

Grimm, Veronika et al.

**Article — Published Version**

## On a tractable single-level reformulation of a multilevel model of the European entry-exit gas market with market power

Journal of Global Optimization

**Provided in Cooperation with:**

Springer Nature

*Suggested Citation:* Grimm, Veronika et al. (2025) : On a tractable single-level reformulation of a multilevel model of the European entry-exit gas market with market power, Journal of Global Optimization, ISSN 1573-2916, Springer US, New York, NY, Vol. 91, Iss. 4, pp. 953-985, <https://doi.org/10.1007/s10898-025-01475-8>

This Version is available at:

<https://hdl.handle.net/10419/323688>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*



<http://creativecommons.org/licenses/by/4.0/>



# On a tractable single-level reformulation of a multilevel model of the European entry-exit gas market with market power

Veronika Grimm<sup>1</sup> · Julia Grübel<sup>1</sup> · Martin Schmidt<sup>2</sup> · Alexandra Schwartz<sup>3</sup> · Ann-Kathrin Wiertz<sup>4</sup> · Gregor Zöttl<sup>4</sup>

Received: 13 November 2023 / Accepted: 20 October 2024 / Published online: 22 March 2025  
© The Author(s) 2025

## Abstract

We propose a framework that allows to quantitatively analyze the interplay of the different agents involved in gas trade and transport in the context of the European entry-exit system. Previous contributions have focused on the case of perfectly competitive buyers and sellers of gas, which allows to replace the respective market equilibrium problem by a single welfare maximization problem. Our novel framework considers the mathematically more challenging case of a monopolistic and thus strategic gas seller. In this framework, the objective functions of the gas sellers and buyers cannot be aggregated into a common objective function, which is why a multilevel formulation is necessary to accurately capture the sequential nature of the decisions taken. For this setup, we derive sufficient conditions that allow for reformulating the challenging four-level model as a computationally tractable single-level reformulation. We prove the correctness of this reformulation and use it for solving several test instances to illustrate the applicability of our approach.

---

✉ Martin Schmidt  
martin.schmidt@uni-trier.de

Veronika Grimm  
veronika.grimm@utn.de

Julia Grübel  
julia.gruebel@utn.de

Alexandra Schwartz  
alexandra.schwartz@tu-dresden.de

Ann-Kathrin Wiertz  
ann-kathrin.wiertz@fau.de

Gregor Zöttl  
gregor.zoetl@fau.de

<sup>1</sup> Energy Systems and Market Design Lab, University of Technology Nuremberg (UTN), Ulmenstrasse 52h, 90443 Nürnberg, Germany

<sup>2</sup> Department of Mathematics, Trier University, Universitätsring 15, 54296 Trier, Germany

<sup>3</sup> Faculty of Mathematics, Technische Universität Dresden, Zellescher Weg 12-14, 01069 Dresden, Germany

<sup>4</sup> Industrial Organization and Energy Markets, Friedrich-Alexander-Universität Erlangen-Nürnberg, Lange Gasse 20, 90403 Nürnberg, Germany

**Keywords** Multilevel optimization · Reformulations · Gas markets · Market power

**Mathematics Subject Classification** 91B24 · 91B74 · 90C26 · 90C35

## 1 Introduction

We analyze the organization of gas trade and transport in the context of the European gas market design. In Europe, the so-called “entry-exit system” aims at largely separating gas trade and transport. Agents have to take an involved sequence of actions, which first lead to gas trading independently of grid restrictions and then transport of the traded volumes through the network. In particular, the transmission system operator (TSO) first announces so-called “technical capacities”, which are capacities available to the buyers and sellers at the different entry and exit nodes. These technical capacities must be chosen such that any balanced set of quantities less than the technical capacity can be transported through the network. In a second step, suppliers and consumers “book” capacities at the nodes to secure access to the network, trade gas at the marketplace (being capacity-constrained by their bookings) and then “nominate” the quantities bought and sold at the market for transport. Finally, the gas is routed through the network by the TSO, where feasibility is guaranteed by the *ex ante* restrictions on technical capacities. For the specific institutional rules we refer to [41] and [32].

It is important to notice that the entry-exit system hinders the efficient use of existing network capacities due to the restrictive requirements on technical capacities. This was not especially harmful when there was abundant network capacity available, as it has been the case for European gas markets. However, in the context of the green transition, a good understanding of the functioning of the entry-exit system in Europe is of increasing importance. Pipeline-bound hydrogen-transport will gain significant importance and future hydrogen networks to a large extent are likely to be built from parts of the existing natural gas networks,<sup>1</sup> which implies that for both, the gas and the hydrogen network, transmission constraints will become more and more relevant. Within the EU, the entry-exit system will also be applied to organize transport for renewable gases and hydrogen; see [40]. Thus, a comprehensive understanding of the limitations and the efficiency losses of the entry-exit system is crucial to assess appropriate regulatory decisions for natural gas networks and the envisioned hydrogen networks in Europe.

The current EU gas market design has already been analyzed in recent scientific contributions that focus on important aspects of properly organizing those markets. A recent strand of literature provides setups that allow to quantitatively assess gas trade and transport in the context of the entry-exit system. Due to the sequential decision-making structure outlined above, this involves multilevel approaches, which, in general, are difficult to tackle as it is known that even bilevel problems are hard to solve; see, e.g., [21, 31] for general hardness results. Recent articles, see, e.g., [7, 19], provide models and computational approaches under the assumption of perfectly competitive behavior of all agents acting on gas markets. In this case, modeling of booking and nomination can be jointly reformulated as decisions taken on the lower level of a bilevel model, whereas all decisions taken by the TSO are modeled in

<sup>1</sup> Compare, e.g., the current plans to build a European hydrogen backbone, which is “based on existing and new pipelines [...]. By 2040 this could include a majority 60 % repurposed pipelines and 40 % new.”; see <https://ehb.eu/newsitems>.

the resulting upper level. The obtained bilevel problem can then be finally reformulated as a single-level optimization problem.

However, market power and strategic behavior of gas suppliers play a crucial role in gas markets; see, e.g., [52]. It is the goal of our analysis to provide and solve a computationally tractable framework, which allows to analyze the interaction of the TSO, price-taking gas buyers, and a strategic monopolistic gas seller in the context of the European entry-exit system, which leads to a setup that significantly differs from the ones discussed in, e.g., [7, 19].

In more detail, as a starting point of our modeling approach, we consider the sequential decisions taken by the TSO (choice of technical capacities and routing of gas transport), a single strategic and profit-maximizing gas seller (booking and nomination) and gas buyers, which are assumed to be price takers (booking and nomination). The objectives of the seller and the buyers differ in this case. Unlike in a setup under the assumption of perfectly competitive agents, see, e.g., [7, 19, 24], the integration of seller and buyer decisions in a single optimization problem is not possible in this case. The resulting interaction of (i) the TSO while setting technical capacities for entry and exit nodes and routing the gas as well as of (ii) the seller and buyers while making their booking and nomination decisions in the context of the entry-exit system thus results in four different levels. The entire model is formulated in Sect. 2. As this cannot be solved directly, we derive a computationally solvable single-level reformulation of the overall problem in Sect. 3. Based on this reformulation, we can solve test instances and conduct performance checks; see Sect. 4.

Our work is related to several strands of the literature. Given the relevance of gas markets in general and given the importance of strategic behavior especially on the supply side, there is a large and well-developed body of literature, which considers many different aspects of strategic supply decisions. Different from our work, which explicitly takes into account real-world market rules arising in the context of the entry-exit system, however, this first strand of literature does not explicitly consider and model the specific institutional market rules of gas trading and transport of the involved companies and network operators.

First, among those contributions we observe highly stylized frameworks that allow for an analytic solution of the considered problem. This makes it possible to identify and discuss general and economically meaningful trade-offs that strategic firms and their customers are facing when trading gas. Prominent examples include [13, 35], which discuss the impact of different network access fees. In [30, 50], the authors consider the geopolitical interests involved in the activities of Gazprom and its short run impact on consumers in Europe. Finally, the studies in [27, 29, 39] rely on setups based on cooperative game theory to discuss the strategic structure of gas export and transit from Russia to Europe.

Second, there are more general setups that provide quantitative results and have to be solved by appropriate computational methods. Those computational approaches often rely on formulations as a complementarity problem, which in principle requires a linear representation of the considered network flow model. Seminal contributions include [6, 8, 17, 26, 51]. Subsequent contributions further provide extensions to include linear approximations of nonlinear gas flow models, which take into account the influence of pressure gradients; e.g., the so-called Weymouth equation [49]. Examples include [36, 37, 43]. Other contributions consider multi-period setups [14, 15], endogenous production capacities [28], or include risk aspects [2]. Strategic supply even on a simple network structure typically leads to multiplicity of equilibria; see, e.g., [4] for methods to deal with such multiplicities. Finally, several approaches have also been used to provide meaningful quantitative results for specific gas markets such as Russia [9, 10] or the impact of shale gas in the US [45].

Both the analytical as well as the computational approaches mentioned above assume that congestion management of scarce network capacities is organized in an efficient and centralized manner. Hence, and in contrast to our approach, these contributions do not explicitly model and analyze the specific rules of congestion management as they arise in the context of the entry-exit system. The explicit consideration of these market rules, as it is done in this article, involves modeling a sequence of decisions taken by the TSO, the supplier, and the consumers.

Another strand of literature explicitly does consider the specific rules of congestion management in the context of the European entry-exit system. Those contributions are typically based on illustrative examples, which discuss the different aspects and implications of the market rules in detail. Those more policy-oriented contributions, however, do not aim at providing computational setups, which would allow to quantify the impact of specific institutional details; see, e.g., [3, 18, 20, 25, 48]. Thus, as compared to those contributions, our analysis provides a detailed and computationally tractable analysis, which explicitly considers market rules arising in the context of the entry-exit system and allows to determine meaningful quantitative results.

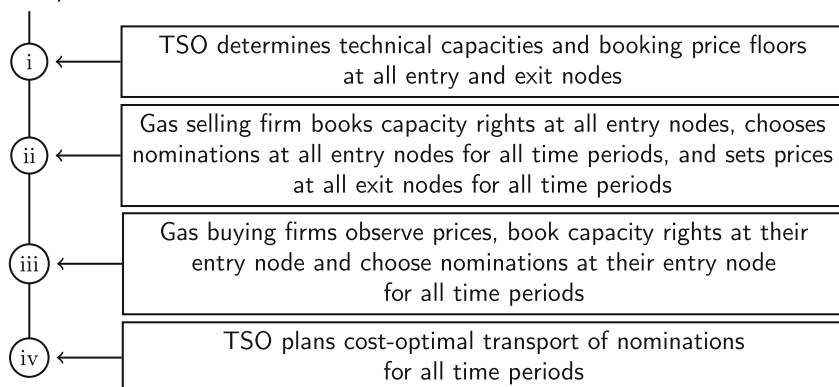
Finally, in a recent strand of literature, several contributions provide setups, which indeed allow for a computational and thus quantitative analysis of the different specific aspects of market interaction in the presence of the entry-exit system. Most of them are based on the model presented in [19]. They all consider setups with perfectly competitive suppliers and consumers and their interaction with the network operator according to the entry-exit system. In [7], the authors exploit several reformulations to obtain an equivalent single-level model that they solve for different institutional setups. Going one step further, load uncertainty has been tackled in [24] by using chance-constrained modeling. For tree-structured networks, highly efficient solution techniques have been developed in [44] and a first step towards addressing active network elements such as compressors are given in [42] for a certain part of the four-level model presented in [19]. All those contributions provide important computational setups that allow to analyze different important aspects of the entry-exit system. In contrast to our contribution, however, they all assume perfectly competitive behavior of gas suppliers. The assumption of perfectly competitive behavior allows to reformulate the originally different levels modeling the interaction of suppliers and consumers as a single welfare maximization problem, which collects the objectives of all suppliers and consumers. In our work, we explicitly consider the important case of strategic gas supply. In this setup, the objectives of the supplier and the consumers are not aligned in this case and therefore cannot be condensed in a joint (welfare maximizing) objective function. Instead, it is necessary to formulate a multilevel model in which, in a sequence, first the strategic gas supplier and then gas consumers take their decisions in the context of the entry-exit system.

The remainder of this paper is structured as follows. In Sect. 2, we present the four-level model with market-power aspects by including a monopolistic gas seller. Afterward, we study the market levels in more detail in Sect. 3 to obtain a single-level reformulation. Numerical results based on this single-level reformulation are presented and discussed in Sect. 4 before we close with some concluding remarks in Sect. 5.

## 2 The four-level model with a monopolistic gas selling firm

It is our scope to analyze the sequential decisions of the TSO, who chooses technical capacities and routes gas, together with the decisions of booking and nomination made by firms. We

decision sequence



**Fig. 1** Sequence of the decisions in the four-level model.

consider the case of strategic booking and nomination decisions of a monopolistic gas seller. The fundamental timing of our setup, in principle, corresponds to the one introduced in [19]. However, as we consider strategic booking and nomination decisions of the gas selling firm, we have to disaggregate seller and buyer decisions into different levels. Formally, this sequential interaction can be described by the following four-level model; see Fig. 1.

Similar to the four-level model proposed in [19], all decisions of the gas sellers and gas buyers are modeled in levels (ii) and (iii). However, in [19], all gas traders book capacities at level (ii) and they nominate quantities in level (iii). In the model we propose, the levels are split according to the different market participants, i.e., the gas seller and the gas buyers. The monopolistic gas selling firm decides on booked and nominated quantities as well as on gas prices at level (ii). The gas buying firms decide on their bookings and nominations at level (iii) based on these gas prices.

We also consider different pricing regimes by introducing price groups. The gas selling firm sets a price for each price group so that all gas buyers in one price group pay the same price. This allows to analyze and compare different pricing regimes like, e.g., nodal pricing, smaller or larger price groups, or a network-wide uniform price.

In the following, we state the multilevel model of the entry-exit market with a monopolistic gas selling firm. We model the gas network as directed and weakly connected graph  $G = (V, E)$ . The node set of this graph is partitioned into entry nodes  $V_+$ , at which the gas selling firm is located, exit nodes  $V_-$ , at which the consumers are located, and the remaining inner nodes  $V_0$ . We consider a finite set of time periods  $t \in T$ .

## 2.1 Level (i): specification of technical capacities and booking price floors by the TSO

The player of level (i) is the transmission system operator (TSO), who specifies technical capacities  $q^{\text{TC}} := (q_u^{\text{TC}})_{u \in V_+ \cup V_-}$  and booking price floors  $\underline{\pi}^{\text{book}} := (\underline{\pi}_u^{\text{book}})_{u \in V_+ \cup V_-}$  for all entry and exit nodes; for a discussion of booking prices and price floors we refer to [19] again. By doing so, the TSO has to ensure that the bookings and nominations that realize in levels (ii) and (iii) can be transported through the network. The optimization problem of the TSO in level (i) reads

$$\max_{q^{\text{TC}}, \underline{\pi}^{\text{book}}} \sum_{t \in T} \left( \sum_{u \in V_-} \int_0^{q_{u,t}^{\text{nom}}} P_{u,t}(s) \, ds - \sum_{u \in V_+} c_u^{\text{var}} q_{u,t}^{\text{nom}} \right) - \varphi^{\text{trans}}(q^{\text{nom}}) - C \quad (1a)$$

$$\text{s.t. } 0 \leq q_u^{\text{TC}}, \quad 0 \leq \underline{\pi}_u^{\text{book}}, \quad u \in V_+ \cup V_-, \quad (1b)$$

$$\sum_{u \in V_+ \cup V_-} \underline{\pi}_u^{\text{book}} q_u^{\text{book}} = \varphi^{\text{trans}}(q^{\text{nom}}) + C, \quad (1c)$$

$$(q_+^{\text{book}}, q_+^{\text{nom}}, \pi^{\text{nom}}) \text{ solves (2),} \quad (1d)$$

$$(p, q) \text{ solves (4).} \quad (1e)$$

The TSO maximizes the welfare, i.e., the sum of the profits of all gas selling and buying firms minus the costs for operating the network and transporting the gas; see Equation (1a). The notation for these lower-level profits will be introduced and explained in the respective sections on levels (ii) and (iii) below. Constraint (1c) models that the transport costs  $\varphi^{\text{trans}}(q^{\text{nom}})$  arising in level (iv) plus additionally given network costs  $C \geq 0$  have to be recovered via the booking fees. Note that, in Constraint (1d), we consider an optimistic setting, i.e., in case of multiple optimal solutions of the lower levels, the TSO chooses the welfare-maximizing solution. As mentioned above, in our setup, the TSO in Constraint (1e) only has to ensure that the realized nominations from levels (ii) and (iii) have to be physically feasible. For further discussion of more robust settings see, e.g., [7, 24].

## 2.2 Level (ii): booking of capacity rights, day-ahead nominations, and setting of gas prices by the gas selling firm

The player of level (ii) is the monopolistic gas selling firm, which is located at the entry nodes  $V_+$  and has variable production costs  $c_u^{\text{var}} > 0$  for all  $u \in V_+$ . This firm has to take into account the technical capacities  $q^{\text{TC}}$  as well as booking price floors  $\underline{\pi}_u^{\text{book}}$  at all entry and exit nodes  $u \in V_+ \cup V_-$ , which are given from level (i). It maximizes its profit by first booking input capacities  $q_+^{\text{book}} := (q_u^{\text{book}})_{u \in V_+}$  and then choosing nominations  $q_+^{\text{nom}} := (q_{u,t}^{\text{nom}})_{u \in V_+, t \in T}$  and setting gas prices  $\pi^{\text{nom}}$  at the exit nodes  $V_-$  for all time periods  $t \in T$ , to which the consumers react as price takers.

We assume that the exit nodes are partitioned into different price groups and that the monopolistic firm sets a gas price being the same for all consumers in each price group. This allows us to consider and compare different pricing regimes. In the literature, this is typically referred to as third-degree price discrimination; see, e.g., [1] for a reference. Let  $V_1, \dots, V_k$  be a partition of the exit nodes  $V_-$  into  $k$  price groups. We use the notation  $[k] := \{1, \dots, k\}$  and denote the gas prices by  $(\pi_{i,t}^{\text{nom}})_{i \in [k], t \in T}$ .

Note that in level (ii), the monopolistic firm decides on bookings  $q_+^{\text{book}}$  and nominations  $q_+^{\text{nom}}$  as well as on gas prices  $\pi^{\text{nom}}$ , while anticipating the bookings  $q_-^{\text{book}}$  and nominations  $q_-^{\text{nom}}$  of the gas buying firms, which are chosen in level (iii).

The monopolistic firm's optimization problem is given by

$$\max_{q_+^{\text{book}}, q_+^{\text{nom}}, \pi^{\text{nom}}} \sum_{t \in T} \left( \sum_{i=1}^k \pi_{i,t}^{\text{nom}} \sum_{u \in V_i} q_{u,t}^{\text{nom}} - \sum_{u \in V_+} c_u^{\text{var}} q_{u,t}^{\text{nom}} \right) - \sum_{u \in V_+} \underline{\pi}_u^{\text{book}} q_u^{\text{book}} \quad (2a)$$

$$\text{s.t. } q_u^{\text{book}} \leq q_u^{\text{TC}}, \quad u \in V_+, \quad (2b)$$

$$0 \leq q_{u,t}^{\text{nom}} \leq q_u^{\text{book}}, \quad u \in V_+, \quad t \in T, \quad (2c)$$

$$\sum_{u \in V_-} q_{u,t}^{\text{nom}} - \sum_{u \in V_+} q_{u,t}^{\text{nom}} = 0, \quad t \in T, \quad (2d)$$

$$(q_-^{\text{book}}, q_-^{\text{nom}}) \text{ solves (3).} \quad (2e)$$

The monopolistic firm's profit (2a) is the difference between the aggregated revenues from selling gas and the costs of the nominated and booked quantities. Constraints (2b)–(2d) model the restriction by technical capacities, compliance with booked capacities, and market clearing.<sup>2</sup> Constraint (2e) models the reactions of the consumers to the market prices  $\pi_{i,t}^{\text{nom}}$  set by the monopolistic firm. The existence of a solution of Problem (2) can be ensured under Assumptions 2.1 and 2.2; see Lemma 3.2 below.

In the context of such a multilevel market model, a common concern is strategic overbooking, which means that a firm or consumer books a capacity that is larger than its maximal nomination in order to prevent its competitors from using this capacity. Note that in our setup, the gas selling firm has no incentive to over-book, i.e., we have  $q_u^{\text{book}} = \max_{t \in T} \{q_{u,t}^{\text{nom}}\}$  for all entry nodes  $u \in V_+$ .

### 2.3 Level (iii): booking of capacity rights and day-ahead nominations by gas buying firms

The players of level (iii) are the gas buying firms, which are located at the exit nodes  $V_-$ . They observe the gas prices  $\pi^{\text{nom}}$ , booking prices  $\pi_u^{\text{book}}$ , and technical capacities  $q_u^{\text{TC}}$  given from levels (i) and (ii), and react to these prices by booking capacities  $q_-^{\text{book}} := (q_u^{\text{book}})_{u \in V_-}$  and by deciding on their nominations  $q_-^{\text{nom}} := (q_{u,t}^{\text{nom}})_{u \in V_-, t \in T}$ . Demand at the exit nodes is modeled by inverse demand functions  $P_{u,t}$ , for which we make the following assumption.

**Assumption 2.1** All inverse demand functions are linear and strictly decreasing. Moreover, the intercepts are the same within each price group, i.e.,

$$P_{u,t}(q_{u,t}^{\text{nom}}) = a_{i,t} - b_{u,t} q_{u,t}^{\text{nom}}$$

holds with  $a_{i,t}, b_{u,t} > 0$  for all nodes  $u \in V_i$  in price group  $i \in [k]$  and all time periods  $t \in T$ .

Note that the specification of market prices depending linearly on demand (inducing quadratic consumer surplus in the objective functions) is very common in the entire literature providing computational approaches for solving equilibrium problems in energy markets; see, e.g., [9, 14, 50] for some examples.<sup>3</sup> Intercepts in our setup can vary with time periods  $t$ . The assumption of equal intercepts  $a_{i,t}$  for all nodes in a price group is worthwhile to note. This does restrict the flexibility of our framework. The assumption is required, however, to guarantee concavity of the maximization problem solved by the seller; see Lemma 3.2 and Problem (13) below.

Regarding symmetry of consumption at different nodes of the same price group, observe that slopes  $b_{u,t}$  of demand can be different at each node and for each time period. This allows

<sup>2</sup> Observe that the monopolistic seller thus faces capacity constraints both regarding its own output at the entry nodes as well as its sales at the different exit nodes. Related problems in which monopolistic sellers face different kinds of capacity constraints are prominent in the recent literature; see, e.g., [46].

<sup>3</sup> Also note that this demand specification results from a framework of quasi-linear utility and thus consumption choices are independent of the specific financial endowment of consumers. This implicitly assumes the access to sufficient financial capacities to finance the desired consumption choices.



for different consumption levels at the different nodes within the same price group in our setup.

All gas buyers are assumed to be price takers as, e.g., in [19]. Given the prices for bookings ( $\underline{\pi}_u^{\text{book}}$ ) and nominations ( $\pi_{i,t}^{\text{nom}}$ ), the booking and nomination decisions of the gas buyers at each exit node  $u \in V_i$  in price group  $i \in [k]$  can thus be obtained as the solution of the optimization problem

$$\max_{q_u^{\text{book}}, q_u^{\text{nom}}} \varphi_u(q_u^{\text{book}}, q_u^{\text{nom}}) \quad (3a)$$

$$\text{s.t. } q_u^{\text{book}} \leq q_u^{\text{TC}}, \quad (3b)$$

$$0 \leq q_{u,t}^{\text{nom}} \leq q_u^{\text{book}}, \quad t \in T, \quad (3c)$$

with

$$\varphi_u(q_u^{\text{book}}, q_u^{\text{nom}}) := \sum_{t \in T} \left( \int_0^{q_{u,t}^{\text{nom}}} P_{u,t}(s) \, ds - \pi_{i,t}^{\text{nom}} q_{u,t}^{\text{nom}} \right) - \underline{\pi}_u^{\text{book}} q_u^{\text{book}}.$$

Here, we use the notation  $q_u^{\text{nom}} := (q_{u,t}^{\text{nom}})_{t \in T}$ . The existence and uniqueness of a solution of these problems under Assumption 2.1 is discussed in Lemma 3.1 below.

We make the assumption that all network fees are collected from the seller. For perfectly competitive sellers and buyers it is irrelevant for the resulting market equilibrium and the tax incidence who nominally pays a tax; see, e.g., [34]. In our setup, which considers the interplay of gas transport with gas trade and a strategic seller, this equivalence does not always hold. Nevertheless, this assumption is crucial in the next step, because it allows us to eliminate  $q_-^{\text{book}}$  and to obtain the reformulated problem (13).

**Assumption 2.2** For all exit nodes  $u \in V_-$  it holds  $\underline{\pi}_u^{\text{book}} = 0$  and  $q_u^{\text{TC}} > 0$ .

## 2.4 Level (iv): cost-optimal transport of the realized nominations by the TSO

The player of level (iv) is the TSO again, who minimizes transport costs resulting from the nominations of the gas seller and the gas buyers. For the modeling of gas physics, we use the nonlinear flow model introduced in [44]. The optimization problem of the TSO in level (iv) is given by

$$\varphi^{\text{trans}}(q^{\text{nom}}) := \min_{p,q} \sum_{t \in T} c_t^{\text{trans}}(p, q; q^{\text{nom}}) \quad (4a)$$

$$\text{s.t. } (p, q) \in \mathcal{F}(q^{\text{nom}}), \text{ i.e., } (p, q) \text{ fulfills (6) – (9)}, \quad (4b)$$

where the transport costs are defined (with a slight abuse of notation) as

$$c_t^{\text{trans}}(p, q; q^{\text{nom}}) = \sum_{e=(u,v) \in E} c_t^{\text{trans}} |p_{u,t}^2 - p_{v,t}^2|. \quad (5)$$

The feasible set  $\mathcal{F}(q^{\text{nom}})$  models all existing restrictions on the gas pressures  $p := (p_{u,t})_{u \in V, t \in T}$  and gas mass flows  $q := (q_{e,t})_{e \in E, t \in T}$ . These are flow bounds

$$-\infty < q_e^- \leq q_{e,t} \leq q_e^+ < \infty, \quad e \in E, \quad t \in T, \quad (6)$$

balance of flows

$$\sum_{e \in \delta^{\text{out}}(u)} q_{e,t} - \sum_{e \in \delta^{\text{in}}(u)} q_{e,t} = q_{u,t}^{\text{nom}}, \quad u \in V_+, \quad t \in T, \quad (7a)$$

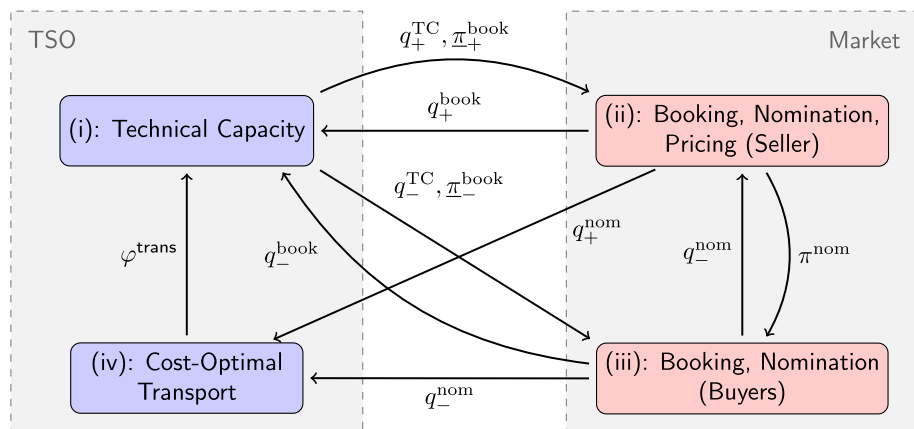


Fig. 2 Illustration of dependencies between the four levels

$$\sum_{e \in \delta^{\text{out}}(u)} q_{e,t} - \sum_{e \in \delta^{\text{in}}(u)} q_{e,t} = -q_{u,t}^{\text{nom}}, \quad u \in V_-, \quad t \in T, \quad (7b)$$

$$\sum_{e \in \delta^{\text{out}}(u)} q_{e,t} - \sum_{e \in \delta^{\text{in}}(u)} q_{e,t} = 0, \quad u \in V_0, \quad t \in T, \quad (7c)$$

as well as pressure loss constraints and pressure bounds

$$p_{u,t}^2 - p_{v,t}^2 = \Lambda_e q_{e,t} |q_{e,t}|, \quad e = (u, v) \in E, \quad t \in T, \quad (8)$$

$$0 < p_u^- \leq p_{u,t} \leq p_u^+ \leq \infty, \quad u \in V, \quad t \in T. \quad (9)$$

Here,  $\delta^{\text{in}}(u)$  and  $\delta^{\text{out}}(u)$  represent in- and outgoing arcs at node  $u$ ,  $\Lambda_e > 0$  is an arc specific constant, and  $c_t^{\text{trans}} > 0$  represents the transport costs in time period  $t$ .

In order to computationally solve instances of this problem, we replace the absolute values in the objective function, see (4a) and (5), and in Constraint (8) of Problem (4) in our implementation as described in the following remark.

**Remark 2.3** We model the absolute values  $|q_e|$  in Constraint (8) of Problem (4) by splitting  $q_e$  into positive and negative parts by adding the constraints  $q_{e,t} = q_{e,t}^{\text{pos}} - q_{e,t}^{\text{neg}}$  and  $q_{e,t}^{\text{pos}} q_{e,t}^{\text{neg}} = 0$ ; see, e.g., [22]. Moreover, we replace the pressure variables by variables for squared pressure  $\tilde{p} := p^2$ . Thus, we replace Constraint (8) by

$$\tilde{p}_{u,t} - \tilde{p}_{v,t} = \Lambda_e q_e (q_{e,t}^{\text{pos}} + q_{e,t}^{\text{neg}}).$$

We also replace the transport costs in the objective function, see (4a) and (5), of Problem (4) by

$$c_t^{\text{trans}} |p_{u,t}^2 - p_{v,t}^2| = c_t^{\text{trans}} \Lambda_e (q_{e,t}^{\text{pos}} + q_{e,t}^{\text{neg}})^2.$$

Bounds for the new flow variables  $q_{e,t}^{\text{pos}}$  and  $q_{e,t}^{\text{neg}}$  are given by  $\max\{0, q_e^-\} \leq q_{e,t}^{\text{pos}} \leq \max\{0, q_e^+\}$  as well as  $\max\{0, -q_e^+\} \leq q_{e,t}^{\text{neg}} \leq \max\{0, -q_e^-\}$  and bounds for the new pressure variables  $\tilde{p}$  are given by  $(p_u^-)^2 \leq \tilde{p}_{u,t} \leq (p_u^+)^2$ .

Figure 2 illustrates the dependencies between the four levels.

### 3 Single-level reformulation

We now provide a reformulation of the four-level model into a single-level optimization problem. To this end, we begin by integrating levels (i) and (iv) in one problem in Sect. 3.1. Then, we reformulate levels (ii) and (iii) as one concave maximization problem in Sect. 3.2, which can be replaced by its KKT conditions. In Sect. 3.3 we combine those results to obtain a single-level optimization problem, whose solutions correspond to the solutions of the four-level model described in Sect. 2.

#### 3.1 Aggregation of Level (i) and Level (iv)

It follows from Theorem 7 of [19], see also Problem (13) in [7], that the TSO levels (i) and (iv) can be aggregated in the following way:

$$\begin{aligned} \max_{q^{\text{TC}}, \underline{\pi}^{\text{book}}, p, q} \quad & \sum_{t \in T} \left( \sum_{u \in V_-} \int_0^{q_{u,t}^{\text{nom}}} P_{u,t}(s) \, ds - \sum_{u \in V_+} c_u^{\text{var}} q_{u,t}^{\text{nom}} \right) \\ & - \sum_{t \in T} c_t^{\text{trans}}(p, q; q^{\text{nom}}) - C \end{aligned} \quad (10a)$$

$$\text{s.t. } 0 \leq q_u^{\text{TC}}, \quad 0 \leq \underline{\pi}_u^{\text{book}}, \quad u \in V_+ \cup V_-, \quad (10b)$$

$$\sum_{u \in V_+ \cup V_-} \underline{\pi}_u^{\text{book}} q_u^{\text{book}} = \sum_{t \in T} c_t^{\text{trans}}(p, q; q^{\text{nom}}) + C, \quad (10c)$$

$$(p, q) \in \mathcal{F}(q^{\text{nom}}), \quad (10d)$$

$$(q_+^{\text{book}}, q_+^{\text{nom}}, \pi^{\text{nom}}) \text{ solves (2)}. \quad (10e)$$

Figure 3 illustrates the dependencies between the three levels.

#### 3.2 Aggregation of Level (ii) and Level (iii)

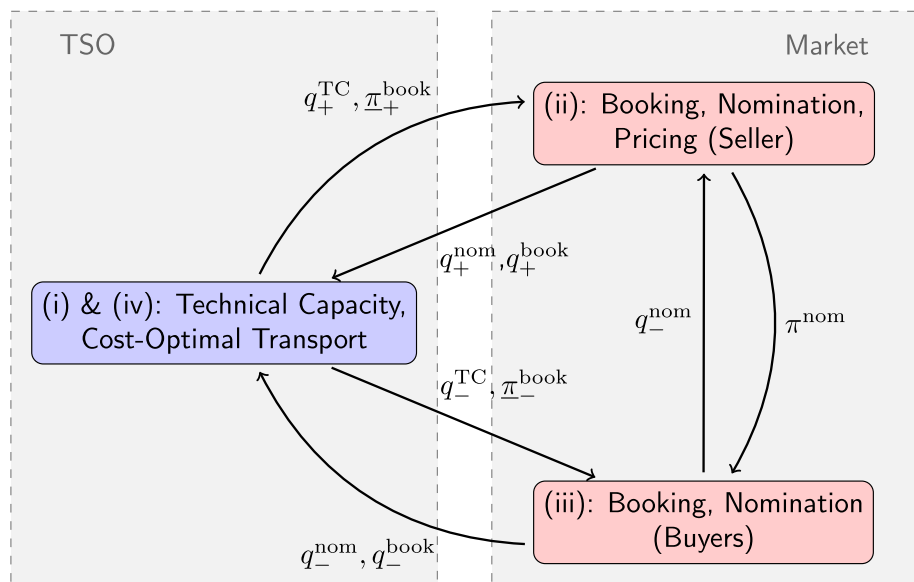
To aggregate level (ii) and (iii) into one optimization problem, we use that the solutions of level (iii) can be computed explicitly.

**Lemma 3.1** *Suppose Assumption 2.1 holds. Then, Problem (3) has an optimal solution for all entry nodes  $u \in V_i$  in all price groups  $i \in [k]$ . The optimal consumers' nominations are unique and given by*

$$q_{u,t}^{\text{nom}} = \max \left\{ 0, \min \left\{ \frac{a_{i,t} - \pi_{i,t}^{\text{nom}}}{b_{u,t}}, q_u^{\text{book}} \right\} \right\} \quad (11)$$

for all  $t \in T$ . For all exit nodes  $u \in V_-$  with  $\underline{\pi}_u^{\text{book}} > 0$ , the optimal consumers' bookings are also unique and given by  $q_u^{\text{book}} = \max_{t \in T} \{q_{u,t}^{\text{nom}}\}$ . For all exit nodes  $u \in V_-$  with  $\underline{\pi}_u^{\text{book}} = 0$ , the optimal consumers' bookings are from the interval  $[\max_{t \in T} \{q_{u,t}^{\text{nom}}\}, q_u^{\text{TC}}]$ .

**Proof** Consider an arbitrary price group  $i \in [k]$  and an entry node  $u \in V_i$ . Then, the existence of a solution follows from the theorem of Weierstraß. The feasible set of Problem (3) is convex and the objective function is concave and strictly concave in  $q_{u,t}^{\text{nom}}$ . Thus, the optimal nominations are unique due to Lemma 1 of [33]. The formula for these optimal nominations follows from fixing  $q_u^{\text{book}}$ , computing the unconstrained global maximum of the quadratic



**Fig. 3** Illustration of dependencies between the three levels

and strictly concave objective function w.r.t.  $q_{u,t}^{\text{nom}}$  and projecting it onto the feasible interval  $[0, q_u^{\text{book}}]$ . The formula for the optimal bookings follows from the fact that the objective function is strictly decreasing in  $q_u^{\text{book}}$  in case  $\pi_u^{\text{book}} > 0$  and from the feasibility constraints in case  $\pi_u^{\text{book}} = 0$ .

From now on, we impose Assumption 2.2, i.e., that the gas buyers do not have to pay anything for their booked capacity. In this case  $q_u^{\text{book}} = q_u^{\text{TC}}$  is always an optimal choice according to Lemma 3.1 and the optimal nominations are given by

$$q_{u,t}^{\text{nom}} = \max \left\{ 0, \min \left\{ \frac{a_{i,t} - \pi_{i,t}^{\text{nom}}}{b_{u,t}}, q_u^{\text{TC}} \right\} \right\} \quad u \in V_i, i \in [k], t \in T. \quad (12)$$

Using this, we will show in Lemma 3.2 that Problem (2) is equivalent to the problem

$$\max_{\substack{q_+^{\text{book}}, \\ q_+^{\text{nom}}, \pi_+^{\text{nom}}, \\ q_-^{\text{nom}}}} \psi(q_+^{\text{book}}, q_+^{\text{nom}}, \pi_+^{\text{nom}}, q_-^{\text{nom}}) \quad (13a)$$

$$\text{s.t. } q_u^{\text{book}} \leq q_u^{\text{TC}}, \quad u \in V_+, \quad [\pi_u^{\text{book}}] \quad (13b)$$

$$0 \leq q_{u,t}^{\text{nom}} \leq q_u^{\text{book}}, \quad u \in V_+, t \in T, \quad [\gamma_{u,t}^-, \gamma_{u,t}^+] \quad (13c)$$

$$\sum_{i=1}^k \sum_{u \in V_i} q_{u,t}^{\text{nom}} - \sum_{u \in V_+} q_{u,t}^{\text{nom}} = 0, \quad t \in T, \quad [\lambda_t] \quad (13d)$$

$$0 \leq q_{u,t}^{\text{nom}} \leq q_u^{\text{TC}}, \quad u \in V_-, t \in T, \quad [\gamma_{u,t}^-, \gamma_{u,t}^+] \quad (13e)$$

$$P_{u,t}(q_{u,t}^{\text{nom}}) - \pi_{i,t}^{\text{nom}} \geq 0, \quad u \in V_i, i \in [k], t \in T, \quad [\alpha_{u,t}] \quad (13f)$$

with objective function

$$\begin{aligned} & \psi(q_+^{\text{book}}, q_+^{\text{nom}}, \pi^{\text{nom}}, q_-^{\text{nom}}) \\ &:= \sum_{t \in T} \left( \sum_{i=1}^k \sum_{u \in V_i} P_{u,t}(q_{u,t}^{\text{nom}}) q_{u,t}^{\text{nom}} - (P_{u,t}(q_{u,t}^{\text{nom}}) - \pi_{i,t}^{\text{nom}}) q_u^{\text{TC}} \right) \\ & \quad - \sum_{t \in T} \left( \sum_{u \in V_+} c_u^{\text{var}} q_{u,t}^{\text{nom}} \right) - \sum_{u \in V_+} \pi_u^{\text{book}} q_u^{\text{book}}. \end{aligned}$$

In the proof of Lemma 3.2 it will also become clear why Constraint (13f) needs to be included in the reformulation. Note that Problem (13) is a concave maximization problem with linear constraints, and therefore, its KKT conditions are necessary and sufficient for global solutions. We use this later to obtain the single-level reformulation of the entire problem described in Sect. 2. For this reason, the dual variables corresponding to the constraints are already indicated in brackets in Problem (13).

**Lemma 3.2** *Suppose Assumptions 2.1 and 2.2 hold. Then, the following is true:*

- (a) *If  $(q_+^{\text{book}}, q_+^{\text{nom}}, \pi^{\text{nom}})$  is feasible for Problem (2), then  $(q_+^{\text{book}}, q_+^{\text{nom}}, \tilde{\pi}^{\text{nom}}, q_-^{\text{nom}})$  with  $q_-^{\text{nom}}$  as defined in (12) and*

$$\tilde{\pi}_{i,t}^{\text{nom}} = \min\{\pi_{i,t}^{\text{nom}}, a_{i,t}\} \quad i \in [k], \quad t \in T,$$

*is feasible for Problem (13) with the same objective function value.*

- (b) *If  $(q_+^{\text{book}}, q_+^{\text{nom}}, \pi^{\text{nom}}, q_-^{\text{nom}})$  is a solution of Problem (13), then  $(q_+^{\text{book}}, q_+^{\text{nom}}, \pi^{\text{nom}})$  is feasible for Problem (2) with the same objective function value.*
- (c) *Problems (2) and (13) both attain a solution and the optimal objective function value of both problems is the same.*

**Proof** We begin the proof with a closer look at the modified part of the objective function of Problem (13). For all  $i \in [k]$ , a slight reformulation shows

$$\begin{aligned} & \sum_{u \in V_i} (P_{u,t}(q_{u,t}^{\text{nom}}) q_{u,t}^{\text{nom}} - (P_{u,t}(q_{u,t}^{\text{nom}}) - \pi_{i,t}^{\text{nom}}) q_u^{\text{TC}}) \\ &= \pi_{i,t}^{\text{nom}} \underbrace{\sum_{u \in V_i} q_{u,t}^{\text{nom}}}_{\geq 0} + \underbrace{\sum_{u \in V_i} (P_{u,t}(q_{u,t}^{\text{nom}}) - \pi_{i,t}^{\text{nom}})}_{\geq 0} \underbrace{(q_{u,t}^{\text{nom}} - q_u^{\text{TC}})}_{\leq 0}. \end{aligned} \quad (14)$$

The signs indicated in the second sum hold for all feasible points of Problem (13).

- (a) Every price  $\pi_{i,t}^{\text{nom}} \geq a_{i,t}$  implies  $q_{u,t}^{\text{nom}} = 0$  for all  $u \in V_i$  by Lemma 3.1. The gas seller is thus indifferent between prices  $\pi_{i,t}^{\text{nom}} \geq a_{i,t}$  and we can switch from  $\pi^{\text{nom}}$  to  $\tilde{\pi}^{\text{nom}}$  in Problem (2) without affecting feasibility or the value of the objective function. Then, the feasibility of  $(q_+^{\text{book}}, q_+^{\text{nom}}, \tilde{\pi}^{\text{nom}})$  for Problem (2) and the formula for  $q_-^{\text{nom}}$  from (12) immediately show that  $(q_+^{\text{book}}, q_+^{\text{nom}}, \tilde{\pi}^{\text{nom}}, q_-^{\text{nom}})$  is feasible for Problem (13). Furthermore, observation (14) together with (12) shows that both objective functions attain the same value, because due to the modified prices  $\tilde{\pi}^{\text{nom}}$  and Assumption 2.2 for all  $u \in V_i$ , one of the two factors in the second sum is zero.
- (b) The constraints (13e) and (13f) are equivalent to

$$0 \leq q_{u,t}^{\text{nom}} \leq \min \left\{ q_u^{\text{TC}}, \frac{a_{i,t} - \pi_{i,t}^{\text{nom}}}{b_{u,t}} \right\} \quad u \in V_i, \quad i \in [k], \quad t \in T. \quad (15)$$

Now, assume that there exists a time period  $t$  and a price group  $i$  with

$$0 \leq q_{u,t}^{\text{nom}} < \min \left\{ q_u^{\text{TC}}, \frac{a_{i,t} - \pi_{i,t}^{\text{nom}}}{b_{u,t}} \right\} \leq \frac{a_{i,t} - \pi_{i,t}^{\text{nom}}}{b_{u,t}} \quad (16)$$

for at least one node  $u \in V_i$  and thus  $a_{i,t} - \pi_{i,t}^{\text{nom}} > 0$  holds. If we increase the corresponding price  $\pi_{i,t}^{\text{nom}}$  to  $\pi_{i,t}^{\text{nom}} + \varepsilon$  with  $\varepsilon > 0$ , in all possibly existing nodes  $u \in V_i$  with

$$q_{u,t}^{\text{nom}} = \frac{a_{i,t} - \pi_{i,t}^{\text{nom}}}{b_{u,t}} > 0,$$

we have to decrease the nomination to  $q_{u,t}^{\text{nom}} - \varepsilon/b_{u,t}$  to retain feasibility. However, since we know that there exists at least one node  $u \in V_i$  satisfying (16), in which we can increase  $q_{u,t}^{\text{nom}}$  slightly, for  $\varepsilon > 0$  sufficiently small it is possible to adjust  $q_{u,t}^{\text{nom}}$ ,  $u \in V_i$ , such that we retain feasibility for the higher price  $\pi_{i,t}^{\text{nom}} + \varepsilon$ , see (15), and do not change  $\sum_{u \in V_i} q_{u,t}^{\text{nom}}$ . This process can be repeated until

$$q_{u,t}^{\text{nom}} = \min \left\{ q_u^{\text{TC}}, \frac{a_{i,t} - \pi_{i,t}^{\text{nom}}}{b_{u,t}} \right\}$$

holds for all  $u \in V_i$ . Since we did not change the sum  $\sum_{u \in V_i} q_{u,t}^{\text{nom}}$  and ensured that the bounds from (15) are still satisfied, the modified point remains feasible for Problem (13). However, the modified point has a better objective function value, because in (14) we increased  $\pi_{i,t}^{\text{nom}}$  by  $\varepsilon > 0$  and we increased the second sum from a negative value to zero. This shows that a solution of Problem (13) has to satisfy (12) and, consequently,  $(q_+^{\text{book}}, q_+^{\text{nom}}, \pi^{\text{nom}})$  is feasible for Problem (2).

As discussed in (a), in points satisfying (12) the objective function values of both problems coincide.

- (c) Problem (13) has a solution according to the theorem of Weierstraß, because its feasible set is a nonempty polytope and its objective function is continuous. Then, using (b), we know that Problem (2) has a feasible point with the same objective function value. If this would not be a solution of Problem (2), there would exist a feasible point of Problem (2) with a larger objective function value. To this point, according to (a), a feasible point of Problem (13) corresponds with the same larger objective function value. This is a contradiction to having started the discussion with a solution of Problem (13).

□

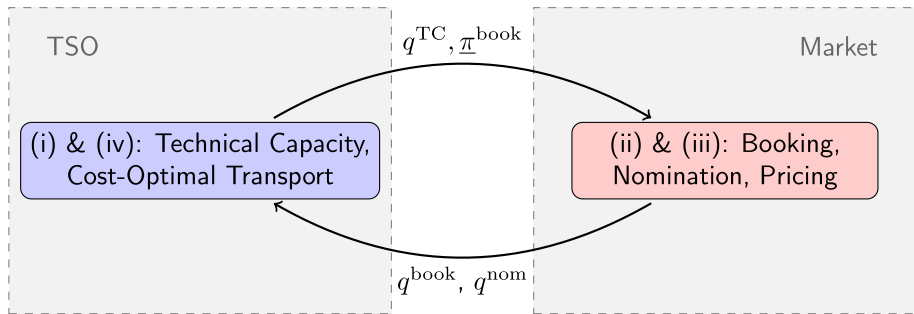
We can thus solve levels (ii) and (iii) by solving Problem (13). Figure 4 illustrates the dependencies between the remaining two levels.

Note that, in Problem (13), the optimal nominations  $q_+^{\text{nom}}$  and the optimal prices  $\pi^{\text{nom}}$  are uniquely determined and, in case the booking costs  $\underline{\pi}^{\text{book}}$  are positive, the optimal bookings  $q_+^{\text{book}}$  are uniquely determined by the nominations  $q_+^{\text{nom}}$ . However, the optimal nominations  $q_+^{\text{nom}}$  are not necessarily unique. For this reason, we would like to recall that we consider an optimistic setting, i.e., in case of multiple solutions of Problem (13) the TSO chooses the welfare-optimal one in the level above.

The KKT conditions for the concave maximization Problem (13) consist of the stationarity conditions

$$-\underline{\pi}_u^{\text{book}} - \pi_u^{\text{book}} + \sum_{t \in T} \gamma_{u,t}^+ = 0, \quad u \in V_+, \quad (17a)$$

$$-c_u^{\text{var}} + \gamma_{u,t}^- - \gamma_{u,t}^+ + \lambda_t = 0, \quad u \in V_+, \quad t \in T, \quad (17b)$$



**Fig. 4** Illustration of dependencies between the two levels

$$\sum_{u \in V_i} q_u^{\text{TC}} - \sum_{u \in V_i} \alpha_{u,t} = 0, \quad i \in [k], t \in T, \quad (17c)$$

$$P_{u,t}(q_{u,t}^{\text{nom}}) + b_{u,t}(q_u^{\text{TC}} - q_{u,t}^{\text{nom}} - \alpha_{u,t}) + \gamma_{u,t}^- - \gamma_{u,t}^+ - \lambda_t = 0, \quad u \in V_i, i \in [k], t \in T, \quad (17d)$$

primal feasibility conditions

$$q_u^{\text{book}} \leq q_u^{\text{TC}}, \quad u \in V_+, \quad (18a)$$

$$0 \leq q_{u,t}^{\text{nom}} \leq q_u^{\text{book}}, \quad u \in V_+, t \in T, \quad (18b)$$

$$\sum_{i=1}^k \sum_{u \in V_i} q_{u,t}^{\text{nom}} - \sum_{u \in V_+} q_{u,t}^{\text{nom}} = 0, \quad t \in T, \quad (18c)$$

$$0 \leq q_{u,t}^{\text{nom}} \leq q_u^{\text{TC}}, \quad u \in V_-, t \in T, \quad (18d)$$

$$P_{u,t}(q_{u,t}^{\text{nom}}) - \pi_{i,t}^{\text{nom}} \geq 0, \quad u \in V_i, i \in [k], t \in T, \quad (18e)$$

dual feasibility conditions

$$\pi_u^{\text{book}} \geq 0, \quad u \in V_+, \quad (19a)$$

$$\gamma_{u,t}^-, \gamma_{u,t}^+ \geq 0, \quad u \in V_- \cup V_+, t \in T, \quad (19b)$$

$$\alpha_{u,t} \geq 0, \quad u \in V_-, t \in T, \quad (19c)$$

and the complementary conditions

$$\pi_u^{\text{book}}(q_u^{\text{TC}} - q_u^{\text{book}}) = 0, \quad u \in V_+, \quad (20a)$$

$$\gamma_{u,t}^- q_{u,t}^{\text{nom}} = 0, \quad u \in V_+ \cup V_-, t \in T, \quad (20b)$$

$$\gamma_{u,t}^+(q_u^{\text{book}} - q_{u,t}^{\text{nom}}) = 0, \quad u \in V_+, t \in T, \quad (20c)$$

$$\gamma_{u,t}^+(q_u^{\text{TC}} - q_{u,t}^{\text{nom}}) = 0, \quad u \in V_-, t \in T, \quad (20d)$$

$$\alpha_{u,t}(P_{u,t}(q_{u,t}^{\text{nom}}) - \pi_{i,t}^{\text{nom}}) = 0, \quad u \in V_i, i \in [k], t \in T. \quad (20e)$$

### 3.3 Single-level optimization problem

Combining the results from Sects. 3.1 and 3.2, we obtain the following single-level reformulation of the four-level model described in Sect. 2 by including the KKT conditions of

Problem (13) into the combined TSO Problem (10):

$$\max_{\substack{q_+^{\text{TC}}, \pi_-^{\text{book}}, p, q, \\ q_+^{\text{book}}, q_-^{\text{nom}}, \pi_-^{\text{nom}}, \\ \pi_-^{\text{book}}, \gamma^-, \gamma^+, \lambda, \alpha}} \sum_{t \in T} \left( \sum_{u \in V_-} \int_0^{q_{u,t}^{\text{nom}}} P_{u,t}(s) \, ds - \sum_{u \in V_+} c_u^{\text{var}} q_{u,t}^{\text{nom}} \right) - \sum_{t \in T} \sum_{(u,v) \in E} c_t^{\text{trans}} |p_{u,t}^2 - p_{v,t}^2| - C \quad (21a)$$

$$\text{s.t. } 0 \leq q_u^{\text{TC}}, \quad 0 \leq \pi_u^{\text{book}}, \quad u \in V_+ \cup V_-, \quad (21b)$$

$$\sum_{u \in V_+} \pi_u^{\text{book}} q_u^{\text{book}} = \sum_{t \in T} \sum_{(u,v) \in E} c_t^{\text{trans}} |p_{u,t}^2 - p_{v,t}^2| + C, \quad (21c)$$

$$(p, q) \in \mathcal{F}(q^{\text{nom}}), \text{ i.e., } (p, q) \text{ fulfills (6) – (9)}, \quad (21d)$$

$$(q_+^{\text{book}}, q_-^{\text{nom}}, \pi_-^{\text{nom}}, \pi_-^{\text{book}}, \gamma^-, \gamma^+, \lambda, \alpha) \text{ fulfills (17) – (20)}. \quad (21e)$$

Recall that we could eliminate the booked quantities  $q_-^{\text{book}}$  of the gas buyers by choosing them as  $q_-^{\text{book}} = q_-^{\text{TC}}$ ; see Lemma 3.1 together with Assumption 2.2. The lemma below shows that we can do something similar with the technical capacities  $q_+^{\text{TC}}$  of the entry nodes in Problem (21).

**Lemma 3.3** *If  $(q_+^{\text{TC}}, x)$  with*

$$x := (q_-^{\text{TC}}, \pi_-^{\text{book}}, p, q, q_+^{\text{book}}, q_-^{\text{nom}}, \pi_-^{\text{nom}}, \pi_-^{\text{book}}, \gamma^-, \gamma^+, \lambda, \alpha)$$

*is feasible for Problem (21), then  $(q_+^{\text{book}}, x)$  is also feasible for Problem (21) with the same objective function value.*

**Proof** The technical capacities  $q_+^{\text{TC}}$  of the entry nodes do not occur in the objective function and only in the constraints (21b), (18a), and (20a). It follows that choosing  $q_+^{\text{TC}} = q_+^{\text{book}}$  retains feasibility and does not change the objective function value.

According to the previous result, we can always choose  $q_+^{\text{TC}} = q_+^{\text{book}}$  and use this to eliminate  $q_+^{\text{TC}}$  from Problem (21). The dual variable  $\pi_-^{\text{book}} \geq 0$  can then also be eliminated, which allows for the following simplifications: Constraints (18a), (19a), (20a), and the first half of (21b) are automatically satisfied and can be omitted. The stationarity equation (17a) turns into the inequality

$$-\pi_u^{\text{book}} + \sum_{t \in T} \gamma_{u,t}^+ \geq 0.$$

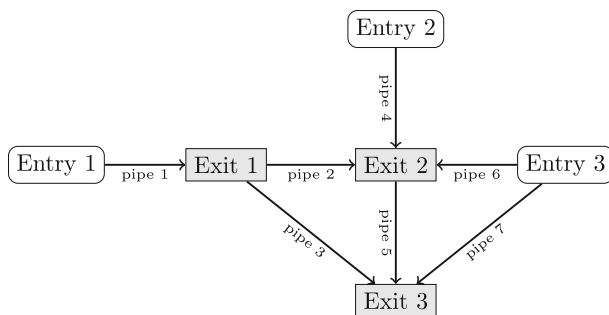
After solving Problem (21), the technical capacities of the entry nodes  $u \in V_+$  can be recovered as

$$q_u^{\text{TC}} \begin{cases} = q_u^{\text{book}}, & \text{if } \sum_{t \in T} \gamma_{u,t}^+ > \pi_u^{\text{book}}, \\ \geq q_u^{\text{book}}, & \text{if } \sum_{t \in T} \gamma_{u,t}^+ = \pi_u^{\text{book}}. \end{cases} \quad (22)$$

## 4 Numerical results

The reformulation derived in Sect. 3 allows to computationally solve instances of our original problem. There, a TSO determines technical capacities as well as controls the gas transport in





**Fig. 5** The 6-node network. The arcs illustrate the direction of positive flow

the network and the gas selling firm books and nominates capacities to supply its customers; see Sect. 2. We solved several test instances by using the reformulation (21), where we removed the variables  $q_+^T C$  according to Lemma 3.3. We implemented the modeling of gas physics and transport costs as described in Remark 2.3.

In the subsequent Sect. 4.1, we provide the technical details regarding calibration as well as the used software and hardware. In Sect. 4.2, we describe the test instances for the economic analysis and some additional test instances that we use to analyze the numerical performance of our approach. We then discuss the economic results in Sect. 4.3 and analyze the numerical performance in Sect. 4.4.

#### 4.1 Computational setup

All optimization problems have been implemented in Python 3.11.1 using Pyomo 6.6.1; see [23]. They have been solved with ANTIGONE 40.1.0. ANTIGONE is a software package for nonconvex mixed-integer nonlinear problems that is based on a branch-and-cut framework using spatial branching to tighten convex envelopes of the present nonconvexities. We refer to [38] for the details. As alternative solvers we also tested BARON, SCIP, and LindoGLOBAL, but all of them were outperformed by ANTIGONE. The computations have been carried out on a ThinkPad T460s laptop running Windows 10 Enterprise with an Intel Core i7-6600U CPU 2.60 GHz processor. We provide the solver ANTIGONE with bounds for all variables. An overview of the variable bounds as well as the proofs of their correctness can be found in Appendix A; see Table 9.

#### 4.2 Test instances

To demonstrate the applicability of our approach and to provide an economic interpretation of our results (see Sect. 4.3), we consider the test instance that is also used in [7]. It contains the 6-node network depicted in Fig. 5.

More specifically, the network consists of 3 entry nodes, 3 exit nodes, and 7 arcs (pipe 1, ..., pipe 7). The pipes' data for the identical pipes  $e \in E$  is given by length  $L_e$ , diameter  $D_e$ , roughness  $k_e$ , and flow bounds  $q_e^\pm$ :

$$L_e = 350\text{km}, \quad D_e = 0.5\text{m}, \quad k_e = 0.1\text{mm}, \quad q_e^\pm = \pm 435\text{kg s}^{-1}.$$

**Table 1** Parameters of test instances: Variable costs  $c_u^{\text{var}}$  (in EUR/(1000  $N^3$ /h)) at entries and slopes  $b_u$  (in EUR/(1000  $N^3$ /h)<sup>2</sup>) and intercepts  $a_t$  (in EUR/(1000  $N^3$ /h)) of inverse demand at exits

Entries		Exits					
	$c_u^{\text{var}}$		$b_u$	$t$	$a_t$		
					6-nodes 70%	6-nodes 100%	6-nodes 140%
Entry 1	63	Exit 1	4.5	1	1330.00	1900.00	2660.00
Entry 2	57	Exit 2	5.0	2	726.83	1038.33	1453.66
Entry 3	71	Exit 3	20.0	3	1225.00	1750.00	2450.00
				4	2030.00	2900.00	4060.00

The pressure bounds are given by

$$\begin{aligned} p_u^- &= 40 \text{ bar}, & p_u^+ &= 65 \text{ bar}, & u &\in V_+, \\ p_u^- &= 40 \text{ bar}, & p_u^+ &= 50 \text{ bar}, & u &\in V_-. \end{aligned}$$

We consider stationary gas flow and assume that the pipes are horizontal. Pressure loss in each pipe  $e = (u, v)$  is described by  $\tilde{p}_{u,t} - \tilde{p}_{v,t} = \Lambda_e q_e (q_{e,t}^{\text{pos}} + q_{e,t}^{\text{neg}})$  with pressure loss coefficient  $\Lambda_e$ ; compare Remark 2.3. In contrast to the modeling in [7], we do not assume a linear transport model. We compute the pressure loss coefficient  $\Lambda_e$  using the formula  $\Lambda_e = (\lambda_e c^2 L_e) / ((0.25\pi)^2 D_e^5)$  with the speed of sound  $c = 340 \text{ms}^{-1}$ . The friction coefficient  $\lambda_e$  is determined with the formula of Nikuradse, i.e.,  $\lambda_e = (2 \log_{10}(D_e/k_e) + 1.138)^{-2}$ ; see [16]. We set the transport cost coefficients to  $c^{\text{trans}} = 1$  and the additional network costs to  $C = 0$ . We consider four time periods that represent the four seasons spring ( $t = 1$ ), summer ( $t = 2$ ), autumn ( $t = 3$ ), and winter ( $t = 4$ ). The economic data for the gas selling and buying firms are given in Table 1. To comply with our setup, the intercepts of the inverse demand functions depend only on the time periods, and do not differ between exit nodes. The slopes  $b_u$ , however, are different at each exit node, they are assumed not to depend on the time periods in our test instances.

To vary the degree of network congestion, we consider different levels of inverse demand denoted by the scenarios 6-nodes 70%, 6-nodes 100%, and 6-nodes 140%. In these scenarios, the physical properties of the network remain unchanged and the intercepts of the inverse demand functions  $a_t$  (see Table 1) at the exit nodes are scaled according to the percentage value in the name of the test instance. In principle, larger demand is likely to create more intensive use of the network and, thus, more congestion.

### Additional test instances

To test the numerical performance of our approach, we consider computationally more complex test instances with 6, 8, 9, and 11 nodes and up to 16 time periods; see Sect. 4.4. The networks are illustrated in Figs. 7, 8, 9, and 10 in Appendix 1. For the instance 6-nodes 50%, the intercepts are given by  $a = (950, 519.17, 875, 1450)$ . The network of the instances 6-nodes with pipe 8 70%, 6-nodes with pipe 8 100%, and 6-nodes with pipe 8 140% is the network given in Fig. 5 with an additional pipe from Entry 2 to Exit 1. The inverse demand functions are the same as in the corresponding instances without the additional pipe; see Table 1.

**Table 2** Intercepts for the instances with  $|T| \in \{8, 12, 16\}$ 

$t$	6-nodes (with pipe 8)				8-nodes	9-nodes	11-nodes
	50%	70%	100%	140%			
1	950.00	1330.00	1900.00	2660.00	1660.00	1530.00	4040.07
2	519.17	726.83	1038.33	1453.66	830.00	826.00	3686.97
3	875.00	1225.00	1750.00	2450.00	1530.00	1350.00	3542.87
4	1450.00	2030.00	2900.00	4060.00	2690.00	1530.00	2907.10
5	1045.00	1463.00	2090.00	2926.00	1000.00	1683.00	2378.13
6	571.08	799.51	1142.16	1599.03	450.00	908.60	2399.77
7	787.50	1102.50	1575.00	2205.00	2000.00	1215.00	2558.60
8	1305.00	1827.00	2610.00	3654.00	1700.00	1377.00	2448.57
9	855.00	1197.00	1710.00	2394.00	1494.00	1377.00	1929.73
10	467.25	654.15	934.50	1308.30	747.00	743.40	3189.77
11	962.50	1347.50	1925.00	2695.00	1683.00	1485.00	3216.93
12	1595.00	2233.00	3190.00	4466.00	2959.00	1683.00	3604.10
13	1045.00	1463.00	2090.00	2926.00	1000.00	1683.00	2378.13
14	467.25	654.15	934.50	1308.30	747.00	743.40	3189.77
15	962.50	1347.50	1925.00	2695.00	1683.00	1485.00	3216.93
16	1305.00	1827.00	2610.00	3654.00	1700.00	1377.00	2448.57

For the 8-nodes instance, we consider a network consisting of 3 entry nodes, 5 exit nodes, and 10 arcs. The economic data differs only in the demand functions of the gas buyers. The intercepts are given by  $a = (1660, 830, 1530, 2690)$  for the four time periods and the slopes are given by  $b = (4.5, 4.8, 5, 20, 10)$  for the five exit nodes.

The network of the 9-nodes instance consists of 4 entry nodes, 5 exit nodes, and 13 arcs. The parameters of the inverse demand functions are given by the intercepts  $a = (1530, 826, 1350, 1530)$  and the slopes  $b = (4.5, 5, 20, 6, 4)$ . Moreover, the variable cost at the additional entry node is given by  $c_{\text{Entry4}}^{\text{var}} = 60$ .

For the configuration price groups of the instances 8-nodes and 9-nodes, the three price groups are given by  $V_1 = \{\text{Exit 1, Exit 2}\}$ ,  $V_2 = \{\text{Exit 3, Exit 4}\}$ , and  $V_3 = \{\text{Exit 5}\}$ .

The network of the 11-nodes instance consists of three entry and three exit nodes as well as of five inner nodes and 11 arcs with pipe lengths varying from 80 to 210km. The variable costs at the entry nodes are given by  $c^{\text{var}} = (274.8, 270.4, 250.3)$ , the intercepts are given by  $a = (3756.64, 2562.33, 2312.3, 3336.93)$ , and the slopes are given by  $b = (16.6, 13.8, 20.7)$ .

All other parameters for the described instances are set according to the data given for the 6-nodes instance. For the analysis of the numerical performance, all test instances are also solved for the cases of 8, 12, and 16 time periods. The intercepts for the additional time periods are given in Table 2. We describe and discuss the results for the analysis of numerical performance in Sect. 4.4.

**Table 3** Overview of analyzed pricing regimes in the 6-node network

Uniform price	$V_1 = \{\text{Exit 1, Exit 2, Exit 3}\}$
Price groups	$V_1 = \{\text{Exit 1, Exit 2}\}, V_2 = \{\text{Exit 3}\}$
Perfect price discr.	$V_1 = \{\text{Exit 1}\}, V_2 = \{\text{Exit 2}\}, V_3 = \{\text{Exit 3}\}$

**Table 4** Comparison of welfare, supplier's profit, and consumer surplus for different levels of demand and different pricing regimes

	Configuration	Welfare	Supplier profit	Cons. surplus
6-nodes 70%	Perfect price discr.	1254099.74	836066.49	418033.25
	Price groups	1254099.74	836066.49	418033.25
	Uniform price	1254099.74	836066.49	418033.25
6-nodes 100%	Perfect price discr.	2366860.28	1694455.94	672404.34
	Price groups	2354966.62	1686683.62	668283.00
	Network price	2349913.42	1690616.30	659297.12
6-nodes 140%	Perfect price discr.	3904386.19	3016880.31	887505.87
	Price groups	3836285.03	3009675.22	826609.81
	Network price	3826532.93	3009261.07	817271.86

### 4.3 Economic interpretation

Our framework allows to analyze different pricing strategies, where the seller might be able to charge different prices at the different exit nodes of the network. First, we consider the case of a uniform price, where prices at all exit nodes coincide. This corresponds to the current market design in the context of the entry-exit system; see, e.g., [41] and [32]. Moreover, we also consider two variants of price discrimination where sellers who exercise market power can charge different prices on some exit nodes (price groups) or on all exit nodes (perfect price discrimination) of the network. For an overview, see Table 3. Such price discrimination is currently not observed in the context of the European entry-exit system. However, in general it is a highly relevant phenomenon in other markets (see, e.g., [5]) and might eventually become relevant also in the case of gas markets. Given that our framework is directly capable to consider this case, we decided to include it in our analysis.

Our main computational results for the 6-nodes 70%, 6-nodes 100%, and 6-nodes 140% test instances are provided in Table 4, which contains the values for welfare, the supplier's profits, and the consumer surplus for the different levels of demand and the different pricing regimes. Figure 6 shows detailed nominations and technical capacities, which we obtain in the different cases of our 6-nodes instances. The nominations in the four time periods at the different entry and exit nodes are depicted as blue columns. The bookings of the supplier are depicted by a dotted red line and the technical capacities, which limit the available bookable capacities, are depicted in gray. In some cases, this technical capacity is not binding; see (22). The cases, in which the technical capacities are not binding for the nominations of the consumers can be determined similarly. If  $P_{u,t}(q_{u,t}^{\text{nom}}) = \pi_{i,t}^{\text{nom}}$  and  $\gamma_{u,t}^+ = 0$  holds for all  $t \in T$ , the technical capacity at exit node  $u \in V_-$  can be chosen arbitrarily with  $q_u^{\text{TC}} \geq \max_{t \in T} q_{u,t}^{\text{nom}}$ . Whenever a technical capacity is binding in a solution, it is depicted as a gray line. Otherwise, it is illustrated by a gray rectangle, meaning that the technical

capacity for this node could be set to any level above the maximal nomination in the optimal solution. We do not depict the bookings of the consumers, because they can be chosen from the set  $[\max_{t \in T} q_{u,t}^{\text{nom}}, q_u^{\text{TC}}]$  and thus either coincide with a binding technical capacity or are not uniquely determined.

Note that we do not have any uniqueness results—neither for the lower-level nor for the upper-level problem of our bilevel reformulation. For the lower-level problem, this aspect is addressed by choosing the optimistic variant of bilevel optimization as it is discussed in Sect. 3.2. However, it can still be the case that the corresponding (optimistic) upper-level solution is not uniquely determined as well. Hence, in what follows, we (as usual) discuss the results obtained by solving the derived single-level reformulation but note here that there might be other welfare-optimal solutions as well.

First of all, observe that both welfare and profits are significantly larger in case of higher inverse demand. Moreover, in case of relatively scarce network capacities, different pricing regimes apparently have a larger impact on welfare, profits, and consumer surplus. This becomes most apparent in case of the 6-nodes 70% case, where technical capacities at the exit nodes are never binding (see Fig. 6a) and the supplier can choose the unconstrained optimum of setting the same price at all exit nodes. For higher levels of demand (instances 6-nodes 100% and 6-nodes 140%), technical capacities are binding at the exit nodes more often. More specifically, technical capacities are binding at node Exit 1 in all 6-nodes 100% and 6-nodes 140% test instances; see Fig. 6b and c.

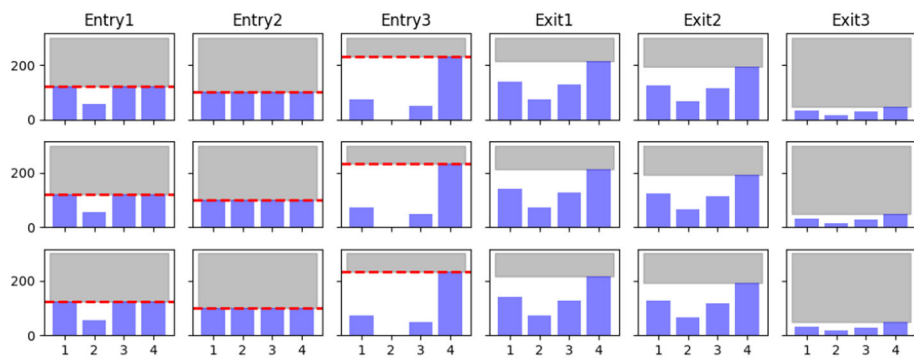
In all these cases, the different pricing regimes indeed deliver different results as the supplier engages in price discrimination among nodes if possible. Interestingly, the possibility of the supplier to choose different prices at the different exit nodes in our results tends to increase both consumer surplus and supplier profits. Apparently, differentiated prices at the different exit nodes allow for a more appropriate representation of the network-congestion situation, which allows for a more favourable choice of technical capacities on the upper level. It is worthwhile to note that this only provides a general intuition of the results. Due to the nonconvex network model considered, see (8), and the nonconvexities induced by the multilevel structure itself, see Sect. 2, we also obtain exceptions to this general intuition; in the present test scenarios the sole exception being the 6-nodes 100%, where supplier profits are lowest in case of price groups.

#### 4.4 Numerical performance

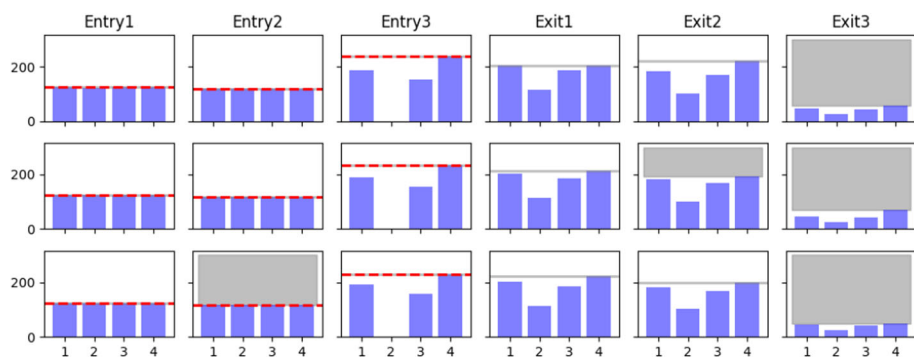
To analyze the numerical performance, we solved additional instances with up to 11 nodes and up to 16 time periods. The instances are described in Sect. 4.2. We limit the run time to 6 h. The run times as well as the number of variables, constraints, and nonlinear terms are depicted in Tables 5, 6, 7, and 8.

As expected, the complexity increases with the number of time periods considered. For the instances with 4 time periods, 26 out of 30 are solved to global optimality within 10 min, and all instances are solved to global optimality in the run time limit of 6 h. For 8 time periods, 28 instances are solved in the run time limit, compared to 24 for 12 time periods, and 23 for 16 time periods.

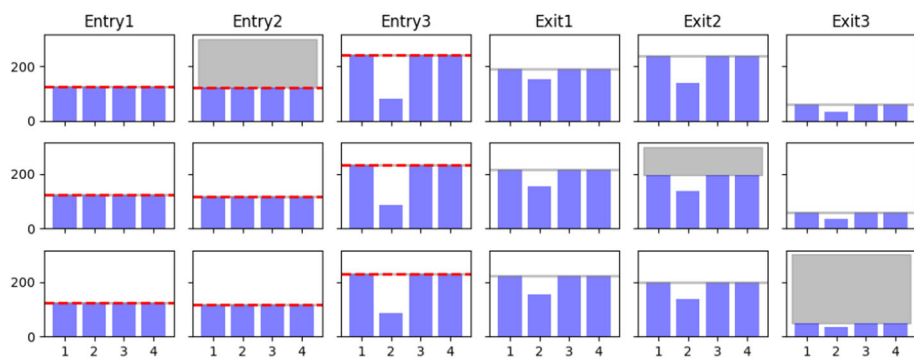
The run times also increase with the size of the considered network; see Table 5. For the instances with 4 time periods, the average run time for the instances with 6 nodes is 4.61s, whereas the average run time for the instances with 8 or more nodes is 1358.32s. The additional pipe in the instances 6-nodes with pipe 8 also increases complexity. The number of variables, constraints, and nonlinear terms is higher in the 6-nodes with pipe 8



(a) 6-nodes 70%



(b) 6-nodes 100%



(c) 6-nodes 140%

**Fig. 6** Illustration of results for the instances 6-nodes 70%, 6-nodes 100%, and 6-nodes 140%: technical capacities (gray), bookings (red), and nominations (blue) in case of perfect price discrimination (top), price groups (middle), and uniform price (bottom)

**Table 5** Data for instances with 4 time periods: run times, number of variables, constraints, and nonlinear terms. Top: 6-nodes, middle: 6-nodes with pipe 8, bottom: 8-nodes, 9-nodes, 11-nodes

Instance	Run time (s)			var.	constr.	nl. terms
	Perf. pr. discr.	Pr. groups	Uniform pr.			
50%	0.53	4.64	15.88	173–177	273–277	243–255
70%	0.64	0.50	3.83	173–177	273–277	243–255
100%	0.42	0.42	0.53	173–177	273–277	243–255
140%	0.74	0.42	0.39	173–177	273–277	243–255
70%	1.54	16.36	40.88	189–193	305–309	267–279
100%	0.67	0.58	4.30	189–193	305–309	267–279
140%	0.48	0.51	2.56	189–193	305–309	267–279
8-nodes	18.37	37.58	65.97	251–159	413–421	359–397
9-nodes	3.17	6516.17	3824.21	297–305	482–490	428–448
11-nodes	18.82	1311.66	19,428.91	225–229	325–329	295–307

instances as for the 6-nodes instances; see Tables 5–8. This also results in longer run times. The average run time for the 6-nodes instances is 2.51s compared to 7.54s for the 6-nodes with pipe 8 instances in the case with 4 time periods.

Also, the instances that are less constrained by the physical constraints have longer run times. This can be observed, i.e., by comparing the run times for the uniform price for the instances 6-nodes 50% (15.88s) and 6-nodes 140% (0.39s). This effect can also be observed for the instances 6-nodes with pipe 8 70% (40.88s) and 6-nodes with pipe 8 140% (2.56s).

The size of the price groups also affect the complexity and therefore the run times. Consider, i.e., the case with 4 time periods. Here, the average run time for the 6-nodes instances with perfect price discrimination is 0.58s. In these instances, the three price groups consist of one exit node each. For the case with price groups, i.e., one group of two exit nodes and one group with one exit node, the average run time increases to 1.50s and to 5.16s for the case with a uniform price, i.e., all three exit nodes are in one price group. This effect is especially pronounced for the larger test instances with 8, 9, and 11 nodes. For the 11-nodes instance, the run time increases by 686.95% from perfect price discrimination to price groups and by 138.12% from price groups to uniform price.

Overall, we see that the instances get very hard to solve already for moderate network sizes and time periods.

## 5 Conclusion

We provide a multilevel setup that allows to quantitatively analyze the interplay of the TSO with gas sellers and gas buyers in the entry-exit system, a market design which is currently used to organize trading and transport of pipeline-bound natural gas in Europe. With the planned onset of pipeline-bound hydrogen transport, see, e.g., [11, 12, 47], the entry-exit system might gain further relevance also in the context of zero-carbon energy supply chains. Moreover, the potential re-dedication of natural gas pipelines to be integrated into a new hydrogen network might also induce tighter network constraints for the natural gas network.

**Table 6** Data for instances with 8 time periods: run times, number of variables, constraints, and nonlinear terms. Top: 6-nodes, middle: 6-nodes with pipe 8, bottom: 8-nodes, 9-nodes, 11-nodes

Instance	Run time (s)			var.	constr.	nl. terms
	Perf. pr. discr.	Pr. groups	Uniform pr.			
50%	0.70	81.78	416.86	337–345	459–549	483–507
70%	0.93	0.83	11.19	337–345	459–549	483–507
100%	4.54	1.74	0.77	337–345	459–549	483–507
140%	0.74	5.65	0.67	337–345	459–549	483–507
70%	1.10	0.91	20,262.77	369–377	605–613	531–555
100%	1.08	0.80	795.93	369–377	605–613	531–555
140%	0.83	0.84	16.45	369–377	605–613	531–555
8-nodes	114.69	302.44	6114.98	491–507	821–837	715–755
9-nodes	1.60	–	10,689.91	581–597	958–974	852–892
11-nodes	935.02	–	83.69	441–449	645–653	587–611

**Table 7** Data for instances with 12 time periods: run times, number of variables, constraints, and nonlinear terms. Top: 6-nodes, middle: 6-nodes with pipe 8, bottom: 8-nodes, 9-nodes, 11-nodes

Instance	Run time (s)			var.	constr.	nl. terms
	Perf. pr. discr.	Pr. groups	Uniform pr.			
50%	1.02	112.05	156.97	501–513	809–821	723–759
70%	1.12	1.04	29.92	501–513	809–821	723–759
100%	0.97	1.12	0.98	501–513	809–821	723–759
140%	1.13	1.05	0.71	501–513	809–821	723–759
70%	1.34	6207.56	1106.88	549–561	905–917	795–831
100%	1.17	1.26	14914.76	549–561	905–917	795–831
140%	1.10	1.26	41.99	549–561	905–917	795–831
8-nodes	114.84	–	–	731–755	1229–1253	1071–1131
9-nodes	3.21	–	–	865–889	1434–1458	1276–1336
11-nodes	13,562.67	–	–	657–669	965–977	879–915

Hence, a careful assessment of the current market design's efficiency to appropriately use physical transport capacities is very important.

Our work crucially builds on the setup provided in [19]. The latter, however, considers perfectly competitive decisions of buyers and sellers. To the best of our knowledge, we are the first ones to propose a framework that allows to analyze strategic seller decisions in this specific context. This is of high practical relevance as gas selling takes place in environments with only few players and substantial market power; see [52]. Introducing a strategic monopolistic seller in our model requires to disaggregate buyer and seller decisions into separate levels. This leads to an overall model with significantly increased complexity and the derivation of a single-level reformulation is much more challenging compared to the case of perfectly competitive markets, which have been discussed in the recent literature. Hence, one of our main contributions is that we provide assumptions under which we can formally show that we can derive a tractable single-level reformulation. Based on our reformulation



**Table 8** Data for instances with 16 time periods: run times, number of variables, constraints, and nonlinear terms. Top: 6-nodes, middle: 6-nodes with pipe 8, bottom: 8-nodes, 9-nodes, 11-nodes

Instance	Run time (s)			var.	constr.	nl. terms
	Perf. pr. discr.	Pr. groups	Uniform pr.			
50%	3.30	82.3	375.87	665–681	1077–1093	963–1011
70%	1.95	1.64	69.97	665–681	1077–1093	963–1011
100%	1.50	2.17	1.54	665–681	1077–1093	963–1011
140%	1.40	1.92	1.10	665–681	1077–1093	963–1011
70%	1.71	15419.72	–	729–745	1205–1221	1059–1107
100%	1.65	1.54	21219.03	729–745	1205–1221	1059–1107
140%	1.95	1.81	120.59	729–745	1205–1221	1059–1107
8-nodes	3999.60	–	–	971–1003	1637–1669	1427–1507
9-nodes	3.77	–	–	1149–1181	1910–1942	1700–1780
11-nodes	–	–	2695.89	873–889	1285–1301	1171–1219

**Table 9** Variable bounds. For a variable  $x$ , we denote the upper (lower) bound of  $x$  by  $x^+(x^-)$ 

Variable	Index set	Lower bound	Upper bound
$q_u^{\text{book}}$	$u \in V_+$	0	$\max_{t \in T} \{(q_{u,t}^{\text{nom}})^+\}$
$q_u^{\text{TC}}$	$u \in V_-$	0	$\max_{t \in T} \{(q_{u,t}^{\text{nom}})^+\}$
$\pi_u^{\text{book}}$	$u \in V_+$	0	$\sum_{t \in T} (\gamma_{u,t}^+)^+$
$p_{u,t}$	$u \in V, t \in T$	$p^-$	$p^+$
$q_{e,t}$	$e \in E, t \in T$	$q_e^-$	$q_e^+$
$q_{u,t}^{\text{nom}}$	$u \in V_+, t \in T$	0	$\sum_{u \in V_-} (q_{u,t}^{\text{nom}})^+$
$q_{u,t}^{\text{nom}}$	$u \in V_-, t \in T$	0	$a_{i,t}/b_{u,t}$
$\pi_{i,t}^{\text{nom}}$	$i \in [k], t \in T$	0	$a_{i,t}$
$\gamma_{u,t}^-$	$u \in V_+, t \in T$	0	$\max\{0, c_u^{\text{var}} - \lambda_t^-\}$
$\gamma_{u,t}^+$	$u \in V_+, t \in T$	0	$\max\{0, -c_u^{\text{var}} + \lambda_t^+\}$
$\gamma_{u,t}^-, \gamma_{u,t}^+$	$u \in V_-, t \in T$	0	see (24)
$\lambda_t$	$t \in T$	$\min_{u \in V_+} \{c_u^{\text{var}}\}$	$\max_{u \in V_-} \{a_{i,t} + b_{u,t} (q_u^{\text{TC}})^+\}$
$\alpha_{u,t}$	$u \in V_-, t \in T$	0	$\sum_{u \in V_i} (q_u^{\text{TC}})^+$

we are then able to solve test instances from the literature. We illustrate the applicability as well as the computational capabilities and limits of our approach using several test instances.

## Appendix A: Finite variable bounds

We present all finite variable bounds in Table 9 and prove their correctness in what follows.

### Bounds for $q_u^{\text{book}}$ , $u \in V_+$ , and $q_u^{\text{TC}}$ , $u \in V_-$

For all nodes  $u \in V_+ \cup V_-$ , we have

$$q_{u,t}^{\text{nom}} \leq q_u^{\text{book}} \leq q_u^{\text{TC}}, \quad t \in T;$$

see Constraints (18b) and (18d). The bound then follows from the bounds on  $q_{u,t}^{\text{nom}}$  for  $u \in V_+ \cup V_-$  using the observation that it is never beneficial to choose  $q_u^{\text{book}}$  or  $q_u^{\text{TC}}$  larger than necessary.

### Bounds for $\pi_{-u}^{\text{book}}$

We use the stationarity conditions (17a) together with (21b) and (19a) to obtain

$$0 \leq \pi_{-u}^{\text{book}} = -\pi_u^{\text{book}} + \sum_{t \in T} \gamma_{u,t}^+ \leq \sum_{t \in T} \gamma_{u,t}^+ \leq \sum_{t \in T} (\gamma_{u,t}^+)^+.$$

### Bounds for $p_{u,t}$ and $q_{e,t}$

These are directly given in (9) and (6).

### Bounds for $q_{u,t}^{\text{nom}}$ , $u \in V_+$

The lower bound is given in (18b) and the upper bound follows from the lower bound together with the market-clearing condition (18c):

$$q_{u,t}^{\text{nom}} \leq \sum_{u \in V_-} q_{u,t}^{\text{nom}} \leq \sum_{u \in V_-} (q_{u,t}^{\text{nom}})^+.$$

### Bounds for $q_{u,t}^{\text{nom}}$ , $u \in V_-$

The lower bound is given in (18d) and the upper bound follows from (18e) together with the lower bound on  $\pi_{i,t}^{\text{nom}}$ :

$$q_{u,t}^{\text{nom}} \leq \frac{a_{i,t} - \pi_{i,t}^{\text{nom}}}{b_{u,t}} \leq \frac{a_{i,t}}{b_{u,t}}.$$

### Bounds for $\pi_{i,t}^{\text{nom}}$

The upper bound follows from Lemma 3.2. The lower bound follows from the fact that  $\pi_{i,t}^{\text{nom}} < 0$  can only be optimal in Problem (2) if  $\sum_{u \in V_i} q_{u,t}^{\text{nom}} = 0$  holds, in which case the gas seller can also choose  $\pi_{i,t}^{\text{nom}} = 0$  without changing the objective value.

### Bounds for $\gamma_{u,t}^-$ , $\gamma_{u,t}^+$ , $u \in V_+$

We start by showing that if Problem (21) has a feasible point, it also has a feasible point with the same objective function satisfying  $\gamma_{u,t}^- \gamma_{u,t}^+ = 0$  for all  $t \in T$  and  $u \in V_+$ . To this end,

consider a feasible point of Problem (21) and an entry node  $u \in V_+$ . If  $q_u^{\text{book}} > 0$ , then for this node  $\gamma_{u,t}^- \gamma_{u,t}^+ = 0$ ,  $t \in T$ , follows immediately from (20b) and (20c). Now, suppose that  $q_u^{\text{book}} = 0$  and  $\gamma_{u,t}^-, \gamma_{u,t}^+ > 0$  holds. Then, we re-define

$$\tilde{\gamma}_{u,t}^- := \max\{\gamma_{u,t}^- - \gamma_{u,t}^+, 0\} \geq 0, \quad (23a)$$

$$\tilde{\gamma}_{u,t}^+ := -\min\{\gamma_{u,t}^- - \gamma_{u,t}^+, 0\} \geq 0, \quad (23b)$$

$$\tilde{\pi}_u^{\text{book}} := 0 \quad (23c)$$

for all  $t \in T$ . This partially modified point is still feasible, because the Constraints (19b), (17a) (in its inequality version after eliminating  $\pi_u^{\text{book}} \geq 0$ ), (17b), (21b) are still satisfied and Constraint (21c) still holds due to  $q_u^{\text{book}} = 0$ . Moreover, none of the modified variables occurs in the objective function (21a).

It thus suffices to consider feasible points with  $\gamma_{u,t}^- \gamma_{u,t}^+ = 0$  for all  $t \in T$  and  $u \in V_+$ . Then, we can use the stationarity condition (17b) together with the bounds on  $\lambda_t$  and obtain

$$\gamma_{u,t}^- = c_u^{\text{var}} - \lambda_t \leq c_u^{\text{var}} - \lambda_t^-, \quad \gamma_{u,t}^+ \leq 0,$$

in the case of  $\gamma_{u,t}^+ = 0$  and

$$\gamma_{u,t}^- \leq 0, \quad \gamma_{u,t}^+ = -c_u^{\text{var}} + \lambda_t \leq -c_u^{\text{var}} + \lambda_t^+$$

in the case of  $\gamma_{u,t}^- = 0$ . We take the maximum of the two possible cases to obtain the upper bound and use the lower bound from (19b).

### Bounds for $\gamma_{u,t}^-, \gamma_{u,t}^+, u \in V_-$

By Assumption 2.2, we have  $q_u^{\text{TC}} > 0$  and thus (20b) and (20d) imply  $\gamma_{u,t}^- \gamma_{u,t}^+ = 0$  for all  $t \in T$ ,  $u \in V_-$ .

For  $\gamma_{u,t}^+ = 0$ , Equation (17d) together with (18d), (21b), and the upper bounds on  $\lambda_t$  and  $\alpha_{u,t}$  implies

$$\begin{aligned} \gamma_{u,t}^- &= -a_{i,t} + 2b_{u,t}q_{u,t}^{\text{nom}} - b_{u,t}q_u^{\text{TC}} + \lambda_t + b_{u,t}\alpha_{u,t} \leq a_{i,t} + \lambda_t^+ + b_{u,t}\alpha_{u,t}^+, \\ \gamma_{u,t}^+ &\leq 0. \end{aligned}$$

For  $\gamma_{u,t}^- = 0$ , Equation (17d) together with (18d), (19c), the upper bound on  $q_u^{\text{TC}}$ , and the lower bound on  $\lambda_t$  implies

$$\begin{aligned} \gamma_{u,t}^- &\leq 0, \\ \gamma_{u,t}^+ &= a_{i,t} - 2b_{u,t}q_{u,t}^{\text{nom}} + b_{u,t}q_u^{\text{TC}} - \lambda_t - b_{u,t}\alpha_{u,t} \leq a_{i,t} + b_{u,t}(q_u^{\text{TC}})^+ - \lambda_t^-. \end{aligned}$$

The lower bounds for  $\gamma_{u,t}^+, \gamma_{u,t}^-$  are given in (19b) and as upper bounds we use the maximum of the two possible cases:

$$(\gamma_{u,t}^-)^+ := \max\{0, a_{i,t} + \lambda_t^+ + b_{u,t}\alpha_{u,t}^+\}, \quad (24a)$$

$$(\gamma_{u,t}^+)^+ := \max\{0, a_{i,t} + b_{u,t}(q_u^{\text{TC}})^+ - \lambda_t^-\}. \quad (24b)$$

## Bounds for $\lambda_t$

Next, we have to make the assumption that there is no time period in which no trade takes place at all. In realistic settings, this extreme situation typically does not occur so we are convinced that the assumption is reasonable.

**Assumption A.1** For all time periods  $t \in T$ , there exists an entry node  $u \in V_+$  with a positive nomination  $q_{u,t}^{\text{nom}} > 0$ .

Suppose that Assumption A.1 holds. Consider a time period  $t \in T$  and an entry node  $u \in V_+$  such that  $q_{u,t}^{\text{nom}} > 0$  holds and thus  $\gamma_{u,t}^- = 0$  follows from (20b). Then, we can use (17b) together with (19b) to obtain

$$\lambda_t = \gamma_{u,t}^+ + c_u^{\text{var}} \geq c_u^{\text{var}} \geq \min_{v \in V_+} \{c_v^{\text{var}}\}.$$

Due to the market-clearing condition (18c), there also exists an exit node  $u \in V_-$  with  $q_{u,t}^{\text{nom}} > 0$  and, thus,  $\gamma_{u,t}^- = 0$  follows from (20b). Using (17d) together with (18d), (19b), and (19c), we then obtain

$$\begin{aligned} \lambda_t &= (a_{i,t} - 2b_{u,t}q_{u,t}^{\text{nom}} + b_{u,t}q_u^{\text{TC}}) - \gamma_{u,t}^+ - \alpha_{u,t}b_{u,t} \\ &\leq a_{i,t} + b_{u,t}(q_u^{\text{TC}})^+ \leq \max_{v \in V_-} \{a_{i,t} + b_{v,t}(q_v^{\text{TC}})^+\}. \end{aligned}$$

## Bounds for $\alpha_{u,t}$

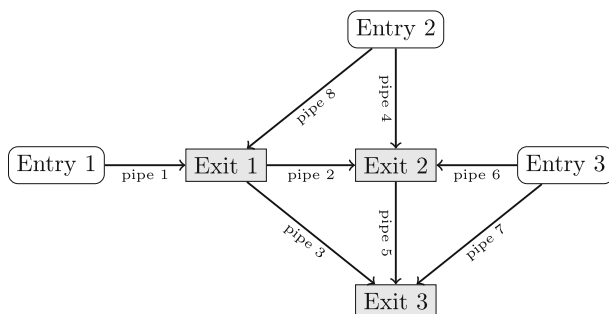
For  $u \in V_i$ ,  $i \in [k]$ , we have

$$0 \leq \alpha_{u,t} \leq \sum_{v \in V_i} \alpha_{v,t} = \sum_{v \in V_i} q_v^{\text{TC}} \leq \sum_{v \in V_i} (q_v^{\text{TC}})^+.$$

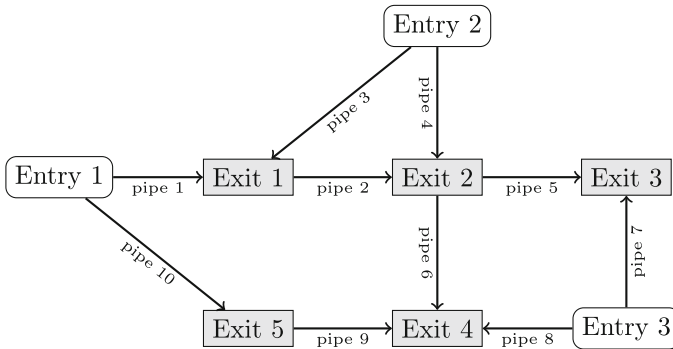
The first inequality holds due to (19c) and the equality holds by (17c).

## Appendix B: Illustration of additional instances

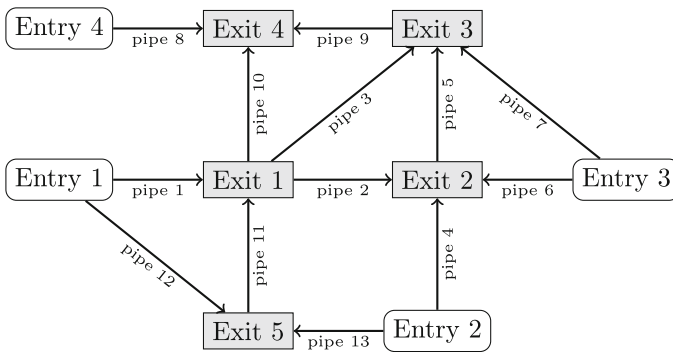
See Figs. 7, 8, 9, and 10.



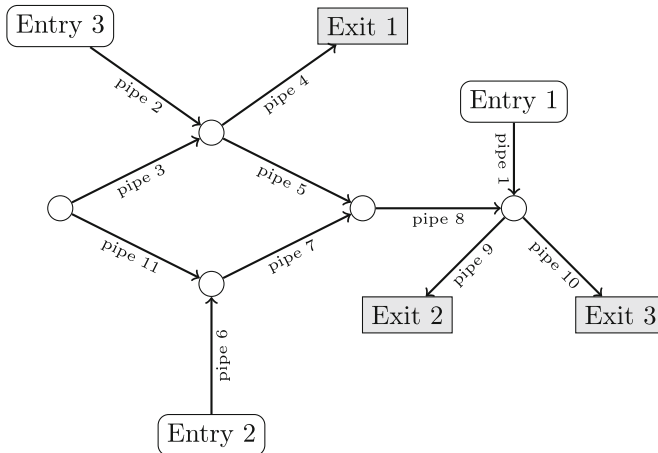
**Fig. 7** The 6-node network with additional “pipe 8”. The arcs illustrate the direction of positive flow



**Fig. 8** The 8-node network. The arcs illustrate the direction of positive flow



**Fig. 9** The 9-node network. The arcs illustrate the direction of positive flow



**Fig. 10** The 11-node network. The arcs illustrate the direction of positive flow

Appendix C: Overview of notation

In this section, we present all sets, parameters, and variables used in the optimization problems (Tables 10, 11, 12, 13). To simplify the notation throughout the paper, we use quantities without an index to denote the vector of indexed quantities, e.g.,  $q_e := (q_{e,t})_{t \in T}$  for the vector of all mass flows on pipe  $e$  over time and  $q := (q_e)_{e \in E}$  for the vector of all mass flows on all pipes over time. In addition, the indices “+” and “−” refer to the entry and exit nodes, respectively. For example,  $q_+^{\text{TC}} := (q_u^{\text{TC}})_{u \in V_+}$  is the vector of all technical capacities at all entry nodes, while  $q_-^{\text{nom}} := (q_{u,t}^{\text{nom}})_{u \in V_-, t \in T}$  is the vector of all nominated gas quantities at all exit nodes over time.

Table 10 Overview of all sets

Set	Explanation
$G$	Graph representing the gas network
$V$	Set of nodes of gas network $G$
$V_0$	Set of inner nodes of gas network $G$
$V_+$	Set of entry nodes of gas network $G$
$V_-$	Set of exit nodes of gas network $G$
$V_i$	$i$ th element of the partition of exit nodes $V_-$ into $k$ price groups
$[k]$	Set of indices for price groups, i.e., $[k] := \{1, \dots, k\}$
$E$	Set of arcs representing the pipes of gas network $G$
$\delta^{\text{in}}(u)$	Set of all ingoing arcs of node $u \in V$
$\delta^{\text{out}}(u)$	Set of all outgoing arcs of node $u \in V$
$\mathcal{F}(\cdot)$	Set of existing restrictions on gas pressures and gas mass flows
$T$	Set of time periods ( $T := \{1, \dots,  T \}$ )

Table 11 Overview of all parameters

Parameter	Explanation
$P_{u,t}$	Inverse demand function of consumers at exit $u \in V_-$ in $t \in T$
$a_{i,t} > 0$	Intercept of $P_{u,t}$ for all exits $u$ in price group $V_i$ , $i \in [k]$ , in $t \in T$
$b_{u,t} > 0$	Slope of $P_{u,t}$ for exit $u \in V_-$ in $t \in T$
$c_u^{\text{var}}$	Variable cost of supply at entry $u \in V_+$
$C$	Fixed cost of gas network $G$
$c_t^{\text{trans}}$	Transport cost in $t \in T$
$q_e^-$	Lower bound on gas mass flow in pipe $e \in E$
$q_e^+$	Upper bound on gas mass flow in pipe $e \in E$
$p_u^-$	Lower bound on gas pressure at node $u \in V$
$p_u^+$	Upper bound on gas pressure at node $u \in V$
$\Lambda_e$	Pressure loss coefficient of pipe $e \in E$ ; computed as described in [7]
$\lambda_e$	Friction coefficient of pipe $e \in E$ ; approximated as described in [7]

**Table 11** continued

Parameter	Explanation
$L_e$	Length of pipe $e \in E$
$D_e$	Diameter of pipe $e \in E$
$k_e$	Roughness of pipe $e \in E$
$c$	Speed of sound in natural gas
$\varphi^{\text{trans}}(\cdot)$	Optimal-value function of Level (iv)
$\varphi_u(\cdot)$	Objective function of consumer at exit $u \in V_-$ on Level (iii)
$\psi(\cdot)$	Objective function of reformulation of Levels (ii) and (iii)

**Table 12** Overview of all primal variables

Variable	Explanation
$q_u^{\text{TC}}$	Technical capacity at entry or exit node $u \in V_+ \cup V_-$
$\pi_u^{\text{book}}$	Price floor for booking at entry or exit node $u \in V_+ \cup V_-$
$q_u^{\text{book}}$	Booked gas quantity at entry or exit node $u \in V_+ \cup V_-$
$q_{u,t}^{\text{nom}}$	Nominated gas quantity at entry or exit node $u \in V_+ \cup V_-$ in $t \in T$
$\pi_{i,t}^{\text{nom}}$	Gas price set by monopolistic firm in price group $V_i$ , $i \in [k]$ , in $t \in T$
$p_{u,t}$	Gas pressure at node $u \in V$ in $t \in T$
$\tilde{p}_{u,t}$	Squared gas pressure at node $u \in V$ in $t \in T$
$q_{e,t}$	Gas mass flow in pipe $e \in E$ in $t \in T$
$q_{e,t}^{\text{pos}}$	Positive part of gas mass flow in pipe $e \in E$ in $t \in T$
$q_{e,t}^{\text{neg}}$	Negative part of gas mass flow in pipe $e \in E$ in $t \in T$

**Table 13** Overview of all dual variables

Variable	Explanation
$\pi_u^{\text{book}}$	Dual variable of Constraint (13b) for all entries $u \in V_+$
$\gamma_{u,t}^-, \gamma_{u,t}^+$	Dual variables of Constraint (13c) for all entries $u \in V_+$ , $t \in T$ and of Constraint (13e) for all exits $u \in V_-$ , $t \in T$
$\lambda_t$	Dual variable of Constraint (13d) for all $t \in T$
$\alpha_{u,t}$	Dual variable of Constraint (13f) for all exits $u$ in price group $V_i$ , $i \in [k]$ , $t \in T$

**Acknowledgements** The authors thank the Deutsche Forschungsgemeinschaft for their support within projects B08 and B09 in the Sonderforschungsbereich/Transregio 154 “Mathematical Modelling, Simulation and Optimization using the Example of Gas Networks”.

**Funding** Open Access funding enabled and organized by Projekt DEAL.

**Open Access** This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

## References

1. Aguirre, I., Cowan, S., Vickers, J.: Monopoly price discrimination and demand curvature. *Am. Econ. Rev.* **100**(4), 1601–1615 (2010). <https://doi.org/10.1257/aer.100.4.1601>
2. Allevi, E., Boffino, L., Giuli, M.E., Oggioni, G.: Evaluating the impacts of the external supply risk in a natural gas supply chain: the case of the Italian market. *J. Global Optim.* **70**(2), 347–384 (2018). <https://doi.org/10.1007/s10898-017-0584-z>
3. Alonso, A., Olmos, L., Serrano, M.: Application of an entry-exit tariff model to the gas transport system in Spain. *Energy Policy* **38**(5), 5133–5140 (2010). <https://doi.org/10.1016/j.enpol.2010.04.043>
4. Baltensperger, T., Füchslin, R.M., Krütti, P., Lygeros, J.: Multiplicity of equilibria in conjectural variations models of natural gas markets. *Eur. J. Oper. Res.* **252**(2), 646–656 (2016). <https://doi.org/10.1016/j.ejor.2016.01.032>
5. Belleflamme, P., Peitz, M.: *Industrial Organization: Markets and Strategies*. Cambridge University Press, Cambridge (2015)
6. Boots, M.G., Rijkers, F.A., Hobbs, B.F.: Trading in the downstream European gas market: a successive oligopoly approach. *Energy J.* **25**(3), 73–102 (2004)
7. Böttger, T., Grimm, V., Kleinert, T., Schmidt, M.: The cost of decoupling trade and transport in the European entry-exit gas market with linear physics modeling. *Eur. J. Oper. Res.* **297**(3), 1095–1111 (2022). <https://doi.org/10.1016/j.ejor.2021.06.034>
8. Boucher, J., Smeers, Y.: Gas trade in the European community during the 1970s. *Energy Econ.* **7**(2), 102–116 (1985). [https://doi.org/10.1016/0140-9883\(85\)90025-8](https://doi.org/10.1016/0140-9883(85)90025-8)
9. Chyong, C.K., Hobbs, B.F.: Strategic Eurasian natural gas market model for energy security and policy analysis: formulation and application to South Stream. *Energy Econ.* **44**, 198–211 (2014). <https://doi.org/10.1016/j.eneco.2014.04.006>
10. Chyong, C. K., Reiner, D., Aggarwal, D.: Market power and long-term gas contracts: the case of Gazprom in Central and Eastern European Gas Markets. *Energy J.* **44**(1) (2023). <https://doi.org/10.5547/01956574.44.1.cchy>
11. Commission, E.: Proposal for a DIRECTIVE OF THE EUROPEAN PARLIAMENT AND OF THE COUNCIL on common rules for the internal markets in renewable and natural gases and in hydrogen COM/2021/803 final (2021). <https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:52021PC0803>
12. Commission, E.: Proposal for a REGULATION OF THE EUROPEAN PARLIAMENT AND OF THE COUNCIL on the internal markets for renewable and natural gases and for hydrogen (recast) COM/2021/804 final. (2021). <https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=COM:2021:804:FIN>
13. Cremer, H., Laffont, J.-J.: Competition in gas markets. *Eur. Econ. Rev.* **46**(4–5), 928–935 (2002). [https://doi.org/10.1016/S0014-2921\(01\)00226-4](https://doi.org/10.1016/S0014-2921(01)00226-4)
14. Egging, R.: Benders Decomposition for multi-stage stochastic mixed complementarity problems—applied to a global natural gas market model. *Eur. J. Oper. Res.* **226**(2), 341–353 (2013). <https://doi.org/10.1016/j.ejor.2012.11.024>
15. Egging, R., Holz, F., Gabriel, S.A.: The World Gas Model: a multi-period mixed complementarity model for the global natural gas market. *Energy* **35**(10), 4016–4029 (2010). <https://doi.org/10.1016/j.energy.2010.03.053>
16. Fügenschuh, A., Geißler, B., Gollmer, R., Morsi, A., Pfetsch, M. E., Rövekamp, J., Schmidt, M., Spreckelsen, K., Steinbach, M. C.: Physical and technical fundamentals of gas networks. In: Koch, T., Hiller, B., Pfetsch, M.E., Schewe, L. (eds.) *Evaluating Gas Network Capacities*. SIAM-MOS series on Optimization. SIAM, Chap. 2, pp. 17–43 (2015). <https://doi.org/10.1137/1.9781611973693.ch2>
17. Gabriel, S.A., Kiet, S., Zhuang, J.: A mixed complementarity-based equilibrium model of natural gas markets. *Oper. Res.* **53**(5), 799–818 (2005). <https://doi.org/10.1287/opre.1040.0199>



18. Glachant, J.-M., Hallack, M., Vazquez, M.: Building Competitive Gas Markets in the EU. Edward Elgar Publishing, Cheltenham (2013)
19. Grimm, V., Schewe, L., Schmidt, M., Zöttl, G.: A multilevel model of the European entry-exit gas market. *Math. Methods Oper. Res.* **89**, 223–255 (2019). <https://doi.org/10.1007/s00186-018-0647-z>
20. Hallack, M., Vazquez, M.: European Union regulation of gas transmission services: challenges in the allocation of network resources through entry/exit schemes. *Util. Policy* **25**(5), 23–32 (2013). <https://doi.org/10.1016/j.jup.2013.01.003>
21. Hansen, P., Jaumard, B., Savard, G.: New branch-and-bound rules for linear bilevel programming. *SIAM J. Sci. Stat. Comput.* **13**(5), 1194–1217 (1992). <https://doi.org/10.1137/0913069>
22. Hante, F.M., Schmidt, M.: Gas Transport Network Optimization: Mixed-Integer Nonlinear Models (2023). <https://optimization-online.org/?p=21705>
23. Hart, W.E., Laird, C.D., Watson, J.-P., Woodruff, D.L., Hackebeil, G.A., Nicholson, B.L., Sirola, J.D.: *Pyomo-Optimization Modeling in Python*, vol. 67. Springer, Cham (2017). <https://doi.org/10.1007/978-3-319-58821-6>
24. Heitsch, H., Henrion, R., Kleinert, T., Schmidt, M.: “On convex lower-level black-box constraints in bilevel optimization with an application to gas market models with chance constraints.” In: *Journal of Global Optimization* **84.3** (2022), pp. 651–685. <https://doi.org/10.1007/s10898-022-01161-z>
25. Hirschhausen, C.: Reform der Erdgaswirtschaft in der EU und in Deutschland: Wieviel Regulierung braucht der Wettbewerb? *Perspekt. Wirtsch.* **7**(1), 89–103 (2006). <https://doi.org/10.1111/j.1465-6493.2006.00200.x>
26. Holz, F., von Hirschhausen, C., Kemfert, C.: A strategic model of European gas supply (GASMOD). *Energy Econ.* **30**(3), 766–788 (2008). <https://doi.org/10.1016/j.eneco.2007.01.018>
27. Hubert, F., Ikonnikova, S.: Investment options and bargaining power: the Eurasian supply chain for natural gas. *J. Ind. Econ.* **59**(1), 85–116 (2011). <https://doi.org/10.1111/j.1467-6451.2011.00447.x>
28. Huppmann, D.: Endogenous production capacity investment in natural gas market equilibrium models. *Eur. J. Oper. Res.* **231**(2), 503–506 (2013). <https://doi.org/10.1016/j.ejor.2013.05.048>
29. Ikonnikova, S., Zwart, G.T.: Trade quotas and buyer power, with an application to the E.U. natural gas market. *J. Eur. Econ. Assoc.* **12**(1), 177–199 (2014). <https://doi.org/10.1111/jeea.12064>
30. Jansen, T., van Lier, A., van Witteloostuijn, A., von Ochsee, T.B.: A modified Cournot model of the natural gas market in the European Union: Mixed-motives delegation in a politicized environment. *Energy Policy* **41**, 280–285 (2012). <https://doi.org/10.1016/j.enpol.2011.10.047>. (Modeling Transport (Energy) Demand and Policies)
31. Jeroslow, R.G.: The polynomial hierarchy and a simple model for competitive analysis. *Math. Program.* **32**(2), 146–164 (1985). <https://doi.org/10.1007/BF01586088>
32. Kema for European Commission. Country Factsheets. Entry-Exit Regimes in Gas, a project for the European Commission-DG ENER under the Framework Service Contract for Technical Assistance TREN/R1/350-2008 Lot 3. Tech. rep. European Commission (2013)
33. Mangasarian, O.: A simple characterization of solution sets of convex programs. *Oper. Res. Lett.* **7**(1), 21–26 (1988). [https://doi.org/10.1016/0167-6377\(88\)90047-8](https://doi.org/10.1016/0167-6377(88)90047-8)
34. Mas-Colell, A., Whinston, M.D., Green, J.R.: *Microeconomic Theory*. Oxford University Press, New York (1995)
35. Meran, G., von Hirschhausen, C., Neumann, A.: Access pricing and network expansion in natural gas markets. *Z. Energiewirtschaft.* **34**(3), 179–183 (2010). <https://doi.org/10.1007/s12398-010-0028-7>
36. Midthun, K.T., Bjørndal, M., Tomasgard, A.: Modeling optimal economic dispatch and system effects in natural gas networks. *Energy J.* **30**(4), 155–180 (2009)
37. Midthun, K.T., Fodstad, M., Hellemo, L.: Optimization model to analyse optimal development of natural gas fields and infrastructure. *Energy Procedia* **64**, 111–119 (2015). <https://doi.org/10.1016/j.egypro.2015.01.014>
38. Misener, R., Floudas, C.A.: ANTIGONE: Algorithms for coNTinuous/Integer Global Optimization of Nonlinear Equations. *J. Global Optim.* **59**, 503–526 (2014). <https://doi.org/10.1007/s10898-014-0166-2>
39. Nagayama, D., Horita, M.: A network game analysis of strategic interactions in the international trade of Russian natural gas through Ukraine and Belarus. *Energy Econ.* **43**, 89–101 (2014). <https://doi.org/10.1016/j.eneco.2014.02.010>
40. E. Parliament and C. of the European Union.: Directive (EU) 2024/1788 of the European Parliament and of the Council of 13 June 2024 on common rules for the internal markets for renewable gas, natural gas and hydrogen, amending Directive (EU) 2023/1791 and repealing Directive 2009/73/EC (2024)
41. E. Parliament and C. of the European Union.: Directive 2009/73/EC of the European Parliament and of the Council concerning common rules for the internal market in natural gas and repealing Directive 2003/55/EC (2009)

42. Plein, F., Thürauf, J., Labbé, M., Schmidt, M.: A bilevel optimization approach to decide the feasibility of bookings in the European gas market. *Math. Methods Oper. Res.* **95**(3), 409–449 (2022). <https://doi.org/10.1007/s00186-021-00752-y>
43. Rømo, F., Tomasgard, A., Hellemo, L., Fodstad, M., Eidesen, B.H., Pedersen, B.: Optimizing the Norwegian natural gas production and transport. *Interfaces* **39**(1), 46–56 (2009). <https://doi.org/10.1287/inte.1080.0414>
44. Schewe, L., Schmidt, M., Thürauf, J.: Global optimization for the multilevel European gas market system with nonlinear flow models on trees. *J. Global Optim.* **82**(3), 627–653 (2022). <https://doi.org/10.1007/s10898-021-01099-8>
45. Siddiqui, S., Gabriel, S.A.: Modeling market power in the U.S. shale gas market. *Optim. Eng.* **18**(1), 203–213 (2017). <https://doi.org/10.1007/s11081-016-9310-9>
46. Somogyi, R.: Monopoly pricing with dual-capacity constraints. *J. Econ. Manag. Strategy* (2023). <https://doi.org/10.1111/jems.12556>
47. van Rossum, R., Jens, J., Guardia, G.L., Wang, A., Kühnen, L., Overgaag, M.: European Hydrogen Backbone: A European Hydrogen Infrastructure Vision Covering 28 Countries (2022). <https://ehb.eu/files/downloads/ehb-report-220428-17h00-interactive-1.pdf> (visited on 10/09/2023)
48. Vazquez, M., Hallack, M., Glachant, J.-M.: Designing the European Gas Market: More Liquid and Less Natural? *Econ. Energy Environ. Policy* **1**(3) (2012). <https://doi.org/10.5547/2160-5890.1.3.3>
49. Weymouth, T.R.: Problems in natural gas engineering. *Trans. Am. Soc. Mech. Eng.* **34**(1349), 185–231 (1912)
50. Yang, Z., Zhang, R., Zhang, Z.: An exploration of a strategic competition model for the European Union natural gas market. *Energy Econ.* **57**, 236–242 (2016). <https://doi.org/10.1016/j.eneco.2016.05.008>
51. Zwart, G., Mulder, M.: NATGAS: a model of the European natural gas market. Tech. rep. 144. CPB Netherlands Bureau for Economic Policy Analysis (2006). <https://ideas.repec.org/p/cpb/memodm/144.html>
52. Zwart, G.T.: European natural gas markets: resource constraints and market power. *Energy J.* **30**, 151–165 (2009)

**Publisher's Note** Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.