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# **Approximating Fixed-Horizon Forecasts Using Fixed-Event Forecasts**

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#### **ABSTRACT**

Many forecast surveys ask their participants for fixed-event forecasts. As fixed-event forecasts have seasonal properties, users often employ an ad hoc approach to approximate fixed-horizon forecasts based on these fixed-event forecasts. We derive an optimal approximation for fixed-horizon forecasts by minimizing the mean-squared approximation error. Like the ad hoc approach, our approximation employs a weighted average of the fixed-event forecasts. The optimal weights tend to differ substantially from those of the ad hoc approach. In empirical applications, the gains from using optimal instead of ad hoc weights turn out to be sizeable. The approximation approach proposed can also be useful in other applications.

JEL Classification: C53, E37

#### 1 | Introduction

Survey data on expectations have become an important ingredient for many empirical analyses in economics and finance. Yet, for many analyses, the expectation data required often do not exactly match the available survey data. A frequently encountered problem is given by the need for fixed-horizon forecasts where surveys can only provide fixed-event forecasts. Fixed-event forecasts are forecasts made at different points in time which have the same target. For instance, if a forecast is made on a monthly basis during the year, but always predicts the annual inflation rate of the next year, it is a fixed-event forecast. A fixed-horizon forecast, in contrast, might target the monthly inflation rate in 12 months, for example.

Fixed-event forecasts have seasonal properties—the dependence of the forecast error's variance on the date of the survey—which hamper many types of empirical analyses. When confronted with this problem, users commonly resort to an ad hoc approach in order to approximate fixed-horizon forecasts from fixed-event

forecasts. This ad hoc approximation uses a weighted average of two fixed-event forecasts to approximate the unknown fixed-horizon forecast, and it has been employed in many studies, including Begg et al. (1998), Alesina et al. (2001), Smant (2002), Heppke-Falk and Hüfner (2004), Gerlach (2007), Kortelainen, Paloviita, and Viren (2011), Dovern, Fritsche, and Slacalek (2012), Siklos (2013), D'Agostino and Ehrmann (2014), de Haan, Hessel, and van den End (2014), Grimme, Henzel, and Wieland (2014), Lamla and Lein (2014), and Hubert (2015). However, concerns about the quality of the approximation sometimes give rise to the need to justify the use of the approach as in Dovern, Fritsche, and Slacalek (2012) and Rossi and Sekhposyan (2017), or to the refusal of its use as in Hubert (2014, 1392). Nowadays, the ad hoc approach is often attributed to Dovern, Fritsche, and Slacalek (2012), for instance in Miranda-Agrippino and Ricco (2021). While all studies mentioned so far use fixed-event forecasts for the current year and the next year, Chun (2011) and Dahlhaus and Sekhposyan (forthcoming) employ the ad hoc approach to approximate interest rate forecasts for fixed monthly horizons based

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on fixed-event forecasts for the current quarter and future quarters provided in the monthly surveys of Blue Chip Financial Forecasts.

In this work, we derive an optimal approximation for fixed-horizon forecasts using fixed-event forecasts by minimizing the mean-squared approximation error. Like the approximation based on the ad hoc approach, our approximation is constructed as a weighted average of the fixed-event forecasts, with easily computable weights. However, these optimal weights tend to differ substantially from those of the ad hoc approach. Moreover, they depend on several known properties of the fixed-event forecasts (e.g., whether these refer to growth rates of annual averages or to growth rates of end-of-current-year on end-of-previous-year values), while these properties are ignored in the ad hoc approach. The optimal weights also depend on an unknown covariance matrix but are found to be very robust with respect to misspecifications of that matrix. While our work focuses on the approximation of fixed-horizon forecasts by fixed-event forecasts, the proposed approximation method is very flexible. The forecast to be approximated can be any linear function of the underlying high-frequency variable. Likewise, the information employed by the approximation can be any linear function of the underlying high-frequency variable. In contrast to the ad hoc approach, the proposed approximation method can make use of more than two such functions.

It should be noted that the proposed approach is optimal in the sense of minimizing the mean-squared approximation error for the forecast of interest for a certain variable based on forecasts made at a certain point in time for that variable. Our approach does not make use of information contained in forecasts for other variables, or in previous or later forecasts. This could be accomplished employing state-space models as in Kozicki and Tinsley (2012), Aruoba (2020), or Clark, Ganics, and Mertens (2022). While state-space models can take additional information into account, they also require additional assumptions about the data-generating process, and their implementation tends to be more involved.

In the empirical applications, we approximate 1-year-ahead inflation and real GDP growth forecasts implied by forecasts for annual inflation and growth in the current and in the next year, which are made on a quarterly basis. We focus on forecast surveys for which 1-year-ahead forecasts are actually available to evaluate the performance of the approximations. In one application, we use forecasts from the US Survey of Professional Forecasters (SPF); in the other, we employ survey forecasts from Consensus Economics for 13 countries. It turns out that the approach based on optimal weights yields considerably lower mean-squared approximation errors than the ad hoc approach.

## 2 | Optimal Approximations

#### 2.1 | Growth Rates

Often, we are interested in the growth rate g of an economic variable. Such growth rates typically have a monthly, quarterly, or annual frequency, and they can be formulated not only with respect to the previous period but also with respect to more

distant past periods. To convey all this information, define the growth rate from period s - n to period s of the variable p by

$$g_{s,s-n}^{u|v} = \frac{p_s^{u|v} - p_{s-n}^{u|v}}{p_{s-n}^{u|v}} \tag{1}$$

with  $s=\ldots,-1,0,1,2,\ldots$  and n being a positive integer. The letters u and v have to do with the fact that we consider situations where the frequency of the variable of interest,  $g^{u|v}_{s,s-n}$ , does not necessarily correspond to the frequency of the data-generating process (DGP) considered, which can have a higher frequency. v denotes the frequency of the DGP, whereas v denotes the frequency of the variable of interest. For instance, we will denote the annual growth rate in year v with respect to year v 1 of a variable with a monthly DGP by v v with respect to year v 1 of a variable with a monthly DGP by v by v v 1. The DGP commonly describes the process for v v 1, that is, for the high-frequency growth rate in period v with respect to period v 1 with v 2. The high-frequency periods v 3 and the low-frequency-periods v 3 are related by

$$s = t/b \tag{2}$$

with  $t = \dots, -b, 0, b, 2b, \dots$ , where b is a positive integer. Hence, the value of b denotes the number of high-frequency periods within one low-frequency period, such that, for example, b = 12 if the low frequency (of the variable of interest) refers to years and the high frequency (of the underlying variable generated by the DGP) to months.

The frequencies we are going to consider in our empirical applications are annual (y), quarterly (q), and monthly (m). The values of n considered are 1, 4, and 12. For instance,  $g_{t,t-12}^{m|m|}$  refers to a monthly growth rate with respect to the same month of the previous year, the so-called monthly year-on-year (y-o-y) rate, whereas  $g_{s,s-4}^{q|q}$  and  $g_{s,s-4}^{q|m}$  both refer to the quarterly growth rate with respect to the same quarter of the previous year, the so-called quarterly y-o-y rate.

We will use the convention that t=1 denotes the first high-frequency period in low-frequency period 1, such that in the previous example, we would have that t=1 corresponds to January of year 1, t=2 corresponds to February of year 1, and so on. In the same example,  $p_1^{m|m}$  refers to the level of the variable in January of year 1,  $p_2^{q|m}$  refers to the level of the variable in the second quarter of year 1,  $p_1^{q|m}$  refers to the level of the variable in year 1, and  $p_2^{y|m}$  refers to the level of the variable in year 2. As we will explain below, the level of a variable in a low-frequency period is defined as the average or the sum over its b levels in the corresponding b high-frequency periods.<sup>3</sup>

The forecasts for growth rates we are going to consider are described by

$$\hat{g}_{s,s-n|\tau}^{u|v} = E_{\tau} \left[ g_{s,s-n}^{u|v} \right]$$

where  $E_{\tau}[ullet]$  denotes the expectation conditional on the information set in high-frequency period  $\tau$ .

A summary of the general notation can be found in Table 1, while Table 2 contains typical cases and examples.

The growth rates of low-frequency variables that are forecasted often refer to the change in the level of these variables between

| Variables  |   |  |  |  |  |  |  |  |  |
|--|---|--|--|--|--|--|--|--|--|
| $p_t^{v v}$  | Level of high-frequency variable  |  |  |  |  |  |  |  |  |
| $p_s^{u v} = c \times \sum_{t=b \times (s-1)+1}^{b \times s} p_t^{v v}$  | Level of low-frequency variable (see below for $c$ )                    |  |  |  |  |  |  |  |  |
| $p_s^{u v} = c \times \sum_{\substack{t=b \times (s-1)+1}}^{b \times s} p_t^{v v}$ $g_{s,s-n}^{u v} = \frac{p_s^{u v} - p_{s-n}^{u v}}{p_{s-n}^{u v}}$ | Growth rate of $p_s^{u v}$ with respect to period $s - n$               |  |  |  |  |  |  |  |  |
| $\hat{g}_{s,s-n \tau}^{u v} = E_{\tau} \left[ g_{s,s-n}^{u v} \right]$   | Forecast for $g_{s,s-n}^{u v}$ conditional on period $\tau$ information |  |  |  |  |  |  |  |  |

| Time indices and parameters |  |  |  |  |  |  |  |  |
|-----------------------------|--|--|--|--|--|--|--|--|
| t                           | Time index of high-frequency periods, where $t = 1$ is first |  |  |  |  |  |  |  |
|                             | high-frequency period in low-frequency period 1              |  |  |  |  |  |  |  |
| b                           | Number of high-frequency periods per low-frequency period    |  |  |  |  |  |  |  |
| S                           | = t/b, time index of low-frequency periods, considered for   |  |  |  |  |  |  |  |
|                             | $t = \ldots, -b, 0, b, 2b, \ldots$                           |  |  |  |  |  |  |  |
| n                           | Lag for growth rate calculation                              |  |  |  |  |  |  |  |
| c                           | A constant, either $c = 1$ or $c = 1/b$                      |  |  |  |  |  |  |  |

a certain low-frequency period and the following low-frequency period. Patton and Timmermann (2011) show that, usually, these growth rates can be well approximated by a linear function of high-frequency growth rates, namely, by

$$g_{s,s-1}^{u|v} \approx \sum_{\kappa=1}^{2b} \omega_{\kappa} g_{t-\kappa+1,t-\kappa}^{v|v}, \tag{3}$$

where as stated in Equation (2),  $t = b \times s$ , and the  $\omega_{\kappa}$  are weights that only depend on known quantities. If one is interested in growth rates that are concerned with the change between nonadjacent periods, that is, if n > 1, it is useful to employ the approximation

$$g_{s,s-n}^{u|v} \approx \sum_{i=1}^{n} g_{s-i+1,s-i}^{u|v}, \tag{4}$$

where the growth rates on the right-hand side can then be approximated by (3) if b>1. A simple case with b=1 is given by monthly y-o-y growth rates which can be calculated as  $g_{t,t-12}^{m|m} \approx \sum_{i=1}^{12} g_{t-i+1,t-i}^{m|m}$ , that is, by the sum of the 12 corresponding month-on-month growth rates.

In what follows, we assume that  $g_{t,t-1}^{v|v}$  is covariance stationary.

# 2.2 | The Case of Inflation Forecasts by Consensus Economics

#### 2.2.1 | Growth Rates

We are first going to focus on a specific example—inflation forecasts by Consensus Economics—in order to illustrate our approach for determining optimal approximations. In the survey, each month two growth rate forecasts are enquired for the annual inflation rate, one for the current and one for the next year. Since the same two low-frequency growth rates are predicted in each month of the year, these are fixed-event forecasts.

Assuming a monthly frequency of the DGP and identifying January of the current year by t = 1, one low-frequency growth rate to be predicted is given by the current-year annual inflation rate

High-frequency period of conditioning information set

$$g_{1,0}^{\nu|m} = \frac{p_1^{\nu|m} - p_0^{\nu|m}}{p_0^{\nu|m}} \tag{5}$$

while the other is given by the next-year annual inflation rate

$$g_{2,1}^{y|m} = \frac{p_2^{y|m} - p_1^{y|m}}{p_1^{y|m}} \tag{6}$$

with

$$p_s^{\nu|m} = \frac{1}{12} \sum_{t=12 \times (s-1)+1}^{12 \times s} p_t^{m|m}.$$
 (7)

If the growth rate of average (or sums of) levels of adjacent low-frequency periods is considered, the weights  $\omega_{\kappa}$  in (3) equal

$$\omega_{\kappa} = 1 - |\kappa - b|/b. \tag{8}$$

with b=12 in the case given here, and  $\kappa=1,2,\ldots,24$ . Hence,  $g_{1,0}^{y|m}$  and  $g_{2,1}^{y|m}$  can be approximated by a weighted sum of 24 monthly month-on-month growth rates, respectively.

# 2.2.2 | Approximating Fixed-Horizon Forecasts

Suppose a survey provides two fixed-event forecasts, one for the annual growth rate in the current year and one for the annual growth rate in the next year. Moreover, let the current year be the first low-frequency period, that is, the period s=1. Suppose further that one is interested in a fixed-horizon forecast for the monthly y-o-y growth rate  $g_{t+12,t}^{m|m}$  that the survey under study does not provide. Ignoring the conditioning information, let this forecast be denoted by  $\hat{g}_{t+12,t}^{m|m}$ . This is the case that was considered in the works by Dovern, Fritsche, and Slacalek (2012) and many

| Case 1   |   |  |  |  |  |  |  |  |
|--|---|--|--|--|--|--|--|--|
| $p_t^{m m}$  | Price index in month t  |  |  |  |  |  |  |  |
| $p_1^{y m} = \frac{1}{12} \times \sum_{t=1}^{12} p_t^{m m}$  | Annual average of monthly price indices in year 1                 |  |  |  |  |  |  |  |
| $p_0^{y m} = \frac{1}{12} \times \sum_{t=-11}^{0} p_t^{m m}$   | Annual average of monthly price indices in year 0                 |  |  |  |  |  |  |  |
| $p_0^{y m} = rac{1}{12} \times \sum_{t=1}^{t=1} P_t$ $p_0^{y m} = rac{1}{12} \times \sum_{t=-11}^{0} p_t^{m m}$ $g_{1,0}^{y m} = rac{p_1^{y m} - p_0^{y m}}{p_0^{y m}}$ | Inflation rate of annual average (briefly: annual inflation rate) |  |  |  |  |  |  |  |
| - 0  | in year 1   |  |  |  |  |  |  |  |
| $\hat{g}_{1,0 10}^{y m}$   | Forecast for annual inflation rate in year 1 conditional on       |  |  |  |  |  |  |  |
|  | information up to October of year 1                               |  |  |  |  |  |  |  |

|  | Case 2   |
|--|--|
| $p_t^{m m}$  | Price index in month t                                     |
| $p_2^{q m} = \frac{1}{3} \times \sum_{t=4}^{6} p_t^{m m}$  | Quarterly average of monthly price indices in quarter 2    |
| $p_{-2}^{q m} = \frac{1}{3} \times \sum_{t=-8}^{-6} p_t^{m m}$   | Quarterly average of monthly price indices in quarter -2   |
| $p_{2}^{q m} = \frac{1}{3} \times \sum_{t=-8}^{-6} p_{t}^{m m}$ $g_{2,-2}^{q m} = \frac{p_{2}^{q m} - p_{-2}^{q m}}{p_{-2}^{q m}}$ | Year-on-year inflation rate of quarterly average (briefly: |
| *  | quarterly y-o-y inflation rate) in quarter 2               |
| $\hat{g}_{2,-2 4}^{q m}$   | Forecast for quarterly y-o-y inflation rate in quarter 2   |
|  | conditional on information up to April in year 1           |

|   | Case 3   |
|---|--|
| $p_t^{q q}$   | Level of GDP in quarter t                                |
| $p_2^{y q} = \sum_{t=5}^8 p_t^{q q}$  | Annual sum of quarterly GDP levels in year 2             |
| $p_1^{y q} = \sum_{t=1}^4 p_t^{q q}$  | Annual sum of quarterly GDP levels in year 1             |
| $p_1^{y q} = \sum_{t=1}^4 p_t^{q q} \ g_{2,1}^{y q} = rac{p_2^{y q} - p_1^{y q}}{p_1^{y q}}$ | Growth rate of annual sum (briefly: annual growth rate)  |
|   | in year 2  |
| $\hat{g}_{2,1 -1}^{y q}$  | Forecast for annual growth rate in year 2 conditional on |
| , I   | information up to quarter 3 in year 0                    |

other studies mentioned in Section 1. Equation (4) implies that this high-frequency growth rate is approximately equal to the sum of twelve monthly month-on-month growth rates.

In order to approximate  $\hat{g}_{t+12,t}^{m|m}$ , the ad hoc approach in Dovern, Fritsche, and Slacalek (2012) uses the weighted average of both fixed-event forecasts given by

$$\hat{g}_{t+12,t}^{m|m} \approx w^{adhoc} \hat{g}_{1,0}^{y|m} + (1 - w^{adhoc}) \hat{g}_{2,1}^{y|m}$$
 (9)

with

$$w^{adhoc} = \frac{12 - t}{12},\tag{10}$$

for  $t=0,1,2,\ldots,12$ . The underlying idea is that  $\hat{g}_{t+12,t}^{m|m}$  is the sum of 12-t month-on-month growth rates from the current year and t month-on-month growth rates from the next year, which makes the weights (12-t)/12 for the current-year forecasts and t/12 for the next-year forecast seem to be natural choices. For example, if we are interested in the forecast for the December y-o-y inflation rate of the current year, that is, in  $\hat{g}_{1,0}^{m|m}$  would receive the weight 1 and the next-year forecast  $\hat{g}_{2,1}^{m|m}$  would receive the weight 1 and the forecast for the y-o-y inflation rate in December of the next year, that is, in  $\hat{g}_{24,12}^{m|m}$ .

the current-year forecast  $\hat{g}_{1,0}^{y|m}$  would receive the weight 0 and the next-year forecast  $\hat{g}_{2,1}^{y|m}$  the weight 1.

To the best of our knowledge, in the setting described, there has been no application of the ad hoc approach with  $n \neq 12$  (or with  $n \neq 4$  in a quarterly setting), as it is unclear how the weights should be chosen in this case. In order to harmonize the scales of the variables under study in the case  $n \neq 12$ , in what follows, we assume that the quantity to be approximated is given by  $(12/n)\hat{g}_{l+n,l}^{m|m}$ . The approximation is denoted by  $\tilde{g}_{l+n,l}^{m|m}$ , and thus, its unconditional expectation has the property

$$E\left[\tilde{g}_{t+n,t}^{m|m}\right] = E\left[(12/n)\hat{g}_{t+n,t}^{m|m}\right]. \tag{11}$$

For our example, this simply means that the approximation always refers to *annualized* inflation rates.

For the ease of exposition, in a first step, we now neglect the fact that we are dealing with forecasts, and simply ask the question of how the high-frequency growth rate  $(12/n)g_{t+n,t}^{m|m}$  can be approximated using the low-frequency growth rates  $g_{1,0}^{y|m}$  and  $g_{2,1}^{y|m}$ . We require the approximation to be unbiased and focus on the linear

function

$$\tilde{g}_{t+n,t}^{m|m} = w g_{1,0}^{y|m} + (1-w) g_{2,1}^{y|m},\tag{12}$$

where both coefficients have to sum to 1 given that  $\tilde{g}_{t+n,t}^{m|m}$  is an annualized inflation rate. For instance,  $\tilde{g}_{12,0}^{m|m} = wg_{1,0}^{y|m} + (1-w)g_{2,1}^{y|m}$  is the approximation of the y-o-y December inflation rate of the current year based on the current year's and the next year's annual inflation rate.

Our aim is the choice of the weight  $\boldsymbol{w}$  that minimizes the expected squared approximation error

$$\min_{w} E\left[\left(g_{t+n,t}^{m|m} - \tilde{g}_{t+n,t}^{m|m}\right)^{2}\right]. \tag{13}$$

To this end, define the  $(36 \times 1)$  vectors G,  $B_1$ ,  $B_2$  and  $A_n$ 

$$G = \begin{bmatrix} g_{24,23}^{m|m} \\ g_{23,22}^{m|m} \\ \vdots \\ g_{-11,-12}^{m|m} \end{bmatrix}, B_1 = \begin{bmatrix} \mathbf{0}_{12} \\ \omega_1 \\ \omega_2 \\ \vdots \\ \omega_{24} \end{bmatrix}, B_2 = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_{24} \\ \mathbf{0}_{12} \end{bmatrix}, A_n = \begin{bmatrix} \mathbf{0}_{24-i} \\ (12/n)\mathbf{1}_n \\ \mathbf{0}_{12+i-n} \end{bmatrix}$$
(14)

where  $\mathbf{0}_j$  denotes a  $j \times 1$  vector of zeros,  $\mathbf{1}_j$  denotes a  $j \times 1$  vector of ones, and we require  $24-i \geq 0$  and  $12+i-n \geq 0$ . The index i is related to the period of the high-frequency growth rate to be approximated. For instance, to approximate the y-o-y growth rate (implying n=12) in January of the next year, we would set i=13, because  $A'_{12}G$  then is equal to this growth rate. The term 12/n appears in  $A_n$  to achieve the property described in (11). The weights  $\omega_\kappa$  follow from Equation (8) with b=12.

The difference  $g_{t+n,t}^{m|m} - \tilde{g}_{t+n,t}^{m|m}$  is given by

$$g_{t+n,t}^{m|m} - \tilde{g}_{t+n,t}^{m|m} = A_n'G - \left(wB_1'G + (1-w)B_2'G\right)$$

$$= \underbrace{\left(A_n' - B_2' + w(B_2' - B_1')\right)}_{:=M}G$$

$$= (M + wN)G$$
(15)

and the quadratic expression to be minimized thus equals

$$E\left[\left(g_{t+n,t}^{m|m} - \tilde{g}_{t+n,t}^{m|m}\right)^{2}\right] = (M+wN)\underbrace{E\left[GG'\right]}_{:=\widetilde{\Omega}}(M+wN)'$$

$$= N\widetilde{\Omega}N'w^{2} + 2M\widetilde{\Omega}N'w + M\widetilde{\Omega}M'.$$
(16)

Obviously, the minimum of (16) is attained when w is set to

$$w^* = -M\widetilde{\Omega}N' / \left(N\widetilde{\Omega}N'\right). \tag{17}$$

The same result is obtained using

$$w^* = -M\Omega N' / (N\Omega N'), \tag{18}$$

with  $\Omega$  being the covariance matrix of G

$$\Omega = E[(G - E[G])(G' - E[G'])], \tag{19}$$

whereas  $\widetilde{\Omega}$  contains the noncentral second moments. In Section I of the supporting information, we briefly explain why both moment matrices,  $\Omega$  and  $\widetilde{\Omega}$ , yield identical results for  $w^*$ . Obviously, the weight  $w^*$  is scale-invariant since the covariance matrix appears in the numerator as well as in the denominator.

Note that  $w^*$  as defined in (18) was derived for the case where the vector G only contains realizations  $g_{t+1,t}^{m|m}$ . However, if G contains realizations and forecasts, this only affects  $\Omega$  (i.e., the covariance matrix of G), but not  $B_1$ ,  $B_2$ , and  $A_n$ , and the formula (18) continues to yield optimal weights. In order to illustrate this fact, assume that  $g_{t+1,t}^{m|m}$  is an independent and identically distributed (iid) random variable with expectation  $\mu$  and variance  $\sigma^2=1$ . Moreover, assume that the last known growth rate was observed in month  $\tau$  and that the forecaster produces optimal mean forecasts. The mean forecasts in this simple case are given by

$$\hat{g}_{\tau+h}^{m|m} = \mu \tag{20}$$

where h is a positive integer denoting the forecast horizon. For the ease of the following exposition concerning G, assume that  $-9 \le \tau \le 20$ . Then the vector G and its covariance matrix  $\Omega$  are given by

$$G = \begin{bmatrix} \hat{g}_{24,23|\tau}^{m|m} \\ \vdots \\ \hat{g}_{\tau+2,\tau+1|\tau}^{m|m} \\ \hat{g}_{\tau+1,\tau|\tau}^{m|m} \\ g_{\tau,\tau-1}^{m|m} \\ \vdots \\ g_{-11,-12}^{m|m} \end{bmatrix} = \begin{bmatrix} \mu \\ \vdots \\ \mu \\ \mu \\ g_{\tau,\tau-1}^{m|m} \\ \vdots \\ g_{-11,-12}^{m|m} \end{bmatrix}, \mathbf{\Omega} = \begin{bmatrix} \mathbf{0}_{24-\tau,24-\tau} & \mathbf{0}_{24-\tau,12+\tau} \\ \mathbf{0}_{12+\tau,24-\tau} & \mathbf{I}_{12+\tau} \end{bmatrix}, (21)$$

where  $\mathbf{0}_{j_1,j_2}$  denotes a  $j_1 \times j_2$  matrix of zeros and  $\mathbf{I}_j$  is the  $j \times j$  identity matrix. The matrix  $\mathbf{0}_{24-\tau,24-\tau}$  is the covariance matrix of the forecasts  $\hat{g}_{\tau+1,\tau|\tau}^{m|m}$ , ...,  $\hat{g}_{24,23|\tau}^{m|m}$ , and the matrix  $\mathbf{I}_{12+\tau}$  is the covariance matrix of the realizations  $g_{-11,-12}^{m|m}$ , ...,  $g_{\tau,\tau-1}^{m|m}$ . The covariances between forecasts and realizations are given by  $\mathbf{0}_{12+\tau,24-\tau}$  and its transpose. If  $g_{\tau+1,t}^{m|m}$  follows an AR(1)-process and forecasts are optimal, the resulting covariance matrix  $\Omega$  has the form described in Section II of the supporting information.

The general case of an optimal approximation of forecasts for variables with  $b \neq 12$  based on more than two available forecasts is presented in Section VI of the supporting information. Also, in the general case, the vector G can contain realizations, forecasts, or a mix of both.

Finally, note that other point forecasts than mean forecasts and other loss functions than the expected squared approximation error could also be minimized using simulations. In general, one would need to assume a DGP including the distribution of the error term and simulate the fixed-event forecasts and the fixed-horizon forecast of interest, for instance for a certain quantile of the distribution. Then, one could simply use a grid search over the set of weights and choose those that minimize the loss.

# 2.2.3 | An Example: Forecasting at the Beginning of the Year

In order to illustrate the differences between the optimal and the ad hoc approach, assume again that  $g_{t+1,t}^{m|m}$  is an iid random variable with expectation  $\mu$  and variance  $\sigma^2=1$ . Moreover, assume that the last known growth rate was observed in December of last year (i.e., in t=0), such that  $\tau=0$ . Optimal mean forecasts are thus given by

$$\hat{g}_{h\,h-1|0}^{m|m} = \mu \tag{22}$$

where h denotes the forecast horizon  $h=1,2,\ldots,24$ , that is, January of the current year until December of next year. Hence, the covariance matrix of  $G=\left[\mu\times\mathbf{1}_{24}'\;g_{0,-1}^{m|m}\;\ldots\;g_{-11,-12}^{m|m}\right]'$  equals

$$\Omega = \begin{bmatrix} \mathbf{0}_{24,24} & \mathbf{0}_{24,12} \\ \mathbf{0}_{12,24} & \mathbf{I}_{12} \end{bmatrix}, \tag{23}$$

which correspond to the vector G and matrix  $\Omega$  in (21) with  $\tau = 0$ .

Assume that we want to approximate the forecast for a monthly y-o-y inflation rate, so that n = 12. If we are interested in the inflation rate in December of the current year, that is, in  $\hat{g}_{12,0|0}^{m|m}$ , using  $A_{12} = \begin{bmatrix} \mathbf{0}'_{12} & \mathbf{1}'_{12} & \mathbf{0}'_{12} \end{bmatrix}'$  yields the optimal weight for the current-year forecast, which actually equals  $w^* = 0$ . This means that the optimal approach puts all the weight on the fixed-event forecast for the next year, whereas the ad hoc approach would put all the weight on the fixed-event forecast for the *current* year. It can easily be seen that here using the optimal weight actually implies an approximation error of 0, because  $\hat{g}_{12,0|0}^{m|m}=\hat{g}_{2,1|0}^{y|m}=$  $12\mu$ . In contrast to that, using the weights of the ad hoc approach and, thus, simply using the fixed-event forecast for the current year, will in general lead to an approximation error, because  $\hat{g}_{1,0|0}^{y|m}$  does not only depend on the forecasts  $\hat{g}_{h,h-1|0}^{m|m}=\mu$ , with  $h=1,2,\,\ldots\,,12,$  but also on the realized growth rates  $g_{1-l,-l}^{m|m}$  with  $l=1,2,\,\ldots\,,12.$  Thus,  $\hat{g}_{1,0|0}^{y|m}$  is "contaminated" by observed growth rates (i.e., realized shocks), whereas  $\hat{g}_{2,1|0}^{\gamma|m}$  only depends on forecasts (i.e., is not "contaminated" by realized shocks).

# 2.3 | A Step-by-Step Procedure to Determine Optimal Weights

In what follows, we describe step by step how to derive optimal weights to approximate a fixed-horizon forecast based on two fixed-event forecasts. The description is kept relatively general to cover a large range of potential applications, for instance with levels instead of growth rates like in Chun (2011).

- 1. Specify the fixed-horizon forecast of interest.
- 2. Determine the high frequency, that is, the number of high-frequency periods *b* within a low-frequency period. The low frequency is determined by the available fixed-event forecasts.

*Remark:* b should be at least as large as the number of forecasts produced in each low-frequency period. Ideally, b

- should be large, such that the conditioning information of a forecast can be captured precisely. Yet it is often convenient to choose b such that it corresponds to the frequency of the fixed-horizon forecast of interest (e.g., b = 12 if monthly inflation rates are to be approximated based on forecasts for annual inflation rates).
- 3. Determine the dimension of the vector G of the high-frequency variable necessary to cover the fixed-event forecasts and the fixed-horizon forecast considered. Determine the vectors  $B_1$  (associated with one of the fixed-event forecast),  $B_2$  (associated with the other fixed-event forecast) and  $A_n$  (associated with the fixed-horizon forecast).
- 4. Specify which elements of *G* are assumed to be known and which are assumed to be forecast.
- 5. Specify a univariate DGP for the high-frequency variable.
- 6. Determine the covariance matrix  $\Omega$  of G assuming optimal forecasts based on the DGP from step 5.
- 7. Apply formula (18) to obtain the optimal weight  $w^*$  for the fixed-event forecast associated with  $B_1$ . The weight for the fixed-event forecast associated with  $B_2$  is given by  $1 w^*$ . Note that these weights are not restricted to lie in the interval [0,1], such that one of them can be negative.
- 8. The sum of  $w^*$  times the fixed-event forecast associated with  $B_1$  and  $1 w^*$  times the fixed-event forecast associated with  $B_2$  yields the approximation of the fixed-horizon forecast.

The steps described are illustrated in Section VII of the supporting information concerning the application to the US SPF forecasts as carried out in Section 4.1.

### 3 | Properties of the Approximations

The quality of the ad hoc approximation and the optimal approximation will be studied for two different cases. Both are related to inflation and GDP growth forecasts from surveys, which are cases often encountered in the literature. The first case aims at investigating the properties of the approximations with respect to the persistence of the underlying DGP and the timing when forecasts are made within the year, that is, the forecast month. The second case studies the robustness of the optimal approximation if it relies on a misspecified covariance matrix  ${\bf \Omega}$ .

### 3.1 | Persistence and Forecast Month

Here, the high-frequency variable has a monthly frequency, and there are two fixed-event forecasts of annual growth rates, the current-year forecast and the next-year forecast, such that b=12. The current-year forecast and the next-year forecast are made from January to December in the current year, that is, in  $t=1,2,\ldots,12$ . Assuming that each monthly forecast is based on observations until the previous month, the fixed-event forecasts are given by  $\hat{g}_{1,0|\tau}^{y|m}$  and  $\hat{g}_{2,1|\tau}^{y|m}$  with  $\tau=t-1$ . We want to approximate the fixed-horizon forecasts for the monthly y-o-y inflation rate (implying n=12) that cover the first 12 months for which

no observations are available yet. Since in January  $g_{1,0}^{m|m}$  has to be forecast, whereas  $g_{0,-1}^{m|m}$  is known, the y-o-y inflation rate in December of the current year is the target of the fixed-horizon forecast in January, that is, in January the fixed-horizon forecast of interest is given by  $\hat{g}_{12,0|0}^{m|m}$ .

The vectors  $A_n$ ,  $B_1$ ,  $B_2$  and G required to implement the approximation are basically those given in (14). Concerning  $A_n$ , we have n=12, and we can directly use t instead of i in its definition, such that we have  $A_{12}=\begin{bmatrix} \mathbf{0}_{13-t} & \mathbf{1}_{12} & \mathbf{0}_{11+t} \end{bmatrix}'$ . For instance, for the forecast from February, we have t=2,  $g_{13,1}^{m|m}$  is the object of interest, and  $g_{13,1}^{m|m}=A'_{12}G=\begin{bmatrix} \mathbf{0}_{11} & \mathbf{1}_{12} & \mathbf{0}_{13} \end{bmatrix}G$ . Of course, G here contains realizations and forecasts, like in (21).

We assume that the DGP is a monthly AR(1)-process given by

$$g_{t,t-1}^{m|m} - \mu = \rho \left( g_{t-1,t-2}^{m|m} - \mu \right) + \varepsilon_t$$
 (24)

with  $\varepsilon_t$  *iid*  $\mathcal{N}\left(0, \sigma_{\varepsilon}^2\right)$ . Without loss of generality, we set  $\sigma_{\varepsilon}^2 = 1 - \rho^2$ , so that the variance of  $g_{t,t-1}^{m|m}$  equals

$$E\left[\left(g_{t,t-1}^{m|m} - \mu\right)^{2}\right] = 1.$$
 (25)

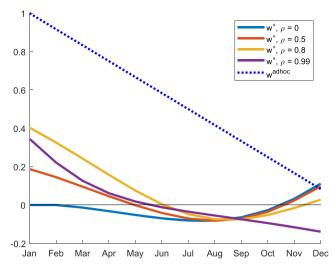
We also assume that, knowing the growth rate  $g_{t-1,t-2}^{m|m}$ , the forecaster makes optimal h-step-ahead forecasts

$$\hat{g}_{t+h-1,t+h-2|t-1}^{m|m} - \mu = \rho^h \left( g_{t-1,t-2}^{m|m} - \mu \right)$$
 (26)

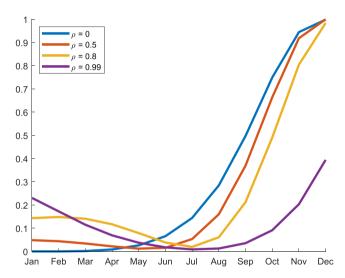
with  $h = 1, 2, \ldots$ . While these assumptions might appear simplistic, they allow us to study the importance of persistence, and they allow the straightforward calculation of the covariance matrix  $\Omega$ , which can be found in Section II of the supporting information.

The optimal weights resulting from this setting are displayed in Figure 1 along with the ad hoc weights.<sup>7</sup> As the target is given by  $\hat{g}_{t+11,t-1|t-1}^{m|m}$ , the ad hoc weights equal  $w^{adhoc} = \frac{12-(t-1)}{12}$ . Obviously, the ad hoc weights  $w^{adhoc}$  for the fixed-event forecast  $\hat{g}_{1,0|0}^{y|m}$ are almost always larger than the optimal weights  $w^*$ . Only at the end of the current year, both weighting schemes can deliver similar values, and  $w^*$  can become marginally larger than the ad hoc weight. Interestingly,  $w^*$  can become negative, implying that the optimal weight for the fixed-event forecast  $\hat{g}_{2.1|0}^{y|m}$  can become larger than 1. Basically, using a negative instead of a zero weight for the current-year forecast can improve the approximation, if the negative weight strongly reduces the approximation error due to realized shocks affecting the current-year and next-year forecasts while introducing only a small approximation error due to realized shocks which affect the current-year forecast only. A detailed explanation of this issue can be found in Section IV of the supporting information.

The expected squared approximation error of the different weights can be calculated employing (16). We define the ratio of the expected squared approximation error with optimal weights  $w^*$  to the expected squared approximation error with ad hoc



**FIGURE 1** | Optimal weights  $w^*$  depending on  $\rho$ , and ad hoc weights  $w^{adhoc}$  for current-year forecasts  $\hat{g}_{1,0|t-1}^{y|m}$ . Target is the fixed-horizon forecast  $\hat{g}_{t+1,t-1|t-1}^{m|m}$ . t ranges from t=1 for forecasts made in January to t=12 for forecasts made in December.



**FIGURE 2** | Ratios  $\phi_t$  of expected squared approximation error with optimal weights to expected squared approximation error with ad hoc weights for different values of  $\rho$ . Weights used and further details are provided in Figure 1.

weights wadhoc as

$$\phi_{t} = \frac{E\left[\left(\hat{g}_{t+11,t-1|t-1}^{m|m} - \tilde{g}_{t+11,t-1|t-1}^{m|m}\right)^{2} \middle| w = w^{*}\right]}{E\left[\left(\hat{g}_{t+11,t-1|t-1}^{m|m} - \tilde{g}_{t+11,t-1|t-1}^{m|m}\right)^{2} \middle| w = w^{adhoc}\right]}.$$
 (27)

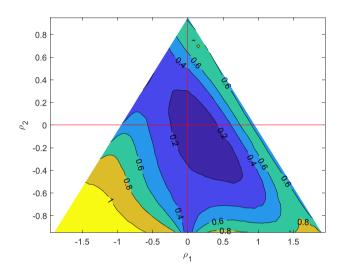
The corresponding values of  $\phi_t$  are displayed in Figure 2 for  $t = 1, 2, \ldots, 12$ , that is, for the forecasts from January to December, for alternative values of the AR(1) coefficient  $\rho$ .<sup>8</sup> It turns out that the mean-squared approximation errors can be reduced substantially by using optimal weights instead of ad hoc weights, especially from the beginning until the middle of the year. At the end of the year, the gains from employing optimal weights are less pronounced, unless the DGP is extremely persistent. The values

of  $\phi_t$  obtained with  $\rho = 0$  are similar to those with  $\rho = 0.5$  and not very different from those with  $\rho = 0.8$ , suggesting that the impact of the DGP's persistence on  $\phi_t$  is small unless the persistence is very strong.

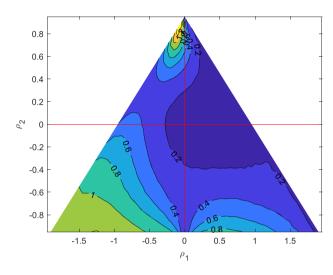
Thus, the question might arise which value of  $\rho$  could be adequate for the inflation example, and it should be noted that an empirically observed large persistence of the monthly y-o-y inflation rate  $g_{t,t-12}^{m|m}$  is not very informative about  $\rho$ . For example, given the DGP from (23) for  $g_{t,t-1}^{m|m}$ , in a regression of  $g_{t,t-12}^{m|m}$  on a constant and  $g_{t-1,t-13}^{m|m}$ , the AR(1)-coefficient would converge to 0.917 with  $\rho=0$ , to 0.969 with  $\rho=0.5$ , and to 0.987 with  $\rho=0.8$ .

# 3.2 | Robustness With Misspecified Covariance Matrix

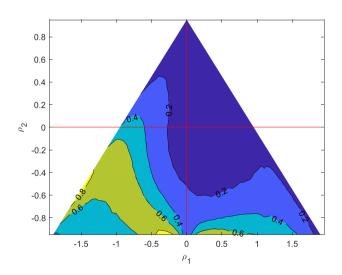
In what follows, we run a simulation exercise to inspect how robust the proposed approach is to misspecifications of the covariance matrix  $\Omega$ . Again, we consider two fixed-event forecasts for annual growth rates, but the high frequency here is the quarterly frequency, such that b=4 (instead of b=12 in the previous case of a monthly frequency). Hence, a current-year and a next-year forecast are made in each quarter t=1,2,3,4 where t=1 corresponds to the first quarter of the current year. Assuming that each quarterly forecast is based on observations until the previous quarter, this corresponds to the current-year forecasts



(a) assuming  $\rho_1 = \rho_2 = 0$ 



(b) assuming AR(1)-process, 
$$\rho = \frac{\rho_1}{1-\rho_2}$$



(c) using correct AR(2)-coefficients

**FIGURE 3** | Isolines show the ratio  $\overline{\phi}$  of the average expected squared approximation error with optimal weights  $w^*$  to the average expected squared approximation error with ad hoc weights  $w^{adhoc}$  for different values of the AR(2)-coefficients  $\rho_1$  and  $\rho_2$ . Values below 1 imply that using optimal weights  $w^*$  yields smaller errors. In panel (a), the covariance matrix  $\Omega$  is based on the assumption of an iid process. In panel (b),  $\Omega$  is based on the assumption of an AR(1)-process with AR-coefficient  $\rho = \rho_1/(1-\rho_2)$ . In panel (c), the correct coefficients  $\rho_1$  and  $\rho_2$  are used to construct  $\Omega$ . Results are based on 5000 simulations for each combination of  $\rho_1$  and  $\rho_2$ .  $\rho_1$  comes from the set  $\{-1.95, -1.90, -1.85, \dots, 1.95\}$  and  $\rho_2$  comes from the set  $\{-0.95, -0.90, -0.85, \dots, 0.95\}$ . We only consider stationary processes.

 $\hat{g}_{1,0|l-1}^{y|q}$  and the next-year forecasts  $\hat{g}_{2,1|l-1}^{y|q}$ . We want to approximate the fixed-horizon forecasts for the quarterly y-o-y growth rates (implying n=4) that cover the first 4 quarters for which no observations are available yet. For instance, in the first quarter,  $g_{1,0}^{q|q}$  has to be forecast, whereas  $g_{0,-1}^{q|q}$  is known, such that the fixed-horizon forecast of interest is given by  $\hat{g}_{4,0|0}$ , that is, by the y-o-y growth rate in the fourth quarter of the current year.

The definitions of  $A_n$ ,  $B_1$ ,  $B_2$  and G required for the approximation are given by quarterly versions of those given in (14), namely, by the (12 × 1) vectors G,  $B_1$ ,  $B_2$  and  $A_4$ 

$$G = \begin{bmatrix} \hat{g}_{8,7|t-1}^{q|q} \\ \vdots \\ \hat{g}_{t,t-1|t-1}^{q|q} \\ g_{t-1,t-2}^{q|q} \\ \vdots \\ g_{-3-4}^{q|q} \end{bmatrix}, B_1 = \begin{bmatrix} \mathbf{0}_4 \\ \omega_1 \\ \omega_2 \\ \vdots \\ \omega_8 \end{bmatrix}, B_2 = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_8 \\ \mathbf{0}_4 \end{bmatrix}, A_4 = \begin{bmatrix} \mathbf{0}_{5-t} \\ \mathbf{1}_4 \\ \mathbf{0}_{t+3} \end{bmatrix}$$
(28)

for t = 1, 2, 3, 4. The weights  $\omega_{\kappa}$  follow from Equation (8) with b = 4 and  $\kappa = 1, 2, \dots, 8$ .

For the first-quarter forecast (i.e., for t=1),  $g_{4,0}^{q|q}$  is the object of interest, such that  $A_4$  is given by  $A_4 = \begin{bmatrix} \mathbf{0}_4 & \mathbf{1}_4 & \mathbf{0}_4 \end{bmatrix}'$ , for the second-quarter forecast (i.e., for t=2), these are  $g_{5,1}^{q|q}$  and  $A_4 = \begin{bmatrix} \mathbf{0}_3 & \mathbf{1}_4 & \mathbf{0}_5 \end{bmatrix}'$ , and so on. We assume that the DGP is a quarterly AR(2)-process given by

$$g_{t,t-1}^{q|q} - \mu = \rho_1 \left( g_{t-1,t-2}^{q|q} - \mu \right) + \rho_2 \left( g_{t-2,t-3}^{q|q} - \mu \right) + \varepsilon_t, \tag{29}$$

with  $\varepsilon_t$  *iid*  $\mathcal{N}\left(0,\sigma_{\varepsilon}^2\right)$ . Again, we assume that the forecaster makes optimal h-step-ahead forecasts, having observed  $g_{t-1,t-2}^{q|q}$  and  $g_{t-2,t-3}^{q|q}$ .

However, we now study the situation where (29) is unknown when constructing the covariance matrix  $\Omega$  of G. In the first case considered, instead of using the correct matrix  $\Omega$ ,  $\Omega$  is based on the assumption of an *iid* process for  $g_{t+1,t}^{q|q}$ , that is, on assuming  $\rho_1 = \rho_2 = 0$ . In the second case considered,  $\Omega$  is based on the assumption of an AR(1)-process for  $g_{t,t-1}^{q|q}$  with the AR-coefficient  $\rho$  of this process given by  $\rho = \rho_1/(1-\rho_2)$ , which is equal to the asymptotic result of a regression of  $(g_{t,t-1}^{q|q} - \mu)$  on  $(g_{t,t-1}^{q|q} - \mu)$ .

We define the ratio of the average expected squared approximation error with optimal weights  $w^*$  to the average expected squared approximation error with ad hoc weights  $w^{adhoc}$  as

$$\overline{\phi} = \frac{\frac{1}{4} \sum_{t=1}^{4} \left( E\left[ \left( \hat{g}_{t+3,t-1|t-1}^{q|q} - \tilde{g}_{t+3,t-1|t-1}^{q|q} \right)^{2} | w = w^{*} \right] \right)}{\frac{1}{4} \sum_{t=1}^{4} \left( E\left[ \left( \hat{g}_{t+3,t-1|t-1}^{q|q} - \tilde{g}_{t+3,t-1|t-1}^{q|q} \right)^{2} \middle| w = w^{adhoc} \right] \right),$$
(30)

where "average" refers to averaging over the four different forecast origins, that is, over the four different quarters. Since the target is given by  $\hat{g}_{l+3,l-1|l-1}$ , the ad hoc weights for the current-year

forecasts are given by  $w^{adhoc} = \frac{4-(t-1)}{4}$ . For instance, for the forecast made in the first quarter, we have t=1. The growth rate of interest is  $\hat{g}_{4,0|0}$ . This growth rate only depends on quarterly growth rates in the current year, such that  $w^{adhoc} = 1$ .

Figure 3 shows the values of  $\overline{\phi}$  for the two misspecified cases and the correctly specified case. Values larger than 1 imply that the ad hoc approach has a smaller approximation error. If  $\rho_1 = \rho_2 = 0$  is assumed, panel (a) shows that the ad hoc approach can give better results only if  $\rho_1$  and  $\rho_2$  are both very small. The underlying AR(2)-process exhibits a persistent zigzagging evolution in this case, which is rather uncommon for economic time series. If an AR(1)-process with  $\rho = \frac{\rho_1}{1-\rho_2}$  is assumed, but  $\rho_1$  and  $\rho_2$  are very small, panel (b) shows that the ad hoc approach can again give better results. Moreover, the same can happen if  $\rho_2$  is very large and  $\rho_1$  is slightly smaller than zero. In both cases,  $\rho$  would become very small, potentially being close to -1. If  $\rho_1$  and hence  $\rho$  is positive, the AR(1)-assumption and optimal weights tend to result in better approximations than the assumption of  $\rho_1 = \rho_2 = 0$ . Of course, with a correctly specified covariance matrix, the optimal approach always gives better approximations than the ad hoc approach, as shown in panel (c).

In summary, the results suggest that using optimal weights  $w^*$  assuming an iid process yields smaller approximation errors than using ad hoc weights for a very large part of the parameter

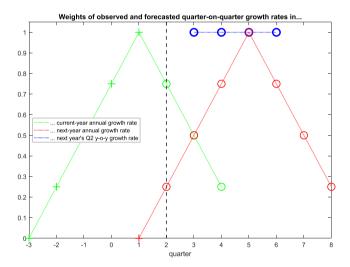


FIGURE 4 | Illustration of setting with US SPF forecasts from May. The dashed vertical line denotes the quarter when the forecast is made. The markers denote the weights of the quarter-on-quarter growth rates used in the calculation of the three growth rates mentioned in the legend. Plus signs ("+") denote weights of observed quarter-on-quarter growth rates, while circles ("o") denote weights of quarter-on-quarter growth rates that have to be forecasted. The weights  $ω_k$  for the annual growth rates  $g_{1,0}^{y|q}$  and  $g_{2,1}^{y|q}$  follow from (3) and (8) with b=4 and  $κ=1,2,\ldots,8$ . The weights for the year-on-year growth rate in the second quarter of the next year  $g_{6,2}^{a|q}$  simply equal 1 and follow from (4). In May, all three growth rates are not yet observed, because at least some of their underlying quarter-on-quarter growth rates have to be forecasted. We aim to approximate the forecast for  $g_{6,2}^{q|q}$ , that is,  $\hat{g}_{6,2|1}^{q|q}$ , based on the forecasts for  $g_{1,0}^{y|q}$  and  $g_{2,1}^{y|q}$ , that is, based on  $\hat{g}_{1,0|1}^{y|q}$  and  $\hat{g}_{2,1|1}^{y|q}$ . The same description applies to the SPF inflation forecasts.

space, including the space relevant for typical economic series. If enough data is available to estimate the AR(1)-coefficient  $\rho$ , using this value can result in even smaller approximation errors. If  $\rho$  turns out to be negative and small, it might be better to set  $\rho=0$ . Of course, ideally, one would fit an appropriate AR(k)-process and determine the respective optimal weights to ensure a better performance than with ad hoc weights.<sup>10</sup> Yet the following applications will show that simply assuming an *iid* process is enough to substantially reduce the approximation error for typical economic time series like inflation and GDP growth.

# 4 | Applications

We evaluate the methods developed employing two different forecast surveys. In both cases, we are going to derive optimal and ad hoc approximations for 1-year-ahead forecasts of quarterly y-o-y growth rates based on quarterly available current-year and next-year forecasts. This is a typical situation, which can be encountered in several of the studies mentioned in the Introduction. Apparently, current-year and next-year forecasts are very common forecasts, while fixed-horizon forecasts are not always available. However, in our applications 1-year-ahead forecasts of quarterly y-o-y growth rates are available, such that we can compare them to the approximations and demonstrate the markedly better performance of the optimal weights proposed.

The first application uses the inflation and growth forecasts from the US Survey of Professional Forecasters (SPF) run by the Federal Reserve Bank of Philadelphia. The SPF provides relatively long time series of inflation and growth forecasts for the current and the next year. Since the sample with these forecasts starts in 1981, it covers at least some quarters before the onset of the Great Moderation (see McConnell and Perez-Quiros 2000). The second application employs Consensus forecasts for inflation and growth. While these forecast samples start later, they cover 13 countries including the US. By using both surveys, we can hope to find results that are robust to changes of the sample concerning time and location.

# 4.1 | US SPF

The US SPF forecasts are a collection of individual forecasts. Every quarter, all panelists surveyed are asked for their forecasts of the levels of the GDP deflator and real GDP in the current and in the next year, as well as for their forecasts of the quarterly levels of these variables from the preceding quarter until four quarters ahead. We focus on the mean of these forecasts across all panelists. We merge these mean forecasts with data from the Real-Time Data Set provided by the Federal Reserve Bank of Philadelphia, enabling us to calculate all growth rates of interest. These are the current-year and next-year growth rate and the 4-quarter-ahead y-o-y growth rate. 12 We choose to approximate the forecasts for the latter growth rate among all available ones, because a 1-year horizon is a common choice in macroeconomic forecasting. The surveys are conducted in the middle of February, May, August, and November, such that the previous quarter's value is known for both variables. While we could assume an underlying monthly process for each variable, we choose the quarterly frequency as the high frequency here. This makes parameter estimation straightforward.<sup>13</sup> Hence, the fixed-horizon forecasts we consider are given by  $\hat{g}_{l+4,t|t-1}^{q|q}$ , where tdenotes the quarter when the forecast is made, and t-1 is the last quarter for which an observation is available. This means that, for instance, we approximate the forecast for the y-o-y growth rate in the second quarter of next year based on the fixed-event forecasts made in May. Figure 4 illustrates this setting. Details on how to determine the optimal approximation weights based on the step-by-step procedure outlined in Section 2.3 can be found in Section VII of the supporting information. This section also contains figures illustrating the settings with forecasts made in February, August, and November.

Our sample starts in 1981Q3 and ends in 2023Q2, and the data for 1985Q1, 1986Q1, and 1990Q1 are excluded from the sample, because the annual forecasts then referred to the previous and current year.<sup>14</sup>

As we approximate the 4-quarter-ahead forecast for the y-o-y growth rate given by  $\hat{g}_{t+4,t|t-1}^{q|q}$  for t=1,2,3,4, the ad hoc weights for the current-year forecast are given by  $w^{adhoc}=\frac{4-t}{4}$ , that is,

**TABLE 3** | Statistical properties of US SPF forecasts.

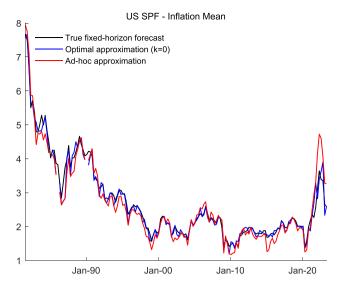
|           | Mean |      | std  | .dev. |       | Mea      | Mean |       | lev. |       |
|-----------|------|------|------|-------|-------|----------|------|-------|------|-------|
| Sample    | cur. | next | cur. | next  | corr. | adhoc    | opt. | adhoc | opt. | corr. |
|           |      |      |      |       | Ir    | nflation |      |       |      |       |
| Full      | 2.63 | 2.68 | 1.45 | 1.19  | 0.91  | 2.64     | 2.69 | 1.24  | 1.19 | 0.98  |
| Pre-COVID | 2.55 | 2.70 | 1.37 | 1.22  | 0.95  | 2.62     | 2.70 | 1.25  | 1.22 | 0.99  |
| Consensus | 2.06 | 2.19 | 0.74 | 0.56  | 0.88  | 2.12     | 2.20 | 0.60  | 0.56 | 0.97  |
|           |      |      |      |       | GD    | P growth |      |       |      |       |
| Full      | 2.43 | 2.69 | 1.83 | 0.74  | 0.28  | 2.58     | 2.70 | 1.01  | 0.76 | 0.69  |
| Pre-COVID | 2.50 | 2.69 | 1.52 | 0.69  | 0.33  | 2.58     | 2.69 | 0.89  | 0.70 | 0.70  |
| Consensus | 2.42 | 2.67 | 1.37 | 0.62  | 0.42  | 2.56     | 2.68 | 0.84  | 0.63 | 0.74  |

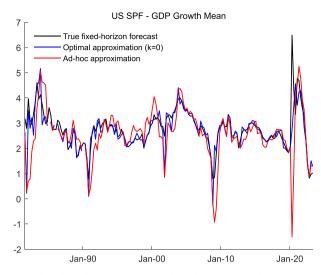
*Note:* The full sample lasts from 1981Q3 to 2023Q2, the pre-COVID sample from 1981Q3 to 2019Q4, and the Consensus sample from 1989Q1 to 2019Q2. "cur." denotes the current-year forecasts, "next" denotes the next-year forecasts, "opt." denotes the optimal approximations of the 4-quarter-ahead forecasts for the y-o-y growth rate determined for k = 0, "adhoc" denotes the corresponding ad hoc approximations. "std.dev." is the standard deviation, "corr." is the correlation between current-year and next-year forecasts, and between the optimal and ad hoc approximations, respectively.

 TABLE 4
 MSE ratios for US SPF mean forecasts.

|       |      |      | Inflation |      |                 |                  | (     | GDP growt | h    |      |
|-------|------|------|-----------|------|-----------------|------------------|-------|-----------|------|------|
|       | All  | Q1   | Q2        | Q3   | Q4              | All              | Q1    | Q2        | Q3   | Q4   |
|       |      |      |           |      | Full sample (19 | 981Q3 to 2023Q2  | 2)    |           |      |      |
|       |      |      |           |      | Ex-ante         | approach         |       |           |      |      |
| k = 0 | 0.34 | 0.35 | 0.10      | 0.19 | 0.86            | 0.26             | 0.20  | 0.18      | 0.09 | 1.08 |
| k = 1 | 0.32 | 0.28 | 0.08      | 0.19 | 0.88            | 0.25             | 0.15  | 0.18      | 0.09 | 1.07 |
| k = 2 | 0.33 | 0.29 | 0.09      | 0.20 | 0.89            | 0.25             | 0.14  | 0.18      | 0.09 | 1.07 |
| k = 4 | 0.35 | 0.30 | 0.09      | 0.24 | 0.94            | 0.25             | 0.13  | 0.19      | 0.09 | 1.08 |
|       |      |      |           |      | Ex-post         | approach         |       |           |      |      |
| k = 1 | 0.32 | 0.28 | 0.08      | 0.19 | 0.88            | 0.26             | 0.20  | 0.17      | 0.09 | 1.08 |
| k = 2 | 0.33 | 0.30 | 0.09      | 0.19 | 0.88            | 0.26             | 0.16  | 0.18      | 0.09 | 1.10 |
| k = 4 | 0.36 | 0.30 | 0.10      | 0.23 | 0.95            | 0.26             | 0.15  | 0.19      | 0.09 | 1.13 |
|       |      |      |           |      | Infeasible      | approaches       |       |           |      |      |
| k = 1 | 0.31 | 0.25 | 0.09      | 0.18 | 0.87            | 0.25             | 0.15  | 0.17      | 0.08 | 1.08 |
| unr.  | 0.28 | 0.25 | 0.08      | 0.18 | 0.70            | 0.19             | 0.13  | 0.10      | 0.08 | 0.99 |
|       |      |      |           | Pre  | -COVID sample   | e (1981Q3 to 201 | .9Q4) |           |      |      |
|       |      |      |           |      | Ex-ante         | approach         |       |           |      |      |
| k = 0 | 0.45 | 0.44 | 0.15      | 0.32 | 0.93            | 0.28             | 0.14  | 0.02      | 0.15 | 1.02 |
| k = 1 | 0.43 | 0.38 | 0.14      | 0.33 | 0.94            | 0.27             | 0.09  | 0.02      | 0.17 | 1.02 |
| k = 2 | 0.43 | 0.40 | 0.15      | 0.33 | 0.94            | 0.27             | 0.08  | 0.02      | 0.17 | 1.02 |
| k = 4 | 0.44 | 0.41 | 0.15      | 0.33 | 0.95            | 0.27             | 0.08  | 0.02      | 0.17 | 1.03 |
|       |      |      |           |      | Ex-post         | approach         |       |           |      |      |
| k = 1 | 0.43 | 0.39 | 0.14      | 0.32 | 0.94            | 0.27             | 0.09  | 0.02      | 0.17 | 1.02 |
| k = 2 | 0.44 | 0.41 | 0.15      | 0.32 | 0.94            | 0.27             | 0.09  | 0.02      | 0.17 | 1.02 |
| k = 4 | 0.46 | 0.41 | 0.16      | 0.33 | 0.99            | 0.27             | 0.09  | 0.02      | 0.17 | 1.03 |
|       |      |      |           |      |                 | approaches       |       |           |      |      |
| k = 1 | 0.42 | 0.37 | 0.14      | 0.34 | 0.93            | 0.27             | 0.09  | 0.02      | 0.17 | 1.02 |
| unr.  | 0.41 | 0.37 | 0.14      | 0.32 | 0.89            | 0.26             | 0.09  | 0.02      | 0.15 | 1.00 |
|       |      |      |           | Co   | _               | e (1989Q1 to 201 | 9Q2)  |           |      |      |
|       |      |      |           |      |                 | approach         |       |           |      |      |
| k = 0 | 0.35 | 0.18 | 0.20      | 0.22 | 0.94            | 0.26             | 0.21  | 0.02      | 0.08 | 1.02 |
| k = 1 | 0.32 | 0.10 | 0.19      | 0.22 | 0.95            | 0.22             | 0.09  | 0.01      | 0.09 | 1.02 |
| k = 2 | 0.33 | 0.13 | 0.20      | 0.22 | 0.95            | 0.21             | 0.07  | 0.01      | 0.09 | 1.02 |
| k = 4 | 0.35 | 0.14 | 0.21      | 0.27 | 0.99            | 0.21             | 0.06  | 0.01      | 0.09 | 1.03 |
|       |      |      |           |      | _               | approach         |       |           |      |      |
| k = 1 | 0.32 | 0.11 | 0.19      | 0.22 | 0.95            | 0.22             | 0.09  | 0.01      | 0.09 | 1.01 |
| k = 2 | 0.33 | 0.14 | 0.20      | 0.22 | 0.95            | 0.21             | 0.08  | 0.01      | 0.09 | 1.02 |
| k = 4 | 0.36 | 0.14 | 0.21      | 0.27 | 0.99            | 0.21             | 0.07  | 0.01      | 0.09 | 1.02 |
| , .   |      |      |           |      |                 | approaches       |       |           |      |      |
| k = 1 | 0.31 | 0.07 | 0.19      | 0.21 | 0.95            | 0.20             | 0.05  | 0.02      | 0.09 | 1.01 |
| unr.  | 0.31 | 0.07 | 0.19      | 0.21 | 0.93            | 0.20             | 0.05  | 0.01      | 0.08 | 1.00 |

Note: The MSE ratios refer to the ratio of the mean-squared approximation error obtained with optimal weights to the mean-squared approximation error obtained with ad hoc weights. "All" is the MSE ratio  $\overline{\phi}$  defined in (31), "Q1" to "Q4" refer to quarter-specific MSE ratios. "Inflation" is GDP deflator growth, "unr." denotes the infeasible unrestricted approach.





**FIGURE 5** | True 4-quarter-ahead US SPF mean forecasts, optimal approximation based on the ex-ante approach with k = 0 and ad hoc approximation for inflation (left panel) and GDP growth (right panel).

by 0.75 (February), 0.5 (May), 0.25 (August), and 0 (November). Concerning the optimal weights, we consider different approaches of specifying the underlying quarterly DGP:

- *Ex-ante approach*: We assume that the quarter-on-quarter (q-o-q) growth rates follow an AR(k)-process with k=0,1,2,4 whose coefficients are estimated recursively, that is, based on the longest time series available at the time of each forecast. Note that no estimation is required if k=0. The results from the ex-ante approach could have been obtained in real-time, because no information beyond what was available at the time of each forecast is employed.
- *Ex-post approach*: We assume that the q-o-q growth rates follow an AR(k)-process with k = 1,2,4 whose coefficients are estimated based on the time series available at the end of the sample. We do not consider k = 0 because, of course, the results would be identical to those of the ex-ante approach.
- *Infeasible approach*, k=1: We assume that the q-o-q growth rates follow an AR(1)-process and that the 4-quarter-ahead forecasts are known. We choose the AR-coefficient  $\rho$  to minimize the mean squared approximation error. We determine this value of  $\rho$  by using a grid search over the interval [-0.99; 0.99].
- Infeasible approach, unrestricted: We assume that the 4-quarter-ahead forecasts are known. For each of the four different quarters, we run a constrained OLS regression of the 4-quarter-ahead forecasts on the current-year and next-year forecasts, where the coefficients are required to sum to 1. Thus, in contrast to the previous AR(k)-process-based approaches, there is no "dynamic restriction" linking the estimation results for the four quarters among each other. Therefore, this approach delivers the weights (i.e., the coefficients for the current-year and next-year forecast, summing to 1), which minimize the mean squared approximation error for each of the four quarters and, hence, for all quarters jointly by construction.  $^{15}$

The latter two approaches are labeled "infeasible" because, in practice, one would not employ an approximation if the fixed-horizon forecast of interest is observable.

In the light of the insights of Clements and Galvão (2013), we use second-vintage time series of GDP deflator growth (henceforth inflation) and real GDP growth (henceforth GDP growth) to estimate the AR-coefficients. These series start in 1965Q2. We consider three different samples to determine and apply the weights, namely, the full sample from 1981Q3 to 2023Q2, the sample before the start of the pandemic, which covers the time span from 1981Q3 to 2019Q4, and a sample corresponding to the Consensus sample for the United States, which lasts from 1989Q1 to 2019Q2.

For these three different samples, Table 3 shows some properties of the current-year forecasts, the next-year forecasts, the approximated fixed-horizon forecasts with ad hoc weights, and the corresponding forecasts with optimal weights determined for the ex-ante approach with k = 0. For inflation and GDP growth, the next-year forecasts have a marginally larger mean than the current-year forecasts. As longer-horizon forecasts tend to predict values close to the unconditional mean, this suggests that forecasters tended to overestimate the unconditional mean of both variables. The mean of the inflation forecasts clearly depends on the sample, because forecasted inflation was very high at the beginning of the 1980s. The standard deviation is always smaller for next-year forecasts than for current-year forecasts, which can again be explained by predicted values close to the unconditional mean for longer horizons.<sup>16</sup> The correlation between current-year and next-year forecasts is considerably larger for inflation than for GDP growth. This high correlation can partly be regarded as spurious because it is affected by the very large inflation forecasts at the beginning of the 1980s. However, even if this high-inflation regime is excluded, as done in the Consensus sample, the correlation still attains almost 0.9.

The approximated fixed-horizon forecasts inherit their properties from the current-year and next-year forecasts. As seen in

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 TABLE 5
 | Current-year optimal weights.

|       |        | Infla  | ition  |                  |                   | rowth  |        |        |
|-------|--------|--------|--------|------------------|-------------------|--------|--------|--------|
|       | Q1     | Q2     | Q3     | Q4               | Q1                | Q2     | Q3     | Q4     |
|       |        |        |        | Ad hoc           | approach          |        |        |        |
|       | 0.75   | 0.50   | 0.25   | 0.00             | 0.75              | 0.50   | 0.25   | 0.00   |
|       |        |        |        | Ex-ante          | approach          |        |        |        |
| k = 0 | 0.00   | 0.00   | -0.06  | -0.06            | 0.00              | 0.00   | -0.06  | -0.06  |
|       |        |        |        | Full sample (19  | 81Q3 to 2023Q2)   |        |        |        |
|       |        |        |        | Ex-ante          | approach          |        |        |        |
| k = 1 | 0.29   | 0.08   | -0.08  | -0.05            | 0.10              | 0.00   | -0.10  | -0.06  |
| k = 2 | 0.32   | 0.09   | -0.07  | -0.05            | 0.13              | 0.01   | -0.10  | -0.06  |
| k = 4 | 0.34   | 0.11   | -0.02  | -0.03            | 0.14              | 0.02   | -0.10  | -0.07  |
|       |        |        |        | Ex-post          | approach          |        |        |        |
| k = 1 | 0.30   | 0.08   | -0.07  | -0.05            | 0.00              | -0.01  | -0.07  | -0.06  |
| k = 2 | 0.33   | 0.09   | -0.06  | -0.05            | 0.05              | 0.01   | -0.08  | -0.07  |
| k = 4 | 0.34   | 0.11   | -0.02  | -0.02            | 0.08              | 0.02   | -0.07  | -0.08  |
|       |        |        |        | Infeasible       | approaches        |        |        |        |
| k = 1 | 0.20   | 0.04   | -0.10  | -0.05            | 0.07              | -0.01  | -0.10  | -0.06  |
| unr.  | 0.20   | 0.06   | -0.11  | -0.22            | 0.16              | -0.20  | -0.09  | 0.03   |
|       | (0.01) | (0.02) | (0.04) | (0.04)           | (0.08)            | (0.11) | (0.03) | (0.03) |
|       |        |        |        | Pre-COVID sample | (1981Q3 to 2019Q  | 4)     |        |        |
|       |        |        |        | Ex-ante          | approach          |        |        |        |
| k = 1 | 0.29   | 0.07   | -0.08  | -0.05            | 0.11              | 0.00   | -0.10  | -0.06  |
| k = 2 | 0.32   | 0.09   | -0.07  | -0.05            | 0.13              | 0.01   | -0.10  | -0.06  |
| k = 4 | 0.34   | 0.11   | -0.02  | -0.03            | 0.14              | 0.02   | -0.10  | -0.07  |
|       |        |        |        | Ex-post          | approach          |        |        |        |
| k = 1 | 0.31   | 0.08   | -0.07  | -0.05            | 0.11              | 0.00   | -0.10  | -0.06  |
| k = 2 | 0.34   | 0.09   | -0.06  | -0.05            | 0.12              | 0.01   | -0.10  | -0.06  |
| k = 4 | 0.35   | 0.11   | 0.00   | -0.01            | 0.14              | 0.02   | -0.10  | -0.07  |
|       |        |        |        | Infeasible       | approaches        |        |        |        |
| k = 1 | 0.20   | 0.04   | -0.10  | -0.05            | 0.14              | 0.01   | -0.10  | -0.06  |
| unr.  | 0.19   | 0.04   | -0.04  | -0.15            | 0.14              | 0.00   | -0.05  | 0.00   |
|       | (0.02) | (0.03) | (0.05) | (0.06)           | (0.08)            | (0.02) | (0.02) | (0.03) |
|       |        |        |        | Consensus sample | (1989Q1 to 2019Q2 | 2)     |        |        |
|       |        |        |        | Ex-ante          | approach          |        |        |        |
| k = 1 | 0.30   | 0.08   | -0.07  | -0.05            | 0.11              | 0.00   | -0.10  | -0.06  |
| k = 2 | 0.33   | 0.09   | -0.06  | -0.05            | 0.13              | 0.01   | -0.10  | -0.06  |
| k = 4 | 0.34   | 0.11   | -0.02  | -0.02            | 0.14              | 0.02   | -0.10  | -0.07  |
|       |        |        |        | Ex-post          | approach          |        |        |        |
| k = 1 | 0.31   | 0.08   | -0.07  | -0.05            | 0.11              | 0.00   | -0.10  | -0.06  |
| k = 2 | 0.34   | 0.09   | -0.06  | -0.05            | 0.12              | 0.01   | -0.10  | -0.06  |
| k = 4 | 0.35   | 0.11   | 0.00   | -0.01            | 0.14              | 0.02   | -0.10  | -0.07  |
|       |        |        |        | Infeasible       | approaches        |        |        |        |
| k = 1 | 0.20   | 0.04   | -0.10  | -0.05            | 0.22              | 0.04   | -0.10  | -0.05  |
| unr.  | 0.19   | 0.05   | -0.10  | -0.11            | 0.22              | 0.02   | -0.06  | -0.02  |
|       | (0.02) | (0.02) | (0.03) | (0.09)           | (0.04)            | (0.01) | (0.01) | (0.04) |

*Note*: The weights shown for the ex-ante approach are averages of the recursively estimated weights except for k = 0. The latter case—like the ad hoc weights—does not require estimations. Numbers in parentheses are Newey-West (1987) standard errors with lag length chosen as proposed in Andrews (1991). For further details, see Table 4.

Figure 1 for the monthly example, for the first three quarters of the year, the ad hoc weights for the current year are much larger than the optimal weights. Therefore, in general, the ad hoc approximations are more strongly related to the current-year forecasts than the optimal approximations, while the latter tend to be very similar to the next-year forecasts. Consequently, as shown in Table 3, the optimal approximations (here determined based on the ex-ante approach with k = 0) have a lower standard deviation than the ad hoc approximations. The approximations are strongly correlated, with correlation coefficients being close to 1 for inflation and around 0.7 for GDP growth.

In analogy to Equation (30), the MSE ratio we consider is

$$\overline{\phi} = \frac{\frac{1}{T} \sum_{t} \left[ \left( \hat{g}_{t+4,t|t-1}^{q|q} - \tilde{g}_{t+4,t|t-1}^{q|q} \right)^{2} | w = w^{*} \right]}{\frac{1}{T} \sum_{t} \left[ \left( \hat{g}_{t+4,t|t-1}^{q|q} - \tilde{g}_{t+4,t|t-1}^{q|q} \right)^{2} | w = w^{adhoc} \right]},$$
(31)

with T denoting the sample size and with  $t = t_0, t_0 + 1, \ldots, T$ , where  $t_0$  is the first period in the sample. We also consider the four quarter-specific MSE ratios with  $t = t_Q, t_Q + 4, t_Q + 8, \ldots$  for  $t_Q = t_0, t_0 + 1, t_0 + 2$ , and  $t_0 + 3$ , respectively.

The values of  $\overline{\phi}$  in Table 4 show that optimal weights always yield substantially better approximations than ad hoc weights. For the three samples and both variables under study, the mean squared approximation error is at least halved if optimal weights are used instead of ad hoc weights. The smallest reduction of approximation errors occurs for inflation in the sample from 1981Q3 to 2019Q4, where the MSE ratios  $\overline{\phi}$  of the ex-ante and ex-post approach range between 0.43 and 0.46. In this sample, the correlation between the ad hoc and the optimal (with k = 0) approximations attains its largest value of 0.99, as reported in Table 3. Therefore, it is not too surprising that the MSE ratio reaches its largest value in this case. In the full sample and in the Consensus sample, the MSE ratios of both approaches vary between 0.32 and 0.36 for inflation. For GDP growth, the MSE ratios of the ex-ante and ex-post approach are between 0.21 and 0.28, that is, the mean squared approximation error is reduced by about three quarters if optimal weights are used instead of ad hoc weights. The smallest reductions are again observed in the sample from 1981Q3 to 2019Q4.

When looking at quarter-specific MSE ratios, the largest values occur for forecasts from the fourth quarter, because the optimal weights are close to the ad hoc weight of 0 (for the current-year forecast) in this case. The MSE ratios for the second quarter are often very small and attain values close to zero for GDP growth except for the full sample. Of course, 2020Q2 was the first quarter of the COVID-19 pandemic, and GDP forecasters faced exceptional challenges then, such that the Q2 results for GDP growth strongly depend on the inclusion of this quarter.

The infeasible unrestricted approach yields the smallest approximation errors by construction. Yet the approximations are only marginally better than those of the infeasible AR(1)-approach in most cases. Moreover, the MSE ratios of the unrestricted approach are at most 7 percentage points lower than the ratios obtained with the simplest (feasible) approach, namely, the

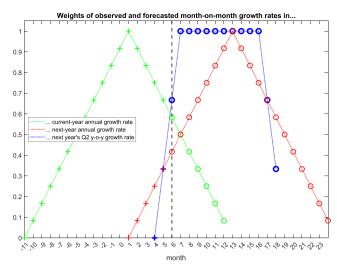


FIGURE 6 | Illustration of setting with Consensus forecasts from June. The dashed vertical line denotes the month when the forecast is made. The markers denote the weights of the month-on-month growth rates used in the calculation of the three growth rates mentioned in the legend. Plus signs ("+") denote weights of observed month-on-month growth rates, while circles ("o") denote weights of month-on-month growth rates that have to be forecasted. The weights  $\omega_k$  for the annual growth rates  $g_{1,0}^{p|m}$  and  $g_{2,1}^{p|m}$  follow from (3) and (8) with b=12 and  $\kappa=1,2,\ldots,24$ . The weights for the year-on-year growth rate in the second quarter of the next year  $g_{6,2}^{q|m}$  are described by (32). In June, all three growth rates are not yet observed, because at least some of their underlying month-on-month growth rates have to be forecasted. We aim to approximate the forecast for  $g_{6,2}^{q|m}$ , that is,  $g_{6,2|5}^{q|m}$ , based on the forecasts for  $g_{1,0|5}^{y|m}$  and  $g_{2,1|5}^{y|m}$ , that is, based on  $g_{1,0|5}^{y|m}$  and  $g_{2,1|5}^{y|m}$ . The same description applies to the Consensus inflation forecasts.

assumption of an *iid* process, that is, the choice of k=0. Assuming k=0 gives the highest MSE ratios for GDP growth, and for both variables, the MSE ratios in the first quarter are virtually always moderately higher than those of the other approaches. However, a high lag order like k=4 can also lead to problems, as observed for inflation, especially in the fourth quarter.

In summary, the empirical results suggest that basing the optimal weights on an AR(1)-process yields good approximations. This holds for the recursive estimation of the ex-ante approach (i.e., an approach feasible in real-time) as well as for the full sample estimation of the ex-post approach. However, if estimating an AR(1)-process is difficult, for instance, due to a lack of data, simply assuming k=0, and hence, an iid process produces only marginally inferior approximations, which are still substantially more precise than those of the ad hoc approach. The true fixed-horizon forecasts, the optimal approximations with k=0 and the ad hoc approximation are displayed in Figure 5. Obviously, the optimal approximation tracks the true forecasts relatively closely, whereas the ad hoc approximation suffers from a too large volatility, which is caused by its excessive reliance on the more volatile current-year forecasts.

Table 5 contains the current-year weights implied by each approach. For the ex-ante approach, these are averages of the

 TABLE 6
 Weights for current-year forecasts.

| Origin    | Target quarter (next year) | <i>w</i> * | $w^{adhoc}$ |
|-----------|----------------------------|------------|-------------|
| March     | Q1                         | 0.04       | 0.75        |
| June      | Q2                         | -0.05      | 0.50        |
| September | Q3                         | -0.07      | 0.25        |
| December  | Q4                         | 0.08       | 0.00        |

*Note:* Optimal weights for the current-year forecasts  $\hat{g}_{1,0|t}$  based on assumptions that inflation and GDP growth are monthly *iid* processes (i.e., based on ex-ante approach with k=0) and that data from previous month are known. Approximation aims at the forecast for the 4-quarter-ahead quarterly year-on-year rate. Origin represents the month when the forecast is made.

**TABLE 7** | Statistical properties of Consensus Economics forecasts.

|         | Mean |      | std. | dev. |       | Mea      | an   | std. o | lev. |       |
|---------|------|------|------|------|-------|----------|------|--------|------|-------|
| Country | cur. | next | cur. | next | corr. | adhoc    | opt. | adhoc  | opt. | corr. |
|         |      |      |      |      | Ir    | ıflation |      |        |      |       |
| US      | 2.44 | 2.49 | 1.10 | 0.69 | 0.64  | 2.43     | 2.49 | 0.80   | 0.68 | 0.90  |
| JP      | 0.31 | 0.48 | 1.07 | 0.90 | 0.93  | 0.39     | 0.48 | 0.94   | 0.90 | 0.98  |
| GE      | 1.82 | 1.90 | 0.96 | 0.70 | 0.83  | 1.85     | 1.90 | 0.78   | 0.70 | 0.95  |
| FR      | 1.61 | 1.67 | 0.85 | 0.60 | 0.86  | 1.63     | 1.67 | 0.68   | 0.60 | 0.96  |
| UK      | 2.59 | 2.80 | 0.81 | 0.62 | 0.62  | 2.70     | 2.81 | 0.59   | 0.63 | 0.88  |
| IT      | 2.55 | 2.39 | 1.65 | 1.36 | 0.96  | 2.43     | 2.38 | 1.47   | 1.36 | 0.99  |
| CA      | 2.03 | 2.15 | 1.03 | 0.76 | 0.72  | 2.08     | 2.15 | 0.83   | 0.76 | 0.93  |
| NL      | 1.86 | 1.91 | 0.88 | 0.59 | 0.76  | 1.89     | 1.91 | 0.67   | 0.59 | 0.93  |
| NO      | 2.06 | 2.01 | 0.78 | 0.38 | 0.25  | 2.02     | 2.01 | 0.46   | 0.38 | 0.70  |
| ES      | 2.45 | 2.35 | 1.40 | 0.85 | 0.89  | 2.36     | 2.35 | 1.04   | 0.85 | 0.96  |
| SE      | 1.38 | 1.83 | 1.02 | 0.57 | 0.75  | 1.63     | 1.83 | 0.70   | 0.57 | 0.89  |
| СН      | 0.54 | 0.88 | 0.77 | 0.43 | 0.80  | 0.73     | 0.88 | 0.54   | 0.43 | 0.91  |
| EZ      | 1.62 | 1.60 | 0.85 | 0.34 | 0.79  | 1.58     | 1.60 | 0.52   | 0.34 | 0.89  |
|         |      |      |      |      | GD    | P growth |      |        |      |       |
| US      | 2.45 | 2.64 | 1.39 | 0.70 | 0.40  | 2.54     | 2.64 | 0.91   | 0.69 | 0.77  |
| JP      | 1.23 | 1.63 | 1.95 | 0.94 | 0.50  | 1.43     | 1.64 | 1.18   | 0.96 | 0.76  |
| GE      | 1.46 | 1.83 | 1.59 | 0.67 | 0.49  | 1.65     | 1.84 | 0.93   | 0.66 | 0.79  |
| FR      | 1.53 | 1.93 | 1.18 | 0.78 | 0.73  | 1.75     | 1.93 | 0.89   | 0.78 | 0.92  |
| UK      | 1.69 | 2.11 | 1.49 | 0.72 | 0.54  | 1.94     | 2.12 | 0.93   | 0.71 | 0.84  |
| IT      | 0.90 | 1.54 | 1.50 | 0.89 | 0.73  | 1.29     | 1.55 | 1.05   | 0.89 | 0.91  |
| CA      | 2.25 | 2.66 | 1.31 | 0.70 | 0.38  | 2.49     | 2.66 | 0.84   | 0.70 | 0.77  |
| NL      | 1.59 | 1.93 | 1.69 | 0.89 | 0.78  | 1.78     | 1.93 | 1.14   | 0.88 | 0.92  |
| NO      | 2.08 | 2.18 | 1.27 | 0.67 | 0.52  | 2.11     | 2.19 | 0.86   | 0.66 | 0.89  |
| ES      | 2.00 | 2.18 | 1.87 | 1.29 | 0.90  | 2.07     | 2.18 | 1.51   | 1.28 | 0.97  |
| SE      | 2.33 | 2.50 | 1.63 | 0.60 | 0.56  | 2.42     | 2.51 | 0.92   | 0.59 | 0.82  |
| СН      | 1.48 | 1.66 | 1.06 | 0.45 | 0.57  | 1.56     | 1.66 | 0.64   | 0.44 | 0.86  |
| EZ      | 1.05 | 1.45 | 1.47 | 0.61 | 0.56  | 1.27     | 1.45 | 0.90   | 0.59 | 0.84  |

Note: Statistics computed for samples starting for each country with the availability of Consensus forecasts and ending in 2019Q1. Periods with VAT changes are excluded from inflation statistics. "cur." denotes the current-year forecasts, "next" denotes the next-year forecasts, "opt." denotes the optimal approximations of the 4-quarter-ahead forecasts for the y-o-y growth rate determined for k = 0, "adhoc" denotes the corresponding ad hoc approximations. "std.dev." is the standard deviation, "corr." is the correlation between current-year and next-year forecasts, and between the optimal and ad hoc approximations, respectively.

recursively estimated weights. Note that for k=0, nothing needs to be estimated, such that the weights do not depend on the sample. The optimal weights are markedly smaller than the ad hoc weights, except for Q4, where the differences are small. Appar-

ently, the weights of the ex-ante approach are very similar to those of the ex-post approach, which suggests that the estimated coefficients are fairly stable over time. An exception is given by the full-sample GDP growth results, where the inclusion of the pan-

**TABLE 8** | Average number of fixed-horizon and fixed-event forecasters.

|               | US               | JP | GE | FR | UK | IT | CA       | NL   | NO | ES | SE | СН | EZ |
|---------------|------------------|----|----|----|----|----|----------|------|----|----|----|----|----|
|               | <u>Inflation</u> |    |    |    |    |    |          |      |    |    |    |    |    |
| Fixed-horizon | 26               | 16 | 16 | 13 | 9  | 6  | 11       | 7    | 4  | 10 | 6  | 10 | 18 |
| Fixed-event   | 27               | 21 | 28 | 19 | 26 | 15 | 16       | 10   | 9  | 15 | 14 | 14 | 29 |
|               |                  |    |    |    |    |    | GDP grov | vth_ |    |    |    |    |    |
| Fixed-horizon | 27               | 17 | 18 | 14 | 14 | 7  | 11       | 8    | 5  | 10 | 6  | 10 | 18 |
| Fixed-event   | 27               | 21 | 28 | 19 | 28 | 15 | 16       | 10   | 9  | 15 | 14 | 14 | 29 |

Note: Average number of forecasters for Consensus fixed-horizon (four-quarter-ahead) and fixed-event forecasts since 2007 (since 2005 for UK). The number displayed for the fixed-event forecasts is the average of the numbers for current- and next-year forecasts which tend to be virtually identical. "EZ" denotes the Euro zone. The other 2-letter country codes are ISO codes.

**TABLE 9** | MSE ratios for Consensus mean forecasts.

| MSE ratio         | Avg  | US               | JP   | GE   | FR   | UK   | IT   | CA     | NL   | NO   | ES   | SE   | СН   | EZ   |
|-------------------|------|------------------|------|------|------|------|------|--------|------|------|------|------|------|------|
|                   |      | <u>Inflation</u> |      |      |      |      |      |        |      |      |      |      |      |      |
| Ex ante, $k=0$    | 0.41 | 0.28             | 0.85 | 0.34 | 0.29 | 0.37 | 0.77 | 0.30   | 0.29 | 0.43 | 0.25 | 0.54 | 0.37 | 0.21 |
| Ex post, $k=1$    | 0.39 | 0.28             | 0.81 | 0.34 | 0.29 | 0.36 | 0.62 | 0.31   | 0.30 | 0.42 | 0.27 | 0.51 | 0.34 | 0.22 |
| Infeasible, $k=1$ | 0.34 | 0.27             | 0.57 | 0.33 | 0.29 | 0.25 | 0.55 | 0.30   | 0.26 | 0.42 | 0.25 | 0.34 | 0.34 | 0.20 |
|                   |      |                  |      |      |      |      | GDP  | growth |      |      |      |      |      |      |
| Ex ante, $k=0$    | 0.38 | 0.36             | 0.28 | 0.27 | 0.30 | 0.35 | 0.31 | 0.41   | 0.43 | 0.52 | 0.81 | 0.23 | 0.47 | 0.17 |
| Ex post, $k=1$    | 0.35 | 0.33             | 0.28 | 0.27 | 0.28 | 0.31 | 0.29 | 0.36   | 0.41 | 0.52 | 0.76 | 0.22 | 0.42 | 0.13 |
| Infeasible, $k=1$ | 0.29 | 0.30             | 0.27 | 0.26 | 0.27 | 0.26 | 0.25 | 0.32   | 0.33 | 0.34 | 0.44 | 0.19 | 0.39 | 0.13 |

*Note*: The MSE ratio refers to the ratio of the mean-squared approximation error obtained with optimal weights to the mean-squared approximation error obtained with ad hoc weights. "EZ" denotes the Euro zone. The other 2-letter country codes are ISO codes. "Avg" denotes the average over all quarterly MSE ratios. "ex ante, k=0" assumes a monthly iid process; "ex post, k=1" uses time series of inflation and GDP growth for each country to estimate the AR(1)-coefficient  $\rho$ ; "infeasible, k=1" sets  $\rho$  to the value that minimizes the MSE ratio for each country and variable.

demic leads to a noticeably lower persistence of the series, resulting in moderately smaller weights of the ex-post approach for the first quarter.

Virtually, all AR(k)-process-based weights for the third and fourth quarter forecasts are negative. Interestingly, almost all (infeasible) unrestricted weights for these quarters are negative as well, while they are almost all positive for the first quarters, being in line with the AR(k)-process-based weights. Most unrestricted weights for the third quarter are actually significantly negative, and none of the unrestricted third- and fourth-quarter weights is significantly positive. These results underline that negative weights are not an implausible implication of our approach, but an empirically desirable feature of optimal weights in the setting under study.

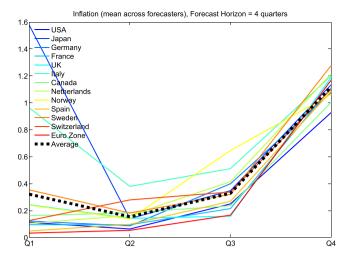
The AR(k)-process-based first-quarter weights with k>0 can differ substantially from those with k=0, while the differences tend to be small for other quarters. Thus, the lower MSE ratios observed with k>0 (as compared with k=0) are at least partly due to the fact that weights moderately larger than zero can be obtained for the first quarter. As the infeasible unrestricted weights show, the optimal first-quarter weights are equal to about 0.2 for inflation and GDP growth. Remarkably, the unrestricted second-quarter weight for GDP growth drops from 0 to -0.2 if the pandemic is included. <sup>18</sup>

## 4.2 | Consensus

In the following, we apply the methods developed to the forecasts published by Consensus Economics for inflation and GDP growth for 13 countries. Like the US SPF forecasts, Consensus forecasts are a collection of individual forecasts. Every month, all forecasters surveyed are asked for their forecasts of annual inflation and GDP growth in the current and in the next year. In addition to these fixed-event forecasts, once per quarter, namely typically in the months March, June, September, and December, the forecasters are asked for their fixed-horizon forecasts of quarterly inflation and GDP growth for the current quarter and the next six or seven quarters. These growth rates are y-o-y rates, that is, the fixed-horizon forecasts are made for growth rates  $g_{s,s-4}^{q|m}$ , where we assume a monthly DGP, such that s=t/3 with  $t=\ldots,-3,0,3,6,\ldots$  <sup>21</sup>

We consider the situation where one only sees the fixed-event forecasts from March, June, September, or December and then attempts to construct fixed-horizon forecasts from these fixed-event forecasts, either using optimal weights or ad hoc weights. The results are compared to the true fixed-horizon forecasts, and the empirical quality of both approximation approaches is evaluated. Again we focus on a fixed-horizon forecast of h=4 quarters ahead, such that, for instance, concerning the March forecast we would like to forecast the y-o-y growth rate of the first quarter of the next year.

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**FIGURE 7** | Quarter-specific ratios of the average squared approximation errors using the optimal weights (ex-ante approach with k=0) to the average squared approximation errors using the ad hoc weights for inflation mean forecasts four quarters ahead. The dashed black line denotes the average over all ratios. Q1 (Q2, Q3, and Q4) corresponds to the Consensus forecasts from March (June, September, and December).

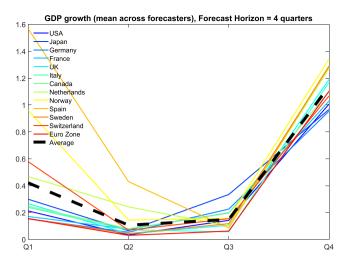
Inflation is measured using some monthly price index for each country, and we assume that inflation data up to the previous month is known when the forecasts are made. Concerning GDP growth, which is a quarterly variable, we assume that: first, forecasters consider this variable as having a monthly frequency, which is based on the fact that they observe monthly data like surveys and industrial production which convey information about the development of GDP growth within a quarter; second, in line with the assumption for inflation, that data on GDP growth is available up to the previous month. Thus, the fixed-horizon forecast to be approximated is given by  $\hat{g}_{s+4,s|t-1}^{q|m}$ , where t=3 refers to March of year 1, t=6 refers to June of year 1, and so on.

The vectors G,  $B_1$  and  $B_2$  are basically as in (14), but G here again contains realizations and forecasts, as in (21). The assumption of a monthly process and a quarterly y-o-y growth rate as the target implies that the  $A_{4,q|m}$ -vector (to be used instead of the  $A_{12}$ -vector that would be employed for a monthly target) is given by

$$A_{4,q|m} = \left[ \mathbf{0}'_{12-t} \ \frac{1}{3} \ \frac{2}{3} \ \mathbf{1}'_{10} \ \frac{2}{3} \ \frac{1}{3} \ \mathbf{0}'_{10+t} \right]'$$
 (32)

with t = 3 (6, 9, 12) for a March (June, September, and December) forecast. More information on these coefficients can be found in equation (3) in the supporting information. Figure 6 illustrates the setting with the Consensus forecasts from June. Section VIII of the supporting information contains figures illustrating the settings with forecasts from March, September, and December.

Concerning the optimal weights, we consider one case of each of the three approaches presented in Section 4.1 to determine the covariance matrix  $\Omega$ . We use the ex-ante approach with k=0, because this is especially easy to implement and suffices to obtain much better results than with ad hoc weights for the US SPF.<sup>22</sup> Moreover, we employ the ex-post approach with k=1, because using a larger value of k does not turn out to be helpful for the US SPF. Finally, we consider the infeasible approach based on an



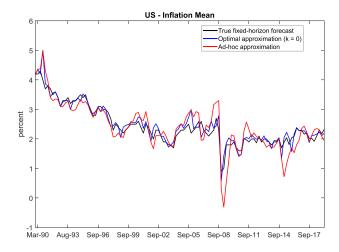
**FIGURE 8** | Quarter-specific ratios of the average squared approximation errors using the optimal weights (ex-ante approach with k = 0) to the average squared approximation errors using the ad hoc weights for GDP growth mean forecasts four quarters ahead. The dashed black line denotes the average over all ratios. Q1 (Q2, Q3, and Q4) corresponds to the Consensus forecasts from March (June, September, and December).

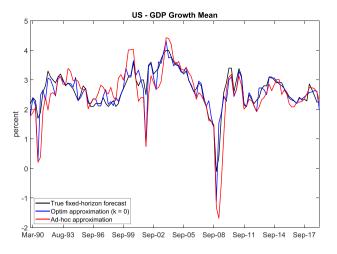
AR-process with k=1, because the corresponding results inform us about the potential to improve upon the results of the ex-post approach given k=1. For the ex-post approach, we derive the monthly AR(1)-coefficients from realized y-o-y inflation data and q-o-q GDP growth data covering the forecast samples.<sup>23</sup> Note that the approach with k=0 could have been used in real-time.

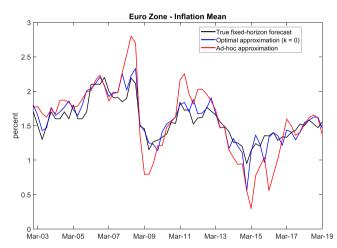
Table 6 shows the weights for the current-year forecasts obtained with the optimal ex-ante approach based on k=0 and with the ad hoc method.<sup>24</sup> Obviously, for the forecasts from March, June and September, the ad hoc approach places much larger weights on the current-year forecasts than the optimal approach.

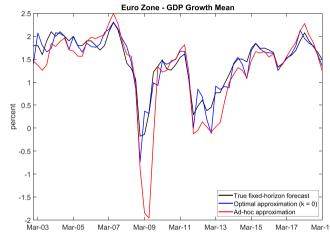
Like for the US SPF, we evaluate the optimal and ad hoc approximations with respect to the mean of the true individual fixed-horizon forecast across all forecasters surveyed, and the approximations are based on the mean of the individual current-year forecasts and the mean of the individual next-year forecasts across all forecasters surveyed. In addition, in Section IX of the supporting information, we investigate which approximation is better suited to approximate the cross-sectional disagreement among forecasters.<sup>25</sup>

As shown in Table 7, the average next-year GDP growth forecasts are higher than the average current-year forecasts across all 13 countries. There is no such systematic difference in the respective mean forecasts of inflation. The standard deviations of the next-year forecasts for inflation and GDP growth are notably smaller than the current-year forecasts across all countries. Like in the US SPF, the correlation between current-year and next-year forecasts tends to be higher for inflation than for GDP growth. The optimal approximations closely match next-year forecasts, whereas the ad hoc approximations align more with current-year forecasts. The optimal approximations for Consensus inflation and GDP growth forecasts have lower standard deviations than the ad hoc approximations in all cases but one. The two approx-









**FIGURE 9** | Quarterly time series of the actual 4-quarter-ahead Consensus mean forecasts and the approximations based on optimal (for the ex-ante approach with k = 0) and ad hoc weights for inflation and GDP growth for the US (upper panels) and the Euro zone (lower panels).

imations are positively correlated, with an average coefficient of about 0.9 for inflation and also for GDP growth.

The average number of forecasters for fixed-horizon and fixed-event forecasts is displayed in Table 8. Apparently, there are many cases where not all fixed-event forecasters also issue fixed-horizon forecasts. <sup>26</sup> The distortions arising from this deviation between the samples of fixed-event and fixed-horizon forecasters can be expected to be of minor importance for larger economies like Japan, where data from many forecasters are available. However, for smaller economies like Norway, these distortions could be relevant, especially for the cross-sectional disagreement among forecasters.

Concerning the true fixed-horizon forecasts, our sample of mean forecasts starts in 1989 for about half of the countries,<sup>27</sup> and it ends in March 2019. For inflation, we drop those time periods from our sample which are associated with changes in the value-added tax rate (VAT) by two percentage points or more. This occurred in Japan, Germany, France, the UK, the Netherlands, and Spain.<sup>28</sup>

Table 9 displays the ratios of the mean-squared approximation errors (MSE ratios) of the optimal approach with respect to the ad hoc approach, corresponding to the empirical analog of the ratio defined in Equation (27) averaged over forecast of all fixed-horizon dates, that is, to

$$\overline{\phi} = \frac{\frac{1}{T} \sum_{t} \left[ \left( \hat{g}_{s+4,s|t-1}^{q|m} - \tilde{g}_{s+4,s|t-1}^{q|m} \right)^{2} | w = w^{*} \right]}{\frac{1}{T} \sum_{t} \left[ \left( \hat{g}_{s+4,s|t-1}^{q|m} - \tilde{g}_{s+4,s|t-1}^{q|m} \right)^{2} \middle| w = w^{ad \, hoc} \right]}$$
(33)

with T denoting the number of fixed-horizon forecasts in the sample, s = t/3, and  $t = t_0, t_0 + 3, t_0 + 6, t_0 + 9, t_0 + 12, \dots$ , where  $t_0$  is the month of the first fixed-horizon forecast.<sup>29</sup> We report the MSE ratios for each of the three specifications of the AR(1)-coefficient  $\rho$  considered for the optimal approach. The values of  $\rho$  obtained for the second and third specifications are shown in Section VIII of the supporting information.

It turns out that the optimal weights yield substantially better approximations for all three specifications considered. On average, the mean-squared approximation errors are more than halved if one uses optimal weights instead of ad hoc weights. Compared to the ex-ante approach with inflation and growth being iid processes, the ex-post estimated  $\rho$  tends to provide small reductions in the MSE ratios. The value of  $\rho$  of the infeasible approach further reduces the mean-squared approximation errors. However, these reductions are also small for most countries and sometimes come at a cost of using implausible values of  $\rho$  due to the flatness of the MSE ratio functions. Therefore, in what follows, we will only consider the optimal approximation method with weights determined for the simple ex-ante approach with k=0.

For inflation, the quarter-specific ratios for each country, that is, for the four quarterly ratios defined by (32) with  $t = t_0, t_0 +$  $12, t_0 + 24, \dots$  for  $t_0 = t_0, t_0 + 3, t_0 + 6$ , and  $t_0 + 9$ , respectively, are shown in Figure 7. Obviously, using optimal weights instead of ad hoc weights can lead to extreme reductions in the approximation errors for the four-quarter ahead forecasts from March, June, and September. For example, for the June forecasts, on average, the optimal approach gives squared approximation errors which are about five times smaller than those of the ad hoc approach. This average refers to the average over the ratios of all countries for a specific quarter. For the March and September forecasts, the squared approximation errors with optimal weights are about three times smaller on average. As to be expected due to the similar weights for the December forecasts, the approximation errors of both approaches attain similar values for these forecasts. Disregarding the December forecasts, the ad hoc approach delivers better approximations only in one case, namely for the March forecast of Japan.

The results for GDP growth shown in Figure 8 resemble those for inflation, with the optimal weights usually leading to much better approximations except for the December forecasts. For the March forecasts, on average, the optimal approach gives squared approximation errors which are about two times smaller than those of the ad hoc approach, with Spain being the only case where the ad hoc approach yields smaller errors than the optimal approach. The squared approximation errors with optimal weights are about ten times smaller on average for the June forecasts and six times smaller for the September forecasts. Like for inflation, for the June and the September forecasts, the optimal approach yields better approximations than the ad hoc approach for each country.

In order to illustrate the time series of the mean forecasts, they are shown for the cases of the US and the Euro zone in Figure 9. It is obvious that the approach based on optimal weights approximates the true fixed-horizon forecasts better than the ad hoc approach. Like in the application to the US SPF, the ad hoc approach puts too much weight on the volatile current-year forecasts, resulting in approximations which are too volatile. The differences between the two methods are particularly sizable around the Great Recession for both jurisdictions.

#### 5 | Conclusions and Outlook

In the empirical literature, one often encounters the need to construct fixed-horizon forecasts from fixed-event forecasts. No well-founded approach has been proposed to perform this construction, and users had to resort to ad hoc approaches. This work attempts to close this gap. We derive easily-computable optimal weights for the fixed-event forecasts which allow the construction of fixed-horizon forecasts that minimize the squared approximation error with respect to the actual fixed-horizon forecasts.

We find that the gains from using optimal instead of ad hoc weights tend to be very large in our empirical applications. The mean-squared approximation errors, on average, are more than halved.

For the often-encountered case of approximating 12-monthahead or 4-quarter-ahead forecasts based on current- and next-year forecasts of inflation and GDP growth, the weights resulting from the assumption of a simple *iid* data-generating process, which can be found in Tables I and II of the supporting information, are a good choice. If an AR(1)-process for the high-frequency variable can be estimated, the corresponding optimal weights can be expected to yield small improvements over the weights based on the *iid* assumption. It goes without saying that especially for other applications, considering data-generating processes more sophisticated than a simple *iid* process could be important.

Several interesting issues could be addressed in future work. For instance, in the US SPF, participants provide histogram forecasts for the current and the next year, from which moments or even density forecasts can be derived. Ganics, Rossi, and Sekhposyan (2024) propose weights to combine such fixed-event density forecasts to obtain fixed-horizon density forecasts. It might be interesting to compare their approach to an approach that uses different weights for the mean forecasts, the variance forecasts, and, potentially, forecasts of higher-order moments, respectively. Likewise, one could investigate quantile forecasts instead of mean forecasts. Moreover, sometimes, fixed-event forecasts and the fixed-horizon forecast of interest are available, but afterward, the fixed-event forecasts will get updated before the fixed-horizon forecast does. It appears promising to think about how to update the fixed-horizon forecast in such a situation.

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This article has been awarded Open Data Badge for making publicly available the digitally-shareable data necessary to reproduce the reported results. Data is available at https://doi.org/10.15456/jae. 2024348.1452008891.

#### **Endnotes**

<sup>1</sup> In practice, our approach can be applied whenever the forecast of interest is unknown, but at least two other forecasts are available. Using our

approach, one could, for instance, easily approximate the 1-year-ahead real GDP growth forecasts from June 2023 for each OECD country based on the OECD forecasts for 2023 and for 2024, which are available at https://www.oecd.org/economic-outlook/june-2023/.

- <sup>2</sup> However, the DGP could also be formulated for the high-frequency levels  $p_s^{v|v}$ , or for  $g_{u|-n}^{v|v}$  with n>1.
- <sup>3</sup> In principle, one could drop the requirement that s is an integer. For example, considering the annual and the monthly frequency, and setting t=1 for January of the current year,  $p_{0.5}^{y|m}$  could refer to the average level of p over the second half of the previous year and the first half of the current year. However, such definitions of low-frequency periods are irrelevant in our empirical applications.
- <sup>4</sup> These weights have a tent-shaped form, as will be illustrated below in Figure 6. Such weights seem to have appeared for the first time in the forecasting literature in the special case of relating monthly and quarterly growth rates, that is, for b = 3, when they were employed in Mariano and Murasawa (2003).
- <sup>5</sup> Note that this approach does not consider the conditioning information used for the forecast. For instance, if we are interested in  $\hat{g}_{12,0}^{m|m}$ ,  $\hat{g}_{1,0}^{y|m}$  receives the weight 1, no matter if the last observed growth rate comes from December or November of the previous year.
- $^6$  The general result obtained does not depend on whether we are considering forecasts, realizations, or a mix of both. This will only play a role for the elements of the covariance matrix  $\Omega$ , which appears in the expression for the optimal weights below, because forecasts and realizations differ with respect to their variances and covariances.
- <sup>7</sup>The numbers used for this figure can be found in Section III of the supporting information.
- 8 The expected squared approximation errors used to produce this figure can be found in Section III of the supporting information.
- <sup>9</sup> We briefly elaborate on this relation in Section V of the supporting information.
- <sup>10</sup> The optimal weights depend on the covariance matrix  $\Omega$ . The fitted AR(k)-process is supposed to result in a good estimate of  $\Omega$ . In principle, one could also consider ARMA-processes or nonlinear processes as DGPs and determine  $\Omega$  based on these. However, in general, AR(k)-processes are more convenient to estimate.
- <sup>11</sup> In addition to the OECD forecast webpage mentioned in footnote 1, this holds, for instance, for the probability forecasts (not for the point forecasts) from the US Survey of Professional Forecasters, for the mean forecasts by Consensus Economics from most months, and for all individual forecasts by Consensus Economics. Both surveys are described below.
- <sup>12</sup> The corresponding formulas for the current-year and next-year growth rates are given by the quarterly analogs of Equations 5, (6), and (7), that is, by  $g_{1,0}^{y|q} = \frac{p_1^{y|q} p_2^{y|q}}{p_0^{y|q}}$ ,  $g_{2,1}^{y|q} = \frac{p_2^{y|q} p_1^{y|q}}{p_1^{y|q}}$ , and  $p_s^{y|q} = \frac{1}{4} \sum_{t=4 \times (s-1)+1}^{4 \times s} p_t^{q|q}$  with  $s = \dots, -1, 0, 1, 2, \dots$
- <sup>13</sup> On the other hand, assuming a monthly process would allow us to calculate month-specific optimal weights. Assuming a quarterly process, for instance, would yield the same weights for forecasts made in February and March. With a monthly process one could account for the fact that, although the variables under study have a quarterly frequency, more information is available in March than in February. We will elaborate on this issue in the application to Consensus forecasts. Note that here, the quarter-on-quarter growth rate in t,  $g_{t,t-1}^{q|q}$ , does not affect the growth rate of interest, which is given by  $g_{t+4,t}^{q|q} = \sum_{i=1}^4 g_{t+i,t+i-1}^{q|q}$ , such that shocks which realized in the first month of t do not have a direct impact on the growth rate of interest.
- <sup>14</sup> Please refer to the US SPF's documentation for additional details on the data used: https://www.philadelphiafed.org/-/media/ frbp/assets/surveys-and-data/survey-of-professional-forecasters/ spf-documentation.pdf.

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- <sup>15</sup> Note that the estimated weights for each quarter can be regarded as optimal combination weights in the spirit of Bates and Granger (1969). As pointed out by a referee, if the fixed-horizon forecasts of interest are not available, in the constrained OLS regression one could also use the corresponding realizations instead. However, as these would be noisy measures of the forecasts and potentially contain outliers like the GDP growth rates at the onset of the COVID-19 pandemic, larger samples would be needed to attain a similar precision of the estimates.
- <sup>16</sup> If we consider each forecast horizon within a year separately, we can see that this pattern continues to hold. That is, the standard deviation for current-year forecasts tends to increase with the quarter of the year in which they are produced, and the same holds for next-year forecasts. Also, next-year forecasts made at the end of the year tend to have a lower standard deviation than current-year forecasts made at the beginning of the year.
- $^{17}$  The exact ad hoc and optimal (with k=0) weights can be found in Table 5 below.
- <sup>18</sup> This is due to 2020Q2, which saw the largest fixed-horizon forecast (6.5%) and the lowest current-year forecast (-5.8%) of the sample. With the next-year forecast of 2.7%, the optimal weight for this single period equals about -0.4. With the full-sample optimal weight of -0.2, the approximation error for 2020Q2 equals about 2 percentage points, which is the largest second-quarter error by far.
- <sup>19</sup> Strictly speaking, the forecasts are made for 12 countries and for the euro area, which is labeled Euro zone by Consensus Economics.
- <sup>20</sup> In the other 8 months, fixed-horizon forecasts are not available for these quantities. Hence, if these forecasts are required, one would need to approximate them, which can be achieved using the optimal approach proposed. The exact definitions of the inflation and GDP growth series forecasted and the corresponding data sources for obtaining the respective realizations are given in Section VIII of the supporting information.
- <sup>21</sup> The assumption of a monthly DGP is convenient for inflation, which is observed monthly. For GDP growth, which is observed quarterly, the assumption of a monthly process somewhat complicates the specification of the DGP. More on this follows below. Note that the time index for the annual variables would again be given by  $\dot{s} = t/12$  for  $t = \dots, -12, 0, 12, 24, \dots$  here.
- <sup>22</sup> Note that, while k=0 for the monthly DGP means that this is a monthly *iid* process, there is a positive autocorrelation of y-o-y as well as of quarter-on-quarter growth (q-o-q) rates. Estimating an AR(1)-model for the q-o-q growth rates of a monthly *iid* process would result in an AR(1)-coefficient of about 0.2 asymptotically. For further details, see Section V in the supporting information.
- <sup>23</sup> Details on the data and on the derivation are provided in Sections V and VIII of the supporting information.
- $^{24}$  Weights for other months and other values of  $\rho$  can be found in Section III of the supporting information.
- <sup>25</sup> This investigation is motivated by the fact that in Dovern, Fritsche, and Slacalek (2012) and other studies, the approximation of fixed-horizon forecasts is an intermediate step to determine the object of interest, which is given by the disagreement among fixed-horizon forecasts.
- <sup>26</sup> Additional data sources suggest that there are no forecasters which only provide fixed-horizon forecasts.
- <sup>27</sup> The forecasts for the Netherlands, Spain and Sweden start in 1995, the forecasts for Norway and Switzerland in 1998, and the forecasts for the Euro zone in 2002. For Germany, the GDP growth forecasts before December 1995 and the inflation forecasts before December 1996 refer to Western Germany. For the UK we focus on the forecasts for the Retail Prices Index, reported since 1989. Before 1994, the fixed-horizon forecasts were not always issued in March, June, September or December. We take the correct dates into account in the empirical analysis, but abstract from this issue in the text for the ease of exposition.

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- <sup>28</sup> We do so because such large changes, which are usually announced well in advance, are at odds with the assumption of a time-invariant DGP. Whether it is better to put a lot of weight on the current-year or on the next-year forecast depends on when the VAT change happens and if the fixed-horizon forecast to be approximated targets a period before or after the change. However, neither the optimal weights nor the ad hoc weights take the timing of VAT changes into account. Therefore, large expected changes would suggest using a two-step approach in order to approximate fixed-horizon forecasts well. First, one would need to subtract the effect of the VAT change from the fixed-event forecasts, then one would use the chosen approximation approach, and then one would add the effect of the VAT change to the approximated fixed-horizon forecasts. The time periods excluded from the analysis can be found in Section VIII of the supporting information. However, even if we include the periods with VAT changes, the MSE ratios defined in Equation (33) below hardly change. This can be seen in Table VI in the supporting information, where we include periods with VAT changes. Yet Figure 7 below would look noisier when including those periods.
- <sup>29</sup> Hence,  $t_0$  refers to March, June, September, or December of the respective first year when forecasts were made. Like (31), this ratio is a small sample version of the ratio in (30).

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#### **Supporting Information**

Additional supporting information can be found online in the Supporting Information section.