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Recourse versus non-recourse mortgage debt and costly state verification

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Recourse versus Non-Recourse Mortgage Debt and Costly State Verification

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August 4, 2025

Abstract

This paper explores how policies that allow lenders to pursue borrowers for remaining debt after mortgage foreclosure (mortgage recourse) influence key aspects of the housing market and the broader macroeconomy. To this end, I develop a dynamic stochastic general equilibrium (DSGE) model featuring savers and borrowers, strategic default behavior on housing debt, and an endogenous loan-to-value (LTV) ratio. The analysis indicates that real house prices, LTV ratios, and mortgage debt levels increase with greater recourse tightness, while mortgage spreads decline. Default rates also increase with stricter recourse, despite more severe penalties targeting borrowers' assets beyond their housing collateral. In addition, mortgage recourse amplifies the volatility of key financial variables — such as LTV ratios, default rates, and mortgage spreads — intensifying financial cycles. Finally, mortgage recourse appears to be welfare-enhancing only for savers, as it provides an insurance-like mechanism in the event of borrower default. These findings underscore the complex trade-offs involved in mortgage recourse policies, offering important insights into their role in shaping housing markets and, more broadly, economic stability.

JEL Classification: E32, E44, G01, R31.

Keywords: DSGE, housing, recourse.

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1 Introduction

Since the onset of the 2008 global financial crisis, loan-to-value (LTV) ratios and mortgage defaults have drawn substantial attention in both academic and policy debates. In most European countries, mortgage recourse is the norm, preventing borrowers from walking away from negative equity without further liability. In contrast, U.S. mortgages are often treated as non-recourse — either *de jure* or *de facto* — meaning that lenders cannot seize borrowers’ assets beyond the housing collateral. The possibility of transitioning toward non-recourse mortgage regimes has generated debate in several European countries, including Ireland, Latvia, and Spain (see Harris and Meir 2015), which were among the most severely affected economies during the Great Recession. This paper contributes to this discussion by developing a theoretical framework that captures the macroeconomic implications of mortgage recourse and quantifies the mechanisms through which it affects key housing market outcomes.

To this end, I build on a two-agent New Keynesian DSGE model with savers and borrowers, endogenous LTV ratios¹, and strategic default, as in Forlati and Lambertini (2011), extending their framework by explicitly modeling recourse in the loan contract. The core contribution of this paper lies in identifying and quantifying two novel transmission mechanisms. First, mortgage recourse operates through a *collateral channel*, whereby stricter recourse increases borrowers’ housing collateral value and thus their borrowing capacity. This relaxation of the borrowing constraint raises the LTV ratio and leads to a greater accumulation of mortgage debt. Second, recourse also operates through a *default channel*, by altering borrowers’ incentives: while it increases the cost of default, the resulting higher leverage — induced by more generous lending terms — can paradoxically raise the overall probability of strategic default. Figure A.3 illustrates that the U.S. experienced a higher mortgage default rate during the financial crisis compared to several recourse countries that also saw significant housing market declines. While cross-country differences in default rates may reflect other institutional and macroeconomic factors, the observed pattern is nonetheless consistent with the model’s predictions and lends support to its implications.

The model is used to perform four main quantitative exercises. First, I conduct a steady-state analysis, comparing key macroeconomic outcomes across recourse and non-recourse regimes. The results indicate that real house prices, LTV ratios, and mortgage debt all increase with tighter recourse, while mortgage spreads decline. Interestingly, in my model, default risk increases with recourse tightness. Previous evidence on the effects of mortgage tightness on default rates has been ambiguous. Feldstein (2008), among others, argues that the prevalence

¹Many theoretical studies assume a fixed LTV ratio at origination. While this modeling choice may be justified by the presence of LTV caps in most mortgage contracts, such caps are rarely binding in practice (See Figure A.1 in the Appendix). Moreover, there is strong evidence that the LTV ratio is endogenous and influenced by the degree of mortgage recourse. For example, countries such as the UK, with prevalent recourse lending, tend to exhibit higher median LTV ratios than the U.S., where non-recourse mortgages are more common (see Figure A.2 and Chart A.1 in *Financial Stability Review* (2020)).

of recourse mortgages in Europe helps explain lower default rates despite similar declines in house prices. On the other hand, Hatchondo, Martinez, and Sánchez (2015) show that in a falling house price environment, mortgage recourse may fail to prevent defaults and can even lead to higher default rates. I find that the default rate rises with stricter recourse despite the harsher penalties, because the collateral-driven increase in leverage dominates the incentive-dampening effect of recourse. This finding highlights the complex interaction between the two transmission channels.

Second, I analyze the dynamic effects of mortgage recourse by simulating impulse responses to standard macroeconomic shocks, including housing preference shocks, monetary policy shocks, and sector-specific technology shocks. A related study by Gete and Zecchetto (2023) examines the business cycle implications of mortgage recourse and finds that it amplifies recessions, particularly in the presence of nominal rigidities and a binding zero lower bound. However, their framework imposes an exogenous loan-to-value (LTV) ratio and therefore omits the endogenous collateral and default channels emphasized in this paper. By contrast, the present study incorporates both channels via a costly state verification framework, allowing mortgage recourse to influence borrower behavior more realistically. The results indicate that recourse increases borrower leverage in steady state and amplifies financial cycles by generating greater volatility in credit variables such as LTV ratios, mortgage debt, and default rates. This amplification effect arises despite the fact that recourse dampens some immediate responses to shocks, especially through improved borrower discipline and persistently lower spreads.

Third, I study the transitional dynamics of a legal reform that eliminates mortgage recourse. Empirical analyses such as Li and Oswald (2017) find that changes to deficiency judgment laws in the U.S. negatively affected borrowers, despite being intended to make the system more borrower-friendly. Likewise, Andries et al. (2021) show that a move from a recourse to a non-recourse system in Romania incentivized strategic default. My theoretical framework supports these findings: even the pre-announcement of such a policy shift significantly alters expectations, causing borrowers to deleverage in advance. Once implemented, the reform leads to a sharp rise in default rates, mortgage spreads, and user costs, along with a decline in borrowers' housing stock and non-durable consumption.

Finally, the model provides normative insights into the distributional consequences of mortgage recourse. I find that recourse is welfare-enhancing only for savers, who benefit from an insurance-like protection against default. Borrowers, by contrast, bear the costs associated with higher debt burdens and greater exposure to house price volatility.

To the best of my knowledge, this paper presents the first state-of-the-art DSGE model that systematically analyzes the implications of mortgage recourse legislation. As such, it contributes to the ongoing debate on the optimal design and regulation of mortgage markets going forward.

The remainder of the paper is structured as follows. Section 2 describes the benchmark model. Section 3 discusses the calibration of the model parameters as well as the steady state

of the model, while Section 4 presents impulse response functions for the stochastic scenarios considered. Section 5 reports the results from the transitional dynamics exercise, and Section 6 evaluates the welfare implications under both recourse and non-recourse mortgage systems. Finally, Section 7 concludes and outlines avenues for future research.

2 Model

The model used in the present study builds on the seminal contributions of Forlati and Lambertini (2011), Iacoviello and Neri (2010) and Monacelli (2009). It is a new Keynesian DSGE model that features patient and impatient households, a three-stage production process for housing, final good firms producing non-durable goods and a central bank setting the risk-free nominal interest rate. Similarly to Forlati and Lambertini (2011), the model's main focus is the *endogenous* borrowing constraint and LTV ratio that are derived from the lenders' participation constraint after accounting for idiosyncratic risk and borrowers' strategic default. Compared to Forlati and Lambertini (2011), however, I derive a borrowing constraint that explicitly allows for a varying degree of mortgage recourse.

2.1 Households

The economy is populated by a continuum of households each with a zero mass and distributed over the $[0, 1]$ interval. A fraction of these households indicated by $\tilde{\alpha}$ are "impatient" whereas $1 - \tilde{\alpha}$ are "patient" households. The latter are the savers in the economy whereas the former are borrowers. The key distinction between the two types lies in the borrowers' lower discount factor relative to that of savers. The heterogeneity in agents' discount factors provides a tractable way to generate financial flows in the steady state: patient households (savers) purchase a positive amount of saving assets and do not borrow. Impatient households, on the other hand, need to borrow in order to finance their purchases of durable goods (housing). The representative household is infinitely-lived and maximizes their expected utility given by the standard CRRA function:

$$\max \mathbb{E}_t \sum_{t=0}^{\infty} \beta_i^t \left[\frac{1}{1-\sigma} (X_{i,t})^{1-\sigma} - \frac{\xi_i}{1+\varphi} (N_{i,t})^{1+\varphi} \right], \quad (1)$$

where $i \in \{b, s\}$ denotes the household group and β_i is the group-specific discount factor where $\beta_b < \beta_s$. Next, $X_{i,t}$ stands for a consumption bundle which consists of both non-durable goods and housing and $N_{i,t}$ indicates the labor supply of the respective household type. In addition, σ stands for the risk-aversion coefficient of households, φ is the inverse of the Frisch elasticity of labor supply and ξ_i captures the agent-specific weight households place on leisure. This leads to the following expression for the consumption bundle:

$$X_{i,t} \equiv \left[(1 - \gamma g_t)^{\frac{1}{\eta_H}} \tilde{C}_{i,t}^{\frac{\eta_H-1}{\eta_H}} + (\gamma g_t)^{\frac{1}{\eta_H}} H_{i,t}^{\frac{\eta_H-1}{\eta_H}} \right]^{\frac{\eta_H}{\eta_H-1}}, \quad (2)$$

where $\tilde{C}_{i,t} = C_{i,t} - hC_{i,t-1}$ with $h \in [0, 1]$ measures the degree of habit persistence in household consumption. Next, $C_{i,t}$ represents the consumption of non-durables while $H_{i,t}$ indicates the consumption of housing services, γ denotes the share of housing in the utility function and $g_t = \exp(\varepsilon_{\gamma,t})$ where

$$\varepsilon_{\gamma,t} = \rho_g \varepsilon_{\gamma,t-1} + \epsilon_{\gamma,t}. \quad (3)$$

This captures the possibility of having exogenous disturbances regarding households' preference for housing services. Additionally, $\eta_H > 0$ stands for the elasticity of substitution between housing services and non-durable goods. As regards the supply of labor, I follow Horvath (2000) and define the aggregate household-specific labor supply index in the following manner:

$$N_{i,t} = \left[N_{i,C,t}^{\frac{1+\eta_N}{\eta_N}} + N_{i,H,t}^{\frac{1+\eta_N}{\eta_N}} \right]^{\frac{\eta_N}{1+\eta_N}}. \quad (4)$$

The CES index given above aggregates the supply of labor of households whereby $N_{i,C,t}$ denotes the labor supply in the non-durable sector and $N_{i,H,t}$ is the household-specific labor services supplied to the housing industry. As regards η_N , it denotes the intratemporal elasticity of substitution across both production sectors. If $\eta_N \rightarrow \infty$, both sectors are perfect substitutes as to how households allocate their labor services across sectors.

2.1.1 The Loan Contract: The Incentive-Compatibility Constraint

The first constraint necessary for deriving an endogenous loan-to-value (LTV) ratio is the incentive-compatibility condition. To this end, I employ the costly state verification framework of Townsend (1979), as adapted to general equilibrium macroeconomic models by Bernanke, Gertler, and Gilchrist (1999). This approach enables the incorporation of *strategic default* as an endogenous choice variable, at both the firm and household levels. While this modeling strategy abstracts from other forms of default, such as those triggered by exogenous life events (e.g., unemployment, illness, divorce), it is particularly well-suited for the analysis of mortgage recourse legislation. As shown by Ganong and Noel (2020), although most delinquencies in practice stem from adverse shocks, strategic default still accounts for a non-negligible share of default behavior. More importantly, when assessing the impact of legal institutions like recourse provisions — which directly affect the costs and benefits of defaulting — it is both appropriate and tractable to model default as an intentional, utility-maximizing choice. In this context, strategic default is not only analytically convenient but also the relevant margin through which recourse legislation operates.

After the house has been purchased, each individual member of the impatient household experiences an idiosyncratic shock to their housing stock. It is traditionally labeled as $\omega^{\tilde{i}}$ which is *i.i.d.* across the members of the borrowing household and is log-normally distributed with an unconditional mean $E(\omega_{t+1}^{\tilde{i}}) = 1$. The household then decides about the shock default threshold level labeled as $\bar{\omega}$. If $\omega^{\tilde{i}} < \bar{\omega}$, the household member defaults whereas in the case of

$\omega^{\tilde{i}} > \bar{\omega}$, the household member pays the full principal and interest. For an individual household member $\tilde{i}$, $\omega_{t+1}^{\tilde{i}} Q_{t+1} H_{b,t}^{\tilde{i}}$ is what the value of the housing stock is after the idiosyncratic shock with Q_{t+1} and $H_{b,t}^{\tilde{i}}$ being the real house price and the housing stock of the individual household member respectively.

In the present model, a borrower defaults only if the benefits of doing so exceed the associated costs, that is, if default is strategic. Let $v \in [0, 1)$ be the *recourse parameter*, representing the degree of mortgage recourse, Z_{t+1} the mortgage interest rate, and $L_t^{\tilde{i}}$ the nominal value of the housing loan. Then, the $\tilde{i}$ -th household member defaults if the following condition holds:

$$\omega_{t+1}^{\tilde{i}}(1 - \delta)Q_{t+1}H_{b,t}^{\tilde{i}} + v \left(Z_{t+1}L_t^{\tilde{i}} - \omega_{t+1}^{\tilde{i}}(1 - \delta)Q_{t+1}H_{b,t}^{\tilde{i}} + M^{\tilde{i}}(\omega^{\tilde{i}}) \right) < Z_{t+1}L_t^{\tilde{i}}. \quad (5)$$

Here, $L_t^{\tilde{i}}$ denotes the nominal value of the mortgage loan taken by the $\tilde{i}$ -th household member, and δ represents the depreciation rate of the housing stock. Furthermore, $M^{\tilde{i}}(\omega^{\tilde{i}}) = \mu\omega_{t+1}^{\tilde{i}}(1 - \delta)Q_{t+1}H_{b,t}^{\tilde{i}}$ denotes the monitoring costs incurred by the lender upon the default of a single borrower, where $\mu \in (0, 1)$ represents the fraction of the ex-post housing value lost to monitoring costs.

When $v = 0$, i.e., when no deficiency judgment exists, the expression above reduces to the standard case of the costly state verification (CSV) framework. Conversely, full recourse on mortgage debt corresponds to a frictionless setup, wherein lenders can recover the entire loan value without incurring strategic default losses.

Unlike the standard non-recourse case, defaulting borrowers must surrender not only the ex-post housing value, but also, depending on the degree of recourse, a fraction of the loan shortfall and associated monitoring costs. It must be noted, however, that while full recourse mortgages provide lenders with broader legal options for recovering foreclosure expenses, the actual allocation of these costs depends on various factors, including contractual terms, jurisdictional differences, and legal enforcement constraints.

It is now worth mentioning that ex-ante, all households are identical. This implies that $H_{b,t}^{\tilde{i}} = H_{b,t}$ since they are of mass 1, i.e., $H_{b,t} = \int_0^1 H_{b,t}^{\tilde{i}} d\tilde{i}$, therefore $H_{b,t}^{\tilde{i}} = H_{b,t}$. The same logic applies to L_t . Furthermore, a threshold idiosyncratic shock value is determined by all members of the borrowing household and called $\bar{\omega}_{t+1}$, that renders the $\tilde{i}$ -th member of the household indifferent between defaulting or not (i.e., the marginal re-paying member). The equation describing the default decision of the borrower thus becomes $\bar{\omega}_{t+1}(1 - \delta)Q_{t+1}H_{b,t} + v(Z_{t+1}L_t - \bar{\omega}_{t+1}(1 - \delta)Q_{t+1}H_{b,t} + \mu\bar{\omega}_{t+1}(1 - \delta)Q_{t+1}H_{b,t}) = Z_{t+1}L_t$ leading to the following incentive-compatibility constraint:

$$\bar{\omega}_{t+1}(1 - \delta)Q_{t+1}H_{b,t} = \frac{1}{\chi} Z_{t+1}L_t, \quad (6)$$

where $\chi \equiv \frac{1-v(1-\mu)}{1-v}$ is the *recourse factor* implying $\chi \in [1, \infty)$. When mortgages are not

recourse (i.e., $v = 0$), then $\chi = 1$ rendering equation 6 identical to standard case of Forlati and Lambertini (2011). As v becomes larger, so does χ by the amount of $\frac{\mu}{1-v} > 0$ where the latter ratio is the slope of χ as a function of v . When recourse becomes very tight (i.e., $v \rightarrow 1$), $\chi \rightarrow \infty$ and $\frac{\mu}{1-v} \rightarrow \infty$. This implies that the right-hand side of equation 6 becomes smaller the tighter the degree of recourse is. It could be thus inferred that a higher recourse tightness, ceteris paribus, reduces the incentives for a strategic default since $\bar{\omega}_{t+1}$ declines. Defining the share of borrowers that choose to strategically default as $F(\bar{\omega}) \equiv \int_0^{\bar{\omega}} f(\omega) d\omega$, where $f(\omega)$ is the *pdf* of the log-normal distribution, the following proposition could be made:

Proposition 1 (Incentive effect): Given $Q_t > 0$ and $H_{b,t} > 0$ and $\mu \in (0, 1)$, full recourse on mortgage loans ($v \rightarrow 1$) ensures strategic default never occurs, i.e., $F(\bar{\omega}) \rightarrow 0$, rendering a frictionless economy.

Proof: See Appendix C.

While the incentive-compatibility condition implies that tighter recourse reduces the default threshold $\bar{\omega}_{t+1}$ (see Proposition 1), this result holds conditional on fixed loan and housing values. As will become clear later in the text, in general equilibrium, tighter recourse raises the LTV ratio, as lenders become more willing to extend credit. The resulting increase in borrower leverage may, in turn, raise the default threshold and default rates — a mechanism referred to as the leverage effect. Initially, the leverage effect can dominate, leading to higher $\bar{\omega}_{t+1}$. However, as recourse tightness approaches its theoretical maximum, the marginal cost of default becomes prohibitively high, reversing the relationship and ultimately reducing default risk.

2.1.2 The Loan Contract: The Participation Constraint

Since the value of the idiosyncratic shock is private information, the lender incurs monitoring costs upon seizing defaulting borrowers' housing stock. Typical costs that the lender incurs after foreclosure include foreclosure sale expenses, property maintenance and preservation, etc. Nevertheless, the fact that Z_{t+1} could be adjusted to reflect the idiosyncratic risk allows lenders to achieve a diversified portfolio of loans whose return is equal to the nominal risk-free rate defined as R_t . Integrating over all household members yields:

$$R_t L_t = \int_0^{\bar{\omega}_{t+1}} \left[\omega_{t+1}(1 - \delta)Q_{t+1}H_{b,t} + v \left(Z_{t+1}L_t - \omega_{t+1}(1 - \delta)Q_{t+1}H_{b,t} \right. \right. \quad (7)$$

$$\left. \left. + \mu \int_0^{\bar{\omega}_{t+1}} \omega_{t+1}(1 - \delta)Q_{t+1}H_{b,t} \right) \right] f(\omega) d\omega + \int_{\bar{\omega}_{t+1}}^{\infty} Z_{t+1}L_t f(\omega) d\omega - M(\bar{\omega}_{t+1}),$$

where $M(\bar{\omega}_{t+1}) \equiv \mu \int_0^{\bar{\omega}_{t+1}} \omega_{t+1}(1 - \delta)Q_{t+1}H_{b,t} f(\omega) d\omega$ is the total value of monitoring costs paid by the lender. Then, combining equations 6 and 7, we obtain the following participation

constraint in real terms:

$$R_t \frac{l_t}{\Pi_{C,t+1}} = \varphi(\bar{\omega}_{t+1})(1 - \delta)Q_{t+1}H_{b,t}, \quad (8)$$

where

$$\varphi(\bar{\omega}_{t+1}) \equiv \chi [1 - (1 - v)F(\bar{\omega}_{t+1})] \bar{\omega}_{t+1} + (1 - v)(1 - \mu)G(\bar{\omega}_{t+1}) \quad (9)$$

is the LTV ratio now dependent on the degree of recourse. The term $G(\bar{\omega}_t)$ represents the fraction of defaulting borrowers' housing stock that has been seized by savers. In addition, l_t stands for real mortgage loans granted to borrowers in period t and $\Pi_{C,t+1}$ is the CPI-inflation in period $t + 1$. Based on the abovementioned expression, I can now make the following proposition:

Proposition 2 (Collateral Effect): An increase in the recourse tightness, ceteris paribus, (weakly) increases the LTV ratio, i.e., $\frac{\partial \varphi(\bar{\omega})}{\partial v} \geq 0$.

Proof: See Appendix E.

Proposition 2 tells us that the effect of mortgage recourse on the LTV ratio is positive. That is, the tighter recourse on mortgage debt, the lower the value of the collateral lenders would demand for granting loans to borrowers. The LTV ratio consequently goes up. On the other hand, the *total* effect of an increase in the degree of recourse tightness includes the effect of a higher rate of default. That is, recourse tightness raises the threshold criterion, and thus the probability of default. Now lenders will demand a higher collateral value in order to lend to borrowers, which lowers the LTV ratio. Overall, the collateral effect dominates the effect of a higher default rate.

Next, I would like to find an expression for the (relative) spread defined as $S = \frac{Z}{R}$. For this purpose one can combine equation 6 and 8 (in nominal terms) and obtain:

$$S(\bar{\omega}_t) = \chi \frac{\bar{\omega}_t}{\varphi(\bar{\omega}_t)}. \quad (10)$$

This implies that the current framework allows to represent the mortgage spread as a direct function of the degree of recourse, the LTV ratio as well as the cutoff criterion whereby the latter is positively correlated with the rate of borrowers' default. Based on this, the following proposition formalizes the impact of mortgage recourse on the mortgage spread:

Proposition 3: An increase in the degree of recourse has an ambiguous effect on the spread, i.e., $\frac{dS(\bar{\omega})}{dv} \leq 0$.

Proof: See appendix F.

The proposition stated above is a result of three effects that have an impact on the relative spread caused by a marginal increase in the recourse tightness. The first is the direct effect,

which could be interpreted as the *direct cost of strategic mortgage default*. It says that when recourse becomes tighter, and the threshold variable remains constant, it is the mortgage rate that has to change in order to keep the incentive-compatibility constraint satisfied. This implies that non-defaulting borrowers will have to pay a higher mortgage rate.

The second effect that has an impact on the mortgage spread works through the cutoff criterion. A higher $\bar{\omega}$ raises the probability of borrowers' default which increases the mortgage spread.

Thirdly, it is the collateral effect that reduces the (relative) spread as borrowers are able to expand their housing stock and consequently the value of their collateral, increasing their borrowing capacity and consumption-smoothing ability. As a result, the mortgage spread declines.

Turning to the empirical evidence, Ghent and Kudlyak (2011), Glancy et al. (2023), and Sá (2023) further support this finding by documenting slightly higher LTV ratios in recourse states within the U.S. The latter provides estimates that align closely with the model's theoretical predictions whereby the author compares the impact of mortgage recourse and judicial foreclosure on the mortgage spread as well as the LTV ratio across U.S. states. Sá (2023) concludes that the effect on the spread is not statistically significant whereas the effect of mortgage recourse on the LTV ratio is strongly positive.

2.1.3 Borrowers

Unlike patient households, borrowers are not able to perfectly smooth consumption due to the presence of a collateral constraint. In what follows, I consider two scenarios and elaborate on each of them in detail. First, mortgage loans are non-recourse which allows lenders to seize only the nominal value of the bankrupt borrowers' housing stock. Against this background, I compare the implications of a recourse on mortgage debt, that is, lenders have the right to seize not only the housing stock of defaulted households, but also borrowers' other income thus making the latter liable for any outstanding difference between the loan value and the value of their collateral ². In what follows, I formally lay out borrowers' program.

Given the above-mentioned description of the costly-state verification framework, the budget constraint of the borrower is given by:

$$\begin{aligned}
& P_{C,t}C_{b,t} + P_{H,t}H_{b,t} + [1 - F(\bar{\omega}_t)]Z_tL_{t-1} + G(\bar{\omega}_t)(1 - \delta)P_{H,t}H_{b,t-1} \\
& + v [F(\bar{\omega}_t)Z_tL_{t-1} - G(\bar{\omega}_t)(1 - \delta)P_{H,t}H_{b,t-1} + \mu G(\bar{\omega}_t)(1 - \delta)P_{H,t}H_{b,t-1}] = \\
& = L_t + (1 - \delta)P_{H,t}H_{b,t-1} + W_{C,t}N_{b,C,t} + W_{H,t}N_{b,H,t},
\end{aligned} \tag{11}$$

²A caveat here is in order. A recourse on mortgage loans, as argued by Aron and Muellbauer (2016) in their study on delinquencies and foreclosures in the UK, normally implies that the lender has the right to seize other assets and / or future income of the defaulted borrower in case the nominal value of the housing stock is smaller than the loan value. Since in the present model there are no other assets borrowers possess, in case of default the borrower has to compensate the lender by means of wage income and pay for the difference between the loan amount and the value of the house pledged as collateral.

where $P_{C,t}$ is the price of non-durable goods, $P_{H,t}$ is the nominal house price and $W_{j,t}$ stands for the nominal wage in sector $j = C, H$. Each period $[1 - F(\bar{\omega}_t)]$ of borrowers repay their mortgage debt amounting to $Z_t L_{t-1}$. By contrast, defaulting borrowers' house gets seized by the lender and, depending on the degree of recourse, lenders are also compensated for the difference between the loan amount plus foreclosure fees and the value of the collateral.

Finally, we can use equations 6 and 7 in the borrowers' budget constraint. Besides, setting the price of non-durable goods as the numéraire yields the following expression for the impatient household's budget constraint in real terms:

$$C_{b,t} + Q_t H_{b,t} + R_{t-1} \frac{l_{t-1}}{\Pi_{C,t}} = l_t + [1 - \mu G(\bar{\omega}_t)] (1 - \delta) Q_t H_{b,t-1} + \frac{W_{C,t}}{P_{C,t}} N_{b,C,t} + \frac{W_{H,t}}{P_{C,t}} N_{b,H,t}, \quad (12)$$

where $Q_t \equiv \frac{P_{H,t}}{P_{C,t}}$ is the real house price and $\Pi_{C,t} \equiv \frac{P_{C,t}}{P_{C,t-1}}$ stands for CPI-inflation in period t .

At the beginning of period t , the representative impatient household decides optimally about $C_{b,t}$, $H_{b,t}$, l_t , $\bar{\omega}_{t+1}$. The respective first-order conditions are as follows:

$$\lambda_{b,t} = (1 - \gamma g_t)^{\frac{1}{\eta_H}} X_{b,t}^{\frac{1}{\eta_H} - \sigma} \tilde{C}_{b,t}^{-\frac{1}{\eta_H}} \quad (13)$$

$$Q_t = \left(\frac{\gamma g_t}{1 - \gamma g_t} \frac{\tilde{C}_{b,t}}{H_{b,t}} \right)^{\frac{1}{\eta_H}} + \beta_b (1 - \delta) \mathbb{E}_t \frac{\lambda_{b,t+1}}{\lambda_{b,t}} [1 - \mu G(\bar{\omega}_{t+1})] Q_{t+1} + \psi_t (1 - \delta) \mathbb{E}_t \varphi(\bar{\omega}_{t+1}) Q_{t+1} \quad (14)$$

$$\frac{R_t}{\Pi_{C,t+1}} \psi_t = 1 - \beta_b \mathbb{E}_t \left[\frac{\lambda_{b,t+1}}{\lambda_{b,t}} \frac{R_t}{\Pi_{C,t+1}} \right] \quad (15)$$

$$\psi_t \mathbb{E}_t \varphi'(\bar{\omega}_{t+1}) Q_{t+1} = \mathbb{E}_t \beta_b \frac{\lambda_{b,t+1}}{\lambda_{b,t}} \frac{1}{\Pi_{C,t+1}} \mu G'(\bar{\omega}_{t+1}) Q_{t+1}, \quad (16)$$

where $\lambda_{i,t}$ stands for the Lagrange multiplier on the agent's i budget constraint. In addition, $\lambda_{b,t} \psi_t$ is the Lagrange multiplier on the borrowing constraint and thus ψ_t stands for the marginal value of borrowing. If $\psi_t = 0$, the borrowing constraint is slack while a rise in ψ_t implies a tightening of the constraint.

As discussed by Monacelli (2009), equation 14 could be also written in a way that equates the marginal rate of substitution between housing and non-durable consumption to the user cost of housing:

$$\frac{U_{H,t}}{U_{C,t}} = J_t, \quad (17)$$

where

$$J_t \equiv Q_t - (1 - \delta) \left\{ \psi_t \mathbb{E}_t \varphi(\bar{\omega}_{t+1}) Q_{t+1} + \beta_b \mathbb{E}_t \frac{\lambda_{b,t+1}}{\lambda_{b,t}} [1 - \mu G(\bar{\omega}_{t+1})] Q_{t+1} \right\} \quad (18)$$

is the user cost of housing. Understanding how recourse impacts the user cost has important implications for the marginal rate of substitution between non-durable consumption and housing.

2.1.4 Savers

Patient households are the so called Ricardian agents in the model as they are able to perfectly smooth their consumption. Unlike borrowers, savers' housing stock is not subject to idiosyncratic risk. As a result, their budget constraint boils down to the constraint of the representative household in standard New Keynesian models with durable goods. The latter is given by the following relationship:

$$C_{s,t} + Q_t H_{s,t} + l_t = (1 - \delta)Q_t H_{s,t-1} + R_{t-1} \frac{l_{t-1}}{\Pi_{C,t}} + \frac{W_{C,t}}{P_{C,t}} N_{s,C,t} + \frac{W_{H,t}}{P_{C,t}} N_{s,H,t}. \quad (19)$$

As shown by equation 19, adjustable mortgage rates ensure that savers always get the risk-free rate when granting mortgage loans to borrowers. In reality, mortgage rates tend to be often fixed. This restricts the ability of lenders to obtain the risk-free rate of their mortgage loan investment period by period. The effects of mortgage recourse with fixed mortgage is, however, beyond the scope of the present study and thus left for future research.

Each period, the patient household decides on the optimal intertemporal allocation of both non-durable and durable consumption as well as on labor and loan supply. The first order conditions associated with savers' optimization problem are given by:

$$\lambda_{s,t} = (1 - \gamma g_t)^{\frac{1}{\eta_H}} X_{s,t}^{\frac{1}{\eta_H} - \sigma} \tilde{C}_{s,t}^{-\frac{1}{\eta_H}} \quad (20)$$

$$Q_t = \left(\frac{\gamma g_t}{1 - \gamma g_t} \frac{\tilde{C}_{s,t}}{H_{s,t}} \right)^{\frac{1}{\eta_H}} + \beta_s (1 - \delta) \mathbb{E}_t \frac{\lambda_{s,t+1}}{\lambda_{s,t}} Q_{t+1} \quad (21)$$

$$1 = \beta_s \mathbb{E}_t \frac{\lambda_{s,t+1}}{\lambda_{s,t}} \frac{R_t}{\Pi_{C,t+1}}. \quad (22)$$

The first relationship presented above equates the marginal utility of non-durable consumption with the shadow price of the budget constraint. Next, equation (21) equates the marginal value of housing in terms of non-durable consumption to its payoff while equation (22) is the standard Euler equation with respect to loans granted to borrowers.

2.2 Firms

The supply side of the economy considers a set of three types of firms. In each sector of the economy there are intermediate firms which operate in a monopolistically competitive environment and produce intermediate products. When pricing the goods they produce, intermediate goods producers account for the possibility that they might be stuck with the price they set for a certain number of periods. I employ the familiar Calvo (1983) pricing framework in order to incorporate staggered prices in the present DSGE model. In the non-durable goods sector, they sell these intermediate goods to firms producing final consumption goods. In the housing

industry, however, I borrow elements from Davis and Heathcote (2005) and Iacoviello and Neri (2010) and assume a three-stage production process. First, intermediate goods firms produce and sell their products (e.g. residential structures) to final goods firms. The latter take intermediate goods as inputs and produce a "differentiated" residential structure that is sold to the housing construction firm. Finally, housing construction producers employ land and differentiated residential structures in order to construct new dwellings and sell them to households.

2.2.1 Housing Construction Firms

The current modeling framework assumes the presence of housing construction firms in order to incorporate the possibility of land restrictions. The latter operate in a similar way to capital adjustment costs.³

The housing construction firms operate in a perfectly competitive environment. To construct new dwellings, they combine final residential structures with the available land through the following Cobb-Douglas production function:

$$Y_{H,t} = Y_{H,t}^{1-\zeta} \mathfrak{L}^\zeta, \quad (23)$$

where $Y_{H,t}$ stands for the newly produced houses, $Y_{H,t}$ is the housing structures used in the dwelling production and $\mathfrak{L}$ is the land available at the beginning of period t . Additionally, ζ denotes the land share in the production function. I furthermore assume that the land size is fixed. The housing construction firm maximizes its profit each period by solving the following static problem:

$$P_{H,t}Y_{H,t} - P_{H,t}Y_{H,t} - P_{L,t}\mathfrak{L}, \quad (24)$$

where $P_{H,t}$ denotes the price of residential structures and $P_{L,t}$ is the price of land. Profit maximization leads to the following expression for the real house price:

$$Q_t = (1 - \zeta)^{-(1-\zeta)} \zeta^{-\zeta} \left(\frac{P_{H,t}}{P_{C,t}} \right)^{1-\zeta} \left(\frac{P_{L,t}}{P_{C,t}} \right)^\zeta. \quad (25)$$

2.2.2 Final Goods Firms

Final goods firms are present in both the non-durable and housing sectors. They operate in a perfectly competitive environment and produce a final good. In the non-durable consumption sector, the final good is sold directly to households. By contrast, in the housing industry the final goods producer takes intermediate goods and inputs and produces a residential structure that is sold to the housing construction firm. The production function of the final goods firm is given by the following CES aggregator:

³For a model with housing adjustment costs, see Jaccard (2011).

$$Y_{j,t} = \left[\int_0^1 Y_{j,t}(z)^{\frac{\epsilon_j}{\epsilon_j-1}} dz \right]^{\frac{\epsilon_j-1}{\epsilon_j}}, \quad (26)$$

where $Y_{j,t}$ is the final good in sector $j \in \{C, H\}$ and $Y_{j,t}(z)$ is the input produced by firm z and ϵ_j is the elasticity of substitution between different varieties. Profit maximization of the final goods firm yields the familiar downward-sloping demand curve:

$$Y_{j,t}(z) = \left(\frac{P_{j,t}(z)}{P_{j,t}} \right)^{-\epsilon_j} Y_{j,t}, \quad (27)$$

where $P_{j,t}(z)$ is the price of the differentiated intermediate product while $P_{j,t} \equiv \left[\int_0^1 P_{j,t}(z)^{1-\epsilon_j} dz \right]^{\frac{1}{1-\epsilon_j}}$ denotes the consumer price index.

2.2.3 Intermediate-Goods Firms

Intermediate good producers operate in a monopolistically competitive market and hence are able to set the price for their differentiated product. The production process is captured by the following linear constant-returns-to-scale production function:

$$Y_{j,t}(z) = A_{j,t} N_{j,t}(z), \quad (28)$$

where $j \in \{C, H\}$, $N_{j,t}(z)$ stands for the firm's labor input, $A_{j,t} = \exp(a_{j,t})$ denotes labor productivity. Technology evolves according to a standard stationary AR(1) process given by:

$$a_{j,t} = \rho_{a_j} a_{j,t-1} + \epsilon_{a_j,t}, \quad (29)$$

where $\epsilon_{a_j,t}$ is a shock process for labor productivity. The Calvo (1983) staggered price framework entails cost-minimization rather than profit maximization whereby the problem is given by:

$$\min_{\{N_{j,t}\}} W_{j,t} N_{j,t}(z) \quad (30)$$

s.t.

$$A_{j,t} N_{j,t}(z) \geq \left(\frac{P_{j,t}(z)}{P_{j,t}} \right)^{-\epsilon_j} Y_{j,t} \quad (31)$$

The first-order condition associated with the cost-minimization problem yields the following expression for the real marginal cost:

$$MC_{j,t} = \frac{1}{A_{j,t}} \frac{W_{j,t}}{P_{j,t}}. \quad (32)$$

Equation (32) indicates that the real marginal cost is independent of z and hence is the same for all intermediate producers.

Price setting. Intermediate firms set nominal prices in a staggered way à la Calvo (1983). That

is, each firm resets its price with probability $1 - \theta_j$ while a fraction θ_j are not able to do so and are stuck with the price they have previously set. Each firm maximizes their price subject to a demand schedule of final good producers,

$$\max_{\{P_{j,t}^*\}} \sum_{k=0}^{\infty} \theta_j^k \mathbb{E}_t \{ \Lambda_{t,t+k} [P_{j,t}^* Y_{t+k}(z) - NMC_{j,t+k} Y_{j,t+k}(z)] \} \quad (33)$$

s.t.

$$Y_{j,t+k}(z) = \left(\frac{P_{j,t}^*}{P_{j,t+k}} \right)^{-\epsilon_j} Y_{j,t+k} \quad (34)$$

and the optimal price of the intermediate firms must satisfy the following relation:

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta_j^k \{ \Lambda_{t,t+k} Y_{t+k|t}(z) [P_{j,t}^* - \mu_j NMC_{j,t+k}] \} = 0, \quad (35)$$

where $\Lambda_{t,t+k}$ is the stochastic discount factor, $NMC_{j,t+k}$ is the nominal marginal cost in sector j and $\mu_j \equiv \frac{\epsilon_j}{\epsilon_j - 1}$ is the average mark-up in the respective industry.

Wage setting. Similar to prices, nominal wages are also set in a staggered fashion. I adopt the Schmitt-Grohe and Uribe (2007) framework and assume households consist of a continuum of members that supply a homogeneous labor service. In addition, insurance within the patient household leads to the same level of consumption across members. This labor service is not directly provided to intermediate good producers. Rather, household members supply their homogeneous labor services to a union which acts as a utility maximizer for its members. It is further assumed that only patient households own labor unions and the rate at which they discount the future is that of the savers. In turn, the union is assumed to have market power and thus supply a differentiated labor service to intermediate good producers. The utility and the relevant part of the budget constraints are given by:

$$\mathbb{E}_t \sum_{k=0}^{\infty} \beta_s^k U(X_{i,t+k}, N_{i,t+k}) \quad (36)$$

s.t.

$$P_{C,t} C_{i,t} \leq \int_0^1 (W_{j,t} N_{i,j,t}^e) de, \quad (37)$$

where $N_{i,j,t}^e$ denotes the differentiated labor services of individual e supplied to the labor union and $N_{i,j,t+k} = \int_0^1 N_{i,j,t+k}^e de$ is the market clearing condition in sector j .

The labor union in each industry chooses the optimal wage $W_{j,t}^*$ respectively in order to maximize its members' utility subject to being able to reset its wage only in a $1 - \theta_j^w$ fraction of markets. After some algebra, one obtains the standard New Keynesian Wage Phillips Curve:

$$\hat{\Pi}_{j,t}^w = \beta_s E_t \hat{\Pi}_{j,t}^w - \frac{(1 - \beta_s \theta_j^w)(1 - \theta_j^w)}{1 - \theta_j^w} \hat{\mu}_{j,t}^w, \quad (38)$$

where $\hat{\mu}_{j,t}^w$ is the wedge between the marginal rate of substitution of the patient household and the real wage in sector j and $\hat{\Pi}_{j,t}^w$ stands for the wage inflation in sector j .⁴ Both variables are represented in log-deviations from their steady state levels.

2.3 Aggregation and Market Clearing

Aggregation of non-durable consumption yields the following relationship:

$$C_t = \tilde{\alpha}C_{b,t} + (1 - \tilde{\alpha})C_{s,t}. \quad (39)$$

In the same fashion, labor supply across sectors is given by the following aggregation equations:

$$N_{C,t} = \tilde{\alpha}N_{b,C,t} + (1 - \tilde{\alpha})N_{s,C,t} \quad (40)$$

and

$$N_{H,t} = \tilde{\alpha}N_{b,H,t} + (1 - \tilde{\alpha})N_{s,H,t}. \quad (41)$$

Market clearing both in the non-durable consumption and housing industries is given by the following two relationships:

$$C_t = Y_{C,t} \quad (42)$$

and

$$Y_{H,t} = \tilde{\alpha}\{H_{b,t} - (1 - \delta)[1 - \mu G(\bar{\omega}_t)]H_{b,t-1}\} + (1 - \tilde{\alpha})[H_{s,t} - (1 - \delta)H_{s,t-1}] \quad (43)$$

Finally, aggregate real output is given by:

$$Y_t = Y_{C,t} + Q_t Y_{H,t}. \quad (44)$$

2.4 Monetary Policy

Closing the model entails defining a rule for the nominal interest rate. Throughout this section, I assume the central bank follows a simple Taylor rule and steers the latter responding to non-durable price inflation. The interest rate rule is thus given by:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_r} \left[\left(\frac{\Pi_{C,t}}{\Pi_c}\right)^{\phi_\Pi}\right]^{1-\rho_r} \varepsilon_{M,t}, \quad (45)$$

where ρ_r is the autoregressive parameter of the interest rate rule, ϕ_Π is the weight the monetary authority places on non-durable price inflation and $\varepsilon_{M,t}$ is an *i.i.d* monetary policy shock.

⁴For a complete discussion and comparison of the different ways to incorporate wage stickiness into a DSGE model, see Born and Pfeifer (2016).

3 Calibrated Parameters

Table 1 presents the parameter values chosen for the current model. The values have been chosen such that the steady-state ratios outlined in Table 2 are matched and are in line with the bulk of the housing DSGE literature.

Starting with the share of borrowers in the economy, a few remarks are in order. This share is usually difficult to obtain from aggregate data. This normally poses a serious obstacle for identifying this parameter in estimated DSGE models with borrowers and savers. Taking this into account, I set $\tilde{\alpha} = 0.5$ which is in line with Forlati and Lambertini (2011). Hence, both agent types have equal weights for the determination of aggregate variables.

The discount factors of savers and borrowers are set according to Forlati and Lambertini (2011), Rubio (2011) and Monacelli (2009). I thus assume $\beta_s = 0.99$ yielding a steady state interest rate of four percent on an annual basis while $\beta_b = 0.98$. Households' risk aversion coefficient is set to 1 implying a log-utility specification of consumption. The habit persistence parameter has been calibrated based on the estimates of Iacoviello and Neri (2010) and Smets and Wouters (2007). The inverse Frisch elasticity is set to 2, in line with standard values in the literature. The elasticity of substitution between hours worked in the two sectors is set to 1.14, as estimated by Iacoviello and Neri (2010). The disutility from labor is calibrated such that both types of agents allocate half of their discretionary time to work in steady state.

Regarding housing parameters, the depreciation rate is set to 0.01 per quarter, corresponding to an annual rate of 4%, consistent with common values in the literature. The housing share in the consumption bundle is set to 0.1, reflecting the empirical share of durables in total U.S. household consumption. The elasticity of substitution between non-durable and durable goods, the land share in housing production, and the land endowment are calibrated to match the empirical housing stock-to-GDP and residential investment-to-GDP ratios.

In both production and labor markets, the elasticity of substitution is set to 7.5, following Forlati and Lambertini (2011) and Iacoviello and Neri (2010), which implies a steady-state markup of 1.15. Calvo price and wage stickiness parameters are set to 0.67, consistent with Carlstrom and Fuerst (2006), except in the housing sector where prices are assumed to be fully flexible. These values imply an average price and wage duration of roughly three quarters in the non-durable goods sector and in both labor markets. The monetary policy parameters follow Iacoviello and Neri (2010), with the inflation response coefficient set to 1.5 and the interest rate smoothing parameter to 0.59.

All parameters associated with the costly state verification framework are chosen in order to match the steady state loan-to-value ratio, the mortgage margin and the rate of default. The average LTV ratio in the period 1999-2007 is around 75%. The average mortgage spread (also called external finance premium) is empirically given by the spread between the thirty-year conventional mortgage rate and the yield on the U.S. Treasury 10-year bonds for the same period. This has been historically around 2%. Furthermore, the model calibration aims at matching the

average delinquency rate on single-family residential mortgages for all commercial banks in the period 1999-2007, which is around 2%. Table 2 illustrates a summary of the steady state ratios providing a direct comparison between data and model. Finally, all autoregressive coefficients for the exogenous shocks are set to 0.9, a standard choice in the literature.

3.1 Steady State Analysis

Starting with the LTV ratio under a non-recourse mortgage system, the chosen calibration produces a slightly lower LTV ratio than observed in the data. However, since not all U.S. states follow non-recourse laws, it is reasonable to assume a lower LTV ratio for a fully non-recourse scenario. Regarding the mortgage spread, the model slightly overestimates the value compared to the data. This discrepancy is expected, as U.S. mortgage rates are typically long-term and influenced by regulatory factors such as initial interest rate caps on adjustable-rate mortgages that are beyond the scope of this model.

In addition to the steady-state ratios discussed above, the model incorporates the response of certain key variables to an increase in the recourse tightness. Figure 1 summarizes the results for the benchmark calibration. The first notable finding is that the default rate rises as mortgage recourse becomes tighter, while the (relative) mortgage spread decreases. This outcome reflects lenders' willingness to accept higher risks when stricter recourse allows them to recover a larger portion of both the difference between the loan amount and the collateral value, as well as foreclosure costs. Consequently, the LTV ratio increases, as creditors are more willing to expand lending, which in turn has a positive effect on the real house price. Interestingly, while the LTV ratio increases and the mortgage spread falls, mortgage recourse raises the user cost. In this case, the direct cost of mortgage default outweighs the collateral effect, leading to higher user costs.

Overall, this steady-state sensitivity analysis paints an intriguing picture. Mortgage recourse, as expected, serves as insurance for creditors against borrower default. However, unless recourse is perfectly tight ($v = 1$, which is ruled out by default), the borrower's default rate increases modestly due to expanded lending and higher leverage.

Table 1: Benchmark Calibration

Parameter	Value	Description
$\tilde{\alpha}$	0.50	Share of borrowers
σ	1.00	Risk aversion coefficient
β_s	0.99	Discount factor (savers)
β_b	0.98	Discount factor (borrowers)
γ	0.10	Share of housing in consumption basket
δ	0.01	Depreciation rate
h	0.65	Habit-persistence in non-durable consumption
ξ_b	16.00	Disutility from work (borrowers)
ξ_s	16.00	Disutility from work (savers)
ϕ	2.00	Inverse of Frisch elasticity of labor supply
ζ	0.20	Share of land in housing production
η_h	1.30	Elasticity of substitution between non-durable and durable goods
η_n	1.14	Elasticity of labor substitution
ϵ_C	7.50	Elasticity of substitution between differentiated nondurable goods
ϵ_H	7.50	Elasticity of substitution between differentiated housing goods
ϵ_{WC}	7.50	Elasticity of substitution between differentiated labor in consumption industry
ϵ_{WH}	7.50	Elasticity of substitution between differentiated labor in housing industry
μ	0.13	Share of monitoring costs
σ_ω	0.16	St. Deviation of idiosyncratic shock
θ_c	0.67	Calvo price parameter for non-durable industry
θ_{wc}	0.67	Calvo wage parameter for non-durable industry
θ_{wh}	0.67	Calvo wage parameter for housing industry
ϕ_Π	1.50	Inflation parameter in Taylor rule
ρ_r	0.59	Taylor rule coefficient on past nominal interest rate
ρ_{a_c}	0.90	Autocorrelation coefficient for a_c
ρ_{a_h}	0.90	Autocorrelation coefficient for a_h
ρ_g	0.90	Autocorrelation coefficient for g

Table 2: Steady State Values of Selected Ratios and Variables

Variable	Data	Model: non-recourse	Model: recourse	Description
ϕ	0.7500	0.7131	0.9498	LTV ratio
Z-R	0.0200	0.0572	0.0547	Mortgage Spread
$F(\bar{\omega})$	0.0220	0.0223	0.0303	Default percentage
CtoY	0.6500	0.6506	0.6483	Non-durable consumption to GDP
QHtoY	1.3500	1.3500	1.3556	Housing stock to GDP
QlhtoY	0.0500	0.0594	0.0617	Housing investment to GDP

Figure 1: Steady State Sensitivity Analysis



4 Stochastic Setup

This section presents impulse response functions (IRFs) to a set of standard aggregate shocks commonly examined in the DSGE housing literature. Specifically, I consider a housing preference shock, a monetary policy shock, and technology shocks affecting both the housing and non-durable goods sectors.⁵

4.1 Housing Preference Shock

Housing demand shocks are widely recognized as a major driver of house price volatility. The standard approach to modeling such a shock is via an exogenous process that affects the utility weight on housing services.⁶ Figure 2 displays the impulse responses to a housing preference shock under both non-recourse and recourse mortgage regimes.

As expected, a housing preference shock does increase the real house price, as borrowers' demand for housing goes up. This, in turn, leads to a rise in the user cost. At the same time, the increase in house prices allows borrowers to obtain a higher level of mortgage debt against their collateral, which further relaxes the collateral constraint. This allows households to de-leverage, which in turn reduces the mortgage spread and the rate of default.

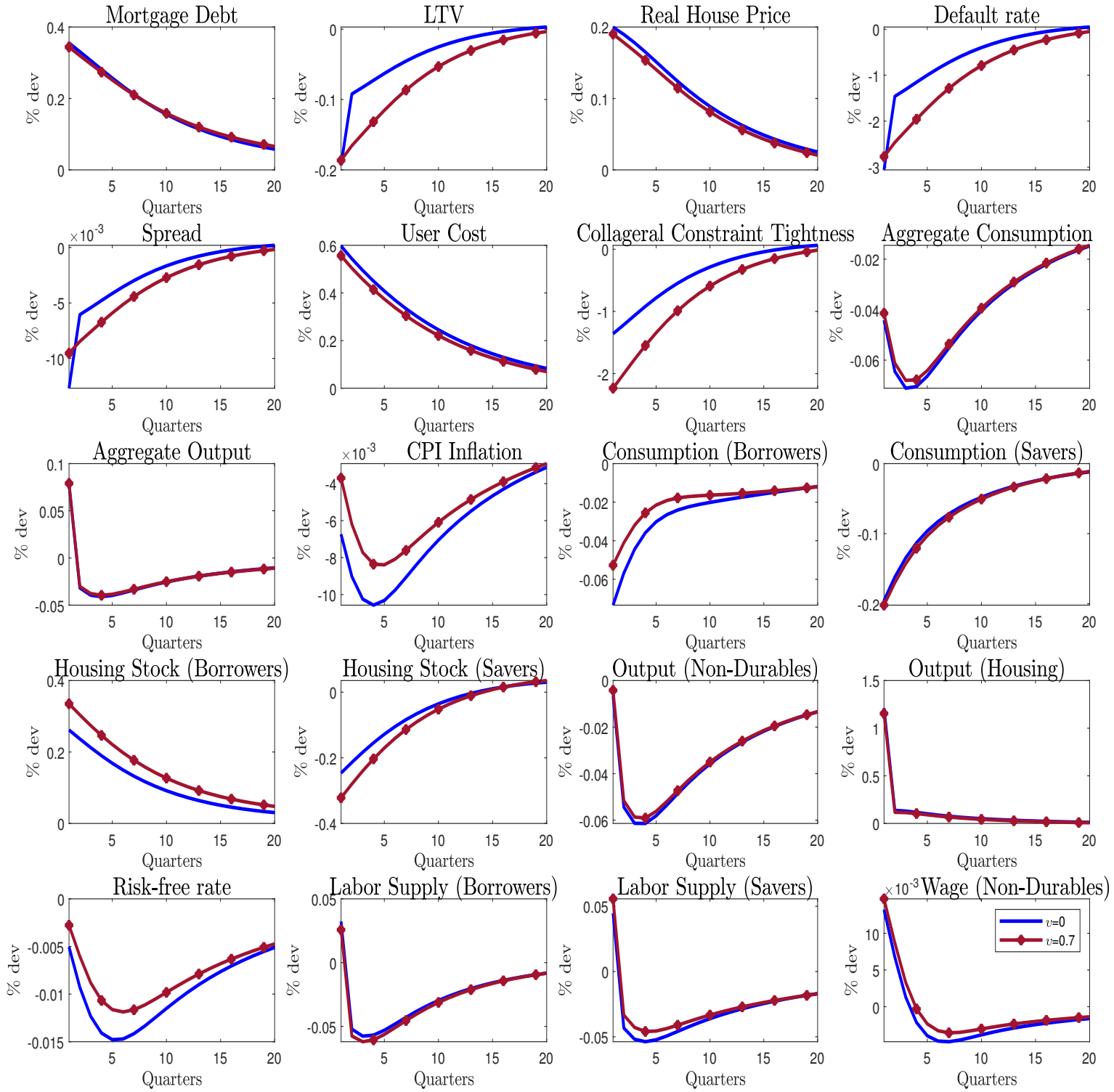
Mortgage recourse strengthens this collateral effect by serving as an insurance-like mechanism for lenders, thereby reducing the risk premium and mortgage spread. This facilitates a larger accumulation of housing stock and greater borrowing capacity for borrowers. Notably, the LTV ratio falls more sharply and remains lower for longer relative to the non-recourse case. The relaxation of the borrowing constraint in the recourse regime mitigates the decline in non-durable consumption and tempers the rise in user cost. Default rates also decline more gradually, reflecting a slower return to the steady state.

For savers, multiple effects influence their choices between non-durable goods and housing. As CPI inflation falls, the associated drop in the risk-free rate increases their average consumption. However, demand for non-durable goods declines as patient households shift their preferences slightly toward housing. Despite this shift in preference, their housing stock ultimately decreases in equilibrium because savers, unlike borrowers, do not benefit from a strong positive collateral effect. Without the collateral boost, savers are able to reduce their stock of housing and non-durable consumption and still satisfy their increased demand for overall consumption basket quality or value.

⁵Table 6 in Iacoviello and Neri (2010) shows that housing-specific demand and supply shocks account for approximately half of the variance in housing investment and house prices.

⁶The present study adopts this reduced-form representation. Alternatively, Dong et al. (2022) propose a heterogeneous-agent framework that provides microfoundations for housing demand shocks, particularly suited to examining the link between house and rental prices. However, this direction lies beyond the scope of the current analysis.

Figure 2: Impulse Responses to a Positive Housing Preference Shock



Mortgage recourse, as already mentioned, allows borrowers to increase their demand for non-durable consumption and housing. Both CPI inflation and the nominal interest rate exhibit thus a small decline relative to the non-recourse scenario. Savers, being able to fully smooth consumption, react to this by lowering their consumption of both goods.

Finally, aggregate consumption falls in both cases but mortgage recourse allows it to have a more muted response. Aggregate output, on the other hand, rises thanks to the increase in housing demand from borrowers. Deficiency judgment, by lowering the mortgage spread, has a small but positive impact on real output.

4.2 Monetary Policy Shock

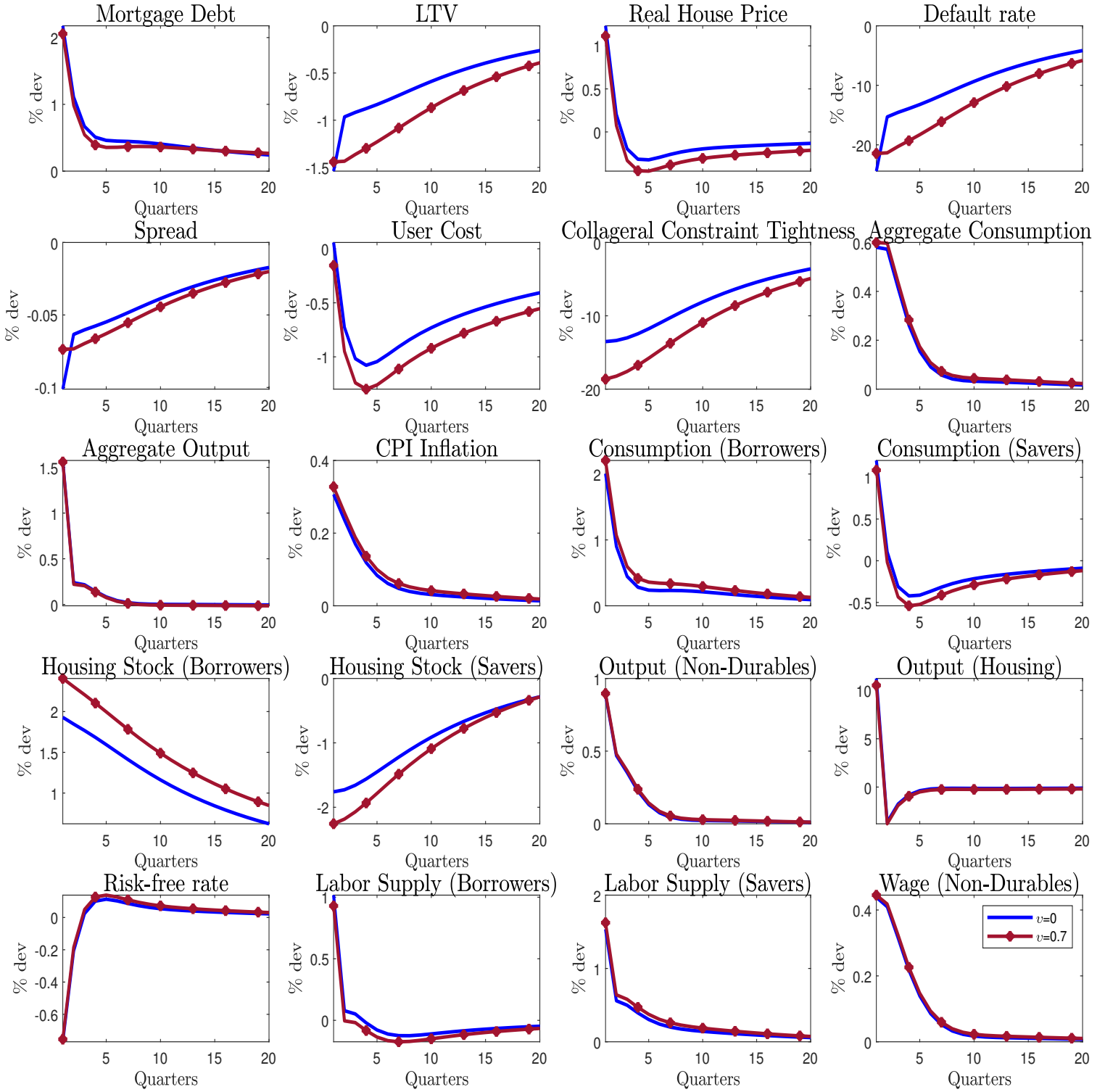
Next the analysis considers a standard monetary policy shock in order to shed light on the interaction between (conventional) monetary policy and mortgage recourse. Figure 3 summarizes the results of one standard deviation decrease in the nominal risk-free rate.

A reduction in the risk-free interest rate, in the context of both sticky prices and wages, induces the expected co-movement between the housing market and non-durable goods production, with both increasing upon impact. Borrowers experience a positive income effect due to the fall in the nominal risk-free interest rate, which reinforces the substitution effect. Consequently, both non-durable consumption and housing demand rise. The increase in house prices relaxes the borrowers' collateral constraint, although this is partially offset by a reduction in the LTV ratio. The latter makes borrowers less leveraged, leading to a decrease in the mortgage spread, the user cost of housing, and the default rate. As a result, mortgage debt increases, but the stronger responses in real house prices and the stock of housing ensure that credit demand remains consistent with a lower LTV ratio.

For savers, the dynamics is somewhat different. The reduction in the interest rate triggers opposing income and substitution effects. On the one hand, the lower interest rate exerts a negative income effect on savers, reducing their income from savings. On the other hand, the substitution effect incentivizes them to consume rather than save, leading to an increase in their non-durable consumption while reducing their housing stock. This divergence in the behavior of borrowers and savers reflects their different responses to changes in interest rates. At the aggregate level, variables such as real output, consumption, and consumer price index (CPI) inflation respond positively to interest rate cuts, as is typically expected in such scenarios.

The introduction of mortgage recourse significantly impacts most of the key variables in the model. While the mortgage spread drops more modestly on impact compared to the non-recourse scenario, it stays persistently low for an extended period. This allows borrowers to accumulate a larger stock of housing while also moderately reducing their mortgage debt relative to the non-recourse case and thus gradually de-leverage. Since housing is also an asset, this increase in the borrowers' housing stock relaxes the collateral constraint allowing both the LTV ratio and default rate to stay lower for a longer time period.

Figure 3: Impulse Responses to an Expansionary Monetary Policy Shock



The relaxation of the collateral constraint enables borrowers to smooth consumption more effectively, which increases further. As borrowers consume more, CPI inflation rises, causing the risk-free interest rate to decline less sharply compared to the non-recourse scenario. In response, savers increase their consumption of non-durables while further reducing their housing consumption. As regards aggregate consumption and output, the overall effect of mortgage recourse is a marginal but positive, primarily driven by the increase in borrowers' non-durable consumption and housing.

4.3 Technology Shock: Housing Industry

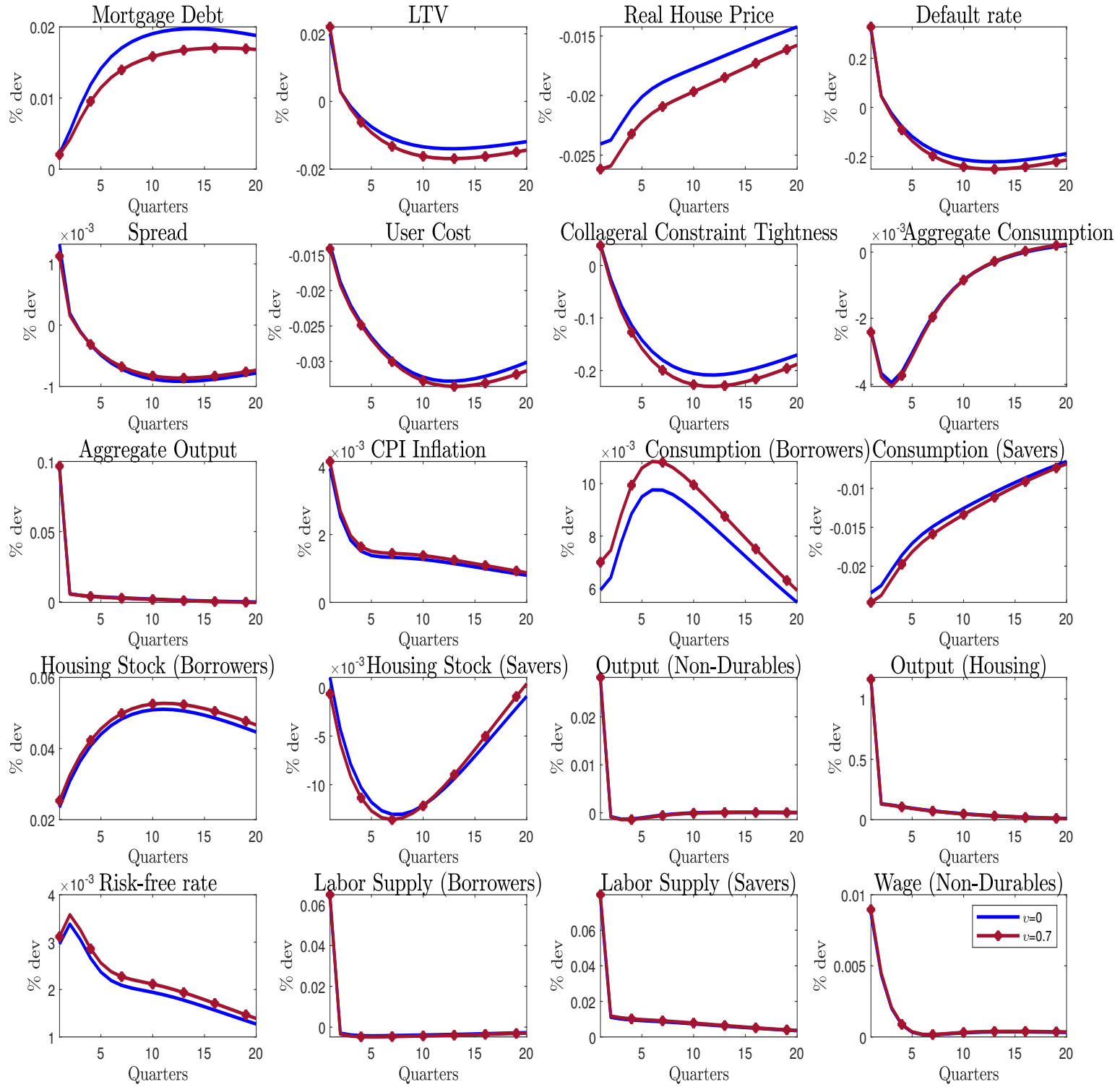
The next scenario considers a positive technology shock specific to the housing sector - an exogenous improvement in the efficiency of intermediate firms involved in housing production. This supply-side shock yields several notable dynamics, as shown in Figure 4.

Greater efficiency in housing production reduces the user cost of housing and leads to an immediate decline in the real house price. While this makes housing more affordable, the fall in house prices also reduces the real value of collateral held by borrowers. Consequently, their effective leverage increases. Simultaneously, productivity gains in the housing sector raise the marginal product of labor, pushing up real wages and generating a positive income effect for borrowers. This allows them to increase consumption of both non-durable goods and housing. However, the lower collateral value combined with higher consumption demand temporarily tightens the borrowing constraint. The LTV ratio rises, strategic default becomes more prevalent, and the mortgage spread increases modestly. These effects, however, are short-lived.

Over time, growing demand for housing from both savers and borrowers reverses the initial decline in real house prices. As the latter recover and rise, the real value of borrowers' collateral improves, easing the borrowing constraint. The LTV ratio, mortgage default rate, and spread all gradually decline. The increase in borrowers' non-durable consumption places upward pressure on CPI inflation and, consequently, on the risk-free interest rate. In response, savers reduce their consumption of both non-durables and housing — and increase saving. The contraction in savers' consumption outweighs the expansion by borrowers, resulting in a temporary decline in aggregate consumption. Nonetheless, the strong housing demand boosts aggregate housing investment and raises output above its steady-state level. The increase in borrowers' housing stock and the lower debt level pushes the LTV ratio downward, which, through the collateral effect, lowers the rate of default.

The introduction of mortgage recourse alters these dynamics in several key ways. Recourse incentivizes lenders to lend more, thereby lowering the mortgage spread, at least in the short run. This allows borrowers to expand their housing stock with relatively lower mortgage debt, supported by improved collateral conditions. As a result, the collateral constraint becomes more relaxed, which enables borrowers to consume more — both in terms of non-durable goods and housing services — relative to the non-recourse regime.

Figure 4: Impulse Responses to a Positive Technology Shock in the Housing Industry



The higher level of borrowers' consumption contributes to a modest rise in CPI inflation, which in turn places upward pressure on the risk-free interest rate. This, in turn, leads to a larger decline in house prices under recourse relative to the non-recourse scenario, as savers respond by further cutting back on consumption and increasing saving. Their housing consumption initially drops but begins to recover around the sixth quarter, supported by the lower user cost of housing.

4.4 Technology Shock: Non-Durable Industry

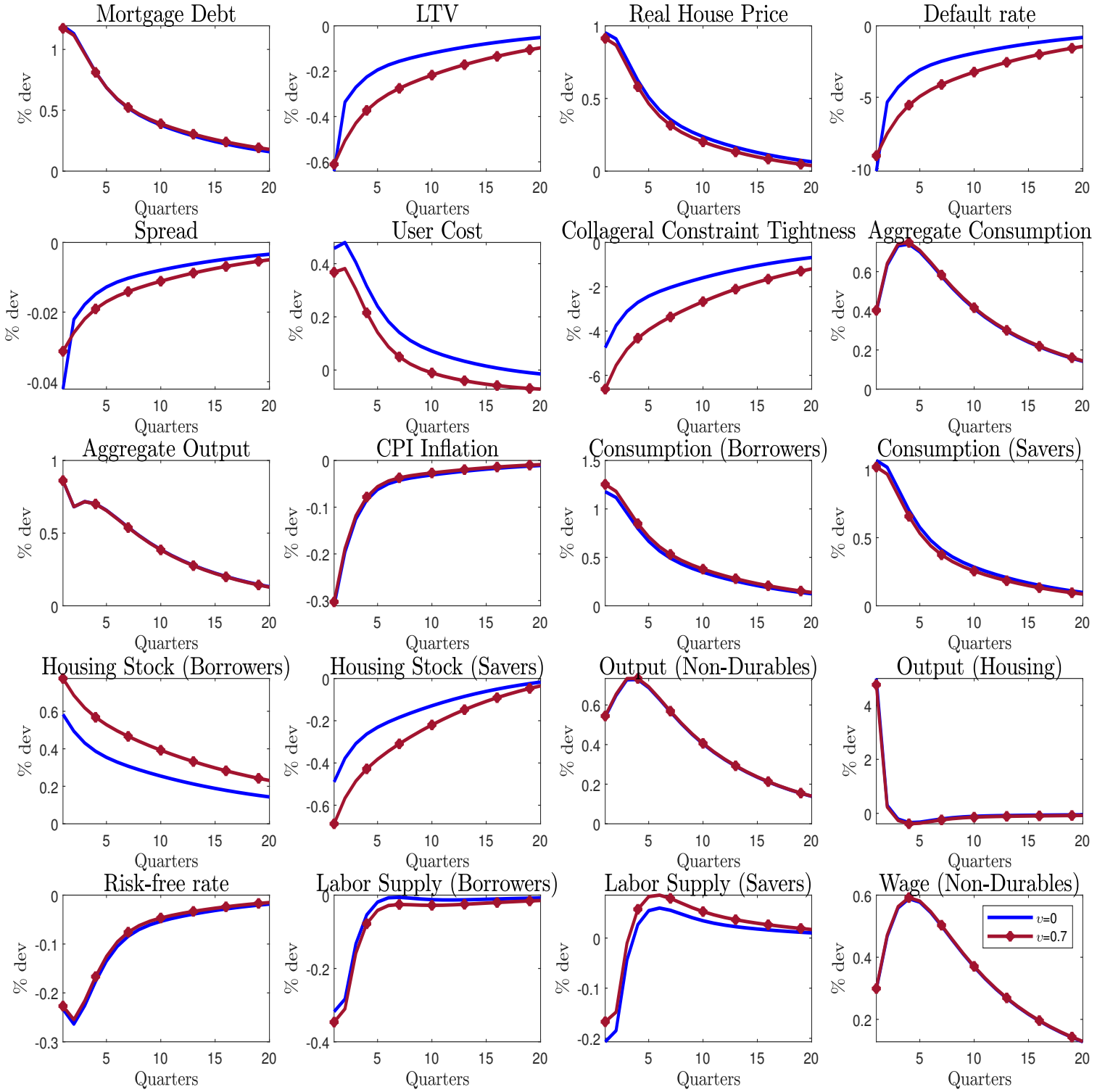
The impulse-response analysis concludes by examining the effects of a standard positive technology shock in the non-durable goods sector. Figure 5 illustrates the main impulse-responses. The increase in labor productivity raises the marginal product of labor, which in turn puts upward pressure on real wages. As production becomes more efficient, CPI inflation declines, allowing the monetary authority to reduce the risk-free interest rate. In response to higher real incomes and lower interest rates, both borrowers and savers expand their consumption of non-durable goods, resulting in a rise in aggregate consumption.

Borrowers also increase their demand for housing, leading to a rise in both mortgage debt and the real house price. The higher house price relaxes the collateral constraint by raising the value of pledged assets. As a result, the LTV ratio declines, reinforcing the positive effect of improved collateral conditions. This enables borrowers to access additional credit, further loosening the collateral constraint and reducing the incentive for strategic default. In contrast, savers reduce their housing demand, driven by a higher user cost of housing and the relative affordability of non-durable goods. The combined effect of higher consumption and increased housing investment by borrowers pushes aggregate output to over 1 percent above its steady-state level.

Introducing mortgage recourse modifies these dynamics in several important ways. Borrowers increase their housing stock even further relative to the non-recourse scenario, while savers reduce their housing holdings more markedly. The mortgage spread begins to decline in the second quarter following the shock and remains persistently lower, facilitating borrower deleveraging. This further reduces the default rate and continues to relax the collateral constraint. As a result, borrowers experience a positive income effect and raise their consumption of non-durable goods.

Despite the fall in inflation, the presence of recourse dampens the deflationary effect compared to the non-recourse case. Consequently, the decline in the risk-free rate is more modest, which encourages savers to postpone consumption and increase saving instead. Their consumption of both non-durable goods and housing remains below the levels observed in the non-recourse scenario. The more muted response in housing demand from savers also contributes to a smaller increase in the real house price. Finally, both aggregate consumption and output rise in both scenarios, whereby the difference across scenarios is rather muted.

Figure 5: Impulse Responses to a Positive Technology Shock in the Non-Durable Goods Sector



4.5 Standard Deviation Analysis

While impulse response analysis provides valuable intuition about dynamic responses to individual shocks, it is also informative to examine the overall volatility patterns in the model. To quantify the extent to which mortgage recourse affects business cycle volatility, Table 3 reports the unconditional standard deviations of key macroeconomic and financial variables under both recourse and non-recourse regimes. The statistics are computed from simulated series using all shocks over 10,000 periods, with a burn-in of 1,000 periods⁷.

Table 3: Standard Deviations (All Shocks)

	No Recourse	Recourse
Mortgage Debt	8.5351	11.0677
LTV	2.2586	4.0648
Real House Price	0.7054	0.7575
Default rate	1.1194	1.9291
Spread	0.2089	0.2196
User Cost	0.0248	0.0300
Aggregate Output	1.4357	1.4330
CPI Inflation	0.6126	0.6371

Table 3 reveals that introducing mortgage recourse significantly increases the volatility of financial and borrower-specific variables such as the loan-to-value (LTV) ratio, mortgage debt, default rates, and borrower consumption, while leaving aggregate output and investment volatility largely unchanged. This outcome stems from the interaction of two transmission mechanisms central to the model. First, the collateral channel implies that tighter recourse improves repayment incentives, encouraging lenders to extend more credit, thereby raising equilibrium leverage. Second, through the default channel, this increased leverage amplifies borrowers' exposure to adverse shocks, particularly housing-related ones. As a result, even though recourse deters default in partial equilibrium, its general equilibrium effect is to exacerbate the cyclical sensitivity of key financial variables. This amplification gives rise to more pronounced financial cycles, with sharper swings in borrowing, default, and borrower consumption in response to macroeconomic shocks. The results underscore that mortgage recourse, while reducing moral hazard, can unintentionally destabilize the financial system by intensifying leverage-driven fluctuations. Finally, recourse also marginally increases the volatility of CPI inflation, suggesting that changes in the structure of mortgage contracts may have non-negligible implications for price stability. This points to a potentially fruitful avenue for future research on the interaction

⁷For a complete table reporting the results for a higher number of variables from simulated series generated under first- and second-order approximations, see Table B.1.

between monetary policy transmission and mortgage market design.

5 Transitional Dynamics

This section develops a quantitative exercise where an economy starts from a state of recourse on mortgage loans and ends in a new state with no mortgage recourse. The advantage of this exercise is to use the full non-linear model. On the other hand, this is a deterministic setup meaning agents do not expect future shocks to occur rendering the model certainty-equivalent. Figure 6 summarizes the results for the major variables of interest.

Initially, the introduction of *Datio in Solutum* is pre-announced, set to take effect in the fifth quarter, after which it remains in place indefinitely. This pre-announcement implies that economic agents are fully aware of the forthcoming legislative change. As discussed by Li and Oswald (2017) and corroborated by Andries et al. (2021), the anticipation of eliminating recourse leads to a reduction in the equilibrium mortgage loan amount. Borrowers' default rates increase in a manner consistent with the findings of the latter study. This expectation triggers borrowers to begin deleveraging even before the law takes effect. Consequently, both the default rate and the mortgage spread decline, albeit temporarily, until the policy is implemented. The tightness of the collateral constraint also diminishes slightly, primarily due to the reduction in the spread and the decrease in non-performing loans. However, anticipating an eventual rise in mortgage spreads and a tightening of the collateral constraint once the law is enacted, borrowers adjust their behavior by reducing their consumption of both non-durable goods and housing. At the same time, they increase their labor supply in order to strengthen their financial positions. This shift in behavior exerts downward pressure on CPI inflation, leading to a reduction in the nominal interest rate.

The period following the enactment of the *Datio in Solutum* law is particularly noteworthy. The default rate surges to more than 10%, triggering a sharp increase in both the mortgage spread and the user cost of housing. As a result, borrowers experience a significant contraction in their housing stock. The real house price begins to fall, which further tightens the borrowing constraint, reducing the consumption-smoothing ability of the borrowers. On the other hand, savers respond differently to these developments. The decline in real house prices, combined with the reduction in the risk-free interest rate, incentivizes savers to expand their housing stock in search of higher returns. Their consumption of non-durable goods also increases, as they are able to smooth consumption effectively.

The lower interest rate reduces the opportunity cost of consumption, prompting savers to front-load their spending and reduce their labor supply. As a result, they increase their consumption of both non-durable goods and housing. In aggregate terms, however, the negative response from borrowers dominates, leading to a decline in non-durable consumption in the quarters preceding the reform. This is followed by a gradual recovery as savers' demand strengthens. Aggregate output, by contrast, rises modestly even before the reform takes effect

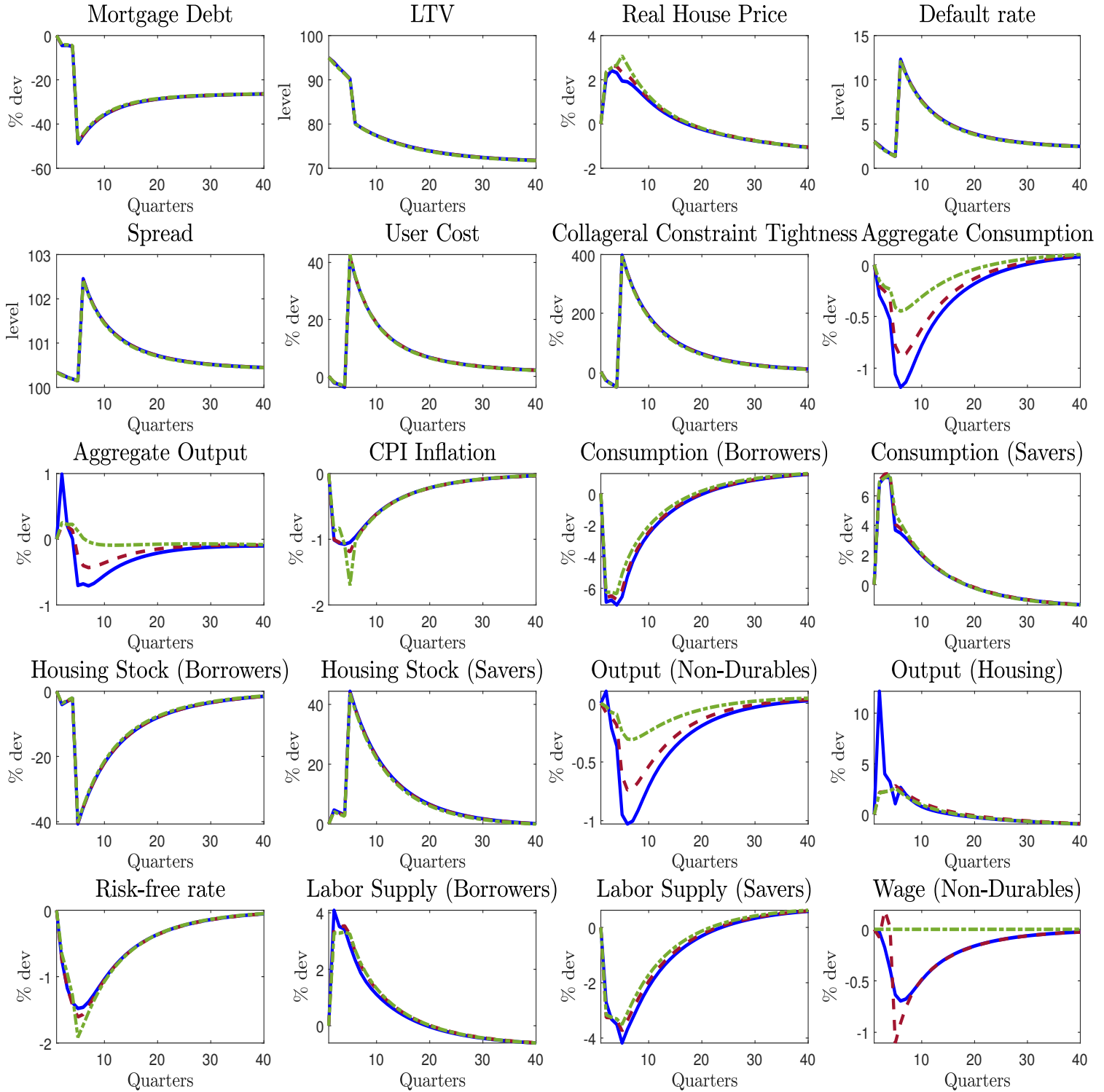
— driven primarily by a pick-up in housing investment fueled by savers’ strong demand.

In conclusion, as supported by the aforementioned studies, a pre-announced transition from a lender-friendly recourse system to a non-recourse mortgage system has negative repercussions for borrowers, particularly when lenders retain the ability to raise mortgage spreads to compensate for the heightened risk of strategic default.

6 Welfare Analysis

The analysis carried out so far sheds light on the positive implications of recourse versus non-recourse household mortgages. Yet no normative conclusions could be drawn. In order to rank the different policies, the present study considers utility-based welfare criteria in order to provide normative assessment on agents’ preferences with respect to the different policy regimes considered in the text. In addition, the heterogeneous-agent structure of the model entails elaborating on the welfare implications of non-recourse versus recourse mortgage debt policy for each single household type (i.e., patient and impatient consumers) and eventually aggregate welfare. The analysis is carried out in a fashion similar to Schmitt-Grohe and Uribe (2007). That is, I take a second order approximation of the entire non-linear model in order to compare both the unconditional and conditional welfare values for each household type in each scenario. This is necessary, since — up to a first-order approximation — the model exhibits certainty equivalence, and the unconditional mean coincides with the steady-state value of the welfare measure. In contrast, with higher-order approximations, the unconditional welfare captures expected lifetime utility, accounting for the stochastic nature of the economy and assuming agents are born into the ergodic distribution. This measure is particularly relevant for comparing long-run average welfare across regimes. Conditional welfare, on the other hand, assesses the utility of agents under uncertainty but starting from a particular point in time, which is often the deterministic steady state. It is especially useful for capturing the short-run and medium-run, state-contingent effects of policy reforms. Nonetheless, the analysis will also report the steady state value of discounted utility since mortgage recourse has an impact on both the steady state of the model and variables’ cyclical deviations from their deterministic trend.

Figure 6: Transitional Dynamics



Blue: Sticky non-durable prices & Sticky Wages; Red: Sticky non-durable prices; Green: no nominal rigidities

To facilitate welfare comparisons across different types of shocks, I simulate the model under each shock individually and then under a scenario in which all four shocks occur simultaneously. Since the analysis focuses on qualitative welfare trade-offs rather than on replicating historical shock frequencies, I assign equal weights to each shock in the multi-shock scenario. Although these weights are not empirically estimated, the equal-weighting scheme serves as a neutral benchmark that would not favor one shock over others. Then the last exercise considers the presence of all four shocks having impact on agents' welfare. The welfare measures used throughout the analysis are given as follows:

$$V_{j,t} \equiv E_t \sum_{t=0}^{\infty} \beta_j^t U(X_{i,t}, N_{i,t}). \quad (46)$$

Formally, maximizing the unconditional welfare would mean finding $E[V_{j,t}]$ whereas the conditional welfare is given by equation 46 starting from the deterministic steady state.⁸ Finally, the average welfare in the economy is given by:

$$V_t = \tilde{\alpha} V_{b,t} + (1 - \tilde{\alpha}) V_{s,t}. \quad (47)$$

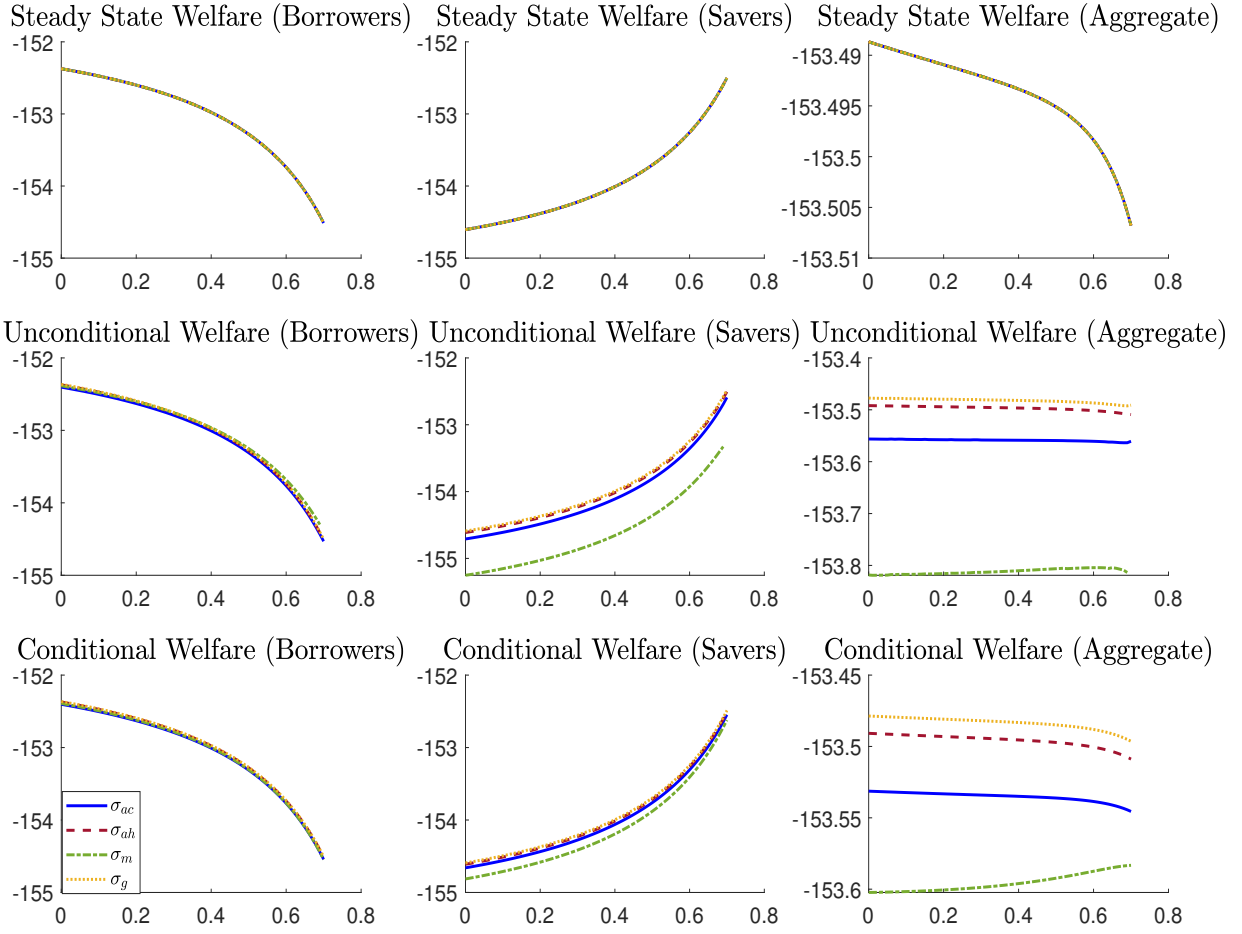
Figure 7 presents the welfare outcomes under varying degrees of mortgage recourse, measured using three different metrics: steady-state welfare, unconditional welfare, and conditional welfare. Each of these captures a distinct dimension of household well-being under alternative policy regimes, providing a multifaceted view of welfare implications.

Across all three metrics, a consistent pattern emerges: increasing mortgage recourse improves the welfare of savers while reducing that of borrowers. The underlying mechanism is intuitive. Recourse reduces lenders' exposure to default risk, lowering the mortgage spread and benefiting savers. However, it imposes stricter consequences on distressed borrowers, thereby reducing their welfare. As a result, recourse regimes disproportionately favor savers. Given the baseline calibration in which borrowers constitute 50% of the population, aggregate welfare typically declines with higher recourse, as the losses incurred by borrowers generally exceed the gains accrued by savers. The sole exception is the case of monetary policy shocks, where aggregate welfare increases under recourse.

Table 4 reports numerical values for the welfare measures considered, presenting both raw utility differences and consumption-equivalent variations following Schmitt-Grohe and Uribe (2007). The latter expresses welfare differences as the percentage of lifetime non-durable consumption a representative agent would be willing to give up to be indifferent between two regimes, enhancing interpretability.

⁸The steady state value of the welfare measure is equal to $V = \frac{1}{1-\beta_j} U(X_i, N_i)$.

Figure 7: Welfare Analysis for a Various Degree of Mortgage Recourse with $\tilde{\alpha} = 0.5$



The results in Table 4 show a clear asymmetry in welfare effects between borrowers and savers. Borrowers consistently fare worse under high-recourse regimes, and this effect intensifies over time. For instance, following a one-time shock, borrowers would be willing to forgo slightly more than 3% of lifetime non-durable consumption to remain in the no-recourse regime. Under unconditional welfare, this figure exceeds 4.5%, highlighting the growing welfare cost of recourse as stochastic fluctuations accumulate. Savers, conversely, benefit from the risk mitigation properties of recourse. They would only be indifferent between regimes if compensated with slightly over 3% of their baseline consumption, implying they are strictly better off under recourse. This pattern holds across all individual shocks and in the joint-shock scenario. The difference between conditional and unconditional welfare is less stark for savers, though recourse still yields a marginally stronger positive effect over time. Finally, with borrowers comprising half the population, it is unsurprising that aggregate welfare tends to decline in most recourse scenarios. However, this result is highly sensitive to the share of borrowers in the economy, suggesting that the welfare evaluation of mortgage recourse policies should consider the population composition explicitly.

Table 4: Welfare Analysis

	Economic Agent	Unconditional Welfare (no recourse)	Unconditional Welfare (recourse)	Conditional Welfare (no recourse)	Conditional Welfare (recourse)	Unconditional Consumption Equivalent	Conditional Consumption Equivalent
St. St.	Borrowers	-152.3729	-154.5135	-152.3729	-154.5135	-4.8237	-4.8237
	Savers	-154.6045	-152.5001	-154.6045	-152.5001	3.2599	3.2599
	Aggregate	-153.4887	-153.5068	-153.4887	-153.5068	-0.7819	-0.7819
σ_γ	Borrowers	-152.3651	-154.5049	-152.3649	-154.5054	-4.8194	-3.2194
	Savers	-154.5935	-152.4887	-154.5924	-152.4874	3.2610	3.0610
	Aggregate	-153.4793	-153.4968	-153.4787	-153.4964	-0.7792	-0.0792
σ_m	Borrowers	-152.3905	-154.4349	-152.3843	-154.5323	-4.6532	-3.1532
	Savers	-155.1303	-153.0661	-154.7966	-152.6313	3.0899	3.1899
	Aggregate	-153.7604	-153.7505	-153.5904	-153.5818	-0.7816	0.0184
σ_{a_h}	Borrowers	-152.3717	-154.5122	-152.3711	-154.5116	-4.8230	-3.2230
	Savers	-154.6130	-152.5086	-154.6104	-152.5061	3.2608	3.0608
	Aggregate	-153.4923	-153.5104	-153.4908	-153.5088	-0.7811	-0.0811
σ_{a_c}	Borrowers	-152.4069	-154.5370	-152.4035	-154.5440	-4.7751	-3.1751
	Savers	-154.7305	-152.6168	-154.6681	-152.5544	3.2723	3.0723
	Aggregate	-153.5687	-153.5769	-153.5358	-153.5492	-0.7514	-0.0514
All	Borrowers	-152.4069	-154.4383	-152.4052	-154.5528	-4.5601	-3.1601
	Savers	-155.2191	-153.0964	-154.8540	-152.6789	3.2561	3.1561
	Aggregate	-153.8130	-153.7673	-153.6296	-153.6158	-0.6520	-0.0020

7 Concluding Remarks

Deficiency judgment is a defining characteristic of the European mortgage market, though it is only present in some U.S. states. While most housing DSGE models assume borrowers always meet their debt obligations and treat the LTV ratio as exogenous, this paper extends the literature by incorporating deficiency judgment into a state-of-the-art DSGE model with strategic default and endogenous LTV ratios. The model explores the implications of mortgage recourse in both stochastic and deterministic settings, focusing on housing market variables such as real house prices, housing investment, user costs, mortgage spreads, and strategic mortgage defaults. In addition, a welfare analysis is conducted to assess the normative implications of mortgage recourse. The key findings are summarized as follows:

Mortgage recourse, as part of the institutional framework, has both steady-state and cyclical implications. It provides insurance for lenders against borrower default, allowing borrowers to expand their debt financing due to higher collateral value. Despite the increased leverage, the mortgage spread declines due to reduced default risk. This leads to higher real house prices and an increase in the value of borrowers' collateral. Echoing the findings of Hatchondo, Martinez, and Sánchez (2015), the study reveals that mortgage recourse tends to *increase* the default rate,

as higher lending activity and LTV ratios offset the deterrent effects of the recourse penalties.

In response to positive demand-side shocks — such as a housing preference shock or expansionary monetary policy — real house prices rise, relaxing the collateral constraint and enabling borrowers to increase mortgage debt while reducing default rates and mortgage spreads. However, the presence of mortgage recourse amplifies the financial transmission of these shocks: borrowers experience deeper and more persistent declines in loan-to-value (LTV) ratios and default rates. This reflects the stronger collateral discipline imposed by deficiency judgments, which — while improving credit quality — leads to more pronounced fluctuations in key financial variables.

Turning to supply-side shocks, the model highlights how mortgage recourse reshapes the transmission of productivity gains in both the housing and non-durable sectors. In the housing sector, recourse mitigates the initial rise in default risk by facilitating faster deleveraging and improving collateral quality. Similarly, in the non-durable sector, recourse supports stronger housing accumulation and borrower consumption despite a more muted deflationary effect.

The standard deviation analysis shows that mortgage recourse amplifies the volatility of key financial variables — such as the LTV ratio, default rate, and mortgage spread — highlighting its role in intensifying financial cycles, even if its effects on real variables like output and housing investment remain relatively modest.

The analysis also considers a transition from a recourse mortgage system to a *Datio in Solutum* (non-recourse) regime, inspired by Romania’s 2016 legislative change. The results indicate that the borrowers’ default rate *increases* following the shift to non-recourse mortgages. This is driven by a rise in mortgage spreads, as lenders respond to the heightened risk of strategic default. Finally, the welfare analysis reveals that while borrowers experience welfare losses under a deficiency judgment regime, savers benefit significantly — particularly when aggregate uncertainty is taken into account.

Following these conclusions, the model can be extended in several directions. First, introducing both fixed and variable long-term mortgage rates would better reflect institutional realities in many OECD countries — particularly the U.S., where 30-year fixed-rate mortgages dominate, as discussed by Rubio (2011) and Pietrunti and Signoretti (2020). Second, incorporating financial intermediaries — who play a central role in risk management and were pivotal in the 2008 crisis as noted by Harris and Meir (2015) — would enrich the analysis of financial stability. Third, examining alternative monetary policy regimes and macroprudential instruments such as LTV caps could offer valuable insights for policymakers.⁹ Finally, a full estimation and model comparison — particularly applied to European data — would further validate the implications of mortgage recourse.¹⁰

⁹See Rubio and Carrasco-Gallego (2014) for a DSGE housing model with macroprudential tools.

¹⁰Key contributions to the estimated DSGE housing literature, among many others, include Iacoviello and Neri (2010), Lambertini, Nuguer, and Uysal (2017), and Darracq Pariès and Notarpietro (2008).

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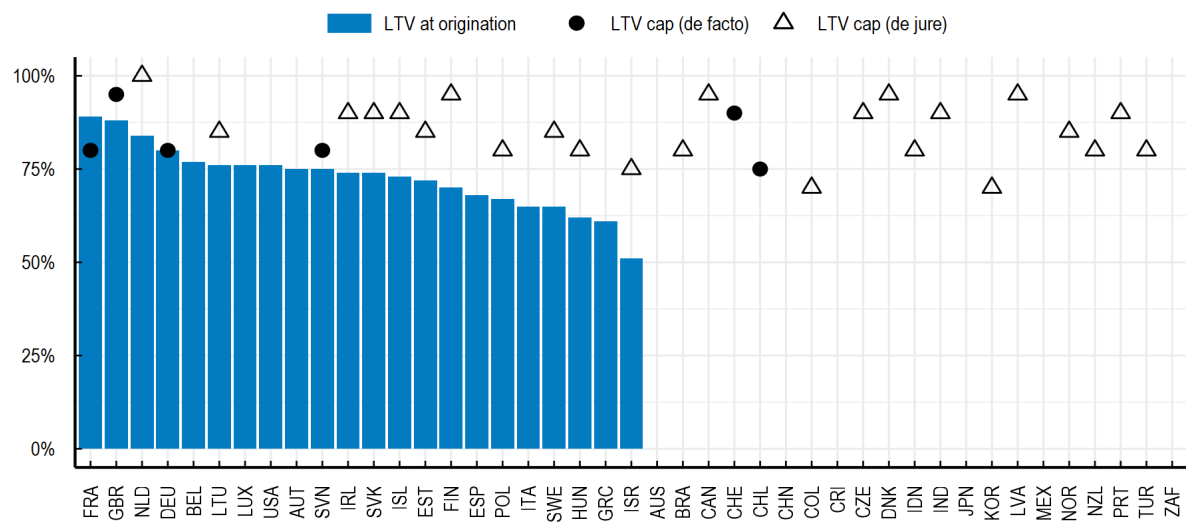
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Appendix A: Figures

Figure A.1: LTV Ratios Across OECD Countries



Source: Hoenselaar et al. (2021)

Figure A.2: Median LTV Ratio

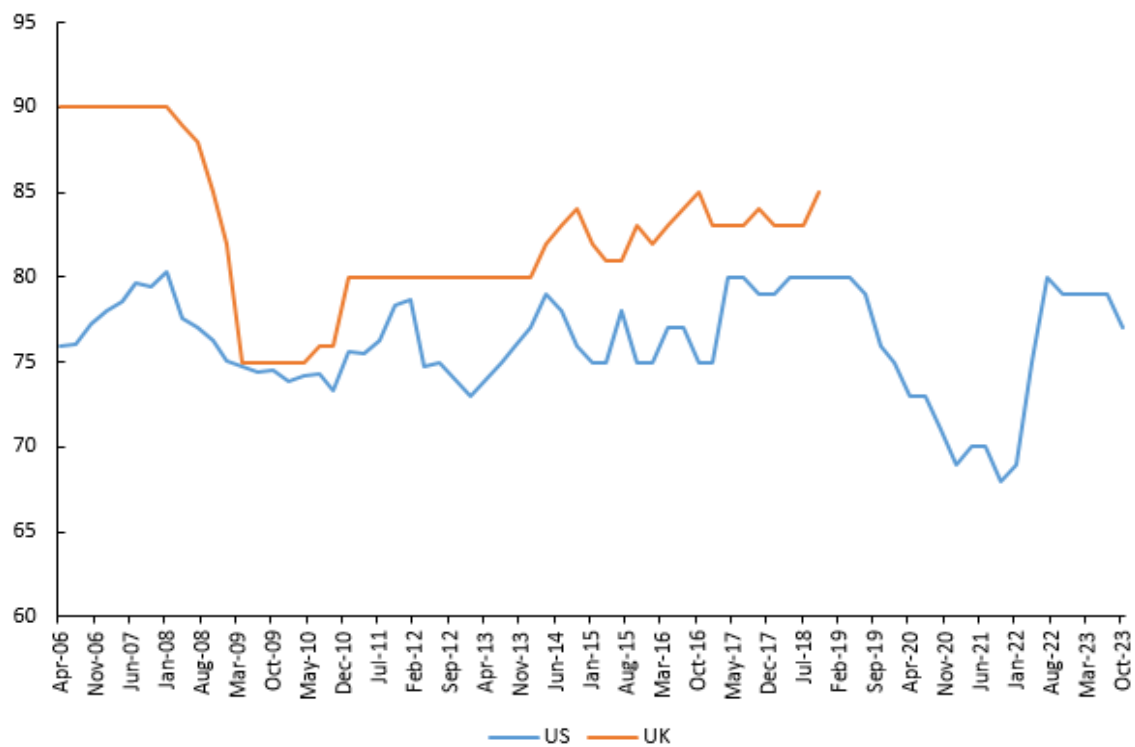


Figure A.3: Mortgage Delinquency Rate

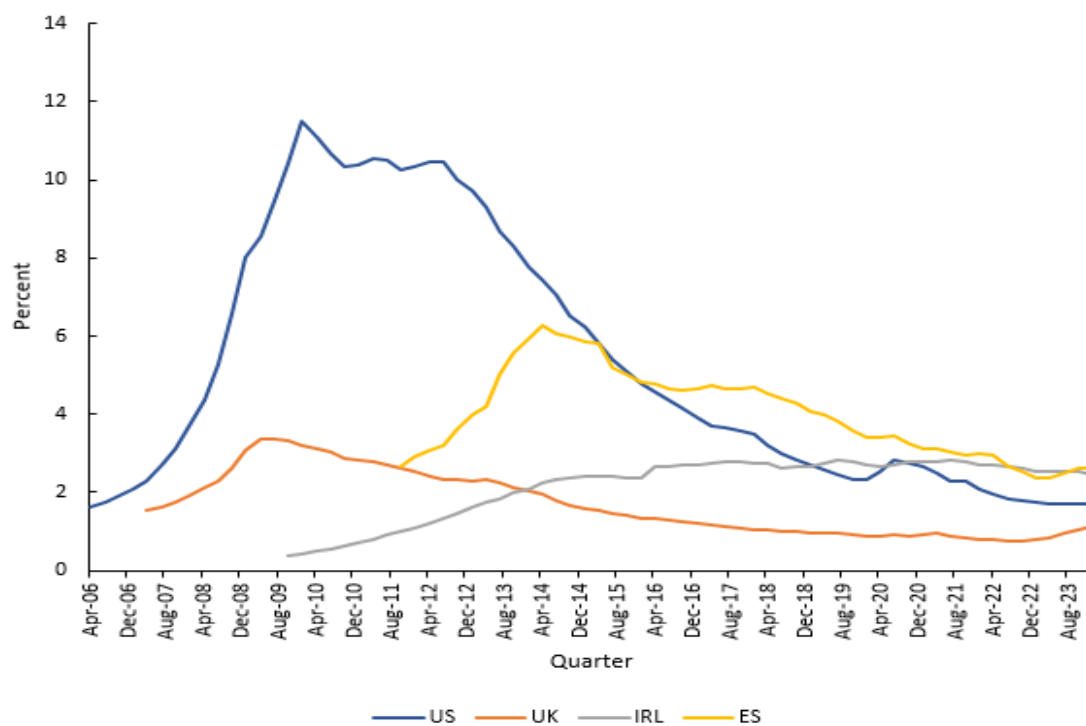
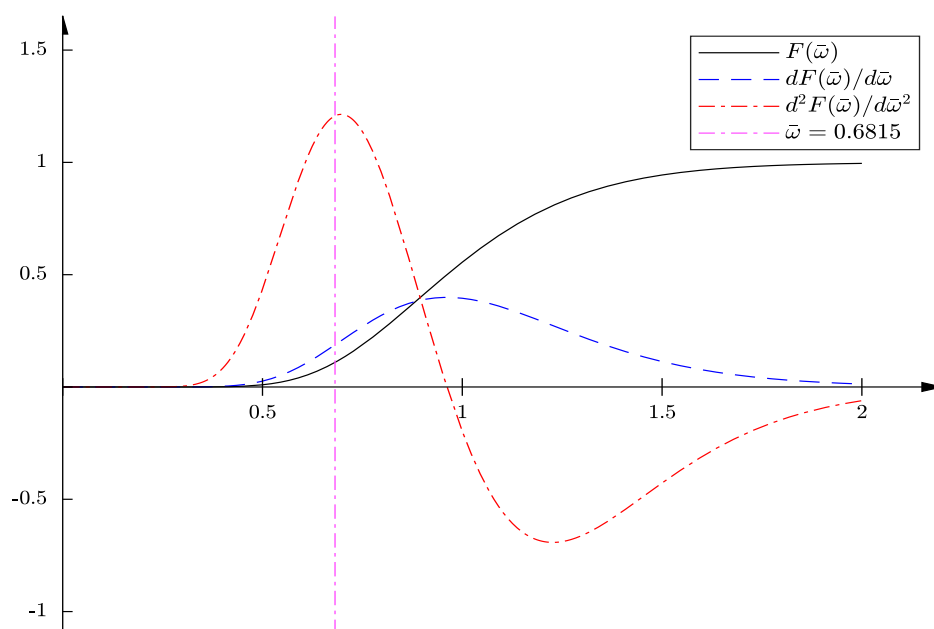


Figure A.4: Log-Normal Distribution



Appendix B: Tables

Table B.1: Standard Deviations (All Shocks)

Variable	No Recourse (order=1)	Recourse (order=1)	No Recourse (order=2)	Recourse (order=2)
Mortgage Debt	8.5351	11.0677	8.2903	10.9497
LTV	2.2586	4.0648	2.2522	4.3548
Real House Price	0.7054	0.7575	0.7051	0.7617
Default rate	1.1194	1.9291	1.0286	1.8119
Spread	0.2089	0.2196	0.1917	0.2061
User Cost	0.0248	0.0300	0.0242	0.0288
Aggregate Output	1.4357	1.4330	1.4571	1.4576
CPI Inflation	0.6126	0.6371	0.6136	0.6316
Aggregate Consumption	0.7955	0.8120	0.7971	0.8060
Consumption (Borrowers)	0.4780	0.5227	0.4746	0.5139
Consumption (Savers)	0.2920	0.2957	0.2945	0.3034
Housing Stock (Borrowers)	64.7581	81.3387	60.6818	75.0574
Housing Stock (Savers)	49.2496	61.2508	47.0877	55.6632
Housing Investment	1.5286	1.4862	1.5358	1.5167
Risk Free Rate	0.9686	0.9596	0.9696	0.9735

Appendix C: Proof of Proposition 1

We see that with full recourse, equation 6 becomes $(1-v)\bar{\omega}_{t+1}Q_{t+1}H_{b,t} + v\mu G(\bar{\omega}_{t+1})Q_{t+1}H_{b,t} = (1-v)Z_{t+1}L_t$. With $v = 1$, $\mu G(\bar{\omega}_{t+1})Q_{t+1}H_{b,t} = 0$. Given the assumption that $Q_t > 0$ and $H_{b,t} > 0$ and $\mu \in (0, 1)$, it must be the case that $G(\bar{\omega}_{t+1}) = 0$. Given the assumptions of the log-normal distribution, this is only true if and only if $\bar{\omega}_{t+1} = 0$. This means, in turn, that strategic default is completely ruled out.

Appendix D: Default Effect (Total Effect of v on $\bar{\omega}$)

The total effect of the degree of mortgage recourse on the cutoff criterion is investigated through totally-differentiating equation 16 in the steady state. The latter is given by:

$$\bar{\omega} = \Theta \frac{\varphi'(\bar{\omega})}{f(\bar{\omega})}, \quad (\text{C.1})$$

where $\Theta \equiv \frac{\beta - \beta_b}{\beta_b} \frac{1}{\mu}$. Totally differentiating equation C.1 yields:

$$d\bar{\omega} = \frac{\partial \bar{\omega}}{\partial \varphi'(\bar{\omega})} d\varphi'(\bar{\omega}) + \frac{\partial \bar{\omega}}{\partial f(\bar{\omega})} df(\bar{\omega}). \quad (\text{C.2})$$

After some algebraic manipulations, the total differential becomes:

$$d\bar{\omega} = \frac{\frac{\partial \bar{\omega}}{\partial \varphi'(\bar{\omega})} \frac{\partial \varphi'(\bar{\omega})}{\partial v}}{1 - \frac{\partial \bar{\omega}}{\partial \varphi'(\bar{\omega})} \frac{\partial \varphi'(\bar{\omega})}{\partial \bar{\omega}} - \frac{\partial \bar{\omega}}{\partial f(\bar{\omega})} \frac{\partial f(\bar{\omega})}{\partial \bar{\omega}}} dv. \quad (\text{C.3})$$

Finally, we obtain the following relationship:

$$d\bar{\omega} = \Omega(\bar{\omega}) dv, \quad (\text{C.4})$$

where

$$\Omega(\bar{\omega}) \equiv \frac{\frac{\Theta}{f(\bar{\omega})} \left[\frac{\mu}{(1-v)^2} + (1-\mu)F(\bar{\omega}) \right]}{1 + \frac{\Theta}{f(\bar{\omega})} \left\{ f(\bar{\omega})[1 + \mu - v(1-\mu)] + f'(\bar{\omega}) \frac{\chi[1-(1-v)F(\bar{\omega})]}{f(\bar{\omega})} \right\}}.$$

The numerator of equation C.4 is always non-negative since all of its components are non-negative. As regards the denominator, it is only $f'(\bar{\omega}) \leq 0$. Then a necessary and sufficient condition for $\frac{d\bar{\omega}}{dv} > 0$ is when $\bar{\omega}$ takes on such a value so that the following condition is fulfilled:

$$\Omega(\bar{\omega}) > 0 \quad \forall \bar{\omega} \in \{\bar{\omega} | f'(\bar{\omega}) > -\tau(\bar{\omega})\}, \quad (\text{C.5})$$

where

$$\tau(\bar{\omega}) \equiv [f(\bar{\omega})]^2 \frac{1 + \Theta[(1+\mu) - v(1-\mu)]}{\chi[1 - (1-v)F(\bar{\omega})]} > 0.$$

Figure A.4 provides a graphical illustration of the effect of $\bar{\omega}$ on $F(\bar{\omega})$, $f(\bar{\omega})$ and $f'(\bar{\omega})$. It is only the latter that could theoretically take on negative values and only when $\bar{\omega}$ takes on values close to 1 or higher. The current calibration yields a value for $\bar{\omega} = 0.6815$ which lies close to the local maximum of $f'(\bar{\omega})$ given the domain.

Appendix E: Effect of Recourse on the LTV Ratio

Partial (Collateral) Effect

Proof of Proposition 2

$$\frac{\partial \varphi(\bar{\omega})}{\partial v} = \frac{\mu}{(1-v)^2} \bar{\omega} + (1-\mu) [\bar{\omega} F(\bar{\omega}) - G(\bar{\omega})]. \quad (\text{D.1})$$

Since $\frac{\mu}{(1-v)^2} \bar{\omega}$ is always positive, all we have to prove is that $F(\bar{\omega})\bar{\omega} \geq G(\bar{\omega})$. Let us prove it

by contradiction. Now we know that $G(\bar{\omega}) \equiv \int_0^{\bar{\omega}} \omega f(\omega) d\omega$ whereas $\bar{\omega}F(\bar{\omega}) \equiv \bar{\omega} \int_0^{\bar{\omega}} f(\omega) d\omega$. Thus $\bar{\omega}F(\bar{\omega})$ represents the maximum possible sum of values of ω weighted by the probability that ω is within the range $[0, \bar{\omega}]$. Thus $G(\bar{\omega})$, being a weighted sum, will always be less than or equal to this upper bound because the weights (given by $f(\omega)$) distribute the probability mass over all ω values from 0 to $\bar{\omega}$, not just at $\bar{\omega}$. Hence, $F(\bar{\omega})\bar{\omega} < G(\bar{\omega})$ if and only if either $v < 0$ and/or $\mu < 0$, which contradicts $\mu \in [0, 1]$. Hence, it must be the case that $F(\bar{\omega})\bar{\omega} \geq G(\bar{\omega})$ for $\forall \bar{\omega} \in [0, \infty)$ *QED*.

Total Effect

Totally differentiating the expression for the LTV ratio in the steady state yields the following expression:

$$d\varphi(\bar{\omega}) = \frac{\partial \varphi(\bar{\omega})}{\partial v} dv + \frac{\partial \varphi(\bar{\omega})}{\partial \chi} d\chi + \frac{\partial \varphi(\bar{\omega})}{\partial F(\bar{\omega})} dF(\bar{\omega}) + \frac{\partial \varphi(\bar{\omega})}{\partial \bar{\omega}} d\bar{\omega} + \frac{\partial \varphi(\bar{\omega})}{\partial G(\bar{\omega})} dG(\bar{\omega}) \quad (D.2)$$

or

$$d\varphi(\bar{\omega}) = \left(\frac{\partial \varphi(\bar{\omega})}{\partial v} + \frac{\partial \varphi(\bar{\omega})}{\partial \chi} \frac{\partial \chi}{\partial v} \right) dv + \left(\frac{\partial \varphi(\bar{\omega})}{\partial \bar{\omega}} + \frac{\partial \varphi(\bar{\omega})}{\partial F(\bar{\omega})} \frac{F(\bar{\omega})}{\partial \bar{\omega}} + \frac{\partial \varphi(\bar{\omega})}{\partial G(\bar{\omega})} \frac{\partial G(\bar{\omega})}{\partial \bar{\omega}} \right) d\bar{\omega}. \quad (D.3)$$

The equation above clearly shows that there are two effects that exert an impact on the LTV ratio. On the one hand, mortgage recourse increases the LTV ratio through the collateral effect. This is the direct effect. On the other hand, a tighter recourse degree impacts the LTV ratio through the effect on $\bar{\omega}$. As mentioned by Bernanke, Gertler, and Gilchrist (1999), the effect of the cutoff criterion on the leverage ratio (in the current model being the LTV ratio) is twofold. On the one hand, a higher $\bar{\omega}$ increases the LTV ratio as lenders obtain a higher payoff from non-defaulting borrowers. On the other hand, however, the probability of default rises.

Moving forward, equation D.3 could be written in the following manner:

$$d\varphi(\bar{\omega}) = \left\{ \frac{\mu}{(1-v)^2} \bar{\omega} + (1-\mu)[F(\bar{\omega})\bar{\omega} - G(\bar{\omega})] \right\} dv + \varphi'(\bar{\omega}) d\bar{\omega} \quad (D.4)$$

and ultimately:

$$d\varphi(\bar{\omega}) = \Phi(\bar{\omega}) dv, \quad (D.5)$$

where

$$\Phi(\bar{\omega}) \equiv \frac{\mu}{(1-v)^2} \bar{\omega} + (1-\mu)[F(\bar{\omega})\bar{\omega} - G(\bar{\omega})] + \varphi'(\bar{\omega})\Omega(\bar{\omega}).$$

A necessary and sufficient condition for $\frac{d\varphi(\bar{\omega})}{dv} > 0$ is given by:

$$\Phi(\bar{\omega}) > 0 \quad \forall \bar{\omega} \in \{\bar{\omega} | f'(\bar{\omega}) > -[\tau(\bar{\omega}) + \Upsilon \varphi'(\bar{\omega})]\}, \quad (D.6)$$

where

$$\Upsilon \equiv \frac{f(\bar{\omega})}{\chi[1 - (1 - v)F(\bar{\omega})]} \frac{\frac{\mu}{(1-v)^2} + (1 - \mu)F(\bar{\omega})}{\frac{\mu}{(1-v)^2}\bar{\omega} + (1 - \mu)[F(\bar{\omega})\bar{\omega} - G(\bar{\omega})]} \geq 0.$$

Appendix F: Effect of Recourse on the (Relative) Mortgage Spread

Partial and Total Effects

Proof of Proposition 3

$$dS(\bar{\omega}) = \frac{\partial S(\bar{\omega})}{\partial \chi} d\chi + \frac{\partial S(\bar{\omega})}{\partial \bar{\omega}} d\bar{\omega} + \frac{\partial S(\bar{\omega})}{\partial \varphi(\bar{\omega})} d\varphi(\bar{\omega}), \quad (\text{E.1})$$

where $\frac{\partial S(\bar{\omega})}{\partial \chi} = \frac{\bar{\omega}}{\varphi(\bar{\omega})}$ and $\frac{\partial S(\bar{\omega})}{\partial \bar{\omega}} = \frac{\chi}{\varphi(\bar{\omega})}$ and $\frac{\partial S(\bar{\omega})}{\partial \varphi(\bar{\omega})} = -\chi \frac{\bar{\omega}}{[\varphi(\bar{\omega})]^2}$.

Furthermore, taking into account all indirect effects, equation E.1 becomes:

$$dS(\bar{\omega}) = \left[\frac{\partial S(\bar{\omega})}{\partial \chi} \frac{\partial \chi}{\partial v} + \frac{\partial S(\bar{\omega})}{\partial \bar{\omega}} \frac{\partial \bar{\omega}}{\partial v} + \frac{\partial S(\bar{\omega})}{\partial \varphi(\bar{\omega})} \frac{\partial \varphi(\bar{\omega})}{\partial v} \right] dv \quad (\text{E.2})$$

and ultimately, considering all effects of the degree of recourse on the spread, the final expression of the total differential has the following form:

$$dS(\bar{\omega}) = \tilde{\zeta}(\bar{\omega}) dv, \quad (\text{E.3})$$

where

$$\tilde{\zeta}(\bar{\omega}) \equiv \frac{1}{\varphi(\bar{\omega})} \left[\frac{\mu}{(1-v)^2} \bar{\omega} + \chi \left(\Omega(\bar{\omega}) - \frac{\bar{\omega}}{\varphi(\bar{\omega})} \Phi(\bar{\omega}) \right) \right] \leq 0. \quad (\text{E.4})$$

It becomes thus obvious that a necessary and sufficient condition for $\tilde{\zeta}(\bar{\omega})$ is:

$$\tilde{\zeta}(\bar{\omega}) > 0 \quad \text{iff} \quad \bar{\omega} > -\frac{\chi \Omega(\bar{\omega})}{\frac{\mu}{(1-v)^2} - \frac{\chi}{\varphi(\bar{\omega})}}.$$