means of stochastic optimization.

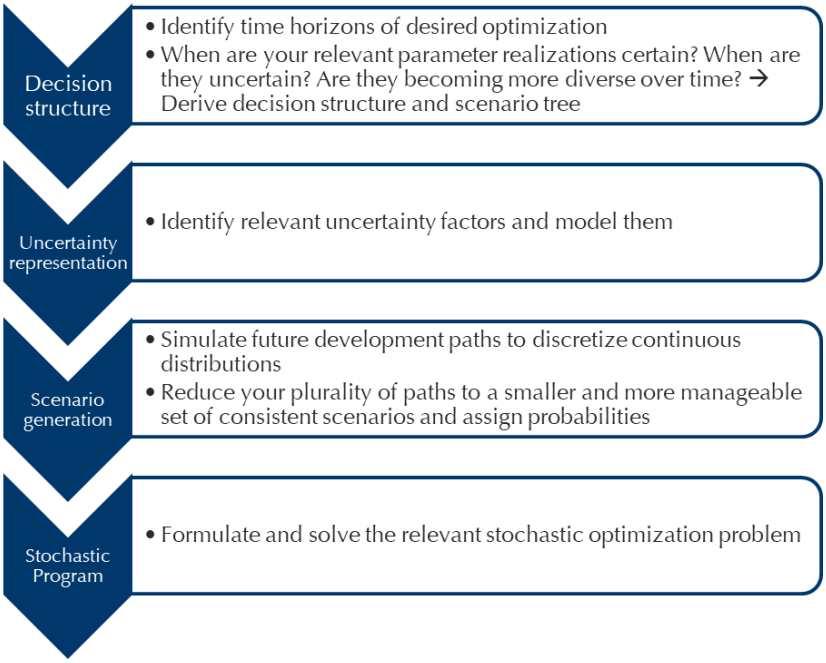

Figure 1: Basic approach to use stochastic optimization for decision support

The remainder of this section is mainly structured according to the mentioned core aspects, whereby the second and third aspect are discussed jointly, and an additional problem-specific aspect (optimal reserve bidding) is included as well: In Section 3.1, the stochastic program structure implemented in this paper is introduced. Subsequently, Section 3.2 describes the modelling of the uncertain parameters of the investigated problem and the scenario reduction. In Section 3.3, the used methodology of optimal reserve bid computation is introduced. Finally, in Section 3.4, we provide insight into the portfolio optimization problem solved by our approach. However, the comprehensive optimization problem description is given in the Appendix of this paper.

3.1 Stochastic Program Structure

The investigated problem in this paper is the determination of an optimal marketing strategy of a small asset portfolio manager, marketing his assets simultaneously in up to three different types of markets, namely short-term wholesale (active) power markets, reserve power markets and a local heat market. Combined heat and power plants may hereby often supply to all three mentioned markets at the same time.

We assume in the following that the portfolio manager has to fulfill heat delivery obligations which are remunerated at a constant heat price exogenously set over a longer period of time[1]. Consequently, heat

[1] A comprehensive overview about different pricing mechanisms in district heating networks is given in Li et al. (2015). While real-time pricing with changes in heat prices during the day is in general possible, it is not the dominant pricing mechanism in heating due to the often-monopolistic ownership structure of local heating provision.

revenues are not impacted by short-term portfolio optimization decisions except for the unlikely case that heat demand is not served. Therefore, heat revenues may be dropped from the optimization model's objective function. However, the restrictions imposed by the heat demand to be covered at each time-step need to be considered explicitly as optimization constraints in the portfolio optimization.

Our model is set in the German market environment. The short-term markets that drive marketing and operation decisions of small portfolio manager examined in the following are

1. Frequency Containment Reserve (FCR) Auction

2. Automatic Frequency Restoration Reserve (aFRR) Auction

3. Manual Frequency Restoration Reserve (mFRR)[2] Auction

4. Day-Ahead (DA) Power Auction

5. Intraday Opening (IDO)[3] Power Auction

These markets are cleared by a one-shot auction each, with different rules regarding bid selection. Most notably, markets 1 to 3 are cleared considering a pay-as-bid allocation rule, markets 4 and 5 implement a uniform pricing regime.

Figure 2 gives an overview on the temporal interdependence of these five markets. The market framework that most of our investigations in this paper are originally based upon are product specifications corresponding to the market situation until July 2019. However, we also include more recent market specifications concerning the FCR auction in our later application section (cf. subsection 4.4). An in-depth description of characteristics of the mentioned reserve market products (1-3) is given by Ocker et al. (2018) and Furtwängler and Weber (2019). However, these markets have been subject to frequent market design changes in the recent past, including changes of procurement period durations of individual reserve products, and changes of the implemented auction clearing mechanisms[4]. What

[2] FCR, aFRR and mFRR are often denoted by their traditional names (primary, secondary and tertiary/minute reserve) in German literature.

[3] There additionally exists a sixth market that represents an opportunity to market electricity shortly before delivery, starting right after the IDO is cleared, the so called continuous intraday market (IDC) of hourly and quarter-hourly electricity contracts. This market is not directly addressed in this paper for various reasons. The three most important aspects are:

(i) This market is a real-time market with a complex market dynamics. Market participants may submit so-called Limit Orders und Market Orders that interact continuously with an Order Book of previous, non-executed Limit Orders of other market participants (and other special products, like so-called Iceberg Orders) – a very dynamic process that is not easily included into the stochastic program developed in this paper.

(ii) There is very limited data available on historical transactions that can reliably inform the stochastic modelling of uncertainties.

(iii) The continuous nature of said markets requires real-time decisions that are not well-compatible with a stochastic optimization approach given the computationally expensive nature of stochastic optimization.

[4] aFRR and mFRR are taking place one day ahead of delivery and cover four-hour time segments, with separate products for positive and negative reserve provision. Until 2019, the FCR auction took place on

remains unchanged is that FCR is a bidirectional reserve product covering both positive and negative reserve and that aFRR and mFRR both encompass dedicated separate positive and negative reserve products with delivery periods of four hours.

The wholesale electricity markets (4 and 5) investigated in this paper represent the most common day-ahead electricity marketing opportunities in Germany. Their most important distinctions are their time of auction (noon for DA, 3 p.m. for IDO) and their covered contracts. While the DA covers hourly products, the IDO represents the first opportunity to trade electricity on a quarter-hourly basis. For a more detailed description of available electricity products, the form of bid submission and evaluation, see (Dietrich et al., 2020).

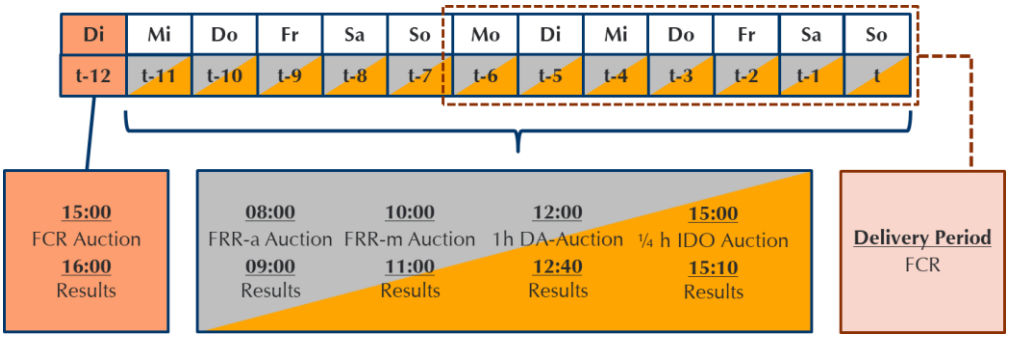

Figure 2: Decision interdependencies between different power markets in Germany before July 2019

The timeframes between result announcements of an auction and the closure of the following auction (minus result processing times by the market participant itself before and after the optimization) represent an upper boundary for acceptable decision (and therefore also computation) times. The shortest timespan between two subsequent decisions until July 2019 was the time in between the aFRR auction result announcement and the mFRR auction, respectively the time in between the mFRR auction result announcement and the DA auction (both one hour). Taking a conservative estimate of 15 minutes of result processing time after the previous decision step and the bidding upload for the following step, the maximum computation time for both mFRR and DA decisions should not exceed 45 minutes if the marketer has participated in the earlier auction(s).

The later the decision is made, the more granular the product is, regarding the length of the traded period. While FCR covers one 168 hour-long period, aFRR and mFRR cover six four-hour periods, the DA Auction 24 one-hour periods and the IDO 96 quarter-hourly periods. Marketing at any step needs to

3 p.m. on Tuesdays and covered the whole following week, so 168 hours in total (Bundesnetzagentur (BNetzA) (2011)). Therefore, determining the optimal marketing opportunity ahead of the FCR auction required the consideration of time periods that were more than twelve days ahead. In July 2019, daily FCR provision was introduced. This auction then initially took place two days ahead of delivery and the FCR product covered the 24 hours of the respective day. Since July 2020, the auctions are held before the aFRR auctions one day before delivery and cover four hour periods BNetzA (2018), just like aFRR BNetzA (2017a) and mFRR BNetzA (2017b). The pay-as-bid auction clearing mechanism first sequentially evaluated power and energy bids, then for a time, a mixed-price evaluation was adopted in 2018, but again halted and abolished in 2019 after a legal decision, reinstating the old clearing mechanism energate messenger (2019).

account for both the decisions that were already made in earlier steps, as well as the anticipation of future steps and their upcoming marketing opportunities.

At all decision steps an individual bidding and operation optimization problem must be solved, which is described in more detail in section 3.4. Thereby, the uncertainty is represented by a scenario tree, consisting of a root node with deterministic information and branches containing individual stochastic scenario paths with a combined probability of 1, thus a scenario fan. As a result, a two-stage optimization model is solved at each decision step. For the FCR step, the optimization horizon consists of a total of 300 hours, whereas the following steps all consist of a total of 240 quarter-hours, or 60 hours. Depending on the decision-step at hand, the optimization considers an hourly or quarter-hourly time resolution, i.e., for weekly FCR optimization an hourly resolution is chosen to limit the computational burden of a stochastic optimization. Quarter-hourly electricity products are not modelled in this case.

For each step, the same toolchain is used, consisting of eight modules. Both one-time auctions and multiple subsequent marketing decisions can be investigated, resulting in one or multiple iterative runs of these modules.

Module 1 loads needed input data, either from external databases or from previous optimization results, if applied in a multi-run setting. Module 2 identifies the current decision step under investigation and sets up relevant parameters. In Module 3, the relevant uncertain parameters are modelled, i.e. the electricity prices from the short-term electricity markets considered (Markets 4 and 5[5]) which are also key inputs for the determination of optimal reserve bids (Module 5). Module 4 reduces the received price scenario paths by applying the scenario reduction algorithm described by Heitsch and Römisch (2003). The methodology for both modules is presented in more detail in section 3.2.

Module 5 determines the reserve marketing bids based on the reduced uncertain price paths. The computed optimal bid prices are entering the optimization as fixed values, while the corresponding quantity of the reserve bid is re-optimized in the stochastic program. This is explained in more detail in section 3.3. Module 6 takes the results of the previous modules and converts them into inputs for a stochastic unit commitment and dispatch model that is called and produces results in Module 7. All results are collected by Module 8. In case of a multi-run, i.e., when model performances are backtested with historical data as in the remainder of this paper, a "Market Simulation" function is called that mimics the market clearing of the decision that was optimized last. In case this is a reserve market (FCR, aFRR, mFRR, 1-3), bid prices are compared to the (historical) marginal clearing price of the market. If the bid price is lower than or equal to the marginal market price, the bid is accepted in its entirety and the portfolio manager receives his/her bidding price for the marketed amount (pay-as-bid). In case of the short-term electricity markets (DA, IDO, 4-5), the portfolio manager's bidding curve that is

[5] Reserve prices of markets 1-3 are not modelled as an uncertain timeseries due to their differing allocation rule. Instead, historical reserve price distributions are considered alongside the opportunity costs induced by the short-term electricity markets 4-5

optimized in Module 7 is evaluated for each hourly or quarter-hourly product, determining the marketed amount using the actual historical price as input.

If multiple days are investigated in a row, i.e., in a backtesting application, an additional optimization is conducted after the IDO decision step that does not actually represent a marketing opportunity but enables the portfolio operator to adjust his/her plant generation schedule one last time after the last market result of the day (IDO) has become available, resulting in the final plant operation schedule used for profit computation. Additionally, the portfolio operator has the opportunity to adjust the market position if technically necessary[6], which reflects possible adjustments in the IDC, but results in a penalty. This "Dispatch" run will be discussed in further detail in section 4.2.

3.2 Modelling of Price Uncertainties

3.2.1 Modelling of Day-Ahead Electricity Prices

To adequately characterize the uncertainty from day-ahead electricity prices, we forecast the underlying multivariate distribution, where the dimensionality of said distribution depends on the decision-step at hand. In the present study, all forecasts are communicated in the form of electricity price ensemble forecasts, that is, a collection of day-ahead electricity price paths with each ensemble member representing a draw from the predicted multivariate distribution.

We base the electricity price ensemble forecasts on the following time-series model, which constitutes a variant of the so-called expert model from the electricity price forecasting literature (Uniejewski et al., 2018; Weron & Ziel, 2019; Ziel & Weron, 2018).

$$p_{t,h} = \beta_{h,0}(t) + \beta_{h,1}(t)p_{t-1,h} + \beta_{h,2}(t)p_{t-2,h} + \beta_{h,3}(t)p_{t-7,h} + \beta_{h,4}(t)p_{t-1,24} + \varepsilon_{t,h} \qquad (1)$$

The electricity price of a particular hour h on day t is modelled as a function of the day-ahead price of the same hour lagged by one, two and seven days as well as the last day-ahead price of the previous day. It should be noted that all coefficients are time-varying, meaning that a coefficient is modelled as the sum of a constant term, a second-order Fourier approximation for seasonal effects and three dummy variables that capture calendar information. The dummies reflect whether the day of the forecast constitutes a Monday, Saturday or Sunday. In addition, all public holidays are modelled as either Saturday or Sunday, depending on whether they constitute local or nationwide public holiday. The days after a public holiday are modelled as Monday.

[6] For example, the *market simulation* evaluation of the IDO piece-wise linear bidding curve may result in marketed quantities leading to a plant schedule incompatible with technical constraints like minimal plant production constraints. The occurrence of this effect cannot be avoided entirely with piece-wise linear curves as bidding amounts are always interpolated between two "kinks" of the bidding curve, and conventional power plant production ranges are usually non-convex due to minimum stable production levels.

$$\beta_{h,i}(t) = \beta_{h,i,0} + \beta_{h,i,1}\sin\left(\frac{2\pi t}{365.24}\right) + \beta_{h,i,2}\cos\left(\frac{2\pi t}{365.24}\right) + \beta_{h,i,3}\sin\left(\frac{4\pi t}{365.24}\right) + \beta_{h,i,4}\cos\left(\frac{4\pi t}{365.24}\right) + \beta_{h,i,5}D_t^{Mo} + \beta_{h,i,6}D_t^{Sa} + \beta_{h,i,7}D_t^{Su} \quad (2)$$

To generate an electricity price ensemble forecast based on the preceding time-series model, a recursive simulation scheme is employed. First, all coefficients are estimated using the LASSO (Least Absolute Shrinkage and Selection Operator) method based on a rolling window of 730 days. The LASSO estimator is used to address the issue of the high dimensionality of the coefficient vector and provides automatic variable selection ((Uniejewski & Weron, 2018; Ziel, 2016).The coefficients and relevant variables are now fixed for the simulation. Second, the one-day-ahead price forecast is calculated from the historical data and a twenty-four-dimensional vector of random disturbances is added to initialize the first ensemble path. The present study considers a parametric approach to generate said disturbances, as they are drawn from a multivariate normal distribution that is fitted to the residuals from the estimation stage to preserve the within-day dependence structure of electricity prices. Third, using the historical data, the one-day-ahead forecast and a newly sampled disturbance vector, the two-day-ahead forecast is calculated, and this recursion is repeated until all prices up to the required horizon of the decision step at hand have been calculated for the first ensemble path. To obtain the electricity price ensemble forecast, the second and third step is repeated until the desired number of ensemble paths has been reached.

3.2.2 Modelling of Intraday Opening (IDO) Electricity Prices

The electricity prices of the IDO power auction and the preceding day-ahead auction are closely related. First, the quarter-hourly resolution of the IDO auction allows for further optimization of hourly schedules resulting from the day-ahead auction. Second, the IDO auction provides the opportunity to react to changes in one's information set from forecast updates received after the day-ahead auction. In the context of stochastic optimization investigated in this study, it is therefore essential to consistently capture the linkage between the electricity prices of the two auctions. We therefore consider an approach that constructs the ensemble forecasts of intraday prices based on the previously calculated ensemble forecast of day-ahead prices. To this end, we do not model intraday prices directly but model the price deviation $d_{t,qh}$ between the intraday price of a particular quarter-hour and the day-ahead price of the corresponding hour; that is, $d_{t,qh} = p_{t,h} - p_{t,qh}$.

The price deviation $d_{t,qh}$ is represented by the following time-series model, inspired by the time-series model for day-ahead prices introduced above.

$$d_{t,qh} = \beta_{qh,0}(t) + \beta_{qh,1}(t)d_{t-1,qh} + \beta_{qh,2}(t)d_{t-2,qh} + \beta_{qh,3}(t)d_{t-7,qh} + \beta_{qh,4}(t)d_{t-1}^{max} + \beta_{qh,5}(t)d_{t-1}^{min} + \beta_{qh,6}(t)d_{t-1,96} + \varepsilon_{t,qh} \quad (3)$$

$$\beta_{qh,i}(t) = \beta_{qh,i,0} + \beta_{qh,i,1}\sin\left(\frac{2\pi t}{365.24}\right) + \beta_{qh,i,2}\cos\left(\frac{2\pi t}{365.24}\right) + \beta_{qh,i,3}\sin\left(\frac{4\pi t}{365.24}\right) + \beta_{qh,i,4}\cos\left(\frac{4\pi t}{365.24}\right) + \beta_{qh,i,5}D_t^{Mo} + \beta_{qh,i,6}D_t^{Sa} + \beta_{qh,i,7}D_t^{Su} \quad (4)$$

The price deviation of a particular quarter-hour qh on day t is modelled as a function of the price deviation of the same quarter-hour lagged by one, two and seven days as well as the maximum, minimum and last price deviation of the previous day. It should be noted that all coefficients are again time-varying.

We estimate the model coefficient using the LASSO method based on a rolling window of 365 days. The ensemble forecast of price deviations is constructed from a recursive simulation scheme as described above with the sole difference that the random disturbances are drawn from a ninety-six-dimensional normal distribution. A combination of paths from the price deviation ensemble with paths from the day-ahead price ensemble lends a consistent intraday price ensemble forecast.

3.2.3 Scenario Reduction

The scenario reduction algorithm was implemented following the concept of Heitsch and Römisch (2003) and Dupačová et al. (2003). We used the variant "forward selection" based on the squared Euclidean distance. This and similar approaches have proven useful and are predominantly used in the literature (Ziel, 2021). In this method, real simulated price paths are selected based on the Wasserstein metric. The use of the Wasserstein metric is supported by stability theory for stochastic programs (Rujeerapaiboon et al. (2018)).

3.3 Modelling of Reserve Bidding

The bid for the reserve market in Germany includes a quantity-price combination for the supply of power and one for the supply of energy. In this setting, it is assumed that the portfolio operator always bids an energy price equal to his costs to reduce the complexity of the decision in the model. To incorporate the reserve market decision into a mixed-integer linear program, some authors have used a discrete set of prices (see Section 2) from which the bid price is selected in the optimisation. To further reduce the complexity, we rely on one single possible bid price which is optimized within Module 5 beforehand the stochastic optimization in Module 7. Hence, the decision is decomposed in two parts. In the first part, the combined price and quantity bid is optimized following Swider and Weber (2007). In the second part, the power price p_B is fixed while the quantity is optimized within a detailed optimization to reassure or adjust the previous optimized quantity of the bid (see section 3.4). In essence, the reserve bidding

tool is used to optimize the bid with a detailed representation of the stochastic properties of the reserve price and a highly simplified representation of the portfolios' capabilities. In this way, we determine a near-optimal bid price while reducing the computational cost by neglecting irrelevant bid price options in the detailed stochastic program.

Swider and Weber (2007) formulated a non-linear optimization problem for the power bid in the German reserve markets. The strengths of the model of are the inclusion of partially accepted bids and including the bidding-price impact on future auctions. The objective function is:

$$\max_{p^B} \Pi = \widetilde{L^A}(p^B; L^B)\left(p^B + \Delta\widetilde{p^v}(p^B) - c^B\right), \tag{5}$$

where Π is the expected profit, $\widetilde{L^A}(p^B; L^B)$ is the expected accepted capacity dependent on the bid price p^B and the bidding capacity L^B, $\Delta\widetilde{p^v}(p^B)$ is the expected loss due to the price-impact on future auctions dependent on the power bid p^B and the expected bid costs c^B which are the costs that arise when the bid is accepted. In this application $\Delta\widetilde{p^v}(p^B)$ is estimated using an ARIMA process. In contrast to Swider and Weber we use discrete probability distributions to estimate the expected accepted reserve capacity L^A to account for the frequent occurrence of reserve power prices of zero. The probability distributions are estimated with an autoregressive quantile regression model (Jahns & Weber, 2019). Within this framework, the costs of c^B are estimated using a simplification of must-run and opportunity costs (see Furtwängler and Weber (2019)). The must-run costs arise if the unit makes a loss while selling the produced electricity on the spot market but still must run at a minimum capacity due to the obligation to provide reserve power. The simplified form of the must-run costs for a unit u at time step t can be defined as follows.

$$C_{u,t}^{Mustrun} = max\left\{P_u^{Min}\left(c_u^{Var} - p_t^{Spot}\right), 0\right\} \tag{6}$$

where P_u^{Min} is the minimum stable power plant production (including the band for negative reserve) of unit u, c_u^{var} represent the variable costs per produced MWh and $p_t^{Spot}$is the realized spot price at time step t. For units that already have a must-run condition, for example because of accepted bids in previous market reserve power auctions or the obligation to produce heat, must-run costs are not considered. The relative opportunity costs per contracted MW in the reserve market can be calculated as follows.

$$c_{u,t}^{Opp} = max\left\{p_t^{Spot} - c_u^{Var}, 0\right\} \tag{7}$$

The opportunity costs do not apply for negative reserve products. Subsequently, we use the expected value of both cost components derived from the simulated price paths described in Section 3.2.

Costs may be reduced by splitting the total bidding capacity L^B between different units and even varying this split across time steps. Therefore, we use the following mixed integer optimization problem to calculate the total expected costs C^B of a marketed capacity L^B. The objective function is:

$$\min_{P_{u,t}^{Res}, o_{u,t}} C^B = \sum_{u,t} P_{u,t}^{Res}\, c_{u,t}^{Opp} + o_{u,t}\, C_{u,t}^{Mustrun} \tag{8}$$

where $P_{u,t}^{Res}$ is the reserve capacity provided by unit u at time step t and $o_{u,t}$ is a binary variable, indicating whether unit u is running. The first constraint ensures that the total reserve power provided by all units in the portfolio is equal to the bidding capacity.

$$\sum_{u} p_{u,t}^{Res} = L^B \;\; \forall t \tag{9}$$

The second constraint enforces that units are on when providing reserve power.

$$P_{u,t}^{Res} \; < o_{u,t}\, P_u^{ResMax} \;\; \forall u, t \tag{10}$$

Using this minimization problem to estimate the costs for an accepted bid with a bidding capacity L^B, we iterate over all possible bidding capacities to find the highest expected profit and the optimal bidding price p_B. The approximation neglects the effects of intertemporal constraints or the usage of storages. Even though the costs are simplified, the power price should still be reliable, as the costs only have a limited effect on the optimal bid when the price impact on future auctions is taken into account (Swider and Weber (2007)).

3.4 Portfolio Optimization Model

The stochastic exogenous parameters obtained from the modelling described in the previous sections, i.e., electricity price scenarios and corresponding consistent reserve price bids, are eventually fed into a mixed-integer linear program, implemented in GAMS. The model implements a unit commitment model within a stochastic optimization framework. Hereby, the prior work of Thorin et al. (2005), Weber and Woll (2006), Kempgens (2018) and Dietrich et al. (2020) is extended. A comprehensive overview over the implemented model restrictions relevant for the following application study is given in Appendix 1.2.

Contrary to its predecessors, the model is now equipped to model all three different European reserve products described in subsection 3.3, namely FCR, aFRR, and mFRR, in addition to the uncertainty modelling of the two electricity wholesale products (DA and IDO) specified in subsection 3.2. Instead of using a standalone GAMS implementation, including backtesting procedures as in Dietrich et al. (2020), the optimization model is embedded within the larger automatized model framework and called by dedicated MATLAB functions. Additionally, the model incorporates new restrictions compared to the previous investigations, enabling the consideration of additional plant types, such as wind production sites.

The timeframe optimized by this model spans 60 hours, respectively 240 quarter-hours for the short-term markets (aFRR, mFRR, DA, IDO). The rolling-horizon approach matches the approach taken in Dietrich et al. (2020), with 12 hours of the auction day (Day 1 in Figure 3) itself being considered, as well as the all 24 hours of the delivery period (Day 2) and the 24 hours of the subsequent day (Day 3).

The latter day is included to achieve a more realistic end-of-day operation of storage technologies for Day 2 and thus a less myopic unit commitment towards the end of the delivery period under investigation. The market results of the remaining hours of Day 1 are already known from previous auctions, therefore the first 12 hours are modelled deterministically.

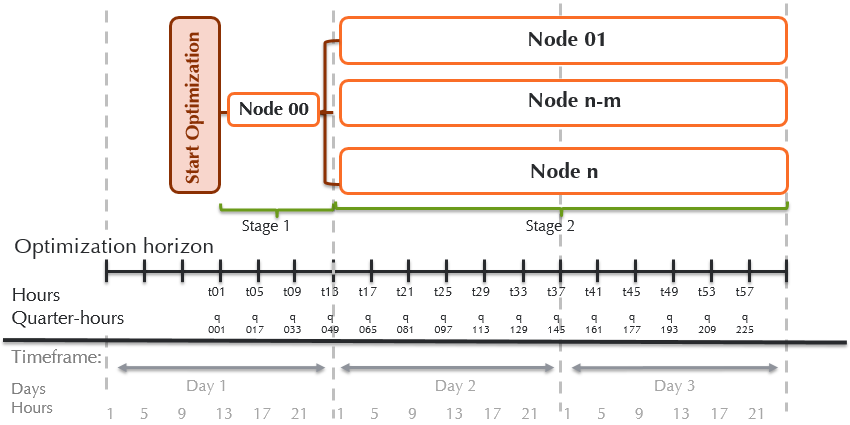

Figure 3: Time period and stochastic tree structure covered by stochastic optimization model (60 hour version) (Dietrich et al., 2020).

Analogously, for the weekly FCR auction, a timeframe of 300 hours is investigated, starting from noon on Tuesday (day of the auction, six days ahead of delivery) until Sunday the week after (last day of the delivery period).

The methodology of calculating reserve bid prices was already discussed in the previous section. Bidding decisions to the wholesale power markets (DA and IDO) are computed by constructing a piece-wise linear, price/quantity-bidding curve for all time-periods of a day, whereby the supporting points of said curve are given by price levels that are previously computed from price scenario simulations. The method applied for their identification (a k-means algorithm) is the same as in Dietrich et al. (2020). The amount that would be bid by the stochastic optimization model in a time step whenever a stochastic price scenario falls into an interval, is optimized jointly with the primal variables describing portfolio operation (like electricity and heat output or charging and discharging of storages). Market rules require the bidding curves to be monotonic increasing for each time-step (EPEXSPOT, 2021), therefore prices within a higher price interval need to result in an equal or higher sold electricity amount.

A problem arises in the extreme parts of the bidding curve. The intervals outside the scenario price range do not contribute to the objective of the optimization and are therefore set randomly by the solver. This leads to expensive or even infeasible schedules if the real price is outside the scenario price range. This happens rarely, but it happens – and if it does, it affects feasibility or, if slack variables are included to ensure feasibility, the objective function value, distorting the optimality of the found solution. It is not possible to put constraints on the curve using an approximation of the maximum capacity, as the unit commitment decision put constraints on the maximum capacity within the optimization. Complementing Dietrich et al. (2020), we introduce new constraints for the construction of the bidding. These ensure a

flat curve on the outer parts of the curve. The outer parts are defined as the intervals outside the simulated price range, including the interval with the smallest and the interval with the highest price, respectively. This is depicted in Figure 4. The left side of the figure shows an example curve without the new constraints, and the right side with the new constraints. The black dotted lines represent the interval boundaries at which the bidding curve may have a kink. The orange dashed lines represent scenario prices. The prices of the scenario are fixed, while the corresponding quantity on the y-axis can be optimized by changing the shape of the bidding curve. Due to the flat curves in the outer parts, every actual realization of the electricity price induces a quantity that has been selected as part of an optimized schedule for at least one scenario price path.

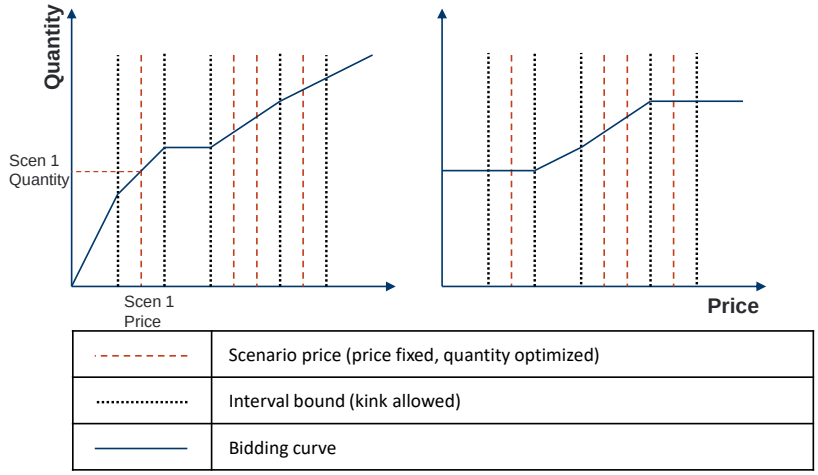

Figure 4: Example for the new constraint that ensure a flat bidding curve in the lateral areas

As a special case, the formulated optimization problem also includes the traditional deterministic optimization. It is obtained by setting up a stochastic optimization with just one scenario.

4 Investigated Cases

In this section, the generation portfolio used for the investigation of different optimization setups is presented in subsection 4.1. Subsection 4.2 then presents the case with stochastic optimization under uncertainty, the case that best reflects the actual decision-making environments of asset portfolio holders. In subsection 4.3, alternative optimization approaches are presented to introduce benchmark cases. Finally, subsection 4.4 reflects on recent market design changes for the FCR product and addresses the expected change in results.

4.1 Case Study Characteristics – Portfolio under Study

We investigate a portfolio close to a real-world application[7], that includes a diverse mix of four plants delivering power and/or heat with different degrees of freedom regarding their generation patterns, as

[7] The portfolio and realized heat demand curves used for validation are the same which were also used in the final report of the research project "StoOpt.NRW" (Beran et al. (2019)), in which the framework presented in this paper was developed (cf. the Acknowledgments).

well as one renewable source and a heat storage. The heat providing technologies of the portfolio, including the heat storage, feed into a local heating grid with a given heat demand that needs to be always satisfied exactly meaning that both under- and overproduction of heat is not allowed. Three plants may provide reserve to one of the reserve markets (FCR, aFRR, mFRR), of which two plants are combined heat and power plants. The renewable plant only participates in the power market and is entitled to feed-in premium payments under the so-called EEG market premium model[8]. Table 1provides an overview of the assumed plant parameters. This portfolio is named *power and heat portfolio P1* for the remainder of this paper.

As a sensitivity, we also investigate a similar portfolio without consideration of the heat side. Thus, the heat storage and heat boiler are deleted from the portfolio and both CHP plants are modelled as simple electric (condensing) turbines, i.e., assuming the same electricity conversion efficiencies, minimum and maximum power restrictions and reserve marketing capabilities as in the reference portfolio P1. This portfolio without consideration of heat demands is in the following referred to as *power portfolio P2*. As no heat delivery obligations need to be fulfilled, this represents a more flexible portfolio, since no heat-induced must-run conditions hamper the electricity and reserve price-driven operation of the CHP plants.

[8] The market premium model under the German Renewable Act ("Erneuerbare-Energien-Gesetz", EEG - (Gesetz für den Ausbau erneuerbarer Energien (Erneuerbare-Energien-Gesetz), 2023)) is part of a mechanism that obliges renewable plants beyond a certain size to market their produced electricity directly to the electricity wholesale market, rather than just receiving a fixed guaranteed feed-in-tariff for their renewable production. A so-called market premium is thereby added to the revenues the plant operator obtains from direct electricity sales. This market premium is computed as the positive difference of the plant's guaranteed remuneration under the EEG and the technology-specific average market value of renewable production in the current month.
This mechanism is intended to prevent overproduction in times of high renewable infeed and low electricity demand, as it implies that plant operators need to react to market signals rather than unconditionally providing their infeed to the grid.

Table 1: Overview of assumed plant characteristics in investigated *power and heat portfolio P1*

		CHP PLANT 1	CHP PLANT 2	CONVENTIONAL POWER PLANT	WIND FARM	HEAT BOILER	HEAT STORAGE
FUEL	Unit	coal	gas	coal	wind	gas	-
CHP-DOF[9]		2	1	-	-	-	-
MAX. POWER	[MW_{el}]	100	120	100	not modelled[10]	-	-
MIN. POWER	[MW_{el}]	40	48	50	0	-	-
MAX. HEAT	[MW_{th}]	220	80	-	-	50	250
MAX. POWER FCR	[MW_{el}]	1	2	1	-	-	-
MAX. POWER AFRR	[MW_{el}]	8	10	8	-	-	-
MAX. POWER MFRR	[MW_{el}]	25	30	25	0	-	-
MAX. CAPACITY	[MWh_{th}]	-	-	-	-	-	250
MEAN EFFICIENCY	[-]	0.26	0.39	0.52	not modelled	0.82	0.9995[11]
START UP COSTS	[€]	7500	1000	3000	0	0	-

4.2 Stochastic Bidding under Uncertainty

The first investigated case is the real-life case of marketing the portfolio assets listed in section 4.1 under uncertainty according to the market rules prior to July 2019 (weekly FCR reserve provision) by making use of stochastic optimization. Different numbers of scenarios are thereby tested and the uncertain parameters are modelled using the methodology described in Section 3.2.

The model is applied in a rolling-horizon back-testing application for four four-week periods within different seasons of the year 2016, using actual data from electricity markets (energate messenger, 2021), reserve markets (50hertz et al., 2021b) and fuel markets (energate messenger, 2021) as well as an authentic heat demand timeseries of the modelled portfolio (Beran et al., 2019). A representative wind production availability timeseries is generated by scaling down the available wind infeed data for Germany published by the German TSOs for 2016 (50hertz et al., 2021a) by the factor 1,000.

[9] DOF = Degrees of Freedom, describing the relationship between electricity and heat output of the CHP plant. A CHP plant with DOF=1 produces electricity and heat at a fixed ratio (e.g. in the case of so-called backpressure turbines), a plant with DOF=2 may alter the combination of electricity and heat output within a technically defined two-dimensional range (e.g. in the case of extraction condensing turbines), cf. Figure 11 in the Appendix for the graphical interpretation of said restrictions.

[10] As the hourly maximum production capability of the wind plant is included directly within the model as a timeseries, there is no need to include the wind plant's generator maximum capacity or efficiency in the model. The maximum hourly value in the used timeseries is 35.5 MW.

[11] The self-discharge of the heat storage per timestep is supposed to amount to 0.05 % of the current energy content. No heat charging or discharging efficiency losses are assumed. Thus, the mean efficiency of the heat storage amounts to almost 1.

The delivery periods chosen for evaluation of the respective seasons are[12]

- Monday, January 4th, 2016 until Sunday, January 31st, 2016 (Winter),
- Monday, April 4th, until Sunday, May 1st, 2016 (Spring),
- Monday, July 4th, until Sunday, July 31st, 2016 (Summer), and
- Monday, September 5th, until Sunday, October 2nd (Autumn).

The sequence of marketing decisions that follow for a back-testing application, building on the market environment and stochastic decision structure described in section 3.1, can be seen in Figure 5. In the dispatch run it is possible to adjust the market position compared to the IDO, yet a penalty of 70 €/MWh is applied.

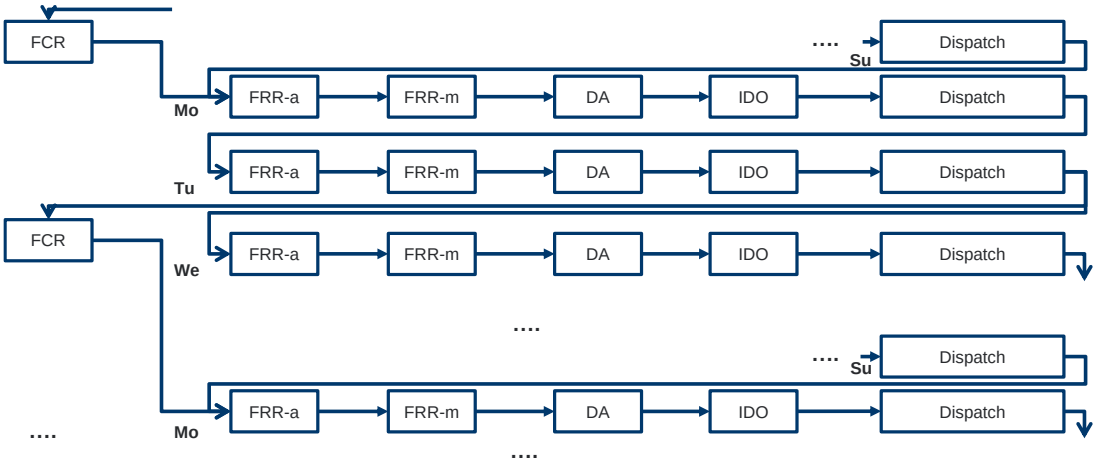

Figure 5: Decision sequence in a long-run (backtesting) application before July 2019

Adjustments to the application case that reflect more recent market developments are addressed in Section 4.4.

Decisions are determined using the stochastic optimization model described in Section 3.4 after reducing the market price simulations to a different number of scenarios n by making use of the methodology described in Section 3.2. A higher n thereby implies a higher diversity of possible outcomes for individual points in time that can be accounted for in constructing the spot market bidding curves, as well as a diverse set of plant schedules that help identifying the optimal dispatch of storage technologies. Therefore, an improvement of the objective function value is expected, if the number of scenarios n is

[12] For the first weekly FCR auction (on Tuesdays in the week before), the respective weekly auction is simulated before the first daily (aFRR) auction, thus dating 5 days before the respective period starts. All daily auctions between this FCR auction and the aFRR auction on the first day of the delivery period are neglected.

increased. To identify the overall benefit of stochastic optimization, however, these results need to be benchmarked to deterministic approaches, which are presented in the following section.

4.3 Alternative Optimization Approaches

4.3.1 Deterministic Bidding with Perfect Foresight (with Rolling Horizon)

Decisions that are made under uncertainty are imperfect by nature. However, stochastic optimization helps to recover some of the objective function losses that cannot be realized given a lack of perfect information.

The first alternative problem that is investigated in this paper answers the question how good the portfolio under study could perform if every decision was made under perfect information in the investigated timeframes (PI^{RH})[13]. Instead of assuming uncertain wholesale prices, actual electricity price realizations are fed into the deterministic (one scenario) version of the optimization model. Similarly, also the tool computing the optimal reserve bids is equipped with perfect information on the resulting reserve prices and thus delivers optimal bid prices for different bid sizes under price certainty.

As a result of the considerations listed above, this approach delivers an objective function value that may be seen as an upper benchmark for the achievable objective function value from stochastic optimization.

4.3.2 Deterministic Bidding under Uncertainty

If perfect information is not available, the optimization may obviously still be carried out in a deterministic manner, by choosing the ex-ante expected value of the price realizations as optimization input. This is also referred to as the "Expected Value Problem" (EV^{RH}) in the literature (Birge, 1982). The objective value that is obtained by solving this problem is by the nature of uncertainty worse than (or at maximum equal to) the objective value of an optimization with perfect information or a stochastic optimization approach including uncertainty parameter information – however, solving it is a lot less computationally expensive than stochastic optimization and might even be the only possible way to obtain results that enable decision-making under uncertainty in real-world applications, notably when decision lead times are very short.

Conceptually, a deterministic optimization under uncertainty assuming the expected value as the parameter's realization can also be framed as a stochastic optimization with $n = 1$ scenarios. As a stochastic optimization with $n > 1$ increases the information contained in the modelled parameters

[13] Note that this case is not a textbook case with perfect foresight, as the limited time horizon of the optimization problem itself and the rolling-window approach applied in the backtesting application still lead to suboptimal intertemporal results compared to an integrated optimization over the whole long-term period studied (here: 28 days). Therefore, the results of an optimization with rolling horizon with perfect information will always be worse than the results of long-term optimization without rolling horizon.
We do not further explore this difference in this paper, as our focus remains on the actual daily marketing decisions of portfolio holders –nevertheless, we would like to point the reader at this distinction. We thus use the superscript RH to reflect this in our following notation.

compared to the deterministic case by including distribution information while maintaining the same expected value, this one scenario case provides a lower benchmark based on which the benefits of stochastic optimization may be assessed.

The difference in objective value that is obtained by solving the problem described in Section 4.3.1 and the deterministic optimization conducted here is also often denoted as the Expected Value of Perfect Information ($EVPI^{RH}$). We define this value following the approach of Dietrich et al. (2020) by[14]:

$$EVPI^{RH} = |objective(PI^{RH}) - objective(EV^{RH})| \tag{11}$$

In Dietrich et al. (2020), a methodology for evaluating the benefits of stochastic optimization (BSO) is proposed that derives information about the merits of stochastic optimization in case stochastic optimization results are compared to deterministic solutions with and without perfect information. The BSO will be used as a measure in absolute values, cf. eq. (12), describing the increase in target function value compared to the expected value problem, but could also be formulated as a target function percentage increase, as in Dietrich et al. (2020).

$$BSO_n^{RH} = objective(SO_n^{RH}) - objective(EV^{RH}) \tag{12}$$

In the following, we further define the *EVPI that can be recovered by stochastic optimization with n scenarios (EVPI recovered)* as a relative parameter that improves the comparability between different stochastic optimization application problems.

$$EVPIrecovered_n^{RH} = \begin{cases} \frac{BSO_n^{RH}}{EVPI^{RH}} \cdot 100\%, & if\ EVPI^{RH} > 0 \\ 0\%, & if\ EVPI^{RH} = 0 \end{cases} \tag{13}$$

4.4 Sensitivity Analysis: Daily Frequency Containment Reserve (FCR) Auction

In this sensitivity analysis, the setting described in Sections 3 and 4.2 is altered by replacing the weekly FCR auction by a daily with a 24-hour delivery period for the next day. The auction happens one hour before the aFRR auction. Therefore, another daily optimization with a horizon of 60 hours for the FCR auction is added and the weekly FCR optimization with a 300 horizon is withdrawn in this case.

As only weekly auction prices are available, daily auction prices must be simulated. We chose a model to simulate prices that are coherent with the mean, variance, and the auction results of the observed weekly auctions, which is described in the following. The price of reserve provision for a week is the sum of the prices for the individual weekdays. Hence, $p_W = \sum_\omega p_\omega$, where ω is a set of all weekdays

[14] The problem investigated in this paper is a profit maximization problem, therefore $EVPI^{RH}$ is always greater than or equal to zero, even without taking the absolute value of the difference $objective(PI^{RH}) - objective(EV^{RH})$. However, by using the absolute value in this definition, the formula may also be directly applied to the case of a minimization problem.

from Monday to Sunday and p_W is the price for the reserve provision for the whole week. It is assumed that the price p_W and p_ω price auction prices are independently normal distributed and that every p_ω for every weekday has the same mean and variance. Therefore, $p_W \sim \mathcal{N}(\mu, \sigma^2)$ and $p_\omega \sim \mathcal{N}(\mu_d, \sigma_d^2)$ where μ_W and σ_W^2 are the mean and variance of the FCR prices for a week and μ_d and σ_d^2 are the mean and variance of the FCR prices in dependence of the weekday respectively. This implies that $7\mu_d = \mu_w$ and $7\sigma_d^2 = \sigma_W^2$. To simulate daily FCR prices we draw random numbers $p_\omega \; \forall \; \omega$ where the sum of the realizations p_ω of the weekdays equals the realization p_w of p_w. Hence, the simulated daily prices are coherent with weekly observations (Appendix). For this we use the conditional multivariate normal distribution that ensures that the sum of the simulated daily prices p_ω matches the weekly prices p_W. This distribution was estimated and was used to simulate the marginal and minimal auction prices separately.

As the costs of provision for a shorter period is lower, while the price level stays the same, we expect that the overall profit remain unchanged or increase compared to the profit with weekly auctions. The smaller planning horizon should lead to a smaller uncertainty regarding the Day-Ahead and Intraday prices. Therefore, we expect that the merit of the stochastic optimization should be reduced.

5 Results

We apply the sequence of models introduced in Section 3 to the portfolio in Section 4.1 in the back-testing applications described in Sections 4.2-4.4.

All runs are carried out on a virtual desktop PC with a processing core of 2.6 GHz, and RAM of 24 GB, whereby no parallel runs are started. The stochastic, mixed-integer optimization problems are solved by using the CPLEX MIP solver version 12.9 with small deviations from standard settings[15]. The GAMS setting *threads*, representing the number of cores used by the program is set to 7 to enable a faster solution of the linear dual problem and hence, the probability that a dual solution is found within the optimization time limit is increased. For the GAMS options, an absolute optimality criterion of *OptCA = 1,000* (euros) is set. The maximum computation time of the optimization model is set to 900 (for the 60-hour model), respectively 1,800 seconds (for the 300-hour model) by use of the option *RESLIM*.

5.1 Stochastic Case

The results for the stochastic optimizations with n=5, n=10, n=15 and n=20 scenarios are depicted in Figure 6 for portfolio P1. The months investigated induce different overall profit levels. During the wintertime, higher electricity prices induce an overall higher profit level, whereas during the summertime, prices, and thus also profits, are substantially lower. As could be expected, the profits rise with an increasing number of scenarios, i.e. stochastic optimization with n=5 leads to lower profits than

[15] The option mipstart=1 is used to generate discrete starting values for the mixed integer program. Rinsheur is set to 50, meaning that Relaxation Induced Neighborhood Search (RINS) is invoked each fiftieth node in the MIP tree.

with n=20 scenarios. For example, portfolio P1 increases its profit by more than 32,000 € during the winter month and more than 22,000 € during the summer. However, the biggest increase in absolute terms occurs in spring, where the profit is increased by more than 46,000 € during the investigated month.

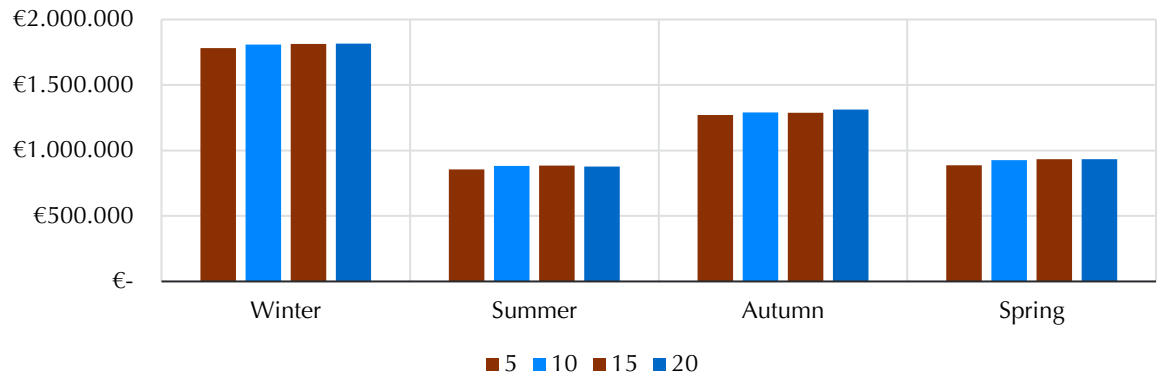

Figure 6: Results for stochastic cases for portfolio P1

However, individual instances where an increase in scenarios does not result in increasing profit can also be observed. In the summer month, the highest profit is achieved with n=15 scenarios, instead of n=20 scenarios. This is due to the combination of an increased computational burden at higher numbers of scenarios and the 15-minute time limit that is set to ensure compliance with the auction schedule. The share of optimizations terminated by the 15-minute resource limit depending on the scenario number is depicted in Figure 7. The number of prematurely terminated optimizations increases substantially for scenario numbers of n=10 or higher and is particularly high in summer[16].

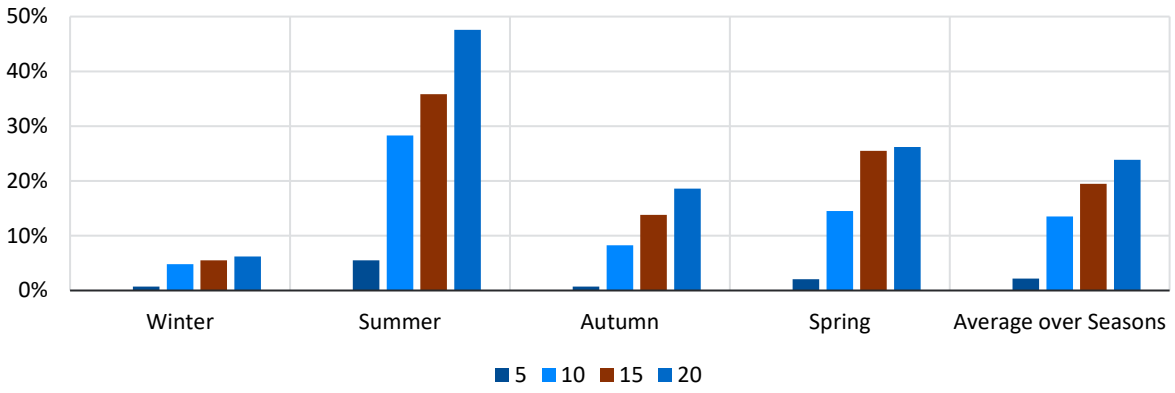

Figure 7: Relative number of 60-hour optimizations terminated by reaching the set maximum time limit

For portfolio P2, the overall profits rise compared to the heat portfolio across all seasons – given that revenues from heat sales are not considered. As for portfolio P1, profits rise when increasing the number

[16] This is a consequence of fewer must-run conditions for the marketed CHP plants due to low heat demand during the summer period. Consequently, a higher number of binary operation variables has to be actively decided upon by the solver, increasing optimization time.

of scenarios fed into the stochastic optimization. The differences between the 5 and 20 scenario cases increase as well, with about 38,000 € for the winter month and even 49,500 € for the summer month. As before, this biggest increase in profit is realized in spring, with more than 56,000 € of additional returns. Contrary to portfolio P1, there is no optimization run for this electricity-only portfolio that reaches the 15-minute optimization time limit.

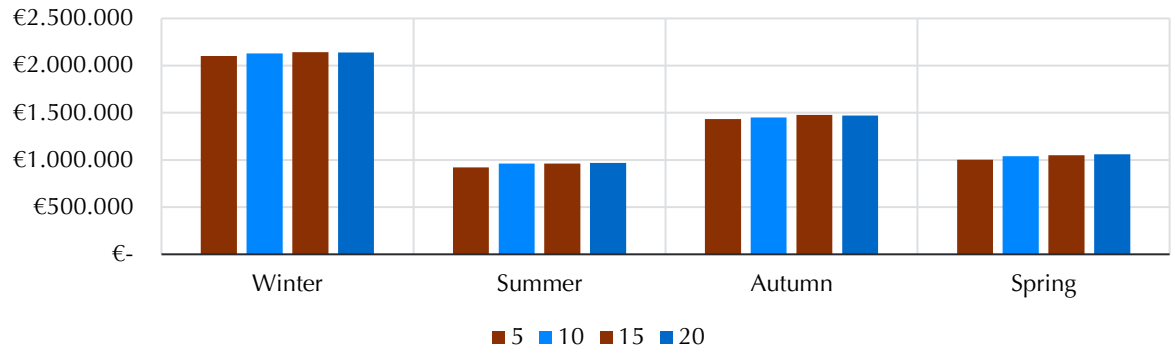

Figure 8: Results for stochastic cases for portfolio P2

5.2 Comparison to Alternative Optimization Approaches

Having assessed the stochastic cases, we now benchmark these results against deterministic representations of the decision problem at hand. Therefore, we compute the deterministic (one-scenario) outcome under uncertainty and the deterministic optimization including perfect information with limited horizon.

Table 2: Profits and benefit of stochastic optimization for P1

Scenarios n	1 ($\boldsymbol{EV^{RH}}$)	5	10	15	20	1 ($\boldsymbol{PI^{RH}}$)
profit [k€]	4,158	4,795	4,910	4,920	4,939	5,119
$EVPI^{RH}$ [k€]	-	-	-	-	-	961
BSO_n^{RH} [k€]	-	636	752	761	781	-
$EVPIrecovered_n^{RH}$ [%]	-	66.3	78.3	79.3	81.3	-

For portfolio P1, the cumulative results over all investigated months are listed in Table 2 and depicted in the left half of Figure 9. The right half of Figure 9 displays the average computation time needed per decision. The optimization time limit of 15 minutes mentioned above prevents an over-proportional increase in average computation time with rising scenario numbers, since this induces an increasing share of optimization runs aborted after 15 minutes. Nevertheless, average computation times are increasing substantially with increasing scenario numbers. The upper computation time boundary for the stochastic optimization thereby induces a sigmoid behavior of average computation time development.

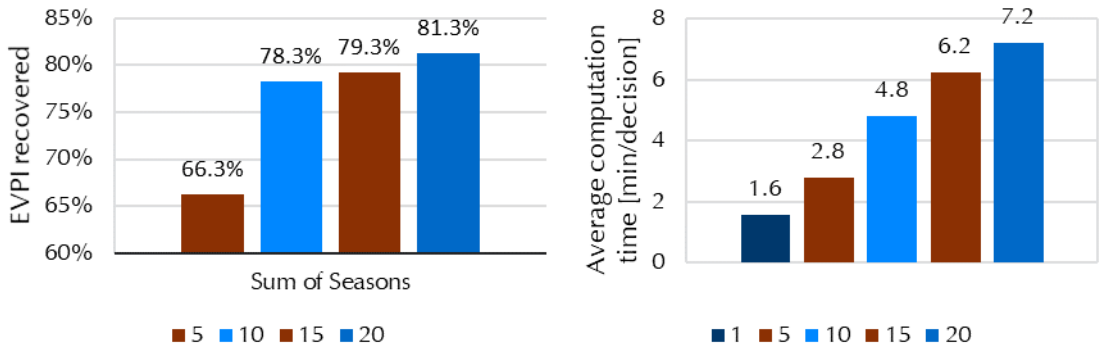

Figure 9: Comparison of EVPI recovered by stochastic optimization and increase in computation time for P1

For the electricity-only portfolio P2, the cumulative results over all investigated months with regards to profits and the benefit of stochastic optimization behave similarly to the case of P1 (cf. Table 3). Notably, the share of EVPI recovered for $n = 20$ scenarios is the same for both portfolios. The computation time, however, shows only a marginal increase for P2, indicating that the computation time is rather driven by overhead processes and not much by the scenario number.

Table 3: Profits and benefit of stochastic optimization for P2

Scenarios n	1 ($\boldsymbol{EV^{RH}}$)	5	10	15	20	1 ($\boldsymbol{PI^{RH}}$)
profit [k€]	4,543	5,458	5,580	5,630	5,640	5,892
$EVPI^{RH}$ [k€]	-	-	-	-	-	1,348
BSO_n^{RH} [k€]	-	914	1,036	1,087	1,096	-
$EVPIrecovered_n^{RH}$ [%]	-	67.8	76.9	80.6	81.3	-

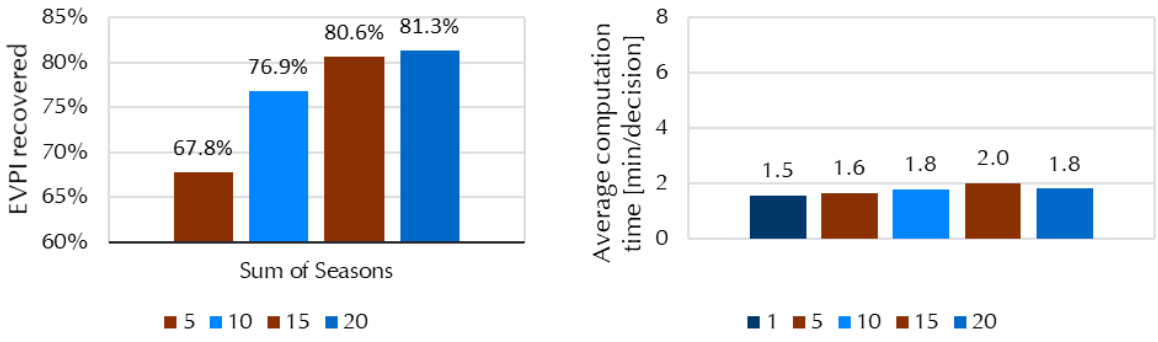

Figure 10: Comparison of EVPI recovered by stochastic optimization and increase in computation time for P2

The comparison of the deterministic benchmarks to the stochastic optimizations of Section 5.1 suggests that the increasing profits obtained with a more detailed representation of uncertainty in the electricity marketing problem at hand are largely independent of the complexity (or rigidity) of the additional constraints that the underlying power plant assets face when heat is considered. The achievable profits

may differ between portfolios P1 and P2 in absolute values, but the share of recoverable information value compared to the deterministic benchmark is similar. However, the computation time that is needed to achieve said increase is much lower for P2. Put differently, the benefits of stochastic optimization relative to its computational expensiveness decline more substantially for P1 than for P2. As a result, a lower scenario number might be the preferable option for decision support in real-world applications, especially if time constraints are tight. In the example of portfolio P1, a further increase of just 3% of recovered EVPI between $n = 10$ and $n = 20$ is paid by an average computation time increase of 50% (7.2 instead of 4.8 minutes).

5.3 Sensitivity Analysis: Alternative Market Design

We additionally perform our analysis for the alternative market design explained in Section 4.4. We find similar results for portfolio P1, if the primary reserve product tenders are limited to daily blocks, as can be seen in Table 4. Interestingly, the overall profits that can be realized under uncertainty for P1 are lower with the daily reserve product – contrary to our initial expectations. In case of perfect foresight, the profits are yet slightly higher, which aligns with the expectation that reserve prices for short-term tenders for single days are more volatile than weekly prices. Therefore, primary reserve provision with perfect foresight may take advantage of hours with high revenues from reserve markets and exploit the opportunity cost differences between reserve market revenues and electricity sales. Managing this trade-off turns out to be much harder under uncertainty, as the lower profits for the scenarios with uncertainty show. Meanwhile, as the number of traded reserve products increases, also the computational expensiveness increases for higher scenario cases. The higher number of prematurely stopped optimization runs may contribute to a lower absolute profit. Both the expected value of perfect information and the value of stochastic optimization as indicated by the BSO_n^{RH} increase with daily reserve auctions - in line with our expectations. Yet, the share of $EVPIrecovered_n^{RH}$ does not exhibit a clear trend.

Table 4: Change in profits and benefits of stochastic optimization compared to Table 2 under the sensitivity analysis: daily primary reserve auction

	Scenarios n	1 ($\boldsymbol{EV^{RH}}$)	5	10	15	20	1 ($\boldsymbol{PI^{RH}}$)
Delta ($Value_{Weekly}$ - $Value_{Daily}$)	profit [k€]	-151	-16	-44	-29	-11	6
	$EVPI^{RH}$ [k€]	-	-	-	-	-	157
	BSO_n^{RH} [k€]	-	135	107	121	140	-
	$EVPIrecovered_n^{RH}$ [%]	-	3	-2	0	1	-

For portfolio P2, the results show overall similar tendencies. The difference between the profits in case of weekly and daily reserve provision are yet somewhat smaller. Moreover, the profits obtained when using stochastic representations now generally outperform those under weekly reserve provision periods

described in Table 3. With portfolio P2, the value of stochastic optimization is clearly higher in terms of the BSO_n^{RH} and the share of $EVPIrecovered_n^{RH}$ in a daily auction scheme.

The direct comparison between the two portfolios indicates that the increase in computation time, coupled with a termination of the optimizations after 15 minutes, likely leads to a performance degradation for P1. The results for P2 with a lower average computation time and almost no termination due to the time limit are in line with the expected results.

The average computation time decreases, as there is no need for weekly optimization anymore. However, the total number of optimizations per week increases from $5 \cdot 7 + 1 = 36$ to $6 * 7 = 42$ under the modified market design. As a result, the relative advantage regarding the total cumulative optimization time for one week is negligible.

Table 5: Changes in profits and benefits of stochastic optimization compared to Table 3 under the sensitivity analysis: daily primary reserve auction

	Scenarios n	1 ($\boldsymbol{EV^{RH}}$)	5	10	15	20	1 ($\boldsymbol{PI^{RH}}$)
Delta ($Value_{weekly}$ - $Value_{Daily}$)	profit [k€]	7	35	31	28	29	3
	$EVPI^{RH}$ [k€]	-	-	-	-	-	5
	BSO_n^{RH} [k€]	-	27	23	20	21	-
	$EVPIrecovered_n^{RH}$ [%]	-	2	2	2	2	-

6 Conclusion

In this paper, we introduce a new stochastic optimization model tailored towards the market structure of the European reserve and electricity markets, including the optimized bidding into three subsequent reserve markets, and the participation in the Day-Ahead and Intraday Opening auctions of EPEX SPOT. Therefore, an elaborate chain of programs is executed to model electricity price uncertainties, perform a scenario reduction of the modelled price paths, derive optimal reserve power bids based on the reduced price paths and finally generate inputs and execute a two-stage stochastic optimization unit commitment model. The toolchain is set up for both weekly and daily optimization cycles, based on underlying weekly or daily reserve product tenders.

We investigate varying numbers of price scenarios and evaluate the resulting benefit of stochastic optimization and the share of the expected value of perfect information that may be recovered, all by applying a rolling-horizon approach. We contrast between plant portfolios with and without heat delivery obligations by investigating both combined heat and power plants as well as condensing turbine power plants with a comparable generation cost structure based on real market data of the year 2016. Furthermore, we investigate the effects of a market design change, reflecting the switch from weekly to

daily reserve tenders for Frequency Containment Reserve in 2019 in a sensitivity analysis. The results indicate a higher value of the stochastic optimization for the new market design.

We find that the underlying electricity price uncertainty can be adequately modelled by a time series model based on a multivariate normal distribution. For the reserve bid modelling, we adapt the approach of Swider and Weber (2007) to generate optimized reserve bids. This approach combines a detailed depiction of the stochastic characteristics of reserve markets with a simplified estimation of the cost of provision. With this, it is possible to neglect irrelevant price options in the detailed stochastic program to substantially decrease the computation time. We find that under the given assumptions and modelling choices, the implemented approach yields substantial benefits of stochastic optimization compared to the deterministic problem under uncertainty, a result in line with previous findings in the literature. With a scenario number of $n = 20$, more than 80% of the value lost from introducing uncertainty in the model framework can be recovered by the stochastic representation of the problem. For the portfolio cases modelled in this paper, significant value can be added for the portfolio managers by introducing stochastic optimization, i.e., monthly profits may be raised by a magnitude of ten thousands of euros. This result is stable across portfolios with and without the inclusion of rigid restrictions such as the modelling of additional constraints regarding heat demand. A sensitivity analysis excluding weekly reserve products and focusing on daily optimization cycles confirms this finding.

In line with prior work (e.g. Dietrich et al. (2020)), we observe that high scenario numbers substantially increase the computational burden of stochastic optimization for diversified portfolios providing electricity, reserve power and heat. We find that this effect is most noticeable in summer periods when heat demand is low. Including a computation time limit of 15 minutes for the stochastic optimization, ensures that a fairly good solution may be found despite not always fulfilling the optimality criterion for higher scenario numbers; however, the relative benefit of stochastic optimization compared to the increase in computational expensiveness is impacted quite negatively. As a result, reducing scenario numbers for the optimization of diversified and complex portfolios is an attractive measure to strike a good balance between exploiting the benefits of stochastic uncertainty representation and an adequate computation time for real-world applications.

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Appendix 1: Portfolio Optimization Model

This section contains the full description of the portfolio optimization model introduced in Section 3.4, which is an extension of the model formulation included in (Dietrich et al., 2020). The implementation of this MILP model in GAMS consists of an objective function and about 50 further types of equality and inequality constraints. For the sake of simplicity, some constraints are hereby displayed jointly that are represented by more than one constraint in the actually implemented model.

Appendix 1.1: Nomenclature

The tables below list all relevant indices, sets, variables and parameters as used in the model description.

Table 6: Nomenclature for indices

Indices	**Description**
n	scenario node
h	hour
t	quarter hour
w	week
tb	(four-hour reserve) time block
u	power/heat unit
f	fuel type
hs	heating system
i	intervals for quarter-hourly bidding curve
ih	intervals for hourly bidding curve
det	deterministic

Table 7: Nomenclature for sets

Index sets	**Description**
N	Set of all scenarios n
W	Set of all distinct weeks w during the investigated period
TB	Set of all (four-hour reserve) time blocks tb
T	Set of all quarter-hourly time-steps t
H	Set of all hourly time-steps h
U	Set of all power or heat producing units u
F	Set of all fuels f
$U^{boiler} \in U$	Set of all heat boilers, subset of U
$U^{heatstorage} \in U$	Set of all heat storages, subset of U
$U^{cond} \in U$	Set of all condensing turbines, subset of U

$U^{chp} \in U$	Set of all combined heat and power units, subset of U
$U^{chp1dof} \in U^{chp}$	Set of all combined heat and power units with a production with one degree of freedom, subset of U^{chp}
$U^{chp2dof} \in U^{chp}$	Set of all combined heat and power units with a production with two degrees of freedom, subset of U^{chp}
$U^{f} \in U$	Set of all units combusting fuel f, subset of U
$U^{EEG} \in U$	Set of all units under the Renewable remuneration scheme, subset of U
NH	Set of consistent hour/scenario combinations (n, h)
NT	Set of consistent quarter-hour/scenario combinations (n, t)
HT	Set of consistent quarter-hour/hour mappings (h, t)
WT	Set of consistent quarter-hour/week mappings (w, t)
TBT	Set of consistent quarter-hour/four-hour time block mappings (tb,t)
TI	Mapping of quarter-hourly price simulations $P^{ID}_{n,t}$ to intervals for quarter-hourly bidding curve i for each scenario n
$TI*$	Mapping of quarter-hourly price simulations $P^{ID}_{n,t}$ to intervals for quarter-hourly bidding curve i for each scenario n. Only intervals on the left and right side of the bidding curve starting with the interval with the highest and lowest simulated price respectively.
HIH	Mapping of hourly price simulations $P^{DA}_{n,h}$ to intervals for hourly bidding curve ih for each scenario n
$HIH*$	Mapping of hourly price simulations $P^{DA}_{n,h}$ to intervals for hourly bidding curve ih for each scenario n. Only intervals on the left and right side of the bidding curve starting with the interval with the highest and lowest simulated price respectively.
$T_{det} \in T$	Quarter-hourly time-steps t belonging to the deterministic time-horizon of the stochastic optimization
$T^{Start} \in T$	First quarter-hourly time-step of the set of time-steps T
$T^{End} \in T$	Last quarter-hourly time-step of the set of time-steps T
H_{det}	Hourly time-steps h belonging to the deterministic time-horizon of the stochastic optimization

Table 8: Nomenclature for variables

Variables	Unit	Range	Description
$p^{DA}_{n,h}$	MW_{el}	$\mathbb{R}$	Net marketed power in DA Auction per hour h in scenario n
$p^{ID}_{n,t}$	MW_{el}	$\mathbb{R}$	Net marketed power in ID Auction per quarter hour t in scenario n
$p^{Power}_{n,u,t}$	MW_{el}	$\mathbb{R}^{+}_{0}$	Produced power by unit u in quarter hour t in scenario n

$p_{n,u,t}^{FCR}$	MW_{el}	$\mathbb{R}_0^+$	FCR power provided by unit u in quarter hour t in scenario n
$p_{n,u,t}^{FRRa,pos}$	MW_{el}	$\mathbb{R}_0^+$	Positive aFRR power provided by unit u in quarter hour t in scenario n
$p_{n,u,t}^{FRRa,neg}$	MW_{el}	$\mathbb{R}_0^+$	Negative aFRR power provided by unit u in quarter hour t in scenario n
$p_{n,u,t}^{FRRm,pos}$	MW_{el}	$\mathbb{R}_0^+$	Positive mFRR power provided by unit u in quarter hour t in scenario n
$p_{n,u,t}^{FRRm,neg}$	MW_{el}	$\mathbb{R}_0^+$	Negative mFRR power provided by unit u in quarter hour t in scenario n
$q_{n,u,t}^{Fuel}$	MW_{th}	$\mathbb{R}_0^+$	Used fuel by unit u in quarter hour t in scenario n
$q_{n,f,t}^{Fuel}$	MW_{th}	$\mathbb{R}_0^+$	Used fuel by fuel type f in quarter hour t in scenario n
$q_{n,u,t}^{Heat}$	MW_{th}	$\mathbb{R}$	Produced heat by unit u in quarter hour t in scenario n (may be negative for charging heat storages)
$vol_{n,u,t}^{Heat}$	MWh_{th}	$\mathbb{R}_0^+$	Heat storage filling level by unit u in quarter hour t in scenario n
$o_{n,u,t}$	-	$\{0; 1\}$	Binary power plant operation variable (1: on, 0: off) for unit u in quarter hour t in scenario n
$up_{n,u,t}$	-	$\{0; 1\}$	Binary power plant operation variable (1: plant starting, 0: plant not starting) for unit u in quarter hour t in scenario n
$p_{t,i}^{ID,lb}$	MW_{el}	$\mathbb{R}$	Power amount bid at the left boundary of the quarter hourly price interval i for quarter hour t
$p_{t,i}^{ID,rb}$	MW_{el}	$\mathbb{R}$	Power amount bid at the right boundary of the quarter hourly price interval i for quarter hour t
$p_{h,ih}^{DA,lbh}$	MW_{el}	$\mathbb{R}$	Power amount bid at the left boundary of the hourly price interval ih for hour h
$p_{h,ih}^{DA,rbh}$	MW_{el}	$\mathbb{R}$	Power amount bid at the right boundary of the hourly price interval ih for hour h
o_w^{FCR}	-	{0;1}	Binary variable indicating whether FCR is marketed / a FCR bid is submitted for week w
o_{tb}^{prod}	-	{0;1}	Binary variable indicating whether reserve product $prod \in FRRa,pos; FRRa,neg; FRRm,pos; FRRm,neg$ is marketed / a bid is submitted for $prod$ in four-hour time block tb
p_w^{FCR}	MW_{el}	$\mathbb{Z}_0^+$	Marketed power in FCR auction per week w
p_{tb}^{prod}	MW_{el}	$\mathbb{Z}_0^+$	Marketed power in reserve product $prod \in FRRa,pos; FRRa,neg; FRRm,pos; FRRm,neg$ per four-hour time block tb

Table 9: Nomenclature for parameters

Parameters	Unit	Range	Description

φ_n	-	[0,1]	Scenario probability of scenario n
$P_{n,h}^{DA}$	€/MWh$_{el}$	$\mathbb{R}$	DA Auction Price of hour h in scenario n
$P_{n,t}^{ID}$	€/MWh$_{el}$	$\mathbb{R}$	ID Auction Price of quarter-hour t in scenario n
P_w^{FCR}	€/MW$_{el}$	$\mathbb{R}_0^+$	FCR price in week w
$P_{tb}^{FRRa,pos}$	€/MW$_{el}$	$\mathbb{R}_0^+$	Positive aFRR price in four-hour time block tb
$P_{tb}^{FRRa,neg}$	€/MW$_{el}$	$\mathbb{R}_0^+$	Negative aFRR price in four-hour time block tb
$P_{tb}^{FRRm,pos}$	€/MW$_{el}$	$\mathbb{R}_0^+$	Positive mFRR price in four-hour time block tb
$P_{tb}^{FRRm,neg}$	€/MW$_{el}$	$\mathbb{R}_0^+$	Negative mFRR price in four-hour time block tb
T_u^{Comp}	€/MW$_{el}$	$\mathbb{R}_0^+$	EEG compensation payment for produced power of unit u
$C_{n,f,t}^{Fuel}$	€/MW$_{th}$	$\mathbb{R}_0^+$	Fuel costs (excl. CO_2) for fuel f in quarter-hour t in scenario n
$C_{n,t}^{CO2}$	€/t CO_2	$\mathbb{R}_0^+$	CO_2 costs in quarter-hour t in scenario n
C_u^{start}	€/ start	$\mathbb{R}_0^+$	Starting costs of plant u
EF_f^{CO2}	t CO_2/ MW$_{th}$	$\mathbb{R}_0^+$	CO_2 emission factor of fuel f
Δh	h		Duration of one hour (1h)
Δt	h		Duration of one quarter-hour (1/4 h)
D_t^{Power}	kW$_{el}$	$\mathbb{R}_0^+$	Power demand position in quarter -hour t (from previous marketing in hourly and quarter hourly markets)
D_t^{Heat}	kW$_{th}$	$\mathbb{R}_0^+$	Local heat demand in quarter hour t
$Q_u^{Heat,max}$	kW$_{th}$	$\mathbb{R}_0^+$	Maximum heat production capacity of unit u
η_u	-	[0,1]	Boiler efficiency of unit u
$Q_u^{Heat,dischargemax}$	kW$_{th}$	$\mathbb{R}_0^+$	Maximum heat storage discharge capacity of unit u
$Q_u^{Heat,chargemax}$	kW$_{th}$	$\mathbb{R}_0^+$	Maximum heat storage charge capacity of unit u
$V_u^{Heat,max}$	kWh$_{th}$	$\mathbb{R}_0^+$	Maximum heat storage filling level of unit u
$V_u^{Heat,Start}$	kWh$_{th}$	$\mathbb{R}_0^+$	Heat storage filling start level of unit u
$V_u^{Heat,End}$	kWh$_{th}$	$\mathbb{R}_0^+$	Heat storage filling end level of unit u
η_u^{Losses}	-	[0,1]	Heat storage efficiency of unit u
P_u^{Max}	MW$_{el}$	$\mathbb{R}_0^+$	Maximum power plant production limit of unit u
P_u^{Min}	MW$_{el}$	$\mathbb{R}_0^+$	Minimum stable power plant production limit (if power plant is running) of unit u
$b_{chp1dof}^{bp}$	MW$_{th}$ /MW$_{el}$	$\mathbb{R}_0^+$	Slope of the backpressure curve (pq-diagram) of one degree-of-freedom CHP unit $chp1dof$
$a_{chp1dof}^{bp}$	MW$_{th}$	$\mathbb{R}$	Section of the backpressure curve (pq-diagram) of one degree-of-freedom CHP unit $chp1dof$
$b_{chp1dof}^{Fuel}$	MW$_{th}$ /MW$_{el}$	$\mathbb{R}_0^+$	Slope of the fuel consumption curve of one degree-of-freedom CHP unit $chp1dof$

$a_{chp1dof}^{Fuel}$	MW_{th}	$\mathbb{R}_0^+$	Minimal fuel consumption, section of the fuel consumption curve of one degree-of-freedom CHP unit $chp1dof$
$b_{chp2dof}^{ec,1}$	MW_{th} /MW_{el}	$\mathbb{R}_0^+$	Slope of the maximum production curve (pq-diagram) of two degree-of-freedom CHP unit $chp2dof$
$a_{chp2dof}^{ec,1}$	MW_{th}	$\mathbb{R}$	Section of the maximum production curve (pq-diagram) of two degree-of-freedom CHP unit $chp2dof$
$b_{chp2dof}^{ec,2}$	MW_{th} /MW_{el}	$\mathbb{R}_0^+$	Slope of the minimum production curve (pq-diagram) of two degree-of-freedom CHP unit $chp2dof$
$a_{chp2dof}^{ec2}$	MW_{th}	$\mathbb{R}$	Section of the minimum production curve (pq-diagram) of two degree-of-freedom CHP unit $chp2dof$
$b_{chp2dof}^{ec,3}$	MW_{th} /MW_{el}	$\mathbb{R}_0^+$	Slope of the backpressire curve (pq-diagram) of two degree-of-freedom CHP unit $chp2dof$
$a_{chp2dof}^{ec3}$	MW_{th}	$\mathbb{R}$	Section of the backpressure curve (pq-diagram) of two degree-of-freedom CHP unit $chp2dof$
$b_{chp2dof}^{Fuel}$	MW_{th} /MW_{el}	$\mathbb{R}_0^+$	Slope of the marginal fuel consumption curve for additional electricity produced of two degree-of-freedom CHP unit $chp2dof$
$c_{chp2dof}^{Fuel}$	MW_{th} /MW_{el}	$\mathbb{R}_0^+$	Slope of the marginal fuel consumption curve for additional electricity produced of two degree-of-freedom CHP unit $chp2dof$
$a_{chp1dof}^{Fuel}$	MW_{th}	$\mathbb{R}_0^+$	Minimal fuel consumption, section of the fuel consumption curve of two degree-of-freedom CHP unit $chp2dof$
OP_u	-	$\mathbb{R}_0^+$	Minimum operation period number of plant u
SD_u	-	$\mathbb{R}_0^+$	Minimum shut-down period number of plant u
$\lambda_{n,t}$	-	[0,1]	Linearization parameter for the position of quarter-hourly spot prices within the quarter-hourly bidding curve in quarter-hour t in scenario n
$\lambda_{n,h}$	-	[0,1]	Linearization parameter for the position of hourly spot prices within the hourly bidding curve in hour h in scenario n
$P^{prod,min}$	MW_{el}	{1}	Minimum bid size for the reserve product $prod \in FCR; FRRa,pos; FRRa,neg; FRRm,pos; FRRm,neg$ auction determined by the market operator

Appendix 1.2: Model formulation

The objective function includes the revenues obtained in the reserve and spot markets, further revenue streams through Renewable infeed premia and the costs associated with power and heat generation, consisting of fuel cost, CO_2 cost and plant start-up costs:

$$\max_{p_{n,h}^{DA},p_{n,t}^{ID},q_{n,u,t}^{Power},q_{n,u,t}^{Fuel},q_{n,t}^{Heat}} \sum_{w\in W} p_w^{FCR}\cdot P_w^{FCR} + \sum_{tb|(tb,t)\in TBT} p_{tb}^{FRRa,pos}\cdot P_{tb}^{FRRa,pos} + p_{tb}^{FRRa,neg} \cdot P_{tb}^{FRRa,neg} + p_{tb}^{FRRm,pos}\cdot P_{tb}^{FRRm,pos} + p_{tb}^{FRRm,neg}\cdot P_{tb}^{FRRm,neg} + \sum_{n\in N}\varphi_n \cdot \left(\sum_{h|(n,h)\in NH} p_{n,h}^{DA}\cdot P_{n,h}^{DA}\cdot \Delta h + \left(\sum_{t|(n,t)\in NT} \left(p_{n,t}^{ID}\cdot P_{n,t}^{ID} + \sum_{u\in U^{EEG}} p_{n,u,t}^{Power}\cdot T_u^{Comp} - \sum_{f\in F} q_{n,f,t}^{Fuel}\cdot C_{n,f,t}^{Fuel} + EF_f^{CO2}\cdot C_{n,t}^{CO2} - \sum_{u\in U} C_u^{start}\cdot up_{n,u,t} \right) \right)\cdot \Delta t \right) \tag{14}$$

There are both electric and heat balance equations implemented in the model. The electric balance equation balances the trading position and the physical fulfilment of said position:

$$\forall\,(n,h)\in NH \wedge \forall\,(n,t)\in NT \wedge \forall\,(h,t)\in HT:$$
$$\sum_{u\in U} p_{n,u,t}^{Power} = D_t^{Power} + D_h^{Power} + p_{n,t}^{ID} + p_{n,h}^{DA} \tag{15}$$

For the local heating system, the following balance equation holds:

$$\forall\,(n,t)\in NT \wedge \forall\, hs\in HS: \sum_{(u)|(u,hs)} q_{n,u,t}^{Heat} = D_t^{Heat} \tag{16}$$

For the individual fuels, the following balance equation holds:

$$\forall\,(n,t)\in NT \wedge \forall\, f\in F, U\in U^f: \sum_{(u)|(u,f)} q_{n,u,t}^{Fuel} = q_{n,f,t}^{Fuel} \tag{17}$$

For heating boilers, the following constraints apply, describing the maximum production capacity (18) and the boiler efficiency (19):

$$\forall\,(n,t)\in NT, u\in U^{boiler}: q_{n,u,t}^{Heat} \le Q_u^{Heat,max} \tag{18}$$

$$\forall\,(n,t)\in NT, u\in U^{boiler}: q_{n,u,t}^{Fuel} = \frac{q_{n,u,t}^{Heat}}{\eta_u} \tag{19}$$

The heat storages are subject to the following constraints, describing maximum storage level changes ((20), (21)), maximum storage level (22), and storage level changes between time-steps (23), as well as start (24) and end storage levels (25):

$$\forall (n,t) \in NT, \forall u \in U^{heatstorage}: q_{n,u,t}^{Heat} \leq Q_u^{Heat,dischargemax} \quad (20)$$

$$\forall (n,t) \in NT, \forall u \in U^{heatstorage}: q_{n,u,t}^{Heat} \geq -Q_u^{Heat,chargemax} \quad (21)$$

$$\forall (n,t) \in NT, \forall u \in U^{heatstorage}: v_{n,u,t}^{Heat} \leq V_u^{Heat,max} \quad (22)$$

$$\forall (n,t) \in NT, \forall u \in U^{heatstorage}: v_{n,u,t}^{Heat} = v_{n,u,t-1}^{Heat} \cdot \eta_u^{Losses} - q_{n,u,t}^{Heat} \cdot \Delta t \quad (23)$$

$$\forall t \in \{T^{Start}\}, \forall u \in U^{heatstorage}: v_{n,u,t}^{Heat} = V_u^{Heat,Start} \cdot \eta_u^{Losses} - q_{n,u,t}^{Heat} \cdot \Delta t \quad (24)$$

$$\forall t \in \{T^{End}\}, (n,t) \in NT, \forall u \in U^{heatstorage}: v_{n,u,t}^{Heat} = V_u^{Heat,End} \quad (25)$$

For all electric units also delivering heat $U^{chp} \in U$, the maximum heat constraint is given by:

$$\forall u \in U^{chp}: q_{n,u,t}^{Heat} \leq Q_u^{Heat,max} \quad (26)$$

The limits for electricity production are defined by the maximum and minimum power capacities P_u^{Max} and P_u^{Min}, as well as by the marketed reserve power that is provided by the respective plant:

$$\forall u \in U^{chp} \cup U^{cond}: p_{n,u,t}^{Power} \leq o_{n,u,t} \cdot P_u^{Max} + p_{n,u,t}^{FCR} + p_{n,u,t}^{FRRa,pos} + p_{n,u,t}^{FRRm,pos} \quad (27)$$

$$\forall u \in U^{chp} \cup U^{cond}: p_{n,u,t}^{Power} \geq o_{n,u,t} \cdot P_u^{Min} + p_{n,u,t}^{FCR} + p_{n,u,t}^{FRRa,neg} + p_{n,u,t}^{FRRm,neg} \quad (28)$$

For all combined heat and power producing units $u \in U^{chp}$ in the system with only one degree of freedom, in this case, CHP with backpressure turbines, the following equations hold to describe the relationship of produced electricity and produced heat (also often denoted as pq-diagram, (29)), as well as the CHP fuel consumption depending on electricity production (30):

$$\forall u \in U^{chp1dof}: q_{n,chp1dof,t}^{heat} = a_{chp}^{bp} \cdot o_{n,chp1dof,t} + b_{chp1dof}^{bp} \cdot p_{n,chp1dof,t}^{elec} \quad (29)$$

$$\forall u \in U^{chp1dof}: q_{n,chp1dof,t}^{Fuel} = a_{chp1dof}^{Fuel} \cdot o_{n,chp1dof,t} + b_{chp1dof}^{Fuel} \cdot p_{n,chp1dof,t}^{elec} \quad (30)$$

In their case, the relationship between a certain electricity output and the corresponding heat output is biunique.

In contrast to this, for CHP plants with two degrees of freedom allow variations of the allowed power and heat output pairs. We model the resulting pq-diagram by the help of three inequations ((31),(32) and (33)) that need to be satisfied simultaneously if the plants are operating.

$$\forall u \in U^{chp2dof}: \; p^{elec}_{n,chp2dof,t} \leq a^{ec,1}_{chp2dof} \cdot q^{heat}_{n,chp2dof,t} + o_{n,chp2dof,t} \cdot b^{ec,1}_{chp2dof} \tag{31}$$

$$\forall u \in U^{chp2dof}: \; p^{elec}_{n,chp2dof,t} \geq a^{ec,2}_{chp2dof} \cdot q^{heat}_{n,chp2dof,t} + o_{n,chp2dof,t} \cdot b^{ec,2}_{chp2dof} \tag{32}$$

$$\forall u \in U^{chp2dof}: \; p^{elec}_{n,chp2dof,t} \geq a^{ec,3}_{chp2dof} \cdot q^{heat}_{n,chp2dof,t} + o_{n,chp2dof,t} \cdot b^{ec,3}_{chp2dof} \tag{33}$$

An illustration of the constraints reflecting CHP production ranges can be found in Figure 11.

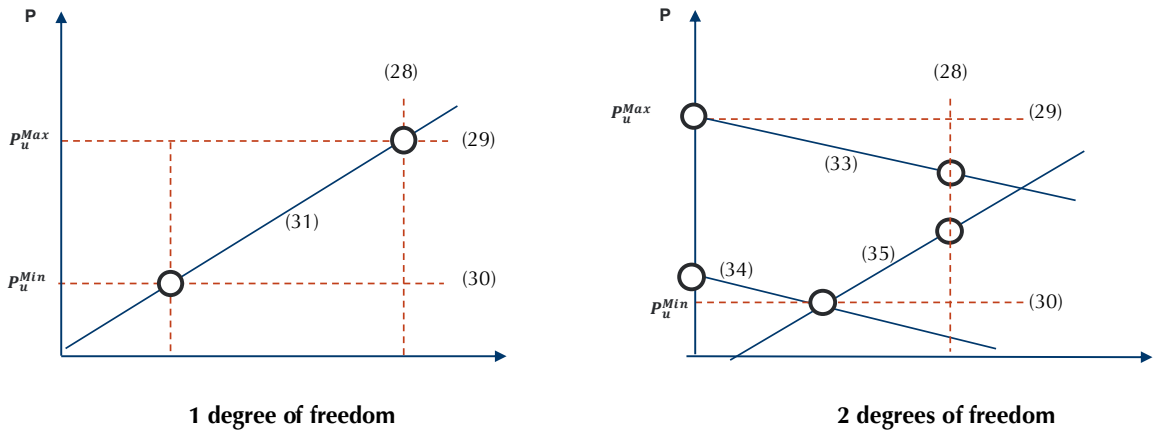

Figure 11: Graphical illustration of pq-diagram constraints

The fuel consumption equation of these power plants needs to be reflect and differentiate between both the marginal fuel consumption by electricity production and the marginal fuel consumption by heat production.

$$\forall u \in U^{chp2dof}: \; q^{Fuel}_{n,chp2dof,t} = a^{Fuel}_{chp2dof} \cdot o_{n,chp2dof,t} + b^{Fuel}_{chp2dof} \cdot p^{elec}_{n,chp2dof,t} + c^{Fuel}_{chp2dof} \cdot q^{Heat}_{n,chp2dof,t} \tag{34}$$

The minimum operation time and minimum shut down times of the CHP plants are modelled by use of further binary variables and by introducing further time-coupling constraints, which describe minimum operation time (35), minimum shut-down time (36) and the definition of a power plant start (37):

$$\forall (n,t) \in NT, \quad (n,t-1) \in NT, u \in U^{chp}: \sum_{t'=t-OP_u+1}^{t} up_{n,u,t'} \leq o_{n,u,t} \tag{35}$$

$$\forall (n,t) \in NT, (n,t-1) \in NT, u \in U^{chp} : \sum_{t'=t-SD_u+1}^{t} up_{n,u,t'} \leq 1 - o_{n,u,t-SD_u} \quad (36)$$

$$\forall (n,t) \in NT, (n,t-1) \in NT, u \in U^{chp} : up_{n,u,t} \geq o_{n,u,t} - o_{n,u,t-1} \quad (37)$$

The electricity production of renewable energy sources (like wind) is restricted by the actual available (wind) potential wp_t at a certain point in time t.

$$\forall u \in U^{chp}: p_{n,u,t}^{Power} \leq wp_t \quad (38)$$

Forbidden marketing in the deterministic part of the optimization ((41), (44)), as well as minimum ((39), (42), (45)) and maximum ((40), (43), (46)) marketing amounts for each interval of the bidding curve for Day Ahead Auction ((39), (40), (41)), Intraday Opening Auction ((42), (43), (45)) and Combined marketing to both auctions ((45), (46)) are described by the following restrictions:

$$\forall (t,i) \in TI \wedge (w,t) \in WT \wedge (tb,t) \in TBT: p_{t,i}^{ID,lb} \geq p_w^{FCR} + p_{tb}^{FRRa,pos} + p_{tb}^{FRRm,pos} \quad (39)$$

$$\forall (t,i) \in TI \wedge (w,t) \in WT \wedge (tb,t) \in TBT: p_{t,i}^{ID,rb} \leq \sum_u P_u^{Max} - p_w^{FCR} - p_{tb}^{FRRa,neg} - p_{tb}^{FRRm,neg} \quad (40)$$

$$\forall t \in T_{det} : p_{n,t}^{ID} = 0 \quad (41)$$

$$\forall (h,ih) \in HIH \wedge (w,t) \in WT \wedge (tb,t) \in TBT: p_{h,ih}^{DA,lbh} \geq p_w^{FCR} + p_{tb}^{FRRa,pos} + p_{tb}^{FRRm,pos} \quad (42)$$

$$\forall (H,ih) \in HIH \wedge (w,t) \in WT \wedge (tb,t) \in TBT: p_{h,ih}^{DA,rbh} \leq \sum_u P_u^{Max} - p_w^{FCR} - p_{tb}^{FRRa,neg} - p_{tb}^{FRRm,neg} \quad (43)$$

$$\forall h \in H_{det} : p_{n,h}^{DA} = 0 \quad (44)$$

$$\forall (h,ih) \in HIH \wedge (h,t) \in HT : p_{h,ih}^{DA,lbh} + p_{t,i}^{ID,lb} \geq p_w^{FCR} + p_{tb}^{FRRa,pos} + p_{tb}^{FRRm,pos} \quad (45)$$

$$\forall (h, ih) \in HIH \wedge (h, t) \in HT : p_{h,ih}^{DA,rbh} + p_{t,i}^{ID,rb} \leq \sum_{u} P_u^{Max} - p_w^{FCR} - p_{tb}^{FRRa,neg} - p_{tb}^{FRRm,neg} \quad (46)$$

The following restrictions model the linearization of the optimized bidding curves within the bidding curve intervals and are defined for both the Day Ahead ((50),(51),(52)) and Intraday Opening Auction ((47),(48),(49)) bidding curves:

$$\forall (t, i) \in TI \wedge (n, t) \in NT : p_{n,t}^{ID} = \left(1 - \lambda_{n,t}\right) \cdot p_{t,i}^{ID,lb} + \lambda_{n,t} \cdot p_{t,i}^{ID,rb} \quad (47)$$

$$\forall (t, i) \in TI : p_{t,i}^{ID,lb} \leq p_{t,i}^{ID,rb} \quad (48)$$

$$\forall (t, i) \in TI, i < I : p_{t,i}^{ID,rb} = p_{t,i+1}^{ID,lb} \quad (49)$$

$$\forall (h, ih) \in HIH \wedge (n, h) \in NH : p_{n,h}^{DA} = \left(1 - \lambda_{n,h}\right) \cdot p_{h,ih}^{DA,lbh} + \lambda_{n,h} \cdot p_{h,ih}^{DA,rbh} \quad (50)$$

$$\forall (n, h) \in NH : p_{h,ih}^{DA,lbh} \leq p_{h,ih}^{DA,rbh} \quad (51)$$

$$\forall (h, ih) \in HIH, ih < IH : p_{h,ih}^{DA,rbh} = p_{h,ih+1}^{DA,lbh} \quad (52)$$

Another restriction ensures that the bidding curve is left and right side of the bidding curve starting with the interval with the highest and lowest simulated price respectively

$$\forall (t, i) \in TI * / : p_{t,i}^{ID,lb} = p_{t,i}^{ID,rb} \quad (53)$$

$$\forall (t, i) \in HIH * : p_{h,ih}^{DA,lbh} = p_{h,ih}^{DA,rbh} \quad (54)$$

Finally, the condition avoiding arbitrage trades between hours and quarter-hours in the 1st Optimization is given by:

$$\forall (n, t) \in NT : \sum_{(h,t) \in HT} p_{n,t}^{ID} = 0 \quad (55)$$

The developed model is executable with both quarter-hourly products and hourly time products, as well as only hourly products considered (for the weekly FCR optimization). In the latter case, the restrictions (39), (40), (41), (45), (46), (47), (48), (49) and (55) are not considered. Additionally, for the former case, there is the option to only consider a quarter-hourly resolution, to reduce computational complexity. In this case, restrictions (42), (43), (44), (45), (46), (50), (51), (52) and (55) are neglected. For the Day-Ahead Auction, the realized marketed amount is then derived from the average marketed amount of the respective quarter-hours belonging to one hour.

The marketing of reserve products is modelled for each reserve product type individually. For the FCR auction, the following four restrictions apply:

If reserve is marketed, the bid needs to exceed the minimum bid size (1 MW for all products, as only one bid is submitted by the portfolio holder). Thus, a binary variable o_w^{FCR} models the submission of an FCR bid.

$$\forall\, w \in W: p_w^{FCR} \geq P^{FCR,min} \cdot o_w^{FCR} \tag{56}$$

Each individual unit u has a maximum FCR power $P_u^{Max,FCR}$ it can deliver within the reserve activation time-period (30 seconds). The reserve power market provision of the overall therefore mustn't exceed the sum of the individual plant's reserve provision capabilities. If a plant may not provide reserve, $P_u^{Max,FCR}$ is set to zero.

$$\forall\, w \in W: p_w^{FCR} \leq o_w^{FCR} \cdot \sum_u P_u^{FCR,max} \tag{57}$$

The sum of all reserve power provision by individual plants in each time-step t needs to equal the total marketed reserve power of the portfolio in the corresponding week w.

$$\forall\, w \in W \wedge (w,t) \in WT: p_w^{FCR} = \sum_{u \in U} p_{n,u,t}^{FCR} \tag{58}$$

The individual plants u are also restricted by their maximum reserve provision capability, thus the following restriction holds:

$$\forall\, (n,t) \in NT, u \in U: p_{n,u,t}^{FCR} \leq P_u^{FCR,max} \tag{59}$$

The aFRR and mFRR auctioning is modelled analogously to FCR provision and do not differ for aFRR and mFRR, as well as for positive and negative reserve direction. Thus, these restrictions are in the following listed jointly, without further differentiation and explanation:

$$\forall\, prod \in \{FRRa,pos; FRRa,neg; FRRm,pos; FRRm,neg\}:$$

$$\forall\, tb \in TB: p_{tb}^{prod} \geq P^{prod,min} \cdot o_{tb}^{prod} \tag{60}$$

$$\forall\, w \in TB: p_{tb}^{prod} \leq o_{tb}^{prod} \cdot \sum_u P_u^{prod,max} \tag{61}$$

$$\forall\, w \in TB \wedge (tb,t) \in TBT: p_{tb}^{prod} = \sum_{u \in U} p_{n,u,t}^{prod} \tag{62}$$

$$\forall\, (n,t) \in NT, u \in U: p_{n,u,t}^{prod} \leq P_u^{prod,max} \tag{63}$$

Besides the mentioned restrictions, the model implements a couple of restrictions that were not considered in the application studies of this paper, including restrictions for the modelling of electric storages such as batteries or pumped hydro storages, restrictions for the modelling of Power-to-Heat plants, restrictions for the modelling of Power-to-Gas and connected gas storage facilities, as well as load change velocity (ramping) restrictions. Readers interested in the implementation are invited to contact the corresponding author of this paper for further information.

Appendix 1.3: Conditional density of daily FCR price simulation

We want to find the conditional distribution of $(\{ P_{Monday} \cdots P_{Sunday} \mid \Sigma_\omega P_\omega = p\_w\}$ where p_w a constant, $P_\omega \sim \mathcal{N}(\mu_d, \sigma_d^2)$ and ω is a set of weekdays[17]. As we lose one degree of freedom this is equivalent to $(\{ P_{Monday} \cdots P_{Saturday} \mid \Sigma_\omega P_\omega = p\_w\}$ with $P_{Sunday} = p_w - \sum_{\omega \setminus \{Sunday\}} p_\omega$

Starting with the vector

$$\begin{pmatrix} P_{Monday} \\ \vdots \\ P_{Saturday} \\ \sum_{\omega} P_\omega \end{pmatrix}. \tag{64}$$

This vector follows a multivariate normal distribution with the mean

$$\mu = \begin{pmatrix} \mu_d \\ \vdots \\ \mu_d \\ 7\mu_d \end{pmatrix} \tag{65}$$

and the variance covariance matrix

$$\Sigma = \sigma_d^2 \begin{pmatrix} I_{6\times6} & \mathbf{1}_{6\times1} \\ \mathbf{1}_{1\times6} & 7 \end{pmatrix}. \tag{66}$$

Where $I_{6\times6}$ is the identity matrix and $\mathbf{1}$ is a matrix filled with ones.

[17] Inspired by https://stats.stackexchange.com/questions/97213/r-pick-10-random-numbers-from-standard-normal-distribution-whose-sum-equals-5

The following is taken from Eaton (1983, pp. 116–117).

If N-dimensional vector x is partitioned as follows,

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

The accordingly μ and Σ are portioned as follows,

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}$$

and

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{21} \\ \Sigma_{12} & \Sigma_{22} \end{bmatrix}.$$

Then the distribution of x_1 conditional on $x_2 = a$ is multivariate normal ($x\_1 \,|x\_2 \sim \mathcal{N}(\bar{\mu}, \bar{\Sigma})$ where

$$\bar{\mu} = \mu_1 + \Sigma_{12}\, \Sigma_{22}^{-1}\, (a - \mu_2)$$

and

$$\bar{\Sigma} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}.$$

Therefore, one can conclude that $(\{ P_{Monday} \cdots P_{Saturday} \mid \Sigma_\omega\, P_\omega = p\_w\}$ is multivariate normal distributed with the mean

$$\bar{\mu} = \mu_d + \mathbf{1}_{1\times 6}\sigma_d^2 \frac{1}{7\sigma_d^2}(p_w - 7\mu_d) \tag{67}$$

And the variance covariance matrix

$$\bar{\Sigma} = I_{6\times 6} - \mathbf{1}_{1\times 6}\sigma_d^2 \frac{1}{7\sigma_d^2}\mathbf{1}_{6\times 1} \tag{68}$$

Which can be simplified to

$$\bar{\mu} = \mu_d + \mathbf{1}_{6\times 1}\left(\frac{1}{7}p_w - \mu_d\right) \tag{69}$$

and

$$\bar{\Sigma} = I_{6\times 6} - \mathbf{1}_{6\times 6}\,\frac{6}{7}\,\sigma_d^2. \tag{70}$$

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