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RESEARCH ARTICLE OPEN ACCESS

Analysing the Compensatory Properties of the Outranking Approach PROMETHEE

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ABSTRACT

The PROMETHEE methods are increasingly applied in environmental and public policy decision-making due to their comprehensiveness and explainability. However, the literature contains differing statements regarding their compensatory properties. Compensation in multiple criteria decision aggregation procedures is commonly understood as allowing a gain in one criterion to offset a loss in another one. In certain domains, such as environmental or public policy decision-making, it may be undesirable, as some impacts may result in losses too severe to be counterbalanced by good performance on other criteria. Therefore, it may be necessary to limit the extent to which an aggregation procedure permits compensation or to explicitly control it as needed. Guidelines and detailed analytical tools, however, that help users and analysts to control compensation in the PROMETHEE methods remain scarce and often lack transparency. In this study, we analyse the compensatory behaviour of the PROMETHEE I and II methods and identify the key determinants for compensation in these methods. Based on these insights, we develop flow insensitivity intervals to assess the sensitivity of a given decision model towards compensatory effects and provide a set of general guidelines for controlling compensation in the PROMETHEE I and II methods for any given pair of criteria. The findings are illustrated at hand of an environmental management case study. By combining the guidelines with flow insensitivity intervals, users and analysts gain access to measures of varying granularity to evaluate and control compensation in a PROMETHEE decision model.

1 | Introduction

An essential part of multiple criteria decision analysis (MCDA) is the formalised procedure to move from a decision model to a synthesis of the information that has been obtained about the different options as well as the objectives and preferences of everyone involved (Belton and Stewart 2002). For this task, a rich set of different multiple criteria aggregation procedures (MCAP) has emerged from the scientific discourse. They are often differentiated into two schools of thought (Vansnick 1990; Roy and Vanderpooten 1996), analogous to the fundamental decision mechanisms developed

by de Borda (1781) and Condorcet (1785). Frequently used approaches from the so-called *American* school of thought (Von Winterfeldt and Edwards 1986; Keeney 1992) are the multiple attribute value and utility theory (MAVT/MAUT) (Keeney 1992; Keeney and Raiffa 1993), while the Preference Ranking Organization METHOD for Enrichment Evaluations (PROMETHEE) (Brans et al. 1986) and the Elimination and Choice Translating Reality (ELECTRE) approach (Roy 1991) are often applied outranking approaches from the *French* or *European* school of thought (Roy and Bouyssou 1993). These two distinct schools are essentially based on different assumptions and axioms when establishing the set of synthesised

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information in the form of preference structures (see e.g., Vansnick 1986; Moulin 1988; Munda 2016; Roy 2016). This leads to a diverging interpretation of criteria weights and different aggregation properties (Munda 2008; Figueira et al. 2016; Brans and De Smet 2016; Martel and Matarazzo 2016).

One aggregation property that distinguishes the various MCAPs is the possible occurrence of compensation. Compensation is commonly understood as allowing a performance gain in one objective or criterion to offset a performance loss on another one (Roy and Słowiński 2013). Due to their different axiomatic foundations, it is often argued that MCAP from the American school of thought are based on a compensatory aggregation logic, while outranking approaches generally consider this property as unwanted in the process of establishing preference relations (Vansnick 1990; Roy and Słowiński 2013). Depending on the decision problem, compensatory effects could either be desired (e.g., for employee performance evaluations) or should be deliberately avoided (e.g., to ensure the concept of strong sustainability) (Cinelli et al. 2014). Especially in contexts where numerous facets of sustainability need consideration and counterbalancing effects between certain dimensions need to be avoided, or limited, the issue of compensation is of particular interest. A few selected examples comprise energy technology assessment (Diakoulaki et al. 2005; Tsoutsos et al. 2009; Oberschmidt et al. 2010; Strantzali and Aravossis 2016), production management (Hämäläinen 2004; Geldermann and Rentz 2001; Tong et al. 2022), environmental management (Kiker et al. 2005; Huang et al. 2011; Lienert et al. 2015), and policy making (Ferretti 2016; Salo and Hämäläinen 2010).

While there is scholarly work available that offers guidance on the choice of aggregation functions to steer compensatory effects in value theory based approaches (e.g., Langhans et al. 2014; Cinelli et al. 2014) provide a comprehensive body of research on the compensation degree of common MCDA methods, research that offers a detailed analysis of the compensation mechanisms of the PROMETHEE methods seems limited. Much more, we find equivocal statements regarding the compensatory behaviour of the PROMETHEE methods in the scientific discourse. In Cinelli et al. (2020, 2022), PROMETHEE I, II, TRI and V (Brans and De Smet 2016; Figueira et al. 2004; Brans and Mareschal 1992) are classified as both not and partially compensatory. Further relevant works characterise the PROMETHEE methods as non-compensatory outranking methods (Pirlot 1997; Greco et al. 2021; Costa and Alves 2021). In other literature it is stated that PROMETHEE methods ‘avoid full compensation’ (Prado et al. 2012) or are either fully, partial or non-compensatory, depending on the selected method and its configuration (Bezerra et al. 2021; Benoit and Rousseaux 2003; Moghaddam et al. 2011; Guitouni and Martel 1998; Ishizaka and Resce 2021). Only recently, Dejaegere and De Smet (2023) proposed the new PROMETHEE γ method, which can deliberately be used in totally compensatory or non-compensatory manner to circumvent unwanted compensatory behaviour of PROMETHEE I in certain situations. Additionally, they remark that ‘[...] it is not clear whether PROMETHEE is considered as non-compensatory or partially compensatory’ (Dejaegere and De Smet 2023, 149). However, we are not aware of any work that discloses the preconditions for compensatory behaviour within

the PROMETHEE methods and provides a comprehensive set of guidelines that allows applicants to steer it in the desired manner.

To complement existing research, this article will provide the following main contributions:

- The compensatory properties of the PROMETHEE methods are highlighted. In particular, the determinants for compensation in the PROMETHEE I and II methods are disclosed.
- A measure to investigate the sensitivity of a given decision model that is aggregated according to PROMETHEE I or II towards compensatory effects is developed. In this way, it is possible to identify the criteria for which compensation can occur and to specify the extent to which compensatory effects are responsible for preserving a preference structure.
- A set of guidelines for the design and parameterisation of a PROMETHEE model to control compensation in a fine-grained resolution, ranging from full compensation to no compensation at all, is proposed.

The remainder of the article is structured as follows. We first define the fundamentals of a multiple criteria decision problem and arrive at definitions for the notion of compensation (Section 2). In Section 3, the PROMETHEE methods are introduced, with particular emphasis on PROMETHEE I and II. Following this, the compensatory behaviour of the PROMETHEE methods is characterised by using a differential calculus approach and showcased by a series of numerical examples (Section 4). The results are discussed in Section 5 and used to elaborate guidelines for applicants and analysts. Conclusions are drawn in Section 6.

2 | Compensation in Multiple Criteria Aggregation Procedures

Generally speaking, MCDA methods apply distinct aggregation procedures to synthesise preference information and evaluate a set of potential courses of action. In this section, the formal notation of these MCAPs is introduced and the notion of compensation is defined.

2.1 | Multiple Criteria Aggregation Procedures

In a multiple criteria decision problem, a set of potential actions $A := \{a_1, a_2, \dots, a_i, \dots, a_m\}$ is to be evaluated on a set of attributes or criteria $G := \{g_1, g_2, \dots, g_j, \dots, g_n\}$ according to the preferences of one or more decision-maker(s) (DM). A decision table then summarises the available alternatives, criteria and the performance of each alternative on each criterion, denoted as $g_j(a_i)$.

An MCAP applies a specified mathematical procedure on the decision table to evaluate the formalised decision problem in order to produce a desired result which allows for the evaluation of alternatives in a comprehensive way (Roy 2016).

Depending on the desired type of result, decision problems are typically distinguished into four different types: *Choice*, *sorting*, *ranking* and *description problematics* (Roy 2016). To reach the desired outcome, MCAPs establish preference structures which formalise the comparison of any pair of alternatives $(a_i, a_x) \in A \times A$ in clear or fuzzy language (Krantz et al. 1971; Fishburn 1999; Roubens and Vincke 1985; Moretti et al. 2016). A preference structure is a set of binary preference relations that describe a DM's attitudes towards a subset of ordered pairs from $A \times A$, that is, alternatives, such that for each pair of alternatives exactly one relation holds (Moretti et al. 2016). Accordingly, binary preference relations R are preference statements on pairs of A . They allow to express situations of preference or indifference, typically denoted $\langle P, I \rangle$. In some cases, weak preference can also be expressed (Vincke 1988). For a more extensive overview on the different types of preference structures that can be established, see for example Moretti et al. (2016).

Each MCAP follows a different logic in the creation of preference structures. Approaches that maximise a value, utility or scoring function are often referred to as aggregating MCAPs (Vincke 1992). They exploit an aggregated score to establish binary preference relations.

In outranking methods, a preference structure is characterised by binary outranking relations, denoted S , that can or can not be transitive and complete, depending on the prevalence of incomparability between alternatives (Bouyssou 1996). The notion of incomparability goes back to the concordance-discordance principle, an essential axiomatic foundation of outranking approaches (Bouyssou and Pirlot 2009; Figueira et al. 2013). The concordance-discordance principle acknowledges that the existence of preference or indifference on a pair of alternatives is not always possible nor desired (Vincke 1992). Therefore, an additional binary relation describing incomparability between alternatives (J) is introduced (Tsoukiàs and Vincke 1995). In $\langle P, I, J \rangle$ preference structures, the relation P describes situations in which one alternative is clearly preferred over another, while the relations I and J both denote that neither alternative is preferred (Moretti et al. 2016). For the reflexive and symmetric indifference relation I , the lack of preference is due to their equivalent valuation. The symmetric and irreflexive incomparability relation J , however, refers to situations in which a lack of information or conflicting evaluations does not allow for the expression of preference and $\neg(a_i Pa_x)$, $\neg(a_x Pa_i)$, and $\neg(a_i Ia_x)$ simultaneously holds (Moretti et al. 2016).

The resulting preference structure can then be defined as follows Dejaegere and De Smet (2023):

$$\begin{aligned} a_i Pa_x &\Leftrightarrow a_i Sa_x \quad \wedge \quad \neg(a_x Sa_i) \\ a_i Ia_x &\Leftrightarrow a_i Sa_x \quad \wedge \quad a_x Sa_i \\ a_i Ja_x &\Leftrightarrow \neg(a_i Sa_x) \quad \wedge \quad \neg(a_x Sa_i) \end{aligned}$$

The outranking relation $a_i Sa_x$ denotes the assertion that ' a_i is at least as good as a_x ' or ' a_i outranks a_x '. That means a_i can be considered at least as good as a_x since there is sufficient evidence

supporting such a statement and no contradicting evidence, given the available information (Bouyssou and Vansnick 1986).

2.2 | Compensatory and Non-Compensatory Multiple Criteria Aggregation Procedures

The notion of compensation in MCAPs has been a subject of extensive study and the academic literature presents a set of definitions. Most commonly, the compensatory character of an MCAP is connected to its axiomatic foundations.

Aggregating MCAPs interpret weights as trade-offs between attributes and typically aggregate the performance of alternatives on these attributes in an additive manner, for example, via simple additive weighting (SAW) in the form $\sum_{j=1}^n w_j \cdot g_j(a_i)$. Such MCAPs are considered inherently compensatory for two analogous reasons. First, the additive aggregation logic implies that a low performance on one attribute can be offset by a relatively good performance on another attribute with regards to the aggregated performance of an alternative (Langhans et al. 2014). Furthermore, the weights that are attached to the different objectives are interpreted as trade-offs between objectives (Moulin 1988). Thus, they are equivalent to the objectives' substitution rates and a fully compensatory aggregation logic is inherently necessary (Munda 2016). An illustrative example which derives this equivalence for the case of SAW is provided by (Munda 2008, chapter 4). The notion of compensation in this case can be defined as the property that the preference relations remain unaffected when a loss on one objective is accompanied by comparable gain on another one, adjusted for the weights (i.e., substitution rate) (Roy and Mousseau 1996; Haag et al. 2019).

In outranking methods, the MCAP is not concerned with maximising an underlying value or utility function. Instead, pairwise performance assessments of alternatives are conducted on the set of criteria to establish preference structures, where the weights only represent the relative importance of each criterion (Figueira et al. 2016). This leads to two fundamental differences compared to aggregating MCAPs:

1. The concordance-discordance principle confines compensatory effects in the construction of preference structures to the intra-criterial comparisons of alternatives and only up to a specified threshold. Within the pairwise comparisons, small performance disadvantages of an alternative that do not surpass this threshold are completely disregarded. The threshold value defines the extent of what can be considered a 'small' difference in performance to distinguish between 'sufficient' and 'insufficient' evidence for preference (Pirlot 1997).
2. The concordance-discordance principle is generally not concerned with performance substitution between the different criteria. The performance differences that exceed the specified thresholds in intra-criterial comparisons are not able to affect the evaluation on another criterion without turning relative disadvantages into advantages and vice versa. It is thus irrelevant for the construction of preference relations by *how much* the performances of alternatives

with regard to a given criterion differ if the threshold is exceeded, and any changes in performance across criteria will not be able to affect the preference structure (Figueira et al. 2010).

Since the preference structures in outranking methods are only affected by whether an alternative outranks another one and vice versa, they are generally classified as non-compensatory MCAPs (Fishburn 1976; Bouyssou and Vansnick 1986). At the same time, the concordance-discordance principle does not strictly imply that outranking relations are non-compensatory, given that small enough disadvantages are not considered in the construction of them (Dejaegere and De Smet 2023). Furthermore, not all outranking methods fully adopt the concordance-discordance principle. The PROMETHEE I and II methods are examples of outranking methods that are not based on the concordance-discordance principle, which leads to different compensatory properties. We characterise their compensatory behaviour after arriving at a set of working definitions.

2.3 | Definitions

So far, no widely accepted and precise definition of compensation in MCAP could be identified in the literature. For the purpose of this work, compensation in MCAP is defined as the possibility for inter-criterial performance substitution (Roy and Mousseau 1996). Inter-criterial performance substitution refers to the balancing or offsetting of a disadvantage on a criterion, in terms of the resulting preference structures, by a sufficient advantage on another criterion.

Therefore, an MCAP is considered compensatory if a preference relation between two alternatives is altered by a change in performance on one criterion and can be reinstated by adjusting the alternatives' performance on another criterion (Roy and Słowiński 2013).

Definition 1. (Compensatory MCAP). Consider a decision problem with multiple criteria $G := \{g_1, \dots, g_n\}$ and alternatives $A := \{a_1, \dots, a_m\}$, where $g_j(a_i)$ denotes the performance of alternative a_i on criterion g_j . Let a_i and a_x then be two distinct alternatives from A with $a_i Ra_x$ derived from the MCAP, where R is the binary preference relation obtained from the MCAP.

We say that the MCAP is compensatory, if there are two criteria g_j and g_k , such that for a change of $g_j(a_i)$ which alters the preference relation $a_i Ra_x$, we can find a corresponding change in $g_k(a_i)$ that reinstates the initial preference relation $a_i Ra_x$.

Definition 2. (Non-compensatory MCAP). Consider a decision problem with multiple criteria $G := \{g_1, \dots, g_n\}$ and alternatives $A := \{a_1, \dots, a_m\}$, where $g_j(a_i)$ denotes the performance of alternative a_i on criterion g_j . Let a_i and a_x then be two distinct alternatives from A with $a_i Ra_x$ derived from the MCAP, where R is the binary preference relation obtained from the MCAP.

We say that the MCAP is non-compensatory, for any two criteria g_j and g_k , and any change of $g_j(a_i)$ which alters the preference relation $a_i Ra_x$, if there is no corresponding change in $g_k(a_i)$ that reinstates the initial preference relation $a_i Ra_x$.

A change in the performance of an alternative a_i on a criterion g_j is henceforth denoted $\Delta g_j(a_i)$. It is considered a 'gain' if it increases the performance difference to the next best alternative on the same criterion or reduces the gap to an alternative that performs better on this criterion. In turn, a change in performance that decreases the performance difference between alternative a_i and a lower performing alternative on the same criterion is considered a 'loss'. These definitions also apply to other initial preference structures, for example, if a_i and a_x are considered indifferent. Compensation can occur if any such performance changes alters a given preference relation between a_i and any other alternative for a sufficiently large value of $\Delta g_j(a_i)$.

In addition, we define a total 'non-compensatory MCAP' since the given definition of non-compensatory MCAPs does not cover cases in which a preference relation can not be destroyed by the stated performance substitutions due to the nature of the aggregation process (Fishburn 1976; Roy and Mousseau 1996).

Definition 3. (Totally non-compensatory MCAP). An MCAP is considered totally non-compensatory if the preference situation between two distinct pairs of alternatives (a_i, a_x) and (a_i', a_x') is considered on an ordinal scale on each criterion g_j (Roy and Mousseau 1996). That is, the preference relation is independent from the difference in performance and there is no possibility of compensation relative to any criterion, so that

$$[a_i Ra_x = a_i' Ra_x' \wedge a_x Ra_i = a_x' Ra_i'] \quad (1)$$

holds for all alternatives a_i' and a_x' that are deduced from (a_i, a_x) by changing their performance on any criterion without altering the ordinal order of alternatives on a criterion. Thus, in a totally non-compensatory MCAP there is no possibility for compensatory effects between different criteria as long as the preferential profile of alternatives is kept (Bouyssou 1986).

3 | The PROMETHEE I and II Methods

Given these definitions, outranking approaches following the concordance-discordance principle generally can be considered non-compensatory. However, PROMETHEE I and II are outranking methods that do not strictly follow the concordance-discordance principle. Instead, they establish a ranking of alternatives based on a set of valued outranking relations (Brans et al. 1986). This means that a numeric value, describing the intensity of a preference relation, is attached to a pair of alternatives (Bouyssou and Pirlot 2009). In PROMETHEE, the valued outranking relations are used to establish scores or *outranking flows*, which represent the performance of an alternative compared to the other alternatives (Brans and De Smet 2016).

To determine the outranking flows, the differences between performances of each pair of alternatives from A are computed for all criteria in the first step:

$$d_j(a_i, a_x) = g_j(a_i) - g_j(a_x), \quad \forall a_i, a_x \in A \quad (2)$$

$$j = 1, \dots, n$$

By means of a preference function \mathcal{P} it is then possible to calculate the individual preferences of the DM:

$$\mathcal{P}_j(a_i, a_x) = \mathcal{P}_j(d_j(a_i, a_x)), \quad \forall a_i, a_x \in A \quad (3)$$

$$j = 1, \dots, n$$

A preference function is a monotonic function of $d_j(a_i, a_x)$ and maps the intra-criterial preferences of the DM, normalised to the interval $[0, 1]$, while Equation (4) holds, so that the preference function for criteria to be minimised is as in Equation (5).

$$\mathcal{P}_j(a_i, a_x) > 0 \Rightarrow \mathcal{P}_j(a_x, a_i) = 0 \quad (4)$$

$$\mathcal{P}_j(a_i, a_x) = F_j - d_j(a_i, a_x) \quad (5)$$

In general, the shape and definition of a preference function can be selected by the DM. To reduce the cognitive load of modelling, a set of six non-decreasing preference functions has been established which are considered suitable for most contexts (Brans and De Smet 2016). These functions and their parameters are depicted in Table A1 in the Appendix A. Depending on the shape of the preference function, additional threshold parameter values may be elicited from the DM. The indifference threshold q_j delimits situations in which the difference in performance between two alternatives is too small to allow any statement of preference for either. Parameter p_j models the threshold for a situation of strict preference. The inflection point σ_j of the Gaussian criterion (type VI) allows to model the DM's sensitivity towards performance differences in a non-linear manner, as depicted in Table 1. A lower value results in greater sensitivity to small differences in performance (single dot-dash line $\cdot -$), while a relatively high value means that preference sensitivity for large differences in performance between two alternatives is higher (double dot-dash line $\cdot \cdot -$).

The DMs intra-criterial preferences are then aggregated to a global preference index for all pairs of alternatives as in Equation (6):

$$\pi(a_i, a_x) = \sum_{j=1}^n w_j \cdot \mathcal{P}_j(a_i, a_x) \quad \forall a_i, a_x \in A \quad (6)$$

A weighting coefficient $w_j \geq 0$ denotes the relative importance of each criterion g_j .

The preference indexes can then be used to compute the out-ranking flow scores of an alternative:

$$\phi^+(a_i) = \frac{1}{m-1} \sum_{a_x \in A} \pi(a_i, a_x) \quad \forall a_i \in A \quad (7)$$

$$\phi^-(a_i) = \frac{1}{m-1} \sum_{a_x \in A} \pi(a_x, a_i) \quad \forall a_i \in A$$

The positive flow score denoted ϕ^+ aggregates the evidence from all pairwise comparisons that reinforce a situation of preference for an alternative over all the other alternatives under consideration; the negative flow score ϕ^- in turn sums up all evidence speaking against such a statement (Linkov et al. 2021, 9).

In PROMETHEE I, $\langle P, I, J \rangle$ preference structures are established based on the positive and negative flow scores.

$$a_i P^I a_x \Leftrightarrow \begin{cases} \phi^+(a_i) \geq \phi^+(a_x) \wedge \phi^-(a_i) < \phi^-(a_x) \\ \text{or} \\ \phi^+(a_i) > \phi^+(a_x) \wedge \phi^-(a_i) \leq \phi^-(a_x) \end{cases}$$

$$a_i I^I a_x \Leftrightarrow \phi^+(a_i) = \phi^+(a_x) \wedge \phi^-(a_i) = \phi^-(a_x) \quad (8)$$

$$a_i J^I a_x \Leftrightarrow \begin{cases} \phi^+(a_i) < \phi^+(a_x) \wedge \phi^-(a_i) < \phi^-(a_x) \\ \text{or} \\ \phi^+(a_i) > \phi^+(a_x) \wedge \phi^-(a_i) > \phi^-(a_x) \end{cases}$$

Thus, the PROMETHEE I ranking of alternatives for a given decision problem is the intersection of the rankings obtained from the positive and negative flow scores and forms a partially ordered set (Brans and De Smet 2016; Dejaegere and De Smet 2023).

The PROMETHEE II method yields a total ranking of alternatives and $\langle P, I \rangle$ preference structures by means of the PROMETHEE net flow:

$$\phi^{net}(a_i) = \phi^+(a_i) - \phi^-(a_i) = \sum_{j=1}^n \phi_j(a_i) \cdot w_j \quad (9)$$

where

$$\phi_j(a_i) = \frac{1}{m-1} \sum_{a_x \in A} [\mathcal{P}_j(a_i, a_x) - \mathcal{P}_j(a_x, a_i)] \quad (10)$$

is the single criterion flow of an alternative with respect to criterion $g_j \in G$.

TABLE 1 | Decision table with three alternatives and four criteria used in the stereotypical cases.

Criterion Name	Performance			Preference information		
	A_1	A_2	A_3	w_j	Polarity	Function
g_1	$g_1(A_1)$	$g_1(A_2)$	$g_1(A_3)$	w_1	Max	\mathcal{P}_1
g_2	$g_2(A_1)$	$g_2(A_2)$	$g_2(A_3)$	w_2	Max	\mathcal{P}_2
g_3	$g_3(A_1)$	$g_3(A_2)$	$g_3(A_3)$	w_3	Max	\mathcal{P}_3
g_4	$g_4(A_1)$	$g_4(A_2)$	$g_4(A_3)$	w_4	Max	\mathcal{P}_4

The total ranking of all alternatives is then given by ordering them according to their net flow scores, while indifference may occur:

$$\begin{aligned} a_i P^I a_x &\Leftrightarrow \phi^{net}(a_i) > \phi^{net}(a_x) \\ a_i I^I a_x &\Leftrightarrow \phi^{net}(a_i) = \phi^{net}(a_x) \end{aligned} \quad (11)$$

The PROMETHEE method family also provides methods for sorting (Figueira et al. 2005; De Smet 2019) and choice problems (Brans and Mareschal 1992). For a complete overview of PROMETHEE methods (Brans and De Smet 2016; Belton and Stewart 2002), or the encompassing survey paper on their development, extensions and future directions (Brans 2015), we kindly refer the reader to the respective literature.

4 | Compensation in the PROMETHEE Methods

Since the PROMETHEE methods do not strictly follow the concordance-discordance principle, they exhibit a varying compensation behaviour that can be either non-compensatory or explicitly allow for compensatory effects between criteria. Specifically, the valued outranking relations that the PROMETHEE methods produce allow for compensation based on the elicitation or selection of preference functions, preference parameters and criteria weights. The compensation behaviour of these methods thus depends on the choices made during the elicitation and modelling of the decision problem. The information that is required to construct these valued outranking relations, namely the outranking flows, also allows one to capture compensatory effects of the PROMETHEE methods and characterise determinants for compensation. This will be the focus of this section.

4.1 | Characterising the Compensation Behaviour of the PROMETHEE II Method

The compensatory behaviour of the PROMETHEE methods is connected to the valued outranking relations that are produced to establish a ranking of alternatives. In order to characterise the compensation behaviour of the PROMETHEE methods, selected stereotypical cases are analysed. We limit the analysis of the compensatory behaviour to PROMETHEE I and II since they are the main methods of this family and are intended to be used for ranking problematics.

The stereotypical cases comprise a set of three decision alternatives that are to be evaluated against a set of four cardinal criteria that are to be maximised, so that:

- $A = \{A_1, A_2, A_3\}$,
- $G = \{g_1, g_2, g_3, g_4\}$, and
- $g_j \in [0, 100]$.

The alternatives' performances, the chosen preference function for each criterion and the corresponding criteria weights are to be varied according to the studied case, leading to the generic decision table presented in Table 1, which summarises the structure of the studied decision problems and the nomenclature adopted in this study. This forms the starting point for the analysis of the PROMETHEE methods' compensatory properties. Subsequently, a change in the performance of a selected alternative with regards to a specific criterion is introduced (formally denoted $\Delta g_j(A_i)$). In the cases presented here, this change represents a performance loss (or gain) whose effect on the outranking flows is to be compensated for by means of a performance increase (decrease) of the same alternative on another criterion (denoted $\Delta g_k(A_i)$). It is also analysed how the selection of preference functions, threshold parameters and criteria weights influence the compensatory behaviour.

4.1.1 | The Equivalence of PROMETHEE II to Additively Aggregating MCAPs

Certain instances of additively aggregating MCAPs (e.g., SAW or MAVT/MAUT with linear value functions and an additive model) are equivalent to PROMETHEE II with type III preference functions and sufficiently large threshold parameters, as shown by Geldermann and Schöbel (2011) or Mareschal (2015). For these instances, both methods also exhibit comparable compensation behaviour, where a loss in performance on one criterion can be fully offset by a corresponding gain on another criterion. It may therefore be argued that the PROMETHEE II method is fundamentally compensatory according to the definition adopted in this work (see Section 2.3).

An exemplary model of PROMETHEE that corresponds to an MAVT model with linear value functions and an additive aggregation function is stated in Table 2. The criteria in the PROMETHEE model are modelled via the widely used linear preference function (type III) and the preference threshold is set to the performance difference between the best and worst performing alternative for each criterion ($p_j = g_j^{max} - g_j^{min}$). Using the piecewise linear preference functions of type V with pertinent parameterisation is likewise possible in this example (Geldermann and Schöbel 2011).

TABLE 2 | Decision table with three alternatives, four criteria and preference information to be used for the application of the PROMETHEE methods.

Criterion Name	Performance				Preference information		
	A_1	A_2	A_3	w_j	Polarity	Function	p_j
g_1	100	10	15	0.25	Max	Type III	90
g_2	0	90	70	0.25	Max	Type III	90
g_3	13	21	66	0.25	Max	Type III	53
g_4	78	100	42	0.25	Max	Type III	58

The alternatives' global values and ϕ^{net} scores after aggregation according to MAVT and PROMETHEE II are shown in Figure 1. They are indicated by the square marks connected by a solid line. According to both methods, the total ranking of alternatives is $A_2 > A_3 > A_1$, while $>$ denotes a situation of preference. Furthermore, Figure 1 displays the equivalent compensation behaviour of both methods. One can identify a performance gain of A_3 on the first criterion that dissolves the preference relation between A_2 and A_3 and yields indifference between these alternatives for both methods. In this case, such a $\Delta g_1(A_3)$ takes the value of 28.6. The global values, as well as the net flows of A_2 and A_3 , denoted V' and ϕ'^{net} are equal after the performance increase of A_3 on criterion g_1 . This is visualised by the two horizontally aligned dotted lines. Given equal criteria weights, a corresponding performance loss on the second criterion, $\Delta g_2(A_3) = -28.6$, reestablishes the original preference relations as well as the exact scores for $V(A_3)$ and $\phi^{net}(A_3)$.

This behaviour can be generalised, so that the extent of required performance gain on criterion g_2 to compensate for a loss on g_1 is

$$\Delta g_2(A_i) = \frac{p_2 \cdot w_1}{p_1 \cdot w_2} \cdot \Delta g_1(A_i) \quad (12)$$

It depends on the ratio of the criteria weights, as well as the preference thresholds of both of the involved criteria to account for different units of measurement. Furthermore, the condition that the linear preference function (type III) is used and parameterised so that it is equally shaped to a linear value function must hold:

$$P_j(d_j) = \begin{cases} 0 & d_j \leq 0 \\ \frac{d_j}{g_j^{max} - g_j^{min}} & 0 \leq g_j^{max} - g_j^{min} \end{cases} \quad (13)$$

We do not recommend to design a type III preference function according to Equation (13) in practical application and further discussing it in Section 5.3. The generalised proof for Equation (12) can be found in the Appendix A.

Despite the adjusted performances, the ϕ^{net} scores for all other alternatives also remain constant after applying performance

changes of $\Delta g_1(A_3) = 28.6$ and $\Delta g_2(A_3) = -28.6$, as the orange bar in the three panels on the right hand side of Figure 2 shows.

Given the methods pairwise-comparison logic to produce a set of valued outranking relations this is not necessarily expected. The compensating effect that is responsible for this circumstance becomes visible upon investigation of corresponding single criterion flows. The bottom right panel shows the single criterion flows for Alternative 3. Compared to the single criterion flows before the performance adjustments, which is indicated by the shaded area, the relative weakness of A_3 regarding criterion g_1 is reduced while the contrary happens for the performance on criterion g_2 . In a non-compensatory setting, this change in single criterion flows would not happen. Logically, the single criterion flows of A_1 and A_2 are also changing on the two performance-adjusted criteria due to the PROMETHEE calculation procedure. This means that across all three alternatives, only the single criterion flows regarding the first two criteria are changing. All other single criterion flows remain constant. The corresponding effect of performance substitution for the MAVT model with additive aggregation function and other value aggregation methods is extensively studied by Langhans et al. (2014).

4.1.2 | Non-Compensatory Modelling in PROMETHEE II

However, there are also modelling variants of PROMETHEE II where compensatory effects are largely or even completely avoided.

If compensation needs to be avoided, this can be done quite intuitively by changing the preference function of the affected criteria. The results of switching to a type I preference function for criterion g_1 and g_2 for the stated decision problem are given in Table 3. Following the definition of this preference function, the performance alterations $\Delta g_1(A_3) = 28.6$ and $\Delta g_2(A_3) = -28.6$ do neither affect the PROMETHEE net flows nor the single criterion flows. Thus, compensatory effects can not occur when confining to this preference function type.

Due to the calculation logic of the outranking flows and the piecewise definition of the remaining preference function types,

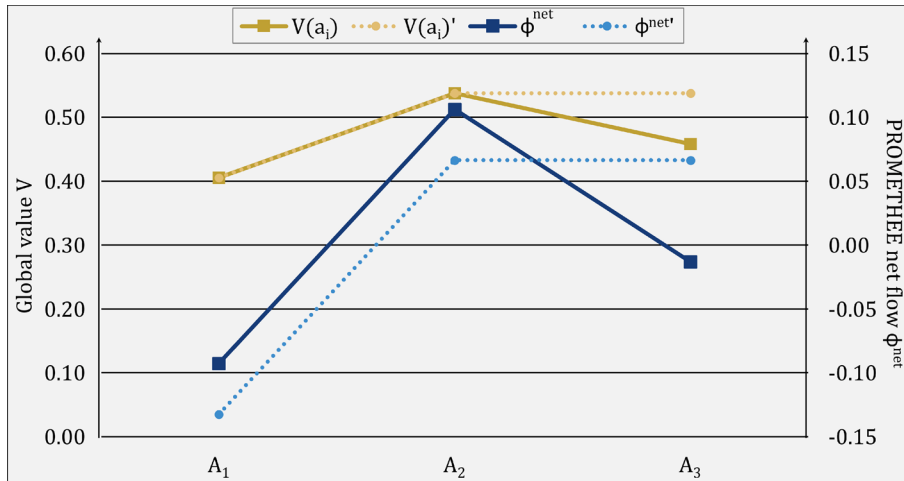


FIGURE 1 | Global values and PROMETHEE net flows for decision problem in Table 2 and effects of changing the performance of A_3 on g_1 .

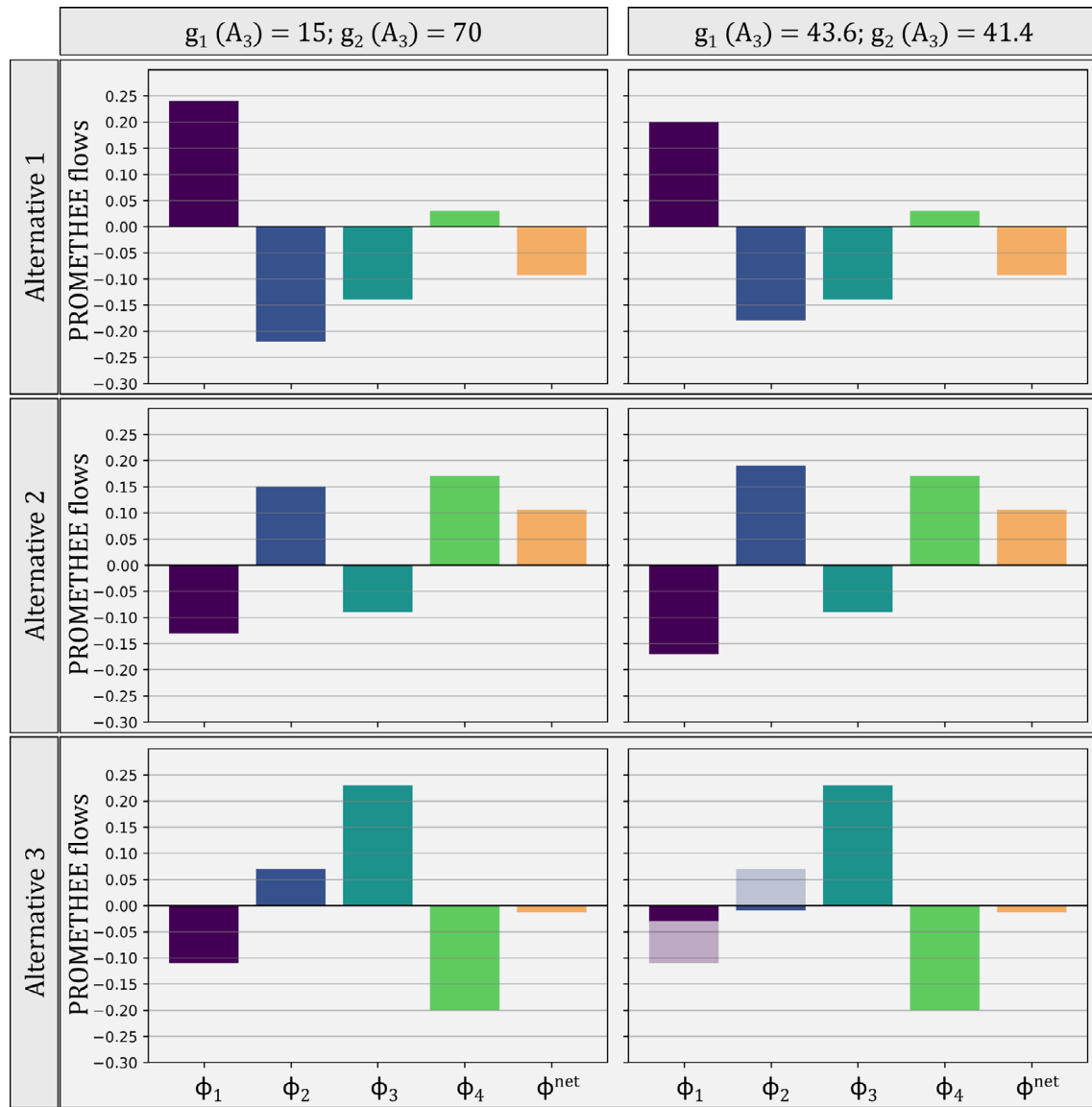


FIGURE 2 | PROMETHEE single criterion flows and net flows for the decision problem in Table 2 (panels on the left half). The three panels on the right side highlight the compensatory effects when decreasing the performance of A_3 on g_1 and compensating for this loss by a gain on criterion g_2 . Although the PROMETHEE net flows of A_3 remain similar since the loss is compensated, the compensation becomes observable in the changed single criterion flows for the two affected criteria (ϕ_1 and ϕ_2 in this instance).

TABLE 3 | PROMETHEE single criterion flows and net flows for the decision problem in Table 2 before and after altering the performance of A_3 on criterion g_1 and g_2 by -28.6 and 28.6 points.

Criteria performance	Alternative	PROMETHEE flows				
		$\phi_1(a_i)$	$\phi_2(a_i)$	$\phi_3(a_i)$	$\phi_4(a_i)$	$\phi^{net}(a_i)$
$g_1(A_3) = 70; g_2(A_3) = 15$	A_1	0.25	-0.25	-0.144	0.03	-0.114
	A_2	-0.25	0.25	-0.087	0.172	0.085
	A_3	0	0	0.231	-0.203	0.029
$g_1(A_3) = 43.6; g_2(A_3) = 41.4$	A_1	0.25	-0.25	-0.144	0.03	-0.114
	A_2	-0.25	0.25	-0.087	0.172	0.085
	A_3	0	0	0.231	-0.203	0.029

Note: Criteria g_1 and g_2 are modelled via the usual preference function of type I. The linear preference function and parameterisation is kept for g_3 and g_4 . The net flows (bold) of an alternative are the sum of the single criterion flows.

it is more difficult to generalise non-compensatory behaviour when they are used. That is because for each criterion an alternative's net flow depends on the pairwise performance difference to all other alternatives on this criterion. This creates a lot of possible cases, depending on the selected preference function for the criteria and the specification of threshold parameters.

For example, the type II preference function introduces an indifference area to the usual preference function of type I. The possible cases resulting from the introduction of this indifference area are generically depicted in Figure 3. Initially, the performance difference between two alternatives $d_j(a_i, a_x)$ is large enough, so that a_i is preferred over a_x on this criterion with a preference index $\mathcal{P}_j = 1$ (Situation 1 in Figure 3, coloured in dark blue).

If the introduced performance loss $\Delta g_j(a_i)$ takes a small enough negative value so that $d_j > q_j$ holds as initially, then the net flow of a_i remains unaffected (Situation 2a, light blue). In this case, the preference function behaves analogously to the type I preference function and no compensation by a performance increase on another criterion is necessary. In turn, if the performance loss leads to a pairwise performance difference of $d_j \leq q_j$ (situation 2b, yellow colour), the value of $\mathcal{P}_j(a_i)$ and thus the net flows can change. Then there can be situations in which a large enough gain on another criterion $\Delta g_k(a_i)$ is able to reinstate the initial net flow of an alternative a_i . Vice versa, criteria that are defined via a preference function of the second type are not necessarily able to compensate for a loss that is occurring on another criterion.

Understanding the effects of performance changes on the pairwise evaluations, as well as the calculation logic of the single criterion net flows is crucial to generalise other possible occurrences of compensation in the PROMETHEE II method.

4.1.3 | Capturing Occurrences of Compensation in PROMETHEE II

The prerequisite for the occurrence of compensation in PROMETHEE II is a change in the single criterion flows of an

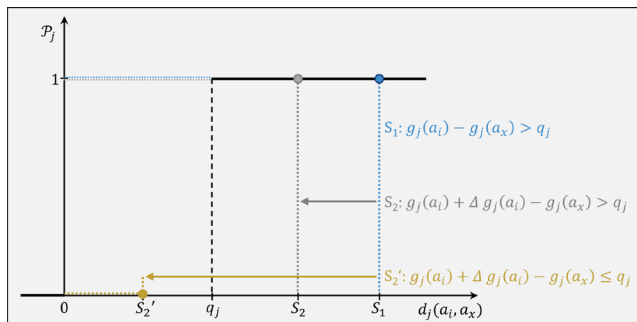


FIGURE 3 | The varying effects after the introduction of a performance loss in the type II criterion. The initial pairwise comparison between two alternatives prior to introducing any performance change (S_1) is highlighted in blue. Introducing a small enough performance change does not lead to a different value on the preference function (situation S_2 , displayed in grey). Only the introduction of a performance change that is large enough to shift the performance difference between two alternatives beyond the indifference threshold (situation S_2' , indicated in yellow colour) can lead to changes in single criterion and thus net flows.

alternative (denoted $\phi_j(a_i)$) following a change in performance scores. If a performance score change does not affect the single criterion flows, it can neither compensate for losses on other criteria nor does it require compensation to maintain the outranking flows at their initial value.

It is possible to determine the bounds of performance changes for which the single criterion flows of a given decision problem remain constant. The intervals that are deduced from these bounds reflect the performance deviations for each alternative and criterion that do not affect the net flows and therefore the PROMETHEE II ranking in any way. Vice versa, they also describe the range in which a performance change of an alternative is not able to compensate for any kind of performance loss on another criterion. This kind of information is beneficial to analyse the susceptibility of a given decision problem towards compensation or to gain insight into its compensatory behaviour.

To calculate these bounds, the derivative of each single criterion net flow is to be determined as in Equation (14).

$$\frac{\partial \phi_j(g_j(a_i))}{\partial g_j(a_i)} = \left[\frac{1}{m-1} \sum_{a_x \in A} [\mathcal{P}_j(g_j(a_i) - g_j(a_x)) - \mathcal{P}_j(g_j(a_x) - g_j(a_i))] \right] \quad (14)$$

Solving the derivative to zero then yields the interval of performance scores where the single criterion net flows remain constant for a given alternative and criterion. The derivatives of all six PROMETHEE preference function types are given in Equations (15–20) of the following paragraph.

4.1.3.1 | Derivatives of the Single Criterion Net Flows for PROMETHEE Preference Functions. Type I (usual criterion):

$$\frac{\partial \phi_j(g_j(a_i))}{\partial g_j(a_i)} = \left[\frac{1}{m-1} \sum_{a_x \in A} \left(\begin{cases} 0 & g_j(a_i) - g_j(a_x) \leq 0 \\ 1 & g_j(a_i) - g_j(a_x) > 0 \end{cases} - \begin{cases} 0 & g_j(a_x) - g_j(a_i) \leq 0 \\ 1 & g_j(a_x) - g_j(a_i) > 0 \end{cases} \right) \right] \quad (15)$$

Type II (quasi-criterion):

$$\frac{\partial \phi_j(g_j(a_i))}{\partial g_j(a_i)} = \left[\frac{1}{m-1} \sum_{a_x \in A} \left(\begin{cases} 0 & g_j(a_i) - g_j(a_x) \leq q_j \\ 1 & g_j(a_i) - g_j(a_x) > q_j \end{cases} - \begin{cases} 0 & g_j(a_x) - g_j(a_i) \leq q_j \\ 1 & g_j(a_x) - g_j(a_i) > q_j \end{cases} \right) \right] \quad (16)$$

Type III (linear criterion):

$$\frac{\partial \phi_j(g_j(a_i))}{\partial g_j(a_i)} = \left[\frac{1}{m-1} \sum_{a_x \in A} \left(\begin{cases} 0 & g_j(a_i) - g_j(a_x) \leq 0 \\ \frac{g_j(a_i) - g_j(a_x)}{p_j} & 0 < g_j(a_i) - g_j(a_x) \leq p_j \\ 1 & g_j(a_i) - g_j(a_x) > p_j \end{cases} - \begin{cases} 0 & g_j(a_x) - g_j(a_i) \leq 0 \\ \frac{g_j(a_x) - g_j(a_i)}{p_j} & 0 < g_j(a_x) - g_j(a_i) \leq p_j \\ 1 & g_j(a_x) - g_j(a_i) > p_j \end{cases} \right) \right] \quad (17)$$

Type IV (level criterion):

$$\frac{\partial \phi_j(g_j(a_i))}{\partial g_j(a_i)} = \left[\frac{1}{m-1} \cdot \sum_{a_x \in A} \begin{cases} 0 & g_j(a_i) - g_j(a_x) \leq q_j \\ \frac{1}{2} & q_j < g_j(a_i) - g_j(a_x) \leq p_j \\ 1 & g_j(a_i) - g_j(a_x) > p_j \end{cases} \right. \\ \left. - \begin{cases} 0 & g_j(a_x) - g_j(a_i) \leq q_j \\ \frac{1}{2} & q_j < g_j(a_x) - g_j(a_i) \leq p_j \\ 1 & g_j(a_x) - g_j(a_i) > p_j \end{cases} \right] \quad (18)$$

Type V (linear criterion with indifference area):

$$\frac{\partial \phi_j(g_j(a_i))}{\partial g_j(a_i)} = \left[\frac{1}{m-1} \cdot \sum_{a_x \in A} \begin{cases} 0 & g_j(a_i) - g_j(a_x) \leq q_j \\ \frac{g_j(a_i) - g_j(a_x) - q_j}{p_j - q_j} & q_j < g_j(a_i) - g_j(a_x) \leq p_j \\ 1 & g_j(a_i) - g_j(a_x) > p_j \end{cases} \right. \\ \left. - \begin{cases} 0 & g_j(a_x) - g_j(a_i) \leq q_j \\ \frac{g_j(a_x) - g_j(a_i) - q_j}{p_j - q_j} & q_j < g_j(a_x) - g_j(a_i) \leq p_j \\ 1 & g_j(a_x) - g_j(a_i) > p_j \end{cases} \right] \quad (19)$$

Type VI (Gaussian criterion):

$$\frac{\partial \phi_j(g_j(a_i))}{\partial g_j(a_i)} = \left[\frac{1}{m-1} \cdot \sum_{a_x \in A} \begin{cases} 0 & g_j(a_i) - g_j(a_x) \leq 0 \\ -\frac{(g_j(a_i) - g_j(a_x))^2}{2\sigma_j^2} & g_j(a_i) - g_j(a_x) > 0 \end{cases} \right. \\ \left. - \begin{cases} 0 & g_j(a_x) - g_j(a_i) \leq 0 \\ -\frac{(g_j(a_x) - g_j(a_i))^2}{2\sigma_j^2} & g_j(a_x) - g_j(a_i) > 0 \end{cases} \right] \quad (20)$$

4.1.3.2 | Compensation Sensitivity Analysis. Table 4 displays these bounds, or flow insensitivity intervals, at hand of an

exemplary decision problem. For demonstrative purposes, different preference functions, compared to the previously discussed cases, are chosen. Two important implications for the occurrence of compensation can be extracted from Table 4. In combination, they provide measures to capture the sensitivity of a given decision model towards compensatory effects.

First, the performance scores are stated along with their bounds. For all performance scores within the bounds, the single criterion net flow is constant since the slope of its tangent equals 0, that is, the flows are not changing when the performance score is altered. This property is independent of the criteria weights. It is to be remarked that only the constant regions for performance scores in the interval [0, 100] are displayed in the table to match the scale of the criterion. Other possible constant regions, for example, for negative performance scores are omitted. The constant regions of all single criterion net flow functions beyond the interval [0, 100] and their calculation are stated in the [Supporting Information](#). It is also observable in Table 4, that for some criteria the intra-criterial dominance relations can be violated without affecting the outranking flows at all. For example, for $g_1(A_2) = 37$, the criterion becomes the second best alternative on criterion g_2 while the (single criterion) net flows and preference relations remain unaffected. This implies that the evaluation of Alternative A_2 is relatively insensitive towards compensatory effects with regards to criterion g_1 . Even relatively large performance changes on this criterion are not able to compensate changes on other criteria.

Second, the resulting range of performance changes that do not affect the single criterion net flows are derived from the performance score bounds and denoted as $\Delta g_j(a_i)$. These intervals provide an essential point of departure to characterise the compensation behaviour in PROMETHEE II since they quantify the (in)sensitivity of a given decision model towards compensatory effects. The general interpretation of these flow insensitivity intervals is further exemplified at hand of Alternative 3 and criterion g_1 in Figure 4.

As depicted, the criterion is defined via the level preference function of type IV with an indifference threshold of 22.5 and

TABLE 4 | Range of performance scores that does not affect the PROMETHEE single criterion flows and resulting range of performance changes that can not compensate or require compensation.

	Criterion			
	g_1	g_2	g_3	g_4
Function	Type IV	Type V	Type II	Type II
p_j	90	90	—	—
q_j	22.5	22.5	13	15
$g_j(A_1)$	100 (37.5,100]	0 { }	13 [8, 34]	78 (57, 85)
$g_j(A_2)$	10 [10,37.5]	90 [90,92.5]	21 [0, 26]	100 (93,100]
$g_j(A_3)$	15 [10,32.5]	70 { }	66 (34,100]	42 [0, 63]
$\Delta g_j(A_1)$	(− 62.5,0]	{ }	[− 5, 21]	(−21, 7)
$\Delta g_j(A_2)$	[0,27.5]	[0,2.5]	[− 21, 5]	(− 7, 0]
$\Delta g_j(A_3)$	[− 5,17.5]	{ }	(− 32, 34]	[− 42, 21)

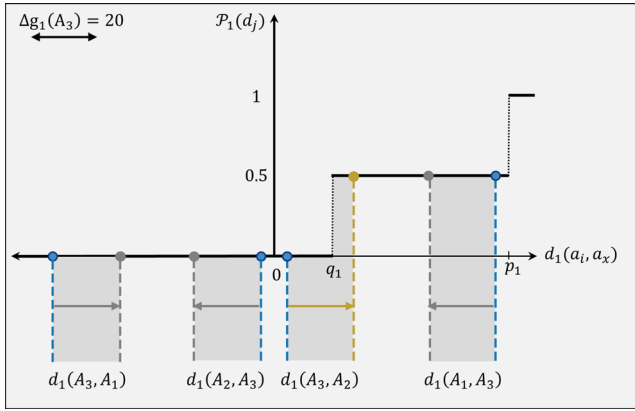


FIGURE 4 | Pairwise evaluation of alternative A_3 on criterion g_1 and effects of introducing performance changes. The dashed blue lines indicate the initial points of pairwise comparisons on the preference functions. An improvement of A_3 on g_1 by 20 units changes its pairwise evaluation with the other alternatives on this criterion (represented by the shaded areas). The direction of the shift (represented by arrows) depends on whether A_3 is being compared with another alternative, or whether one of the other alternatives is being compared against A_3 . Depending on the location on P , this may affect the single criterion flows (dashed yellow line) or not (dashed grey lines).

a preference threshold of 90. Furthermore, the differences from the pairwise performance comparisons of alternatives that are relevant for the calculation of the net flows are indicated in blue colour. The performance difference of A_3 to A_2 is not sufficient to pass the indifference threshold of 22.5 ($d_1(A_3, A_2)$), while the relative weakness of the third alternative compared to the first one ($d_1(A_1, A_3)$) negatively affects the single criterion flow of A_3 on criterion g_1 . These properties hold as long as $g_1(A_3)$ is not decreased by more than 5 and increased by more than 17.5. In this interval, all pairwise comparisons remain in the same region of the piecewise defined preference function.

Assume the performance score of A_3 on criterion g_1 increases by 20 units, affecting the pairwise comparisons as indicated in Figure 4. For most pairwise evaluations this causes no change in single criterion flows (indicated in grey colour), whereas the pairwise comparison between A_3 and A_2 reaches the next part of the piecewise defined preference function (yellow colour). This increases $\phi^{net}(A_3)$ from 0.025 to 0.088.

In turn, if this performance score increase of A_3 on the first criterion is accompanied by a corresponding performance decrease on the second criterion, the net flow of A_3 can be maintained. This is possible because criterion g_2 is defined via the fifth preference function type and the performance scores are shifted in the linear part of the preference function. The effect of a performance change $\Delta g_2(A_3) = -18.3$ that re-establishes the original net flow of 0.025 is visualised in Figure 5. The two changes that are marked in yellow colour are jointly responsible for the reduction in single criterion net flow of A_3 . Although it may appear counter-intuitive, both of these observed shifts lead to a reduction in net flows of the third alternative. As indicated, the pairwise comparison (A_2, A_3) leads to a marked increase in

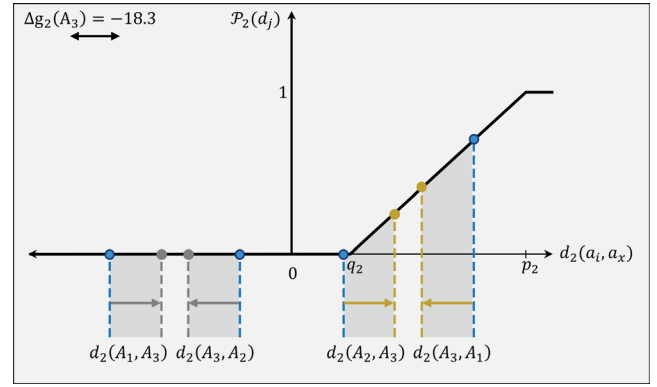


FIGURE 5 | Pairwise evaluation of alternative A_3 on criterion g_2 and effects of introducing performance changes. Pairwise evaluations of A_3 against A_1 and A_2 (and vice versa) before the performance change are indicated in blue. Shifts on P as a results of decreasing the performance of A_3 on criterion g_2 (shaded grey areas) that lie on the linear part of the preference functions lead to changes in single criterion flows (highlighted in yellow). Since the relative position of A_3 against A_2 and A_1 resp. is weakened in both cases, both of these shifts decrease the net flow of A_3 . Shifts represented via dashed grey lines do not alter the flows.

arguments against the preference of A_3 compared to A_2 , passing the indifference threshold of 22.5. In turn, the pairwise comparison (A_3, A_1) weakens the favourable evaluation of the third alternative.

While these changes make it possible to restore the original net flow of A_3 , the net flows of A_1 and A_2 are different from what they were before these interventions. In contrast to the fully compensatory case shown in Section 4.1.1, the net flows of all alternatives can not be reinstated given the selected preference functions in this case. It resembles an instance of partial compensation in PROMETHEE II.

In addition, some performance scores are affecting pairwise comparisons which are located at a linear part of a preference function, for example, as for $g_2(A_2)$. For them, every change would result in a change in outranking flows, leading to an empty interval of possible performance changes. The effects of all performance changes that are stated in Table 4 on the pairwise evaluations are visualised in the [Supporting Information](#).

4.2 | Compensation in PROMETHEE I

The PROMETHEE I preference relations are established based on a comparison of positive and negative flow scores for each pair of alternatives (see Equation 8). For calculating the positive flows, the preference indices from the pairwise comparisons of a given alternative with all other alternatives are summed up (Equation 7). In other words, only the flows that favour this alternative are able to increase the positive flow. The negative flows are obtained from the pairwise comparisons of all other alternatives with a given one, that is, only the pairwise comparisons where this alternative performs worse do increase the negative flow. Since the PROMETHEE net flow is the difference between the positive and negative flows (Equation 9), the compensatory properties determined in the

previous section also hold for the PROMETHEE I ranking. Within the bounds that are obtained via Equation (14), compensatory effects can also not occur in PROMETHEE I.

Beyond these bounds, compensatory effects can also occur in PROMETHEE I. That means, the partial preorder that is created based on the positive and negative flow scores in PROMETHEE I is susceptible to compensatory effects. There can occur performance changes on a criterion for a given alternative that are able to reinstate a preference structure that has been previously destroyed by a change in that alternative's performance on another criterion in PROMETHEE I. However, the extent of the performance change that is required to destroy and subsequently reestablish a PROMETHEE I preference structure does not necessarily match the extent of changes that is required to alter the PROMETHEE II ranking. For example, a given $\Delta g_j(a_i)$ may therefore be high or low enough to suggest that two alternatives become *incomparable*, while the PROMETHEE II preference relations still hold for the same two alternatives. The same holds for the corresponding performance change $\Delta g_k(a_i)$ that is able to reestablish the original PROMETHEE I preference relation. While it definitely will affect the single criterion flows, it may or may not be high enough to affect the PROMETHEE II preference relations.

5 | Implications and Discussion

The results presented in the previous section demonstrate that a decision problem evaluated using the PROMETHEE methods can be subject to compensatory effects. In this section, the practical implications of the analysis are examined at hand of a real-world environmental management case study. Furthermore, the results are discussed in relation to the central assumptions and definitions of this work.

5.1 | Guidelines to Control Compensation in the PROMETHEE Methods

Compensation manifests in the single criterion flows and can influence the PROMETHEE net flows, potentially affecting the preference structures derived from them. Under full compensation, changes in single criterion flows do not affect the net flows. Conversely, in a non-compensatory setting, any loss or gain in

single criterion flows is fully transferred to an alternative's net flow. In a partially compensatory setting, changes in net flow can be counterbalanced to some extent by adjustments in other single criterion flows. Compensatory effects may either be deliberately incorporated or only partly desired for certain criteria, or they may be explicitly avoided, depending on the nature of the decision problem and the preferences of the decision maker (DM).

Table 5 proposes a general set of guidelines for designing and parameterising a PROMETHEE model to control compensation. These guidelines assist analysts and practitioners during the modelling stage in addressing compensation issues. Special emphasis is placed on selecting the six preference functions provided by PROMETHEE and on the compensatory interactions between pairs of criteria (or preference functions).

For criteria modelled using the type I preference function, no change in performance score will lead to changes in the outranking relations, unless an alternative gets better compared to any other one with regard to this criterion. In these cases (marked with an asterisk* in Table 5), compensation is possible, and whether partial or full compensation can be achieved depends on the criteria weights. This requires careful handling by the analyst to avoid unwanted compensatory effects. We will underpin this special case and its practical implications at hand of a real-world case study in Section 5.2.2. Otherwise, compensation is neither required nor is it possible for criteria modelled with a type I preference function to compensate for losses in other criteria.

If an indifference area is introduced to the type I preference function, as in the type II criterion, compensating effects may immediately occur. A performance change that is large enough to affect the evaluation of an alternative can then be compensated by a large enough performance change on another criterion. If this other criterion is also modelled as type II, full compensation is possible (adjusted for the criteria weights). Criteria of type III, IV, V or VI are able to at least partially compensate for the loss in single criterion flows of a type II criterion. The precondition for the occurrence of partial compensation is that the performance change in the compensating criterion is substantial enough to shift a pairwise comparison of alternatives to a different piece of the function. The initial outranking flows of all alternatives, however, will most likely not be fully reestablished by this change.

TABLE 5 | Type of preference function and possible extent of compensation in the PROMETHEE methods.

		Type of compensating criterion					
		Type I	Type II	Type III	Type IV	Type V	Type VI
Type of criterion to be compensated	Type I	—*	—*	—*	—*	—*	—*
	Type II	—*	—/●	●	—/●	—/●	●
	Type III	—*	—/●	●	—/●	—/●	●
	Type IV	—*	—/●	●	—/●/●	—/●	●
	Type V	—*	—/●	●	—/●	—/●/●	●
	Type VI	—*	—/●	●	—/●	—/●	●/●

Note: ●: Full compensation; ●: Partial compensation; —: No compensation; *: Compensation possible in special cases. Type I: Usual criterion; Type II: Usual criterion with indifference; Type III: Linear criterion; Type IV: Level criterion; Type V: Linear criterion with indifference; Type VI: Gaussian criterion.

Changes in performance on type III criteria induce outranking flow alterations in most situations. A corresponding change on a second criterion of type II that is substantial enough may then be able to partially compensate for this loss. For a pair of two type III criteria, even full compensation is possible, as demonstrated in Section 4.1.1. A special case is the interaction between criteria of type III and V. Since an indifference area is introduced in the latter, compensation does not necessarily occur and it may also not be guaranteed to identify a performance change that fully reinstates the initial outranking flows, although this is theoretically possible.

Given the distinct shape of the Gaussian criterion, it does mostly lead to partially compensatory characteristics. It is not possible to reinstate the outranking flows of all alternatives in decision problems with at least three alternatives unless the interacting criteria pair is modelled by two equally shaped Gaussian criteria. The same reasoning applies to other combinations of equally shaped criteria. Although arriving at equal preference function configurations is rather unlikely in practical applications, occurrences of full compensation cannot be completely disregarded.

5.2 | Case Study: Capturing Occurrences of Compensation in an Environmental Decision Problem

These guidelines provide an initial orientation for practical purposes to control compensation in the PROMETHEE methods. However, they also demonstrate that the compensatory properties strongly depend on the structure of the decision problem and the PROMETHEE model. The outranking logic of PROMETHEE I and II and the piecewise definition of the six preference functions make it difficult to derive generalisable recommendations. At hand of a case study, we now demonstrate

how the insights of the compensation sensitivity analysis and these guidelines can be integrated to attain a finer level of control over the issue of compensation.

5.2.1 | Flow Insensitivity Intervals

The decision table stated in Table 6 stems from a real-world environmental management problem (de Bourgoing et al. 2022). Four different infrastructure concepts for drinking water production through seawater desalination and subsequent long-range transfer are assessed according to PROMETHEE. The objectives are from the technical, economic, social, environmental and political domain. The 11 respective criteria are measured in different units of measurement and have been assigned different weights w_j . A detailed description of the decision problem, the alternatives, criteria and the preference elicitation process can be found in Schär (2024). Besides the alternatives' performance scores, the polarity of criteria, the selected PROMETHEE preference functions, threshold parameters and the flow insensitivity intervals (see Section 4.1.3.2) are displayed in Table 6.

According to PROMETHEE II, Alternative A_4 is ranked first, followed by A_1 , A_3 and A_2 (Table 7). As shown before, the outranking flows, and thus the ranking, will not be affected by performance changes Δg_j within the flow insensitivity intervals. For example, an increase of $g_1(A_1)$ will have not any effect on the outranking flows, as long as $1021.58 < g_1(A_1) < 1033.99$. It is also observable that the outranking flows are very sensitive towards performance score changes in a lot of criteria, that is, where the flow insensitivity interval is empty and any change in performance would affect the flows.

In the presented case, the ranking is highly susceptible to changes in input data. Although A_4 is preferred over A_1 according to both

TABLE 6 | Decision table for an environmental management decision problem.

	Technical			Economic		Social		Environmental		Political	
Criterion	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
w_j	0.08	0.11	0.06	0.12	0.05	0.09	0.09	0.11	0.07	0.05	0.17
Function type	V	IV	IV	III	III	IV	IV	V	V	IV	IV
Polarity	Min	Max	Max	Min	Min	Max	Max	Min	Min	Max	Max
p_j	74.38	2	2	0.10	528.64	2	2	5.8	0.09	2	2
q_j	37.19	1	1	—	—	1	1	2.9	0.05	1	1
$g_j(A_1)$	937.4	5	5	1.52	3340	5	5	70	3.91	5	5
$g_j(A_2)$	947.2	4	5	1.74	4682	7	7	95	3.89	7	7
$g_j(A_3)$	996.8	2	4	1.50	3873	6	5	87	3.67	6	6
$g_j(A_4)$	1028.2	3	5	1.34	3022	6	6	80	3.47	5	5
$\Delta g_j(A_1)$	{}	{}	$[-1, 0]$	{}	{}	{}	{}	$[-70, 4.2]$	$(-0.07, 0.03)$	{}	{}
$\Delta g_j(A_2)$	{}	{}	$[-1, 0]$	$(-0.12, \infty)$	$(-280.36, \infty)$	{}	{}	$(-2.2, 5)$	$(-0.03, 0.07)$	{}	{}
$\Delta g_j(A_3)$	{}	{}	$[0, 2]$	{}	{}	{}	{}	$(-1.2, 2.2)$	$(-0.11, 0.13)$	{}	{}
$\Delta g_j(A_4)$	$(-6.62, 5.79)$	{}	$[-1, 0]$	$[-1.34, 0.06]$	{}	{}	{}	$(-4.2, 1.2)$	$[-3.47, 0.11]$	{}	{}

PROMETHEE I and PROMETHEE II, the flows of both alternatives are relatively close to each other. In such a situation, there is a risk that undesired compensatory effects between criteria from different objective domains have influence on the ranking. Thus, in addition to a cautious interpretation of the ordinal PROMETHEE II ranking, particular attention should be paid to the issue of compensation.

Assume the performance of the originally preferred alternative A_4 on the social criterion g_6 reduces by 2. This performance change also changes the evaluation, so that A_1 is now preferred over A_4 . At the same time, there is an improvement with regard to a criterion of another dimension, for example, in g_1 , which can compensate for this effect. In this instance, g_6 is of type IV and a criterion to be compensated, while g_1 is a compensating criterion of type V. This constellation allows for partial compensation (see Table 5). A performance increase of A_4 on g_1 , for example, by 20 units, could then reinstate the original ranking of alternatives. This situation is summarised in the first part of Table 8.

In general, there are two different modelling approaches to avoid compensation between selected criteria:

1. Modifying threshold parameters. This allows retaining the preference function types of the criteria concerned. The preference thresholds are modified only for selected interactions of performance scores to shift from the partially compensatory to the non-compensatory area of a single criterion flow function.
2. Changing the preference function type of criteria that should not be able to compensate for other criteria. In the

most far-reaching case, preference function type I can be selected for such criteria. This drastically reduces the possibility of compensation.

These general approaches, particularly the latter, should not be adopted unconditionally, as an incautious application can substantially impact the aggregation results. In certain cases, it may even totally counteract the initial intention to avoid compensation.

Modifying threshold parameters, while keeping the shape of the preference function, could be sufficient to reflect a DMs preferences on the issue of compensation. For this, the compensation sensitivity analysis provides helpful starting points. Modifications of preference functions, however, should always be accompanied by consistency checks with the DM. Referring to criterion g_1 in this case, this would mean to shift the non-compensatory areas of the type V preference function. To showcase, the outranking flows for the performance score $g_1(A_4)$ are insensitive down to a value of 1021.58 (and up to 1033.99). Beyond these bounds, the preference threshold of 74.38 is exceeded in the pairwise comparisons of A_4 with A_2 on criterion g_1 . This means single criterion flows change and performance changes of A_4 on g_1 can compensate for performance changes on other criteria. Let us assume that this is unwanted. Reducing the preference threshold p_1 , for example, by 20 units to 54.38, increases the insensitivity interval of A_4 on g_1 . This also reduces the occurrence of undesired compensation mechanism when the performance of A_4 changes on this criterion. Although this approach is not entirely free from interactions, for example, in the pairwise comparison of $g_1(A_1)$ with $g_1(A_3)$, these interactive effects on the overall evaluation of the alternatives are in most cases much smaller compared to those resulting from a change in preference functions types.

A change in preference function types represents a more drastic measure to prevent compensatory effects in a PROMETHEE model. Indeed, defining criterion g_1 as a usual criterion of type I may totally prevent changes in single criterion flows when the performance of an alternative on g_1 changes and thus the ability of this criterion to compensate for performance changes on other criteria. Nevertheless, changing preference function types also fundamentally alters the original outranking flows before

TABLE 7 | Outranking flows of the decision problem in Table 6 and ranking according to PROMETHEE II.

Rank	Alternative	ϕ^+	ϕ^-	ϕ^{net}
1	Alternative A_4	0.307	0.145	0.162
2	Alternative A_1	0.281	0.156	0.125
3	Alternative A_3	0.148	0.265	-0.117
4	Alternative A_2	0.157	0.327	-0.169

TABLE 8 | Compensation in an environmental management decision problem and one approach to control it.

Performance	Function	Parameters	Ranking
$g_1(A_4) = 1028.2$	Type V	$p_1 = 74.38$ $q_1 = 37.19$	$A_4 > A_1 > A_3 > A_2$
$g_6(A_4) = 6$	Type IV	$p_6 = 2$ $q_6 = 1$	
$g_1(A_4) = 1028.2$	Type V	$p_1 = 74.38$ $q_1 = 37.19$	$A_1 > A_4 > A_3 > A_2$
$g_6(A_4) = 4$	Type IV	$p_6 = 2$ $q_6 = 1$	
$g_1(A_4) = 1008.2$	Type V	$p_1 = 74.38$ $q_1 = 37.19$	$A_4 > A_1 > A_3 > A_2$
$g_6(A_4) = 4$	Type IV	$p_6 = 2$ $q_6 = 1$	
$g_1(A_4) = 1008.2$	Type V	$p_1 = 54.38$ $q_1 = 37.19$	$A_1 > A_4 > A_3 > A_2$
$g_6(A_4) = 4$	Type IV	$p_6 = 2$ $q_6 = 1$	

possible compensatory effects and may not adequately map the preferences of the DM. In addition, there is even a risk to allow for full compensation on criteria where the alternatives perform relatively similar. The following example will highlight the benefits and drawbacks of this approach to control compensation.

5.2.2 | Compensatory Properties of the Type I Preference Function

As noted above, changing to a type I preference function does not necessarily eliminate the issue of compensation in a PROMETHEE model. It does, however, substantially reduce the sensitivity of it towards compensatory effects. Consider the same decision problem as in Table 8 with the only modifications being (i) criteria g_1 and g_9 are both of type I and (ii) all criteria weights are equal. These changes lead to larger flow insensitivity intervals compared to the initial model, as Table 9 shows. An analogous pattern could be observed if other criteria than g_1 or g_9 were changed to type I.

At the same time, changing to a type I preference function can also create unwanted instances of full compensation. This happens when the alternatives' performances markedly change beyond the flow insensitivity intervals. Figure 6 displays the single

criterion and net flows when the performance of A_4 on criteria g_1 and g_9 changes beyond the flow insensitivity intervals displayed in Table 9. In a first step, $g_9(A_4)$ is increased by 0.2 units. G_9 is an environmental criterion and to be minimised, so that this change reflects a performance loss. The striped bars indicate the new flows after this change is applied. The performance loss of A_4 on the environmental criterion g_9 leads to a rank change so that A_1 becomes the preferred alternative. In a second step, the performance score of A_4 on the technical criterion g_1 is reduced by 31.4 units. Since g_1 is also to be minimised, it reflects a performance gain. The effects that this change has are visualised by the dotted bars in the figure. The introduced performance gain on g_1 fully compensates the loss in flows on g_9 and reinstates the initial net flows and rank order. The single criterion flows on all other criteria are not affected by the performance changes and thus not displayed.

This example highlights that the previously introduced guidelines offer starting points for considering the issue of compensation in the PROMETHEE modelling process. In addition, the flow insensitivity intervals offer a more detailed means to analyse the occurrence and extent of compensation in a targeted manner. They are particularly potent if additional information on the uncertainty in the performance scores is known. In this case, preference thresholds can be modified considering this

TABLE 9 | Flow insensitivity intervals for an environmental management decision problem with adapted preference functions.

Criterion	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
Function type	I	IV	IV	III	III	IV	IV	V	I	IV	IV
$\Delta g_j(A_1)$	[- 937.4, 9.8)	{}	[- 1, 0]	{}	{}	{}	{}	[- 70, 4.2)	(- 0.02, ∞)	{}	{}
$\Delta g_j(A_2)$	(- 9.8, 49.6)	{}	[- 1, 0]	(- 0.12, ∞)	(- 280.36, ∞)	{}	{}	(- 2.2, 5)	(- 0.22, 0.02)	{}	{}
$\Delta g_j(A_3)$	(- 49.6, 31.4)	{}	[0, 2]	{}	{}	{}	{}	(- 1.2, 2.2)	(- 0.2, 0.22)	{}	{}
$\Delta g_j(A_4)$	(- 31.4, ∞)	{}	[- 1, 0]	[- 1.34, 0.06)	{}	{}	{}	(- 4.2, 1.2)	[- 3.47, 0.2)	{}	{}

Note: Bold-faced intervals indicate changes.

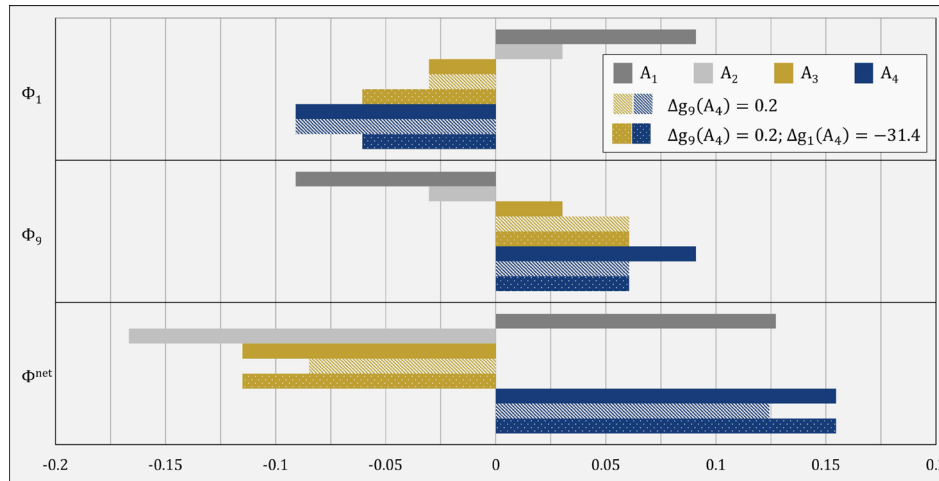


FIGURE 6 | PROMETHEE single criterion flows and net flows for the decision problem in Table 9. Criteria g_1 and g_9 modelled via type I preference function. Initial net flows are indicated by solid bars. Striped bars (performance of A_4 on g_9 changed beyond insensitivity boundaries) indicate the change in net flows and ranking between A_1 and A_4 . Dotted bars (performance of A_4 on g_1 changed in opposite polarity) showcase full compensation between both criteria.

uncertainty to circumvent unwanted compensatory effects in a targeted and granular manner.

5.3 | Discussion

The previous analysis is based on a number of deliberate assumptions regarding the definition of compensation and the nuances in which it can occur. We now discuss these assumptions and the practical implications that can be drawn from them.

For the purpose of this work, the general notion of compensation in an MCAP has been defined as the property that preference relations, which are dissolved by changes in an alternative's performance, can be reinstated by a performance change on any other criterion for the same alternative (see Section 2.3).

A definition analogous to other ranking approaches, such as MAUT/MAVT, could refer to the net flows instead of the ranking of alternatives derived from comparing their net flows. Adapted to the PROMETHEE methods, compensation would then be defined as the possibility of net flow substitution between criteria. According to this revised definition, an instance of PROMETHEE is fully compensatory if there exists a pair of performance score changes for a given alternative that maintains its net flow. However, such a definition may not be a reasonable choice in light of the method's different axiomatic foundations compared to the MAUT/MAVT (see Section 2.1). Additionally, the actual net flows are typically not used when interpreting a ranking that has been conducted according to PROMETHEE II, as often done for value theory-based approaches (Coquelet et al. 2024). We therefore argue that the broad definition of a compensating MCAP adopted in this work, based on the fundamental ability of altering or preserving preference structures (Section 2.3) is more reasonable.¹ This fundamental definition of compensation can then be operationalised for the PROMETHEE methods by means of the single criterion flows. It allows for nuance between partial and full compensation based on the ability of single criterion flow substitution and is of higher use for practical application of PROMETHEE I and II.

In this regard, we presented a fully compensatory case of PROMETHEE II in Section 4.1.1. It is based on a set of artificial assumptions that appear not very realistic for practical application. We especially do not recommend setting the preference threshold of a type III preference functions in such a way since we believe that it contradicts the conceptual foundations of the PROMETHEE methods. We only use it here to showcase that there can be instances of compensation in the PROMETHEE methods. More precisely, this case shows that PROMETHEE II can be fully compensatory, according to how we define the notion of compensation in MCAP. It provides contradicting evidence to some statements in the literature on the compensatory characteristics of the PROMETHEE methods.

The theoretical insights gained from this analysis lead to several practical considerations, especially in managing compensatory effects within PROMETHEE models and facilitating communication with DMs. As highlighted in the case study (Section 5.2), the type I function can be an approach to circumvent compensation between criteria. However, it requires careful attention

since there may be performance changes that introduce full compensation into a relationship of two criteria (see Section 5.2.2). Resorting to a type I preference function also comes at the expense of being unable to model the DMs preferences in finer resolution, which is a distinct feature of the PROMETHEE methods compared to other outranking methods. The type I criterion is often criticised because it does not take into account imperfect information, ambiguity or other factors that might cause a DM to hesitate in expressing a clear preference for one action over another (Brans et al. 1986; Roy et al. 2014). In turn, when the DM requests another preference modelling, for example, via the type III preference function, but simultaneously desires no compensation for this criterion, careful communication and adjustments are required. To reconcile their preference for no compensation, it would be necessary to switch to other functions and introduce preference or indifference thresholds that model this purpose. This requires a dialogue with the DM and a case-specific and deliberate trade-off between modelling choices.

6 | Conclusion

In this work, we conducted an in-depth analysis of the compensatory properties of the PROMETHEE outranking methods. Our focus has been on characterising the determinants of compensation within PROMETHEE I and II. Compensatory effects can preserve a preference structure when performance on multiple objectives changes at the same time. Since this behaviour may be undesired for certain objectives, for example, to enforce the principle of strong sustainability, a preservation of the ranking due to compensatory mechanisms may not reflect the DMs actual preferences. In particular, we have revealed how the choice of preference functions and elicitation of their parameters can allow for instances of full or partial compensation regarding the derived preference structure. Ultimately, there are instances where PROMETHEE II and also PROMETHEE I can be compensatory for any type of preference function. The findings contribute to the existing research by disclosing determinants for compensation in PROMETHEE I and II and offering mechanisms of different granularity to capture and control compensatory effects in these methods.

We have introduced a compensation sensitivity analysis as a novel tool to investigate the sensitivity of a PROMETHEE decision model towards compensatory effects and highlighted different approaches to control compensation. Flow insensitivity intervals help decision-makers (DMs) and analysts to identify components of a decision model that may trigger compensatory effects and conduct appropriate changes. Specifically, they allow determining the changes in performance scores that will affect the outranking flows and thus can compensate for changes on other criteria. This information can be utilised to control compensatory effects in a fine-grained resolution by means of adjusting the preference functions or the threshold parameters.

An environmental management case study demonstrates that controlling compensation in a PROMETHEE model can have a substantial impact on the final ranking. By deliberately controlling for compensation, it is possible to avoid unintended shifts in the ranking of alternatives that might happen due to compensatory effects between a particular pair of criteria. Controlling

for compensation is enabled by referring to the set of general guidelines and by using the results of the compensation sensitivity analysis developed in this study. It allows one to take the aspiration of a non-compensatory decision model into account in the design and parameterisation of a PROMETHEE model.

The question of how to deal with compensatory properties in MCAP and the recognition of incomparability has contributed markedly to the emergence of some outranking methods. As highlighted by Dejaegere and De Smet (2023), compensatory properties could be linked to the existence of incomparability. In further research, a detailed analysis of the links between compensation and (in)comparability in PROMETHEE I, II could provide interesting insights. In particular, the investigation of whether and when compensation in PROMETHEE II leads to incomparability in PROMETHEE I and comparisons with the newly proposed PROMETHEE γ method. Another central characteristic of outranking procedures is the logic of pairwise comparisons and the much-discussed rank reversal phenomenon, which is strongly related to it. The relations between rank reversal and compensation, however, are not yet disclosed.

Future research could also explore further applications of the compensation sensitivity analysis across different problem contexts and settings. An analytical study to explore the characteristics of a decision problem, in terms of the number of alternatives and criteria, could be an interesting point of departure. In addition, an investigation into what extent and how the presented findings on the compensation behaviour of the PROMETHEE methods can systematically inform the preference elicitation process also poses an intriguing avenue of research.

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Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

The data that support the findings of this study are available in the [Supporting Information](#) of this article.

Endnotes

¹ Even though, putting increased attention on the actual flow scores that PROMETHEE yields could also yield important insights in the analysis of results (Dejaegere et al. 2022; Dejaegere and De Smet 2023).

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Supporting Information

Additional supporting information can be found online in the Supporting Information section.

Appendix A

Generalisation of the Compensatory Case for PROMETHEE II

Theorem 1. *If the preference functions of two criteria in PROMETHEE are defined as in Equation (13), the required gain in criterion g_k to achieve compensation w. r. t. to the PROMETHEE II net flow of Alternative a_i for a loss in criterion g_j is given by*

$$\Delta g_k(a_i) = - \frac{p_k \cdot w_j}{p_j \cdot w_k} \cdot \Delta g_j(a_i) \quad (A1)$$

Proof. The PROMETHEE net flow of an alternative is determined according to Equations (9) and (10). Any value of $\Delta g_j(a_i) \neq 0$ then alters the PROMETHEE net flow of an Alternative by $\Delta \phi^{net}(a_i)$, while

$$|\Delta \phi^{net}(a_i)| = \left| \phi^{net}(a_i) - \left[\frac{1}{m-1} \sum_{j=1}^n \sum_{a_x \in A} \underbrace{\Delta \mathcal{P}_j(a_i, a_x) - \Delta \mathcal{P}_j(a_x, a_i)}_{g_j(a_i) = g_j(a_i) + \Delta g_j(a_i)} \cdot w_j \right] \right|$$

The PROMETHEE net flow of an alternative after the performance on two criteria, criterion g_j and any other criterion g_k , has been altered is denoted $\phi^{net}(a_i)'$. The performance change that is required on any other criterion g_k to reinstate the original PROMETHEE net flow and thus verify that

$$\phi^{net}(a_i)' \stackrel{!}{=} \underbrace{\phi^{net}(a_i) + \Delta \phi^{net}(a_i)}_{=0}$$

is then given by

$$\begin{aligned}
& \phi^{net}(a_i)' = \phi^+(a_i)' - \phi^-(a_i)' \\
&= \frac{1}{m-1} \sum_{\substack{a_x \in A, \\ i \neq x}} \pi(a_i, a_x) - \frac{1}{m-1} \sum_{\substack{a_x \in A, \\ i \neq x}} \pi(a_x, a_i) \\
&= \frac{1}{m-1} \sum_{\substack{a_x \in A, \\ i \neq x}} \sum_{j=1}^n \mathcal{P}_j(d_j(a_i, a_x)) \cdot w_j - \frac{1}{m-1} \sum_{\substack{a_x \in A, \\ i \neq x}} \sum_{j=1}^n \mathcal{P}_j(d_j(a_x, a_i)) \cdot w_j \\
&= \frac{1}{m-1} \cdot \sum_{a_x \in A} \left[\frac{g_j(a_i) + \Delta g_j(a_i) - g_j(a_x)}{p_j} \cdot w_j + \frac{g_k(a_i) + \Delta g_k(a_i) - g_k(a_x)}{p_k} \cdot w_k + \dots \frac{g_n(a_i) - g_n(a_x)}{p_n} \cdot w_n \right] \\
&\quad - \frac{1}{m-1} \cdot \sum_{a_x \in A} \left[\frac{g_j(a_x) - g_j(a_i) + \Delta g_j(a_i)}{p_j} \cdot w_j + \frac{g_k(a_x) - g_k(a_i) + \Delta g_k(a_i)}{p_k} \cdot w_k + \dots \frac{g_n(a_x) - g_n(a_i)}{p_n} \cdot w_n \right].
\end{aligned}$$

Under the condition that $\Delta \phi^{net}(a_i) = 0$, we then have that

$$\begin{aligned}
& \Delta \phi^{net}(a_i) = \phi^{net}(a_i) - \phi^{net}(a_i)' \\
&= \frac{1}{m-1} \cdot \left[\frac{w_j \cdot (g_j(a_i) - g_j(a_x))}{p_j} + \frac{w_k \cdot (g_k(a_i) - g_k(a_x))}{p_k} - \frac{w_j \cdot (g_j(a_i) + \Delta g_j(a_i) - g_j(a_x))}{p_j} + \frac{w_k \cdot (g_k(a_i) + \Delta g_k(a_i) - g_k(a_x))}{p_k} \right] \\
&= p_k \cdot w_j (g_j(a_i) - g_j(a_x) - g_j(a_i) + g_j(a_x) + \Delta g_j(a_i)) \\
&\quad - p_j \cdot w_k (g_k(a_x) - g_k(a_i) - g_k(a_x) + g_k(a_i) + \Delta g_k(a_i)) \\
&= p_k \cdot w_i \cdot \Delta g_j(a_i) - p_j \cdot w_k \cdot \Delta g_k(a_i) \\
&= \Delta g_k(a_i) = - \frac{p_k \cdot w_j}{p_j \cdot w_k} \cdot \Delta g_j(a_i)
\end{aligned}$$

TABLE A1 | Overview on the six generalised preference functions in the PROMETHEE methods (Brans and De Smet 2016).

Criterion	Graph	Definition	Parameter
Type I: Usual criterion		$p_j(d_j) = \begin{cases} 0 & d_j \leq 0 \\ 1 & d_j > 0 \end{cases}$	—
Type II: Usual criterion with indifference area		$p_j(d_j) = \begin{cases} 0 & d_j \leq q_j \\ 1 & d_j > q_j \end{cases}$	q_j
Type III: Criterion with linear preference		$p_j(d_j) = \begin{cases} 0 & d_j \leq 0 \\ \frac{d_j}{p_j} & 0 \leq d_j \leq p_j \\ 1 & d_j > p_j \end{cases}$	p_j
Type IV: Level criterion		$p_j(d_j) = \begin{cases} 0 & d_j \leq q_j \\ \frac{1}{2} & q_j < d_j \leq p_j \\ 1 & d_j > p_j \end{cases}$	p_j, q_j
Type V: Criterion with linear preference and indifference area		$p_j(d_j) = \begin{cases} 0 & d_j \leq q_j \\ \frac{d_j - q_j}{p_j - q_j} & q_j < d_j \leq p_j \\ 1 & d_j > p_j \end{cases}$	p_j, q_j
Type VI: Gaussian criterion		$p_j(d_j) = \begin{cases} 0 & d_j \leq 0 \\ 1 - e^{-\frac{d_j^2}{2\sigma_j^2}} & d_j > 0 \end{cases}$	σ_j