

Korn, Ralf

Article — Published Version

A framework for optimal portfolios with sustainable assets and climate scenarios

European Actuarial Journal

Provided in Cooperation with:

Springer Nature

Suggested Citation: Korn, Ralf (2024) : A framework for optimal portfolios with sustainable assets and climate scenarios, European Actuarial Journal, ISSN 2190-9741, Springer, Berlin, Heidelberg, Vol. 15, Iss. 1, pp. 1-13,
<https://doi.org/10.1007/s13385-024-00406-3>

This Version is available at:

<https://hdl.handle.net/10419/323662>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



<http://creativecommons.org/licenses/by/4.0/>



A framework for optimal portfolios with sustainable assets and climate scenarios

Ralf Korn^{1,2}

Received: 7 August 2024 / Revised: 17 November 2024 / Accepted: 3 December 2024 /
Published online: 27 December 2024
© The Author(s) 2024

Abstract

With the increasing impact of global warming and climate change, governments must take actions to reduce CO₂ output and to encourage more environmentally friendly industrial processes. As the use of climate scenarios is the common method in illustrating possible consequences of the climate changes, we assume that it will also be the basis for political decisions with respect to future sustainability requirements on companies and thus (directly or indirectly) for sustainability constraints for life insurance companies and pension funds. Hence, we assume that bonus/malus measures by the government will already be laid out conditioned on the finally realized climate scenario. We will therefore present a corresponding framework for optimal investment under agreed government measures for the realization of future climate scenarios. This framework is particularly suited for strategies of large institutional investors such as life insurances. It will also be illustrated by explicit examples.

Keywords Sustainable investment · Large investors · Climate scenarios · Conditional political actions · Portfolio framework

1 Introduction

In [4], we considered the optimal investment problem for a life insurer (or more general, a fund provider) that is facing a demand for sustainable investment by its customers. Here, we pick up this problem, but consider the biggest possible *customer*, the state/government in its particular role as provider of regulations and requirements.

To cope with the increasing impact of global warming and climate change, governments must take actions to reduce CO₂ output and to encourage more environmentally friendly industrial processes. By looking at so-called climate scenarios (i.e. hypothetical future evolutions of climate parameter sets, see [6] for an introduction on their

✉ Ralf Korn
korn@mathematik.uni-kl.de

¹ Fachbereich Mathematik, RPTU Kaiserslautern-Landau, 67653 Kaiserslautern, Germany

² Fraunhofer Institute ITWM, Fraunhofer Platz 1, 67663 Kaiserslautern, Germany

relevance in the insurance business), possible consequences of the climate changes can be illustrated and form the basis for political and public discussions. They should therefore also be the basis for decisions with respect to future sustainability requirements on companies and thus (directly or indirectly) for sustainability constraints for life insurance companies and pension funds. Hence, we assume that bonus/malus measures by the government will already be laid out conditioned on the realized climate scenario. They could be directly placed on the companies, its dividends or—indirectly—on funds and products offered for old age provision.

As one of our main contributions, we will present a framework for optimal investment under agreed government measures for the realization of future climate scenarios. This framework is particularly suited for strategies of large institutional investors such as life insurers. Its main advantages are

- the possibility for explicit solutions of portfolio problems under sustainability constraints;
- the transformation of the estimation of the drift parameters b of the stocks to the conditional sustainability requirements of the government;
- a conceptual approach to avoid the consideration of transaction costs in the portfolio problem.

We will recall the basic setting and results from [4] in the next section. This will be followed by a section that discusses various conceptual issues such as climate scenarios, conditional political decisions and the specific characteristics of the portfolio problem of a large, institutional investor. Section 4 will then contain our finally suggested framework, the solution of the portfolio problem and an explicit example. Finally, Sect. 5 summarizes our intentions and hints at future aspects of research and applications.

2 Mathematical setting and the portfolio framework

Our model framework will be based on the one introduced in [4], i.e. we consider an investor who wants to maximize the expected utility from wealth at a terminal time T . We assume that the market contains a money market account with price evolution

$$dB(t) = rB(t)dt, B(0) = 1$$

and d stocks with price dynamics given by

$$dS_i(t) = S_i(t) \left[b_i dt + \sum_{j=1}^d \sigma_{ij} dW_j(t) \right], S_i(0) = s_i, i = 1, \dots, d.$$

Here, $W(t)$, $t \in [0, T]$ denotes a standard d –dimensional Brownian motion. Further, we assume the matrix $\sigma\sigma'$ to be positive definite and the initial stock prices s_i to be positive. In addition to the usual setting, the investor has to satisfy a sustainability

constraint of the form

$$D(t) \leq R^\pi(t)$$

where the portfolio sustainability rating for the portfolio process $\pi(t)$, $t \in [0, T]$ is defined via

$$R^\pi(t) = \sum_{i=1}^d \pi_i(t) R_i(t) + (1 - \pi(t)' \underline{1}) R_0(t)$$

with $R(t) := (R_1(t), \dots, R_d(t))'$ being the vector containing the sustainability ratings $R_i(t)$ of the stocks. $R_0(t)$ denotes the sustainability rating of the money market account or—specifically for a life insurer—the actuarial reserve fund for which every year a constant return will be declared. Of course, we assume $R_i(t) \in [0, 1]$, $i = 0, 1, \dots, d$ with 1 being the highest possible sustainability rating. Further, the higher the rating, the more sustainable is the company deemed underlying the stock. Finally, we denote the demand for sustainable assets expressed in percentage of the wealth at time t by $D(t)$.

As shown in [4], we have a lot of explicit solutions for various tasks when choosing the log-utility function and considering the portfolio problem under sustainability requirements of the form

$$\max_{\pi(\cdot) \in A(x)} \mathbb{E}_{0,x} (\ln(X^\pi(T))) \quad \text{s.t. } D(t) \leq R^\pi(t) \quad \forall t \in [0, T]. \quad (1)$$

One could also generalize our setting to a geometric Lévy process framework when using the log-utility function and still get relatively explicit solutions for the unconstrained portfolio problem (see e.g. [1]). However, we do not consider such a generalization here as we are more interested in conceptual issues than in the most possible generality. For this reason, we also do not consider regime-switching models here that could also be a promising framework.¹

Constant asset ratings As we will in the following not need the time dependence of the ratings of the securities explicitly, we will drop it for notational simplicity and restrict ourselves to constant sustainability ratings for the assets and the actuarial reserve fund, i.e. we assume

$$R_i(t) = R_i, \quad i = 0, 1, \dots, d, \quad t \in [0, T].$$

Further, this is also not far from practical applications as the ratings of stocks are not updated quite often.² Here, as in [4], we also assume that the max-offer condition

$$D(t) \leq R^* := \max \{R_0, \dots, R_d\} \quad \forall t \in [0, T] \quad (2)$$

¹ I thank an anonymous referee for pointing out the two alternative modelling settings.

² I thank an anonymous referee for pointing this out.

will be cared for by the sales coordination, i.e. the demand will only be satisfied until $D(t) = R^*$ is reached. In later sections, we assume this condition to be satisfied in the form that the government's regulation requirements on sustainability also respects the max-offer condition.

Results regarding the optimal portfolio process under sustainability constraints are then typically of the following form (see [4]):

Proposition 1 *If under the assumption (2) the bond/actuarial reserve fund possesses a sustainability rating $R_0 \geq 0$, then the optimal portfolio process for problem (1) is given by*

$$\pi_S^{opt}(t) = \begin{cases} (\sigma\sigma')^{-1} (b - r\mathbb{1}), & \text{if } R_0 + (R - R_0\mathbb{1})' (\sigma\sigma')^{-1} (b - r\mathbb{1}) \geq D(t) \\ (\sigma\sigma')^{-1} \left[(b - r\mathbb{1}) + \frac{D(t) - R_0 - (b - r\mathbb{1})' (\sigma\sigma')^{-1} (R - R_0\mathbb{1})}{(R - R_0\mathbb{1})' (\sigma\sigma')^{-1} (R - R_0\mathbb{1})} (R - R_0\mathbb{1}) \right], & \text{else} \end{cases}$$

Remark 1 (Some comments)

- Of course, the use of the logarithm as utility function results in a simple solution method of the portfolio problem as the use of the HJB-Equation of stochastic control can be avoided. However, we would also point out that the log-optimal strategy in the unconstrained problem is even asymptotically path-wise optimal (see [2] or Chapter 6 of [3] for details on the definition of asymptotic optimality).
- Various numerical examples in [4] illustrate the influence of the sustainability constraint on the form of the optimal portfolio process. It can be best understood in the case of independent asset returns, i.e. a diagonal volatility matrix $\sigma\sigma'$. Then, we see that in the second case (i.e. when the sustainability constraint is active) only those asset positions will be increased where the asset has a higher sustainability rating than the actuarial reserve fund. The others will be decreased.
- We further look at the case of just one stock and constant sustainability ratings for the actuarial reserve fund (the money market account) and the stock. Then, if the unconstrained log-optimal portfolio does not satisfy the sustainability condition the optimal sustainable portfolio is given by

$$\pi^*(t) = \frac{D(t) - R_0}{R_1 - R_0}$$

i.e. either the portfolio process will be increased (in the case of $R_1 > R_0$) or decreased (in the case of $R_1 < R_0$) only as much such that the sustainability constraint is satisfied. For this, the volatility of the stock plays no role. This, however, is slightly different in the multi-asset case.

3 Climate scenarios and political measures—consequences for the financial market

Political decisions about a (future) strategy to influence the climate evolution in a positive way are highly likely to affect the industry and should then also have consequences for the financial market, in particular through the evolution of the underlying

stock prices. We will now look at the necessary ingredients to model this chain of actions and their impact.

Climate scenarios As the evolution of global warming and climate change are highly complicated processes that live on a long term scale, it is hard to describe them via a simple stochastic process of the (jump-) diffusion type as those being popular in financial markets.³

An alternative look into the future consists of so-called *climate scenarios* (see [6] for an introduction to climate scenarios for actuaries). A climate scenario consists of fixing some key quantities such as e.g. the temperature or the CO₂ output at the end of a given period. Conditional on those assumptions, one tries to predict consequences for the living conditions, the occurrence of catastrophes or the impact on flora and fauna.

For our considerations, we assume that for each scenario the governments have already agreed on consequences such as e.g. highly taxing high-CO₂-output companies or subsidizing companies for introducing measures or production methods that avoid a certain percentage of their carbon footprint. As an example for climate scenarios, we will in Sect. 4 consider two possible scenarios for the global warming $K(t)$ in 2030, i.e. if it will be above or below 1.5 C.

Conditional political decisions We will assume that the government of the financial market that is relevant for our investor has already announced its decisions conditioned on the occurrence of scenarios SC_1, \dots, SC_m . We further assume that there is a real-valued random variable $K(t)$ that is observable and that scenario SC_i is described by the event $\{K(T) \in I_i\}$ with

$$I_i = (\alpha_i, \beta_i], \quad -\infty \leq \alpha_i < \beta_i \leq \alpha_{i+1} < \beta_{i+1} \leq \infty, \quad i = 1, 2, \dots, m-1. \quad (3)$$

Conditioned on the occurrence of scenario SC_k , we assume that the political requirements for a life insurer are to hold a portfolio with a sustainability rating of (at least) D_k , $k = 1, \dots, m$. Typically, we assume D_k to form a strictly increasing sequence.

Remark 2 (*Probability of occurrence of the different scenarios*) As we have already said that the dynamics of the climate evolution are hard to predict, we should leave it to experts to predict probabilities of the occurrence of the different scenarios. We will later give a simplified example for the evolution of global warming $K(t)$ explicitly, but this will be more for illustrative purposes.

Modelling assumptions We have assumed that $K(t)$ is publicly observable. One can think of e.g. the (average) temperature at a fixed place. As thus, we model its influence on the stock prices in the form of

$$dS_i(t) = S_i(t) \left(b_i(t, K(t))dt + \sum_{j=1}^i \sigma_{ij} dW_j(t) \right), \quad S_i(0) = s_i,$$

³ Still, we will do this in Sect. 4 for illustrational purposes.

leading to a wealth equation of

$$dX^\pi(t) = X^\pi(t) \left((r + \pi(t)'(b(t, K(t)) - r\mathbf{1})) dt + \pi(t)' \sigma dW(t) \right), \quad X^\pi(0) = x$$

if the portfolio process $\pi(t)$, $t \in [0, T]$ is followed. Although the drift parameter may be stochastic, the wealth equation still is linear in $X^\pi(t)$ and can be solved explicitly. Of course, we make the usual assumptions on the portfolio process to be a.s. square integrable and progressively measurable. Note, however, that the underlying filtration now also has to incorporate the information on $K(t)$, $t \in [0, T]$.

Consequences for large investors In [4], it was possible to switch from one portfolio to any other one without causing transaction costs. Thus, in this setting it might still be optimal to ignore the conditional measures of the government until just before the moment when it becomes clear which climate scenario actually will be realized. If we would use the model without climate effect in the drift parameter, then this would directly lead to the fact that the investor can follow the same strategy as in [4] until time T and then at this time instant switch to the optimal one in the climate scenario that turns out to be realized.

However, institutional investors and in particular life insurers have huge portfolios. Thus, a significant shift of their proportions in different assets will lead to enormous liquidity discounts to be accepted if a position has to be reduced and to liquidity premia to be paid if a position has to be increased. Thus, we can assume that a life insurer will already make suitable adjustments of the different positions well ahead and in a smooth way to avoid those losses caused by a discontinuity in the portfolio positions.

For guaranteeing appropriate measurability requirements, we have to make particular assumptions on the form of $b(t, K)$ and on the randomness behind the dynamics of $K(t)$ itself. This will be detailed when we look at specific choices for $b(t, K)$ and $K(t)$ in the next section.

Transaction costs Referring to the above stated large investor problem, one could argue for the explicit introduction of transaction costs. While there is nothing to say against that from a conceptual point of view, portfolio problems under transaction costs are notoriously hard to deal with. In particular, explicit solutions only occur in very rare cases. And even the numerical treatment is by far not easy due to the curse of dimensionality, i.e. the originally one-dimensional portfolio problem (the state of the investor without transaction costs is fully described by the current wealth of the investor) turns into a $(d+1)$ -dimensional problem (the state now depends on the different amounts of money invested in the $d+1$ assets).

Although not visible at first sight, the introduction of climate dependent drift parameters is a kind of way out of this dilemma. In Sect. 4, we will give an example of climate dependent drift parameters that lead to an optimal portfolio process where the final wealth satisfies the sustainability constraints and large re allocations of funds can be avoided.

4 Log-utility, conditional political measures, diffusion dynamics for the temperature

In this section, we will give an explicit example to demonstrate how our framework can be applied. This serves to set the scene for more detailed sophisticated frameworks to be developed in future research. We will comment on some resulting challenges in the second part of this section and in the conclusion.

4.1 An explicit example: a diffusion framework

In addition to the requirements in the standard model of [4], let us assume the following setting:

- All companies in our financial market have a constant sustainability rating R_i , $i = 1, \dots, d$.
- Let the variable $K(t)$ be a publicly available and continuously observable temperature index (such as a running average temperature for a certain month).
- The conditional political decisions are based on given intervals I_j , $j = 1, \dots, m$, (as described in (3)) for the temperature $K(T)$ at the decision horizon T : If we have $K(T) \in I_j$ then the life insurers have to satisfy

$$R^\pi(T) \geq D_j \quad \forall j = 1, \dots, m \quad (4)$$

with $0 < D_1 < \dots < D_m$.

- As a model for the evolution of $K(t)$ we use a geometric Brownian motion of the form

$$dK(t) = K(t) (\bar{\alpha} dt + \bar{\beta} dW_{d+1}(t)), \quad K(0) = K_0$$

with given parameters $\bar{\alpha}, \bar{\beta} \in \mathbb{R}$ and $W_{d+1}(t)$ a one-dimensional Brownian motion which—for simplicity—is independent of those driving the securities market. I.e. we now consider a $(d+1)$ -dimensional Brownian motion $W(t) = (W_1(t), \dots, W_{d+1}(t))$ with the corresponding Brownian filtration which we still denote by F_T .

Note first that now our admissible portfolio processes $\pi \in A(x)$ have to be progressively measurable with respect to the new, enlarged Brownian filtration. Further, we only require the sustainability constraint at the final time instant T . Thus, we are not yet in the exact situation of [4].

Remark 3 (*Scenario requirement and implicit taxation/subsidies*) Note that from the view of a life insurer having an actuarial reserve fund with a sustainability rating of R_0 , the required sustainability condition (4) can be interpreted as an implicit taxation/subsidy vector of the form

$$\tilde{b}^{(j)} := \frac{D_j - R_0 - (R - R_0 \underline{1}) (\sigma \sigma')^{-1} (b - r \underline{1})}{(R - R_0 \underline{1}) (\sigma \sigma')^{-1} (R - R_0 \underline{1})} (R - R_0 \underline{1}) \quad (5)$$

that has to be added to the original drift vector such that the corresponding optimal strategy for the life insurer would be automatically admissible with regard to the sustainability requirement. To see this, simply solve the representation of the optimal constrained portfolio for \tilde{b} (see [5] for more details on this aspect).

Still considering the log-utility function $U(x) = \ln(x)$, the usual sustainability constraint with some progressively measurable $D(t)$ that will be specified later, and $b(t, k)$ being a bounded measurable function, we can parallel the combination of the ω -wise optimization with elementary Lagrangian considerations as in [4] to obtain:

Proposition 2 *Under the assumption (2) and the above stated requirements on $b(t, k)$ and on $K(t)$, the portfolio problem (1) admits the optimal portfolio process*

$$\pi_S^{opt}(t) = \begin{cases} (\sigma\sigma')^{-1} (b(t, K(t)) - r\mathbb{1}), & \text{if } R_0 + (R(t) - R_0\mathbb{1})' (\sigma\sigma')^{-1} (b(t, K(t)) - r\mathbb{1}) \geq D(t) \\ (\sigma\sigma')^{-1} \left[(b(t, K(t)) - r\mathbb{1}) \right. \\ \quad \left. + \frac{D(t) - R_0 - (b(t, K(t)) - r\mathbb{1})' (\sigma\sigma')^{-1} (R(t) - R_0\mathbb{1})}{(R(t) - R_0\mathbb{1})' (\sigma\sigma')^{-1} (R(t) - R_0\mathbb{1})} (R(t) - R_0\mathbb{1}) \right], & \text{else} \end{cases}$$

Thus, what remains is to choose the exact forms of the drift coefficient $b(t, k)$, the sustainability constraint $D(t)$ and to justify their use.

Remark 4 (*Suggestion for the choice of the drift parameter and for $D(t)$.*) At first sight, it seems a bit strange that we are going to *suggest* a form of the drift parameter in dependence of the the evolution of global warming. Drift parameters are determined by supply and demand at the stock market, by announced dividends and by the performance of the underlying company. However, what we have to keep in mind is that on one hand we are considering a life insurer, i.e. a big investor, and on the other hand the sustainability rating of a particular stock investment is a kind of performance index that we have to keep in mind, too.

As an institutional investor, a life insurer will not be able to make sufficiently large sales and purchases at or just before the time T when the final decision about the political consequences (i.e. its exact form) of the value of $K(T)$ are revealed. Thus, for an institutional investor it is mandatory to predict the final value of $K(T)$ (or at least the interval I_j that contains $K(T)$)) as good as possible.

With the introduction of

$$b^{(j)} := b + \tilde{b}^{(j)}$$

(see (5) for the definition of $\tilde{b}^{(j)}$) and the conditional probabilities of $K(T) \in I_j$ given $K(t) = k$ as

$$p_j(t, k) := P(K(T) \in I_j | K(t) = k) \\ = \Phi \left(\frac{\ln(\beta_j/k) - (\bar{\alpha} - \frac{1}{2}\bar{\beta}^2)(T-t)}{|\bar{\beta}|\sqrt{T-t}} \right) - \Phi \left(\frac{\ln(\alpha_j/k) - (\bar{\alpha} - \frac{1}{2}\bar{\beta}^2)(T-t)}{|\bar{\beta}|\sqrt{T-t}} \right),$$

our final choice will be to use the subjective drift functions

$$b(t, k) := \sum_{j=1}^m b^{(j)} p_j(t, k).$$

It remains to specify sustainability constraints⁴

$$R^\pi(t) \geq D(t), \quad 0 \leq t < T$$

to be able to apply Proposition 2. The choice that is compatible to our desire to end exactly in the unconstrained optimal portfolio related to the appearing scenario at time T is

$$D(t) = E(D|F_t) = \sum_{j=1}^m D_j p_j(t, K(t)), \quad 0 \leq t < T,$$

i.e. the corresponding unconstrained optimal portfolio in our artificial market is admissible and it is given as

$$\pi^*(t) = (\sigma\sigma')^{-1} (b(t, K(t)) - r\mathbf{1}).$$

Note also that these sustainability conditions only have the purpose to finally reach the optimal unconstrained portfolio in the appearing scenario. They are now not related to temporal sustainability demand of the customers, but are only present as an artificial aid for making use of the method in [4].

Example 1 Let us for simplicity look at the case of $d = 1$, i.e. one stock (or stock index), a fixed interest rate (declaration) of $r = 0.02$, $b = 0.05$, $\sigma = 0.3$, $R_0 = 0.1$, $R_1 = 0.4$, $T = 6$ (i.e. our decision time is the year 2030). In the unconstrained case and having thus all $b^{(j)} = b$, this would yield

$$\pi^* = 1/3, \text{ and thus } R^* = 0.2.$$

We consider the following two global climate scenarios given by⁵:

- We assume a global warming in 2024 of $K(0) = 1^\circ\text{C}$.
- Scenario SC1: $K(6) > 1.5^\circ\text{C}$, Scenario SC2: $K(6) \leq 1.5^\circ\text{C}$.

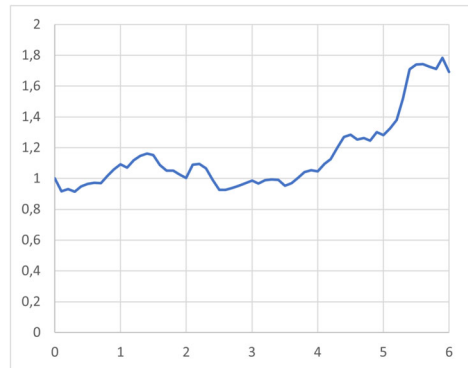
As conditional consequences, the government has requested each pension provider to hold portfolios that have to fulfill

$$D(T) = \begin{cases} 0.3 & \text{if scenario SC1 occurs,} \\ 0.2 & \text{if scenario SC2 occurs.} \end{cases}$$

⁴ A careful reading and a resulting question by an anonymous reviewer has lead to the reformulation of this section.

⁵ The chosen parameters only serve illustrational purposes and are not taken from any source.

Fig. 1 A path of the evolution of the global warming $K(t)$ over the next 6 years



For the choice of the parameters in our example this yields the requirements that the considered pension provider has to hold a portfolio of $\pi(T) = \pi^{(1)} = 2/3$ in case that scenario SC1 materializes, while for SC2 the portfolio of $\pi(T) = \pi^{(2)} = 1/3$ coincides with the actually held optimal portfolio. To see this, simply solve the relevant equation

$$D(T) = 0.1 \cdot (1 - \pi(T)) + 0.4 \cdot \pi(T)$$

in the two scenarios SC1 and SC2. Looking at the corresponding drift coefficients, we realize that for the optimal unconstrained portfolio in scenario SC1 to equal $2/3$, it needs implicit drift coefficients of $b_1 = 0.08$, and $b_2 = 0.05$ for the optimal unconstrained portfolio satisfying the sustainability constraint (i.e. no drift change at all as the originally unconstrained portfolio already satisfied the sustainability requirement for SC2).

We further assume the following global warming dynamics

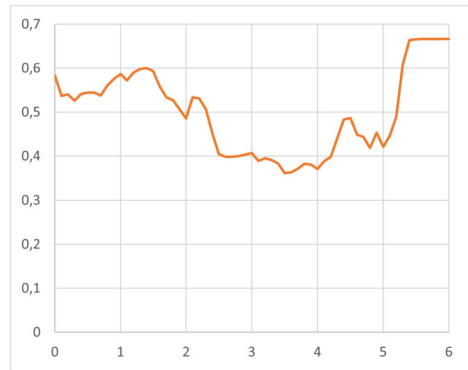
$$dK(t) = K(t)[0.1dt + 0.1dW_{d+1}(t)], \quad K(0) = 1$$

which in particular yield

$$\begin{aligned} E(K(t)) &= e^{0.1t}, \\ P(SC1|K(t)) &= P(K(T) > 1.5|K(t)) = \Phi\left(\frac{\ln(K(t)/1.5) + 0.95(6-t)}{0.1\sqrt{6-t}}\right), \\ \pi^*(t, K(t)) &= \frac{1}{0.09} \left[0.03 + 0.03 \Phi\left(\frac{\ln(K(t)/1.5) + 0.95(6-t)}{0.1\sqrt{6-t}}\right) \right] \end{aligned}$$

To illustrate this strategy, we give a path of the temperature evolution in Fig. 1 together with the evolution of the optimal portfolio process in Fig. 2. Note that the very moderate increase of global warming in the first three years—which is well behind its mean—leads to a decrease of the initially high portfolio value. However, the crossing of the 1.6°C -level in the first half of the last year lets the portfolio increase

Fig. 2 The corresponding path of the optimal portfolio process $\pi^*(t)$ over the next 6 years



strongly and pretty close to its maximum level already half a year before the decision time.

4.2 Challenges for the practical application

We will of course not hide that we have made some simplifying assumptions to obtain explicit results. Thus, we want to point out some challenges for the practical application of our framework and the corresponding optimal portfolios.

Optimal portfolios as an orientation We have motivated our choice of the climate-dependent drift parameters by the fact that an institutional investor—such as a life insurer—has to be prepared in time for the consequences of new regulations as the large investor will otherwise suffer from liquidity discounts on the sell side and liquidity premia on the buy side.

However, this argument is also a valid one for the change to our initially optimal portfolio process. But in contrast to the final time, the regulations are not yet in charge. There is no need to switch initially to the optimal portfolio process in our offered framework. The institutional investor can take the optimal portfolios as an orientation and can change swiftly over time towards them and thus avoids huge position shifts.

Constant interest rate r Note that our actions of managing the actuarial reserve fund in the investment period $[0, T]$ under consideration do not affect the declared interest rate r as this has its reason in past performance. So, assuming that there is an opportunity for achieving a constant rate of return is justified for the coming year.

However, on its end, a new declaration of the constant interest rate takes place. Thus, if we look at a longer horizon than just one year, we have to assume a model for the declaration of interest rate. On the other hand, as the duration of the dominating bond component and of the illiquid investment part in the actuarial reserve fund is quite long, the order of the size of the declaration is much better predictable than the evolution of the bond market.

5 Conclusion and further aspects

As the use of climate scenarios is the state of the art in discussing and predicting global warming and its consequences, we have given a framework for optimal investment of an institutional investor such as a life insurer that is suited for dealing with climate scenarios and its possible regulatory consequences.

To avoid being forced to make big and costly re-allocations of funds at the time when the actual scenario materializes, we have suggested to introduce climate dependent drift parameters that ensure that the optimal portfolio under sustainability constraints will eventually be reached.

Although we have given an example with explicit solutions, there remain issues that have to be dealt with when applying our framework in practical applications such as e.g.

- modelling the future declaration of interest rate for the actuarial reserve fund,
- finding a strategy how to close the initial gap over time between the optimal portfolio fractions and the actual fractions held by the life insurer,
- fitting a realistic stochastic process to the climate scenarios or—at least—a stochastic process for the evolution of the conditional probability of the scenarios to finally happen.

A further challenge is to include a management strategy for the actuarial research fund with respect to both its annual declaration of interest rate and its sustainability rating.

Acknowledgements I thank two anonymous reviewers for their very careful reading and constructive comments that have helped to improve my work, and I thank the editor for handling my contribution in a careful way.

Funding Open Access funding enabled and organized by Projekt DEAL.

Data availability There are no data used in this contribution, numbers are only chosen for illustrating purposes.

Declarations

Conflict of interest I declare that I have no conflict of interest.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

References

1. Goll T, Kallsen J (2003) A complete explicit solution to the log-optimal portfolio problem. *Ann Appl Probab* 13:774–799

2. Jamshidian F (1992) Asymptotically optimal portfolios. *Math Financ* 2(2):131–150
3. Korn R (1997) *Optimal portfolios*. World Scientific, Singapore
4. Korn R, Nurkanovic A (2023) Optimal portfolios with sustainable assets—aspects for life insurers. *Eur Actuar J* 13:125–145
5. Korn R, Nurkanovic A (2024) Drift adjustments and more general utility functions for optimal portfolios under sustainability constraints. Working paper
6. SAA Sustainability Working Group (2024) Introduction to climate scenarios. Suisse Actuarial Association

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.